Bits, Bytes and Integers – Part 1

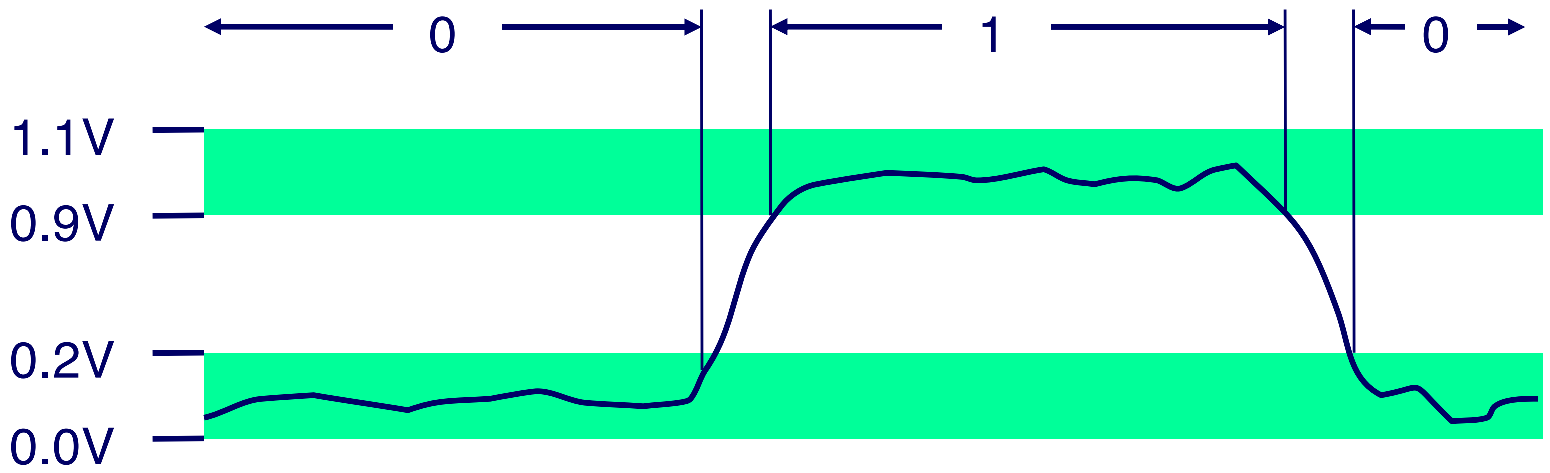
Computer Systems
Friday, September 22 2023

Today: Bits, Bytes, and Integers

- **Representing information as bits**
- **Bit-level manipulations**
- **Integers**
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - Addition, negation, multiplication, shifting
  - Summary
- **Representations in memory, pointers, strings**

Everything is bits

- **Each bit is 0 or 1**
- **By encoding/interpreting sets of bits in various ways**
  - Computers determine what to do (instructions)
  - … and represent and manipulate numbers, sets, strings, etc…
- **Why bits? Electronic Implementation**
  - Easy to store with bistable elements
  - Reliably transmitted on noisy and inaccurate wires



Encoding Byte Values

- **Byte = 8 bits**
  - Binary 00000000_2 to 11111111_2
  - Decimal: 0_{10} to 255_{10}
  - Hexadecimal 00_{16} to FF_{16}
    - Base 16 number representation
    - Use characters ‘0’ to ‘9’ and ‘A’ to ‘F’
    - Write $FA1D37B_{16}$ in C as
      - 0xFA1D37B
      - 0xfa1d37b

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

$$15213: \underbrace{0011}_{3}\ \underbrace{1011}_{B}\ \underbrace{0110}_{6}\ \underbrace{1101}_{D}$$

Example Data Representations

C Data Type	Typical 32-bit	Typical 64-bit
`char`	1	1
`short`	2	2
`int`	4	4
`long`	4	8
`float`	4	4
`double`	8	8
pointer	4	8

Example Data Representations

C Data Type	Typical 32-bit	Typical 64-bit
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`int`	4	4
`long`	4	8
`float`	4	4
`double`	8	8
pointer	4	8
	"ILP32"	"LP64"

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Boolean Algebra

- **Developed by George Boole in 19th Century**
  - Algebraic representation of logic
  - Encode "True" as 1 and "False" as 0

And

A&B = 1 when **both** A=1 and B=1

&	0	1
0	0	0
1	0	1

Or

A|B = 1 when **either** A=1 or B=1 **or both**

|	0	1
0	0	1
1	1	1

Not

~A = 1 when A=0

~	0	1
	1	0

Exclusive-Or (Xor)

A^B = 1 when A=1 or B=1, **but not both**

^	0	1
0	0	1
1	1	0

General Boolean Algebras

- **Operate on Bit Vectors**
  - Operations applied bitwise

```
  01101001     01101001     01101001
& 01010101   | 01010101   ^ 01010101   ~ 01010101
  01000001     01111101     00111100     10101010
```

- **All of the Properties of Boolean Algebra Apply**

Example: Sets of Small Integers

- **Width w bit vector represents subsets of $\{0, 1, \dots, w-1\}$**
  - Let a be a bit vector representing set A, then bit $a_j = 1$ if $j \in A$
  - Examples:
    - 01101001 { 0, 3, 5, 6 }
      76543210
    - 01010101 { 0, 2, 4, 6 }
      76543210

- **Operations**
  - & Intersection 01000001 { 0, 6 }
  - | Union 01111101 { 0, 2, 3, 4, 5, 6 }
  - ^ Symmetric difference 00111100 { 2, 3, 4, 5 }
  - ~ Complement 10101010 { 1, 3, 5, 7 }

Bit-Level Operations in C

- **Operations &, |, ~, ^ Available in C**
  - Apply to any “integral” data type
    - `long, int, short, char, unsigned`
  - View arguments as bit vectors
  - Arguments applied bit-wise

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Contrast: Logic Operations in C

- **Contrast to Bit-Level Operators**
  - **Logic Operations: &&, ||, !**
    - View 0 as "False"
    - Anything nonzero as "True"
    - Always return 0 or 1
    - Early termination
- **Examples (char data type)**
  - `!0x41 → 0x00`
  - `!0x00 → 0x01`
  - `!!0x41→ 0x01`
  - `0x69 && 0x55 → 0x01`
  - `0x69 || 0x55 → 0x01`
  - `p && *p` (avoids null pointer access)

Watch out for && vs. & (and || vs. |)…
Super common C programming pitfall!

Shift Operations

- **Left Shift: `x << y`**
  - Shift bit-vector **x** left **y** positions
    - – Throw away extra bits on left
    - Fill with 0's on right
- **Right Shift: `x >> y`**
  - Shift bit-vector **x** right **y** positions
    - Throw away extra bits on right
  - Logical shift
    - Fill with 0's on left
  - Arithmetic shift
    - Replicate most significant bit on left
- **Undefined Behavior**
  - Shift amount < 0 or ≥ word size

Argument x	01100010
<< 3	00010*000*
Log. >> 2	*00*011000
Arith. >> 2	*00*011000

Argument x	10100010
<< 3	00010*000*
Log. >> 2	*00*101000
Arith. >> 2	*11*101000

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Encoding Integers

Unsigned

$$B2U(X) = \sum_{i=0}^{w-1} x_i \cdot 2^i$$

Two's Complement

$$B2T(X) = -x_{w-1} \cdot 2^{w-1} + \sum_{i=0}^{w-2} x_i \cdot 2^i$$

Sign Bit

```
short int x =  15213;
short int y = -15213;
```

- **C does not mandate using two's complement**
  - But, most machines do, and we will assume so
- **C `short` 2 bytes long**

	Decimal	Hex	Binary
`x`	15213	`3B 6D`	`00111011 01101101`
`y`	-15213	`C4 93`	`11000100 10010011`

- **Sign Bit**
  - For 2's complement, most significant bit indicates sign
    - 0 for nonnegative
    - 1 for negative

Two-complement: Simple Example

	-16	8	4	2	1	
10 =	0	1	0	1	0	8+2 = 10

	-16	8	4	2	1	
-10 =	1	0	1	1	0	-16+4+2 = -10

Two-complement Encoding Example (Cont.)

```
x =      15213: 00111011 01101101
y =     -15213: 11000100 10010011
```

Weight	15213		-15213	
1	1	1	1	1
2	0	0	1	2
4	1	4	0	0
8	1	8	0	0
16	0	0	1	16
32	1	32	0	0
64	1	64	0	0
128	0	0	1	128
256	1	256	0	0
512	1	512	0	0
1024	0	0	1	1024
2048	1	2048	0	0
4096	1	4096	0	0
8192	1	8192	0	0
16384	0	0	1	16384
-32768	0	0	1	-32768
Sum		**15213**		**-15213**

Numeric Ranges

- **Unsigned Values**
  - *UMin* = 0
    000…0
  - *UMax* = $2^w - 1$
    111…1
- **Two's Complement Values**
  - *TMin* = -2^{w-1}
    100…0
  - *TMax* = $2^{w-1} - 1$
    011…1
  - Minus 1
    111…1

Values for *W* = 16

	Decimal	Hex	Binary
UMax	65535	FF FF	11111111 11111111
TMax	32767	7F FF	01111111 11111111
TMin	-32768	80 00	10000000 00000000
-1	-1	FF FF	11111111 11111111
0	0	00 00	00000000 00000000

Values for Different Word Sizes

	W			
	8	**16**	**32**	**64**
UMax	255	65,535	4,294,967,295	18,446,744,073,709,551,615
TMax	127	32,767	2,147,483,647	9,223,372,036,854,775,807
TMin	-128	-32,768	-2,147,483,648	-9,223,372,036,854,775,808

- **Observations**
  - $|TMin| = TMax + 1$
    - Asymmetric range
  - $UMax = 2 * TMax + 1$
  - Question: abs(TMin)?

- **C Programming**
  - #include <limits.h>
  - Declares constants, e.g.,
    - ULONG_MAX
    - LONG_MAX
    - LONG_MIN
  - Values platform specific

Unsigned & Signed Numeric Values

X	B2U(*X*)	B2T(*X*)
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	8	–8
1001	9	–7
1010	10	–6
1011	11	–5
1100	12	–4
1101	13	–3
1110	14	–2
1111	15	–1

- **Equivalence**
  - Same encodings for nonnegative values
- **Uniqueness**
  - Every bit pattern represents unique integer value
  - Each representable integer has unique bit encoding
- **⇒ Can Invert Mappings**
  - U2B(x) = B2U^{-1}(x)
    - Bit pattern for unsigned integer
  - T2B(x) = B2T^{-1}(x)
    - Bit pattern for two's comp integer

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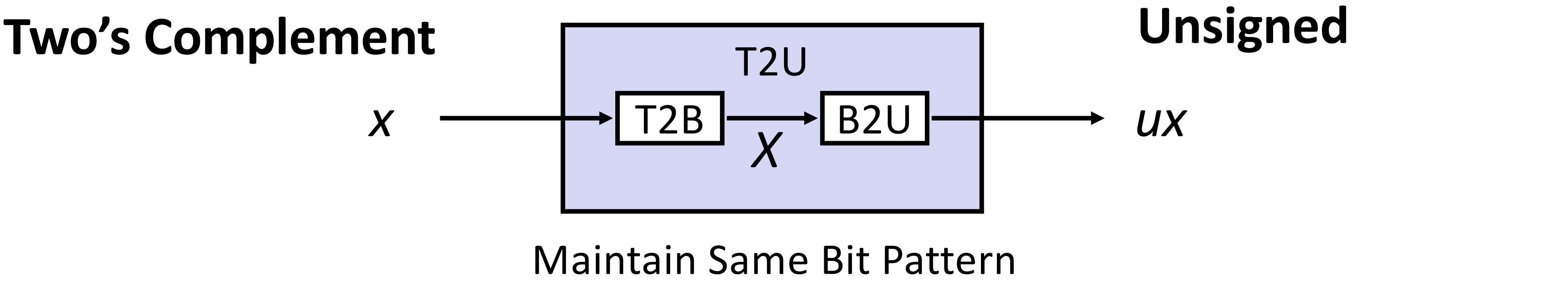
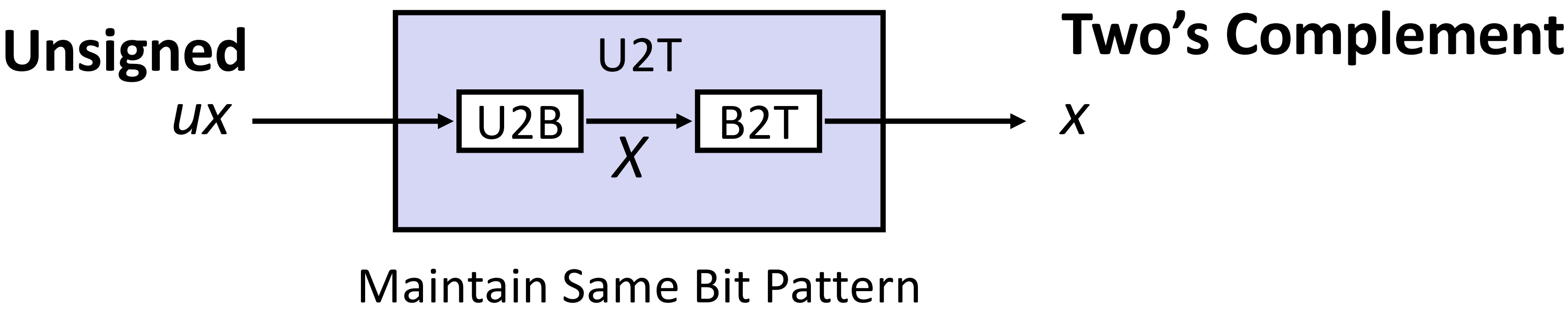
Mapping Between Signed & Unsigned





