Name: Sasha Farhat

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CSCI 3104, Algorithms **Escobedo & Jahagirdar**
Mid Term exam Summer 2020 (30 points) **Summer 2020, CU-Boulder**

Advice 1: For every problem in this class, you must justify your answer: show how you arrived at it and why it is correct. If there are assumptions you need to make along the way, state those clearly.

Advice 2: Verbal reasoning is typically insufficient for full credit. Instead, write a logical argument, in the style of a mathematical proof.

Honor code: On my honor as a University of Colorado at Boulder student, I have neither given nor sought unauthorized assistance in this work

Initials S.F.

Date 06/27/2020

If you violate the CU **Honor Code**, you will receive a 0.

Instructions for submitting your solution:

- The solutions **should be typed**, we cannot accept hand-written solutions. Here's a short intro to **Latex**.
- In this homework we denote the asymptomatic *Big-O* notation by $\mathcal{O}$ and *Small-O* notation is represented as o.
- We recommend using online Latex editor **Overleaf**. Download the **.tex** file from Canvas and upload it on overleaf to edit.
- You should submit your work through **Gradescope** only.
- If you don't have an account on it, sign up for one using your CU email. You should have gotten an email to sign up. If your name based CU email doesn't work, try the identikey@colorado.edu version.
- Gradescope will only accept **.pdf** files (except for code files that should be submitted separately on Canvas if a problem set has them) and **try to fit your work in the box provided**.
- You cannot submit a pdf which has less pages than what we provided you as Gradescope won't allow it.

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Master Method: Consider a recurrence relation of the form

$$T(n) = aT(n/b) + f(n)$$

where $a \geq 1$ and $b > 1$ are constants, and $T(n) = \textit{constant}$ for $n \leq 1$.
The asymptotic growth of T(n) is bounded as follows:

- **Case 1** $f(n) = \mathcal{O}(n^{\log_b a - \epsilon})$ for some constant $\epsilon > 0 \implies T(n) = \Theta(n^{\log_b a})$
- **Case 2** $f(n) = \Theta(n^{\log_b a}) \implies T(n) = \Theta(n^{\log_b a} \log(n))$
- **Case 3** $f(n) = \Omega(n^{\log_b a + \epsilon})$ for some constant $\epsilon > 0$ and if $af(\frac{n}{b}) \leq cf(n)$ for some constant $c < 1$ and all sufficiently large n. $\implies T(n) = \Theta(f(n))$

Formulae

- $1 + 2 + 3 + ... + n = \frac{n(n+1)}{2}$
- $1^2 + 2^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$
- **Sequences** The formulae for the sum of an Arithmetic Progression (AP) and Geometric Progression (GP) are available here

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1. (3 pts) Which of $\mathcal{O}$, Ω or Θ is the correct asymptotic relationship for $f(n) = n^{\frac{1}{3}} \log_3(n)$ compared to $g(n) = \sqrt{n}$? Write your answer as $f(n) = \tau(g)$ where τ should be replaced by the appropriate symbol $(\mathcal{O}, \Omega, \Theta)$, show the necessary work to justify your answer.

$f(n) = n^{\frac{1}{3}} \log_3(n)$
$g(n) = \sqrt{n}$
$f(n) = \Omega(g(n))$

$0 \leq c \cdot g(n) \leq f(n)$

$0 \leq c \cdot n^{\frac{1}{2}} \leq n^{\frac{1}{3}} \log_3 n$

$0 \leq c \cdot n^{\frac{1}{6}} \leq \log_3 n$

Setting arbitrary value for n_0 we can prove that there is a constant c where the ineualities hold true.
$n_0 = 3^6$
$0 \leq c \cdot \sqrt[6]{3^6} \leq log_3(3^6)$
$0 \leq c \cdot 3 \leq 6$
$c = 1$
Since there is a value n_0 and c that make the statement true: $f(n) = \Omega(g(n))$

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2. (2 pts) Which of $\mathcal{O}$, Ω or Θ is the correct asymptotic relationship for $f(n) = \frac{n^2-8n+15}{n-5}$ compared to $g(n) = \frac{1^2+2^2+....+n^2}{1+2+...+n}$? Write your answer as $f(n) = \tau(g)$ where τ should be replaced by the appropriate symbol $(\mathcal{O}, \Omega, \Theta)$, show the necessary work to justify your answer

