Deep Learning

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https://www.aparat.com/mehran.safayani

https://github.com/safayani/deep_learning_course

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Supervised Learning

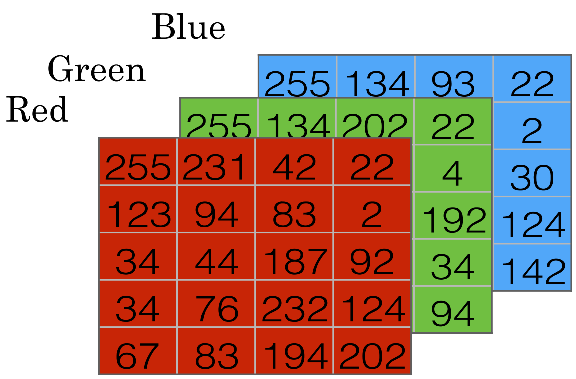
Input(x)	Output (y)	Application
Home features	Price	Real Estate
Ad, user info	Click on ad? (0/1)	Online Advertising
Image	Object (1,…,1000)	Photo tagging
Audio	Text transcript	Speech recognition
English	Chinese	Machine translation
Image, Radar info	Position of other cars	Autonomous driving

Basics of Neural Network Programming

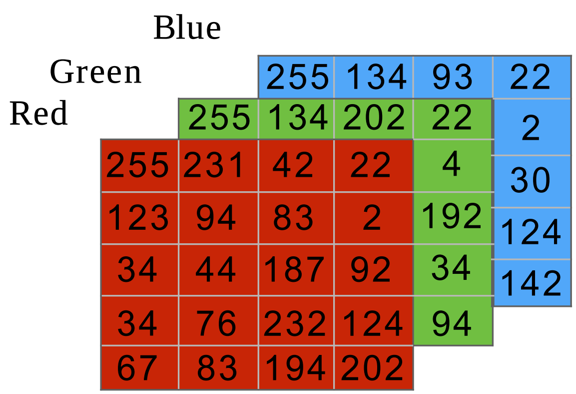
Binary Classification

Binary Classification

→ 1 (cat) vs 0 (non cat)



Binary classification



$$\vec{x} = \begin{bmatrix} 255 \\ 231 \\ \vdots \\ 254 \\ 253 \\ 250 \\ 220 \end{bmatrix}$$

$$64 \times 64 \times 3 = \underbrace{12288}_{n_x}$$

$$\vec{x} \rightarrow \boxed{\text{model}} \rightarrow \hat{y}$$

- Notation

  $(\vec{x}, y) \qquad x \in R^{n_x}, \; y \in \{0,1\}$

Binary classification

- m training example: $\{(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}, \ldots, (x^{(m)}, y^{(m)})\}$

- $$\mathrm{X} = \begin{array}{c} \left[\begin{array}{cccc} | & | & & | \\ x^{(1)} & x^{(2)} & \ldots\ldots\ldots & x^{(\mathrm{m})} \\ | & | & & | \end{array}\right] \\ \mathrm{m} \end{array} \; n_x$$

