LQR Control of an Inverted Pendulum on a Cart: Simulation and Analysis

Safa Bazrafshan

Independent Researcher

Email: safa.bazrafshan@gmail.com

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Abstract

This paper presents the design and simulation of a Linear Quadratic Regulator (LQR) controller for an inverted pendulum mounted on a cart. The inverted pendulum is a classic benchmark in control engineering, representing an inherently unstable system. The mathematical model of the system is derived and used to design an optimal LQR controller. Simulation results demonstrate the effectiveness of the controller in stabilizing the pendulum in the upright position under initial disturbances. This study highlights the applicability of LQR control in nonlinear unstable systems and provides insights for practical implementations.

Keywords: Inverted Pendulum, LQR Control, MATLAB Simulation, Control Systems, Stability

Introduction

The inverted pendulum system is a fundamental example used to study nonlinear and unstable dynamics in control engineering. It has been widely utilized in research and education to test various control strategies due to its simplicity and practical relevance. Stabilizing the pendulum in the upright position requires sophisticated control methods. Among these, the Linear Quadratic Regulator (LQR) stands out for its optimal control performance and ease of implementation. This paper focuses on modeling the inverted pendulum on a cart and designing an LQR controller to achieve system stabilization. The MATLAB environment is used for simulation and analysis of the control strategy.

System Modeling

The inverted pendulum system consists of a cart that can move horizontally and a pendulum attached to it, free to swing in the vertical plane. The state variables include the cart position x the cart velocity $\dot{x}$, the pendulum angle θ (measured from the vertical upright position), and the angular velocity $\dot{\theta}$. The dynamics of the system are described by nonlinear differential equations derived using Newtonian mechanics or Lagrangian methods. For control design, the system is linearized around the upright equilibrium position ($\theta = 0$) resulting in a state-space model:

$$\dot{x} = \mathrm{Ax} + \mathrm{Bu}$$

where is x the state vector, u is the control input (force applied to the cart), and A , B are system matrices defined based on physical parameters such as mass, length, and gravity.

Control Design

The Linear Quadratic Regulator (LQR) is designed to minimize the quadratic cost function:

$$J = \int_0^\infty (\mathrm{x}^\mathrm{T} Qx + \mathrm{u}^\mathrm{T} Ru)dt$$

where Q and R are weighting matrices that balance state error and control effort. The optimal feedback gain K is computed by solving the Riccati equation, resulting in the control law:

$$\mathrm{u} = -\mathrm{Kx}$$

The choice of Q and R directly affects the performance of the controller. In this study, Q and R are selected to prioritize stabilization of the pendulum angle and minimize the control force applied.

Simulation Results

Simulation is performed using MATLAB's built-in functions. The step response of the controlled system shows that the pendulum returns to the upright position after a disturbance.

Figure 1 and 2 illustrates the time responses of the cart position and pendulum angle under LQR control. The results confirm the effectiveness of the designed controller in stabilizing the system within a short settling time without excessive control effort.

