

# CS201: Data Structures and Discrete Mathematics I

Linked List, Stacks and Queues

# Data Structure

- A *construct* that can be defined within a programming language to store a collection of data
  - one may store some data in an array of integers, an array of objects, or an array of arrays

# Abstract Data Type (ADT)

- Definition: a collection of *data* together with a set of *operations* on that data
  - specifications indicate *what* ADT operations do, but not *how* to implement them
  - data structures are part of an ADT's implementation
- Programmer can use an ADT without knowing its implementation.

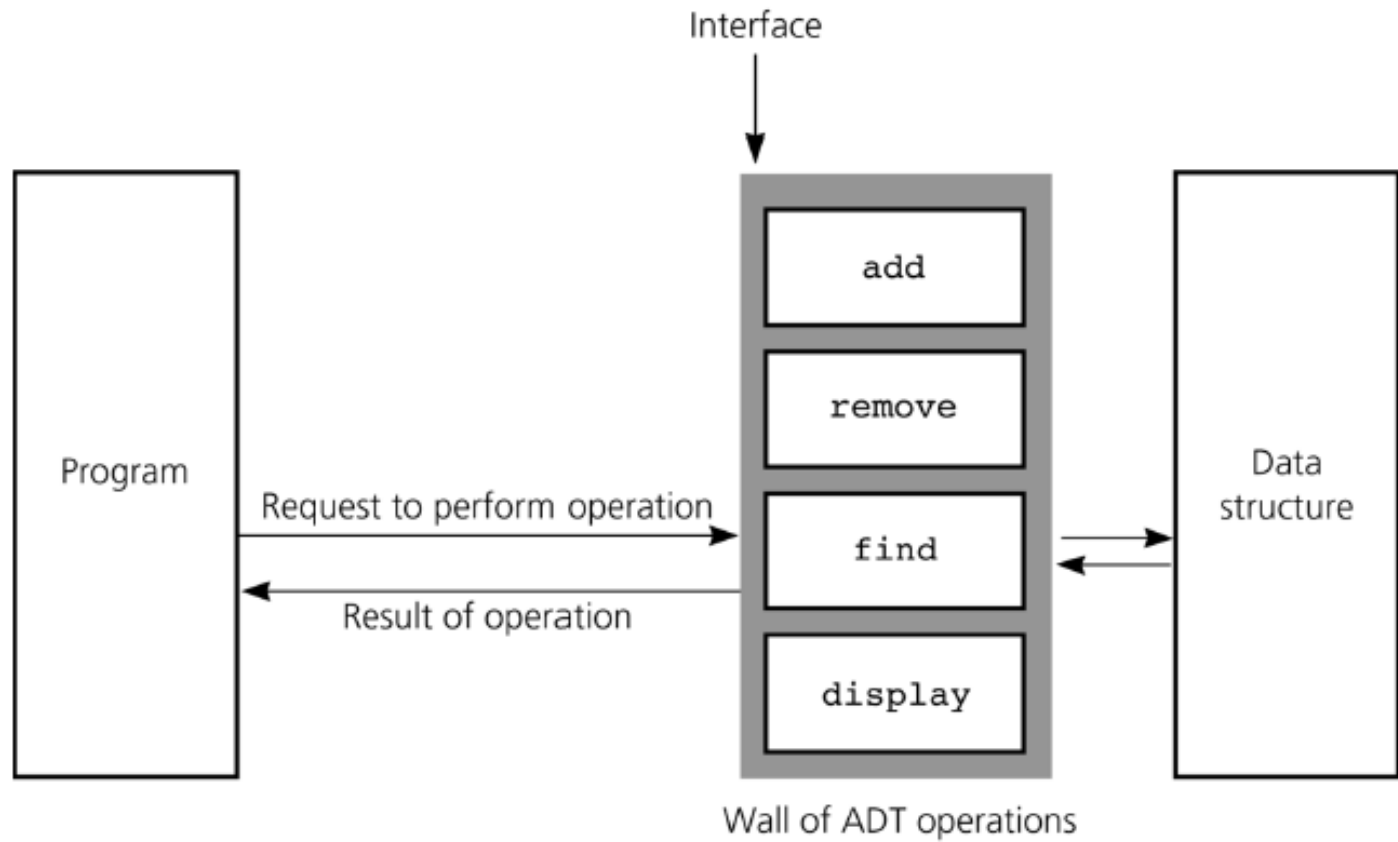
# Typical Operations on Data

- Add data to a data collection
- Remove data from a data collection
- Ask questions about the data in a data collection. E.g., what is the value at a particular location.

# Why ADT

- Hide the unnecessary details
- Help manage software complexity
- Easier software maintenance
- Functionalities are less likely to change
- Localised rather than global changes

# Illustration



# Linked Lists

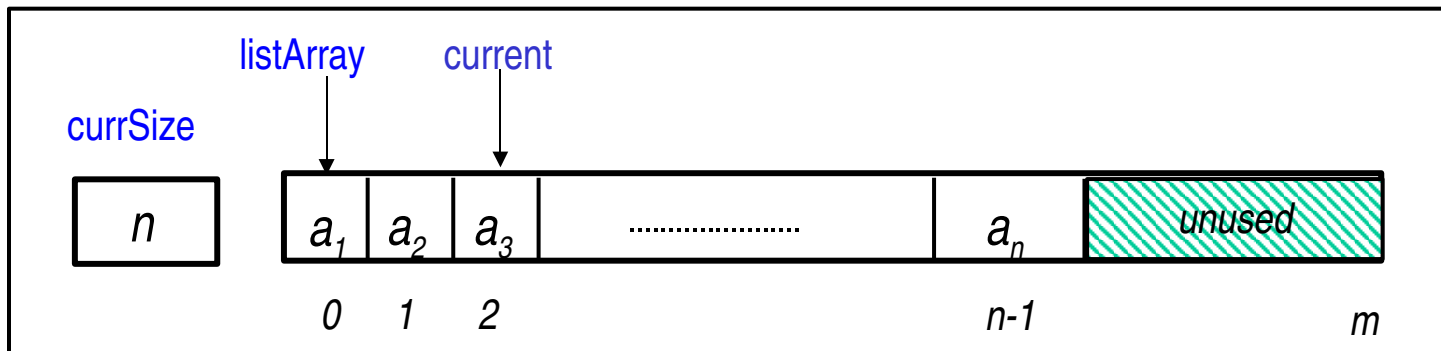
# Lists

- List: a finite sequence of data items  
     $a_1; a_2; a_3; \dots; a_n$
- Lists are pervasive in computing
  - e.g. class list, list of chars, list of events
- Typical operations:
  - Creation
  - Insert / remove an element
  - Test for emptiness
  - Find an element
  - Current element / next / previous
  - Find k-th element
  - Print the entire list



# Array-Based List Implementation

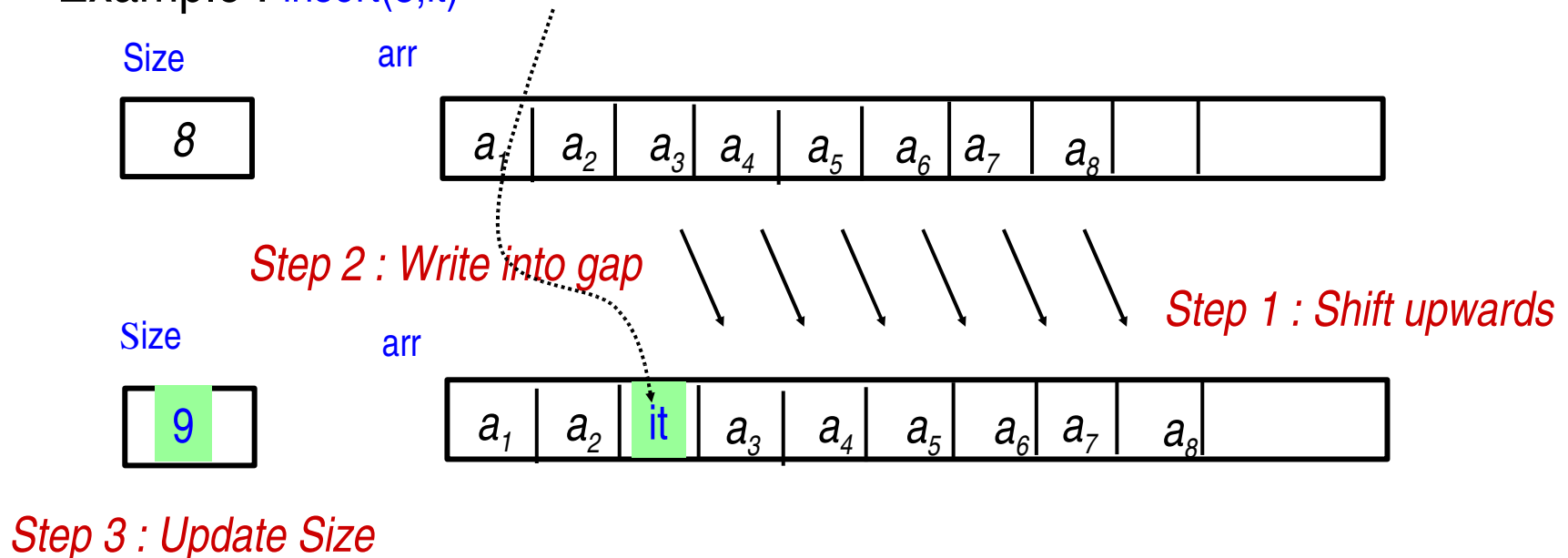
- One simple implementation is to use java arrays
  - A sequence of  $n$ -elements
- Maximum size must be anticipated a priori.
- Internal variables:
  - Maximum size *maxSize* ( $m$ )
  - Current size *currSize* ( $n$ )
  - Current index *current*
  - Array of elements *listArray*



# Inserting Into an Array

- While retrieval is very fast, insertion and deletion are very slow
  - Insert has to shift upwards to create gap

