Multilayer Perceptron

ITM528 Deep learning

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Multilayer Perceptron

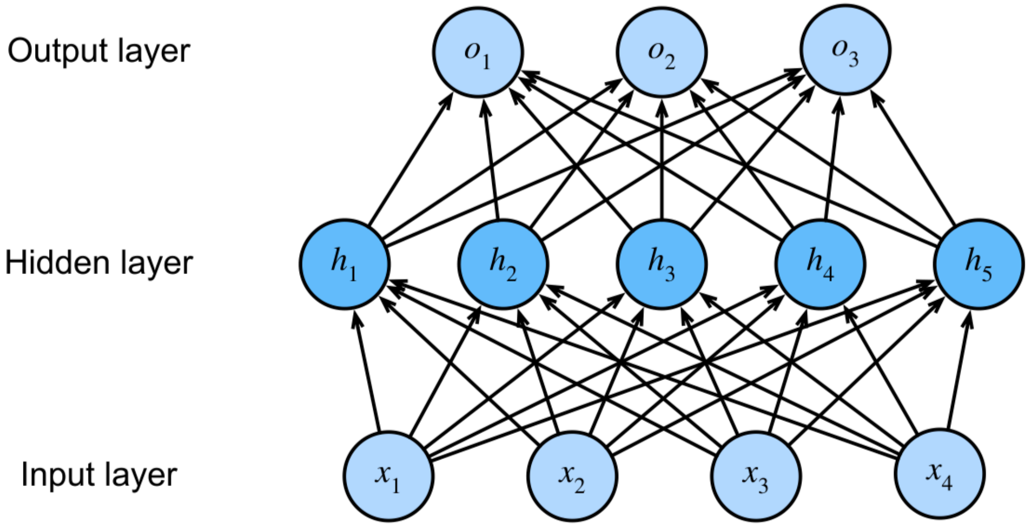
Multilayer Perceptrons

- We move on to truly (and the simplest) deep neural networks called **multilayer perceptrons** (MLPs).
  - They consist of multiple layers of neurons fully connected to those in the layer below and those above.
- Although automatic differentiation significantly simplifies the implementation, we will see how the gradients are calculated in deep networks.
- MLPs are high-capacity models and they are often prone to overfitting.
  - Thus, we will revisit regularization and generalization for deep networks.

Multilayer Perceptrons

- Our previous linear classification model simply maps inputs directly to outputs via a single affine transformation followed by a softmax operation.
- However, **linearity** (in affine transformation) is a strong assumption.
    - For example, linearity implies the weaker assumption of monotonicity.
    - However, most real-world problems are **nonlinear** (e.g., health assessments as a function of body temperature or dog classifier).
- One way to overcome this is to find a suitable **representation** of inputs, on top of which a single linear model would suffice.

Hidden Layers

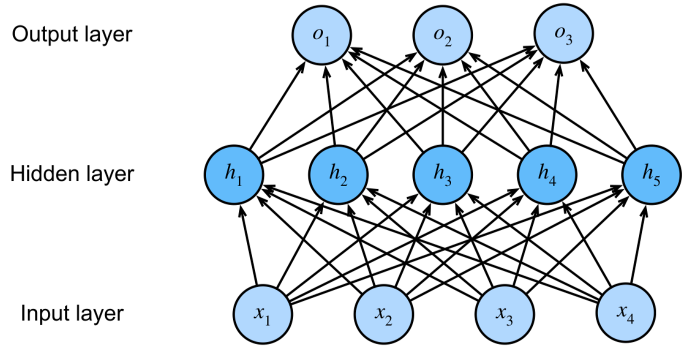


- We can overcome the limitations of linear models by incorporating **one or more hidden layers**.
  - This architecture is commonly called a multilayer perceptron, aka **MLP**.

From Linear to Nonlinear

- Suppose we denote the matrix $\mathbf{X} \in \mathbb{R}^{n \times d}$ a minibatch of n examples with d inputs (features).
- For a one-hidden-layer MLP whose hidden layer has h hidden units, we denote $\mathbf{H} \in \mathbb{R}^{n \times h}$ the outputs of the hidden layer (hidden representations).
- Then, the output of the MLP is calculated by:

$$\mathbf{H} = \mathbf{X}\mathbf{W}^{(1)} + \mathbf{b}^{(1)}$$
$$\mathbf{O} = \mathbf{H}\mathbf{W}^{(2)} + \mathbf{b}^{(2)}$$



- However, simply adding the hidden layer does not change anything!
  - Why?

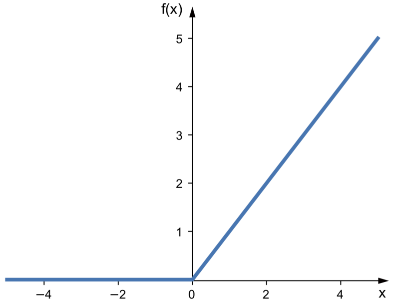
$$\mathbf{O} = (\mathbf{X}\mathbf{W}^{(1)} + \mathbf{b}^{(1)})\mathbf{W}^{(2)} + \mathbf{b}^{(2)} = \mathbf{X}\mathbf{W} + \mathbf{b}$$

From Linear to Nonlinear

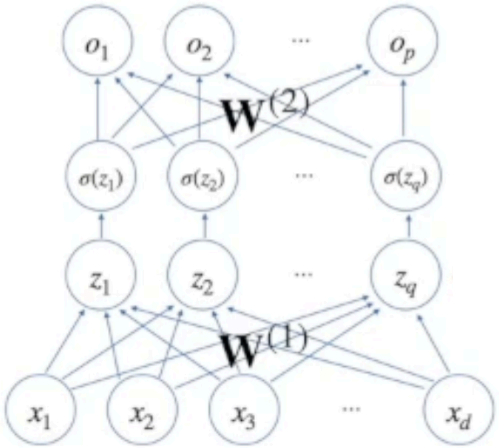
- Hence, we need a **nonlinear activation** function σ to be applied to each hidden unit following the affine transformation.
  - One popular choice is the ReLU (Rectified Linear Unit), $\sigma(x) = \max(0, x)$.
- Combining all together,

$$\mathbf{H} = \sigma(\mathbf{X}\mathbf{W}^{(1)} + \mathbf{b}^{(1)})$$

$$\mathbf{O} = \mathbf{H}\mathbf{W}^{(2)} + \mathbf{b}^{(2)}$$



- Of course, we can build more general MLPs by continuously stacking such hidden layers.



Universal Approximators

- It is worth asking just **how powerful** a deep network could be.
- A universal approximation property states that
- George Cybenko proved that an **MLP** with one hidden layer and sigmoid activations has the **universal approximation** property [1].
    - Polynomial functions also satisfy this property [2]. In fact, **polynomial functions** are the first function classes to have this property:

Fundamental Theorem of Approximation Theory

Let $f \in C[a, b]$, $-\infty < a < b < \infty$. Given $\epsilon > 0$, there exists an algebraic polynomial p for which

$$|f(x) - p(x)| < \epsilon$$

for all $x \in [a, b]$.

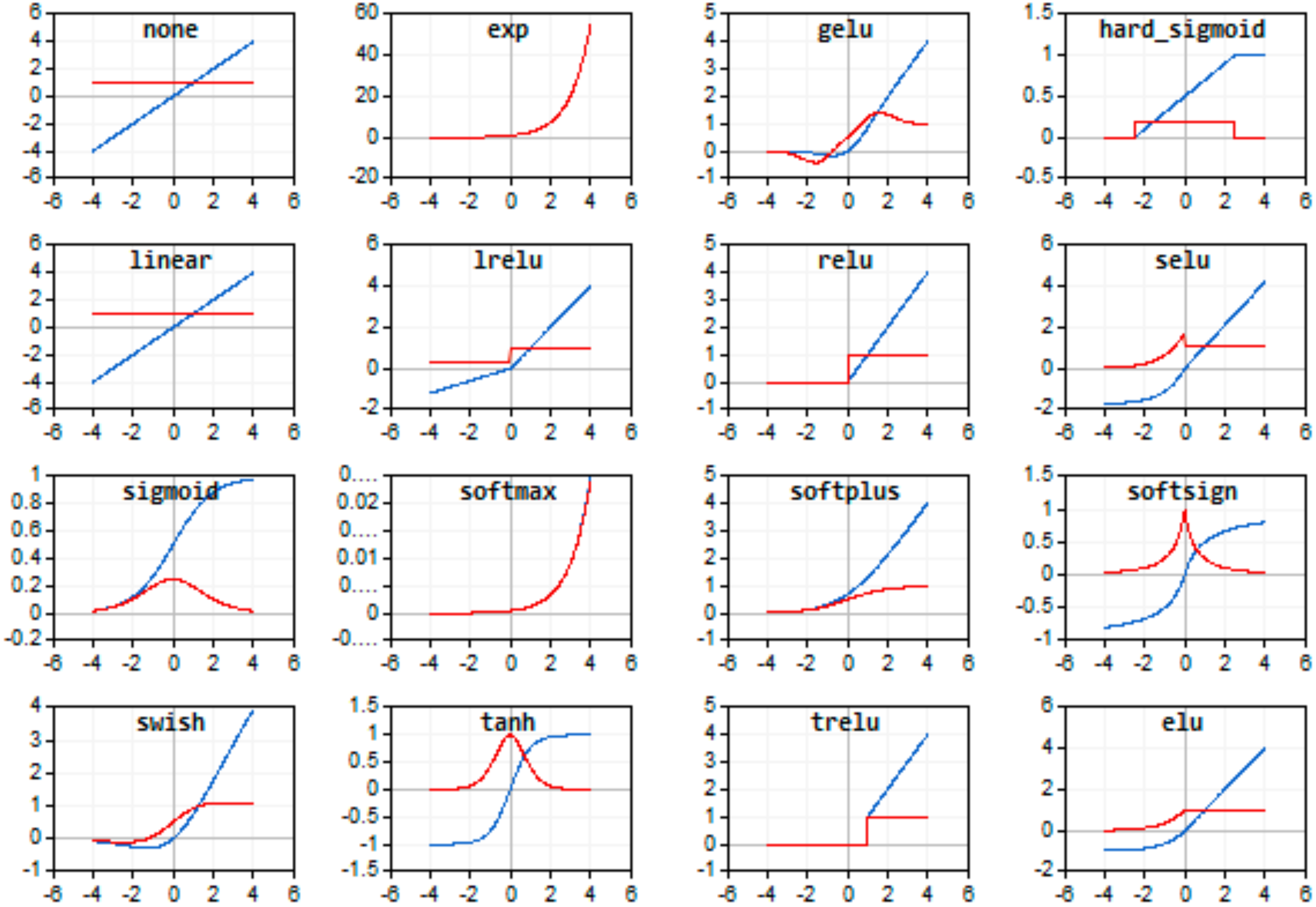
- In fact, if one can show that other families of functions (e.g., deep networks) behave like polynomials, then such families also have universal approximation properties.
- **Kernel methods** also satisfy this property [3].

