

## Numerical Method for Analysis of Laminated Elastomer Bearings

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### ABSTRACT

Numerical analysis of laminated elastomer were conducted using ABAQUS and MARC code. (1)Mooney Rivlin model and (2)the one obtained from bi-axial loading test of rubber material were utilized as the constitutive equations of rubber. The Numerical simulation of laminated elastomer bearing were conducted and the following were observed.

(1)ABAQUS and MARC give practically identical results.

(2)The numerical results using the second model agrees well with the experimental ones up to large shear strain, whereas the results of first model agrees only up to moderate strains.

(3)Vertical displacements are difficult to predict. So, development of new numerical method is desired.

A numerical methods to be developed is described. Functional for mixed finite element method are introduced, where material nonlinearities are considered for both shear and bulk strain.

### 1. INTRODUCTION

The laminated elastomer bearings are considered to compose one of the most promising type of seismic isolation system of nuclear power facilities.

The adequate numerical methodology is needed to evaluate the stiffness and ultimate displacement of the bearing for the design of elastomer bearings. In this paper, numerical simulation of the bearings were conducted using the general-purpose computer code based on Finite Element Method, i.e. ABAQUS and MARC. The results were compared to the experimental ones to evaluate the applicability of the codes for the design of the bearings. The special purpose computer code to be developed, is also discussed.

### 2. NUMERICAL MODEL

The numerical method for the bearing must be able to handle geometrical and material nonlinearities, since maximum strain of rubber can be as several hundred percent and elastic moduli are strain dependent.

General purpose finite element programs MARC and ABAQUS have such capabilities and were applied to the analysis of the bearings. For simplicity, rubber is assumed to be isotropic, hyperelastic and nearly incompressible. Element type used in the analysis by MARC and ABAQUS are listed in Table 1.

Constitutive equations used were as follows.

Model A (Mooney Rivlin Model)

$$W = C_{10}(I_1 - 3) + C_{01}(I_2 - 3) \quad (1)$$

Where,

$$\begin{aligned}I_1 &= \lambda_1^2 + \lambda_2^2 + \lambda_3^2 \\I_2 &= \lambda_2^2 \lambda_3^2 + \lambda_3^2 \lambda_1^2 + \lambda_1^2 \lambda_2^2 \\I_3 &= \lambda_1^2 \lambda_2^2 \lambda_3^2 \\I_3 &= 1\end{aligned}$$

$W$  = energy density function,  
and  $\lambda_i$  ( $i=1, 2, 3$ ) = the principal extension ratios.

Coefficients of Eq.(1) were determined to fit the shear test of rubber bearing[2].

Model B(Ref[1])

$$\frac{\partial W}{\partial I_i} = a_i + b_i(I_i - 3) + c_i(I_i - 3)^2 + d_i \exp[e_i(I_i - 3)] \quad (2)$$

Coefficients of Eq.(2) were determined to fit the bi-axial test of rubber material[1](Fig.1)

### 3. SIMULATION OF LAMINATED ELASTOMER BEARING

#### 3.1 Laminated Elastomer bearing and FE mesh.

The bearing which support  $4.9 \times 10^5 N(500t_r)$  of weight were simulated(Ref.[2]). At first, vertical load was applied, and then forced horizontal displacement was given to the upper flange. Considering the symmetry of the geometry of bearing, loading, and displacements, only a half of the bearing was modelled. Linear hexahedral elements were used for steel plated and rubber.

The number of meshes in circumferential direction and in radial direction were 8 and 2, respectively. Each rubber sheet or steel plate is divided into two meshes in thickness direction. The number of layers were 25. FEM mesh is shown in Fig.1.

#### 3.2 Comparison of the Results of MARC and ABAQUS

Model A was adopted for the constitutive equation of rubber. The results of MARC and ABAQUS were compared, and they show good agreement [Fig.2.]

