There are 7 questions in this paper. You need to answer 2 questions out of the question numbers 1, 2 & 3 and 3 questions from question numbers 4, 5, 6 & 7.
(Each question carries 20 marks)
Time-2hrs
Use Jupiter Notebook to write answers and upload the answer sheet as .html format.
Calculators are not allowed.

1)

a) Consider the following two matrices for the sub question.

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 56 \\ 1 & 34 & 4 & 1 & 4 \\ 5 & 1 & 4 & 56 & 2 \\ 4 & 2 & 4 & 5 & 67 \end{pmatrix}$$

i) Print the matrix.
ii) Print 1^{st} row.
iii) Print 5^{th} column.
iv) Print the 3^{rd} row 4^{th} column element from the above matrix.
v) Find the order of the matrix.

b) Write the property or properties of matrix subtraction.

c) Find the final result.

i)
$$\begin{pmatrix} 1 & 2 \\ 3 & 1 \\ 3 & 1 \end{pmatrix} \cdot \begin{pmatrix} 4 & 1 & 4 & 2 \\ 1 & 3 & 1 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 23 & 1 & 31 \\ 43 & 1 & 4 & 1 \\ 13 & 1 & 13 & 1 \end{pmatrix}$$

ii)
$$\begin{pmatrix} 1 & 3 \\ 4 & 2 \\ 2 & 4 \end{pmatrix} + 4 \cdot \left(\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \end{pmatrix} + transpose \begin{pmatrix} 1 & 2 & 3 \\ 4 & 3 & 2 \end{pmatrix} \right)$$

2)

a) Find the final outputs of the following.

i)
$$det\left(\begin{pmatrix} 1 & 2 & 3 \\ 4 & 1 & 3 \\ 1 & 2 & 3 \\ 1 & 4 & 3 \end{pmatrix} \cdot \begin{pmatrix} 5 & 3 & 5 & 2 \\ 2 & 5 & 6 & 3 \\ 2 & 5 & 5 & 7 \end{pmatrix} \right)$$

ii) $$det\left(\begin{pmatrix} 12 & 2 \\ 1 & 2 \\ 2 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 & 3 & 1 \\ 3 & 1 & 4 \end{pmatrix} + 6 \cdot transpose\begin{pmatrix} 1 & 7 & 8 \\ 6 & 7 & 5 \\ 7 & 4 & 7 \end{pmatrix}\right)$$

iii) $$\left(det\begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 4 \\ 7 & 5 & 4 \end{pmatrix}\right) \cdot \left(\begin{pmatrix} 1 & 3 \\ 1 & 7 \\ 5 & 89 \end{pmatrix} \cdot \begin{pmatrix} 5 & 4 & 8 & 0 \\ 5 & 75 & 43 & 4 \end{pmatrix}\right)$$

b) Find the final outputs of the following.

i) $$transpose\left(\begin{pmatrix} 1 & 9 & 2 \\ 4 & 5 & 4 \\ 2 & 6 & 8 \end{pmatrix} \cdot \begin{pmatrix} 2 & 4 \\ 6 & 9 \\ 0 & 6 \end{pmatrix}\right) + \begin{pmatrix} 1 & 4 & 21 \\ 35 & 6 & 6 \end{pmatrix}$$

ii) $$3 \cdot transpose\left(\begin{pmatrix} 1 & 3 \\ 2 & 98 \end{pmatrix} \cdot \begin{pmatrix} 1 & 43 & 6 & 8 \\ 4 & 6 & 8 & 6 \end{pmatrix}\right) + 3 \cdot \begin{pmatrix} 1 & 3 \\ 6 & 8 \\ 0 & 8 \\ 7 & 5 \end{pmatrix}$$

iii) $$transpose\left(transpose\begin{pmatrix} 1 & 2 & 3 \\ 6 & 7 & 8 \end{pmatrix} \cdot \begin{pmatrix} 1 & 8 \\ 9 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 3 \\ 5 & 7 \\ 9 & 6 \end{pmatrix}\right)$$

3)

a) Find the inverse of the following matrices.

i) $$inverse\left(\begin{pmatrix} 2 & 3 & 5 \\ 1 & 4 & 6 \end{pmatrix} \cdot transpose\begin{pmatrix} 2 & 4 & 7 \\ 4 & 5 & 8 \end{pmatrix}\right)$$

ii) $$transpose\left(inverse\begin{pmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \\ 1 & 2 & 0 \end{pmatrix} + 7 \cdot \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix} \cdot \begin{pmatrix} 3 & 2 & 1 \\ 4 & 3 & 2 \end{pmatrix}\right)$$

iii)

$$inverse\left(\begin{pmatrix} 1 & 3 \\ 4 & 5 \end{pmatrix} + 6 \cdot inverse\begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix}\right)$$

b) Find the values of the unknown terms in the given linear equation system using the both methods.

i) 2x+3y = 13
y+z = 7
x+y-4z = -11

ii) x+y = 3
z+x = 4
x-3z+3y = -2

4) Prove following first order differentiation problems.

