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STRUCTURAL SEISMIC ISOLATION BY ANTIRESONANCE LOCY

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ABSTRACT

The antiresonance and resonance phenomena in structures are investigated by the method of dynamic condensation and by means of the frequency approach. Antiresonance and resonance frequencies are obtained from the structure transfer function. The main structural parameters alteration leads to change of these frequencies. They describe paths in the parameter space. These loci are used for seismic isolation by an optimal choice of the structural parameters. An illustrative example is given.

1. INTRODUCTION.

The structural dynamical behaviour can be investigated in the frequency domain through the well known relation between Fourier spectra of the input excitation $X(j\omega)$, the output reaction $Y(j\omega)$ and the system frequency responce function $W(j\omega)$:

(1) $$Y(j\omega) = W(j\omega)X(j\omega).$$

From the amplification function:

(2) $$A(j\omega) = |W(j\omega)|,$$

the resonance frequenceis ω_{rj} can be obtained (fig.1^a.)

There are frequencies in which oppositte phenomena can be observed - considerable vibration damping. Some times such antiresonance frequencies ω_{ai} can be seen in the amplification function $A(\omega)$ [Savidis,1992,fig.6.]. But it is easier to detect them from the logarithmic amplificatin function - fig.1 :

(3) $$L(\omega) = 20\lg A(\omega).$$

The system property to damp fluctuation in determined frequency range can be used for structure protection from harmonic, seismic or wind loadings. Such an idea is proposed in this paper.

2. ANTIRESONANCE AND RESONANCE FREQUENCIES DETERMINATION.

By means of the finite element method the structures are described by the following equation:

(4) $$M\ddot{Y} + C\dot{Y} + KY = -MR\ddot{X}_g,$$

where M,C,K are mass, damping and stiffnes matrices,R is distributive vector, Y is system displacement vector and $\ddot{X}_g$ is ground acceleration in the place.

After Laplace transformation, equation (4) becomes:

(5) $$[Ms^2 + Cs + K]\,Y(s) = -MR\ddot{X}_g(s),$$

The relation between Laplace transform $Y(s)$ and $\ddot{X}_g(s)$ is:

(6) $$Y(s) = -W(s)\ddot{X}_g,$$

where $W(s)$ is a matrix transfer function:

(7) $$W(s) = \left[Ms^2 + Cs + K\right]^{-1} MR.$$

Usually only a few degrees of freedom are important for a dynamical analysis. In this case it is convenient to obtain the matrix transfer function through a dynamical con densation from equation (5) [Ishtev,1985]. For an arbitrary degree of freedom through single dynamical condensation, $W(s)$ is fraction rational function:

(8) $$W(s) = \frac{b_0 s^{2n-2} + b_1 s^{2n-3} + \cdots + b_{2n-2}}{a_0 s^{2n} + a_1 s^{2n-1} + \cdots + a_{2n}},$$

where n is the initial system degree of freedom number.

For antiresonance and resonance frequencies obtaining, the factorization form of $W(s)$ it is more conveinient:

(9) $$W(s) = \frac{b_0}{a_0} \frac{\prod_{i=1}^{n-1} (s^2 + g_i s + d_i)}{\prod_{l=1}^{n} (s^2 + e_l s + f_l)}.$$

The frequency response function can be obtained by substituting s with $j\omega$. According to (2) and (9) the amplification function is:

(10) $$A(\omega) = \frac{b_0}{a_0} \frac{\prod_{i=1}^{n-1} A_i(\omega)}{\prod_{l=1}^{n} A_l(\omega)}$$

The logarithmic function according to (3) and (10) assumes the form of the elementary term sums:

(11) $$L(\omega) = \sum_{i=1}^{n-1} L_i(\omega) - \sum_{l=1}^{n-1} L_l(\omega) + \lg \frac{b_0}{a_0}.$$

The antiresonance frequencies are determined from each of the first sum terms. Indeed, the terms of the logarithmic function:

(12) $$L_i(j\omega) = 20 \lg \sqrt{(d_i - \omega^2)^2 + \omega^2 g_i^2},$$

when $g_i < 2d_i$ have minimum in frquency.

