# Macro Data Analysis of M4 and GDP

Sam Forbes

## Contents

# 1 GDP

## 1.1 GDP Estimates

GDP is integral to the national accounts collected by the Office for National Statistics (ONS) [8]. The national accounts provide a description of economic activity in the UK.

In the UK there are three different GDP estimations:

1. GDP from the output or production approach which measures the sum of value added through the production of goods and services within the economy.
2. GDP from the income approach which measures the total income generated by the production of goods and services within the economy. Figures broken down into income earned by companies (corporations), employees and the self employed.
3. GDP from the expenditure approach which measures the total expenditures on all finished goods and services produced within the economy.

Three estimates of GDP are produced by ONS. The preliminary estimate is based on output figures published 3.5 weeks after the end of the quarter, the second estimate is based on all three approaches published 8 weeks after the end of the quarter and the final estimate is the UK quarterly national accounts published 12 weeks after the end of the quarter.

GDP is measured quarterly or annually.

## 1.2 Formulae for GDP

GDP = private consumption spending + investments + government spending + (exports - imports)

[4]

## 1.3 Data

Figure 1 shows UK real (in 2016 prices) and nominal quarterly GDP from 1955-2018.

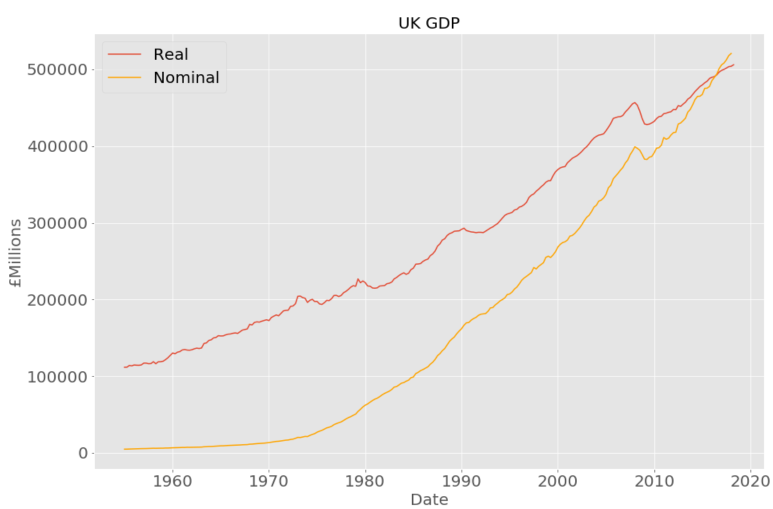


Figure 1: UK real and nominal GDP per quarter (1955-2018) [5]

Figure 2 shows the UK nominal GDP per quarter from 1955-2018 with a non-linear least squares exponential function fit and a Gaussian process fit and Figure 3 shows the same but for real GDP.

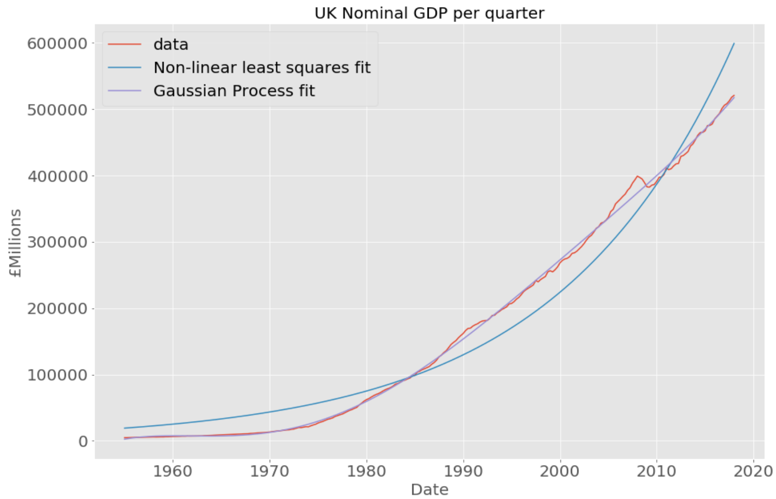


Figure 2: UK nominal GDP per quarter with non-linear least squares and Gaussian process fits (1955-2018)

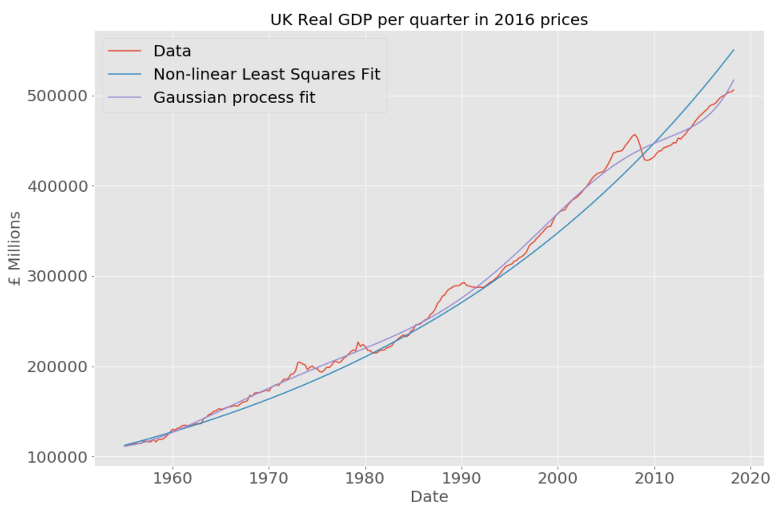


Figure 3: UK real GDP per quarter with non-linear least squares and Gaussian process fits (1955-2018)

Figure 4 shows the first difference of quarterly real GDP from 1955-2018 in 2016 prices. The series is weakly stationary according to the augmented Dickey-Fuller test [12].

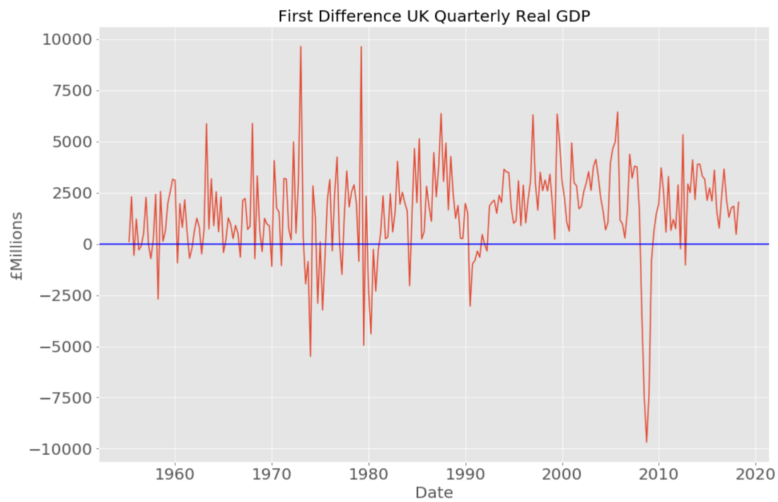


Figure 4: First difference of quarterly real GDP in 2016 prices (1955-2018)

### 1.4 GDP Breakdown

The output approach breaks GDP down into different sectors. Gross Value Added (GVA) defined as output less intermediate consumption is used as a measure of the contribution made by an individual producer, industry or sector [7]. Most sectors of GDP output classified by ONS are broadly defined to be in the manufacturing or services category. The two sectors agriculture, forestry and fishing and construction belong to neither manufacturing or services and are in the category of 'other'. Figure 5 shows the proportions of total GVA of the sectors. A further breakdown of total GVA for sectors categorised as services is given in Figure 6.

