Name:	Sasha Farhat
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CSCI 3104, Algorithms **Escobedo & Jahagirdar**

Final Exam Summer 2020 (60 points) **Summer 2020, CU-Boulder**

Advice 1: For every problem in this class, you must justify your answer: show how you arrived at it and why it is correct. If there are assumptions you need to make along the way, state those clearly.

Advice 2: Verbal reasoning is typically insufficient for full credit. Instead, write a logical argument, in the style of a mathematical proof.

Honor code: On my honor as a University of Colorado at Boulder student, I have neither given nor sought unauthorized assistance in this work

Initials	SF
Date	07/21/2020

If you violate the CU **Honor Code**, you will receive a 0.

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Instructions for submitting your solution:

- The solutions **should be typed**, we cannot accept hand-written solutions. Here's a short intro to **Latex**.
- In this homework we denote the asymptomatic *Big-O* notation by $\mathcal{O}$ and *Small-O* notation is represented as o.
- We recommend using online Latex editor **Overleaf**. Download the **.tex** file from Canvas and upload it on overleaf to edit.
- You should submit your work through **Gradescope** only.
- If you don't have an account on it, sign up for one using your CU email. You should have gotten an email to sign up. If your name based CU email doesn't work, try the identikey@colorado.edu version.
- Gradescope will only accept **.pdf** files (except for code files that should be submitted separately on Canvas if a problem set has them) and **try to fit your work in the box provided**.
- You cannot submit a pdf which has less pages than what we provided you as Gradescope won't allow it.

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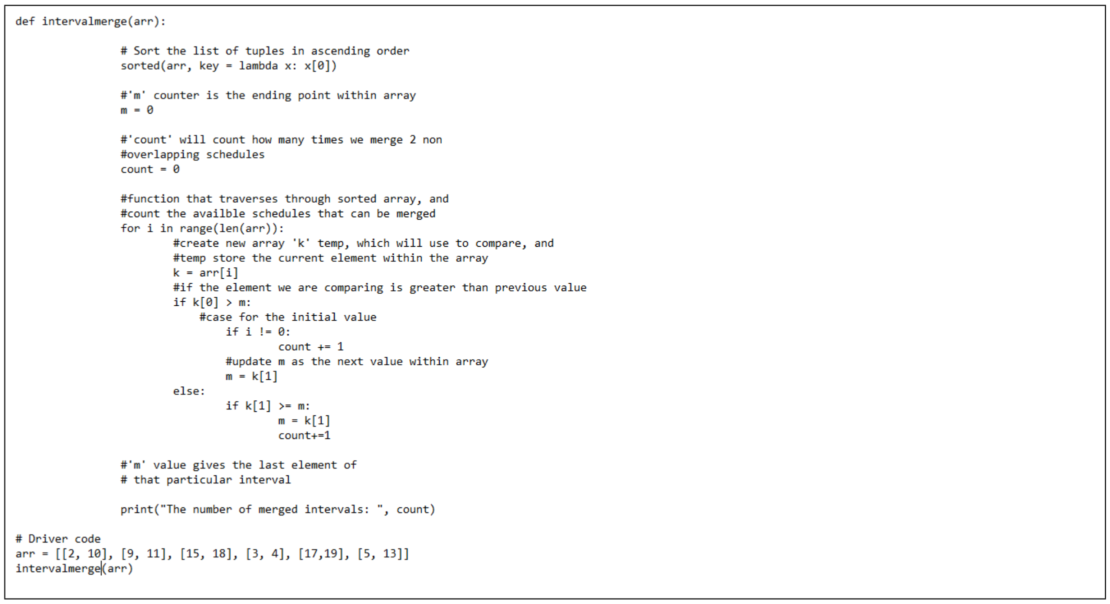
1. (5 pts) You are given a list of jobs with start and end times. Assume that only one job can be executed at a time on a processor. Your task is to find out the minimum number of processors required so that the jobs are executed as soon as they arrive (no waiting time for a process). The input to your algorithm will be a list of tuples indicating the start and end times of the respective jobs. The output should be a number indicating the minimum number of processors required. Your algorithm should have a runtime of $\mathcal{O}(n \log(n))$, where n is the number of jobs. Assume you do not have access to any auxiliary functions.

   Example
   Input: $(2, 10), (9, 11), (15, 18), (3, 4), (17, 19), (5, 13)$
   Output: 3
   Explanation: Between the interval (9, 10) it can be seen that there will be 3 { (2, 10) , (9, 11), (5, 13)} jobs that need to be executed simultaneously. Thus at the very least 3 processors are required.

