Machine Learning

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https://www.aparat.com/mehran.safayani

https://github.com/safayani/machine_learning_course

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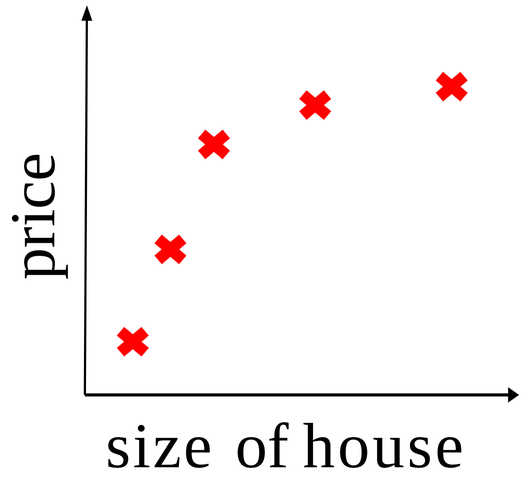
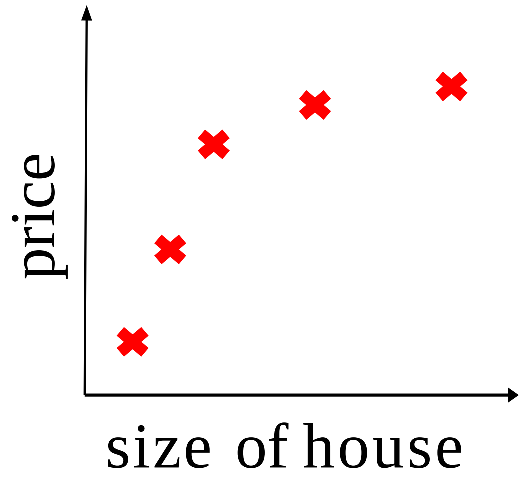
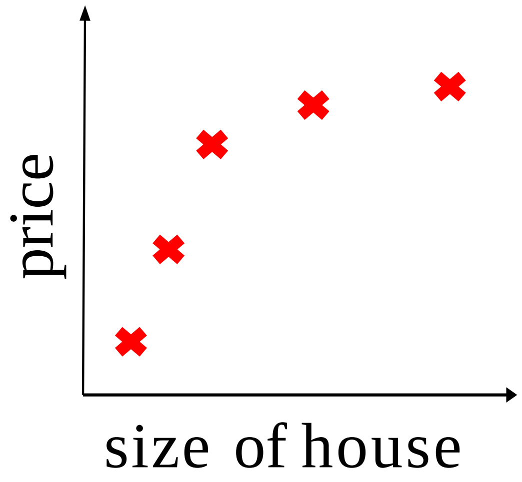
Example: Linear regression (housing prices)



$\theta_0 + \theta_1 x$

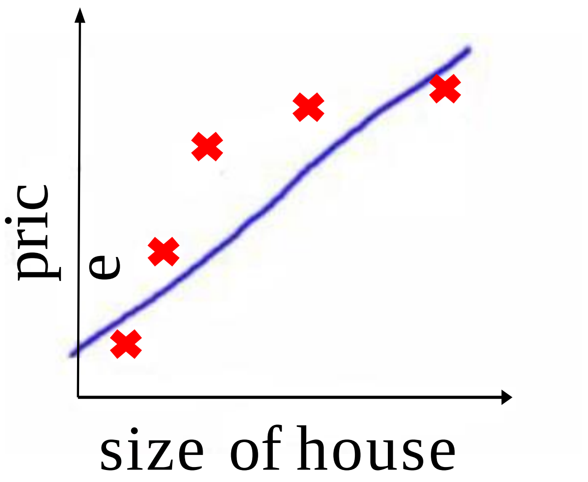


$\theta_0 + \theta_1 x + \theta_2 x^2$



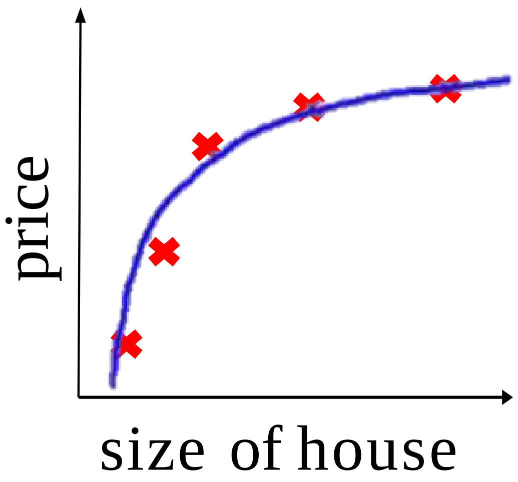
$\theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \theta_4 x^4$

Example: Linear regression (housing prices)



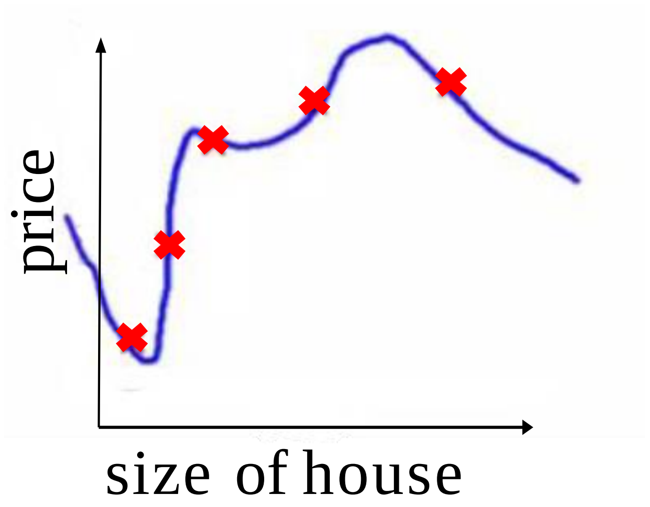
$\theta_0 + \theta_1 x$

"Underfit" **"High bias"**



$\theta_0 + \theta_1 x + \theta_2 x^2$

"Just right"