- **Mappings between unsigned and two's complement numbers:**
  Keep bit representations and reinterpret

Mapping Signed ↔ Unsigned

Bits	Signed	Unsigned
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	-8	8
1001	-7	9
1010	-6	10
1011	-5	11
1100	-4	12
1101	-3	13
1110	-2	14
1111	-1	15

$\longrightarrow$ T2U $\longrightarrow$

$\longleftarrow$ U2T $\longleftarrow$

Mapping Signed ↔ Unsigned

Bits	Signed	Unsigned
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	-8	8
1001	-7	9
1010	-6	10
1011	-5	11
1100	-4	12
1101	-3	13
1110	-2	14
1111	-1	15

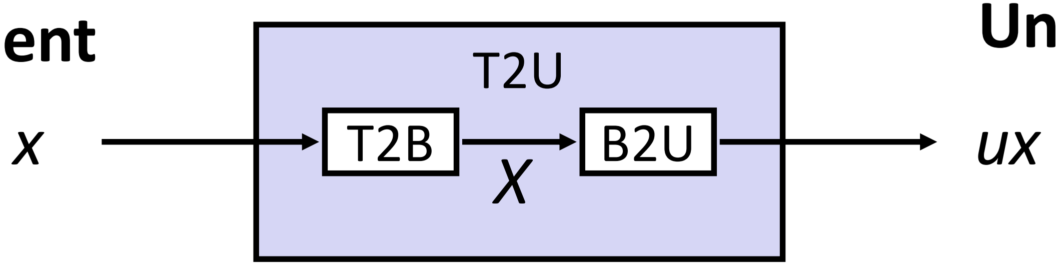
=

+/- 16

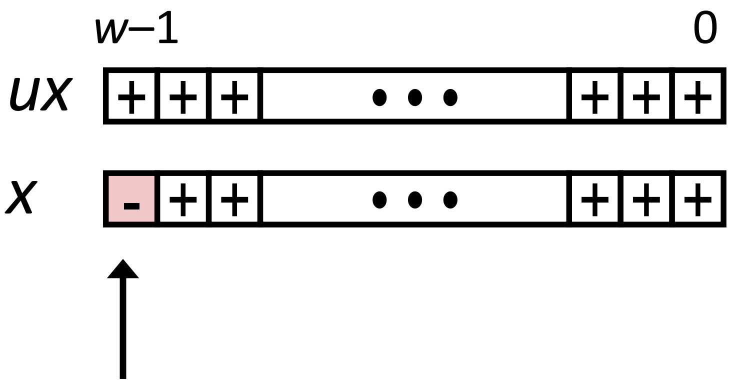
Relation between Signed & Unsigned

Two's Complement

Unsigned



Maintain Same Bit Pattern



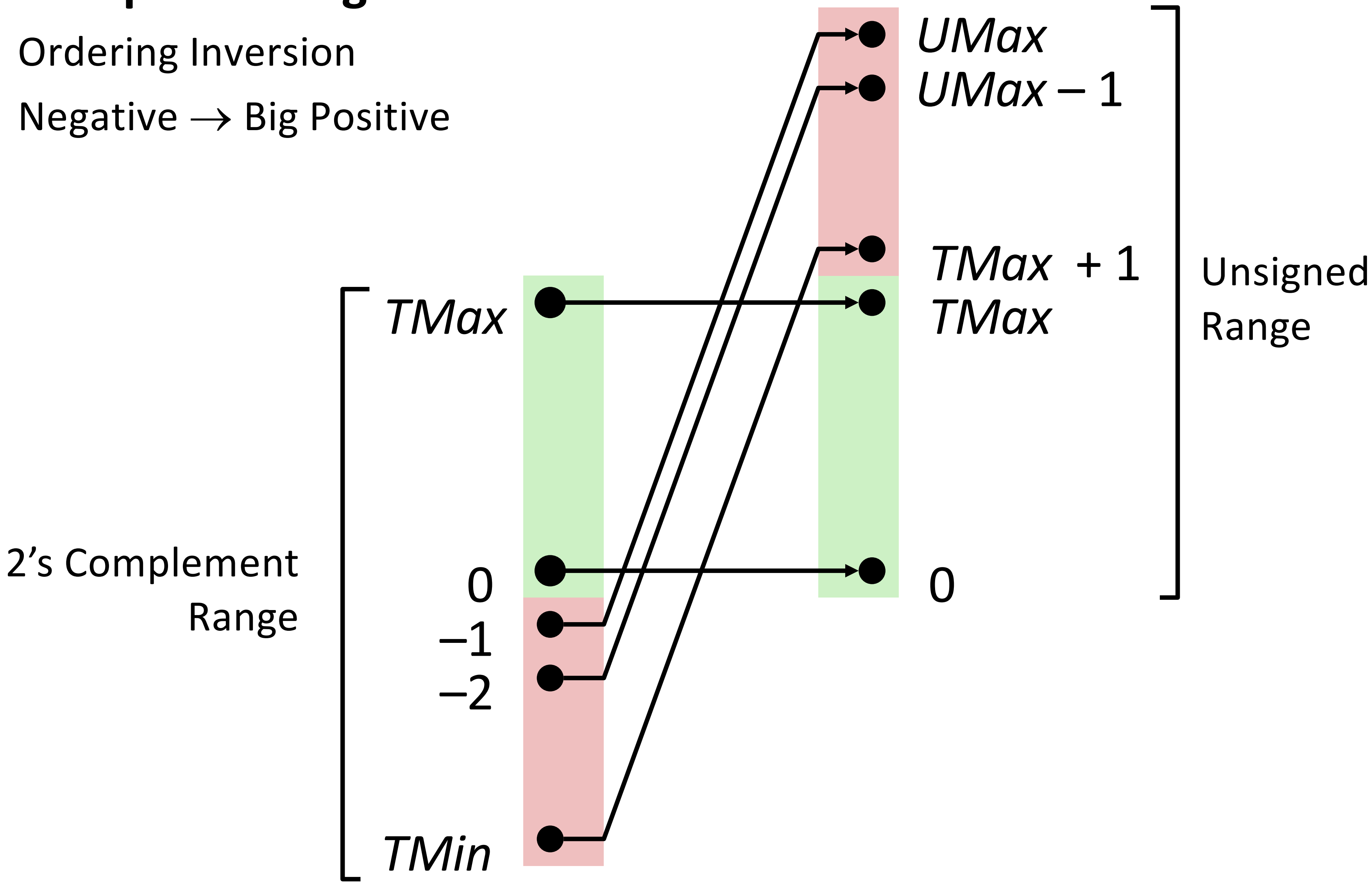
Large negative weight

becomes

Large positive weight

Conversion Visualized

- **2's Comp. → Unsigned**
  - Ordering Inversion
  - Negative → Big Positive



Signed vs. Unsigned in C

- **Constants**
  - By default are considered to be signed integers
  - Unsigned if have “U” as suffix

```
0U, 4294967259U
```

- **Casting**
  - Explicit casting between signed & unsigned same as U2T and T2U

```
int tx, ty;
unsigned ux, uy;
tx = (int) ux;
uy = (unsigned) ty;
```

  - Implicit casting also occurs via assignments and procedure calls

```
tx = ux;                    int fun(unsigned u);
uy = ty;                    uy = fun(tx);
```

Casting Surprises

- **Expression Evaluation**
  - If there is a mix of unsigned and signed in single expression, ***signed values implicitly cast to unsigned***
  - Including comparison operations **<, >, ==, <=, >=**
  - Examples for *W* = 32: **TMIN = -2,147,483,648 ,** **TMAX = 2,147,483,647**

Constant$_1$	Constant$_2$	Relation	Evaluation
0	0U	**==**	**unsigned**
-1	0	**<**	**signed**
-1	0U	**>**	**unsigned**
2147483647	-2147483647-1	**>**	**signed**
2147483647U	-2147483647-1	**<**	**unsigned**
-1	-2	**>**	**signed**
(unsigned)-1	-2	**>**	**unsigned**
2147483647	2147483648U	**<**	**unsigned**
2147483647	(int) 2147483648U	**>**	**signed**

Summary
Casting Signed ↔ Unsigned: Basic Rules

- Bit pattern is maintained
- But reinterpreted
- Can have unexpected effects: adding or subtracting 2^w

- **Expression containing signed and unsigned int**
  - `int` is cast to `unsigned`!!

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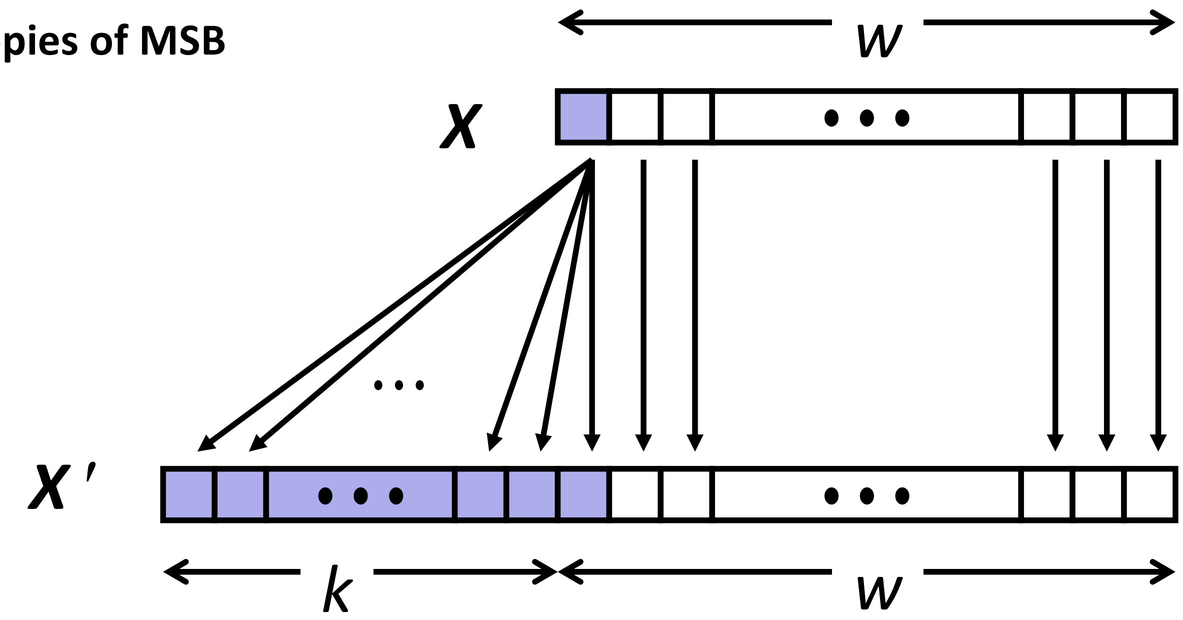
Sign Extension

- **Task:**
  - Given *w*-bit signed integer *x*
  - Convert it to *w*+*k*-bit integer with same value
- **Rule:**
  - Make *k* copies of sign bit:
  - $X' = \underbrace{x_{w-1}, \ldots, x_{w-1}}_{k \text{ copies of MSB}}, x_{w-1}, x_{w-2}, \ldots, x_0$



