Deep Learning

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https://github.com/safayani/deep_learning_course

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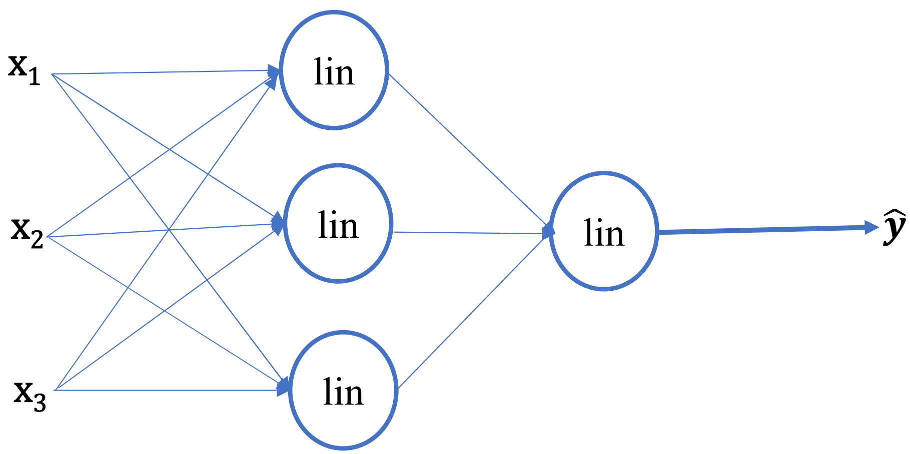
Activation function

Non-linear activation function



- چرا به توابع فعال‌ساز غیرخطی نیاز داریم؟

- با فرض تابع فعال‌ساز خطی

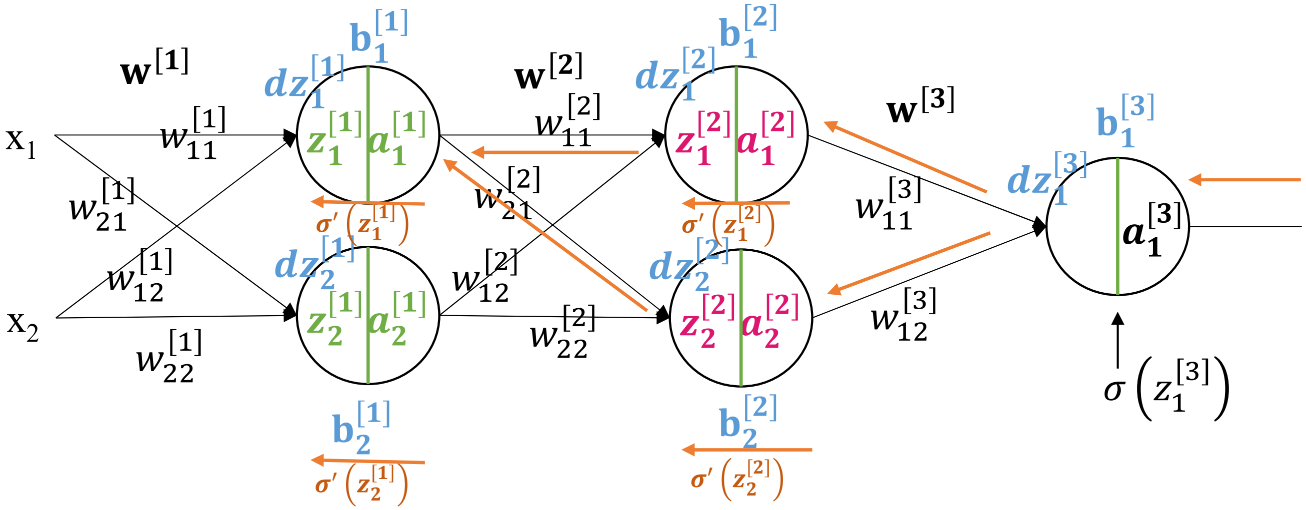
- $z^{[1]} = w^{[1]}x + b^{[1]}$
- $a^{[1]} = g\left(z^{[1]}\right) = z^{[1]}$
- $z^{[2]} = w^{[2]}a^{[1]} + b^{[2]}$
- $a^{[2]} = g(z^{2}) = z^{[2]}$

$$a^{[1]} = z^{[1]} = w^{[1]}x + b^{[1]}$$
$$a^{[2]} = z^{[2]} = w^{[2]}a^{[1]} + b^{[2]}$$
$$= w^{[2]}(w^{[1]}x + b^{[1]}) + b^{[2]}$$
$$= \underbrace{w^{[2]}w^{[1]}}_{w'}x + \underbrace{w^{[2]}b^{[1]} + b^{[2]}}_{b'}$$
$$= w'x + b'$$