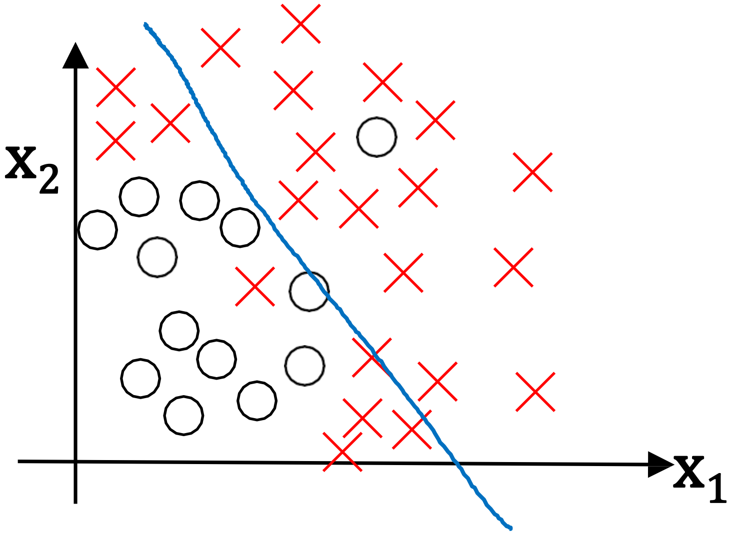


$\theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \theta_4 x^4$

"overfit" **"High variance"**

Overfitting: If we have too many features, the learned hypothesis may fit the training set very well ($J(\theta) = \frac{1}{2m}\sum_{i=1}^{m}\left(h_\theta\left(x^{(i)}\right) - y^{(i)}\right)^2 \approx 0$), but fail to generalize to new examples (predict prices on new examples).

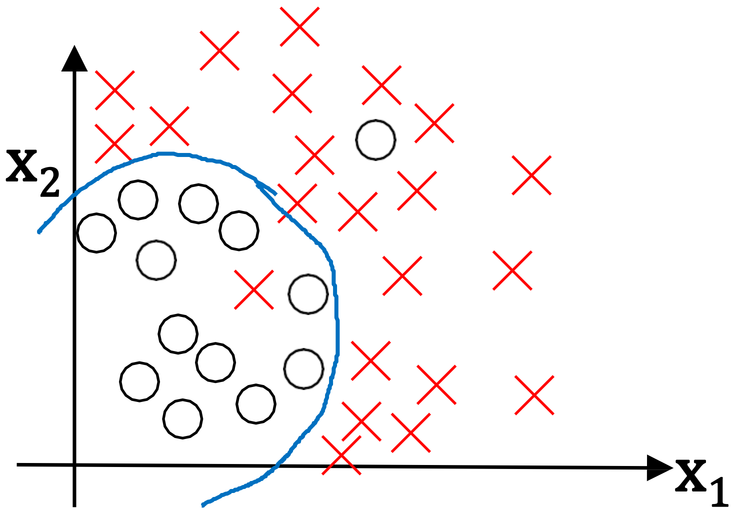
Example: Logistic regression



$$h_\theta(x) = g(\theta_0 + \theta_1 x_1 + \theta_2 x_2)$$

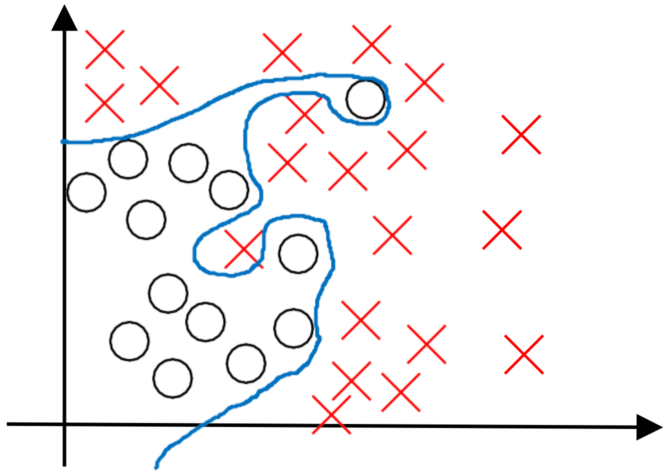
high bias

underfitting



$$g(\theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_1^2 + \theta_4 x_2^2 + \theta_5 x_1 x_2)$$

"just right"



$$g\begin{pmatrix} \theta_0 + \theta_1 x_1 + \theta_2 x_1^2 + \theta_3 x_1^2 x_2 \\ +\theta_4 x_1^2 x_2^2 + \theta_5 x_1^2 x_2^3 + \theta_6 x_1^3 x_2 + \cdots \end{pmatrix}$$

high variance

overfitting

Addressing overfitting:

x_1 = size of house

x_2 = no. of bedrooms

x_3 = no. of floors

x_4 = age of house

x_5 = average income in neighborhood

x_6 = kitchen size

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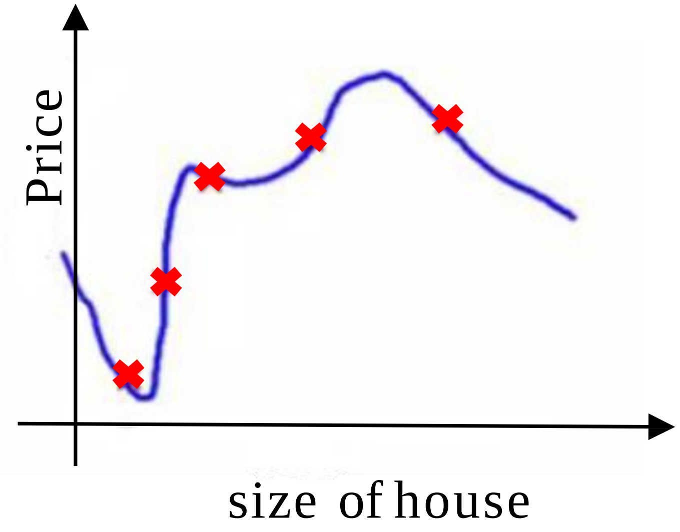
x_{100}



Evaluating your hypothesis

- Dataset:

Size	Price
2104	400
1600	330
2400	369
1416	232
3000	540
1985	300
1534	315
1427	199
1380	212
1494	243

60% Training set

$$\begin{array}{c} \left(x^{(1)}, y^{(1)}\right) \\ \left(x^{(2)}, y^{(2)}\right) \\ \vdots \\ \left(x^{(m)}, y^{(m)}\right) \end{array}$$

20% Cross validation set (cv)

$$\begin{array}{c} \left(x_{cv}{}^{(1)}, y_{cv}{}^{(1)}\right) \\ \left(x_{cv}{}^{(2)}, y_{cv}{}^{(2)}\right) \\ \vdots \\ \left(x_{cv}{}^{(m_{cv})}, y_{cv}{}^{(m_{cv})}\right) \end{array}$$

20% Test set

$$\begin{array}{c} \left(x_{test}{}^{(1)}, y_{test}{}^{(1)}\right) \\ \left(x_{test}{}^{(2)}, y_{test}{}^{(2)}\right) \\ \vdots \\ \left(x_{test}{}^{(m_{test})}, y_{test}{}^{(m_{test})}\right) \end{array}$$

Train/validation/test error

- Training error:

$$J_{train}(\theta) = \frac{1}{2m}\sum_{i=1}^{m}\left(h_\theta\left(x^{(i)}\right) - y^{(i)}\right)^2$$

- Cross Validation error:

$$J_{cv}(\theta) = \frac{1}{2mcv}\sum_{i=1}^{m_{cv}}\left(h_\theta\left(x_{cv}{}^{(i)}\right) - y_{cv}{}^{(i)}\right)^2$$

- Test error:

$$J_{test}(\theta) = \frac{1}{2mtest}\sum_{i=1}^{m_{test}}\left(h_\theta\left(x_{test}{}^{(i)}\right) - y_{test}{}^{(i)}\right)^2$$

Model Selection

$h_{\theta_1}(x) = \theta_0 + \theta_1 x$ $\quad J_{cv}(\theta^1)$

$h_{\theta_2}(x) = \theta_0 + \theta_1 x + \theta_2\, x^2$ $\quad J_{cv}(\theta^2)$

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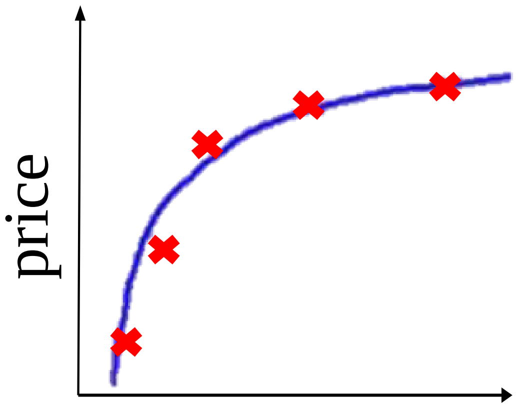
$h_{\theta_{10}}(x) = \theta_0 + \theta_1 x + \ldots + \theta_{10}\, x^{10}$ $\quad J_{cv}(\theta^{10})$

$$i^* = \underset{i}{\text{argmin}}\; J_{cv}(\theta^{i})$$

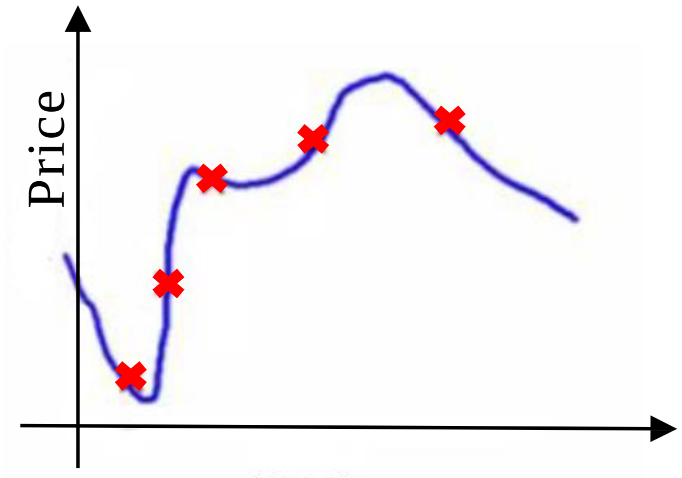
$$J_{test}(\theta^{i^*})$$

Regularization Intuition



size of house

$\theta_0 + \theta_1 x + \theta_2 x^2$



size of house

$\theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \theta_4 x^4$

- Suppose we penalize and make θ_3, θ_4 really small.

$$\min_{\theta} \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta\left(x^{(i)}\right) - y^{(i)}\right)^2 + \underbrace{1000\theta_3^2 + 1000\theta_4^2}_{\theta_3 \approx 0 \quad \theta_4 \approx 0}$$

Regularization

- Small values for parameters $\theta_0, \theta_1, \dots, \theta_n$
  - "Simpler" hypothesis
  - Less prone to overfitting
- Housing:
  - Features: $x_1, x_2, \dots, x_{100}$
  - Parameters: $\theta_0, \theta_1, \theta_2, \dots, \theta_{100}$

$$J(\theta) = \frac{1}{2m}\left[\sum_{i=1}^{m}\left(h_\theta\left(x^{(i)}\right) - y^{(i)}\right)^2 + \lambda \sum_{j=1}^{n} \theta_j^2\right]$$

