Machine Learning

Dr. Mehran Safayani

safayani@iut.ac.ir

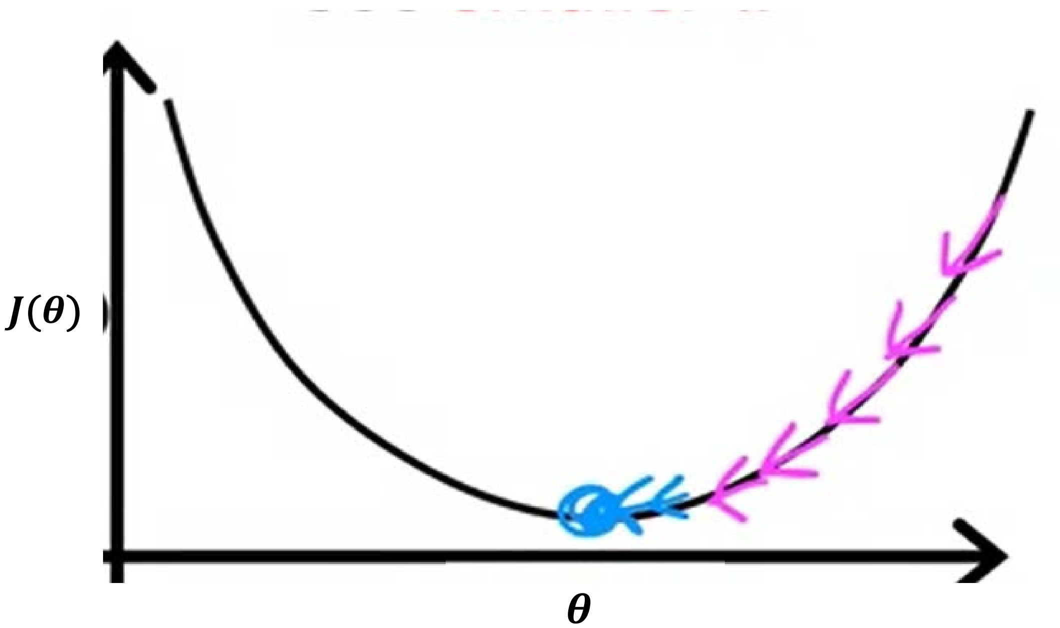
safayani.iut.ac.ir

https://www.aparat.com/mehran.safayani

https://github.com/safayani/machine_learning_course

Department of Electrical and computer engineering, Isfahan university of technology, Isfahan, Iran

Gradient Descent



$$h_\theta(x) = \theta_0 + \theta_1 x$$

$$\boldsymbol{\theta_1} = \boldsymbol{\theta_1} - \alpha \frac{d\boldsymbol{J}(\boldsymbol{\theta_1})}{d\boldsymbol{\theta_1}}$$

number	size	#bedrooms	# floors	Price(y)
1	100	2	1	10000
2	150	3	2	175000
…	…	…	…	…
m	…	…	…	…

N: #features = 3

M: #training data

x_i: i th data in training set

x_j^i: j th feature of i th data in training set

$$h_\theta(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \dots + \theta_n x_n$$

$$y = [y^1, y^2, \dots, y^m]^T \in R^{m*(n+1)}$$

$$X = [x^1, x^2, \dots, x^m]^T \in R^{m*(n+1)}$$

$$\vec{x} = \begin{bmatrix} \boldsymbol{x_0} \\ \boldsymbol{x_1} \\ \boldsymbol{x_2} \\ . \\ . \\ . \\ \boldsymbol{x_n} \end{bmatrix} \in R^{n+1} \quad x_0 = 1 \quad , \quad \overrightarrow{\theta} = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ . \\ . \\ . \\ \theta_n \end{bmatrix} \in R^{n+1} \quad \theta_0 \textit{ is bias}$$

$$\Rightarrow \quad h_\theta(x) = x^T\theta = \theta^T x$$

Cost function

$J(\overrightarrow{\theta}) = \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x_i) - y_i)^2$

