Tree

Ja-Hee Kim

Agenda

01 Introduction

Terminology, size

02 Binary Tree

Definition, examples

03 Tree Traversal

Implementing a binary tree using an array

04 Implementation of Tree

Array Tree and Linked Tree

Introduction of Tree

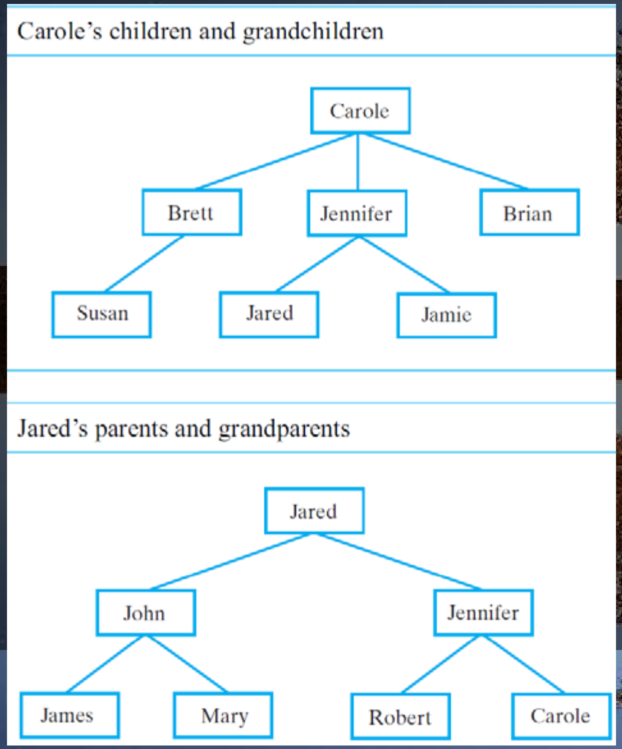
Introduction of a Tree

- Definition

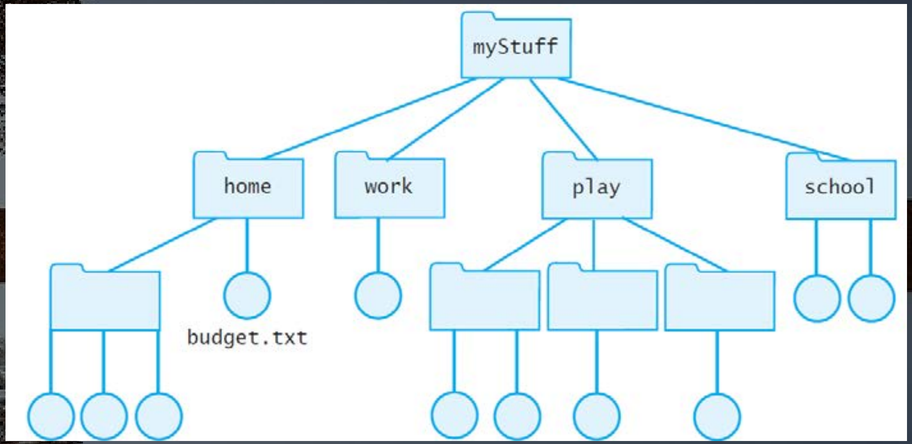
  a non-linear structure based on nodes and links

- Example

  Family tree



  Files in folders



Rooted Tree

- Empty tree is a tree.

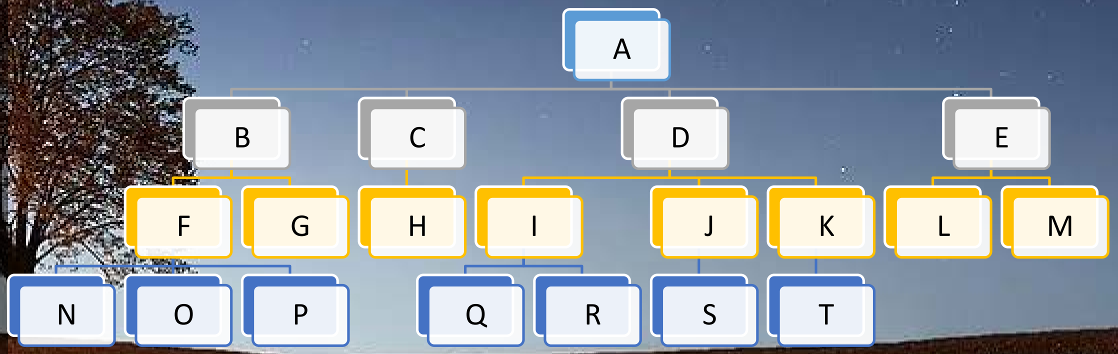
- If S is a set of trees
  - any trees of S do not share a node.
  - T = (r, S) is a tree
    - r is a root
    - a tree in S is a sub-tree of T

Terminologies of a Tree

- Node
- Edge
- Level
- Root
- Parent
- Children
- Ancestor
- Descendant
- Subtree
- Degree
- Leaf
- Interior node
- Path
- Height
- Depth
- Full
- Size

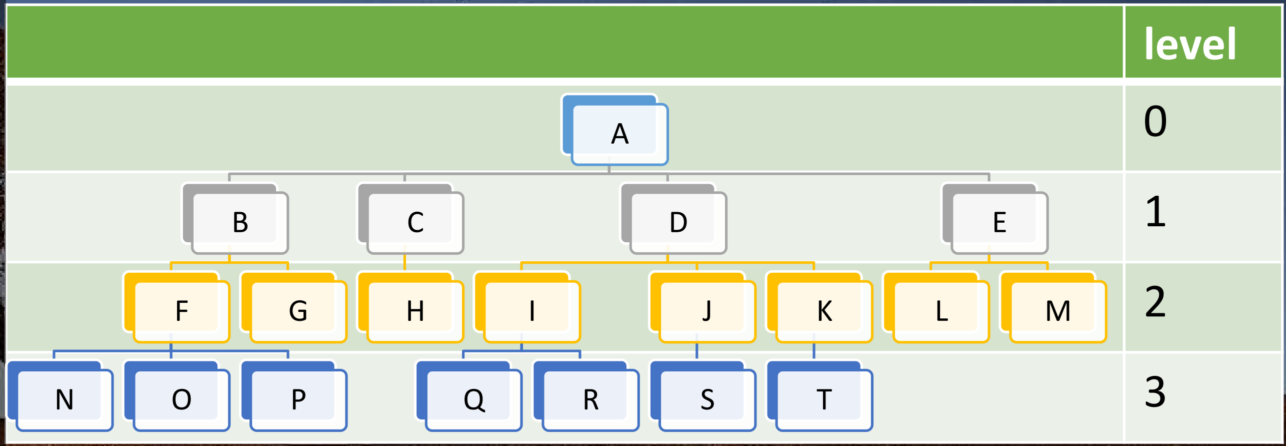
Node and edges

- A tree = nodes + edges
- Node = element
  - It usually contains data.
- Edge
  - the connecting link between any two nodes



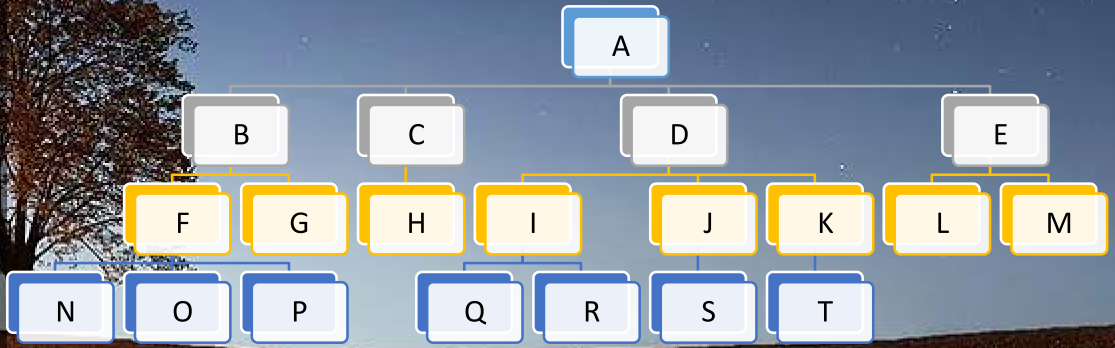
Level and Root

- Level
  - Node's hierarchy
- Root
  - A node in the top level
  - A single node in a tree



Parent & Children

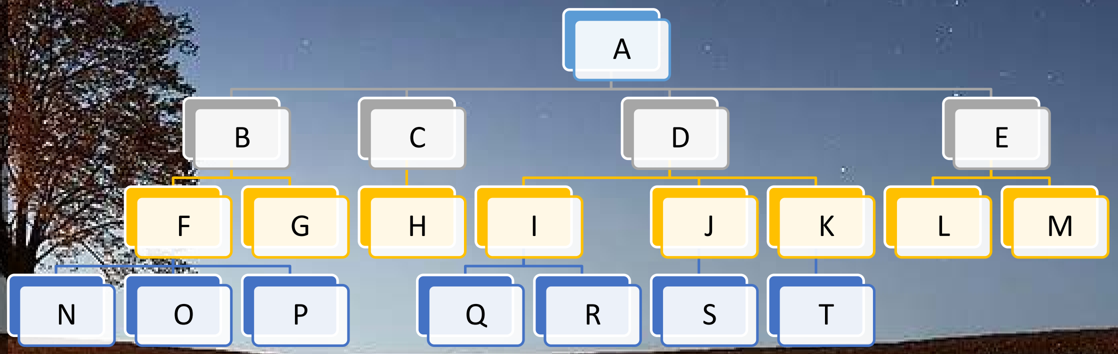
- A node which has branch from it to any other node
- Parent node
  - A node which has child / children
  - If two nodes are connected, the closer to the root is a parent.
- Children node
  - If two nodes are connected, the closer to the root is a parent.
- Sibling
  - Nodes with the same parent.



Ancestor vs descendant

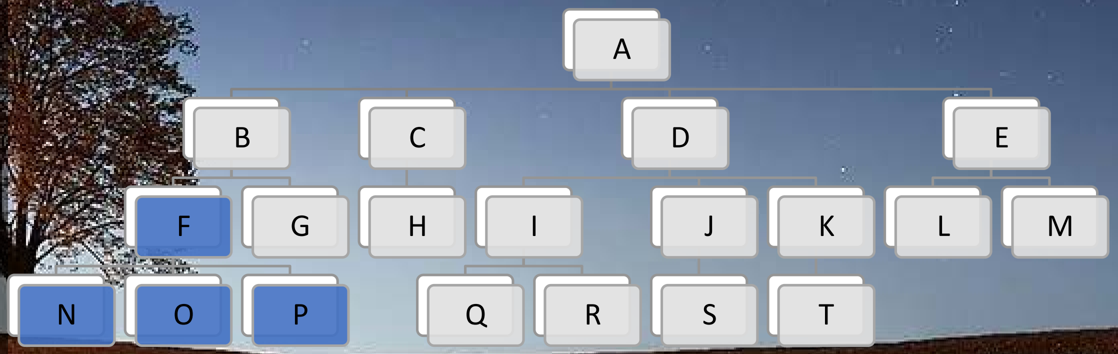
- descendant
  - either one of its children or a child of another descendant.

- Ancestor
  - If a node is an ancestor of an element x is an element of whom x is a descendant.



Subtree of a tree

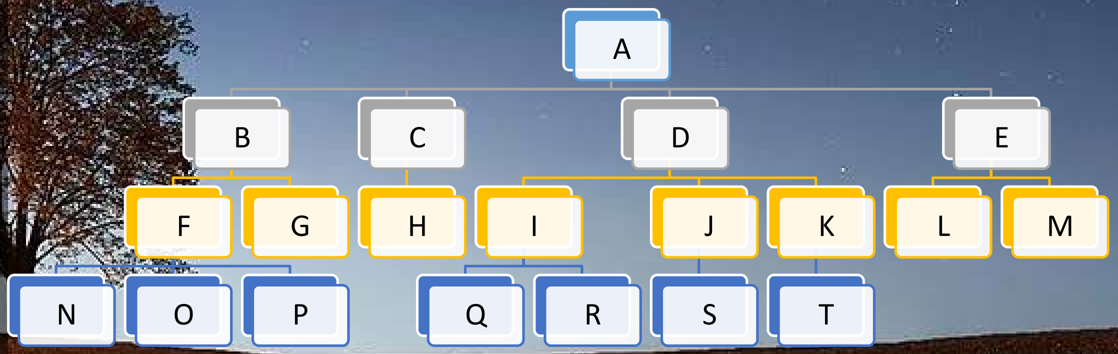
- Subtree
  - Any node and its descendants form a subtree of the original tree
  - a tree rooted at a child of that node.



Degree

- The degree
  - the size of its subtree set
  - its number of children.
  - Ex: deg(D) = 3

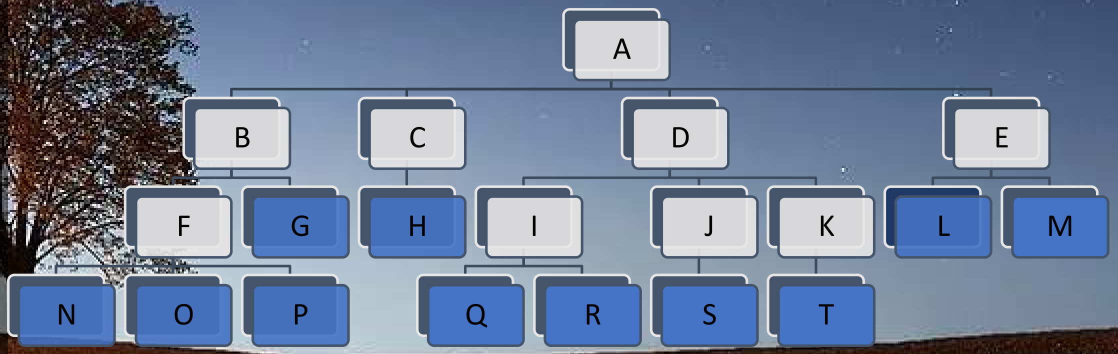
- The degree of a tree
  - the largest degree of its elements.
  - Ex: the degree of the right tree is 4 because of A.



