



Machine Learning

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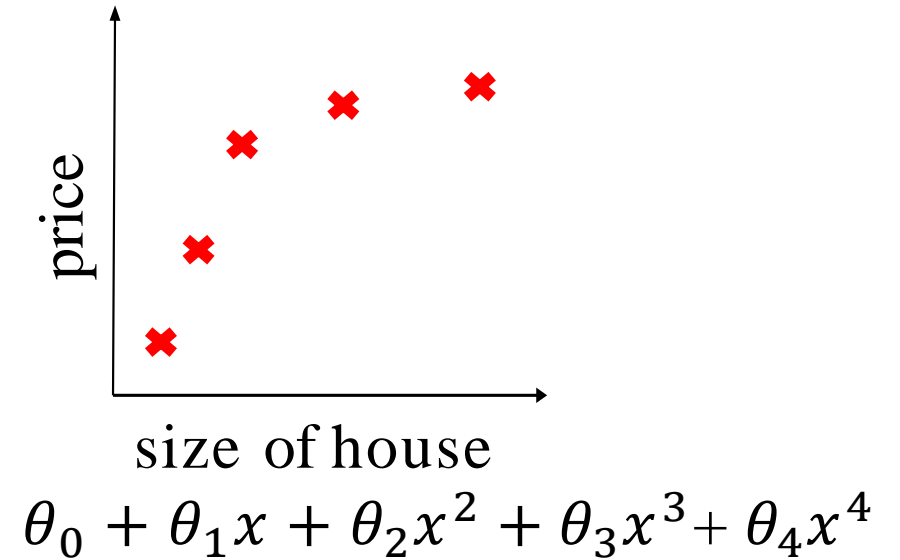
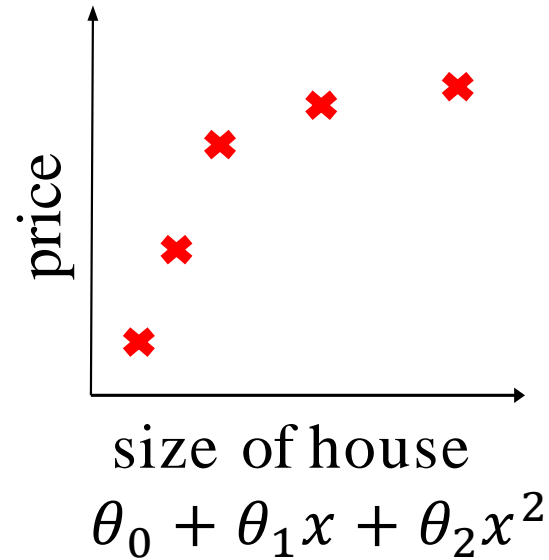
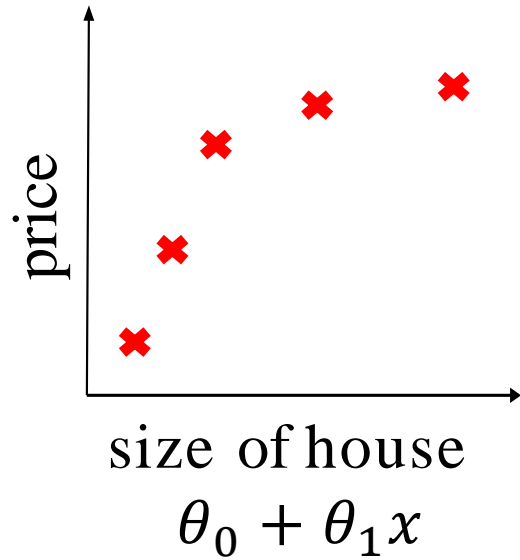
<https://www.aparat.com/mehran.safayani>



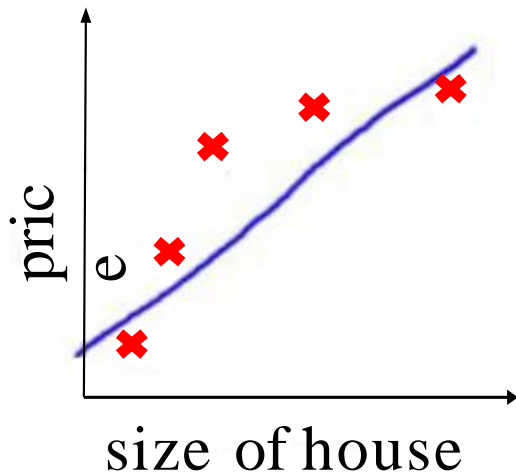
https://github.com/safayani/machine_learning_course



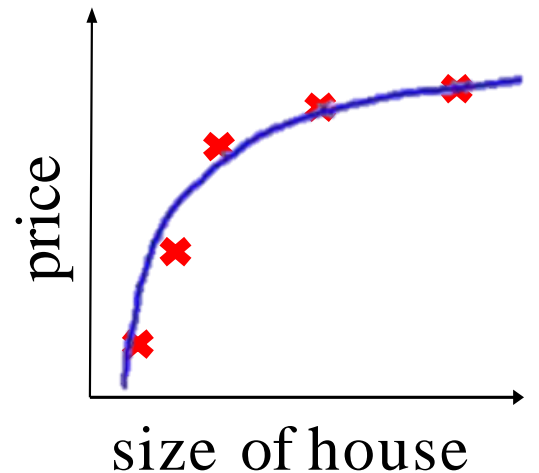
Example: Linear regression (housing prices)



