

# Optimization of mechanical properties of structures from the point of aseismic design

N. Model

*Central Institute of Cybernetics and Informations Processes, Berlin, GDR*

P. Dineva & L. Hadjиков

*Institute of Mechanics and Biomechanics, Sofia, Bulgaria*

## I. INTRODUCTION

The vibroinsulation problem is solved by passive and active dynamic systems control methods. The attacking of the problem by design of vibroisolators that passively extinguish the harmful vibration has begun historically earlier (Piculev 1975, Petersen 1979). Quite in the latest years some attention has been drawn on the possibility about dynamical system's active control. But the question about the technical realization at the optimal and the modal regulators making the behaviour safe during earthquake remains open until now.

The optimal control theory application to the aims of the dynamical systems stability motion (systems being described as rigid, nondeformable solids) by a passive vibroinsulation is done (Model 1979). Thus the optimal dynamic system properties are identified without one having to solve the problem about a technical realization of active control equipments.

An aim of this paper is an application of an optimal control theory to obtaining optimal elastic and dissipative characteristics of the buildings at their aseismic design.

## 2. MECHANO-MATHEMATICAL DESCRIPTION OF THE STRUCTURE MOTION UNDER SEISMIC LOAD

The motion equations of a structure under seismic load have the form:

$$(I) \quad M \ddot{X} + C \dot{X} + K X = -M b \ddot{x}_0 \\ X(0) = \dot{X}(0) = 0$$

where:  $M, C, K$  is the mass, damping and stiffness matrix,  $X, \dot{X}, \ddot{X}$  are vectors of displacement, velocity and acceleration,  $b$  is a cosine-director's vector of external action,  $\ddot{x}_0$  is the external seismic action's acceleration,  $n$  is the number of equations as well as the number of the system's degrees of freedom.

### 3. THEORY OF OPTIMAL CONTROL APPLICATION TO OPTIMIZATION OF THE MECHANICAL PROPERTIES OF BUILDING STRUCTURES AT THEIR ASEISMIC DESIGN

#### 3.1 Formulation of the active control problem

The structure dynamics description in the case of a free motion and controlling active forces present in the state space has the form:

$$(2) \quad \dot{y}(t) = Ay(t) + Bu(t)$$

there:

$y(t) = \begin{Bmatrix} X \\ \dot{X} \end{Bmatrix}$  is a state vector with a dimension  $2n$

$A = \begin{bmatrix} [0] & [I] \\ -M^{-1}.K & -M^{-1}.C \end{bmatrix}$  is a matrix of mechanical

properties with a dimension  $2n \times 2n$ ,  $[0]$  and  $[I]$  are a zero and a unit matrix correspondingly with a dimension  $n \times n$ ,  $B$  is a vector showing the places of action of the controlling forces,  $u(t)$  is a vector of control having a dimension  $m$ , where  $m$  is the number of control inputs. All components of the  $X$  vector are assumed directly measurable.

The task for solving is set to find such a control  $u(t)$  for the system (2) that the quadratic functional of quality:

$$(3) \quad J(u) = \frac{1}{2} \int_0^\infty ((y(t) - w(t))^T \cdot Q \cdot (y(t) - w(t)) + u^T \cdot R \cdot u) dt$$

to be minimized with respect to all smooth and continuous  $u(t)$ . There  $R$  is a positively defined weight matrix with a dimension  $m \times m$ ,  $Q$  is a nonnegative weight matrix with a dimension  $2n \times 2n$ ,  $w(t)$  is a function of assign.

The problem of vibroisolation (active and passive) coincides with the problem about synthesis of an optimal regulator  $u(t)$  with  $w(t) = 0$ .

The optimal control has the form:

$$(4) \quad u(t) = -R^{-1} \cdot B^T \cdot P \cdot y(t)$$

where  $P$  is determined by the matrix algebraic equation of Riccati:

$$(5) \quad A^T \cdot P + P \cdot A - P \cdot B \cdot R^{-1} \cdot B^T \cdot P + Q = 0$$

Following this formulation the authors (Hadjikov, Dineva, Ivanova 1986) obtain numerical results for examples of real buildings showing effective decreasing of the dangerous peaks in their seismic motion. As we have already mentioned in part I., the difficulty of the active control problem consists in its technical realization.

### 3.2 Optimization of the mechanical properties of structures from the point of view of aseismic design

We are going to consider the control forces from (4)  $u(t)$  as hypothetical control forces whose representation in the form of a feed-back leads us to the task about an identification of the dynamic optimal system. After replacing (4) in (2) we obtain a motion equation for the system have already been optimized:

$$(6) \dot{y}(t) = (A - BR^{-1} \cdot B^T \cdot P)y(t) = \tilde{A}_{opt} y(t) = \begin{bmatrix} [0] & [I] \\ -\tilde{M}^{-1} \tilde{K}_{opt} & -\tilde{M}^{-1} \tilde{C}_{opt} \end{bmatrix} y(t)$$