   (a) (4 pts) Write down well commented pseudo-code or paste real code to solve the above problem.



```
def intervalmerge(arr):

                # Sort the list of tuples in ascending order
                sorted(arr, key = lambda x: x[0])

                #'m' counter is the ending point within array
                m = 0

                #'count' will count how many times we merge 2 non
                #overlapping schedules
                count = 0

                #function that traverses through sorted array, and
                #count the availble schedules that can be merged
                for i in range(len(arr)):
                        #create new array 'k' temp, which will use to compare, and
                        #temp store the current element within the array
                        k = arr[i]
                        #if the element we are comparing is greater than previous value
                        if k[0] > m:
                            #case for the initial value
                                if i != 0:
                                        count += 1
                                #update m as the next value within array
                                m = k[1]
                        else:
                                if k[1] >= m:
                                        m = k[1]
                                        count+=1

                #'m' value gives the last element of
                # that particular interval

                print("The number of merged intervals: ", count)

# Driver code
arr = [[2, 10], [9, 11], [15, 18], [3, 4], [17,19], [5, 13]]
intervalmerge(arr)
```

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(b) (1 pts) Explain your algorithms runtime and space complexity by analyzing your code. For example, stating that a sorting algorithm runs in $\mathcal{O}(n\log(n))$ without any justification is insufficient.

Runtime: $\mathcal{O}(n\log n)$
Space Complexity: $\mathcal{O}(1)$
(1) Sort all intervals in decreasing order based on their start time.
  Time Complexity: $\mathcal{O}(n\log n)$
(2) Traverse through the array of sorted tuples and do:
    if the element we are comparing is greater than previous value, and is not the first interval within the array, merge and increment count
  Space Complexity: $\mathcal{O}(n)$
Analyzing the algorithm we see that our time complexity is there for $\mathbf{O}(n\log n)$. The space complexity is $\mathbf{O}(n)$ because we need the extra space for our array m when comparing the previous and current element in the array.

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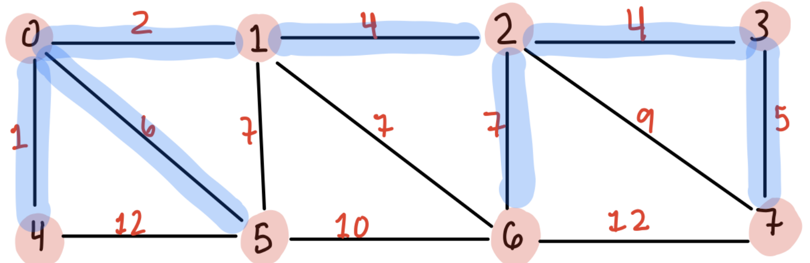
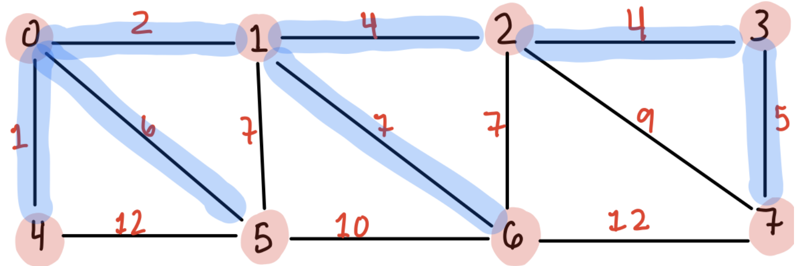
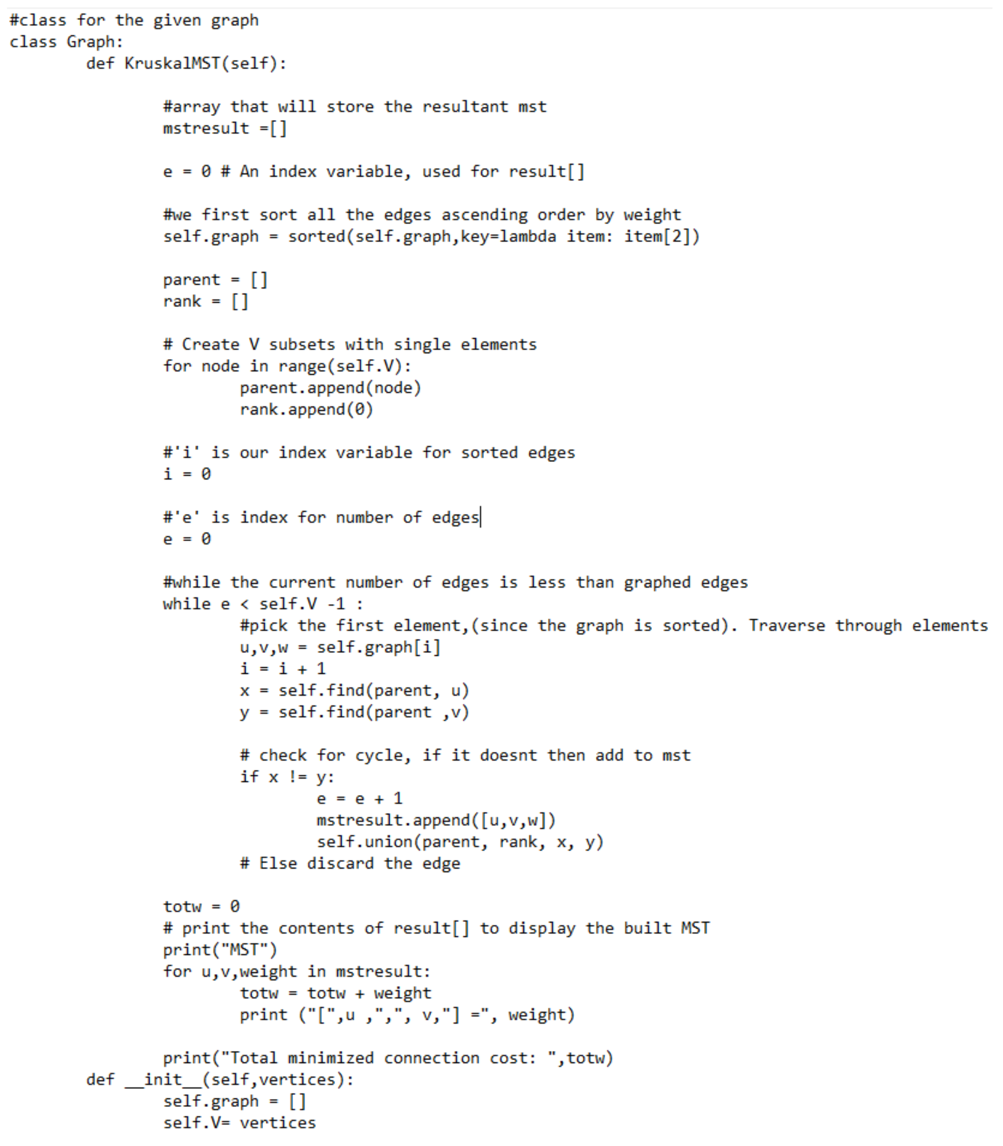
2. (5 pts) You are designing a network, to connect several offices in different locations. You can contact your network cable provider about the cost of connecting any pair of offices. Your task is to design an algorithm, so that all offices are connected and the cost associated with building the network is minimized.
The input to your algorithm is a Graph in the form of an adjacency list or matrix. The nodes in this graph represent the offices and edge weights represent the cost associated with connecting a pair of offices. The output of the algorithm will be a list of office pairs such that the total connection cost is minimized.