#### 3.3 Comparison of computed and experimental results

The results of the ABAQUS code were compared with the experimental ones. Fig.4 shows the computed results using Mooney-Rivlin model, together with experimental ones. The agreement is good only up to medium shear strain. In Fig.5, computed results using model(2) are compared with experimental ones. Numerical results were obtained up to 400% of nominal shear strain.(Horizontal displacement divided by total rubber thickness.) The agreement is fairly good even in large strain. In Fig.6, the computed deformation are shown.

#### 3.4 The results of compressible material model

The compressible model with constant bulk modulus were also used for the simulation of forced horizontal displacement of the elastomer bearings. The bulk modulus was calculated from the value of Mooney-Rivlin constants assuming poisson's ratio  $\nu = 0.498$ . The constitutive model(2) were adopted for shear strain. Shear forces computed using compressible and incompressible models are shown in Fig.7. The difference of the two model is small. In Fig.8, the computed vertical displacements using both models are shown together with experimental ones.

The difference between the computed results are large, and neither results agrees with experimental ones. The vertical displacement obtained using compressible model decrease as horizontal displacement increases. It is qualitatively contrary to experimental results. C3D8H element of ABAQUS may not be adequate for the evaluation of vertical displacement of laminated elastomer. The following computer codes are now being developed.

### 4. NUMERICAL METHOD FOR 3-D ANALYSIS

#### 4.1 Functional

Whereas rubber can be considered to be nearly incompressible material, compressibility of

the rubber must be taken into account to evaluate vertical stiffness of elastomer bearings. The bulk stiffness of rubber is also known to be bulk strain dependent(Ref.[2]). The bulk modulus is due to the changing of interatomic spacing, where shear modulus of rubber is mainly attributable to entropy. It is, therefore, reasonable to assume that the energy density function has the following separated form.

$$W = W_B(I_1, I_2) + W_T(I_3) \quad (3)$$

Since the bulk modulus is by far larger than shear modulus, it is still considered to be adequate to adopt mixed formulation with displacements and pressure as independent variables. The functional for mixed formulation is expressed as follows.

$$\Pi = W_B(I_1^*, I_2^*) + f(I_3)P + g(p) \quad (4)$$

where,

$$I_1^* = I_1 / I_{30}^{1/3} \quad \text{and} \quad I_2^* = I_2 / I_{30}^{2/3}$$

$$f(I_3) = \sqrt{I_3 - 1}$$

$g(p)$  : complementary energy due to bulk strain

#### 4.2 Constitutive equations

Eq.(2) is adopted for shear strain, and the following the constitutive equation is assumed for bulk strain.(Ref.[3]).

$$P = \left( \frac{C}{\sqrt{I_3} - \sqrt{I_{30}}} \right)^n - \left( \frac{C}{1 - \sqrt{I_{30}}} \right)^n \quad (5)$$

Where,  $I_{30}$  is ultimate deformation, and  $C$ , and  $n$  are material constants.

#### 4.3 Derivation of Finite Element Scheme

Applying the variational principle on the above functional, the following weak form equation is obtained.

$$\begin{aligned} & \iiint_V t^{ij} \delta r^{ij} dv - \iiint_V \rho_0 F^q \delta U_q - \iint \partial V_T T^q \delta U_q ds \\ & - \iiint_V f(I_3) \delta P dv - \iiint_V \frac{\partial g(p)}{\partial P} \delta P dv = 0 \quad (6) \end{aligned}$$

Where,  $t^{ij}$  is 2nd Piola-Kirchhoff stress tensor,  $\gamma^{ij}$  Green's strain tensor,  $\rho_0$  is density,  $F^q$  is internal force, and  $T^q$  is surface stress.

Introducing adequate discrete independent variables and interpolation function, the equations to be solved are obtained.

#### 4.4 Miscellaneous

Besides the elements described in table 1, it is added the 27 nodes element with 3 components of displacement at each node and 4 degree of freedom for pressure for a element. Total Lagrangian formulation is considered to be adequate for hyperelastic large deformation analysis. Rezoning algorithm is desired for the analysis of extremely large deformation.