[1] "Approximation by superpositions of a sigmoidal function," 1989
[2] "A generalized Weierstrass approximation theorem," 1948
[3] "Universal Approximation Using Radial-Basis-Function Networks," 1991

Universal Approximators

- Note that these results only suggest that a single-hidden-layer network with enough nodes can model any continuous function.
  - Hence, just because a single-hidden-layer network can learn any function, it does not mean that you should try to solve all of your problems with this.

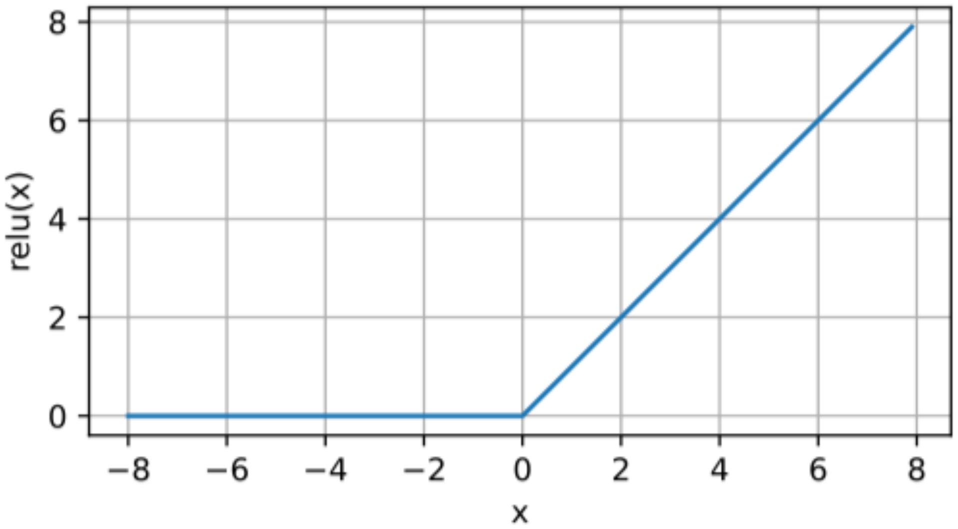
Activation Functions



https://www.mql5.com/en/articles/12627

Activation Functions

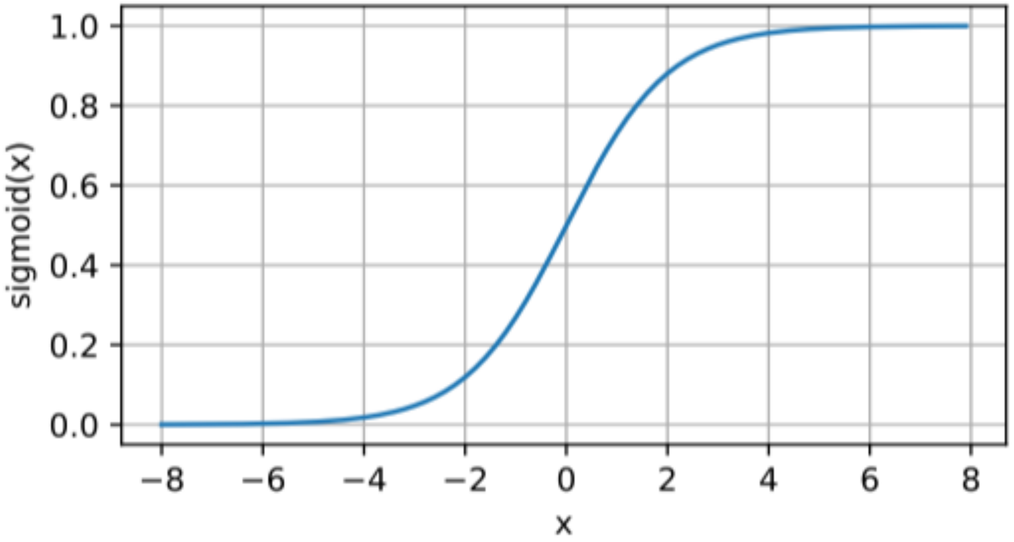
- Activation functions are differentiable operators to transform input signals to outputs to add **non-linearity**.
- ReLU (rectified linear unit) Function: $\mathrm{ReLU}(x) = \max(x, 0)$



- A ReLU activation function is one of the most popular activation functions in deep learning and has its strength in handling the **vanishing gradient issue**.
- However, a dead relu problem might occur.

Activation Functions

- Sigmoid Function: $\text{sigmoid}(x) = \dfrac{1}{1 + \exp(-x)}$

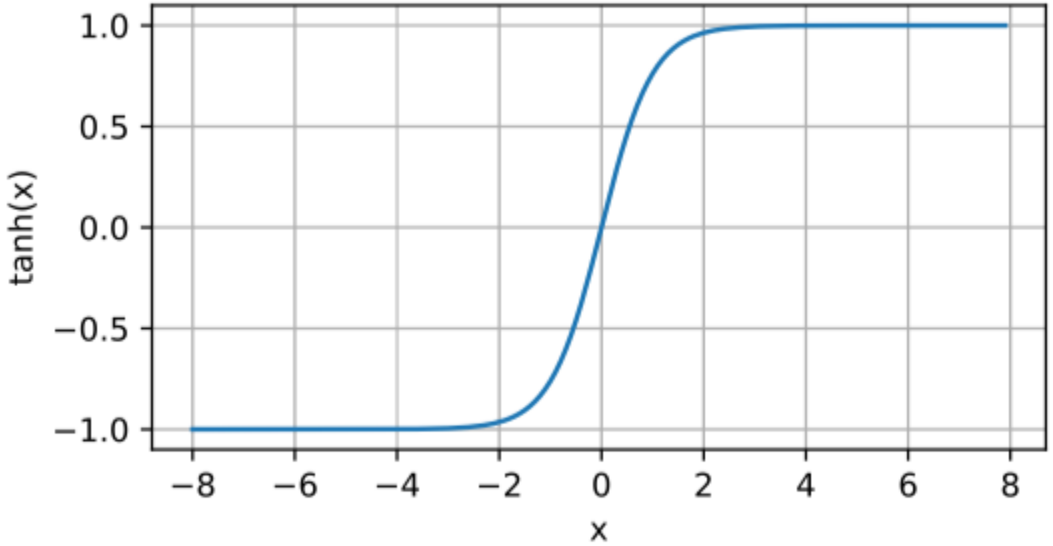


- The sigmoid function squashes the output to lie on the interval $(0,1)$.
- The gradient vanishing problem might happen, hindering its performance on deep networks.
- Also, the outputs are not zero-centered.
- One useful property of the sigmoid is that

