Deep Learning

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https://www.aparat.com/mehran.safayani

https://github.com/safayani/deep_learning_course

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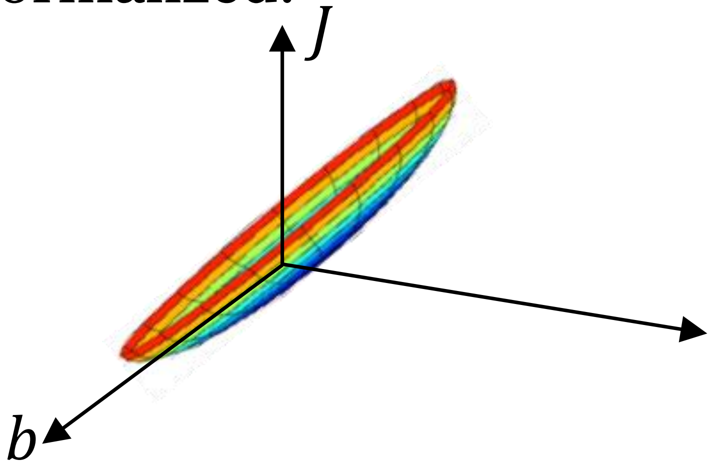
Improving deep neural networks

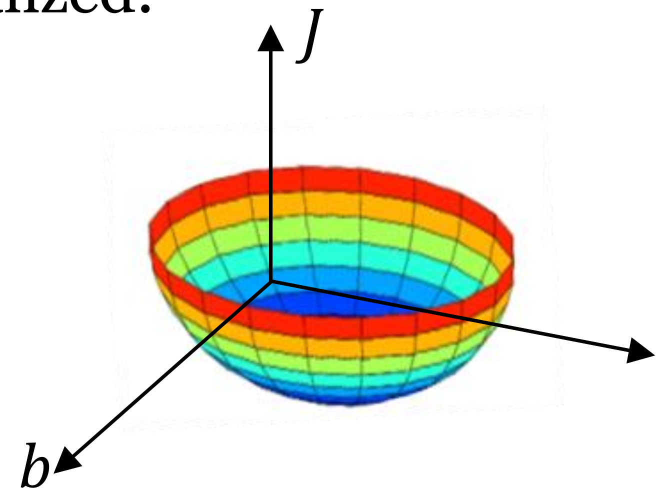
Why normalize inputs?

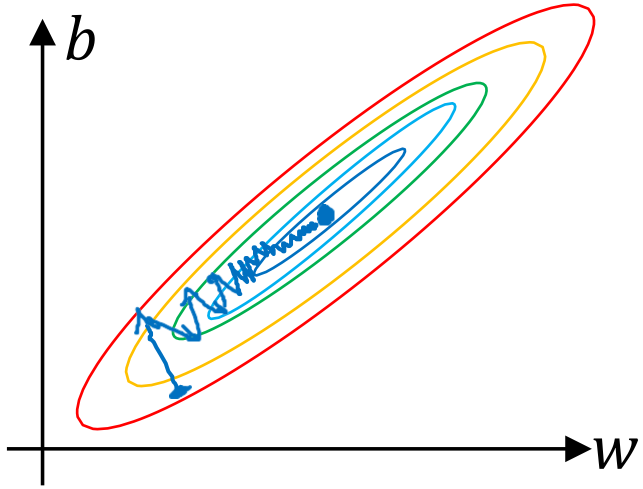
$$J(w, b) = \frac{1}{m}\sum_{i=1}^{m} L(\hat{y}^{(i)}, y^{(i)})$$

Unnormalized:

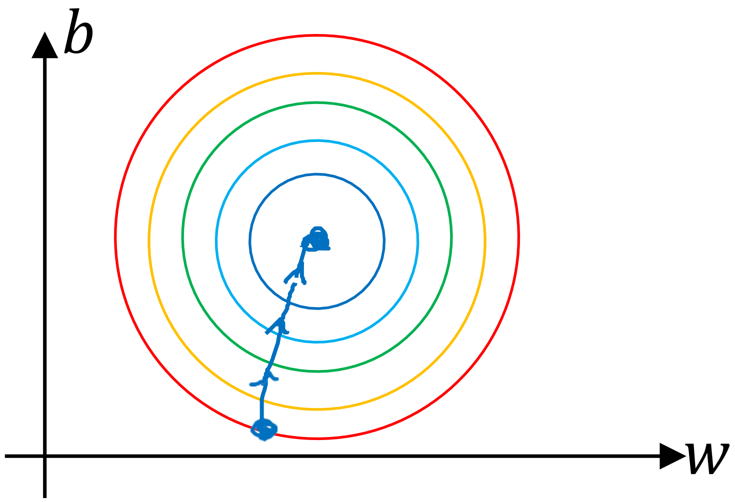


Normalized:





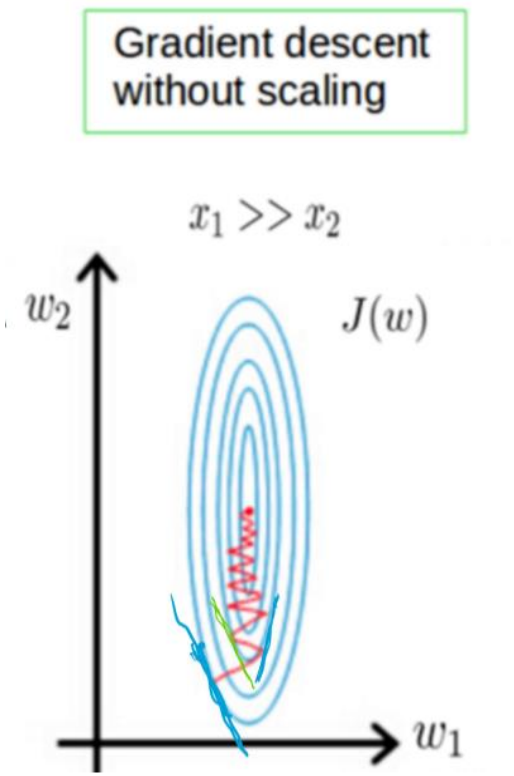
x_1: 0_100   0…1
x_2: -1,1   -1…1
x_3: 0.001_0.005   1…2

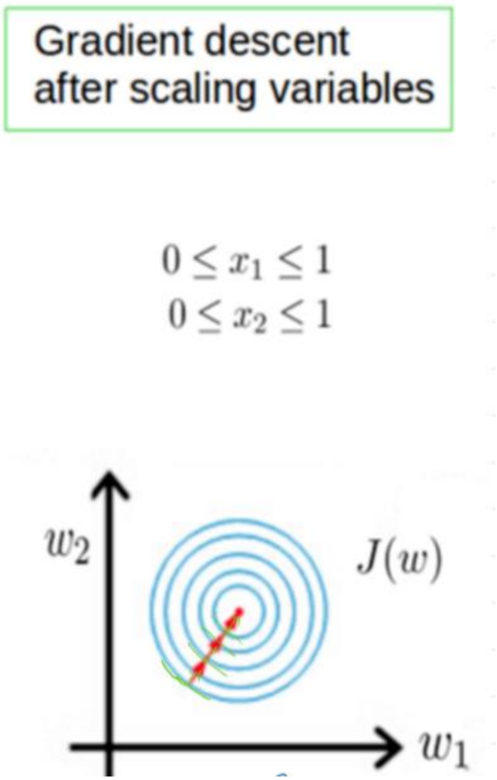


Contour Plot

$$\frac{w_1^2}{b^2} + \frac{w_2^2}{a^2} = 1$$

2a: قطر بزرگ
2b: قطر کوچک





$$\frac{w_1^2}{a^2} + \frac{w_2^2}{a^2} = 1$$

Feature Scaling

Scaled features:

- $0 \leq x_1 \leq 3$ ✓
- $-3 \leq x_1 \leq 3$ ✓
- $-2 \leq x_2 \leq 0.5$ ✓
- $-\frac{1}{3} \leq x_2 \leq \frac{1}{3}$ ✓

Need scaling:

$-100 \leq x_3 \leq 100$ ✗

$-0.001 \leq x_4 \leq 0.001$ ✗

Feature Scaling

$$x_1^* = \frac{x_1 - \mu_1}{standard\_deviation}$$

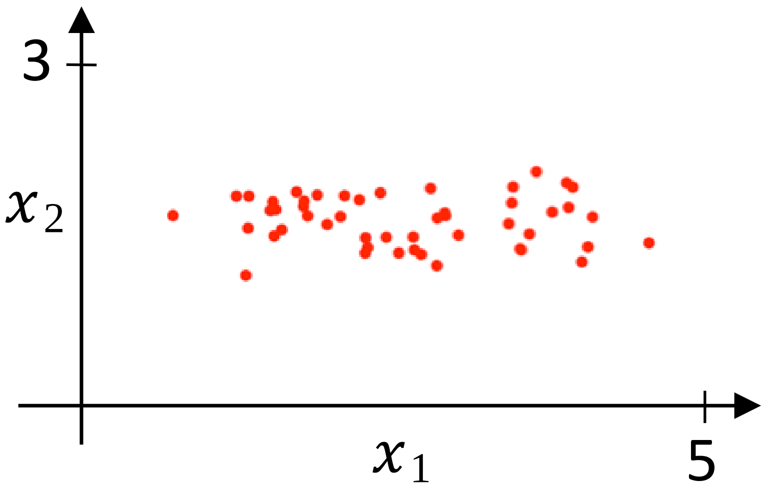
$$x_{scaled} = \frac{x - x_{min}}{x_{max} - x_{min}}$$

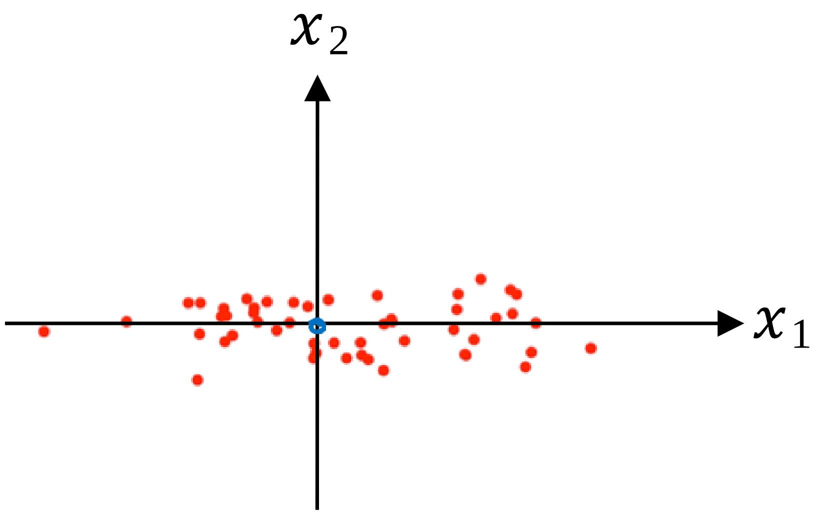
$$\mu_j = \frac{1}{m} \sum_{i=1}^{m} x_j^i$$