**Link between GDP and GVA: [16]**

$$\text{GDP} = \text{GVA} + \text{taxes on products} - \text{subsidies on products}$$

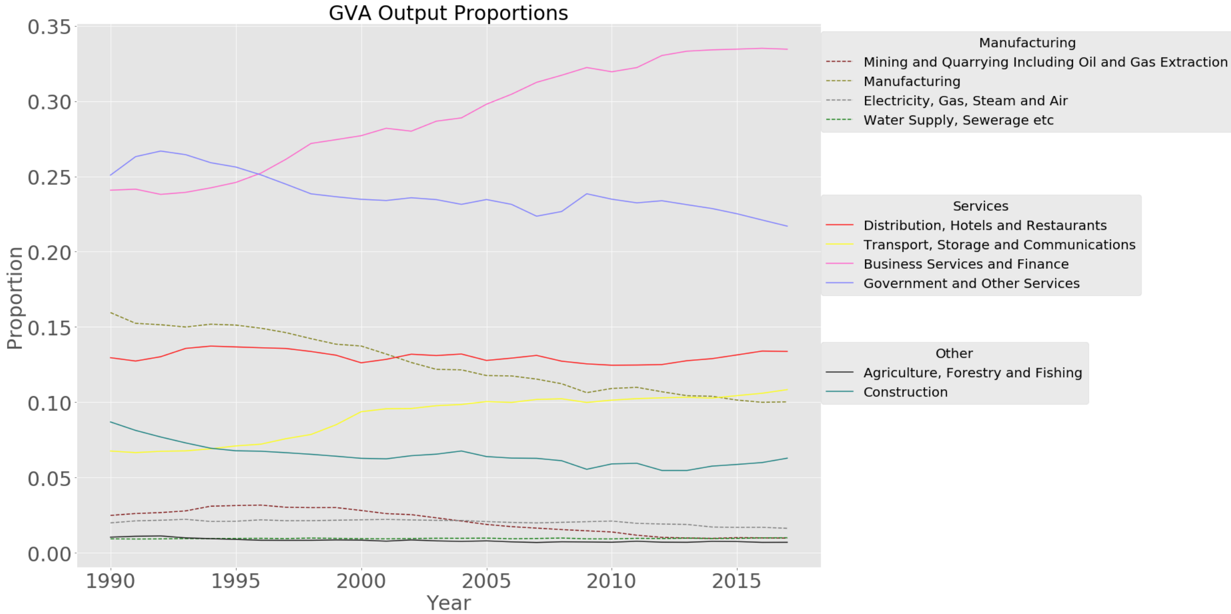


Figure 5: UK Yearly GVA Proportions (1990-2017)

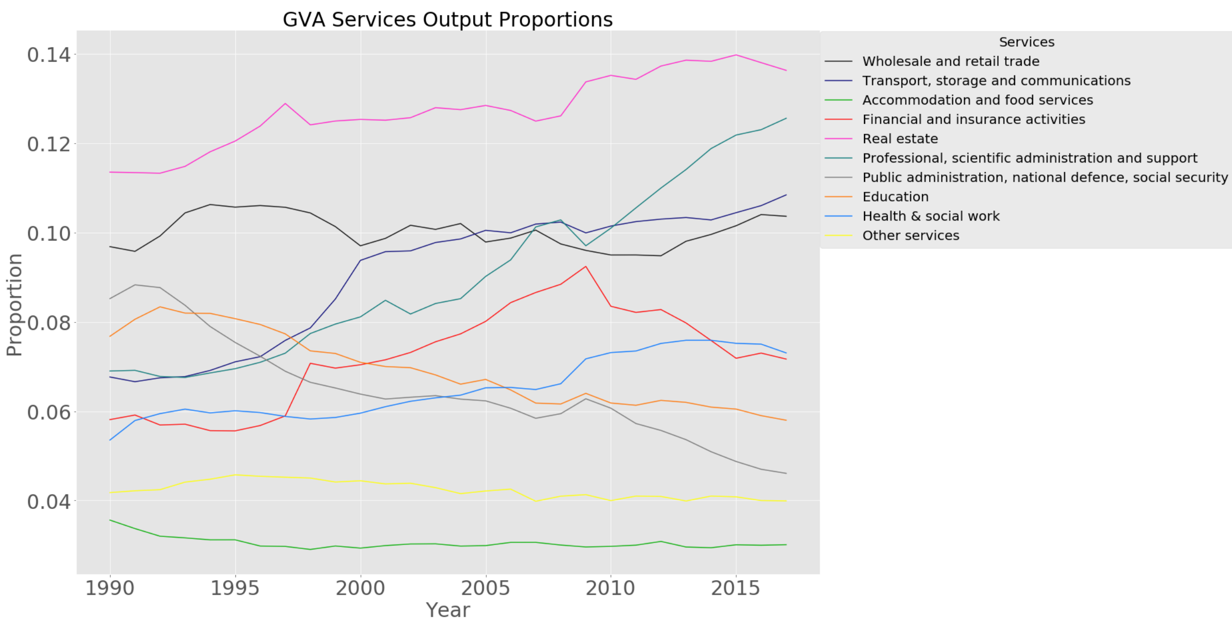


Figure 6: UK Yearly GVA Services Proportions (1990-2017)

## 2 M4

M4 data derived from the consolidated balance sheet of UK monetary financial institutions (MFIs) [2]. Data provided by

1. UK resident banks,
2. UK resident building societies,

3. the Bank of England Banking and Issue Departments.

M4 comprises of

1. the UK private sector other than MFIs holdings of sterling notes and coin,
2. sterling deposits, including certificates of deposit; commercial paper, bonds, FRNs (Floating Rate Note, which is a debt instrument) and other instruments of up to and including five years' original maturity issued by UK MFIs; claims on UK MFIs arising from repos (repurchase agreement: form of short term borrowing for dealers in government securities) (from December 1995),
3. estimated holdings of sterling bank bills,
4. 35% of the sterling inter-MFI difference (added to OFC (other financial corporations deposits, within wholesale M4).

## 2.1 Data

Figure 7 shows quarterly M4 with non-linear least squares exponential and Gaussian process fits from 1963-2018. Figure 8 shows the first difference of M4. Second difference of M4 is weakly stationary according to the augmented Dickey Fuller test.

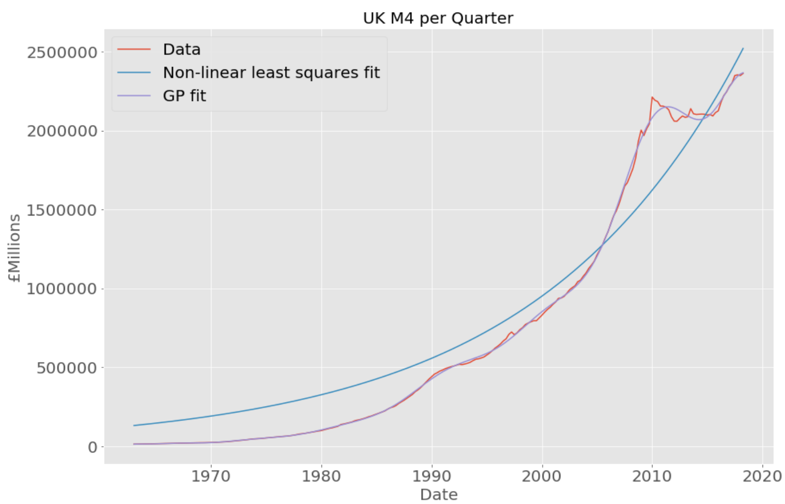


Figure 7: UK quarterly M4 with non-linear exponential least squares and Gaussian process fits (1963-2018) [1]