Leaf vs Interior node

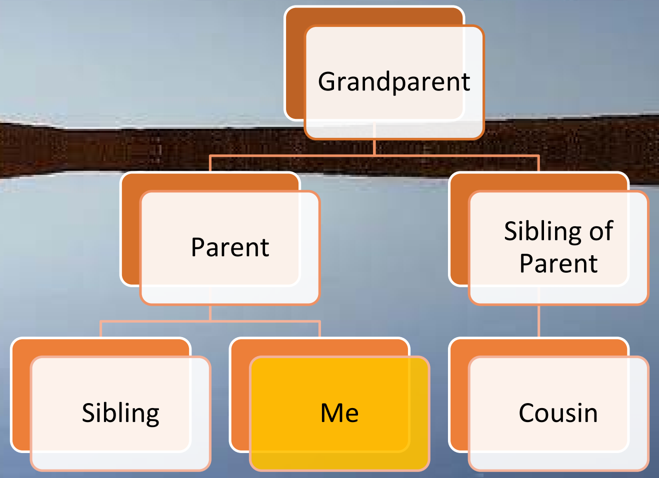
- Leaf
  - A node of degree 0
  - A node with no children.

- Interior node
  - A node is not a leaf.



Path

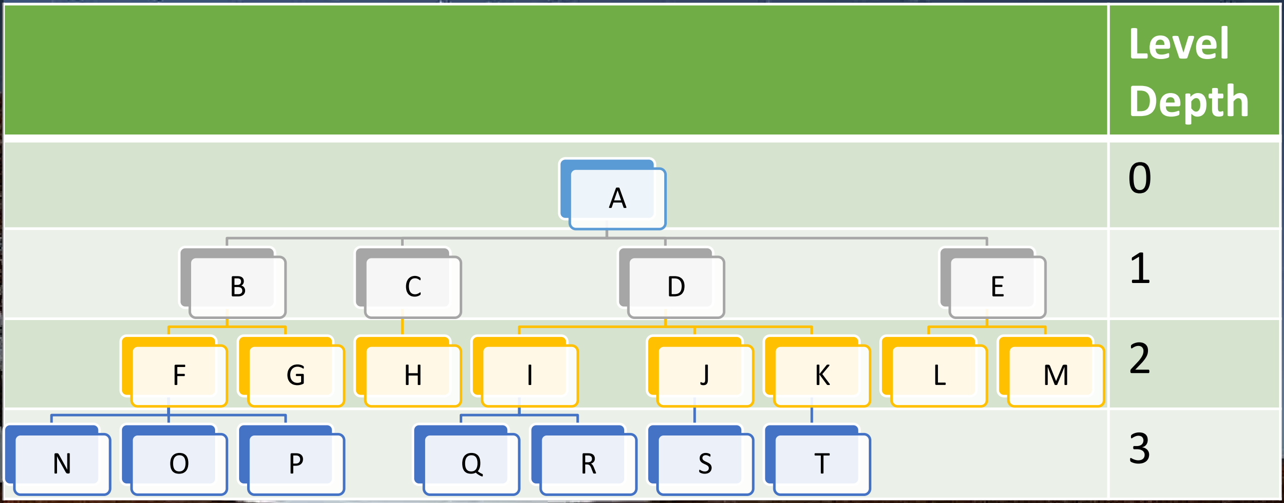
- Path
  - a sequence of elements, each the parent of the next element in the sequence.
- root path
  - a path that begins at the root.
- root-to-leaf path
  - a root path that ends at a leaf.
- The length of a path
  - the number of adjacent pairs in the path; i.e., the number of steps in the path.



Depth and Height

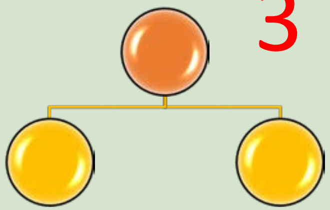
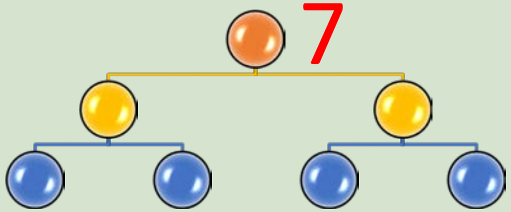
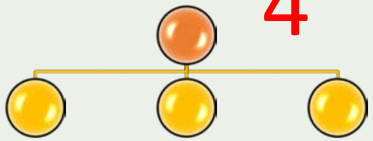
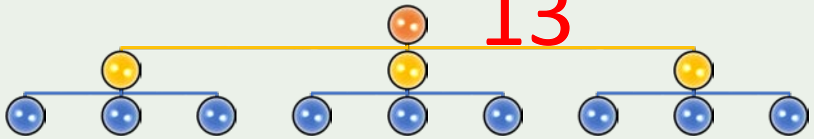
- The height of a tree
  - its greatest level; the length of its longest root-to-leaf path.

- The depth
  = Level
  - the length of its root path.



Full Tree

- A tree is full if all of its leaves are at the same level and all of its other nodes have the same degree.

Height / Degree	0	1	2
2	 1	 3	 7
3	 1	 4	 13

$$n = \frac{d^{h+1} - 1}{d - 1}$$

$$\frac{3^{2+1} - 1}{3 - 1} = \frac{27 - 1}{2} = 13$$

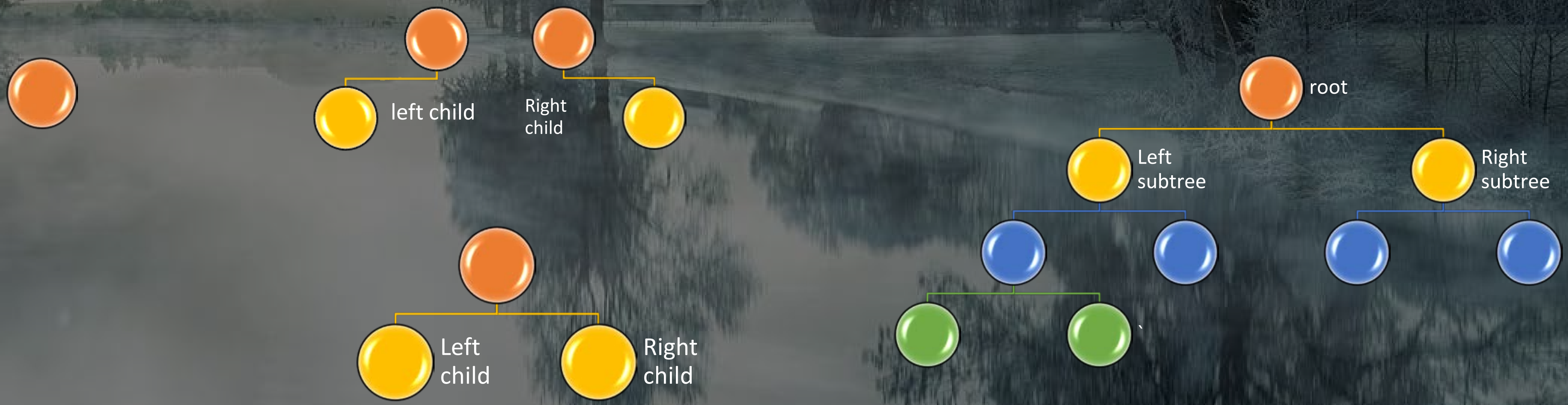
Binary Tree

Binary Tree

- A tree of which the maximum degree is two

  ≠ a tree of 2 degree

- Recursive definition of a binary tree
  - Empty tree is a binary tree.
  - Each tree has two subtrees whose root nodes are the nodes pointed to by the leftLink and rightLink of the root node.



Ordered Tree

- An ordered tree is an oriented tree in which the children of a node are somehow "ordered."

Ordered tree

Unordered tree

Terminologies

- Full binary tree
- Complete binary tree
- Skewed binary tree

Full Binary Tree

- Every node in the tree has either 0 or 2 children.
- The number of node of full binary tree

$$2^{h+1} - 1$$

  - Where h = the height of the tree

Complete Binary Tree

- In a complete binary tree every level, except possibly the last, is completely filled, and all nodes in the last level are as far left as possible.

Skewed Binary Tree

- Every node has only left (or right) sub

Left Skewed Binary Tree

Right Skewed Binary Tree

The number of nodes

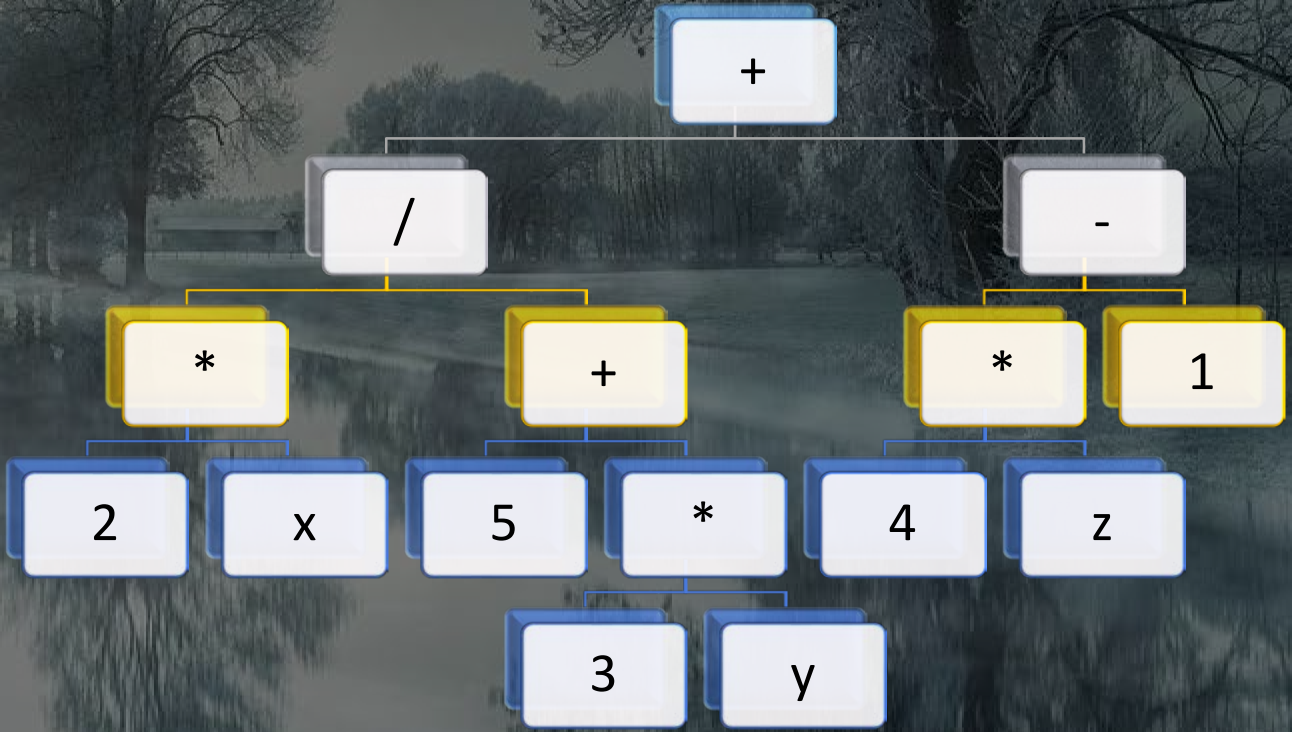
- The number of node of which tree (n)
  - Where the height is h
  - Is more than or equal to a skewed tree
  - Is less than or equal to a full tree

$$h + 1 \le n \le 2^{h+1} - 1$$

h=3

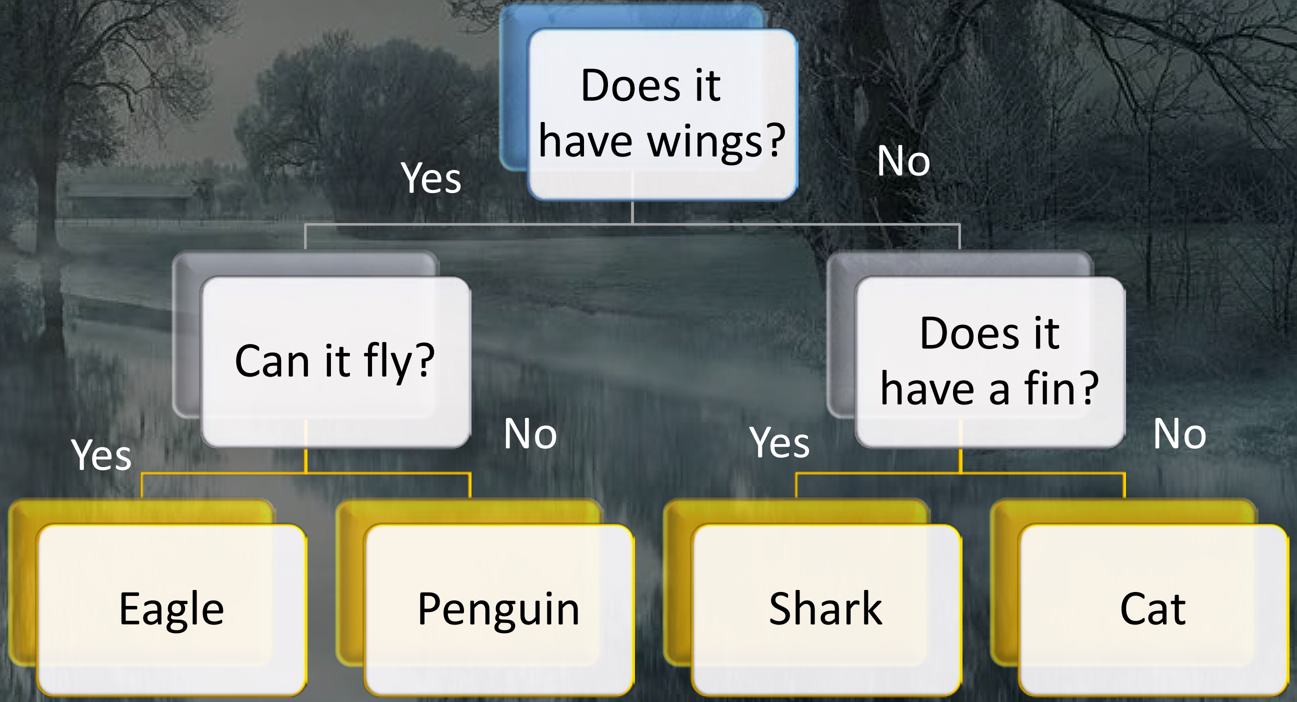
Example1: Expression Tree

- an arithmetic expression such that Each operand is in a distinct leaf.
- Each binary operator is in a distinct internal node whose two subtrees represent the two values upon which it operates.
- Ex: $\frac{2x}{5+3y} + (4z - 1)$