$$\boldsymbol{\theta_0, \theta_1, \theta_2, \dots, \theta_{100}}$$

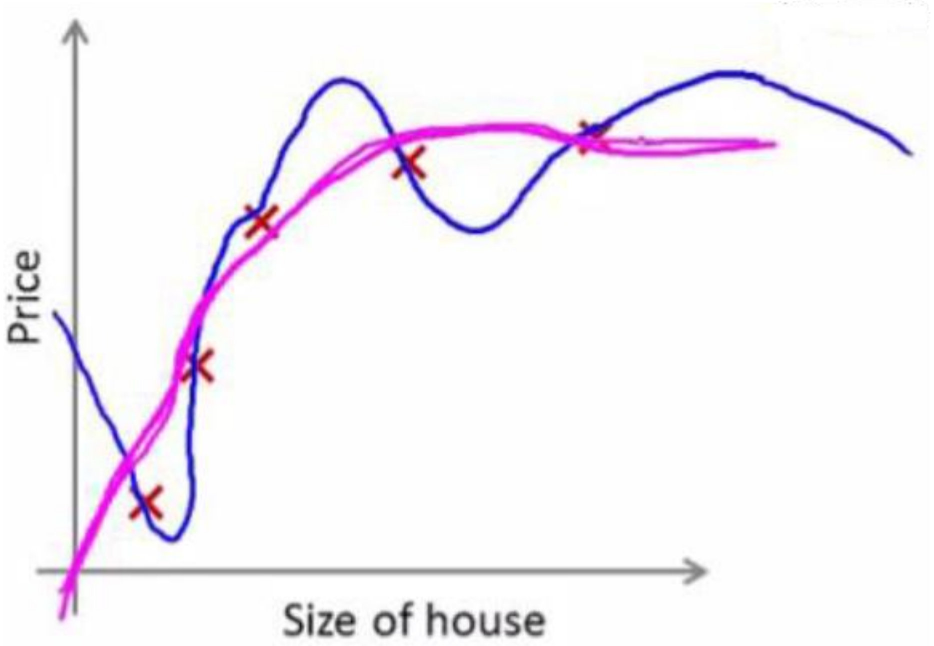
λ = regularization parameter

lambda lambd

Regularization

- $J(\theta) = \frac{1}{2m}\left[\sum_{i=1}^{m}\left(h_\theta\left(x^{(i)}\right) - y^{(i)}\right)^2 + \lambda \sum_{j=1}^{n} \theta_j^2\right]$

$$\min_{\theta} J(\theta)$$

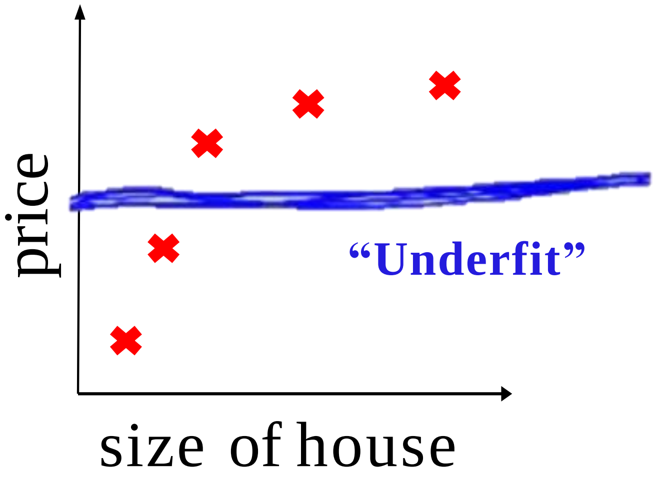


Regularization

- In regularized linear regression, we choose θ to minimize

$$J(\theta) = \frac{1}{2m}\left[\sum_{i=1}^{m}\left(h_\theta\left(x^{(i)}\right) - y^{(i)}\right)^2 + \lambda\sum_{j=1}^{n}\theta_j^2\right]$$

- What if λ is set to an extremely large value (perhaps for too large for our problem, say / = 1010)?



$$\theta_1, \theta_2, \theta_3, \theta_4$$

$$\theta_1 \approx 0, \theta_2 \approx 0$$

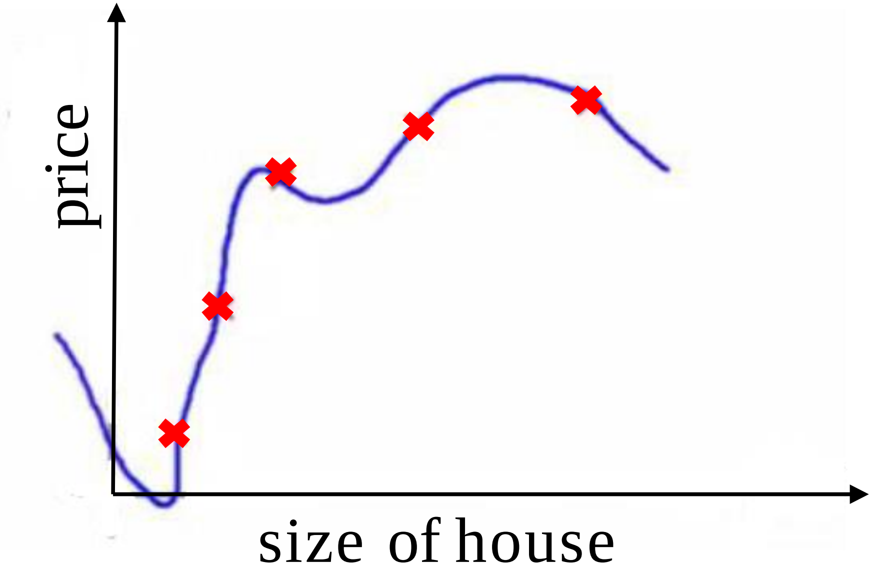
$$\theta_3 \approx 0, \theta_4 \approx 0$$

$$\boxed{h_\theta(x) = \theta_0}$$

$h_\theta(x)$

$$\theta_0 + \cancel{\theta_1 x} + \cancel{\theta_2 x^2} + \cancel{\theta_3 x^3} + \cancel{\theta_4 x^4}$$

Evaluating your hypothesis



$\theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \theta_4 x^4$

- Fails to generalize to new examples not in training set.

  x_1 = size of house

  x_2 = no. of bedrooms

  x_3 = no. of floors

  x_4 = age of house

  x_5 = average income in neighborhood

  x_6 = kitchen size

  .