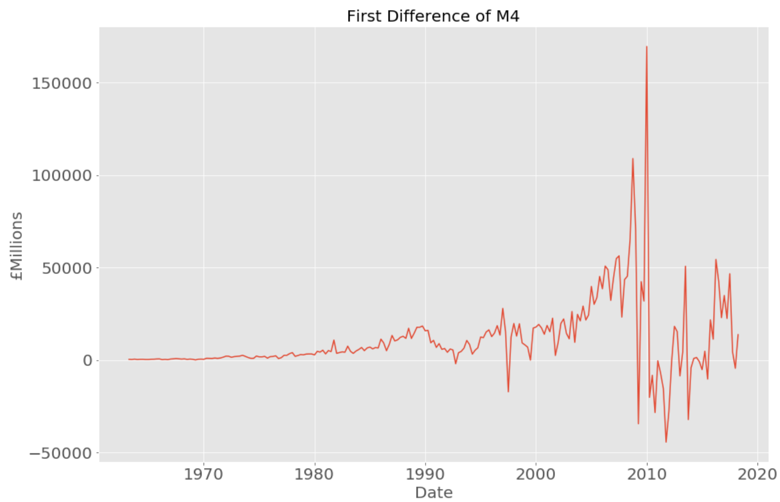


Figure 8: First difference of M4

# 3 More M4 Statistics

## 3.1 M4ex

In a 2007 Bank of England report a new measure of broad money was formulated that excluded the money holdings of some other financial corporations (OFCs). This may provide a better estimate of the money supply used as a medium of exchange [3]. Figure 9 shows quarterly M4 against M4ex.

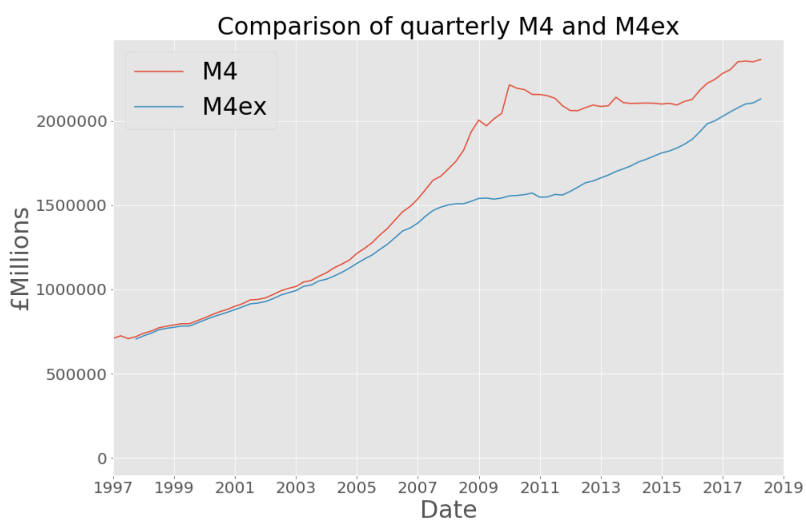


Figure 9: Quarterly M4 and M4ex 1997-2018

## 3.2 M4 Lending

M4 lending to the private sector, other financial institutions, non-financial corporations and households as well as M4 and M4ex is shown in Figure 10. Private sector lending is the sum of other financial institutions, non-financial corporations and household lending. M4 lending is considered an asset whereas M4 is a deposit and considered a liability [17].

The pink line in Figure 10 shows a theoretical M4 if it was rising twice as fast as NGDP (a constant velocity of 2).

**Formula relating M4 and M4L in [17]**

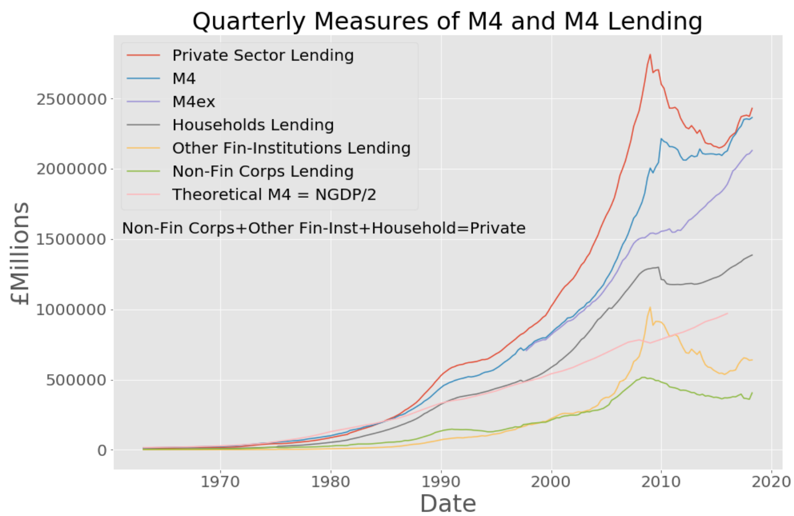


Figure 10: Quarterly M4 and M4 Lending [2]

# 4 V4

The velocity of M4 is defined

$$\mathrm{V4} = \frac{\mathrm{M4}}{\mathrm{NGDP}} \tag{1}$$

and is shown in Figure 11. From 1980-2008 nominal GDP (NGDP) increased by £336924 million and M4 increased by ~4.8 times as much at £1614420 million which is illustrated in the decrease in V4 during this time.

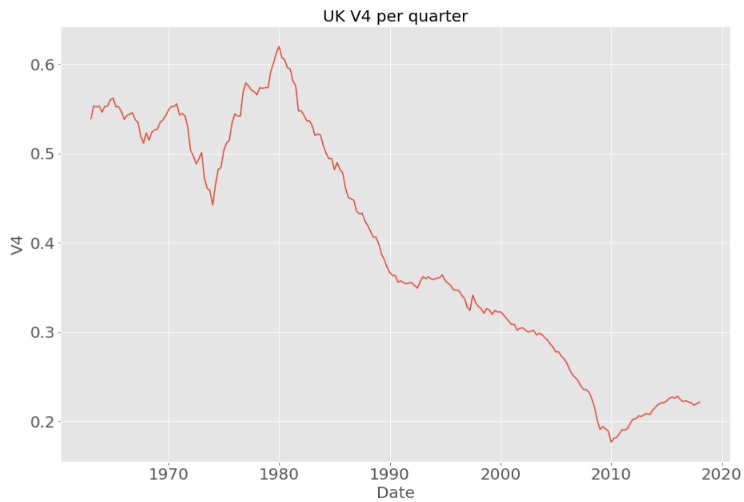


Figure 11: UK V4

## 4.1 Long Run Velocity

A synthetic estimate of M4 has been made from 1880-1962 and there exists estimates of nominal yearly GDP back to 1700. Figure 12 shows the yearly V4 from 1880-2016. Velocity goes down before both the two largest recessions in the 1930s and 2008.

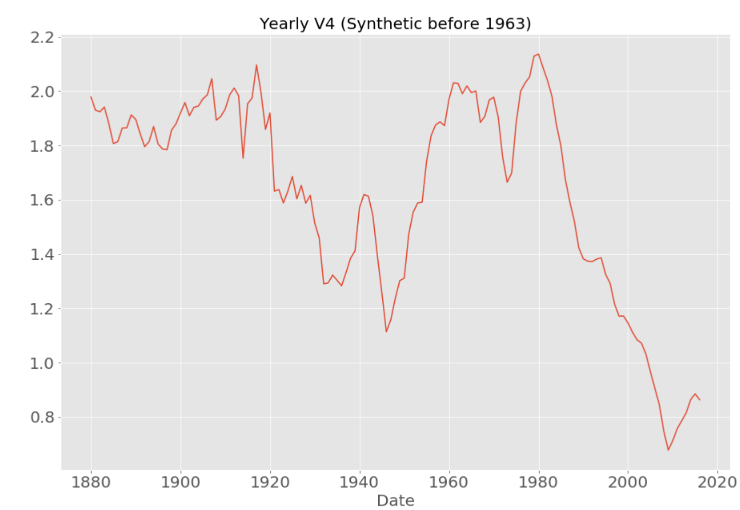


Figure 12: Yearly V4 from 1880-2016. Before 1963 synthetic M4 data is used.

