Wave energy delivery to multiple subsurface targets using time-reversal method

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Abstract

We discuss the applicability of a time-reversal concept to the focusing of wave energy to multiple subsurface targets embedded within an arbitrarily heterogeneous three-dimensional elastic host. The motivation stems from an interest in facilitating oil ganglia mobility in support of enhanced oil recovery (EOR) methods. We quantify the focusing by a suitable motion metric, and provide numerical evidence supportive of the method's efficacy in illuminating the targets even when embedded within randomized media.

Keywords: wave energy focusing, time-reversal, subsurface targets, EOR, fracking

1 Introduction

There are a few engineering applications, where there is interest in focusing wave energy to targets embedded within heterogeneous hosts. Lithotripsy, i.e., the breaking of kidney stones, has been a long-standing application of wave focusing in therapeutics; similarly motivated applications can be found in cancer treatment. In therapeutics, the typical setting involves a closed-cavity, or equivalently, the surrounding of the target with sources/receivers that direct energy to the target. Here, we are interested in exploring wave focusing to select targets embedded within the subsurface, i.e., hosted by a semi-infinite heterogeneous elastic domain, which poses challenges not encountered in closed-cavity or waveguide settings. The application is motivated by enhanced oil recovery needs, where there is interest in facilitating the mobility of oil ganglia in reservoir subregions typically bypassed by primary modes of recovery.

To this end, we numerically evaluate the potential of the application of a time-reversal concept to illuminate the targets, and assess its effectiveness by computing suitable motion metrics. Specifically, we consider the setting depicted in Fig. 1: we assume that there are multiple targets embedded within an arbitrarily heterogeneous semi-infinite host, with, in general, contrasting properties with the host. We assume further that a single source (or more) is present within each target: the sources are triggered, and a time-reversal (TR) mirror (e.g., geophones) records the response on the surface of the half-space. The receiver signals are time-reversed and the interest is in assessing the wave energy refocusing potential to the targets, given the presence of multiple challenges, which include the limited extent of the mirror, the unboundedness of the host, the lack of a sink, and others.

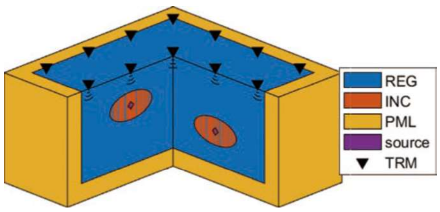

Figure 1: Model configuration: REG, INC, and PML stand for host (regular) domain, inclusions (targets), and perfectly-matched-layers, respectively; the sources are located within the targets, and TRM stands for the time-reversal mirror

2 Mathematical background

To numerically simulate the refocusing experiment, we consider the two steps typically involved in a time-reversal application. In a first, or forward step, the sources are triggered, and the receiver array (the TR mirror) records. This phase is governed by the Navier equations of motion, i.e.,

$$\nabla \cdot [\mu (\nabla \mathbf{u} + \nabla \mathbf{u}^{\mathsf{T}}) + \lambda \{(\nabla \cdot \mathbf{u}) \mathbf{I}\}] - \rho \ddot{\mathbf{u}} = \mathbf{f} \quad (1)$$

where $\mu(\mathbf{x})$ and $\lambda(\mathbf{x})$ are the Lamé parameters, and $\rho(\mathbf{x})$ denotes mass density; $\mathbf{u}(\mathbf{x}, t)$ is the displacement field and $\mathbf{f}(\mathbf{x}, t)$ denotes the source/force vector ($\mathbf{x} \in \Omega_{\mathrm{REG}}$). To properly account for the unboundedness of the domain, the physical domain is truncated through the introduction of perfectly-matched-layers (PMLs); the PMLs form the buffer zone Ω_{PML} around Ω_{REG}, per Fig. 1. The details of the numerical treatment for the forward step are given in [2].

Following the forward step, the recorded response at the mirror is time-reversed. If the domain of interest were a closed cavity, if the mirror were to envelope the domain, and if the forcing term in (1) were to be also time-reversed (sink), then perfect refocusing to the original source locations within the targets is expected [1] (for an infinite aperture and for the continuous problem). Due to practical limitations, the aforementioned ideal conditions are impossible to attain, and, consequently the focusing is expected to degrade. In addition: while the receivers record Dirichlet data (displacements), equipment limitations allow only the application of Neumann data (tractions), which could further degrade the focusing. With the above conditions, the time-reversal phase is numerically simulated using (1), without the forcing term, and subject to the free-surface mirror conditions, i.e., applied tractions implemented by time-reversing the recorded Dirichlet data. Mathematically, the time-reversal of recorded Dirichlet data as Neumann data resembles steps in subsurface imaging processes; however, the difference in the illumination zone between a migration approach and the inclusion-originating data would likely not allow for the imaging of the targets using a migration method.

To quantify the focusing, we introduce the time-averaged kinetic energy for $\mathbf{x} \in \Omega_{\text{REG}}$ [3]:

$$\text{KE}^{\text{TA}}(\mathbf{x}) = \frac{1}{2T} \int_0^T \mathbf{u}^{\intercal}(\mathbf{x}, t)\rho(\mathbf{x})\mathbf{u}(\mathbf{x}, t)\, dt. \quad (2)$$

3 Numerical experiment

The model is 80m×80m×40m (depth), with a 6.25m-thick PML buffer enveloping all sides except the top surface. The model is divided into two layers, with the interface at 20m depth. Two spheroidal, relatively soft, targets (semi-axes 7.5m, 7.5m, and 3.75m), are placed at two different depths, with one centered at (-15m,-15m,-20m) and the other at (15m,15m,-30m), respectively. The physical properties are summarized in table 1; in order to introduce further heterogeneity, the values of table 1 were spatially randomized; c_p and c_s indicate P- and S-wave speed, respectively; the shear wave map is shown in Fig. 2. The resulting time averaged kinetic energy is shown in Fig. 3: despite the various limitations, the targets are clearly illuminated.

	c_p(m/s)	c_s(m/s)
targets	387.30	223.61
top layer	670.82	387.30
bottom layer	866.03	500.00

Table 1: Model physical properties

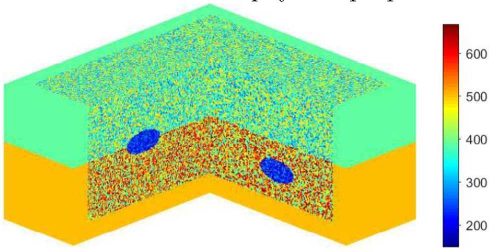

Figure 2: Shear wave speed distribution

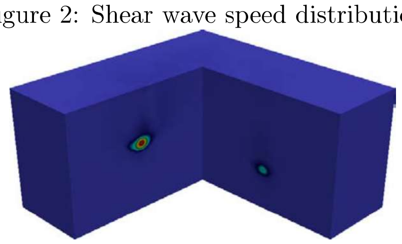

Figure 3: Time-averaged kinetic energy KE^{TA}

4 Conclusion

We quantified and demonstrated the applicability of time-reversal in the focusing of wave energy at multiple subsurface targets embedded within heterogeneous elastic hosts, without resolution loss of practical significance. The approach is a good candidate for EOR.

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