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ESTIMATION OF STIFFNESS CHARACTERISTICS OF LRB BEARINGS BY FINITE ELEMENT ANALYSIS

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ABSTRACT

The material property of the rubber has been studied in order to improve the reliability of the finite element model of a lead rubber bearing (LRB) which is a typical base isolator. Rubber exhibits elastic behavior even within the large strain range, unlike the general structural material, and has a hyper-elastic characteristics that shows non-linear relationship between load and deformation. This study represents the mechanical characteristics of the rubber by strain energy function in order to develop a finite element (FE) model of LRB. For the study, several strain energy functions were selected and mechanical properties of the rubber were estimated with the energy functions. A FE model of LRB has been developed by using material properties of rubber and lead which were identified by stress tests. This study estimated the horizontal and vertical force-displacement relationship of a LRB with the FE model. The adequacy of the FE model was validated by comparing the analytical results with the experimental data.

INTRODUCTION

Lead rubber bearing (LRB) is a type of seismic isolation bearing that has dynamic function with interaction of rubber elasticity and lead's plasticity. To design base isolated structure, it is important to explain characteristics of LRB against external loading. To ensure the reliability of LRB made of rubber as its main material, it is an important prerequisite to figure out nonlinear characteristics. If LRB can be expressed as finite element (FE) model accurately, it can figure out dynamic characteristics of device instead of high cost and difficult test, it can secure many advantages in practical level. Considering such advantages, it is necessary to develop FE model of LRB.

To analyze mechanical behavior of LRB, it must firstly explain nonlinear characteristics of rubber and lead. It needs to depend on experiments with much effort and time and if size increases, it is more difficult to perform experiments. In order to solve this problem requires a state identified by the analysis. To increase reliability of analysis result, more effort is required to analyze non-linear characteristics. Rubber is a special material having large deformation and non-linearity at the same time. Accordingly, it is important to decide accurate constitutive equation of rubber for reasonable realization of characteristics the study examined strain energy function based on hyper elastic theory on rubber by various researchers and selected some equations to perform non-linear static analysis and to calculate horizontal and vertical stiffness. The stiffness extracted by analysis was compared with performance test result of LRB.

MATERIAL NONLINEARITY OF RUBBER

LRB base isolated bearing consists of rubber, steel plate and lead. Among them, rubber is a main material to control characteristics and functions of LRB. Rubber is a hyper-elastic material maintaining elasticity at large deformation and having non-linearity in relation between force and deformation. When analyzing a rubber is interpreted by FE method, it is important to determine suitable strain energy function to express the relation between stress and strain.

Constitutive Equation of Rubber

Rubber controls dynamic behavior of LRB. In general elasticity, if load is applied on an object, it causes deformation and if load is removed, it restores to its original condition. Thus, the relation between load and deformation is linear. To the contrary, rubber shows elastic behavior in the non-linear relation between loading and deformation. Characteristics of rubber material can be explained based on the concept that change rate of strain energy per unit volume is same with power by stress against finite deformation or elastic behavior of large deformation. Coefficient values of strain energy function are determined by minimizing the differences with testing values after joining stress- strain test data obtained through material test with stress- strain ratio induced from test result data.
In finite element analysis, hyper elastic material model is expressed as strain energy function and has the following relation (Rivlin, 1948).

$$\sigma_{ij} = \partial W / \partial E_{ij} \tag{1}$$

Where, σ_{ij} is second order Piola-Kirchhoff stress tensor, W is strain energy function, E_{ij} is Green-Lagrange Strain ratio tensor which is expressed as below.

$$E_{ij} = 0.5\left(F^T F - I\right) \tag{2}$$

Where, F is deformation gradient tensor and $F_{ij} = \partial x_i / \partial X_j$. Where, x_i and X_j are linear element vendor before and after deformation. I is unit tensor.
Strain energy function expressing rubber behavior is studied in various types. Generally, it is expressed in principal invariant (Rivlin, 1948). Generally, material behavior to express rubber deformation is assumed to be elastic, isotropic and non-compression. From the first and second assumptions, strain energy W can be expressed in strain invariant function.

