



# Machine Learning

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[https://github.com/safayani/machine\\_learning\\_course](https://github.com/safayani/machine_learning_course)



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# Supervised Learning

- Regression
- Classification

# example

## Notation:

m: number of training samples

x: input variable

y: output variable

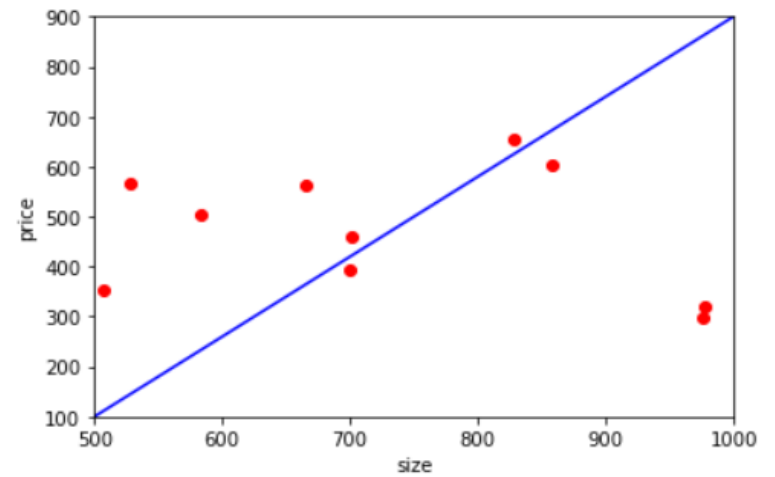
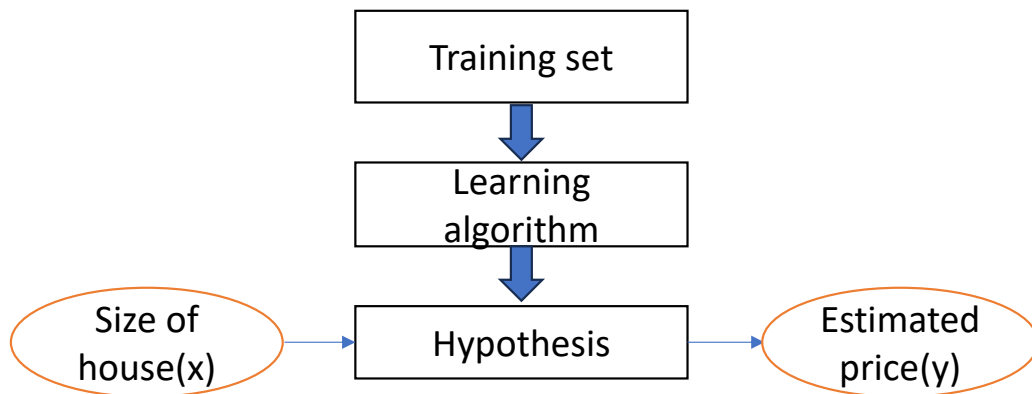
Or

target variable

$(x_i, y_i)$ : i th training sample

number	Size (x variable)	Price (y variable)	
1	100	500	$(x_1, y_1)$
2	750	2000	$(x_2, y_2)$
3	852	178	$(x_3, y_3)$
	...	...	
m	3210	870	$(x_m, y_m)$

