2103-496 Applied Dynamics/2103-615 Advanced Dynamics

Project Assignment #1 (Group of Two)

Due: March 1, 2019

GenPath is a current topic of the senior project in Mechanical Engineering Department, Chulalongkorn University. GenPath is an equipment prototype that embedded under a floor tile, capable of harvesting kinetic energy from people's footsteps and converting it to electrical energy. In the process of energy generation, the rotational-electromagnetic generator is deployed. The harvested energy is then stored and used in low energy-consumption devices.

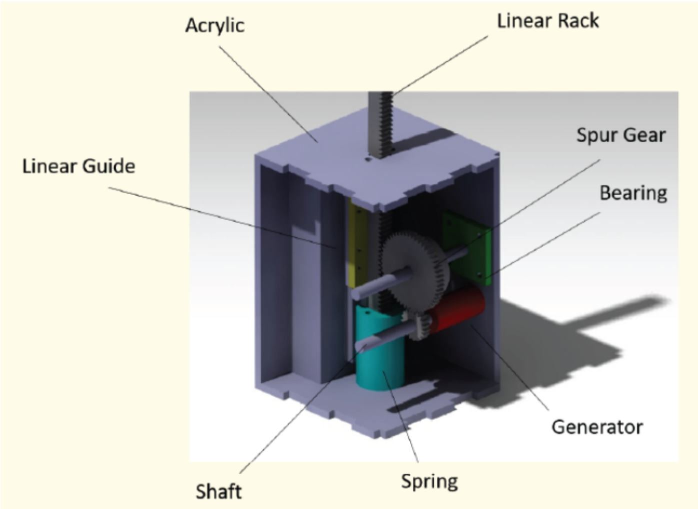

Figure 1: Mock Up of the GenPath

In this project, we will analyze dynamics of the electro-mechanical system in the GenPath shown in Figure 1, using Matlab/Simulink. The system consists of a rack/pinion mechanism to convert the rack's translation from a footstep to the pinion's rotation. The footstep force is modeled as a ramp function with a magnitude of 600 N within 0.8 s. The pinion then drives the DC-generator shaft through the gear train which transforms low-speed power to high-speed power. With rotation of the generator, the electrical current can be generated. In addition, to restore the rack back to the equilibrium position, the lower end of the rack is connected to a spring with the maximum compression of 20 mm. The whole system is limited by the space of $20\times10\times10$ cm^3, therefore the small size of 12-24 V-DC motor is used as a DC generator.

Given Simulink model of the rack/pinion mechanism (model.xls), here are your tasks:

1. Construct the Simulink model for the combined electro-mechanical system that described above.
2. Then run the simulation using your choice of designed parameters, to simulate the power that generated in one cycle at no load and at various load resistances. Also give a discussion on your design system, i.e. how much the power can be generated, what kinds of applications the generated electrical energy can be used, and etc.
3. Extra credit for showing the results and discussion for the parametric study in order to maximize your harvesting energy.

Derivation

The dynamic equations governing the electro-mechanical model of the Genpath are formulated as follows. Figure 2 shows the physical model of the GenPath system consisting of the elements of rack, pinion and gear train on mechanical side, and also DC-generator connected to the load R_L on electrical side.