$$W = W\left(I_1, I_2, I_3\right) = W\left(\lambda_1, \lambda_2, \lambda_3\right) \tag{3}$$

$$I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2 \tag{4a}$$

$$I_2 = \lambda_1^2\lambda_2^2 + \lambda_2^2\lambda_3^2 + \lambda_1^2\lambda_3^2 \tag{4a}$$

$$I_3 = \lambda_1^2\lambda_2^2\lambda_3^2 \tag{4a}$$

Where, W is strain energy density I_1, I_2, I_3 are as in Formula (4a) ~ (4c) invariants on principal stretches λ_1 , λ_2 , λ_3 . As rubber is non-compressive material $\lambda_1\lambda_2\lambda_3$ i.e., $I_3 = 1$, $I_2 = 1/\lambda_1^2 + 1/\lambda_2^2 + 1/\lambda_3^2$.

Polynomial Strain Energy Function

The basic type of strain energy function expressing characteristics of rubber is as follows (ABAQUS, 2012).

$$W = \sum_{i+j=1}^{N} C_{ij}\left(\bar{I}_1 - 3\right)^i \left(\bar{I}_2 - 3\right)^j \qquad (5)$$

Where, $\bar{I}_1$ and $\bar{I}_2$ are the first and second deviatoric strain invariants and it is expressed as principal stretch value as in equation (2). J^{el} is the elastic volume ratio, C_{ij} and D_i are temperature-dependent material parameters and N is a disparity of energy equation. D_i is constant expressing rubber compressibility. If D_i is 0, the material is considered to be non-compressibility. C_{ij} can be figured out from rubber test data having deformation and stress distribution as simple border condition and from the relation of differential of strain energy function of $\bar{I}_1$, $\bar{I}_2$ and J^{el}.

Strain energy function has been developed in slightly different ways by proposers. Stain energy function that has been regarded as high practicality and reliability by previous researchers is the function suggested by Mooney-Rivlin(1948), Neo-Hookean(1948), Polynomial(1951), Ogden(1972) and Yeoh(1993). The types of these functions are aligned in Table 1.

Compare of Strain Energy Function

Material characteristics of rubber samples are defined by uniaxial, equibiaxial, pure shear and volumetric test. In calculating strain energy function, at least two test data among uniaxial, equibiaxial and pure shear are required. And to figure out deformation characteristic accurately, volumetric data is required further.

This study cited test data performed by Italy ENEA(1996) which is shown in Figure 1. Material constant was calculated based on the selected formula using rubber material test data and curve fitting was used to figure out coefficient. Figure 2~6 shows coefficient on stress-strain ratio curve and each formula against strain energy function mentioned in Table 1 targeted to test data in Figure 1.

According to the comparison result of strain energy function, Mooney-Rivlin formula and Neo-Hookean formula had great differences from test result. Polynomial, Ogden formula and Yeoh formula assumed the test results well at large deformation zone. For volumetric test on vertical stiffness, polynomial function and Ogden function model were very close to test results, but Mooney-Rivlin formula had large differences.

Table 1: Formulas of strain energy functions.

Proposer	Formula
Mooney-Rivlin	$W = C_{10}\left(\bar{I}_1 - 3\right) + C_{01}\left(\bar{I}_2 - 3\right) + \frac{1}{D}\left(J^{el} - 1\right)^2$
Neo-Hookean	$W = C_{10}\left(\bar{I}_1 - 3\right) + \frac{1}{D}\left(J^{el} - 1\right)^2$
Ogde	$W = \sum_{i=1}^{N} \frac{2\mu_i}{\alpha_i^2}\left(\bar{\lambda}_1^{\alpha_i} + \bar{\lambda}_2^{\alpha_i} + \bar{\lambda}_3^{\alpha_i} - 3\right) + \sum_{i=1}^{N} \frac{1}{D_i}\left(J^{el} - 1\right)_{2i}$
Yeoh	$W = C_{10}\left(\bar{I}_1 - 3\right) + C_{20}\left(\bar{I}_1 - 3\right)^2 + C_{30}\left(\bar{I}_1 - 3\right)^3 + \frac{1}{D_1}\left(J^{el} - 1\right)^2 + \frac{1}{D_2}\left(J^{el} - 1\right)^4 + \frac{1}{D_3}\left(J^{el} - 1\right)^6$

