



## Optimal Seismic Design Method for Base-Isolated Pool Structure by Minimizing Life Cycle Cost

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### ABSTRACT

An optimal seismic design method for base-isolated and fluid-filled pool structures subjected to random ground excitation is studied. The criterion selected for the optimization is minimum life-cycle cost. The damage cost is estimated based on the structural failure probability calculated by the stochastic response analysis. Prescribed values of the isolator displacement and the wall base shear force are predetermined to constitute the limit-state levels. Added mass matrix derived by FEM modeling is able to consider fluid-structure interaction effects between the flexible walls and contained fluid. Design variables for optimization are the wall thickness and isolator stiffness. Flexible isolator allows us to choose relatively thinner lateral wall with similar failure probability. The more flexible isolator is, the smaller failure probability and total life-cycle cost are. This optimal design case is insensitive to assumed damage cost scale.

### 1. INTRODUCTION

Civil structures should be designed to maximize their profit to the society by minimizing life-cycle cost, which consists of initial construction cost and expected damage cost under natural hazards such as earthquake. Numerous researches and applications on optimal seismic design and economical efficiency evaluation method have been performed based on the life-cycle cost concept for various structural systems, e.g. buildings, bridges, active and passive control systems (Ang and Leon, 1996; Koh and Song 1998; Wen, 1991).

But there are few researches on optimal seismic design method for seismic-isolated pool structure. To define and evaluate the life-cycle cost function, the method should consider the properties of input ground motions, limit states of pool structure, properties of seismic isolation, and failure probabilities. In this study, we aim to develop an optimal seismic design method for base-isolated fluid-filled pool structures that can take the aforementioned features into account.

### 2. INPUT GROUND MOTION MODELING

To consider conveniently acceleration and site conditions in optimal design and cost effectiveness evaluation, we model input ground motion as spectral density function compatible with response spectra. Acceleration coefficient  $A$  and site coefficient  $S$  in

response spectrum reflect on acceleration and site condition of the construction site, respectively. We can obtain the compatible spectral density by upgrading spectral density repeatedly while comparing the target response spectrum with the response spectrum simulated from the upgraded spectral density (Deodatis, 1996). The results are as follows (Figs.1, 2). The acceleration coefficient describes the scale of the input ground motion, and the site coefficient reflects on frequency properties of ground motion and amplification of displacement by soil effect.

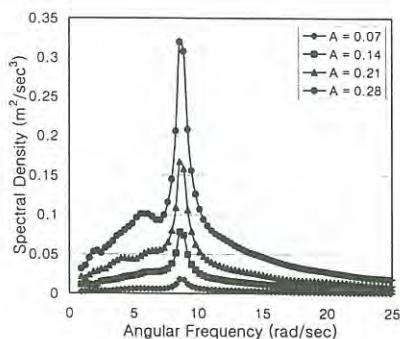


Figure 1. Variation of Spectral Density with Acceleration Coefficient

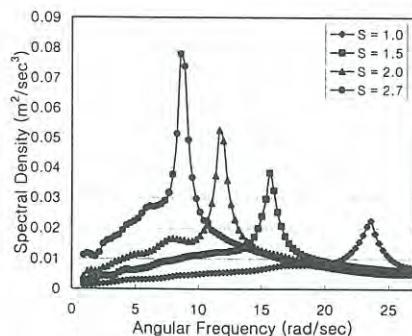


Figure 2. Variation of Spectral Density with Site Coefficient

### 3. FLUID-STRUCTURE INTERACTION MODELING

To describe the interaction between the flexible wall and fluid motion, fluid dynamic pressure at the wall is considered in the form of added matrix. The added mass matrix is derived by FEM of fluid motion. Irrotational flow of incompressible and inviscid ideal fluid can be described with velocity potential  $\phi(z,t)$ , which satisfies the following equations (Fig. 3).

