Saemix 3 - count data models

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Version

Use saemix version ≥ 3.2

Objective

Run binary and categorical models in saemix

This notebook uses additional code from the **saemix** development github, not yet integrated in the package. The *workDir* folder in the next chunk of code points to the folder where the user stored this code, and is needed to run the notebook (*workDir* defaults to the current working directory). Specifically, the notebook loads:

- code for different bootstraps in non-linear mixed effect models (Comets et al. 2021 and submitted)
 - the bootstrap runs have been performed previously and are stored in files to be read
 - * bootstraps can be run instead by switching the runBootstrap variable to TRUE in the first chunk of code
 - * in the code, the number of bootstraps is set to 10 for speed but we recommend to use at least 200 for a 90% CI.
 - this can be changed in the following change of code by uncommenting the line *nboot*<-200 and setting the number of bootstrap samples (this may cause memory issues in **Rstudio** with older machines, if this is the case we recommend executing the code in a separate script)
- code for the MC/AGQ provided by Sebastian Ueckert (Ueckert et al. 2017)
 - again if memory issues arise the code can be run in a separate script.

The current notebook can be executed to create an HMTL or PDF output with comments and explanations. A script version containing only the R code is also given as $saemix3_categoricalModel.R$ in the same folder.

Count data model

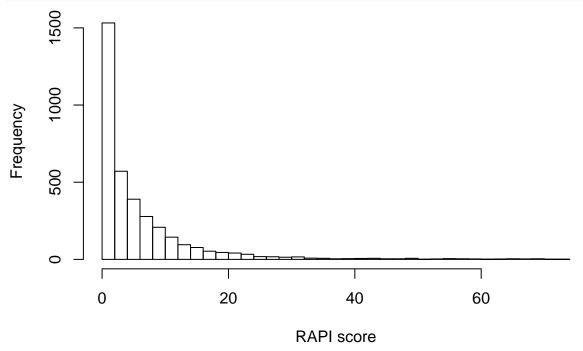
Data description The rapi.saemix dataset in the saemix package contains count data kindly made available by David Atkins (University of Washington) in his tutorial on modelling count data (Atkins et al. 2013). The data comes from a randomised controlled trial assessing the effectiveness of web-based personalised normative feedback intervention on alcohol consumption (Neighbors et al. 2010a, 2010b). The rapi.saemix dataset records alcohol-related problems, as measured by the Rutgers Alcohol Problem Index (RAPI) (White et al. 1989), in freshmen at risk for heavy drinking behaviours. Students were asked to report every six months the number of alcohol-related problems, and the dataset includes 3,616 repeated measures of these counts in 818 subjects, 561 of whom had the full 5 measurements over a period of 2 years. Interesting features of this dataset are first, the longitudinal aspect which allow to evaluate changes over time, and second, the shape of the distribution of counts. Counts are often positively skewed, bounded by zero, with a large stack of data points at zero, indicating individuals and/or occasions without drinking, use, or related problems. This dataset was used in Atkins et al. (2013) to illustrate mixed effects count regression using the qlmer() function from the lme4 package.

Similarly to categorical data, we need the value of the outcome to compute the associated likelihood. Therefore, to create the data object using saemixData, we need to specify the response column both as a response (name.response="rapi") and as a predictor (here, time is the first predictor and we add the response in the argument name.predictors).

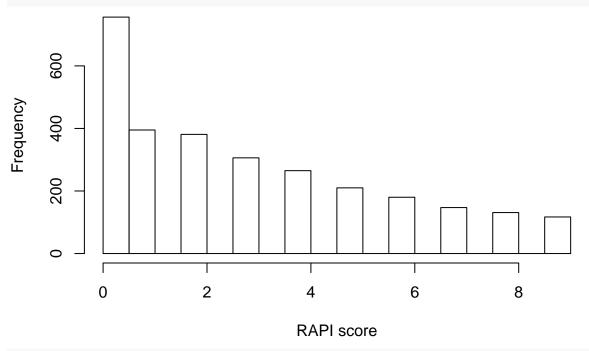
```
##
##
## The following SaemixData object was successfully created:
##
##
  Object of class SaemixData
##
       longitudinal data for use with the SAEM algorithm
  Dataset rapi.saemix
##
##
       Structured data: rapi ~ time + rapi | id
##
       X variable for graphs: time (months)
##
       covariates: gender ()
##
         reference class for covariate gender : Men
```

Exploring data The distribution of count data can be visualised as a histogram. The over-representation of low scores can be seen when zooming on the early part of the histogram. We can also tabulate the data by stratifying on men and women to realise that there seems to be a gender difference, with more women not reporting any episode of drinking than men.

```
# Simple histogram
hist(rapi.saemix$rapi, main="", xlab="RAPI score", breaks=30)
```



```
# Zooming on small values of scores
hist(rapi.saemix$rapi[rapi.saemix$rapi < 10], main="", xlab="RAPI score", breaks=30)</pre>
```



table(rapi.saemix\$gender, as.integer(rapi.saemix\$rapi > 2))

```
## ## 0 1
## Men 548 938
## Women 984 1146
```

Statistical model Several models can be fit to the data.

The simplest one is a Poisson model which assumes that the probability to observe a count value equal to n is given by:

$$P(Y=n) = \frac{\lambda^n e^{-\lambda}}{n!} \tag{1}$$

where $\lambda > 0$ is the parameter of the model. We assume a time effect λ , which is defined as a linear function of time.

For the statistical model, we assume a normal distribution for intercept and slope, so that the distribution of λ is log-normal.

For the simulation function, we add a test to detect outlier values (above the maximum observed value in the original dataset) - not doing this leads to unrealistic simulated values orders of magnitude larger than the maximum observed value, and causes in particular the residual bootstraps to fail - disclaimer: this is a workaround and may not be the proper statistical solution! (if you have an idea about how to avoid aberrant values when simulating from a Poisson model, feel free to drop me a line at emmanuelle.comets@inserm.fr)

```
## Poisson with a time effect
# Model
count.poisson<-function(psi,id,xidep) {
   time<-xidep[,1]
   y<-xidep[,2]
   intercept<-psi[id,1]
   slope<-psi[id,2]
   lambda<- exp(intercept + slope*time)</pre>
```

```
logp <- -lambda + y*log(lambda) - log(factorial(y))
return(logp)
}
# Simulation function
countsimulate.poisson<-function(psi, id, xidep) {
   time<-xidep[,1]
   y<-xidep[,2]
   ymax<-max(y)
   intercept<-psi[id,1]
   slope<-psi[id,2]
   lambda<- exp(intercept + slope*time)
   y<-rpois(length(time), lambda=lambda)
   y[y>ymax]<-ymax+1 # truncate to maximum observed value to avoid simulating aberrant values return(y)
}</pre>
```

Fitting models

Poisson model The code below fits a base Poisson model without correlation between the parameters, as well as a model with a covariance between the two parameters and a gender effect on both parameters.

```
## Poisson
### Model without covariate
saemix.model.poi<-saemixModel(model=count.poisson, description="Count model Poisson", simulate.function=c</pre>
                               modeltype="likelihood",
                               psi0=matrix(c(log(5),0.01),ncol=2,byrow=TRUE,dimnames=list(NULL, c("inter
                                transform.par=c(0,0), omega.init=diag(c(0.5, 0.5)))
##
##
## The following SaemixModel object was successfully created:
##
## Nonlinear mixed-effects model
##
     Model function: Count model Poisson
##
     Model type: likelihood
## function(psi,id,xidep) {
     time<-xidep[,1]
##
     y < -xidep[,2]
##
     intercept<-psi[id,1]</pre>
##
##
     slope<-psi[id,2]</pre>
     lambda<- exp(intercept + slope*time)</pre>
##
##
     logp <- -lambda + y*log(lambda) - log(factorial(y))</pre>
##
     return(logp)
## }
##
     Nb of parameters: 2
##
         parameter names: intercept slope
##
         distribution:
##
        Parameter Distribution Estimated
## [1,] intercept normal
                                Estimated
                                Estimated
## [2,] slope
                  normal
##
     Variance-covariance matrix:
##
             intercept slope
## intercept
                            0
                      1
                      0
## slope
                            1
```

```
##
       No covariate in the model.
##
       Initial values
                intercept slope
##
## Pop.CondInit 1.609438 0.01
### Gender effect on intercept and slope
saemix.model.poi.cov2<-saemixModel(model=count.poisson,description="Count model Poisson",simulate.funct
                                   modeltype="likelihood",
                                   psi0=matrix(c(log(5),0.01),ncol=2,byrow=TRUE,dimnames=list(NULL, c("
                                    transform.par=c(0,0), omega.init=diag(c(0.5, 0.5)),
                                    covariance.model =matrix(data=1, ncol=2, nrow=2),
                                    covariate.model=matrix(c(1,1), ncol=2, byrow=TRUE))
##
##
## The following SaemixModel object was successfully created:
##
## Nonlinear mixed-effects model
##
     Model function: Count model Poisson
##
    Model type: likelihood
## function(psi,id,xidep) {
##
    time<-xidep[,1]
##
    y < -xidep[,2]
##
     intercept<-psi[id,1]</pre>
##
     slope<-psi[id,2]</pre>
     lambda<- exp(intercept + slope*time)</pre>
##
     logp <- -lambda + y*log(lambda) - log(factorial(y))</pre>
##
##
     return(logp)
## }
##
    Nb of parameters: 2
##
         parameter names: intercept slope
##
         distribution:
        Parameter Distribution Estimated
## [1,] intercept normal
                           Estimated
## [2,] slope
                               Estimated
                normal
##
     Variance-covariance matrix:
##
           intercept slope
## intercept
                   1
## slope
                     1
##
    Covariate model:
##
       intercept slope
## [1,]
               1
##
       Initial values
##
               intercept slope
## Pop.CondInit 1.609438 0.01
## Cov.CondInit 0.000000 0.00
saemix.options<-list(seed=632545,save=FALSE,save.graphs=FALSE, displayProgress=FALSE, fim=FALSE)</pre>
### Fit with saemix
poisson.fit<-saemix(saemix.model.poi,saemix.data,saemix.options)</pre>
## Nonlinear mixed-effects model fit by the SAEM algorithm
```

