•Question: The light-curve shows substantial variations outside of eclipses – why?

Due to both stars having very high fill factors, the stars are similar in shape to a Roche-lobe (rather than a perfect sphere). This means that slight eclipsing can still occur even after the spherical outlines of the star no longer overlap due to the excess material forming the Roche-lobe eclipsing the star behind it in a smaller, less distinct and clear way.

Sensitivity tests

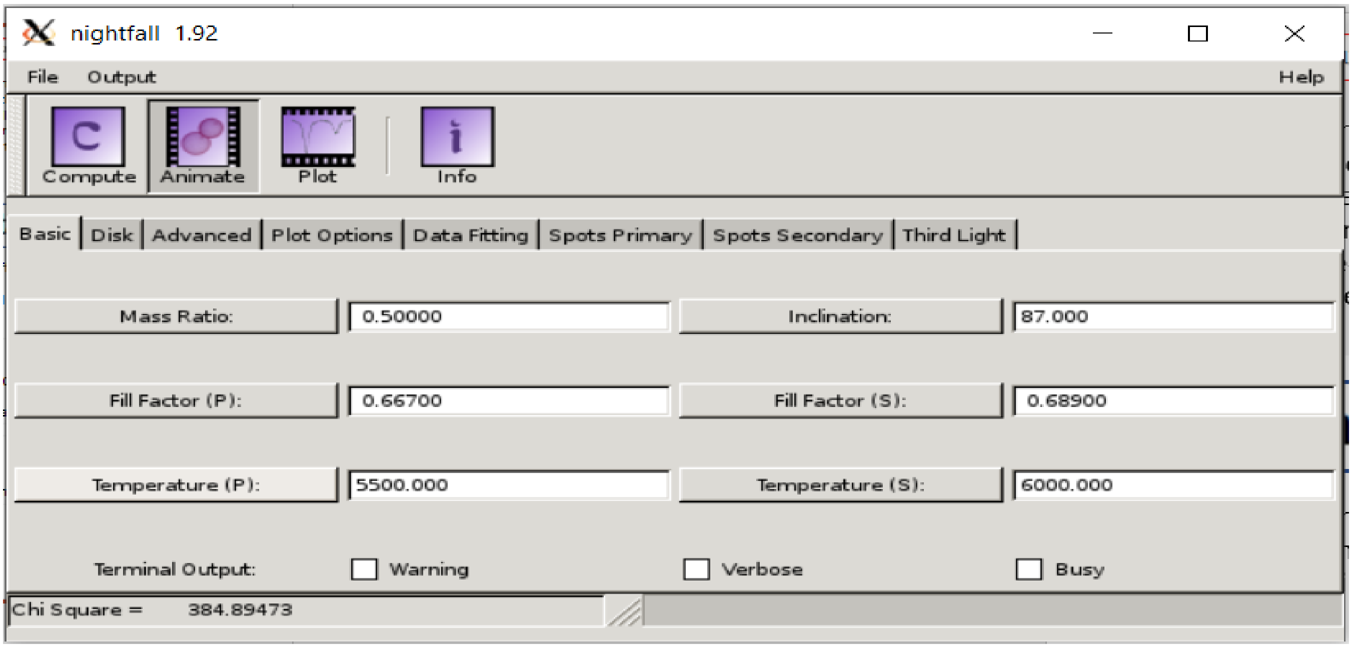
Here I will be summarizing the differences observed by changing some of the parameters from the original parameters preloaded in the uclo_test.cfg file. You can find the images of the graphs of light curves and R-V after the summary.

Original

<u>Width</u> = 0.2

<u>R-V graph</u>: P-star ranges from -100 to +100 km/s, while S-star ranges from -150 to 150 km/s.

<u>Light curve</u>: First dip was = -0.7, and second dip = -0.5.