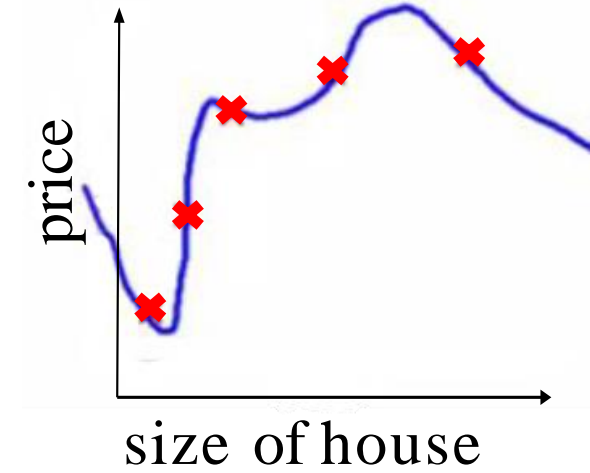
Example: Linear regression (housing prices)



$\theta_0 + \theta_1 x$
"Underfit" "High bias"



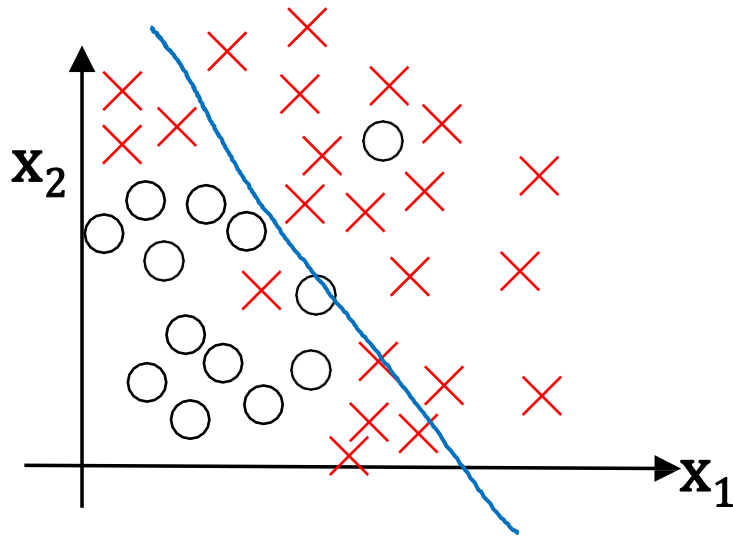
$\theta_0 + \theta_1 x + \theta_2 x^2$
"Just right"



$\theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \theta_4 x^4$
"overfit" "High variance"

Overfitting: If we have too many features, the learned hypothesis may fit the training set very well ($J(\theta) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2 \approx 0$), but fail to generalize to new examples (predict prices on new examples).

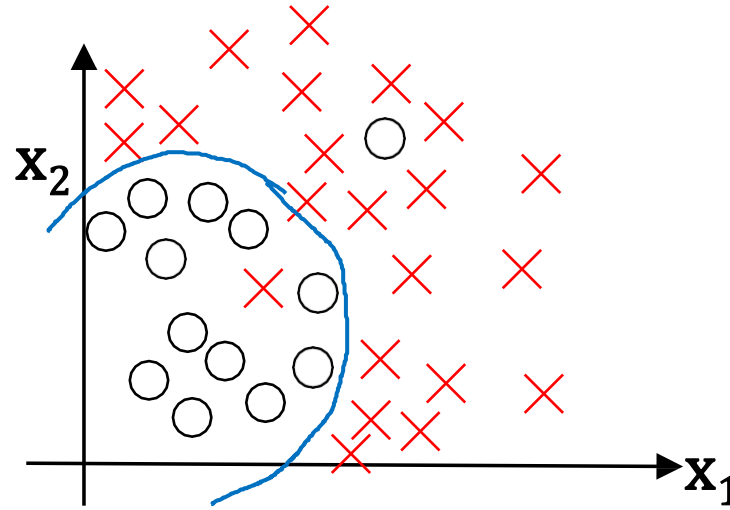
Example: Logistic regression



$$h_{\theta}(x) = g(\theta_0 + \theta_1 x_1 + \theta_2 x_2)$$

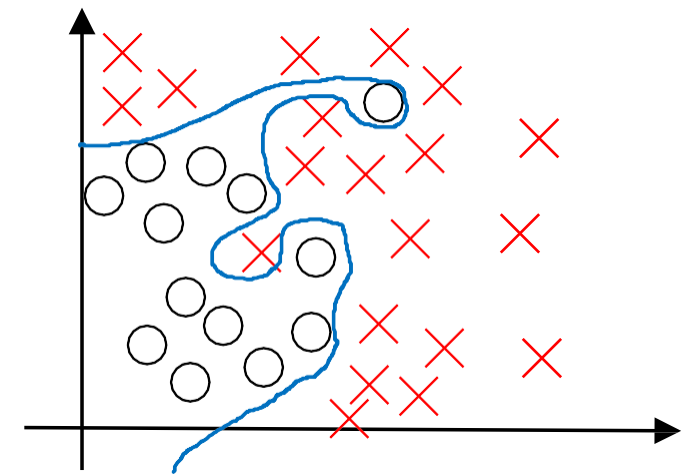
high bias

underfitting



$$g(\theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_1^2 + \theta_4 x_2^2 + \theta_5 x_1 x_2)$$