$e^i = x^T \theta - y_i \Longrightarrow e = X\theta - y \Longrightarrow J(\theta) = \frac{1}{2m} e^T e$

$e \,,\ X\theta \,, y \in R^m$

Gradient Descent

Repeat until convergence: {

For j=0,…,n

$$\boldsymbol{\theta_j} = \boldsymbol{\theta_j} - \alpha \frac{d\boldsymbol{J}(\boldsymbol{\theta_0}, \boldsymbol{\theta_1}, \ldots, \boldsymbol{\theta_n})}{d\boldsymbol{\theta_j}}$$

}

$$\frac{d\boldsymbol{J}(\boldsymbol{\theta_0}, \boldsymbol{\theta_1})}{d\boldsymbol{\theta_0}} = \frac{1}{m}\sum_{i=1}^{m}(h_\theta(x_i) - y_i)$$

$$\frac{d\boldsymbol{J}(\boldsymbol{\theta_0}, \boldsymbol{\theta_1}, \ldots, \boldsymbol{\theta_n})}{d\boldsymbol{\theta_j}} = \frac{1}{m}\sum_{i=1}^{m}(h_\theta(x_i) - y_i)\, x_j^i$$

(j=0,…,n , x_0^i = 1)

(n+1)*1

$$\frac{d\boldsymbol{J}(\boldsymbol{\theta})}{d\boldsymbol{\theta}} = \frac{1}{m} X^T \mathrm{e}$$

m*1

حجم محاسبات ضرب ماتریس

$A \in R^{m*p}$, $B \in R^{p*n}$ $\Rightarrow$ $AB \in R^{m*n}$ *(2p – 1) mn flops*

Calculating $e^{O(pmn)}$:

$m(2n+1)$ (ضرب و جمع)

m (تفریق)

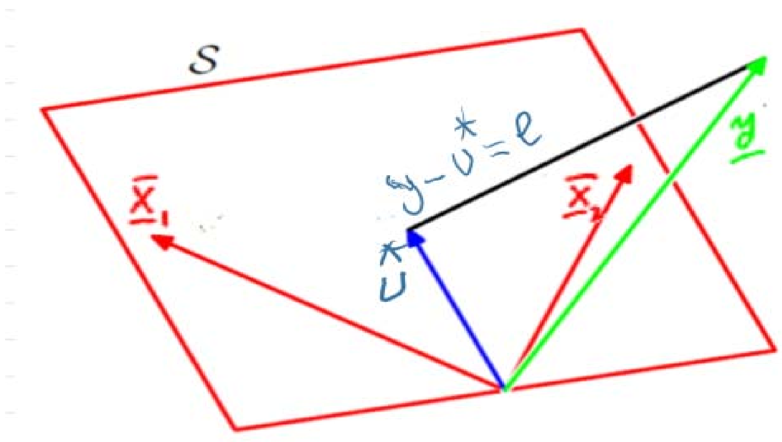
$\Big\}$ 2m(n+1) (مجموع) $\Rightarrow$ O(mn)

$$\frac{dJ(\theta)}{d\theta} : (2\mathrm{m} - 1)(\mathrm{n}+1) + (\mathrm{n}+1) = 2\mathrm{m}(\mathrm{n}+1)$$ $\Rightarrow$ O(mn) در مجموع

تقسیم بر m

مفهوم هندسی

$$\min_{W} \|y - XW\|_2 = \min \|e\|_2$$



تعداد ویژگی * تعداد داده

Span of X:

فضایی که توسط ستون های X پوشش داده می شود. هر بردار در این فضا به صورت U = XW نشان داده می شود. و به آن span(X) می گویند. U بهینه که به صورت U* نشان داده می شود برداری است که e = y – U* بر span(X) عمود باشد. یا به عبات دیگر U* ای باید انتخاب شود که برابر با نگاشت y در span(X) باشد.