Data

```
## Object of class SaemixData
      longitudinal data for use with the SAEM algorithm
## Dataset rapi.saemix
      Structured data: rapi ~ time + rapi | id
##
      X variable for graphs: time (months)
      covariates: gender ()
        reference class for covariate gender : Men
##
## Dataset characteristics:
##
      number of subjects:
                            818
      number of observations: 3616
##
      average/min/max nb obs: 4.42 / 1 / 5
## First 10 lines of data:
     id time rapi rapi.1 gender mdv cens occ ytype
## 1
          0
               0
                     0
                          Men
      1
                               Ω
## 2
      1
          6
               0
                     0
                          Men
                                    0
## 3
        18
             0
                     0
                        Men
                              0
                                    0
      1
                                        1
## 4
     2 0 3
                     3 Women
## 5 2 6 6
                     6 Women 0
                                  0 1
    2 12 5
                     5 Women 0
## 6
                                    0
                                  0
## 7
      2 18 4
                     4 Women 0
## 8 2 24 5
                   5 Women 0
                                    0 1
## 9 3 0 9
                     9 Men O
## 10 3
        12
             1
                   1 Men O
## -----
              Model
## -----
## Nonlinear mixed-effects model
    Model function: Count model Poisson
    Model type: likelihood
## function(psi,id,xidep) {
##
    time<-xidep[,1]
##
    y < -xidep[,2]
##
    intercept<-psi[id,1]</pre>
##
    slope<-psi[id,2]</pre>
    lambda<- exp(intercept + slope*time)</pre>
##
##
    logp <- -lambda + y*log(lambda) - log(factorial(y))</pre>
##
    return(logp)
## }
## <bytecode: 0x55a65e996378>
    Nb of parameters: 2
        parameter names: intercept slope
##
        distribution:
##
       Parameter Distribution Estimated
## [1,] intercept normal
                          Estimated
## [2,] slope
                            Estimated
              normal
    Variance-covariance matrix:
##
           intercept slope
## intercept
                 1
                        0
## slope
                   0
                        1
##
      No covariate in the model.
##
      Initial values
##
              intercept slope
## Pop.CondInit 1.609438 0.01
```

```
## ----
      Key algorithm options ----
## -----
##
    Estimation of individual parameters (MAP)
    Estimation of log-likelihood by importance sampling
##
    Number of iterations: K1=300, K2=100
##
    Number of chains: 1
##
    Seed: 632545
##
    Number of MCMC iterations for IS: 5000
##
    Simulations:
##
       nb of simulated datasets used for npde: 1000
##
       nb of simulated datasets used for VPC: 100
##
    Input/output
       save the results to a file: FALSE
##
##
       save the graphs to files: FALSE
## -----
                 Results
## -----
## ----- Fixed effects -----
## -----
     Parameter Estimate
## [1,] intercept 1.577
## [2,] slope
           -0.033
## -----
## ----- Variance of random effects -----
## -----
        Parameter
                     Estimate
## intercept omega2.intercept 0.9039
## slope omega2.slope 0.0039
## ----- Correlation matrix of random effects -----
              omega2.intercept omega2.slope
## omega2.intercept 1
## omega2.slope 0
## -----
## ----- Statistical criteria -----
## -----
##
## Likelihood computed by importance sampling
    -2LL= 21486.75
##
     AIC = 21496.75
     BIC = 21520.29
## -----
poisson.fit.cov2<-saemix(saemix.model.poi.cov2,saemix.data,saemix.options)</pre>
## Nonlinear mixed-effects model fit by the SAEM algorithm
## -----
## ----
           Data
## -----
## Object of class SaemixData
    longitudinal data for use with the SAEM algorithm
## Dataset rapi.saemix
    Structured data: rapi ~ time + rapi | id
```

```
##
      X variable for graphs: time (months)
##
      covariates: gender ()
       reference class for covariate gender : Men
##
## Dataset characteristics:
##
      number of subjects:
                           818
##
      number of observations: 3616
      average/min/max nb obs: 4.42 / 1 / 5
## First 10 lines of data:
     id time rapi rapi.1 gender mdv cens occ ytype
## 1
          0
                         Men
     1
              0
                    0
                              0
## 2
     1
          6
               0
                     0
                         Men
## 3
                       Men
      1
        18
              0
                     0
                             0
                                   0
        0
                     3 Women
## 4
      2
              3
                             0
                                   0
## 5
     2 6 6
                     6 Women
## 6
     2 12 5
                     5 Women
                              0
                                   0 1
      2 18 4
## 7
                    4 Women
                              0
                                   0
## 8
      2
         24 5
                     5 Women 0
                                   0 1
## 9
      3 0 9
                       Men
## 10 3 12 1
                             0
                                   0 1
                    1 Men
## -----
## ----
              Model
## -----
## Nonlinear mixed-effects model
    Model function: Count model Poisson
##
    Model type: likelihood
## function(psi,id,xidep) {
##
    time<-xidep[,1]
##
    y<-xidep[,2]
##
    intercept<-psi[id,1]</pre>
##
    slope<-psi[id,2]</pre>
##
    lambda<- exp(intercept + slope*time)</pre>
##
    logp <- -lambda + y*log(lambda) - log(factorial(y))</pre>
##
    return(logp)
## }
## <bytecode: 0x55a65e996378>
##
    Nb of parameters: 2
##
       parameter names: intercept slope
##
       distribution:
##
      Parameter Distribution Estimated
## [1,] intercept normal Estimated
## [2,] slope normal
                          Estimated
    Variance-covariance matrix:
           intercept slope
## intercept 1
## slope
##
    Covariate model:
##
        [,1] [,2]
## gender
         1
##
      Initial values
##
              intercept slope
## Pop.CondInit 1.609438 0.01
## Cov.CondInit 0.000000 0.00
## -----
## ---- Key algorithm options ----
```

```
##
     Estimation of individual parameters (MAP)
##
     Estimation of log-likelihood by importance sampling
     Number of iterations: K1=300, K2=100
##
##
     Number of chains: 1
##
     Seed: 632545
##
     Number of MCMC iterations for IS: 5000
##
     Simulations:
##
        nb of simulated datasets used for npde: 1000
##
        nb of simulated datasets used for VPC: 100
##
     Input/output
##
        save the results to a file: FALSE
        save the graphs to files: FALSE
## -----
                   Results
## ----- Fixed effects -----
## -----
      Parameter
                       Estimate
## [1,] intercept
                        1.683
## [2,] beta_gender(intercept) -0.196
## [3,] slope
## [4,] beta_gender(slope)
                     -0.017
## -----
## ----- Variance of random effects -----
## -----
##
         Parameter
                       Estimate
## intercept omega2.intercept 0.9179
## slope omega2.slope
                     0.0039
## ----- Correlation matrix of random effects -----
               omega2.intercept omega2.slope
## omega2.intercept 1.00
                          -0.14
## omega2.slope -0.14
                             1.00
## -----
## ----- Statistical criteria -----
## -----
##
## Likelihood computed by importance sampling
     -2LL= 21454.94
##
      AIC = 21470.94
      BIC = 21508.59
           ______
### Results
if(FALSE) {
 cat("Poisson parameter at time 0 in base model: lambda_0=", exp(poisson.fit@results@fixed.effects[1])
 cat("Poisson parameter at time 24 in base model: lambda_24=", exp(poisson.fit@results@fixed.effects[1
}
# print(exp(poisson.fit@results@fixed.effects))
# exp(poisson.fit.cov2@results@fixed.effects)
```

• Results

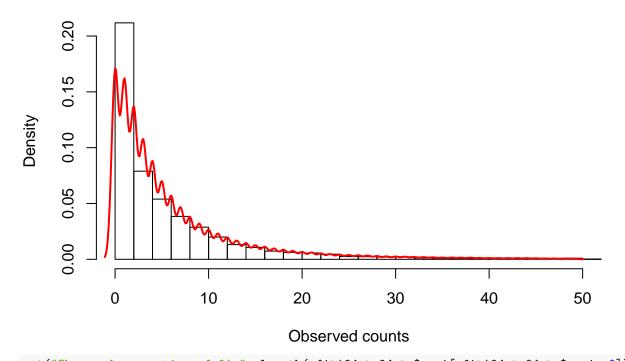
- numerical output
 - * the population value of the Poisson parameter at baseline is 4.8
 - * we see a decreasing trend with time, with the population value of λ after 2 years decreasing to 2.2
- convergence plots show good convergence for all parameters

Comparing the parameter estimates from this fit to the estimates obtained by **glmer()** using a Laplace approximation in Table~2 of (Atkins et al. 2013), we find very good agreement with the SAEM algorithm.

Diagnostics Some diagnostics for this model can be obtained by simulating from the model. To to this we need to define a simulation function associated with the structural model, with the same arguments as the model function, and returning simulated responses.

- Simulation function to simulate from a count model
 - the model function defines directly the log-pdf, so the user needs to define a function to simulate from the appropriate function
 - note the similarities between the model function ($count.poisson()\!)$ and the simulation function ($countsimulate.poisson()\!)$
 - * same setting of dependent variables (time and rapi) from xidep and parameters (inter and slope) from psi
 - · note that we don't use rapi in countsimulate.poisson()
 - * same definition of pevent ($=P(Y_{ij}=k)$, the probability of observing k counts)
 - * in count.poisson() we then compute the probability of the observed outcome using the observed value of Y_{ij} contained in rapi for each observation
 - * in count simulate.poisson(), we use the individual value of $\lambda(Y_{ij})$ to simulate from a Poisson distribution using the rpoisson() function
- once the simulation function has been defined, we use the *simulateDiscreteSaemix()* function from the {saemix} package to simulate *nsim* values (here 100) with the population parameters estimated in *poisson.fit*
 - this adds a *simdata* element to the *poisson.fit*
 - we extract dataframe with the simulated data (poisson.fit@sim.data@datasim) and add a column gender to stratify the plots

```
### Simulations
nsim<-100
yfit1<-simulateDiscreteSaemix(poisson.fit.cov2, nsim=nsim)
hist(yfit1@data@data$rapi, xlim=c(0,50), freq=F, breaks=30, xlab="Observed counts", main="")
lines(density(yfit1@sim.data@datasim$ysim[yfit1@sim.data@datasim$ysim<50]), lwd = 2, col = 'red')</pre>
```



cat("Observed proportion of 0's", length(yfit1@data@data\$rapi[yfit1@data@data\$rapi==0])/yfit1@data@ntot
Observed proportion of 0's 0.2090708

cat(" Poisson model, p=",length(yfit1@sim.data@datasim\$ysim[yfit1@sim.data@datasim\$ysim==0])/length

```
# Checking proportion of zeroes
yfit<-yfit1
simdat <-yfit@sim.data@datasim
simdat$time<-rep(yfit@data@data$time,nsim)</pre>
```

```
ytab<-NULL
for(irep in 1:nsim) {
  xtab<-simdat[simdat$irep==irep,]
  suppressMessages(
  xtab1 <- xtab %>%
    group_by(time, gender) %>%
```

Poisson model, p= 0.1518501

simdat\$gender<-rep(yfit@data@data\$gender,nsim)</pre>

##

summarise(nev = sum(ysim==0), n=n()) %>%
 mutate(freq = nev/n)
)
 ytab<-rbind(ytab,xtab1[,c("time","gender","freq")])
}
gtab <- ytab %>%

group_by(time, gender) %>%
summarise(lower=quantile(freq, c(0.05)), median=quantile(freq, c(0.5)), upper=quantile(freq, c(0.95))
mutate(gender=ifelse(gender==0, "Men", "Women"))

mutate(gender=ifelse(gender==0,"Men","Women"))
`summarise()` has grouped output by 'time'. You can override using the `.groups` argument.