Primary Temp = 10000K
Width = Same as original
R-V graph: Same as original
Light curve: Since primary is much brighter due to high temp, secondary being eclipsed of low relative effect. First dip = -0.1, while second dip = -0.9

Secondary Temp = 10000K
Width = Same as original
R-V graph: Same as original
Light curve: Since secondary is much brighter due to high temp, primary being eclipsed of low relative effect. First dip = -2.2, while second dip = -0.2

Switching primary and secondary temps
Width = Same as original
R-V graph: Same as original
Light curve: Reverse of original. First dip = -0.4, while second dip = -0.7

Increasing both temps by factor of 10
Width = Same as original
R-V graph: Same as original
Light curve: Dips got closer in depth to each other than original. First dip = -0.55, while second dip = -0.5

Angle of inclination = 90 , 80 , 70 ,60

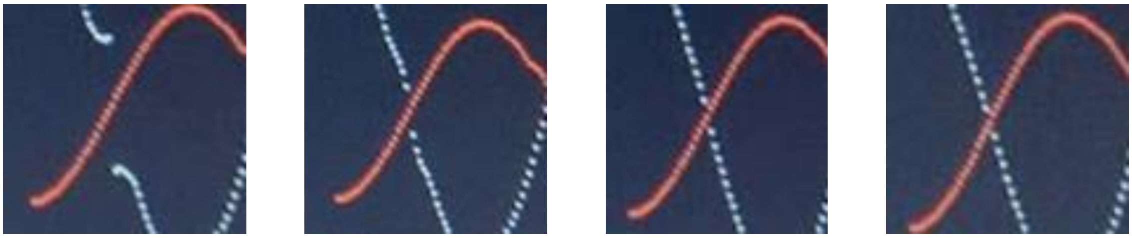
<u>Width</u> = Increased as angle decreased

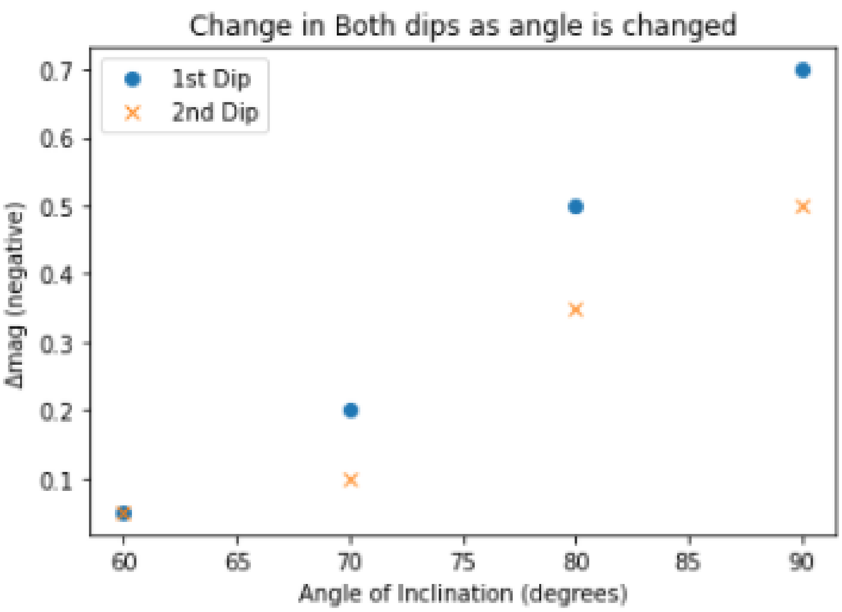
<u>R-V graph</u>: Breakage of curves at eclipse reduced as angle reduced.