$$\frac{d}{dx}\text{sigmoid}(x) = \text{sigmoid}(x)(1 - \text{sigmoid}(x))$$

Activation Functions

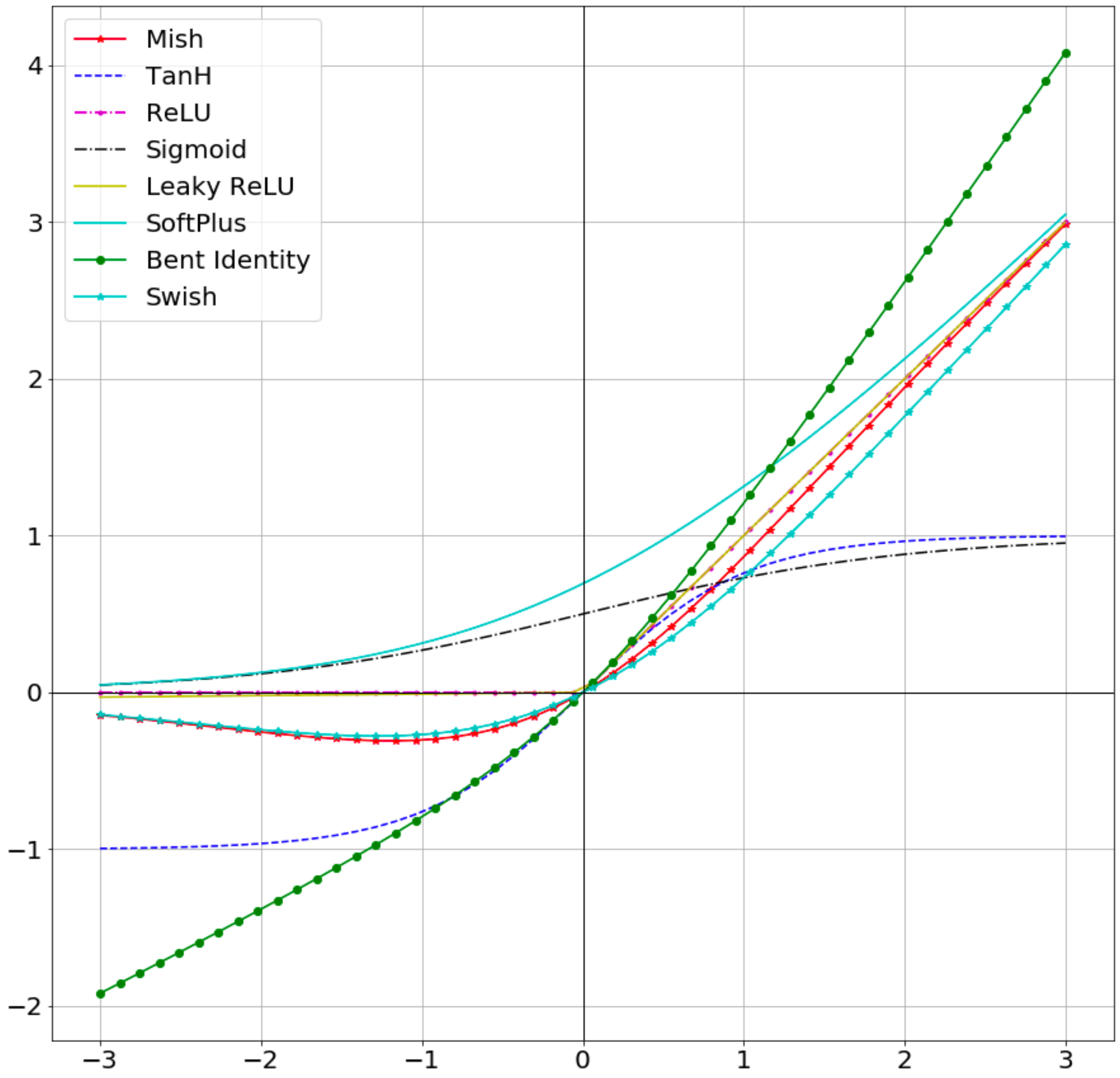
- Tanh Function: $\tanh(x) = \dfrac{1 - \exp(-2x)}{1 + \exp(-2x)}$



- The tanh function squashes the output to lie on the interval $(-1,1)$, hence symmetric.
- The vanishing gradient problem still exists.
- One useful property of the tanh is that

$$\frac{d}{dx}\tanh(x) = 1 - \tanh^2(x)$$

Activation Functions



- The **relu** is notorious for the dead relu problem.
- To handle this, the **elu** function was proposed. However, it introduces a longer computation time due to the exponential operation included.

$$\text{elu}(x) = \begin{cases} x, & \text{if } x > 0 \\ \alpha(e^x - 1), & x < 0 \end{cases}$$

- The **leaky relu** function also avoids the dead relu problem and is fast. However, we have to tune the parameter α.

$$\text{lrelu}(x) = \begin{cases} x, & \text{if } x > 0 \\ \alpha x, & x < 0 \end{cases}$$

- The **gelu** function works well in NLP, specifically Transformer models, as it is fast.

$$\text{gelu}(x) = 0.5x(1 + \tanh(\sqrt{2/\pi}(x + 0.044715x^3)))$$

- The **swish** function is continuous and differentiable at all points. And it works well on standard image datasets (CIFAR or ImageNet) compared to others (relu, lrelu, elu, gelu).

$$\text{swish}(x) = x(1 + e^{-x})^{-1}$$

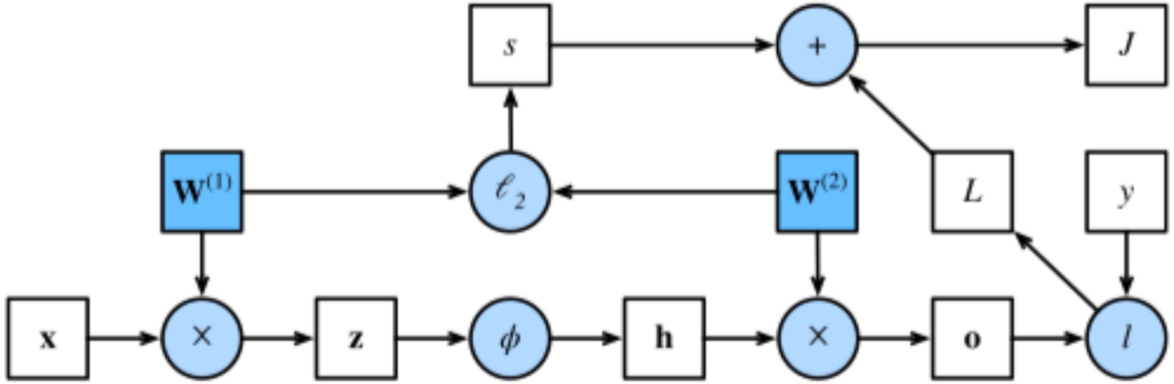
- The **mish** function is C^∞-continuous and approximates identity near the origin. In some experiments, the **mish** works better than **swish** activations.

$$\text{mish}(x) = x\tanh(\text{softplus}(x))$$

https://data-newbie.tistory.com/297

Backpropagation

Forward Propagation



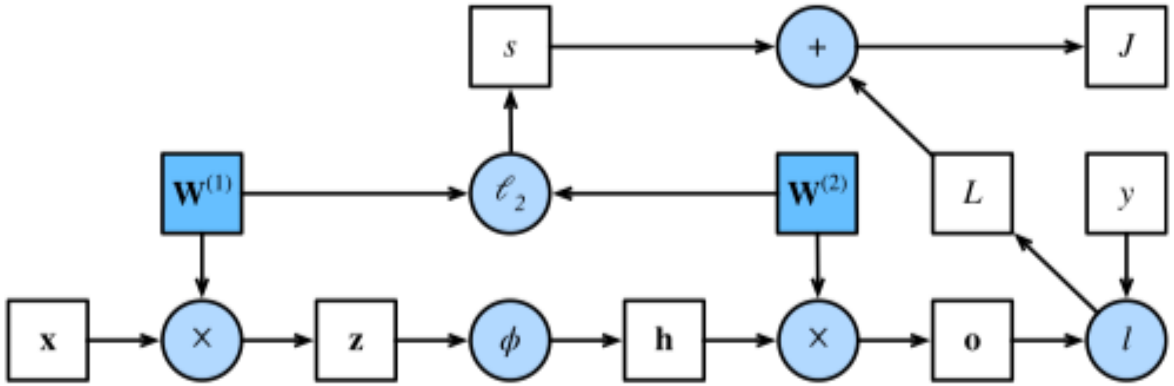
- [1] For the sake of simplicity, an input example is $\mathbf{x} \in \mathbb{R}^d$ and no bias term exists.
- [2] Then, the intermediate variable is $\mathbf{z} = \mathbf{W}^{(1)}\mathbf{x} \in \mathbb{R}^h$ where $\mathbf{W}^{(1)} \in \mathbb{R}^{h \times d}$ is the weight parameter of the hidden layer.
- [3] Our hidden activation vector is $\mathbf{h} = \phi(\mathbf{z}) \in \mathbb{R}^h$.
- [4] The hidden layer output becomes $\mathbf{o} = \mathbf{W}^{(2)}\mathbf{h} \in \mathbb{R}^q$ where $\mathbf{W}^{(2)} \in \mathbb{R}^{q \times h}$ is the weight parameter of the hidden layer.
- [5] The loss term for a single data becomes $L = l(\mathbf{o}, y)$.
- [6] Also, the regularization term becomes $s = \frac{\lambda}{2}(\|\mathbf{W}^{(1)}\|_F^2 + \|\mathbf{W}^{(2)}\|_F^2)$.
- [7] Finally, the model's regularized loss on a given data example is: $J = L + s$.

