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Utilization of the S-T Viscoelastic Constitutive Model for the Simulation of Isolation Bearings

M. J. KIM, A. GUPTA, A. H. MARCHERTAS
Northern Illinois University, Dekalb, IL USA

1 INTRODUCTION

The traditional methods of earthquake protection have in the past relied on strength and ductility of the structural materials. Such means could prevent the collapse of structures, but would not guard the contents from destruction. The development of the so-called isolation bearings, to be placed between the structure and the ground, has increased the prospect of protection by damping the earthquake effects. These bearings support the weight of the structures and are able to sustain very large relative horizontal displacements. Thus, they are specifically designed for very high vertical stiffness and very low horizontal stiffness for isolation purposes.

The isolation bearings are rather simple in design and offer many additional benefits which are attractive for nuclear reactor design. Those include sufficient internal damping without need for additional components to absorb imposed energy, the ability to support the dead weight safely, sufficient stiffness at low displacements to resist wind loads without instability, self-centering after a seismic event, and bearing properties unaffected by cycling. Furthermore, long experience with similar material bearings in civil engineering applications indicates that the elastomer compound used is extremely durable and thus requires minimum in-service inspection and maintenance.

It certainly appears that the future of the isolation bearing as a means to mitigate horizontal earthquake loads is bright. It is also important, therefore, that the analytical treatment of this practical alternative be feasible as well. For this purpose the imposed displacement at the ground level must be mathematically related to the resultant load on the structure. This appears to be feasible by a constitutive relation for viscoelastic materials introduced by Simo and Taylor [1-3]. The adaptability of this relation to the isolation bearing is the subject of this paper.

2 ALGORITHM OF THE CONSTITUTIVE RELATION

A three-dimensional constitutive relation was adapted for the analytical simulation of viscoelastic materials, materials which are being utilized to dampen the seismic effects on nuclear reactors. The dampers are the seismic isolation bearings specifically designed to guard the nuclear facilities from damaging earthquakes. The constitutive formulation

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used was that derived by Simo and Taylor [1-3], where the volumetric response of the material was assumed to be purely elastic and the viscoelastic effects were embodied in the deviatoric component.

The most significant contribution stems from the deviatoric component which is in the form of a convolution integral. The stress is expressed as follows:

$$\sigma(t) = K \ln J + \int_0^t \mu(t-s)\, \dot{\pi}(s)\, ds \,, \tag{1}$$

where the first term on the left refers to the volumetric and the second to the deviatoric stress component. The volumetric term is expressed by the bulk modulus K and the determinant of the deformation gradient J. The deviatoric stress component is represented by the integral of the product of the relaxation function $\mu(t)$ and the strain history function $\pi(s)$. Simo and Taylor express the relaxation function as follows:

$$\mu(t) = G_\infty + \left(G_o - G_\infty\right) e^{-t/v} \,, \tag{2}$$

where G_∞ and G_o are the long-term and short-term shear moduli of the material, respectively, v is the relaxation time constant and t is time. Similarly the strain history function is reduced to:

$$\pi(s) = \left[\beta + (1-\beta)\frac{1-e^{-\varphi_s/\alpha}}{\varphi_s/\alpha}\right] \mathrm{dev}\bar{C}(s) \,, \tag{3}$$

where α and β are parameters which are referred to as the damage exponent and damage limit, respectively, and the damage parameter φ_s is expressed as:

$$\varphi_s = \max||\mathrm{dev}\bar{C}(s)|| \,. \tag{4}$$

In physical terms φ_s is the maximum value attained by the norm of $\mathrm{dev}\bar{C}(s)$ during the history of loading, where $\bar{C}$ is the volume-preserving tensor in terms of the deformation gradient F, namely:

$$\bar{C} = F^T F J^{-2/3} \,. \tag{5}$$

By employing a generalized midpoint rule and the mean value theorem to resolve the convolution integral, the deviatoric stress component is expressed in terms of recursive relations. This leads to an algorithm for calculating the shear stress of the seismic bearing. Thus, starting from the deformation gradient the calculation of stress requires the following sequence:*

