## Saemix 3 - time-to-event data models

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09/2022

#### Version

Use saemix version  $\geq 3.2$ 

## Objective

Run TTE and RTTE models in saemix

This notebook uses additional code from the **saemix** development github (https://github.com/saemixdevel opment/saemixextension), not yet integrated in the package. The *workDir* folder in the next chunk of code points to the folder where the user stored this code, and is needed to run the notebook (*workDir* defaults to the current working directory). Specifically, the notebook loads:

- code for the MC/AGQ provided by Sebastian Ueckert (Ueckert et al. 2017)
  - if memory issues arise the code can be run in a separate script.
- the results for the bootstrap runs performed using different approaches (see Comets et al. Pharm Res 2021)
  - bootstraps can be run instead by switching the runBootstrap variable to TRUE in the first chunk of code
  - in the code, the number of bootstraps is set to 10 for speed but we recommend to use at least 200 for a 90% CI.
  - this can be changed in the following change of code by uncommenting the line *nboot*<-200 and setting the number of bootstrap samples (this may cause memory issues in **Rstudio** with older machines, if this is the case we recommend executing the code in a separate script)

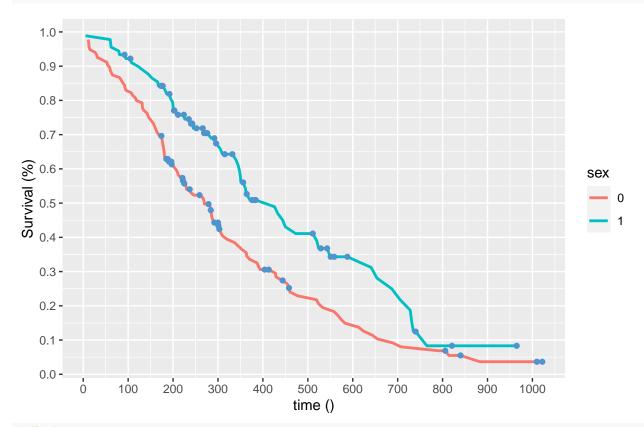
The current notebook can be executed to create an HMTL or PDF output with comments and explanations. A script version containing only the R code is also given as  $saemix3\_tteModel.R$  in the same folder.

#### TTE data

Data description - lung cancer The example chosen to illustrate the analysis of time-to-event data in saemix is the NCCTG Lung Cancer Data, describing the survival in patients with advanced lung cancer from the North Central Cancer Treatment Group (Loprinzi et al. 1994). Covariates measured in the study include performance scores rating how well the patient can perform usual daily activities. We reformatted the cancer dataset provided in the survival package in R in SAEM format: patients with missing age, sex, institution or physician assessments were removed from the dataset. Status was recoded as 1 for death and 0 for a censored event, and a censoring column was added to denote whether the patient was dead or alive at the time of the last observation. A line at time=0 was added for all subjects. Finally, subjects were numbered consecutively from 0 to 1.

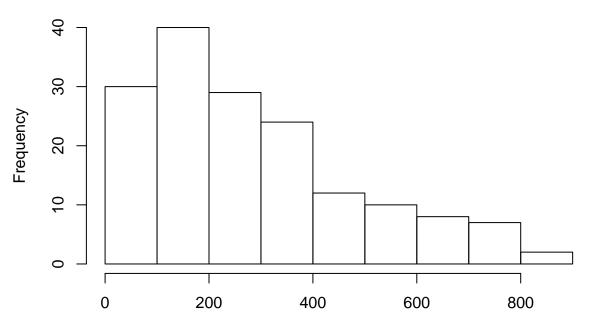
We can plot the distribution of times as a histogram.

```
name.covariates=c("age", "sex", "ph.ecog", "ph.karno", "pat.karno", "wt.loss", "meal.cal"),
    units=list(x="days",y="",covariates=c("yr","","-","%","%","cal","pounds")), verbose=FALSE)
plotDiscreteData(saemix.data, outcome="tte", which.cov="sex")
```



# Histogram
hist(lung.saemix\$time[lung.saemix\$status==1])

# Histogram of lung.saemix\$time[lung.saemix\$status == 1]



lung.saemix\$time[lung.saemix\$status == 1]