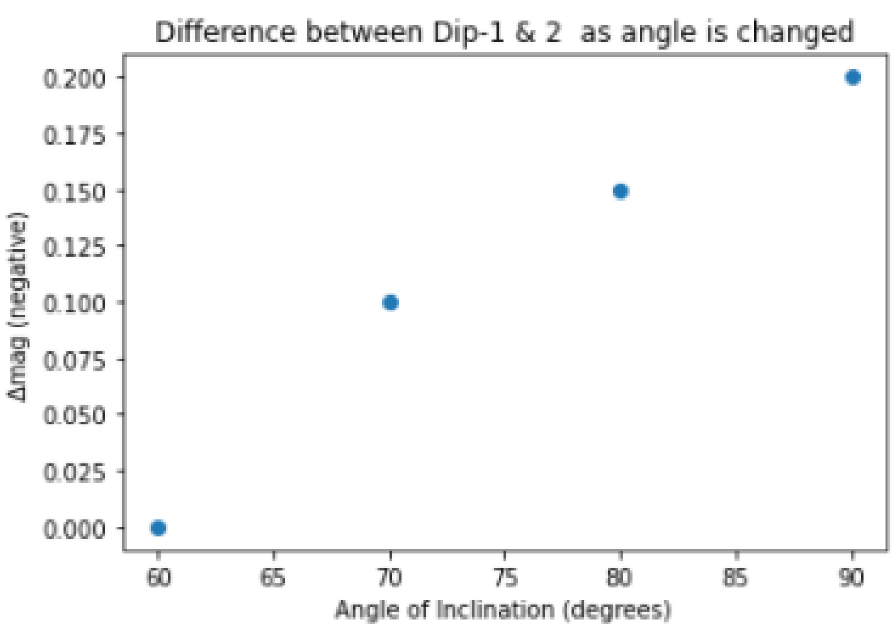
<u>Light curve</u>: Both dips got smaller, but also got closer in value as angle reduced.

	90°	80°	70°	60°

Breakage of lines





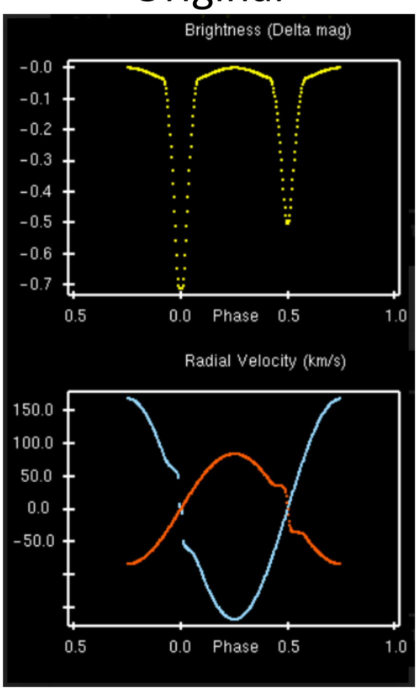


Primary’s Fill Factor = 0.5
Width = Same as original
R-V graph: Same as original
Light curve: First dip = -0.85, while second dip = -0.5

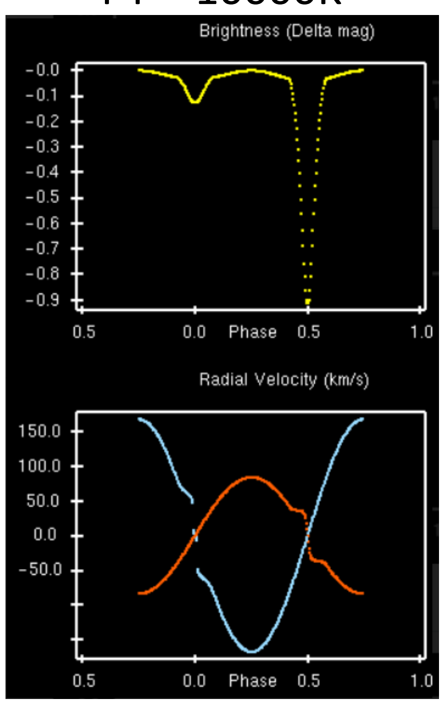
Secondary’s Fill Factor = 0.5
Width = Same as original
R-V graph: Same as original, but slightly less broken at eclipse points.
Light curve: First dip = -0.48, while second dip = -0.38