# example

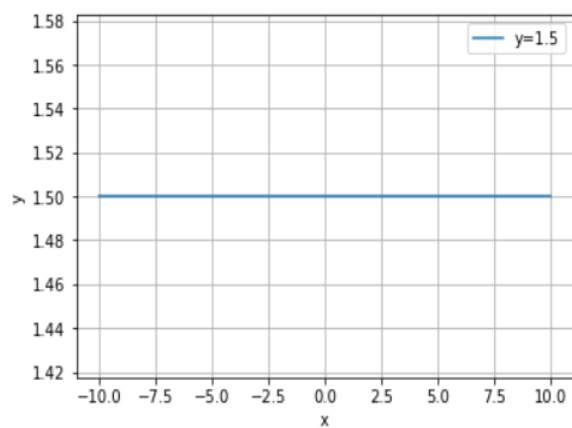


$$h(x) = \theta_0 + \theta_1 x$$

$$\text{parameters} = \left\{ \theta_0, \theta_1 \right\}$$

$$h(x) = \theta_0 + \theta_1 x$$

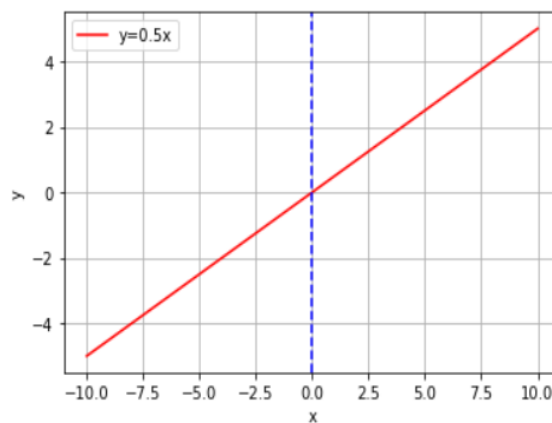
parameters=



$$h(x) = 1.5$$

$$\theta_0 = 1.5$$

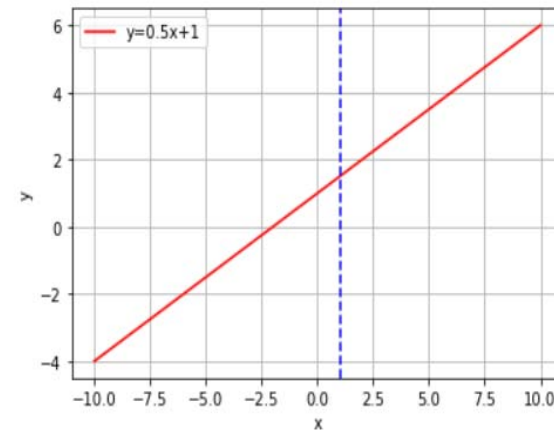
$$\theta_1 = 0$$



$$h(x) = 0.5x$$

$$\theta_0 = 0$$

$$\theta_1 = 0.5$$



$$h(x) = 0.5x + 1$$

$$\theta_0 = 1$$

$$\theta_1 = 0.5$$

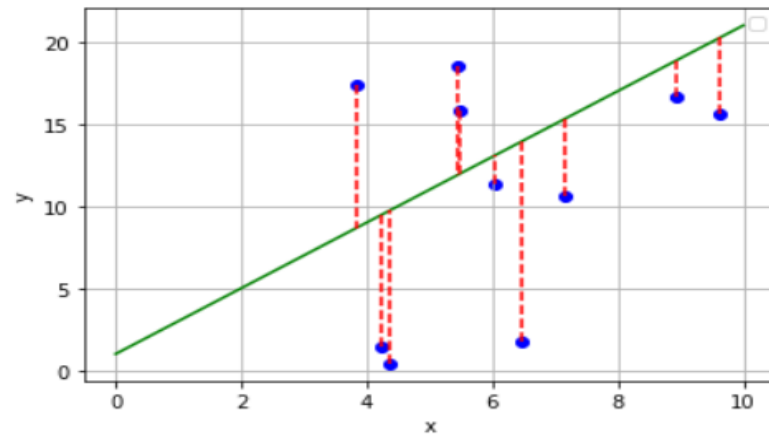
# Cost Function

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^m (h(x_i) - y_i)^2$$

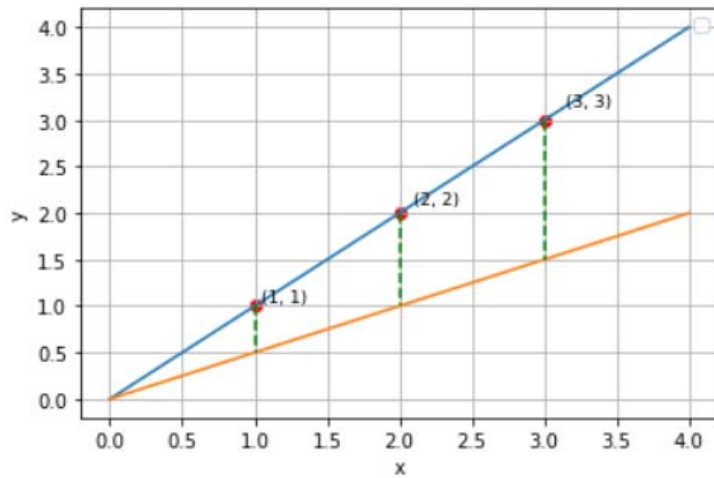
Mean square error(MSE)