$$\dot{u}_i = \frac{\partial \phi}{\partial z_i}, \quad \nabla^2 \phi = \left( \frac{\partial^2}{\partial z_1^2} + \frac{\partial^2}{\partial z_2^2} \right) \phi(z, t) = 0 \quad (1)-(2)$$

$$\frac{\partial \phi}{\partial t} + \frac{p}{\rho} = 0 \quad \text{in } \Omega, \quad \frac{\partial \phi}{\partial t} + g\eta = 0 \quad \text{on } S_f, \quad \frac{\partial \phi}{\partial n} = \dot{\eta} \quad \text{on } S_f \quad (3)-(5)$$

where  $\mathbf{z} = (z_1, z_2)$  is the location vector in inertia reference frame,  $\dot{u}_i = \dot{u}_i(\mathbf{z})$  is an  $i$ -direction component of fluid velocity vector in inertial reference frame,  $p = p(\mathbf{z})$  is the pressure of fluid,  $\eta = \eta(\mathbf{z}, t)$  is the height of free surface,  $\mathbf{n} = \mathbf{n}(\mathbf{z})$  is the outward normal vector,  $\rho$  is the fluid density,  $g$  is the acceleration of gravity,  $\Omega$  is the fluid domain, and  $S_f$  is the free surface.

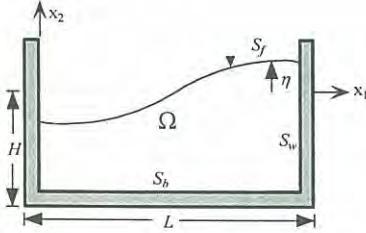


Figure 3. Domain and Boundary of Fluid

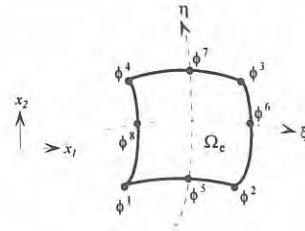


Figure 4. Nodal Velocity Potential

Velocity potential  $\phi$  in each element is represented by nodal velocity potential  $\phi_e^j$ .

$$\phi = \sum_{j=1}^8 N^j \phi_e^j \quad (6)$$

where  $N^j$  is the shape function of 8-node isoparametric element. When Galerkin method is applied to Eq. 2 with vector identity and Eq. 6, weighted residuals of each shape function are

$$\begin{aligned} R_i &= \int_{\Omega_e} N^i \nabla^2 \phi dV = \int_{\Omega_e} (-\nabla N^i \cdot \nabla \phi) dV + \int_{\Omega_e} \nabla (N^i \nabla \phi) dV \\ &= -\sum_{j=1}^8 \phi_e^j \int_{\Omega_e} (N_{i,k}^j N_{j,k}^i) dV + \sum_{j=1}^4 \int_{S_e^j} N^i q dS = 0 \end{aligned} \quad (7)$$

where  $q = \nabla \phi \cdot \mathbf{n} = \phi_{,n}$ ,  $\Omega_e$  is the element domain, and  $S_e^j$  is the element boundary.

Normal derivative of potential,  $q$  is also discretized using the same shape function, and then Eq. 7 is transformed into the following matrix equation at each element.

$$\mathbf{A}_e \phi_e = \mathbf{H}_e \mathbf{q}_e \quad (8)$$

where  $\phi_e$  is the nodal velocity potential vector of element,  $\mathbf{q}_e$  is the normal derivative of potential at the nodes, and  $\mathbf{A}_e, \mathbf{H}_e$  are coefficient matrices about potential and flux. We can obtain the governing equation of the entire fluid domain by assembling Eq.8 using direct

stiffness method. The equation describing the relation between nodal potential vector and nodal velocity vector is derived by static condensation. If we differentiate the equation by time and apply the conditions of Eqs. 1, 4, and 5, equation about the relation between nodal pressure vector and nodal acceleration vector comes out. Using virtual work principle, the equation can be transformed into the following equation (Koh et al, 1994).

$$\mathbf{M}' \ddot{\mathbf{u}} = -\mathbf{f}' \quad (9)$$

where  $\mathbf{f}'$ ,  $\ddot{\mathbf{u}}$ ,  $\mathbf{M}'$  are respectively the nodal force vector acting on one side of the wall, the nodal acceleration vector, and the added mass matrix. We can consider the interaction between fluid and structure by adding this mass matrix to the corresponding DOFs of structure.