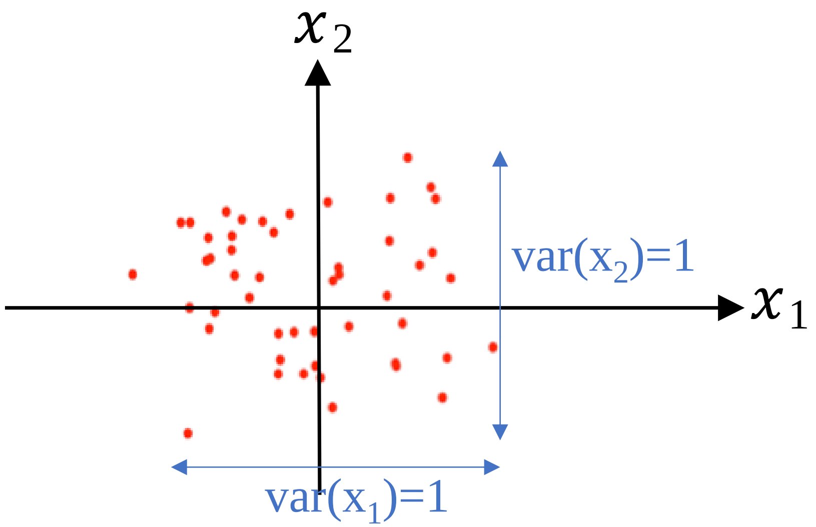
$$bedroom^* = \frac{bedroom - 2.5}{5}$$

$$size^* = \frac{size - 300}{2000}$$

Normalizing training set







Subtract mean:

$$\mu = \frac{1}{m}\sum_{i=1}^{m} \vec{x}^{(i)}$$

$$x = x - \vec{\mu}$$

Normalize variance:

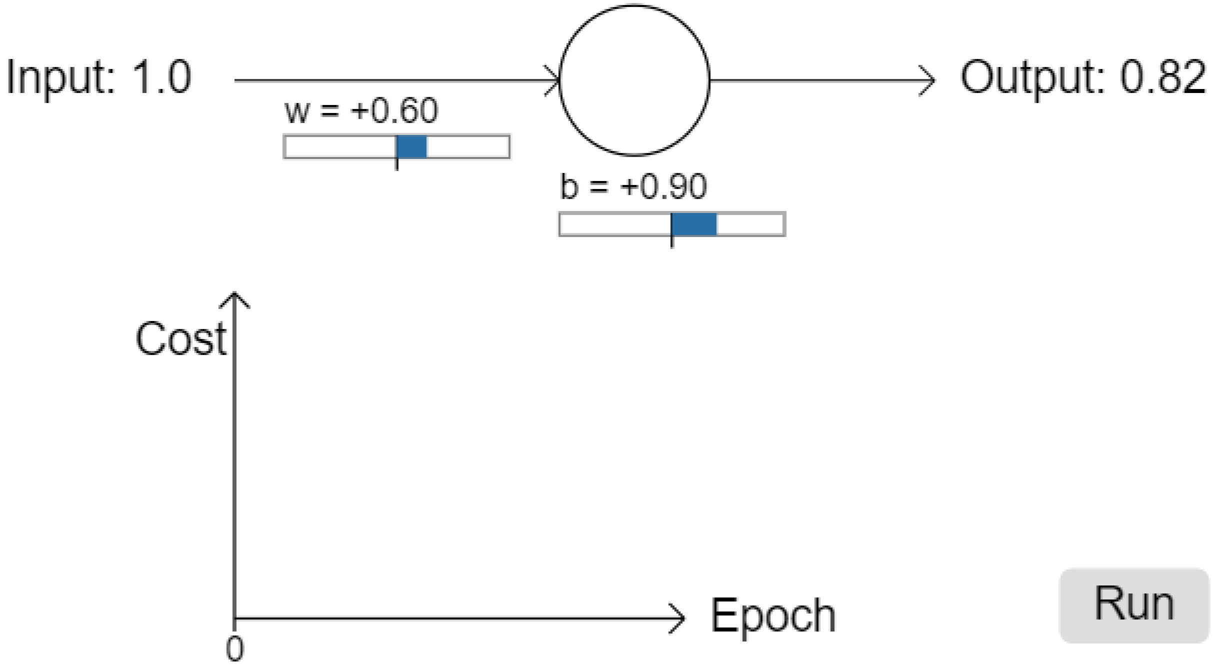
$$\sigma^2 = \frac{1}{m}\sum_{i=1}^{m} x^{(i)} ** 2$$

Element-wise

$$x /= \sigma^2$$

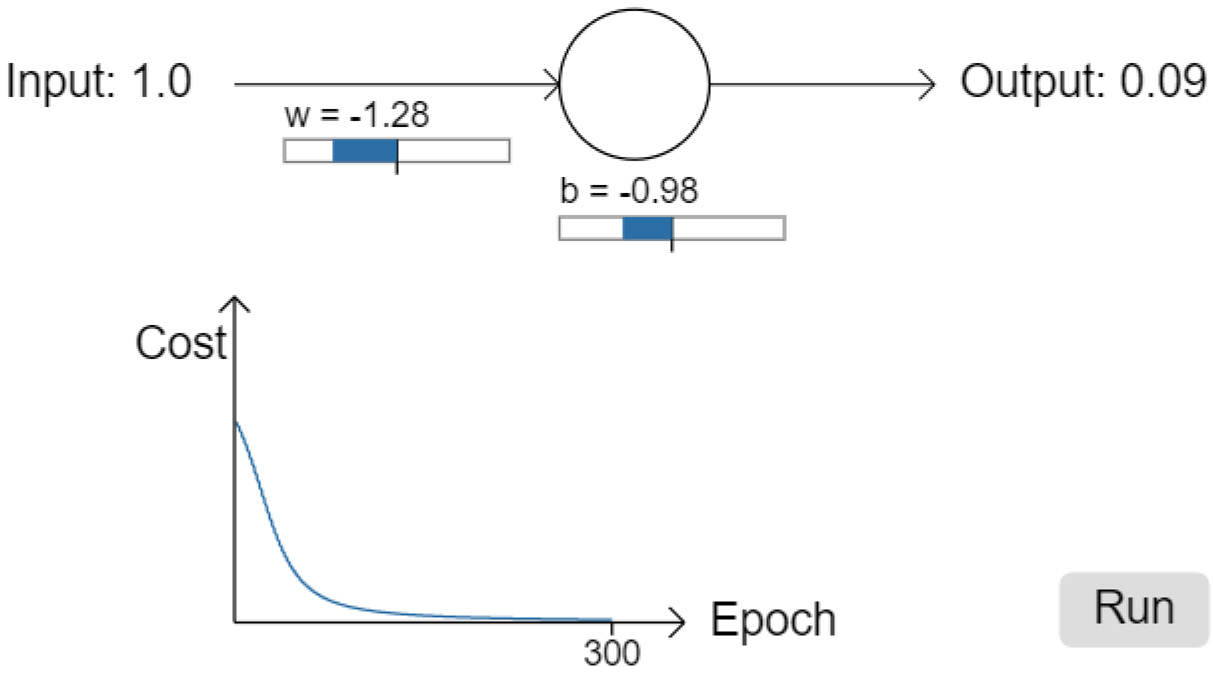
Use same $\mu\,,\sigma^2$ *to normalize test set.*

