1. Discrete Probability Distributions

Used when the random variable takes distinct values. They are defined by the **Probability Mass Function (PMF)**, which gives the probability of each value.

Common Discrete Distributions:

- **Discrete Uniform Distribution**: All possible values have equal probabilities.
- **Bernoulli Distribution**: Represents a single trial with two possible outcomes (success = 1, failure = 0).
- **Binomial Distribution**: Describes the number of successes in nnn independent trials.
- **Poisson Distribution**: Models the number of events occurring in a fixed time or space, given a constant average rate.

2. Continuous Probability Distributions

Used when the random variable takes an infinite number of possible values within a range. They are defined by the **Probability Density Function (PDF)**, which represents the likelihood of values occurring within an interval.

Common Continuous Distributions:

- **Continuous Uniform Distribution**: All values in a range have equal probabilities.
- **Normal Distribution (Gaussian Distribution)**: The famous bell-shaped curve, characterized by mean and standard deviation.
- **Exponential Distribution**: Describes the time between events in a Poisson process.
- **Chi-Square Distribution**: Used in statistical hypothesis testing.
- **Student's t-Distribution**: Similar to the normal distribution but with heavier tails. Used for hypothesis testing and confidence intervals when the sample size is small, and the population standard deviation is unknown. As the degrees of freedom increase, it approaches the normal distribution.

Probability

Probability is the measure of how likely an event is to occur, ranging from **0** (impossible) to **1** (certain). It is the foundation of statistics and is used in decision-making, predictions, and data analysis.

1. Basic Probability Concepts

- **Experiment**: A process with uncertain outcomes (e.g., rolling a die).
- **Sample Space (S)**: The set of all possible outcomes.
- **Event (E)**: A subset of the sample space (e.g., rolling an even number).
- **Probability of an Event (P(E))**: Given by: P(E)=Number of favorable outcomes/ Total outcomes in the sample space.

2. Probability Rules

- **Rule of Complements**: P(Not E)=1−P(E)P(\text{Not E}) = 1 - P(E)P(Not E)=1−P(E)
- **Addition Rule (For Mutually Exclusive Events A and B)**: P(A∪B)=P(A)+P(B)P(A \cup B) = P(A) + P(B)P(A∪B)=P(A)+P(B)
- **Multiplication Rule (For Independent Events A and B)**: P(A∩B)=P(A)×P(B)P(A \cap B) = P(A) \times P(B)P(A∩B)=P(A)×P(B)

Conditional Probability

- The probability of an event **A** occurring given that another event **B** has already occurred.
- Formula: P(A ∣ B)=P(B)P(A∩B).

Independent Events

- Two events **A** and **B** are independent if the occurrence of one does not affect the probability of the other.
- Formula: P(A∩B)=P(A)×P(B)
- Example: Rolling two dice—one die's result does not affect the other.

Bayes' Theorem

- Used to update probabilities based on new evidence.
- Formula: P(A ∣ B)=P(B)P(B ∣ A)⋅P(A)
- Example: Used in spam filters to determine the probability of an email being spam based on certain words.

Random Variables

- A function that assigns numerical values to outcomes of a random process.
- **Types:**
  - **Discrete Random Variable**: Takes distinct values (e.g., number of heads in coin flips).
  - **Continuous Random Variable**: Takes an infinite number of values within a range (e.g., height of students).

Poisson Distribution

- Models the number of events occurring in a fixed interval of time or space when events happen independently at a constant rate.
- Formula: P(X=k)=(e^(−λ) λ^k)/k!
- Example: Number of calls received at a call center per hour.

Expected Value (Mean, E(X)E(X)E(X))

- The average value a random variable takes over many trials.
- Formula for discrete variables: E(X)=∑xiP(xi)

- Example: Expected number of heads in 10 coin flips = 10×0.5=510 \times 0.5 = 510×0.5=5.

Probability Density Function (PDF)

- Defines the likelihood of a continuous random variable falling within an interval.
- The area under the curve of a PDF over an interval gives the probability of that range.
- Example: The normal distribution's bell curve is a common PDF.

Binomial Distribution

- Models the number of successes in **n** independent Bernoulli trials (success/failure).
- Formula: P(X=k)=(n/k)p^k (1−p)^n−k where p is the probability of success, and k is the number of successes.
- Example: Probability of getting exactly 3 heads in 5 coin flips.