Active Optimal Loop Control for Non-linear Isolation System by Canonical Transformations

POTERASU V. F.
Polytechnic Institute of Jassy, Iasi, Romania

ABSTRACT

The canonical transformations and perturbation theory are applied for the active optimal loop control for nonlinear isolation system modelled by a linear structure and nonlinear interaction with the soil. As an example is considered the base excitation of a shear beam with a cubic hardening constraint.

1 INTRODUCTION

In many structural systems significant earthquake damage is confined to one particular location. Examples include reactor building with nonlinear soil interaction, adjacent structures with a flexible seismic connection and equipment mounted on flexible supports. Generally speaking, the nonlinear behaviours occurs in that portion of the system which is more flexible than the rest, either by intentional design or by prevailing circumstances. The active control scheme developed uses a system of force actuators (most probable hydraulic) to counter the action of forcing input due to earthquake. The design problem concerns the optimal control input so that the system responds favourably. For to obtain the optimal value are used the canonical transformations and perturbation theory together Pontryagin's maximum principle which defines also a hamiltonian system of ordinary differential equations generally non-linearly solved numerically. The partial differential equation of dynamic programing (Bellman's equation) is the corresponding Hamilton-Jacobi equation and the canonical transformation method can be used to solve the problem, to rewrite it in a convenient set of new variables.

2 CANONICAL TRANSFORMATIONS AND PERTURBATION THEORY

Let $H(q,p,t)$ be the Hamiltonian function of a mechanical system with n degrees of freedom, q^T being the vector of the generalized coordinates and p the vector of generalized momenta. The

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equations of motion are then the form

$$\dot{q} = \frac{\partial H}{\partial p} \qquad \dot{p} = -\frac{\partial H}{\partial q} \tag{1}$$

and if H does not explicitely depend on t it is easily seen that it is a first integral. A transformation

$$q^* = q^*(q,p,t) \qquad p^* = p^*(q,p,t) \tag{2}$$

is said to be canonical if it preserves the form of all Hamiltonian systems. This means that for each $H(q,p,t)$ there can be found a function $H^*(q^*,p^*,t)$ so that the equation (1) are transformed into

$$\dot{q}^* = \frac{\partial H^*}{\partial p^*} \qquad \dot{p}^* = -\frac{\partial H^*}{\partial q^*} \tag{3}$$

A necessary and sufficient condition for a coordonate transformation of the type (2) to be canonical, can be written in terms of Poisson brackets. The Poisson brackets of two functions $f(q,p,t)$ and $g(q,p,t)$ are defined as

$$(f,g) = \sum_{i=1}^{n} \left(\frac{\partial f}{\partial q_i} \frac{\partial g}{\partial p_i} - \frac{\partial f}{\partial p_i} \frac{\partial g}{\partial q_i} \right) \tag{4}$$

and the conditions for (2) being canonical are

$$(q_i^*, p_j^*) = \delta_{ij} \qquad (q_i^*, q_j^*) = 0 \qquad (p_i^*, p_j^*) = 0; \, i,j = 1,\ldots,n$$

where δ_{ij} is the Kronecker symbol. Poisson brackets have a number of useful applications; it can for instance be shown, that if $\varphi(q,p,t)$ and $\psi(q,p,t)$ are first integrals of (1) and are of class C_2, then (φ, ψ) will also be a first integral of (1).

Canonical transformations are usually obtained from "generating functions", which are in general functions of one of the old variables, one of the new variables and of time. If we use a generating function of the type $V(q,p^*,t)$ it can be shown that the transformation defined by

$$p^* = \frac{\partial V}{\partial q} \qquad q^* = \frac{\partial V}{\partial p^*} \tag{5}$$

is canonical and moreover that H^* is given by

$$H^*(q^*,p^*,t) = H(q(q^*,p^*,t), p(q^*,p^*,t)) + \frac{\partial V}{\partial t} \tag{6}$$

It is easily seen that the first of the equations (5) gives p^* as a function of p,q,t and with that result $q^*(q,p,t)$ can be obtained from the second equation provided

$$\det\left(\frac{\partial^2 V}{\partial q \partial p^*} \right) \neq 0$$

A particular case of canonical transformations occur if the first equation (2) is of the type

$$q^{*} = q^{*}(q,t) \tag{7}$$

i.e. the new coordinates depend only on the old coordinates and time, not on the momenta. Such a transformation is called a "point transformation" and the corresponding generating function is

$$V(q,p^{*},t) = p^{*T}q^{*}(q,t) \tag{8}$$

so that the new momenta are obtained from

$$p^{*} = \frac{\partial V}{\partial q} = \left(\frac{\partial q^{*}(q,t)}{\partial q}\right)^{T} p^{*} \tag{9}$$

