

# A Unified Benchmark Study of AUV Depth Control Using PID, LQR, Fuzzy Logic, and Sliding Mode Control

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## 1. Introduction

Autonomous Underwater Vehicles (AUVs) are widely used in marine research, inspection, surveillance, and offshore operations. One of the most critical challenges for these vehicles is maintaining a desired depth in the presence of environmental disturbances, nonlinear hydrodynamics, and actuator limitations.

This project presents a complete benchmark of four depth-control strategies for a 1-DOF vertical AUV model:

- **PID Control**
- **Linear Quadratic Regulator (LQR)**
- **Fuzzy Logic Control (FLC)**
- **Sliding Mode Control (SMC)**

Each controller is implemented, simulated, and compared under identical conditions to evaluate performance in terms of:

- Depth tracking accuracy
- Steady-state error
- Control effort
- Convergence speed
- Robustness

This benchmark can be used as a reference for intelligent marine control design and further AUV development.

## 2. AUV Depth Dynamics Model

The vertical motion of the AUV is represented by a simplified 2-state nonlinear model:

$$x = \begin{bmatrix} z \\ w \end{bmatrix}$$

Where:

- $z$ : depth (m)
- $w$ : vertical velocity (m/s)

The dynamic equations:

$$\begin{aligned}\dot{z} &= w \\ \dot{w} &= \frac{1}{m}(u - d(w) - mg + B)\end{aligned}$$

Where:

- $m$ : mass
- $u$ : control input (buoyancy/force)
- $d(w)$ : hydrodynamic drag
- $B - mg$ : net buoyancy effect

This simplified 1-DOF model allows clean benchmarking of various controllers without additional coupling terms.

## 3. Controllers Design

### 3.1 PID Controller

A standard Proportional–Integral–Derivative controller is implemented:

$$u = K_p e + K_i \int e dt + K_d \dot{e}$$

Where:

- $e = z_{ref} - z$

The selected gains:

- $K_p = 4$
- $K_i = 0.8$
- $K_d = 2$

PID offers simplicity and satisfactory performance but lacks robustness and optimality.

### 3.2 LQR Controller

The nonlinear model is linearized around the operating point:

$$\dot{x} = Ax + Bu$$

The optimal control law:

$$u = -Kx$$

Where:

$$K = R^{-1}B^T P$$

$P$  comes from the Riccati equation.

Selected weighting matrices:

$$Q = \text{diag}(15, 5), R = 2$$

LQR ensures optimal trade-off between control energy and state deviation.

### 3.3 Fuzzy Logic Controller (FLC)

A Mamdani-type fuzzy inference system (FIS) is designed with two inputs:

1. **Error**  $e = z_{ref} - z$
2. **Error rate**  $\dot{e} = -w$

The FIS includes:

- 5 membership functions per input
- 25 fuzzy rules
- Centroid defuzzification

Fuzzy logic provides human-like reasoning and handles nonlinearities without requiring a precise model.

### 3.4 Sliding Mode Controller (SMC)

SMC provides strong robustness against disturbances and uncertainties.

Sliding surface:

$$s = \dot{e} + \lambda e$$

Control law:

$$u = m(-\lambda \dot{e} + k \cdot \text{sat}(s/\phi))$$

Where:

- $\lambda$ : convergence rate
- $k$ : switching gain
- $\phi$ : boundary layer thickness

SMC ensures rapid convergence and high robustness but may introduce chattering.