Vanishing gradient problem
Loss: MSE

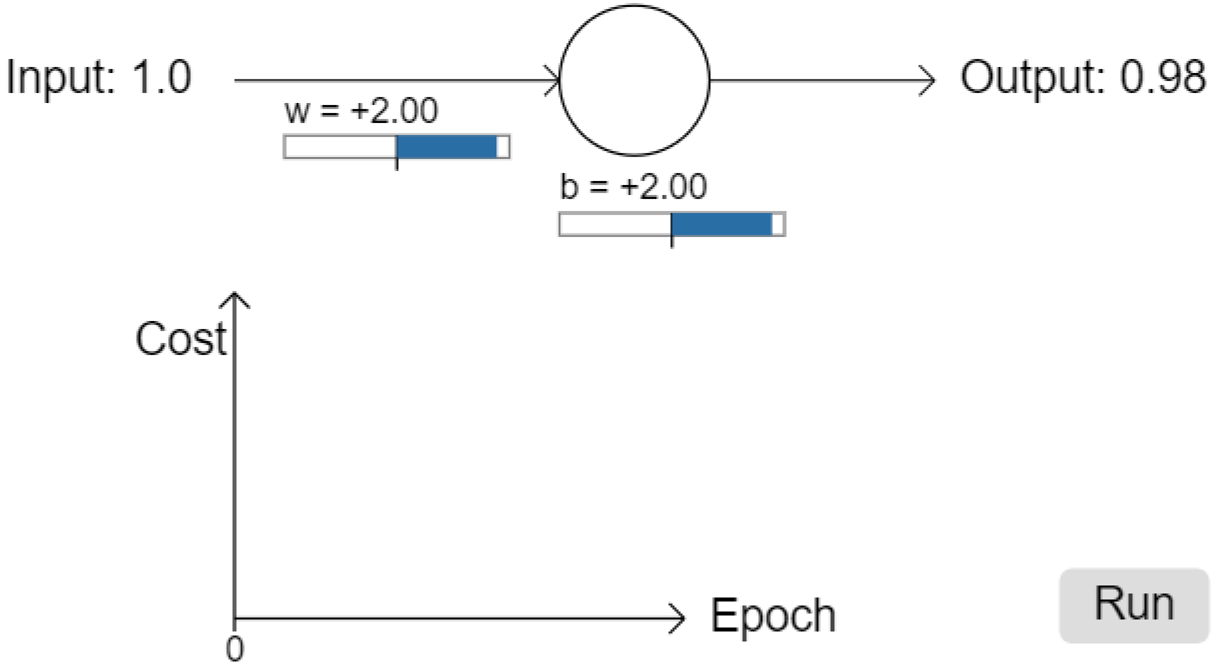


http://neuralnetworksanddeeplearning.com/chap3.html

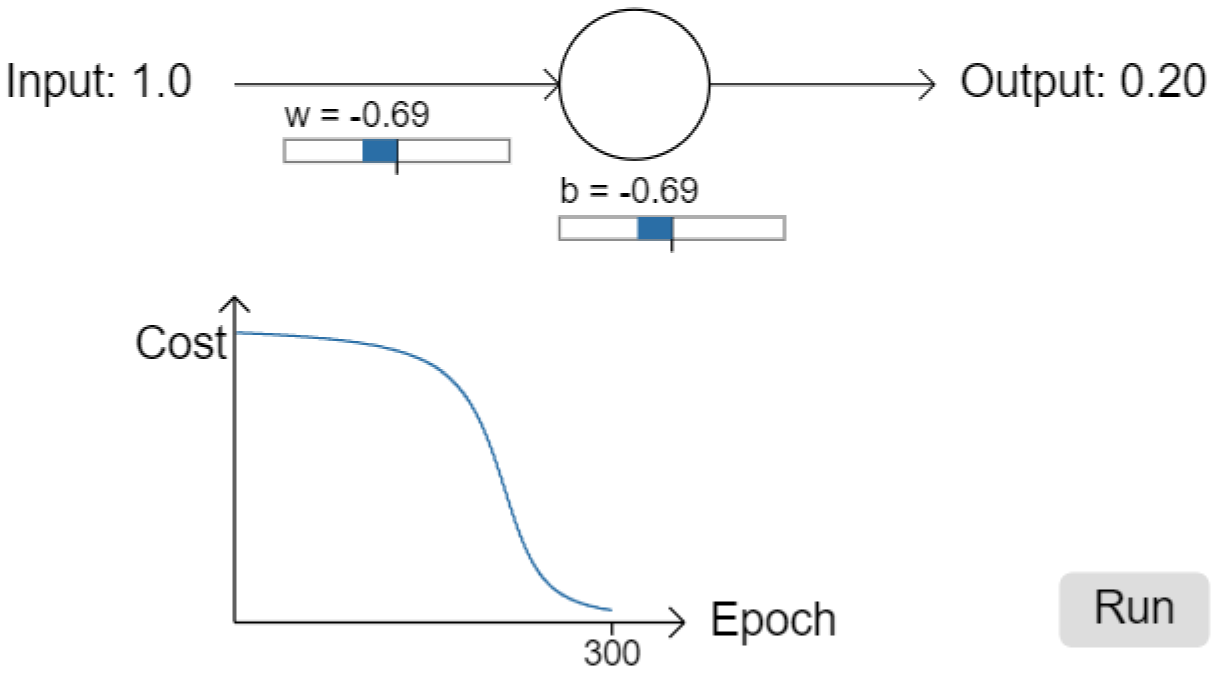
Loss:MSE



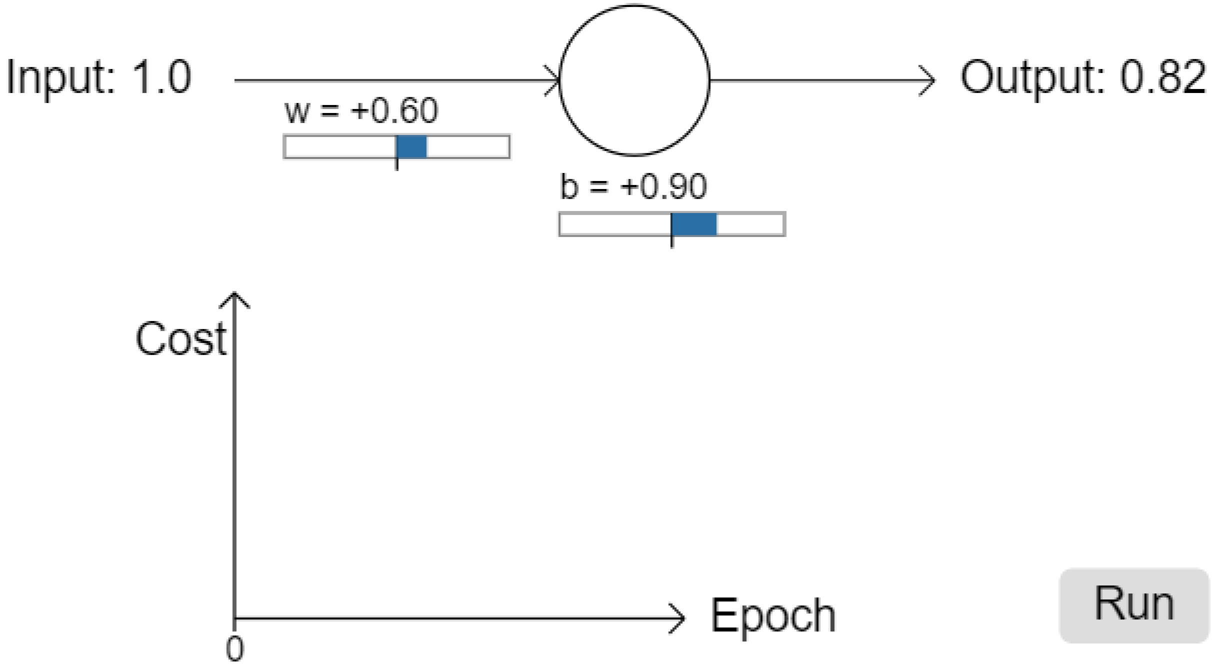
Loss:MSE



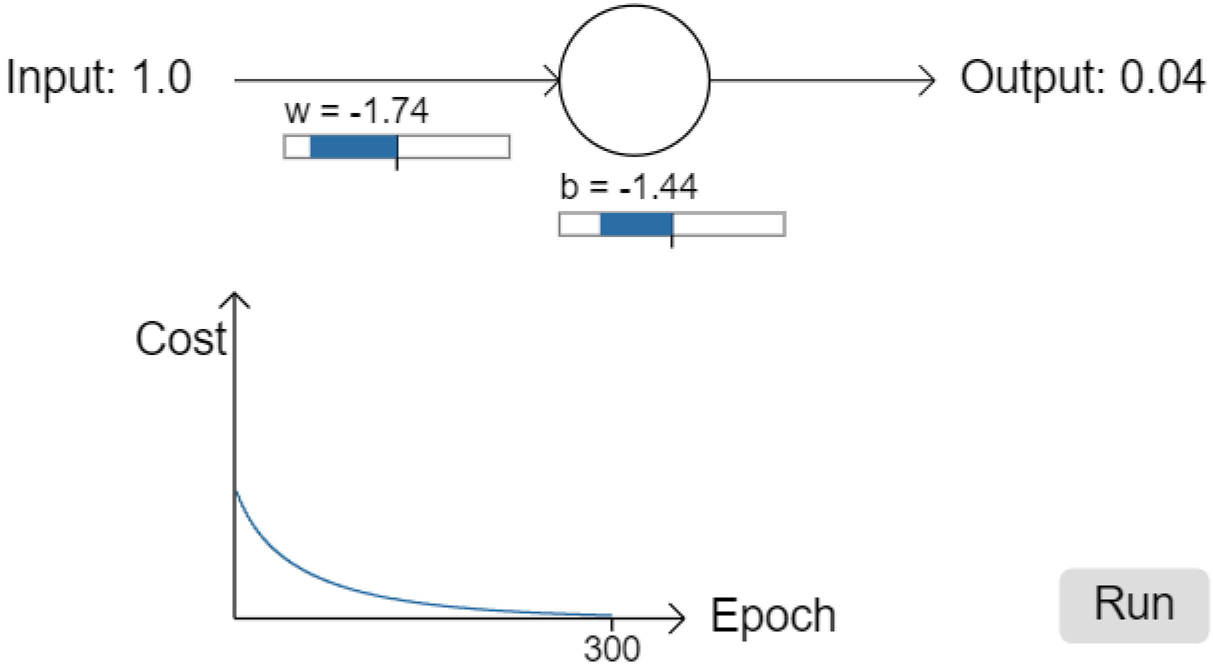
Loss:MSE



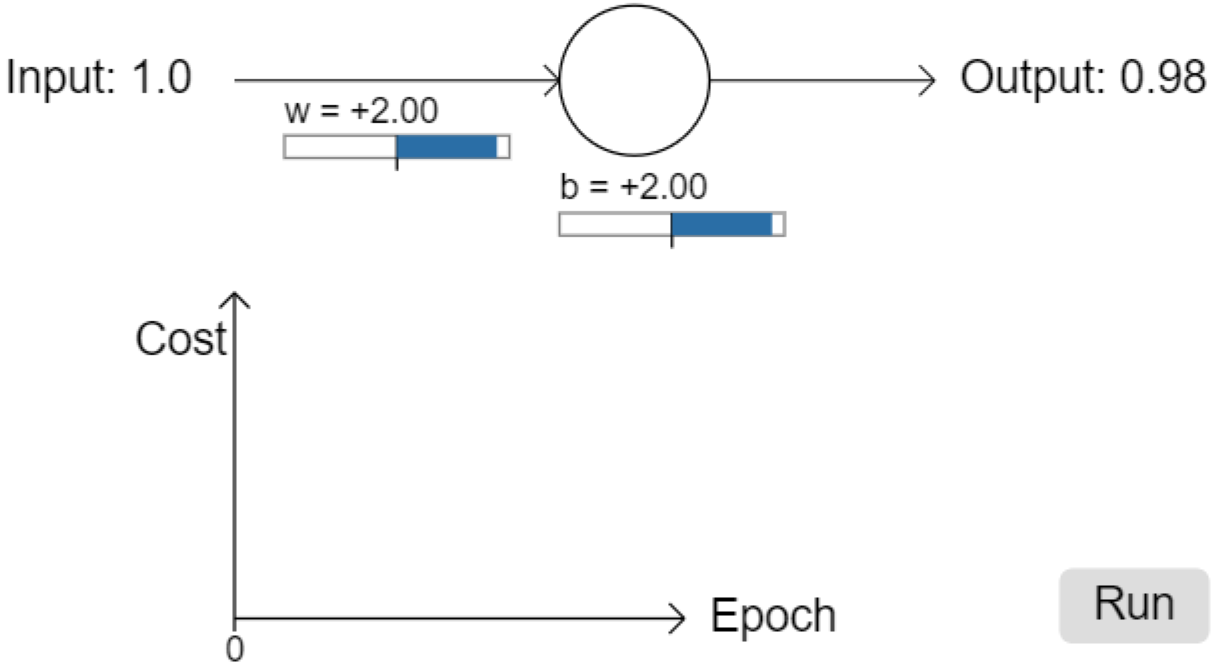
Loss:CE



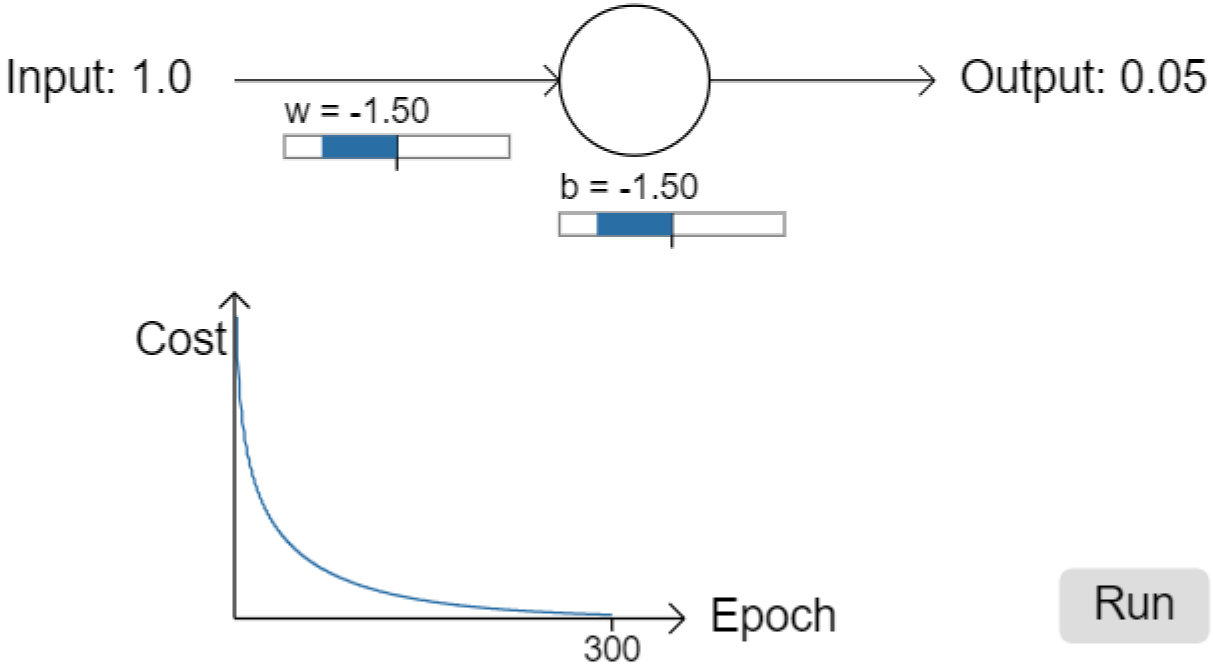
Loss:CE



Loss:CE

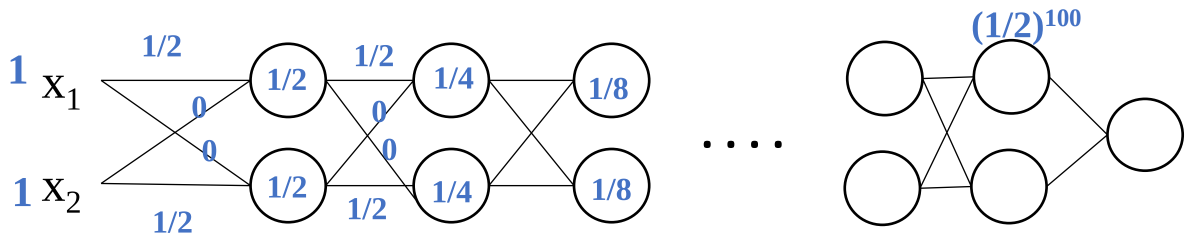


Loss:CE



Vanishing/exploding gradients

- $$\boxed{\begin{array}{c}\text{gradient clipping}\\ \text{If } \|g\| > \text{threshold}\\ g \leftarrow \dfrac{threshold \times g}{\|g\|}\end{array}}$$