   (a) (2 pts) Provide an example with at least 3 offices and 3 connections. Your example must include at least 2 unique solutions.

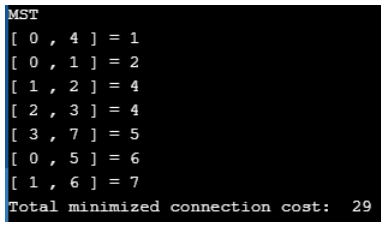




(b) (3 pts) Provide well commented pseudo code or actual code to solve the above problem.



```
#class for the given graph
class Graph:
        def KruskalMST(self):

                #array that will store the resultant mst
                mstresult =[]

                e = 0 # An index variable, used for result[]

                #we first sort all the edges ascending order by weight
                self.graph = sorted(self.graph,key=lambda item: item[2])

                parent = []
                rank = []

                # Create V subsets with single elements
                for node in range(self.V):
                        parent.append(node)
                        rank.append(0)

                #'i' is our index variable for sorted edges
                i = 0

                #'e' is index for number of edges
                e = 0

                #while the current number of edges is less than graphed edges
                while e < self.V -1 :
                        #pick the first element,(since the graph is sorted). Traverse through elements
                        u,v,w = self.graph[i]
                        i = i + 1
                        x = self.find(parent, u)
                        y = self.find(parent ,v)

                        # check for cycle, if it doesnt then add to mst
                        if x != y:
                                e = e + 1
                                mstresult.append([u,v,w])
                                self.union(parent, rank, x, y)
                        # Else discard the edge

                totw = 0
                # print the contents of result[] to display the built MST
                print("MST")
                for u,v,weight in mstresult:
                        totw = totw + weight
                        print ("[",u ,",", v,"] =", weight)

                print("Total minimized connection cost: ",totw)
        def __init__(self,vertices):
                self.graph = []
                self.V= vertices
        # function to add an edge to graph
        def addEdge(self,u,v,w):
                self.graph.append([u,v,w])

        # path comparison to find the element i
        def find(self, parent, i):
                if parent[i] == i:
                        return i
                return self.find(parent, parent[i])

        #union function for x and y
        def union(self, parent, rank, x, y):
                xroot = self.find(parent, x)
                yroot = self.find(parent, y)

                # Attach smaller rank tree under root of higher rank tree
                if rank[xroot] < rank[yroot]:
                        parent[xroot] = yroot
                elif rank[xroot] > rank[yroot]:
                        parent[yroot] = xroot

                # if ranks are the same, increment its rank by one
                else :
                        parent[yroot] = xroot
                        rank[xroot] += 1

        # The main function to construct MST using Kruskal's algorithm


# Driver code
#weighted undirected graph (picture drawn in 2a)
g = Graph(8)
g.addEdge(0, 1, 2)
g.addEdge(0, 5, 6)
g.addEdge(0, 4, 1)
g.addEdge(1, 5, 7)
g.addEdge(1, 2, 4)
g.addEdge(1, 6, 7)
g.addEdge(2, 6, 7)
g.addEdge(2, 3, 4)
g.addEdge(2, 7, 9)
g.addEdge(3, 7, 5)
g.addEdge(6, 7, 12)
g.addEdge(5, 6, 10)
g.addEdge(4, 5, 12)

g.KruskalMST()
```



```
MST
[ 0 , 4 ] = 1
[ 0 , 1 ] = 2
[ 1 , 2 ] = 4
[ 2 , 3 ] = 4
[ 3 , 7 ] = 5
[ 0 , 5 ] = 6
[ 1 , 6 ] = 7
Total minimized connection cost:  29
```