Example : `insert(3,it)`



# Coding

```
class list {  
    private int size  
    private Object[] arr;  
  
    public void insert(int j, Object it)  
    { // pre : 1<=j<=size+1  
  
        for (i=size; i>=j; i=i-1)  
            { arr[i+1]=arr[i]; }; // Step 1: Create gap  
  
        arr[j]=it; // Step 2: Write to gap  
  
        size = size + 1; // Step 3: Update size  
    }
```

# Deleting from an Array

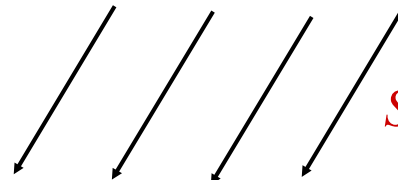
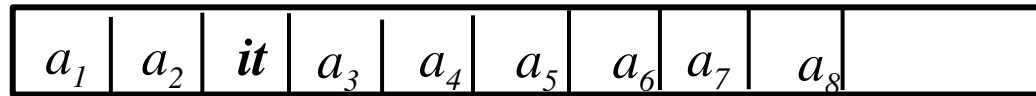
- Delete has to shift downwards to close gap of deleted item

Example: `delete(5)`

size

9

arr

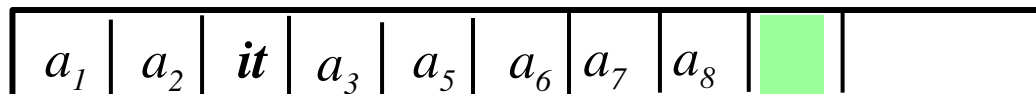


*Step 1 : Close Gap*

size

8

arr



*Step 2 : Update Size*

*Not part of list*

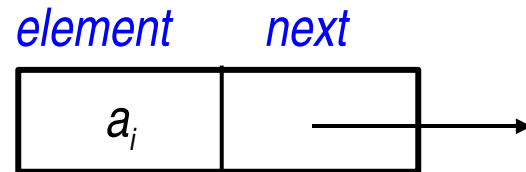
# Coding

```
public void delete(int j)
{
    // pre : 1<=j<=size
    for (i=j+1; i<=size; i=i+1)
        { arr[i-1]=arr[i]; }; // Step1: Close gap

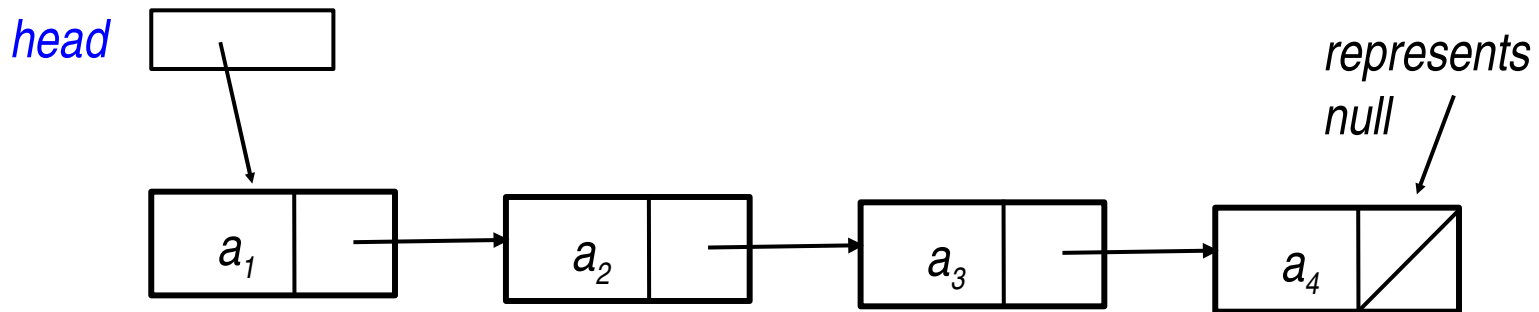
    size = size - 1; // Step 2: Update size
}
```

# Linked List Approach

- Main problem of array is deletion/insertion slow since it has to shift items in its *contiguous* memory
- **Solution:** linked list where items need *not be contiguous* with nodes of the form



- Sequence (list) of four items  $\langle a_1, a_2, a_3, a_4 \rangle$  can be represented by:



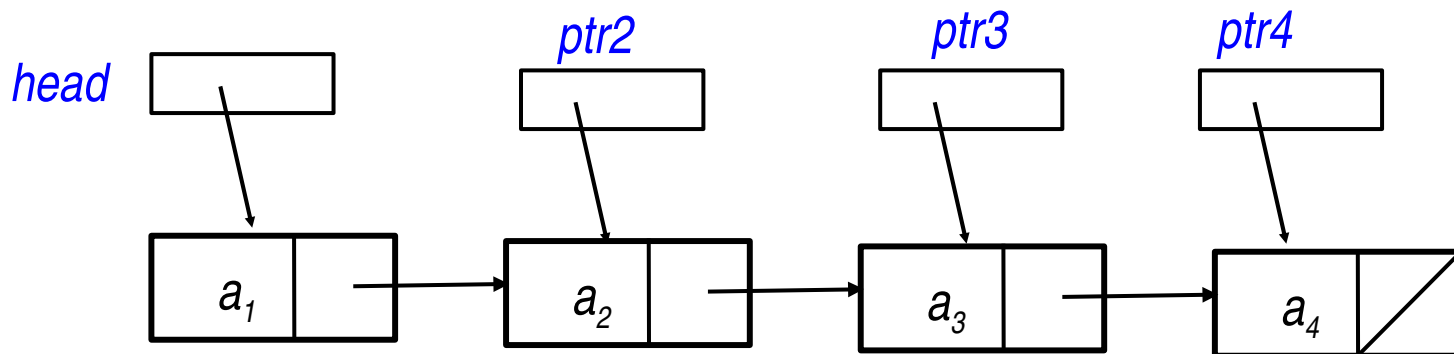
# Coding

```
class ListNode {  
    Object element;  
    ListNode next;  
  
    public ListNode(Object o)  
        { element = o;  
          next = null; }  
    public ListNode(Object o, ListNode n)  
  
        { element = o;  
          next = n; }  
}
```

# Add elementz to list

The earlier sequence can be built by:

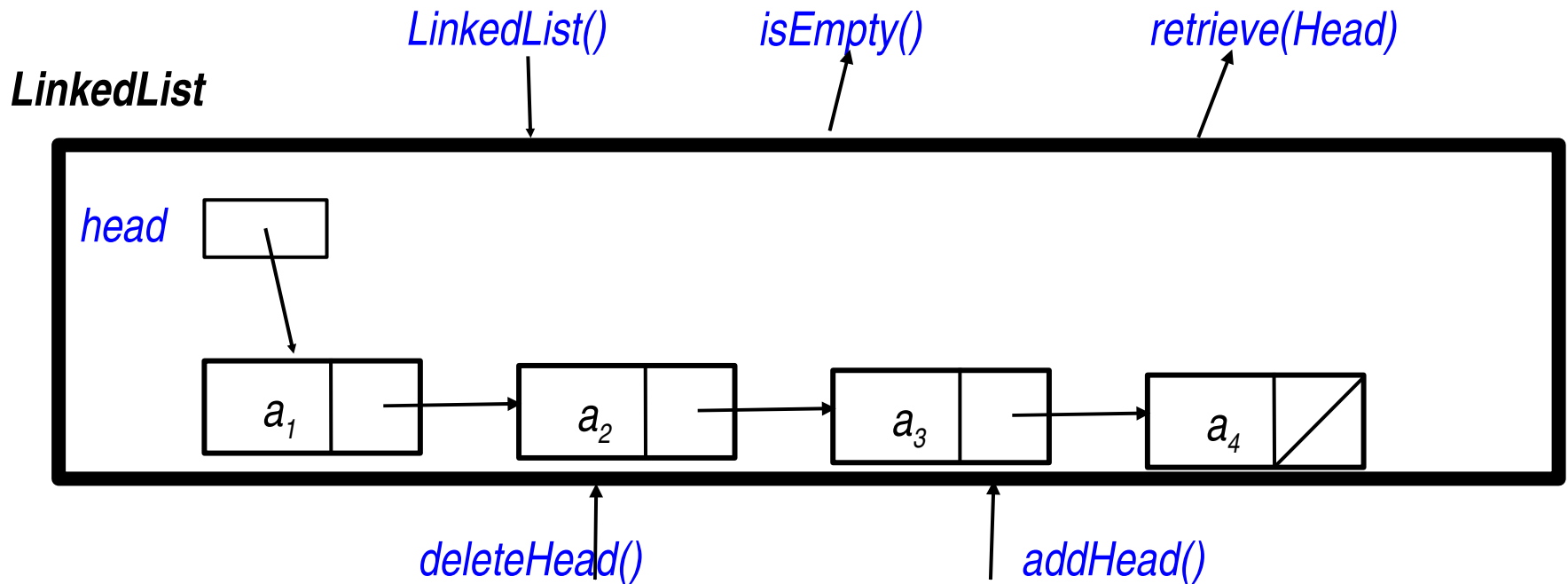
- ➡ `ListNode ptr4 = new ListNode("a4", null);`
- ➡ `ListNode ptr3 = new ListNode("a3", ptr4);`
- ➡ `ListNode ptr2 = new ListNode("a2", ptr3);`
- ➡ `ListNode head = new ListNode("a1", ptr2);`





# Linked List ADT

- We can provide an ADT for linked-list.
  - This can help hide unnecessary internal details