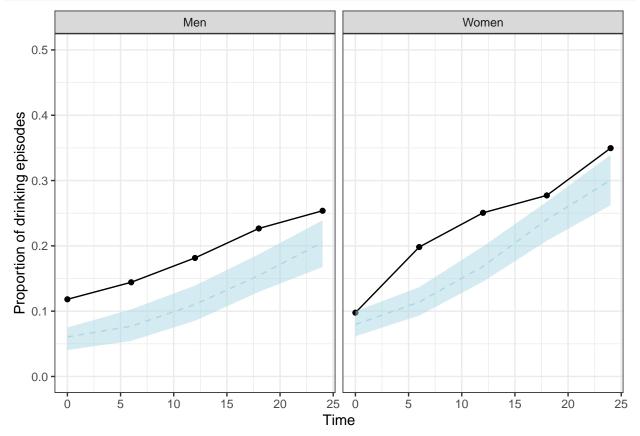
gtab\$freq<-1
gtab1<-cbind(gtab, model="Poisson")</pre>

```
rapipl <- rapi.saemix %>%
    group_by(time, gender) %>%
summarise(nev = sum(rapi==0), n=n()) %>%
mutate(freq = nev/n, sd=sqrt((1-nev/n)/nev)) %>%
mutate(lower=freq-1.96*sd, upper=freq+1.96*sd)
```

```
## `summarise()` has grouped output by 'time'. You can override using the `.groups` argument.
rapipl$lower[rapipl$lower<0] <-0 # we should use a better approximation for CI

plot2 <- ggplot(rapipl, aes(x=time, y=freq, group=gender)) + geom_line() +
    geom_point() +
    geom_line(data=gtab, aes(x=time, y=median, group=gender), linetype=2, colour='lightblue') +
    geom_ribbon(data=gtab, aes(ymin=lower, ymax=upper, group=gender), alpha=0.5, fill='lightblue') +
    ylim(c(0,0.5)) + theme_bw() + theme(legend.position = "none") + facet_wrap(.~gender) +
    xlab("Time") + ylab("Proportion of drinking episodes")

print(plot2)</pre>
```



Dealing with overdispersion The Poisson model in the previous section predicts a lower proportion of subjects without alcohol-related problems than we observe in data, a sign of overdispersion (with a Poisson model, the mean of the Poisson distribution, λ , is equal to the variance, an assumption which is violated here). Several models can be used to take this feature into account.

First, we can use the Zero-Inflated Poisson model, where the number of counts equal to 0 is increased. This model can be built as a mixture between a distribution of 0's with probability p_0 and a standard Poisson model. No variability is set on p_0 which represents a proportion at the level of the population and we use a logit-normal distribution for this parameter to ensure it remains between 0 and 1.

Other potential models include:

- the Generalised Poisson distribution
- the Negative Binomial model
- the Hurdle model (next section)