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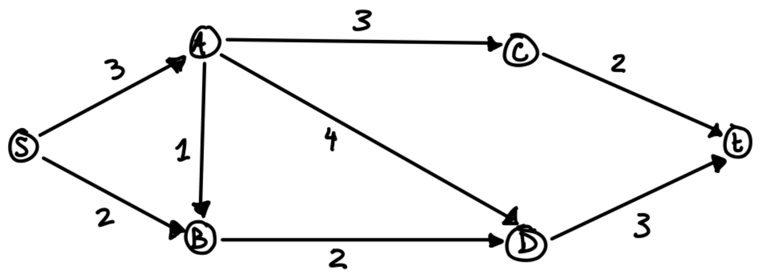
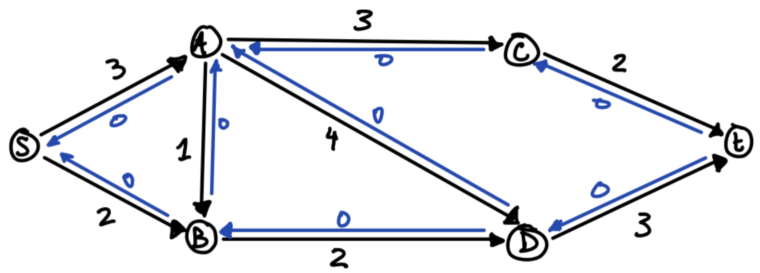
3. (10 pts) Consider the graph $G = (V, E)$ with edge capacities $c(e)$, $\forall e \in E$, the edge capacity represents the maximum amount of flow that can pass through the edge. In addition to edge capacities, the above graph has vertex capacities $c(v)$, $\forall v \in V - \{s, t\}$ (The source and the sink vertex do not have vertex capacities). Each vertex capacity represents the maximum amount of flow that can pass through the vertex.

   (a) (4 pts) Given a source vertex $s \in V$ and a sink $t \in V$. How will you modify the given graph G, so that you can find the max-flow from s to t with a known algorithm? Also, show an example input and modified graph with at least 5 vertices. This must be a drawing!

We will modify the graph in a way that the algorithm can undo its decisions by sending flow back in the opposite direction. We create a residual graph with maximum flow.





(b) (4 pts) Provide an algorithm to find the max-flow in the above graph from s to t.

```
#class graph for matrix representation
class Graph:

        def __init__(self,graph):
                #xreate residual graph
                self.graph = graph
                self. ROW = len(graph)


        # using BFS we will check if there is a path from source to sink,
        # within our residual graph G
        def BFS(self,s, t, parent):

                # create boolean array (visited) and
                # initially mark all the vertices as false or unvisted
                visited = [False] * (len(self.graph))

                # this will be our queue for bfs
                queue = []

                # Mark the given node as true
                # or visited and enqueue it
                queue.append(s)
                visited[s] = True

                while queue:

                        # Dequeue vertex from queue
                        s = queue.pop(0)

                        # enque through all the vertices within the neighboring visited node s
                        # using our boolean array we traverse through the node,weight
                        for i,w in enumerate(self.graph[s]):
                            if visited[i] == False and w > 0:
                                #only count the vertices that are in range of k
                                queue.append(i)
                                visited[i] = True
                                #update parent node to current visited
                                parent[i] = s
                return True if visited[t] else False
```