Derivatives of activation functions

- Sigmoid:
- $g(z) = \frac{1}{1+e^{-z}}$ $\quad g'(z) = \frac{dg(z)}{dz} = \frac{1}{1+e^{-z}}\left(1 - \frac{1}{1+e^{-z}}\right) = g(z)(1 - g(z))$
- Tanh:
- $g(z) = \frac{e^{z}-e^{-z}}{e^{z}+e^{-z}}$ $\quad g'(z) = 1 - \tanh^2(z) = 1 - g^2(z)$
- Relu:
- $g(z) = \max(0, z) = \begin{cases} z & z \geq 0 \\ 0 & z < 0 \end{cases}$ $\quad g'(z) = \begin{cases} 1 & if\ z \geq 0 \\ 0 & if\ z < 0 \end{cases}$
- Leaky Relu:
- $g(z) = \max(z, 0.01z) = \begin{cases} z & z \geq 0 \\ 0.01z & z < 0 \end{cases}$ $\quad g'(z) = \begin{cases} 1 & if\ z \geq 0 \\ 0.01 & if\ z < 0 \end{cases}$

Back propagation(Bp)



- پس‌انتشار

$\mathbf{w}_{\mathbf{ij}}^{[l]}$

$$\frac{\boldsymbol{dL}}{\boldsymbol{dw_{ij}^{l}}} \quad \text{for } \; l = 1 \ldots L$$

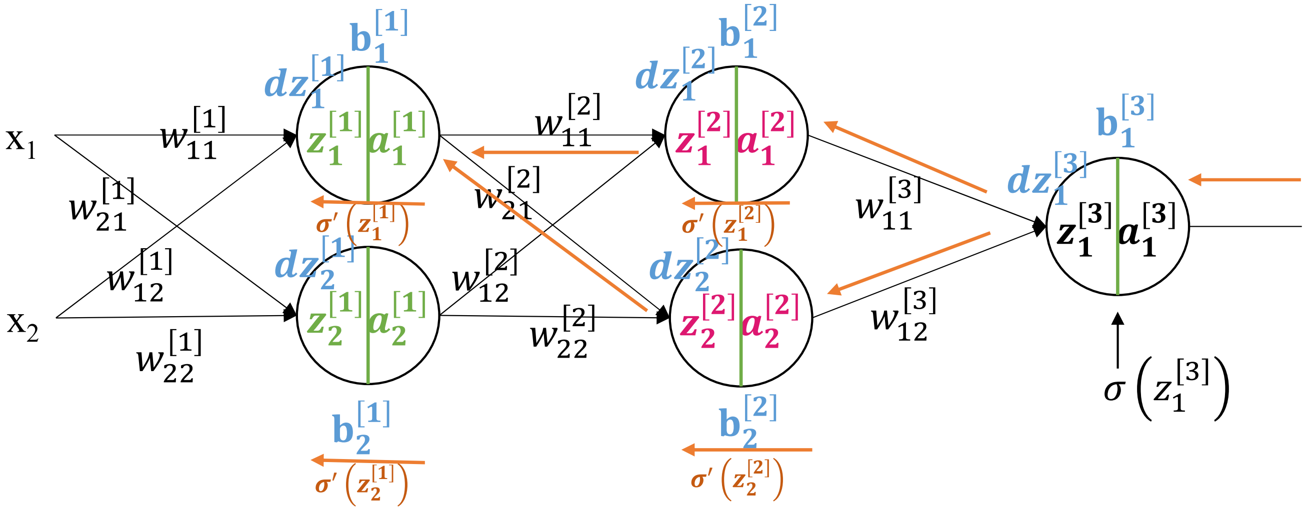
for each i,j

$$\mathbf{w}_{\mathbf{ij}}^{[l]} = \mathbf{w}_{\mathbf{ij}}^{[l]} - \boldsymbol{\alpha} \frac{\boldsymbol{dL}}{\boldsymbol{dw_{ij}^{l}}}$$