#### 4. ISOLATED POOL STRUCTURE MODELING

Isolated pool structure can be simplified as Fig. 5, and its equation may be condensed into Eq. 10 by neglecting the slight effect of rotational inertia.

$$\mathbf{M}'' \ddot{\mathbf{x}} + \mathbf{C}'' \dot{\mathbf{x}} + \mathbf{K}'' \mathbf{x} = -\mathbf{M}'' \mathbf{1} (\ddot{x}_h + \ddot{x}_g) + \mathbf{f}'' \quad (10)$$

where  $\mathbf{M}''$ ,  $\mathbf{C}''$ ,  $\mathbf{K}''$  are respectively the condensed mass, damping, stiffness matrix of wall, and  $\mathbf{f}''$  is the external force vector acting on the nodes.

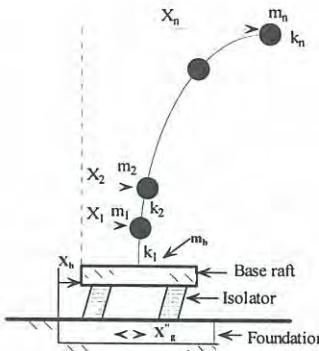


Figure 5. Lumped Parameter Model of Structure

If DOFs of nodes at the base raft are separated, added mass matrix, nodal acceleration vector, and nodal force vector in Eq. 9 can be expressed as follows.

$$\mathbf{M}' = \begin{bmatrix} m_{11} & \cdots & m_{1n} & m_{1b} \\ \vdots & \ddots & \vdots & \vdots \\ m_{n1} & \cdots & m_{nn} & m_{nb} \\ m_{b1} & \cdots & m_{nb} & m_{bb} \end{bmatrix} = \begin{bmatrix} \mathbf{M}'_{ww} & \mathbf{m}'_{wh} \\ \mathbf{m}'_{bw} & \mathbf{M}'_{hh} \end{bmatrix} \quad (11)$$

$$\mathbf{f}' = \begin{pmatrix} \mathbf{f}_w' \\ f_b' \end{pmatrix}, \quad \ddot{\mathbf{u}}' = \begin{pmatrix} \ddot{\mathbf{x}} + \mathbf{1}(\ddot{x}_b + \ddot{x}_g) \\ \ddot{x}_b + \ddot{x}_g \end{pmatrix} \quad (12)-(13)$$

Arranged equations of motion considering interaction between pool structure and base isolator are as follows (Koh et al., 1994).

$$\begin{cases} \mathbf{M}_s \ddot{\mathbf{x}} + \mathbf{C}_s \dot{\mathbf{x}} + \mathbf{K}_s \mathbf{x} + \mathbf{q}_s \ddot{x}_b = -\mathbf{q}_s \ddot{x}_g \\ M_b \ddot{x}_b + c_b \dot{x}_b + k_b x_b + \mathbf{q}_b \ddot{\mathbf{x}} = -M_b \ddot{x}_g \end{cases} \quad (14)$$

where  $\mathbf{M}_s = \mathbf{M}^w + \mathbf{M}'_{ww}$ ,  $\mathbf{C}_s = \mathbf{C}^w$ ,  $\mathbf{K}_s = \mathbf{K}^w$ ,  $\mathbf{q}_s = \mathbf{M}_s \mathbf{1} + \mathbf{m}'_{wh}$ ,  $M_b = m_b + M'_{bb} + \mathbf{m}'_{bw} \mathbf{1} + \mathbf{1} \mathbf{q}_s$ ,  $\mathbf{q}_b = \mathbf{1} (\mathbf{M}^w + \mathbf{M}'_{ww}) + \mathbf{m}'_{bw}$ .

## 5. FAILURE PROBABILITY ESTIMATION

Two equations in Eq. 14 can be merged into the following unique equation.

