Active optimal loop control to reduce the seismic response of a nonlinear isolation system

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1 INTRODUCTION

Many systems are characterized by the fact that non-linear behaviour is restricted to certain points,particularly in the soil-interaction cases.The structure considered as a single degree of freedom is nonlinearly attached in a singular point,practically simulating the structure soil-interaction.In the first part of the paper we present the general problem for the situation when the dimensions of the state space and of the control are the same.Simple computations using Lie brackets of vector fields show that the optimal feedback law satisfies a system of quasi-linear first order partial differential equations.When the control does not appear in the criterion,this system degenerates into algebraic equations.For the polynomial nonlinearity and quadratic performance index we obtain by the original procedure the optimal law in the analytical form.The theoretical study is applied for to minimize the displacement of an idealized reactor building which is subject to horizontal earthquake-induced accelerations.The active control scheme developed uses a system of force actuators(most probable hydraulic)to counter the action of forcing input due to earthquake.The design problem concerns the optimal control input so that the system responds favourably.The paper is in a certain meaning an extension for the nonlinear optimal cases,soil-structure interaction,of the results of (Wolf,Madden 1981).As a practical examples we present the behaviour of a structure of shear beam linear type with a cubic nonlinearity confined to the connection between the foundation and the moving base.

2 OPTIMAL LOOP CONTROL LAW

We consider the system

$$(1) \qquad \dot{q}(t) = F(t,q(t),u(t)) \ , \ y(t) = h(q(t)).$$

where the state vector $q \in \mathbb{R}^n$,the control vector $u \in \mathbb{R}^m$, $m,n \in \mathbb{N}$.

The function $F:\mathbb{R}^{1+n+m}$ and the output function $h:\mathbb{R}^n \to \mathbb{R}^n$ are indefinitely derivables belonging C^∞. The optimal control problem considered has the Mayer's form without restrictions, with the initial state known, the final end free i.e. we optimize the output function $y(T) = h(q(T))$. We seek the optimal control under the feedback law form

$$(2) \qquad u = u(t,q), \quad u:\mathbb{R}^{1+n} \to \mathbb{R}^m$$

Pseudo-hamiltonian of the system (1)

$$(3) \qquad H = \langle p, F(t,q,u) \rangle = \sum_{k=1}^{n} p_k F^k(t,q,u)$$

where $p \in \mathbb{R}^n$ is the adjoint vector. The Hamilton's equations are given by

$$(4) \qquad \dot{q} = \frac{\partial H}{\partial p}, \qquad \dot{p} = -\frac{\partial H}{\partial q}$$

with the end conditions $q(0) = q_0, p(T) = -h_q(q(T))$.
According to Maximum principle, we have

$$(5) \qquad \frac{\partial H}{\partial u} = 0 \quad (0 \le t \le T), \; H_{u_i} = \langle p, F_{u_i}(t,q,u) \rangle = 0 \quad i=1,\dots,m$$

Since the optimal control has the form (2), in (4)

$$(6) \qquad \frac{\partial H}{\partial q^k} = \sum_{i=1}^{n} p_i \left[F^i_{q^k}(t,q,u) + \sum_{j=1}^{m} F^i_{u_j}(t,q,u) \frac{\partial u_j}{\partial q^k} \right]$$

Let $F = (F^1, F^2, \dots, F^n)$ where $F^k:\mathbb{R}^{1+n+m} \to \mathbb{R}, k=\overline{1,n}$, and the fields vectors defined by the differential operators of the first order

$$(7) \qquad A, F_{u_i} : \mathbb{R}^{n+1} \to \mathbb{R}^{n+1}, \quad A = \frac{\partial}{\partial t} + \sum_{k=1}^{n} F^k(t,q,u) \frac{\partial}{\partial q^k}$$
$$F_{u_i} = \sum_{k=1}^{n} F^k_{u_i}(t,q,u) \frac{\partial}{\partial q^k}, \quad i=\overline{1,m}$$