Backpropagation

- **Backpropagation** refers to the method of calculating the gradient of neural network parameters.
- In short, this method traverses the network in reverse order, from the output to the input layer, according to the **chain rule** from calculus.
    - Assume that we have $Y = f(X)$ and $Z = g(Y)$.
    - By using the chain rule:

$$\frac{\partial Z}{\partial X} = \prod\left(\frac{\partial Z}{\partial Y}, \frac{\partial Y}{\partial X}\right)$$

Backpropagation



$$\mathbf{o} = \mathbf{W}^{(2)}\mathbf{h} \in \mathbb{R}^q$$

$$s = \frac{\lambda}{2}\left(\|\mathbf{W}^{(1)}\|_F^2 + \|\mathbf{W}^{(2)}\|_F^2\right)$$

- [1] The first step is to calculate the gradients of the objective $J = L + s$ w.r.t. the loss term L and the regularization term s.

$$\frac{\partial J}{\partial L} = 1 \text{ and } \frac{\partial J}{\partial s} = 1$$

- [2] Next, we compute the gradient of J w.r.t. the output layer $\mathbf{o}$:

$$\frac{\partial J}{\partial \mathbf{o}} = \prod\left(\frac{\partial J}{\partial L}, \frac{\partial L}{\partial \mathbf{o}}\right) = \frac{\partial L}{\partial \mathbf{o}} \in \mathbb{R}^q$$

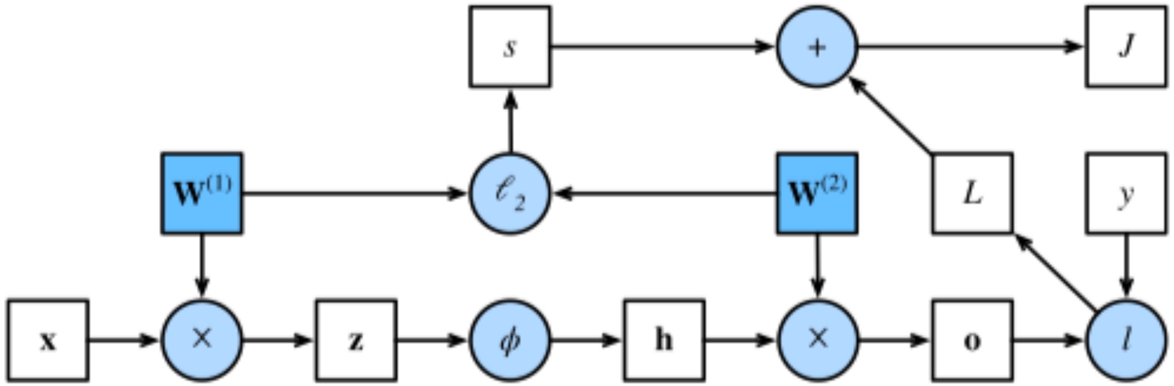
- [3] Next, we calculate the gradients of s w.r.t. parameters $\mathbf{W}^{(1)}$ and $\mathbf{W}^{(2)}$:

$$\frac{\partial s}{\partial \mathbf{W}^{(1)}} = \lambda \mathbf{W}^{(1)} \text{ and } \frac{\partial s}{\partial \mathbf{W}^{(2)}} = \lambda \mathbf{W}^{(2)}$$

- [4] Now, we are able to calculate the gradient $\partial J / \partial \mathbf{W}^{(2)} \in \mathbb{R}^{q \times h}$:

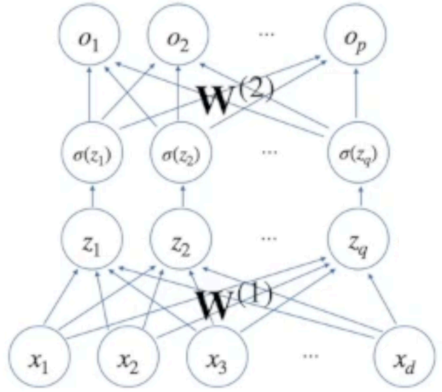
$$\frac{\partial J}{\partial \mathbf{W}^{(2)}} = \prod\left(\frac{\partial J}{\partial \mathbf{o}}, \frac{\partial \mathbf{o}}{\partial \mathbf{W}^{(2)}}\right) + \prod\left(\frac{\partial J}{\partial s}, \frac{\partial s}{\partial \mathbf{W}^{(2)}}\right) = \frac{\partial J}{\partial \mathbf{o}} \mathbf{h}^T + \lambda \mathbf{W}^{(2)}$$

Backpropagation



$$\mathbf{o} = \mathbf{W}^{(2)}\mathbf{h} \in \mathbb{R}^q$$

$$s = \frac{\lambda}{2}\left(\|\mathbf{W}^{(1)}\|_F^2 + \|\mathbf{W}^{(2)}\|_F^2\right)$$



- [1]To obtain the gradient w.r.t. $\mathbf{W}^{(1)}$, we need to continue backpropagation along the output layer to the hidden layer

$$\frac{\partial J}{\partial \mathbf{h}} = \prod\left(\frac{\partial J}{\partial \mathbf{o}}, \frac{\partial \mathbf{o}}{\partial \mathbf{h}}\right) = \mathbf{W}^{(2)^T}\frac{\partial J}{\partial \mathbf{o}}$$

- [2] Since the activation function ϕ applies elementwise:

$$\frac{\partial J}{\partial \mathbf{z}} = \prod\left(\frac{\partial J}{\partial \mathbf{h}}, \frac{\partial \mathbf{h}}{\partial \mathbf{z}}\right) = \frac{\partial J}{\partial \mathbf{h}} \odot \phi'(\mathbf{z})$$

- [3] Finally, we can obtain the gradient $\partial J/\partial \mathbf{W}^{(1)} \in \mathbb{R}^{h\times d}$:

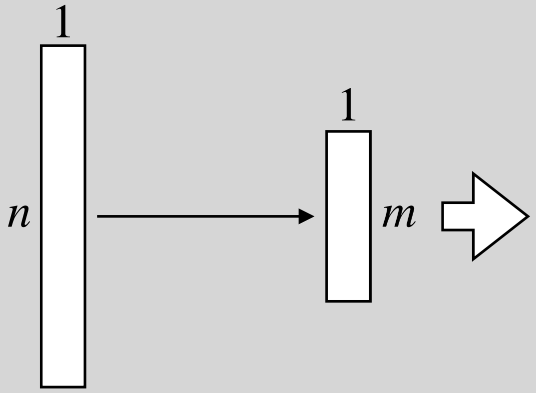
$$\frac{\partial J}{\partial \mathbf{W}^{(1)}} = \prod\left(\frac{\partial J}{\partial \mathbf{z}}, \frac{\partial \mathbf{z}}{\partial \mathbf{W}^{(1)}}\right) + \prod\left(\frac{\partial J}{\partial s}, \frac{\partial s}{\partial \mathbf{W}^{(1)}}\right) = \frac{\partial J}{\partial \mathbf{z}}\mathbf{x}^T + \lambda \mathbf{W}^{(1)}$$

- [4] We can further express this with:

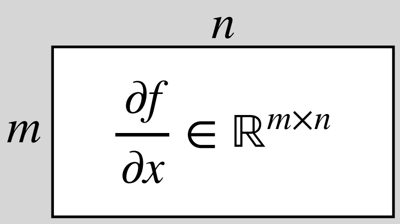
$$\frac{\partial J}{\partial \mathbf{W}^{(1)}} = \left(\left(\mathbf{W}^{(2)^T}\frac{\partial L}{\partial \mathbf{o}}\right) \odot \phi'\left(\mathbf{W}^{(1)}\mathbf{x}\right)\right)\mathbf{x}^T + \lambda \mathbf{W}^{(1)}$$

Matrix Calculus

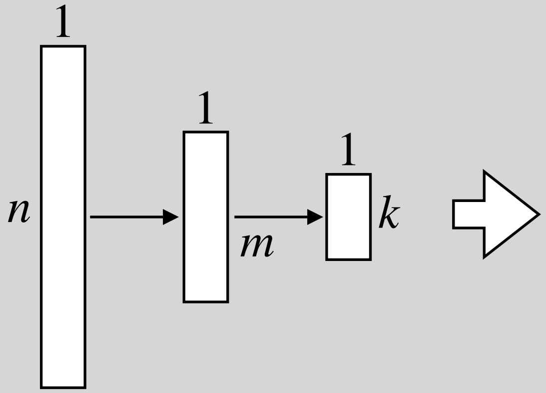
$x \in \mathbb{R}^n \mapsto f \in \mathbb{R}^m$



$$\frac{\partial f}{\partial x} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_m}{\partial x_1} & \cdots & \frac{\partial f_m}{\partial x_n} \end{bmatrix}$$