“just right”



$$g\left(\begin{array}{l} \theta_0 + \theta_1 x_1 + \theta_2 x_1^2 + \theta_3 x_1^2 x_2 \\ + \theta_4 x_1^2 x_2^2 + \theta_5 x_1^2 x_2^3 + \theta_6 x_1^3 x_2 + \dots \end{array}\right)$$

high variance

overfitting

Addressing overfitting:

x_1 = size of house

x_2 = no. of bedrooms

x_3 = no. of floors

x_4 = age of house

x_5 = average income in neighborhood

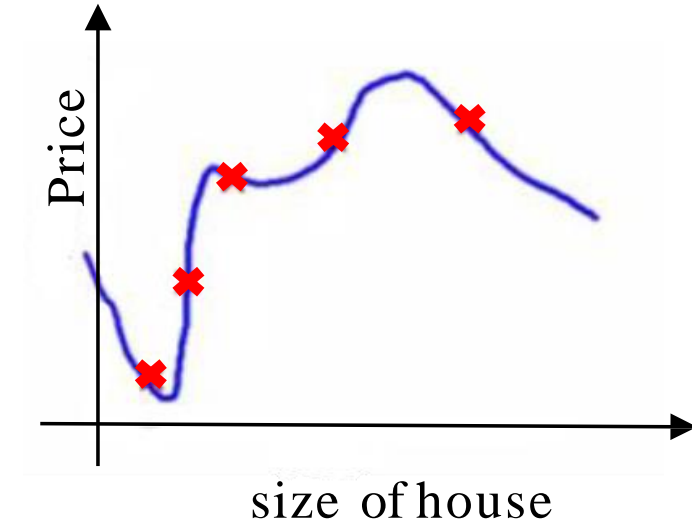
x_6 = kitchen size

.

.

.

x_{100}



Evaluating your hypothesis

- Dataset:

Size	Price
2104	400
1600	330
2400	369
1416	232
3000	540
1985	300
1534	315
1427	199
1380	212
1494	243

60%
Training set

20%
Cross validation set (cv)

20%
Test set

$(x^{(1)}, y^{(1)})$

$(x^{(2)}, y^{(2)})$

.

.

.

$(x^{(m)}, y^{(m)})$

$(x_{cv}^{(1)}, y_{cv}^{(1)})$

$(x_{cv}^{(2)}, y_{cv}^{(2)})$

.

.

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$(x_{cv}^{(m_{cv})}, y_{cv}^{(m_{cv})})$

$(x_{test}^{(1)}, y_{test}^{(1)})$

$(x_{test}^{(2)}, y_{test}^{(2)})$

.

.

.

$(x_{test}^{(m_{test})}, y_{test}^{(m_{test})})$

Train/validation/test error

- Training error:

$$J_{train}(\theta) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2$$

- Cross Validation error:

$$J_{cv}(\theta) = \frac{1}{2m_{cv}} \sum_{i=1}^{m_{cv}} (h_{\theta}(x_{cv}^{(i)}) - y_{cv}^{(i)})^2$$

- Test error:

$$J_{test}(\theta) = \frac{1}{2m_{test}} \sum_{i=1}^{m_{test}} (h_{\theta}(x_{test}^{(i)}) - y_{test}^{(i)})^2$$

Model Selection

$$h_{\theta_1}(x) = \theta_0 + \theta_1 x \quad J_{cv}(\theta^1)$$

$$h_{\theta_2}(x) = \theta_0 + \theta_1 x + \theta_2 x^2 \quad J_{cv}(\theta^2)$$

.

