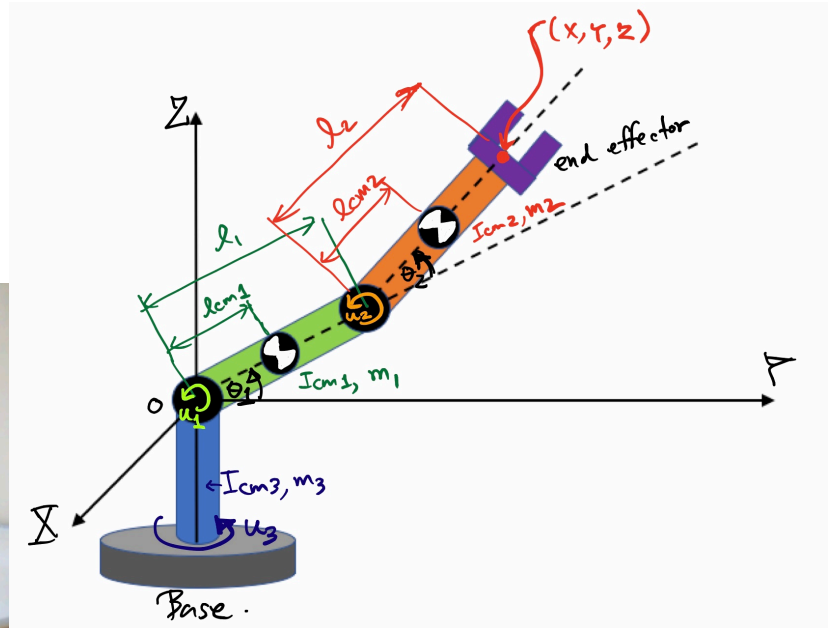
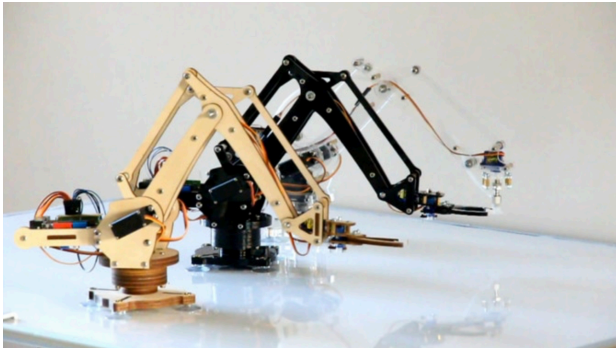


## Advanced Dynamics Project (Spring 2019)

### Final Project: 3-DOFs Scalar Robot Manipular Dynamics and Control Created by Kan Kanjanapas (Ph.D.)

In this project, we will explore the dynamics and control of 3-DOFs Scalar Robot Manipulator which is commonly used in many industrial applications. The robot in figure below describes how a scalar robot operates in workspace given the reference coordinate frame XYZ. The robot has 3-DOFs including rotation at joint 1 (along X axis), rotation at joint 2 (along X axis) and rotation at joint 3 (along Z axis). The end effector position is described by  $(x,y,z)$ .



a) uARM robot manipulator (left) b) diagram and related parameters (right)

#### Parameters:

|                               |   |
|-------------------------------|---|
| $l1$ :                        | Length of link 1 (Green) [1 m]  |
| $l2$ :                        | Length of link 2 (Orange) [1 m]   |
| $lcm1$ :                      | Center of mass position of link 1 with respect to joint 1 (@ $u1$ ) [0.5 m]     |
| $lcm2$ :                      | Center of mass position of link 2 with respect to joint 2 (@ $u2$ ) [0.5 m]     |
| $\theta1, \theta2, \theta3$ : | Angle of joint 1,2 and 3 respectively [rad, deg]                                |
| $m1, m2, m3$ :                | Mass of joint 1,2 and 3 respectively [50, 50, 50 Kg]                            |
| $lcm1, lcm2, lcm3$ :          | Moment of inertia @ CM [10, 10, 10 Kg.m <sup>2</sup> ]                          |
| $(x,y,z)$ :                   | End effector position [m]   |
| XYZ:                          | Global coordinate system as world reference                                     |
| $u1, u2, u3$ :                | Control inputs = Motor torque [N.m]   |
| $kr1, kr2, kr3$ :             | Gear ratio of motor at joint 1,2,3 [ $kri = 100/1$ ]                            |
| $Imotor1, Imotor2, Imotor3$ : | Moment of inertia of motor at joint 1,2,3 [0.01, 0.01, 0.01 Kg.m <sup>2</sup> ] |
| $g$ :                         | Gravity (9.81 m/s <sup>2</sup> )  |

Q1: Derive forward kinematics relationship, given all joint angles  $\theta_1, \theta_2, \theta_3$ , what's the end effector position  $(x, y, z)$  as a function of joint angles and the robot parameters.

Q2: Derive inverse kinematics relationship, given the end effector position  $(x, y, z)$ , what're the joint angles  $(\theta_1, \theta_2, \theta_3)$  as a function of  $(x, y, z)$  and the robot parameters?

[50 Points] Must include a plot to verify whether the forward and inverse kinematics mappings are correct. For example, you may create desired trajectory of the end effector  $(x, y, z)$ , then calculate the joint angles, then cross check again whether the calculated joint angles give the same end effector  $(x, y, z)$  or not.

Q3 [50 Points]: Derive the equations of motions (EOMs) of this 3-DOFs Scalar Robot Manipulator. Let define the state as  $X = [\theta_1, \theta_2, \theta_3]'$ ;  $\dot{X} = f(X, u, t, \text{robot parameters}) = ?$  Note that  $\dot{X}$  dimension is  $6 \times 1$ .

Simulate open loop response by

- Let initial posture is  $\theta_1 = 90^\circ$ ,  $\theta_2 = 0^\circ$ ,  $\theta_3 = 0^\circ$ .
- All control inputs are inactive ( $u_i = 0$ )
- Don't forget about the gravity.
- Create a Simmechanics model to demonstrate this motion in 3D, submit Simmechanics model, Plots of all states, and recorded robot motion video.

Q4 [Extra Credit 30 Points]: Create the controller for trajectory tracking. You may create the desired trajectory of the end effector. You should adapt the control law derivation from the case of 2-DOFs Scalar robot manipulator. First thing to check is that the simulation result should be the same if there's no rotation at joint 3 ( $\theta_3$  is fixed). If your model is valid from this point, then let explore the controller performance when operating in 3D space. A well-designed controller should be robustly stable and achieve the tracking performance (small tracking error). Note that tracking error = desired goal – actual response (i.e. error of  $\theta_1 = \text{desired } \theta_1 - \text{actual } \theta_1$ ).

- Show your work on the control law derivation! Stability Check?
- Given the desired trajectory  $(x, y, z) \rightarrow$  derived the desired joint angles  $(\theta_{1d}, \theta_{2d}, \theta_{3d})$
- Given the states are all zeros, simulate the closed loop response. Submit Matlab, Simmechanics models, comparison plots between desired  $(x, y, z)$  vs actual  $(x, y, z)$ , comparison plots between desired joint angles vs actual joint angles. Error plots are optional. Also submit the recored robot motion video)