$$\mathbf{M}_e \ddot{\mathbf{x}}' + \mathbf{C}_e \dot{\mathbf{x}}' + \mathbf{K}_e \mathbf{x}' = -\mathbf{m}_g \ddot{x}_g \quad (15)$$

where  $\mathbf{M}_e = \begin{bmatrix} \mathbf{M}_s & \mathbf{q}_s \\ \mathbf{q}_b & M_b \end{bmatrix}$ ,  $\mathbf{C}_e = \begin{bmatrix} \mathbf{C}_s & \mathbf{0} \\ \mathbf{0} & c_b \end{bmatrix}$ ,  $\mathbf{K}_e = \begin{bmatrix} \mathbf{K}_s & \mathbf{0} \\ \mathbf{0} & k_b \end{bmatrix}$ ,  $\mathbf{m}_g = \begin{pmatrix} \mathbf{q}_s \\ M_b \end{pmatrix}$ ,  $\ddot{\mathbf{x}}' = \begin{pmatrix} \ddot{\mathbf{x}} \\ \ddot{x}_b \end{pmatrix}$ ,  $\dot{\mathbf{x}}' = \begin{pmatrix} \dot{\mathbf{x}} \\ \dot{x}_b \end{pmatrix}$ , and  $\mathbf{x}' = \begin{pmatrix} \mathbf{x} \\ x_b \end{pmatrix}$ .

From this equation, we can derive the transfer function vector.

$$\mathbf{h}(\omega) = (\omega^2 \mathbf{M}_e - i\omega \mathbf{C}_e - \mathbf{K}_e)^{-1} \mathbf{m}_g \quad (16)$$

Spectral density function of the wall base shear is calculated by the transfer function vector, lateral stiffness vector, and input spectral density function.

$$S_V(\omega) = |\mathbf{k}_w \mathbf{h}_w(\omega)|^2 S_g(\omega) \quad (17)$$

where  $\mathbf{k}_w$  is the row vector of lateral stiffness of wall,  $\mathbf{h}_w(\omega)$  is the reduced vector from Eq. 16 only for wall lateral direction DOFs, and  $S_g(\omega)$  is the spectral density function of input ground motion.

Standard deviations of shear force and its time rate are calculated assuming that the system is narrow-banded.

$$\sigma_V^2 = \int_{-\infty}^{\infty} S_V(\omega) d\omega, \quad \sigma_{V'}^2 = \int_{-\infty}^{\infty} \omega^2 S_V(\omega) d\omega \quad (18)-(19)$$

The crossing rate is estimated on the assumption that the input ground motion is subjected to the normal distribution (Newland, 1993).

$$\nu_a = 2\nu_a^+ = \frac{1}{\pi} \frac{\sigma_\nu}{\sigma_\nu} \exp(-a^2/2\sigma_\nu^2) \quad (20)$$

where  $a$  is the limit state level of wall base shear force.

Supposing the number of crossing events is subjected to Poisson distribution, the failure probability during life-cycle is

$$P_f = P_{f/eq} \cdot P_{eq} = P_{eq} \left\{ 1 - \exp(-\nu_a \cdot T_d^s) \right\} \quad (21)$$

where  $P_{f/eq}$  is the conditional probability of failure given the earthquake occurrence,  $P_{eq}$  is the probability of earthquake occurrence, and  $T_d^s$  is the duration time of strong excitation.

## 6. OPTIMAL SEISMIC DESIGN

The total cost function of base-isolated pool structure is defined for optimal seismic design. Wall thickness  $t_w$  and isolator stiffness  $k_h$  are chosen as design variables for optimization (Song, 1999).

$$C_t(t_w, k_h) = C_i + E[C_d] = C_{uv} \cdot \{2t_w w_w h_w\} + C_d^s \sum_k r_k P_{f_k}(t_w, k_h) \quad (22)$$

where  $C_t$  is the total life-cycle cost,  $C_i$  is the initial construction cost,  $E[C_d]$  is the expected value of damage cost during life-cycle,  $C_{uv}$  is the initial cost per unit volume of wall,  $C_d^s$  is the assumed damage scale,  $r_k$  is the weight of  $k^{\text{th}}$  limit state,  $P_{f_k}$  is the failure probability of  $k^{\text{th}}$  limit state,  $t_w$  is the wall thickness,  $w_w$  is the width of wall, and  $h_w$  is the height of wall.