For the first two, we give expressions for the corresponding **saemix** models in the chunk code below, but not for the simulation functions which are more complicated (see package **RNGforGPD** for an implementation). The Hurdle model is given in the next section for comparison with the ZIP-Poisson model.

```
## Zero-inflated Poisson model
# Model
count.poissonzip<-function(psi,id,xidep) {</pre>
  time<-xidep[,1]</pre>
  y < -xidep[,2]
  intercept<-psi[id,1]</pre>
  slope<-psi[id,2]</pre>
  p0<-psi[id,3] # Probability of zero's
  lambda<- exp(intercept + slope*time)</pre>
  logp <- log(1-p0) -lambda + y*log(lambda) - log(factorial(y)) # Poisson</pre>
  logp0 \leftarrow log(p0+(1-p0)*exp(-lambda)) # Zeroes
  logp[y==0] < -logp0[y==0]
  return(logp)
}
# Simulation function
countsimulate.poissonzip<-function(psi, id, xidep) {</pre>
  time<-xidep[,1]
  y < -xidep[,2]
  ymax<-max(y)</pre>
  intercept<-psi[id,1]</pre>
  slope<-psi[id,2]</pre>
  p0<-psi[id,3] # Probability of zero's
  lambda<- exp(intercept + slope*time)</pre>
  prob0<-rbinom(length(time), size=1, prob=p0)</pre>
  y<-rpois(length(time), lambda=lambda)</pre>
  y[prob0==1]<-0
  y[y>ymax] < -ymax+1 # truncate to maximum observed value to avoid simulating aberrant values
  return(y)
## Generalized Poisson model with time effect
count.genpoisson<-function(psi,id,xidep) {</pre>
  time<-xidep[,1]</pre>
  y < -xidep[,2]
  intercept<-psi[id,1]</pre>
  slope<-psi[id,2]</pre>
  lambda<- exp(intercept + slope*time)</pre>
  delta<-psi[id,3]
  logp <- log(lambda) + (y-1)*log(lambda+y*delta) - lambda - y*delta - log(factorial(y))</pre>
  return(logp)
# Simulation function - TBD, see RNGforGPD ?
## Negative binomial model with time effect
```

```
count.NB<-function(psi,id,xidep) {</pre>
  time<-xidep[,1]
  y < -xidep[,2]
  intercept<-psi[id,1]</pre>
  slope<-psi[id,2]</pre>
  k<-psi[id,3]
  lambda<- exp(intercept + slope*time)</pre>
  logp \leftarrow log(factorial(y+k-1)) - log(factorial(y)) - log(factorial(k-1)) + y*log(lambda) - y*log(lambda)
  return(logp)
}
# Simulation function - TBD ?
We then fit the ZIP model without and with covariates.
## ZIP base model
saemix.model.zip<-saemixModel(model=count.poissonzip,description="count model ZIP",modeltype="likelihoo")</pre>
                                simulate.function = countsimulate.poissonzip,
                                psi0=matrix(c(1.5, 0.01, 0.2), ncol=3, byrow=TRUE, dimnames=list(NULL, c("in
                                transform.par=c(0,0,3), covariance.model=diag(c(1,1,0)), omega.init=diag(
##
##
## The following SaemixModel object was successfully created:
##
## Nonlinear mixed-effects model
     Model function: count model ZIP
##
##
     Model type: likelihood
## function(psi,id,xidep) {
##
     time<-xidep[,1]</pre>
##
     y < -xidep[,2]
##
     intercept <- psi [id, 1]
##
     slope<-psi[id,2]</pre>
##
     p0<-psi[id,3] # Probability of zero's
##
     lambda<- exp(intercept + slope*time)</pre>
     logp \leftarrow log(1-p0) - lambda + y*log(lambda) - log(factorial(y)) # Poisson
##
##
     logp0 \leftarrow log(p0+(1-p0)*exp(-lambda)) # Zeroes
##
     logp[y==0] < -logp0[y==0]
##
     return(logp)
## }
##
     Nb of parameters: 3
##
         parameter names: intercept slope p0
##
         distribution:
        Parameter Distribution Estimated
##
## [1,] intercept normal
                                 Estimated
## [2,] slope
                   normal
                                 Estimated
## [3,] p0
                                 Estimated
                   logit
##
     Variance-covariance matrix:
##
              intercept slope p0
## intercept
                      1
## slope
                      0
                             1 0
## p0
                      0
                             0 0
##
       No covariate in the model.
##
       Initial values
```

```
##
                intercept slope p0
                      1.5 0.01 0.2
## Pop.CondInit
### ZIP Poisson with gender on both intercept
saemix.model.zip.cov1<-saemixModel(model=count.poissonzip,description="count model ZIP",modeltype="like
                               simulate.function = countsimulate.poissonzip,
                                    psi0=matrix(c(1.5, 0.01, 0.2),ncol=3,byrow=TRUE,dimnames=list(NULL,
                                    transform.par=c(0,0,3), covariance.model=diag(c(1,1,0)), omega.init=
                                    covariate.model = matrix(c(1,0,0),ncol=3, byrow=TRUE))
##
##
## The following SaemixModel object was successfully created:
## Nonlinear mixed-effects model
##
     Model function: count model ZIP
##
     Model type: likelihood
## function(psi,id,xidep) {
##
     time<-xidep[,1]
##
     y<-xidep[,2]
##
     intercept <- psi[id,1]
##
     slope<-psi[id,2]</pre>
##
     p0<-psi[id,3] # Probability of zero's
##
     lambda<- exp(intercept + slope*time)</pre>
##
     logp <- log(1-p0) -lambda + y*log(lambda) - log(factorial(y)) # Poisson</pre>
     logp0 \leftarrow log(p0+(1-p0)*exp(-lambda)) # Zeroes
##
##
     logp[y==0] < -logp0[y==0]
     return(logp)
##
## }
##
     Nb of parameters: 3
##
         parameter names: intercept slope p0
##
         distribution:
        Parameter Distribution Estimated
## [1,] intercept normal
                               Estimated
## [2,] slope
                               Estimated
                  normal
## [3,] p0
                  logit
                               Estimated
##
    Variance-covariance matrix:
##
             intercept slope p0
                           0 0
## intercept
                     1
## slope
                           1 0
## p0
                     0
##
    Covariate model:
##
        intercept slope p0
## [1,]
       Initial values
##
##
                intercept slope p0
## Pop.CondInit
                      1.5 0.01 0.2
## Cov.CondInit
                      0.0 0.00 0.0
### ZIP Poisson with gender on both intercept and slope
saemix.model.zip.cov2<-saemixModel(model=count.poissonzip,description="count model ZIP",modeltype="like
                               simulate.function = countsimulate.poissonzip,
                                    psi0=matrix(c(1.5, 0.01, 0.2),ncol=3,byrow=TRUE,dimnames=list(NULL,
                                    transform.par=c(0,0,3), covariance.model=diag(c(1,1,0)), omega.init=
                                    covariate.model = matrix(c(1,1,0),ncol=3, byrow=TRUE))
```

```
##
##
## The following SaemixModel object was successfully created:
##
## Nonlinear mixed-effects model
    Model function: count model ZIP
##
    Model type: likelihood
## function(psi,id,xidep) {
##
    time<-xidep[,1]
##
    y < -xidep[,2]
    intercept<-psi[id,1]</pre>
    slope<-psi[id,2]</pre>
##
    p0<-psi[id,3] # Probability of zero's
##
    lambda<- exp(intercept + slope*time)</pre>
##
##
    logp <- log(1-p0) -lambda + y*log(lambda) - log(factorial(y)) # Poisson</pre>
    logp0 \leftarrow log(p0+(1-p0)*exp(-lambda)) # Zeroes
##
##
    logp[y==0] < -logp0[y==0]
##
    return(logp)
## }
##
    Nb of parameters: 3
##
        parameter names: intercept slope p0
##
        distribution:
##
       Parameter Distribution Estimated
## [1,] intercept normal Estimated
## [2,] slope normal
                            Estimated
## [3,] p0
                logit
                            Estimated
##
   Variance-covariance matrix:
         intercept slope p0
## intercept 1
                          0 0
## slope
                    0
                          1 0
## p0
                          0 0
##
    Covariate model:
##
       intercept slope p0
                    1 0
## [1,]
           1
##
      Initial values
##
              intercept slope p0
## Pop.CondInit 1.5 0.01 0.2
## Cov.CondInit
                    0.0 0.00 0.0
zippoisson.fit<-saemix(saemix.model.zip,saemix.data,saemix.options)</pre>
## Nonlinear mixed-effects model fit by the SAEM algorithm
## -----
              Data
## -----
## Object of class SaemixData
      longitudinal data for use with the SAEM algorithm
## Dataset rapi.saemix
##
      Structured data: rapi ~ time + rapi | id
##
      X variable for graphs: time (months)
##
      covariates: gender ()
        reference class for covariate gender : Men
##
## Dataset characteristics:
##
      number of subjects:
                              818
      number of observations: 3616
```

```
average/min/max nb obs: 4.42 / 1 / 5
## First 10 lines of data:
     id time rapi rapi.1 gender mdv cens occ ytype
         0
                   0 Men
## 1
            0
                             0
                                 0
                        Men 0
## 2
     1
         6
              0
                    0
                                 0
                                0
## 3
     1
       18 0
                    Ω
                      Men 0
## 4 2 0 3
                  3 Women 0
                  6 Women 0
## 5 2 6 6
                                0 1
                   5 Women 0
                                0
## 6
     2 12 5
## 7
     2 18 4
                  4 Women 0
                               0 1
## 8
     2 24 5
                  5 Women 0 0 1
     3 0 9
                      Men 0
                                0 1
                                         1
## 9
                  9
## 10 3 12 1
                1 Men 0
                                 0
## -----
         Model ----
## -----
## Nonlinear mixed-effects model
    Model function: count model ZIP
    Model type: likelihood
## function(psi,id,xidep) {
##
   time<-xidep[,1]
##
    y < -xidep[,2]
##
    intercept<-psi[id,1]</pre>
##
    slope<-psi[id,2]</pre>
##
    p0<-psi[id,3] # Probability of zero's
##
    lambda<- exp(intercept + slope*time)</pre>
##
    logp <- log(1-p0) -lambda + y*log(lambda) - log(factorial(y)) # Poisson</pre>
    logp0 \leftarrow log(p0+(1-p0)*exp(-lambda)) # Zeroes
##
    logp[y==0] < -logp0[y==0]
##
    return(logp)
## }
## <bytecode: 0x55a6604fb908>
##
   Nb of parameters: 3
##
       parameter names: intercept slope p0
##
       distribution:
      Parameter Distribution Estimated
## [1,] intercept normal Estimated
## [2,] slope
            normal
                         Estimated
## [3,] p0
              logit
                          Estimated
## Variance-covariance matrix:
     intercept slope p0
## intercept 1 0 0
                 0
                      1 0
## slope
## p0
                 0
##
     No covariate in the model.
##
     Initial values
##
             intercept slope p0
## Pop.CondInit 1.5 0.01 0.2
## -----
        Key algorithm options ----
     Estimation of individual parameters (MAP)
##
##
     Estimation of log-likelihood by importance sampling
##
     Number of iterations: K1=300, K2=100
```

```
##
    Number of chains: 1
##
    Seed: 632545
##
    Number of MCMC iterations for IS: 5000
    Simulations:
##
##
       nb of simulated datasets used for npde: 1000
##
       nb of simulated datasets used for VPC: 100
##
    Input/output
##
       save the results to a file: FALSE
       save the graphs to files: FALSE
## -----
                 Results
## -----
## ------ Fixed effects ------
## -----
     Parameter Estimate
## [1,] intercept 1.657
## [2,] slope -0.029
## [3,] p0
            0.076
## -----
## ----- Variance of random effects -----
## -----
        Parameter
                    Estimate
## intercept omega2.intercept 0.7977
       omega2.slope 0.0032
## slope
## -----
## ----- Correlation matrix of random effects -----
             omega2.intercept omega2.slope
                         Ο
## omega2.intercept 1
## omega2.slope 0
## -----
## ----- Statistical criteria -----
## -----
##
## Likelihood computed by importance sampling
     -2LL= 20479.88
##
     AIC = 20491.88
     BIC = 20520.12
## -----
zippoisson.fit.cov1<-saemix(saemix.model.zip.cov1,saemix.data,saemix.options)</pre>
## Nonlinear mixed-effects model fit by the SAEM algorithm
## -----
           Data
## -----
## Object of class SaemixData
    longitudinal data for use with the SAEM algorithm
## Dataset rapi.saemix
    Structured data: rapi ~ time + rapi | id
##
##
     X variable for graphs: time (months)
##
     covariates: gender ()
      reference class for covariate gender : Men
## Dataset characteristics:
    number of subjects:
                      818
```

```
##
      number of observations: 3616
##
      average/min/max nb obs: 4.42 / 1 / 5
## First 10 lines of data:
     id time rapi rapi.1 gender mdv cens occ ytype
## 1
          0
               0
                      0 Men 0
## 2
      1
          6
               0
                      0
                         Men
                                0
                                     0
                                         1
## 3
      1
        18
               0
                      0
                        Men
## 4
      2
                      3 Women
          0
               3
                                0
                                     0
## 5
      2
          6
               6
                      6 Women
                                0
                                     0
## 6
      2 12 5
                              0
                                   0 1
                      5 Women
## 7
      2 18 4
                     4 Women
      2 24 5
                                     0 1
                                             1
## 8
                      5 Women
                              0
                                             1
## 9
      3
         0 9
                      9
                        Men
                              0
                                     0
                                       1
## 10 3 12 1
                                     0
                     1
                          Men
## ----
               Model
## Nonlinear mixed-effects model
    Model function: count model ZIP
    Model type: likelihood
##
## function(psi,id,xidep) {
    time<-xidep[,1]
##
    y < -xidep[,2]
##
    intercept<-psi[id,1]</pre>
##
    slope<-psi[id,2]</pre>
    p0<-psi[id,3] # Probability of zero's
##
    lambda<- exp(intercept + slope*time)</pre>
##
    logp <- log(1-p0) -lambda + y*log(lambda) - log(factorial(y)) # Poisson</pre>
##
    logp0 \leftarrow log(p0+(1-p0)*exp(-lambda)) # Zeroes
    logp[y==0] < -logp0[y==0]
##
    return(logp)
## }
## <bytecode: 0x55a6604fb908>
##
    Nb of parameters: 3
##
        parameter names: intercept slope p0
##
        distribution:
##
       Parameter Distribution Estimated
## [1,] intercept normal
                          Estimated
             normal
## [2,] slope
                            Estimated
## [3,] p0
                logit
                            Estimated
    Variance-covariance matrix:
           intercept slope p0
                        0 0
## intercept 1
                   0
                         1 0
## slope
## p0
    Covariate model:
##
        [,1] [,2] [,3]
##
## gender 1 0
##
      Initial values
##
              intercept slope p0
## Pop.CondInit 1.5 0.01 0.2
## Cov.CondInit 0.0 0.00 0.0
## -----
## ---- Key algorithm options ----
```

```
##
     Estimation of individual parameters (MAP)
##
     Estimation of log-likelihood by importance sampling
     Number of iterations: K1=300, K2=100
##
##
     Number of chains: 1
##
    Seed: 632545
##
    Number of MCMC iterations for IS: 5000
##
     Simulations:
##
       nb of simulated datasets used for npde: 1000
##
       nb of simulated datasets used for VPC: 100
##
     Input/output
##
       save the results to a file: FALSE
       save the graphs to files: FALSE
## -----
                Results
## -----
## ----- Fixed effects -----
## -----
    Parameter
                     Estimate
## [1,] intercept
                      1.786
## [2,] beta_gender(intercept) -0.226
## [3,] slope
## [4,] p0
                     0.076
## -----
## ----- Variance of random effects -----
## -----
##
        Parameter
                     Estimate
## intercept omega2.intercept 0.7849
## slope omega2.slope 0.0033
## ----- Correlation matrix of random effects -----
              omega2.intercept omega2.slope
## omega2.intercept 1
## omega2.slope 0
## -----
## ----- Statistical criteria -----
## -----
##
## Likelihood computed by importance sampling
    -2LL= 20469.41
##
     AIC = 20483.41
     BIC = 20516.35
## -----
zippoisson.fit.cov2<-saemix(saemix.model.zip.cov2,saemix.data,saemix.options)
## Nonlinear mixed-effects model fit by the SAEM algorithm
## -----
## ----
          Data
## -----
## Object of class SaemixData
     longitudinal data for use with the SAEM algorithm
## Dataset rapi.saemix
     Structured data: rapi ~ time + rapi | id
```

```
##
      X variable for graphs: time (months)
##
      covariates: gender ()
        reference class for covariate gender : Men
##
## Dataset characteristics:
##
      number of subjects:
                              818
##
      number of observations: 3616
      average/min/max nb obs: 4.42 / 1 / 5
## First 10 lines of data:
##
      id time rapi rapi.1 gender mdv cens occ ytype
## 1
           0
                0
                            Men
      1
                       0
                                  0
                                       0
## 2
      1
           6
                0
                       0
                            Men
                                           1
## 3
                            Men
                                       0
      1
          18
                0
                       0
                                  0
                                           1
                                                 1
## 4
      2
           0
                3
                       3 Women
                                  0
                                       0
                                           1
                                                 1
## 5
      2
         6
                       6 Women
                                       0
                6
## 6
      2 12
                5
                       5 Women
                                  0
                                       0
                                           1
## 7
      2
         18
                4
                       4 Women
                                  0
                                       0
## 8
      2
          24
                5
                       5 Women
                                  0
                                       0
                                           1
## 9
      3
          0
                9
                            Men
## 10 3
          12
                                  0
                                       0
                1
                       1
                            Men
## -----
## ----
                Model
## -----
## Nonlinear mixed-effects model
    Model function: count model ZIP
    Model type: likelihood
##
## function(psi,id,xidep) {
##
    time<-xidep[,1]</pre>
##
    y<-xidep[,2]
##
    intercept<-psi[id,1]</pre>
##
    slope<-psi[id,2]</pre>
##
    p0<-psi[id,3] # Probability of zero's
##
    lambda<- exp(intercept + slope*time)</pre>
##
    logp <- log(1-p0) -lambda + y*log(lambda) - log(factorial(y)) # Poisson</pre>
##
    logp0 \leftarrow log(p0+(1-p0)*exp(-lambda)) # Zeroes
##
    logp[y==0] < -logp0[y==0]
##
    return(logp)
## }
## <bytecode: 0x55a6604fb908>
##
    Nb of parameters: 3
##
        parameter names: intercept slope p0
##
        distribution:
##
       Parameter Distribution Estimated
## [1,] intercept normal
                              Estimated
## [2,] slope
                 normal
                              Estimated
## [3,] p0
                 logit
                              Estimated
##
    Variance-covariance matrix:
##
            intercept slope p0
                          0 0
## intercept
                 1
## slope
                    0
                          1 0
                          0 0
## p0
                    0
##
    Covariate model:
##
         [,1] [,2] [,3]
## gender
            1
                 1
      Initial values
##
```

```
##
      intercept slope p0
## Pop.CondInit 1.5 0.01 0.2
               0.0 0.00 0.0
## Cov.CondInit
## -----
        Key algorithm options ----
## -----
     Estimation of individual parameters (MAP)
     Estimation of log-likelihood by importance sampling
##
##
     Number of iterations: K1=300, K2=100
##
     Number of chains: 1
##
     Seed: 632545
     Number of MCMC iterations for IS: 5000
##
##
     Simulations:
        nb of simulated datasets used for npde: 1000
##
##
        nb of simulated datasets used for VPC: 100
##
     Input/output
##
       save the results to a file: FALSE
        save the graphs to files: FALSE
## -----
                  Results
## -----
## ----- Fixed effects -----
## -----
     Parameter
                       Estimate
## [1,] intercept
                       1.773
## [2,] beta_gender(intercept) -0.197
## [3,] slope
                       -0.020
## [4,] beta_gender(slope)
                       -0.016
## [5,] p0
                       0.075
## ----- Variance of random effects -----
        Parameter
                     Estimate
## intercept omega2.intercept 0.7826
## slope omega2.slope 0.0033
## ----- Correlation matrix of random effects -----
## -----
##
              omega2.intercept omega2.slope
                           0
## omega2.intercept 1
## omega2.slope 0
## -----
## ----- Statistical criteria -----
## Likelihood computed by importance sampling
      -2LL= 20459.27
##
##
      AIC = 20475.27
     BIC = 20512.93
exp(zippoisson.fit@results@fixed.effects)
```

