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Homework #3
1.7.4, 1.7.5, 1.7.6, 1.7.7, 1.8.1, 1.8.3

1.7.4) Show each of the following:

a.) $\{e\}^* = \{e\}$

$\{e\}^* = \{e, ee, eee, eeee, \ldots\}$ since $\{ee\} = \{e\}$, $\{eee\} = \{e\}$ etc, then the set $\{e\}^* \subseteq \{e\}$ and $\{e\} \subseteq \{e\}^*$ hence, $\{e\}^* = \{e\}$

b.) For any alphabet Σ and any $L \subseteq \Sigma^*$, $(L^*)^* = L^*$

Let $L = \{a,b,c\}$ $L \subseteq \Sigma^*$
$L^* = \{e,a,b,c,aa,ab,ac,\ldots\}$
$(L^*)^*$ is redundant since every possible pair is already contained inside L^* adding another Kleene star will generate the same result. Making $L^* \subseteq (L^*)^*$ and $(L^*)^* \subseteq L^*$ making $L^* = (L^*)^*$

c.) If a and b are distinct symbols, then $\{a,b\}^* = \{a\}^*(\{b\}\{a\}^*)^*$

$\{a,b\}^*$ is the language over a,b that contains every combination of a,b include e
$(\{b\}\{a\}^*)^*$ is the language over a,b that contains every combination of a,b including e. concatenating it with $\{a\}^*$ makes $\{a\}^*$ irrelevant since every occurrence of $\{a\}^*$ already occurs in $(\{b\}\{a\}^*)^*$ and in $\{a,b\}^*$
$\{a\}^*(\{b\}\{a\}^*)^* \subseteq \{a,b\}^*$ and $\{a,b\}^* \subseteq \{a\}^*(\{b\}\{a\}^*)^*$ hence $\{a,b\}^* = \{a\}^*(\{b\}\{a\}^*)^*$

d.) If Σ is any alphabet, $e \in L_1 \subseteq \Sigma^*$ and $e \in L_2 \subseteq \Sigma^*$, then $(L_1\Sigma^*L_2)^* = \Sigma^*$

Σ^* is the language obtained by concatenating zero or more strings. Since $L_1 \subseteq \Sigma^*$ and $L_2 \subseteq \Sigma^*$ hence $L_1\Sigma^*L_2 = \Sigma^*$
based on the proof in 1.7.4(b) we proved that $(L^*)^* = L^*$ using the same idea the kleene star is redundant in this case as well, since every combination in $(\Sigma^*)^*$ is already in Σ^* making:
$(L_1\Sigma^*L_2)^* \subseteq \Sigma^*$ and $\Sigma^* \subseteq (L1\Sigma^*L2)^*$ therefore, $(L_1\Sigma^*L_2)^* = \Sigma^*$

e.) For any language, L, 0L = L0 = 0 (0 is empty set)

Let L= {a,b}
0L = {e, a, b}
L0 = {e,a,b}
since 0 is an empty set
L0 ≠0 and 0L ≠ 0 since L can contain some elements which are not e or 0 the statement is false.

1.7.5) Give some examples of strings in and not in these sets, where $\Sigma = \{a,b\}$.

a. {w: for some u ∈ $\Sigma\Sigma$, w=uu^ru}

in: aaaaaa, abbaab

not in: ababab

b. {w: ww=www}

in: let w=e ee = eee

not in: let w=a aa ≠ aaa

c. {w: for some u,v ∈ Σ*, uvw = wvu}

in: aaaa, abab = abba

not in: ababab, aaaa

d. {w: for some u ∈ Σ*, www=uu}

in: u=aaa aaaaaa is in

not in: u=ab, ababab is not in since www≠ uu

1.7.6)Under what circumstances is L+ = L* - {e}?

L+ is defined as: L+ = L*L

L+ = L* - {e} is always true since L+ is defined as L+=L*L which never contains {e} therefore, L* - {e} will always be equal to L+

1.7.7)The Kleene star of a language L is the closure of L under which relations?

The Kleene start of a language L (ie. L*) is the closure of L under concatenation.

L $_0$ L* closes the language.

1.8.1) What language is represented by the regular expression $(((a^*a)b) \cup b)$?

The language of L over zero or more a's followed by b

1.8.3)Let $\Sigma = \{a,b\}$. Write regular expressions for the following sets:

a.) All strings in Σ^* with no more then three a's

$b^*ab^*ab^*ab^*$

b.) All strings in Σ^* with a number of a's divisible by three

$(ab^*ab^*ab^*)^*$

c.) All strings in Σ^* with exactly one occurrence of the substring aaa

(b^*aaab^*)