Sign Extension: Simple Example

Positive number

	-16	8	4	2	1
10 =	0	1	0	1	0

	-32	16	8	4	2	1
10 =	0	0	1	0	1	0

Negative number

	-16	8	4	2	1
-10 =	1	0	1	1	0

	-32	16	8	4	2	1
-10 =	1	1	0	1	1	0

Larger Sign Extension Example

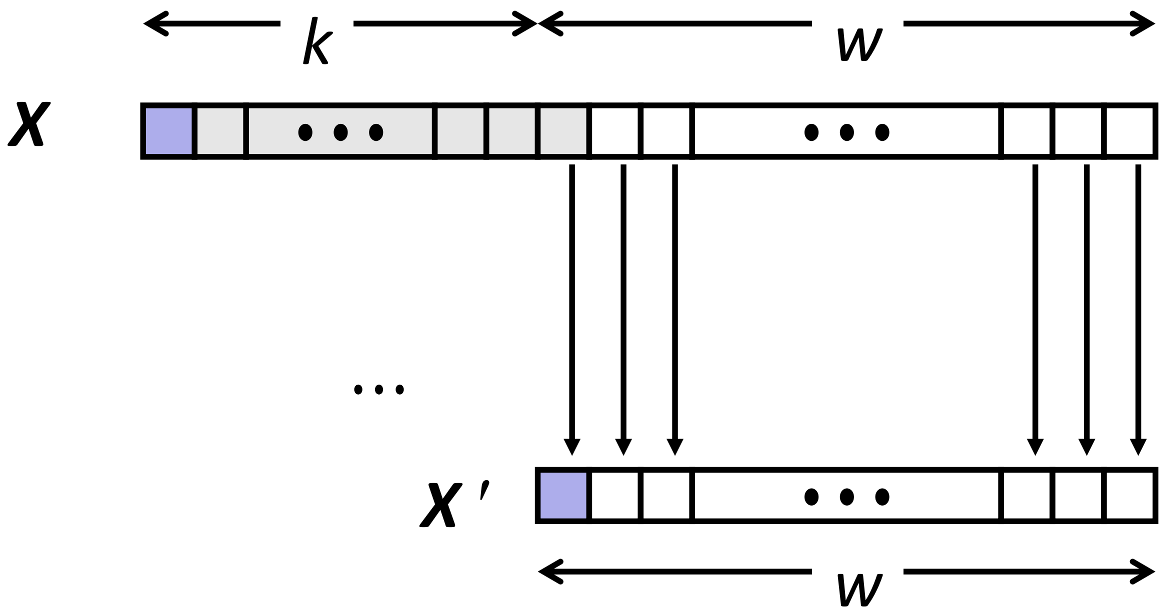
```
short int x =  15213;
int       ix = (int) x;
short int y = -15213;
int       iy = (int) y;
```

	Decimal	Hex	Binary
x	15213	3B 6D	00111011 01101101
ix	15213	00 00 3B 6D	00000000 00000000 00111011 01101101
y	-15213	C4 93	11000100 10010011
iy	-15213	FF FF C4 93	11111111 11111111 11000100 10010011

- Converting from smaller to larger integer data type
- C automatically performs sign extension

Truncation

- **Task:**
  - Given k+*w*-bit signed or unsigned integer *X*
  - Convert it to *w*-bit integer X' with same value for "small enough" X
- **Rule:**
  - Drop top *k* bits:
  - $X' = x_{w-1}, x_{w-2}, \ldots, x_0$



Truncation: Simple Example

No sign change

	-16	8	4	2	1
2 =	0	0	0	1	0

	-8	4	2	1
2 =	0	0	1	0

2 mod 16 = 2

	-16	8	4	2	1
-6 =	1	1	0	1	0

	-8	4	2	1
-6 =	1	0	1	0

-6 mod 16 = 26U mod 16 = 10U = -6

Sign change

	-16	8	4	2	1
10 =	0	1	0	1	0

	-8	4	2	1
-6 =	1	0	1	0

10 mod 16 = 10U mod 16 = 10U = -6

	-16	8	4	2	1
-10 =	1	0	1	1	0

	-8	4	2	1
6 =	0	1	1	0

-10 mod 16 = 22U mod 16 = 6U = 6

Summary: Expanding, Truncating: Basic Rules

- **Expanding (e.g., short int to int)**
  - Unsigned: zeros added
  - Signed: sign extension
  - Both yield expected result

- **Truncating (e.g., unsigned to unsigned short)**
  - Unsigned/signed: bits are truncated
  - Result reinterpreted
  - Unsigned: mod operation
  - Signed: similar to mod
  - For small (in magnitude) numbers yields expected behavior

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Bits, Bytes, and Integers – Part 2

Computer Systems
Friday, September 22 2023

Today: Bits, Bytes, and Integers

- Representing information as bits
- Bit-level manipulations
- Integers
  - Representation: unsigned and signed; negation and addition
  - Conversion, casting, extension, truncation
  - Multiplication, division, shifting
- Byte order in memory, pointers, strings

Encoding "Integers"

Unsigned | **Signed (twos complement)**

Given a bit vector x, w bits long…

$$\mathrm{B2U}(x) = \sum_{i=0}^{w-1} x_i \cdot 2^i$$

$$\mathrm{B2T}(x) = -x_{w-1} \cdot 2^{w-1} + \sum_{i=0}^{w-2} x_i \cdot 2^i$$

Sign Bit

Examples (w = 5)

±16	8	4	2	1
0	1	0	1	0

$0 + 8 + 0 + 2 + 0 =$ **10**

16	8	4	2	1
1	0	1	1	0
-16	8	4	2	1

$16 + 8 + 0 + 2 + 0 =$ **26**

$-16 + 8 + 0 + 2 + 0 =$ **-10**

Negation: Complement & Increment

Negate through complement and increase

`~x + 1 == -x`

Why?

- `-x + x         == 0` *(by definition)*
- `~x + x         == 1111…111 == -1`
- `~x + x + 1 == 0`
- `(~x+1) + x == 0`
- `~x+1          == -x`

x	1 0 0 1 1 1 0 1
+ ~x	0 1 1 0 0 0 1 0
−1	1 1 1 1 1 1 1 1

Example: x = 15213

	Decimal	Hex	Binary
`x`	15213	3B 6D	00111011 01101101
`~x`	-15214	C4 92	11000100 10010010
`~x+1`	-15213	C4 93	11000100 10010011
`y`	-15213	C4 93	11000100 10010011

Complement & Increment Examples

$x = 0$

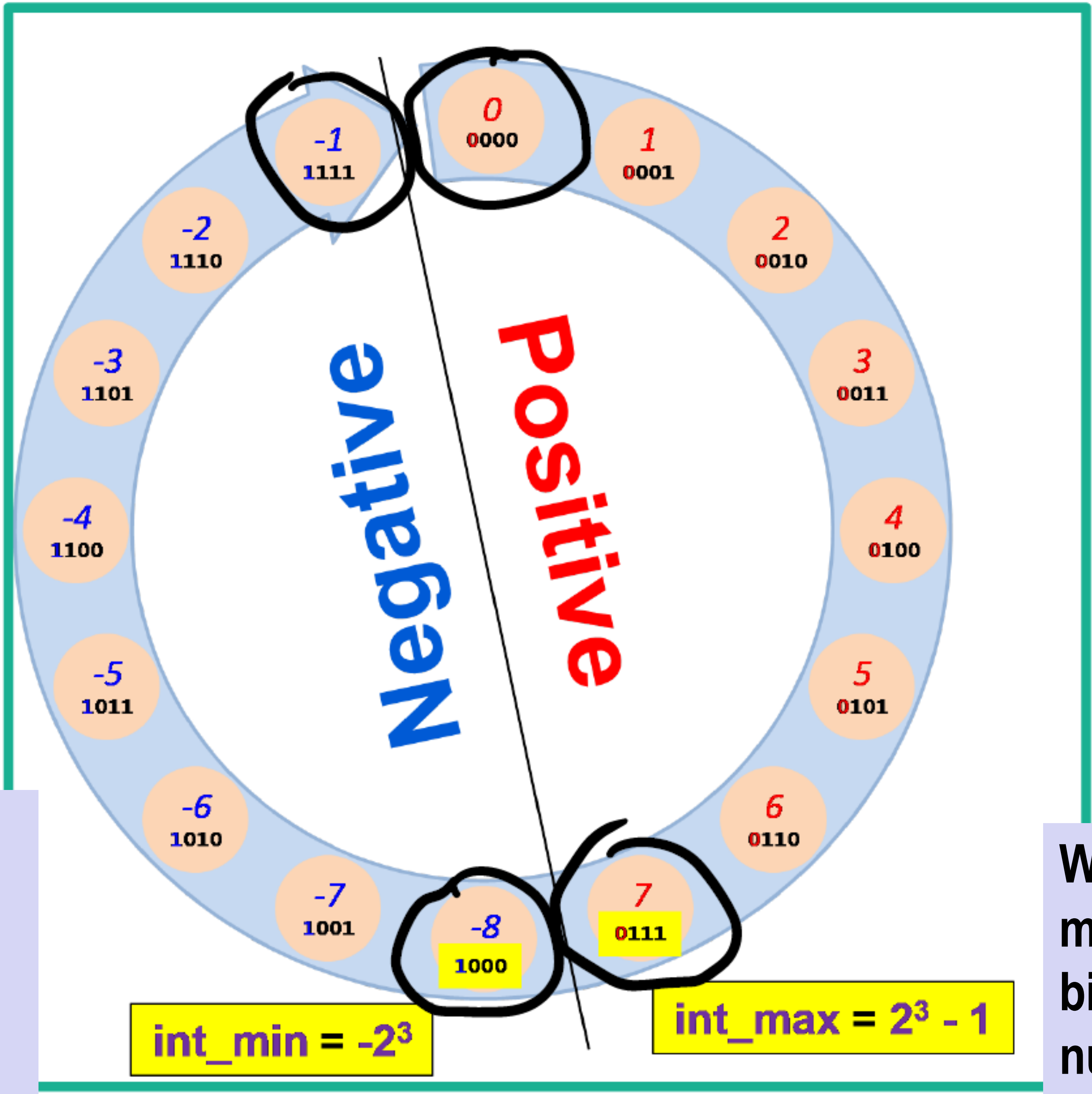
	Decimal	Hex	Binary
0	**0**	00 00	00000000 00000000
~0	**-1**	FF FF	11111111 11111111
~0+1	**0**	00 00	00000000 00000000

$x = T_{\min}$

	Decimal	Hex	Binary
x	**-32768**	80 00	10000000 00000000
~x	**32767**	7F FF	01111111 11111111
~x+1	**-32768**	80 00	10000000 00000000

Oops! It's still negative!

