Seismic Analysis of Sliding Structures

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1. INTRODUCTION

To limit the seism effects, structures may be base isolated. A sliding system located between the structure and the support allows differential motion between them.

The aim of this paper is the presentation of the method to calculate the response of the structure when the structure is represented by its eigenmodes, and the sliding phenomenon by the Coulomb friction model. Finally, an application to a simple structure shows the influence on the response of the main parameters (friction coefficient, stiffness,...).

2. COULOMB FRICTION MODEL

Let us consider a stiff mass, layed on an horizontal support and submitted to an external force F_e (parallel to the support). When F_e is smaller than a limit force F_ℓ, there is no differential motion between the support and the mass and the friction force balances the external force. The limit force is written: $F_\ell = \mu F_n$ where μ is the friction coefficient and F_n the modulus of the normal force applied by the mass to the support (in this case, F_n is equal to the weight of the mass). When F_e increases and becomes equal to F_1, sliding begins and the friction force F_t is directed in the opposite sense of the velocity, and is expressed as:

$$|F_t| = \mu F_n = F_\ell$$

3. EQUATIONS OF MOTION

Let us study the behaviour of a structure sliding on its seismically excited support. We are studying the case of a one contact point between the structure and its support.

The motion (absolute displacement X_a) of the structure may be expressed in the following way:

$$X_a(t) = Y_o(t)\ U + X(t)$$

where $Y_o(t)$ is the support motion, U a unit vector in the direction of the excitation and $X(t)$ the relative displacement (including the sliding) of the structure with respect to the support, which will be expanded on the basis of the eigenmodes of the structure. The boundary condition changes whether there is

sliding or not and we have chosen to use the basis corresponding to free boundary conditions at the contact point.

The equations of motion are derived in the same way as for the impact problems ([2] and [3]). For the modal coefficients, we obtain harmonic oscillators equations, coupled through the projection of the link force on each eigenmode. As for the seismic analysis, we are mainly interested by the low frequency behaviour of the structure, the modal basis is truncated and we keep only the first few eigen modes. The neglected high-order modes are supposed to have a static response. The set of equations to be solved are the following:

$$M \ddot{\alpha} + C\dot{\alpha} + K\alpha + B^T F_t = - \gamma_o Q \qquad (\gamma_o = \ddot{y}_o)$$

- during a sliding phase:

$$F_T = \mu F_n \, \text{Sign}(B\dot{\alpha})$$

- during a grip phase:

$$|F_t| \leqslant \mu F_n$$

$$\text{and} \quad B\alpha + K_L^{-1} F_T = X_G$$

where: - M (resp K) is the diagonal generalised (resp stiffness) matrix
- C is the modal damping matrix (assumed to be diagonal)
- F_T is the link force
- B a line matrix allowing to calculate the displacement of the contact point
- Q is the vector of the generalised displacements
- α the vector of the modal coefficients
- K_L is the stiffness of the neglected high order modes, evaluated at the contact point
- X_G is the sliding of the contact point at the end of the former sliding phase.

The last equation means that during the grip phase, due to the truncation, there is a relative displacement between the structure and its support at the contact point. The amplitude of this displacement will decrease when the number of modes in the basis will increase. Such effects due to the truncation has been observed for impact problems [2], [3], [4].

4. APPLICATION

The aim of this application is to study the behaviour of a simple structure sliding on its support (fig. 1) and to determine the influence of some important parameters like the friction coefficient, the stiffness and the characteristics of the structure.

First, we shall consider a rigid structure which will be represented by a mass m_t sliding on a support. Then we will study the behaviour of a structure which response is mainly influenced by one eigenmode. The structure will be representated by a 2 DOF model (fig. 1) composed of a sliding mass m_2 linked by a spring of stiffness k to another mass m_1. The mass m_2 represents the mass of the

sliding stiff basis and the frequency $f_o = \frac{1}{2\pi} \sqrt{k/m_1}$ is this first eigenfrequency of the structure with its basis clamped on the support, m_1 being the associated modal mass. Such model is fully described by its total mass $m_1 + m_2$, the ratio $r = m_2/m_1$ which represents the mass repartition and the frequency f_o. When the mass m_2 is free, the model has two eigenmodes, the first one is the

rigid body motion (translation parallel to the support) and the second one corresponds to an opposite vibration of m_1 and m_2 with a frequency equal to: $f_1 = f_o[(1+r)/r]^{0.5}$.