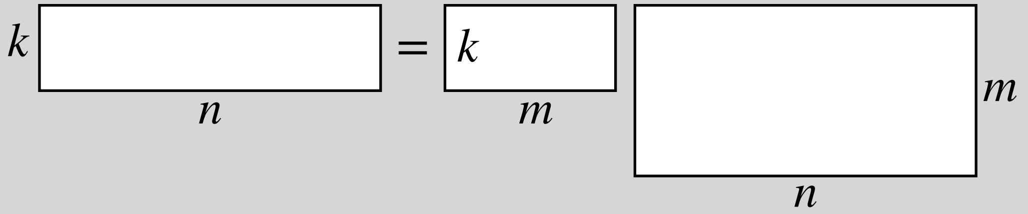


$x \in \mathbb{R}^n \mapsto f \in \mathbb{R}^m \mapsto g \in \mathbb{R}^k$



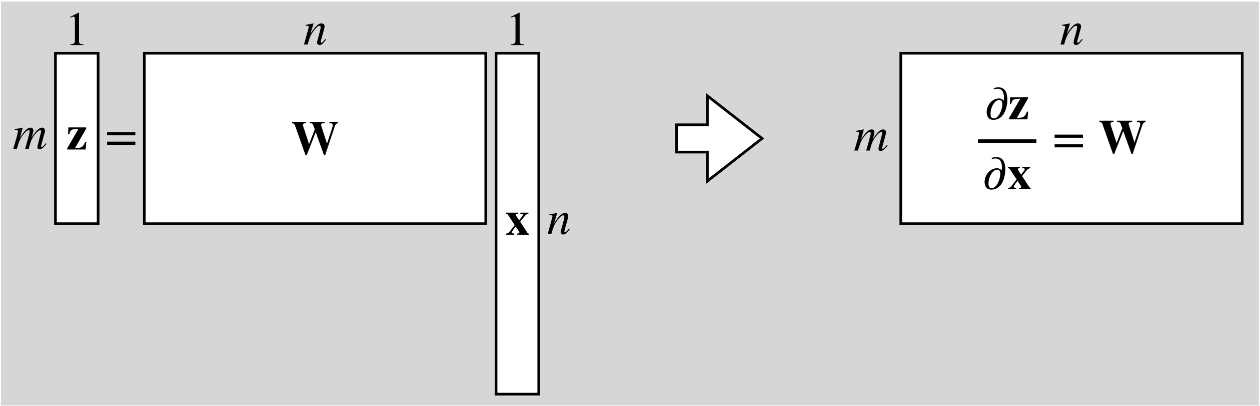
$$\frac{\partial g}{\partial x} = \frac{\partial g}{\partial f} \frac{\partial f}{\partial x}$$

$\frac{\partial g}{\partial x} \in \mathbb{R}^{k \times n}$ $\frac{\partial g}{\partial f} \in \mathbb{R}^{k \times m}$ $\frac{\partial f}{\partial x} \in \mathbb{R}^{m \times n}$

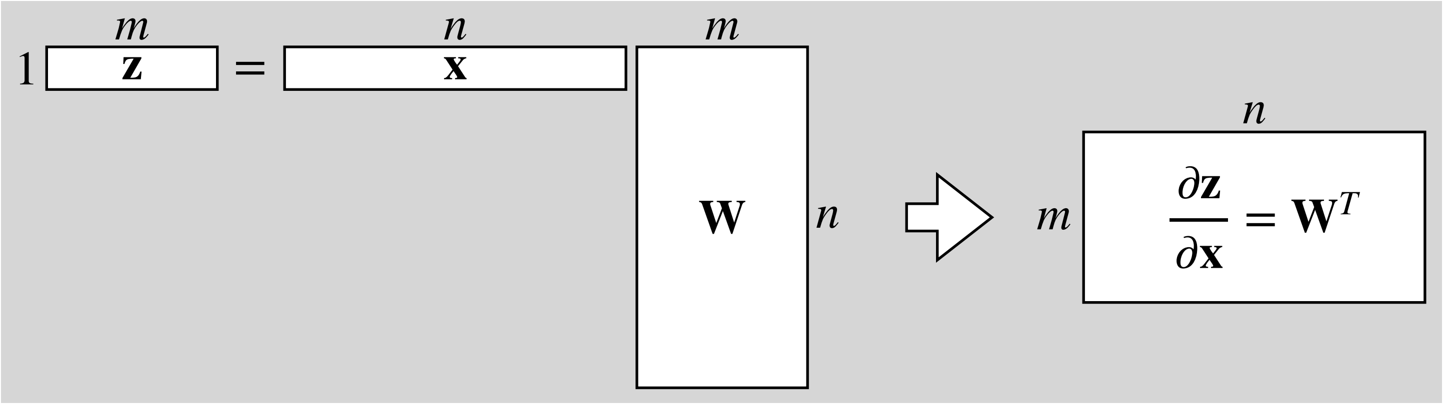


Matrix Calculus

- Matrix times column vector: $\mathbf{z} = \mathbf{W}\mathbf{x}$ where $\mathbf{z} \in \mathbb{R}^{m\times 1}$, $\mathbf{x} \in \mathbb{R}^{n\times 1}$, and $\mathbf{W} \in \mathbb{R}^{m\times n}$. Then, $\dfrac{\partial \mathbf{z}}{\partial \mathbf{x}} = \mathbf{W} \in \mathbb{R}^{m\times n}$.

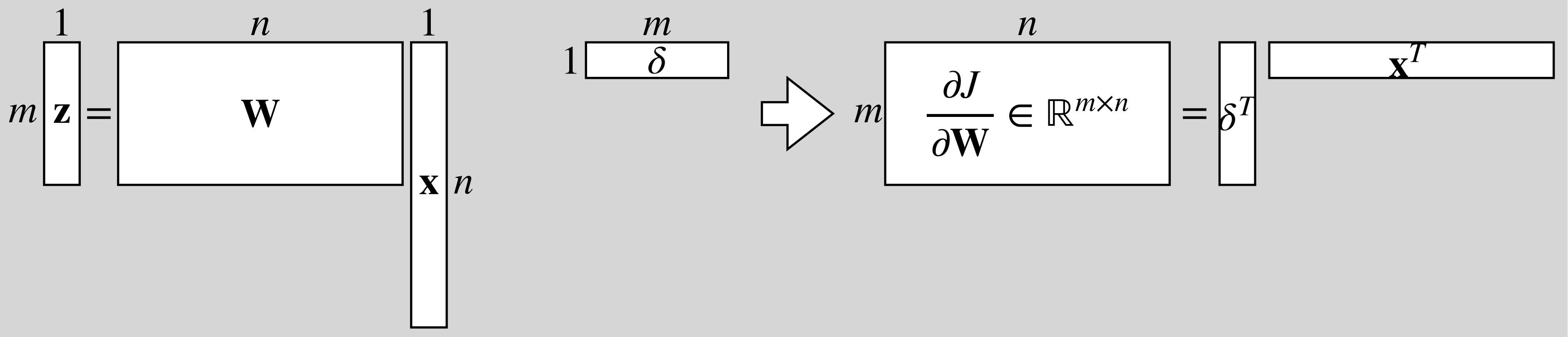


- Row vector times matrix: $\mathbf{z} = \mathbf{x}\mathbf{W}$ where $\mathbf{z} \in \mathbb{R}^{1\times m}$, $\mathbf{x} \in \mathbb{R}^{1\times n}$, and $\mathbf{W} \in \mathbb{R}^{n\times m}$. Then, $\dfrac{\partial \mathbf{z}}{\partial \mathbf{x}} = \mathbf{W}^T \in \mathbb{R}^{m\times n}$.



Matrix Calculus

- A vector with itself: $\mathbf{z} = \mathbf{x}$. Then, $\dfrac{\partial \mathbf{z}}{\partial \mathbf{x}} = \mathbf{I}$.
- An elementwise function applied to a vector: $\mathbf{z} = f(\mathbf{x})$ where $z_i = f(x_i)$. Then, $\dfrac{\partial \mathbf{z}}{\partial \mathbf{x}} = \mathrm{diag}(f'(\mathbf{x}))$.
  - We can also write $\odot f'(\mathbf{x})$ when applying the chain rule!
- Matrix times column vector: $\mathbf{z} = \mathbf{W}\mathbf{x}$ and $\delta = \dfrac{\partial J}{\partial \mathbf{z}} \in \mathbb{R}^{1\times m}$. Then, $\dfrac{\partial J}{\partial \mathbf{W}} = \dfrac{\partial J}{\partial \mathbf{z}} \dfrac{\partial \mathbf{z}}{\partial \mathbf{W}} = \delta \dfrac{\partial \mathbf{z}}{\partial \mathbf{W}} = \delta^T \mathbf{x}^T \in \mathbb{R}^{m\times n}$



- Row vector times Matrix: $\mathbf{x} \in \mathbb{R}^{1\times n}$, $\mathbf{W} \in \mathbb{R}^{n\times m}$, $\mathbf{z} = \mathbf{x}\mathbf{W} \in \mathbb{R}^{1\times m}$, and $\delta = \dfrac{\partial J}{\partial \mathbf{z}} \in \mathbb{R}^{1\times m}$.
  - Then, $\dfrac{\partial J}{\partial \mathbf{W}} = \dfrac{\partial J}{\partial \mathbf{z}} \dfrac{\partial \mathbf{z}}{\partial \mathbf{W}} = \delta \dfrac{\partial \mathbf{z}}{\partial \mathbf{W}} = \mathbf{x}^T \delta \in \mathbb{R}^{n\times m}$

Matrix Calculus

- (Recall) Cross-entropy loss w.r.t. logits:
  - Suppose $J = \mathrm{CE}(\mathbf{y}, \hat{\mathbf{y}}) \in \mathbb{R}$ and $\hat{\mathbf{y}} = \mathrm{softmax}(\mathbf{z}) \in \mathbb{R}^{1\times k}$.
    - Then, $\dfrac{\partial J}{\partial \mathbf{z}} = \hat{\mathbf{y}} - \mathbf{y} \in \mathbb{R}^{1\times k}$ where k is the number of classes.