$f(n) = \frac{n^2-8n+15}{n-5}$
$g(n) = \frac{1^2+2^2+....+n^2}{1+2+...+n}$

$f(n) = \Theta(g(n))$
$0 \leq c_1 \cdot g(n) \leq f(n) \leq c_2 \cdot g(n)$

$0 \leq c_1 \cdot \frac{1^2+2^2+....+n^2}{1+2+...+n} \leq \frac{n^2-8n+15}{n-5} \leq c_2 \cdot \frac{1^2+2^2+....+n^2}{1+2+...+n}$

Both f(n) abd g(n) have arbitrary constants added to their equations. By keeping the largest variable and removing the added constants without changing f(n) and g(n) graph trend, we can evaluate the inequalities easier.
$0 \leq c_1 \cdot \frac{n^2}{n} \leq \frac{n^2}{n} \leq c_2 \cdot \frac{n^2}{n}$

$0 \leq c_1 \cdot n \leq n \leq c_2 \cdot n$
We can clearly see that f(n) is tightly bound by g(n) for there is a constant c such that $0 \leq c_1 \cdot g(n) \leq f(n) \leq c_2 \cdot g(n)$ holds true. Therefore $f(n) = \Theta(g(n))$

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3. (1 pts) State True or false without justification
   If the running time of an algorithm satisfies the recurrence relation
   $T(n) = T(n/18) + T(17n/18) + cn$, then $T(n) = \mathcal{O}(n \log(n))$.

   True

4. (1 pts) Let H be a hash table with 2020 slots with a hash function $h(x)$ that satisfies **uniform hashing** property. Given two items x_1, x_2, what is the probability that they do not hash to the same location?

   The probability that the two items hash into the same location is $\frac{1}{m}$, where m is the table size. The probability that they do not hash to the same location is $1-\frac{1}{m} = \frac{2019}{2020}$.

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5. (3 pts) Suppose $T(n) = T(n-5) + n$, where $T(n) = \Theta(1)$ if $n \leq 5$. Find a function $g(n)$ such that $T(n) = \Theta(g(n))$. Clearly justify your answer.

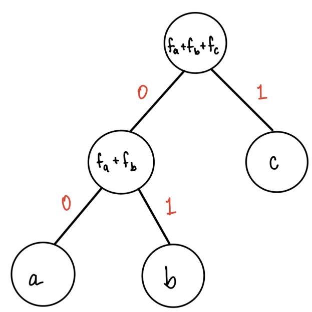
Given: $T(n) = T(n-5) + n \rightarrow n > 5$, and $= 1 \rightarrow n \leq 5$

$T(n) = T(n-5) + n$
$= T(n-10) + (n-5) + n$
$= T(n-15) + (n-10) + (n-5) + n$
$= T(n-20) + (n-15) + (n-10) + (n-5) + n$
.
.
.
$T(1) + ... + (n-10) + (n-5) + n$

we see that we have $\frac{n}{5}$ terms
We get the equeation $T(n) = T(1) + (\frac{n}{5})(n)$
$= 1 + n \cdot \frac{n}{5}$

ignoroing the constants we get $g(n) = n^2$ such that $T(n) = \Theta(g(n))$

6. (1 pt) For the set $\Sigma = \{a, b, c\}$, give the set of inequalities on the frequencies f_a, f_b, f_c that would yield the corresponding codewords $00, 01, 1$ respectively under Huffman's algorithm. Assume that while constructing the tree, we merge two nodes such that the node with least frequency is the left child. In your tree branching left corresponds to 0 and branching right corresponds to 1. List the frequencies f_a, f_b, f_c below that produce the specified codewords. Your choice of frequencies should **always** produce the same tree/codes provided.



$f_a < f_b < f_c$

7. (3 pts) For the given algorithm, solve the following.