- $dw_{11}^{[2]} = \dfrac{L\left(\ldots, w_{11}^{[2]} + \varepsilon, \ldots\right) - L\left(\ldots, w_{11}^{[2]} - \varepsilon, \ldots\right)}{2\varepsilon}$

Back propagation(Bp)

- پس‌انتشار



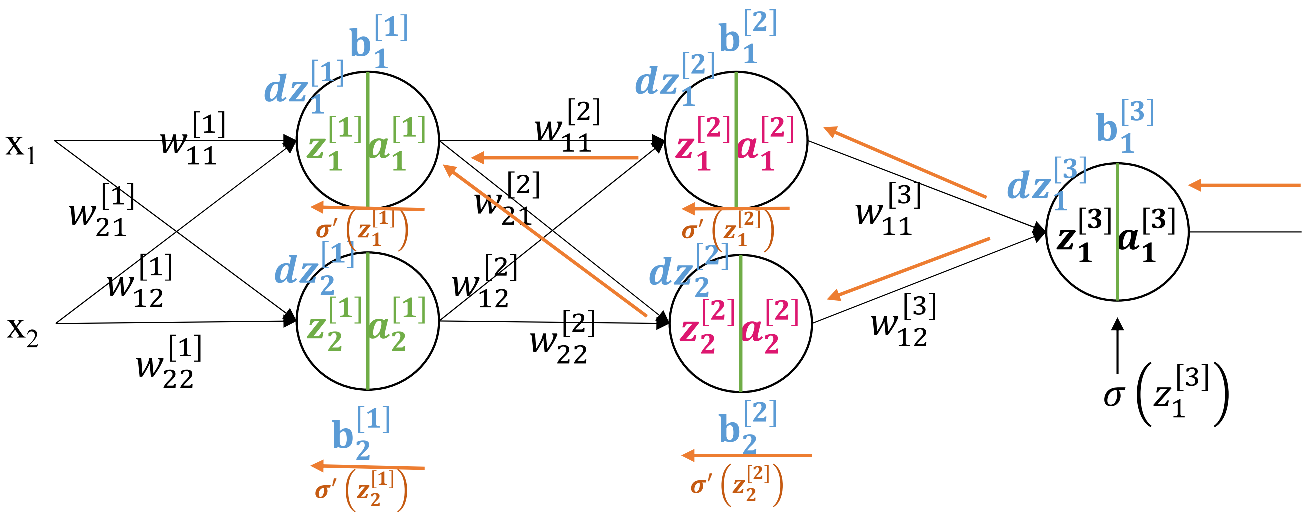
$$\frac{dL}{dw_{11}^{[3]}} = \underbrace{\frac{dL}{da_1^{[3]}} \times \frac{da_1^{[3]}}{dz_1^{[3]}}}_{\frac{dL}{dz_1^{[3]}} = dz_1^{[3]}} \times \frac{dz_1^{[3]}}{dw_{11}^{[3]}} = \left(a_1^{[3]} - y\right) a_1^{[2]}$$

$$z_1^{[3]} = a_1^{[2]} w_{11}^{[3]} + a_2^{[2]} w_{12}^{[3]} + b_1^{[3]}$$

$$\frac{dL}{db_1^{[3]}} = \frac{dL}{dz_1^{[3]}} \times \underbrace{\frac{dz_1^{[3]}}{db_1^{[3]}}}_{1} = dz_1^{[3]} \times 1 = dz_1^{[3]}$$

Back propagation(Bp)

- پس‌انتشار



$$dz_1^{[2]} = \frac{dL}{dz_1^{[2]}} = \underbrace{\frac{dL}{dz_1^{[3]}}}_{dz_1^{[3]}} \times \underbrace{\frac{dz_1^{[3]}}{da_1^{[2]}}}_{w_{11}^{[3]}} \times \underbrace{\frac{da_1^{[2]}}{dz_1^{[2]}}}_{\sigma'\left(z_1^{[2]}\right)} \qquad a_1^{[2]} = \sigma\left(z_1^{[2]}\right) \qquad \frac{da_1^{[2]}}{dz_1^{[2]}} = \sigma'\left(z_1^{[2]}\right)$$

$$dz_1^{[2]} = dz_1^{[3]} \times w_{11}^{[3]} \times \sigma'\left(z_1^{[2]}\right)$$