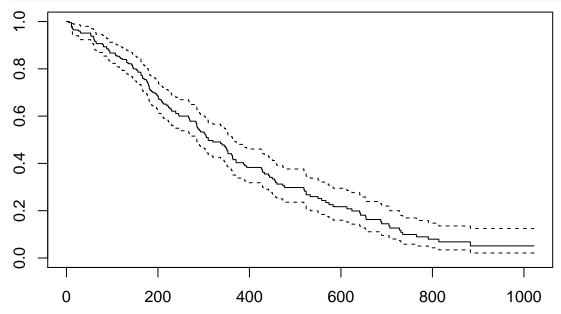
```
# Note: missing data in pat.karno, wt.loss and meal.cal
if(FALSE)
    print(summary(lung.saemix))
```

```
lung.surv<-lung.saemix[lung.saemix$time>0,]
lung.surv$status<-lung.surv$status+1
Surv(lung.surv$time, lung.surv$status) # 1=censored, 2=dead</pre>
```

## Kaplan-Meier plot

```
[1]
           306
                                             1022+
                                                                                  170
                                                                                         654
##
                  455
                        1010+
                                210
                                       883
                                                     310
                                                            361
                                                                    218
                                                                           166
    [13]
           728
                         144
                                       707
                                                61
                                                      88
                                                                          624
                                                                                  371
                                                                                         394
##
                  567
                                613
                                                            301
                                                                     81
##
    [25]
           520
                  574
                         118
                                390
                                        12
                                               473
                                                       26
                                                            533
                                                                    107
                                                                            53
                                                                                  122
                                                                                         814
           965+
                                460
                                                             583
##
    [37]
                   93
                         731
                                       153
                                               433
                                                     145
                                                                     95
                                                                          303
                                                                                  519
                                                                                         643
##
    [49]
           765
                  735
                         189
                                 53
                                       246
                                               689
                                                      65
                                                               5
                                                                    132
                                                                          687
                                                                                  345
                                                                                         444
    [61]
           223
                  175
                          60
                                         65
                                               208
                                                     821+
                                                                    230
                                                                          840+
                                                                                  305
##
                                163
                                                            428
                                                                                          11
##
    [73]
           132
                  226
                         426
                                705
                                       363
                                               11
                                                     176
                                                            791
                                                                     95
                                                                          196+
                                                                                  167
                                                                                         806+
           284
##
    [85]
                  641
                         147
                                740+
                                       163
                                               655
                                                     239
                                                              88
                                                                    245
                                                                          588+
                                                                                   30
                                                                                         179
    [97]
           310
                  477
                         166
                                559+
                                       450
                                               364
                                                     107
                                                             177
                                                                          529+
                                                                                         429
##
                                                                    156
                                                                                   11
   [109]
           351
                                       201
                                               524
##
                   15
                         181
                                283
                                                       13
                                                            212
                                                                    524
                                                                          288
                                                                                  363
                                                                                         442
##
   [121]
           199
                  550
                          54
                                558
                                       207
                                               92
                                                      60
                                                            551+
                                                                    543+
                                                                          293
                                                                                  202
                                                                                         353
   [133]
           511+
                  267
                         511+
                                371
                                       387
                                               457
                                                             201
                                                                    404+
                                                                          222
                                                                                   62
                                                                                         458+
##
                                                     337
   [145]
           356+
                  353
                         163
                                 31
                                       340
                                               229
                                                     444+
                                                            315+
                                                                    182
                                                                          156
                                                                                  364+
                                                                                         291
##
   [157]
           179
                  376+
                         384+
                                268
                                       292+
                                               142
                                                                          320
                                                                                         285
##
                                                     413+
                                                            266+
                                                                    194
                                                                                  181
   [169]
           301+
                  348
                         197
                                382+
                                       303+
                                              296+
                                                     180
                                                             186
                                                                    145
                                                                          269+
                                                                                  300+
                                                                                         284+
##
   [181]
           350
                  272+
                         292+
                                332+
                                       285
                                               259+
                                                     110
                                                             286
                                                                    270
                                                                            81
                                                                                  131
                                                                                         225+
   [193]
           269
                  225+
                         243+
                                279+
                                       276+
                                               135
                                                      79
                                                              59
                                                                    240+
                                                                          202+
                                                                                  235+
                                                                                         224+
##
##
   [205]
           239
                  237+
                         173+
                                252+
                                       221+
                                               185+
                                                       92+
                                                              13
                                                                    222+
                                                                          192+
                                                                                  183
                                                                                         211+
## [217]
           175+
                  197+
                         203+
                                116
                                        188+
                                               191+
                                                     105+
                                                            174+
                                                                    177+
```

```
nonpar.fit <- survfit(Surv(time, status) ~ 1, data = lung.surv)
plot(nonpar.fit)</pre>
```