Feature Scaling

$$x_1^i \ , x_2^i \ , \ \dots , x_n^i$$

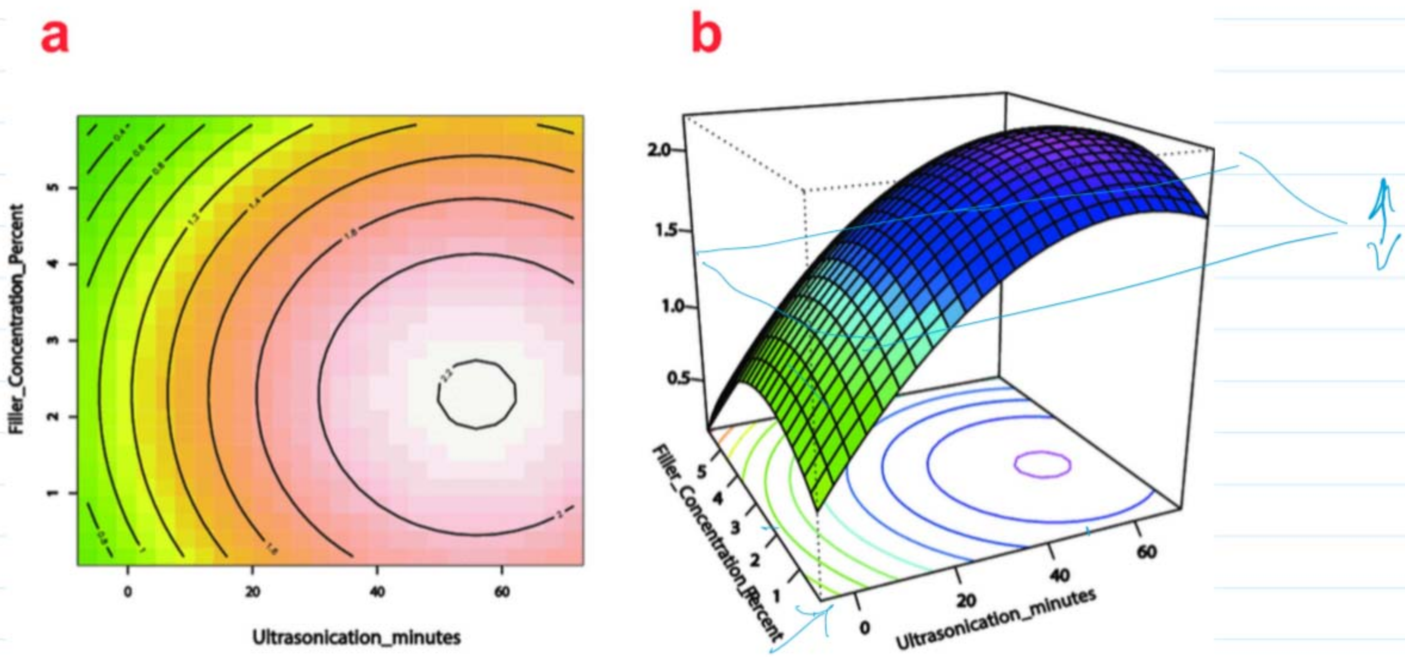
$$-1 \leq x_j \ \leq 1$$

$$0 < \ x_1 \ < 1000$$

$$0 < \ x_2 \ < 5$$

$$x_1 : \frac{size}{1000}$$

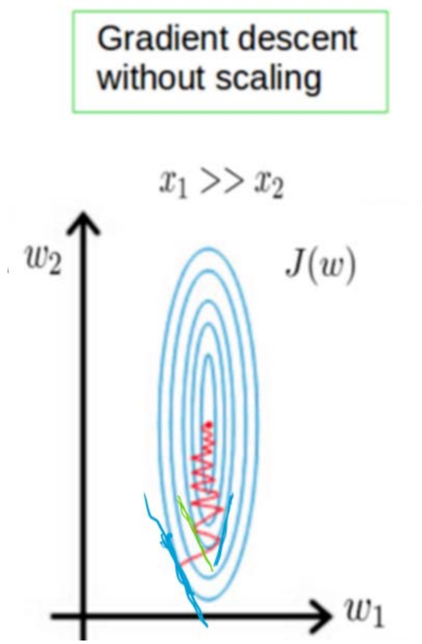
$$x_2 : \frac{\#bedrooms}{5}$$

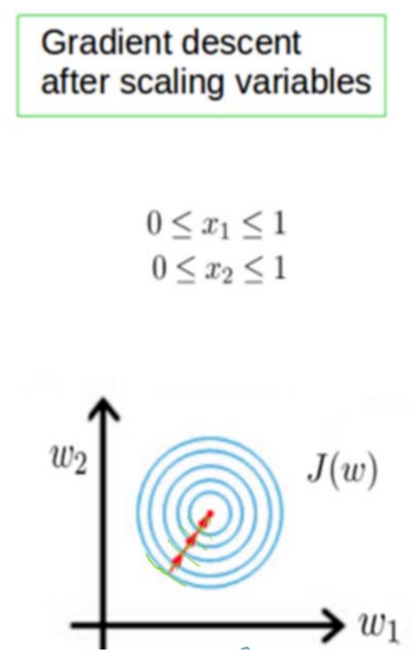

a
b
Filler_Concentration_Percent
Ultrasonication_minutes
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0
20
40
60
1
2
3
4
5
2.0
1.5
1.0
0.5
Filler_Concentration_Percent
Ultrasonication_minutes

Contour Plot

$$\frac{w_1^2}{b^2} + \frac{w_2^2}{a^2} = 1$$

2a: قطر بزرگ
2b: قطر کوچک





$$\frac{w_1^2}{a^2} + \frac{w_2^2}{a^2} = 1$$

Feature Scaling

Scaled features:

- $0 \leq x_1 \leq 3$ ✓
- $-3 \leq x_1 \leq 3$ ✓
- $-2 \leq x_2 \leq 0.5$ ✓
- $-\frac{1}{3} \leq x_2 \leq \frac{1}{3}$ ✓