[1] 5.2450012 0.9714983 1.0793068

```
exp(zippoisson.fit.cov1@results@fixed.effects)

## [1] 5.9656256 0.7975888 0.9714754 1.0793259

exp(zippoisson.fit.cov2@results@fixed.effects)
```

[1] 5.8872267 0.8213610 0.9797720 0.9844017 1.0783237

saemix.model.qen<-saemixModel(model=count.qenpoisson,description="count model Generalised Poisson",mo

The results from the covariate model are very close to the previous ones for the common parameters (intercept, slope and gender effects). The proportion of overinflated 0.075.

Diagnostics We compare the previous fit from the Poisson model with the ZIP model using the same approach as before, and find a much better agreement between the predictions and the fit in both genders.

```
### Simulations
ysim.zip2<-simulateDiscreteSaemix(zippoisson.fit.cov2, 100)

cat("Observed proportion of 0's", length(yfit1@data@data$rapi[yfit1@data@data$rapi==0])/yfit1@data@ntot

## Observed proportion of 0's 0.2090708

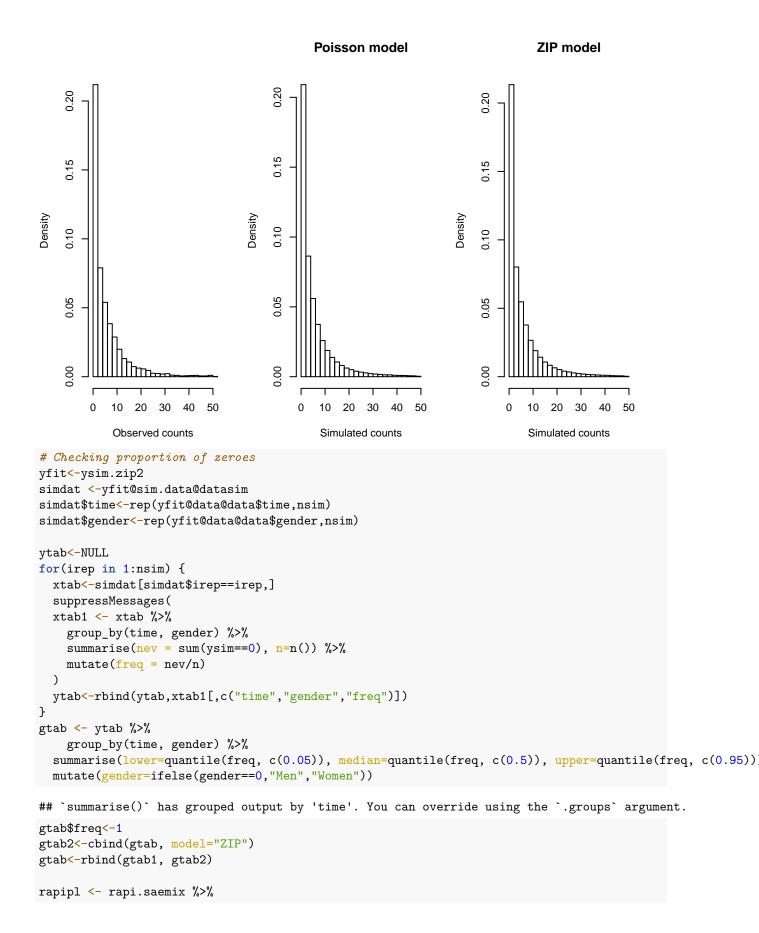
cat(" Poisson model, p=",length(yfit1@sim.data@datasim$ysim[yfit1@sim.data@datasim$ysim==0])/lengti

## Poisson model, p= 0.1518501

cat(" ZI-Poisson model, p=",length(ysim.zip2@sim.data@datasim$ysim[ysim.zip2@sim.data@datasim$ysim==0]

## ZI-Poisson model, p= 0.1957329

par(mfrow=c(1,3))
hist(yfit1@data@data$rapi, xlim=c(0,50), freq=F, breaks=30, xlab="Observed counts", main="")
hist(yfit1@sim.data@datasim$ysim[yfit1@sim.data@datasim$ysim<50], xlim=c(0,50), freq=F, breaks=20, xlab
hist(ysim.zip2@sim.data@datasim$ysim[ysim.zip2@sim.data@datasim$ysim<50], xlim=c(0,50), freq=F, breaks=</pre>
```



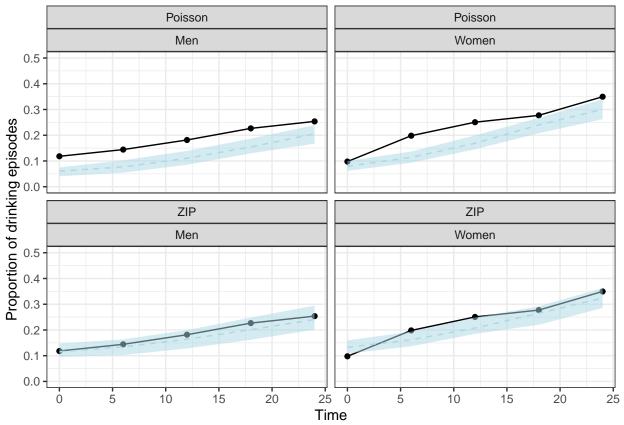
```
group_by(time, gender) %>%
summarise(nev = sum(rapi==0), n=n()) %>%
mutate(freq = nev/n, sd=sqrt((1-nev/n)/nev)) %>%
mutate(lower=freq-1.96*sd, upper=freq+1.96*sd)

## `summarise()` has grouped output by 'time'. You can override using the `.groups` argument.
rapipl$lower[rapipl$lower<0] <-0 # we should use a better approximation for CI

plot2 <- ggplot(rapipl, aes(x=time, y=freq, group=gender)) + geom_line() +
    geom_point() +
    geom_line(data=gtab, aes(x=time, y=median, group=gender), linetype=2, colour='lightblue') +
    geom_ribbon(data=gtab,aes(ymin=lower, ymax=upper, group=gender), alpha=0.5, fill='lightblue') +
    ylim(c(0,0.5)) + theme_bw() + theme(legend.position = "none") + facet_wrap(model~gender) +</pre>
```

xlab("Time") + ylab("Proportion of drinking episodes")

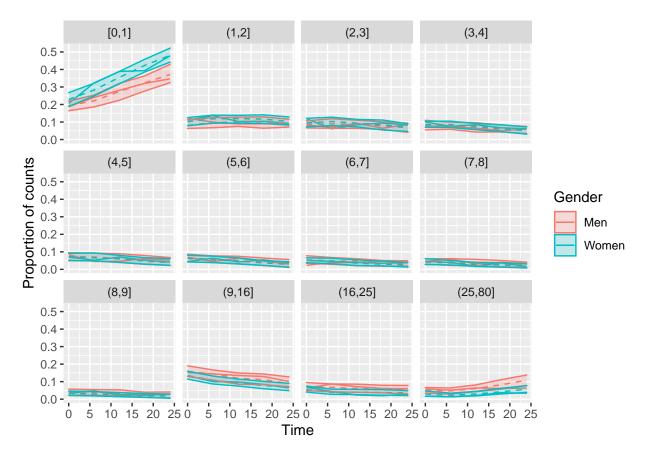
print(plot2)



To diagnose the model further, we can look at VPC plots for the different values of the score. Here, we plot separate VPC for scores 0 to 9 then regroup the higher scores into 3 categories as the number of subjects with more than 10 drinking episodes drops quickly. The code below can be tweaked to adjust to different score categories if needed (changing the *mybreaks* line). The ZIP model is able to predict the probability of observing different values of the score in both men and women over time.

```
# Grouping data by time and score
yfit <- ysim.zip2
ydat <- yfit@data
ysim <- yfit@sim.data@datasim$ysim
nsim<-length(ysim)/dim(ydat@data)[1]</pre>
```

```
obsmat<-data.frame(id=ydat@data[,ydat@name.group], x=ydat@data[,ydat@name.X], y=ydat@data[,ydat@name.re
# Regrouping times - not needed here as everyone has the same times
# Regrouping scores - observed data
mybreaks \leftarrow c(0:9, 16, 25, 80)
x <- cut(obsmat$y, breaks=mybreaks, include.lowest = TRUE)</pre>
obsmat$score.group <- x
# With tidyverse
counting.scores <- obsmat %>%
  group_by(x, covariate.group) %>%
  count(score.group)
number.samples <- obsmat %>%
  group_by(x, covariate.group) %>%
  summarise(n=n())
## `summarise()` has grouped output by 'x'. You can override using the `.groups` argument.
freq.scores <- number.samples %>%
  left_join(counting.scores,
            by = c("x","covariate.group")) %>%
  mutate(freq=n.y/n.x) %>%
  mutate(covariate.group=ifelse(covariate.group==0,"Men","Women"))
\# qqplot(data=freq.scores, aes(x=x, y=freq, qroup=covariate.qroup, colour=as.factor(covariate.qroup)))
# Regrouping scores - simulated data
ysim.tab <- data.frame(irep=rep(1:nsim, each=dim(ydat@data)[1]), x=rep(obsmat$x, nsim), covariate.group
sim.scores <- ysim.tab %>%
  group_by(irep, x, covariate.group) %>%
  count(score.group)
simfreq.scores <- number.samples %>%
  left_join(sim.scores,
            by = c("x","covariate.group")) %>%
  mutate(freq=n.y/n.x) %>%
  mutate(covariate.group=ifelse(covariate.group==0, "Men", "Women"))
simfreq.bands <- simfreq.scores %>%
  group_by(x, covariate.group, score.group) %>%
  summarise(lower=quantile(freq, c(0.05)), median=quantile(freq, c(0.5)), upper=quantile(freq, c(0.95))
## `summarise()` has grouped output by 'x', 'covariate.group'. You can override using the `.groups` arg
plot.counts <- ggplot(data=freq.scores, aes(x=x, y=freq, group=covariate.group, colour=as.factor(covari
  geom_line(data=simfreq.bands, aes(x=x, y=median, group=covariate.group, colour=as.factor(covariate.gr
  geom_ribbon(data=simfreq.bands, aes(ymin=lower, ymax=upper, group=covariate.group, fill=as.factor(co
  xlab("Time") + ylab("Proportion of counts") + guides(fill=guide_legend(title='Gender'), colour=guide_
  facet_wrap(.~score.group, ncol=4)
print(plot.counts)
```