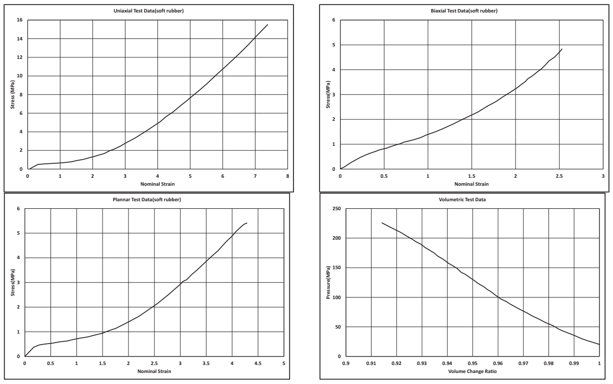


Figure 1. Soft Rubber Compounds Material Properties(ENEA HDRB)

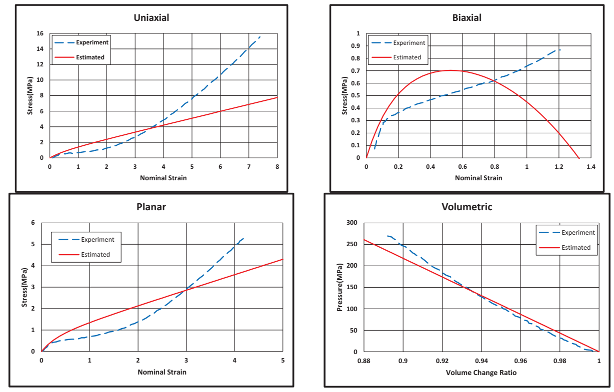


Figure 2. Properties Estimated by Mooney-Rivlin Equation

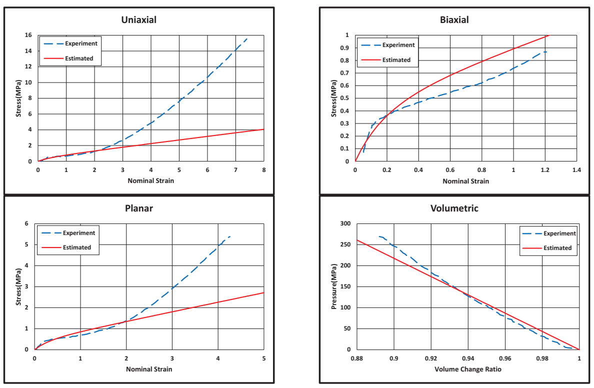


Figure 3. Properties Estimated by Neo-Hookean Equation

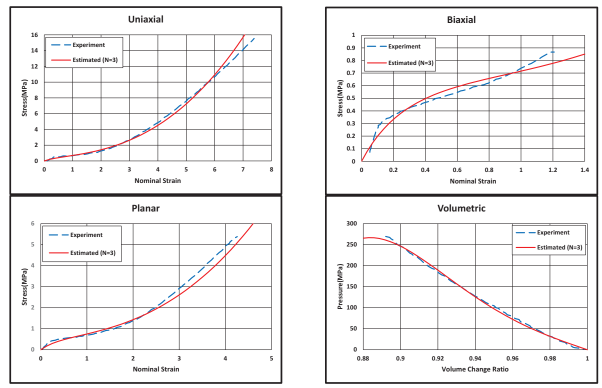


Figure 4. Properties Estimated by Ogden Equation

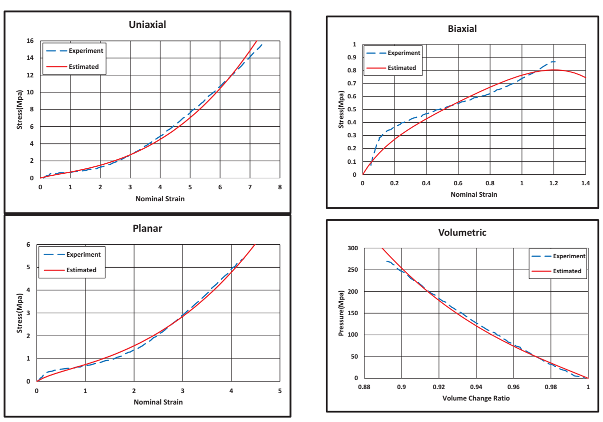


Figure 5. Properties Estimated by Polynomial Equation

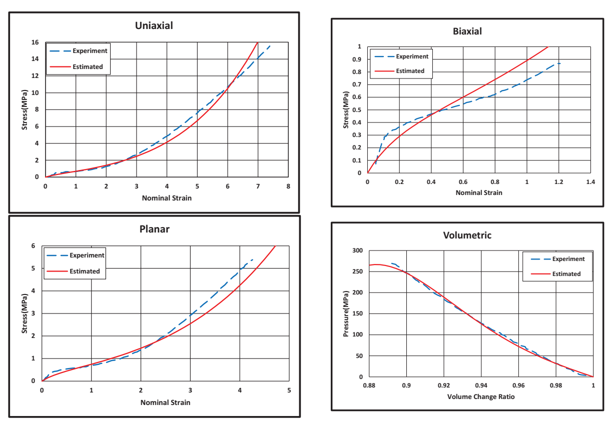


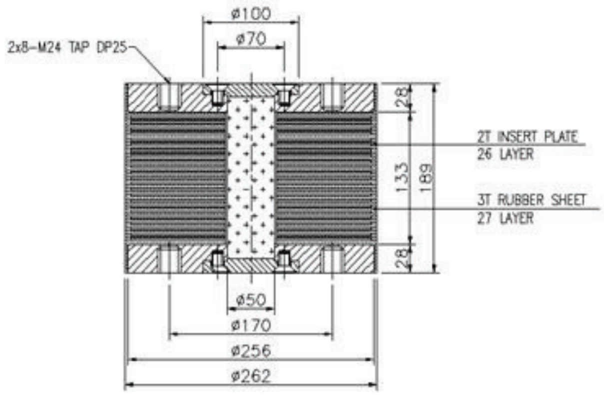
Figure 6. Properties Estimated by Yeoh Equation

CYCLING LOADING TEST OF LRB SPECIMENS

Dimension

To evaluate horizontal and vertical stiffnesses of LRB, LRB specimen was manufactured and loading test was performed. The design pressure of LRB was 5 MPa. Specification and design value of LRB are determined in accordance with JEAG 4614 (Japanese electricity association,

