



## Pseudo-dynamic test and nonlinear numerical simulation of isolated structure

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### Abstract

A computer code based on the finite element method has been developed for analyzing the response of isolated structure subjected to the dynamically excited loading induced by the operation of machines or the earthquake. A finite element program has the four different elements; beam, plate/shell, boundary spring and spring elements. The spring element represents the seismic or vibration isolator that is used to lower the fundamental frequency of structure, and thus it would take a large portion of nonlinear deformation of structures. As result, the spring element only is considered to be nonlinear in the program. In order to verify the performance of the developed code, the pseudo-dynamic tests of an isolated steel structure under two different earthquakes have been carried out. The layered rubber springs are used as the isolation devices, of which mechanical behavior is assumed to be bi-linear, and the mechanical properties are obtained from the experiments. Non-linear responses of an isolated steel structure subjected to two different earthquakes are simulated and compared with the results of pseudo-dynamic tests. Simulations of structural responses show good agreement with the results of the pseudo-dynamic tests.

### 1. Introduction

If a program is developed for analyzing the structural responses caused by seismic, shock, and vibration loading, it may be necessary that the program be evaluated by comparing the numerical results with those of experimental tests. In the study a program is coded on the base of the finite element method to estimate the nonlinear dynamic responses of isolated structures, and its validity is evaluated by the pseudo-dynamic tests of isolators, LRB(Layered Rubber Bearing), and a proper test model of isolated structure.

The finite element program includes three types of element which are the most common elements in analysis of structures; beam, plate/shell and spring elements. The spring element represents the isolator and it only is considered to be nonlinear. The reason is that its stiffness is the smaller than that of other element. The isolator shifts down the fundamental frequency to less than the frequency spectrum of the dynamic loading[1]. The stiffness matrix of spring element in the study is formulated by using the bi-linear load-deflection relation and the kinetic hardening rule, which are reflected from the experimental hysteretic loops.

The pseudo-dynamic test are carried out after investigating the mechanical properties of LRB under the various conditions such as the maximum shear strain, the vertical loads and the loading rates. Three sizes of LRBs are fabricated and tested to prove the similitude rule

and to observe the static properties. The equivalent stiffness and the effective damping ratios can be obtained from the hysteretic load-displacement loops, and they are used as the input of the finite element program to directly construct the stiffness matrix of isolator.

A five-story two-bay steel building is chosen to evaluate the home made program NAIS (Nonlinear Analysis for Isolated Structure) through the pseudo-dynamic test[2,3]. The pseudo-dynamic test is more economical method, utilize the conventional laboratory test equipment and permit the detailed observation of specimens during test. It is well known that a pseudo-dynamic test model is combination of the actual structural model to be tested and the numerical model which does not exist. LRBs of the steel structure in tests are the actual structural elements, while the superstructure above isolators is replaced by the numerical models. The seismic responses caused by El Centro and Taft earthquakes are measured by the pseudo-dynamic test and they are simulated by NAIS, and two results are compared.

## 2. Non-linear Finite Element Analysis

Since the stiffness of isolator could be so small compared with that of any structural member, the natural frequency of isolated structure is far less than that of the un-isolated structure. If the earthquake excitation or the machine load are enough for nonlinear behavior of isolated structure, it may be due to isolator as expected.

As a result, a nonlinear finite element program has been coded not only for the estimation of the structural behavior but also for the design of isolated foundation of structure and machine. In the section, one describes the formulation of stiffness matrix of the spring properties element in the finite element program.

The potential energy of structure can be expressed as

$$\Pi = \int_V \sigma \varepsilon dV - \int_V \mathbf{u}(\mathbf{f}_b - \rho \ddot{\mathbf{u}} - c\dot{\mathbf{u}}) dV - \int_V \mathbf{u} \mathbf{q} dV$$

in which  $\mathbf{f}_b$  is the body force,  $\rho$  is the density,  $c$  the damping coefficient, and  $(\bullet)$  indicates the derivatives with respect to time. Thus  $\dot{\mathbf{u}}$  and  $\ddot{\mathbf{u}}$  represent the acceleration and the velocity, respectively. By applying the principle of virtual work, the equation of motion then can be written as

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{F}$$

where  $\mathbf{M}$ ,  $\mathbf{C}$ ,  $\mathbf{K}$  are the matrices of mass, damping and stiffness, respectively. The matrices in the finite element analysis can be expressed in a form

$$\mathbf{M} = \int_V \mathbf{N}^T \rho \mathbf{N} dV, \quad \mathbf{C} = \int_V \mathbf{N}^T c \mathbf{N} dV, \quad \mathbf{K} = \int_V \mathbf{B}^T \mathbf{D} \mathbf{B} dV$$

in which  $\mathbf{N}$  is the interpolation matrix,  $\mathbf{B}$  is the strain displacement matrix, and  $\mathbf{D}$  is the stress-strain relation or moduli matrix dependent upon the type of element.