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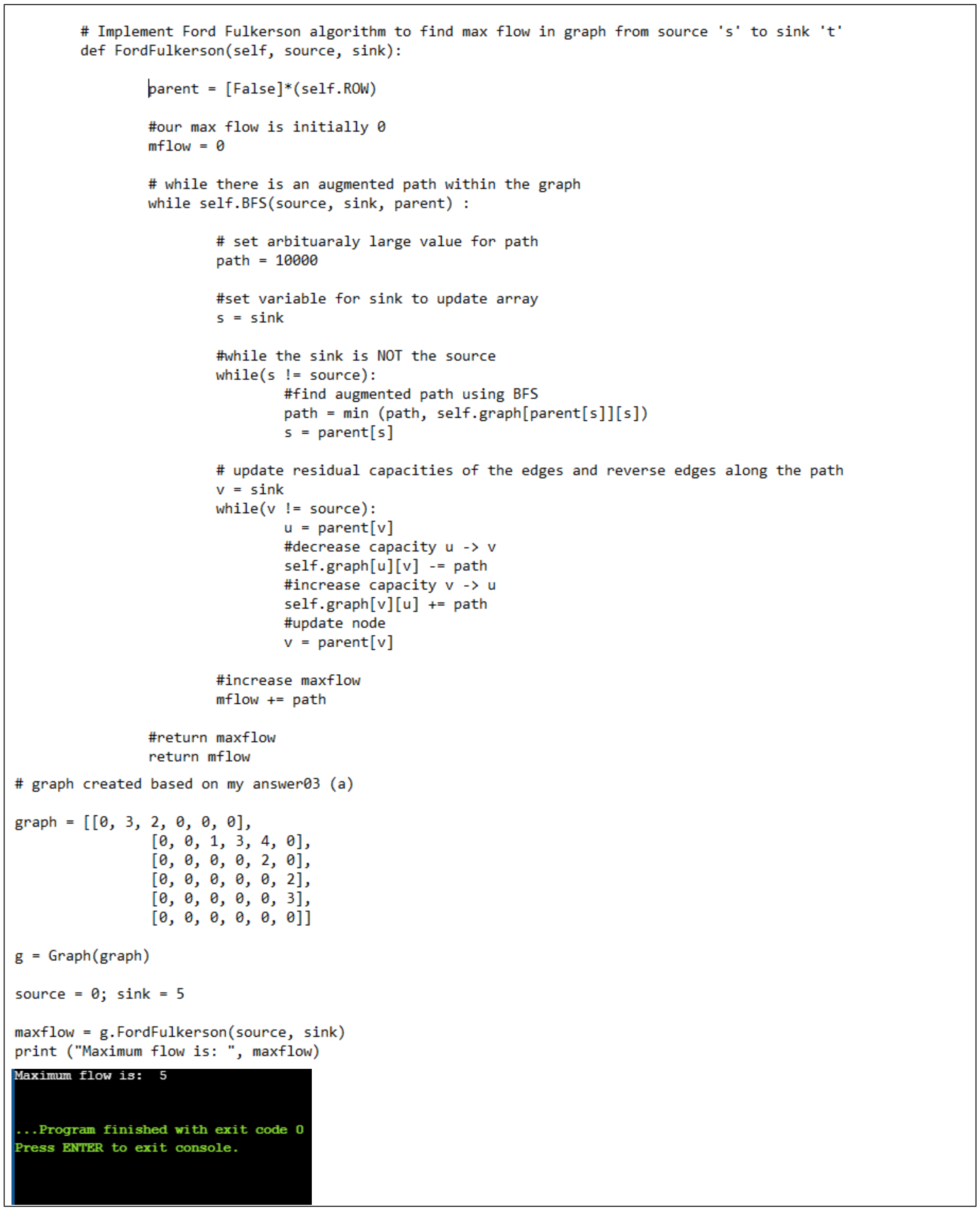

```
        # Implement Ford Fulkerson algorithm to find max flow in graph from source 's' to sink 't'
        def FordFulkerson(self, source, sink):

                parent = [False]*(self.ROW)

                #our max flow is initially 0
                mflow = 0

                # while there is an augmented path within the graph
                while self.BFS(source, sink, parent) :

                        # set arbituaraly large value for path
                        path = 10000

                        #set variable for sink to update array
                        s = sink

                        #while the sink is NOT the source
                        while(s != source):
                                #find augmented path using BFS
                                path = min (path, self.graph[parent[s]][s])
                                s = parent[s]

                        # update residual capacities of the edges and reverse edges along the path
                        v = sink
                        while(v != source):
                                u = parent[v]
                                #decrease capacity u -> v
                                self.graph[u][v] -= path
                                #increase capacity v -> u
                                self.graph[v][u] += path
                                #update node
                                v = parent[v]

                        #increase maxflow
                        mflow += path

                #return maxflow
                return mflow
# graph created based on my answer03 (a)

graph = [[0, 3, 2, 0, 0, 0],
                [0, 0, 1, 3, 4, 0],
                [0, 0, 0, 0, 2, 0],
                [0, 0, 0, 0, 0, 2],
                [0, 0, 0, 0, 0, 3],
                [0, 0, 0, 0, 0, 0]]

g = Graph(graph)

source = 0; sink = 5

maxflow = g.FordFulkerson(source, sink)
print ("Maximum flow is: ", maxflow)
```

```
Maximum flow is:  5


...Program finished with exit code 0
Press ENTER to exit console.
```

