Vibration Control of Structure Using Dynamic Absorber Including Lever and Pendulum Mechanism

T. FUJINAMI, S. YAMAMOTO
Kyoto Institute of Technology, Kyoto, Japan

ABSTRACT

The new dynamic absorber system using lever and pendulum mechanism has been proposed for controlling the displacement of high-rise structures due to wind or earthquakes (Fujinami and Yamamoto; 1990). The optimum condition for design of this system and its characteristics for vibration control of structure are investigated. Due to the effect of the lever and pendulum mechanism, the displacement of the dynamic absorber can be several times less than that of ordinary dynamic absorber, and the spring of dynamic absorber can be designed easily from the point of view of its strength.

1 INTRODUCTION

Recently, several mechanisms of dynamic absorbers have been installed to the top of the high-rise structures such as towers to control the displacement generated by wind. In general, the design of such dynamic absorbers have been made by using the method introduced by Den Hartog (1947). Therefore, the natural period of these dynamic absorbers should be tuned nearly to the first natural period of the structures. Consequently, to apply them to high rise structures, the natural period of the dynamic absorbers must be lengthened, and their amplitudes of the vibration becomes tremendously large. They have been the obstacles to apply dynamic absorbers, since the springs of the dynamic absorbers suffer the great amplitude of vibration and they must have been the linear characteristics even in the range of large amplitude.

To eliminate this obstacles, a new dynamic absorber system using lever and pendulum mechanism is proposed to control the displacement of structures (Fujinami and Yamamoto; 1990)

In this paper, the motion of this system is evaluated in the point of view of the distributed mass of lever, and the effect of the mass of the lever on the characteristics of vibration control is described.

2 MECHANISM AND ITS OPTIMUM CONDITION

The schematic view of the proposed dynamic absorber system is shown in Fig. 1. The ordinal dynamic absorber (A) is installed to the top of the structure (G) by a spring (B) and a damper (C). The fulcrum (D) is set on the structure, and the lever (E) is connected from the fulcrum to the pendulum (F) passing through the mass of dynamic absorber. When the mass of the dynamic absorber begins to vibrate, the amplitude of the pendulum is enlarged by being amplified by the lever ratio λ $(=l_2/l_1)$.

The dynamic model of structure and the proposed system is simplified by using the first modal mass m_1, the first modal damping c_1 and the first modal spring k_1 as shown in Fig.2. Mass of the dynamic absorber and the pendulum are modeled as mass m_2 and m_3, and the spring of the dynamic absorber is modeled as k_2, respectively. From this figure, the equation of motion of this system and structure becomes as follows.

$$[M]\{\ddot{X}\} + [C]\{\mathring{X}\} + [K]\{X\} = -[\bar{M}]\{I\}\ddot{U}g \tag{1}$$

where,

$$[M]=\begin{bmatrix} m_1 + m_2 + m_3 + ml_1(1+\lambda) & m_2 + (1+\lambda)\, m_3 + ml_1(1+\lambda)^2/2 \\ m_2 + (1+\lambda)\, m_3 + ml_1(1+\lambda)^2/2 & m_2 + (1+\lambda)^2\, m_3 + ml_1\,(1+\lambda)^3/3 \end{bmatrix}$$

$$[\bar{M}]=\begin{bmatrix} m_1 + m_2 + m_3 + ml_1(1+\lambda) & 0 \\ 0 & m_2 + (1+\lambda)\, m_3 + ml_1(1+\lambda)^2/2 \end{bmatrix}$$

$$[C]=\begin{bmatrix} c_1 & 0 \\ 0 & c_2 \end{bmatrix}, \quad [K]=\begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix}, \quad \{I\}=\{1 \;\; 1\}^T$$

By using the method introduced by Den Hartog, the optimum condition of this system can be obtained. Assuming sinusoidal ground motion $(U_o e^{i\omega t})$ and neglecting the critical damping ratio of structure ζ_1, the relative displacement X_1 of mass m_1 and the relative displacement X_2 of mass m_2 are given by following equations (Fujinami and Yamamoto; 1990).