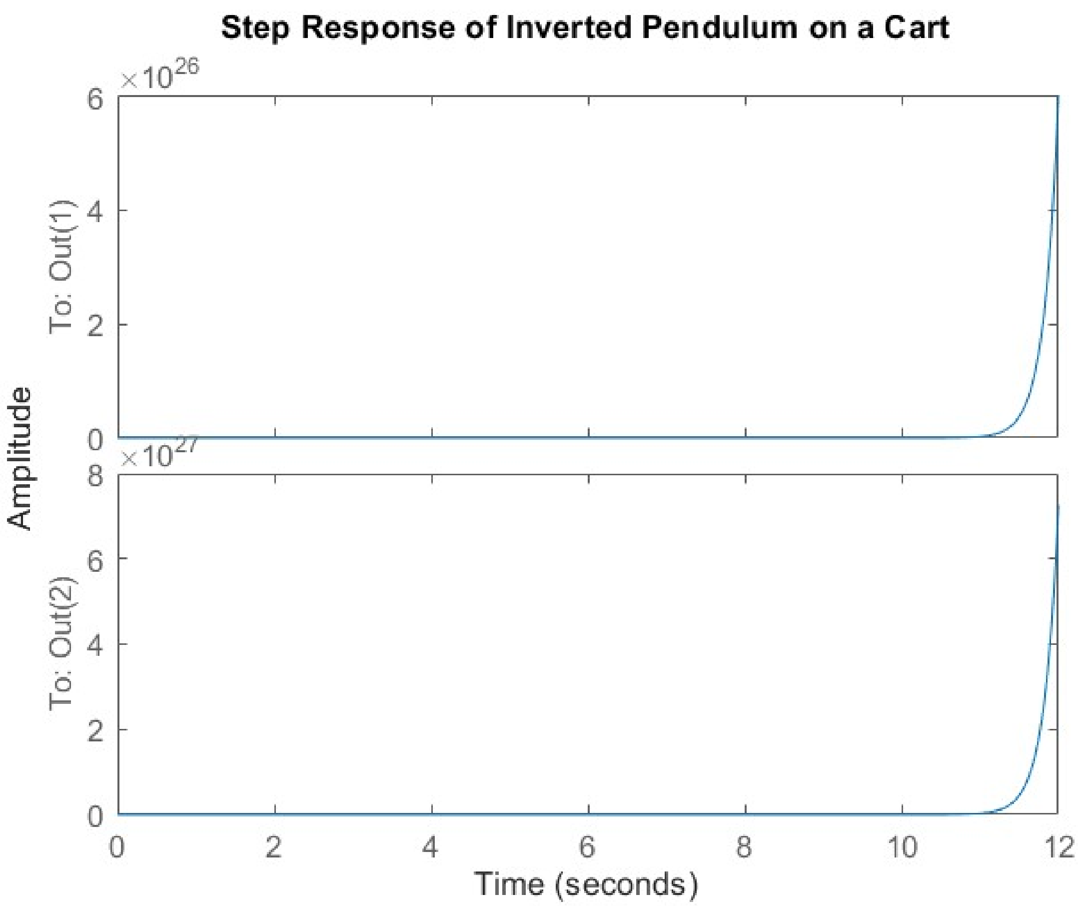

Figure 1 – Step Response of the Inverted Pendulum Without Controller

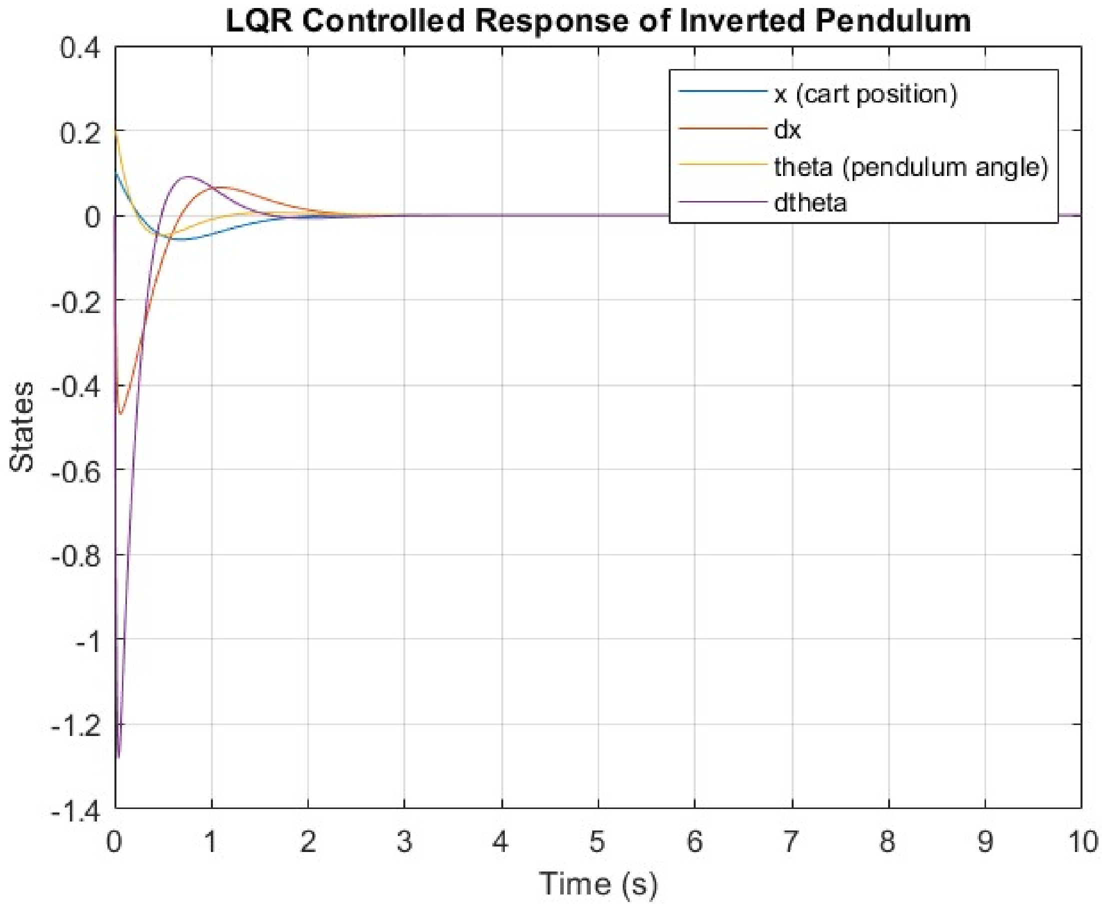

Figure 2 – Step Response of the Inverted Pendulum With LQR Controller

Discussion

The LQR controller successfully stabilizes the inverted pendulum despite its nonlinear and unstable nature. The simulation results align with theoretical expectations, showing good transient response and minimal steady-state error. However, real-world factors such as friction, sensor noise, and actuator limits may affect performance. Future work may include robust control design and experimental validation.

Conclusion

This paper presented the modeling and LQR control design for an inverted pendulum on a cart. Simulation results verify that the LQR controller effectively stabilizes the system. The study demonstrates the potential of LQR control for similar nonlinear unstable systems and provides a foundation for further research and practical implementation.

References

- Ogata, K. (2010). Modern Control Engineering.