- $g(z) = z$

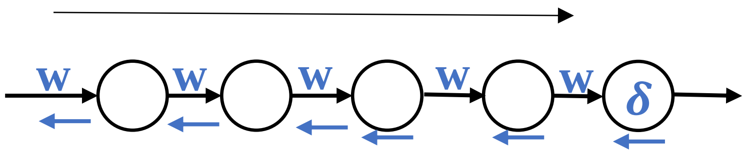
$b^{[l]} = 0$

$$\hat{y} = w^{[l]} w^{[l-1]} \ldots w^{[3]} w^{[2]} w^{[1]} x$$

$$w = \begin{bmatrix} 0 \cdot 9 & 0 \\ 0 & 0 \cdot 9 \end{bmatrix}$$

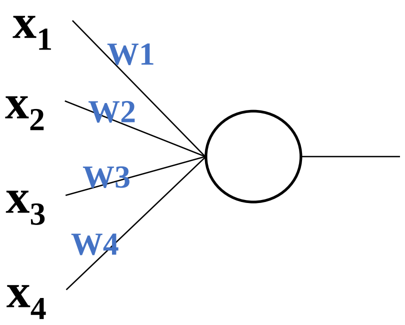
$$\hat{y} = w^{[l]} \begin{bmatrix} 0 \cdot 9 & 0 \\ 0 & 0 \cdot 9 \end{bmatrix}^{[l\text{-}1]}$$

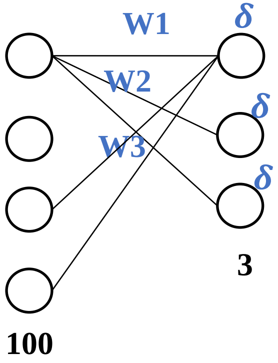
$$\begin{bmatrix} 1 \cdot 5 & 0 \\ 0 & 1 \cdot 5 \end{bmatrix}^{100}$$



Single neuron example

- $W \approx N(0,1) * 0.01$





- *larger n* $\longrightarrow$ *smaller* w_i

Xavier initialization

- $\mathrm{z} = w_1 x_1 + w_2 x_2 + \cdots + w_n x_n$ اثبات کنید.
- $W_i \approx N(0, \sigma^2)$ $z \approx N(0, \underbrace{n\sigma^2}_{1})$
- $E(x_i) = 0$ $n\sigma^2 = 1$
- $var(x_i) = 1$ $\sigma^2 = 1/n$
- $w^{[l]} = np \cdot random.randn(shape) * np.sqrt\left(\frac{1}{n^{[l-1]}}\right)$
- $w^{[l]} = np \cdot random.randn(shape) * np.sqrt(\frac{2}{n^{[l-1]}})$

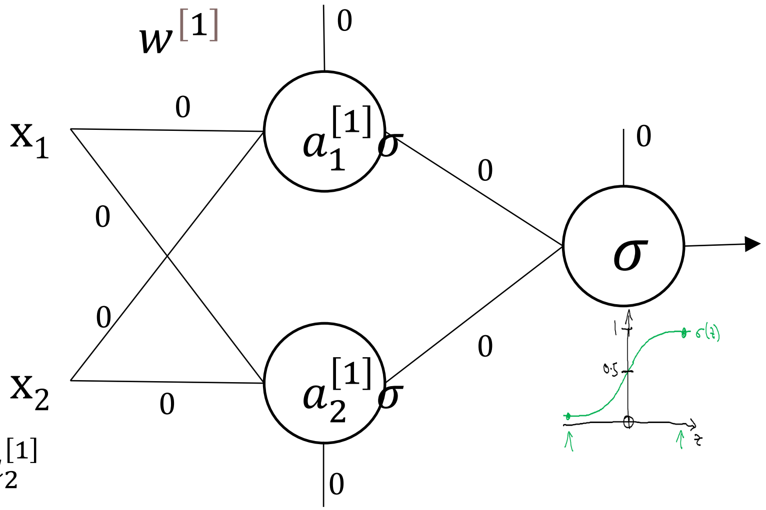
Xavier initialization

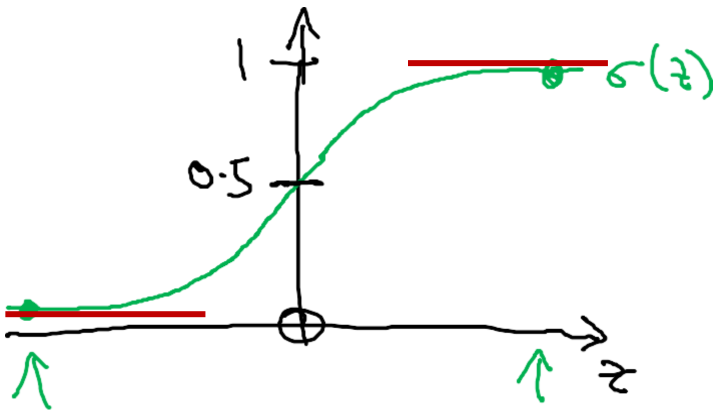
- tanh: $\sqrt{\frac{1}{n^{[l-1]}}}$
- Relu: $\sqrt{\frac{2}{n^{[l-1]}}}$
- Relu* $= \sqrt{\frac{2}{n^{[l-1]}+n^{[l]}}}$ →begin

Random Initialization

- **Restricted Boltzmann Machine(RBM): A restricted Boltzmann machine** is a generative stochastic artificial neural network that can learn a probability distribution over its set of inputs.

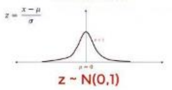
- اگر همه وزن‌های اولیه و بایاس‌ها صفر باشند:





Vanishing gradient

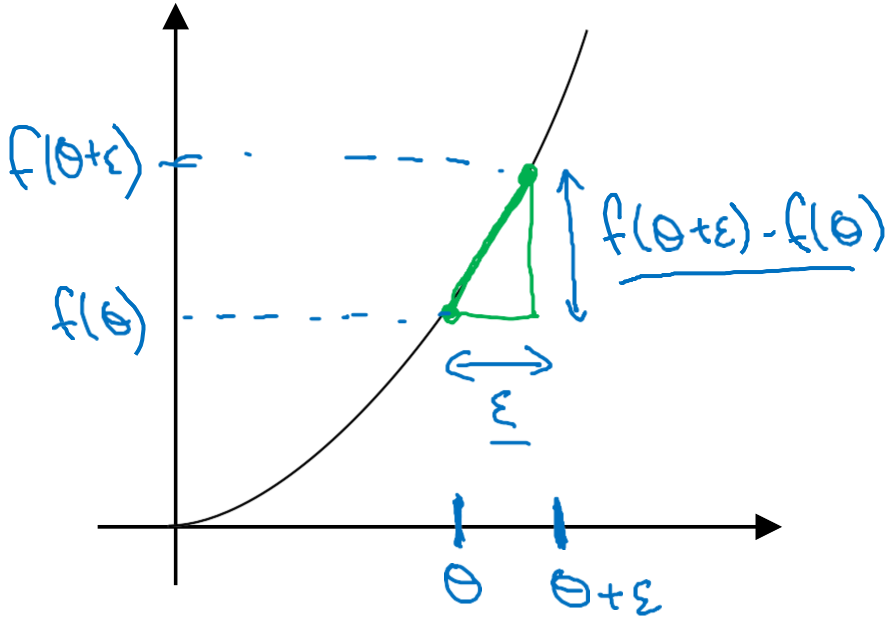
- $a_1^{[1]} = a_2^{[1]}$
- $w^{[1]} = np.random.randn((2,2)) * 0.01$
- $b^{[1]} = np.zeros((2,1))$ $N(0,1)$



- $w^{[2]} = np.random.randn((1,2)) * 0.01$
- $b^{[2]} = 0$

Numerical approximation of gradients

- $\frac{f(\theta+\varepsilon)-f(\theta-\varepsilon)}{2\varepsilon} \approx g(\theta)$



1.0

ε = 0.01

$$f(\theta) = \theta^3$$
$$\theta = 1$$

$$\frac{f(1.01)-f(0.99)}{2\times 0.01} = \frac{(1.01)^3-(0.99)^3}{0{\cdot}02} = 3.0001$$

θ = 1

θ + ε = 1.01

$$\frac{f(\theta+\varepsilon)-f(\theta-\varepsilon)}{2\varepsilon}$$

Gradient Checking

- $for\ each\ i$

$$d\theta_{approx}^{[i]} = \frac{\mathrm{J}(\theta_1,\theta_2,\ldots,\theta_{i+\varepsilon},\ldots) - \mathrm{J}(\theta_1,\theta_2,\ldots,\theta_{i-\varepsilon},\ldots)}{2\varepsilon}$$

$$\begin{bmatrix} \times \\ \times \\ \times \\ \times \end{bmatrix}$$

- $d\theta[i]_{approx} = \frac{\mathrm{d}J}{\mathrm{d}\theta_i} \approx \mathrm{d}\theta_i$

$$\frac{\left\| d\theta_{approx} - d\theta \right\|_2}{\left\| d\theta_{approx} \right\|_2 + \| d\theta \|_2} \ll 10^{-7} \qquad\qquad 10^{-3}$$

Mini-batch gradient descent

Batch, mini-batch

- $X = \left[x^{(1)}, x^{(2)}, \ldots, x^{(1000)} | , x^{(1001)} \ldots\ x^{(2000)} | \ldots\ x^{(m)}\right]$

$X^{\{1\}}_{(n_x,1000)}$ $\quad X^{\{2\}}_{(n_x,1000)}$ $\quad X^{\{5000\}}_{(n_x,m)}$ m=5000,000

$$Y = \left[\underbrace{y^{(1)}, y^{(2)} \ldots, y^{(1000)}}_{Y^{\{1\}}} | , \underbrace{y^{(1001)} \ldots\ y^{(2000)}}_{Y^{\{2\}}} | \ldots | \underbrace{\ldots , y^{(m)}}_{Y^{\{5000\}}}\right]$$

X^t, Y^t

Mini-batch GD

- Repeat{

For t=1, …, 5000 {

forward prop. on $x^{\{t\}}$

$z^{[1]} = w^{[1]} x^{\{t\}} + b^{[1]}$

$A^{[1]} = g^{[1]}(z^{[1]})$

.

.

.