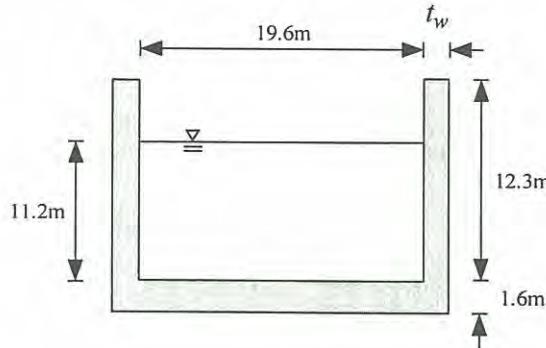


Figure 6. Fluid-Filled Pool Structure Model for Optimal Seismic Design

We applied our optimization method to fluid-filled pool structure shown in Figure 6. Its modulus of elasticity is  $20.79 \times 10^9 N/m^2$ , density is  $2300 kg/m^3$ , and Poisson ratio is 0.17. Spectral density with  $A = 0.14$  and  $S = 1.0$  is used for input ground motion model. Two LRB (Laminated Rubber Bearing) isolators are installed on one side, whose stiffness varies from  $6.28 \times 10^4$  to  $3.14 \times 10^5 N/m$ .

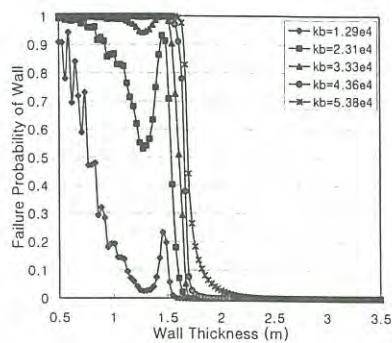


Figure 7. Failure Probability of Lateral Wall

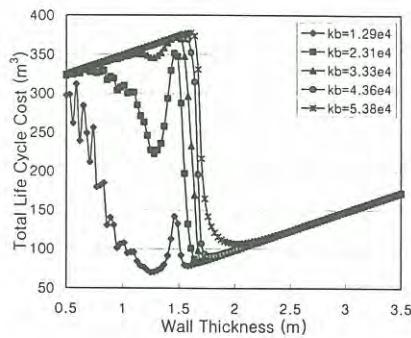


Figure 8. Total Life-Cycle Cost

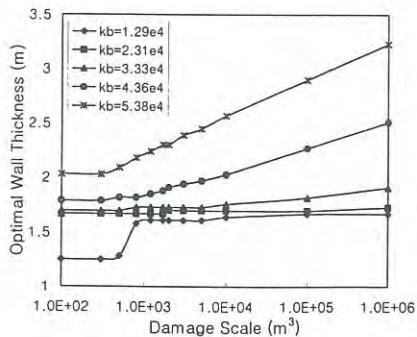


Figure 9. Sensitivity of Optimal Wall Thickness

Fig.7 shows failure probability of wall about wall thickness and isolator stiffness. Flexible isolator reduces significantly probability of failure even at very thin wall, but this effect of isolation decreases suddenly as the isolator gets stiffer. Therefore we should choose isolator stiffness properly while considering excessive displacement of isolator.

Isolator stiffness affects also the total life-cycle cost (Fig.8). Flexible isolator reduces total cost for identical wall thickness and this effect disappears when using thick wall.

Pool structure used as a container for radioactive material will lead to huge social cost if failure occurs. Therefore, the sensitivity of the resulting optimized design to any assumed damage scale should be verified (Fig. 9). Values obtained for pool structure with flexible isolator are nearly insensitive to variations of damage scale.

## 7. CONCLUSIONS

We developed a seismic design optimization method for fluid-filled and base-isolated pool structures based on minimum life-cycle cost concept. Fluid-structure interaction, base isolation effect, site condition of input ground motion are taken into account.

Results show that flexible isolator makes it possible to reduce wall thickness and total life-cycle cost significantly. The more flexible base isolator is, the smaller failure probability and total life-cycle cost are. Moreover, the case of optimal seismic design with flexible isolator is considerably insensitive to the assumed damage scale. This constitutes a very appreciable property because pool structures will be utilized to contain radioactive material and their failure will lead to immeasurable damage.

This method relies on linearity of system and stationarity of input ground motion. Therefore it presents some limit in considering nonlinear behavior of isolator and fluid and their effects.

If we compare minimum life-cycle cost of base-isolated pool structure with that of non-isolated and utilize the developed spectral density models, the economical efficiency evaluation of base isolation according to various conditions can be performed hereafter.

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