Eight negative values: –1, – 2, …, – 8



Eight *non*-negative values: 0, 1, …, 7

Mathematicians would prefer it if a 4-bit signed

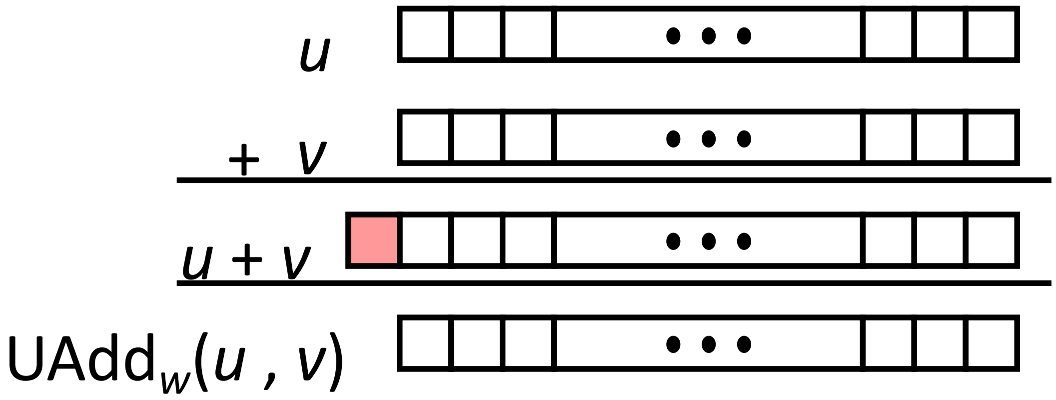
What if we made a 4-bit signed number only

Unsigned Addition

Operands: w bits

True Sum: w+1 bits

Discard Carry: w bits



- **Standard Addition Function**
  - Ignores carry output
- **Implements Modular Arithmetic**

$s \quad = \quad \text{UAdd}_w(u\ ,\ v) \quad = \quad u + v \mod 2^w$

```
unsigned char      1110 1001        E9         233
                 +  1101 0101     + D5       + 213
                 ────────────     ────       ─────

                 ────────────     ────       ─────
```

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
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C	12	1100
D	13	1101
E	14	1110
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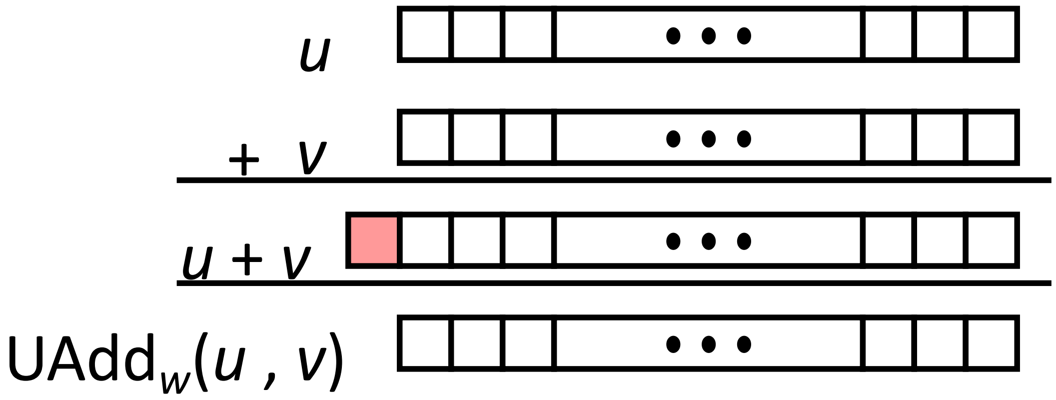


- **Standard Addition Function**
  - Ignores carry output
- **Implements Modular Arithmetic**

$$s \quad = \quad \mathrm{UAdd}_w(u\ ,\ v) \quad = \quad u + v \mod 2^w$$

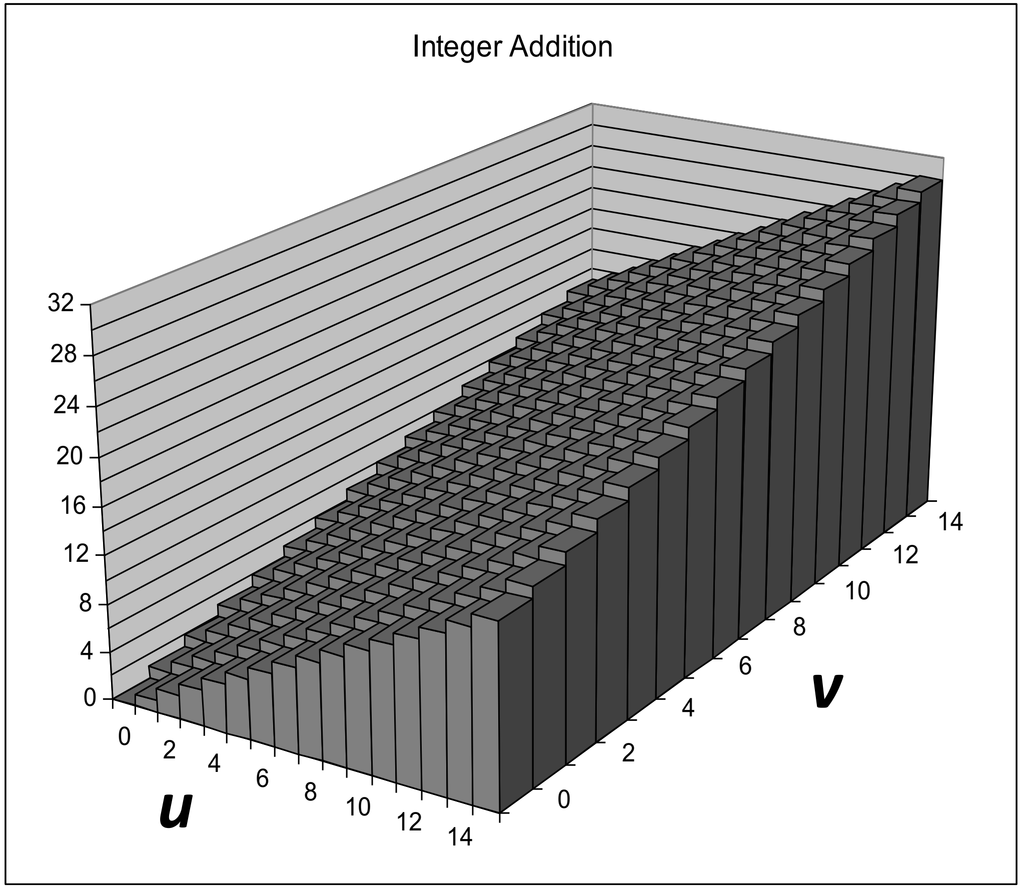
unsigned char			
	1110 1001	E9	233
	+ 1101 0101	+ D5	+ 213
	1 1011 1110	1BE	446
	1011 1110	BE	190

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Visualizing (Mathematical) Integer Addition

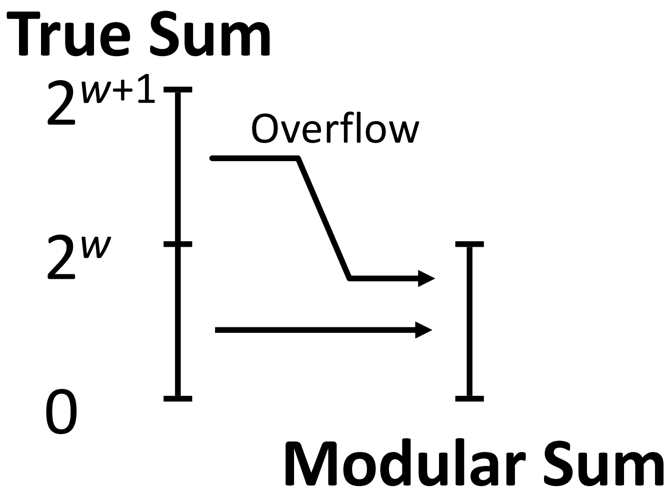
- **Integer Addition**
  - 4-bit integers *u*, *v*
  - Compute true sum $\text{Add}_4(u\ ,\ v)$
  - Values increase linearly with *u* and *v*
  - Forms planar surface

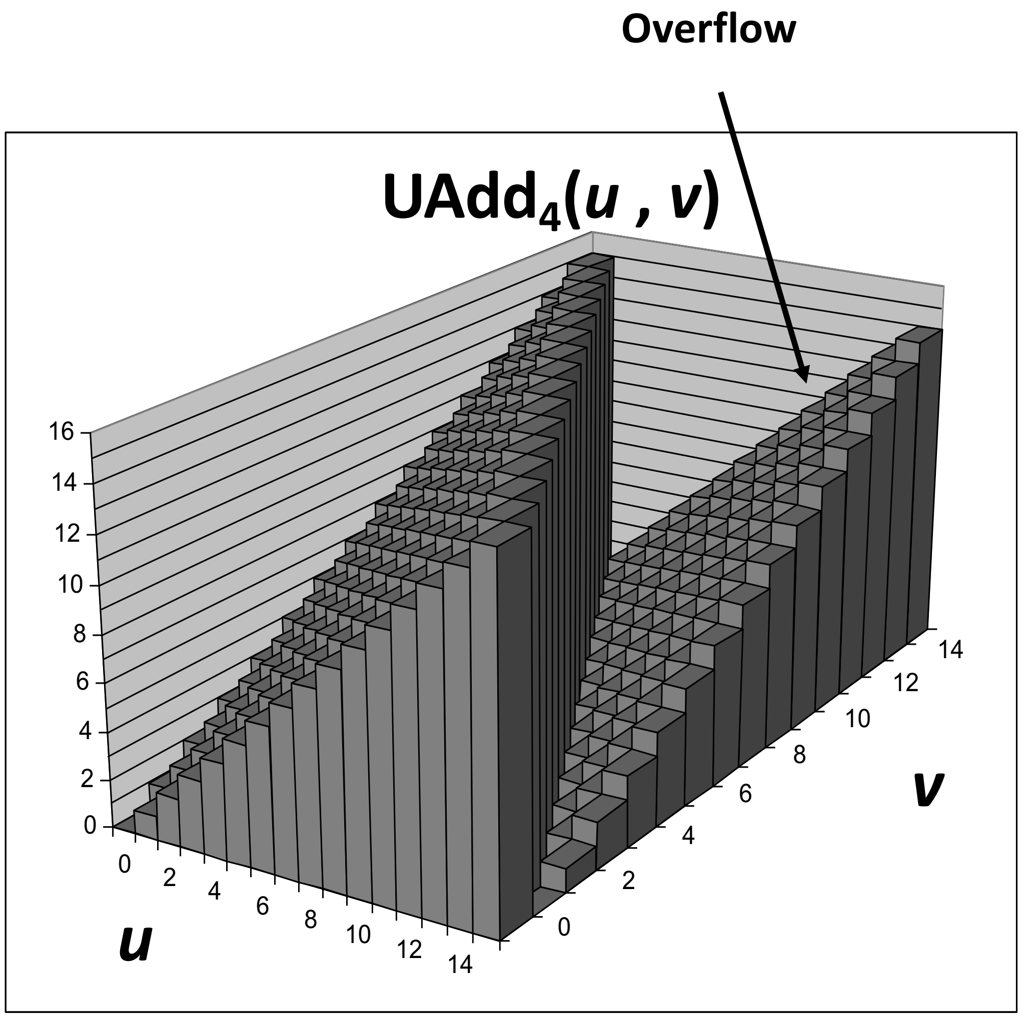
$\text{Add}_4(u\ ,\ v)$



Visualizing Unsigned Addition

- **Wraps Around**
  - If true sum ≥ 2^w
  - At most once



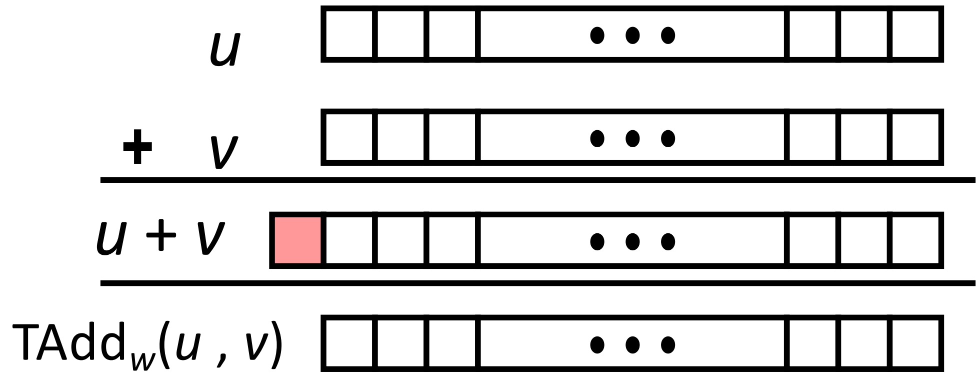


Two's Complement Addition

Operands: *w* bits

True Sum: *w*+1 bits

Discard Carry: *w* bits



- **TAdd and UAdd have Identical Bit-Level Behavior**
  - Signed vs. unsigned addition in C:

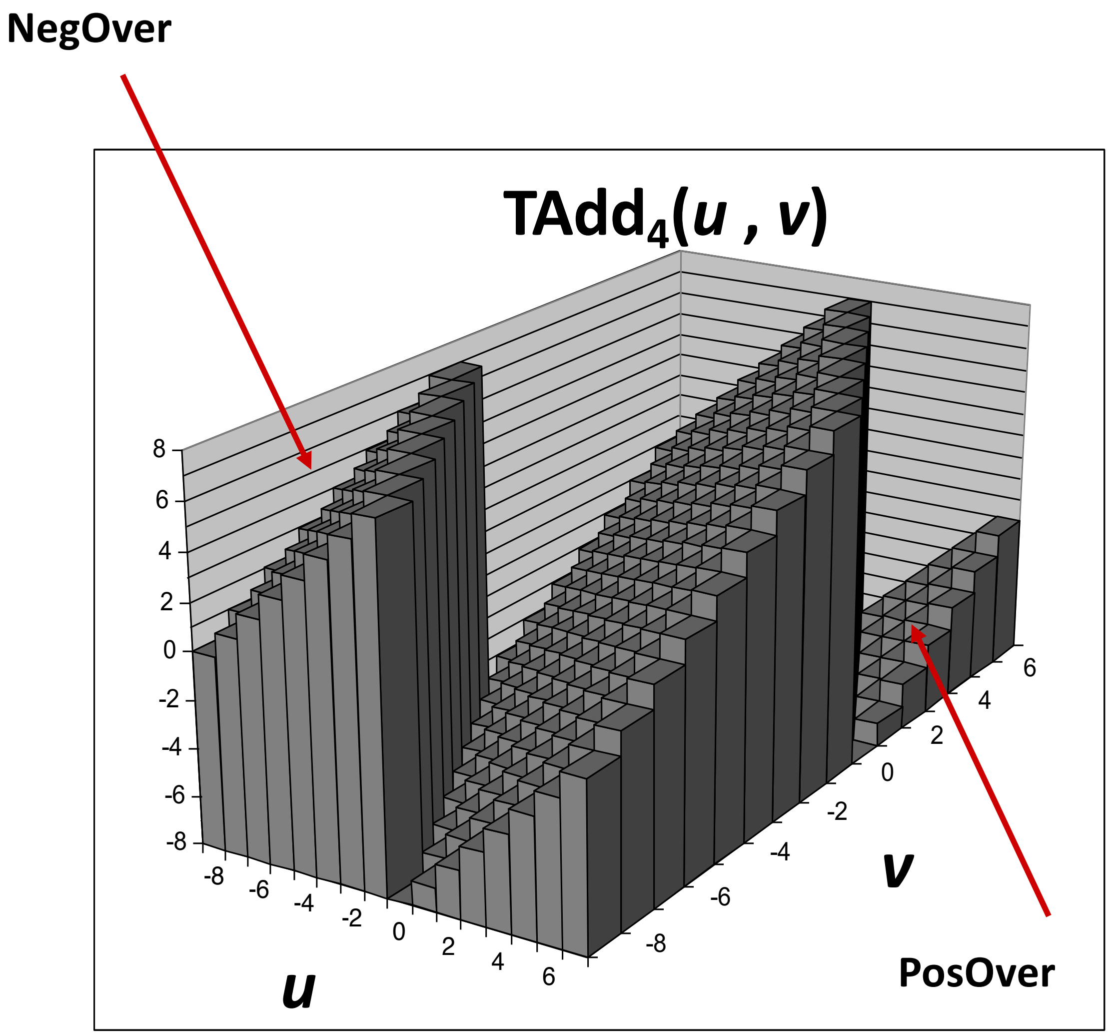
```
int s, t, u, v;
s = (int) ((unsigned) u + (unsigned) v);
t = u + v
```

  - Will give `s == t`

```
     1110 1001      E9      -23
+    1101 0101    + D5    + -43
   1 1011 1110     1BE      -66
     1011 1110      BE      -66
```

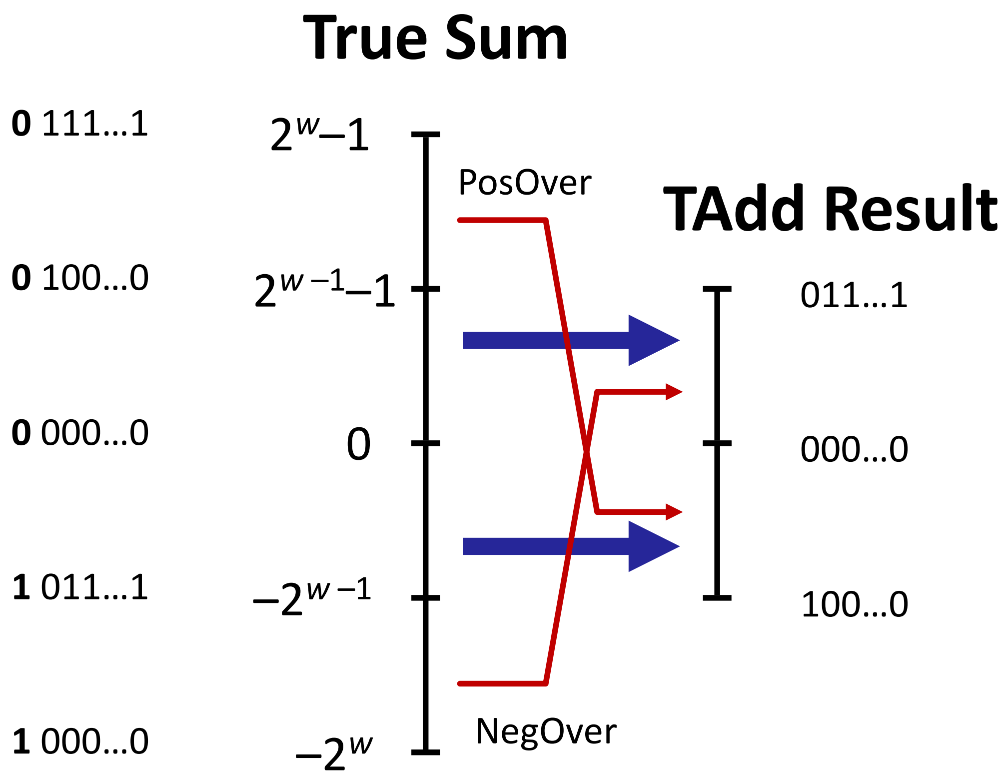
Visualizing 2's Complement Addition

- **Values**
  - 4-bit two's comp.
  - Range from -8 to +7
- **Wraps Around**
  - If sum $\geq 2^{w-1}$
    - Becomes negative
    - At most once
  - If sum $< -2^{w-1}$
    - Becomes positive
    - At most once



TAdd Overflow

- **Functionality**
  - True sum requires w+1 bits
  - Drop off MSB
  - Treat remaining bits as 2's comp. integer



Today: Bits, Bytes, and Integers

- Representing information as bits
- **Bit-level manipulations**
- Integers
  - Representation: unsigned and signed; negation and addition
  - Conversion, casting, extension, truncation
  - Multiplication, division, shifting
- Byte order in memory, pointers, strings

Boolean Algebra

Developed by George Boole in 19th Century

- Algebraic representation of logic
  - Encode "True" as 1 and "False" as 0

And

- **A&B = 1 when both A=1 and B=1**

&	0	1
0	0	0
1	0	1

Or

- **A|B = 1 when either A=1 or B=1**

|	0	1
0	0	1
1	1	1

Not

- **~A = 1 when A=0**

~	
0	1
1	0

Exclusive-Or (Xor)

- **A^B = 1 when either A=1 or B=1, but not both**

^	0	1
0	0	1
1	1	0

General Boolean Algebras

Operate on Bit Vectors

- Operations applied bitwise

```
  01101001      01101001      01101001
& 01010101    | 01010101    ^ 01010101    ~ 01010101
  01000001      01111101      00111100      10101010
```

All of the Properties of Boolean Algebra Apply

Example: Representing & Manipulating Sets

Representation

- Width w bit vector represents subsets of {0, …, w–1}
- $a_j = 1$ if $j \in A$

    - 01101001 { 0, 3, 5, 6 }
    - *76543210*

    - 01010101 { 0, 2, 4, 6 }
    - *76543210*

Operations

&	Intersection	01000001	{ 0, 6 }
|	Union	01111101	{ 0, 2, 3, 4, 5, 6 }
^	Symmetric difference	00111100	{ 2, 3, 4, 5 }
~	Complement	10101010	{ 1, 3, 5, 7 }

Bit-Level Operations in C

Operations &, |, ~, ^ Available in C

- Apply to any “integral” data type
  - long, int, short, char, unsigned
- View arguments as bit vectors
- Arguments applied bit-wise

Examples (Char data type)

- ~0x41 →
- ~0x00 →
- 0x69 & 0x55 →
- 0x69 | 0x55 →

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Bit-Level Operations in C

Operations &, |, ~, ^ Available in C

- Apply to any "integral" data type
  - long, int, short, char, unsigned
- View arguments as bit vectors
- Arguments applied bit-wise

Examples (Char data type)

- ~0x41 → 0xBE
  - ~0100 0001_2 → 1011 1110_2
- ~0x00 → 0xFF
  - ~0000 0000_2 → 1111 1111_2
- 0x69 & 0x55 → 0x41
  - 0110 1001_2 & 0101 0101_2 → 0100 0001_2
- 0x69 | 0x55 → 0x7D
  - 0110 1001_2 | 0101 0101_2 → 0111 1101_2

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Contrast: Logic Operations in C

Contrast to Bit-Level Operators

- **Logic Operations: &&, ||, !**
  - View 0 as "Fals
  - Anything nonze
  - Alway
  - Early

Example

- !0x41 →
- !0x00 →
- !!0x41→ 0x01

- 0x69 && 0x55 → 0x01
- 0x69 || 0x55 → 0x01
- p && *p (avoids null pointer access)

Watch out for && vs. & (and || vs. |)… one of the more common oopsies in C programming

Logical versus Bitwise

X	!X	!!X	!!X == X
-1	0	1	No
0	1	0	Yes
1	0	1	Yes
2	0	1	No

X	~X	~~X	~~X == X
-1	0	-1	Yes
0	-1	0	Yes
1	-2	1	Yes
2	-3	2	Yes

`!!x != x`

`~~x == x`

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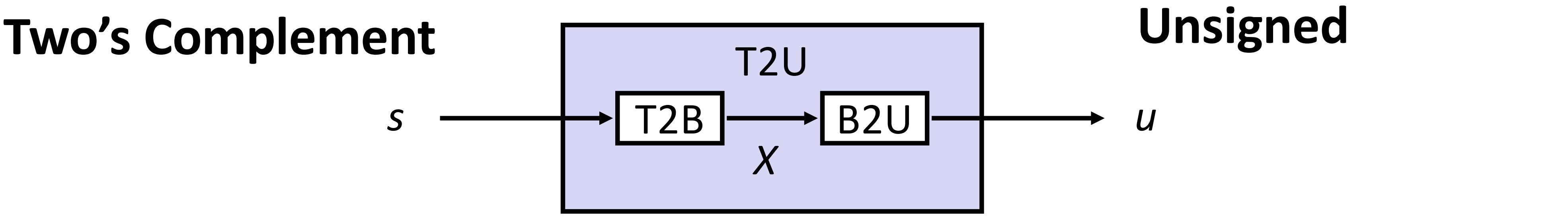
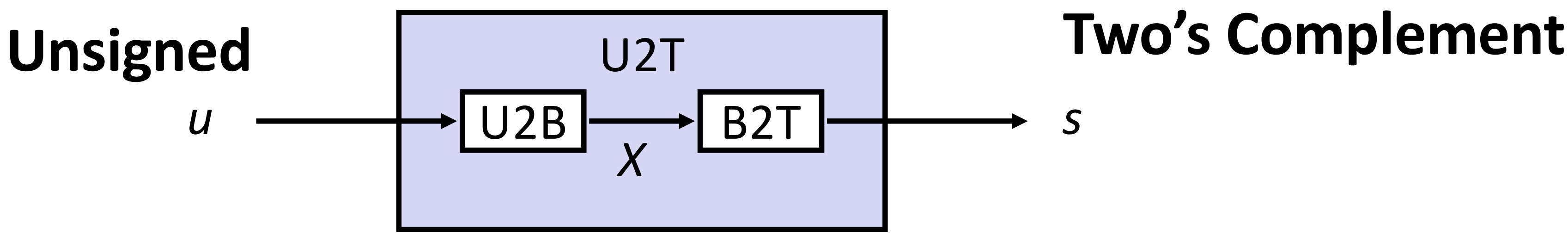
Mapping Between Signed & Unsigned