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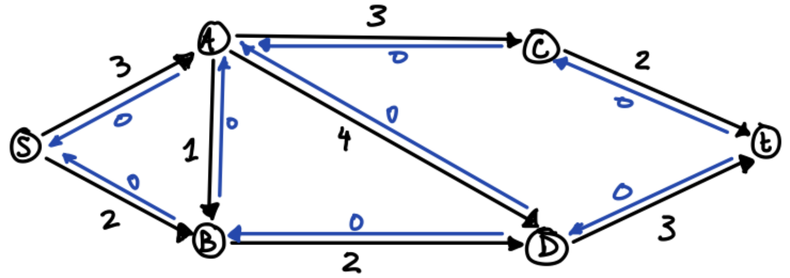
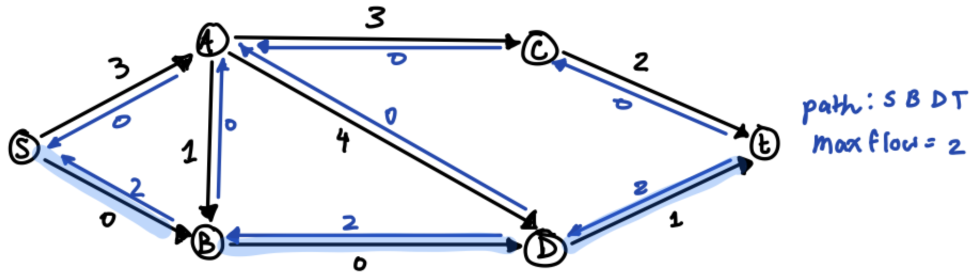
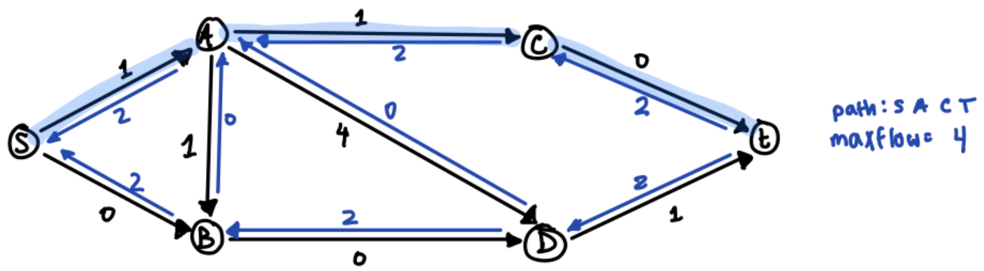
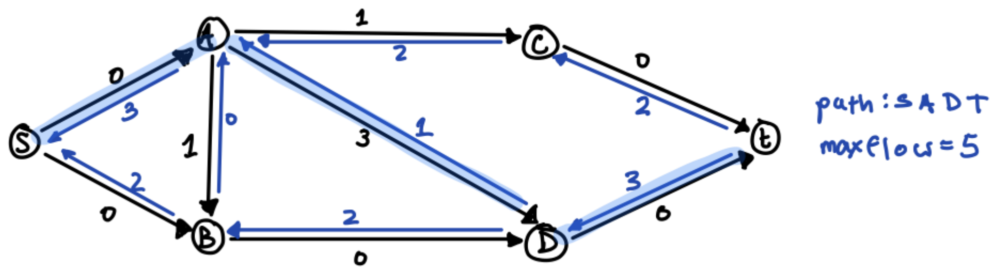
(c) (2 pts) Find the max-flow in your example graph by stepping though your algorithm. Provide a new graph image each time the flow though an edge is altered.









4. (10 pts) There are n rooms numbered from $\{1, 2, ..n\}$ in Williams Village. Each of the rooms has a teleportation device with a value $v_i \ \ \forall i \in \{1, 2, ...n\}$. The teleportation device placed in the room i with value v_i teleports to a room numbered $(v_i \ \% \ n) + 1$.

   A room in Williams Village is said to be beautiful if you can get back to the room you started using the teleportation devices. The task is to count the number of beautiful rooms in Williams Village.
   The input to your algorithm is the list of values associated with the teleportation device placed in the rooms. The output should be a number indicating the number of beautiful rooms.

   Example 1
   Input: [2, 3, 4, 5]
   Output: 4
   Explanation: All the 4 rooms that are beautiful and the corresponding paths to get back to them are as follows.

   - **Room 1** The teleportation path for Room1 is Room1, Room3, and back to Room1
   - **Room 2** The teleportation path for Room2 is Room2, Room4, and back to Room2
   - **Room 3** The teleportation path for Room3 is Room3, Room1, and back to Room3
   - **Room 4** The teleportation path for Room4 is Room4, Room2, and back to Room4

   Example 2
   Input: [1, 2, 3, 6]
   Output: 2
   Explanation: It can be seen that only Room3 and Room4 are beautiful.