Hurdle model Another way to handle the excess of 0's is the so-called hurdle model. In this model, we first fit a binary model to the dichotomised data (no event versus an event), then a Poisson model to the subjects who experienced at least one event. Compared to the ZIP model we fitted previously, this model lets the proportion of subjects without alcohol-related events vary with time through the binary logistic regression part (see more details on binary/categorical models in the corresponding notebook).

```
## Hurdle - 2 models
saemix.data1<-saemixData(name.data=rapi.saemix[rapi.saemix$rapi>0,], name.group=c("id"),
                          name.predictors=c("time", "rapi"), name.response=c("rapi"),
                          name.covariates=c("gender"),
                          units=list(x="week",y="",covariates=c("")))
##
##
  The following SaemixData object was successfully created:
##
##
##
   Object of class SaemixData
##
       longitudinal data for use with the SAEM algorithm
##
   Dataset rapi.saemix[rapi.saemix$rapi > 0, ]
##
       Structured data: rapi ~ time + rapi | id
##
       X variable for graphs: time (week)
##
       covariates: gender ()
##
         reference class for covariate gender :
rapi.saemix$y0<-as.integer(rapi.saemix$rapi==0)</pre>
saemix.data0<-saemixData(name.data=rapi.saemix, name.group=c("id"),</pre>
                          name.predictors=c("time","y0"),name.response=c("y0"),
                          name.covariates=c("gender"),
```

```
units=list(x="week",y="",covariates=c("")))
##
##
## The following SaemixData object was successfully created:
##
## Object of class SaemixData
##
       longitudinal data for use with the SAEM algorithm
## Dataset rapi.saemix
##
       Structured data: y0 ~ time + y0 | id
##
       X variable for graphs: time (week)
##
       covariates: gender ()
         reference class for covariate gender : Men
##
# Fit Binomial model to saemix.data0
binary.model<-function(psi,id,xidep) {</pre>
  tim<-xidep[,1]</pre>
  y < -xidep[,2]
  inter<-psi[id,1]</pre>
  slope<-psi[id,2]</pre>
  logit<-inter+slope*tim</pre>
  pevent<-exp(logit)/(1+exp(logit))</pre>
  pobs = (y==0)*(1-pevent)+(y==1)*pevent
  logpdf <- log(pobs)</pre>
  return(logpdf)
}
# Associated simulation function
simulBinary<-function(psi,id,xidep) {</pre>
  tim<-xidep[,1]</pre>
  y < -xidep[,2]
  inter<-psi[id,1]</pre>
  slope<-psi[id,2]</pre>
  logit<-inter+slope*tim</pre>
  pevent<-exp(logit)/(1+exp(logit))</pre>
  ysim<-rbinom(length(tim),size=1, prob=pevent)</pre>
  return(ysim)
saemix.hurdle0<-saemixModel(model=binary.model,description="Binary model",</pre>
                              modeltype="likelihood",simulate.function=simulBinary,
                              psi0=matrix(c(-1.5,-.1,0,0),ncol=2,byrow=TRUE,dimnames=list(NULL,c("theta1"
                              transform.par=c(0,0), covariate.model=c(1,1),
                              covariance.model=matrix(c(1,0,0,1),ncol=2), omega.init=diag(c(1,0.3)))
##
##
## The following SaemixModel object was successfully created:
##
## Nonlinear mixed-effects model
     Model function: Binary model
##
##
     Model type: likelihood
## function(psi,id,xidep) {
##
     tim<-xidep[,1]</pre>
##
     y < -xidep[,2]
     inter<-psi[id,1]</pre>
##
```

```
##
    slope<-psi[id,2]</pre>
##
    logit<-inter+slope*tim</pre>
##
    pevent<-exp(logit)/(1+exp(logit))</pre>
    pobs = (y==0)*(1-pevent)+(y==1)*pevent
##
    logpdf <- log(pobs)</pre>
##
##
    return(logpdf)
## }
##
    Nb of parameters: 2
##
        parameter names: theta1 theta2
##
        distribution:
##
       Parameter Distribution Estimated
## [1,] theta1 normal Estimated
## [2,] theta2 normal
                           Estimated
##
   Variance-covariance matrix:
##
        theta1 theta2
## theta1 1 0
## theta2
             0
   Covariate model:
      theta1 theta2
##
## [1,]
       1
##
      Initial values
##
              theta1 theta2
## Pop.CondInit -1.5 -0.1
## Cov.CondInit 0.0
                       0.0
saemix.options<-list(seed=1234567,save=FALSE,save.graphs=FALSE, displayProgress=FALSE, nb.chains=10, fix</pre>
hurdlefit0<-saemix(saemix.hurdle0,saemix.data0,saemix.options)</pre>
## Nonlinear mixed-effects model fit by the SAEM algorithm
## -----
             Data
## -----
## Object of class SaemixData
      longitudinal data for use with the SAEM algorithm
## Dataset rapi.saemix
      Structured data: y0 ~ time + y0 | id
##
##
      X variable for graphs: time (week)
##
      covariates: gender ()
       reference class for covariate gender : Men
## Dataset characteristics:
      number of subjects:
                            818
##
      number of observations: 3616
##
      average/min/max nb obs: 4.42 / 1 / 5
## First 10 lines of data:
     id time y0 y0.1 gender mdv cens occ ytype
##
## 1
          0 1
                1
                     Men
                           0
## 2
          6 1
                          0
                                0
                                   1
      1
                      Men
                                          1
                 1
## 3
      1
        18 1
                 1
                      Men
                           0
                                0
                                   1
## 4
     2
          0 0
                0 Women
                           0 0 1
                                         1
## 5
     2 6 0 0 Women
                           0 0 1
     2 12 0 0 Women
                           0 0 1
## 6
                                        1
## 7
      2 18 0
                0 Women
                           0
                               0 1
                                         1
```