We consider the Lie brackets recufrsive iterative

$$(8) \qquad ad^0_A F_{u_i} = F_{u_i}, \; ad^1_A F_{u_i} = [A, F_{u_i}], \; ad^{\nu+1}_A F_{u_i} = [A, ad^\nu_A F_{u_i}] \quad \nu \ge 0$$

where $[A, F_{u_i}]$ are of the type

$$Z^k = \sum_{i=1}^{n} X^i \frac{\partial Y^k}{\partial q^i} - Y^i \frac{\partial X^k}{\partial q^i}$$

From the conditions, it results

(9) $$\frac{d^\nu}{dt^\nu} H_{u_i} = 0 \ , i=\overline{1,m} \ ; \ \nu=0,1,2,\ldots$$

Using (3),(8),(9) we obtain the necessary optimum conditions for the feedback law u(t,q)

(10) $$\langle p, ad_A^\nu F_{u_i} \rangle = 0.$$

If we eliminate the adjoint vector p from (10) it obtains a system of quasi-linear partial differential equations. For a Bolza problem where the dimensions of the state space and of the control are the same it obtainsa system of equations of the first degree.Thus, the system with feedback loop law

(11) $$\dot{q}(t)=F(t,q,u), \quad J = \phi(q(T)) + \int_0^T F^0(t,q,u)\, dt \qquad m=n$$

The system (11)is changed in a Mayer form with the new coordinate q^0:

$$\dot{q}^0(t)=F^0(t,q,u), \dot{q}(t)=F(t,q,u), y(t)=\phi(q)+q^0$$
$$q^0(0)=0$$

In this case the field vectors A is given by

$$\tilde{A} = \frac{\partial}{\partial t} + F^0(t,q,u)\frac{\partial}{\partial q^0} + \sum_{k=1}^{n} F^k(t,q,u)\frac{\partial}{\partial q^k} =$$
$$= \frac{\partial}{\partial t} + \tilde{F}, \qquad \tilde{F} = \sum_{l=0}^{n} F^l \frac{\partial}{\partial q^l}$$

For $\nu=0,1$ the necessary conditions of optimum (10)are

$$\sum_{l=0}^{n} p_l F^l_{u_i}(t,q,u)=0, \qquad \sum_{l=0}^{n} p_l \left[\tilde{A},\tilde{F}_{u_i}\right]^l = 0$$

where $[\cdot]^l$ is the l-th components of Lie brackets.
If the optimal problem has the Lagrange type

$$\dot{q}(t) = F(t,q,u), \quad J = \int_0^T F^0(t,q)\, dt$$

the feedback optimal loop law is given by the algebraic equations system:

(12) $$\sum_{k=1}^{n} F^0_{q^k} F^k_{u_i} = 0, \quad i=1,\ldots,n$$

3 DYNAMIC MODEL OF SOIL-STRUCTURE INTERACTION PROBLEM

A major problem in the design of nuclear facilities is the satisfaction of the seismic response criteria. In this aim the active control concepts go further than passive control in significantly reducing the dynamic response. Reactor design dependence on the uncertainties of the amplitudes and of the frequency content of an actual earthquake is diminished. The forces can be applied by hydraulic or electromagnetic actuators. In this paper we consider an one-dimensional linear system attached at a point P to an external non-linear compliant constraint, as indicated in Figure 1.

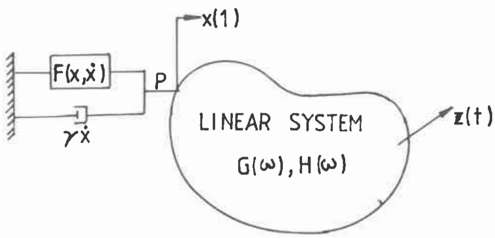

Figure 1. General linear system attached to a non-linear constraint

The constraint force is assumed to consist of separate contribution from a linear viscous element and a nonlinear elastic or hysteretic element modelling soil-structure interaction. The linear system is assumed to be undamped. The motion of point P, including the control input (earthquake action) and white noise disturbance is governed by the equation

$$(13) \qquad m\ddot{x} + F(x,\dot{x}) + \gamma\dot{x} = u(x,t) + w(t), \quad x(0)=0, x(0)=v_o$$

or equivalently

$$(14) \qquad \ddot{x} + f(x,\dot{x}) = u(x,t) + w(t)$$

The control u consists of a feedback and constitutes the basic active controller structure (Wolf, Madden 1981).