$\mathrm{X} \in \mathbb{R}^{n_x \times m}$

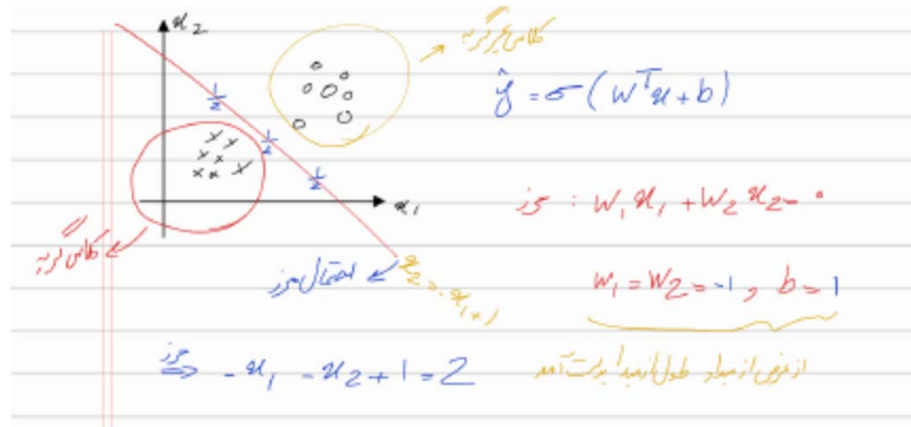
$\mathrm{X.shape} = (n_x, m)$

$\mathrm{Y} = [y^{(1)}, y^{(2)}, \ldots, y^{(m)}]$

$\mathrm{Y} \in \mathbb{R}^{1 \times m}$

$\mathrm{Y.shape} = (1, m)$

Logistic Regression

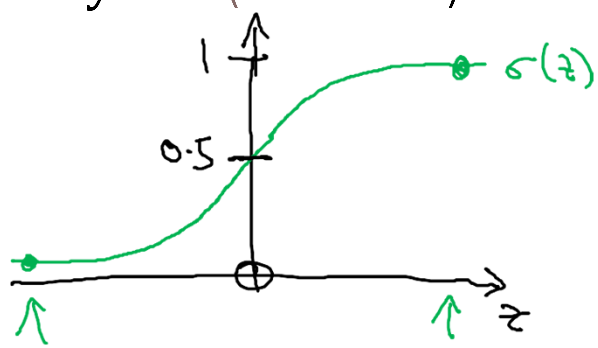


- Given x , output $\hat{y} = P(y=1|x)$ $\quad 0 \le \hat{y} \le 1$

$x \in \mathbb{R}^{n_x}$ $\qquad$ parameters: $w \in \mathbb{R}^{n_x}$, $b \in \mathbb{R}$

$$\hat{y} = w^T x + b$$

$$\hat{y} = \sigma(\overbrace{w^T x + b}^{z})$$



Sigmoid function $\quad \sigma(z) = \frac{1}{1+e^{-z}}$

if z large $\sigma(z) \approx 1$

if z large negative $\sigma(z) \approx 0$

Logistic Regression

- $\hat{y} = \sigma(\underbrace{w^T x + b}_{z})$

$$\hat{y} = w^T x$$

$x_0 = 1 \, , x \in \mathbb{R}^{n_x+1}$

$$W = \begin{array}{c} \begin{bmatrix} b = w_0 \\ w_1 \\ w_2 \\ w_3 \\ . \\ . \\ . \\ w_{n_x} \end{bmatrix} \end{array} \begin{array}{l} \} \; b \\ \left.\begin{array}{l} \\ \\ \\ \\ \\ \\ \\ \end{array}\right\} w \end{array}$$

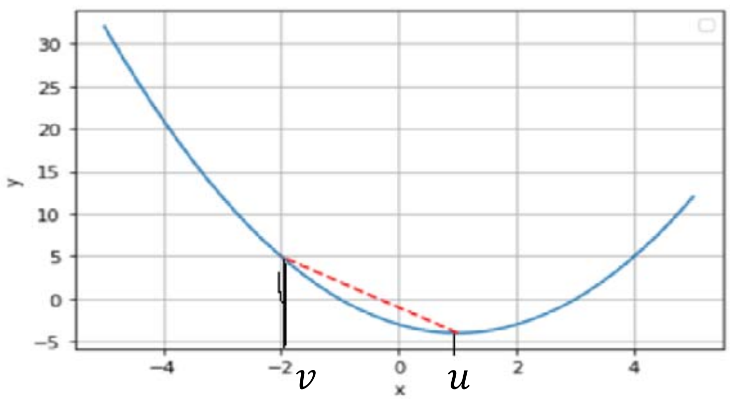
Logistic Regression cost function

- Loss (error) function: $L(\hat{y}, y) = \frac{1}{2}(\hat{y} - y)^2 = \frac{1}{2}\left(\sigma\left(w^T x + b\right) - y\right)^2$ SE: Square Error

- What is the problem?

- .

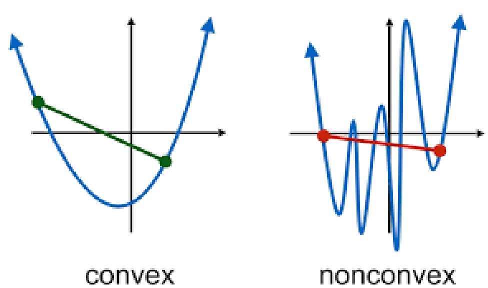
Convexity



Function h(u) with u$\in$ X is convex if for any u, v $\in$ X and for any $0 \leq \lambda \leq 1$ we have:

$$\mathbf{h(\lambda u + (1- \lambda)v) \leq \lambda\, h(u) + (1- \lambda)\, h(v)}$$

برای توابع محدب هر بهینه محلی یک بهینه سراسری است.