**Model for TTE data** We can use a Weibull model for the hazard, parameterised as  $\lambda$  and  $\beta$ . For individual i, the hazard function of this model is:

$$h(t) = \frac{\beta}{\lambda} \left( \frac{t}{\lambda} \right)^{\beta - 1}$$

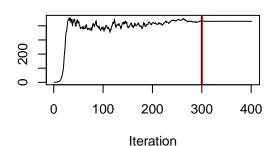
And the parametric survival function is given by:

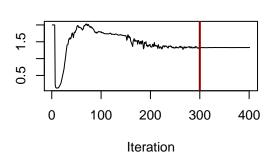
$$S(t) = e^{-\left(\frac{t}{\lambda}\right)^{\beta}}$$

```
weibulltte.model<-function(psi,id,xidep) {</pre>
  T < -xidep[,1]
  y<-xidep[,2] # events (1=event, 0=no event)
  cens<-which(xidep[,3]==1) # censoring times (subject specific)</pre>
  init <- which(T==0)</pre>
  lambda <- psi[id,1] # Parameters of the Weibull model</pre>
  beta <- psi[id,2]
  Nj <- length(T)
  ind <- setdiff(1:Nj, append(init,cens)) # indices of events</pre>
  hazard <- (beta/lambda)*(T/lambda)^(beta-1) # ln(H')
  H \leftarrow (T/lambda)^beta # ln(H)
  logpdf \leftarrow \text{rep}(0,Nj) \# ln(l(T=0))=0
  logpdf[cens] <- -H[cens] + H[cens-1] # ln(l(T=censoring time))</pre>
  logpdf[ind] <- -H[ind] + H[ind-1] + log(hazard[ind]) # ln(l(T=event time))</pre>
  return(logpdf)
}
saemix.model<-saemixModel(model=weibulltte.model, description="time model", modeltype="likelihood",</pre>
  psi0=matrix(c(1,2),ncol=2,byrow=TRUE,dimnames=list(NULL, c("lambda","beta"))),
  transform.par=c(1,1),covariance.model=matrix(c(1,0,0,0),ncol=2, byrow=TRUE), verbose=FALSE)
saemix.options<-list(seed=632545,save=FALSE,save.graphs=FALSE, displayProgress=FALSE, print=FALSE)</pre>
```

```
tte.fit<-saemix(saemix.model,saemix.data,saemix.options)
plot(tte.fit, plot.type="convergence")</pre>
```

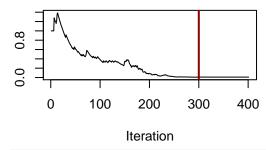






beta

### omega2.lambda



### summary(tte.fit)

```
## -----
## ----- Fixed effects -----
## -----
##
                 SE CV(%)
   Parameter Estimate
## 1
     lambda 431.81 51.60 11.95
## 2
            1.33 0.19 14.27
      beta
  ----- Variance of random effects -----
  _____
        Parameter Estimate SE CV(%)
##
## lambda omega2.lambda
                0.009 0.17 1857.95
 ---- Correlation matrix of random effects ---
  ______
##
##
          omega2.lambda
## omega2.lambda 1.00
## ----- Statistical criteria -----
 _____
## Likelihood computed by linearisation
##
     -2LL= 5189.352
##
     AIC = 5197.352
##
     BIC = 5211.017
##
```

**Simulation function** Simulating from a TTE model is slightly more complicated than for the other non Gaussian models. When the hazard function has an inverse, we can use the inverse CDF technique (or inverse transformation algorithm) for generating a random sample. The method uses the fact that a continuous cumulative density function, F, is a one-to-one mapping of the domain of the cdf into the interval (0,1). Therefore, if U is a uniform random variable on (0,1), then  $X = F^{-1}(U)$  has the distribution F.