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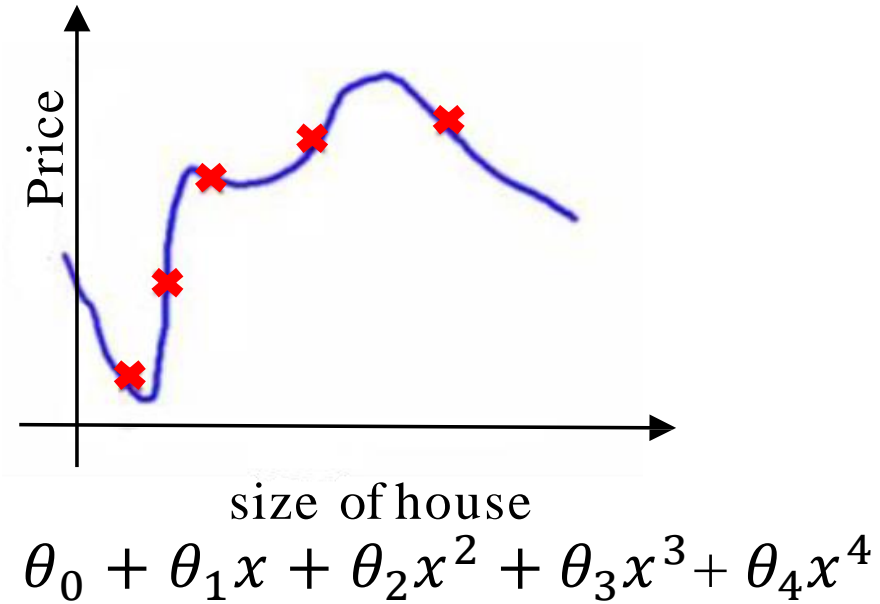
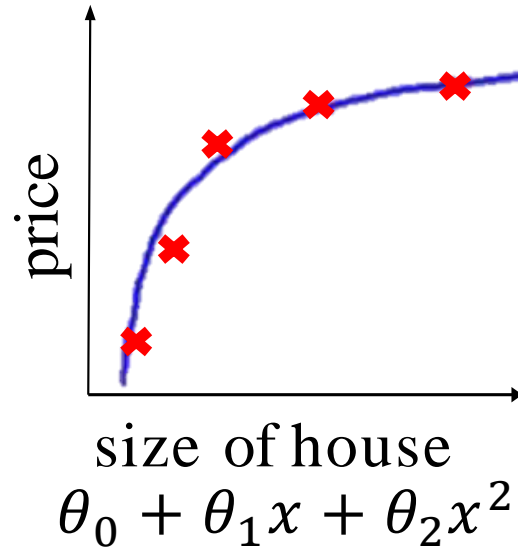
$$h_{\theta_{10}}(x) = \theta_0 + \theta_1 x + \dots + \theta_{10} x^{10} \quad J_{cv}(\theta^{10})$$

$$i^* = \underset{i}{\operatorname{argmin}} J_{cv}(\theta^i)$$



$$J_{test}(\theta^{i^*})$$

Regularization Intuition



- Suppose we penalize and make θ_3, θ_4 really small.

$$\min_{\theta} \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2 + \underbrace{1000\theta_3^2}_{\theta_3 \approx 0} + \underbrace{1000\theta_4^2}_{\theta_4 \approx 0}$$

Regularization

- Small values for parameters $\theta_0, \theta_1, \dots, \theta_n$
 - "Simpler" hypothesis
 - Less prone to overfitting
- Housing:
 - Features: x_1, x_2, \dots, x_{100}
 - Parameters: $\theta_0, \theta_1, \theta_2, \dots, \theta_{100}$

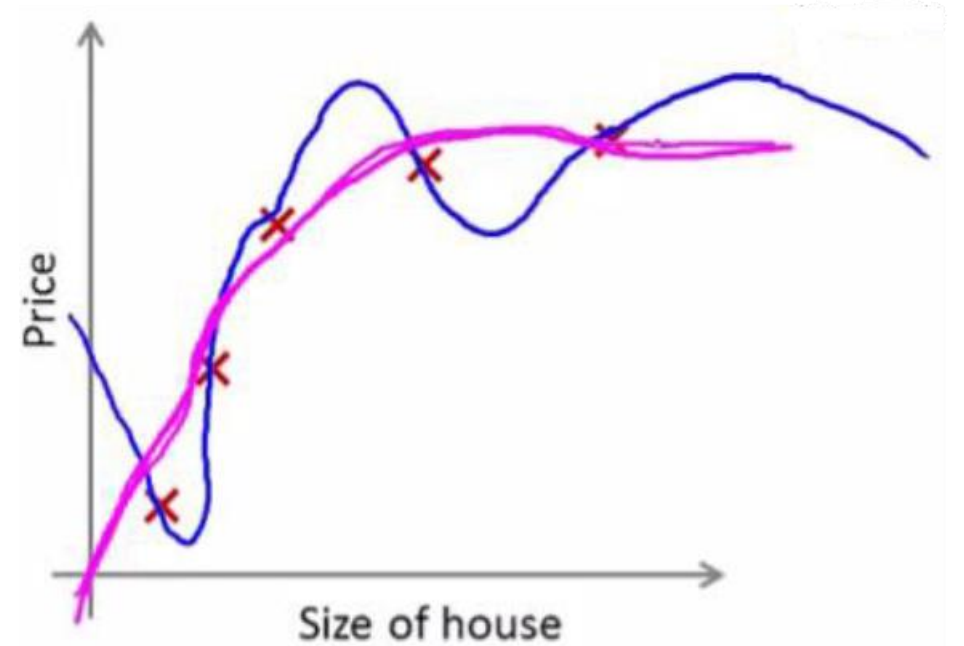
$$J(\theta) = \frac{1}{2m} \left[\sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2 + \lambda \sum_{j=1}^n \theta_j^2 \right]$$

$\theta_0, \theta_1, \theta_2, \dots, \theta_{100}$

λ = regularization parameter
lambda lambda

Regularization

- $J(\theta) = \frac{1}{2m} \left[\sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2 + \lambda \sum_{j=1}^n \theta_j^2 \right]$
 $\min_{\theta} J(\theta)$