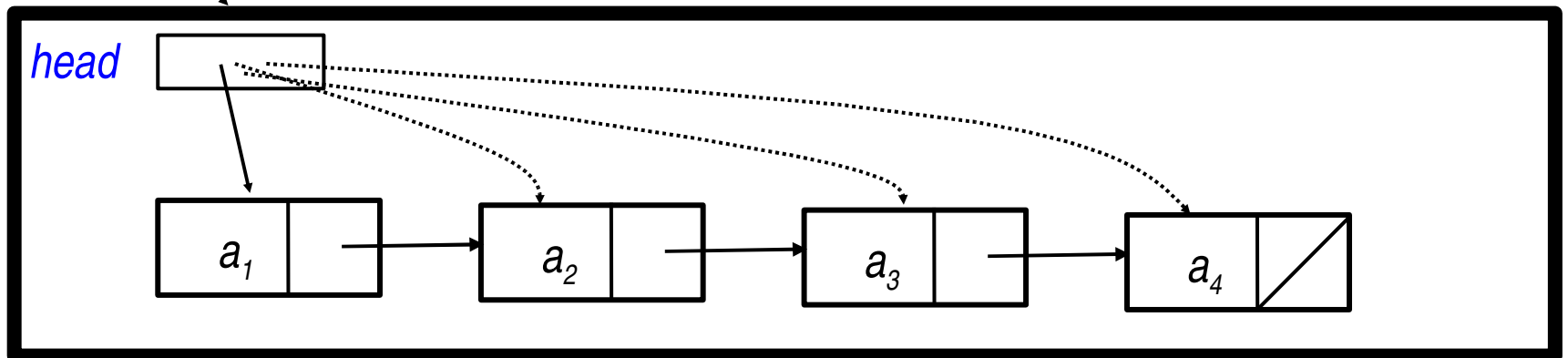
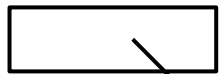


# Sample

Sequence of four items  $\langle a_1, a_2, a_3, a_4 \rangle$  can be built, as follows:

```
➡ LinkedList list = new LinkedList();  
➡ list.addHead("a4");  
➡ list.addHead("a3");  
➡ list.addHead("a2");  
➡ list.addHead("a1");
```

*list*



# Coding

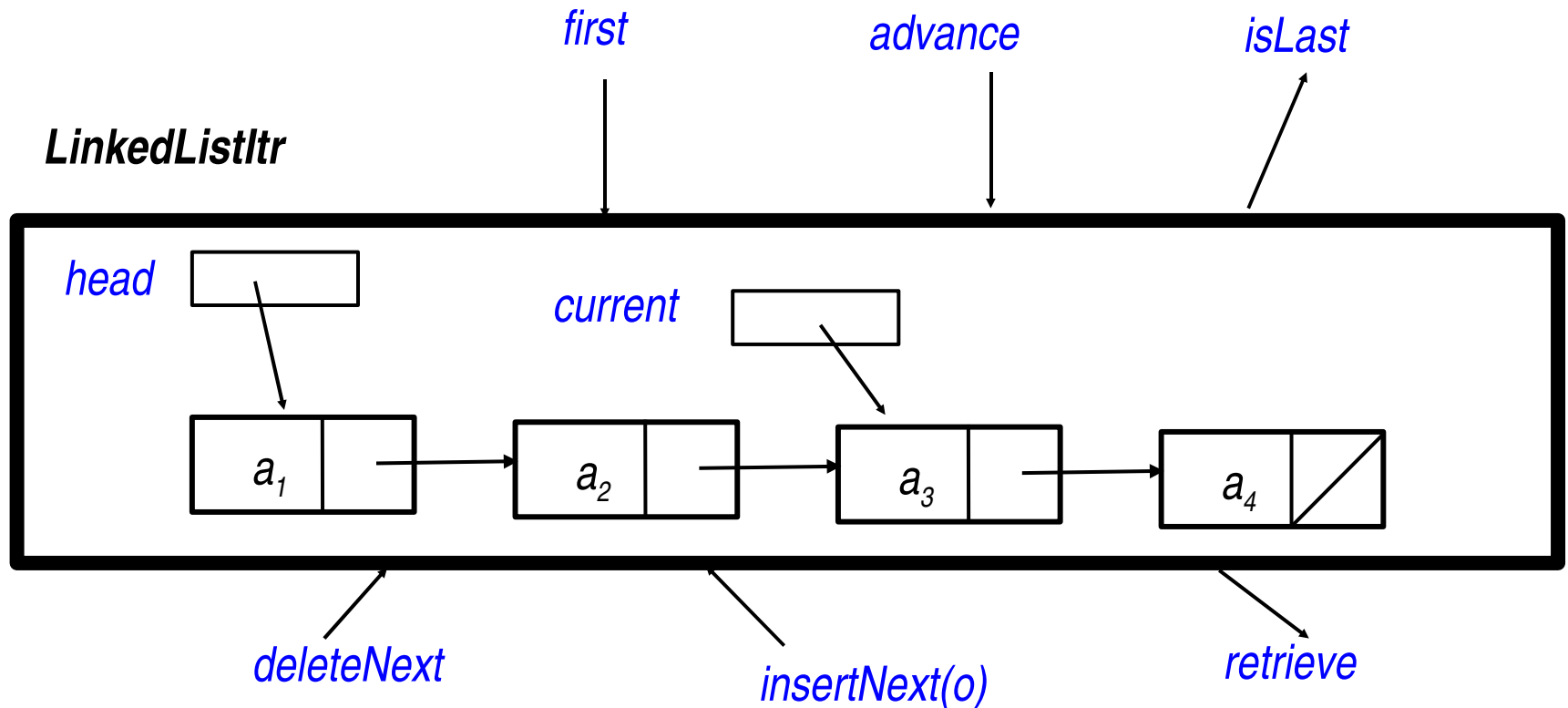
```
class LinkedList {
    protected ListNode head ;

    public LinkedList()
        {head = null; }
    public boolean isEmpty()
        {return (head == null); }
    public void addHead(Object o)
        {head = new ListNode(o,head); }
    public void deleteHead() throws ItemNotFound
        {if (head ==null)
            {throw new ItemNotFound("DeleteHead
fails");}
            else head = head.next;}
        };
}

class ItemNotFound extends Exception {
    public ItemNotFound(String msg)
        {super(msg); }
}
```

# Linked List Iteration ADT

- To support better access to our linked-list, we propose to build a linked-list iteration ADT



# Declaration

```
class LinkedListItr extends LinkedList{  
    private ListNode current;  
    private boolean zeroflag;  
  
    public LinkedListItr() { ... }  
    public void zeroth() { ... }  
    public void first() { ... }  
    public void advance() { ... }  
    public boolean isLast() { ... }  
    public boolean isInList() { ... }  
  
    public Object retrieve() { ... }  
    public void insertNext(Object o) { ... }  
    public void deleteNext() { ... }  
}
```

} *data structure*

} *access or change  
current pointer*

} *access or change  
nodes*

# Coding

```
public void zeroth()           // set position prior to  
    first element  
    { current = null;  
      zeroflag = true;  
    }
```

- Why zeroflag? – To distinguish “zeroth position” from “beyond list position”

Zeroth Position

(current == null) && (zeroflag==true)

InList

(current != null)

Beyond List

(current == null) && (zeroflag==false)

# More coding

```
public void first() throws ItemNotFound
    // set position to first element
    { current = head;
      zeroflag = false;
      if (current == null)
          {throw new ItemNotFound("No first element");};
    }
public void advance()          // advance to next item
    {if (current != null)
      {current = current.next }
    else {if (zeroflag)
          {current = head;
            zeroflag = false;}
        else {}
    };
    }
```

# More coding

```
public boolean isLast() // check if it current is at the last node

    {if (current!=null)
        return (current.next == null);
    else return false;
    }

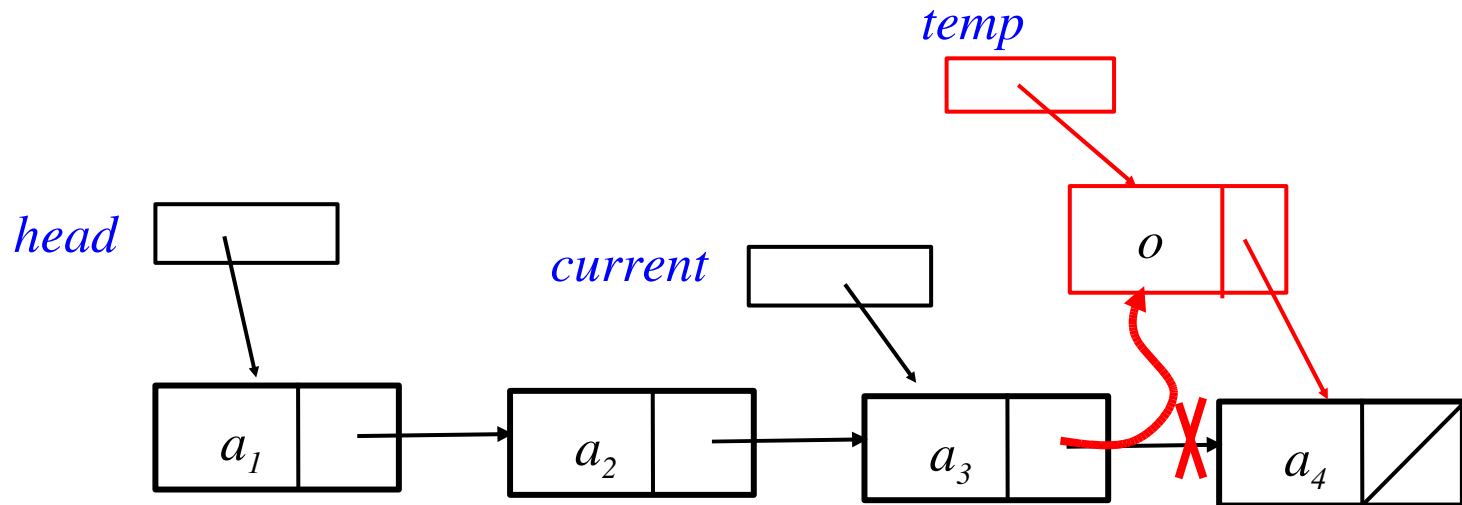
public boolean isInList() // check if current is at some node
    (return (current != null);
    }

public Object retrieve() throws ItemNotFound
    // current object at current position
    { if (current!=null)
        { return current.element; }
      else { throw new ItemNotFound("retrieve fails"); };
    }
```