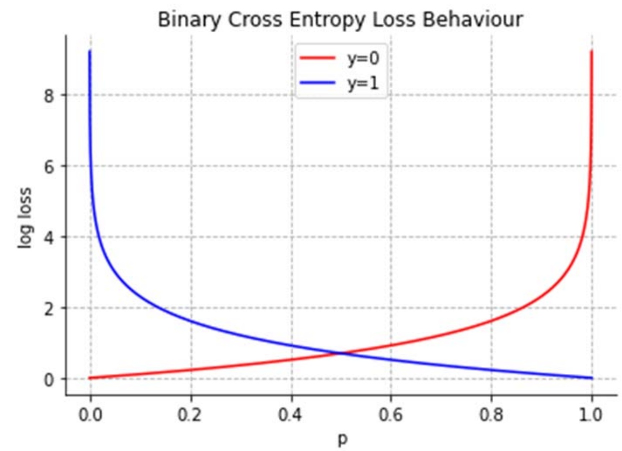
https://mlstory.org/optimization.html

Cross Entropy

- $L(\hat{y}, y) = -(y \log \hat{y} + (1-y)\log(1-\hat{y}))$

if $y=1$: $L(\hat{y}, y) = -\log \hat{y}$

if $y=0$: $L(\hat{y}, y) = -\log(1-\hat{y})$



https://datamonje.com/classification-loss-functions/

- Cost function: $J(w,b) = \frac{1}{m}\sum_{i=1}^{m} L\left(y^{(i)}, \hat{y}^{(i)}\right)$

$$= -\frac{1}{m}\sum_{i=1}^{m}[y^{(i)}\log \hat{y}^{(i)} + (1-y^{(i)})\log(1-\hat{y}^{(i)})]$$

Gradient Descent

Cost Function

Minimize $\boldsymbol{J(b, w_1)}$

$\boldsymbol{b, w_1}$

If $J(w_1) = (w_1 - 2)^2$

$$\frac{dJ(w_1)}{dw_1} = 0 \Rightarrow \frac{dJ(w_1)}{dw_1} = 2\,(w_1 - 2) = 0 \Rightarrow w_1 = 2$$

Gradient Descent

Minimize $\boldsymbol{J}(\boldsymbol{b}, \boldsymbol{w_1})$

$\boldsymbol{b}, \boldsymbol{w_1}$

Minimize $\boldsymbol{J}(\boldsymbol{b}, \boldsymbol{w_1}, \dots, \boldsymbol{w_n})$

$\boldsymbol{b}, \boldsymbol{w_1}, \dots, \boldsymbol{w_n}$

Repeat until convergence: {

For j=0,...,n

$$\boldsymbol{w_j} = \boldsymbol{w_j} - \alpha \frac{d\boldsymbol{J}(\boldsymbol{b}, \boldsymbol{w_1}, \dots, \boldsymbol{w_n})}{d\boldsymbol{w_j}}$$

b= $\boldsymbol{w_0}$

}

α **is learning rate**

Updating all w_j ***Simultaneously***

Convergence condition:

$$\|W^{t+1} - W^t\|_2 \leq \varepsilon$$

Gradient Descent

Correct form

$$temp0 = b - \alpha \frac{dJ(b, w_1)}{db}$$

$$temp1 = w_1 - \alpha \frac{dJ(b, w_1)}{dw_1}$$

$\boldsymbol{b}$ = temp0

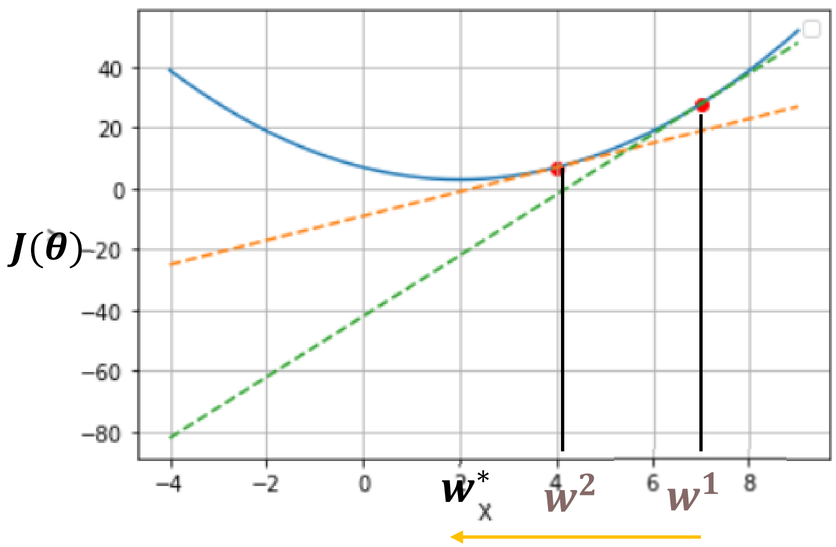
$\boldsymbol{w_1}$ = temp1

Incorrect form

$$b = b - \alpha \frac{dJ(b\ , w_1)}{db}$$

$$w_1 = w_1 - \alpha \frac{dJ(b, w_1)}{dw_1}$$

Gradient Descent



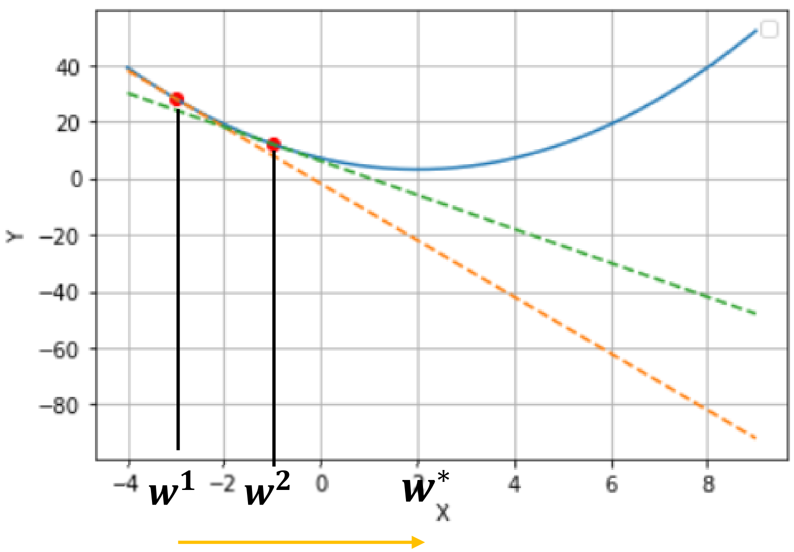
خطوط مماس نشان داده شده دارای شیب یا مشتق مثبت هستند.
درنتیجه:

$$\frac{dJ(w^1)}{dw^1} > 0\,,\ \alpha > 0 \Rightarrow \alpha\frac{dJ(w^1)}{dw^1} > 0$$

$$\Rightarrow w^2 = w^1 - \alpha d w^1$$

w کوچکتر میشود و به سمت چپ حرکت میکنیم.

Gradient Descent



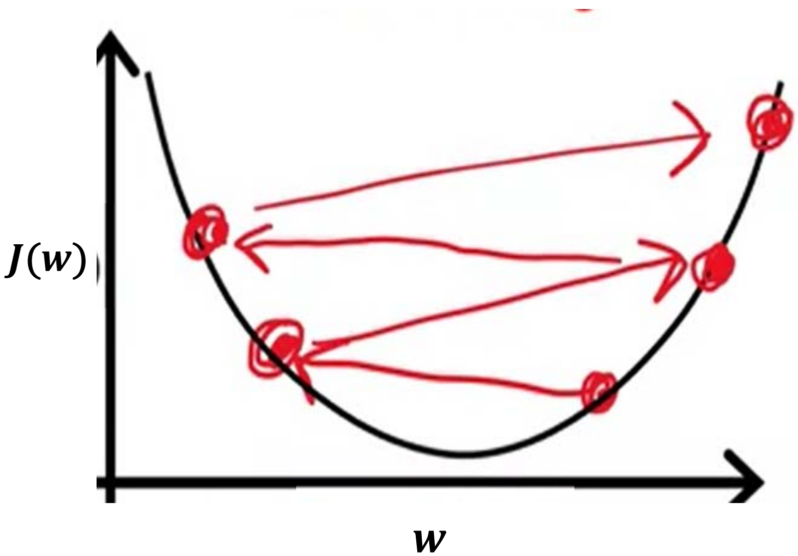
خطوط مماس نشان داده شده دارای شیب یا مشتق مثبت هستند.
درنتیجه:

$$\frac{dJ(w^1)}{dw^1} < 0 \,,\ \alpha > 0 \Rightarrow \alpha\frac{dJ(w^1)}{dw^1} < 0$$

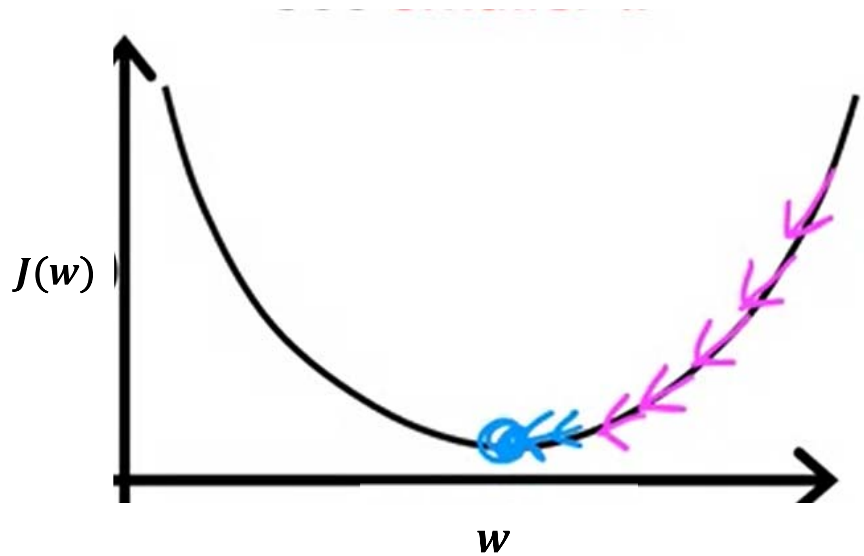
$$\Rightarrow w^2 = w^1 - \alpha d w^1$$