(a) $C_{n+1} = F^T_{n+1} F_{n+1}$,

(b) $\bar{C}_{n+1} = C_{n+1} J^{-2/3}_{n+1}$,

(c) $||\mathrm{dev}\bar{C}_{n+1}|| = \left\{\mathrm{dev}\bar{C}_{n+1} \cdot \left[\mathrm{dev}\bar{C}_{n+1}\right]^T\right\}^{1/2}$,

(d) $\varphi_{n+1} = \max\left\{||\mathrm{dev}\bar{C}_{n+1}||, \varphi_n\right\}$,

(e) $\pi_{n+1} = \left[\beta + (1-\beta)\dfrac{1-e^{\varphi_{n+1}/\alpha}}{\varphi_{n+1}/\alpha}\right] \mathrm{dev}\bar{C}_{n+1}$,

(f) $\Delta\pi_{n+1} = \pi_{n+1} - \pi_n$,

(g) $\Delta h_{n+1} = \left(e^{-\Delta t/v} - 1\right)\left[h_n - \dfrac{G_o - G_\infty}{\Delta t/v}\Delta\pi_{n+1}\right]$,

(h) $h_{n+1} = h_n + \Delta h_{n+1}$,

(i) $\sigma_{n+1} = \sigma_n + K\dfrac{\Delta J_{n+1}}{J_{n+1}} + \Delta h_{n+1} + G_\infty \Delta\pi_{n+1}$.

*The above algorithm, with minor changes, was reproduced from references [1-3].

The last step of the sequence pertains to the shear stress as calculated from the given displacement in step one. For pure shear (or simple shear) the volumetric term need not be calculated because then there is no volume change and the second term on the right should be zero. The third term refers to the effective increment (Δh) of the history stress component due to relaxation; the last term pertains to the contribution of the reduction of shear stiffness with increasing displacement.

In order to utilize the algorithm for calculating shear stress five input parameters (G_0, G_∞, ν, α, β) are required. With these parameters and the initial displacement given, the resultant stress (or force) in the bearing can hence be calculated.

3 PARAMETRIC STUDY OF THE CONSTITUTIVE RELATION

The properties of the isolation bearing which are relevant in the damping process are the damping capacity and the shear stiffness. Both of these properties can be discerned from the load-deflection diagrams of the bearing. The 2-D plot of the horizontal shear displacement at one end of the bearing and the corresponding shear force on the opposite end of the bearing, as determined from the experiment of cyclic loading, is the required data. This information, when cast in the non-dimensional units of strain and stress, is usually referred to as the hysteresis loop. One example of the recorded test information, as retrieved from the 1/4 - scale PRISM bearing model [4] is shown in Figure 1. The slope of this loop is a measure of the shear stiffness of the bearing tested and the area enclosed by the loop is a measure of the energy stored in the bearing during one cycle of loading. Another example of the hysteresis loops performed on Argonne-Sendai bearings at the Tohoku University [5] is shown in Figure 2.

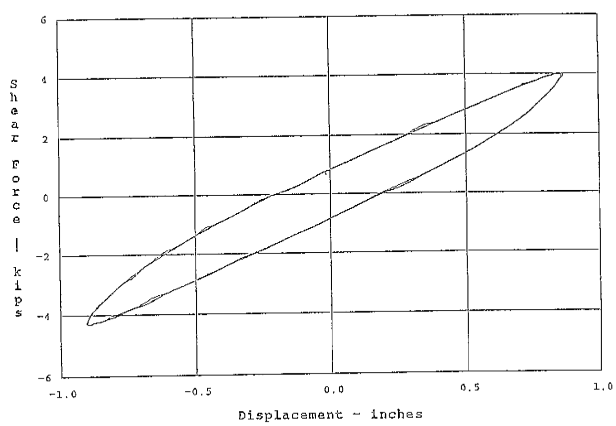

Fig.1 Horizontal Shear Test of a ¼-Scale Dowelled PRISM Bearing, Axial Load = 52 kips, Maximum Strain 25% [4]