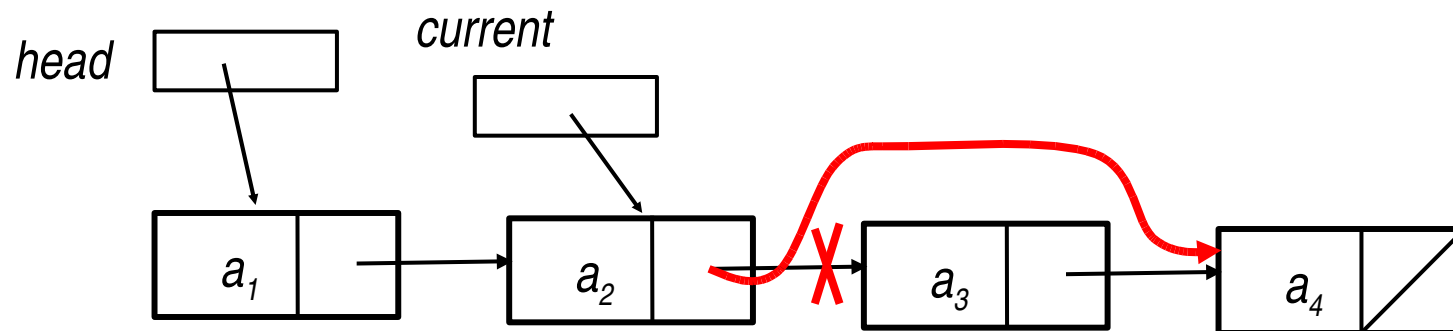
# Insertion

```
public void insertNext(Object o) throws ItemNotFound
    // insert after current position
{
    ListNode temp;
    if (current != null)
    {
        temp = new ListNode(o, current.next);
        current.next = temp;
    }
    else if (zeroflag) { head = new ListNode(o, head); }
    else { throw new ItemNotFound("insert fails"); }
}
```



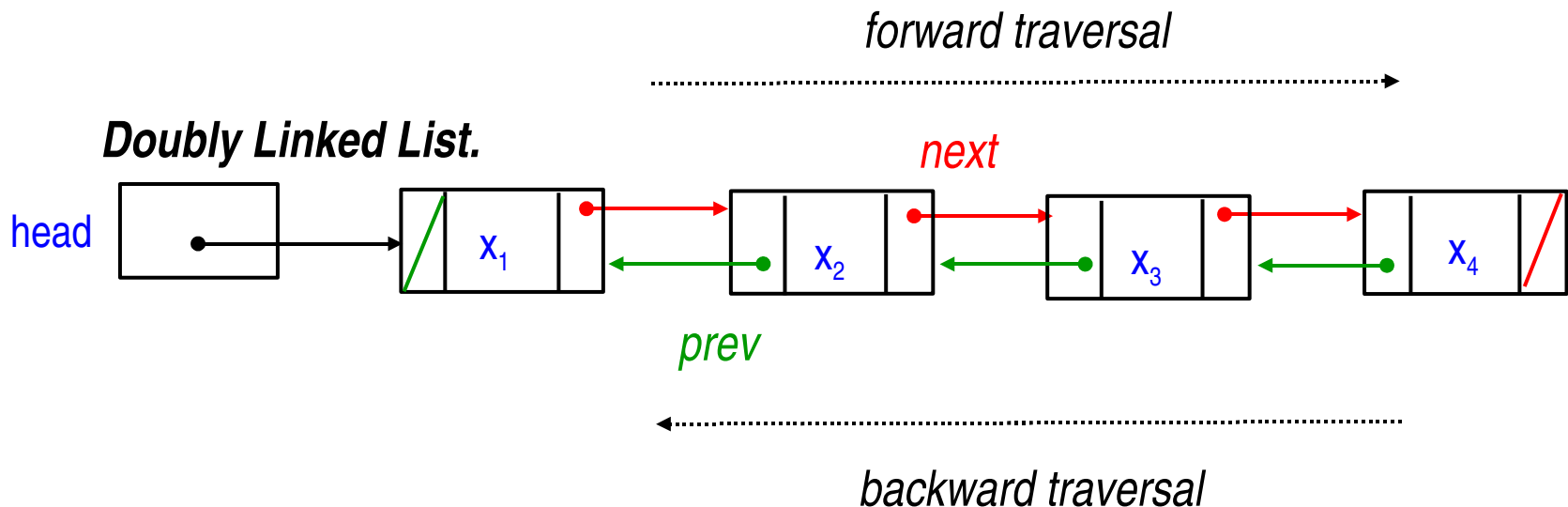
# Delete

```
public void deleteNext() throws ItemNotFound
    // delete node after current position
{ if (current!=null)
    {if (current.next!=null)
        {current.next = current.next.next;}
    else {throw new ItemNotFound("No Next Node to Delete");}}
    }
    else if (zeroflag && head!=null) {head=head.next;}
    else {throw new ItemNotFound("No Next Node to Delete");}}
}
```



# Doubly Linked Lists

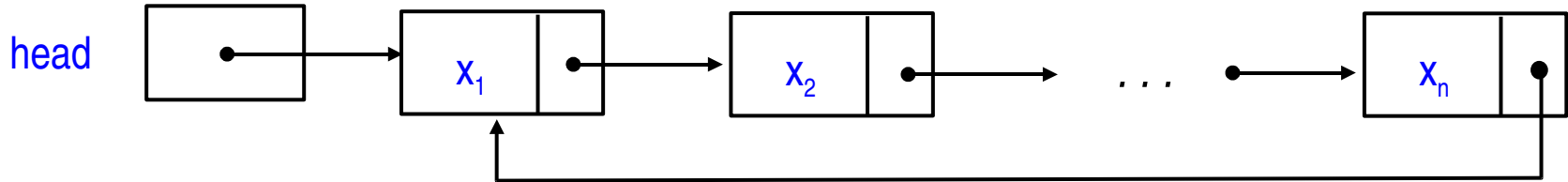
- Frequently, we need to traverse a sequence in BOTH directions efficiently
- *Solution* : Use doubly-linked list where each node has two pointers



# Circular Linked Lists

- May need to cycle through a list repeatedly, e.g. round robin system for a shared resource
- *Solution* : Have the last node point to the first node

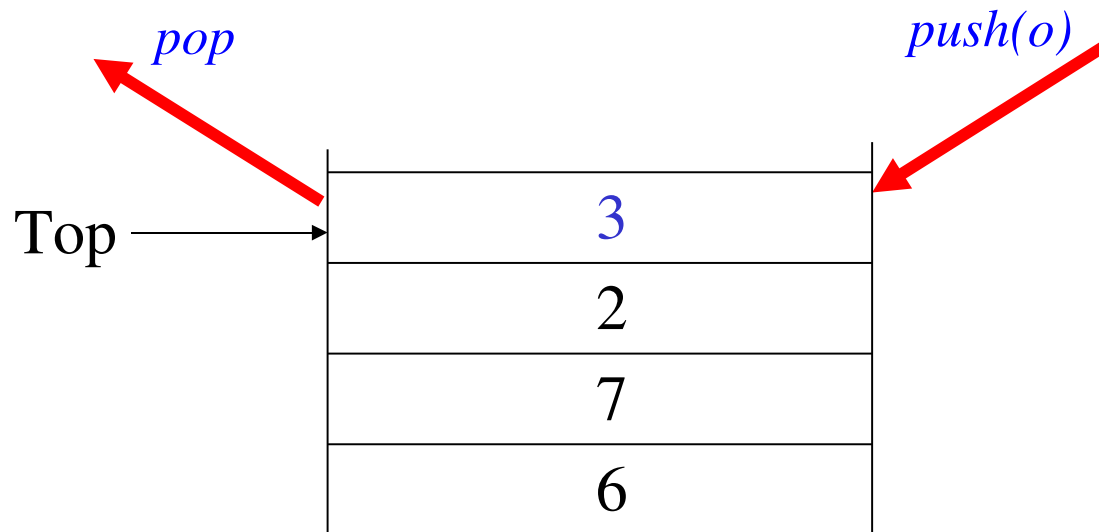
## ***Circular Linked List.***



# Stacks

# What is a Stack?

- A *stack* is a list with the restriction that insertions and deletions can be performed in only one position, namely, the end of the list, called the *top*.
- The operations: push (insert) and pop (delete)



# Stack ADT Interface

- We can use Java Interface to specify Stack ADT Interface

```
interface Stack {  
  
    boolean isEmpty();                // return true if empty  
  
    void push(Object o);              // insert o into stack  
  
    void pop() throws Underflow;      // remove most recent item  
  
    void popAll();                    // remove all items from stack  
  
    Object top() throws Underflow;    // retrieve most recent item  
  
    Object topAndPop() throws Underflow; // return & remove most recent item  
}
```