$A^{[L]} = g^{[L]}(z^{[L]})$

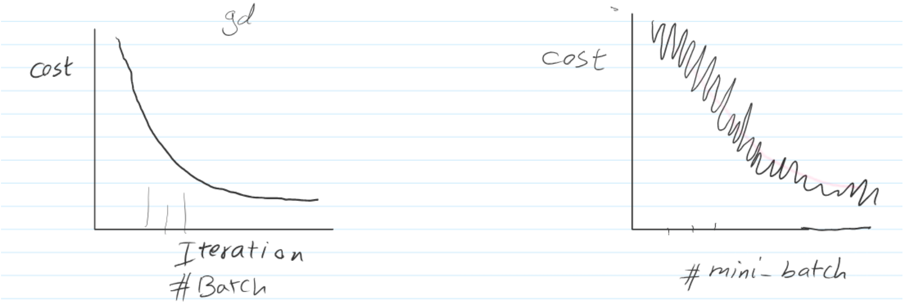
compute cost $J^{\{t\}} = \frac{1}{1000} \sum_{i=1}^{1000} L\left(\hat{y}^{(i)}, y^{(i)}\right) + \frac{\lambda}{2\times 1000} \sum_{l} \left\| w^{[l]} \right\|_F^2$

back propagation: $dw^{[l]}, db^{[l]}$

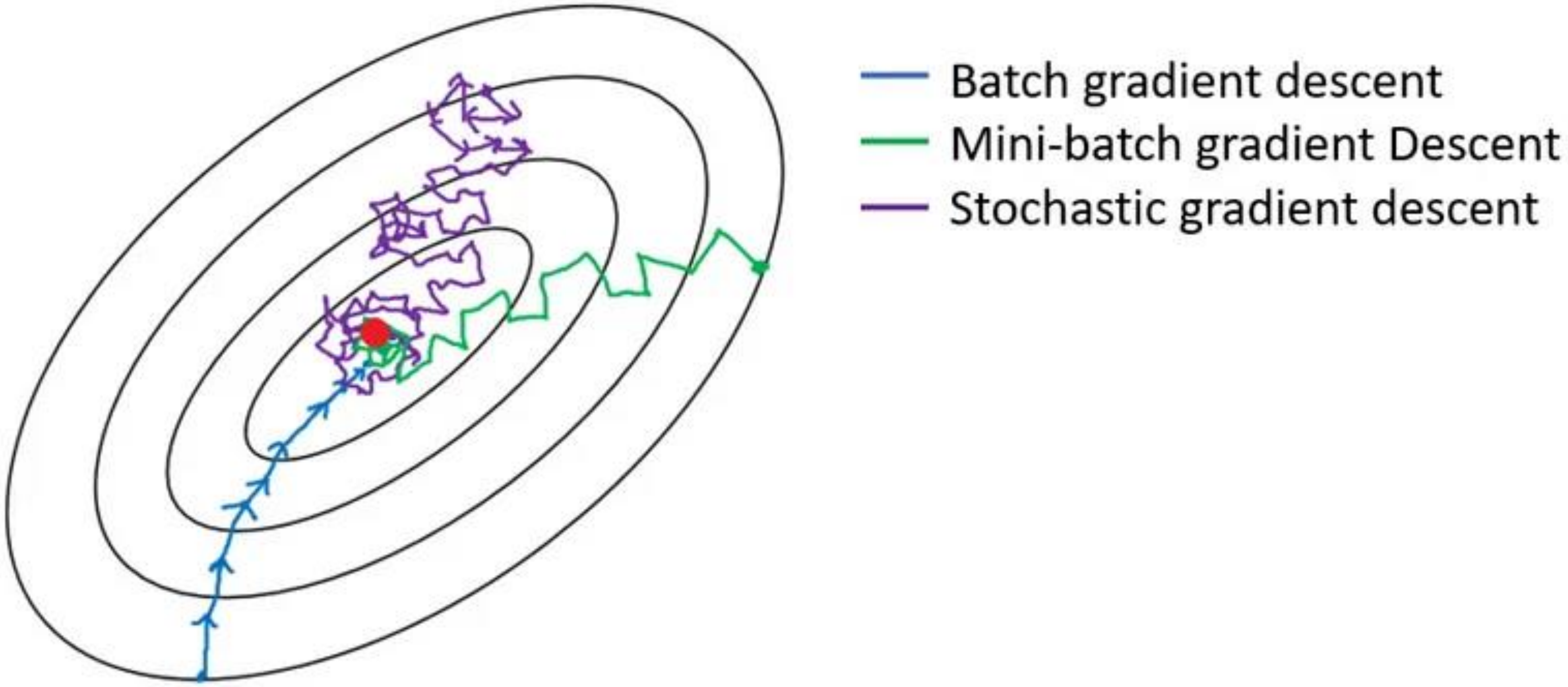
$w^{[l]} = w^{[l]} - \alpha dw^{[l]}, b^{[l]} = b^{[l]} - \alpha db^{[l]}$

}

} Until covergence



Mini-batch GD



https://medium.com/analytics-vidhya/gradient-descent-vs-stochastic-gd-vs-mini-batch-sgd-fbd3a2cb4ba4

Mini-batch GD

- min-batch size=1 stochastic gradient descent (SED)

$\{x^{(1)}, x^{(2)}, \dots, x^{(1000)}\}$

$$\frac{1}{100}\sum_{i=1}^{100} L\left(y^{(i)}, \hat{y}^{(i)}\right) = \frac{1}{1000} \times 1000\ L\left(y^{(1)}, \hat{y}^{(1)}\right)$$

$x^{(1)} = x^{(2)} = \cdots = x^{(1000)}$

1000 epoch

$\left.\begin{matrix} \boldsymbol{x}^{(1)} \\ \boldsymbol{x}^{(2)} \\ \vdots \\ \boldsymbol{x}^{(1000)} \end{matrix}\right\} w^{(1)}$updated

$w^{(1)} = w^{*(\mathbf{1})}$

$\left.\begin{matrix} \boldsymbol{x}^{(1)} \\ \boldsymbol{x}^{(2)} \\ \vdots \\ \boldsymbol{x}^{(1000)} \end{matrix}\right\} w^{(2)}$updated

$\equiv$

$\vdots$

$\left.\begin{matrix} \boldsymbol{x}^{(1)} \\ \boldsymbol{x}^{(2)} \\ \vdots \\ \boldsymbol{x}^{(1000)} \end{matrix}\right\} w^{(1000)}$updated

SGD:

$x^{(1)} \rightarrow w^{*(\mathbf{1})}$ updated

$x^{(\mathbf{2})} \rightarrow w^{*(\mathbf{2})}$ updated

$x^{(\mathbf{3})} \rightarrow w^{*(\mathbf{3})}$ updated

$\vdots$

$x^{(\mathbf{1000})} \rightarrow w^{*(\mathbf{1000})}$ updated

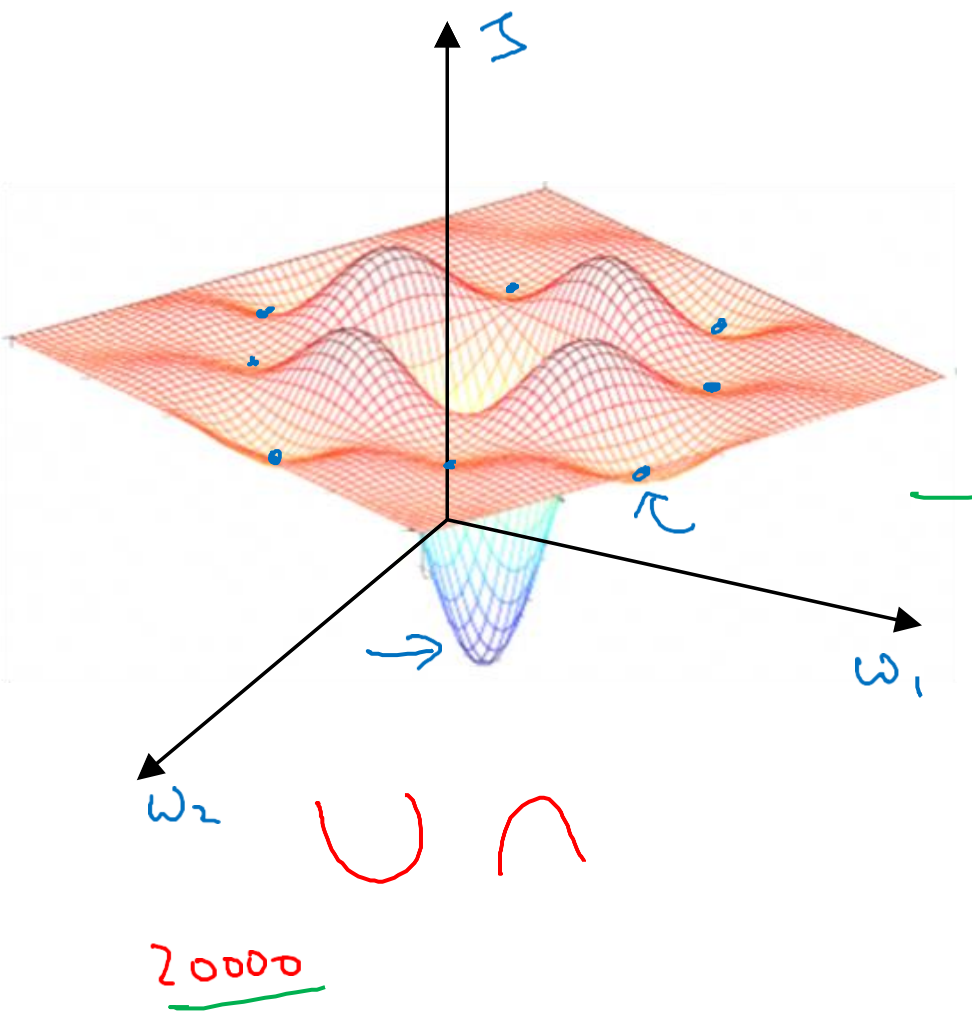
Mini-batch GD

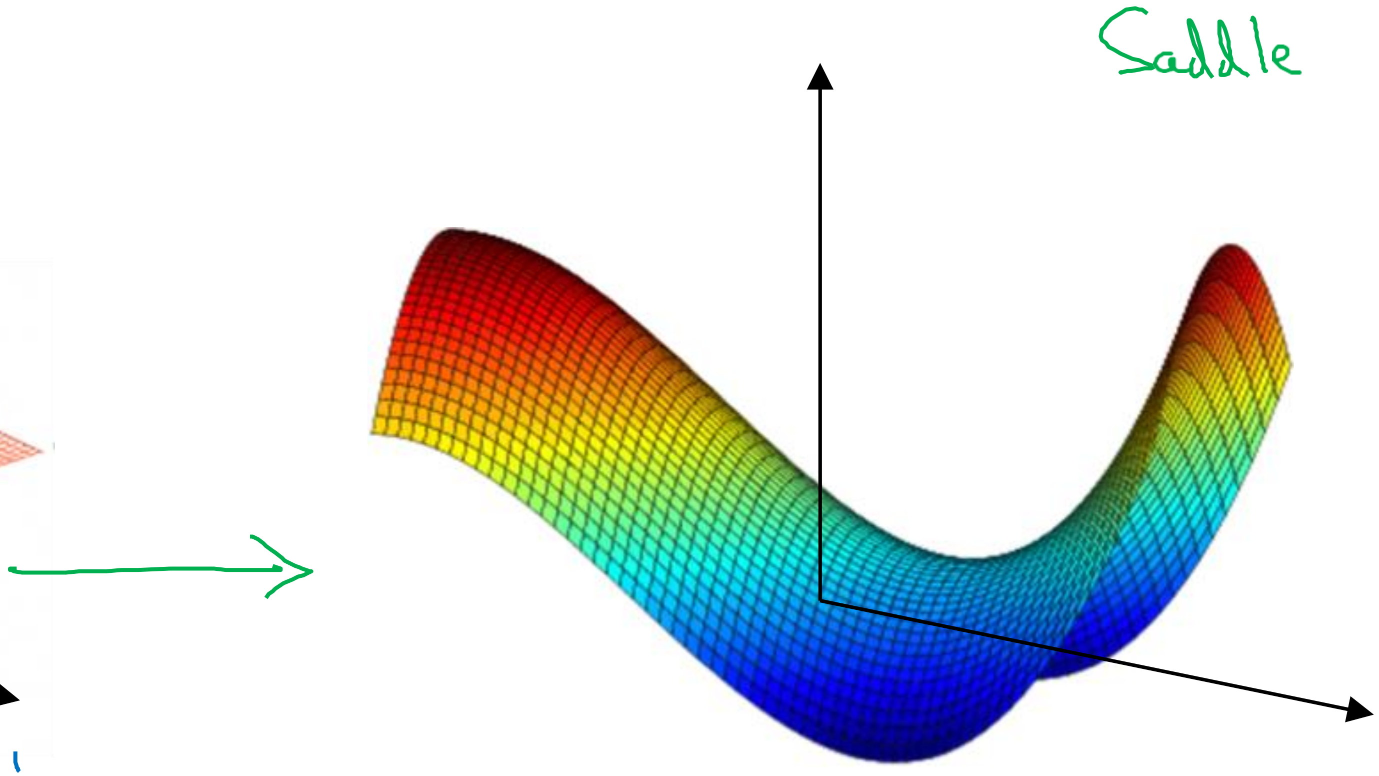
SGD	mini-batch GD	Batch GD
Noisy Lack of vectorization	Fastest learning	too long per iteration

