Deep Learning

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https://www.aparat.com/mehran.safayani

https://github.com/safayani/deep_learning_course

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Supervised Learning

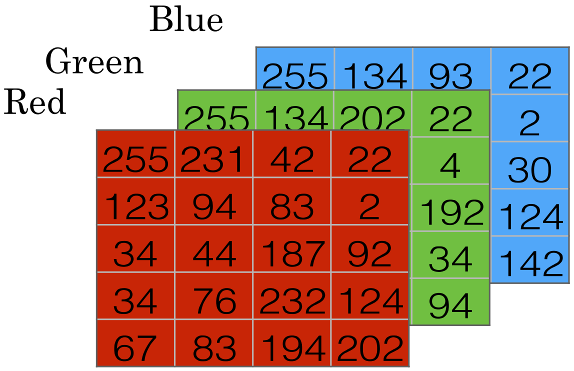
Input(x)	Output (y)	Application
Home features	Price	Real Estate
Ad, user info	Click on ad? (0/1)	Online Advertising
Image	Object (1,…,1000)	Photo tagging
Audio	Text transcript	Speech recognition
English	Chinese	Machine translation
Image, Radar info	Position of other cars	Autonomous driving

Basics of Neural Network Programming

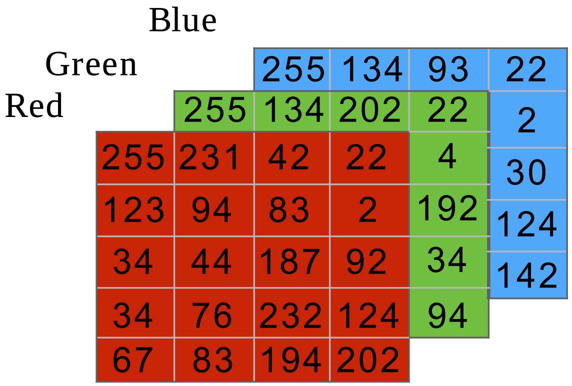
Binary Classification

Binary Classification

$\longrightarrow$ 1 (cat) vs 0 (non cat)



Binary classification



$$\vec{x} = \begin{bmatrix} 255 \\ 231 \\ \vdots \\ 254 \\ 253 \\ 250 \\ 220 \end{bmatrix} \qquad 64 \times 64 \times 3 = \underbrace{12288}_{n_x}$$

$$\vec{x} \rightarrow \boxed{\text{model}} \rightarrow \hat{y}$$

- Notation

$(\vec{x}, y) \qquad x \in R^{n_x},\ \ y \in \{0,1\}$

Binary classification

- m training example: $\{(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}, \dots, (x^{(m)}, y^{(m)})\}$

- $X = \underbrace{\left[\begin{array}{cccc} | & | & & | \\ x^{(1)} & x^{(2)} & \dots\dots\dots & x^{(\mathrm{m})} \\ | & | & & | \end{array}\right]}_{\mathrm{m}} \updownarrow n_x$

