DSA - Seminar 6

- Deadline for "Project stage" – for projects without trees: today, until 11:59 PM

1. **Iterator for a SortedMap represented on a hash table, collision resolution with separate chaining.**

- Assume
  - We memorize only the keys from the Map
  - Keys are integer numbers

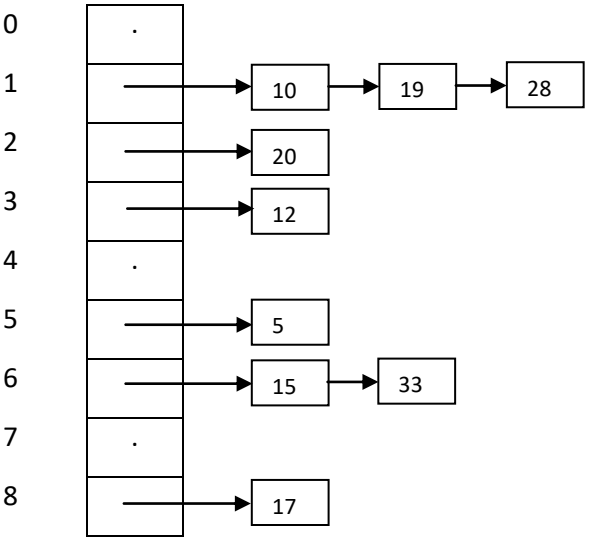
For ex:

- Keys from the map: 5, 28, 19, 15, 20, 33, 12, 17, 10 – Keys have to be unique!
- HT
  - m = 9
  - Hash function defined with the division method
    - h(k) = k mod m

k	5	28	19	15	20	33	12	17	10
h(k)	5	1	1	6	2	6	3	8	1

- h(k) can contain duplicates – they are called collisions

HD:



Iterator:

- If we iterate through the elements using the iterator, they should be visited in the following order: 5, 10, 12, 15, 17, 19, 20, 28, 33
- If we use the iterator -> complexity of the whole iteration to be Θ(n)

Representation:

TNode:
e: TElem // key, value
next: ↑TNode

SortedMap:
m: Integer
T : (↑TNode)[]
h: TFunction
R: relation

IteratorSortedMap:
sm: SortedMap
l: TList
currentNode: ↑TNode

```
subalgorithm init(it, sm):
      it.sm ← sm
      mergeLists (sm, it.l)
      it.currentNode ← it.l.head
end-subalgoritm
```

- mergeLists merges the separate linked lists:
  - first with the second, the result with the third, etc.
  - all lists using a binary heap
- Operations *valid, next, getCurrent* have a complexity of Θ(1)

Complexity of merging:

$$\left.\begin{matrix}\text{HT with m positions}\\ SortedMap\ with\ n\ elems\end{matrix}\right\} \Rightarrow average\ number\ of\ elems\ in\ a\ list: \frac{n}{m} = \alpha\ (load\ factor)$$

Merge the first list with the second, the result with the third, etc.

- list1 + list2 => list12 => α + α = 2α
- list12 + list3 => list123 => 2α + α = 3α
- list123 + list4 => list1234 => 3 α + α = 4 α
- …

Total merging: $2\alpha + 3\alpha + \cdots + m\alpha \approx \left.\begin{matrix}\frac{m*(m+1)}{2}\alpha\\ \alpha = \frac{n}{m}\end{matrix}\right\} \rightarrow \frac{m\,(m+1)}{2}\,\frac{n}{m} \Rightarrow \in \theta\,(n*m)$

All lists using a binary heap:

- Add from each list the first node to the heap
- Remove the minimum from the heap, and add to the heap the next of the node (if exists)
- The heap will contain at most k elements at any given time (k is the number of the listst, $1 \leq k \leq$ m) => height of the heap is $O(\log_2 k)$

- Merge complexity:
  - $O(n \log_2 k)$, if k > 1
  - Θ(n), if k = 1

2. Map – reprezentation on a hash table – collision resolution with coalesced chaining

- Assume:
  - We memorize only the keys
  - The keys are integer numbers

For ex:

- 5, 18, 16, 15, 13, 31, 26
- HT:
  - m = 13
  - Hash function defined with the division method

k	5	18	16	15	13	31	26
h(k)	5	5	3	2	0	5	0

	0	1	2	3	4	5	6	7	8	9	10	11	12
t	18	13	15	16	31	5	26						
next	~~-1~~ 1	~~-1~~ 4	-1	-1	~~-1~~ 6	~~-1~~ 0	-1	-1	-1	-1	-1	-1	-1

firstFree = ~~0~~ ~~1~~ ~~4~~ ~~6~~ 7

- firstFree is considered to be the first empty position from left to right (empty positions are no longer linked)
- One „linked list" can contain elements belonging to different collisions: for ex. the list starting at position 5: 5 (5) – 18 (5) – 13 (0) – 31 (5) – 26 (0)

Reprezentation:

TElem:
k: TKey
v: TValue

Map:
m: Integer
t: TElem[]
next:Integer[]
firstFree: Integer
h: TFunction

```
subalgorithm init (map):
      @ initialize the hash function
      @ initialize the value of m
      for i ← 0, m-1 execute
            map.t[i] ← -1
            map.next[i] ← -1
      end-for
      map.firstFree ← 0
end-subalgorithm
Complexity: Θ(m)
```

```
Function search(map, k):
// for simplicity we return the position where the key was found, or -1
// in case of a real map, you return the value associated to the key
      i ← map.h(k)
      while (i ≠ -1 and map.t[i] ≠ k) execute
            i ← map.next[i]
      end-while
      if i = -1 then
            search ← -1
      else
            search ← i
end-function
Complexity: O(m) in worst case, but on average Θ(1)

subalgorithm insert – discussed in Lecture 10
```

Remove: remove key 5

- **Problem:** we might lose links to other elements
- Cannot just do a remove like in case of a linked list on array, because not every element can be at any position in the table. No element can be "before" (considering the links) the position to which it hashes. For example, we cannot move 26 to replace 5 (because 26 hashes to 0, and a search starting from position 0 does not go through position 5).
-

	0	1	2	3	4	5	6	7	8	9	10	11	12
T	~~18~~ 13	13	15	16	31	~~5~~ 18	26						
Next	~~1~~ 4	~~4~~ -1	-1	-1	6	0	-1	-1	-1	-1	-1	-1	-1

firstFree: 7

Steps:

1. We cannot just set t[5] = -1 and next[5] = -1 because we lose the link to 18 (and a search for 18 or 31 will not find these elements)
2. Search for elements (following the links) that hash to the position from which I am removing an elements (position 5 in our example)
    a. If no element is found, we remove the element as we remove an element from a singly linked list on array.
    b. If an element is found, it is moved to the position where we delete from, and the process of removal is repeated with the position from which we moved the element.

- Remove key 5, which is at position 5
- Search for the first key that hashes to position 5 => 18
- move 18 on position 5
- Now we want to remove key 18, which is at position 0
- Search for the first key that hashes to position 0 => 13
- move 13 on position 0
- Now we want to remove key 13, which is at position 1
- Search for the first key that hashes to position 1 => no such key
- Remove key 13,modifying the links

```
subalgorithm remove(map, k) is
    i ← map.h(k)
    j ← -1 {previous of i, when we want to remove node from pos i, we need
its previous node}
    {parse the table to check if i has any previous element}
    k ← 0
    while (k < map.m and j = -1) execute
        if map.next[k] = i then
            j ← k
        end-if
    end-while
    {find the key to be removed. Set its previous as well}
    while i ≠ -1 and map.t[i] ≠ c execute
        j ← i
        i ← map.next[i]
    end-while
    if i= -1 then
        @key does not exist
    else
        {find another key that hashes to i}
        over ← false {becomes true when nothing hashes to i}
        repeat
            p ← map.next[i] {first position to be checked}
            pp ← i {previous of p}
            while p ≠ -1 and map.h(map.t[p]) ≠ i execute
                pp ← p
                p ← map.next[p]
            end-while
            if p = -1 then
                over ← true
            else
                map.t[i] ← map.t[p]
                j ← pp
                i ← p
            end-if
        until over
        {remove key from position i}
        if j ≠ -1 then
            map.next[j] ← map.next[i]
        end-if
        map.t[i] ← -1
        map.next[i] ← -1
        if map.firstFree > i then
            map.firstFree ← i
        end-if
    end-if
end-subalgorithm
```