The system is excited by the San Francisco seism normalized to 0.1 g. The parametric study has been performed for different values of the friction coefficient μ, the ratio r, and the frequency f_o.

The modal damping coefficients of the free structure are equal to 2% and as the modal basis is complete, the stiffness K_L has no physical sense, and is chosen so that the frequency of the system linked with the spring K_L is far enough from the frequency range of interest.

Results

Only the main results are presented in this paper (see [5] for a detailed analysis).

We can observe that:

- it exists a residual sliding at the end of the excitation (fig. 2).

- the maximal value of the sliding decreases when μ increases; the stiffness of the structure increases the sliding in comparison with a sliding mass alone. This phenomenon is more sensitive for r small (fig. 3).

- in the case of the sliding mass alone for $\mu \geqslant 0.1$, there is no sliding and the Response Spectrum (fig. 4) of the sliding mass is equal to the excitation spectrum (except the peak at 55 Hz due to the modelisation of the grip phase with a spring). For $\mu = 0.01$, the spectrum is below the excitation spectrum and for intermediate values the spectrum of the sliding mass may be over the excitation one.

- for a deformable structure for small values of μ (0.01) a peak appears at the frequency f_1 of the 2nd eigenmode of the free structure (fig. 5, for $f_o = 6$ Hz and $r = 0.5$, this peak is at 10.4 Hz). This peak may be observed on the response spectrum of the mass m_2.

- the maximal value γ_{m_1} of the absolute acceleration of the mass m_1 as a function of the frequency f_o represents the spectrum of an equivalent excitation taking into account the sliding phenomenon [6], [7]. γ_{m_1} is always smaller than the value corresponding to the clamped basis and the reduction increases when the ratio r becomes small (fig. 6).

5. CONCLUSION

This study has shown in a simple case the influence of the sliding and of the characteristics of structure on its response. The sliding reduces strongly the level of response and the stiffness of the structure amplifies the sliding.

For future studies, it would be interesting first to calculate the response of a structure having some high order modes located in the frequency range where there is an amplification due to the non linearity and to analyse the case of a more complex structure having several points of contact with the support where the sliding may occur independently.

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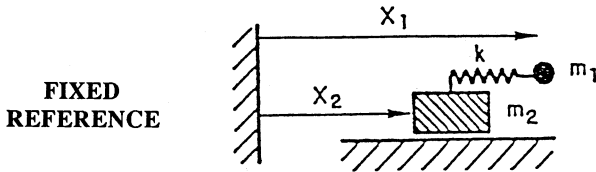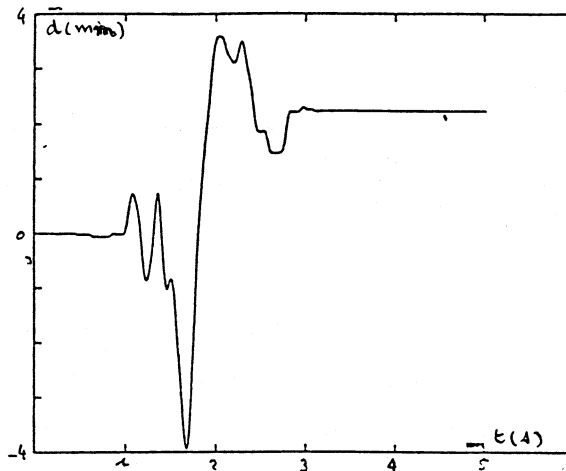
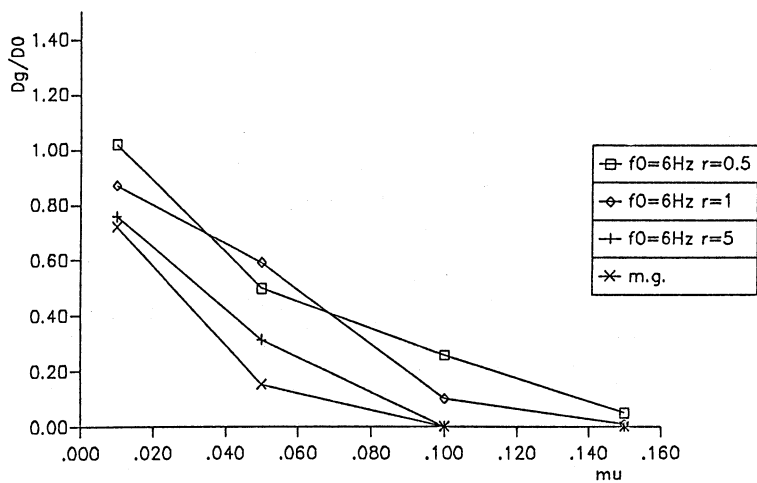Fig.1 2 DOF model