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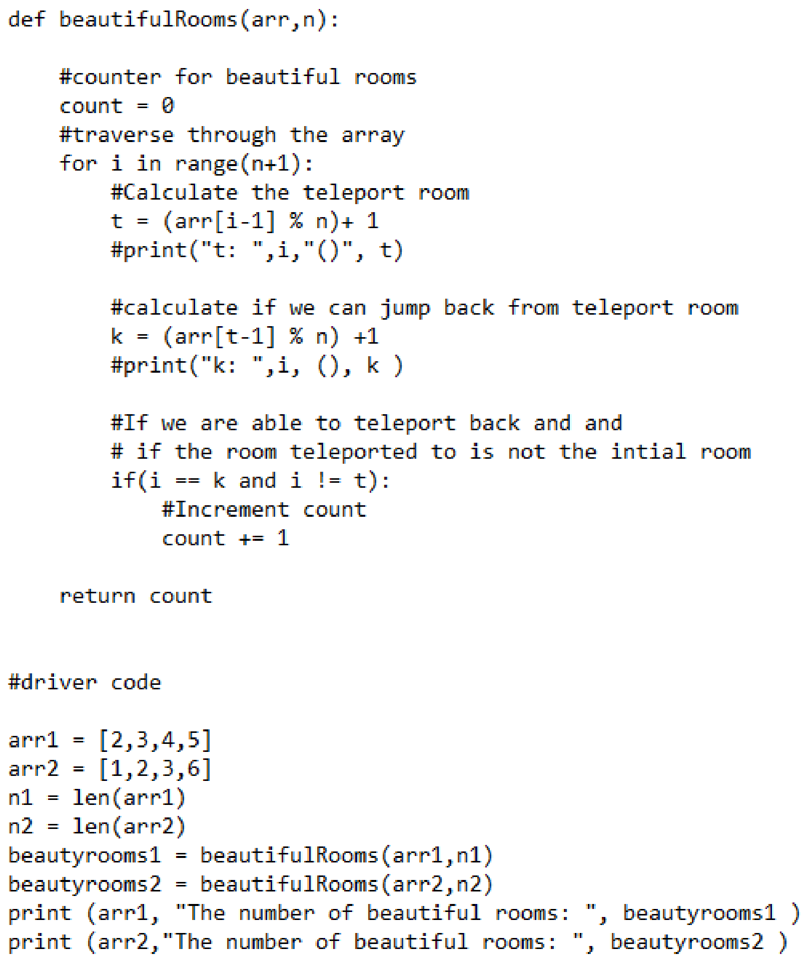
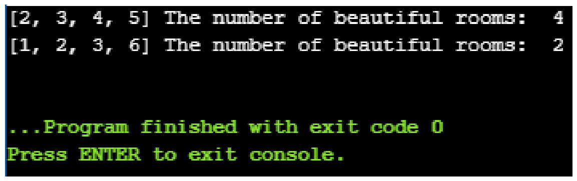
(a) (2 pts) Give a high-level explanation of how you plan to solve this problem. (How would you explain your potential solution to a younger sibling or someone who has never taken a computer science class?) Minimum 1 paragraph.

I would approach the problem in a mathematical approach. In the problem we are given the equation $(v_i \ \% \ n) + 1$ where $v_i = arr[i]$. I know that this equation helps to find the next room to teleport to. So i set an unknown variable to the equation to solve, $t = (v_i \ \% \ n) + 1$. Next we want to see if the room we teleported to can be teleported back to the original. Using the same equation we set another unknown variable and solve for it, $k = (v_t \ \% \ n) + 1$. Using the two equations we can now compare the room i to k. If i == k this means that the room is we can get back to the room we started at using the teleported device thus we increment count for beautiful rooms.

(b) (5 pts) Provide well commented pseudo code or actual code to solve the above problem.



```
def beautifulRooms(arr,n):

    #counter for beautiful rooms
    count = 0
    #traverse through the array
    for i in range(n+1):
        #Calculate the teleport room
        t = (arr[i-1] % n)+ 1
        #print("t: ",i,"()", t)

        #calculate if we can jump back from teleport room
        k = (arr[t-1] % n) +1
        #print("k: ",i, (), k )

        #If we are able to teleport back and and
        # if the room teleported to is not the intial room
        if(i == k and i != t):
            #Increment count
            count += 1

    return count


#driver code

arr1 = [2,3,4,5]
arr2 = [1,2,3,6]
n1 = len(arr1)
n2 = len(arr2)
beautyrooms1 = beautifulRooms(arr1,n1)
beautyrooms2 = beautifulRooms(arr2,n2)
print (arr1, "The number of beautiful rooms: ", beautyrooms1 )
print (arr2,"The number of beautiful rooms: ", beautyrooms2 )
```



```
[2, 3, 4, 5] The number of beautiful rooms:  4
[1, 2, 3, 6] The number of beautiful rooms:  2


...Program finished with exit code 0
Press ENTER to exit console.
```

(c) (3 pts) Give an example input with at least 5 rooms and structure of how your algorithm explores the rooms. This must be an image! Your input can **not** be an example already given.

Given the array $[2, 3, 5, 6, 7]$, my algorithm returns 4 rooms as beautiful.
My algorithm will calculate the first room and check if the value of room teleported to can be teleported back.
$R1 : (2\%5) + 1 = 3$ and $R3 : (5\%5) + 1 = 1$
$R2 : (2\%5) + 1 = 4$ and $R4 : (6\%5) + 1 = 2$
$R5 : (7\%5) + 1 = 3$
The algorithm thus explores the room as such:

$$\begin{tikzcd}
1 \arrow[d] & 4 \arrow[l] \\
3 \arrow[r] & 2 \arrow[u] \\
5 \arrow[u] &
\end{tikzcd}$$

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5. (10 pts) You are at your home in Boulder enjoying the summer and your friend challenges you to ride your bike for at least s miles starting from your home and ending at any of the scenic outlooks. You have a map of Boulder that has information about roads connecting various view points and your house. The task is to determine if there is a sequence of scenic outlooks that you can travel to starting from your house such that you have travelled at least s miles. You can not travel on the same edge more than 1 time and once you visit an outlook you cannot travel to it again.
The input to your problem is a graph $G = (V, E)$ in the form of an adjacency list or adjacency matrix and the source vertex s corresponding to your house. Your algorithm should output a boolean value indicating if it's possible to complete the challenge of riding at least s miles starting from your house.

   (a) (2 pts) Give a high-level explanation of how you plan to solve this problem. (How would you explain your potential solution to a younger sibling or someone who has never taken a computer science class?) Minimum 1 paragraph.