1

1

0 0 1

8 2 24 0 0 Women

9 3 0 0 0 Men 0 0 1

```
## 10 3 12 0 0 Men 0 0 1
        Model
## -----
## Nonlinear mixed-effects model
   Model function: Binary model
    Model type: likelihood
## function(psi,id,xidep) {
    tim<-xidep[,1]</pre>
##
##
    y < -xidep[,2]
    inter<-psi[id,1]</pre>
##
    slope<-psi[id,2]</pre>
##
    logit<-inter+slope*tim</pre>
##
    pevent<-exp(logit)/(1+exp(logit))</pre>
##
    pobs = (y==0)*(1-pevent)+(y==1)*pevent
##
    logpdf <- log(pobs)</pre>
##
   return(logpdf)
## }
## <bytecode: 0x55a65ee426c0>
##
   Nb of parameters: 2
      parameter names: theta1 theta2
##
##
       distribution:
##
      Parameter Distribution Estimated
## [1,] theta1 normal Estimated
## [2,] theta2 normal
                         Estimated
    Variance-covariance matrix:
##
     theta1 theta2
## theta1 1 0
## theta2 0
  Covariate model:
##
     [,1] [,2]
## gender 1 1
##
     Initial values
##
            theta1 theta2
## Pop.CondInit -1.5 -0.1
## Cov.CondInit 0.0 0.0
## -----
## ---- Key algorithm options ----
## -----
##
     Estimation of individual parameters (MAP)
     Estimation of log-likelihood by importance sampling
##
     Number of iterations: K1=300, K2=100
##
     Number of chains: 10
##
     Seed: 1234567
##
     Number of MCMC iterations for IS: 5000
##
     Simulations:
##
         nb of simulated datasets used for npde: 1000
##
         nb of simulated datasets used for VPC: 100
##
     Input/output
##
         save the results to a file: FALSE
         save the graphs to files: FALSE
## -----
                   Results
## -----
```

```
## ----- Fixed effects -----
## -----
      Parameter
##
                      Estimate
## [1,] theta1
                      -2.796
## [2,] beta_gender(theta1) 0.132
## [3,] theta2
                        0.036
## [4,] beta_gender(theta2) 0.030
## -----
## ----- Variance of random effects -----
        Parameter
                   Estimate
## theta1 omega2.theta1 2.4033
## theta2 omega2.theta2 0.0062
## -----
## ----- Correlation matrix of random effects -----
##
              omega2.theta1 omega2.theta2
## omega2.theta1 1
## omega2.theta2 0
                         1
## -----
## ----- Statistical criteria -----
## -----
##
## Likelihood computed by importance sampling
##
       -2LL= 3249.132
       AIC = 3263.132
##
       BIC = 3296.08
cat("Expected proportion of 0's at time 0:",1/(1+exp(-hurdlefit0@results@fixed.effects[1])),"\n")
## Expected proportion of 0's at time 0: 0.05753853
table(rapi.saemix$rapi[rapi.saemix$time==0] == 0) # 10.6%
##
## FALSE TRUE
   731
         87
# Fit Poisson model to saemix.data1
saemix.hurdle1.cov2<-saemixModel(model=count.poisson,description="Count model Poisson",modeltype="likel
                           simulate.function = countsimulate.poisson,
                           psi0=matrix(c(log(5),0.01),ncol=2,byrow=TRUE,dimnames=list(NULL, c("in
                           transform.par=c(0,0), omega.init=diag(c(0.5, 0.5)),
                           covariance.model =matrix(data=1, ncol=2, nrow=2),
                           covariate.model=matrix(c(1,1), ncol=2, byrow=TRUE))
##
##
## The following SaemixModel object was successfully created:
## Nonlinear mixed-effects model
    Model function: Count model Poisson
##
    Model type: likelihood
## function(psi,id,xidep) {
   time<-xidep[,1]
```

```
##
     v < -xidep[,2]
##
     intercept<-psi[id,1]</pre>
     slope<-psi[id,2]</pre>
##
     lambda<- exp(intercept + slope*time)</pre>
##
##
     logp <- -lambda + y*log(lambda) - log(factorial(y))</pre>
##
     return(logp)
## }
## <bytecode: 0x55a65e996378>
##
     Nb of parameters: 2
##
        parameter names: intercept slope
##
        distribution:
##
       Parameter Distribution Estimated
## [1,] intercept normal
                              Estimated
## [2,] slope
                 normal
                              Estimated
##
    Variance-covariance matrix:
##
            intercept slope
## intercept
                   1
                           1
## slope
                           1
##
    Covariate model:
##
        intercept slope
## [1,]
               1
##
       Initial values
##
               intercept slope
## Pop.CondInit 1.609438 0.01
## Cov.CondInit 0.000000 0.00
saemix.options<-list(seed=632545,save=FALSE,save.graphs=FALSE, displayProgress=FALSE)</pre>
hurdlefit1<-saemix(saemix.hurdle1.cov2,saemix.data1,saemix.options)
## Error in solve.default(F0) :
    routine Lapack dgesv : le système est exactement singulier : U[2,2] = 0
## Nonlinear mixed-effects model fit by the SAEM algorithm
## -----
## ----
                Data
## -----
## Object of class SaemixData
##
       longitudinal data for use with the SAEM algorithm
## Dataset rapi.saemix[rapi.saemix$rapi > 0, ]
       Structured data: rapi ~ time + rapi | id
##
       X variable for graphs: time (week)
##
       covariates: gender ()
        reference class for covariate gender : Men
##
## Dataset characteristics:
##
      number of subjects:
                               802
##
      number of observations: 2860
       average/min/max nb obs: 3.57 / 1 / 5
## First 10 lines of data:
##
      id time rapi rapi.1 gender mdv cens occ ytype
## 4
      2
           0
                3
                        3 Women
                                  0
                                       0
                                            1
## 5
      2
           6
                6
                        6 Women
                                       0
## 6
                5
                       5 Women
                                  0
                                       0
      2
          12
                                            1
                                                  1
## 7
      2
          18
                4
                                  0
                                       0
                        4 Women
                                            1
      2
                5
                                  0
                                       0
## 8
          24
                        5 Women
                                           1
                                                  1
## 9
           0
                9
                          Men
                                       0
```

```
12 1 1 Men 0
## 10 3
## 12 4 0 3
                   3 Women 0
                                   0
## 13 4
       6 2
                   2 Women
         0 35
                  35 Women 0
                                   0 1
## 14 5
                                           1
## -----
             Model
## -----
## Nonlinear mixed-effects model
    Model function: Count model Poisson
##
    Model type: likelihood
## function(psi,id,xidep) {
##
    time<-xidep[,1]
##
    y < -xidep[,2]
##
    intercept<-psi[id,1]
##
    slope<-psi[id,2]</pre>
##
    lambda<- exp(intercept + slope*time)</pre>
##
    logp <- -lambda + y*log(lambda) - log(factorial(y))</pre>
##
    return(logp)
## }
## <bytecode: 0x55a65e996378>
##
    Nb of parameters: 2
##
       parameter names: intercept slope
##
       distribution:
      Parameter Distribution Estimated
## [1,] intercept normal Estimated
## [2,] slope
            normal
                         Estimated
##
   Variance-covariance matrix:
           intercept slope
## intercept 1
## slope
                 1
##
   Covariate model:
##
        [,1] [,2]
## gender
          1
##
      Initial values
             intercept slope
## Pop.CondInit 1.609438 0.01
## Cov.CondInit 0.000000 0.00
         Key algorithm options ----
## -----
      Estimation of individual parameters (MAP)
##
      Estimation of standard errors and linearised log-likelihood
      Estimation of log-likelihood by importance sampling
##
##
      Number of iterations: K1=300, K2=100
##
      Number of chains: 1
      Seed: 632545
##
      Number of MCMC iterations for IS: 5000
##
##
      Simulations:
##
         nb of simulated datasets used for npde: 1000
##
         nb of simulated datasets used for VPC: 100
##
      Input/output
##
         save the results to a file: FALSE
##
         save the graphs to files: FALSE
## -----
```

```
Results
## -----
## ----- Fixed effects -----
## -----
    Parameter
                  Estimate SE CV(%) p-value
## [1,] intercept
                   1.8656 0.066 3.5 -
## [2,] beta_gender(intercept) -0.1972  0.089  44.9  0.013
                   -0.0059 0.057 955.8 -
## [3,] slope
## [4,] beta_gender(slope) -0.0085 0.075 881.7 0.455
## -----
## ----- Variance of random effects -----
## -----
       Parameter
                   Estimate SE CV(%)
## intercept omega2.intercept 0.6000 NA NA
      omega2.slope 0.0017 NA NA
## slope
## covar
       cov.intercept.slope -0.0103 NA NA
## -----
## ----- Correlation matrix of random effects -----
## -----
           omega2.intercept omega2.slope
## omega2.intercept 1.00
                   -0.32
          -0.32
## omega2.slope
## -----
## ----- Statistical criteria -----
## -----
## Likelihood computed by linearisation
##
     -2LL= 437509.5
     AIC = 437525.5
##
     BIC = 437563
##
## Likelihood computed by importance sampling
##
     -2LL= 17628.18
##
     AIC = 17644.18
##
     BIC = 17681.67
summary(hurdlefit0)
## -----
## ----- Fixed effects ------
## -----
         Parameter Estimate
          theta1 -2.796
## 2 beta_gender(theta1) 0.132
## 3
           theta2
                 0.036
## 4 beta_gender(theta2)
                 0.030
## -----
## ----- Variance of random effects -----
## -----
##
        Parameter Estimate
## theta1 omega2.theta1 2.4033
## theta2 omega2.theta2 0.0062
## -----
## ----- Correlation matrix of random effects -----
```

```
##
            omega2.theta1 omega2.theta2
## omega2.theta1 1.00 0.00
## omega2.theta2 0.00
                     1.00
## -----
## ----- Statistical criteria -----
## -----
## Likelihood computed by importance sampling
      -2LL= 3249.132
##
##
      AIC = 3263.132
     BIC = 3296.08
summary(hurdlefit1)
## ----- Fixed effects -----
## -----
## Warning in .local(object, ...): NAs introduits lors de la conversion automatique
             Parameter Estimate
                            SE CV(%) p-value
## 1
             intercept 1.8656 0.066 3.53
## 2 beta_gender(intercept) -0.1972 0.089 44.92
                                     0.013
               slope -0.0059 0.057 955.79
     beta_gender(slope) -0.0085 0.075 881.67 0.455
## -----
## ----- Variance of random effects -----
## -----
##
              Parameter Estimate SE CV(%)
## intercept omega2.intercept 0.6000 NA
## slope omega2.slope 0.0017 NA
## ----- Correlation matrix of random effects -----
## -----
##
             omega2.intercept omega2.slope
## omega2.intercept 1.00 -0.32
            -0.32
## omega2.slope
## ------ Statistical criteria -----
## -----
## Likelihood computed by linearisation
      -2LL= 437509.5
     AIC = 437525.5
##
##
     BIC = 437563
##
## Likelihood computed by importance sampling
    -2LL= 17628.18
##
      AIC = 17644.18
##
      BIC = 17681.67
## -----
# Simulate binary data
# proportion of 0's in the data
rapi.tab <- table(rapi.saemix$rapi == 0)</pre>
```

```
nsim<-100
ysim.hurdle0 <- simulateDiscreteSaemix(hurdlefit0, nsim=nsim)</pre>
cat("Observed proportion of 0's overall:",rapi.tab[2]/sum(rapi.tab),"\n")
## Observed proportion of 0's overall: 0.2090708
cat("Simulated proportion of 0's overall:",sum(ysim.hurdle0@sim.data@datasim$ysim)/length(ysim.hurdle0@
## Simulated proportion of 0's overall: 0.2069994
ysim.hurdle1 <- simulateDiscreteSaemix(hurdlefit1, nsim=nsim)</pre>
# Graph of proportion of O's with time
yfit<-ysim.hurdle0
simdat <-yfit@sim.data@datasim</pre>
simdat$time<-rep(yfit@data@data$time,nsim)</pre>
simdat$gender<-rep(yfit@data@data$gender,nsim)</pre>
ytab<-NULL
for(irep in 1:nsim) {
  xtab<-simdat[simdat$irep==irep,]</pre>
  suppressMessages(
  xtab1 <- xtab %>%
    group_by(time, gender) %>%
    summarise(nev = sum(ysim), n=n()) %>%
    mutate(freq = nev/n)
  ytab<-rbind(ytab,xtab1[,c("time","gender","freq")])</pre>
gtab <- ytab %>%
    group_by(time, gender) %>%
  summarise(lower=quantile(freq, c(0.05)), median=quantile(freq, c(0.5)), upper=quantile(freq, c(0.95))
  mutate(gender=ifelse(gender==0, "Men", "Women"))
## `summarise()` has grouped output by 'time'. You can override using the `.groups` argument.
gtab$freq<-1
gtab3<-cbind(gtab, model="Hurdle")</pre>
gtab<-rbind(gtab1, gtab2, gtab3)</pre>
gtab <- gtab %>%
  mutate(model=factor(model, levels=c("Poisson", "ZIP", "Hurdle")))
rapipl <- rapi.saemix %>%
    group_by(time, gender) %>%
  summarise(nev = sum(y0), n=n()) \%>\%
  mutate(freq = nev/n, sd=sqrt((1-nev/n)/nev)) %>%
  mutate(lower=freq-1.96*sd, upper=freq+1.96*sd)
## `summarise()` has grouped output by 'time'. You can override using the `.groups` argument.
rapipl$lower[rapipl$lower<0] <-0 # we should use a better approximation for CI
# Table form - compare to column B in Table 2
yfit0<-hurdlefit0
yfit1<-hurdlefit1
```

```
rr.tab<-data.frame(param=c("intercept", "beta.Male.inter", "slope", "beta.Male.slope", "omega.inter", "o
                   poissonNoZero=c(yfit1@results@fixed.effects, c(sqrt(diag(yfit1@results@omega)))),
                   logistic=c(yfit0@results@fixed.effects, c(sqrt(diag(yfit0@results@omega)))))
print(rr.tab)
##
               param poissonNoZero
                                      logistic
## 1
                       1.865583452 -2.79604024
           intercept
## 2 beta.Male.inter
                      -0.197211376 0.13215067
## 3
               slope
                      -0.005943599
                                    0.03642832
## 4 beta.Male.slope
                     -0.008525854
                                    0.02950090
## 5
         omega.inter
                       0.774608111
                                    1.55026563
```

Comparing the proportion of 0's for the different models We have already seen a clear model misfit for Poisson, on the other hand the other two models predict the proportion of subjects without drinking episodes much more accurately, with no clear difference between them.