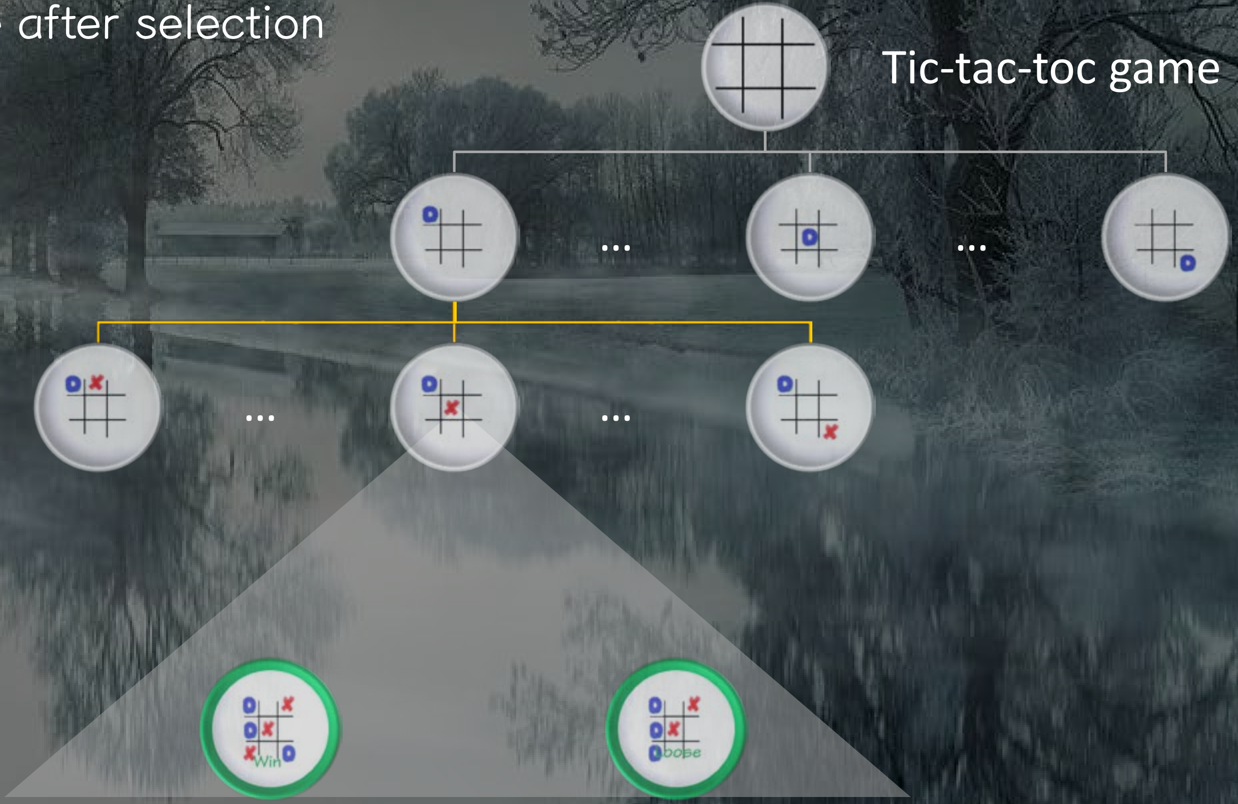
Example2: Decision Tree

- Expert System
  - System for helping its user to solve problems or make decisions
- Decision Tree
  - The basis of an expert system
  - Interior node: question
  - Leaf: conclusion



Example3: Game Tree

- A kind of a decision tree
- Interpretation
  - Node: a state of the game
  - Children: possible next state after selection
  - Leaf: win or lose



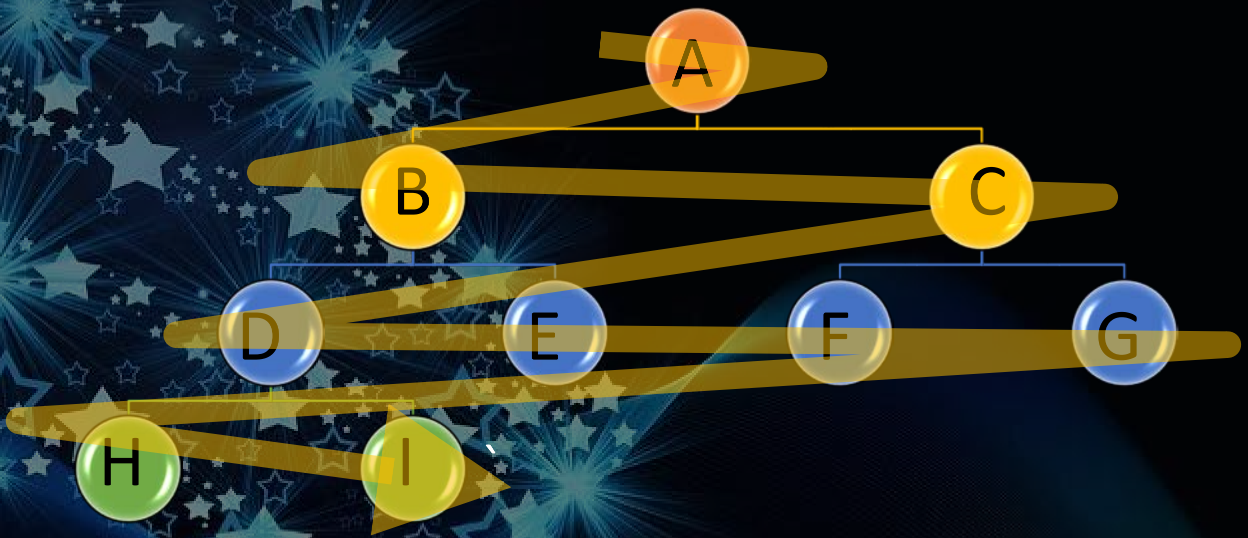
Tree Traversal

Traversals for Ordered Tree

- BFS (Breadth First Search)
  - Level Order Traversal
- DFS (Depth First Search)
  - Pre-order Traversal
  - In-order Traversal (Only for a binary tree)
  - Post-order Traversal

BFS

1. Initialize a queue
2. Add the root to the queue
3. Repeat until the queue is empty
    1. Remove x from the queue
    2. Visit x
    3. Add its children



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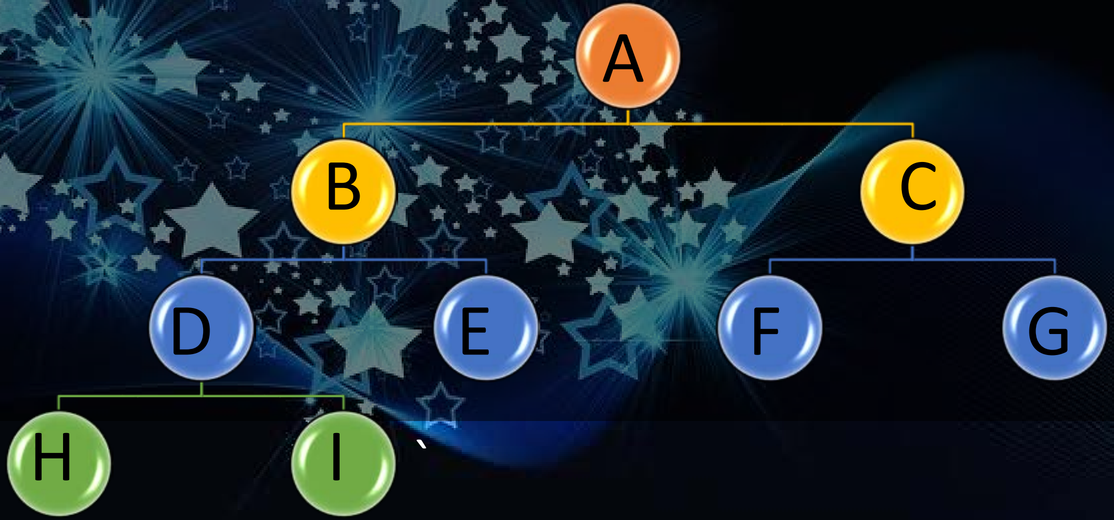
queue



BFS

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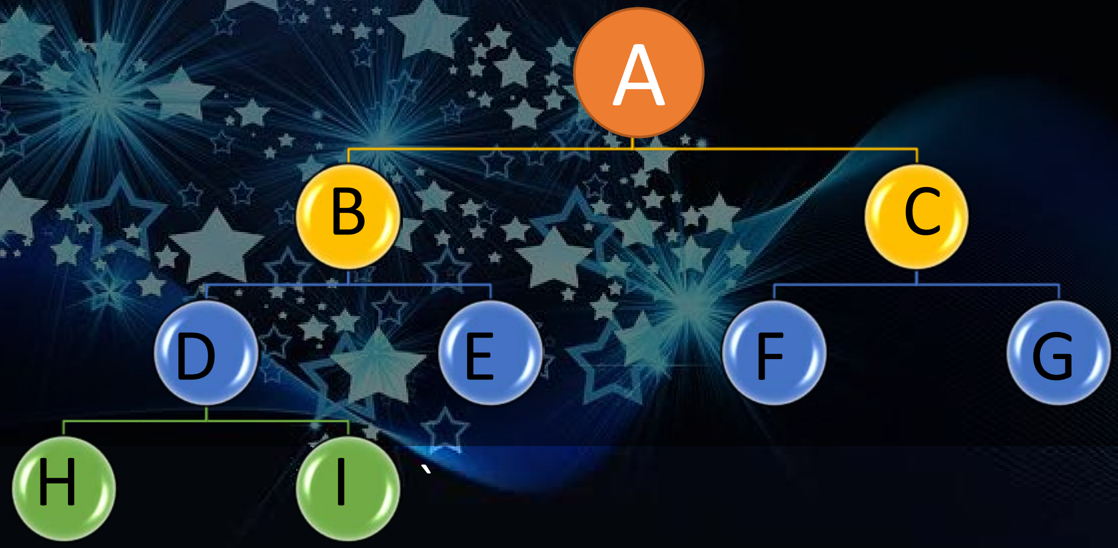

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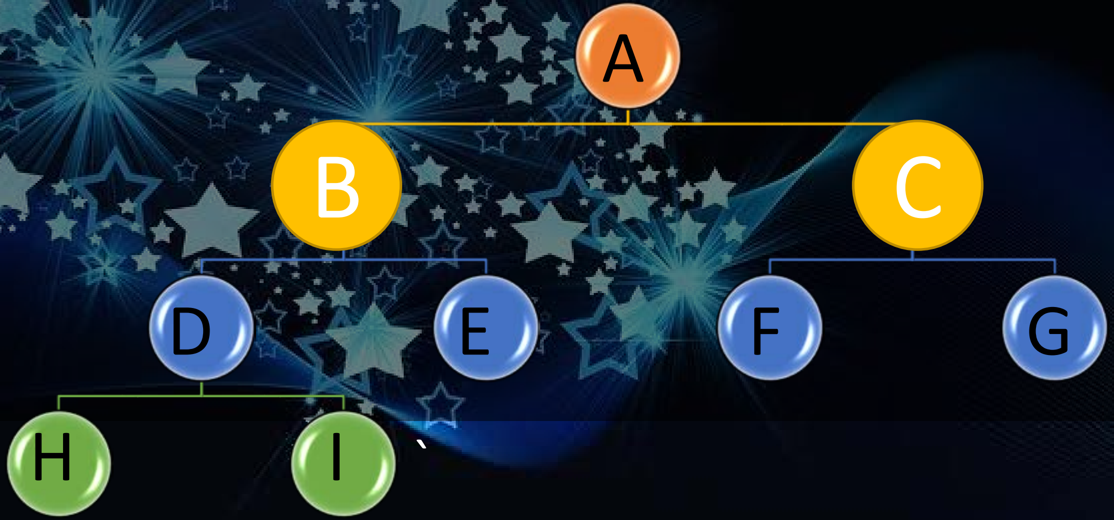