Graph Dump

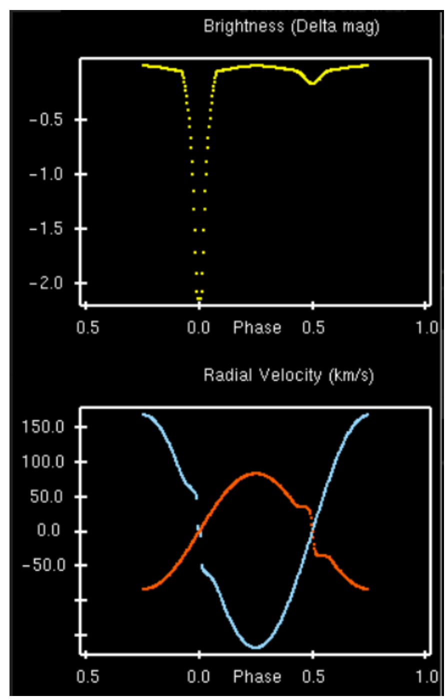
Original



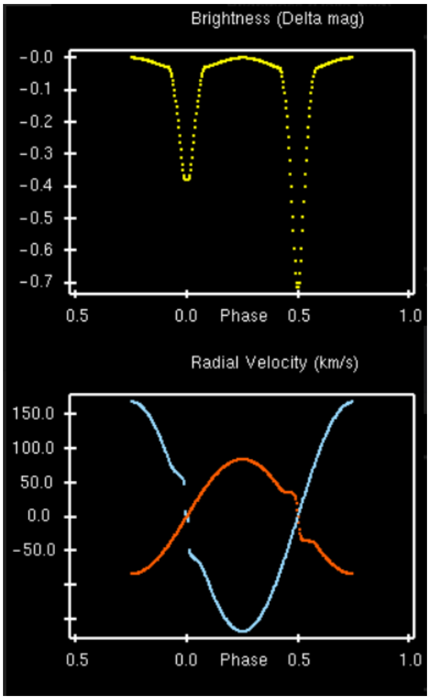
PT = 10000K



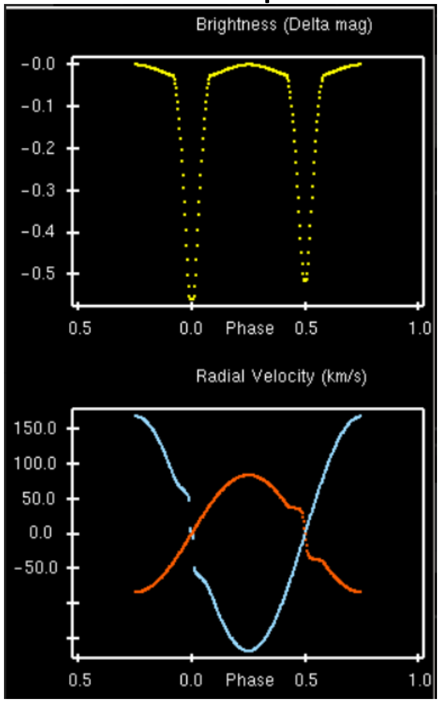
ST = 10000K



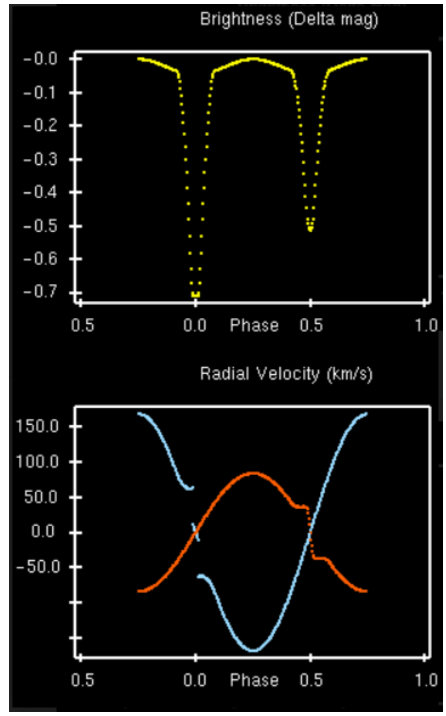
Switched Temp



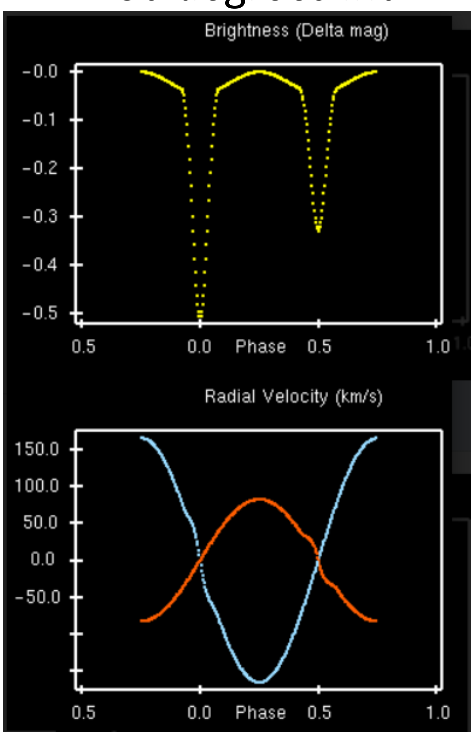
X10 Temp



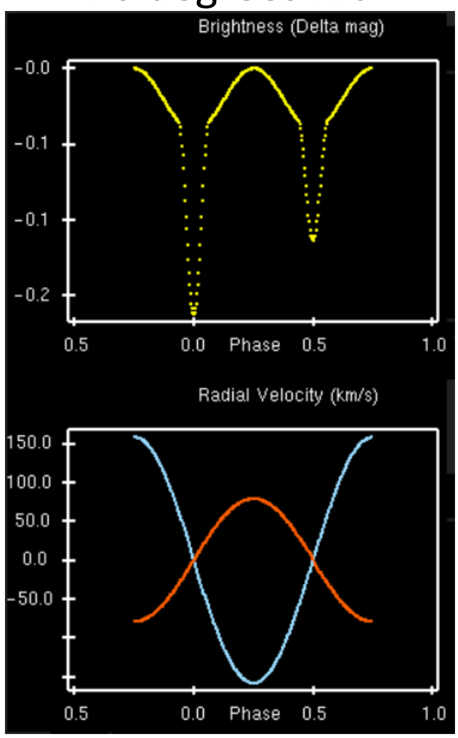
90 degrees inc



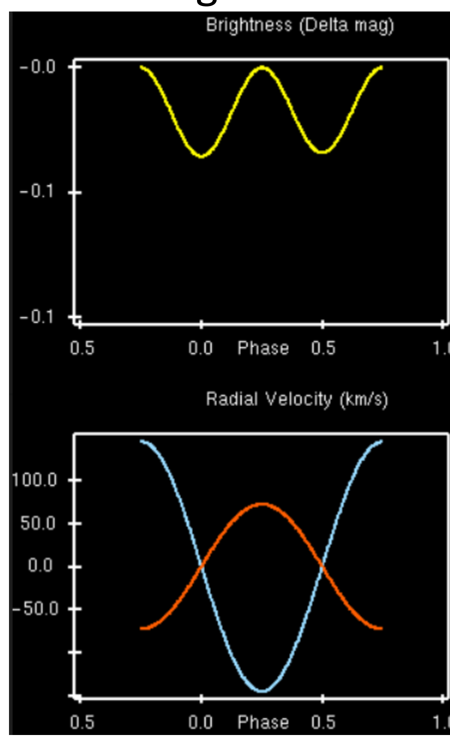
80 degrees inc



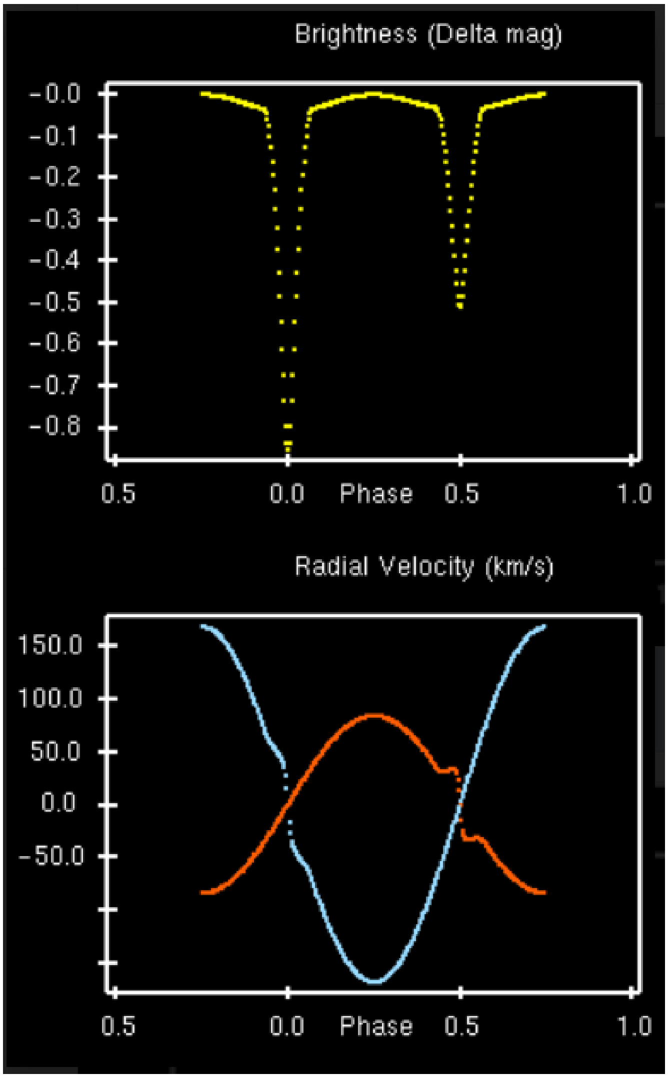
70 degrees inc



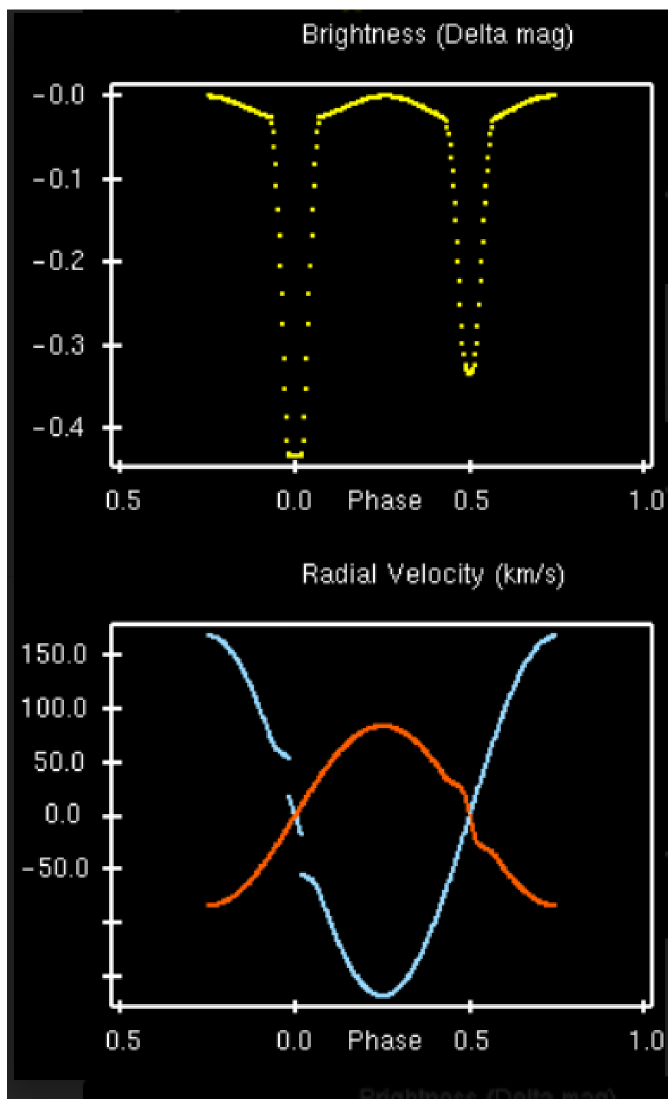
60 degrees inc



PFF = 0.5


Brightness (Delta mag)
-0.0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
0.5
0.0
Phase
0.5
1.0
Radial Velocity (km/s)
150.0
100.0
50.0
0.0
-50.0
0.5
0.0
Phase
0.5
1.0

SFF = 0.5


Brightness (Delta mag)
-0.0
-0.1
-0.2
-0.3
-0.4
0.5
0.0
Phase
0.5
1.0
Radial Velocity (km/s)
150.0
100.0
50.0
0.0
-50.0
0.5
0.0
Phase
0.5
1.0

Qualitative conclusions of sensitivity tests

Affects from Temperature