Eq.(6) gives us the new elastic properties  $\tilde{K}_{opt}$  and the new dissipative  $\tilde{C}_{opt}$  at the dynamic system having already been optimized. With the obtained  $\tilde{K}_{opt}$  and  $\tilde{C}_{opt}$  we can predict how a given structure must change its elastic and dissipative characteristics in order to have a more stable behaviour during an earthquake. For the purpose it is necessary the reverse task to be solved, namely, out of the new optimal matrices of stiffness and damping the new optimal mechanical characteristics of the system to be obtained. Using the complex eigenvalue frequencies  $\omega_i^*$  and modes of the optimized system (6), following Sotirov, Dineva, Hadjиков (1984) we can obtain an optimal complex stiffness matrix  $\tilde{K}_{opt}^*$  by the formula:

$$(7) \tilde{K}_{opt}^* = \text{Diag}(\omega_i^*) \cdot M$$

Out of  $\tilde{K}_{opt}^*$  it is easy to find the optimal complex stiffnesses of the separate finite elements having the form:

$$(8) k_e^* = k_e + i \zeta_e k_e, \quad k_e - \text{real stiffness of the "e" finite element, } \zeta_e \text{ is the damping coefficient of the same element.}$$

From (7) and (8) the optimal elastic moduluses of the structure material as well as the optimal damping coefficient for the separate element are to be simultaneously determined.

Some interest offer the case when in the building's motion under a seismic action, the soil mechanic properties influence is being into account. Then in the matrices  $K$  and  $C$  from eq.(1) the elastic and damping soil characteristics will participate. After implementing the presented methods of optimization we are going to obtain the optimal complex stiffnesses of the soil.

A numerical example has been solved about a mechanical parameter optimization of the soil-structure system for an illustrative nuclear power plant shown on Fig.1. It is shown on Fig.2 the displacement of mass N7 from the dynamic scheme before and after optimization of the system. In table I the mechanical parameters of the soil under the structure are shown before and after the optimal vibroprotection.

### 3.3 Theory of modal control application to mechanical characteristic optimization of structures

This part deals with the modal control theory for obtaining of the new mechanical parameters of a dynamic system having a new, beforehand assigned eigenvaluespectrum. This is especially important to avoid the dangerous resonance cases.

If the system  $X(t)=AX(t)+Bu(t)$  is controllable and  $\varphi(\lambda)=\lambda^N+f_1\lambda^{N-1}+\dots+f_N$  is a given arbitrary polynome of order  $N$  and with real coefficients, then there exists a feedback vector  $u(t)=-KX(t)$  such, that  $\varphi(\lambda)$  stands for a characteristic polynome of the closed system  $y(t)=(A-BK)y(t)$ . The control vector  $u(t)$  can change both a part of the eigen oscillation and the whole eigen spectrum of the dynamic system. The system obtained has a new optimal matrix  $\hat{A}_{opt}$  and new elastic  $\hat{K}_{opt}$  and damping  $\hat{C}_{opt}$  properties, as well as a new eigenvaluespectrum identical to the one beforehand given.

### 4. CONCLUSION

A new way is to apply the active optimal and modal control theory to optimization of the elastic and dissipative characteristics of structures at their aseismic design. For this purpose is created a fortran code that allows computing the optimal parameter of arbitrary building or defining the optimal vibroinsolation characteristics of additional devices. The code can be employed also for identifying the optimal complex stiffnesses of the soil in the case of soil-structure interaction problem.

The task has been solved in the case of a free motion at the building. A future work is going to be dedicated to a passive control of a building subjected to a dynamic stochastic action where in the structure's optimized mechanical properties, the stochastic model parameters of the concrete geological column will participate too.

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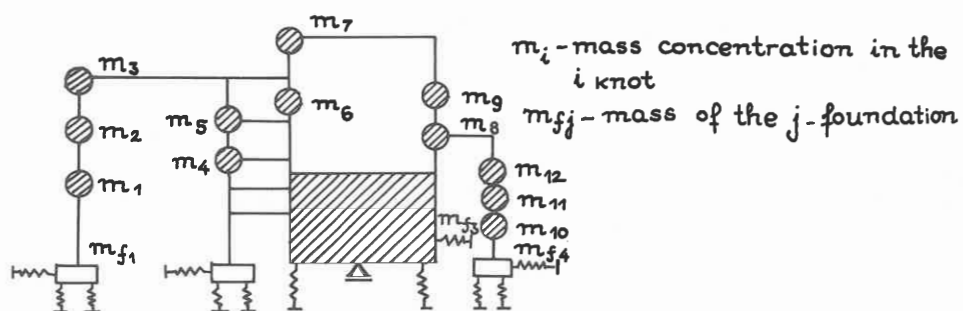