For the single event Weibull model:

$$F = 1 - e^{-\int_0^T h(u)du} = 1 - e^{-\left(\frac{T}{\lambda}\right)^{\beta}} \sim \mathcal{U}(0, 1)$$

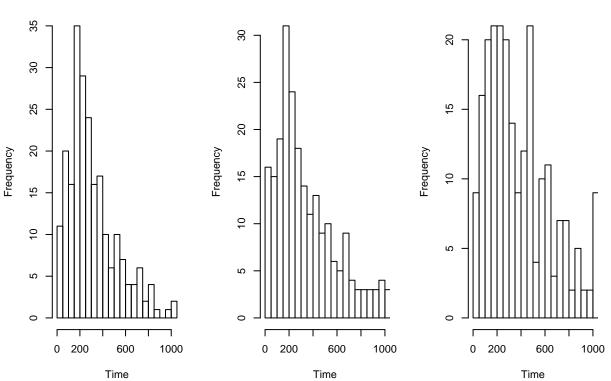
Assuming we simulate U = 1 - V from  $\mathcal{U}(0,1)$ , we can obtain a sample from the Weibull parametric model as:

$$T = \lambda \left( -\ln(V) + \left(\frac{T}{\lambda}\right)^{\beta} \right)^{1/\beta}$$

In the following we assume the first column of *xidep* contains the observed times, and that there is a common censoring time (the maximum observed time). We could also assume a common censoring (function *simulateWeibullTTE.maxcens()* below) but simulating from this function shows an excess of times simulated at the censoring limit compared to the original dataset.

```
# Simulate events based on the observed individual censoring time
simulateWeibullTTE <- function(psi,id,xidep) {</pre>
  T \leftarrow xidep[,1]
  y<-xidep[,2] # events (1=event, 0=no event)
  cens<-which(xidep[,3]==1) # censoring times (subject specific)</pre>
  init <- which(T==0)</pre>
  lambda <- psi[,1] # Parameters of the Weibull model</pre>
  beta <- psi[,2]
  Ni <- length(T)
  ind <- setdiff(1:Nj, append(init,cens)) # indices of events
  tevent<-T
  Vj<-runif(dim(psi)[1])</pre>
  tsim<-lambda*(-log(Vj))^(1/beta) # nsuj events</pre>
  tevent[T>0]<-tsim</pre>
  tevent[tevent[cens]>T[cens]] <- T[tevent[cens]>T[cens]]
  return(tevent)
# Checking the simulation function
xidep1<-saemix.data@data[,saemix.data@name.predictors]</pre>
nsuj <- saemix.data@N
psiM<-data.frame(lambda=rnorm(nsuj, mean=tte.fit@results@fixed.effects[1], sd=2), beta=tte.fit@results@
id1<-rep(1:nsuj, each=2)</pre>
simtime<-simulateWeibullTTE(psiM, id1, xidep1)</pre>
par(mfrow=c(1,2))
hist(saemix.data@data$time[saemix.data@data$time>0], breaks=30, xlab="Time", main="Original data")
hist(simtime[simtime>0], breaks=30, xlim=c(0,1000), xlab="Time", main="Simulated data")
```

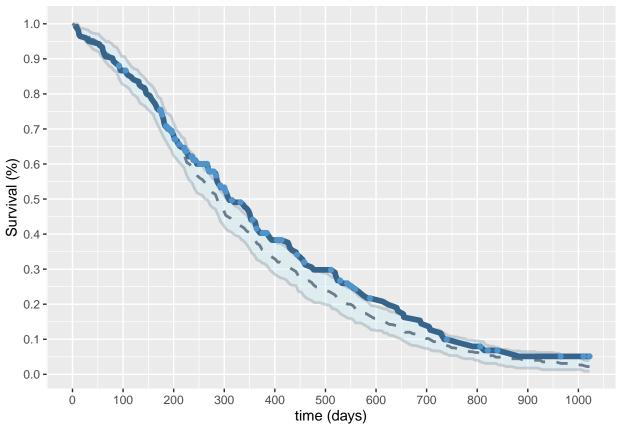
```
# Ignoring the cens column and assuming a common censoring time instead
simulateWeibullTTE.maxcens <- function(psi,id,xidep) {</pre>
  etime<-xidep[,1]
  censoringtime <- max(etime)</pre>
  lambda <- psi[,1]</pre>
  beta <- psi[,2]
  N<-dim(psi)[1]</pre>
  Vj<-runif(N)
  T<-lambda*(-log(Vj))^(1/beta)
  T[T>censoringtime] <-censoringtime</pre>
  etime[etime>0]<-T
  return(etime)
simtime.maxcens<-simulateWeibullTTE.maxcens(psiM, id1, xidep1)</pre>
par(mfrow=c(1,3))
hist(saemix.data@data$time[saemix.data@data$time>0], breaks=30, xlab="Time", main="Original data")
hist(simtime[simtime>0], breaks=30, xlim=c(0,1000), xlab="Time", main="Simulated data")
hist(simtime.maxcens[simtime.maxcens>0], breaks=30, xlim=c(0,1000), xlab="Time", main="Simulated data")
           Original data
                                          Simulated data
                                                                          Simulated data
```