## 4.2 GVA Proportions of V4

Taking the sector proportions of GVA as a proxy for the sector proportions of GDP we can calculate sector proportions of V4. For years $i = 1, 2, \ldots, N$ and a given sector A with GVA proportion for that year $(p_A)_i$ and mean sector proportion over the time period $\overline{p_A} = \dfrac{1}{N}\sum_{i=1}^{N}(p_A)_i$ we can define the yearly sector proportion of V4 as

$$(\mathrm{V4}_{p_A})_i = \frac{(p_A)_i}{\overline{p_A}}(\mathrm{V4})_i.$$

Dividing by the mean sector proportion makes sure the velocity proportions for each sector are on a comparable scale (deviation away from the mean).

These velocity proportions are shown in Figure 13. Some manufacturing sectors proportional velocity have a large comparable drop before the crisis, while some stay quite flat especially business services and finance.

A breakdown of service velocity proportions is shown in Figure 14. Similarly there is a large drop in some sectors notably public administration, national defence and social security and education. Financial and insurance activities and transport, storage and communications only service velocities exhibiting any notable upwards trend before the crisis.

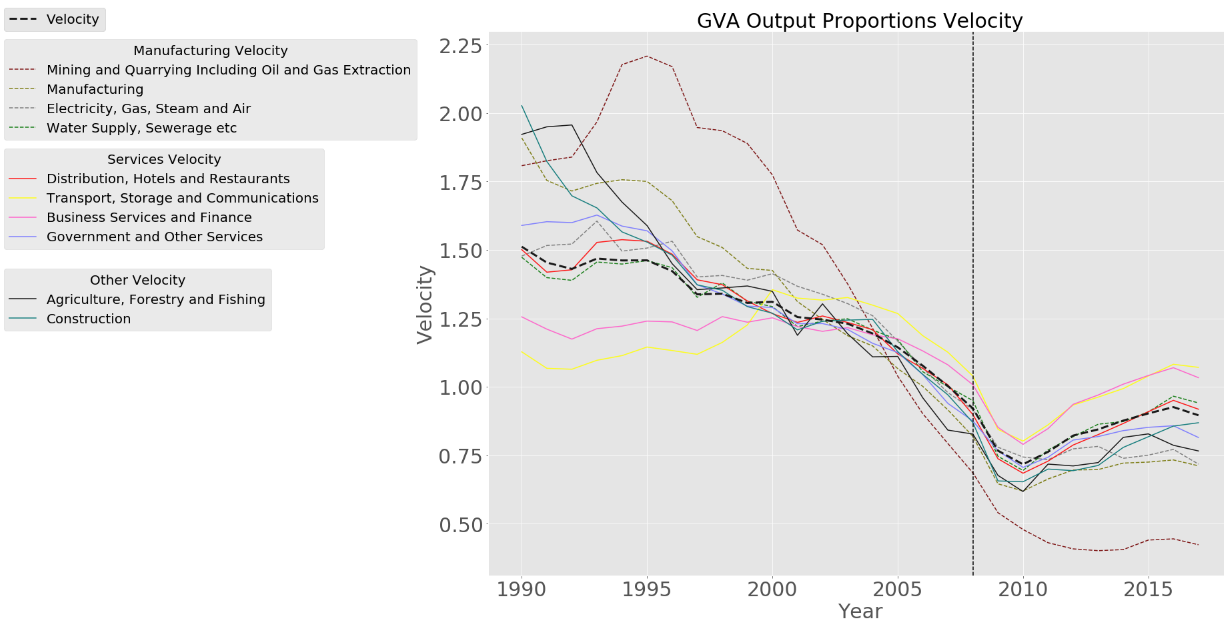


Figure 13: GVA sector yearly velocity proportions for 1990-2017. Original velocity black dotted line.

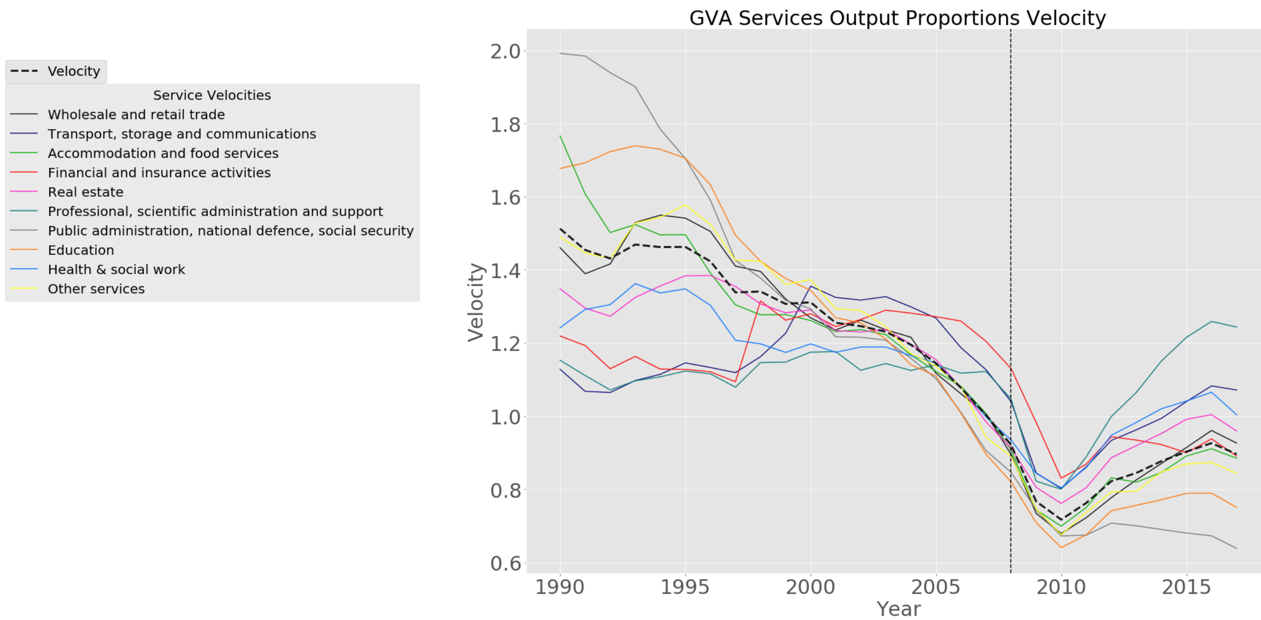


Figure 14: GVA sector yearly velocity proportions for Services for 1990-2017. Original velocity black dotted line.

# 5 Mortgage Lending

Quarterly mortgage lending data is presented in Figure 15 from 1997-2018 (seems not seasonally adjusted).

There is a dramatic spike around the 2008 crisis.