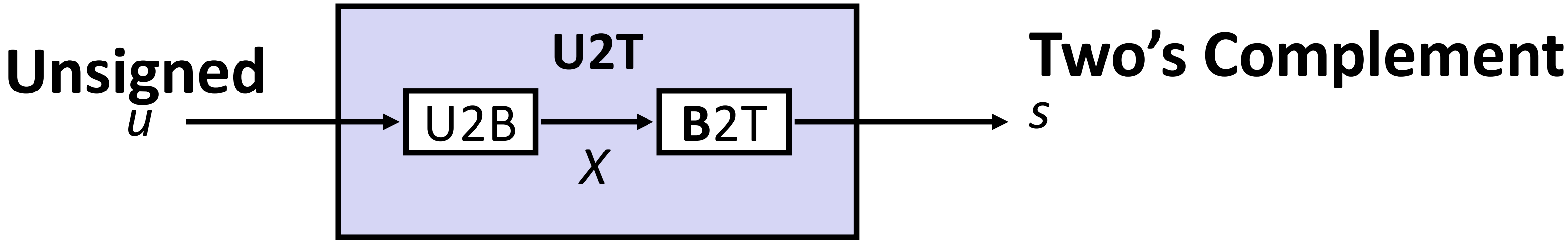
Maintain Same Bit Pattern



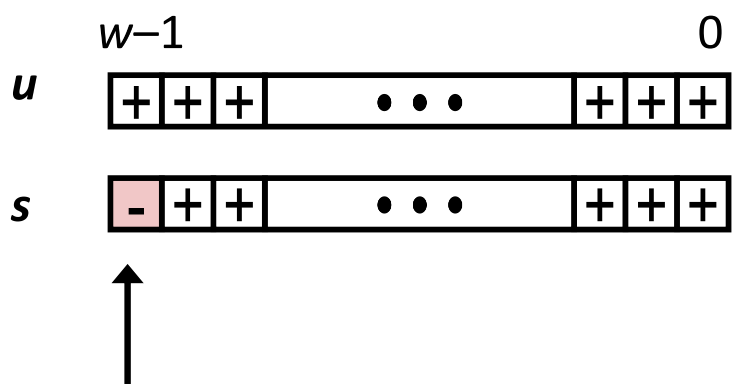
Maintain Same Bit Pattern

- **Mappings between unsigned and two's complement numbers:**
  Keep bit representations and reinterpret

Relation between Signed & Unsigned



Maintain Same Bit Pattern



Large positive weight

becomes

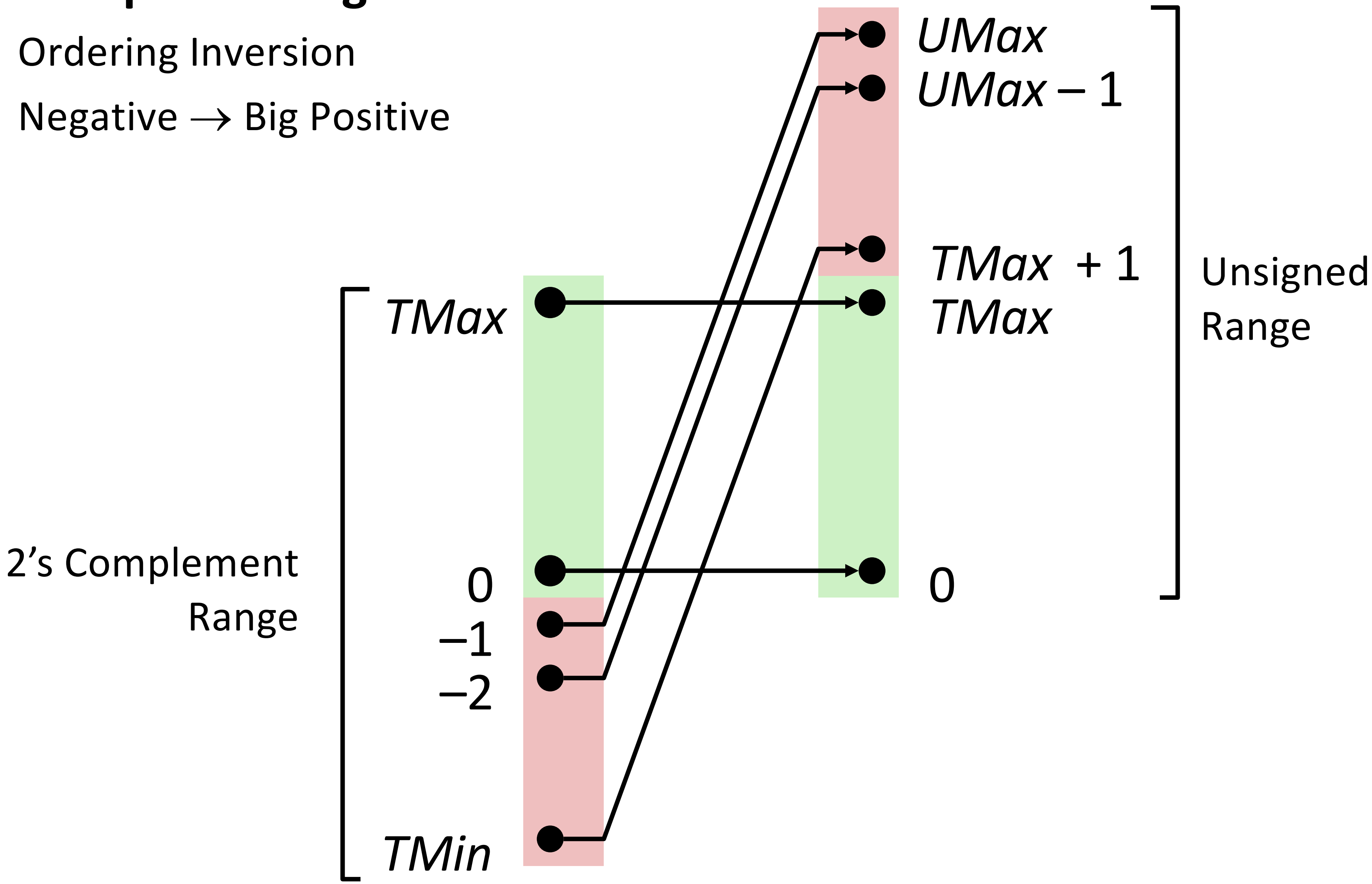
Large negative weight

Mapping Signed ↔ Unsigned

Bits	Signed		Unsigned
0000	0		0
0001	1		1
0010	2		2
0011	3	=	3
0100	4		4
0101	5		5
0110	6		6
0111	7		7
1000	-8		8
1001	-7		9
1010	-6	± 16	10
1011	-5		11
1100	-4		12
1101	-3		13
1110	-2		14
1111	-1		15

Conversion Visualized

- **2's Comp. → Unsigned**
  - Ordering Inversion
  - Negative → Big Positive



Signed vs. Unsigned in C

- **Constants**
  - By default are considered to be signed integers
  - Unsigned if have "U" as suffix

```
0U, 4294967259U
```

- **Casting**
  - Explicit casting between signed & unsigned same as U2T and T2U

```
int tx, ty;
unsigned ux, uy;
tx = (int) ux;
uy = (unsigned) ty;
```

  - Implicit casting also occurs via assignments and procedure calls

```
tx = ux;                    int fun(unsigned u);
uy = ty;                    uy = fun(tx);
```

Casting Surprises

- **Expression Evaluation**
  - If there is a mix of unsigned and signed in single expression, ***signed values implicitly cast to unsigned***
  - Including comparison operations **<, >, ==, <=, >=**
  - Examples:

Constant 1	Constant 2	Relation	Evaluation
`0`	`0U`	`==`	Unsigned
`-1`	`0`	`<`	Signed
`-1`	`0U`	`>`	Unsigned
`INT_MAX`	`INT_MIN`	`>`	Signed
`(unsigned)INT_MAX`	`INT_MIN`	`<`	Unsigned
`-1`	`-2`	`>`	Signed
`(unsigned)-1`	`-2`	`>`	Unsigned
`INT_MAX`	`((unsigned)INT_MAX) + 1`	`<`	Unsigned
`INT_MAX`	`(int)(((unsigned)INT_MAX) +`	`>`	Signed

Summary
Casting Signed ↔ Unsigned: Basic Rules

- **Bit pattern is maintained**
- **But reinterpreted**
- **Can have unexpected effects: adding or subtracting 2^w**

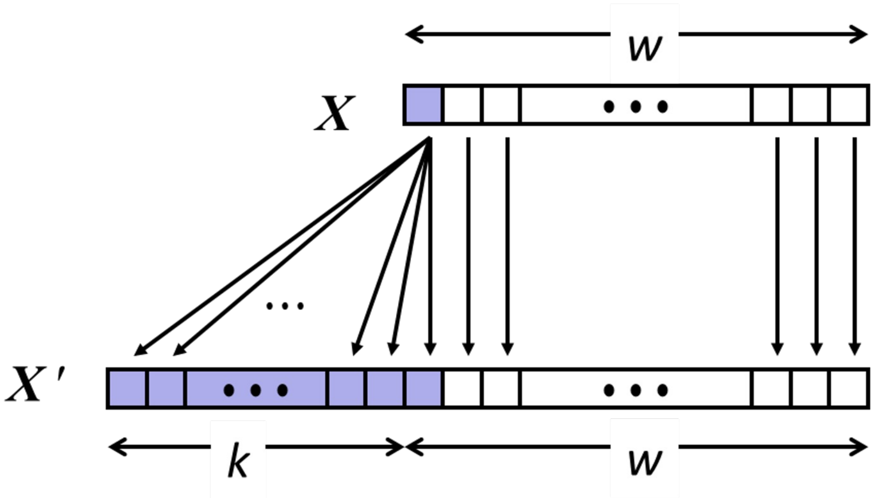
- **Expression containing signed and unsigned int**
  - `int` is cast to `unsigned`!!

Today: Bits, Bytes, and Integers

- Representing information as bits
- Bit-level manipulations
- **Integers**
  - Representation: unsigned and signed; negation and addition
  - Conversion, casting, extension, truncation
  - **Multiplication, division, shifting**
- Byte order in memory, pointers, strings

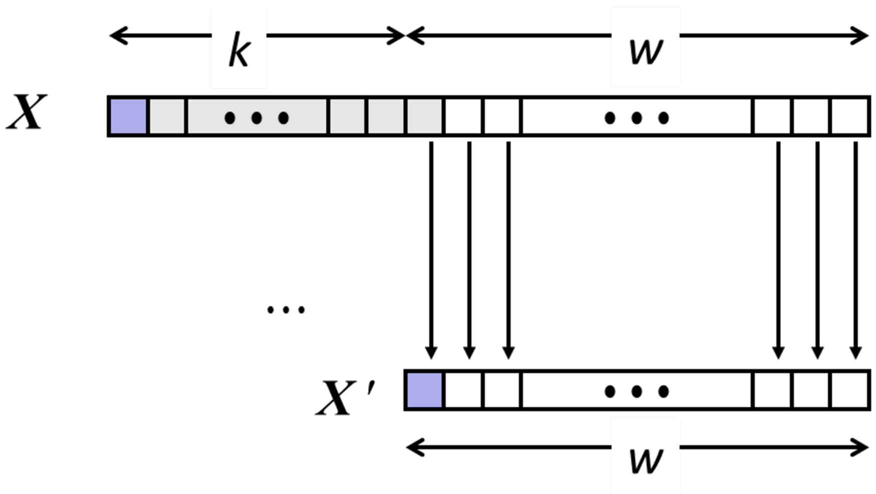
Sign Extension and Truncation

Sign Extension



Make κ copies of sign bit

Truncation



Chop off κ highest bits

Sign Extension: Simple Example

Positive number

10 =

-16	8	4	2	1
0	1	0	1	0

10 =

-32	16	8	4	2	1
0	0	1	0	1	0

Negative number

-10 =

-16	8	4	2	1
1	0	1	1	0

-10 =

-32	16	8	4	2	1
1	1	0	1	1	0

Truncation: Simple Example

No sign change

	-16	8	4	2	1
2 =	0	0	0	1	0

	-8	4	2	1
2 =	0	0	1	0

`2 mod 16 = 2`

	-16	8	4	2	1
-6 =	1	1	0	1	0

	-8	4	2	1
-6 =	1	0	1	0

`-6 mod 16 = 26U mod 16 = 10U = -6`

Sign change

	-16	8	4	2	1
10 =	0	1	0	1	0

	-8	4	2	1
-6 =	1	0	1	0

`10 mod 16 = 10U mod 16 = 10U = -6`

	-16	8	4	2	1
-10 =	1	0	1	1	0

	-8	4	2	1
6 =	0	1	1	0

`-10 mod 16 = 22U mod 16 = 6U = 6`

Today: Bits, Bytes, and Integers

- Representing information as bits
- Bit-level manipulations
- Integers
  - Representation: unsigned and signed; negation
  - Conversion, casting
  - Extension, truncation, shifting
  - **Addition, multiplication**
- Representations in memory, pointers, strings