Fig. 1

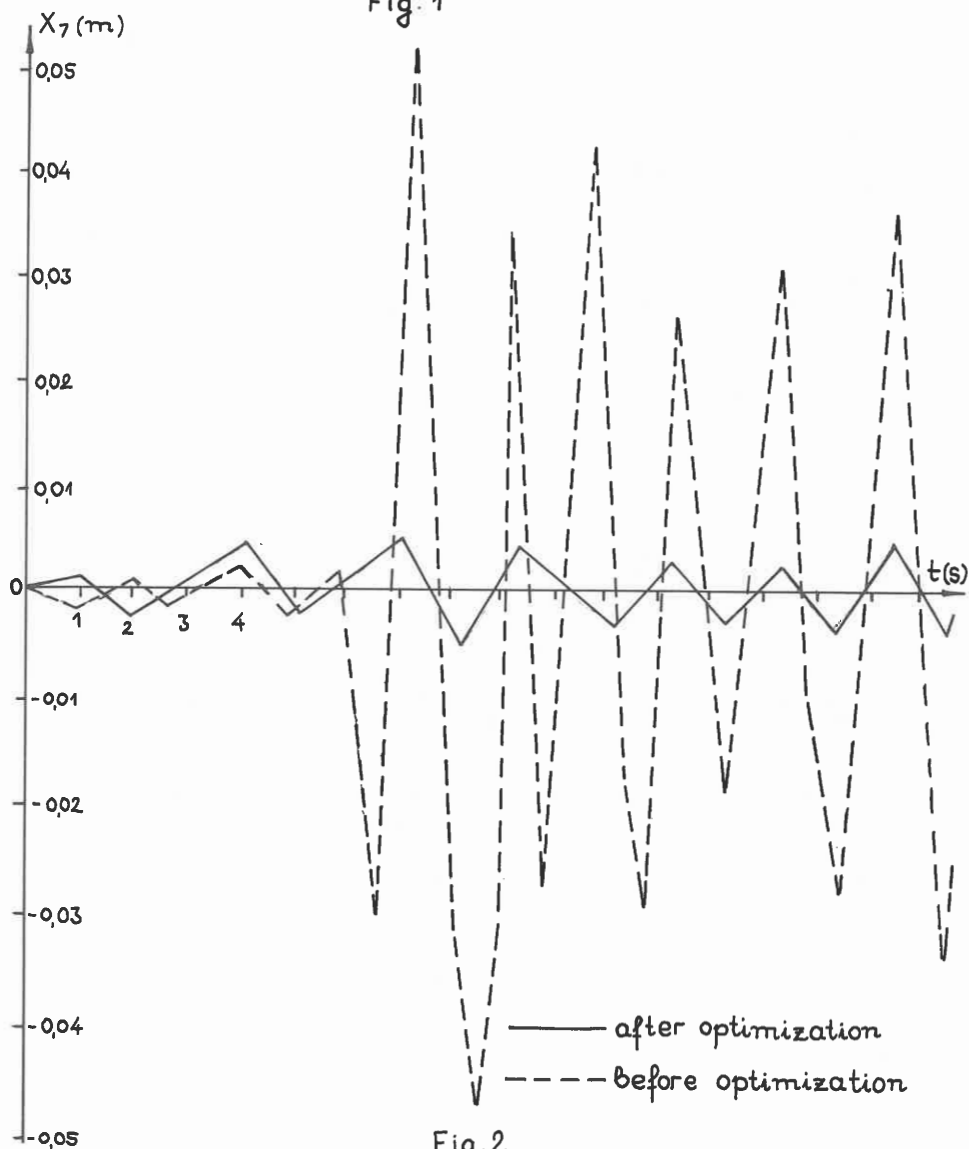


Fig. 2

Table I. The mechanical properties of soil before and after optimization

Soil properties	$C_{f_1} = C_{f_2} = C_{f_4} [t/m^3]$ $1t = 10^3 kgf$	$C_{\varphi_1} = C_{\varphi_2} = C_{\varphi_4} [t/m^3]$	$\alpha_{f_1} = \alpha_{f_2} = \alpha_{f_3} = \alpha_{f_4}$	$\alpha_{\varphi_1} = \alpha_{\varphi_2} = \alpha_{\varphi_3} = \alpha_{\varphi_4}$	$C_f^3 [t/m^3]$	$C_\varphi^3 [t/m^3]$
Before optimization	572	1195	0,3	0,2	777	1204
After optimization	5726	12325	0,3	0,2	8123	12039

$$\text{here: } \left| \begin{array}{l} k_{fj}^* = k_{fj} + \alpha_{fj} k_{fj} ; \\ k_{fj} = C_f \cdot F_j \end{array} \right| \left| \begin{array}{l} k_{\varphi j}^* = k_{\varphi j} + \alpha_{\varphi j} k_{\varphi j} ; \\ k_{\varphi j} = C_\varphi \cdot J_{\varphi j} \end{array} \right|$$

$j$  - number of foundation

$j = 1, 2, 3, 4$  (see fig.1)

$k_{fj}$  - stiffness for horizontal displacement of the  $j$  foundation with cross section  $F_j$

$k_{\varphi j}$  - stiffness for the rotation of the  $j$  foundation with geometric inertial moment  $J_j$

$C_f, C_\varphi$  - soil characteristics of an ideal elastic foundation that undergoes displacement and rotation respectively

$\alpha_{fj}, \alpha_{\varphi j}$  - damping hysteresis coefficients