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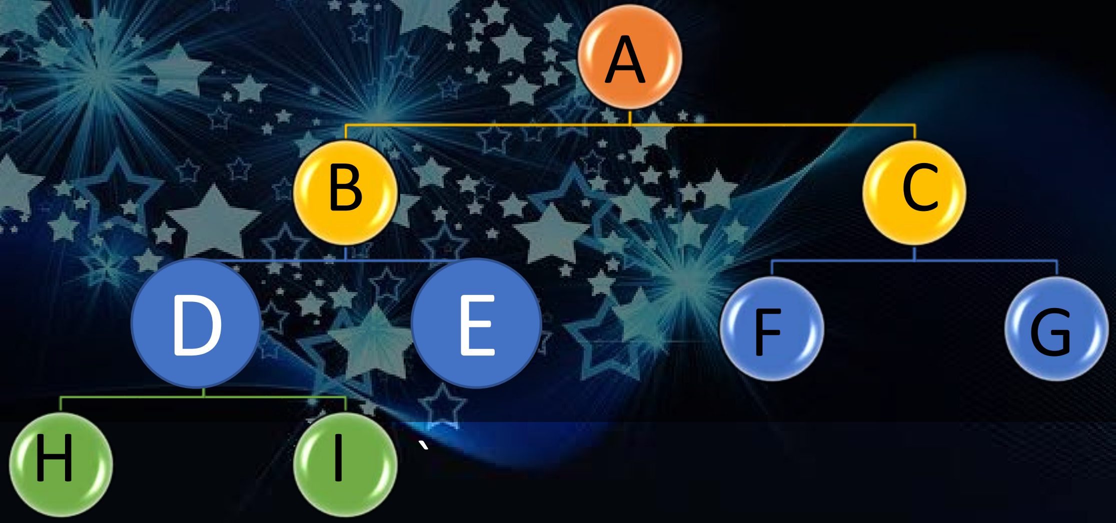
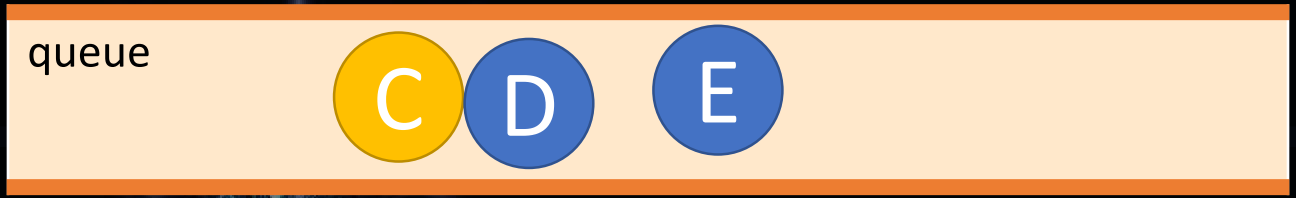


BFS



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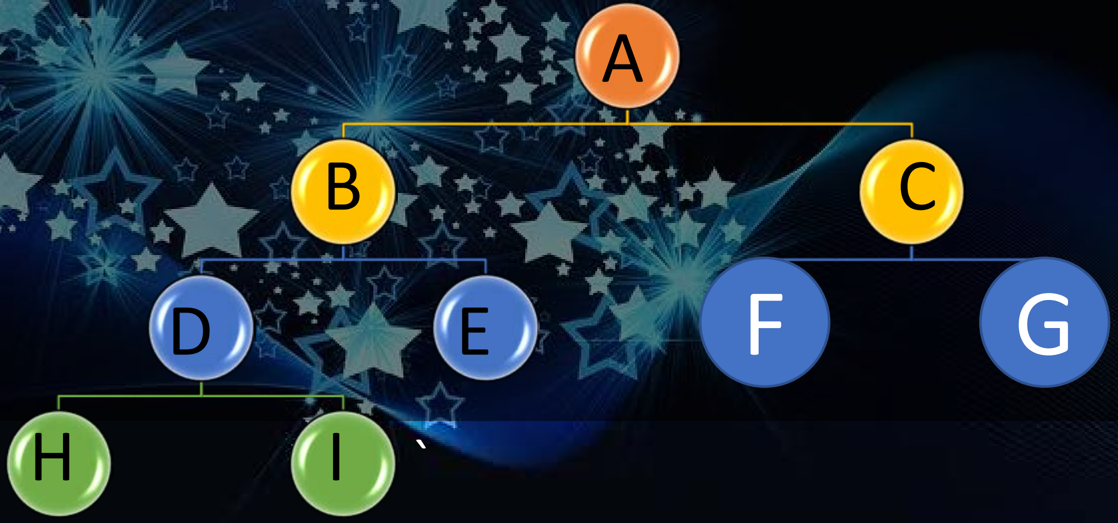
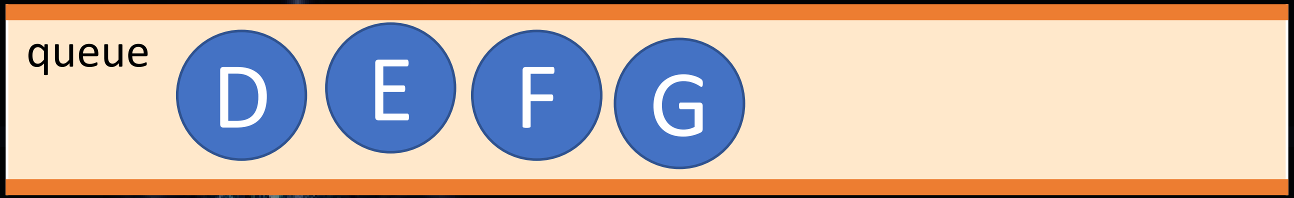


BFS



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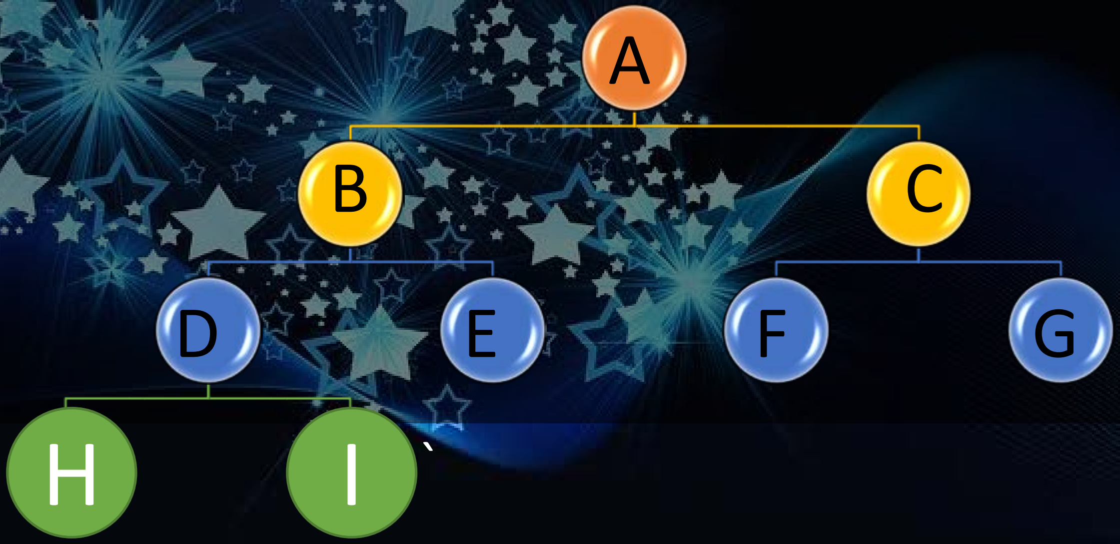
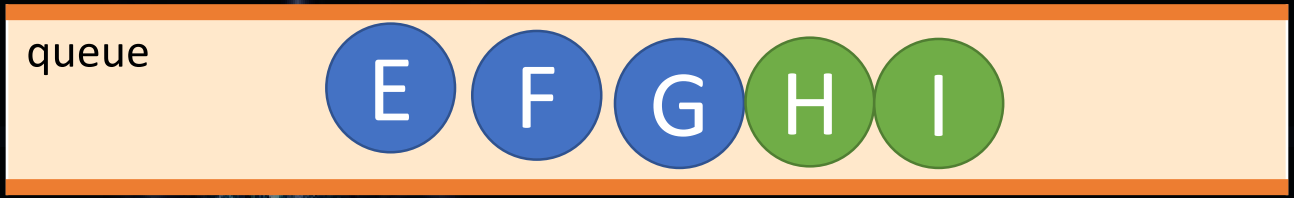


BFS



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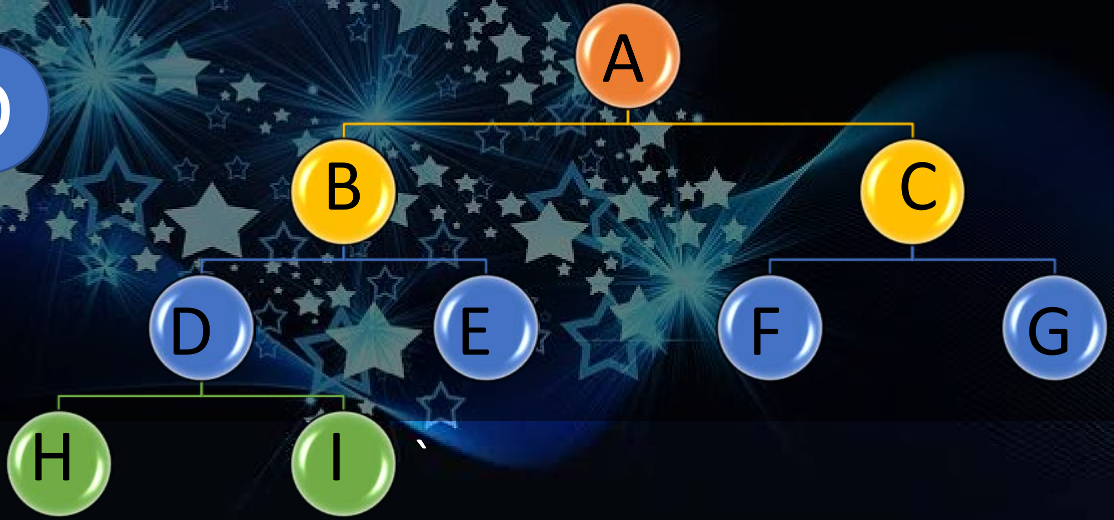
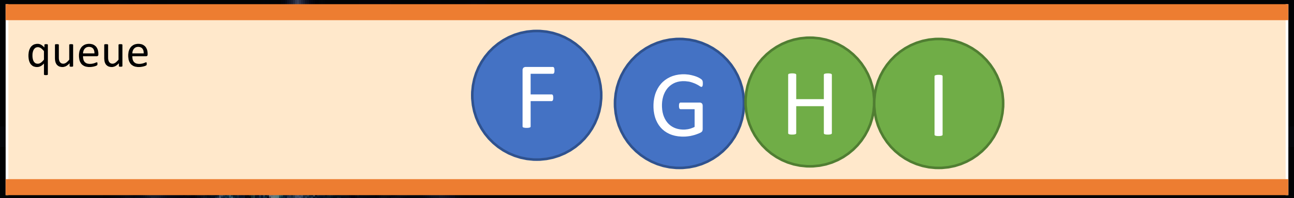


BFS



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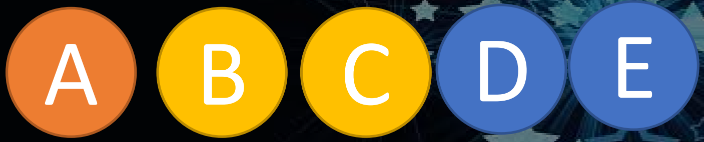

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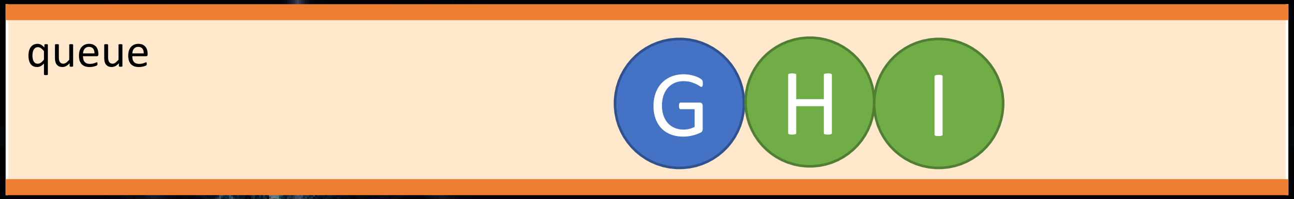

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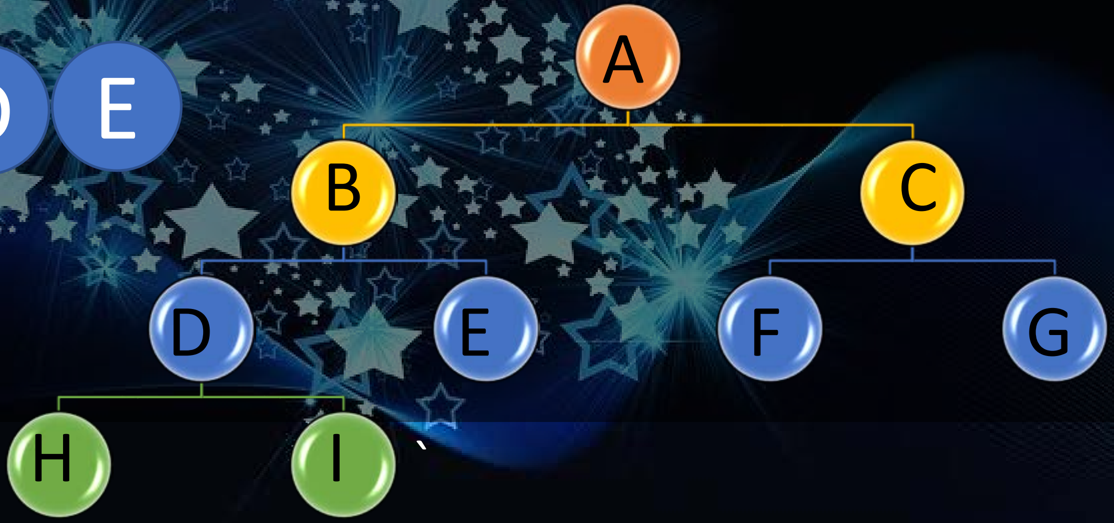