2013) standard and through repetitive processes. Figure 7 shows design drawing and shape of LRB used for the test. Elastic coefficient of steel plate inserted between rubbers is 200 GPa, Poisson's ratio is 0.3, lead's elastic coefficient is16 MPa and Poisson's ratio is 0.44. Table 2 and Table 3 are design of devices.



(a) Dimension

(b) Shape

Figure 7. Specimen of LRB

Table 2. LRB Isolator Data

Bearing stress	σ	5.0 MPa	Displacement	δ	81.0 mm
Shear modulus	G	0.4418 MPa	Rubber thickness	tr	3.0 mm
Diameter	Do	250 mm	Rubber layer	nr	27EA
Lead diameter	Dp	50 mm	Steel plate thickness	ts	2.0 mm
Shear strain ratio	γ	100% ratio	End plate thickness	te	28 mm

Table 3. LRB Isolator Design Parameters

Horizontal Characteristics		γ=100%	Vertical Characteristics		σ=5 MPa
1st stiffness	Ku	26.47 kN/mm	Vertical stiffness	Kv	418 kN/mm
2nd stiffness	Kd	0.268 kN/mm	Design load	Pu	236 kN
Characteristics	Qd	15.65 kN		Pu+30%	306.8 kN
Equivalent stiffness	Keq	0.461 kN/mm		Pu-30%	165.2 kN
Equivalent Damping	heq	0.251	Deformation	δv	0.564 mm