The ability of a constitutive relation to reproduce the experimental hysteresis loop is its main objective. The degree to which this relation replicates the hysteresis loop will be a measure of success of the constitutive model. At this time only qualitative comparisons will be made to observe the predicted trends of the relevant properties with the change of sinusoidal loading. The inherent characteristics of the constitutive relation are therefore tested for comparison with the experimental data. For this purpose the derived input parameter data for PRISM bearings [3] will be used, namely:

$$G_0 = 336 \text{ psi},\ G_\infty = 144 \text{ psi},\ \nu = 0.786 \text{ sec},\ \alpha' = 0.1,\ \beta = 0.3\ .$$

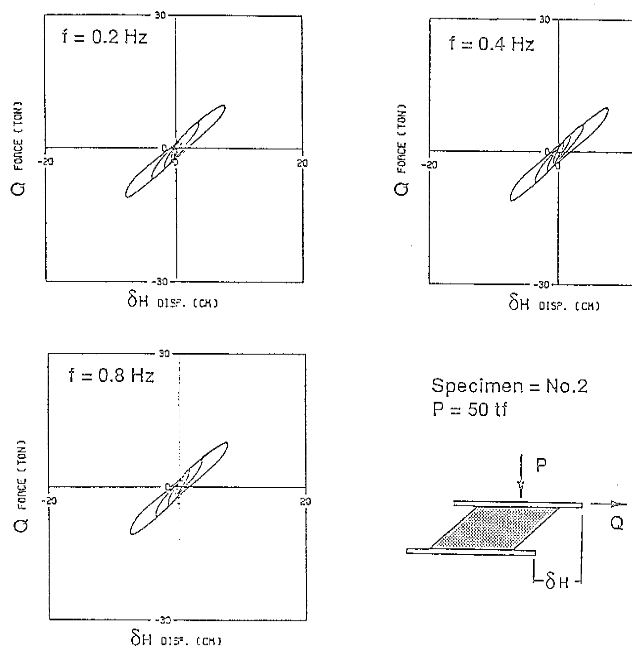

Fig.2 Changes in Hysteresis Characteristics With Speed [5]

Using the above input the following hysteresis loops shown in Figure 3 was obtained [6]. Different hysteresis loops for different maximum strains and constant frequency are indicated. In this figure the sequence of loading proceeds from small to higher displacements. As observed, the sequence of the first and the subsequent hysteresis loops at the same maximum strain are different. This characteristic behavior is substantiated by experimental evidence of the bearings, where the steady state process of the bearing is attained in a few cycles provided the maximum strain amplitude remains constant.

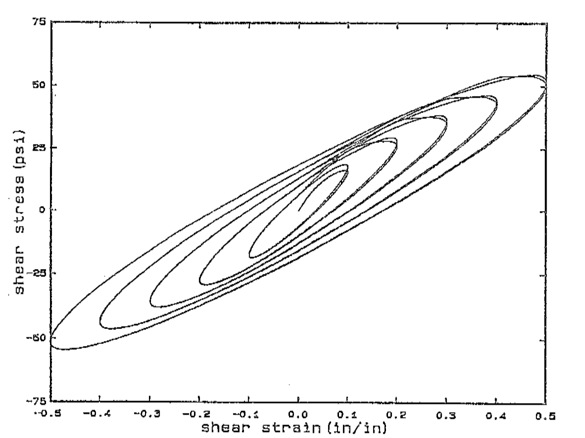

Fig.3 Reproduction of Shear Sinusoidal Loading to 50% Strain Using Present Constitutive Formulation

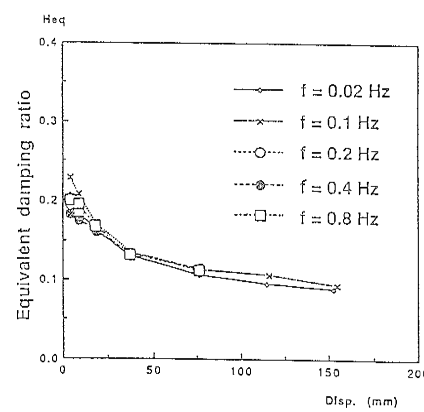

Fig.4 Change in Equivalent Damping Ratio With Displacement [5]

The other observation of Figure 3 is the geometric similarity of all the hysteresis loops ranging from small to large displacement. The area of the loops indicates the damping capacity of the bearings. When the damping of the analytical predictions are calculated, their variations are constant with respect to the maximum strain. The experimental data provided by Kuroda et al. [5] show variations, as depicted in Figure 4. This indicates that at small strains damping is maximum (about 28%) and at 100% strain the damping is only about 10%. Thus, the present constitutive relation does not account for the variation of damping with the change in the maximum strain amplitude of the loops.