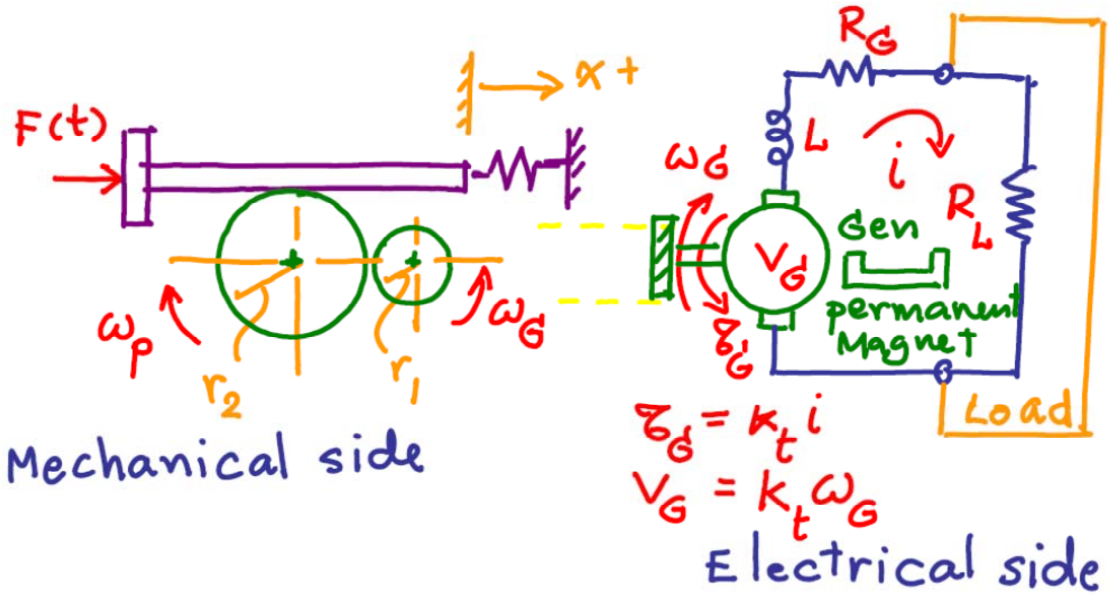

Figure 2: Physical Model of the GenPath

For the electrical side, Kirchhoff's voltage law yields

$$V_G = V_L + V_{RG} + V_{RL} \tag{1}$$

where V_G is the back emf of the generator; i.e. $V_G = K_t \omega_G$. In addition, V_L, V_{RG} and V_{RL} are the voltages across the generator's inductor, generator's resistor and load's resistor, respectively. With the voltage-current relations, (1) becomes

$$K_t \omega_G = L\frac{di}{dt} + R_G i + R_L i \tag{2}$$

where i is the current, K_t is the back emf (torque) constant, ω_G is the generator speed (in rad/s), L is the inductance generator (in H), R_G is the resistance of the generator (in Omh) and R_L is the resistance of the load (in Omh). Rewrite (2), we the differential equation describing the armature winding of the generator as

$$\frac{di}{dt} + \left(\frac{R_G + R_L}{L}\right) i - \left(\frac{K_t}{L}\right)\omega_G = 0 \tag{3}$$

From FBD of the mechanical system in Figure 3, Newton's second law and the law of angular momentum describe the translation of the rack, and rotations of the pinion and the generator rotor as follows

Rack: $$F - F_r - F_s = m\ddot{x} \tag{4}$$

Pinion: $$(F_r - f_r')r_2 = J_p \dot{\omega}_p \tag{5}$$

Generator rotor: $$-\tau_G + f_r' r_1 = J_G \dot{\omega}_G \tag{6}$$

where m is mass of the rack, $F(t)$ and $F_s(t)$ are applied force and spring force, $F_r(t)$ and f_r' are frictions. τ_G is the generator's electromagnetic torque where $\tau_G = K_t i$. Also J_G and J_p are mass moments of inertia, r_1 and r_2 are radius, and ω_G and ω_p are angular speeds for the transmission gear and the pinion, respectively.

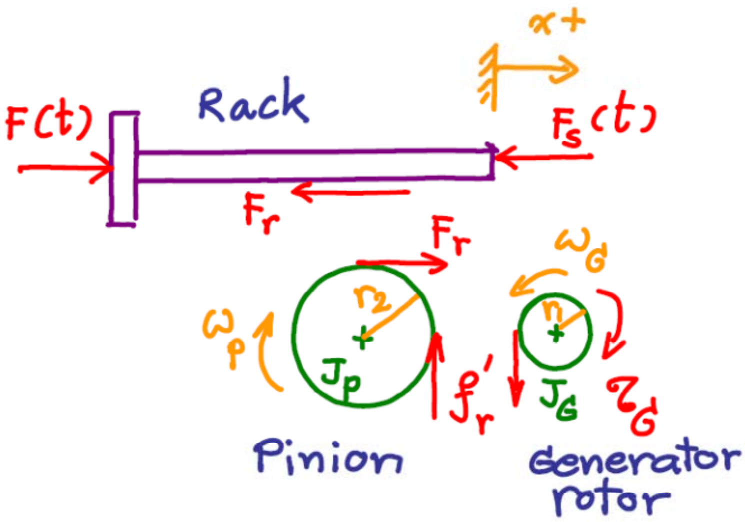

Figure 3: Free Body Diagram (FBD) of the mechanical components

Substitute kinematic relations $\dot{\omega}_p = \ddot{x}/r_2$ and $\dot{\omega}_G = \ddot{x}/r_1$ into (5)-(6), and eliminate F_r and f_r' in (4)-(6), we get the differential equation governing dynamics of the mechanical system as

$$M\ddot{x} + \frac{K_t}{r_1} i + F_s(t) = F(t) \tag{7}$$

where $M = (m + \frac{J_p}{r_2^2} + \frac{J_G}{r_1^2})$. Eqns (3) and (7) are the governing equations of the whole electro-mechanical system. Let $x_1 = x$ and $x_2 = \dot{x}$, the governing equations can be derived in state form as

$$\frac{d}{dt}\begin{pmatrix} i \\ x_1 \\ x_2 \end{pmatrix} = \begin{bmatrix} -\left(\frac{R_G + R_L}{L}\right) & 0 & \frac{K_t}{Lr_1} \\ 0 & 0 & 1 \\ \frac{-K_t}{Mr_1} & 0 & 0 \end{bmatrix} \begin{pmatrix} i \\ x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \frac{-F_s + F(t)}{M} \end{pmatrix}$$

The parameters in Table 1 might be used for your simulation, if you cannot find them.

Table 1: Parameters

r_1	0.75 cm
r_2	3 cm
k (stiffness coefficient of spring)	350 N/m
J_p	10^{-5} kg•m^2
m	0.16 kg
R_G	50 Ω
L	1 mH
K_t	0.048 V•s/rad