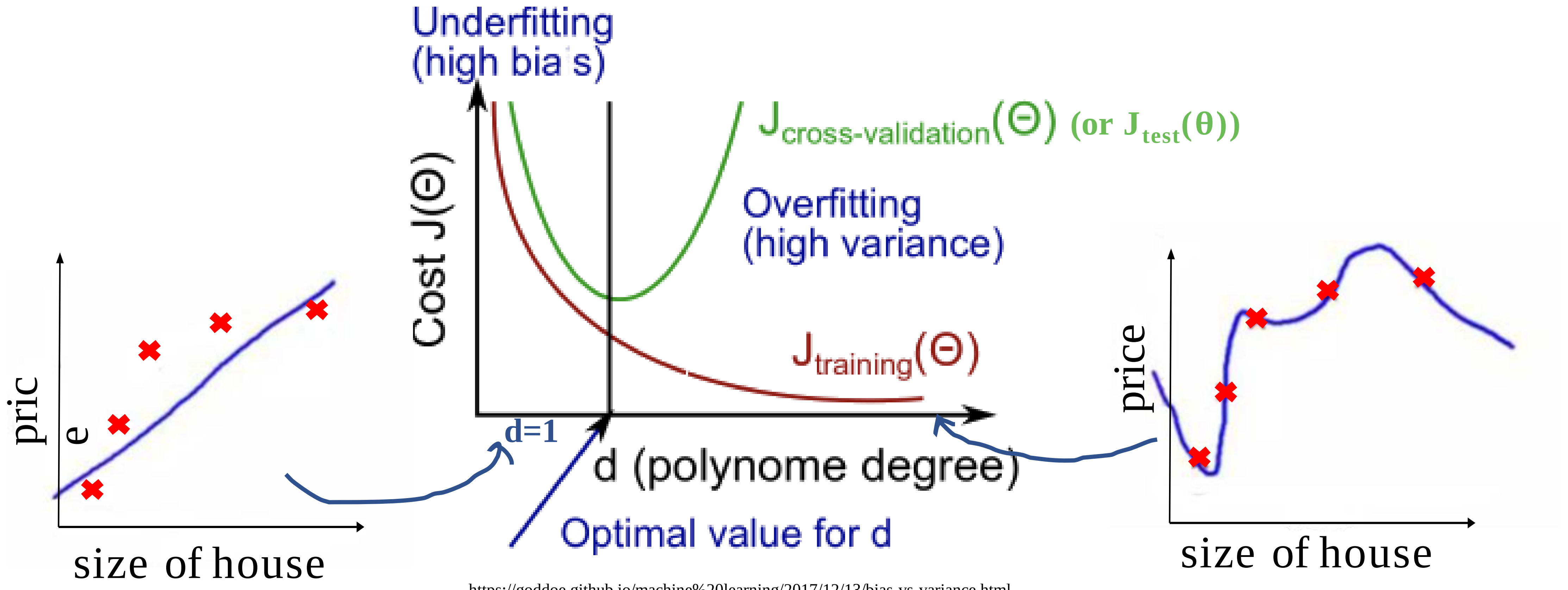
  .

  .

  x_{100}

Bias/variance

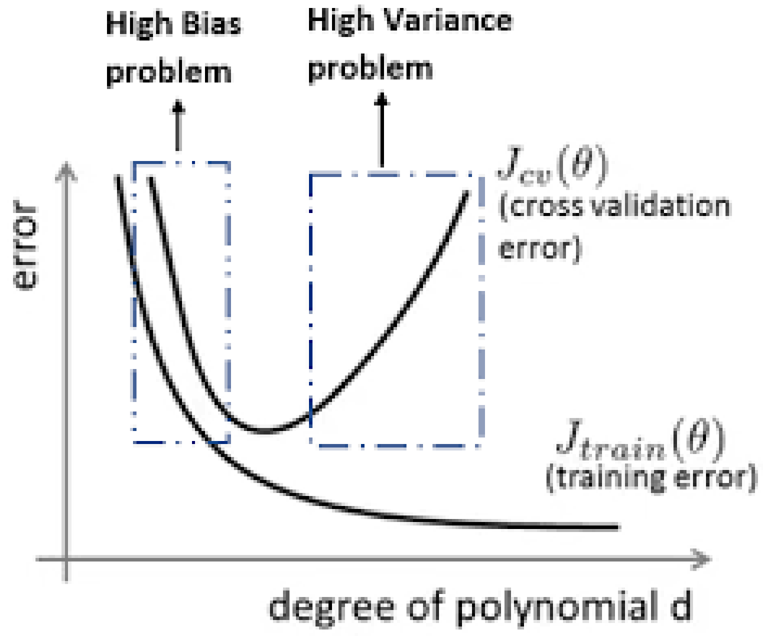
- Training error: $J_{train}(\theta) = \frac{1}{2m}\sum_{i=1}^{m}\left(h_\theta\left(x^{(i)}\right) - y^{(i)}\right)^2$
- Cross validation error: $J_{cv}(\theta) = \frac{1}{2mcv}\sum_{i=1}^{m_{cv}}\left(h_\theta\left(x_{cv}{}^{(i)}\right) - y_{cv}{}^{(i)}\right)^2$ **(or $J_{test}(\theta)$)**



https://goddoe.github.io/machine%20learning/2017/12/13/bias-vs-variance.html

Diagnosing bias vs. variance

- Suppose your learning algorithm is performing less well than you were hoping. ($J_{cv}(\theta)$ or $J_{test}(\theta)$) is high.) Is it a bias problem or a variance problem?



https://towardsdatascience.com/

Bias (underfit):

$J_{train}(\theta)$ *will be high*

$J_{cv}(\theta) \approx J_{train}(\theta)$

Variance (overfit):