Vanishing and Exploding Gradients

- Consider a deep network with L layers, input $\mathbf{x}$, and output $\mathbf{o}$.
- With each layer l defined by a transformation f_l parametrized by weights $\mathbf{W}^{(l)}$, whose hidden layer output is $\mathbf{h}^{(l)}$ (let $\mathbf{h}^{(0)} = \mathbf{x}$), our network can be expressed as:

$$\mathbf{h}^{(l)} = f_l(\mathbf{h}^{(l-1)}) \text{ and thus } \mathbf{o} = f_L \circ \cdots \circ f_1(\mathbf{x})$$

- If all the hidden layer output and the input are vectors, we can write the gradient of $\mathbf{o}$ with respect to $\mathbf{W}^{(l)}$:

$$\partial_{\mathbf{W}^{(l)}} \mathbf{o} = \underbrace{\partial_{\mathbf{h}^{(L-1)}} \mathbf{h}^{(L)}}_{=M^{(L)}} \cdots \underbrace{\partial_{\mathbf{h}^{(l)}} \mathbf{h}^{(l+1)}}_{=M^{(l+1)}} \underbrace{\partial_{\mathbf{W}^{(l)}} \mathbf{h}^{(l)}}_{=\mathbf{v}^{(l)}}$$

- In other words, this gradient is the **product of $L-1$ matrices** and the gradient vector.

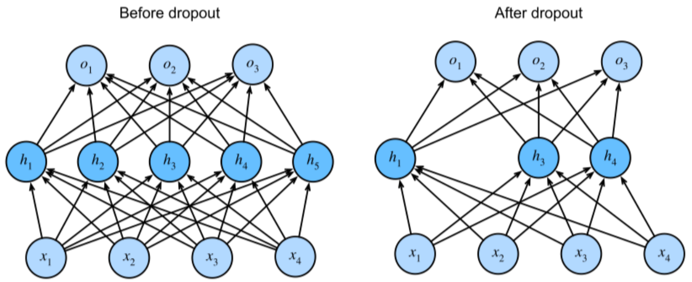
Other Issues

Nonparametrics?

- Are MLPs parametric models?
    - The models do have millions of parameters.
- While neural networks clearly have parameters, in some ways, it can be more fruitful to think of them as behaving like nonparametric models.
- So what precisely makes a model nonparametric?
    - While the name covers a diverse set of approaches, one common theme is that nonparametric methods tend to have a level of complexity that grows as the amount of available data grows.
    - Nonparametric methods include 1) k-nearest neighbor algorithm and 2) kernel-based methods.
- In a sense, because neural networks are over-parametrized, they tend to interpolate the training data (fitting it perfectly) having the same property as nonparametric models.

Regularization in Neural Networks

- Early stopping
  - While deep neural networks are capable of fitting arbitrary labels, early stopping becomes an efficient method for regularizing networks.
  - In other words, whenever a model has fitted the cleanly labeled data but not randomly labeled examples, it has, in fact, been generalized.
- Weight decay
  - Depending on which weight norm is penalized, it is known either as ridge regularization (for l_2-norm) or lasso regularization (for l_1-norm).
  - While it still remains a popular tool, researchers have noted that typical strengths of weight decay are insufficient for generalization.
- Dropout
  - Randomly replace some portion of nodes into 0.



Code Implementation (MLP)

Colab Link:

https://colab.research.google.com/drive/13JTcJ1mBCk5ZZH68yRb3LqmnxTLrz4Ua?authuser=2#scrollTo=uaokkwJwsN5I

1. Library Import

1. Library Import

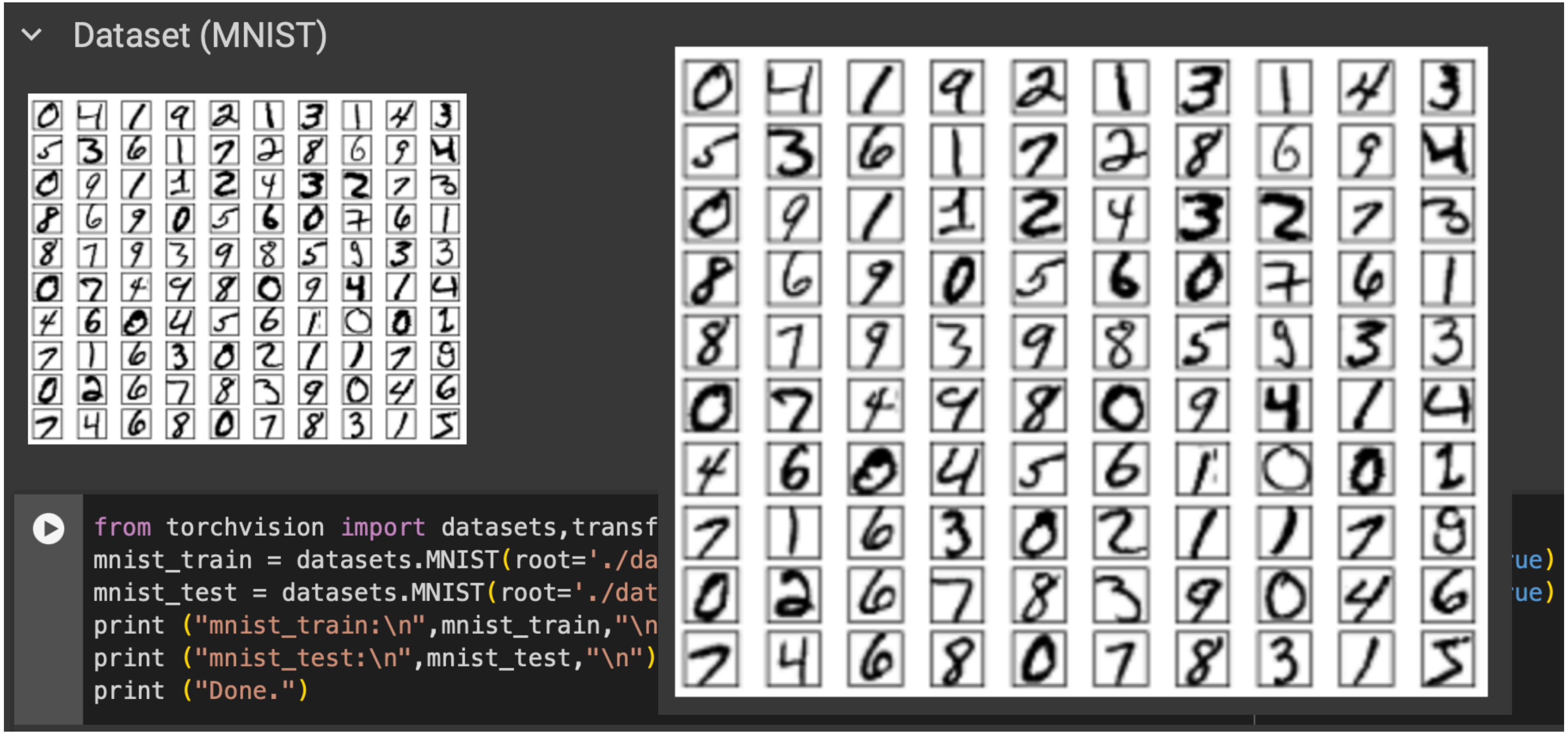
Colab

Multilayer Perceptron (MLP)

```
import numpy as np
import matplotlib.pyplot as plt
import torch
import torch.nn as nn
import torch.optim as optim
import torch.nn.functional as F
%matplotlib inline
%config InlineBackend.figure_format='retina'
print ("PyTorch version:[%s]."%(torch.__version__))
device = torch.device('cuda:0' if torch.cuda.is_available() else 'cpu')
print ("device:[%s]."%(device))
```

Importing Libraries

2. Load the dataset (MNIST)



2. Load the dataset (MNIST)

Dataset (MNIST)

```
from torchvision import datasets,transforms
mnist_train = datasets.MNIST(root='./data/',train=True,transform=transforms.ToTensor(),download=True)
mnist_test = datasets.MNIST(root='./data/',train=False,transform=transforms.ToTensor(),download=True)
print ("mnist_train:\n",mnist_train,"\n")
print ("mnist_test:\n",mnist_test,"\n")
print ("Done.")
```