$$\frac{X_1}{X_{st}} = \sqrt{\frac{(2\zeta_2)^2 A + B}{(2\zeta_2)^2 C + D}} \tag{2}$$

$$\frac{X}{X_{st}} = \sqrt{\frac{b^2}{(2\zeta_2)^2 C + D}} \tag{3}$$

where,

$$A = a^2 f^2 g^2, \quad B = \{a(f^2-dg^2) + \mu_{21} b^2 g^2\}^2, \quad C = f^2(1-ag^2)^2 g^2, \quad D = \{(1-ag^2)(f^2-dg^2) - b^2 g^4 \mu_{21}\}^2$$

$$\mu_{21} = m_2/m_1, \quad \mu_{31} = m_3/m_1, \quad \mu_D = ml_1/m_1, \quad \zeta_1 = c_1/2\sqrt{m_1 k_1}, \quad \zeta_2 = c_2/2\sqrt{m_1 k_2},$$

$$f = \omega_2/\omega_1, \quad g = \omega/\omega_1, \quad Xst = m_1\omega^2 U_o/k_1, \quad \omega_1^2 = k_1/m_1, \quad \omega_2^2 = k_2/m_2,$$

$$a = 1 + \mu_{21} + \mu_{31} + (1+\lambda)\mu_D, \quad b = \mu_{21} + (1+\lambda)\,\mu_{31} + (1+\lambda)^2 \mu_D/2$$

$$d = \mu_{21} + (1+\lambda)^2 \mu_{31} + (1+\lambda)^3 \mu_D/3$$

From the condition that the points P and Q, which are the points of the response curve for displacement of modal mass, are independent from magnitude of damping ζ_2 and the amplitudes of P and Q are on the condition to be equal, the following relation between the frequency ratio $f(=\omega_2/\omega_1)$, lever ratio λ, mass ratio $\mu_{21}(=m_2/m_1)$, $\mu_{31}(=m_3/m_1)$ and $\mu_D(=ml_1/m_1)$ are obtained as shown in Fig.4. Where, ω_1 and ω_2 are the uncoupled natural circular frequency of first modal mass and dynamic absorber respectively, and m is the distributed mass of the lever.

$$f^2 = \frac{2ad - 3b^2}{2\,\mu_{21} a^2} \tag{4}$$

Placing the condition of having the maximum amplitude at points P and Q, the damping ratio can be given after differentiating Eq. (2) by the frequency ratio g $(=\omega/\omega_1)$, as following equation. Where, ω is the circular frequency of ground motion.

$$\zeta_2{}^2 = \frac{b^2}{8a}\left\{ 3 + \frac{3b^2 \mp \sqrt{2\,(ad - b^2)\,b^2}}{2ad - 3b^2} \right\} \tag{5}$$

After substituting lever ratio λ, mass ratio μ_{21}, μ_{31} and μ_D which satisfy the Eq. (4) and the damping ratio given by Eq. (5) into Eq. (2), the maximum amplitude of the mode mass is obtained as follows.

$$\frac{X_1}{X_{st}} = \frac{a}{1-ag^2} = \frac{2a^2(ad-b^2)}{b^2} \tag{6}$$

3 CALCULATION RESULT

The non dimensional value of frequency ratio f of proposed system divided by the value of the ordinary one is plotted in Fig.3 against the lever ratio λ, mass ratio μ_{31} and μ_D. The value of mass ratio μ_{21},is treated as 0.002. By the virtual mass effect of lever and pendulum mechanism, the optimum frequency ratio of proposed system becomes higher than that of ordinary one. This tendency can be observed clear in the figure for the larger value of lever ratio λ, mass ratio μ_{31} and μ_D. It means that in the case of applying this system for the longer period structure, the stiffness of the spring k_2 can be set several times harder than that of ordinary one.