   You may assume the existence of a `max` function taking $\mathcal{O}(1)$ time, which accepts at most four arguments and returns the highest of the four.

```
FindMax(A[0, ..., n-1]):
     if A.length == 1:
         return A[0]
    count = 0
    for i = 1 to n/2:
        count++

     return max( FindMax(A[0, ..., floor(n/4)],
                 FindMax(A[floor(n/4) + 1, ..., floor(n/2)],
                 FindMax(A[floor(n/2) + 1, ..., floor(3n/4)],
                 FindMax(A[floor(3n/4) + 1, ..., n-1]))
```

   Find a recurrence for the worst-case runtime complexity of this algorithm. You can assume that n is a multiple of four. Solve the recurrence found to obtain worst-case runtime complexity.

Recurrence relation for the following code:
Let $T(n)$ be the worst-case runtime where $T(n) = 4T(\frac{n}{4}) + \mathcal{O}(n) + \mathcal{O}(1)$.
$= 4T(\frac{n}{4}) + n$

Using the master's method
$a = 4$, $b = 4$, $k = 1$
$a = b^k \rightarrow 4 = 4^1$

Therefore we apply the second case of master theorem
$T(n) = \Theta(n^k \log n)$
The worst case runtime is $T(n) = \Theta(n \cdot \log n)$

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8. (4 pts) Assume there are n items $\{item_1, item_2, ...\ item_n\}$, each item has a weight w_i and value v_i associated with it. You have a bag that can carry a maximum load of weight W. Each of the n items can be divided into **fractions** such that the value and weight associated with the item decreases proportionally.
The inputs to your function will be values v, weights w, number of items n and capacity W.
Provide well commented pseudo or actual code, that returns the maximum total value of all items that can be carried.
Also briefly discuss the space and runtime complexity of your pseudo-code.

Input
$v = [20, 27, 18]$
$w = [2, 3, 3]$
$W = 3$
Output
29
The Algorithm should pick $item_1$ and $\frac{1}{3}^{rd}$ of $item_2$ leading to a total value of $20 + 27\frac{1}{3} = 29$.

pseudo code on next page

```
bubblesort(v,w)
        n = len(v)
        \\instead of rearranging value and weight it keeps it ordered in a seperate array
        sorted = list(range(n))

        for i in range(n to 1)
                for j in range(0, i)
                        \\if the ratio of the jth item > j+1 item then
                        if (v[sorted[j]] / w[sorted[j]]) < (v[sorted[j+1]] / w[sorted[j+1]])
                                temp = sorted[j]
                                sorted[j] = sorted[j+1]
                                sorted[j+1] = temp \\swap the items
        return order

maxWeight(v,w,n,W)
        sorted = bubblesort(v,w) \\sort the items based on decreasing order ratio v/w

        weight = 0
        max_value = 0

        \\looping through all items
        for i =1  to n
                \\ if the added weight isn't more than the maximum weight W
                if weight <= W
                        max_value = max_value + sorted[i].v
                        weight = weight + sorted[i].w

                \\if current item can't be added, add fractional part of it
                else
                        remainingWeight = W - weight
                        //taking the fraction and adding to max_value
                        max_value = max_value + sorted[i].v * (remainingWeight / sorted[i].w)
        return  max_value
```

Time Complexity Average: O(n^2). If the items are already sorted into decreasing order then the Algorithm would have time complexity O(n).The space complexity is O(n) due to an array for storing the fractional ratios of items.

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9. (6 pts) Given an array A and a value k, design a divide and conquer algorithm to find the k^{th} smallest element in the array. The algorithm proposed should not use extra space i.e the space complexity of the algorithm should be constant $\Theta(1)$. Your algorithm must have an average case runtime of $\mathcal{O}(n \log(n))$. You can assume the access to a function which returns a random number within a range in constant time. You are allowed to modify the array passed as input.
   The inputs to your function will be an array A and value k.
   Provide well commented pseudo or actual code for the algorithm.

   Assume that you have access to a function *rand(min, max)* which will return a random integer between *min* and *max* in constant time inclusive of both min and max.