# Sample Operation

➔ `Stack s = makeStack();`

➔ `s.push("a");`

➔ `s.push("b");`

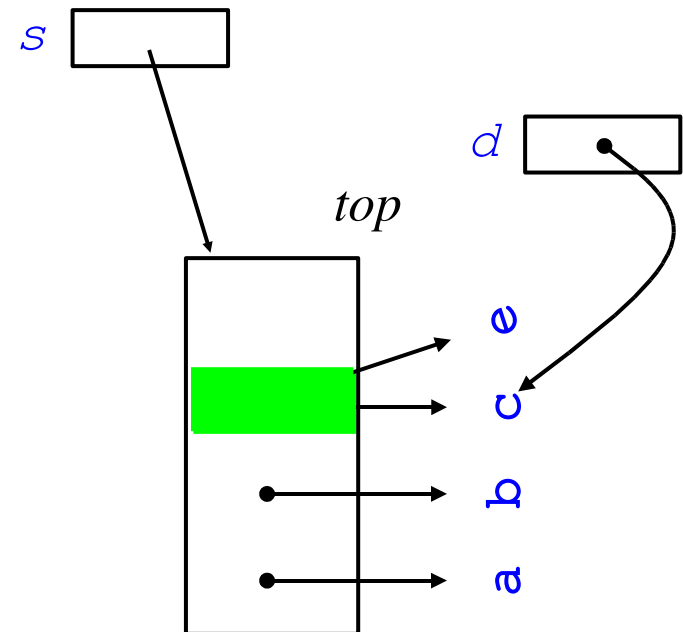
➔ `s.push("c");`

➔ `d=s.top();`

➔ `s.pop();`

➔ `s.push("e");`

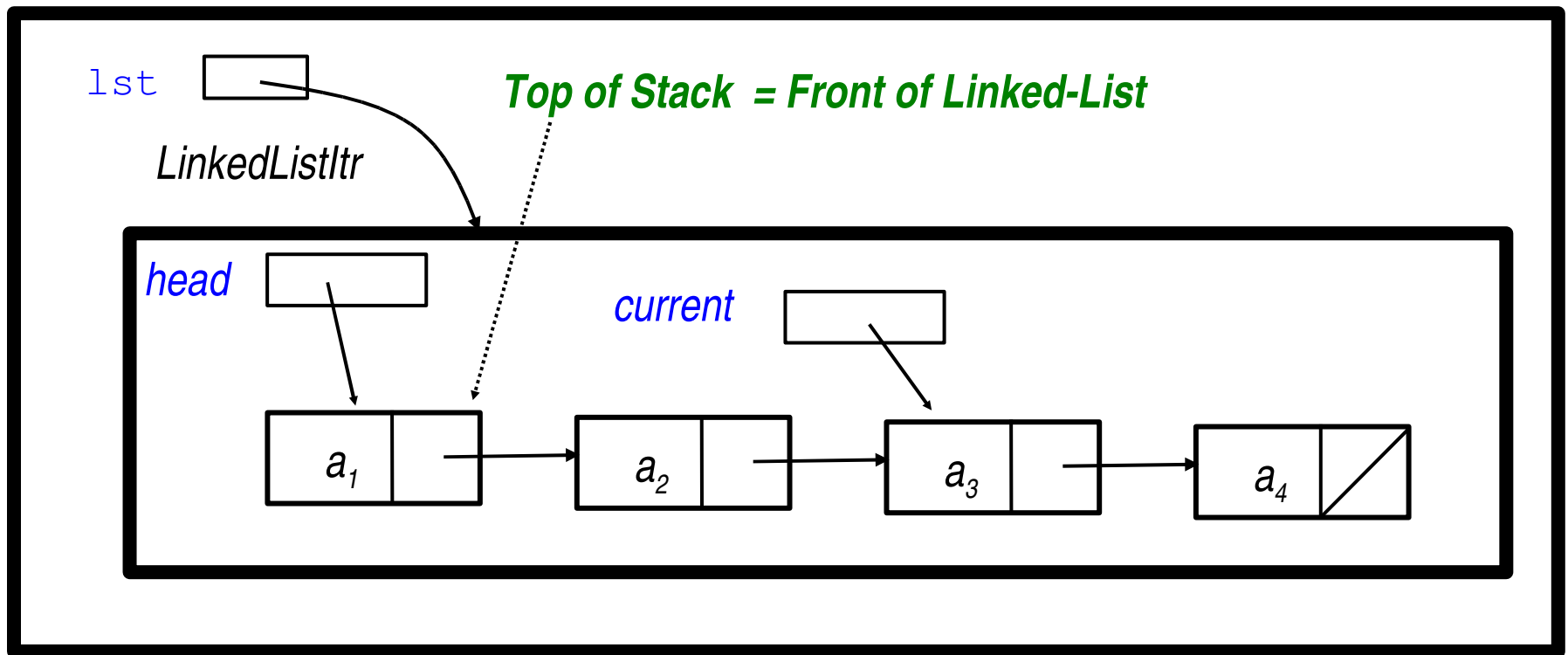
➔ `s.pop();`





# Implementation by Linked Lists

- Can use `LinkedListtr` as implementation of stack  
*StackLL*



# Code

```
Class StackLL implements Stack {  
  
    private LinkedListItr lst;  
  
    public StackLL() { lst = new LinkedListItr(); }  
  
    public Stack makeStack() { return new StackLL(); }  
  
    public boolean isEmpty()                // return true if empty  
    { return lst.isEmpty(); };  
  
    public void push(Object o)                // add o into the stack  
    { lst.addHead(o); }  
  
    public void pop() throws Underflow // remove most recent item  
    { try {lst.deleteHead();}  
      catch (ItemNotFound e)  
      {throw new Underflow("pop fails - empty stack");}  
    }  
}
```

# More code

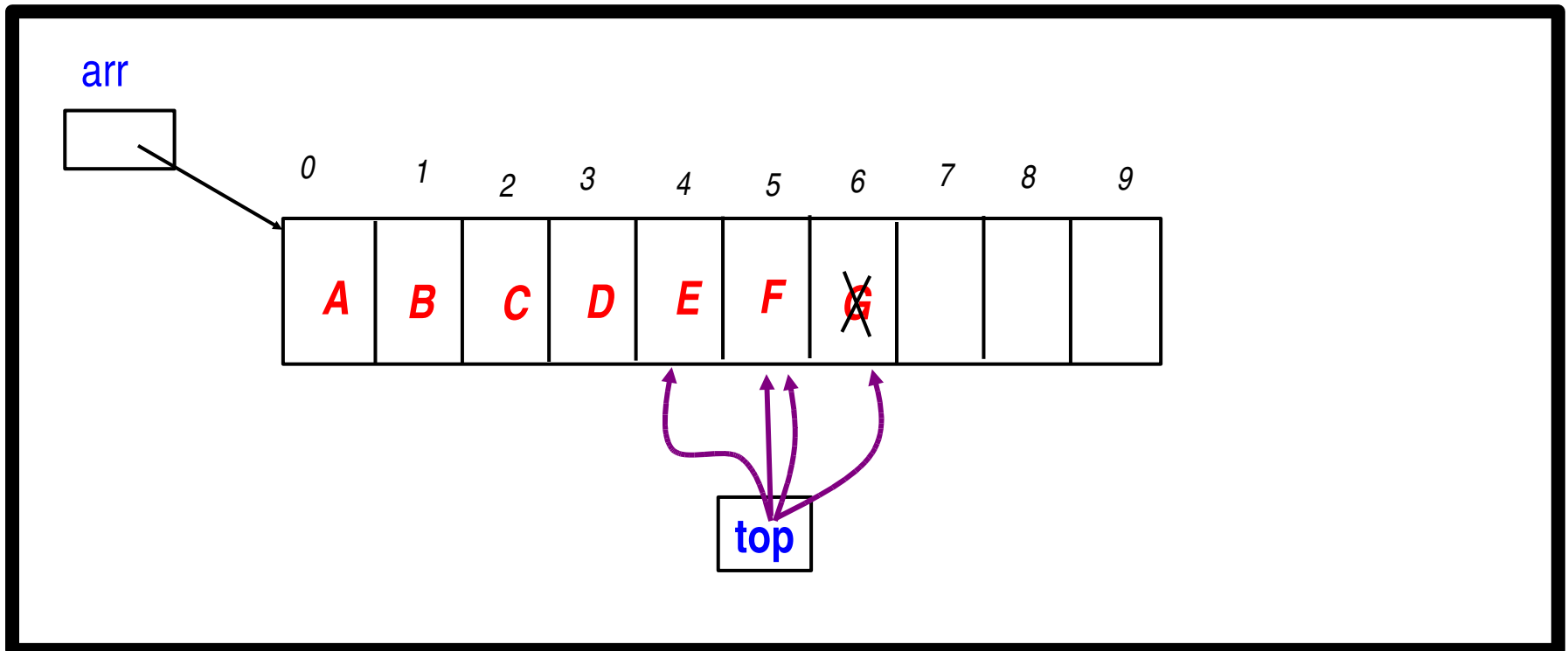
```
public Object top() throws Underflow; // retrieve most recent
    item
{ try { lst.first();
    return lst.retrieve();
    } catch (ItemNotFound e)
    {throw new Underflow("top fails - empty stack");};
}

public Object topAndPop() throws Underflow;
    // return & remove most recent item
{ try { lst.first();
    Object p=lst.retrieve();
    lst.deleteHead();
    return p;
    } catch (ItemNotFound e)
    {throw new Underflow("topAndPop fails - empty stack");};
}
```

# Implementation by Array

- Can use Array with a **top** index pointer as an implementation of stack

*StackAr*



# Code

```
class StackAr implements Stack {  
  
    private Object [] arr;  
    private int top;  
    private int maxSize;  
    private final int initSize = 1000;  
    private final int increment = 1000;  
  
    public StackAr() { arr = new Object[initSize];  
        top = -1;  
        maxSize=initSize }  
  
    public Stack makeStack() { return new StackAr(); }  
  
    public boolean isEmpty()  
        { return (top<0); }  
  
    private boolean isFull()  
        { return (top>=maxSize); }  
}
```

# More code

```
public void push(Object o)           // insert o
{ top++;
  if (this.isFull()) this.enlargeArr() ;
  arr[top]=o; }

private void enlargeArr()           // enlarge the array
{int newSize = maxSize+increment;
  Object [] barr = new Object[newSize];
  for (int j=0;j<maxSize;j++) {barr[j]=arr[j];};
  maxSize = newSize;   arr = barr;
}
```

# More code

```
public void pop() throws Underflow
{ if (!this.isEmpty()) {top--;}
  else {throw new Underflow("pop fails - empty stack");};
}
```

```
public Object top() throws Underflow
{ if (!this.isEmpty()) {return arr[top];}
  else {throw new Underflow("top fails - empty stack");};
}
```

```
public Object topAndPop() throws Underflow
{ if (!this.isEmpty()) { Object t = arr[top];
                        top--;
                        return t;
                      };
  else {throw new Underflow("top&pop fails - empty stack");};
}
```