Horizontal Direction Cyclic Load Test

LRB characteristic test was performed based on ISO 22762 standard (International Organization for Standardization, 2010). With design pressure of 236 kN being loaded in vertical, horizontal load increased gradually until it reached target displacement of 81 mm. Using displacement control method, the specimen of LRB was tested at horizontal direction and Figure 8 (a) shows its results. Horizontal stiffness of sample was 0.460 kN/mm and it had - 0.18% difference against design sample.

Vertical Direction Cyclic Load Test

In vertical direction test, stiffness was measured by repeated loading of 306.8 kN and 165.2 kN which were ±30% of design loading of 236 kN. Stiffness was shown in Figure 8 (b) by calculating difference between loading and displacement different at maximum loading and minimum loading. The vertical stiffness of sample was 494.53 kN/mm and it had +18.31% difference against design sample.

(a) Horizontal (b) Vertical

Figure 8. Load-Displacement curves of LRB Specimen

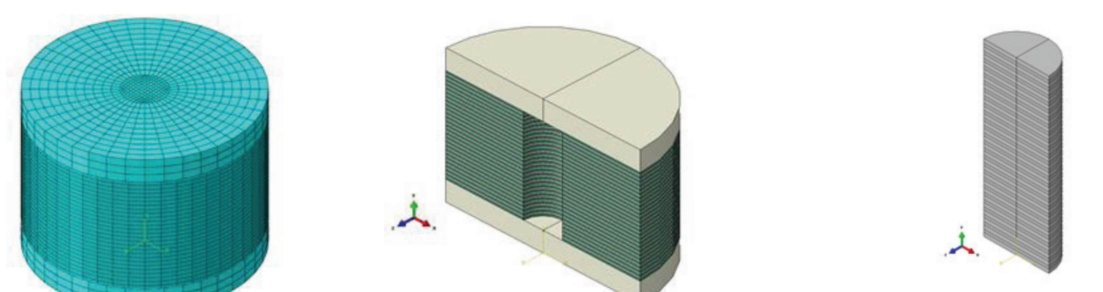
FINITE ELEMENT ANALYSIS OF LRB ISOLATOR

Modeling

FE analysis performed by using commercial program ABAQUS (ABAQUS, 2012) and composed of 3D model as in Figure 9. Rubber part of model was assumed to be non-compressibility and metal plate was assumed to be steel body as it had almost no deformation compared with rubber. Model consists of 4 types including rubber, lead, inserted steel plate and finishing plate, all of which were expressed in 3D solid. Model diameter was Φ250 mm except for LRB external sheath. Internal lead rod diameter was 50 mm and height was 189 mm.

As rubber has hyper elastic behavior, hybrid element was additionally applied to ABAQUS to control deformation of rubber. This study modelled rubber as 3D solid element for detailed analysis instead of axisymmetric model. Each rubber layer of LRB was divided into 2 elements with thickness direction

- Steel plate: Solid element (C3D8R), Do = Φ250mm, Di = Φ50mm, Thickness = 2mm with 26 sheets
- Rubber: Solid element (C3D8RH), Do = Φ250mm, Di = Φ50mm, Thickness = 3mm with 27 sheets
- Finishing steel plate: Solid element (C3D8R), Diameter Φ250mm, Height 28mm with 2 sheets of top and bottom
- Lead: Solid element (C3D8R), Diameter Φ50mm, Height 133mm



(a) LRB (b) Rubber & Steel (c) Lead core

Figure 9. FE Model of LRB specimen

Boundary & Load Condition

In horizontal analysis, the boundary condition was 3-directional fixed of nodal points forming bottom steel plate. The top plate bound vertical displacement ingredient to prevent buckling distortion and to make uniform deformation and satisfied axial symmetric conditions. Lead and steel plate are completely adhered and rigid body connection was used to express unified behavior for lead, rubber and steel plate.