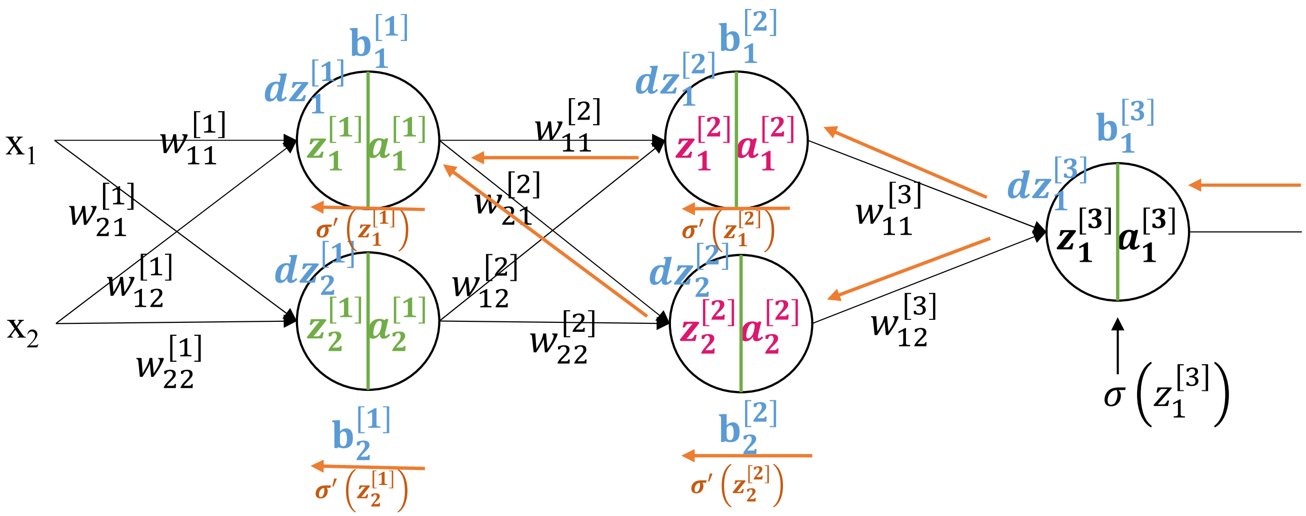
$$dz_2^{[2]} = dz_1^{[3]} \times w_{12}^{[3]} \times \sigma'\left(z_2^{[2]}\right)$$

$$dw_{11}^{[2]} = dz_1^{[2]} \times a_1^{[1]} \qquad db_1^{[2]} = dz_1^{[2]}$$

$$dw_{12}^{[2]} = dz_1^{[2]} \times a_2^{[1]} \qquad db_2^{[2]} = dz_2^{[2]}$$

Back propagation(Bp)