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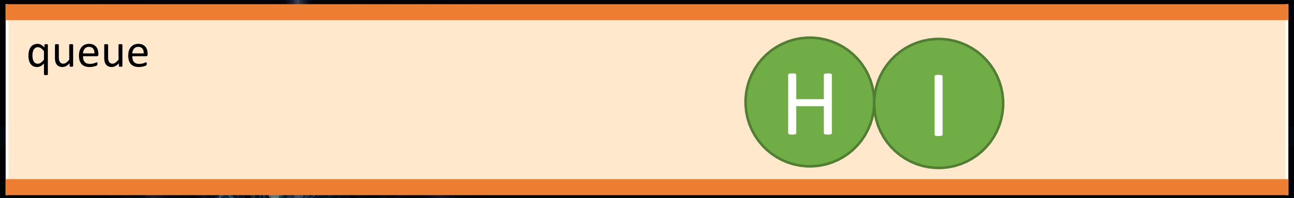

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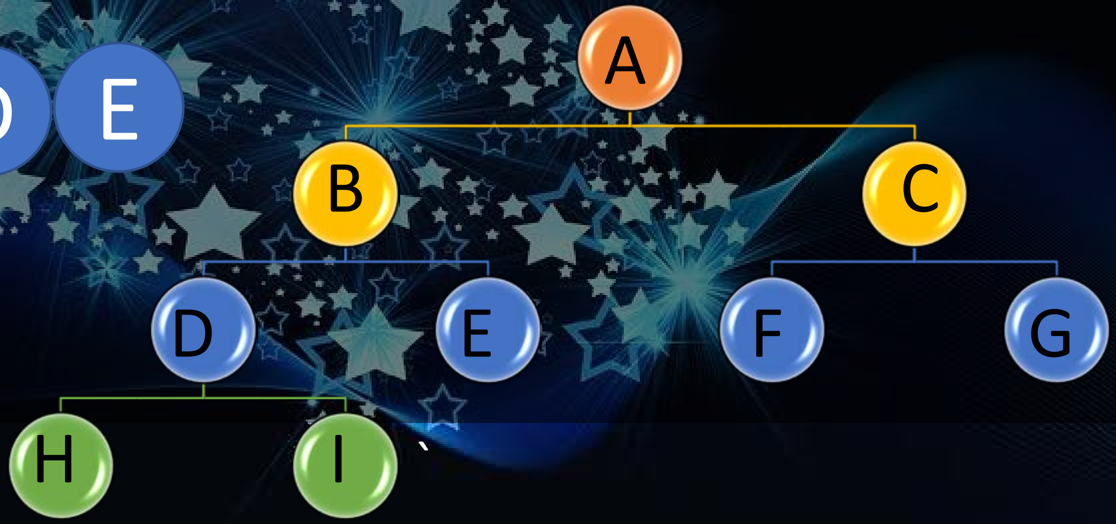

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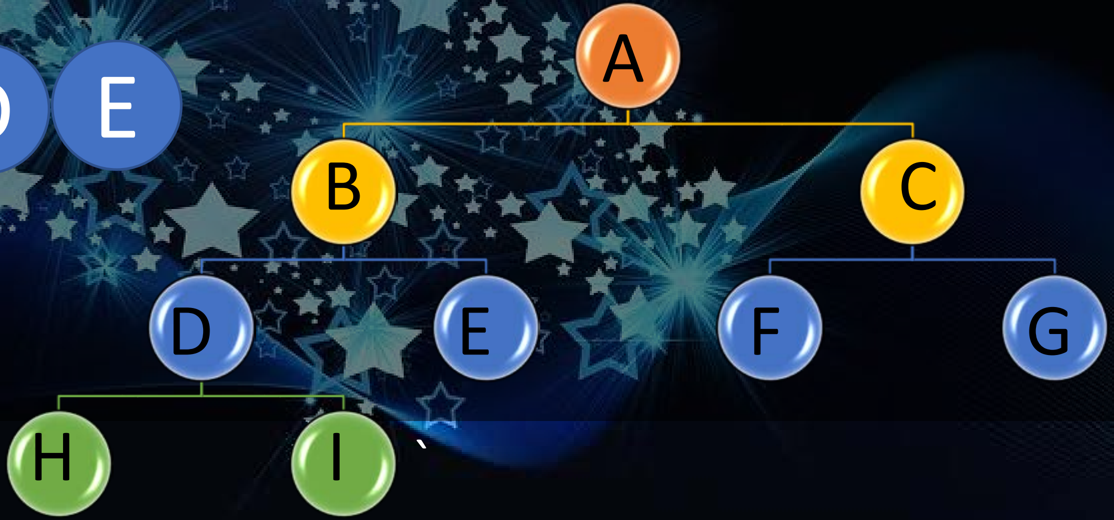
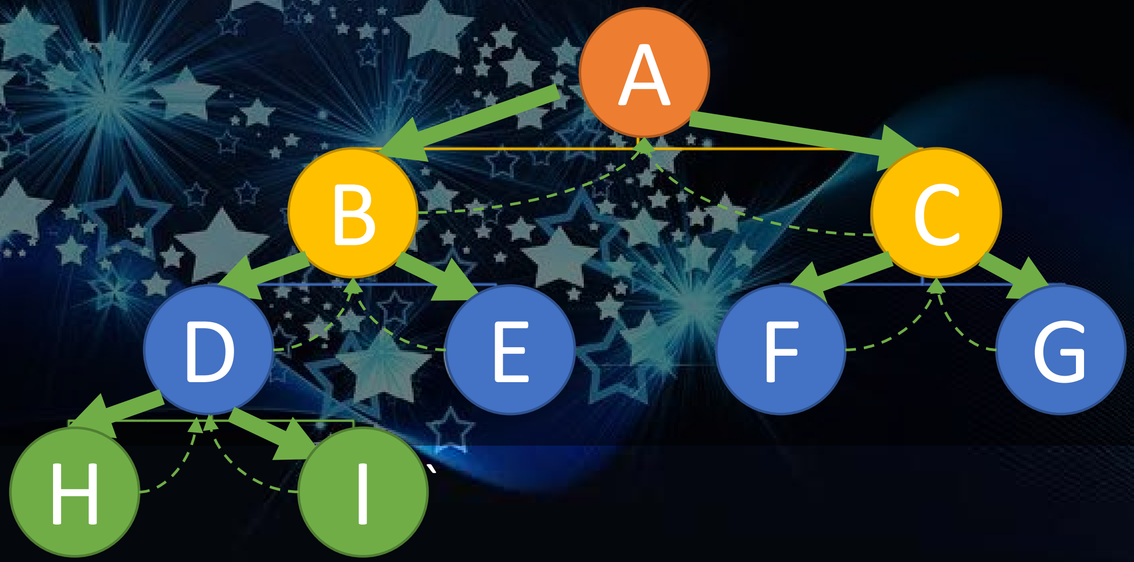


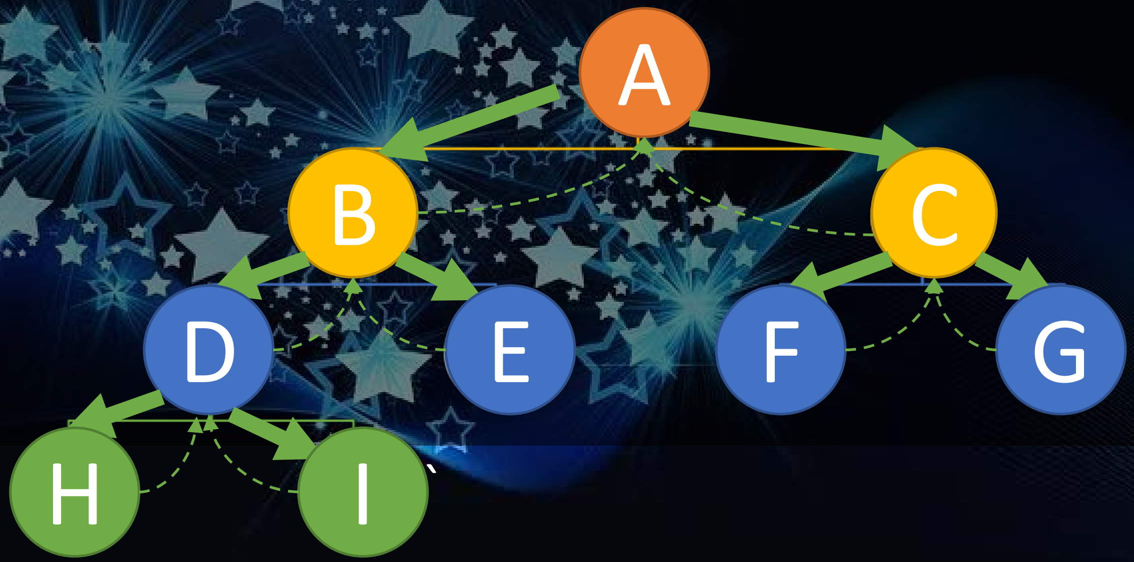
DFS(Pre-order)

1. Visit me
2. Pre-order left subtree
3. Pre-order right subtree



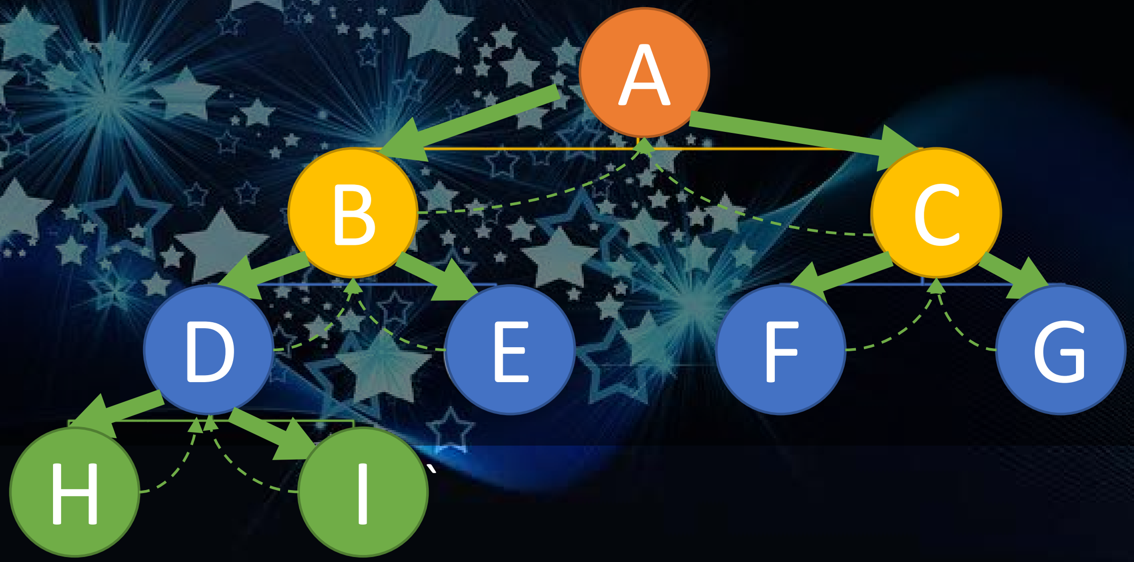
DFS(in-order)

1. in-order left
2. Visit me
3. in-order right



DFS(Post-order)

1. Post-order left
2. Post-order right
3. Visit me



Array Tree

Two ways of Implementing a Tree

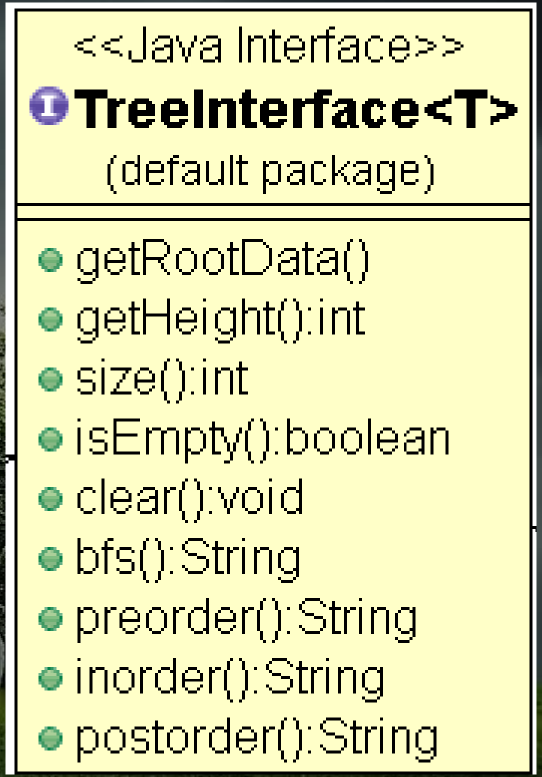
- Assume that the tree is a binary tree.