We then use the simulation function defined above to simulate from the fitted model, adding it first to the model component, and plot VPC (we can also include the simulation function when creating the model by adding the argument *simulate.function=simulateWeibullTTE* to saemixModel in the code above).

```
tte.fit@model@simulate.function <- simulateWeibullTTE
simtte.fit <- simulateDiscreteSaemix(tte.fit, nsim=100)</pre>
```

```
gpl <- discreteVPC(simtte.fit, outcome="TTE")
## Error in exists(object) : premier argument incorrect
plot(gpl)</pre>
```

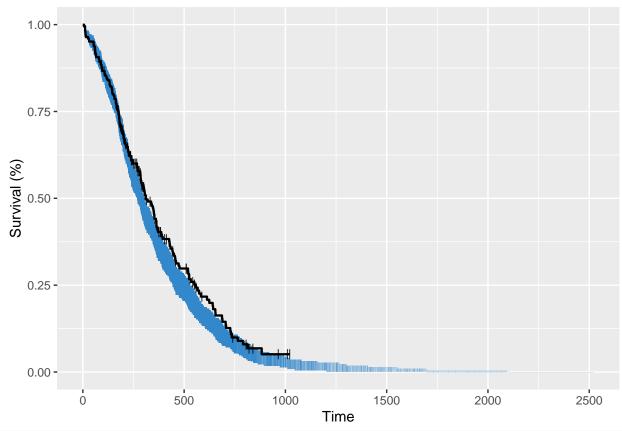


Note that there are some specialised packages such as the **survsim** and the **simsurv** package that could be leveraged for this exercise.

**VPC using Ron's package** A recent package was developed by Ron Keizer to implement VPC for different types of data. For survival data, we can also use the  $vpc\_tte()$  function from this package to produce the KM-VPC plot.

## Initializing.

## Detected column 'cens' with censoring information in observation data, assuming 1=censored event, 0=
## Warning: `group\_by\_()` was deprecated in dplyr 0.7.0.



Exact FIM by AGQ (code by Sebastian Ueckert)

For non-Gaussian models, the exact FIM should be computed, and two approaches have been proposed using

either numerical integration by a combination of MC and adaptive Gaussian quadrature (MC/AGQ, Ueckert et al 2017) or stochastic integration by MCMC (Rivière et al. 2017).

Both these approaches are computationally intensive.

**Eco** pas sûre que la méthode pour calculer la FIM exacte dans un protocole de population soit utilisable pour calculer la FIM empirique d'un jeu de données...

Eco no example for TTE models in Sebastian's code so need to define the different functions by hand...

Here we use code provided by Sebastian Ueckert implementing the MC/AGQ approach, as the MCMC requires the installation of rStan. In this approach, the information matrix (FIM) over the population is first decomposed the sum of the individual FIM:

$$FIM(\Psi,\Xi) = \sum_{i=1}^{N} FIM(\Psi,\xi_i)$$

where  $\xi_i$  denotes the individual design in subject i. Assuming Q different elementary designs, the FIM can also be summed over the different designs weighted by the number of subjects  $N_q$  in design q as:

$$FIM(\Psi,\Xi) = \sum_{q=1}^{Q} N_q FIM(\Psi,\xi_q)$$

In the following, we first load the functions needed to compute the exact FIM. We then define a model object with the following components:

- parameter\_function: a function returning the list of parameters as the combination of fixed and random effects
- log likelihood function: using the parameters, computes the log-likelihood for all y in the dataset
- simulation\_function: using the parameters, computes the log-likelihood and produces a random sample from the corresponding distribution
- inverse\_simulation\_function: supposed to be the quantile function but not quite sure :-/ (here, returns the category in which is urand)
- mu: the fixed parameters
- omega: the variance-covariance matrix

For mu and omega, we use the results from the saemix fit.