A nodal point in the spring element has 6 degree of freedom same as beam element; Three( $u, v, w$ ) are from the translation, and the three ( $\theta_x, \theta_y, \theta_z$ ) are from the rotation. In the study, a spring element is composed of 2 nodes, and thus the rank of stiffness or mass matrix is 12.

The nodal displacement vectors can be expressed as

$$\mathbf{u}^T = [u \ v \ w \ \theta_x \ \theta_y \ \theta_z]$$

and the corresponding nodal force vector can be written as

$$\mathbf{f}^T = [f_x \ f_y \ f_z \ m_x \ m_y \ m_z]$$

where  $m_i$  denotes the moment about the  $i$ -axis in the local coordinate. If the nodal variables

are assumed to be uncoupled, the force vector can be determined by the difference of displacements at two nodes in element, and the relationship between the nodal force vector and the nodal displacement vector can be established as

$$f_k = k_k(u_{kj} - u_{ki})$$

where the subscript  $k$  denotes the  $k^{th}$  degree of freedom in element. The local coordinate system of spring element is defined as the similar manner for beam element. The stiffness matrix of spring element can be expressed

$$\mathbf{K} = \begin{bmatrix} \mathbf{K}_s & -\mathbf{K}_s \\ -\mathbf{K}_s & \mathbf{K}_s \end{bmatrix}, \quad \mathbf{K}_s = \begin{pmatrix} k_x & 0 & 0 & 0 & 0 & 0 \\ 0 & k_y & 0 & 0 & 0 & 0 \\ 0 & 0 & k_z & 0 & 0 & 0 \\ 0 & 0 & 0 & k_{\theta x} & 0 & 0 \\ 0 & 0 & 0 & 0 & k_{\theta y} & 0 \\ 0 & 0 & 0 & 0 & 0 & k_{\theta z} \end{pmatrix}$$

As mentioned before, the rank of stiffness matrix of spring element is 12, the stiffness in  $\mathbf{K}_s$  are directly formulated by the input stiffness.

### 3. Pseudo-Dynamic Test

For the safety and integrity of structure, the dynamic behavior of structure at the stage of the conceptual design has been experimentally observed using the appropriate test methods such as the shaking table test, the pseudo-dynamic test and so on. If the structure in the nuclear power plant are designed by introducing new concepts, the prototype or the scale model of structure should be tested in order to investigate its dynamic properties and behaviors under the severe loading conditions such as earthquake motions.

The quasi-static test method has been used to investigate the nonlinear characteristics of the whole or the parts of structures. The method is simple and is adopted independent of the size of structure, but the dynamic effects may not be well reflected because the nonlinear force-deformation characteristics of structure should be known or assumed before testing.

In 1969 the pseudo-dynamic test method was introduced by Takanashi[4] for investigating the large structure under the seismic ground motions. Since the test is the combination of the shaking table test and the quasi-static test, the test facility is relatively more simple than that of the shaking table test, but the testing results may be comparable to the results of the shaking table test. Also, the pseudo-dynamic test is relatively inexpensive method when the full-scale structure is tested, because the structural elements whose mechanical properties are not well defined are tested actual but the remaining structural elements are modeled numerically. The mechanical properties of elements numerically modeled are considered to be well defined rather than the elements subjected to the actual forces.

The pseudo-dynamic test constitutes of two parts; One is the computational part and the other is the experimental part under the actual forces, as shown in Fig. 1. The computational part can be performed by the computer, and the experimental part includes the servo valves, load cells, and/or oil pressure transducers. The reaction forces from the structure are measured by load cell and used as the input of the numerical integration via the data acquisition system by which the analog signal are filtered and converted to the digital signal. The structure then is deformed by actuator as much as displacement from the numerical integration.