In the previous analytical simulation loading was applied by gradually increasing the amplitude of the hysteresis loops. In the next analytical model the loading with decreasing amplitude is illustrated. Two loops are given for each maximum amplitude to show the initial and steady state conditions. Thus, beginning with 100% strain, followed by 50% and ending with 10% maximum strain is shown in Figure 5. The slopes of the hysteresis loops are constant, which is confirmed by the experimental data.

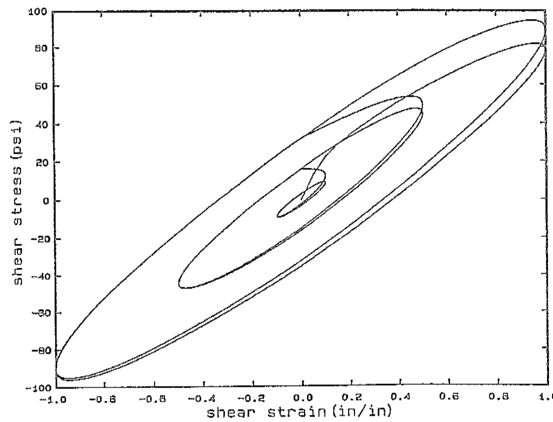

Fig.5 Hysteresis Plots Showing the Effect of Decreasing Maximum Displacements

At very large strains experiments indicate distorted hysteresis loops. While at low strains the hysteresis loop show elliptical shapes; at large amplitudes they become increasingly distorted as shown in Figure 6. There is nothing in the analytical formulation which would tend to change the shape of the hysteresis loops to conform to the test data: the analytical hysteresis loops, after the first cycle assume an elliptical configuration.

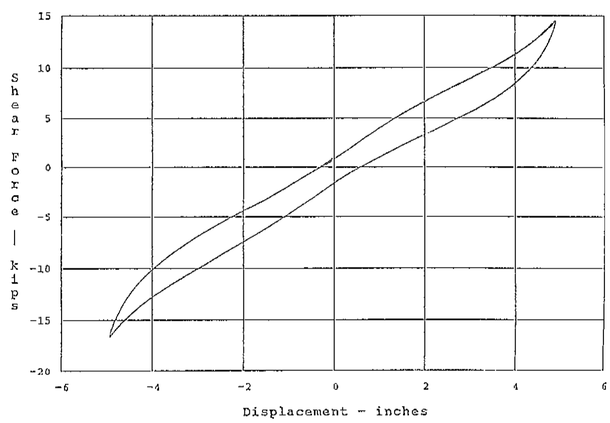

Fig.6 Horizontal Shear Test of a ¼-Scale Dowelled PRISM Bearing, Axial Load = 52 kips, Maximum Strain 137.4% [4]

4 CONCLUDING REMARKS

The validity of the Simo-Taylor constitutive relation for the simulation of the response of isolation bearings was evaluated in this paper. This was investigated on the basis of how closely the experimentally derived hysteresis loop of the bearings were reproduced by the constitutive relation. It was determined that the given relation may well be used for that purpose, subject to certain conditions [6]. The main conclusions were the following:

1. The hysteresis loops are of elliptical configuration as exhibited by experimental data on bearings for small displacements,
2. Change in shape of the hysteresis loops for the same maximum strain is also observed in tests under cyclic loading,
3. The experimental variation of shear modulus with the maximum strain in the bearing tests is reproduced by the results of the constitutive relation,
4. At large strain the shape of the hysteresis loop of the bearing becomes distorted and could not be duplicated by the current constitutive relation,
5. The variation of damping in the bearings with change in maximum shear is not compatible with experimental observations.

The results reached in this investigation apply only if the input of the constitutive relation is known. Thus far, these input data were available from tests on reduced scale models of bearings. It is not certain at this time if sufficient input information required for the constitutive model could be secured purely from coupon tests.

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