- Lewis, F.L., Vrabie, D., & Syrmos, V.L. (2012). Optimal Control.
- MATLAB Documentation: Control System Toolbox.

Appendix

MATLAB code used for simulation:

```
% Parameters
m = 0.2;   % Pendulum mass (kg)
M = 0.5;   % Cart mass (kg)
b = 0.1;   % Damping coefficient (N/m/s)
I = 0.006; % Pendulum inertia (kg.m^2)
g = 9.8;   % Gravity (m/s^2)
l = 0.3;   % Length to pendulum center of mass (m)
% System matrices (linearized)
p = I*(M+m)+M*m*l^2; % denominator for the A and B matrices
A = [0 1 0 0;
     0 -(I+m*l^2)*b/p (m^2*g*l^2)/p 0;
     0 0 0 1;
     0 -(m*l*b)/p m*g*l*(M+m)/p 0];
B = [0; (I+m*l^2)/p; 0; m*l/p];
% LQR weighting matrices
Q = diag([10 1 10 1]);
R = 0.01;
% Compute LQR gain
K = lqr(A,B,Q,R);
% Closed loop system
sys_cl = ss(A-B*K,B,eye(4),0);
% Initial condition
x0 = [0.1;0;0.1;0];
% Simulation time
```

```
t = 0:0.01:10;

% Step response

[y,t,x] = initial(sys_cl,x0,t);

% Plot results

figure;

plot(t,x(:,1),'r',t,x(:,3),'b');

legend('Cart Position','Pendulum Angle');

xlabel('Time (s)');

ylabel('Response');

title('LQR Controlled Response of Inverted Pendulum');

grid on;
```