Loading the MNIST Dataset

3. Batch size and Data Iterator

Data Iterator

1. Setting Batch size

```
[ ] BATCH_SIZE = 256
    train_iter = torch.utils.data.DataLoader(mnist_train,batch_size=BATCH_SIZE,shuffle=True,num_workers=1)
    test_iter = torch.utils.data.DataLoader(mnist_test,batch_size=BATCH_SIZE,shuffle=True,num_workers=1)
    print ("Done.")
```

2. Creating Data Loaders

4. Define the MLP model

Define the MLP model

```python
class MultiLayerPerceptronClass(nn.Module):
    """
        Multilayer Perceptron (MLP) Class
    """
    def __init__(self,name='mlp',xdim=784,hdim=256,ydim=10):
        super(MultiLayerPerceptronClass,self).__init__()
        self.name = name
        self.xdim = xdim
        self.hdim = hdim
        self.ydim = ydim
        self.lin_1 = nn.Linear(
            # FILL IN HERE
            self.xdim, self.hdim
        )
        self.lin_2 = nn.Linear(
            # FILL IN HERE
            self.hdim, self.ydim
        )
        self.init_param() # initialize parameters
```

```python
    def init_param(self):
        nn.init.kaiming_normal_(self.lin_1.weight)
        nn.init.zeros_(self.lin_1.bias)
        nn.init.kaiming_normal_(self.lin_2.weight)
        nn.init.zeros_(self.lin_2.bias)

    def forward(self,x):
        net = x
        net = self.lin_1(net)
        net = F.relu(net)
        net = self.lin_2(net)
        return net

M = MultiLayerPerceptronClass(name='mlp',xdim=784,hdim=256,ydim=10).to(device)
loss = nn.CrossEntropyLoss()
optm = optim.Adam(M.parameters(),lr=1e-3)
print ("Done.")
```

4. Define the MLP model

Define the MLP model

```
class MultiLayerPerceptronClass(nn.Module):
    """
        Multilayer Perceptron (MLP) Class
    """
    def __init__(self,name='mlp',xdim=784,hdim=256,ydim=10):
        super(MultiLayerPerceptronClass,self).__init__()
        self.name = name
        self.xdim = xdim
        self.hdim = hdim
        self.ydim = ydim
        self.lin_1 = nn.Linear(
            # FILL IN HERE
            self.xdim, self.hdim
        )
        self.lin_2 = nn.Linear(
            # FILL IN HERE
            self.hdim, self.ydim
        )
        self.init_param() # initialize parameters
```

Class Definition

4. Define the MLP model

Define the MLP model

```
class MultiLayerPerceptronClass(nn.Module):
    """
        Multilayer Perceptron (MLP) Class
    """
    def __init__(self,name='mlp',xdim=784,hdim=256,ydim=10):
        super(MultiLayerPerceptronClass,self).__init__()
        self.name = name
        self.xdim = xdim
        self.hdim = hdim
        self.ydim = ydim
        self.lin_1 = nn.Linear(
            # FILL IN HERE
            self.xdim, self.hdim
        )
        self.lin_2 = nn.Linear(
            # FILL IN HERE
            self.hdim, self.ydim
        )
        self.init_param() # initialize parameters
```

initial setting

4. Define the MLP model

Define the MLP model

```python
class MultiLayerPerceptronClass(nn.Module):
    """
        Multilayer Perceptron (MLP) Class
    """
    def __init__(self,name='mlp',xdim=784,hdim=256,ydim=10):
        super(MultiLayerPerceptronClass,self).__init__()
        self.name = name
        self.xdim = xdim
        self.hdim = hdim
        self.ydim = ydim
        self.lin_1 = nn.Linear(
            # FILL IN HERE
            self.xdim, self.hdim
        )
        self.lin_2 = nn.Linear(
            # FILL IN HERE
            self.hdim, self.ydim
        )
        self.init_param() # initialize parameters
```

Linear Layers

4. Define the MLP model

Define the MLP model

```
    def init_param(self):
        nn.init.kaiming_normal_(self.lin_1.weight)
        nn.init.zeros_(self.lin_1.bias)
        nn.init.kaiming_normal_(self.lin_2.weight)
        nn.init.zeros_(self.lin_2.bias)

    def forward(self,x):
        net = x
        net = self.lin_1(net)
        net = F.relu(net)
        net = self.lin_2(net)
        return net

M = MultiLayerPerceptronClass(name='mlp',xdim=784,hdim=256,ydim=10).to(device)
loss = nn.CrossEntropyLoss()
optm = optim.Adam(M.parameters(),lr=1e-3)
print ("Done.")
```

Parameter Initialization

```
        )
        self.init_param() # initialize parameters
```

4. Define the MLP model

Define the MLP model

```python
    def init_param(self):
        nn.init.kaiming_normal_(self.lin_1.weight)
        nn.init.zeros_(self.lin_1.bias)
        nn.init.kaiming_normal_(self.lin_2.weight)
        nn.init.zeros_(self.lin_2.bias)

    def forward(self,x):
        net = x
        net = self.lin_1(net)
        net = F.relu(net)
        net = self.lin_2(net)
        return net

M = MultiLayerPerceptronClass(name='mlp',xdim=784,hdim=256,ydim=10).to(device)
loss = nn.CrossEntropyLoss()
optm = optim.Adam(M.parameters(),lr=1e-3)
print ("Done.")
```

Forward propagation

```python
        )
        self.init_param() # initialize parameters
```

4. Define the MLP model

Define the MLP model

```
    def init_param(self):
        nn.init.kaiming_normal_(self.lin_1.weight)
        nn.init.zeros_(self.lin_1.bias)
        nn.init.kaiming_normal_(self.lin_2.weight)
        nn.init.zeros_(self.lin_2.bias)

    def forward(self,x):
        net = x
        net = self.lin_1(net)
        net = F.relu(net)
        net = self.lin_2(net)
        return net
```

MLP Model, Loss Function, and Optimizer Setup

```
M = MultiLayerPerceptronClass(name='mlp',xdim=784,hdim=256,ydim=10).to(device)
loss = nn.CrossEntropyLoss()
optm = optim.Adam(M.parameters(),lr=1e-3)
print ("Done.")
```

```
        )
        self.init_param() # initialize parameters
```

5. Evaluation Function

Evaluation Function

```
[ ]  def func_eval(model,data_iter,device):
         with torch.no_grad():
             model.eval() # evaluate (affects DropOut and BN)
             n_total,n_correct = 0,0
             for batch_in,batch_out in data_iter:
                 y_trgt = batch_out.to(device)
                 model_pred = model(
                     # FILL IN HERE
                     batch_in.view(-1, 28*28).to(device)
                 )
                 _,y_pred = torch.max(model_pred.data,1)
                 n_correct += (
                     # FILL IN HERE
                     y_pred == y_trgt
                 ).sum().item()
                 n_total += batch_in.size(0)
             val_accr = (n_correct/n_total)
             model.train() # back to train mode
         return val_accr
     print ("Done")
```

5. Evaluation Function

Evaluation Function

Switching to Evaluation Mode

```
[ ] def func_eval(model,data_iter,device):
        with torch.no_grad():
            model.eval() # evaluate (affects DropOut and BN)
            n_total,n_correct = 0,0
            for batch_in,batch_out in data_iter:
                y_trgt = batch_out.to(device)
                model_pred = model(
                    # FILL IN HERE
                    batch_in.view(-1, 28*28).to(device)
                )
                _,y_pred = torch.max(model_pred.data,1)
                n_correct += (
                    # FILL IN HERE
                    y_pred == y_trgt
                ).sum().item()
                n_total += batch_in.size(0)
            val_accr = (n_correct/n_total)
            model.train() # back to train mode
        return val_accr
    print ("Done")
```

5. Evaluation Function

Evaluation Function

```
[ ] def func_eval(model,data_iter,device):
        with torch.no_grad():
            model.eval() # evaluate (affects DropOut and BN)
            n_total,n_correct = 0,0
            for batch_in,batch_out in data_iter:
                y_trgt = batch_out.to(device)
                model_pred = model(
                    # FILL IN HERE
                    batch_in.view(-1, 28*28).to(device)
                )
                _,y_pred = torch.max(model_pred.data,1)
```

Looping Through Data and Making Predictions

```
                    # FILL IN HERE
                    y_pred == y_trgt
                ).sum().item()
                n_total += batch_in.size(0)
            val_accr = (n_correct/n_total)
            model.train() # back to train mode
        return val_accr
    print ("Done")
```

5. Evaluation Function

Evaluation Function

```
[ ]  def func_eval(model,data_iter,device):
         with torch.no_grad():
             model.eval() # evaluate (affects DropOut and BN)
             n_total,n_correct = 0,0
             for batch_in,batch_out in data_iter:
                 y_trgt = batch_out.to(device)
                 model_pred = model(
                     # FILL IN HERE
```