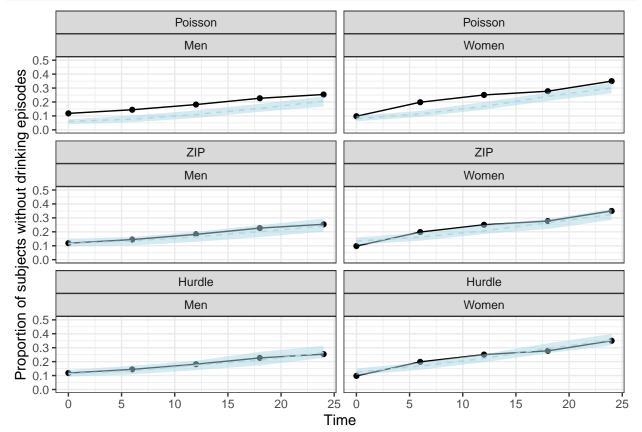
0.07889691

6

omega.slope

0.041313987

```
plot.prop0 <- ggplot(rapipl, aes(x=time, y=freq, group=gender)) + geom_line() +
    geom_point() +
    geom_line(data=gtab, aes(x=time, y=median, group=gender), linetype=2, colour='lightblue') +
    geom_ribbon(data=gtab, aes(ymin=lower, ymax=upper, group=gender), alpha=0.5, fill='lightblue') +
    ylim(c(0,0.5)) + theme_bw() + theme(legend.position = "none") + facet_wrap(model~gender, ncol=2) +
    xlab("Time") + ylab("Proportion of subjects without drinking episodes")</pre>
```



We can also produce VPC for the counts as previously for the ZIP model.

```
nsim<-100
ysim.hurdle1 <- simulateDiscreteSaemix(hurdlefit1, nsim=nsim)</pre>
# Grouping data by time and score
yfit <- ysim.hurdle1
ydat <- yfit@data
ysim <- yfit@sim.data@datasim$ysim
nsim<-length(ysim)/dim(ydat@data)[1]</pre>
obsmat<-data.frame(id=ydat@data[,ydat@name.group], x=ydat@data[,ydat@name.X], y=ydat@data[,ydat@name.re
# Regrouping times - not needed here as everyone has the same times
# Regrouping scores - observed data
mybreaks \leftarrow c(0:9, 16, 25, 80)
x <- cut(obsmat$y, breaks=mybreaks, include.lowest = TRUE)
obsmat$score.group <- x
# With tidyverse
counting.scores <- obsmat %>%
  group_by(x, covariate.group) %>%
  count(score.group)
number.samples <- obsmat %>%
  group_by(x, covariate.group) %>%
  summarise(n=n())
## `summarise()` has grouped output by 'x'. You can override using the `.groups` argument.
freq.scores <- number.samples %>%
  left_join(counting.scores,
            by = c("x","covariate.group")) %>%
  mutate(freq=n.y/n.x) %>%
  mutate(covariate.group=ifelse(covariate.group==0,"Men","Women"))
\# ggplot(data=freq.scores, aes(x=x, y=freq, group=covariate.group, colour=as.factor(covariate.group)))
# Regrouping scores - simulated data
ysim.tab <- data.frame(irep=rep(1:nsim, each=dim(ydat@data)[1]), x=rep(obsmat$x, nsim), covariate.group
sim.scores <- ysim.tab %>%
  group_by(irep, x, covariate.group) %>%
  count(score.group)
simfreq.scores <- number.samples %>%
  left_join(sim.scores,
            by = c("x","covariate.group")) %>%
  mutate(freq=n.y/n.x) %>%
  mutate(covariate.group=ifelse(covariate.group==0,"Men","Women"))
simfreq.bands <- simfreq.scores %>%
  group_by(x, covariate.group, score.group) %>%
  summarise(lower=quantile(freq, c(0.05)), median=quantile(freq, c(0.5)), upper=quantile(freq, c(0.95))
## `summarise()` has grouped output by 'x', 'covariate.group'. You can override using the `.groups` arg
plot.counts <- ggplot(data=freq.scores, aes(x=x, y=freq, group=covariate.group, colour=as.factor(covari
  geom_line(data=simfreq.bands, aes(x=x, y=median, group=covariate.group, colour=as.factor(covariate.gr
```

geom_ribbon(data=simfreq.bands, aes(ymin=lower, ymax=upper, group=covariate.group, fill=as.factor(co



Standard errors of estimation The computation of the FIM in saemix uses the so-called FOCE method, an approximation where the model function f is linearised around the conditional expectation of the individual parameters. This approximation is particularly bad for discrete data models, which is why currently saemix doesn't provide estimation errors for count data models. For non-Gaussian models, the exact FIM should be computed, and two approaches have been proposed using either numerical integration by a combination of MC and adaptive Gaussian quadrature (MC/AGQ, Ueckert et al 2017) or stochastic integration by MCMC (Rivière et al. 2017). Both these approaches are computationally intensive. They require to define the elementary FIM associated to the observed design (here, the number of times and the gender which enters the model), and sum over all the subjects sharing the same design. Here, as in the toenail example, the subjects in the real dataset tapi.saemix were not all seen at the same times. This makes the computation of an exact FIM somewhat cumbersome, especially when stratifying on gender, as we need to compute the FIM in each combination of gender and observation times and sum the elementary FIM. We leave that to the

5

Time

10 15 20 25 0

5

10 15 20 25 0

0.0

10 15 20 25 0

Alternatively, different bootstrap approaches can be used in non-linear mixed effect models and have been implemented for **saemix** in Comets et al. 2021, with code available on the github for the package (https://github.com/saemixdevelopment/saemixextension).

```
# Time-patterns
time.pattern <- tapply(rapi.saemix$time, rapi.saemix$id, function(x) paste(x,collapse="-"))
table(time.pattern)
## time.pattern
## 0 0-12 0-12-18 0-12-18-24 0-12-24 0-18</pre>
```

```
##
                25
                                                   4
                                                                  18
                                                                                    3
                                                                                                     4
                                  4
##
          0 - 18 - 24
                                                0 - 6
                                                                          0-6-12-18 0-6-12-18-24
                               0 - 24
                                                             0 - 6 - 12
##
                 6
                                  3
                                                 27
                                                                  36
                                                                                   54
                                                                                                  561
##
       0-6-12-24
                            0-6-18
                                        0-6-18-24
                                                             0-6-24
##
                28
                                  7
                                                 28
                                                                  10
```

```
# Time-patterns by gender
print(table(time.pattern, rapi.saemix$gender[!duplicated(rapi.saemix$id)]))
```

```
##
##
   time.pattern
                     Men Women
##
                      15
                             10
                       2
##
     0 - 12
                               2
##
     0-12-18
                       2
                               2
     0-12-18-24
                       8
##
                             10
                       2
##
     0-12-24
                               1
                       2
                               2
##
     0 - 18
##
     0-18-24
                       4
                               2
##
     0 - 24
                       0
                               3
##
     0-6
                      15
                             12
     0-6-12
##
                      17
                             19
##
     0-6-12-18
                      22
                             32
##
     0-6-12-18-24 213
                            348
##
     0-6-12-24
                      15
                             13
##
     0-6-18
                       4
                              3
                               5
##
     0-6-18-24
                      23
                       3
                               7
##
     0-6-24
```

for(i in 1:2) {

Case bootstrap: The first bootstrap approach we can use is case bootstrap, where we resample at the level of the individual. We plot the bootstrap distribution for the 4 parameters (intercept, slope, treatment effect on slope, and variability of intercept). The red vertical line represents the estimate obtained on the original data while the blue line shows the mean of the bootstrap distribution.

Conditional bootstrap: We can also use conditional bootstrap, a non-parametric residual bootstrap which bootstraps samples from the conditional distributions and preserves the exact structure of the original dataset.

Bootstrap CI Here we load the results from the two bootstrap files prepared beforehand by running the *saemix.bootstrap* code with 500 simulations for the ZIP model with covariates. We compute the bootstrap quantiles for the 95% CI, as well as the SD of the bootstrap distribution, corresponding to a normal approximation of the SE. Both bootstrap methods give similar estimates for the CI's of the different parameters.

```
if(runBootstrap) {
   case.count <- saemix.bootstrap(zippoisson.fit.cov2, method="case", nboot=nboot)
   cond.count <- saemix.bootstrap(zippoisson.fit.cov2, method="conditional", nboot=nboot)
} else {
   case.count <- read.table(file.path(saemixDir, "bootstrap", "results", "rapi_caseBootstrap.res"), header=T
   cond.count <- read.table(file.path(saemixDir, "bootstrap", "results", "rapi_condBootstrap.res"), header=T
   nboot<-dim(case.count)[1]
}
case.count <- case.count[!is.na(case.count[,2]),]
cond.count <- cond.count[!is.na(cond.count[,2]),]

par.estim<-c(zippoisson.fit.cov2@results@fixed.effects,diag(zippoisson.fit.cov2@results@omega)[zippoiss
df2<-data.frame(parameter=colnames(case.count)[-c(1)], saemix=par.estim)</pre>
```

```
if(i==1) {
    resboot1<-case.count
    namboot<-"case"
  } else {
    resboot1<-cond.count
    namboot <-"cNP"
  mean.bootDist<-apply(resboot1, 2, mean)[-c(1)]</pre>
  sd.bootDist<-apply(resboot1, 2, sd)[-c(1)]</pre>
  quant.bootDist<-apply(resboot1[-c(1)], 2, quantile, c(0.025, 0.975))
  11<-pasteO(format(mean.bootDist, digits=2)," (",format(sd.bootDist,digits=2, trim=T),")")</pre>
  12<-paste0("[",format(quant.bootDist[1,], digits=2),", ",format(quant.bootDist[2,],digits=2, trim=T),
  df2<-cbind(df2, 11, 12)
  i1<-3+2*(i-1)
  colnames(df2)[i1:(i1+1)]<-paste0(namboot,".",c("estimate","CI"))</pre>
}
print(df2)
##
                                                                        case.CI
                  parameter
                                   saemix
                                              case.estimate
## 1
                  intercept 1.772785031 1.7677 (0.05781)
                                                              [ 1.6532, 1.8840]
## 2 beta_gender.intercept. -0.196792601 -0.1991 (0.07404) [-0.3434, -0.0565]
## 3
                      slope -0.020435421 -0.0196 (0.00384) [-0.0276, -0.0121]
## 4
         beta gender.slope. -0.015721267 -0.0155 (0.00473) [-0.0248, -0.0064]
## 5
                         p0 0.075407726 0.0763 (0.00765)
                                                             [ 0.0627, 0.0916]
                                                              [ 0.6890, 0.8964]
## 6
           omega2.intercept 0.782636954 0.7908 (0.05303)
## 7
               omega2.slope 0.003255963
                                           0.0032 (0.00037)
                                                             [ 0.0024, 0.0039]
##
          cNP.estimate
     1.7825 (0.05076) [ 1.6793, 1.8798]
## 2 -0.1960 (0.06865) [-0.3315, -0.0722]
## 3 -0.0202 (0.00393) [-0.0275, -0.0122]
## 4 -0.0148 (0.00496) [-0.0255, -0.0055]
                        [ 0.0663, 0.0872]
## 5 0.0770 (0.00553)
## 6 0.7552 (0.04654)
                        [ 0.6702, 0.8484]
## 7 0.0028 (0.00024)
                        [ 0.0023, 0.0032]
```

References

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