### 5. CONCLUSION

The relationship between the shear restoring force and shear displacement of rubber bearings can be predicted fairly well by general purpose finite element program up to large shear displacement.

For the prediction of vertical displacement a special computer program such as described in 4, may be needed.

## ACKNOWLEDGMENT

The authors wish to thank Dr.Y.Fukahori of Bridgestone Corporation for his valuable data and advises.

## REFERENCES

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- [2]Mazda,T., et.al., 10th SMiRT, K Division, Anaheim, USA(1989)
- [3]Kawabata,S., the 3rd US-Japan Conference on Elastomer, Akron, USA(1988)

Table 1 Type of elements

	ABAQUS	MARC
Steel	C3D8R C3D8	7
Rubber	C3D8H	84

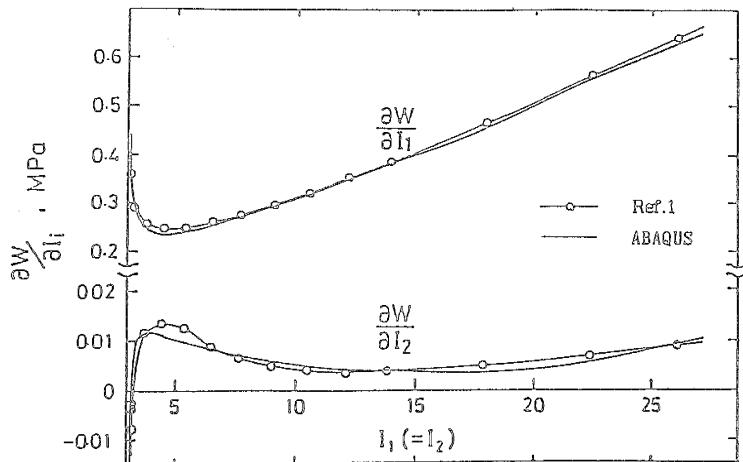


Fig.1 Relationship between  $I_1 (= I_2)$  and  $\partial W / \partial I_1$ ,  $\partial W / \partial I_2$

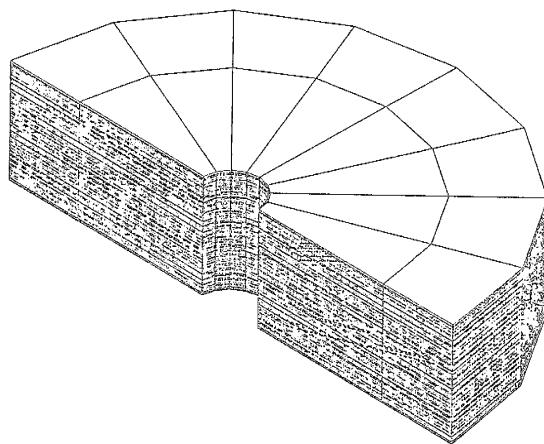


Fig.2 FEM mesh

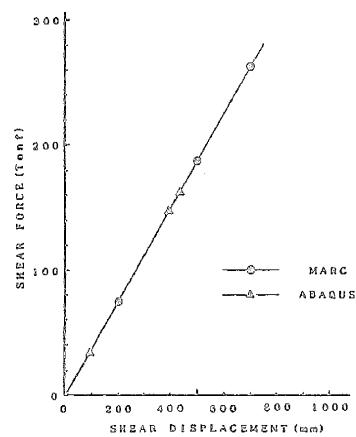


Fig.3 Relationship between shear force and shear displacement  
(Comparison of ABAQUS and MARC)