# Applications

- Many application areas use stacks:
  - line editing
  - bracket matching
  - postfix calculation
  - function call stack



# Line Editing

- A line editor would place characters read into a buffer but may use a backspace symbol (denoted by  $\leftarrow$ ) to do error correction
- *Refined Task*
  - read in a line
  - correct the errors via backspace
  - print the corrected line in reverse

Input : `abc_defgh $\leftarrow$ 2klhgr $\leftarrow$  $\leftarrow$ wxyz`

Corrected Input : `abc_defg2klpwxz`

Reversed Output : `zyxwplk2gfed_cba`

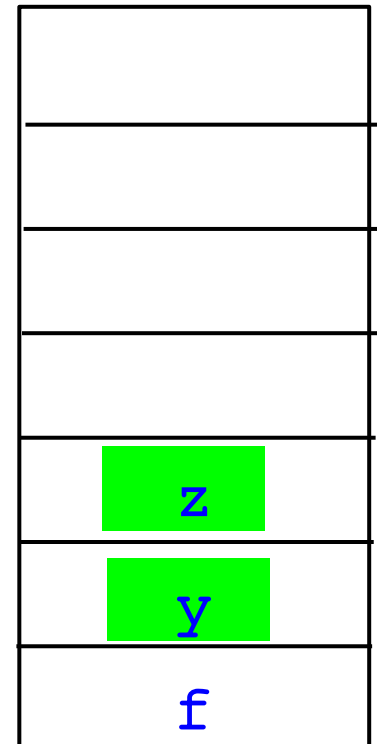
# The Procedure

- Initialize a new stack
- For each character read:
  - if it is a backspace, *pop out last char entered*
  - if not a backspace, *push the char into stack*
- To print in reverse, pop out each char for output

Input : fgh←r←←yz

Corrected Input : fyz

Reversed Output : zyf

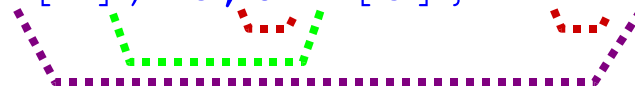


*Stack*

# Bracket Matching Problem

- Ensures that pairs of brackets are properly matched

- An Example:  $\{ a, (b+f[4])*3, d+f[5] \}$



- Bad Examples:

$(...)..)$

// too many closing brackets

$(...(.)$

// too many open brackets

$[...(.)..)$

// mismatched brackets



# Informal Procedure

Initialize the stack to empty

For every char read

if open bracket then *push onto stack*

if close bracket, then

return & remove most recent item  
from *the stack*

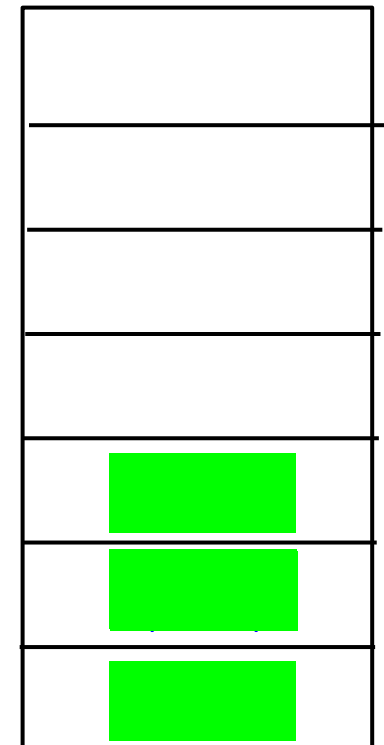
if doesn't match then *flag error*

if non-bracket, *skip the char read*

Example

{ a , ( b + f [ 4 ] ) \* 3 , d + f [ 5 ] }

↑   ↑   ↑   ↑   ↑   ↑   ↑   ↑   ↑

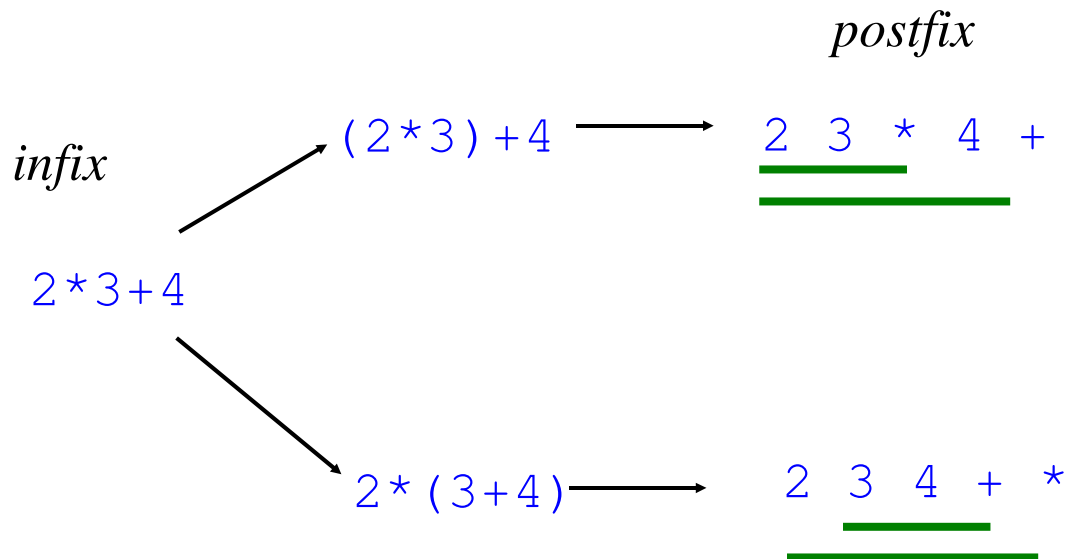


*Stack*

# Postfix Calculator

- Computation of arithmetic expressions can be efficiently carried out in Postfix notation with the help of a stack.

Infix            -  $\text{arg1 op arg2}$   
Prefix          -  $\text{op arg1 arg2}$   
Postfix        -  $\text{arg1 arg2 op}$



# Informal Procedure

Initialise stack

For each item read.

If it is an operand,

*push* on the stack

If it is an operator,

*pop* arguments from stack;

*perform operation*;

*push* result onto the stack

Expr

```
2      s.push(2)
3      s.push(3)
4      s.push(4)
+      arg2=s.topAndPop()
      arg1=s.topAndPop()
      s.push(arg1+arg2)
*      arg2=s.topAndPop()
      arg1=s.topAndPop()
      s.push(arg1*arg2)
```



*Stack*

# Summary

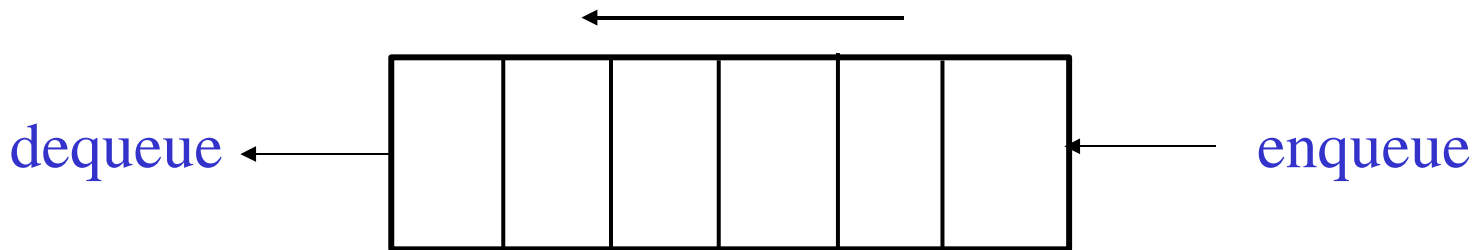
- The ADT stack operations have a last-in, first-out (LIFO) behavior
- Stack has many applications
  - algorithms that operate on algebraic expressions
  - a strong relationship between recursion and stacks exists
- Stack can be implemented by arrays and linked lists

# Queues



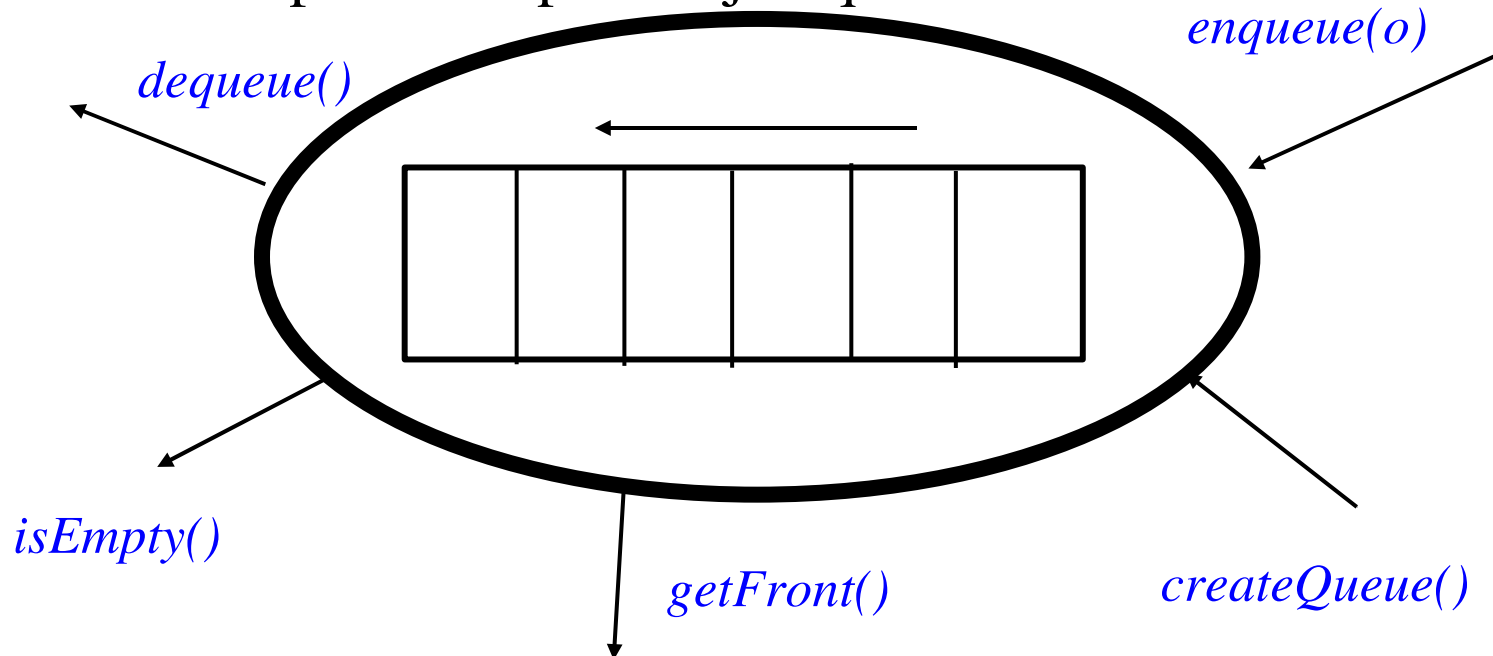
# What is a Queue?