#### TODO Alexandra?

```
# Code Sebastian
source(file.path(dirAGQ, "default_settings.R"))
source(file.path(dirAGQ, "helper_functions.R"))
source(file.path(dirAGQ, "integration.R"))
source(file.path(dirAGQ, "model.R"))
saemix.fit <- tte.fit</pre>
# TODO - adapt to TTE model ???
# model <- Model$new(</pre>
   parameter_function = function(mu, b) list(alp1=mu[1]+b[1], alp2=mu[2], alp3=mu[3], alp4=mu[4], beta
    log_likelihood_function = function(y, design, alp1, alp2, alp3, alp4, beta) {
      logit1<-alp1 + beta*design$time</pre>
#
      logit2<-logit1+alp2
#
      logit3<-logit2+alp3
      logit4<-logit3+alp4
#
      pge1<-exp(logit1)/(1+exp(logit1))</pre>
```

```
pge2<-exp(logit2)/(1+exp(logit2))
#
      pqe3<-exp(logit3)/(1+exp(logit3))
#
#
      pqe4<-exp(logit4)/(1+exp(logit4))
      pobs = (y=-1)*pqe1+(y=-2)*(pqe2 - pqe1)+(y=-3)*(pqe3 - pqe2)+(y=-4)*(pqe4 - pqe3)+(y=-5)*(1 - pqe3)
#
#
      log(pobs)
#
   },
#
   simulation_function = function(design, alp1, alp2, alp3, alp4, beta) {
#
      logit1<-alp1 + beta*design$time</pre>
      logit2<-logit1+alp2
#
#
      logit3<-logit2+alp3
#
      logit4<-logit3+alp4
      pge1<-exp(logit1)/(1+exp(logit1))</pre>
#
      pge2<-exp(logit2)/(1+exp(logit2))
#
      pge3<-exp(logit3)/(1+exp(logit3))
#
#
      pqe4<-exp(logit4)/(1+exp(logit4))
#
      x<-runif(length(time))</pre>
#
      ysim < -1 + as.integer(x > pqe1) + as.integer(x > pqe2) + as.integer(x > pqe3) + as.integer(x > pqe4)
   },
#
#
    inverse_simulation_function = function(design, urand, alp1, alp2, alp3, alp4, beta) {
      if(is.null(urand)) return(seq_along(design$time))
#
#
      logit1<-alp1 + beta*design$time</pre>
#
      logit2 < -logit1 + alp2
#
      logit3<-logit2+alp3
#
      logit4<-logit3+alp4
#
      pqe1<-exp(logit1)/(1+exp(logit1))</pre>
#
      pge2<-exp(logit2)/(1+exp(logit2))
#
     pge3<-exp(logit3)/(1+exp(logit3))
#
      pge4<-exp(logit4)/(1+exp(logit4))
      1+as.integer(urand>pge1)+as.integer(urand>pge2)+as.integer(urand>pge3)+as.integer(urand>pge4)
#
   },
#
   mu = saemix.fit@results@fixed.effects,
#
#
   omega = saemix.fit@results@omega[c(1,5),c(1,5)]
#
# TBContinued
```

## Diagnostics

Comparison to the KM fit With TTE data the First-Order approximation for the FIM doesn't seem to perform too badly. We can use the delta-method to obtain standard errors around the value of the survival function, using the following vector of derivatives:

$$\begin{pmatrix} \frac{\delta S}{\delta \lambda} \\ \frac{\delta S}{\delta \beta} \end{pmatrix} = \begin{pmatrix} \frac{\beta}{\lambda} \left( \frac{t}{\lambda} \right)^{\beta} e^{-\left( \frac{t}{\lambda} \right)^{\beta}} \\ -\ln\left( \frac{t}{\lambda} \right) \left( \frac{t}{\lambda} \right)^{\beta} e^{-\left( \frac{t}{\lambda} \right)^{\beta}} \end{pmatrix}$$