w بزرگتر میشود و به سمت راست حرکت میکنیم.

Choosing Learning Rate

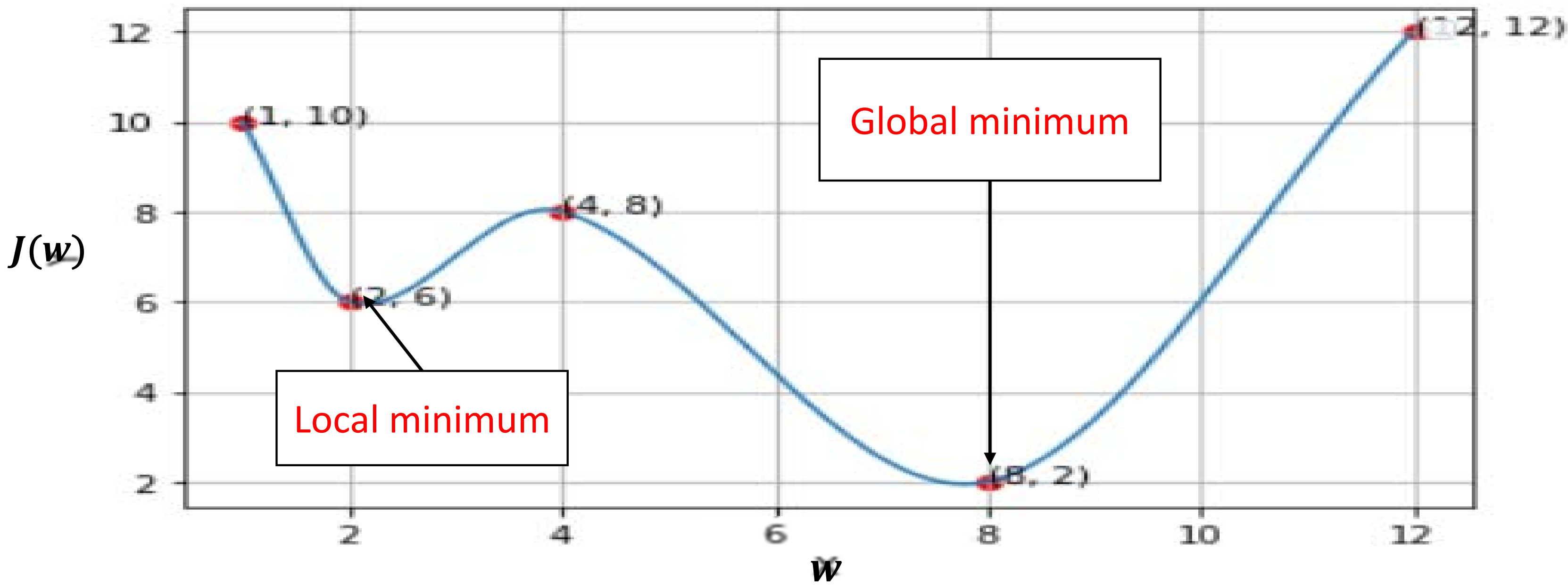


α is too large

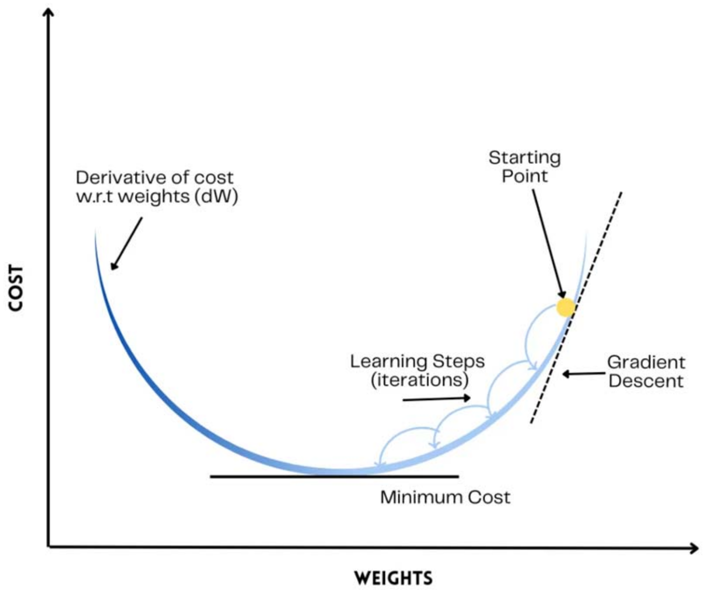


α is small

Gradient Descent Weakness



Gradient Descent



- 1) $\alpha > 0$

Repeat{

$$w = w - \alpha \underbrace{\frac{dJ(w)}{d(w)}}_{dw}$$

}until convergence

$$w = w - \alpha dw$$

$$z = w^T x + b$$

- $\hat{y} = a = \sigma(z)$
- $L(a, y) = -(y \log a + (1 - y) \log(1 - a))$

https://www.askpython.com/

Gradient Descent

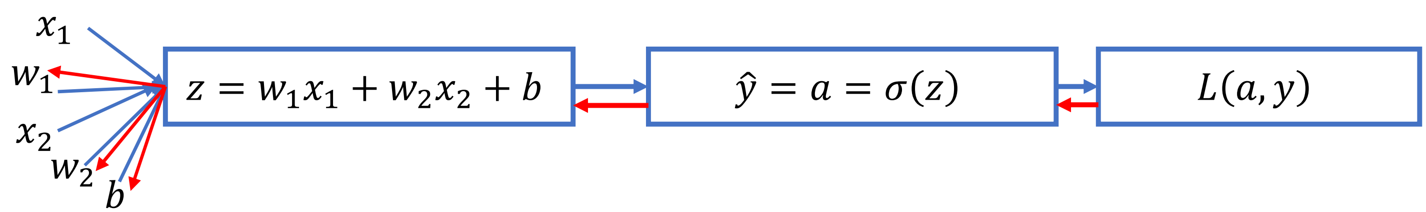
Computational Graph

$$L(a,y) = -(y\log a + (1-y)\log(1-a))$$

$$\frac{da}{dz} = \acute{\sigma}(z) = \underbrace{\sigma(z)}_{a}\,(\underbrace{1-\sigma(z)}_{1-a})$$

$$da = \frac{dL}{da} = \frac{-y}{a} + \frac{1-y}{1-a} = \frac{a-y}{a(1-a)}$$

$$a = \sigma(z) = \frac{1}{1+e^{-z}}$$



$$dz = \frac{dL}{dz} = \frac{dL(a,y)}{dz} = \frac{dL}{da}\times\frac{da}{dz} = \frac{a-y}{a(1-a)}\times a(1-a) = a-y$$