$X \in \mathbb{R}^{n_x \times m}$

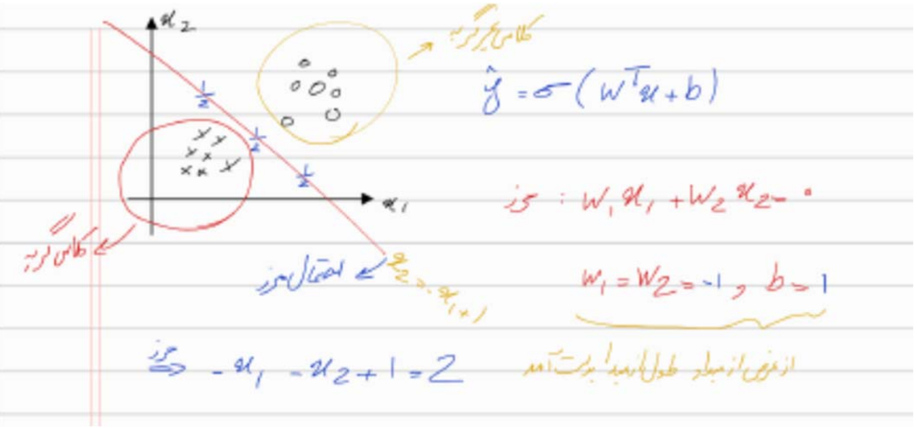
X.shape $= (n_x, m)$

$Y = [y^{(1)}, y^{(2)}, \dots, y^{(m)}]$

$Y \in \mathbb{R}^{1 \times m}$

Y.shape $= (1, m)$

Logistic Regression

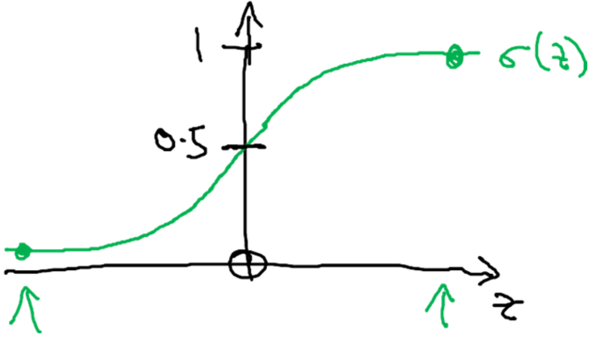

x_2
x_1
کلاس بزرگتر
کلاس کوچکتر
$\hat{y} = \sigma(w^T x + b)$
$\frac{1}{2}$
$\frac{1}{2}$
$\frac{1}{2}$
مرز : $w_1 x_1 + w_2 x_2 = 0$
احتمال مرز
$x_2 = -x_1 + 1$
$w_1 = w_2 = -1 , b = 1$
مرز $\Rightarrow -x_1 - x_2 + 1 = z$
از فرض از مبدا طول از مبدا بدست آمد

- Given x , output $\hat{y} = P(y=1|x)$ $0 \leq \hat{y} \leq 1$

$x \in \mathbb{R}^{n_x}$ parameters: $w \in \mathbb{R}^{n_x}$, $b \in \mathbb{R}$

$$\hat{y} = w^T x + b$$

$$\hat{y} = \sigma(\overbrace{w^T x + b}^{z})$$



Sigmoid function $\sigma(z) = \frac{1}{1+e^{-z}}$

if z large $\sigma(z) \approx 1$

if z large negative $\sigma(z) \approx 0$

Logistic Regression

- $\hat{y} = \sigma(\underbrace{w^T x + b}_{z})$

$$\hat{y} = w^T x$$

$x_0 = 1 \, , x \in \mathbb{R}^{n_x + 1}$

$$W = \begin{bmatrix} b = w_0 \\ w_1 \\ w_2 \\ w_3 \\ . \\ . \\ . \\ w_{n_x} \end{bmatrix} \begin{matrix} \}\; b \\ \left.\begin{matrix} \\ \\ \\ \\ \\ \\ \\ \end{matrix}\right\} w \end{matrix}$$

Logistic Regression cost function

- Loss (error) function: $L(\hat{y}, y) = \frac{1}{2}(\hat{y} - y)^2 = \frac{1}{2}\left(\sigma\left(w^T x + b\right) - y\right)^2$ SE: Square Error

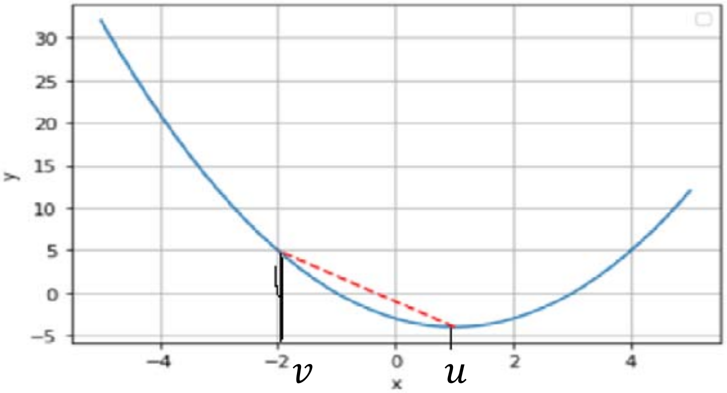
- What is the problem?

- .

Convexity

Function h(u) with u∈ X is convex if for any u, v ∈ X and for any $0 \leq \lambda \leq 1$ we have:

$$\mathbf{h(\lambda u + (1- \lambda)v) \leq \lambda\, h(u) + (1- \lambda)\, h(v)}$$



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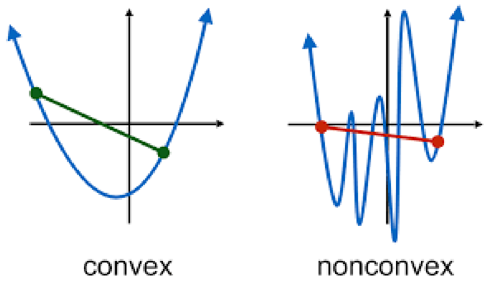


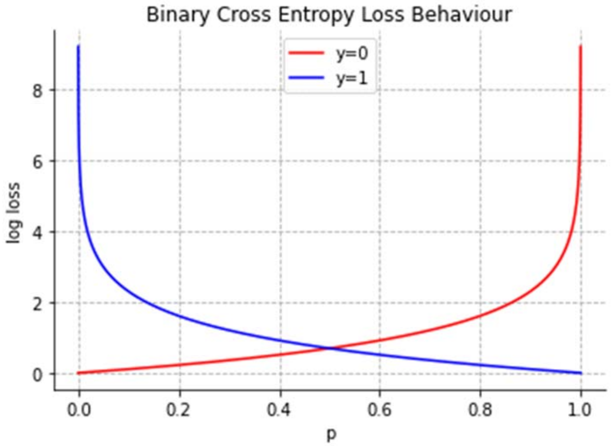
https://mlstory.org/optimization.html

Cross Entropy

- $L(\hat{y}, y) = -(y \log \hat{y} + (1-y)\log(1-\hat{y}))$

if $y=1$: $L(\hat{y}, y) = -\log \hat{y}$

if $y=0$: $L(\hat{y}, y) = -\log(1-\hat{y})$



https://datamonje.com/classification-loss-functions/

- Cost function: $J(w,b) = \frac{1}{m}\sum_{i=1}^{m} L\left(y^{(i)}, \hat{y}^{(i)}\right)$

$$= -\frac{1}{m}\sum_{i=1}^{m} [y^{(i)}\log \hat{y}^{(i)} + (1-y^{(i)})\log(1-\hat{y}^{(i)})]$$

