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CS301

1.1.1)

a.) $\emptyset \subseteq \emptyset$ true

b.) $\emptyset \in \emptyset$ false

c.) $\emptyset \in \{\emptyset\}$ true

d.) $\emptyset \subseteq \{\emptyset\}$ true

e.) $\{a,b\} \in \{a,b,c,\{a,b\}\}$ true

f.) $\{a,b\} \subseteq \{a,b,\{a,b\}\}$ true

g.) $\{a,b\} \subseteq 2^{\{a,b,\{a,b\}\}}$ true

h.) $\{\{a,b\}\} \in 2^{\{a,b,\{a,b\}\}}$ true

i) $\{a,b,\{a,b\}\} - \{a,b\} = \{a,b\}$ false

1.1.2)

a.) $(\{1,3,5\} \cup \{3,1\}) \cap \{3,5,7\} =$
$\{1,3,5\} \cap \{3,5,7\} = \{3,5\}$

b.) $\bigcup \{\{3\},\{3,5\}, \bigcap \{\{5,7\},\{7,9\}\}\}$
$= \bigcup \{\{3\},\{3,5\},\{7\}\}$
$= \{3,5,7\}$

c.) $(\{1,2,5\} - \{5,7,9\}) \cup (\{5,7,9\} - \{1,2,5\}) =$
$\{1,2\} \cup \{7,9\} = \{1,2,7,9\}$

1.2.1) (continued)

d.) $2^{\{7,8,9\}} - 2^{\{7,9\}} =$
$\{\{7\}, \{8\}, \{9\}, \{7,8\}, \{8,9\}, \{7,9\}, \{7,8,9\}\} -$
$\{\{7\}, \{9\}, \{7,9\}\}$

$= \{\{8\}, \{7,8\}, \{8,9\}, \{7,8,9\}\}$

e.) $2^{\emptyset} = \emptyset$

1.1.3) a.) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

* $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$

suppose $x \in A \cup (B \cap C)$ by def of union $x \in A$ or $x \in B \cap C$

since $x \in A$ then $x \in A \cup B$ by def of union and also $x \in A \cup C$ by def. of union. hence $x \in (A \cup B) \cap (A \cup C)$ by def of intersection.

therefore $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$ by def of a subset

* $(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$

suppose $x \in (A \cup B) \cap (A \cup C)$ by def of intersection, $x \in A \cup B$ and $x \in A \cup C$

since $x \in A$ we can immediatoly conclude that $x \in A \cup (B \cap C)$ by def. of union.

since both subset relations have been proven, it follows that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ by def. of set equality.

1.1.3) (continued)

b.) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

* $A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$

suppose $x \in A \cap (B \cup C)$ by def of intersection

$x \in A$ and $x \in (B \cup C)$

since $x \in A$ then $x \in (A \cap B)$ by def. of intersect

and $x \in (A \cap C)$ by def of intersection and

$x \in (A \cap B) \cup (A \cap C)$ by def of union.

therefore since $x \in (A \cap B) \cup (A \cap C)$ and $x \in A \cap (B$

it holds that $A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$

* $(A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$

suppose $x \in (A \cap B) \cup (A \cap C)$ by def of union

$x \in (A \cap B)$ or $x \in (A \cap C)$,

since $x \in A$ we can conclude that $x \in A \cap (B$

by def of ~~union~~ intersection.

Since $x \in A \cap (B \cup C)$ then by def.

$(A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$

since both subset relations hold, it follows that

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

c.) $A \cap (A \cup B) = A$

$A \cup B = \{x : x \in A \text{ or } x \in B\}$

$A \cap B = \{x : x \in A \text{ and } x \in B\}$

if any $\exists x \in A$ or $x \in B$ then $x \in (A \cup B)$

let $A \cup B$ be defined as C (ie. $C = A \cup B$)

since $A \subseteq C$ and $A \cap C$ is defined as any $x \in A$ and $x \in C$ the $A \cap C = A$

therefore $A \cap (A \cup B) = A$

d.) $A \cup (A \cap B) = A$

if $x \in A$ ~~and~~ then by def. of union

$x \in (A \cup \text{anything})$ therefore $x \in A$.

e.) $A - (B \cap C) = (A - B) \cup (A - C)$

let $L = A - (B \cap C)$ and $R = (A-B) \cup (A-C)$

a.) let $x \in L$ then $x \in A$ but $x \notin B$ or $x \notin C$

hence x is an element of either $A - B$ or $A - C$

and thus an element of R ie. $L \subseteq R$

b.) let $x \in R$ then x is an element of either $(A-B)$ or $A - C$ by def of substraction, it has to b in A, $(x \in A)$ but $x \notin (B \cap C)$ by def of substraction and union on R

so $x \in L$, therefore $R \subseteq L$

by definition of set equality $L = R$

$A - (B \cap C) = (A - B) \cup (A - C)$