$$dw_1 = \frac{dL}{dw_1} = \underbrace{\frac{dL}{dz}}_{dz}\times\underbrace{\frac{dz}{dw_1}}_{x_1} = x_1\,dz \qquad dw_2 = x_2\,dz \qquad db = \frac{dL}{db} = \frac{dL}{dz}\times\frac{\overbrace{dz}^{1}}{db} = dz$$

Gradient Descent

- $J(w,b) = \frac{1}{\mathrm{m}} \sum_{\mathrm{i}=1}^{\mathrm{m}} \mathrm{L}\left(a^{(\mathrm{i})}, \mathrm{y}^{(\mathrm{i})}\right)$
- $a^{(i)} = \hat{y}^{(i)} = \sigma\left(z^{(i)}\right) = \sigma\left(wx^{(i)} + b\right)$

- $dw_j = \frac{1}{\mathrm{m}} \sum_{\mathrm{i}=1}^{\mathrm{m}} \frac{d\mathrm{L}\left(a^{(\mathrm{i})}, \mathrm{y}^{(\mathrm{i})}\right)}{dw_j}$

Logistic regression on m examples

$J = 0;\quad dw_1 = 0;\quad dw_2 = 0;\quad db = 0;$

$w_1 \leftarrow \text{random}\quad w_2 \leftarrow \text{random}\quad b \leftarrow \text{random}$

Repeat{

For *i=1* *to* *m*

$z^{(i)} = w^T x^{(i)} + b$

$a^{(i)} = \sigma\left(z^{(i)}\right)$

$J \mathrel{+}= \left[y^{(i)} Log\, a^{(i)} + \left(1 - y^{(i)}\right) Log\left(1 - a^{(i)}\right)\right]$

$dz^{(i)} = a^{(i)} - y^{(i)}$

$dw_1 \mathrel{+}= x_1^{(i)}\, dz^{(i)}\qquad dw_2 \mathrel{+}= x_2^{(i)}\, dz^{(i)}\qquad db \mathrel{+}= dz^{(i)}$