Need scaling:

$-100 \leq x_3 \leq 100$ ✗

$-0.001 \leq x_4 \leq 0.001$ ✗

Feature Scaling

$$x_1^* = \frac{x_1 - \mu_1}{\text{بازه عدد}}$$

Max_min

standard_deviation

$$bedroom^* = \frac{bedroom - 2.5}{5}$$

$$size^* = \frac{size - 300}{2000}$$

$$\mu_j = \frac{1}{m} \sum_{i=1}^{m} x_j^i$$

Creating New Features

$h_\theta(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2$

قیمت خانه

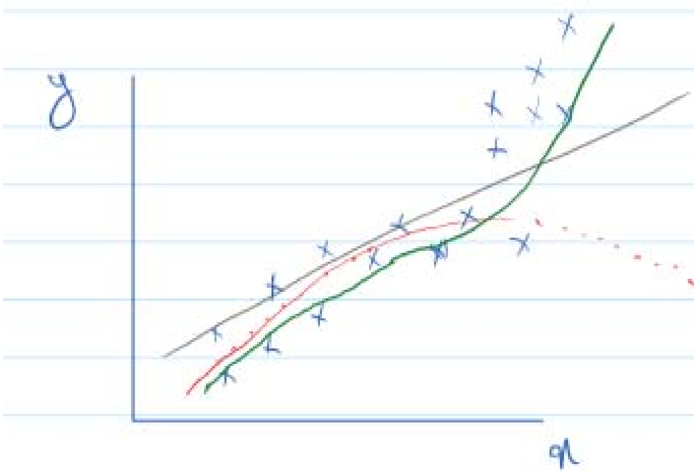
طول خانه

عرض خانه

(مساحت خانه) $x^* = x_1 * x_2$

$h_\theta(x) = \theta_0 + \theta_1 x^*$

Creating New Features



درجه ۲:

$$\theta_0 + \theta_1 x + \theta_2 x^2$$

درجه 3:

$$\theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3$$

Need scaling:

x: 0,…,1000
x^2: 0,…, 10^6
x^3: 0,…, 10^9

We can use:
x , x^2 , x^3 , $\sqrt{x}$
$$\theta_0 + \theta_1 x + \theta_2 \sqrt{x}$$