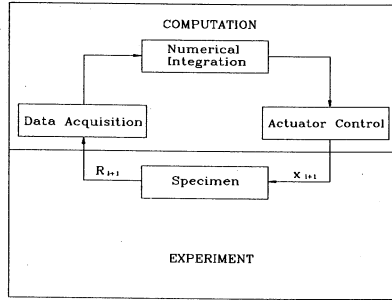


Fig. 1 Schematic Diagram of Pseudo-Dynamic Test

Since the explicit time integration scheme is adopted in test, the equation of motion is solved corresponding to the restoring force which is measured on the structure, and also the stiffness matrix are established at every time step. The computational procedure[5] are as follows;

1. Calculate the initial condition using equation,

$$\ddot{\mathbf{x}}_0 = \mathbf{M}^{-1}(\mathbf{F}_0^e - \mathbf{F}_0^i)$$

2. Evaluate the displacement using the equation

$$\mathbf{x}_{n+1} = \mathbf{x}_n + \Delta t \dot{\mathbf{x}}_n + \frac{\Delta t^2}{2} \ddot{\mathbf{x}}_n$$

3. Impose the displacement  $\mathbf{x}_{n+1}$  on the structure to be tested.
4. Measure the restoring force  $\mathbf{R}_{n+1}$  at the designated points of structure.
5. Correct the restoring force
6. Compute the acceleration and the corresponding using the equations

$$\ddot{\mathbf{x}}_{n+1} = \left( \mathbf{M} + \frac{\Delta t}{2} \mathbf{C} \right)^{-1} \cdot \left( \mathbf{F}_{n+1} + \mathbf{R}_{n+1} - \mathbf{C} \dot{\mathbf{x}}_{n+1} - \frac{\Delta t}{2} \mathbf{C} \ddot{\mathbf{x}}_n \right), \quad \mathbf{x}_{n+1} = \mathbf{x}_n + \frac{\Delta t}{2} (\ddot{\mathbf{x}}_{n+1} + \ddot{\mathbf{x}}_n)$$

7. If the step  $n$  is less than the total number of steps  $N$ , return to 2 after setting  $n = n + 1$ . Otherwise, stop.

The steps 3 and 4 are included in the experimental part as mentioned before, and others are included in the numerical part.

#### 4. Experimental Tests of LRB

The cylindrical LRBs whose diameters are 75, 150 and 300 mm are designed and fabricated for investigating the similitude rule and the effective damping ratio and the equivalent stiffness under various conditions. The schematic shape are shown in Fig. 2 and the their specification are listed in Table 1. Table 2 shows the mechanical properties of rubber, and Table 3 shows the design values of stiffness and buckling loads of three LRBs.

Fig. 3 shows the schematic diagram of LRB test equipment and facility. Two hydraulic actuators are used to apply loads in the horizontal and vertical directions simultaneously. The capacities of vertical actuator which controls the vertical load are  $\pm 50\text{ton}$  and  $\pm 125\text{mm}$ , and those of horizontal actuator which controls the shear deformation are  $\pm 25\text{ton}$  and  $\pm 125\text{mm}$ .

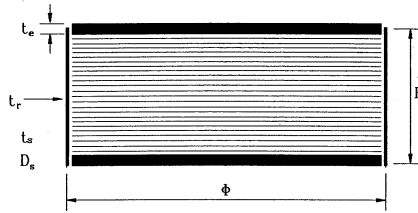


Fig. 2 LRB being tested;  $\Phi$ : diameter, H: height,  $D_s$ : diameter of steel plate,  $t_r$  and  $t_s$ : thickness of rubber and steel plate,  $t_e$ : thickness of end plate, n: number of layers

Table 1 Dimensions of LRB( unit:mm)

	Type I	Type II	Type III
$\Phi$	150	75	300
H	87	64.9	147
$D_s$	142	71	284
$t_r$	3.7	1.85	7.4
$t_s$	2	1.6	3
n	16	16	23
$t_e$	10	10	10
scale	1/2	1/4	1

Table 2 Material Properties of Synthetic Rubber

shear modulus	$G = 1.0 \text{ MPa}$
bulk compressive modulus	$E_0 = 1.0 \text{ GP}$
apparent compressive modulus	$E_C = 0.87 \text{ GPa}$
Poisson's ratio	$\nu = 0.5$
chemical parameter	$\beta = 3.53$