To see the effect of four parameters (three mass ratio and a lever ratio) for vibration control, the maximum displacement of modal mass and the dynamic absorber applying the proposed system is plotted in Fig. 4 through Fig. 6. They are normalized by dividing by the value of ordinaly one and their total mass ratio (total mass of proposed system to modal mass) is set as 0.01. In Fig.4, the relation between mass ratio μ_{21}, μ_{31} and non-dimensional displacement is shown for several lever ratio λ. In Fig.5, the relation between mass ratio μ_{21}, μ_D and non-dimensional displacement is shown for several lever ratio λ. .In Fig.6, the relation between mass ratio μ_{31}, μ_D and non-dimensional displacement is shown for several lever ratio λ. In Fig.4, the maximum displacement of X_1 is observed around the region where μ_{21} is 0.008 to 0.009 and this suggests that μ_{31} shall be selected as large as possible within the limitation to get effect of controlling the vibration of X_1. The phenomenon giving the maximum non-dimensional displacement of X_1 for the region where μ_{21} is 0.008 to 0.009 can be explained physically as follows. That is, the reaction force from this absorber system transmit through fulcrum, spring and damper. However, the force through spring and damper have an opposit sign to the force through fulcrum. Then, the effective force of vibration control to X_1 is obtained by substructing the reaction force of fulcrum from the force through spring and damper. When this substruction of the force becomes minimum, the displacement of X_1 becomes maximum. This condition for this substruction of the force becoming minimum shall be explained as approaching to the phenomenon of a balancing toy.

As numerical examples, the resonance curves of X_1/X_{st} and X_2/X_{st} of structure with this system are shown for mass ratios μ_{21} =0.001 and 0.005, μ_{31}=0.007 and 0.003 and $(1+\lambda)\mu_D$ =0.002 (total mass = 0.01) in Fig. 7. In this figure, it is observed that the displacement of proposed dynamic absorber (X_2/X_{st}) can be reduced to 1/3 - 1/4 of ordinaly one, though the displacement of modal mass (X_1/X_{st}) is almost the same value for the case of ordinaly dynamic absorber.

4 CONCLUSION

In order to reduce the response of structure against dynamic disturbances, the dynamic absorber using lever and pendulum mechanism was proposed, and its optimum tuning condition with considering its distributed mass of lever was studied. From the several numerical result, following conclusion were obtained.

1) The displacement of dynamic absorber using lever and pendulum could be several times less than that of ordinary dynamic absorber without loosing the ability of vibration control of modal mass.

2) Mass ratio μ_{31} (mass of pendulum to modal mass ratio) should be selected as large as possible within the limit of total mass ratio.

REFERENCES

Yamamoto,S. and Nagai,Y. etal. (1975). An Earthquake Isolating and Vibration Absorbing Equipment for Spherical Tanks. Journal of Japan High Pressure Institute, Vol.13, No.5, pp235-243.

Shimizu,N. and Yamamoto,S. etal. (1983). Theoretical Study on Single Story Structure Installed Earthquake Isolation Equipment. Transaction of Architectural Institute of Japan, No.327, pp29-39.

Den Hartog. (1947). Mechanical Vibrations. McGRAW-Hill, pp119-133

Seto,K. (1985). A method of the Optimal Design for a Pendulum-Type Vibration Controlling Equipment. Transaction of the Japan Society of Mechanical Engineers, No.467, pp1454-1462

Fujinami,T. and Yamamoto,S. (1990). Dynamic absorber Using Lever and Pendulum Mechanism and Installed to Top of Structure. ASME PVP, Vol. 197, pp143-148.