(b) (6 pts) Provide well commented pseudo code or actual code to solve the above problem.

(c) (2 pts) Explain your algorithms runtime and space complexity by analyzing your code. For example, stating that a sorting algorithm runs in $\mathcal{O}(n\log(n))$ without any justification is insufficient.

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6. (20 pts) There are an even number of items arranged along a line. Each item has a value v_i associated with it. You are competing against an opponent to collect items such that total value of your items collected is maximized. At each round of the game, one player is allowed to pick an item from either end of the line.

   Determine the maximum total value that can be obtained, if you are allowed to play first assuming that the opponent is equally clever. The input to your algorithm will be a list of values. The output should be a number indicating the maximum amount of value that you can obtain.

   Example
   Input: $22, 43, 10, 20$
   Output: 63
   Explanation: As the first player I would first pick 20 from the right end. The opponent would then pick 22. Out of the two elements 43, 10 remaining, I can pick 43. Thus 20+43=63 is the maximum total value that can be obtained.
   Note: Observe in the above example that the greedy choice of picking the max valued item does not work. If I had picked 22 instead of 20 in the first round, the maximum value that I could have obtained is 32.

   (a) (5 pts) State the base case and recursive relation that can be used to solve the above problem using dynamic programming.

   Base Case 01: $T(0) = 0$
   Base Case 02: $T(1) = c_1$
   note: c_i equivalent to v_i
   Recursive Relation: $T(n) = max(c_n + T(n-2), T(n-1))$ for $n \geq 1$

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(b) (5 pts) Provide an example input consisting of at least 5 items. Show how the dynamic programming data structure used to store previous computations is built based on the recursive relation.

c	0	1	2	3	4	5	6
arr	0	12	8	14	15	13	11

T[0]	0
T[1]	$c_1 = 12$
T[2]	$\max(c_2 + T[0], T[1]) = \max(8+0, 12) = 12$
T[3]	$\max(c_3 + T[1], T[2]) = \max(14+12, 12) = 26$
T[4]	$\max(c_4 + T[2], T[3]) = \max(15+12, 26) = 27$
T[5]	$\max(c_5 + T[3], T[4]) = \max(13+26, 27) = 39$
T[6]	$\max(c_6 + T[4], T[5]) = \max(11+27, 39) = 39$

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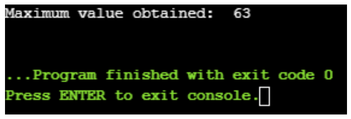
(c) (8 pts) Write down well commented pseudo-code or paste real code to solve the above problem using Dynamic Programming or Memoization based on the above recurrence relation.

```
def maxValueGame(arr, i, j,maxval):
        # base case 01
        if j == i:
            return max(arr[0], arr[1])

        # base case 02
        if j == i + 1:
                return max(arr[i], arr[j])

        # we maximize our decisions and minimize the opponents
        if maxval[i][j] == 0:
            # 'start' represents the maxiumum value from picking from the start of the arr
            start = arr[i] + min(maxValueGame(arr, i + 2, j,maxval), maxValueGame(arr, i + 1, j - 1,maxval))

            # 'end' represents the maximum value from picking at the end of the arr
            end = arr[j] + min(maxValueGame(arr, i + 1, j - 1,maxval), maxValueGame(arr, i, j - 2,maxval))

            # the maximum of the two choices is the optimal solution
            maxval[i][j] = max(start, end)
        return maxval[i][j]


# driver Code
arr = [22,43,10,20]
n = len(arr)-1
maxval = [[0 for x in range(len(arr))] for y in range(len(arr))]
value = maxValueGame(arr,0, n, maxval)
print("Maximum value obtained: ", value)
```



```
Maximum value obtained:  63


...Program finished with exit code 0
Press ENTER to exit console.
```

(d) (2 pts) Discuss the space and runtime complexity of the code, providing necessary justification.

Time complexity: $\mathcal{O}(n^2)$
Space Complexity: $\mathcal{O}(n^2)$
Using memoization and dynamic programming we solve algorithm has a time and space complexity of $\mathcal{O}(n^2)$. Here we solve for subproblems and instead of resolving, our solution is memoized. In my code i create an extra empty array to store my previously solved recursed values T(n). There is $\mathcal{O}(n^2)$ subproblems because there is at most (i, j) indcice such that 1 i j n. Therefore, computing a subproblem is $\mathcal{O}(1)$ since we use memoization to compute the previous value intead of having to re-solve recurrence everytime. This therefore gives a $\mathcal{O}(n^2)$ instead of $\mathcal{O}(nm)$ exponential time complexity.

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CSCI 3104, Algorithms **Escobedo & Jahagirdar**

Final Exam Summer 2020 (60 points) **Summer 2020, CU-Boulder**

7. **Extra Credit (3 pts) will only be considered if your final exam score is less than 100%**

   Write two double spaced pages about the job or position you hope to have when you graduate from college (If you already have a job lined up, write about your next career goal). Your essay must include i) a tractable career goal, ii) a reason why you want to be hired/accepted to this position, iii) a step-by-step plan of how you intend to achieve your goal.