$m \leq 2000$ Full batch

#mini batch sizes : 64, 128, 256, 512

Local optima in neural networks



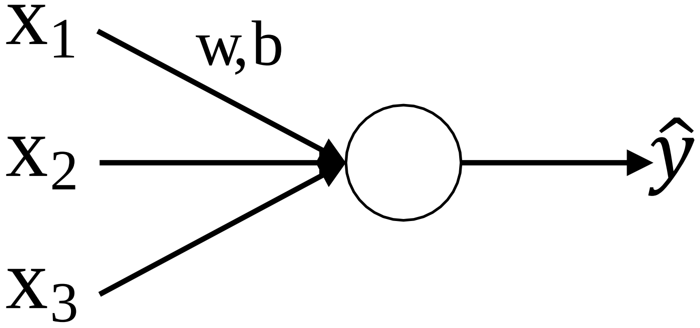


Batch Normalization

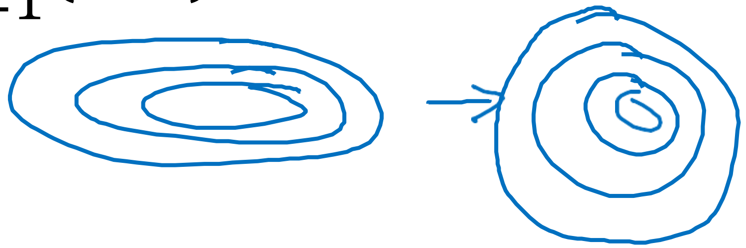
Normalizing activations in a network

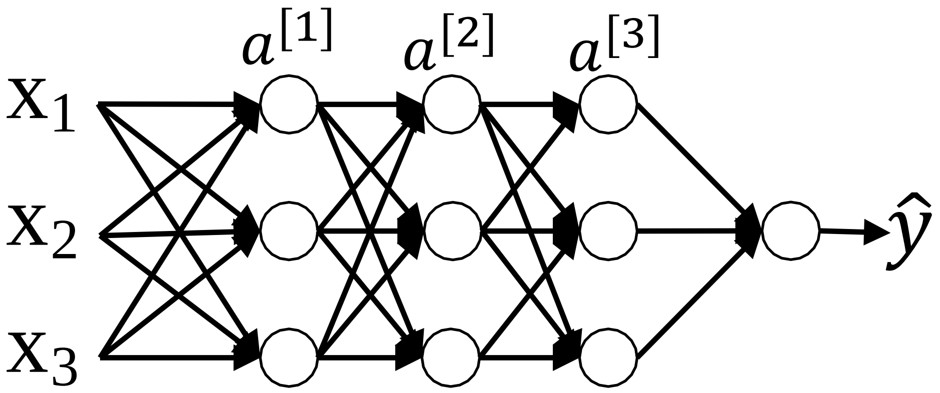
Batch Normalization

Normalizing inputs to speed up learning



- $\mu = \frac{1}{m}\sum_{i=1}^{m} x^{i}$
- $X = X - \mu$
- $\sigma^2 = \frac{1}{m}\sum_{i=1}^{m}(x^{(i)})^2$
- $X = X/\sigma^2$





Batch Normalization

In an intermediate layer: Implementing Batch Norm

- $\mu = \frac{1}{m}\sum_{i=1}^{m} z^{(i)}$
- $\sigma^2 = \frac{1}{m}\sum_{i=1}^{m}(z^{(i)} - \mu)^2$
- $Z_{norm}^{(i)} = \frac{z^{(i)} - \mu}{\sqrt{\sigma^2 + \varepsilon}}$

$$\gamma = \sqrt{\sigma^2 + \varepsilon}$$

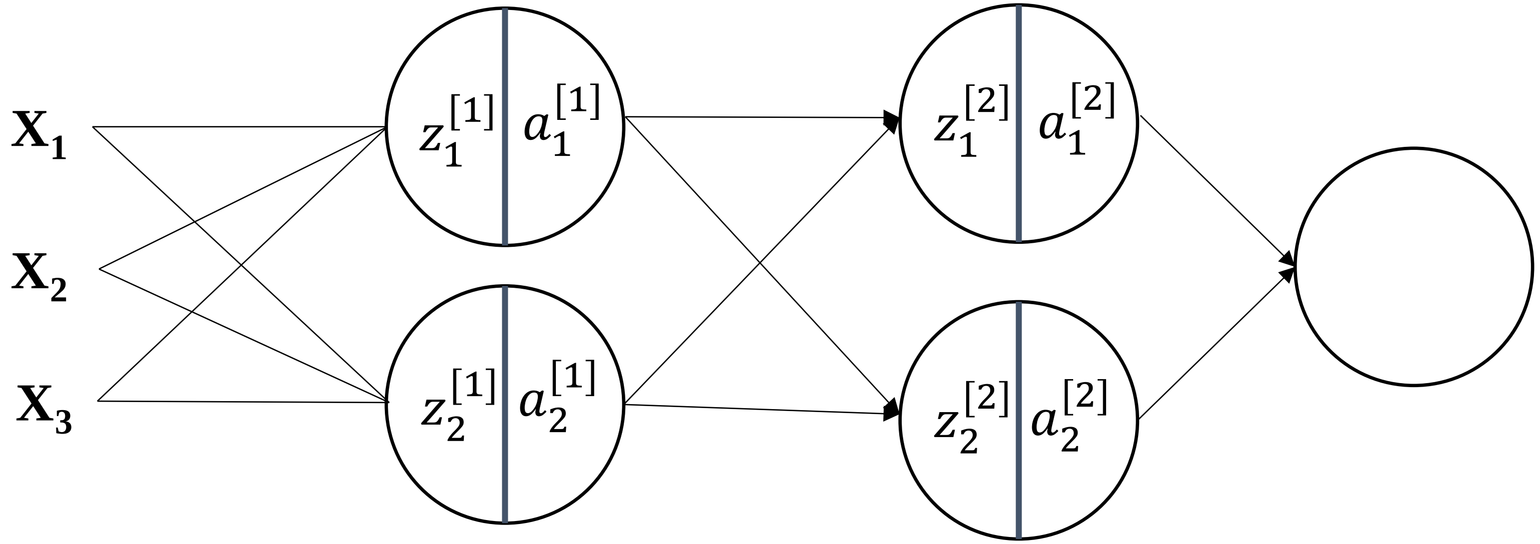
$$\beta = \mu$$

$$\tilde{z}^{(i)} = \gamma Z_{norm}^{(i)} + \beta$$

Learnable Parameters

Batch Normalization

Fitting Batch Norm into a neural network



- $X \xrightarrow{w^{[1]}, b^{[1]}} z^{[1]} \xrightarrow[\text{Batch Norm (BN)}]{\beta^{[1]}, \gamma^{[1]}} \tilde{Z}^{[1]} \longrightarrow a^{[1]} = g^{[1]}(\tilde{Z}^{[1]}) \xrightarrow{w^{[2]}, b^{[2]}} Z^{[2]} \xrightarrow[\text{BN}]{\beta^{[2]}, \gamma^{[2]}} \tilde{Z}^{[2]} \longrightarrow a^{[2]}$

  $w^{[1]}, b^{[1]}, w^{[2]}, b^{[2]}, \dots, w^{[L]}, b^{[L]}$

  $\beta^{[1]}, \gamma^{[1]}, \beta^{[2]}, \gamma^{[2]}, \dots, \beta^{[L]}, \gamma^{[L]}$

  $d\beta^{[l]}$ $\quad\quad$ $\beta^{[l]} = \beta^{[l]} - \alpha d\beta^{[l]}$

Batch Normalization

Working with mini-batches

- $z^{[l]} = w^{[l]} a^{[l-1]} + \cancel{b^{[l]}}$

  $z^{[l]} = w^{[l]} a^{[l-1]}$

  $z_{norm}^{[l]}$

  $\tilde{z}^{[l]} = \gamma^{[l]} z_{norm}^{[l]} + \beta^{[l]}$

  $w^{[l]}, \underbrace{\beta^{[l]}}_{(n^{[l]} \times 1)}, \underbrace{\gamma^{[l]}}_{(n^{[l]} \times 1)}$

Implementing gradient descent

- for t=l ... #num MiniBatches

  forward Propagation on $X^{\{t\}}$

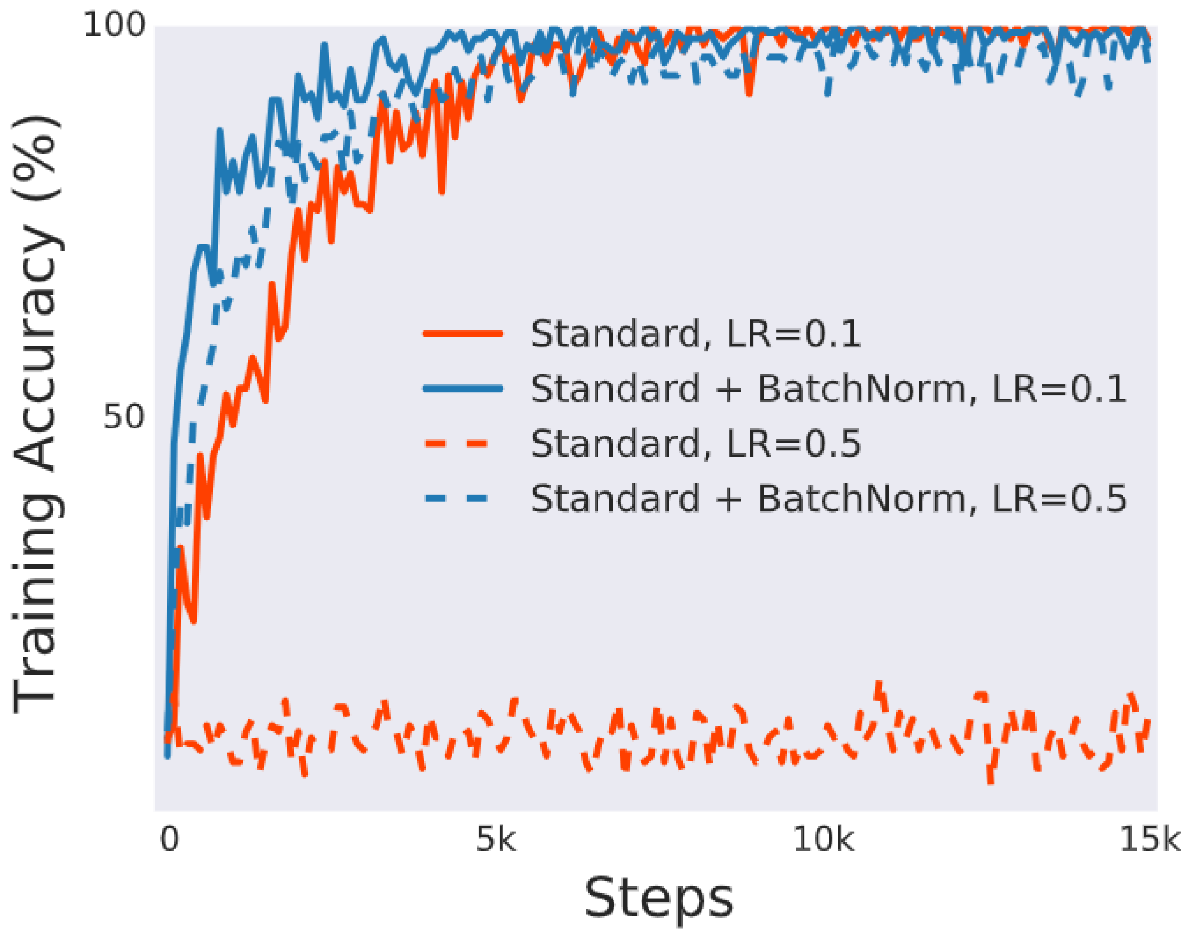
  use BN in each layer to compute $\tilde{z}^{[l]}$