- پس‌انتشار



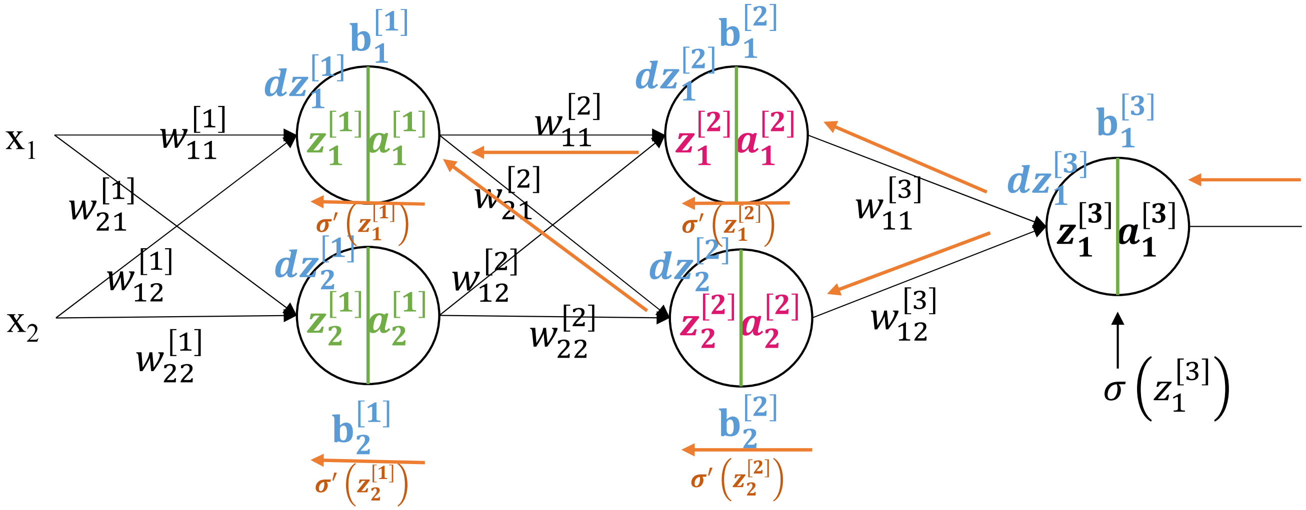
$$dz_1^{[1]} = \frac{dL}{dz_1^{[1]}} = \underbrace{\frac{dL}{dz_1^{[2]}}}_{dz_1^{[2]}} \times \underbrace{\frac{dz_1^{[2]}}{da_1^{[1]}}}_{w_{11}^{[2]}} \times \underbrace{\frac{da_1^{[1]}}{dz_1^{[1]}}}_{\sigma'\left(z_1^{[1]}\right)} + \underbrace{\frac{dL}{dz_2^{[2]}}}_{dz_2^{[2]}} \times \underbrace{\frac{dz_2^{[2]}}{da_1^{[1]}}}_{w_{21}^{[2]}} \times \underbrace{\frac{da_1^{[1]}}{dz_1^{[1]}}}_{\sigma'\left(z_1^{[1]}\right)}$$

$$dz_1^{[1]} = (dz_1^{[2]} w_{11}^{[2]} + dz_2^{[2]} w_{21}^{[2]})\, \sigma'\left(z_1^{[1]}\right) \qquad db_1^{[1]} = dz_1^{[1]}$$

$$dz_2^{[1]} = (dz_1^{[2]} w_{12}^{[2]} + dz_2^{[2]} w_{22}^{[2]})\, \sigma'\left(z_2^{[1]}\right) \qquad db_2^{[1]} = dz_2^{[1]}$$

Back propagation(Bp)

• پس‌انتشار



$$dw_{11}^{[1]} = dz_1^{[1]} \times x_1$$

$$dw_{21}^{[1]} = dz_2^{[1]} \times x_1$$

$$dw_{12}^{[1]} = dz_1^{[1]} \times x_2$$

$$dw_{22}^{[1]} = dz_2^{[1]} \times x_2$$

Gradient descent

Gradient descent

- Parameters: $\underbrace{w^{[1]}}_{(n^{[1]}, n^{[0]})}$, $\underbrace{b^{[1]}}_{(n^{[1]}, 1)}$, $\underbrace{w^{[2]}}_{(n^{[2]}, n^{[1]})}$, $\underbrace{b^{[2]}}_{(n^{[2]}, 1)}$
- $w^{[1]}, b^{[1]}, w^{[2]}, b^{[2]} \leftarrow$ random initialization
- Repeat{

  →Forward propagation to compute $\hat{y}^{(i)}$ i=1, …, m

  Backward prop

  $\rightarrow dw^{[1]} = \frac{dJ}{dw^{[1]}}, db^{[1]}, dw^{[2]}, db^{[2]}$

  $$\text{Update}\begin{cases} w^{[1]} = w^{[1]} - \alpha\, dw^{[1]} \\ w^{[2]} = w^{[2]} - \alpha\, dw^{[2]} \\ b^{[1]} = b^{[1]} - \alpha\, db^{[1]} \\ b^{[2]} = b^{[2]} - \alpha\, db^{[2]} \end{cases}$$