- Static memory
  - Array
- Dynamic memory
  - Linked Tree

Interface of a Tree

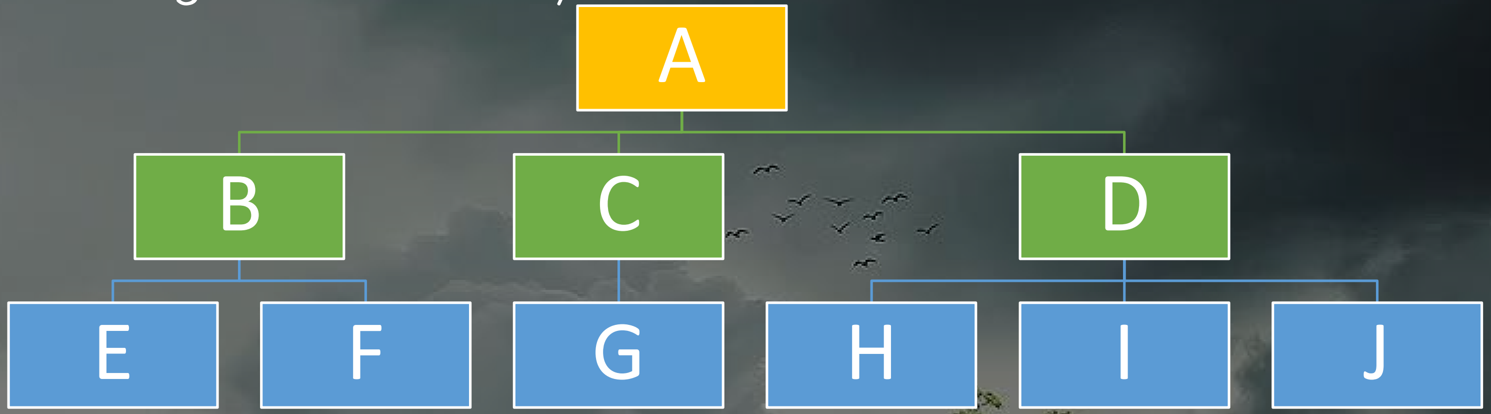
- Generic interface

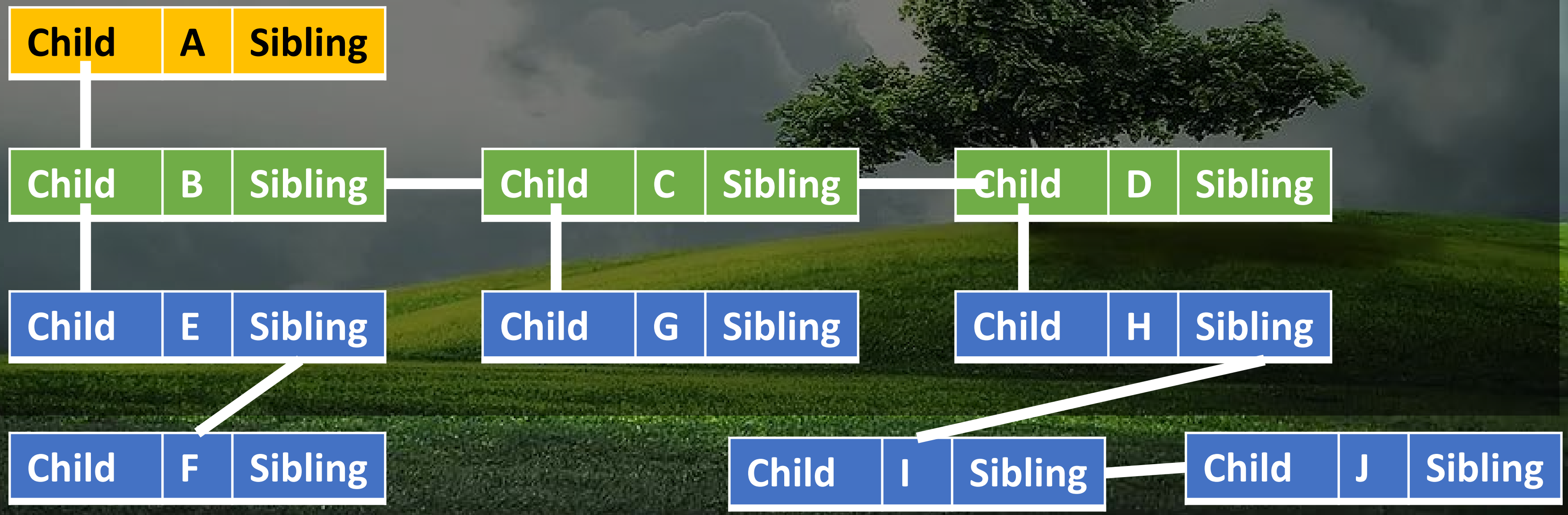
- Get Information
  - getRootData
  - getHeight
  - size
  - isEmpty
- Clear
- Traversal of tree
  - bfs: level order traversal
  - preorder: pre-order traversal
  - inorder: in-order traversal
  - postorder: post-order traversal



General tree → binary tree

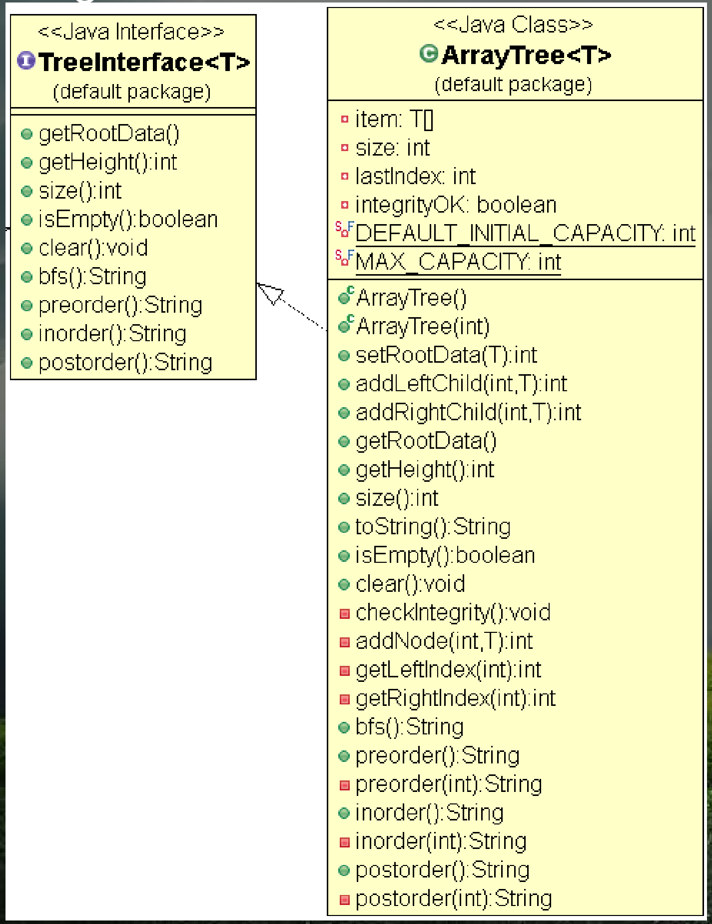
- Every general tree can be changed into a binary tree.
- Rule
  - Child → left child
  - Sibling →right child

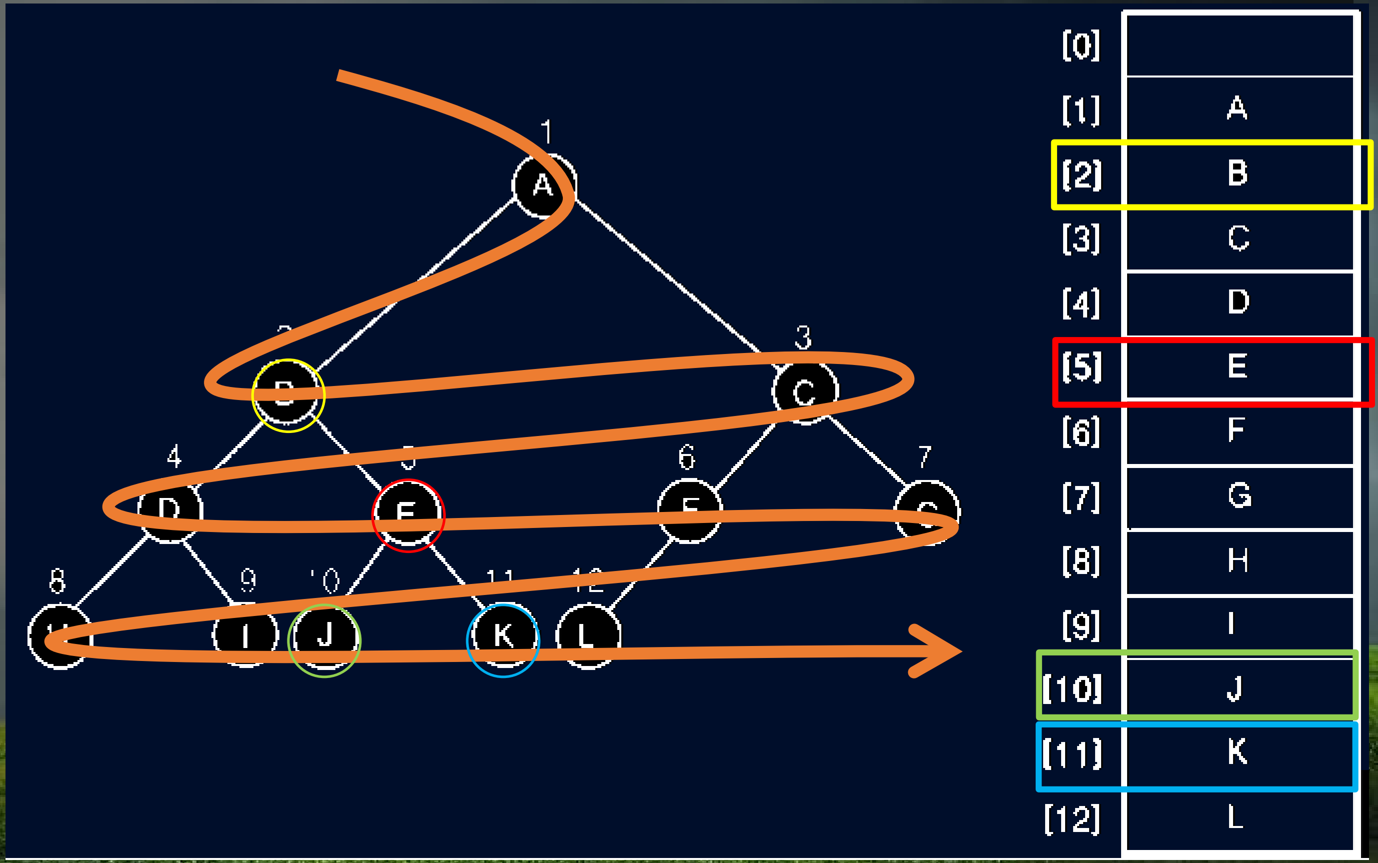




Complete Binary Tree

- Index of node i
  - Parent : $\lfloor i/2 \rfloor$
  - Left child : index*2
  - Right child : index*2+1



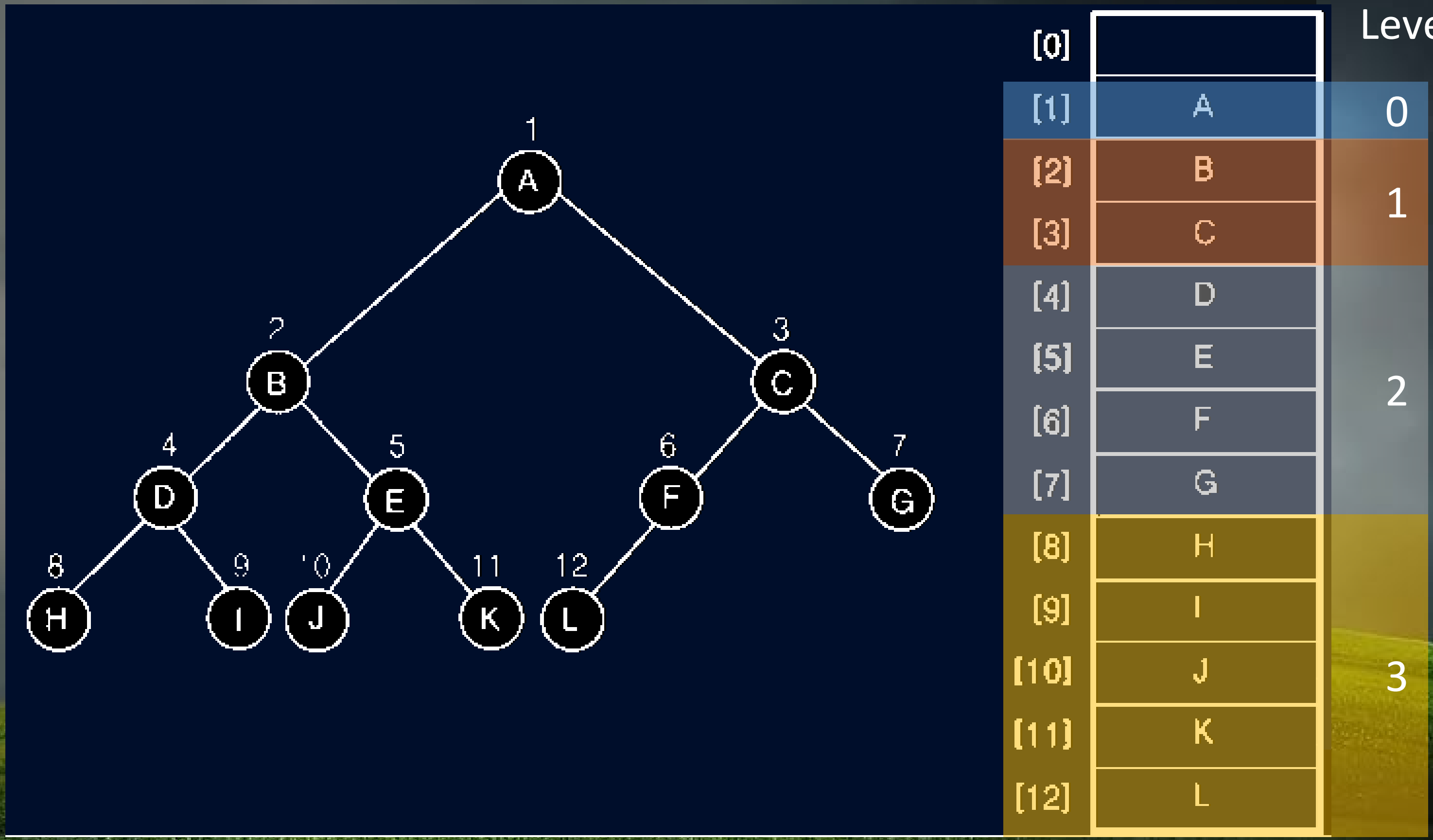


UML model of a Tree

<<Java Class>>
ArrayTree<T>
(default package)