Gradient Descent

Cost Function

$$\underset{b, w_1}{\textbf{Minimize}}\; J(b, w_1)$$

If $J(w_1) = (w_1 - 2)^2$

$$\frac{dJ(w_1)}{dw_1} = 0 \;\Longrightarrow\; \frac{dJ(w_1)}{dw_1} = 2\,(w_1 - 2) = 0 \;\Longrightarrow\; w_1 = 2$$

Gradient Descent

Minimize $\boldsymbol{J}(\boldsymbol{b}, \boldsymbol{w_1})$

$\quad \boldsymbol{b}, \boldsymbol{w_1}$

Minimize $\boldsymbol{J}(\boldsymbol{b}, \boldsymbol{w_1}, \dots, \boldsymbol{w_n})$

$\quad \boldsymbol{b}, \boldsymbol{w_1}, \dots, \boldsymbol{w_n}$

Repeat until convergence: $\{$

b= $\boldsymbol{w_0}$

For j=0,…,n

$$\boldsymbol{w_j} = \boldsymbol{w_j} - \alpha \frac{d\boldsymbol{J}(\boldsymbol{b}, \boldsymbol{w_1}, \dots, \boldsymbol{w_n})}{d\boldsymbol{w_j}}$$

$\}$

α **is learning rate**

Updating all w_j ***Simultaneously***

Convergence condition:

$$\|W^{t+1} - W^{t}\|_2 \leq \varepsilon$$

Gradient Descent

Correct form

temp0 = $\boldsymbol{b} - \alpha \frac{d\boldsymbol{J}(\boldsymbol{b}, \boldsymbol{w_1})}{d\boldsymbol{b}}$

temp1 = $\boldsymbol{w_1} - \alpha \frac{d\boldsymbol{J}(\boldsymbol{b}, \boldsymbol{w_1})}{d\boldsymbol{w_1}}$

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$\boldsymbol{b}$ = temp0

$\boldsymbol{w_1}$ = temp1

✓

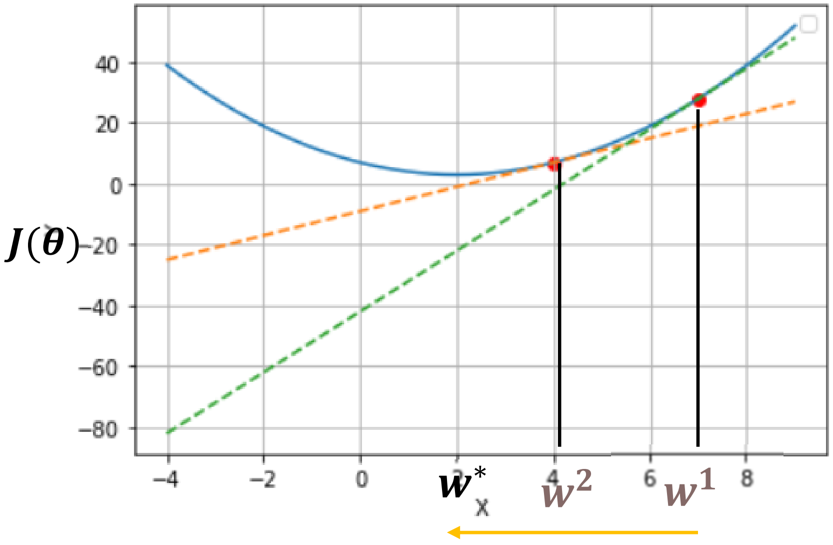
Incorrect form

$$\boldsymbol{b} = \boldsymbol{b} - \alpha \frac{d\boldsymbol{J}(\boldsymbol{b}\ , \boldsymbol{w_1})}{d\boldsymbol{b}}$$

$$\boldsymbol{w_1} = \boldsymbol{w_1} - \alpha \frac{d\boldsymbol{J}(\boldsymbol{b}, \boldsymbol{w_1})}{d\boldsymbol{w_1}}$$

✗

Gradient Descent



خطوط مماس نشان داده شده دارای شیب یا مشتق مثبت هستند.