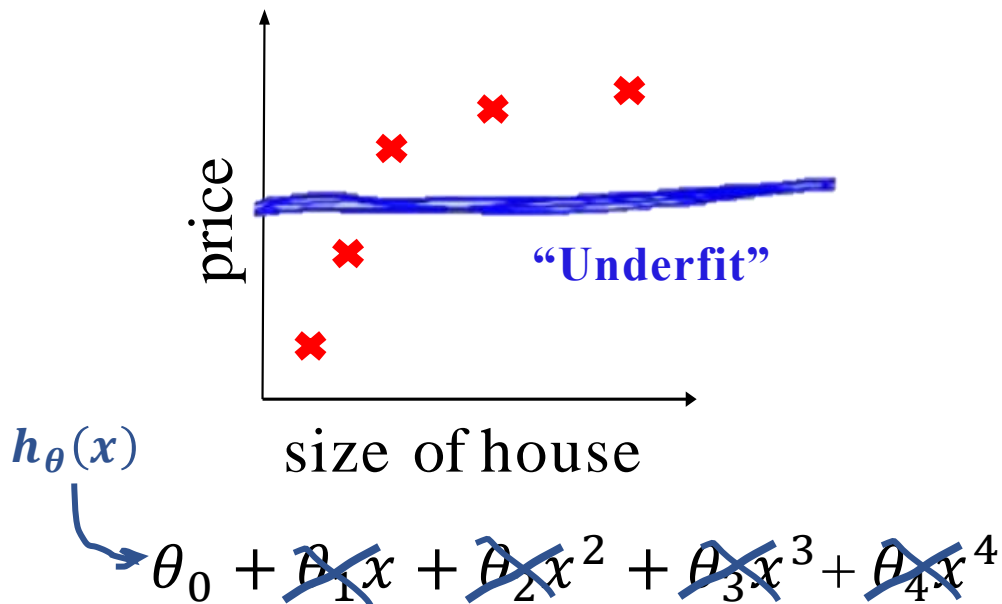


Regularization

- In regularized linear regression, we choose θ to minimize

$$J(\theta) = \frac{1}{2m} \left[\sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2 + \lambda \sum_{j=1}^n \theta_j^2 \right]$$

- What if λ is set to an extremely large value (perhaps far too large for our problem, say $\lambda = 1010$)?



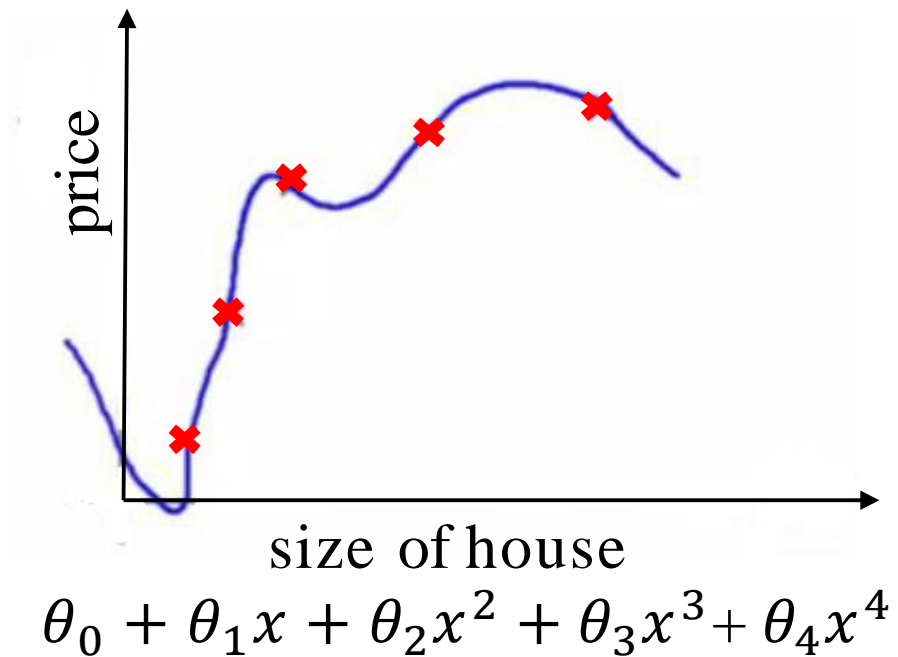
$$\theta_1, \theta_2, \theta_3, \theta_4$$

$$\theta_1 \approx 0, \theta_2 \approx 0$$

$$\theta_3 \approx 0, \theta_4 \approx 0$$

$$h_{\theta}(x) = \theta_0$$

Evaluating your hypothesis



- Fails to generalize to new examples not in training set.

x_1 = size of house

x_2 = no. of bedrooms

x_3 = no. of floors

x_4 = age of house

x_5 = average income in neighborhood

x_6 = kitchen size

.

