Late 29.5/2 = 14.75

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CS361

hw2- 1.3.5, 1.3.6, 1.5.4, 1.5.3

1.3.5

Let $f: A \to B$ show that the following relation R is an equivalence relation on A: $(a,b) \in R$ iff $f(a) = f(b)$

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equivalence = reflexive, transitive, and symmetric

relation R is a bisection which means that it's a one-to-one function and onto. Since R is a bisection, it has to be reflexive, transitive, and symmetric hence an equivalence.

1.3.6 Let $R \subseteq A \times A$ be a binary relation as defined below. In which cases is R a partial order? a total order?

a.) A = the positive ints; $(a,b) \in R$ iff b is divisible by a
partial order

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b.) $A = N \times N$; $((a,b),(c,d)) \in R$ iff $a \leq c$ or $b \leq d$
total

c.) $A = N$; $(a,b) \in R$ iff $b = a$ or $b = a+1$
total order

d.) A = all English words; $(a,b) \in R$ iff a is no longer than b
total order

e.) A = all English words; $(a,b) \in R$ if a is the same as b or occurs more frequently than b in the present work.
Poset

2 please give reasons......

1.5.2

Show that $n^4 - 4n^2$ is divisible by 3 for all $n \geq 0$

base case: $n = 0$ $\quad 0^4 - 4\,0^2 = 0$ $\quad$ 0 is divisible by 3

inductive hypothesis:

$k^4 - 4k^2 = 3r$ is divisible by 3 for $n \geq 0$

$$\underbrace{(k+1)^4}_{a^2} - \underbrace{4(k+1)^2}_{b^2} = \left[(k+1)^2 + 2(k+1)\right]\left[(k+1)^2 - 2(k+1)\right]$$

$$= k^4 + 4k^3 + 2k^2 - 4k - 3$$

$$= k^4 + 6k^2 - 4k^2 + 4k^3 - 4k - 3$$

$$= (k^4 - 4k^2) + 6k^2 + 4k^3 - 4k - 3 = 3r + 6k^2 + 4k^3 - 4k - 3$$

$$= 3(r-1) + 2k(2k^2 + 3k - 2)$$

$$= 3(r-1) + \underbrace{2k}_{a}\,\underbrace{(2k-1)}_{b}\,\underbrace{(k+2)}_{c}$$

for x to be divisible by 3 it has to be one of 3 cases:

$3S$

$3S+1$

$3S+2$

A) let $k = 3S$

$2(3S) = 6S$ ✓

B) let $k = 3S+2$

$2(3S+2) - 1$

$6S + 3$ ✓

C) let $k = 3S+1$

$3S+1+2$

$3S + 3$ ✓

since $3(r-1)$ is divisible by 3, and the product of $2k(2k-1)(k+2)$ has to be divisible by 3 as well, the inductive hypothesis is proven true which makes it hold for all $n \geq 0$

②

1.5.3.

Basic step: there is only one horse then clearly all horses are the same color

Induction Hypothesis: In any group of n horses, all horses are the same color

Induction Step: Consider a group of n+1 horses. Discard one horse; by the induction hypothesis, all the remaining horses have the same color. Now, put that horse back and discard another; again all the remaining horses have the same color. So all the horses have the same color as the ones that were discarded either time, so they all have the same color.

the induction step is incorrect. it proves that each individual horse is one color but makes no correlation to the previous one or the series. Further more colors are not quantifiable and induction cannot be applied to this case.

partially correct, you are on the right track.

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$p(n) = p(n+1)$