Minimize  $J(\theta_0, \theta_1)$

$\theta_0, \theta_1$



# example

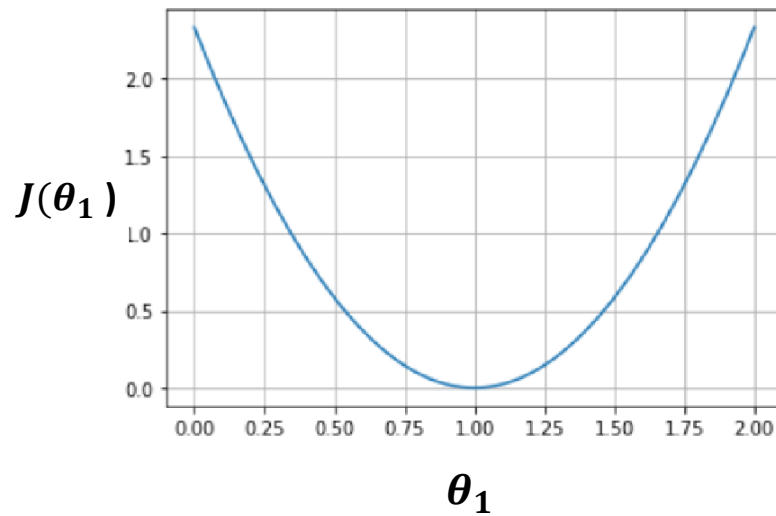


$$\begin{aligned} J(\theta_0=0, \theta_1=0.5) &= \frac{1}{2m} \sum_{i=1}^m (0.5x_i - y_i)^2 \\ &= \frac{1}{2 \cdot 3} [(0.5 - 1)^2 + (1 - 2)^2 + (1.5 - 3)^2] \\ &= \frac{1}{6} (3.5) = 0.58 \end{aligned}$$

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$$\begin{aligned} J(\theta_0=0, \theta_1=1) &= \frac{1}{2m} \sum_{i=1}^m (x_i - y_i)^2 \\ &= \frac{1}{2 \cdot 3} [(1 - 1)^2 + (2 - 2)^2 + (3 - 3)^2] \\ &= \frac{1}{6} (0) = 0 \end{aligned}$$

# example



$\theta_1$	$J(\theta_1)$
0	14/6
0.5	0.58
1	0
1.5	0.58
2	14/6

- Plotting the cost for each value of  $\theta_1$
- The minimum point:  $\theta_1=1$
- Using **Grid Search** to find best values of parameters



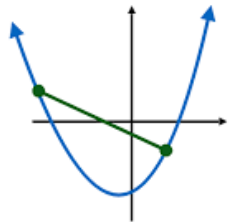
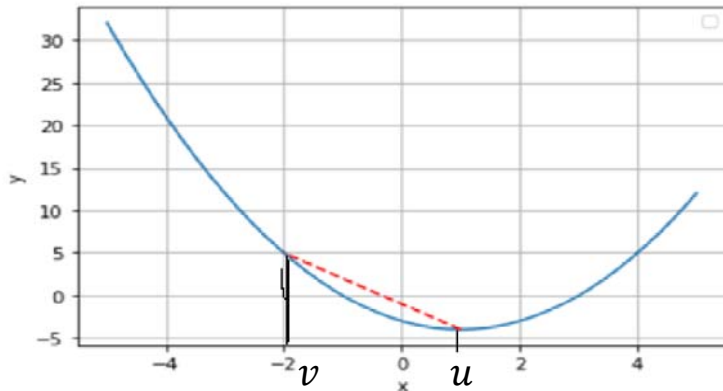
# Cost Function