Table 3 Designed Stiffness and Buckling Load

	vertical stiffness (ton/cm)	horizontal stiffness (ton/cm)	rotational stiffness (ton-cm/rad)	buckling load (ton)
Type I	225.15	436.75	1376.20	75.06
Type II	112.58	218.38	171.65	12.56
Type III	450.31	873.51	10985.6	301.4

The load-displacement relation in the vertical direction is shown in Fig. 4. The vertical stiffness from this test is about 193.88 ton/cm, while the designed vertical stiffness in Table 3 is 225.15 ton/cm. Hence the difference between two results is 13.9%, which is considered to be in the acceptable range. The offset in Fig 4. is considered to be due to the loose of test fixture.

The horizontal hysteretic loops dependent upon the maximum shear deformation are shown in Fig. 5, and the effective stiffness and the damping ration with respect to the maximum shear strain are shown in Fig. 6. The horizontal hysteretic tests are carried out under the constant vertical load of 11.2 ton. As shown in Fig. 5, the stiffness varies dependent upon load, and thus the enough interval between subsequent tests were provided for the settlement.

Fig. 5 shows that the stiffness increases at the tips of hysteretic loops if the maximum shear strain are more than 150%. But the phenomena do not appear for relatively small strain.

Fig. 7 shows the stiffness and the damping ratio referring to the rate of load in the range of 0~2Hz. The effective stiffness gradually increases with the loading rate, and the effective damping ratio also increases with rather larger slope than stiffness.

It is considered that the effective stiffness and the effective damping ratio as shown in Fig. 8 are not much affected by the vertical loads.

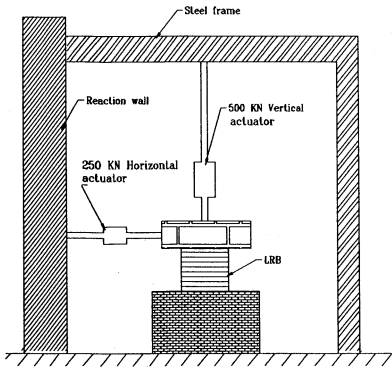


Fig. 3 Testing Machine

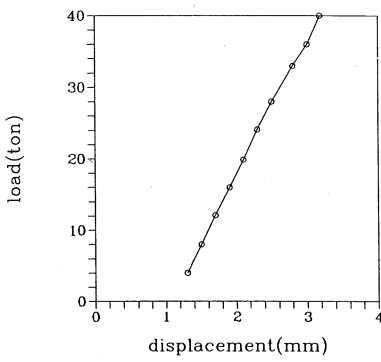


Fig. 4 Vertical Load-Displacement Relation

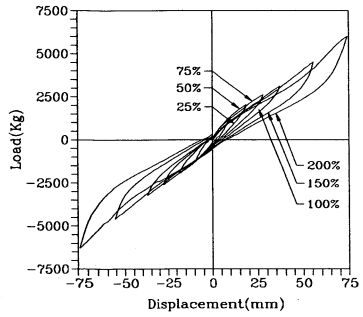


Fig. 5 Hysteretic Loops According to Maximum Shear Strains

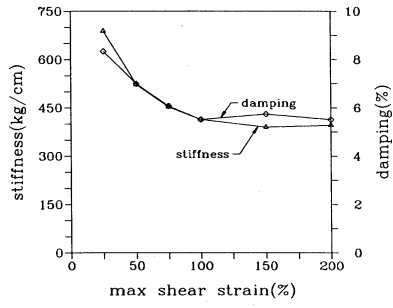


Fig. 6 Equivalent stiffness and effective damping ratio

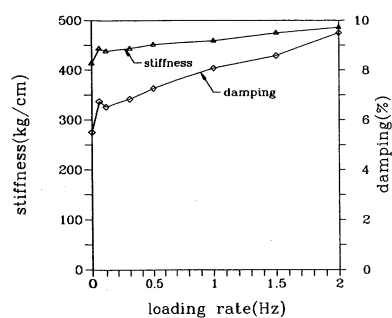


Fig. 7 Equivalent stiffness and effective damping ratio with respect to loading rate