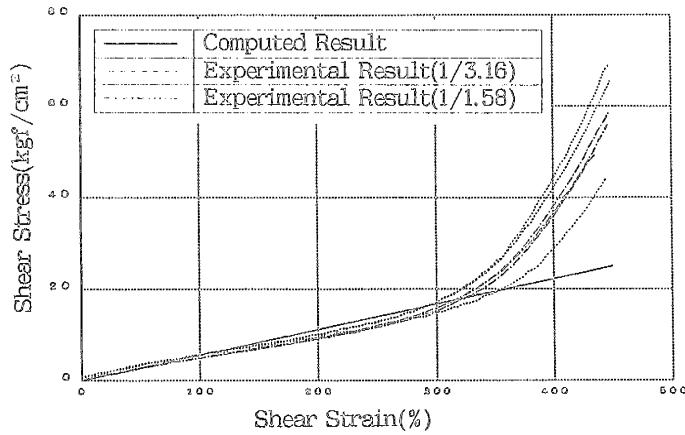


Fig.4 Comparison of computed results using Mooney-Rivlin model and experimental results

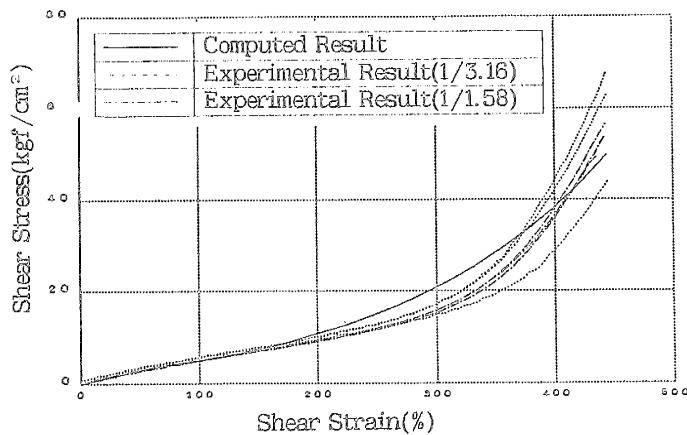


Fig.5 Comparison of computed results using model (2) and experimental results



(a) Horizontal Displacement(25cm)



(b) Horizontal Displacement(50cm)

Fig.6 Computed deformation

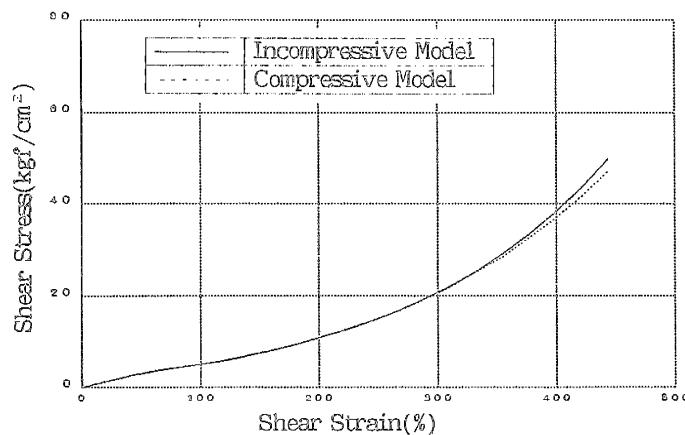


Fig.7 Comparison of compressive and incompressive models ( horizontal direction)

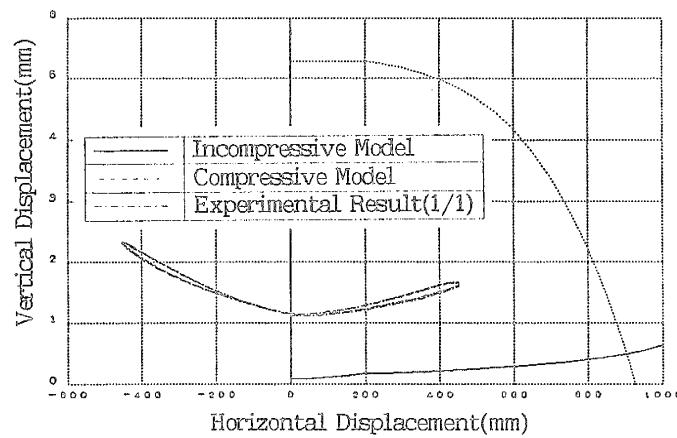


Fig.8 Comparison of compressive and incompressive models (vertical direction)