$J /= m;\qquad dw_1 /= m;\qquad dw_2 /= m;\qquad db /= m;$

$w_1 = w_1 - \alpha\, dw_1\qquad w_2 = w_2 - \alpha\, dw_2\qquad b = b - \alpha\, db$

} until convergence

$$w^t = \begin{bmatrix} w_1^t \\ w_2^t \\ b^t \end{bmatrix} \quad w^{t+1} = \begin{bmatrix} w_1^{t+1} \\ w_2^{t+1} \\ b^{t+1} \end{bmatrix}$$

$$\|w^{t+1} - w^t\|_2 \le \varepsilon$$

$$dw = \begin{bmatrix} dw_1 \\ dw_2 \\ db \end{bmatrix}$$

$$\|dw\| \le \varepsilon = 10^{-4}$$

What's wrong with the code?

Logistic regression on m examples

$w_1 \leftarrow \text{random} \quad w_2 \leftarrow \text{random} \quad b \leftarrow \text{random}$

$$w^t = \begin{bmatrix} w_1^t \\ w_2^t \\ b^t \end{bmatrix} \quad w^{t+1} = \begin{bmatrix} w_1^{t+1} \\ w_2^{t+1} \\ b^{t+1} \end{bmatrix}$$

Repeat{

$J = 0; \quad dw_1 = 0; \quad dw_2 = 0; \quad db = 0;$

$$\|w^{t+1} - w^t\|_2 \leq \varepsilon$$

For i=1 to m

$z^{(i)} = w^T x^{(i)} + b$

$$dw = \begin{bmatrix} dw_1 \\ dw_2 \\ db \end{bmatrix}$$

$a^{(i)} = \sigma\left(z^{(i)}\right)$

$J \mathrel{+}= \left[y^{(i)} Log a^{(i)} + \left(1 - y^{(i)}\right) Log(1 - a^{(i)})\right]$

$$\|dw\| \leq \varepsilon = 10^{-4}$$

$dz^{(i)} = a^{(i)} - y^{(i)}$

$dw_1 \mathrel{+}= x_1^{(i)}\, dz^{(i)} \quad dw_2 \mathrel{+}= x_2^{(i)}\, dz^{(i)} \quad db \mathrel{+}= dz^{(i)}$

$J /= m; \quad dw_1 /= m; \quad dw_2 /= m; \quad db /= m;$

$w_1 = w_1 - \alpha\, dw_1 \quad w_2 = w_2 - \alpha\, dw_2 \quad b = b - \alpha\, db$

} until convergence

Basics of Neural Network Programming

Vectorizing Logistic Regression’s Gradient Computation

Vectorizing Logistic Regression

- *z=0;*
  For i in range(n_x)
  $$z += w[i] * x[i]$$
  $$z += b$$
  First method

- *z=0;*
  z =np.dot(w, x)+b
  Second method
  SIMD
  GPU

Vectorizing Logistic Regression

$z^{(1)} = w^T x^{(1)} + b$ $z^{(2)} = w^T x^{(2)} + b$ $z^{(m)} = w^T x^{(m)} + b$

$a^{(1)} = \sigma\left(z^{(1)}\right)$ $a^{(2)} = \sigma\left(z^{(2)}\right)$ $a^{(m)} = \sigma\left(z^{(m)}\right)$

- $X = \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \dots\dots\dots & x^{(\mathrm{m})} \\ | & | & & | \end{bmatrix} \in R^{n_x \times m}$ $\underbrace{[w_1 \dots w_{n_x}]}_{W^T} \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \dots\dots\dots & x^{(\mathrm{m})} \\ | & | & & | \end{bmatrix}$
- $Z = \begin{bmatrix} \underbrace{z^{(1)}}_{w^T x^{(1)} + b} & \underbrace{z^{(2)}}_{w^T x^{(2)} + b} & \dots & \underbrace{z^{(m)}}_{w^T x^{(m)} + b} \end{bmatrix} = w^T X + [b \; b \cdots b]_{1 \times m}$
- $Z = \underbrace{np \cdot dot(w \cdot T, X)}_{1_{\times} m} + \underbrace{b}_{(1,1)}$ *"broadcasting"* $[b, b \dots, b]_{1 \times m}$