**Most of mortgage lending not contributing to GDP but does to M4?**

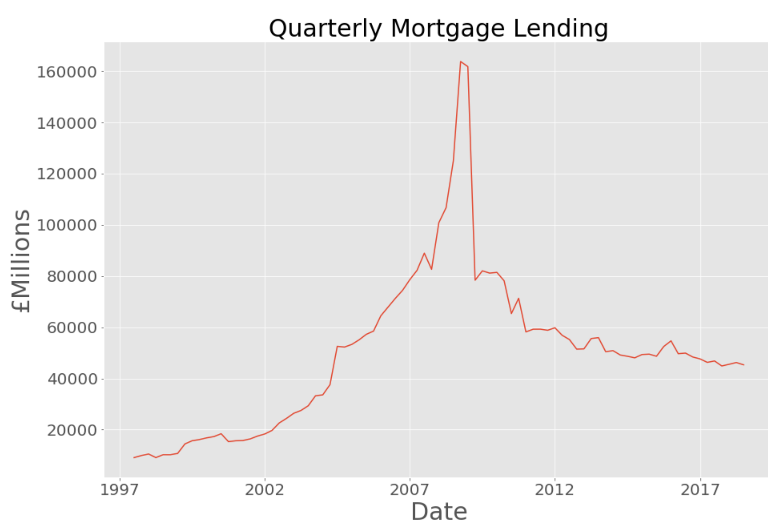


Figure 15: UK quarterly total market gross mortgage lending (1997-2018) [6]

[11] relates wealth which includes housing wealth to M4 indicating that mortgages are in M4. They do a vector autoregressive model (VAR) on real personal sector M4 deposits with real consumption, real disposable income, real total (financial and tangible) gross personal sector wealth, the 91-day Sterling Treasury bill rate

as a proxy for the return on alternative assets, an own-weighted average interest rate on personal sector M4 deposits and the inflation rate. They split M4 into personal and corporate sector balances and conclude wealth is an important indicator of M4.

# 6 Trade

The trade balance is defined as total exports minus total imports. Total quarterly exports and imports are shown in Figure 16.

Figure 17 illustrates the UK quarterly trade balance from 1998-2018. The trade balance fell dramatically since the late 1990s and has been in a deficit since. It has been oscillating substantially since the 2008 crash.

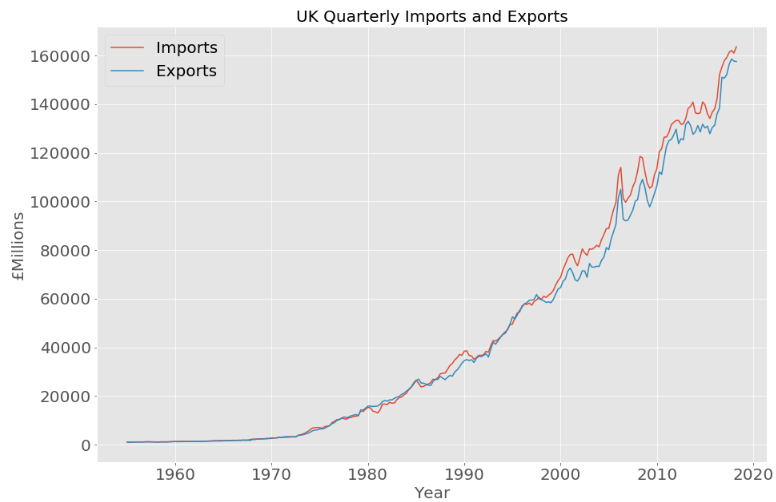


Figure 16: UK quarterly imports and exports (1955-2018)

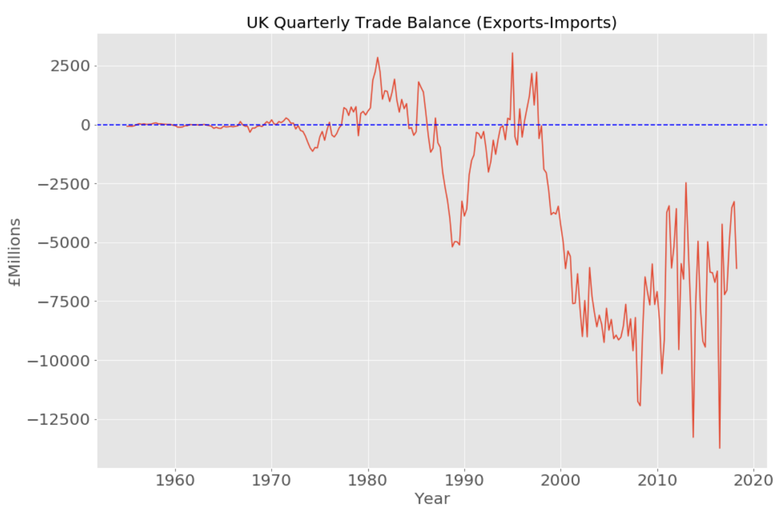


Figure 17: UK quarterly trade balance (1955-2018)

# 7 V4 Adjusted for Mortgages, Trade and Money Supply

Velocity can be adjusted to account for financial OFCs, prices, trade balance and mortgages. Two adjustments are listed below

$$V4_1 = \frac{\text{NGDP} - \text{BAL} + \text{IMP}}{\text{M4} - \text{MOR}} \tag{2}$$

$$\text{V4ex} = \frac{\text{NGDP}}{\text{M4ex}} \tag{3}$$

where BAL is the trade balance or deficit, IMP is the total imports and MOR is total mortgage lending.

Figure 18 shows V4 adjusted only for the trade balance for the years 1963-2018. Figure 19 shows four adjustments for the years 1997-2018. It seems that using M4ex is the only factor that takes away a significant amount of the trend of V4.

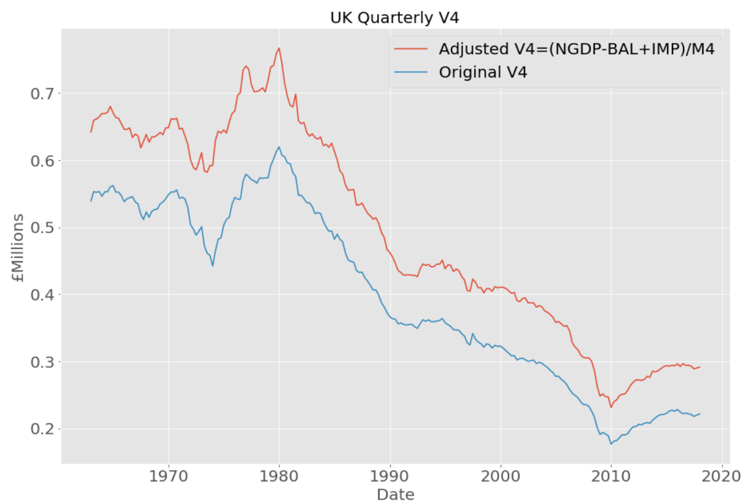


Figure 18: Adjusted Quarterly V4 1963-2018

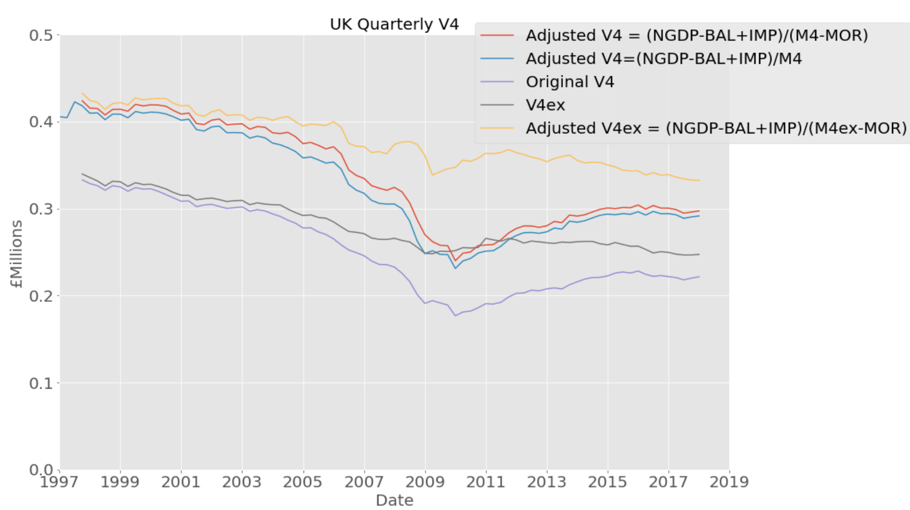


Figure 19: Adjusted Quarterly V4 1997-2018

Instead of using nominal GDP we can use real GDP in the velocity equation to fix prices as shown in Figure 20. The downwards trend shows a decrease in the quantity of products compared to the money supply.