Calculating Predictions and Comparing Accuracy

```
                 )
                 _,y_pred = torch.max(model_pred.data,1)
                 n_correct += (
                     # FILL IN HERE
                     y_pred == y_trgt
                 ).sum().item()
                 n_total += batch_in.size(0)
             val_accr = (n_correct/n_total)
             model.train() # back to train mode
         return val_accr
     print ("Done")
```

5. Evaluation Function

Evaluation Function

```python
def func_eval(model,data_iter,device):
    with torch.no_grad():
        model.eval() # evaluate (affects DropOut and BN)
        n_total,n_correct = 0,0
        for batch_in,batch_out in data_iter:
            y_trgt = batch_out.to(device)
            model_pred = model(
                # FILL IN HERE
                batch_in.view(-1, 28*28).to(device)
            )
            _,y_pred = torch.max(model_pred.data,1)
            n_correct += (
                # FILL IN HERE
                y_pred == y_trgt
            ).sum().item()
        val_accr = (n_correct/n_total)
        model.train() # back to train mode
    return val_accr
print ("Done")
```

Returning to Training Mode

5. Training model

Train

```
[ ] print ("Start training.")
    M.init_param() # initialize parameters
    M.train()
    EPOCHS,print_every = 10,1
    for epoch in range(EPOCHS):
        loss_val_sum = 0
        for batch_in,batch_out in train_iter:
            # Forward path
            y_pred = M.forward(batch_in.view(-1, 28*28).to(device))
            loss_out = loss(y_pred,batch_out.to(device))
            # Update
            # FILL IN HERE      # reset gradient
            optm.zero_grad()
            # FILL IN HERE      # backpropagate
            loss_out.backward()
            # FILL IN HERE      # optimizer update
            optm.step()
            loss_val_sum += loss_out
        loss_val_avg = loss_val_sum/len(train_iter)
        # Print
        if ((epoch%print_every)==0) or (epoch==(EPOCHS-1)):
            train_accr = func_eval(M,train_iter,device)
            test_accr = func_eval(M,test_iter,device)
            print ("epoch:[%d] loss:[%.3f] train_accr:[%.3f] test_accr:[%.3f]."%
                   (epoch,loss_val_avg,train_accr,test_accr))
    print ("Done")
```

5. Training model

Train

```
[ ] print ("Start training.")
    M.init_param() # initialize parameters
    M.train()
    EPOCHS,print_every = 10,1
    for epoch in range(EPOCHS):
```

Starting Training and Initializing Parameters

```
        for batch_in,batch_out in train_iter:
            # Forward path
            y_pred = M.forward(batch_in.view(-1, 28*28).to(device))
            loss_out = loss(y_pred,batch_out.to(device))
            # Update
            # FILL IN HERE      # reset gradient
            optm.zero_grad()
            # FILL IN HERE      # backpropagate
            loss_out.backward()
            # FILL IN HERE      # optimizer update
            optm.step()
            loss_val_sum += loss_out
        loss_val_avg = loss_val_sum/len(train_iter)
        # Print
        if ((epoch%print_every)==0) or (epoch==(EPOCHS-1)):
            train_accr = func_eval(M,train_iter,device)
            test_accr = func_eval(M,test_iter,device)
            print ("epoch:[%d] loss:[%.3f] train_accr:[%.3f] test_accr:[%.3f]."%
                   (epoch,loss_val_avg,train_accr,test_accr))
    print ("Done")
```

5. Training model

Train

```
[ ] print ("Start training.")
    M.init_param() # initialize parameters
    M.train()
    EPOCHS,print_every = 10,1
    for epoch in range(EPOCHS):
        loss_val_sum = 0
        for batch_in,batch_out in train_iter:
            # Forward path
            y_pred = M.forward(batch_in.view(-1, 28*28).to(device))
            loss_out = loss(y_pred,batch_out.to(device))
            # Update
            # FILL IN HERE      # reset gradient
```

Epoch Loop and Batch Processing

```
            # FILL IN HERE      # backpropagate
            loss_out.backward()
            # FILL IN HERE      # optimizer update
            optm.step()
            loss_val_sum += loss_out
        loss_val_avg = loss_val_sum/len(train_iter)
        # Print
        if ((epoch%print_every)==0) or (epoch==(EPOCHS-1)):
            train_accr = func_eval(M,train_iter,device)
            test_accr = func_eval(M,test_iter,device)
            print ("epoch:[%d] loss:[%.3f] train_accr:[%.3f] test_accr:[%.3f]."%
                   (epoch,loss_val_avg,train_accr,test_accr))
    print ("Done")
```

5. Training model

Train

```
print ("Start training.")
M.init_param() # initialize parameters
M.train()
EPOCHS,print_every = 10,1
for epoch in range(EPOCHS):
    loss_val_sum = 0
    for batch_in,batch_out in train_iter:
        # Forward path
        y_pred = M.forward(batch_in.view(-1, 28*28).to(device))
        loss_out = loss(y_pred,batch_out.to(device))
        # Update
        # FILL IN HERE      # reset gradient
        optm.zero_grad()
        # FILL IN HERE      # backpropagate
        loss_out.backward()
        # FILL IN HERE      # optimizer update
        optm.step()
        loss_val_sum += loss_out
    loss_val_avg = loss_val_sum/len(train_iter)
    # Print
```

Backpropagation and Weight Update

```
-1)):
)
        test_accr = func_eval(M,test_iter,device)
        print ("epoch:[%d] loss:[%.3f] train_accr:[%.3f] test_accr:[%.3f]."%
               (epoch,loss_val_avg,train_accr,test_accr))
print ("Done")
```

6. Test model

Test

```
[ ] n_sample = 25
    sample_indices = np.random.choice(len(mnist_test.targets), n_sample, replace=False)
    test_x = mnist_test.data[sample_indices]
    test_y = mnist_test.targets[sample_indices]
    with torch.no_grad():
        y_pred = M.forward(test_x.view(-1, 28*28).type(torch.float).to(device)/255.)
    y_pred = y_pred.argmax(axis=1)
    plt.figure(figsize=(10,10))
    for idx in range(n_sample):
        plt.subplot(5, 5, idx+1)
        plt.imshow(test_x[idx], cmap='gray')
        plt.axis('off')
        plt.title("Pred:%d, Label:%d"%(y_pred[idx],test_y[idx]))
    plt.show()
    print ("Done")
```

6. Test model

Test

```python
n_sample = 25
sample_indices = np.random.choice(len(mnist_test.targets), n_sample, replace=False)
test_x = mnist_test.data[sample_indices]
test_y = mnist_test.targets[sample_indices]
with torch.no_grad():
                                        (test_x.view(-1, 28*28).type(torch.float).to(device)/255.)
y_pred = y_pred.argmax(axis=1)
plt.figure(figsize=(10,10))
for idx in range(n_sample):
    plt.subplot(5, 5, idx+1)
    plt.imshow(test_x[idx], cmap='gray')
    plt.axis('off')
    plt.title("Pred:%d, Label:%d"%(y_pred[idx],test_y[idx]))
plt.show()
print ("Done")
```

Selecting Test Samples

6. Test model

Test

```
[ ] n_sample = 25
    sample_indices = np.random.choice(len(mnist_test.targets), n_sample, replace=False)
    test_x = mnist_test.data[sample_indices]
    test_y = mnist_test.targets[sample_indices]
    with torch.no_grad():
        y_pred = M.forward(test_x.view(-1, 28*28).type(torch.float).to(device)/255.)
    y_pred = y_pred.argmax(axis=1)
    plt.figure(figsize=(10,10))
                         n_sample):
        plt.subplot(5, 5, idx+1)
        plt.imshow(test_x[idx], cmap='gray')
        plt.axis('off')
        plt.title("Pred:%d, Label:%d"%(y_pred[idx],test_y[idx]))
    plt.show()
    print ("Done")
```

Model Prediction

6. Test model

Test

```
[ ] n_sample = 25
    sample_indices = np.random.choice(len(mnist_test.targets), n_sample, replace=False)
    test_x = mnist_test.data[sample_indices]
    test_y = mnist_test.targets[sample_indices]
    with torch.no_grad():
                                      st_x.view(-1, 28*28).type(torch.float).to(device)/255.)
        y_pred = y_pred.argmax(axis=1)
    plt.figure(figsize=(10,10))
    for idx in range(n_sample):
        plt.subplot(5, 5, idx+1)
        plt.imshow(test_x[idx], cmap='gray')
        plt.axis('off')
        plt.title("Pred:%d, Label:%d"%(y_pred[idx],test_y[idx]))
    plt.show()
    print ("Done")
```

Visualizing the Predictions

6. Test model

Test

```
[ ] n_sample = 25
    sample_indices = np.random.choic
    test_x = mnist_test.data[sample_
    test_y = mnist_test.targets[samp
    with torch.no_grad():
        y_pred = M.forward(test_x.vi
    y_pred = y_pred.argmax(axis=1)
    plt.figure(figsize=(10,10))
    for idx in range(n_sample):
        plt.subplot(5, 5, idx+1)
        plt.imshow(test_x[idx], cmap
        plt.axis('off')
        plt.title("Pred:%d, Label:%c
    plt.show()
    print ("Done")
```