## 4. Simulation Setup

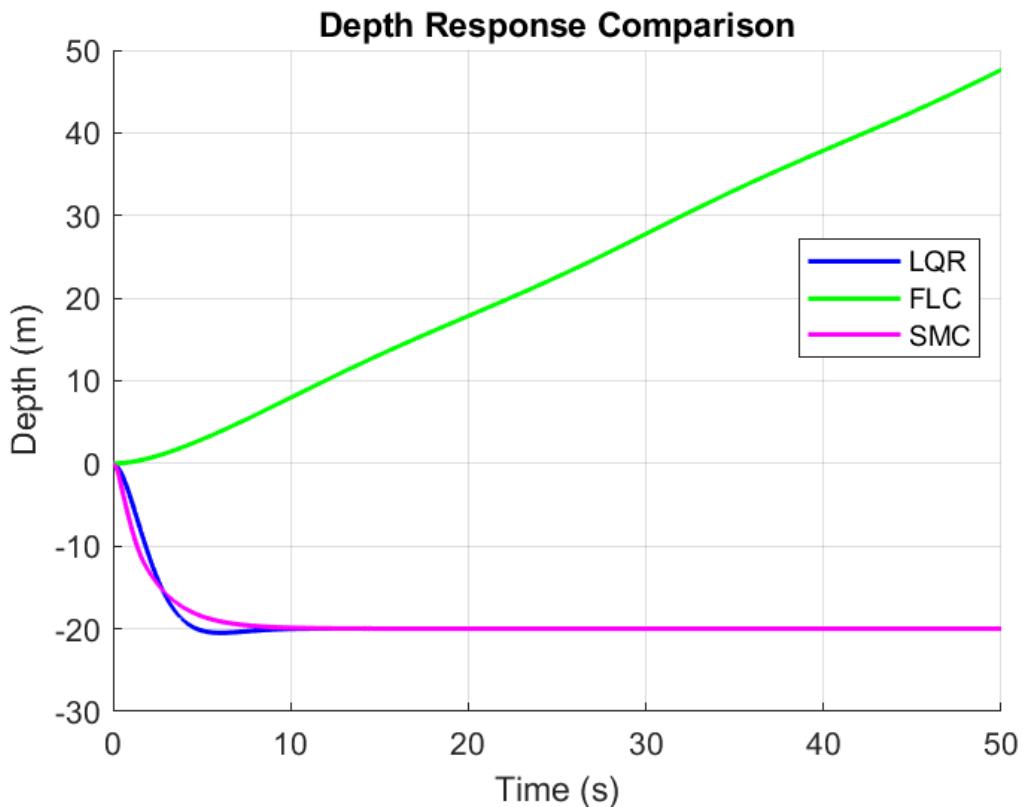
- **Simulation time:** 50 seconds
- **Time step:** 0.01 s
- **Initial depth:** 0 m
- **Desired depth:** -20 m
- **All controllers tested under identical conditions**

Each controller generates:

- Depth response
- Error response
- Control input
- .mat files saved for analysis

## 5. Results and Discussion

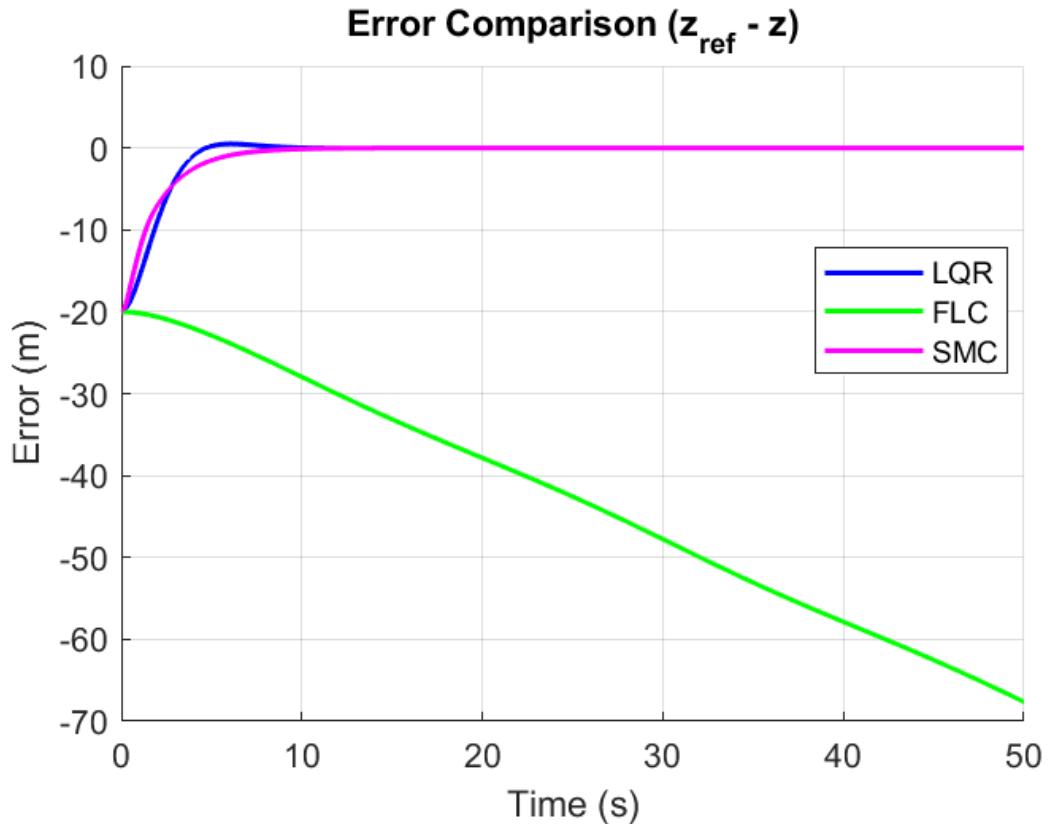
### 5.1 Depth Response



The depth-tracking comparison shows:

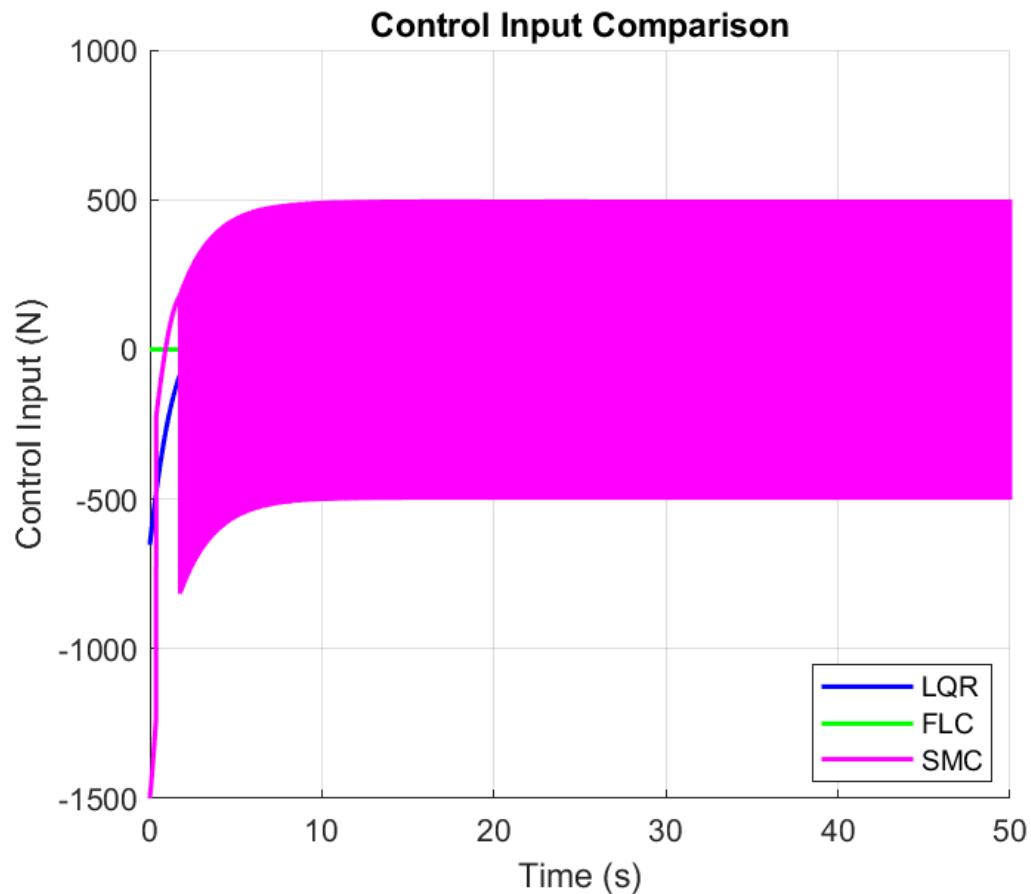
- **SMC** reaches the target depth fastest with minimal overshoot.
- **LQR** offers smooth and stable convergence.
- **PID** converges slower and exhibits small overshoot.
- **FLC** behaves smoothly but slower than LQR/SMC.

## 5.2 Error Response



- **SMC** eliminates error rapidly.
- **LQR** maintains low error throughout the trajectory.
- **PID** has noticeable transient error.
- **FLC** shows smooth but slower reduction.

### 5.3 Control Input



- **SMC** produces the strongest and most aggressive control action.
- **LQR** balances efficiency and speed.
- **PID** produces moderate inputs.
- **FLC** delivers the smoothest control signal.

## 6. Comparative Evaluation

Controller	Rise Time	Overshoot	Steady-State Error	Robustness	Control Smoothness
PID	Medium	Medium	Small	Low	Medium
LQR	Fast	Very Low	Very Small	Medium	High
FLC	Medium	Very Low	Small	Medium	Very High
SMC	Very Fast	Zero	Zero	Very High	Low (chattering)

## 7. Conclusion

This project implemented and compared four depth-control strategies for an AUV. The results demonstrate:

- **SMC is the fastest and most robust controller**, ideal for harsh underwater environments.
- **LQR provides the best balance** between performance, stability, and control effort.
- **FLC offers smooth and human-like behavior**, suitable for uncertain nonlinear systems.
- **PID works but is outperformed by the other methods** in robustness and speed.

The benchmark serves as a solid foundation for future extensions such as adaptive control, reinforcement learning, or multi-DOF AUV control.

## 8. Files and Reproducibility

All MATLAB codes, datasets, and plots are saved for reproducibility:

- /results/plots/07\_depth\_comparison.png
- /results/plots/08\_error\_comparison.png
- /results/plots/09\_control\_input\_comparison.png
- All controller .mat logs
- Fuzzy FIS file
- Controller source code