4 DIMINISHING OF THE SEISMIC RESPONSE OF NONLINEAR ISOLATION SYSTEM

A general performance index has the usual form:

$$(15) \qquad J = \int_0^T (a_1^2x^2 + a_2^2x^2 + \rho^2u^2)\, dt$$

where ρ is a scalar weighting parameter. The performance index is an acceptable compromise between achieved system response and control energy expended.

The optimization problem (14) and (15) is equivalently with Mayer's problem

$$(16)\qquad \dot{x}_0 = a_1^2x_1^2 + a_2^2x_2^2 + \rho^2u^2 \ , \ \dot{x}_1 = x_2,$$
$$\dot{x}_2 = u(x,t) + w(t) - f(x_1,x_2),\ y(t) = x_0(t),$$
$$x_0(0) = 0,\ x_1(0) = x_0\ ,\ x_2(0) = v_0\ .$$

With notations

$$(17)\qquad F^0 = a_1^2x_1^2 + a_2^2x_2^2 + \rho^2u^2,\ F^1 = x_2,$$
$$F^2 = u(x,t) + w(t) - f(x_1,x_2)\ .$$

The conditions (10) wrote for $i=1,2$ and $\vartheta=0,1,2$ are

$$(18)\qquad \sum_{k=0}^{2} p^k ad^{\vartheta} F_u^k = 0, \qquad \vartheta=0,1,2$$

Eliminating the components of the adjoint vector $p^k, k=0,2$ between equations(18)we obtain the necessary optimum condition

$$(19)\qquad \begin{vmatrix} ad^0F_u^0 & ad^0F_u^1 & ad^0F_u^2 \\ ad^1F_u^0 & ad^1F_u^1 & ad^1F_u^2 \\ ad^2F_u^0 & ad^2F_u^1 & ad^2F_u^2 \end{vmatrix} = 0$$

With (8),(9) and (17) from (19)we obtain the following partial differential equation of second order

$$(20)\qquad a_{11}\frac{\partial^2 u}{\partial x_1^2} + 2a_{12}\frac{\partial^2 u}{\partial x_1 \partial t} + a_{22}\frac{\partial^2 u}{\partial t^2} + a_{10}\frac{\partial u}{\partial x_1} + a_{10}\frac{\partial u}{\partial t} + a_{00} = 0$$

for the optimal control u.In (20)we have:

$$a_{11}=\rho^2x_2^2,\ a_{12}=\rho^2x_2^2,\ a_{22}=\rho^2\ ,\ a_{10}=\rho^2(u-w-f-x_2\frac{\partial f}{\partial x_2}),$$
$$a_{01}=-\rho^2\frac{\partial f}{\partial x_2}, a_{00}=a_1^2x_1^2 - a_2^2(u+w-f)+\rho^2u\frac{\partial f}{\partial x_1} - \rho^2u(u+w-f)\frac{\partial^2 f}{\partial x_1^2} - \rho^2ux_2\frac{\partial^2 f}{\partial x_1\partial x_2}$$

The initial conditions from differential equation (20) it results from $u(T,x)$ calculated with $p(T)=0$.

If in the functional (15) $\rho=0$,(20) degenerates in the following algebraic equation:

$$(21) \qquad u = \frac{a_1^2}{a_1^2} x_1^2 + f - w$$

For the linear system, a shear beam and for the non-linear soil-structure interaction, the function with a cubic elastic part

$$f(x,x) = kx_1(1+\varepsilon x_1^2) + \gamma x_2$$

we obtain the differential equation, similarly with (20), for the optimal control of non-linear problem. The equation (20) can be solved numerically and the results for many practical situations will be prepared and published in the future.

5 CONCLUSIONS

In the paper is given a general method for to obtain the optimal control for the non-linear differential systems, particularly, for to determine the active optimal loop control for to reduce the seismic response of reactor component. In the example, the nonlinearity is confined to soil-structure interaction. The effectivity of the method proposed will be proof by numerical results.

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