For vertical load, design load of 236 kN was loaded at the top surface with distributed pressure of 5 MPa. As for the application order of loading, as in Table 4, design load was given to vertical direction. And, then, 81 mm which considered maximum shear strain ratio of 100% was added to both directions horizontally. The vertical load was repeated loading of ± 30% 165.2kN and 306.8kN the design vertical loads.

Table 4. Load Condition

	Horizontal Analysis		Vertical Analysis
	Vertical Load (kN)	Horizontal Load(mm)	Vertical Load (kN)
Step 1	236	-	306.8
Step 2	-	81	165.2
Step 3	-	-81	306.8
Step 4	-	81	165.2
Step 5		-81	306.8
Step 6		81	165.2
Step 7		-81	306.8

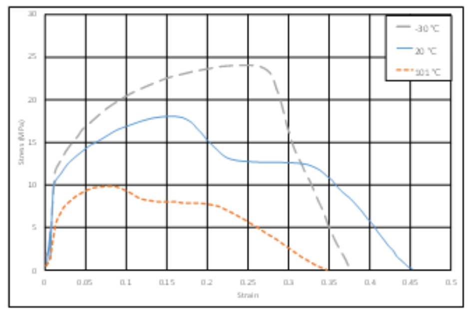


Figure 10. Stress-Strain Relation of Lead at 0.0075/sec Strain Rate

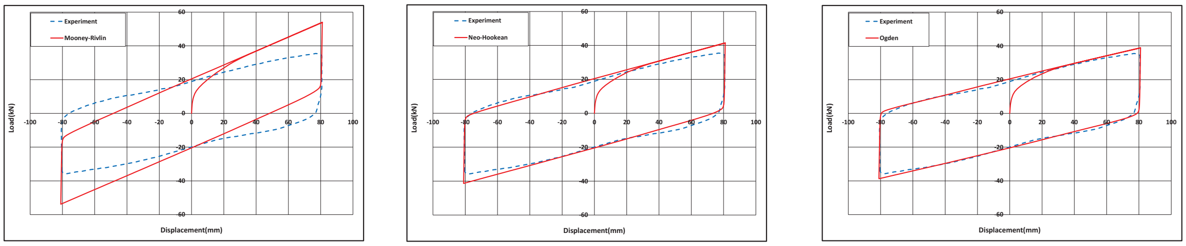
Material Properties

Rubber characteristics were expressed in 5 strain energy functions with the assumption of hyper elastic material. Material characteristics of standard steel were applied for inserted steel plate and finishing plate. Lead molecule becomes recrystallized to original molecular structure at 20℃.

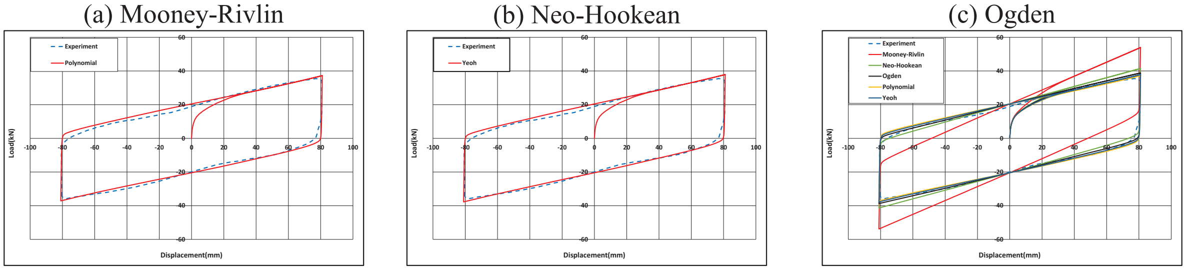
After finishing earthquake, LRB applied structure restores structure to the original condition with rubber elastic restoring force, and lead returns to original molecular structure at room temperature. Different from rubber having many variables in production, lead is pure material that does not require mixture or cure and that can be commonly applied. Yield stress of lead is subject to change by shearing strain ratio. According to the analysis data by MCEER (Ioannis, et al., 2008), stress-strain ratio of lead is as in Figure 10 by temperature. This study applied 20℃ curve of MCEER data as material characteristic value of lead.