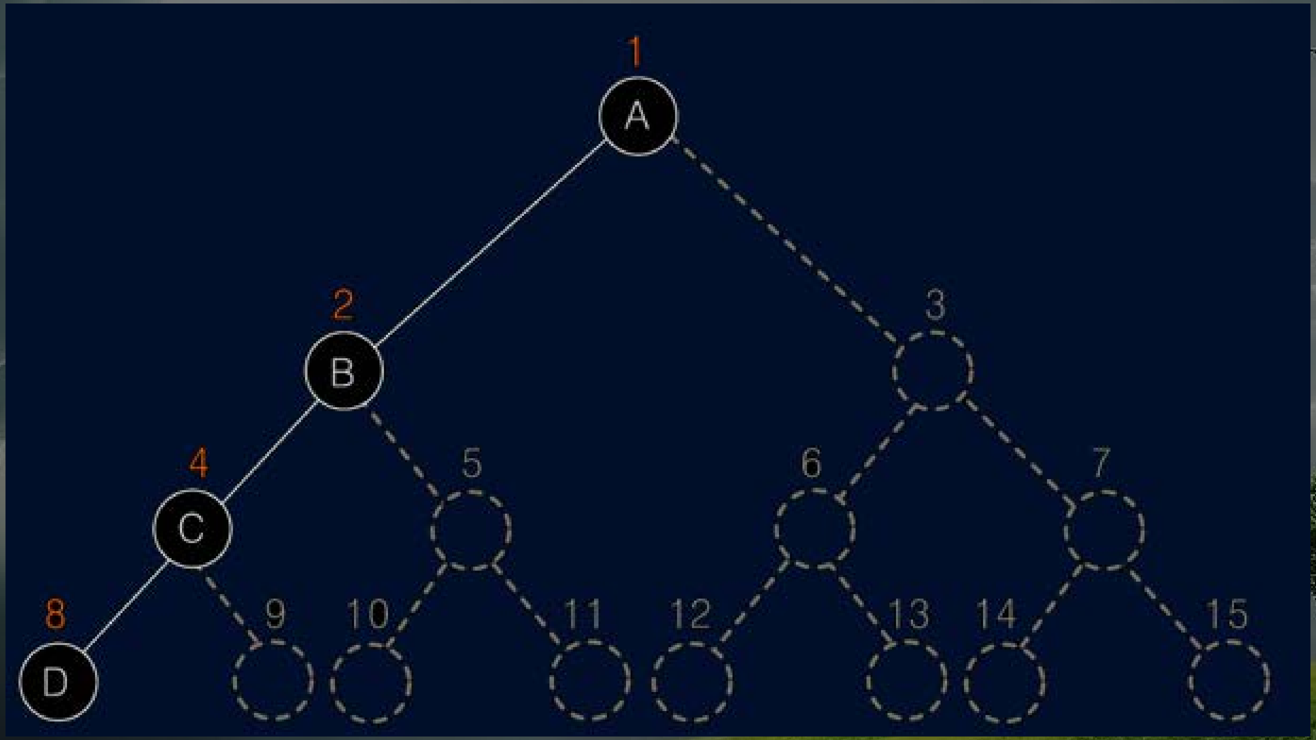
- item: T[]
- size: int
- lastIndex: int
- integrityOK: boolean
- DEFAULT_INITIAL_CAPACITY: int
- MAX_CAPACITY: int

- ArrayTree()
- ArrayTree(int)
- setRootData(T):int
- addLeftChild(int,T):int
- addRightChild(int,T):int
- getRootData()
- getHeight():int
- size():int
- toString():String
- isEmpty():boolean
- clear():void
- checkIntegrity():void
- addNode(int,T):int
- getLeftIndex(int):int
- getRightIndex(int):int
- bfs():String
- preorder():String
- preorder(int):String
- inorder():String
- inorder(int):String
- postorder():String
- postorder(int):String



Disadvantage of ArrayTree

- a skewed tree wastes the memory



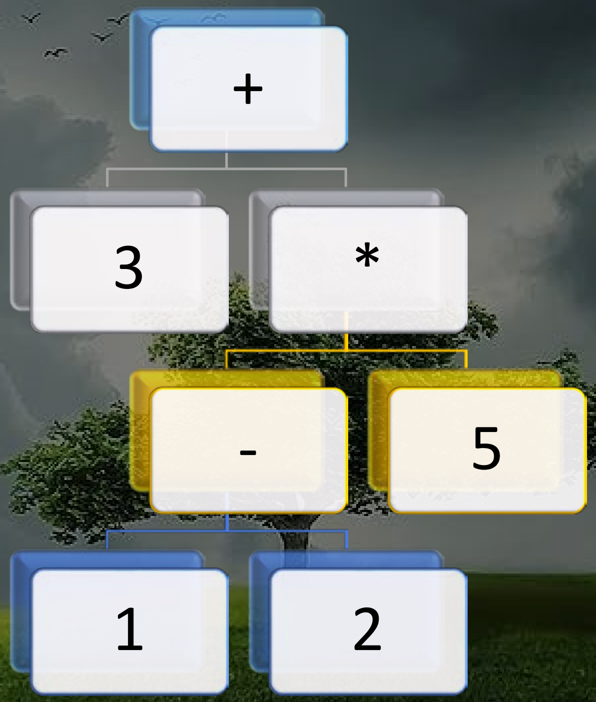
[0]	
[1]	A
[2]	B
[3]	
[4]	C
[5]	
[6]	
[7]	
[8]	D
[9]	
[10]	
[11]	
[12]	
[13]	
[14]	
[15]	

Test Program

```java
public class ArrayTest {
    public static void main(String[] args) {
        ArrayTree<Character> exTree = new ArrayTree<>(15);
        System.out.println("Is the tree empty? "+ exTree.isEmpty());
        int index = exTree.setRootData('+');
        exTree.addLeftChild(index, '3');
        index = exTree.addRightChild(index, '*');
        exTree.addRightChild(index, '5');
        index = exTree.addLeftChild(index, '-');
        exTree.addLeftChild(index, '1');
        exTree.addRightChild(index, '2');
        System.out.println("After adding, is the tree empty? "+ exTree.isEmpty());
        System.out.println("The current height is "+exTree.getHeight());
        System.out.println(exTree.getRootData()+"["+exTree.size()+"]: "+exTree.toString());
        System.out.println("BFS: "+exTree.bfs());
        System.out.println("pre-order traversal: "+exTree.preorder());
        System.out.println("in-order traversal: "+exTree.inorder());
        System.out.println("post-order traversal: "+exTree.postorder());
        exTree.clear();
        System.out.println("After clearing, is the tree empty? "+ exTree.isEmpty());
        if(!exTree.isEmpty()) System.out.println("The current height is "+exTree.getHeight());
        System.out.println(exTree.getRootData()+"["+exTree.size()+"]: "+exTree.toString());
    }
}
```

Problems Javadoc Declaration Console

```
<terminated> ArrayTest [Java Application] C:₩Program Files₩Java₩jdk-14.0.2₩bin₩javaw.exe (2020. 10. 20. 오후 1:00:51 – 오후 1:00:52)
Is the tree empty? true
After adding, is the tree empty? false
The current height is 3
+[7]: [null, +, 3, *, null, null, -, 5, null, null, null, null, 1, 2, null]
BFS: + 3 * - 5 1 2
pre-order traversal: + 3 * - 1 2 5
in-order traversal: 3 + 1 - 2 * 5
post-order traversal: 3 1 2 - 5 * +
After clearing, is the tree empty? true
null[0]: [null, null, null, null, null, null, null, null, null, null, null, null, null, null, null
```

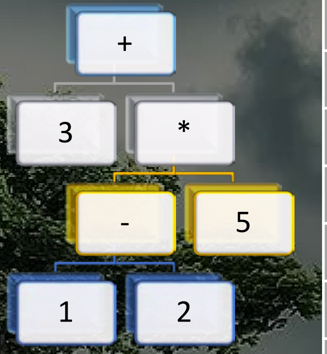


[0]	
[1]	+
[2]	3
[3]	*
[4]	
[5]	
[6]	-
[7]	5
[8]	
[9]	
[10]	
[11]	
[12]	1
[13]	2
[14]	

Add Root

Client program

```java
public class ArrayTest {
    public static void main(String[] args) {
        ArrayTree<Character> exTree = new ArrayTree<>(15);
        System.out.println("Is the tree empty? "+ exTree.isEmpty());
        int index = exTree.setRootData('+');
        exTree.addLeftChild(index, '3');
        index = exTree.addRightChild(index, '*');
        exTree.addRightChild(index, '5');
        index = exTree.addLeftChild(index, '-');
        exTree.addLeftChild(index, '1');
        exTree.addRightChild(index, '2');
```



[0]	
[1]	+
[2]	3
[3]	*
[4]	
[5]	
[6]	-
[7]	5
[8]	
[9]	
[10]	
[11]	
[12]	1
[13]	2
[14]	

In ArrayTree<T>

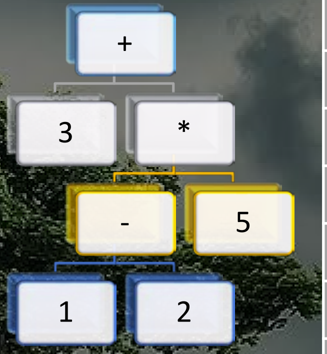
```java
public int setRootData(T data) {
    return addNode(1, data);
}
public int addLeftChild(int parentIndex, T data) {
    return addNode(parentIndex*2, data);
}
public int addRightChild(int parentIndex, T data) {
    return addNode(parentIndex*2+1, data);
}
```

Add Children

Be careful not to create a child node without a parent node

Client program

```
exTree.addLeftChild(index, '3');
index = exTree.addRightChild(index, '*');
exTree.addRightChild(index, '5');
index = exTree.addLeftChild(index, '-');
exTree.addLeftChild(index, '1');
exTree.addRightChild(index, '2');
```



[0]	
[1]	+
[2]	3
[3]	*
[4]	
[5]	
[6]	-
[7]	5
[8]	
[9]	
[10]	
[11]	
[12]	1
[13]	2
[14]	

In ArrayTree⟨T⟩

```
private int addNode(int index, T data) {
    checkIntegrity();
    if(index < item.length) {
        if(item[index]==null)size++;
        item[index] = data;
        if(index>lastIndex) lastIndex = index;
    } else {
        index=-1;
        System.err.println(data+" cannot be added to "+(index*2)+" becase of the array size");
    }
    return index;
}
```

Get information

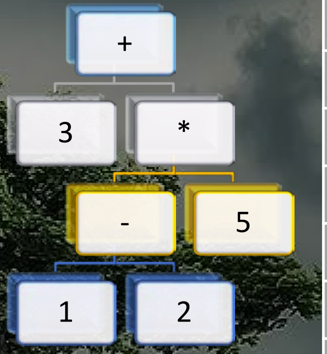
height, the number of nodes, root data

Client program

```java
System.out.println("After adding, is the tree empty? "+ exTree.isEmpty());
System.out.println("The current height is "+exTree.getHeight());
System.out.println(exTree.getRootData()+"["+exTree.size()+"]: "+exTree.toString());
```

In ArrayTree⟨T⟩

```java
public T getRootData() {
    checkIntegrity();
    T result = item[1];
    return result;
}
public int getHeight() {
    return (int)(Math.log(lastIndex)/Math.log(2.));
}
public int size() {
    return size;
}
public String toString() {
    checkIntegrity();
    return Arrays.toString(item);
}
public boolean isEmpty() {
    return size == 0;
}
```



[0]	
[1]	+
[2]	3
[3]	*
[4]	
[5]	
[6]	-
[7]	5
[8]	
[9]	
[10]	
[11]	
[12]	1
[13]	2
[14]	

traversal

bfs: using array

in order traversal

```
public String inorder() {
    return inorder(1);
}
private String inorder(int i) {
    String s = "";
    int left = getLeftIndex(i);
    int right = getRightIndex(i);
    if(left>0) s += inorder(left);
    s += item[i] +" ";
    if(right>0) s += inorder(right);
    return s;
}
```

Pre order traversal

```
public String preorder() {
    return preorder(1);
}
private String preorder(int i) {
    String s = "";
    int left = getLeftIndex(i);
    int right = getRightIndex(i);

    s += item[i] +" ";
    if(left>0) s += preorder(left);
    if(right>0) s += preorder(right);
    return s;
}
```

post order traversal

```
public String postorder() {
    return postorder(1);
}
private String postorder(int i) {
    String s = "";
    int left = getLeftIndex(i);
    int right = getRightIndex(i);
    if(left>0) s += postorder(left);
    if(right>0) s += postorder(right);
    s += item[i] +" ";
    return s;
}
```

Linked Tree

Design Issue

Tree is defined

as a composition of nodes

LinkedTree<T>

- root : TreeNode<T>

LinkedNode<T>

- data : T
- left : TreeNode<T>
- right: TreeNode<T>

the definition of a rooted tree

- Empty tree is a tree.
- If S is a set of trees

  any trees of S do not share a node.