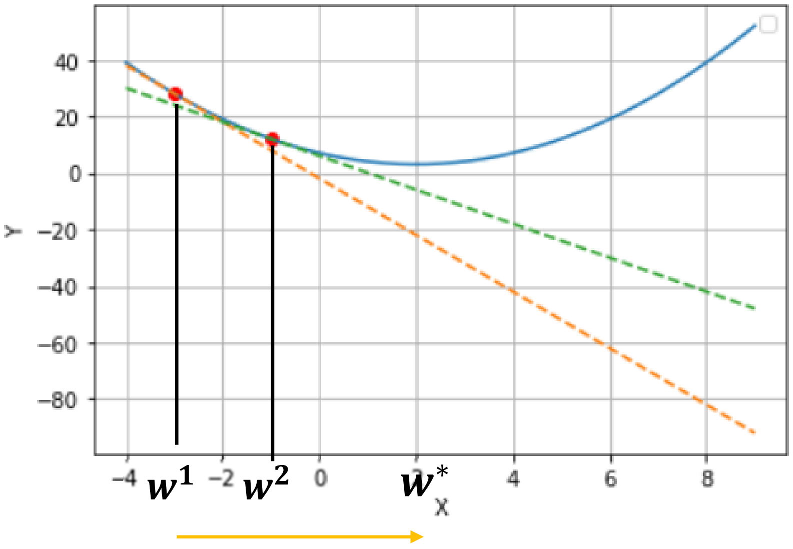
درنتیجه:

$$\frac{dJ(w^1)}{dw^1} > 0\,,\ \ \alpha > 0 \ \Rightarrow\ \alpha\frac{dJ(w^1)}{dw^1} > 0$$

$$\Rightarrow w^2 = w^1 - \alpha d w^1$$

w کوچکتر میشود و به سمت چپ حرکت میکنیم.

Gradient Descent



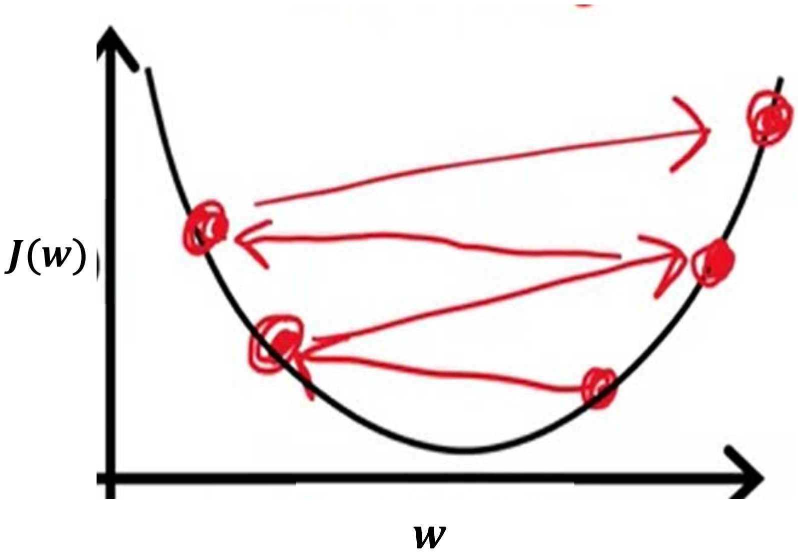
خطوط مماس نشان داده شده دارای شیب یا مشتق مثبت هستند.
درنتیجه:

$$\frac{dJ(w^1)}{dw^1} < 0\ ,\ \alpha > 0 \quad ➡ \quad \alpha\frac{dJ(w^1)}{dw^1} < 0$$

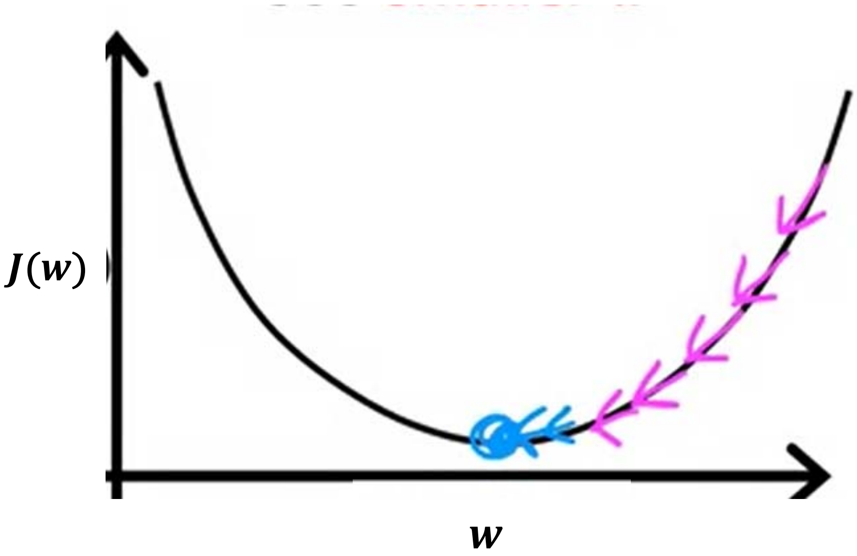
$$➡ \quad w^2 = w^1 - \alpha d w^1$$

w بزرگتر میشود و به سمت راست حرکت میکنیم.

