

```
#Evaluate the following
```

```
# a.
x=7503
y=81
print (x % y)

51
```

```
# b.
x=-7503
y=81
print (x % y)

30
```

```
#c.
x=81
y=7503
print (x % y)
```

 81

```
#d
x=-81
y=7503
print (x % y)
```

7422

```
#### Exercie 2.5
```

```
###Use exhaustive key search to decrypt the following ciphertext, which was encrypted using a Shift Cipher:
```

```
# BEEAKFYDJXUQYHYJIQRYHTYJIFBQDUYJIIKFUHCQD.
```

```
# Let's define the characters that we have in our alphabets and assumption is we are first changing into lowercase
```

```
letters = "abcdefghijklmnopqrstuvwxyz"
```

```
# Initialize variable for encrypted text
```

```
ciphertext = "BEEAKFYDJXUQYHYJIQRYHTYJIFBQDUYJIIKFUHCQD"
```

```
# Initialize integer for loop execution
```

```
x = 0
```

```
# Enter in the loop
```

```
for a in range(1,26):
```

```
    x = x+1
```

```
    to_decrypt=ciphertext
```

```
    alphabets = letters.upper()
```

```
    to_decrypt=to_decrypt.upper()
```

```
    shift=int(x)
```

```
    decrypted=""
```

```
    for character in to_decrypt:
```

```
        position = alphabets.find(character)
```

```
        newposition = position-shift
```

```
        if character in alphabets:
```

```
            decrypted = decrypted + alphabets[newposition]
```

```
        else:
```

```
            decrypted = decrypted + character
```

```
    shift =str(shift)
```

```
    print(" Key="+shift )
```

```
    print("Plain text:")
```

```
    print(decrypted)
```

```
    print("#####\n")
```

```
    Key=1
```

```
    Plain text:
```

```
    ADDZJEXCIWTPXGXIHPPXGSIHPEAPCTXIHHJETGBPC
```

```
    #####
```

```
    Key=2
```

```
    Plain text:
```

```
    ZCCYIDWBHVSOWFWHGOWFRWHGODZOBZWHGGIDSFAOB
```

```
    #####
```

```

Key=3
Plain text:
YBBXHCVAGURNVEVGFNOVEQVGFNCYNARVGFHCREZNA
#####

```

```

Key=4
Plain text:
XAAWGBUZFTQMUDUFEMNUDPUFEMBMZQUFEEGBQDYMZ
#####

```

```

Key=5
Plain text:
WZZVFATYESPLTCTEDLMTCTEDLAWLYPTEDDFAPCXLY
#####

```

```

Key=6
Plain text:
VYYUEZSXDROKSBSDCCKLSBNSDCKZVKXOSDCCEZOBWKX
#####

```

```

Key=7
Plain text:
UXXTDYRWQCNJRARCBJKRAMRCBJYUJWNRCBBDYNAVJW
#####

```

```

Key=8
Plain text:
TWWSXCQVBPMIQZQBAlJQZLQBAlXTIVMQBAACXMZUIV
#####

```

```

Key=9
Plain text:
SVVRBWPUAOLHPYPAPYKPAZHWSHULPAZZBWLYTHU
#####

```

```

Key=10
Plain text:
RUUQAVOTZNGOXOZYGHGXJOZYGVGRGKZYYAVKXSGT
#####

```

```

Key=11
Plain text:
QTPZUNSYMJFNWNYXFGNWINYXFUQFSJNYXXZUJWRFS
#####

```

```

Key=12
Plain text:
RSCQVTPMXLTFMVMUEFMVMUEFTREPTMVLNLTUQER
#####

```

##### 2.8 List all the invertible elements in  $Z_m$  for  $m = 28, 33$ , and  $35$ .

```

# When m=28
import math
m=28
for a in range (1,m):
    gcd =math.gcd(a,m) # we are checking if there is common divisor or not
    if gcd==1:
        print(a)

```

```

1
3
5
9
11
13
15
17
19
23
25
27

```

```

# When m=33
import math
m=33
for a in range (1,m):
    gcd =math.gcd(a,m)
    if gcd==1:
        print(a)

```

```

1
2

```

```

4
5
7
8
10
13
14
16
17
19
20
23
25
26
28
29
31
32

```

```

# When m=35
import math
m=35
for a in range (1,m):
    gcd =math.gcd(a,m)
    if gcd==1:
        print(a)

```

```

1
2
3
4
6
8
9
11
12
13
16
17
18
19
22
23
24
26
27
29
31
32
33
34

```

```
# 2.9 For  $1 \leq a \leq 28$ , determine  $a^{-1} \pmod{29}$  by trial and error
```

```
# we can find this by finding out relatively prime
```

```

m = 29
for a in range(1, 29):
    i = 1
    while (a * i) % m != 1: # We are checking if the remainder is 1 which means if it is perfectly divisible or not
        i += 1
    print(" When a = " +str(a) +" value is "+ str(i))

```

```

When a = 1 value is 1
When a = 2 value is 15
When a = 3 value is 10
When a = 4 value is 22
When a = 5 value is 6
When a = 6 value is 5
When a = 7 value is 25
When a = 8 value is 11
When a = 9 value is 13
When a = 10 value is 3
When a = 11 value is 8
When a = 12 value is 27
When a = 13 value is 9
When a = 14 value is 20
When a = 15 value is 2
When a = 16 value is 20

```

```
When a = 17 value is 12
When a = 18 value is 21
When a = 19 value is 26
When a = 20 value is 16
When a = 21 value is 18
When a = 22 value is 4
When a = 23 value is 24
When a = 24 value is 23
When a = 25 value is 7
When a = 26 value is 19
When a = 27 value is 14
When a = 28 value is 28
```

2.3 Prove that  $a \bmod m = b \bmod m$  if and only if  $a \equiv b \pmod{m}$ .

Here we need to prove that  $a \bmod m = b \bmod m$

Given,  $a \equiv b \pmod{m}$  [congruency] or,  $m \mid (a-b)$  or,  $(a-b) = m(x_1 - x_2)$  [  $x_1$  and  $x_2$  are integers] or,  $a - b = x_1m - x_2m$   
or,  $a = x_1m + r$  and  $b = x_2m + r$  [they will have common remainder. Adding common remainder  $r$ ] which implies,  $a \bmod m = b \bmod m$

Proved//

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