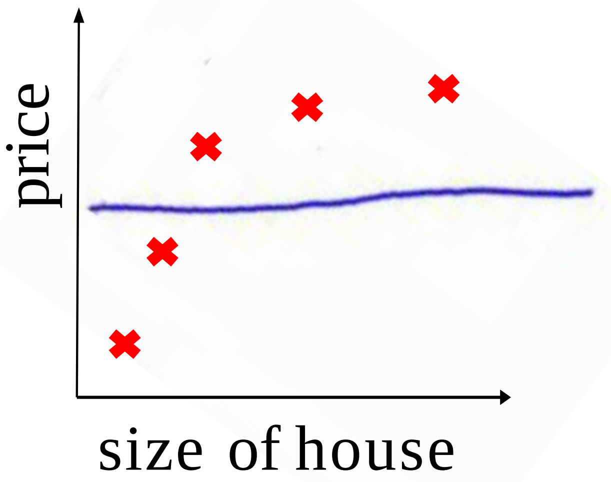
$J_{train}(\theta)$ will be low

$J_{cv}(\theta) \gg J_{train}(\theta)$

Linear regression with regularization

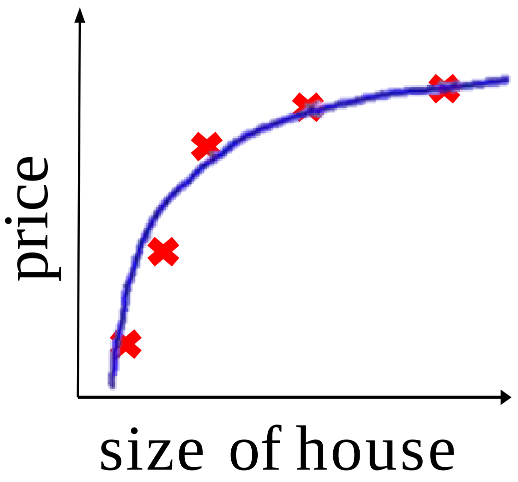
Model: $h_\theta(x) = \theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \theta_4 x^4$

$$J(\theta) = \frac{1}{2m}\sum_{i=1}^{m}\left(h_\theta\left(x^{(i)}\right) - y^{(i)}\right)^2 + \frac{\lambda}{2m}\sum_{j=1}^{n}\theta_j^2$$



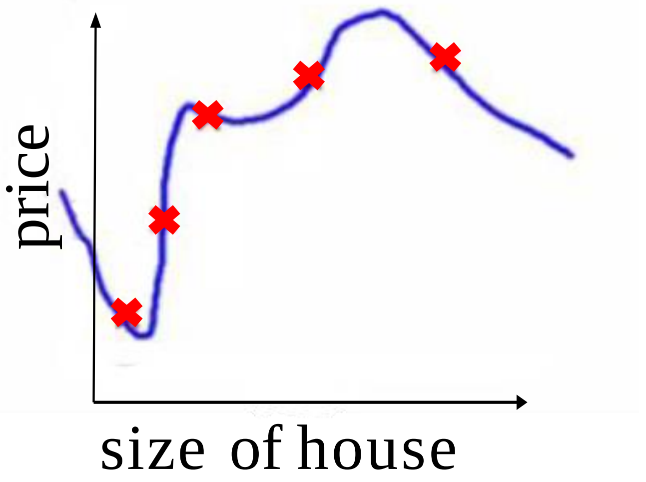
Large λ

High bias(underfit)



Intermediate λ

"Just right"



Small λ

High variance (overfit)

$\lambda = 10000.\ \theta_1 \approx 0, \theta_2 \approx 0, \ldots$

$h_\theta(x) \approx \theta_0$

$\lambda = 0$

Choosing the regularization parameter λ

Model: $h_\theta(x) = \theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \theta_4 x^4$

$$J(\theta) = \frac{1}{2m}\sum_{i=1}^{m}\left(h_\theta\left(x^{(i)}\right) - y^{(i)}\right)^2 + \frac{\lambda}{2m}\sum_{j=1}^{n}\theta_j^2$$

1. Try $\lambda = 0 \rightarrow \min J(\theta) \rightarrow \theta^{(1)} \rightarrow J_{cv}(\theta^{(1)})$
2. Try $\lambda = 0.01 \rightarrow \min J(\theta) \rightarrow \theta^{(2)} \rightarrow J_{cv}(\theta^{(2)})$
3. Try $\lambda = 0.02 \rightarrow \min J(\theta) \rightarrow \theta^{(3)} \rightarrow J_{cv}(\theta^{(3)})$
4. Try $\lambda = 0.04$
5. Try $\lambda = 0.08 \rightarrow \min J(\theta) \rightarrow \theta^{(5)} \rightarrow J_{cv}(\theta^{(5)})$

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12. Try $\underbrace{\lambda = 10}_{10.24} \rightarrow \min J(\theta) \rightarrow \theta^{(12)} \rightarrow J_{cv}(\theta^{(12)})$