  T = (r, S) is a tree

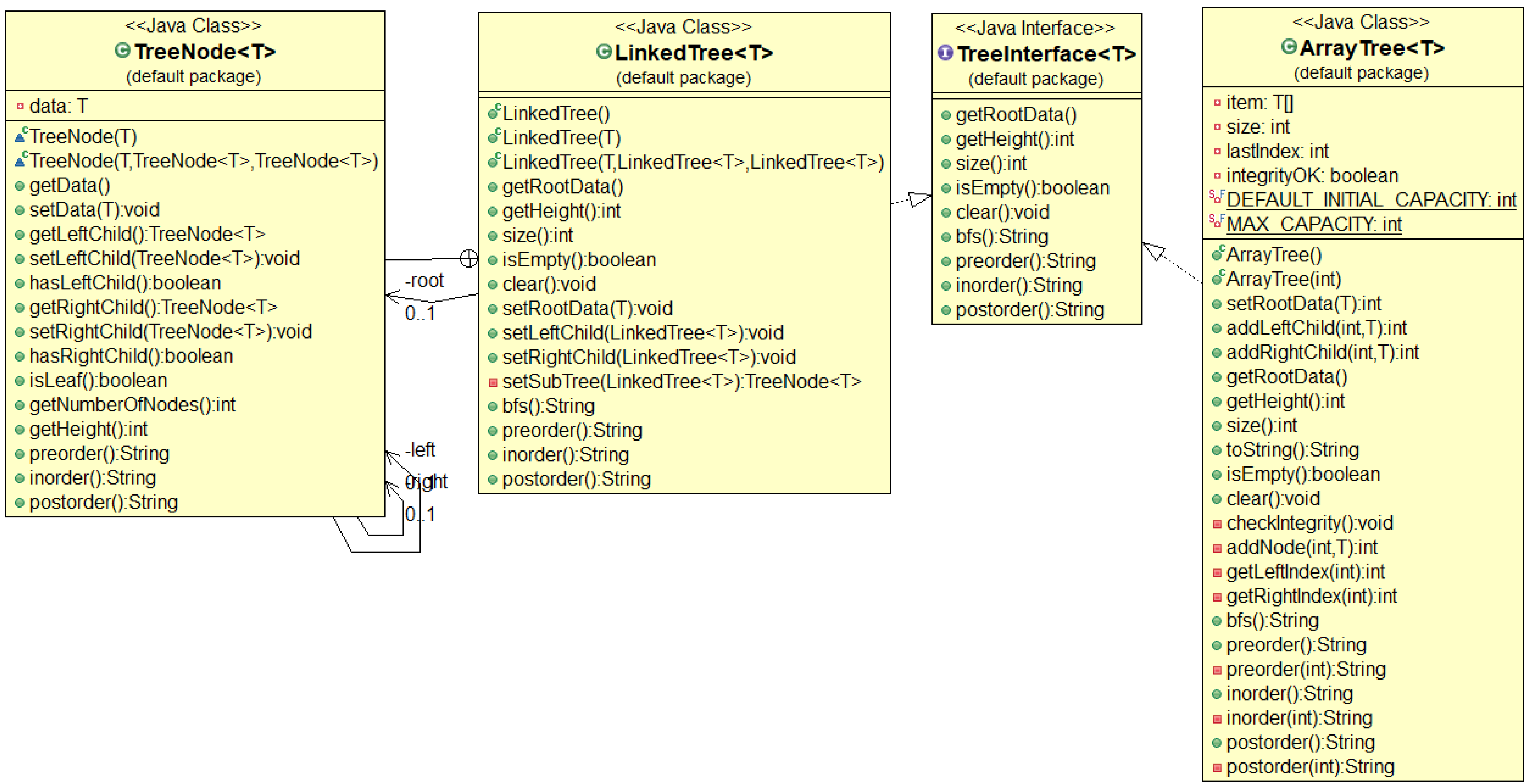
  r is a root

  a tree in S is a sub-tree of T

LinkedTree<T>

- data: T
- left: LinkedTree<T>
- right: LinkedTree<T>

UML class diagram



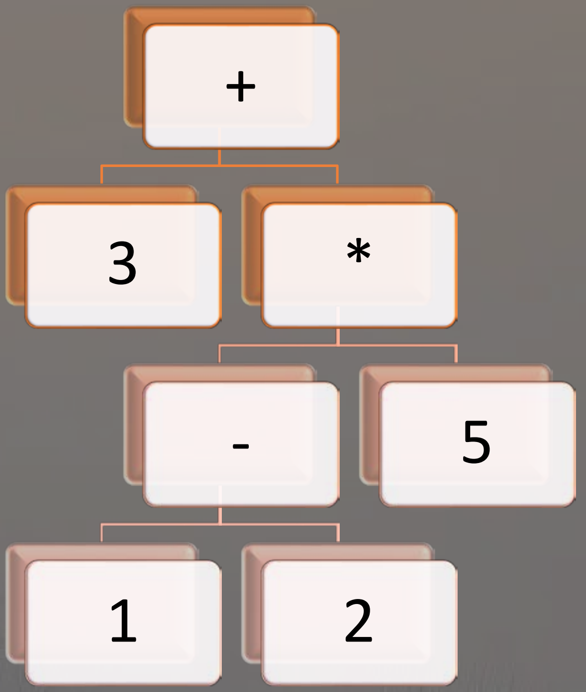
Test Program

```java
public class LinkedTest {
    public static void main(String[] args) {
        LinkedTree<Character> exTree = new LinkedTree<>();
        System.out.println("Is the tree empty? "+ exTree.isEmpty());
        LinkedTree<Character> minus
            = new LinkedTree<>('-', new LinkedTree<Character>('1'), new LinkedTree<Character>('2'));
        LinkedTree<Character> five = new LinkedTree<Character>('5');
        LinkedTree<Character> times = new LinkedTree<Character>('*', minus, five);
        exTree.setRootData('+');
        exTree.setLeftChild(new LinkedTree<Character>('3'));
        exTree.setRightChild(times);
        System.out.println("After adding, is the tree empty? "+ exTree.isEmpty());
        System.out.println("The current height is "+exTree.getHeight());
        System.out.println(exTree.getRootData()+"["+exTree.size()+"]: "+exTree.toString());
        System.out.println("BFS: "+exTree.bfs());
        System.out.println("pre-order traversal: "+exTree.preorder());
        System.out.println("in-order traversal: "+exTree.inorder());
        System.out.println("post-order traversal: "+exTree.postorder());
        exTree.clear();
        System.out.println("After clearing, is the tree empty? "+ exTree.isEmpty());
        if(!exTree.isEmpty()) System.out.println("The current height is "+exTree.getHeight());
        System.out.println(exTree.getRootData()+"["+exTree.size()+"]: "+exTree.toString());
    }
}
```

Problems Javadoc Declaration Console

<terminated> LinkedTest [Java Application] C:₩Program Files₩Java₩jdk-14.0.2₩bin₩javaw.exe (2020. 10. 20. 오후 9:03:14 – 오후 9:03:14)

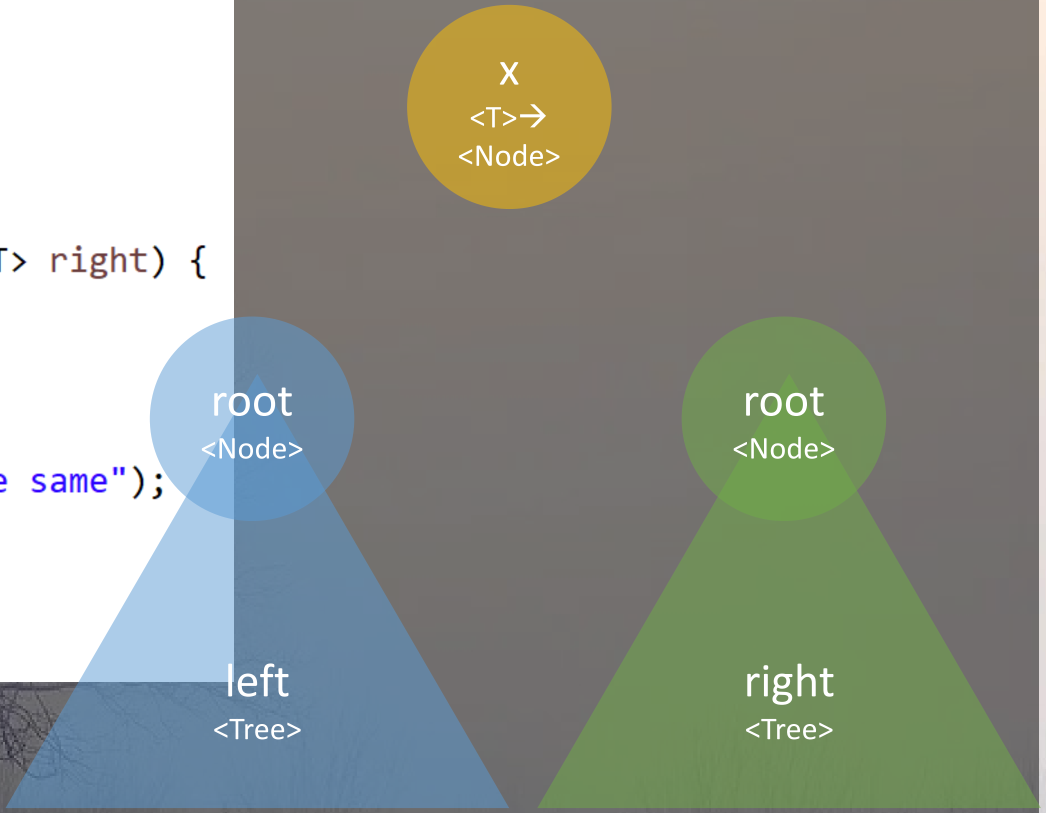
```
Is the tree empty? true
After adding, is the tree empty? false
The current height is 3
+[7]: LinkedTree@1f32e575
BFS: + 3 * - 5 1 2
pre-order traversal: + 3 * - 1 2 5
in-order traversal: 3 + 1 - 2 * 5
post-order traversal: 3 1 2 - 5 * +
After clearing, is the tree empty? true
null[0]: LinkedTree@1f32e575
```



Add Node

Three kind of constructors

```java
public LinkedTree() {
    root = null;
}
public LinkedTree(T x) {
    setRootData(x);
}
public LinkedTree(T x, LinkedTree<T> left, LinkedTree<T> right) {
    setRootData(x);
    root.left = setSubTree(left);
    TreeNode<T> temp = setSubTree(right);
    if(temp != null && temp == root.left ) {
        System.err.println("Both children cannot be the same");
        temp = null;
    }
    root.right = temp;
}
```



Get Information

TreeNode calculates the information recursively.

In class LinkedTree

```java
public T getRootData() {
    if(root==null) return null;
    return root.getData();
}
public int getHeight() {
    if(root == null) {
        System.err.println("You try to know the height of an empty tree");
        return -1;
    }
    return root.getHeight();
}
public int size() {
    if(root == null) return 0;

    return root.getNumberOfNodes();
}

public boolean isEmpty() {
    return root==null;
}
```

In inner class TreeNode

```java
public int getNumberOfNodes() {
    if(isLeaf()) return 1;

    int num = 1;
    if(hasLeftChild()) num += left.getNumberOfNodes();
    if(hasRightChild()) num += right.getNumberOfNodes();
    return num;
}
public int getHeight() {
    if(isLeaf()) return 0;
    int leftHeight=0, rightHeight=0;
    if(hasLeftChild()) leftHeight = left.getHeight();
    if(hasRightChild()) rightHeight = right.getHeight();

    return Math.max(leftHeight, rightHeight)+1;
}
```

BFS Traversal

It uses queue for TreeNode.

```java
public String bfs() {
    Queue<TreeNode<T>> q = new ArrayDeque();
    String s = "";
    if(!isEmpty()) {
        q.offer(root);
        for(TreeNode<T> node = q.poll() ; node!=null ; node = q.poll()) {
            s+=node.data + " ";
            if(node.hasLeftChild()) q.offer(node.left);
            if(node.hasRightChild()) q.offer(node.right);
        }
    }
    return s;
}
```

DFS Traversal

In class LinkedTree

```java
public String preorder() {
    return root.preorder();
}
public String inorder() {
    return root.inorder();
}
public String postorder() {
    return root.postorder();
}
```

In inner class TreeNode

```java
public String preorder() {
    String s = "";
    s+=data + " ";
    if(hasLeftChild())s+=left.preorder();
    if(hasRightChild())s+=right.preorder();
    return s;
}
public String inorder() {
    String s = "";
    if(hasLeftChild())s+=left.inorder();
    s+=data + " ";
    if(hasRightChild())s+=right.inorder();
    return s;
}
public String postorder() {
    String s = "";
    if(hasLeftChild())s+=left.postorder();
    if(hasRightChild())s+=right.postorder();
    s+=data + " ";
    return s;
}
```


Thank you