The system (1) is solved completely if we find a transformation that transforms H into $H^{*}=0$ because then the new variables q^{*}, p^{*} are constants, as can be seen from (3). Suppose V is such a generating function, depending on q, on the constant new momenta $p^{*}=\beta$ and on t. Then V has to satisfy (6) with $p=\partial V/\partial q$ and with $H^{*}=0$, so that it is a solution to the partial differential equation

$$H(q,\frac{\partial V}{\partial q},t) + \frac{\partial V}{\partial t} = 0 \tag{10}$$

which is the Hamilton-Jacobi equation. To solve the system (1) corresponds to the solution of (10) by the method of characteristics. We do not need the general solution of (10) but only a complete solution which is a solution of (10) depending on n constants $\beta_1, \beta_2, \ldots, \beta_n$ in such a way that

$$\det\left(\frac{\partial^2 V}{\partial q \partial \beta}\right) \neq 0 \tag{11}$$

If a non-linear system of type (1) is to be solved by perturbation theory this can be done in the following way. First the Hamiltonian H(q,p,t) is split up into two parts

$$H(q,p,t) = H_0(q,p,t) + H_1(q,p,t) \tag{12}$$

such that a complete solution $V(q,\beta,t)$ of

$$\frac{\partial V}{\partial t} + H_0(q,\frac{\partial V}{\partial q},t) = 0 \tag{13}$$

can be found. Let us use this function $V(q,\beta,t)$ as a generating function for the transformation of the complete system described by $H = H_0 + H_1$. Then H^{*} is given by

$$H^{*} = H_0(q,\frac{\partial V}{\partial q},t) + H_1(q,\frac{\partial V}{\partial q},t) + \frac{\partial V}{\partial t} = H_1(q(q^{*},\beta,t),p(q^{*},\beta,t),t) \tag{14}$$

and the transformed equations (1) are written

$$\dot{q}^* = \frac{\partial H^*}{\partial \beta} \qquad \dot{\beta} = -\frac{\partial H^*}{\partial q^*} \tag{15}$$

No approximation has been made in the derivation of (15). This This procedure can now be repeated, i.e. H can be split up into H_o and H_1 and so on. At some point the computations are interrupted and the remaining part of the Hamiltonian is disregarded. This method has been used for a long time in celestial mechanics and more recent are given in quantum mechanics. Instead of solving (13) we can also find the solutions to

$$\dot{q} = \frac{\partial H_o}{\partial p} \qquad \dot{p} = -\frac{\partial H_o}{\partial q} \tag{16}$$

in the form

$$q = q(q_o, p_o, t) \qquad p = p(q_o, p_o, t) \tag{17}$$

It can be shown that the transformation from the initial values q_o, p_o to q and p given by (17) is always canonical and the transformed equations (1) are

$$\dot{q}_o = \frac{\partial H^*}{\partial p_o} \qquad \dot{p}_o = -\frac{\partial H^*}{\partial q_o} \tag{18}$$

with $H^* = H_1(q(q_o, p_o, t), p(q_o, p_o, t), t)$.

In optimal control we face the problem of finding solutions of

$$\dot{x} = f(x, u, t) \tag{19}$$

minimizing a functional if the type

$$J = \int_{t_o}^{t_1} f_o(x, u, t)\, dt \tag{20}$$

Here $x^T = (x_1, \ldots, x_n)$ is the "state vector" and $u^T = (u_1, \ldots, u_m)$ the control function to be determined. Pontryagin s maximum principle tells us that the value of the optimal control function $u(t)$ is at each instant the one which minimizes

$$H(x, u, y, t) = f^T(x, u, t)y + y_o f_o(x, u, t) \tag{21}$$

with $y_o \leq 0$, where $y^T = (y_1, \ldots, y_n)$ is the vector of the adjoint variables, which satisfy the differential equation

$$\dot{y} = -\frac{\partial H}{\partial x} \tag{22}$$

while the equation (19) can be written as

$$\dot{x} = \frac{\partial H}{\partial y} \tag{23}$$

Through the maximum principle u can be expressed as a function of x, y and t, so that also H depends only on these variables. The method of canonical perturbation can of course be used in the form outlined above for the solution of the Hamiltonian system (22),(23).