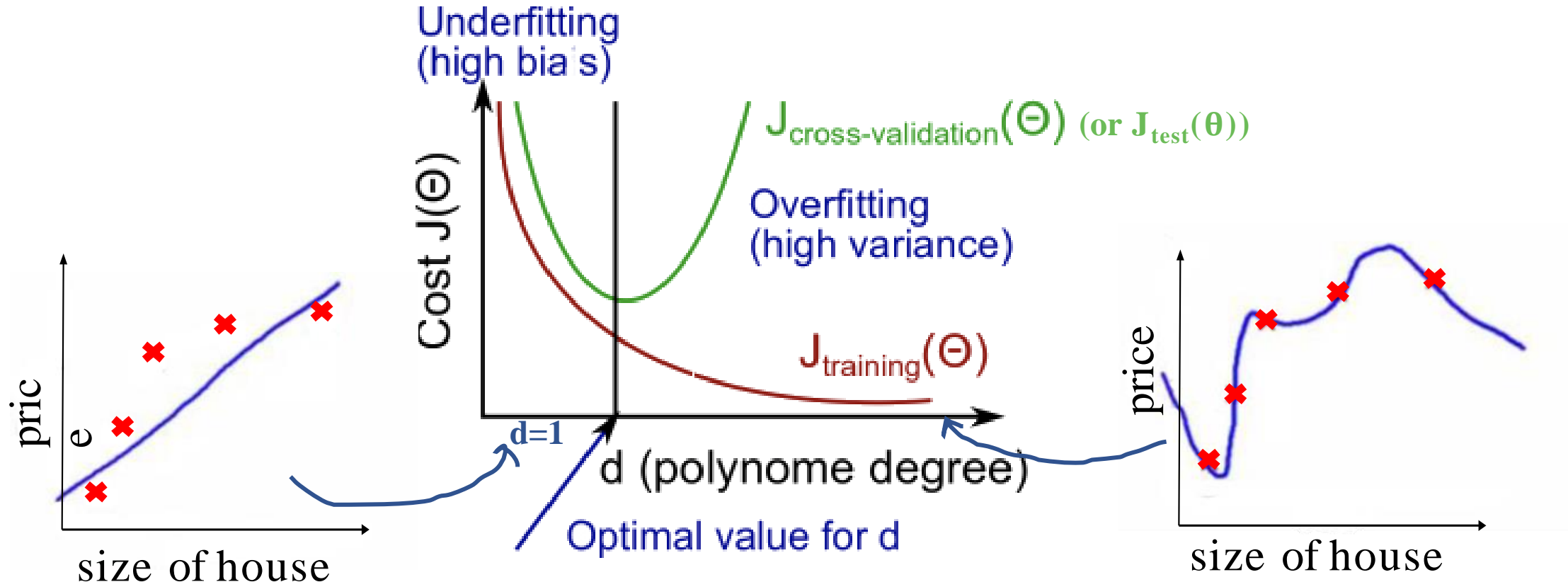
.

.

x_{100}

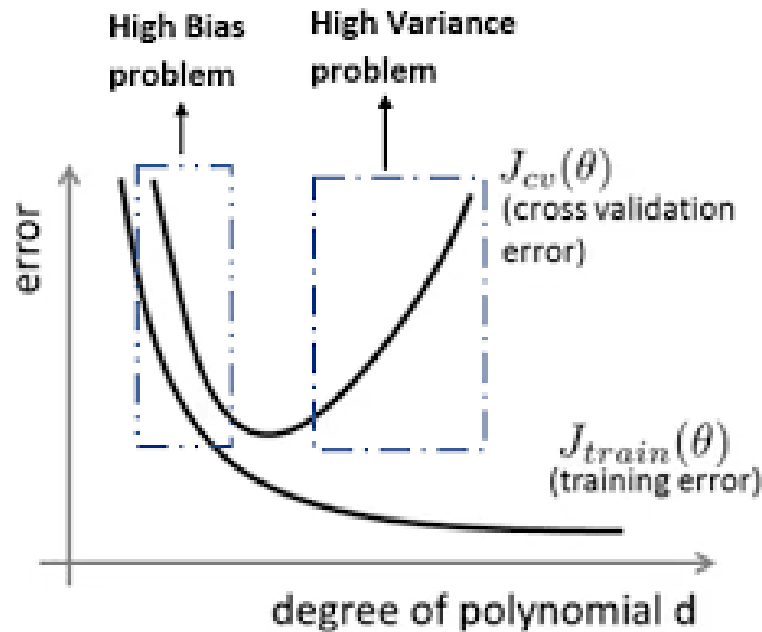
Bias/variance

- Training error: $J_{train}(\theta) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2$
- Cross validation error: $J_{cv}(\theta) = \frac{1}{2m_{cv}} \sum_{i=1}^{m_{cv}} (h_{\theta}(x_{cv}^{(i)}) - y_{cv}^{(i)})^2$ (or $J_{test}(\theta)$)



Diagnosing bias vs. variance

- Suppose your learning algorithm is performing less well than you were hoping. ($J_{cv}(\theta)$ or $J_{test}(\theta)$) is high.) Is it a bias problem or a variance problem?