Fig. 2 Sliding mass displacement

Fig. 3   Maximal sliding

(D_0 evaluation of max(Y_0(t))

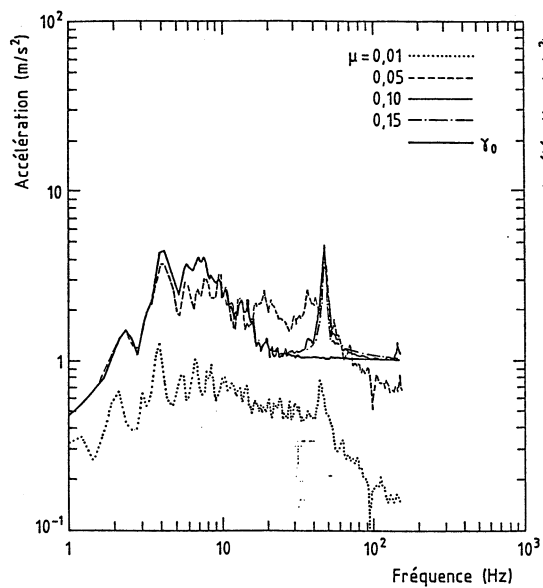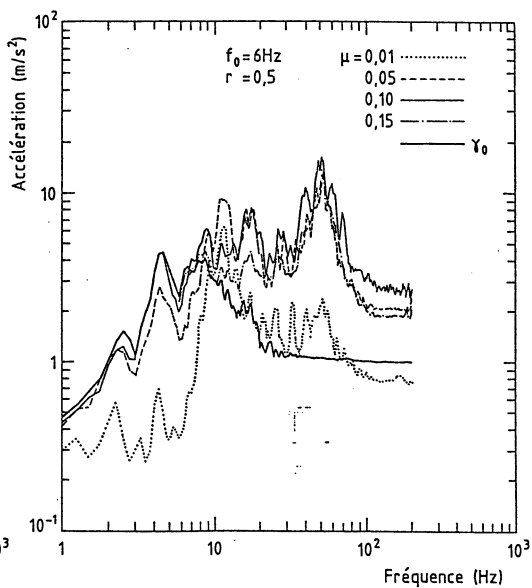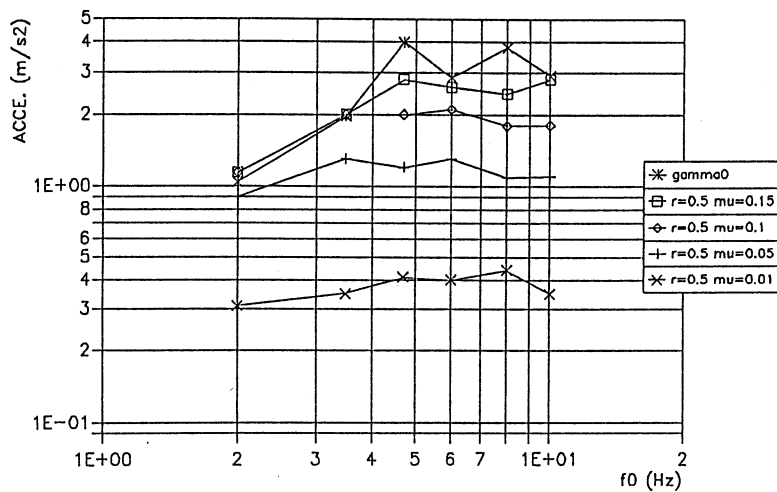Fig. 4 Response spectrum of sliding mass.

Fig. 5 Response spectrum of mass m_2

Fig. 6 Maximal absolute acceleration m_1