3 ACTIVE OPTIMAL CONTROL FOR NONLINEAR ISOLATION SYSTEM

To illustrate the application of the canonical transformations and perturbation theory it is considered the steady state oscillation of the nonlinear continuous system indicated in Fig.1.

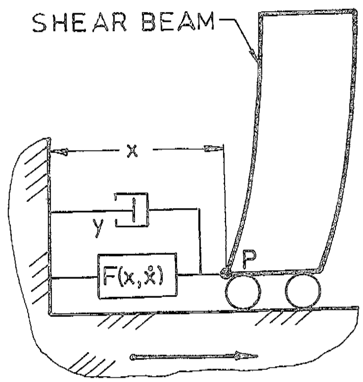

Fig.1 Base excitation of a shear beam with a cubic hardening constraint

The nonlinearity is confined to the connection between the structure and the moving base. Furthermore, let the nonlinearity be of a cubic hardening type. This system might be a highly idealized model for a reactor structure including nonlinear seismic isolation effects. For this example, the nonlinear restoring force $F(x,\dot{x})$ is independent of $\dot{x}$ and may be expressed as

$$F(x,\dot{x}) = kx(1+\varepsilon x^2) \qquad (24)$$

where ε is a nonlinearity parameter. The motion of the point P including the control input is governed by the equation

$$m\ddot{x} + F(x,\dot{x}) + \gamma\dot{x} = u(x,t) \qquad (25)$$

The control u consists of a feedback and constitutes the basic active controller structure(Wolf,Madden 1981). The general performance index considered has the usual form

$$J = \int_0^T (a_1^2x^2 + a_2^2x^2 + \rho^2u^2)\, dt \qquad (26)$$

where ρ is a scalar weighting parameter. The performance index is an acceptable compromise between achieved system response and control energy expended. The differential equation (25) become in transformed form the following

$$\dot{x}_1 = x_2 \qquad \dot{x}_2 = -\frac{1}{m}F(x_1,x_2) - \frac{1}{m}\gamma x_2 + \frac{1}{m}u \qquad (27)$$

With $y_o = -1$ we have

$$H = -(a_1^2 x_1^2 + a_2^2 x_2^2 + \rho^2 u^2) + y_1 x_2 + \frac{1}{m}(u - \gamma x_2 - F(x_1,x_2)) \qquad (28)$$

In (28) is replaced $u = \frac{1}{2m\rho^2} y_2$ from $\partial H/\partial u = 0$. Hamilton's equations are

$$\dot{x}_1 = x_2;\ \dot{x}_2 = \frac{1}{m}(u - \gamma x_2 - F(x_1,x_2));\ \dot{y}_1 = 2a_1^2 x_1 + \frac{1}{m}\frac{\partial F}{\partial x_1} y_2$$

$$\dot{y}_2 = 2a_2^2 x_2 - y_1 + \frac{1}{m}\left(\gamma y_2 + \frac{\partial F}{\partial x_2} y_2\right) \qquad (29)$$

It is considered

$$H_o = y_1 x_2 - \frac{1}{m}(\gamma x_2 + kx_1) y_2;\ H_1 = -(a_1^2 x_1^2 + a_2^2 x_2^2) - \frac{1}{m} k \varepsilon y_2 x_1^3 + \frac{1}{4m^2 \rho^2} y_2^2 \qquad (30)$$

A complete solution to

$$x_2 \frac{\partial V}{\partial x_1} - \frac{m}{x_2 + kx_1} \frac{\partial V}{\partial x_2} + \frac{\partial V}{\partial t} = 0 \qquad (31)$$

is

$$V = e^{\gamma t}\left(\frac{1}{\omega} x_1 \sin(\omega t + p_2) + \frac{1}{k} x_2 \cos(\omega t + p_2)\right) p_1 \qquad (31)$$

from which is generated the transformation $q_1 = \frac{\partial V}{\partial p_1}, q_2 = \frac{\partial V}{\partial p_2}$ q_1, q_2 being the new"state variables"and p_1, p_2 the new adjoint variables.

4 CONCLUSIONS

One optimal solution for the non-linear isolation system in soil-structure interaction may be obtained by canonical transformation using the numerical results and the standard computer programs and will be presented late.

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