Pick (say) $\theta^{(5)}$. **Test error:** $J_{test}(\theta^{(5)})$

Bias/variance as a function of the regularization parameter λ

- **Training error:**

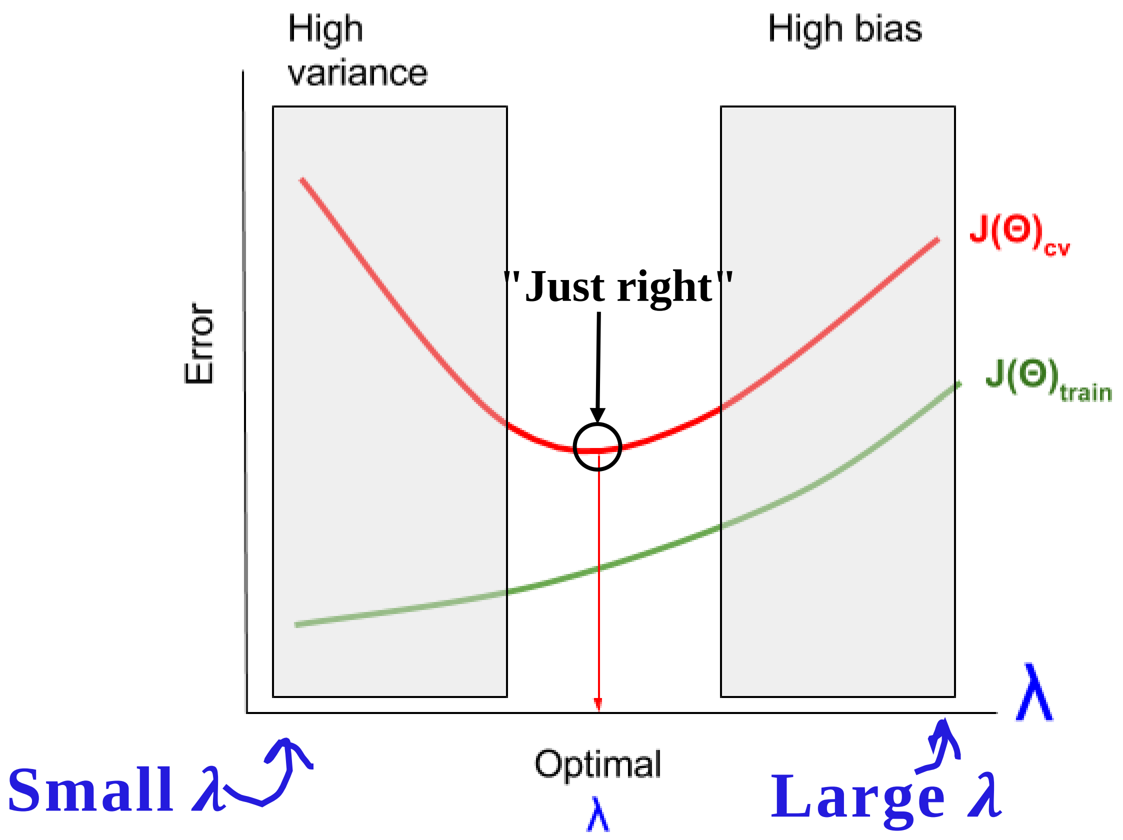
$$J_{train}(\theta) = \frac{1}{2m}\sum_{i=1}^{m}\left(h_\theta\left(x^{(i)}\right) - y^{(i)}\right)^2 + \lambda\sum_{j=1}^{n}\theta_j^2$$

- **Cross Validation error:**

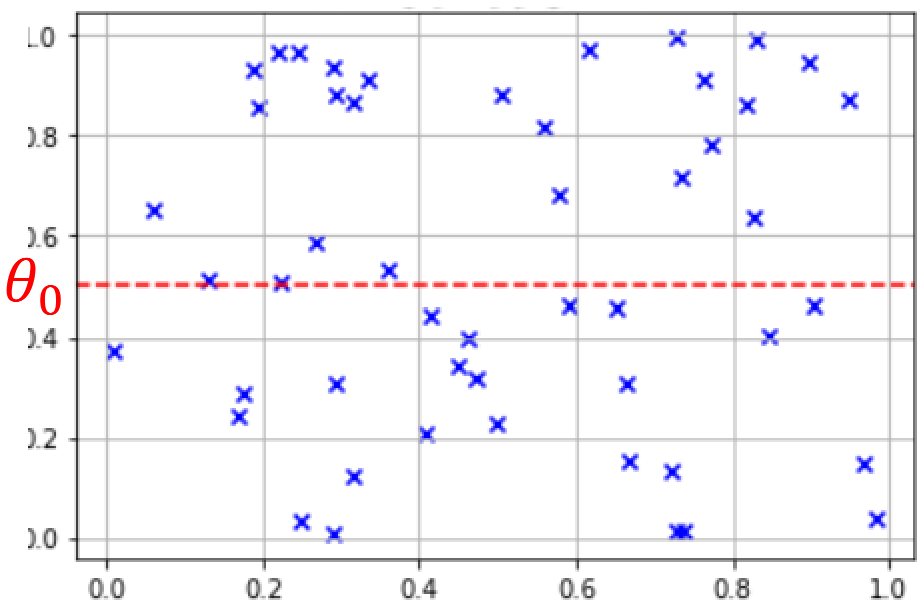
$$J_{cv}(\theta) = \frac{1}{2mcv}\sum_{i=1}^{m_{cv}}\left(h_\theta\left({x_{cv}}^{(i)}\right) - {y_{cv}}^{(i)}\right)^2$$

- **Test error:**

$$J_{test}(\theta) = \frac{1}{2mtest}\sum_{i=1}^{m_{test}}\left(h_\theta\left({x_{test}}^{(i)}\right) - {y_{test}}^{(i)}\right)^2$$



https://stats.stackexchange.com/questions/222493/why-do-we-use-the-unregularized-cost-to-plot-a-learning-curve



GD:

Repeat until convergence{

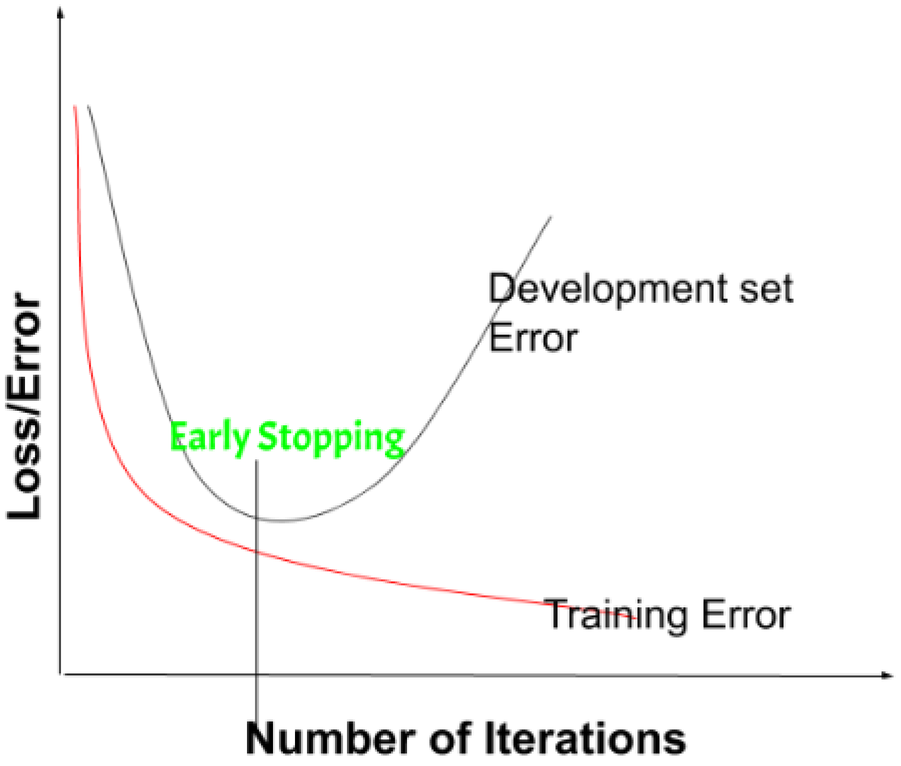
$$\theta_0 = \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x_i) - y_i) \qquad (x_0^i = 1)$$

$$\theta_j = \theta_j - \alpha \left[\frac{1}{m} \sum_{i=1}^{m} (h_\theta(x_i) - y_i)\, x_j^i + \lambda\theta_j\right]$$