- Like stacks, queues are lists. With a queue, however, insertion is done at one end whereas deletion is done at the other end.
- Queues implement the FIFO (first-in first-out) policy. E.g., a printer/job queue!
- Two basic operations of queues:
  - **dequeue**: remove an element from front
  - **enqueue**: add an element at the back



# Queue ADT

- Queues implement the FIFO (first-in first-out) policy
  - An example is the printer/job queue!



# Sample Operation

➔ `Queue q = createQueue();`

➔ `q.enqueue("a");`

➔ `q.enqueue("b");`

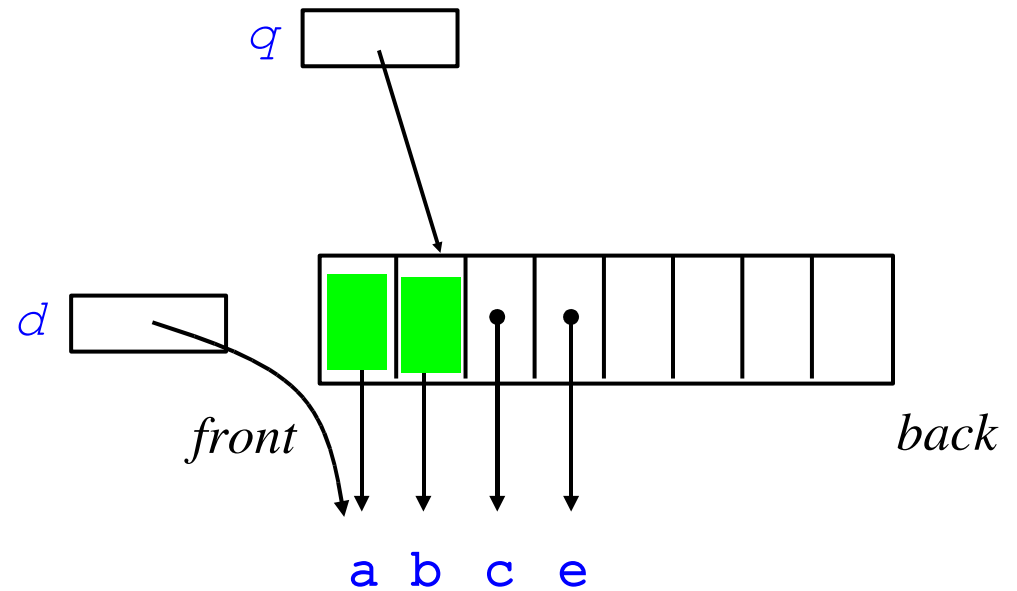
➔ `q.enqueue("c");`

➔ `d=q.getFront();`

➔ `q.dequeue();`

➔ `q.enqueue("e");`

➔ `q.dequeue();`



# Java Interface

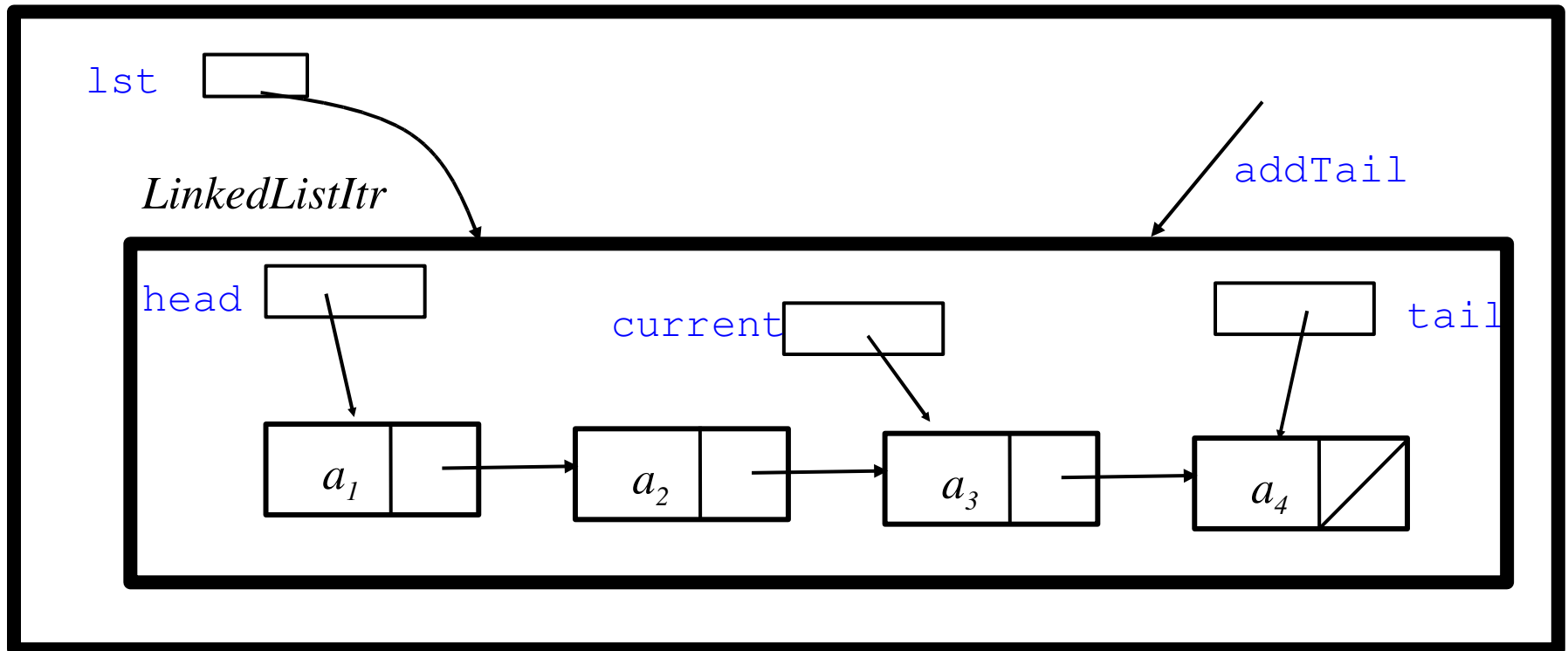
- We can also use Java Interface to specify Queue ADT Interface. This provides a more abstract mechanism to support simultaneous implementations

```
interface Queue {  
  
    void enqueue(Object o)                // insert o to back of Q  
  
    void dequeue() throws Underflow;      // remove oldest item  
  
    Object getFront() throws Underflow;   // retrieve oldest item  
  
    boolean isEmpty();                    // checks if Q is empty  
  
}
```

# Implementation of Queue (Linked List)

- Can use LinkedListItr as underlying implementation of Queues

*Queue*



# Code

```
class QueueLL implements Queue {  
    private LinkedListItr lst;  
  
    public QueueLL() { lst = new LinkedListItr(); }  
  
    public static Queue makeQueue()                // return a new empty queue  
    { return new QueueLL(); }  
  
    public void enqueue(Object o)                    // add o to back of queue  
    { lst.addTail(o); }  
  
    public void dequeue() throws Underflow           // remove oldest item  
    { try {lst.deleteHead();  
      catch (ItemNotFound e)  
      {throw new Underflow("dequeue fails - empty q");}  
    }  
}
```

# More code

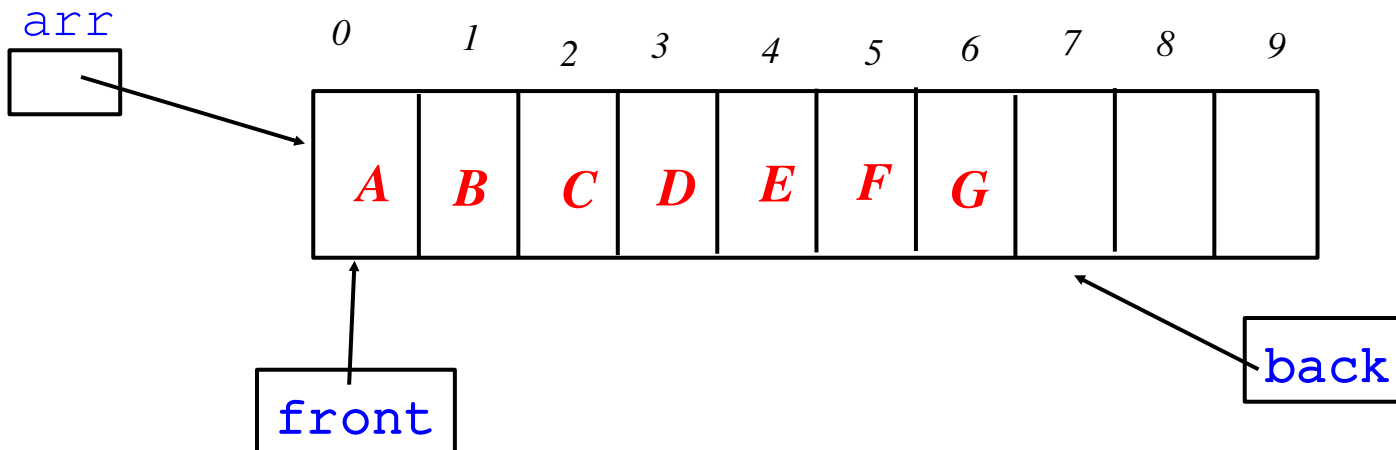
```
public Object getFront() throws Underflow;    // retrieve
    oldest item
{ try { lst.first();
      return lst.retrieve();
    } catch (ItemNotFound e)
    {throw new Underflow("getFront fails - empty queue");};
}

public boolean isEmpty()                      // return true if empty
{ return lst.isEmpty(); }
```