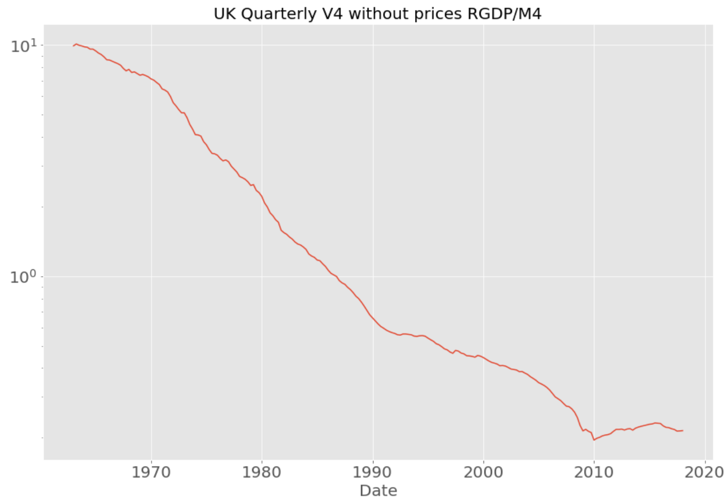


Figure 20: Adjusted Quarterly V4 1963-2018

# 8 Change in Trends of Rate of Change of M4 and GDP

Since the industrial revolution money supply and GDP have increased at an exponential rate. Figure 21 shows quarterly M4 on a log scale from 1963-2018 and Figure 22 shows quarterly NGDP on a log scale from 1955-2018.

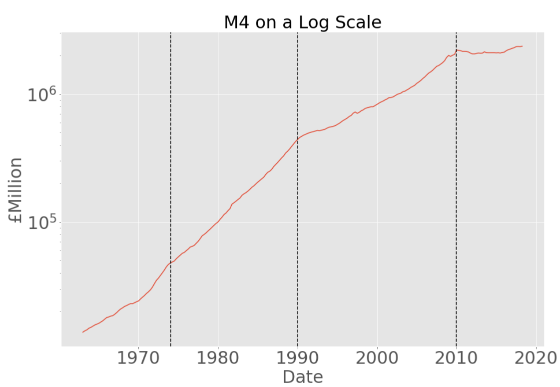


Figure 21: Quarterly M4 on a Log Scale (1963-2018)

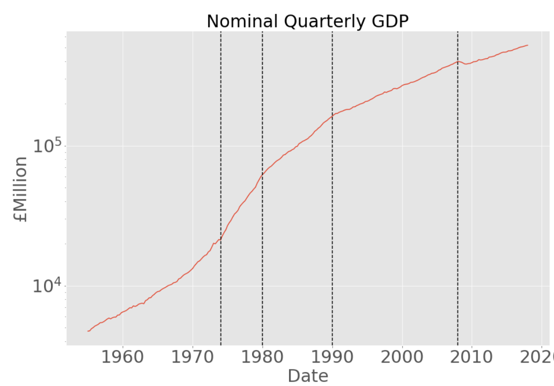


Figure 22: Quarterly NGDP on a Log Scale (1955-2018)

The black dotted lines show a possible visual change in the rate of growth. These occur at roughly the same time in both M4 and NGDP and happen at similar periods as recessions in the UK (note the absence of a change in the rate of change in the early 1980s in M4). Time of last four UK recessions detailed in [15].

For a time series $\{x_i\}_{i=1}^{N}$ we define the rate of change as $\{\frac{x_{i+1}}{x_i}\}_{i=1}^{N-1}$.

Figures 23 and 24 show the rate of change for quarterly M4 and NGDP respectively. The blue lines are a smoothing of the data via a four fold average of nearest points.

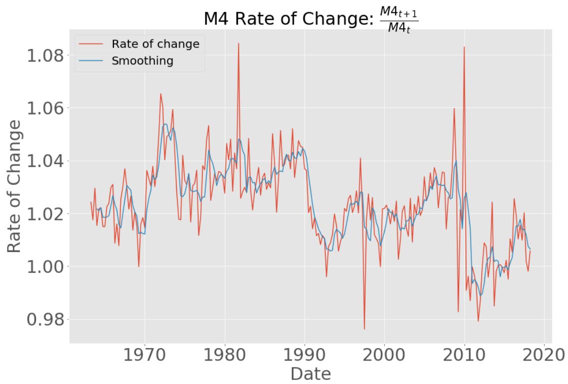


Figure 23: Quarterly M4 Rate of Change (1963-2018)

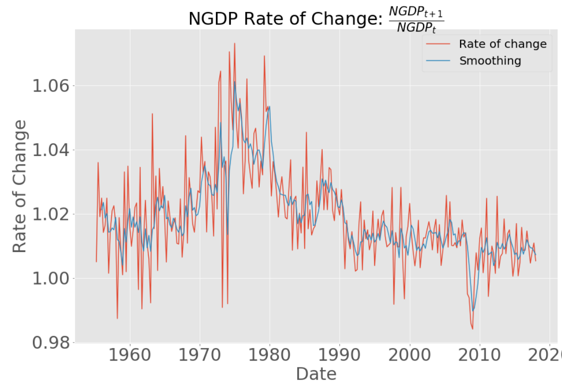


Figure 24: Quarterly NGDP Rate of Change (1955-2018)

We use decision tree regression [14] (**discuss in appendix**) on the smoothed rate of change data to find periods of time of similar rate of change. A decision tree is characterised by the depth of the tree. A higher depth gives a larger number of intervals. **Use errors on training and testing set to give theoretical depth to avoid overfitting**.

Figures 25 and 26 show the results of regression trees of max depth 2 and 3 for the smoothed M4 and NGDP rate of change data respectively. Figure 27 shows a comparison of the regression trees of depth 2 of NGDP and M4 rate of change. Note the changes occur at similar times as the UK recessions in 1973,1990 and 2008 however there is no change in the recession of the early 1980s. However this is picked up with a regression tree of max-depth 3 in NGDP as shown in the purple line in Figure 26.

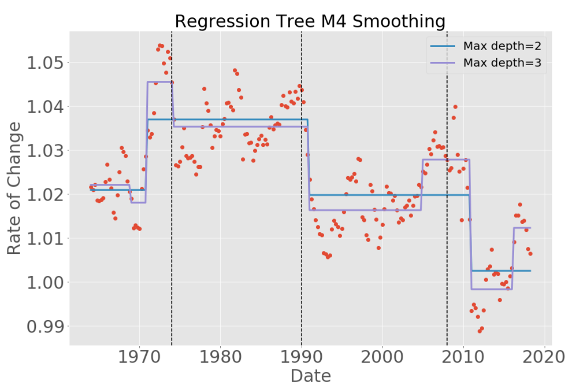


Figure 25: Decision Tree Regression of Quarterly M4 Rate of Change (1963-2018)

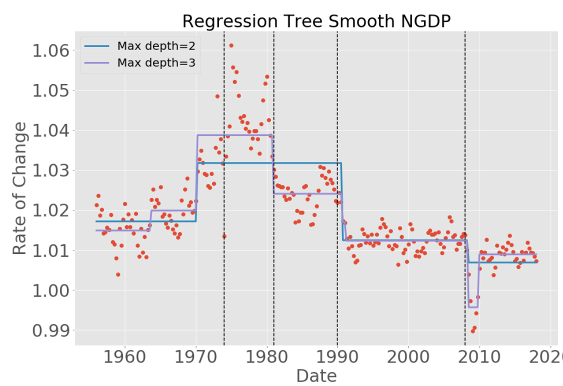


Figure 26: Decision Tree Regression of Quarterly NGDP Rate of Change (1955-2018)