We overlay the parametric fit and its confidence interval in red over the previous non-parametric KM estimate, and find a good concordance between the two.

```
# Use survival package to assess Survival curve
xtim<-seq(0,max(lung.saemix$time), length.out=200)
estpar<-tte.fit@results@fixed.effects</pre>
```

```
estse<-tte.fit@results@se.fixed
ypred<-exp(-(xtim/estpar[1])^(estpar[2]))</pre>
# Computing SE for the survival curve based on linearised FIM (probably not a good idea) through the de
invfim<-solve(tte.fit@results@fim[1:2,1:2])</pre>
xcal<- (xtim/estpar[1])^estpar[2]</pre>
dsdbeta<- -log(xtim/estpar[1]) * xcal *exp(-xcal)</pre>
dsdalpha<- estpar[2]/estpar[1] * xcal *exp(-xcal)</pre>
xmat<-rbind(dsdalpha, dsdbeta)</pre>
     x1 < -t(xmat[,1:3]) %*% invfim %*% xmat[,1:3]
sesurv<-rep(0,length(xcal))</pre>
for(i in 1:length(xcal))
  sesurv[i]<-sqrt(t(xmat[,i]) %*% invfim %*% xmat[,i])</pre>
# Comparison between KM and parametric fit
plot(nonpar.fit, xlab = "Days", ylab = "Overall survival probability")
lines(xtim,ypred, col="red",lwd=2)
lines(xtim,ypred+1.96*sesurv, col="red",lwd=1, lty=2)
lines(xtim,ypred-1.96*sesurv, col="red",lwd=1, lty=2)
Overall survival probability
       Ö
      9.0
      0.4
      0.2
      0.0
              0
                            200
                                          400
                                                         600
                                                                        800
                                                                                      1000
```

#### RTTE model

In this section we simulate repeated time-to-event data from a Weibull model and fit it. To simulate from a RTTE model, we simulate repeated events starting from the previous one using the inverse CDF technique. Because we don't know in advance the number of events in each subject, we lose the efficient vectorisation from  $\mathbf{R}$  and this function can be considerably slower than the single event TTE.

