# Mathematical Techniques in Evolution and Ecology

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# Contents

# Introduction

To come.

Chapter 1

# How to construct a model

## 1.1 Problems

P.1.1 In the squirrel example, assume that death through cyclists occurs before immigration, and immigration is followed by birth. Derive the discrete-time recursion equation and compare it to the one we obtained above, where we assumed that death is followed by birth and immigration. Start from the life-cycle diagram. Interpret the difference.

P.1.2 [Problem 2.5 in OD2007] Consider a model of disease transmission with the following equations:

$$\frac{dS}{dt} = \theta - dD - \beta SI + \gamma I, \tag{1.1a}$$

$$\frac{dI}{dt} = \beta SI - (d + \nu + \gamma)I. \tag{1.1b}$$

The variables $S$ and $I$ denote the number of susceptible and infected individuals. (a) Draw and label a flow diagram for these two variables. (b) Suggest a plausible biological interpretation of the parameters $\gamma$ and $\nu$.

P.1.3 [Problem 2.6 in OD2007] In the flu model, suppose that after contracting the flu, people are initially resistant to reinfection, but this immunity eventually wanes. Alter the flow diagram for the flu model that we derived to include a "recovered and immune" class with these properties. (b) Suppose that immune individuals have a constant per capita rate of losing immunity. What are the continuous-time equations for this modified model?

# Chapter 2

# Solutions to problems

P.1.1 The life-cycle diagram is as given in Fig. 2.1. The recursion equation is

$$n(t+1) = (1+b)\left[(1-d)n(t)+m\right], \tag{2.1}$$

as compared to

$$n(t+1) = (1+b)(1-d)n(t)+m \tag{2.2}$$

for the case of immigration after death and birth. The first equation differs from the second one by an amount of $bm$, which is the fraction of squirrels born by immigrating (female) squirrels.

P.1.2 The flow diagram is shown in Fig. 2.2. The parameter $\gamma$ is the rate at which infected individuals recover and become susceptible again. The parameter $\nu$ denotes an additional death rate experienced by infected individuals as compared to the death rate $d$ of susceptible ones.

P.1.3 The flow diagram is as given in Fig. 2.3, and the continuous-time differential equations are

$$\frac{ds}{dt} = \sigma r(t) - acs(t)n(t) \tag{2.3a}$$

$$\frac{dn}{dt} = acs(t)n(t) - \rho n(t) \tag{2.3b}$$

$$\frac{dr}{dt} = \rho n(t) - \sigma r(t) \tag{2.3c}$$

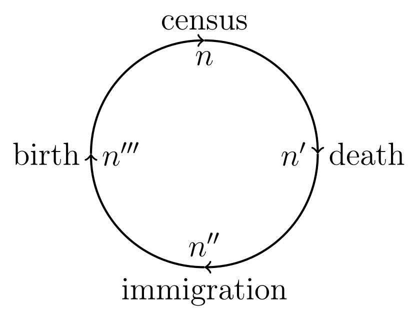


**Figure 2.1** Life-cycle diagram for a squirrel model with alternative order of events.

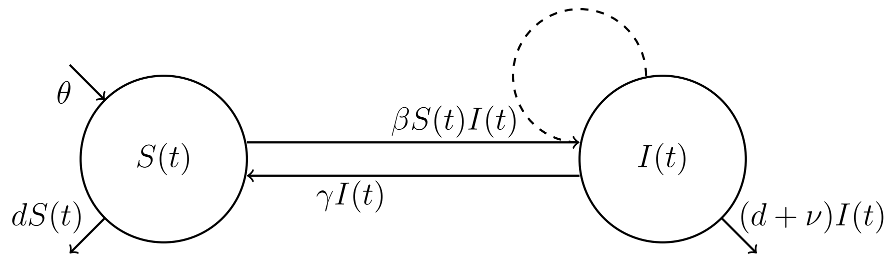


**Figure 2.2** Flow diagram for the continuous-time model of Problem P.1.2.

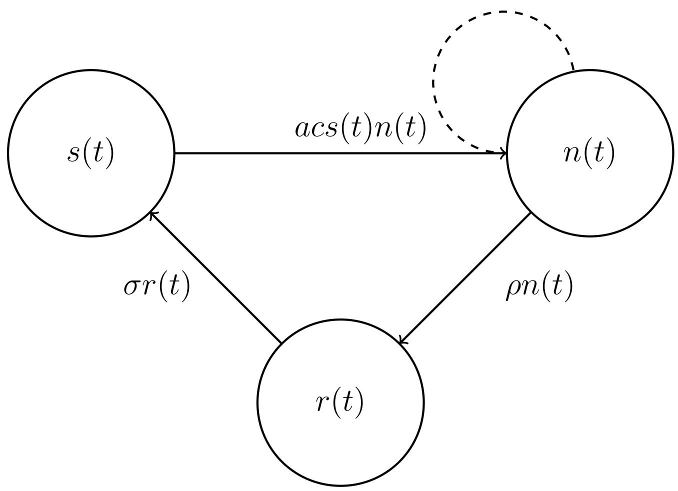


**Figure 2.3** Flow diagram for the continuous-time model of Problem P.1.3, with an additional class of resistant individuals.