Vectorizing Logistic Regression

- $A = [a^{(1)}, a^{(2)}, \dots, a^{(m)}] = \sigma(\underbrace{Z}_{1\times m})$
- $dz^{(1)} = a^{(1)} - y^{(1)}$ $\quad dz^{(2)} = a^{(2)} - y^{(2)}$
- $dZ = [dz^{(1)}\ dz^{(2)}\ \dots\ dz^{(m)}]_{1 \times m}$
- $A = [a^{(1)}\ a^{(2)}\ \dots\ a^{(m)}]$ $\quad Y = [y^{(1)}\ y^{(2)}\ \dots\ y^{(m)}]$
- $dZ = A - Y = [a^{(1)} - y^{(1)}\quad a^{(2)} - y^{(2)}\ \dots\ a^{(m)} - y^{(m)}]$
- $dw=0$ $\quad db=0$

$$1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ \vdots \\ \vdots \\ 1 \end{bmatrix}_{m\times 1}$$

- $\begin{bmatrix} dw_1 \\ dw_2 \\ dw_3 \\ \vdots \\ dw_{nx} \end{bmatrix}_{nx \times 1}$

$$\begin{cases} dw \mathrel{+}= \underbrace{x^1}_{\text{بردار}}\ \overbrace{dz^1}^{\overset{\text{عدد}}{a^1-y^1}} \\ dw \mathrel{+}= x^2 dz^2 \\ dw \mathrel{+}= x^m dz^m \end{cases} \qquad \begin{cases} db \mathrel{+}= \overbrace{dz1}^{\overset{\text{عدد}}{a1-y1}} \\ db \mathrel{+}= dz2 \\ db \mathrel{+}= dz^m \end{cases}$$

$dw /= m$ $\quad db /= m$ $\quad dz = [dz^1\ \ dz^2\ \dots\ dz^m]_{1 \times m}$

$$db = \frac{1}{m}\sum_{i=1}^{m} dz^{(i)} = \frac{1}{m} np.sum(dz) \qquad db = \frac{1}{m} dz \times 1$$

$$dw = \frac{1}{m}[x^{(1)}dz^1 + x^{(2)}dz^2 + \dots + x^{(m)}dz^m]$$

$$\boxed{dw = \frac{1}{m} X\, dz^T} = \frac{1}{m}\begin{bmatrix} \\ x^{(1)} x^{(2)}\ \dots\dots\dots\ x^{(m)} \\ \\ \end{bmatrix} \times \begin{bmatrix} dz^1 \\ dz^2 \\ dz^3 \\ \vdots \\ \vdots \\ \vdots \\ dz^m \end{bmatrix}$$

Vectorizing Logistic Regression

- $w_1, w_2, b \leftarrow random$

For iter in range(1000)

1 epoch

$$\left\{\begin{array}{l} Z = W^T X + b = np \cdot dot(w.T, X) + b \\ A = \sigma(Z) \\ dZ = A - Y \\ dw = \frac{1}{m} X\, dz^T \\ db = \frac{1}{m} np.sum(dz) \\ w = w - \alpha\, dw \\ b = b - \alpha\, db \end{array}\right.$$

$$w = np \cdot random \cdot \mathrm{rand}n(n_x, 1)$$

Logistic Regression Cost function

- $\hat{y} = \sigma(w^T x + b) \qquad 0 < \sigma(z) = \frac{1}{1+e^{-z}} < 1$
- $\hat{y} = p(y = 1|x)$

$$\left.\begin{array}{l} \bullet\ if \quad y = 1 \ : \quad p(y|x) = \hat{y} \\ \bullet\ if \quad y = 0 \ : \quad p(y|x) = 1 - \hat{y} \end{array}\right\} \boldsymbol{p(y|x)}$$

- $\boxed{p(y|x) = \hat{y}^y (1 - \hat{y})^{1-y}}$ $\qquad$ *distribution? Bernoulli*
- $if \quad y = 1 \ : \quad p(y|x) = \hat{y}$
- $if \quad y = 0 \ : \quad p(y|x) = 1 - \hat{y}$
- $\mathrm{logp(y|x)} = \log[\hat{y}^y (1 - \hat{y})^{1-y}] = y \log \hat{y} + (1 - y) \log(1 - \hat{y})$

$$= -L(\hat{y}, y) \quad \textit{Max Likelihood}$$

Logistic Regression Cost function

- $\log P(\textit{ labels in trainingset}) = \log \prod_{i=1}^{m} P\left(y^{i} \mid x^{i}\right)$
- $\log P(\cdots) = \sum_{i=1}^{m} \log P\left(y^{(i)} \mid x^{(i)}\right) = -\sum_{i=1}^{m} L\left(\hat{y}^{(i)}, y^{(i)}\right)$

- Cost function $\underbrace{J(w,b)}_{\boldsymbol{minimize}} = \frac{1}{m}\sum_{i=1}^{m} L\left(\hat{y}^{(i)}, y^{(i)}\right)$

Neural Networks