   Input
   $A = [10, 13, 20, 8, 7, 6, 100]$
   $k = 4$
   Output
   10

   10 is the 4th smallest value in the given array.

```
randQuicksort(A, p, r)
        if (p < r)
                \\q is the partition index
                q = randPartition(A, p, r)

                \\seperatly sort elements before and after partition
                randQuicksort(A, p, q - 1)
                randQuicksort(A, q + 1, r)
        return


randPartition(A, p, r)
        i = rand(p , r) // random integer between p and r
        swap(A[p], A[r]) // swap corresponding element with last element
        i = p-1 // index the smaller element
        x = A[r] // set the pivot

        for (p <=r-1)
                //if the element is smaller than the pivot
                if A[p]<=x
                        i = i + 1
                        swap(A[i],A[p])
        swap(A[i+1],A[r])
        return i+1

kthsmallest(A, k)
        n = len(A)
        A = randQuickSort(A, 0, n-1)

        return A[k-1]
```

10. (6 pts) Assume there are n carrots and n rabbits along a straight line. Each rabbit needs to eat a carrot. Rabbits can move in either direction, simultaneously along the line and travelling 1 unit of distance takes 1 minute. Design a greedy algorithm that takes $\mathcal{O}(n \log(n))$ to assign carrots to rabbits such that the time taken to eat the last carrot is minimized. The algorithm should return the value of the time taken to eat last carrot.
You will be given the position of rabbits and carrots along the straight line.

Expectations

- You should clearly describe the greedy choice that the algorithm makes in assigning carrots to rabbits.

- Provide well commented pseudo or actual code for the algorithm.

- Discuss the space and runtime complexity of the pseudo or actual code.

Example 1:
rabbits = [7, 3, 2, 13, 2]
carrots = [1, 3, 5, 14, 21]
output : 8
In this example the assignment is as follows.

- The carrot at distance 1 (index 0) is eaten by rabbit at distance 2 (index 4).
- The carrot at distance 3 (index 1) is eaten by rabbit at distance 2 (index 2).
- The carrot at distance 5 (index 2) is eaten by rabbit at distance 3 (index 1).
- The carrot at distance 14 (index 3) is eaten by rabbit at distance 7 (index 0).
- The carrot at distance 21 (index 4) is eaten by rabbit at distance 13 (index 3), with 21-13=8 minutes being the longest time.

Example 2:
rabbits = [84, 15, 15, 161, 187, 9, 66, 1]
carrots = [92, 103, 163, 119, 63, 117, 144, 172]
output : 102

```
randQuicksort(A, p, r)
        if (p < r)
                \\q is the partition index
                q = randPartition(A, p, r)

                \\seperatly sort elements before and after partition
                randQuicksort(A, p, q - 1)
                randQuicksort(A, q + 1, r)
        return


randPartition(A, p, r)
        i = rand(p , r) // random integer between p and r
        swap(A[p], A[r]) // swap corresponding element with last element
        i = p-1 // index the smaller element
        x = A[r] // set the pivot

        for (p<=r - 1)
                \\if the element is smaller than the pivot
                if A[p]<=x
                        i = i + 1
                        swap(A[i],A[p])
        swap(A[i+1],A[r]) |
        return i+1


rabbitTime(r, c)
        n = len(r)

        r = randQuickSort(r, 0, n-1) \\sorts rabbits
        c = randQuickSort(c, 0, n-1) \\sorts the carrots

        for i in range(n)
                temp = abs(r[i] - c[i]) \\subtract distances
                time = temp \\last elemented in list subtracted is the final time
        return time
```

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11. **For extra credit pick one of the two questions provided. Extra credit will only be considered if your midterm score less than 100% (2 pts)**

    Please provide your solution with proper comments, solutions without proper comments will not be considered.

    https://leetcode.com/problems/gas-station/

    OR

    https://leetcode.com/problems/maximal-square/

```
class Solution(object):
    def canCompleteCircuit(self, gas, cost):
        """TIME COMPLEXITY O(n)
            SPACE COMPLEXITY O(1)"""
        #we calculate the surpless and deficeit amounts to reach the station
        tank = currtank = start = 0 #starting
        n = len(gas)
        for i in range(n):
            #refuel and drive to next station
            tank += gas[i] - cost[i]
            #if the sum the tank is a negative value
            if tank < 0:
                #set the value of tank to currtank
                currtank += tank
                """"print("tank1", currtank)"""
                #reset the value of tank
                tank = 0
                start = i + 1
        #currtank value is added to check if the refeul was enough
        currtank += tank
        """print(currtank, tank)"""
        #check to see if the currtank value is less than 0
        if currtank < 0: return -1
        #if we completed the full circle then return the the last value of start
        else: return start
```