(13) $$\omega_{ai} = \sqrt{d_i - \frac{g_i^2}{2}}$$

If $g_i = 0$, L_i in (12) and respectively L in (11) gets $-\infty$ values which corespond to absolute antiresonance (zero amplification) in the respective frequency ω_{ai}. This means that the described degree of freedom is immovable. In the building structure attenuation $g_i \neq 0$ but normaly $g_i^2 \ll d_i$. In this case the antiresonance is not absolute (the amplification is different from zero) but it exists in frequencies determined by equation (13).

Resonance frequencies are determinated by analogy from the second sum terms in equation (11). These terms, render an account of the minus sign before the sum, obtain a very big (if $e_l = 0$ - infinite) value in the well known resonance frequencies:

(14) $$\omega_{rl} = \sqrt{f_l - \frac{e_l^2}{2}}$$

3. ANTIRESONANCE AND RESONANCE LOCI.

The structures frequency response function depends on its parameters (dimensions, reinforcement and so on). This parameter alteration leads to respective modification in M, C, K matrices. In the final reckoning ω_{ai} and ω_{rl} are alterating - describing loci in the parameter space. Convenient graphics visualization can be received when there is only one essential structure parameter (p) alteration - (fig.2.).

The loci can be used for solving the inverse problem - from desirable antiresonances frequencies value, to obtain essential structure parameters. This problem solution will be important for example in the case of passive seismic isolation of the very important structure element such as a nuclear reactor and so on, when the predominant seismic signal frequency ω_s in the place is known. For this frequency (fig.2.), the searched for parameter values p_{a1} or p_{a2} which ensure antiresonance can be obtained. On the opposite,the parameters p_{r1} or p_{r2} are hardly undesirable bacause of the resonance pheno me na arise risk.

In the case of more than one structure parameters choosing the graphical interpretation is difficult and the solution of this problem can be searched for through the medium of expressions (13) and (14).

4. ILLUSTRATIVE EXAMPLE.

The structure under consideration and designing is shown in fig.3.

In the case of damping neglecting ($C = 0$) equation (4) becomes:

$$mM\ddot{Y} + kKY = -mMR\ddot{X}_g. \tag{15}$$

where $k = \frac{24EJ}{H^3}$ and m are structural parameters for chosing in the design process. The transfer function W_4 for an important degree of freedom Y_4 received by the dynamical condensation is:

$$\frac{s^6 + 6\frac{k}{m}s^4 + 10\left(\frac{k}{m}\right)^2 s^2 + 4\left(\frac{k}{m}\right)^3}{s^8 + 7\frac{k}{m}s^6 + 15\left(\frac{k}{m}\right)^2 s^2 + 10\left(\frac{k}{m}\right)^3 s^2 + \left(\frac{k}{m}\right)^4} \tag{16}$$

The transfer function (16) presented in the form (9) is:

$$\frac{\left(s^2 + 0,5858\frac{k}{m}\right)\left(s^2 + 2,0\frac{k}{m}\right)\left(s^2 + 3,4142\frac{k}{m}\right)}{\left(s^2 + 0,1206\frac{k}{m}\right)\left(s^2 + \frac{k}{m}\right)\left(s^2 + 2,3473\frac{k}{m}\right)\left(s^2 + 3,532\frac{k}{m}\right)}. \tag{17}$$

The antiresonance and resonanse frequences are:

$$\begin{aligned} \omega_{a1} &= 0,7654\sqrt{\frac{k}{m}} & \omega_{r1} &= 0,3473\sqrt{\frac{k}{m}} \\ \omega_{a2} &= 1,4142\sqrt{\frac{k}{m}} & \omega_{r2} &= 1,0000\sqrt{\frac{k}{m}} \\ \omega_{a3} &= 1,8478\sqrt{\frac{k}{m}} & \omega_{r3} &= 1,5321\sqrt{\frac{k}{m}} \\ & & \omega_{r4} &= 1,8794\sqrt{\frac{k}{m}} \end{aligned} \tag{18}$$

In fact there is only one generalized parameter $p = \frac{k}{m}$ - for loci analysis. The antiresonance and resonance loci are shown in fig.4.