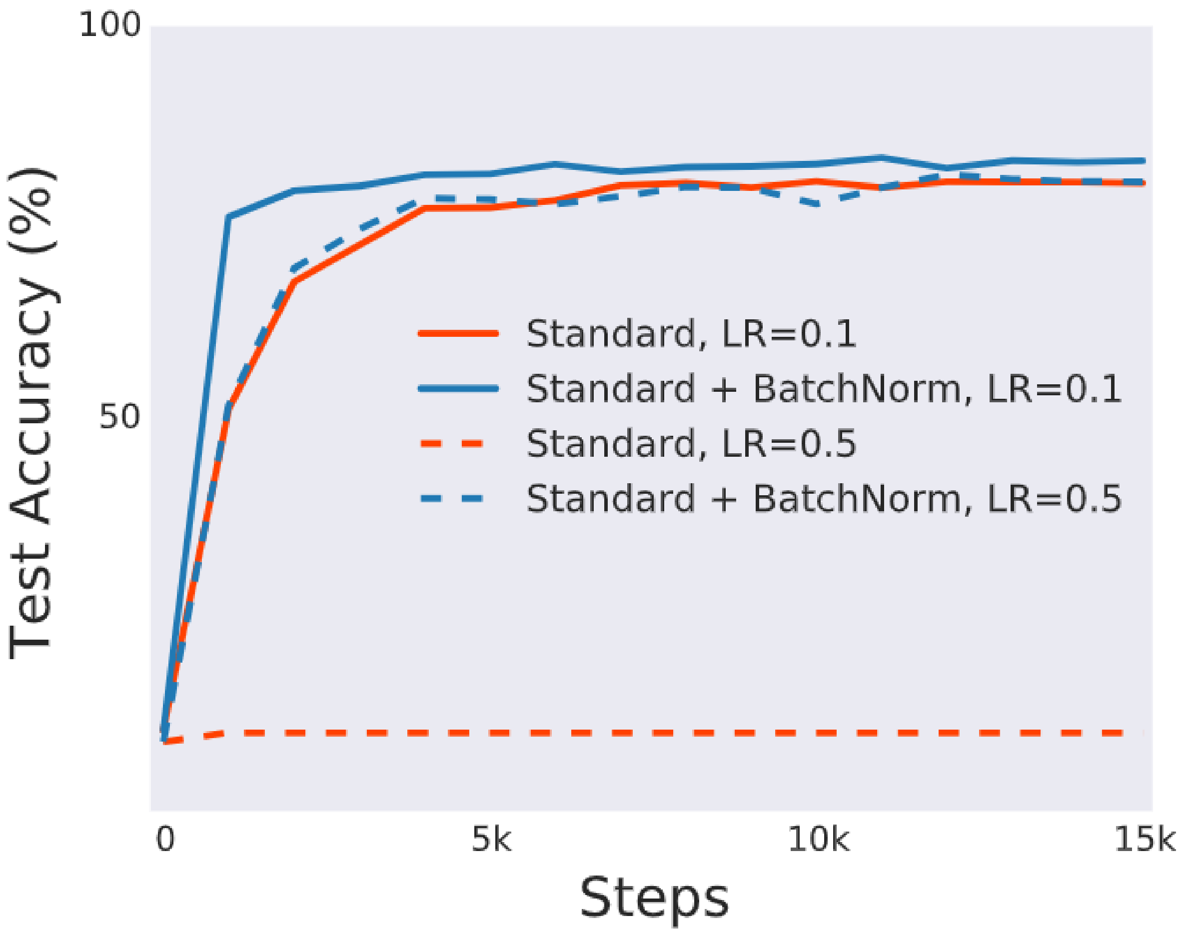
  use Backprop to compute $dw^{[l]}, d\beta^{[l]}, d\gamma^{[l]}$

  update parameters:

$$w^{[l]} = w^{[l]} - \alpha dw^{[l]}$$
$$\beta^{[l]} = \beta^{[l]} - \alpha d\beta^{[l]}$$
$$\gamma^{[l]} = \gamma^{[l]} - \alpha d\gamma^{[l]}$$
$$.$$
$$.$$
$$.$$

Batch normaliztion





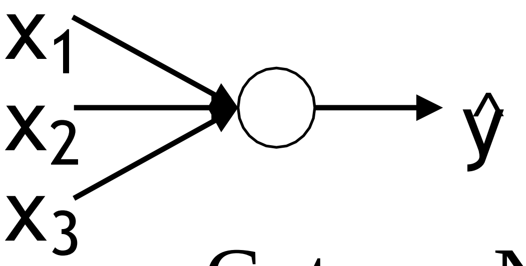
VGG network on CIFAR

https://arxiv.org/pdf/1805.11604, How Does Batch Normalization Help Optimization?

How Does Batch Normalization Help Optimization?

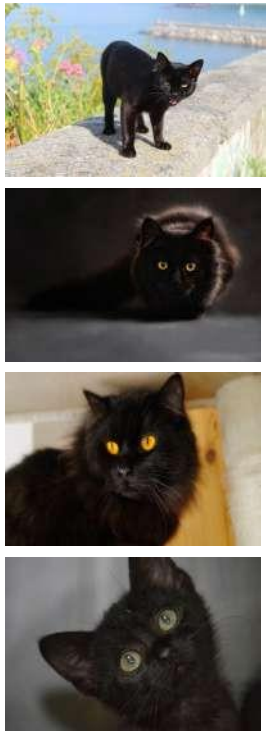
- prevention of exploding or vanishing gradients
- robustness to different settings of hyperparameters such as learning rate and initialization scheme
- keeping most of the activations away from saturation regions of non-linearities
- Smoothing effect of batch norm; the loss changes at a smaller rate and the magnitudes of the gradients are smaller too.
- the key implication of Batch Norm’s reparametrization is that it makes the gradients more reliable and predictive.

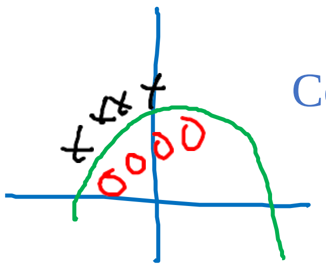
Learning on shifting input distribution



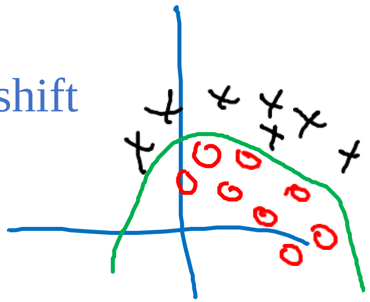
Cat | Non-Cat

$y = 1$ | $y = 0$





Covariate shift

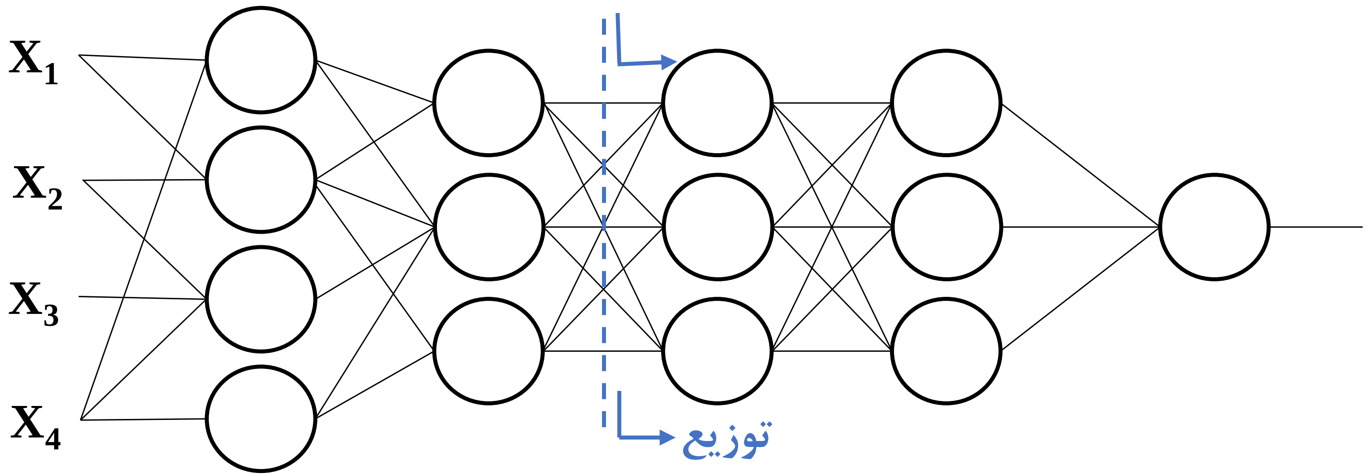


$y = 1$ | $y = 0$

"Covariate shift"

$X \rightarrow y$

Why this is a problem with neural networks?



https://arxiv.org/pdf/1805.11604

Batch Norm as regularization

- Each mini-batch is scaled by the mean/variance computed on just that mini-batch.
- This adds some noise to the values $z^{[l]}$ within that minibatch. So similar to dropout, it adds some noise to each hidden layer's activations.
- This has a slight regularization effect.