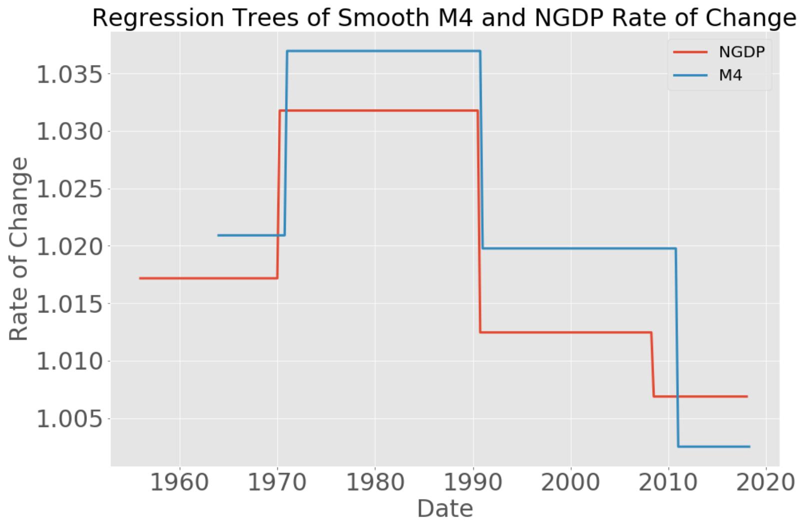


Figure 27: Comparison of NGDP and M4 rate of change decision trees

# 9 Periodicity of Rate of Change of M4 and GDP

For a time series $\{x_n\}_{n=0}^{N-1}$ we define the discrete Fourier transform as $\{X_k\}_{k=0}^{N-1}$ where

$$X_k := \sum_{n=0}^{N-1} x_n e^{-\frac{2\pi ikn}{N}}.$$

The periodogram is defined by the points $\{I(\omega_k)\}_{k=0}^{N-1}$ with frequency $\omega_k = \frac{2\pi k}{N}$ and

$$I(\omega_k) := \frac{|X_k|^2}{N}.$$

The discrete inverse Fourier transform is the inverse of the Fourier transform $\{X_k\}_{k=0}^{N-1}$ and returns the original series $\{x_n\}_{n=0}^{N-1}$. It is defined by

$$x_n := \frac{1}{N} \sum_{k=0}^{N-1} X_k e^{\frac{2\pi ikn}{N}}.$$

The periodogram [9] describes the magnitude of the amplitudes at each of the frequencies $\omega_k$ of the inverse Fourier transform.

The highest values of the periodogram of a time series indicate possible frequencies of periodicities in the series. Non-stationary time series often have high values of lower frequencies in the periodogram which do not tell much about the presence of 'hidden' frequencies in the data. Therefore before periodogram analysis it is important to transform non-stationary data to stationary.

Both the M4 and NGDP rate of change data illustrated in Figures 23 and 24 respectively are non-stationary. One method to transform the data to stationary is to divide the rate of change data by their corresponding regression trees found in the last section. If $x_t$ is a rate of change data point then the transformation is $\frac{x_t}{c_t}$ where $c_t$ is the decision tree point corresponding to $x_t$.

(Stationarity tested by augmented Dickey-Fuller test).

Another method for transforming non-stationary data to stationary is using the Hodrick-Prescott filter [13] which removes the cyclic part of the data leaving the stationary trend to analyse for periodicity. **Discuss Hodrick-Prescott filter in appendix**.

Figures 28-31 show the plotted periodograms for both transformations of the smoothed M4 and NGDP rate of change data. Note these plots are $I(\omega_k)$ against the ordinary frequency $k$ ($k$ cycles in the time period).

As can be seen in Figures 28 and 29 the highest vlaue of the periodogram for both the regression tree and Hodrick-Prescott filter for M4 corresponds to an (ordinary) frequency of 6. If the periods are equal sized this corresponds to a period of 9.04 years. **Periods of cycles in actual data might vary, how to find period lengths?**

For the NGDP rate of change data there is not as obviously a clear frequency which dominates the periodogram. The highest value of the periodogram for the NGDP regression tree transformation is 10 whereas for the Hodrick-Prescott transformation it is 16. **Similar for real GDP.**

This suggests that periodicity in NGDP is not as well defined as M4. The Fourier inverse transform of the first 6 Fourier components of the smoothed M4 rate of change regression tree transform is shown in Figure 32. Similarly Figure 33 shows the inverse Fourier transform for the first 10 Fourier components for the regression tree transformed NGDP smoothed data.

Figure 34 shows the two inverse Fourier transforms from Figure 32 and Figure 33 on one plot. As can be seen there does not seem to be a large correlation between them suggesting that periodic changes in M4 rate of change don't always occur with periodic changes in NGDP rate of change.

**Test for correlation on the transformed rate of change of M4 and NGDP: Pearson and Spearman rank give results near zero but assumptions of tests I think broken for application here.**

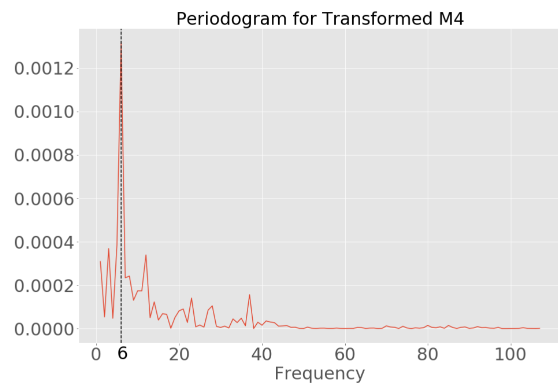


Figure 28: Periodogram of regression tree transform of smoothed M4 rate of change

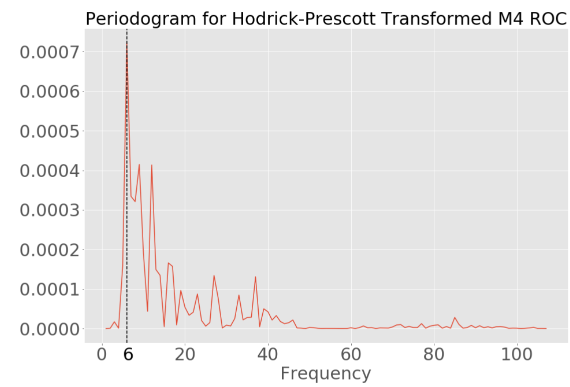


Figure 29: Periodogram of Hodrick-Prescott transform of smoothed M4 rate of change

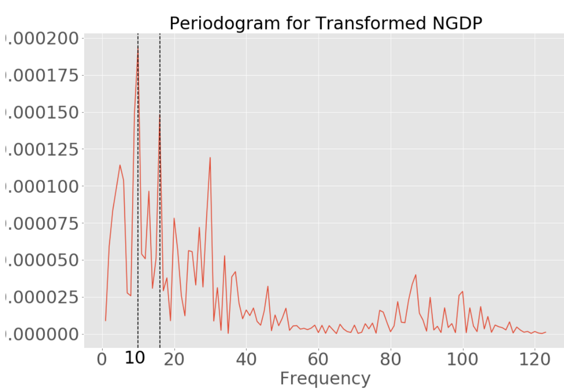


Figure 30: Periodogram of regression tree transform of smoothed NGDP rate of change

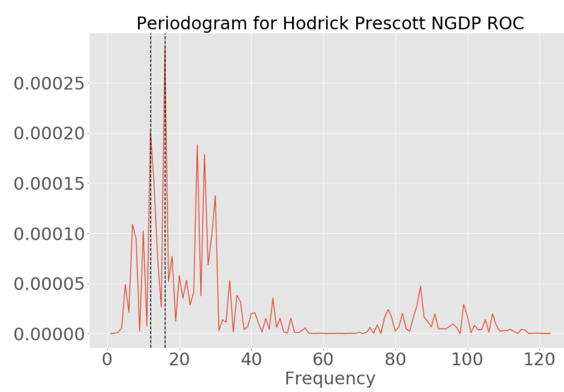


Figure 31: Periodogram of Hodrick-Prescott transform of smoothed NGDP rate of change

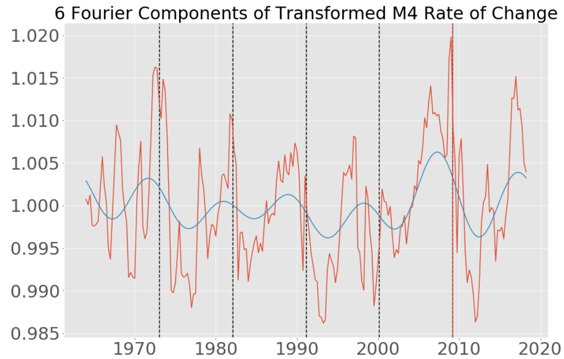


Figure 32: Inverse Fourier transform of first 6 components of smoothed M4 rate of change regression tree transform.