# Implementation of Queue (Array)

- Can use Array with **front** and **back** pointers as implementation of queue

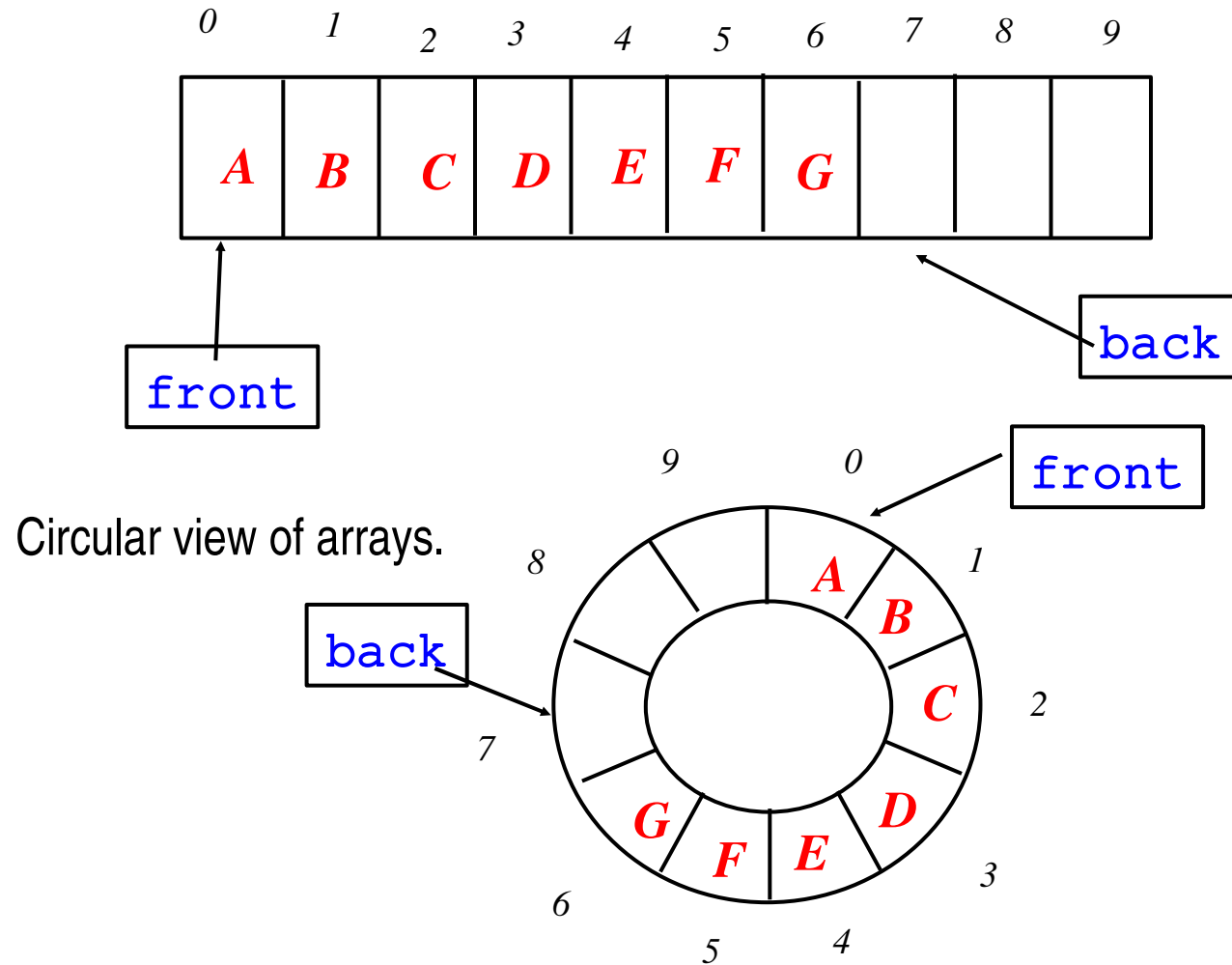
*Queue*





# Circular Array

- To implement queue, it is best to view arrays as circular structure



# How to Advance

- Both front & back pointers should make advancement until they reach end of the array. Then, they should re-point to beginning of the array

```
front = adv(front);  
back  = adv(back);
```

```
public static int adv(int p)  
{ int r = p+1;  
  if (r<maxsize) return r;  
  else return 0;  
}
```

*upper bound of the array*

*Alternatively, use modular arithmetic:*

```
public static int adv(int p)  
{ return ((p+1) % maxsize);  
}
```

*mod operator*

# Sample

➔ `Queue q = QueueAR.makeQueue();`

➔ `q.enqueue("a");`

➔ `q.enqueue("b");`

➔ `q.enqueue("c");`

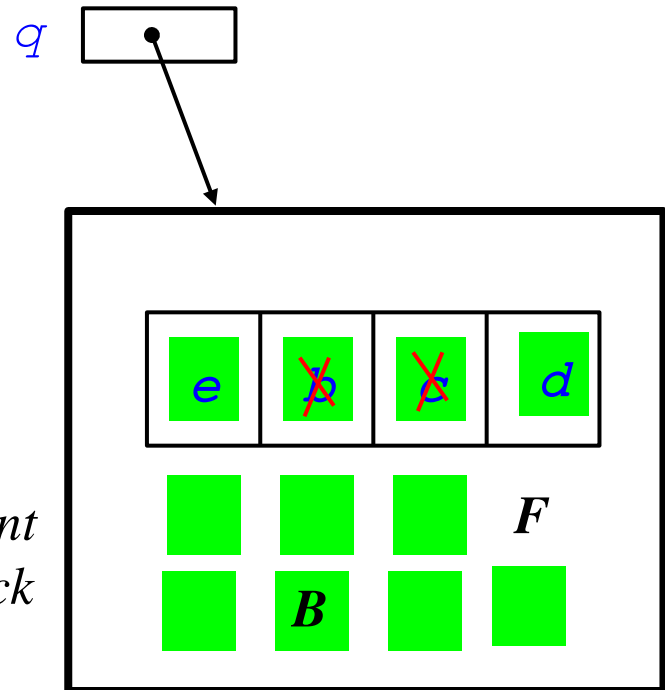
➔ `q.dequeue();`

➔ `q.dequeue();`

➔ `q.enqueue("d");`

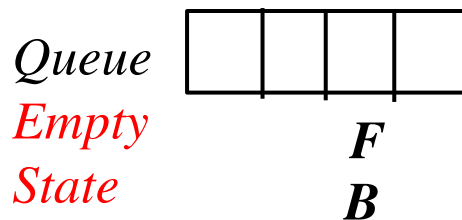
➔ `q.enqueue("e");`

➔ `q.dequeue();`



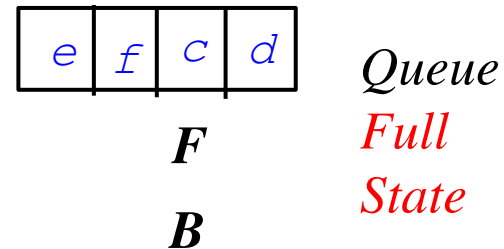
# Checking for Full/Empty State

What does  $(F==B)$  denote?



*size*

0
---

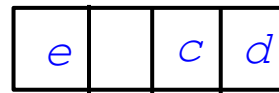


*size*

4
---

***Alternative - Leave a Deliberate Gap!***

*No need for size field.*



*Full Case :  $(\text{adv}(B) == F)$*

**B F**

# Code

```
class QueueAr implements Queue {  
  
    private Object [] arr;  
    private int front,back;  
    private int maxSize;  
    private final int initSize = 1000;  
    private final int increment = 1000;  
  
    public QueueAr()  
        {arr = new Object[initSize]; front = 0; back=0; }  
  
    public static Queue makeQueue() {return new QueueAr(); }  
  
    public boolean isEmpty()          // check if queue is empty  
        { return (front==back); }  
  
    private boolean isFull()         // check if queue overflows  
        { return (adv(back)==front); }  
}
```

# More code

```
public void enqueue(Object o) // add o to back of queue
{ if (this.isFull()) this.enlarge();
  arr[back]=o;
  back=adv(back); }

private void enlargeArr() // enlarge the array
{int newSize = maxSize+increment;
  Object [] barr = new Object[newSize];
  for (int j=0,k=front;j<=maxSize;j++,k=adv(k))
    {barr[j]=arr[k];};
  front=0; back=maxSize-1;
  maxSize = newSize; arr = barr;
}
```

# More code

```
public void dequeue() throws Underflow
{ if this.isEmpty()
    {throw new Underflow("dequeue fails - empty
q");}
    else front=adv(front);
}
```

```
public Object getFront() throws Underflow
{ if this.isEmpty()
    {throw new Underflow("getFront fails - empty
q");}
    else return arr[front];
}
```

# Summary

- The definition of the queue operations gives the ADT queue first-in, first-out (FIFO) behavior
- The queue can be implemented by linked lists or by arrays
- There are many applications
  - Printer queues,
  - Telecommunication queues,
  - Simulations,
  - Etc.