Choosing Learning Rate

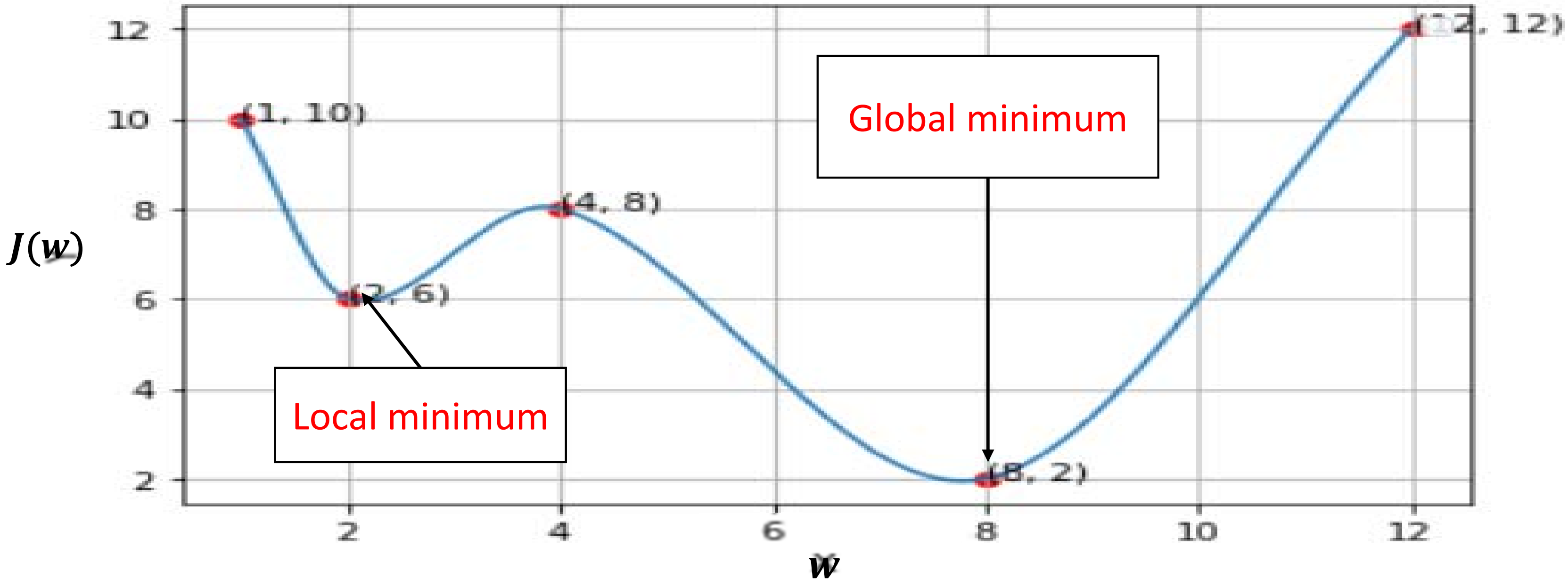


α is too large

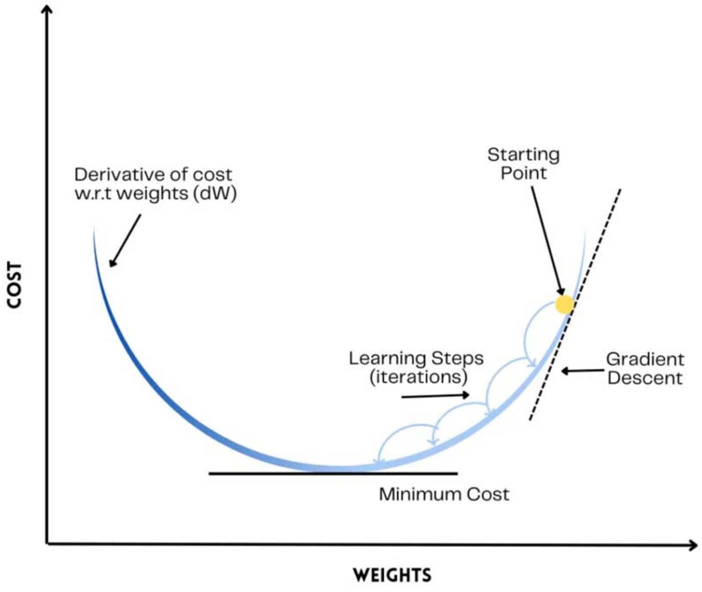


α is small

Gradient Descent Weakness



Gradient Descent


Derivative of cost w.r.t weights (dW)
Starting Point
Learning Steps (iterations)
Gradient Descent
Minimum Cost
COST
WEIGHTS