J = 1 , 2 , … , n

}

Early stopping



https://www.geeksforgeeks.org/regularization-by-early-stopping/

Old way of splitting data

- Deep learning

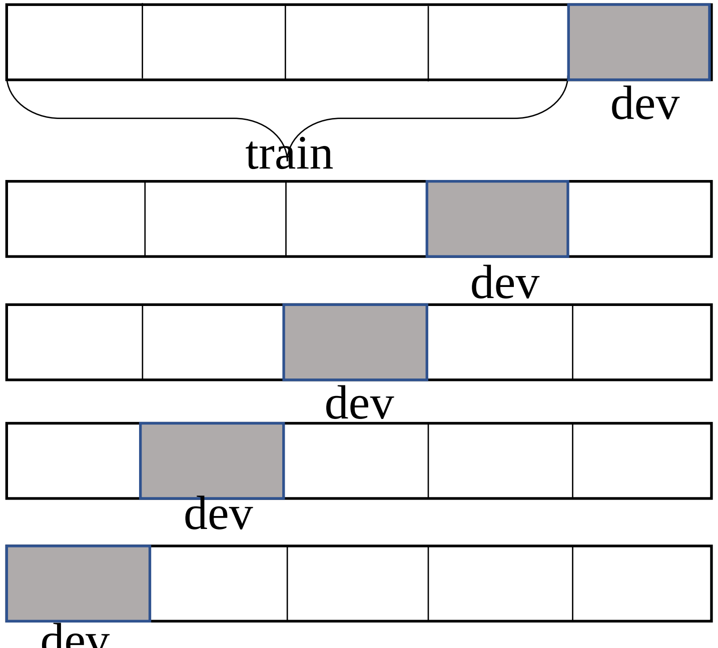
60%   20%   20%

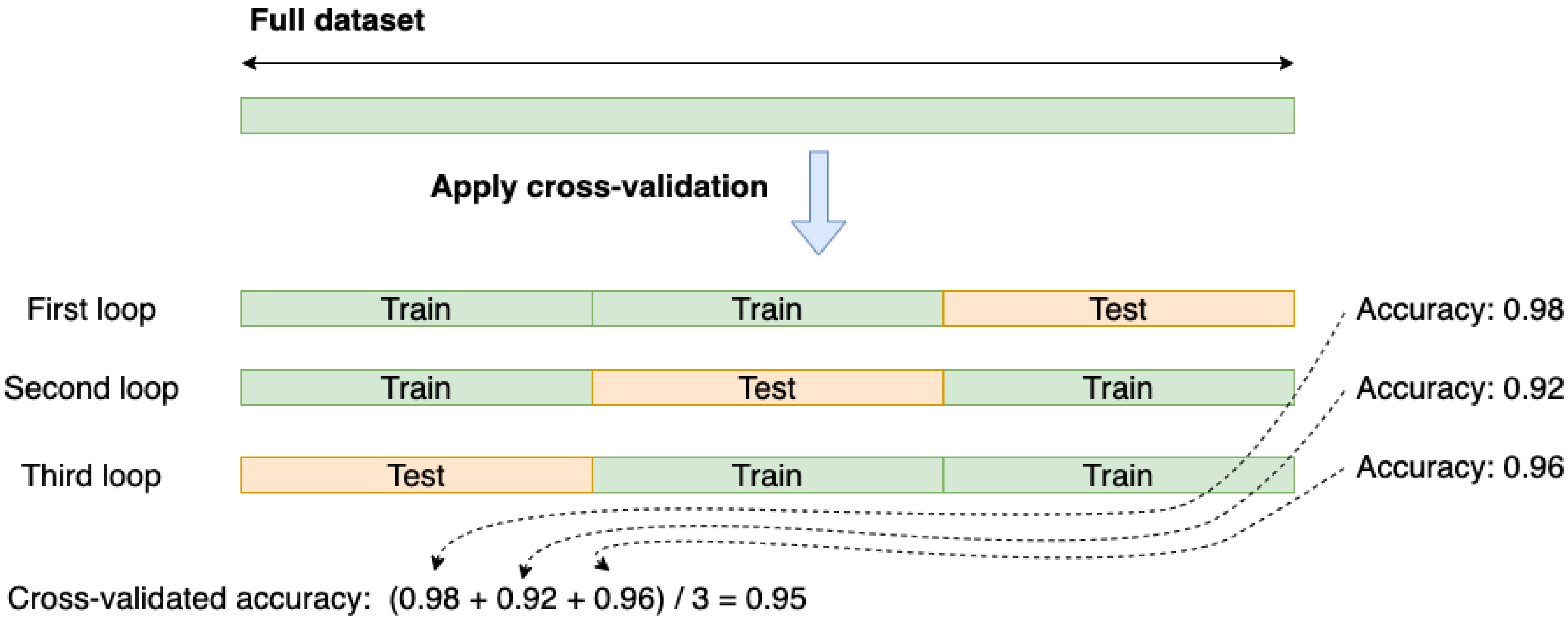
1000,000 data   1%   10,000

98%   1%   1%

99.5% 0.4% 0.1%

- K-fold cv




Full dataset
Apply cross-validation
First loop
Train
Train
Test
Accuracy: 0.98
Second loop
Train
Test
Train
Accuracy: 0.92
Third loop
Test
Train
Train
Accuracy: 0.96
Cross-validated accuracy: (0.98 + 0.92 + 0.96) / 3 = 0.95