Days

```
# Simulating RTTE data by simulating from U(0,1) and inverting the cdf
simul.rtte.unif<-function(psi) { # xidep, id not important, we only use psi
  censoringtime <- 3
  maxevents <- 30
  lambda <- psi[,1]</pre>
```

```
beta <- psi[,2]
  simdat<-NULL
  N<-nrow(psi)
  for(i in 1:N) {
    eventTimes<-c(0)
    T<-0
    Vj<-runif(1)</pre>
          T \leftarrow (-log(Vj)*lambda[i])^(beta[i])
    T<-lambda[i]*(-log(Vj))^(1/beta[i])</pre>
    nev<-0
    while (T < censoringtime & nev<maxevents){</pre>
      eventTimes <- c(eventTimes, T)</pre>
      nev<-nev+1
      Vj<-runif(1)</pre>
              T \leftarrow T + (-\log(V_j) * lambda[i]) \hat{beta[i]}
              T \leftarrow (-\log(V_j) * lambda[i] + T^(1/beta[i]))^(beta[i])
      T \leftarrow lambda[i] * (-log(Vj) + (T/lambda[i])^(beta[i]))^(1/beta[i])
    if(nev==maxevents) {
      message("Reached maximum number of events\n")
    eventTimes<-c(eventTimes, censoringtime)</pre>
    cens<-rep(1,length(eventTimes))</pre>
    cens[1] <-cens[length(cens)] <-0</pre>
    simdat<-rbind(simdat,</pre>
                    data.frame(id=i, T=eventTimes, status=cens))
  }
  return(simdat)
}
# Subjects
set.seed(12345)
param < -c(2, 1.5, 0.5)
\# param < -c(4, 1.2, 0.3)
omega < -c(0.25, 0.25)
nsuj<-200
risk<-rep(0,nsuj)
risk[(nsuj/2+1):nsuj]<-1
psiM<-data.frame(lambda=param[1]*exp(rnorm(nsuj,sd=omega[1])), beta=param[2]*exp(param[3]*risk+rnorm(nsuj,sd=omega[1]))
simdat <- simul.rtte.unif(psiM)</pre>
## Reached maximum number of events
simdat$risk<-as.integer(simdat$id>(nsuj/2))
saemix.data<-saemixData(name.data=simdat, name.group=c("id"), name.predictors=c("T"), name.response="st</pre>
rtte.model<-function(psi,id,xidep) {</pre>
  T < -xidep[,1]
  N <- nrow(psi) # nb of subjects
  Nj <- length(T) # nb of events (including 0 and censoring times)
  # censoringtime = 6
  censoringtime = max(T) # same censoring for everyone
  lambda <- psi[id,1]</pre>
```

```
beta <- psi[id,2]
     tinit <- which(T==0) # indices of beginning of observation period
     tcens <- which(T==censoringtime) # indices of censored events
     tevent <- setdiff(1:Nj, append(tinit,tcens)) # indices of non-censored event times
     hazard <- (beta/lambda)*(T/lambda)^(beta-1)</pre>
     H <- (T/lambda)^beta</pre>
     logpdf <- rep(0,Nj)</pre>
     logpdf[tcens] <- -H[tcens] + H[tcens-1]</pre>
     logpdf[tevent] <- -H[tevent] + H[tevent-1] + log(hazard[tevent])</pre>
     return(logpdf)
}
saemix.model.base<-saemixModel(model=rtte.model,description="Repeated TTE model",modeltype="likelihood"
                                                                               psi0=matrix(c(1,2),ncol=2,byrow=TRUE,dimnames=list(NULL, c("lambda","be
                                                                               transform.par=c(1,1),covariance.model=matrix(c(1,0,0,1),ncol=2, byrow=TR
saemix.model <-saemixModel (model=rtte.model, description="Repeated TTE model", model type="likelihood", model type="li
                                                                  psi0=matrix(c(1,2),ncol=2,byrow=TRUE,dimnames=list(NULL, c("lambda","beta"))
                                                                  transform.par=c(1,1),covariate.model=matrix(c(0,1),ncol=2),
                                                                  covariance.model=matrix(c(1,0,0,1),ncol=2, byrow=TRUE), verbose=FALSE)
saemix.options<-list(seed=632545,save=FALSE,save.graphs=FALSE, fim=FALSE, displayProgress=FALSE, print=</pre>
rtte.fit<-saemix(saemix.model,saemix.data,saemix.options)</pre>
plot(rtte.fit, plot.type="convergence")
                         lambda
                                                                                                                                                                          beta_risk(beta)
                                                                                                        beta
<del>6</del>.
<u>4</u>.
                                                                                                                                                       0.0
0
         0
                  100
                            200
                                       300
                                                  400
                                                                                             100
                                                                                                        200
                                                                                                                              400
                                                                                                                                                                0
                                                                                                                                                                         100
                                                                                                                                                                                   200
                                                                                                                                                                                                          400
                                                                                                                   300
                                                                                                                                                                                              300
                         Iteration
                                                                                                    Iteration
                                                                                                                                                                                Iteration
                 omega2.lambda
                                                                                                omega2.beta
0.
0.2
         0
                 100
                          200
                                       300
                                                                                             100
                                                                                                        200
                                                                                                                   300
                                                                                    0
                         Iteration
                                                                                                    Iteration
print(rtte.fit@results)
```

----- Fixed effects ------

```
##
     Parameter
                 Estimate
## [1,] lambda
                 2.1
  [2,] beta
                 1.6
  [3,] beta_risk(beta) 0.4
  _____
  ----- Variance of random effects -----
##
       Parameter
                 Estimate
## lambda omega2.lambda 0.1125
  beta
      omega2.beta 0.0015
  ----- Correlation matrix of random effects -----
  ._____
##
            omega2.lambda omega2.beta
## omega2.lambda 1
## omega2.beta 0
  ----- Statistical criteria -----
##
## Likelihood computed by importance sampling
      -2LL= 690.2485
##
##
      AIC = 702.2485
      BIC = 722.0384
  -----
```

Work in progress: currently, no diagnostic plots available for RTTE, stay tuned for progress.

**Statistical model** A nice review of the more frequent hazard functions used in parametric models of TTE data has recently been van Wijk and Simonsson (*CPT:PSP* 2022), including a Shiny app to explore their shape and how to set initial parameters. These models are very sensitive to the initial parameter estimates and their variance, therefore using

## References

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