<https://towardsdatascience.com/>

Bias (underfit):

$J_{train}(\theta)$ will be high

$$J_{cv}(\theta) \approx J_{train}(\theta)$$

Variance (overfit):

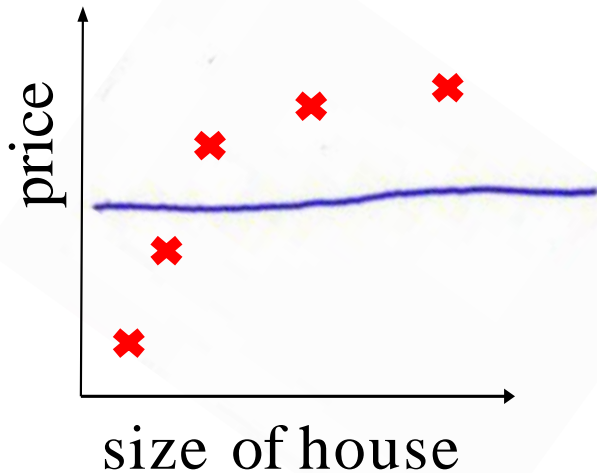
$J_{train}(\theta)$ will be low

$$J_{cv}(\theta) \gg J_{train}(\theta)$$

Linear regression with regularization

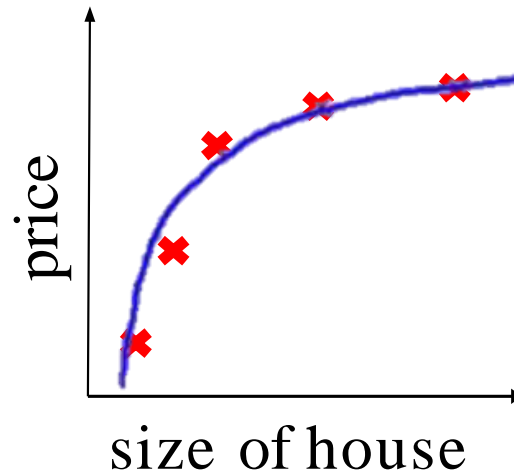
Model: $h_{\theta}(x) = \theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \theta_4 x^4$

$$J(\theta) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2 + \frac{\lambda}{2m} \sum_{j=1}^n \theta_j^2$$

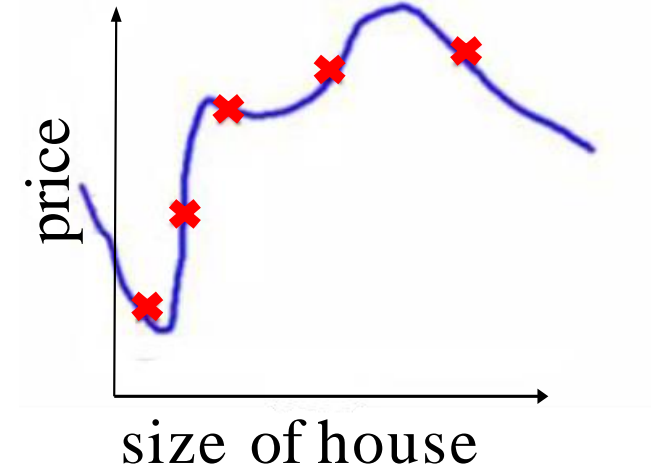


Large λ
High bias(underfit)

$$\lambda = 10000. \theta_1 \approx 0, \theta_2 \approx 0, \dots$$
$$h_{\theta}(x) \approx \theta_0$$



Intermediate λ
"Just right"



Small λ
High variance (overfit)

$$\lambda = 0$$

Choosing the regularization parameter λ

Model: $h_{\theta}(x) = \theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \theta_4 x^4$

$$J(\theta) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2 + \frac{\lambda}{2m} \sum_{j=1}^n \theta_j^2$$

1. Try $\lambda = 0 \rightarrow \min J(\theta) \rightarrow \theta^{(1)} \rightarrow J_{cv}(\theta^{(1)})$
2. Try $\lambda = 0.01 \rightarrow \min J(\theta) \rightarrow \theta^{(2)} \rightarrow J_{cv}(\theta^{(2)})$
3. Try $\lambda = 0.02 \rightarrow \min J(\theta) \rightarrow \theta^{(3)} \rightarrow J_{cv}(\theta^{(3)})$
4. Try $\lambda = 0.04$
5. Try $\lambda = 0.08 \rightarrow \min J(\theta) \rightarrow \theta^{(5)} \rightarrow J_{cv}(\theta^{(5)})$
- .
- .
- .
12. Try $\lambda = 10$ $\rightarrow \min J(\theta) \rightarrow \theta^{(12)} \rightarrow J_{cv}(\theta^{(12)})$
10.24 **Pick (say) $\theta^{(5)}$. Test error: $J_{test}(\theta^{(5)})$**