Analysis result of Horizontal Direction

In analyzing horizontal stiffness analysis, loading was added in two stages. To make same condition with test, vertical force was added with 236kN in the 1st step, and load was added with 81㎜, which is total thickness of rubber layer for displacement of vertical direction by means of rubber displacement control. In the same condition with test, FE analysis was performed according to strain energy function in order to analyze behavior and stiffness of horizontal direction. When the analysis result in Figure 11 was compared with stiffness of Figure 8 (a), there was the greatest difference of the horizontal direction force-deformation behavior of LRB with test result if Mooney-Rivlin formula was applied. In Figure 2~6, Ogden formula and polynomial formula that assumed characteristics of uniaxial and equibiaxial well could estimate the horizontal force-deformation behavior of LRB relatively well.



(a) Mooney-Rivlin (b) Neo-Hookean (c) Ogden

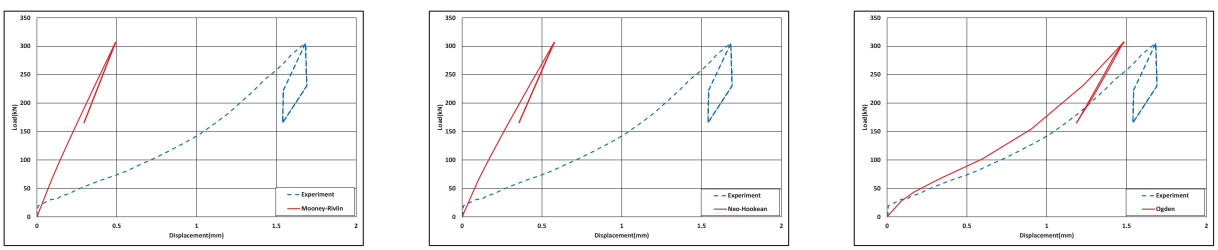


(d) Polynomial (e) Yeoh (f) Comparison

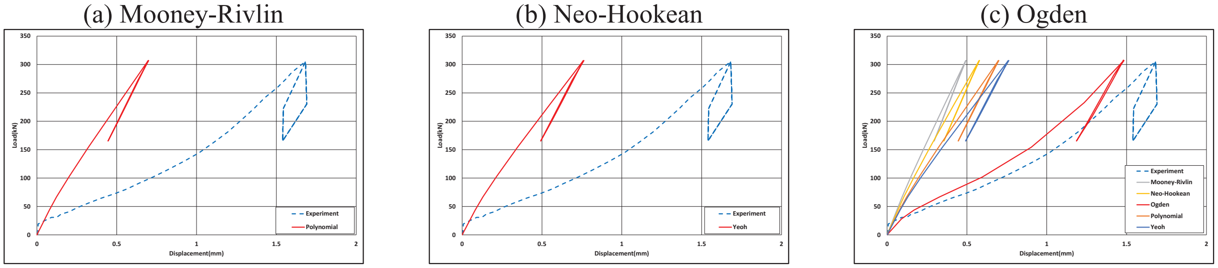
Figure 11. Hysteresis Loops from Horizontal FE Analysis

Analysis result of Vertical Direction

Vertical stiffness analysis was designed with the same condition with test method applying design load of ±30%. Like horizontal direction, it analyzed vertical direction behavior by strain energy function under same conditions. For vertical stiffness in Figure 12, Ogden formula had the closest stiffness to Figure 8 (b) value. Like in Figure 2~6, it is because Ogden formula was closely suitable for volumetric test result. While the difference of curve fitting result was slight, it had great difference in behavior and stiffness value in actual analysis.