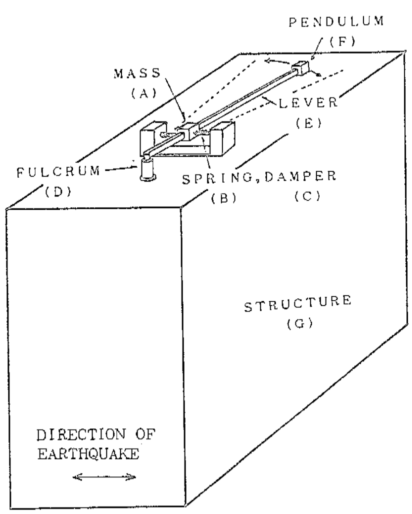


Fig.1 Schematic View of Proposed Dynamic Absorber System

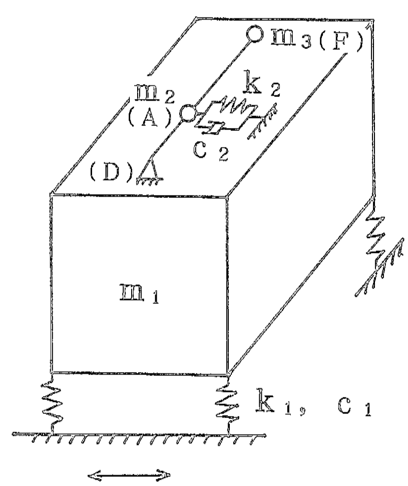


Fig.2 Dynamic Model of Structure and Proposed Dynamic Absorber

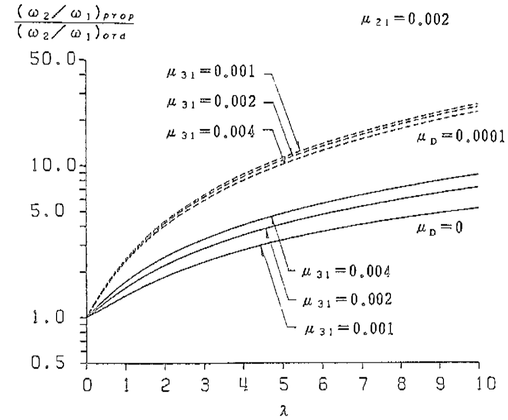


Fig.3 Tuned Frequency Ratio f in Optimum for Proposed Dynamic Absorber

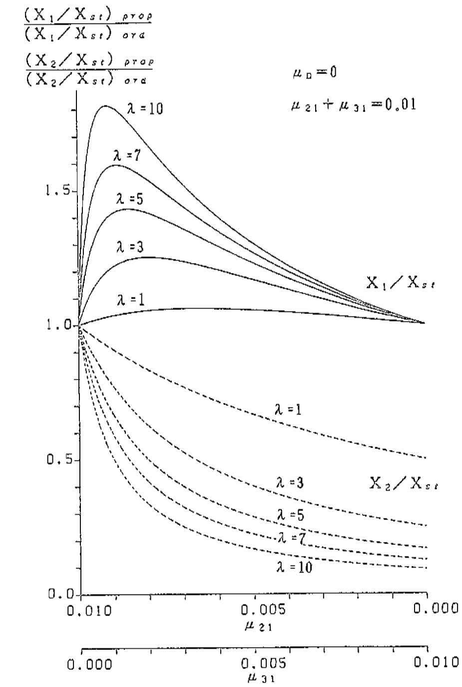


Fig.4 Maximum Displacement of Modal Mass and Proposed Dynamic Absorber