The soil amplification function of the geological column from fig.3. is shown in fig.5. If the bed rock signal $\ddot{X}_b$ is white noise, from the relation between the spectral densities in the bed rock $S_{\ddot{x}_b}(\omega)$ and surface signal $S_{\ddot{x}_g}(\omega)$:

$$S_{\ddot{x}_g}(\omega) = \left|W_{soil}(j\omega)\right|^2 S_{\ddot{x}_b}(\omega) \tag{19}$$

it follows that the predominant seismic signal density is concentrated around $15[s^{-1}](\approx 2.39Hz)$. For ensuaring the antiresonance frequency $\omega_a = \omega_s$ the generalized parameter $p = \frac{k}{m}$ can be chosen: $p_1 = 384s^{-1}$ (from locus ω_{a1}) or $p_2 = 112s^{-1}$ (from locus ω_{a2}).

or $p_3 = 66s^{-1}$ (from locus ω_{a3}). In fig.6. are shown the structures logarithmic amplification functions for the parameter $p_1 = 384s^{-1}$ and the spectral density of the seismic signal $\ddot{X}_g$.

5.CONCLUSION.

The design of structures and equipment is usually realized in accordance with standards based on the modal analysis. By this method the attenuation is focused on resonance phenomena while the antiresonance phenomena are hardly included. At the antiresonances a significant decreasing of the movement is observed. This can be used for structure stability improvement against seismic effects.

This report is devoted to some studies connected with the antiresonance phenomena and application in design. For investigation of these phenomena the transfer function is used. The numerator of the transfer function determines the frequencies in which a decrease of the system output signal is observed (antiresonance frequencies). The denominator defines natural resonance frequencies. Antiresonance loci can be used in the design process. The essential structure parameters may be determined in a way to ensure minimum dynamical loading in the responsible elements.

Although the enclosed example is illustrative, it shows the potentialities of the considered method for structural design. It is connected whith one typical computational model, which is applied in the majority of the codes. The parameter $\frac{k}{m}$ chosen in the example is of a physical meaning and can by modified in large ranges by the designer. For example inreinforced concrete structures the modification of the $\frac{k}{m}$ can be carried out by chosing of the armature.

The application of the proposed method is connected with a large volume of calculations, which are possible if the respectiv software is available. The elements of such a sortware are in the elaboration and they will be added to the part of CAD system, described in [Ishtev,1993].

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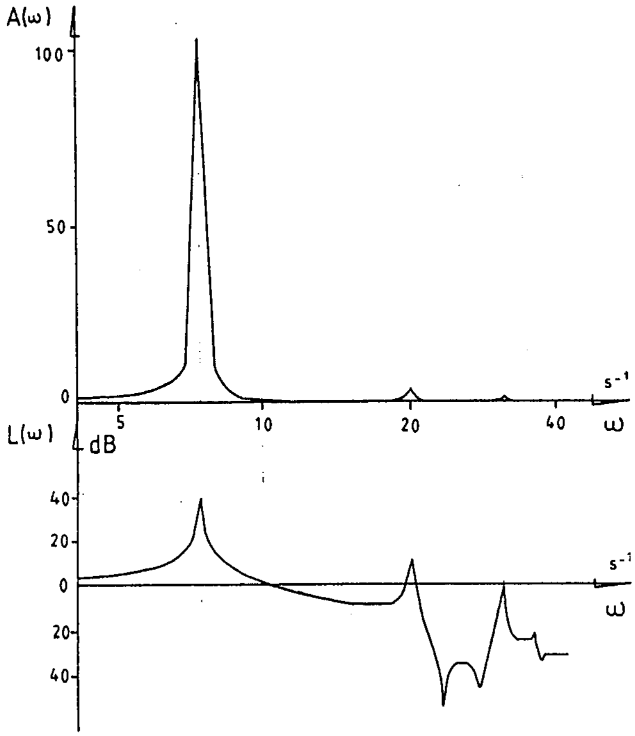
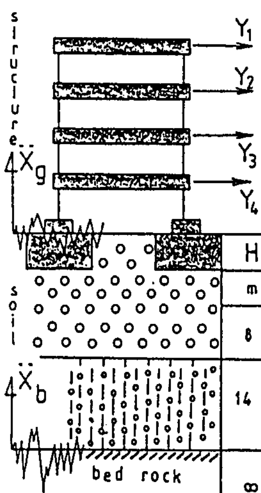


FIG.1.



$$R=\begin{bmatrix}1\\1\\1\\1\end{bmatrix}\qquad M=m\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}\qquad K=k\begin{bmatrix}1&-1&0&0\\-1&2&-1&0\\0&-1&2&-1\\0&0&-1&2\end{bmatrix}$$

H	ρ	G	G_r	T_m, T_r
m	kNm/s²	kN/m²	kN/m²	s, s
8	20	$3{,}6\cdot10^{6}$	$1{,}1\cdot10^{6}$	100
14	20	$2{,}3\cdot10^{8}$	$3{,}5\cdot10^{4}$	10^{-6}
∞	20	$1{,}7\cdot10^{7}$	$2{,}1\cdot10^{8}$	

FIG.3.