Bias/variance as a function of the regularization parameter λ

- **Training error:**

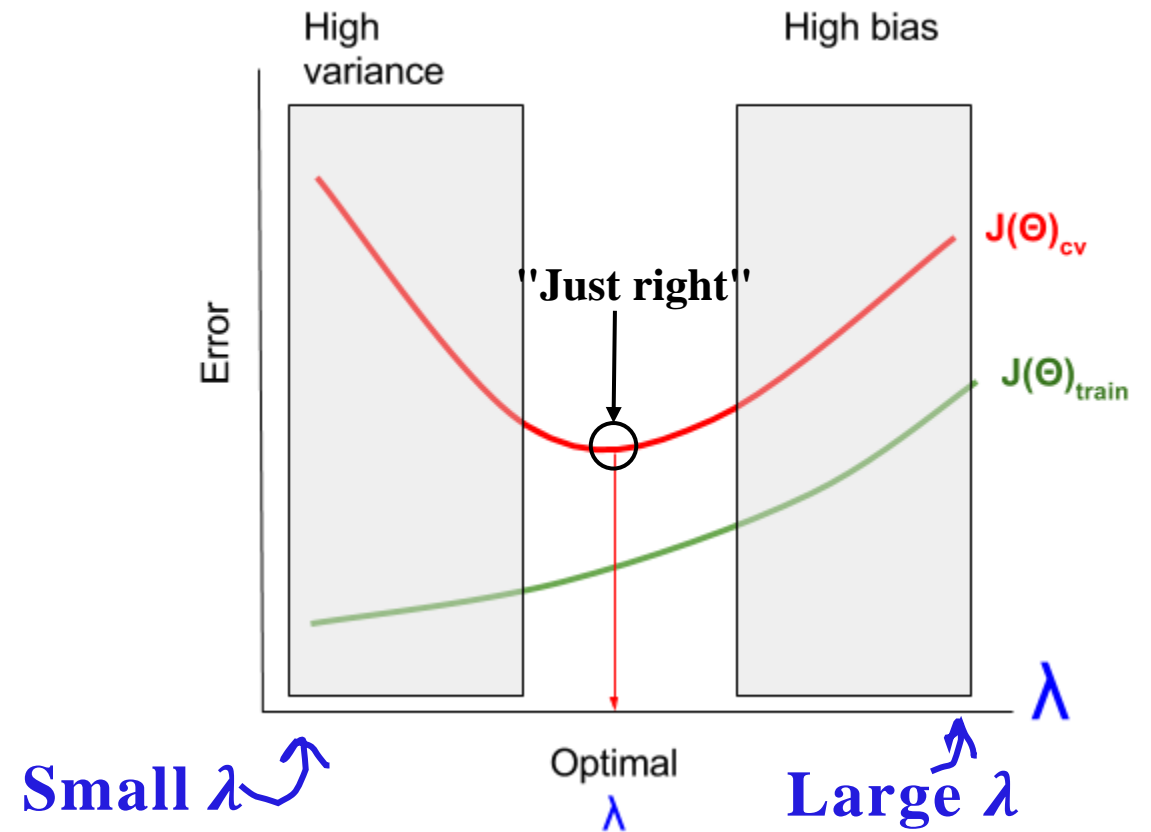
$$J_{train}(\theta) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2 + \lambda \sum_{j=1}^n \theta_j^2$$

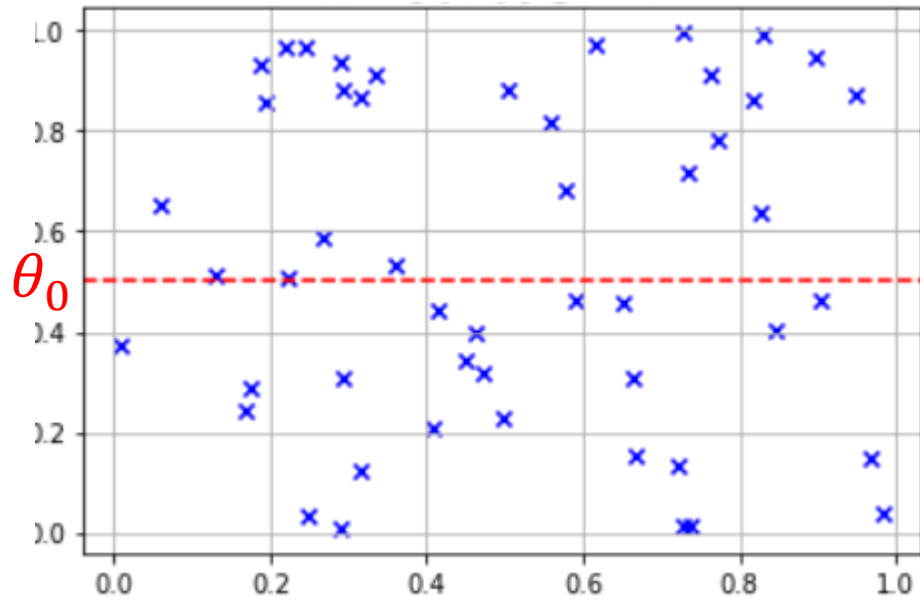
- **Cross Validation error:**

$$J_{cv}(\theta) = \frac{1}{2m_{cv}} \sum_{i=1}^{m_{cv}} (h_{\theta}(x_{cv}^{(i)}) - y_{cv}^{(i)})^2$$

- **Test error:**

$$J_{test}(\theta) = \frac{1}{2m_{test}} \sum_{i=1}^{m_{test}} (h_{\theta}(x_{test}^{(i)}) - y_{test}^{(i)})^2$$





GD:

Repeat until convergence{

$$\theta_0 = \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x_i) - y_i) \quad (x_0^i = 1)$$

$$\theta_j = \theta_j - \alpha \left[\frac{1}{m} \sum_{i=1}^m (h_{\theta}(x_i) - y_i) x_j^i + \lambda \theta_j \right]$$

$$j = 1, 2, \dots, n$$

}