(a) Mooney-Rivlin (b) Neo-Hookean (c) Ogden



(d) Polynomial (e) Yeoh (f) Comparison

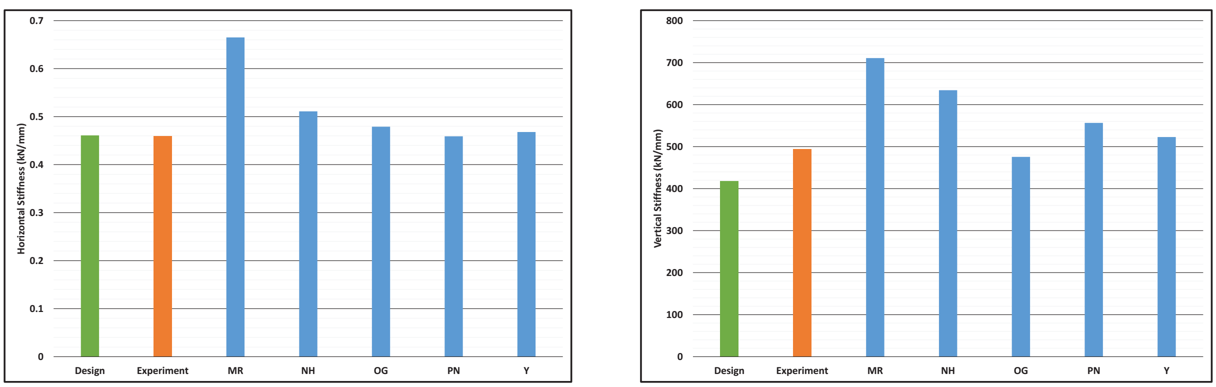
Figure 12. Hysteresis Loops from Vertical FE Analysis

Discussion

Design value and analysis value on horizontal and vertical stiffness on LRB are shown in Table 5 and Figure 13. For horizontal stiffness, if strain energy function that accurately expresses uniaxial and equibiaxial is applied, it could express horizontal stiffness of LRB accurately. For vertical stiffness, it had great differences in stiffness value by types of strain energy function. To figure out deformation characteristics of rubber toward vertical direction, volumetric test data of rubber is essential. And, as a result of curve fitting of strain energy function, Ogden could assume vertical stiffness that expresses volumetric characteristics accurately. This study found that if rubber characteristics of LRB are expressed in Ogden formula, both horizontal and vertical stiffness could be assumed relatively accurately.

Table 5. Comparison of Stiffness Values

		Stiffness (kN/mm)		Design / Exp. & Analysis (%)		Experiment / Result of Analysis (%)	
		Hor.	Ver.	Hor.	Ver.	Hor.	Ver.
Design Parameter		0.461	418	-	-	-	-
Experiment		0.460	494.53	100.22	84.52	-	-
Analysis	Mooney-Rivlin(MR)	0.665	710.80	69.33	58.8	69.17	69.57
	Neo-Hookean(NH)	0.511	634.40	90.22	65.89	90.02	77.95
	Ogden(OG)	0.479	475.73	96.24	87.86	96.03	103.95
	Polynomial(PN)	0.459	556.62	100.44	75.10	100.21	88.85
	Yeoh(Y)	0.468	522.77	98.5	79.96	98.29	94.60



(a) Horizontal (b) Vertical

Figure 13. Comparison of Stiffness Values

CONCLUSION

To improve reliability of finite element analysis on LRB base isolated system, each analysis was performed using strain energy function and the test value and analysis value were compared. The results are as follows.

(1) Nonlinear dynamics characteristic of the rubber contained in the LRB govern the force-deformation behavior of LRB finite element model.

(2) Mechanical properties of the rubber are shown the characteristics of hyper-elastic material, when creating a finite element model of the LRB, it is important to determine the type of strain energy function and coefficients of function that can be expressed exactly the hyper-elastic deformation of rubber.

(3) This study determines the coefficient of some strain energy function for the standard properties of the rubber, and the results compared to the experimental results, Ogden

function expression was confirmed to be the most representative of the characteristics of a standard rubber.

(4) Through the analysis of the finite element analysis results the vertical stiffness of LRB, the results with the most of the strain energy function similar force-deformation relationship curves showed. However, as an experimental results is a big difference.

(5) This study from the Ogden function using function seems the nearest match to results and to express the experimental results in vertical direction behavior of the LRB.

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