- 1) $\alpha > 0$

$$\text{Repeat}\{$$

$$\text{w} = \text{w} - \alpha \underbrace{\frac{\text{dJ(w)}}{\text{d(w)}}}_{\text{dw}}$$

$$\}\text{until convergence}$$

$$\text{w} = \text{w- } \alpha\text{dw}$$

$$z = w^T x + b$$

- $\hat{y} = a = \sigma(z)$
- $L(a, y) = -(y \log a + (1 - y) \log(1 - a))$

https://www.askpython.com/

Gradient Descent

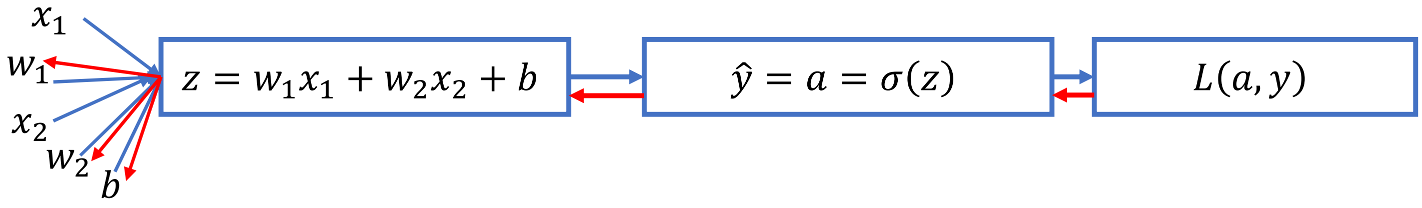
Computational Graph

$$L(a,y) = -(y\log a + (1-y)\log(1-a))$$

$$da = \frac{dL}{da} = \frac{-y}{a} + \frac{1-y}{1-a} = \frac{a-y}{a(1-a)}$$

$$\frac{da}{dz} = \acute{\sigma}(z) = \underbrace{\sigma(z)}_{a}\,\underbrace{(1-\sigma(z))}_{1-a}$$

$$a = \sigma(z) = \frac{1}{1+e^{-z}}$$



$$dz = \frac{dL}{dz} = \frac{dL(a,y)}{dz} = \frac{dL}{da} \times \frac{da}{dz} = \frac{a-y}{a(1-a)} \times a(1-a) = a-y$$

$$dw_1 = \frac{dL}{dw_1} = \frac{dL}{\underbrace{dz}_{dz}} \times \underbrace{\frac{dz}{dw_1}}_{x_1} = x_1\,dz$$

$$dw_2 = x_2\,dz$$

$$db = \frac{dL}{db} = \frac{dL}{dz} \times \frac{\overbrace{dz}^{1}}{db} = dz$$

Gradient Descent

- $J(w,b) = \frac{1}{\mathrm{m}} \sum_{\mathrm{i=1}}^{\mathrm{m}} \mathrm{L}\left(a^{(\mathrm{i})}, \mathrm{y}^{(\mathrm{i})}\right)$
- $a^{(i)} = \hat{y}^{(i)} = \sigma\left(z^{(i)}\right) = \sigma\left(wx^{(i)} + b\right)$

- $dw_j = \frac{1}{\mathrm{m}} \sum_{\mathrm{i=1}}^{\mathrm{m}} \frac{d\mathrm{L}\left(a^{(\mathrm{i})},\mathrm{y}^{(\mathrm{i})}\right)}{dw_j}$

Logistic regression on m examples

```
$J = 0$;   $dw_1 = 0$;   $dw_2 = 0$;   $db = 0$;
$w_1 \leftarrow \text{random}$   $w_2 \leftarrow \text{random}$   $b \leftarrow \text{random}$
Repeat{
    For   i=1   to   m
        $z^{(i)} = w^T x^{(i)} + b$
        $a^{(i)} = \sigma\left(z^{(i)}\right)$
        $J \mathrel{+}= \left[y^{(i)} Log a^{(i)} + \left(1 - y^{(i)}\right) Log\left(1 - a^{(i)}\right)\right]$
        $dz^{(i)} = a^{(i)} - y^{(i)}$
        $dw_1 \mathrel{+}= x_1^{(i)}\, dz^{(i)}$   $dw_2 \mathrel{+}= x_2^{(i)}\, dz^{(i)}$   $db \mathrel{+}= dz^{(i)}$

$J /= m$;   $dw_1 /= m$;   $dw_2 /= m$;   $db /= m$;

$w_1 = w_1 - \alpha\, dw_1$   $w_2 = w_2 - \alpha\, dw_2$   $b = b - \alpha\, db$
    } until convergence
```

$$w^t = \begin{bmatrix} w_1^t \\ w_2^t \\ b^t \end{bmatrix} \quad w^{t+1} = \begin{bmatrix} w_1^{t+1} \\ w_2^{t+1} \\ b^{t+1} \end{bmatrix}$$

$$\|w^{t+1} - w^t\|_2 \leq \varepsilon$$

$$dw = \begin{bmatrix} dw_1 \\ dw_2 \\ db \end{bmatrix}$$

$$\|dw\| \leq \varepsilon = 10^{-4}$$

What's wrong with the code?