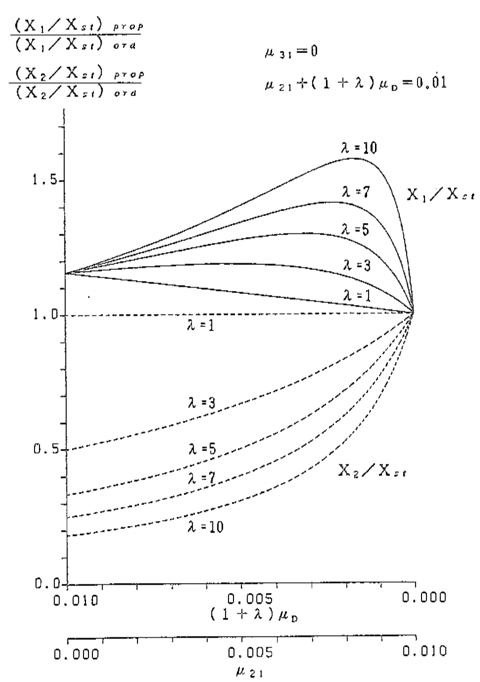


Fig.5 Maximum Displacement of Modal Mass and Proposed Dynamic Absorber

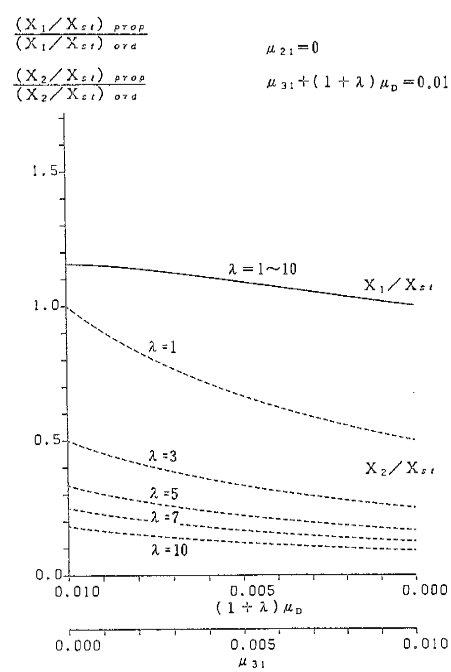
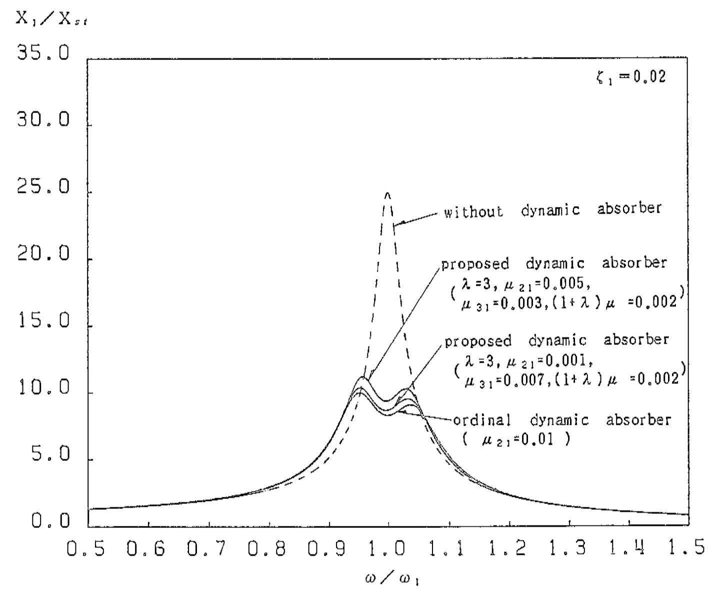
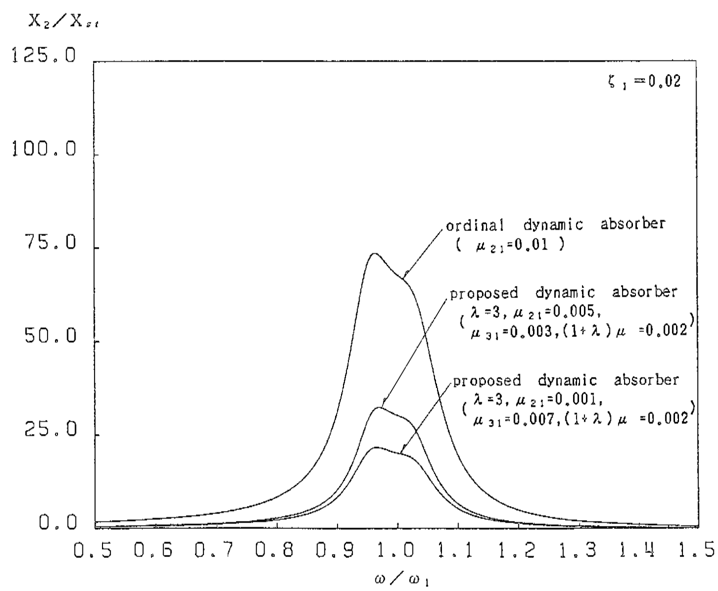


Fig.6 Maximum Displacement of Modal Mass and Proposed Dynamic Absorber



(a) Displacement of First Mode of Structure



(b) Relative Displacement of Dynamic Absorber

Fig.7 Response Curves of Structure and Dynamic Absorber