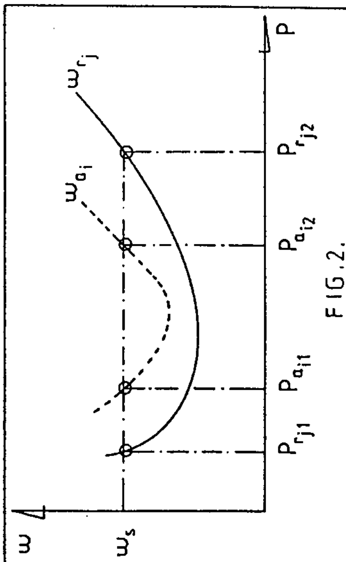


FIG.2.

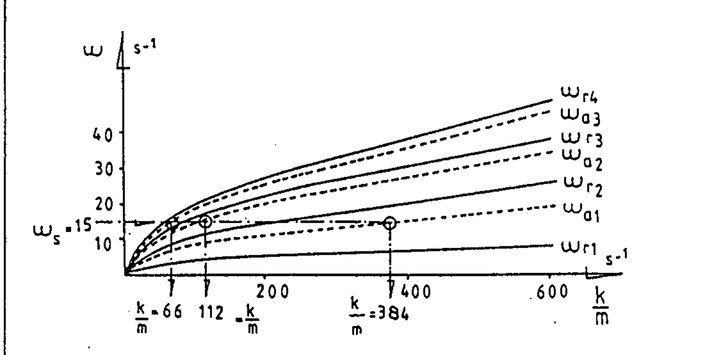


FIG.4.

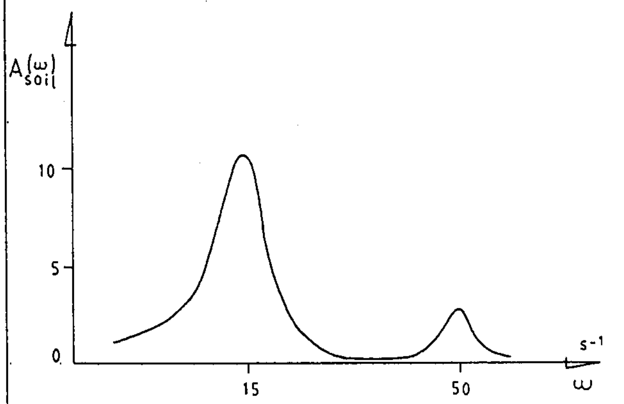


FIG.5.

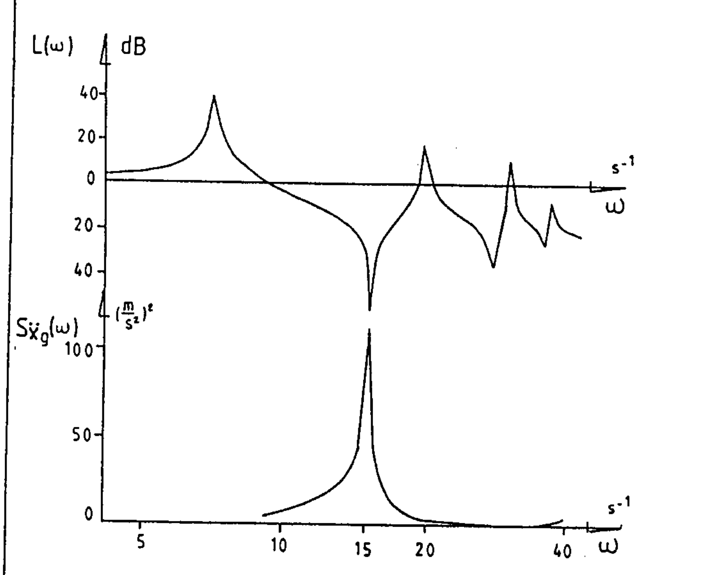


FIG.6.