- $J(\theta_0, \theta_1) = \frac{1}{m} \sum_{i=1}^m |h(x_i) - y_i|$  Mean absolute error(MAE)
- Better for outliers compared with MSE

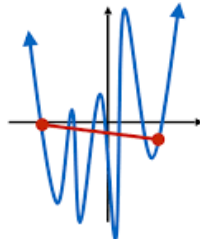
# Convexity

Function  $h(u)$  with  $u \in X$  is **convex** if for any  $u, v \in X$  and for any  $0 \leq \lambda \leq 1$  we have:

$$h(\lambda u + (1 - \lambda)v) \leq \lambda h(u) + (1 - \lambda) h(v)$$



convex

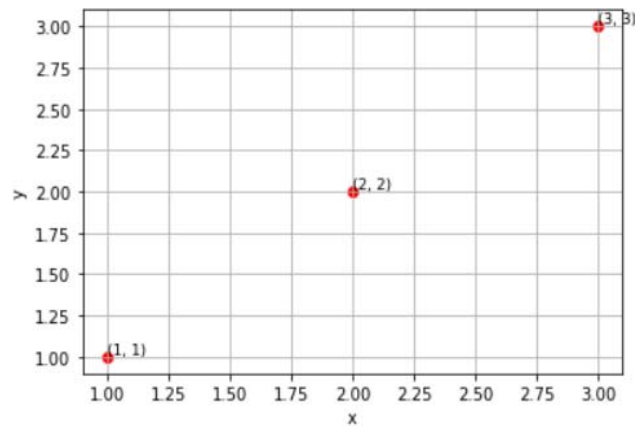


nonconvex

$f(y) \sim y^2$  is convex,  $f(y) \sim y^3$  is nonconvex

/.

# example



*if  $\theta_1 = -1$ :*

$$\text{MAE} = \frac{1}{3} [|1 - (-1)| + |2 - (-2)| + |3 - (-3)|] = 4$$

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*if  $\theta_1 = 0$ :*

$$\text{MAE} = \frac{1}{3} [|1 - 0| + |2 - 0| + |3 - 0|] = 2$$

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*if  $\theta_1 = 1$ :*

$$\text{MAE} = \frac{1}{3} [|1 - 1| + |2 - 2| + |3 - 3|] = 0$$

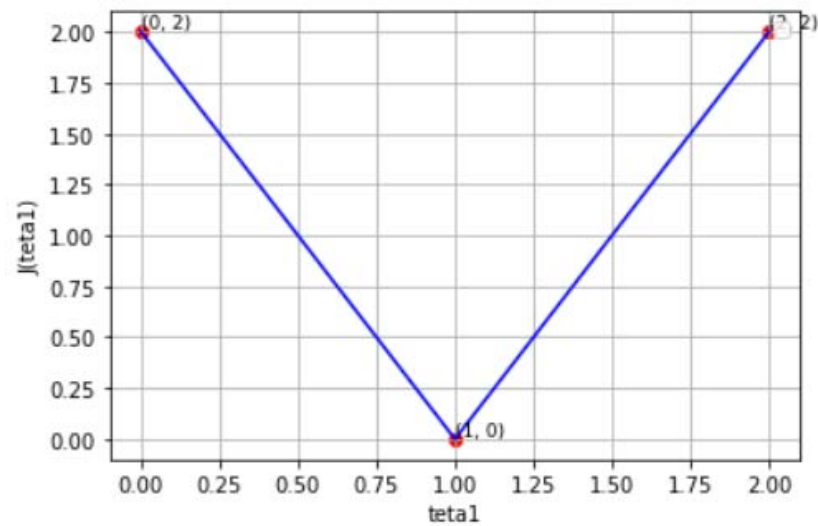
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*if  $\theta_1 = 2$ :*

$$\text{MAE} = \frac{1}{3} [|1 - 2| + |2 - 4| + |3 - 6|] = 2$$

//

# example



MAE is **convex**

$\theta_1$	$J(\theta_1)$
-1	4
-0.5	3
0	2
0.5	1
1	0
1.5	1
2	2
2.5	3
3	4