Logistic regression on m examples

```
$w_1 \leftarrow \text{random}$    $w_2 \leftarrow \text{random}$    $b \leftarrow \text{random}$
Repeat{
$J = 0;$    $dw_1 = 0;$    $dw_2 = 0;$    $db = 0;$
        For    i=1    to    m
                $z^{(i)} = w^T x^{(i)} + b$
                $a^{(i)} = \sigma\left(z^{(i)}\right)$
                $J \mathrel{+}= \left[y^{(i)} Log a^{(i)} + \left(1 - y^{(i)}\right) Log(1 - a^{(i)})\right]$
                $dz^{(i)} = a^{(i)} - y^{(i)}$
                $dw_1 \mathrel{+}= x_1^{(i)}\, dz^{(i)}$    $dw_2 \mathrel{+}= x_2^{(i)}\, dz^{(i)}$    $db \mathrel{+}= dz^{(i)}$

$J/=m;$    $dw_1/=m;$    $dw_2/=m;$    $db/=m;$

$w_1 = w_1 - \alpha\, dw_1$    $w_2 = w_2 - \alpha\, dw_2$    $b = b - \alpha\, db$
        } until convergence
```

$$w^t = \begin{bmatrix} w_1^t \\ w_2^t \\ b^t \end{bmatrix} \quad w^{t+1} = \begin{bmatrix} w_1^{t+1} \\ w_2^{t+1} \\ b^{t+1} \end{bmatrix}$$

$$\|w^{t+1} - w^t\|_2 \le \varepsilon$$

$$dw = \begin{bmatrix} dw_1 \\ dw_2 \\ db \end{bmatrix}$$

$$\|dw\| \le \varepsilon = 10^{-4}$$

Basics of Neural Network Programming

Vectorizing Logistic Regression’s Gradient Computation

Vectorizing Logistic Regression

- *z=0;*

  For i in range(n_x)

  $z \mathrel{+}= w[i] * x[i]$

  $z \mathrel{+}= b$

  õ²ý#E ²¯

- *z=0;*

  z =np.dot(w, x)+b

  ô²`#E ²¯

  SIMD

  GPU

Vectorizing Logistic Regression

$z^{(1)} = w^T x^{(1)} + b$ $z^{(2)} = w^T x^{(2)} + b$ $z^{(m)} = w^T x^{(m)} + b$

$a^{(1)} = \sigma\left(z^{(1)}\right)$ $a^{(2)} = \sigma\left(z^{(2)}\right)$ $a^{(m)} = \sigma\left(z^{(m)}\right)$

- $\mathrm{X} = \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \dots\dots\dots & x^{(\mathrm{m})} \\ | & | & & | \end{bmatrix} \in R^{n_x \times m}$ $\underbrace{[w_1 \dots w_{n_x}]}_{w^T} \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \dots\dots\dots & x^{(\mathrm{m})} \\ | & | & & | \end{bmatrix}$
- $Z = \left[\underbrace{z^{(1)}}_{w^T x^{(1)} + b} \quad \underbrace{z^{(2)}}_{w^T x^{(2)} + b} \quad \dots \quad \underbrace{z^{(m)}}_{w^T x^{(m)} + b} \right] = w^T X + [b\ b \cdots b]_{1 \times m}$
- $Z = \underbrace{np \cdot dot(w \cdot T, X)}_{1_{\times} m} + \underbrace{b}_{(1,1)}$ $"broadcasting"$ $[b, b \dots, b]_{1\ \times\ m}$

Vectorizing Logistic Regression

- $A = [a^{(1)}, a^{(2)}, \dots, a^{(m)}] = \sigma(\underbrace{Z}_{1\times m})$
- $dz^{(1)} = a^{(1)} - y^{(1)}$ $\quad dz^{(2)} = a^{(2)} - y^{(2)}$
- $dZ = [dz^{(1)}\ dz^{(2)} \dots\ dz^{(m)}]_{1\times m}$
- $A = [a^{(1)}\ a^{(2)} \dots\ a^{(m)}]$ $\quad Y = [y^{(1)}\ y^{(2)} \dots\ y^{(m)}]$
- $dZ = A - Y = [a^{(1)} - y^{(1)}\ \ a^{(2)} - y^{(2)}\ \dots\ a^{(m)} - y^{(m)}]$
    - $dw=0$ $\quad db=0$
- $\begin{bmatrix} dw_1 \\ dw_2 \\ dw_3 \\ \vdots \\ dw_{nx} \end{bmatrix}_{nx\times 1}$

$$\begin{cases} dw{+}{=}\ \overbrace{x^1}^{\text{بردار}}\ \ \overbrace{dz^1}^{\overset{\text{عدد}}{a^1-y^1}} \\ dw{+}{=}x^2dz^2 \\ dw{+}{=}x^m dz^m \end{cases} \qquad \begin{cases} db{+}{=}\ \overbrace{dz1}^{\overset{\text{عدد}}{a1-y1}} \\ db{+}{=}dz2 \\ db{+}{=}dz^m \end{cases}$$