Shifting

- **Left Shift: `x << y`**
  - Shift bit-vector `x` left `y` positions
  - Throw away extra bits on left
  - Fill with 0's on right
  - Equivalent to multiplying by 2^y
- **Right Shift: `x >> y`**
  - Shift bit-vector `x` right `y` positions
  - Throw away extra bits on right
  - Two kinds:
    - "Logical": Fill with 0's on left
    - "Arithmetic": Replicate most significant bit on left
  - *Almost* equivalent to dividing by 2^y
- **Undefined Behavior (in C)**
  - Shift amount < 0 or ≥ word size

Argument `x`	01100010
<< 3	00010000
Logical >> 2	00011000
Arithmetic >> 2	00011000

Argument `x`	10100010
<< 3	00010000
Logical >> 2	00101000
Arithmetic >> 2	11101000

Multiplication

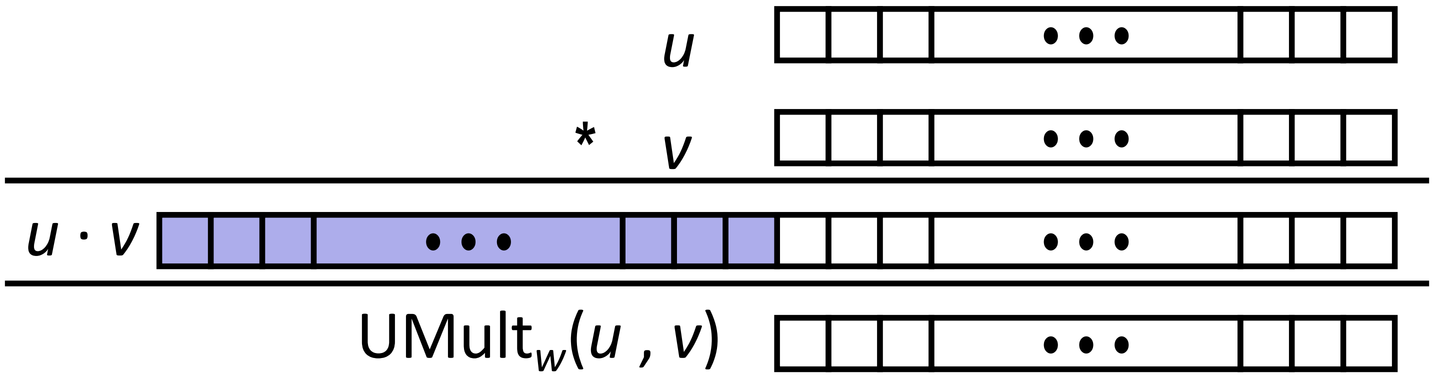
- **Goal: Computing Product of *w*-bit numbers *x*, *y***
  - Either signed or unsigned
- **But, exact results can be bigger than *w* bits**
  - Unsigned: up to 2*w* bits
    - Result range: $0 \leq x * y \leq (2^w - 1)^2 = 2^{2w} - 2^{w+1} + 1$
  - Two's complement min (negative): Up to 2*w*-1 bits
    - Result range: $x * y \geq (-2^{w-1})*(2^{w-1}-1) = -2^{2w-2} + 2^{w-1}$
  - Two's complement max (positive): Up to 2*w* bits, but only for $(TMin_w)^2$
    - Result range: $x * y \leq (-2^{w-1})^2 = 2^{2w-2}$
- **So, maintaining exact results…**
  - would need to keep expanding word size with each product computed
  - is done in software, if needed
    - e.g., by "arbitrary precision" arithmetic packages

Unsigned Multiplication in C

Operands: *w* bits

True Product: 2**w* bits

Discard *w* bits: *w* bits



- **Standard Multiplication Function**
  - Ignores high order *w* bits
- **Implements Modular Arithmetic**

  $\text{UMult}_w(u, v) = u \cdot v \bmod 2^w$

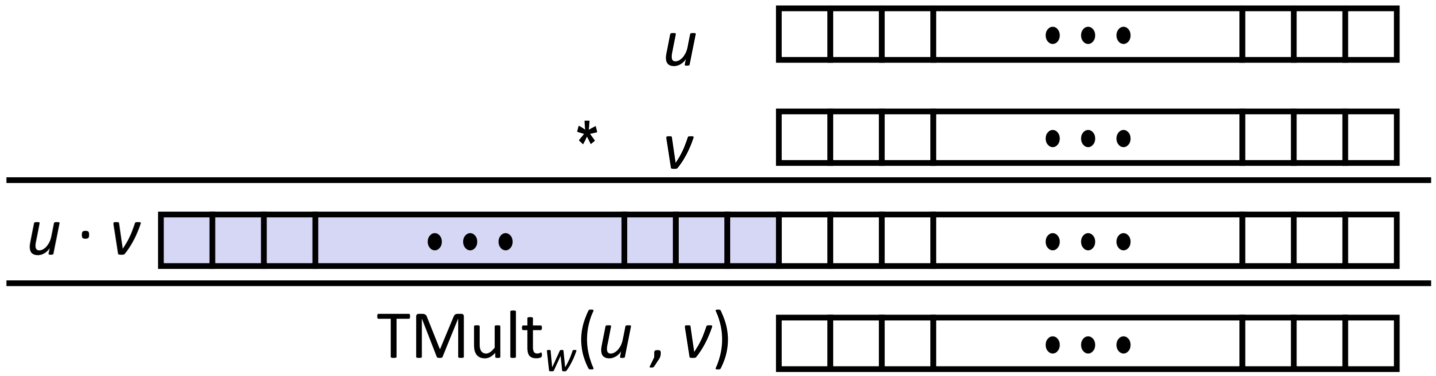
	1110 1001	E9	233
*	1101 0101	* D5	* 213
	1100 0001 1101 1101	C1DD	49629
	1101 1101	DD	221

Signed Multiplication in C

Operands: *w* bits

True Product: 2**w* bits

Discard *w* bits: *w* bits

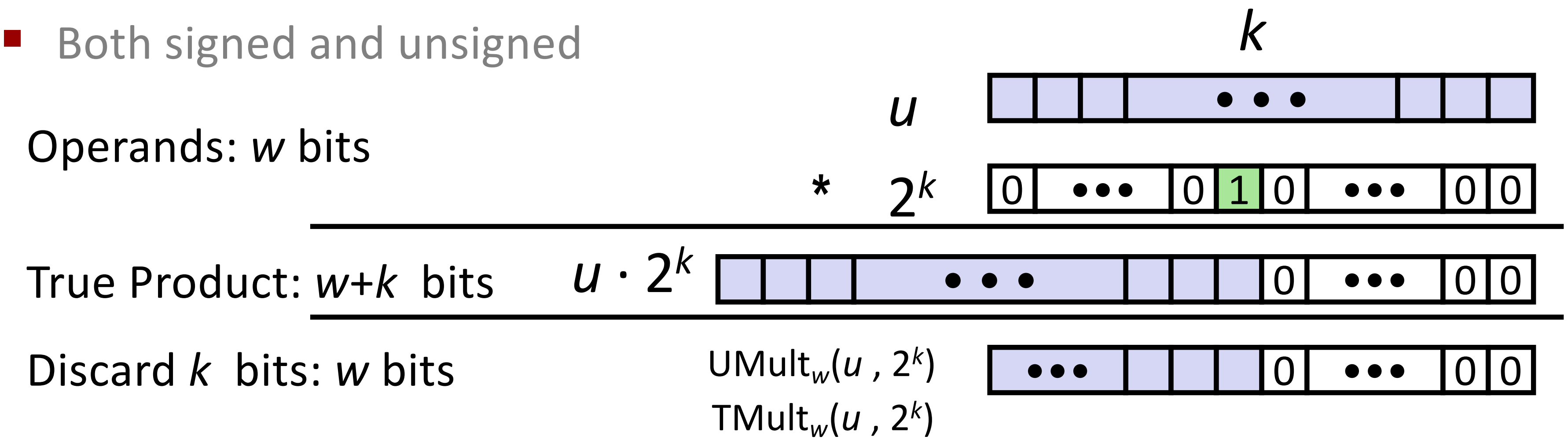


- **Standard Multiplication Function**
  - Ignores high order *w* bits
  - Some of which are different for signed vs. unsigned multiplication
  - Lower bits are the same

```
               1110 1001       E9         -23
*              1101 0101   *   D5   *     -43
0000 0011 1101 1101          03DD         989
               1101 1101       DD         -35
```

Power-of-2 Multiply with Shift

- **Operation**
  - `u << k` gives `u * 2`^{k}
  - Both signed and unsigned

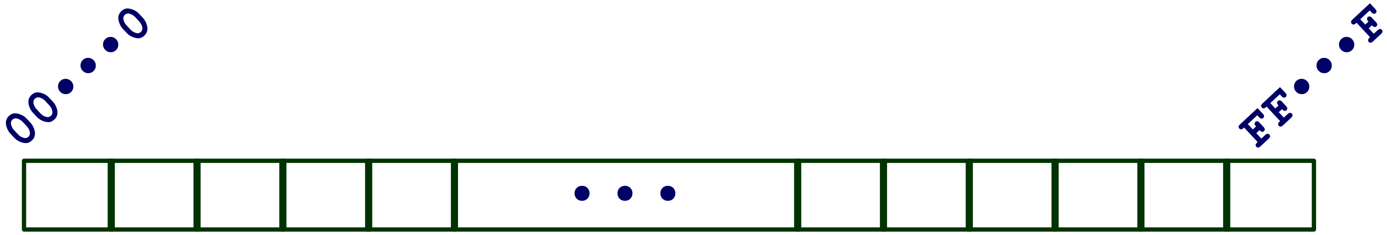


- **Examples**
  - `u << 3 == u * 8`
  - `(u << 5) - (u << 3)== u * 24`
  - Most machines shift and add faster than multiply
    - Compiler generates this code automatically

Today: Bits, Bytes, and Integers

- Representing information as bits
- Bit-level manipulations
- Integers
  - Representation: unsigned and signed; negation and addition
  - Conversion, casting, extension, truncation
  - Multiplication, division, shifting
- **Byte order in memory, pointers, strings**

Byte-Oriented Memory Organization



Programs refer to data by address

- Imagine all of RAM as an enormous array of bytes
- An address is an index into that array
  - A pointer variable stores an address

System provides a private *address space* to each "process"

- A process is an instance of a program, being executed
- An address space is one of those enormous arrays of bytes
- Each program can see only its own code and data within its enormous array
- We'll come back to this later ("virtual memory" classes)

Machine Words

Any given computer has a "Word Size"

- Nominal size of integer-valued data
  - and of addresses

- Until recently, most machines used 32 bits (4 bytes) as word size
  - Limits addresses to 4GB (2^{32} bytes)