$\tilde{z} = \frac{z-\mu}{\sigma}$

$\bar{z} \times \varepsilon$ slight regularization

Batch Norm at test time

- $\mu = \frac{1}{m}\sum_i z^i$
- $\sigma^2 = \frac{1}{m}\sum_i \left(z^i - \mu\right)^2$
- $z_{norm}^i = \frac{z^i - \mu}{\sqrt{\sigma^2 + \varepsilon}}$
- $\tilde{z}^i = \underbrace{\gamma}_{scale} \; z_n^i + \underbrace{\beta}_{shift}$
- $\mu \xrightarrow{\text{در زمان آموزش}}$
  $\sigma^2 \longrightarrow$

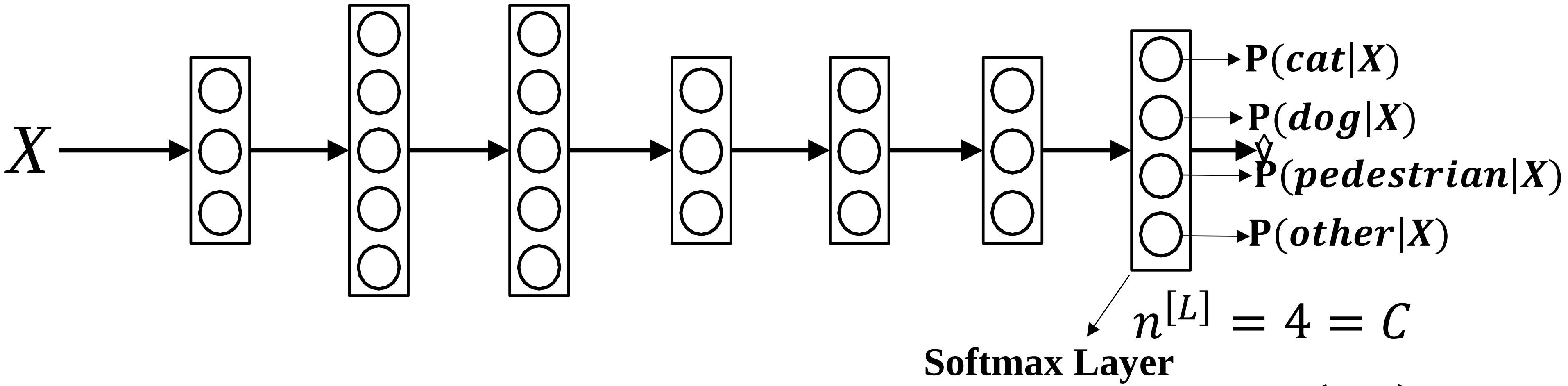
$$\mu_1^{[l]}, \mu_2^{[l]}, \mu_3^{[l]} \ldots \rightarrow moving\ Average$$

$$Z_{norm}^{[l]} = \frac{Z^{[l]} - \mu^{[l]}}{\sqrt{{\sigma^{[l]}}^2 + \varepsilon}}$$

Multi-class classification

Softmax regression

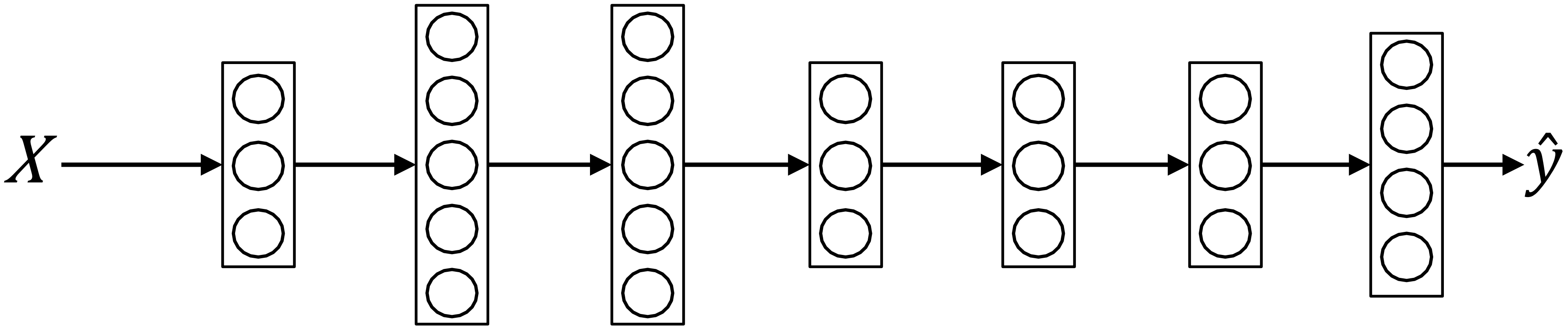
- C=#classes=4



$n^{[L]} = 4 = C$

$\hat{y}: (4,1)$

Softmax layer



- $z^{[L]} = w^{[L]} a^{[L-1]} + b^{[L]}$
- Softmax Activation function

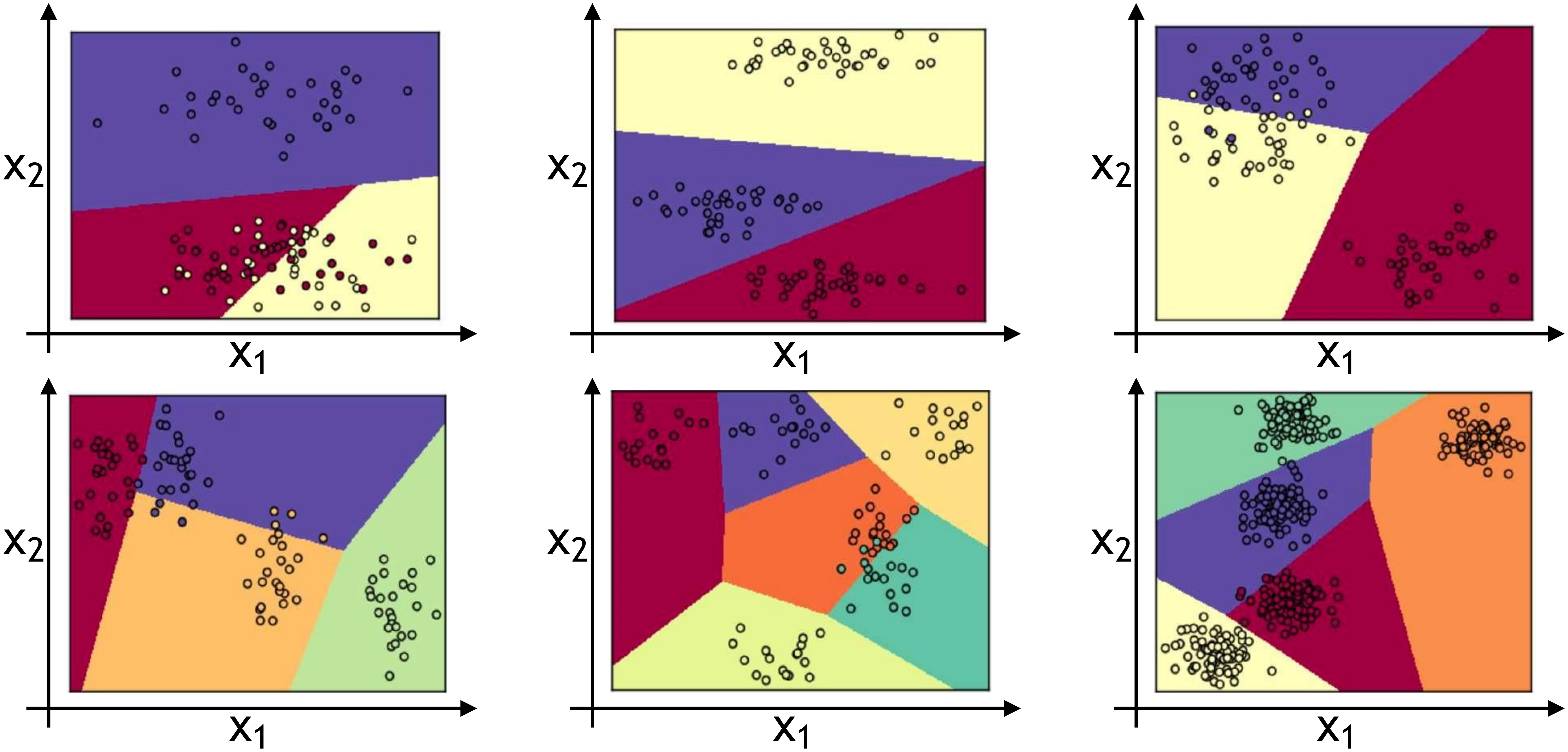
$t = e^{(z^{[L]})} \qquad (4,1)$

$(4,1) \quad a^{[L]} = \frac{e^{z^{[L]}}}{\sum_{j=1}^{4} tj} \Rightarrow a_i^{[L]} = \frac{t_i}{\sum_{j=1}^{4} tj}$

$a^{[L]} = g^{[L]}\left(z^{[L]}\right)$

Softmax Activation

Softmax examples



Softmax examples

- $z^{[L]} = \begin{bmatrix} 5 \\ 2 \\ -1 \\ 3 \end{bmatrix}$ $\qquad t = \begin{bmatrix} e^5 \\ e^2 \\ e^{-1} \\ e^3 \end{bmatrix} = \begin{bmatrix} 148.4 \\ 7.4 \\ 0.4 \\ 20.1 \end{bmatrix}$

$$\sum_{j=1}^{4} t_j = 176.3$$

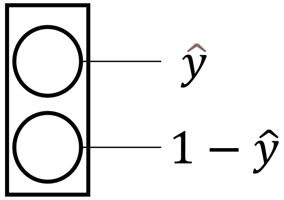
$$a^{[L]} = \frac{t}{176 \cdot 3}$$

$$\frac{1}{176 \cdot 3} \begin{bmatrix} 148.4 \\ 7.4 \\ 0.4 \\ 20.1 \end{bmatrix} = \begin{bmatrix} 0.842 \\ 0.042 \\ 0.002 \\ 0.114 \end{bmatrix} = \hat{y}^{(i)} \qquad \underset{(4,1)}{y^{(i)}} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

one hot vector

Loss function

- $-[y \log y + (1 - y) \log(1 - y)]$
- $L(\hat{y}, y) = -\sum_{j=1}^{4} y_j \log \hat{y}_j$



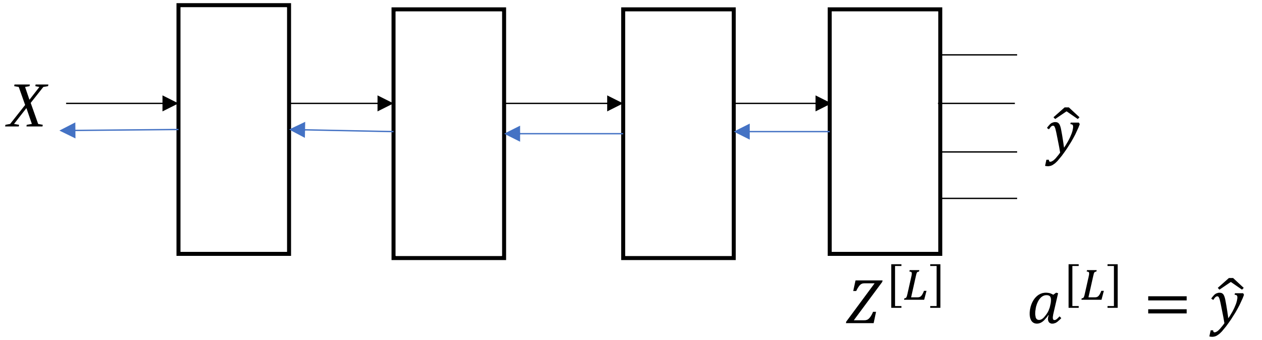
$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$-y_2 \log \hat{y}_2 = -\log \hat{y}_2$ $\qquad$ $\hat{y}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

$Y = [y^{(1)}, y^{(2)}, \dots, y^{(m)}]$ $\quad$ $\hat{Y} = [\hat{y}^{(1)}, \hat{y}^{(2)}, \dots, \hat{y}^{(m)}]$

$$\begin{bmatrix} 0 & 1 & 1 & 0 & \\ 1 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 1 & \end{bmatrix} \qquad \begin{bmatrix} 0.3 & \\ 0.2 & \\ 0.3 & \cdots \\ 0.2 & \end{bmatrix}$$

Gradient descent with softmax



$L(\hat{y}, y)$

- اثبات کنید:

$$dz^{[L]}_{4\times1} = \hat{y}_{4\times1} - y_{4\times1}$$