$dw/{=}m$ $\quad db/{=}m$

$$db = \frac{1}{m}\sum_{i=1}^{m} dz^{(i)} = \frac{1}{m} np.sum(dz)$$

$$1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ \vdots \\ \vdots \\ 1 \end{bmatrix}_{m\times 1} \qquad db = \frac{1}{m} dz \times 1$$

$$dz = [dz^1\ \ dz^2 \dots\ dz^m]_{1\times m}$$

$$dw = \frac{1}{m}[x^{(1)}dz^1 + x^{(2)}dz^2 + \dots + x^{(m)}dz^m]$$

$$\boxed{dw = \frac{1}{m} X\, dz^T} = \frac{1}{m}\begin{bmatrix} x^{(1)} x^{(2)} & \dots\dots\dots & x^{(m)} \end{bmatrix} \times \begin{bmatrix} dz^1 \\ dz^2 \\ dz^3 \\ \vdots \\ \vdots \\ \vdots \\ dz^m \end{bmatrix}$$

Vectorizing Logistic Regression

- $w_1, w_2, b \leftarrow random$

For iter in range(1000)

$$\textbf{1 epoch}\left\{\begin{aligned}
&Z = W^T X + b = np \cdot dot(w.T, X) + b \\
&A = \sigma(Z) \\
&dZ = A - Y \\
&dw = \frac{1}{m} X\, dz^T \\
&db = \frac{1}{m} np.sum(dz) \\
&w = w - \alpha\, dw \\
&b = b - \alpha\, db
\end{aligned}\right.$$

$$w = np \cdot random \cdot \mathrm{rand}n(n_x, 1)$$

Logistic Regression Cost function

- $\hat{y} = \sigma(w^T x + b)$ $\qquad 0 < \sigma(z) = \frac{1}{1+e^{-z}} < 1$
- $\hat{y} = p(y = 1|x)$

$$\left.\begin{array}{l} \bullet\ if \quad y = 1 \ : \quad p(y|x) = \hat{y} \\ \bullet\ if \quad y = 0 \ : \quad p(y|x) = 1 - \hat{y} \end{array}\right\} \boldsymbol{p(y|x)}$$

- $\boxed{p(y|x) = \hat{y}^y (1 - \hat{y})^{1-y}}$ $\qquad$ *distribution? Bernoulli*
- $if \quad y = 1 \ : \quad p(y|x) = \hat{y}$
- $if \quad y = 0 \ : \quad p(y|x) = 1 - \hat{y}$
- $\mathrm{logp(y|x)} = \log[\hat{y}^y (1 - \hat{y})^{1-y}] = y \log \hat{y} + (1 - y) \log(1 - \hat{y})$

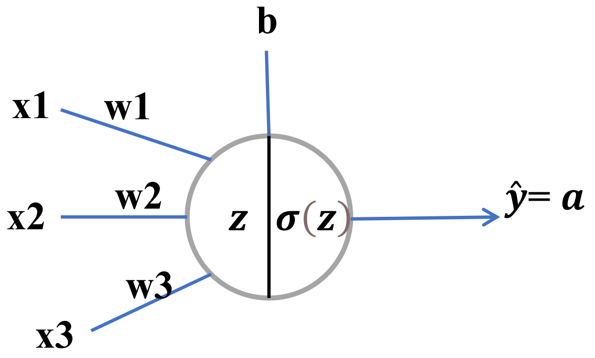
$$= -L(\hat{y}, y) \ \ \textit{Max Likelihood}$$

Logistic Regression Cost function

- $\log P(\,\textit{labels in trainingset}\,) = \log \prod_{i=1}^{m} P\left(y^{i} \mid x^{i}\right)$
- $\log P(\cdots) = \sum_{i=1}^{m} \log P\left(y^{(i)} \mid x^{(i)}\right) = -\sum_{i=1}^{m} L\left(\hat{y}^{(i)}, y^{(i)}\right)$

- Cost function $\underbrace{J(w,b)}_{\boldsymbol{minimize}} = \frac{1}{m}\sum_{i=1}^{m} L\left(\hat{y}^{(i)}, y^{(i)}\right)$

Neural Networks



b

x1

w1

x2

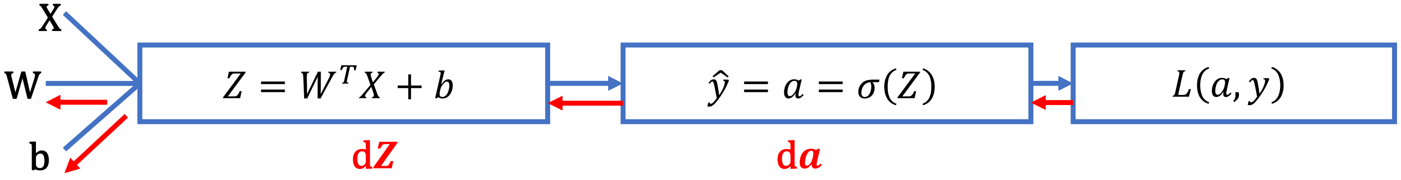
w2

z | $\sigma(z)$

$\hat{y} = a$

w3

x3



X

W

b

$Z = W^T X + b$

$\hat{y} = a = \sigma(Z)$

$L(a, y)$

dZ

da