I.

$$\frac{d}{dx}(\log(4x^x) + 2\sin(\tan(x))) = 2\left(\tan^2(x) + 1\right)\cos(\tan(x)) + \log(x) + 1$$

II. Derivative of $\sin(xy(x)) = \cos^{\cos(x)+\tan(y(x))}(x)$ is

$$\frac{y(x)\cos(x)\cos(xy(x)) + \log(\cos(x))\sin(x)\cos^{\cos(x)+\tan(y(x))+1}(x) + \sin(x)\cos^{\cos(x)+\tan(y(x))}(x)\tan(y(x)) + \sin(x)\cos^{\cos(x)+\tan(y(x))+1}(x)}{\left(-x\cos(xy(x)) + \log(\cos(x))\cos^{\cos(x)+\tan(y(x))}(x)\tan^2(y(x)) + \log(\cos(x))\cos^{\cos(x)+\tan(y(x))}(x)\right)\cos(x)}$$

III.

$$\frac{d}{dx}(\log(\tan(x)) + \sin(\sin(\tan(x))))\Big|_{x=1} = \left(1 + \tan^2(1)\right)\cos(\sin(\tan(1)))\cos(\tan(1)) + \frac{1 + \tan^2(1)}{\tan(1)}$$

IV. $xy(x) + y(x)e^x = x^{y(x)e^x}$ at x = 1 is $\frac{d}{dx}y(x)\Big|_{x=1} = -\frac{1}{(1+e)^2}$

V. Prove $\frac{\partial}{\partial x}5x^2\sin(y) = 10x\sin(y)$

VI. Prove that partial differentiation of $y f(x, y) = x y^2 + \sin(y)$ with respect to x gives,

$$y \frac{\partial}{\partial x} f(x, y) = y^2$$

5.

I. Plot the function y = cos(x) in the range (-1,5)

II. Find optimum points for the above function (Hint: 2 points)

III. Find global maximum point for the above function.

6.

I. Prove analytically integration of

$$\int \left(x^3 \sin(x) + e^x \cos(x)\right) dx = -x^3 \cos(x) + 3x^2 \sin(x) + 6x \cos(x) + \frac{e^x \sin(x)}{2} + \frac{e^x \cos(x)}{2} - 6 \sin(x)$$

II. Prove analytical integration of

$$\int_1^5 \left(x^3 e^x + x^3 \cos(x) + \sin(x)\right) dx = 95 \sin(5) + 4 \cos(1) + 5 \sin(1) + 2e + 68 \cos(5) + 74 e^5$$

III. Prove numerical integration of

$$\int_1^5 \left(x^3 \cos(x) + x^2 e^x + \sin(x) \cos(x)\right) dx = \left(2454.71430063408,\ 2.7252803354406 \cdot 10^{-11}\right)$$

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Intergal is 2454.714300634084
Error is 2.7252803354405985e-11
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IV. Prove $\frac{y(x) \frac{d}{dx} y(x)}{x} = e^x$ ODE has two solutions of

$$\left[y(x) = -\sqrt{2}\sqrt{C_1 + x e^x - e^x},\ y(x) = \sqrt{2}\sqrt{C_1 + x e^x - e^x}\right]$$

where C_1 & C_2 are arbitrary constants.

V. Prove $y(x)\frac{d}{dx}y(x) = \tan(x)$ ODE has two solutions that passes through (0,0) point of

$$\left[y(x) = -\sqrt{2}\sqrt{-\log(\cos(x))},\ y(x) = \sqrt{2}\sqrt{-\log(\cos(x))}\right]$$

7.

A company makes two products (X and Y) using two machines (A and B). Each unit of X that is produced requires 50 minutes processing time on machine A and 30 minutes processing time on machine B. Each unit of Y that is produced requires 24 minutes of processing time on machine A and 33 minutes of processing time on machine B.

At the start of the current week there are 30 units of X and 90 units of Y in stock. Available processing time on machine A is forecast to be 40 hours and on machine B is forecast to be 35 hours.

The demand for X in the current week is forecasted to be 75 units and for Y is forecasted to be 95 units. Company policy is to maximize the combined sum of the units of X and the units of Y in stock at the end of the week.

Formulate the problem of deciding how much of each product to make in the current week as a linear program.

Solve this linear program.