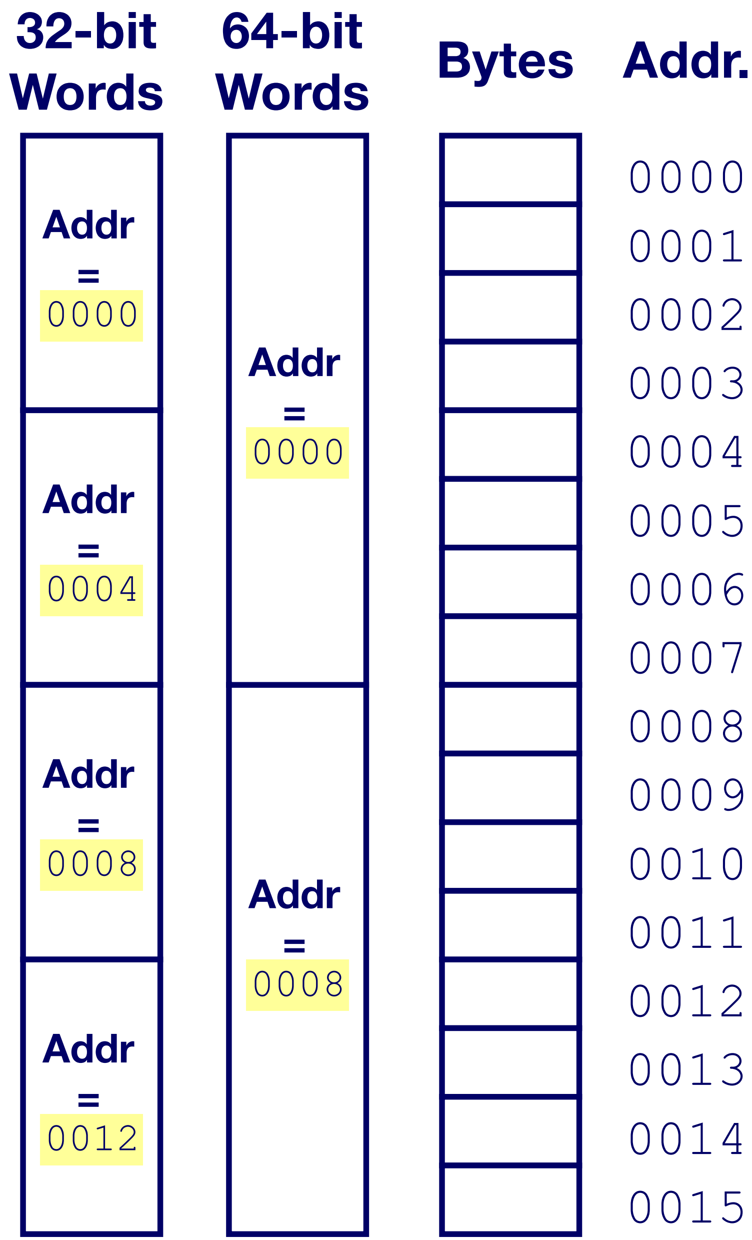
- Increasingly, machines have 64-bit word size
  - Potentially, could have 16 EB (exabytes) of addressable memory
  - That's 18.4×10^{18} bytes

- Machines still support multiple data formats
  - Fractions or multiples of word size
  - Always integral number of bytes

Yes, both of these numbers are correct. This discrepancy is known as the Great Storage Industry Marketing Lie. Ask me about it after class if you really want to know.

Addresses *Always* Specify Byte Locations

- Address of a word is address of the first byte in the word
- Addresses of successive words differ by 4 (32-bit) or 8 (64-bit)



Example Data Representations

C Data Type	Typical 32-bit	Typical 64-bit	x86-64
`char`	1	1	1
`short`	2	2	2
`int`	4	4	4
`long`	4	8	8
`float`	4	4	4
`double`	8	8	8
pointer	4	8	8

Byte Ordering

So, how are the bytes within a multi-byte word ordered in memory?

Conventions

- Big Endian: Sun, PPC Mac, *network packet headers*
  - Least significant byte has highest address
- Little Endian: *x86*, ARM processors running Android, iOS, and Windows
  - Least significant byte has lowest address

Byte Ordering Example

Example

- Variable x has 4-byte value of 0x01234567
- Address given by &x is 0x100

Big Endian

		0x100	0x101	0x102	0x103		
		01	23	45	67		

Little Endian

		0x100	0x101	0x102	0x103		
		67	45	23	01		

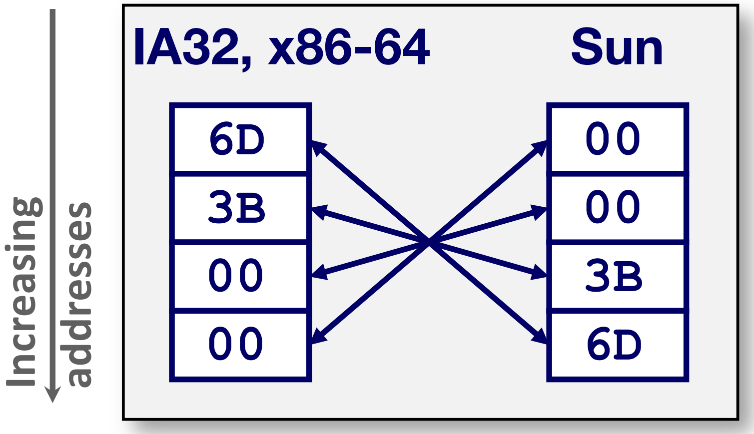
Representing Integers

Decimal: 15213

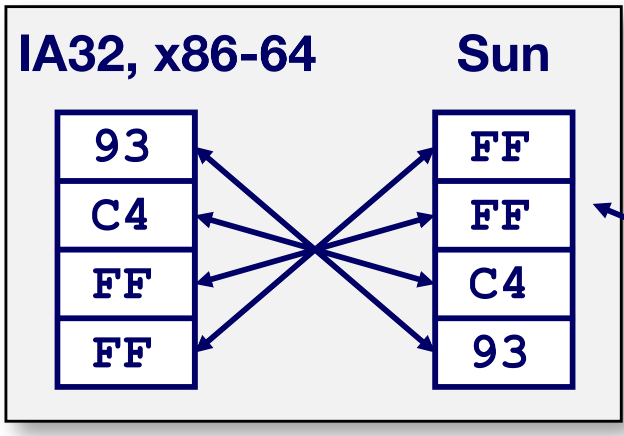
Binary: 0011 1011 0110 1101

Hex: 3 B 6 D

```
int A = 15213;
```

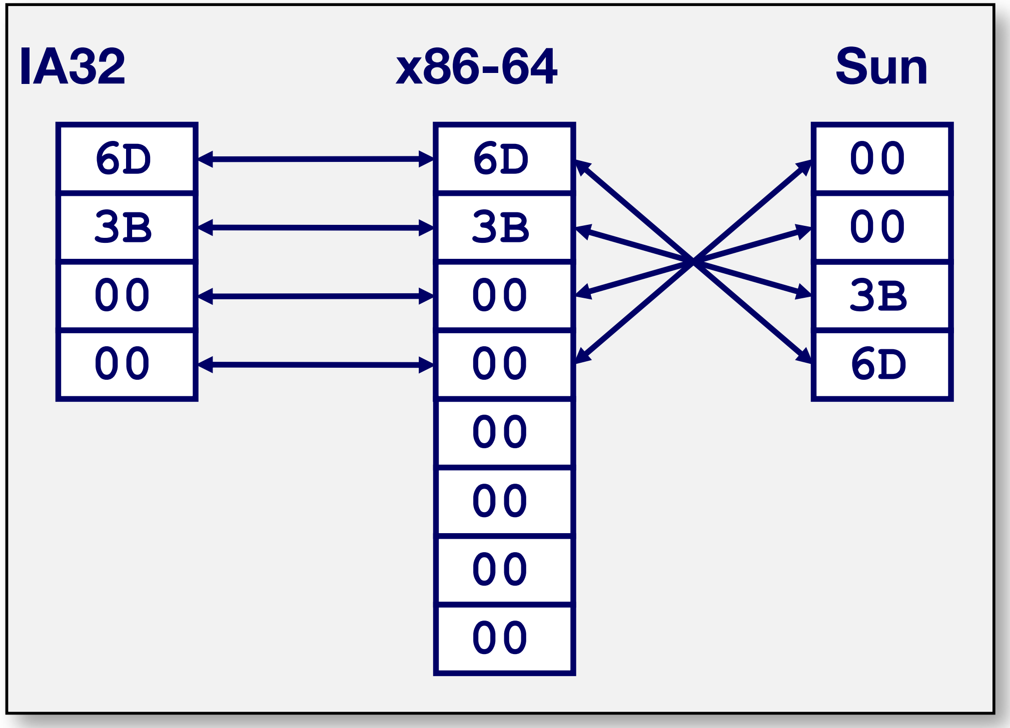



```
int B = -15213;
```



Two’s complement representation

```
long int C = 15213;
```



Examining Data Representations

Code to Print Byte Representation of Data

- Casting pointer to unsigned char * allows treatment as a byte array

```
typedef unsigned char *pointer;

void show_bytes(pointer start, size_t len){
  size_t i;
  for (i = 0; i < len; i++)
    printf(”%p\t0x%.2x\n",start+i, start[i]);
  printf("\n");
}
```

Printf directives:

%p: Print pointer

%x: Print Hexadecimal

`show_bytes` Execution Example

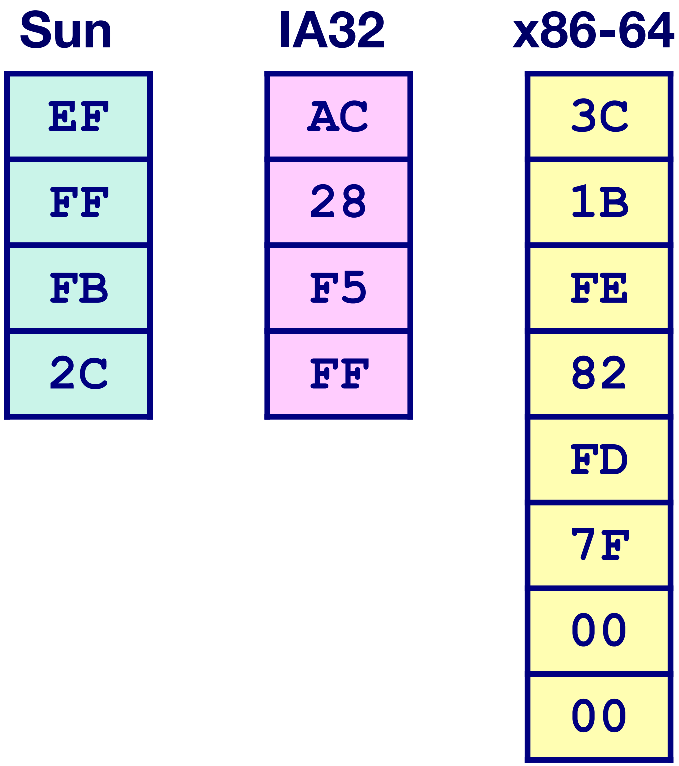
```
int a = 15213;
printf("int a = 15213;\n");
show_bytes((pointer) &a, sizeof(int));
```

Result (Linux x86-64):

```
int a = 15213;
0x7fffb7f71dbc    6d
0x7fffb7f71dbd    3b
0x7fffb7f71dbe    00
0x7fffb7f71dbf    00
```

Representing Pointers

```
int B = -15213;
int *P = &B;
```



Different compilers & machines assign different locations to objects

Even get different results each time run program

Representing Strings

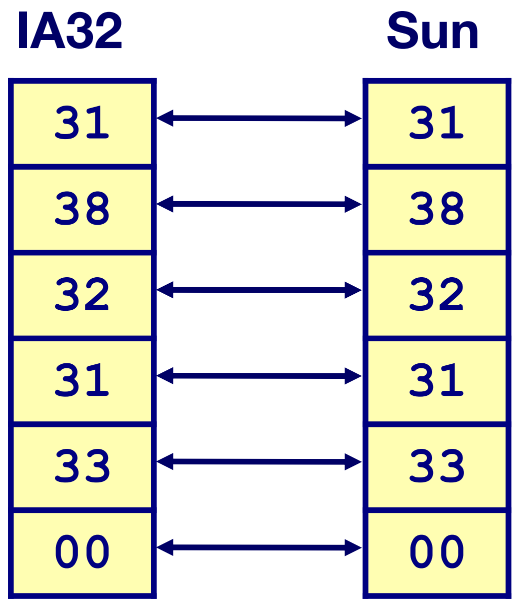
```
char S[6] = "18213";
```

Strings in C

- Represented by array of characters
- Each character encoded in ASCII format
  - Standard 7-bit encoding of character set
  - Character "0" has code 0x30
    - Digit *i* has code 0x30+*i*
- String should be null-terminated
  - Final character = 0

Compatibility

- Byte ordering not an issue



Representing x86 machine code

- **x86 machine code is a sequence of *bytes***
  - Grouped into variable-length instructions, which look like strings…
  - But they contain embedded little-endian numbers…
- **Example Fragment**

Address	Instruction Code	Assembly Rendition
`8048365:`	`5b`	`pop %ebx`
`8048366:`	`81 c3 ab 12 00 00`	`add $0x12ab,%ebx`
`804836c:`	`83 bb 28 00 00 00 00`	`cmpl $0x0,0x28(%ebx)`

- **Deciphering Numbers**
  - Value: `0x12ab`
  - Pad to 32 bits: `0x000012ab`
  - Split into bytes: `00 00 12 ab`
  - Reverse: `ab 12 00 00`