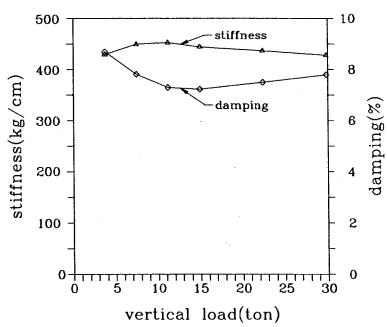


Fig. 8 Equivalent stiffness and effective damping ration with respect to vertical load

## 5. Numerical Simulations and Pseudo-Dynamic Tests

Two-bay five-story steel building as shown in Fig. 9 are chosen to evaluate the home made finite element analysis program NAIS through the pseudo-dynamic tests. The behaviors of steel structure subjected to the two different earthquake excitations, El Centro and Taft, are simulated by NAIS and are observed by the pseudo-dynamic tests.

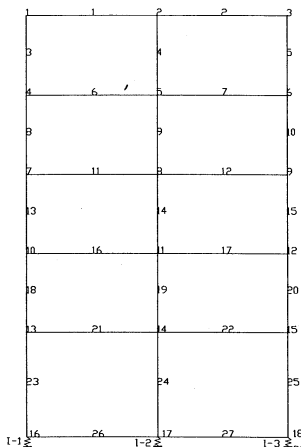


Fig. 9 Finite element model of isolated steel building

In experimental tests, the isolators LRB are real but the rest of structure is modeled by the finite element model as defined in the fundamental of pseudo-dynamic test. The nodal displacements calculated by computer are imposed to the designated points of structure using the hydraulic actuator, and the reaction measured by load-cell is used as the input of a numerical program which is different from NAIS.

In the finite element analysis, the number of nodes can be determined by the expected behavior of structure. The nodes of the finite element model are located at the junction points as shown in Fig. 9, because the fundamental frequency of steel elements is far less than the major frequency spectrum of excitations. The recommended minimum number of nodes to be placed along the members should be larger than the number of mode of a simply supported beam. As a result the computational efforts can be reduced. The super structure model are supported by three LRB isolator as indicated I-1, I-2 and I-3. The finite element model is composed of 31 nodes, 27 beam elements and 3 spring elements as shown in Fig. 9.

The load-displacement relation of spring is assumed to be bi-linear which can be determined by elastic and elasto-plastic stiffnesses and yield strength. Since the stiffness of LRB depends upon the shear strain, the behavior of isolated structure can not be estimated precisely by one simulation. Thus at least two numerical simulations should be required to determine two stiffness and yield strength.

Taft and El Centro earthquakes are adopted as the base motion of experimental tests and numerical simulations. The seismic responses from the numerical simulations are agreed well with those of pseudo-dynamic tests as shown in Fig. 10 and 11. Therefore, it is considered that the computer program NAIS can be useful to simulate the dynamic responses of isolated structure with accuracy.

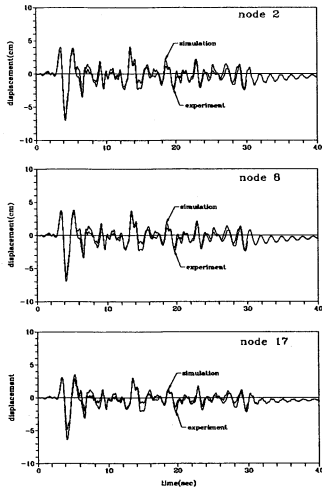


Fig. 10 Responses of earthquake Taft

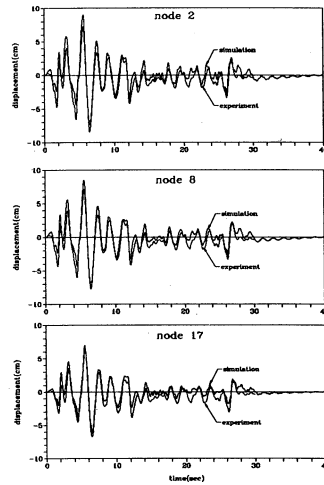


Fig. 11 Responses of earthquake El Centro

## 6. Conclusion

A finite element program NAIS for analyzing the dynamic nonlinear behavior of isolated structures has been verified through the pseudo-dynamic tests of a isolated steel structure subjected to two earthquake motions, and the program could be useful for the analysis of the dynamic behavior appropriately.

## References

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