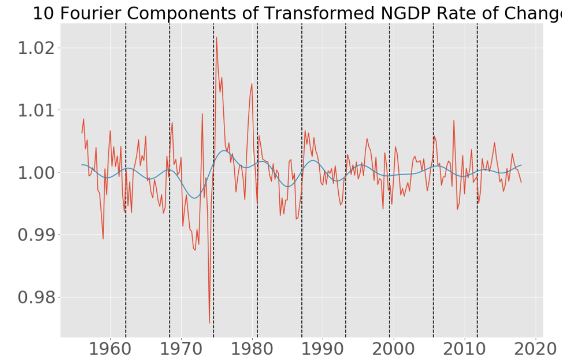


Figure 33: Inverse Fourier transform of first 10 components of smoothed NGDP rate of change regression tree transform.

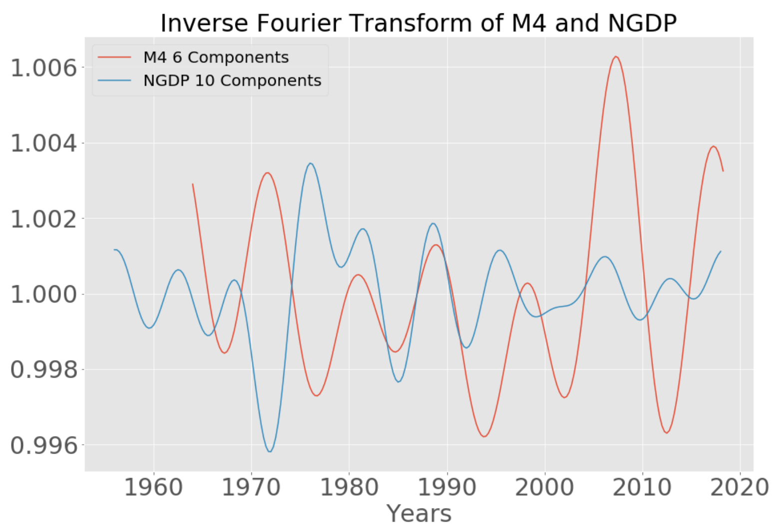


Figure 34: Comparison of inverse Fourier transform of highest frequency Fourier components of smoothed NGDP and M4 rate of change.

Another method for detecting periodicities is looking at the plot of the correlogram [10]: a plot of the autocorrelation coefficient $r_k$ with lag $k$. $r_k$ is defined

$$r_k = \frac{\sum_{t=1}^{N-k} (x_k - \bar{x})(x_{t+k} - \bar{x})}{\sum_{t=1}^{N} (x_t - \bar{x})^2}.$$

Figures 35 and 36 show the correlograms of the regression tree transformed rate of change smoothed data for M4 and NGDP respectively. The lags $k$ are converted to time in years. Notice again the clearer frequency of 6 in the M4 data compared to the more erratic NGDP data.

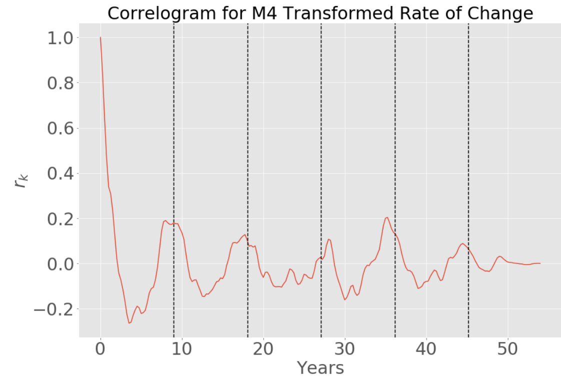


Figure 35: Correlogram of smoothed M4 decision tree transform rate of change

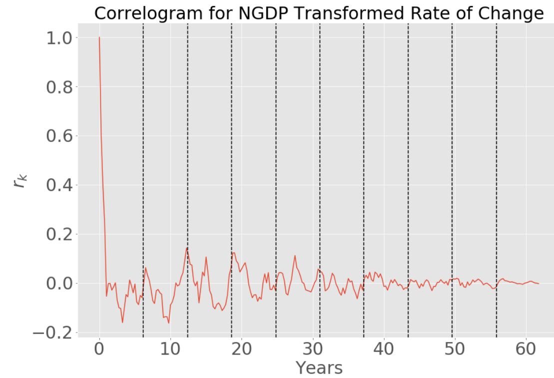


Figure 36: Correlogram of smoothed NGDP decision tree transform rate of change

**Look at periodicities in long range M4 and GDP data and unemployment.**

# References


[1] Bank of England database. https://www.bankofengland.co.uk/boeapps/database/default.asp.

[2] Further details about M4 data. https://www.bankofengland.co.uk/statistics/details/further-details-about-m4-data.

[3] Further details about M4 excluding intermediate other financial corporations (OFCs) data. https://www.bankofengland.co.uk/statistics/details/further-details-about-m4-excluding-intermediate-other-financial-corporations-data.

[4] Investipedia article of GDP and trade balance. https://www.investopedia.com/ask/answers/061515/what-impact-does-balance-trade-have-gdp-calculations.asp.

[5] Office for National Statistics (ONS) gdp data. https://www.ons.gov.uk/economy/grossdomesticproductgdp.

[6] UK finance mortgage lending data. https://www.ukfinance.org.uk/statistics/mortgages/.

[7] (2008). System of national accounts 2008.

[8] (2016). National Accounts. *Office for National Statistics*.

[9] Brockwell, P. J. and Davis, R. A. (1991). *Time Series: Theory and Methods*. Springer, 2nd edition.

[10] Chatfield, C. (1999). *The Analysis of Time Series*. Chapman & Hall, 5th edition.

[11] Fisher, P. V. and Vega, J. L. (1994). An emperical analysis of M4 in the United Kingdom.

[12] Greene, W. H. (2012). *Econometric Analysis*. Prentice Hall, 7th edition.

[13] Hodrick, R. J. and Prescott, E. C. (1997). Postwar U.S. business cycles: An emperical investigation. *Journal of Money, Credit and Banking, Vol.29, No. 1*.

[14] Kubat, M. (2017). *An Introduction to Machine Learning*. Springer, 2nd edition.

[15] Lea, R. (2018). The past four UK recessions compared, and no recession expected on Brexit. *Arbuthnot Banking Group Plc Article*.

[16] (ONS), M. B. (2006). Methodology notes: Links between gross domestic product (GDP) and gross value added (GVA).

[17] Power, J. and Andrew, P. (2001). Explaining the difference between the growth of M4 deposits and M4 lending: implications of recent developments in public finance. *Bank of England quarterly bulliten*.