  }Until Convergence

Gradient descent

- *Forward propagation:*

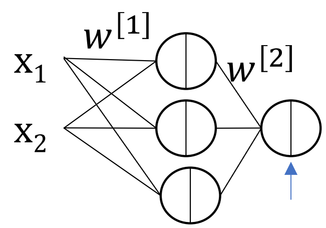
$$z^{[1]} = w^{[1]}X + b^{[1]}$$

$$A^{[1]} = g^{[1]}(z^{[1]})$$

$$z^{[2]} = w^{[2]}A^{[1]} + b^{[2]}$$

$$A^{[2]} = g^{[2]}(z^{[2]})$$

- *Back prop. :*



$$dz^{[2]} = A^{[2]}{}_{1\times m} - Y_{1\times m}$$

$$\underbrace{dw^{[2]}}_{1\times n^{[1]}} = \frac{1}{m}\underbrace{dz^{[2]}}_{1\times m}\underbrace{A^{[1]T}}_{m\times n^{[1]}}$$

$$\underbrace{db^{[2]}}_{1\times 1} = \frac{1}{m} np \cdot sum\left(\underbrace{dz^{[2]}}_{1\times m}, axis = 1\right) (1,1)$$

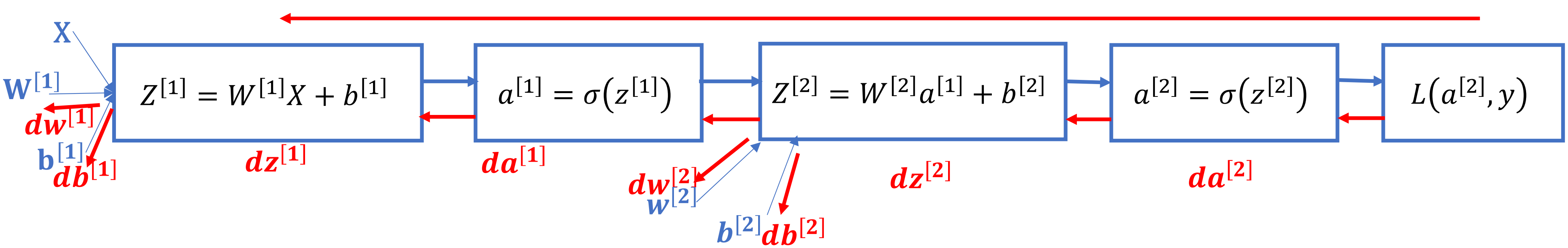
Gradient descent

- $\underbrace{dz^{[1]}}_{\mathrm{n}^{[1]}\times m} = \underbrace{w^{[2]T}}_{\mathrm{n}^{[1]}\times 1}\underbrace{dz^{[2]}}_{1\times m} * \underbrace{g^{[1]'}(\underbrace{z^{[1]}}_{\mathrm{n}^{[1]}\times m})}_{\mathrm{n}^{[1]}\times m}$

  Element wise

- $\underbrace{dw^{[1]}}_{\mathrm{n}^{[1]}\times \mathrm{n}^{[0]}} = \frac{1}{m}\underbrace{dz^{[1]}}_{\mathrm{n}^{[1]}\times m}\underbrace{X^T}_{m\times \mathrm{n}^{[0]}}$

- $\underbrace{db^{[1]}}_{\mathrm{n}^{[1]}\times 1} = \frac{1}{m}np\cdot sum\left(\underbrace{dz^{[1]}}_{\mathrm{n}^{[1]}\times m}, axis = 1, keepdims = True\right)$ $(\mathrm{n}^{[1]}, 1)$



Gradient descent

- $dz^{[1]} = w^{[2]T}\, dz^{[2]} * g^{[1]'}(z^{[1]})$ m=1
- $dz^{[2]} = a^{[2]} - y$
- $dw^{[1]} = dz^{[1]} X^T$
- $dw^{[2]} = dz^{[2]} {a^{[1]}}^T$
- $db^{[1]} = dz^{[1]}$
- $db^{[2]} = dz^{[2]}$

Summary of gradient descent

- $dz^{[2]} = A^{[2]} - Y$
- $dw^{[2]} = \frac{1}{m} dz^{[2]} A^{[1]T}$
- $db^{[2]} = \frac{1}{m} np \cdot sum\left(dz^{[2]}, axis = 1, keepdims = True\right)$
- $dz^{[1]} = w^{[2]T} dz^{[2]} * g^{[1]'}(z^{[1]})$
- $dw^{[1]} = \frac{1}{m} dz^{[1]} X^T$
- $db^{[1]} = \frac{1}{m} np \cdot sum\left(dz^{[1]}, axis = 1, keepdims = True\right)$