Increasing temp of smaller star resulted in the biggest change as it being eclipsed resulted in a big dip in brightness, (the biggest encountered in this experiment). This is due to it being very bright while also being able to be fully eclipsed by the dimmer star.

Increasing the temp of the big star had a smaller effect as it was out shining the smaller star when being eclipsed and when not being eclipsed.

Switching the temperatures resulted in is essentially the reverse of the original scenario. this is because the temps are very similar and so the mass ratio takes the bulk of the credit for eclipse effects.

When temp *10, they become so bright that mass or temp diff have negligible effects on the light curve.

Affects from Angle of Inclination

As the angle decreases, the breaks in the R-V graph when eclipses occur become reduced.

As the angle decreases, the depth of the dips in the light curve reduce in size. Also the difference in size between the two dips in a light curve also decreases, so that at 60 degrees, they were approximately the same depth.

The widths decreased while also becoming harder to discern due to the eclipses not being very complete.

Affects from Radii (changing fill factors)

There was no change in the width as the angle of inclination was unchanged.

Only Dip-1 increased when the primary's fill factor was decreased. When secondary's fill factor was increased, both dips' depth decreased.

The reason why only dip-1's depth increases when the PFF is decreased is because the brightness emitted by the primary decreases, as its temp and luminosity remain the same but its surface area for radiation decreases. Thus, when the brighter secondary is eclipsed, the change is more noticeable. Dip-2 doesn't change because the secondary is brighter than the primary, so the brightness detected regardless of the PFF is the same.

The reason why both dips' depth decreased when SFF decreased is because this reduced the radiative area while the luminosity was the same, so the total flux was reduced. Thus, when the secondary is eclipsed, the dip isn't that big since the star being eclipsed isn't as bright as it was. And when the primary is eclipsed, the depth is also lesser as the secondary is eclipsing a smaller part of the primary than the original.

•**Question:** Why does the mass ratio matter as far as the light-curve is concerned?

As the mass ratio is, for example, decreased, the primary gets more massive relatively, and thus has a larger Roche-lobe size. Thus, with the same fill factor, it can eclipse more of the star behind when it is eclipsing, while making it harder for the primary to be eclipsed. Also its flux increases, affecting the light curve. Also(in real-life), the more massive a star is, the faster it evolves and fills up a bigger Roche-lobe, causing changes in the light curve as matter from it starts moving towards the less massive star.

•Question: Why do the radial-velocity curves depend on parameters determined from the light-curve? (Hint: what would the radial-velocity curves look like if you view the system at 90° to the orbital plane?)

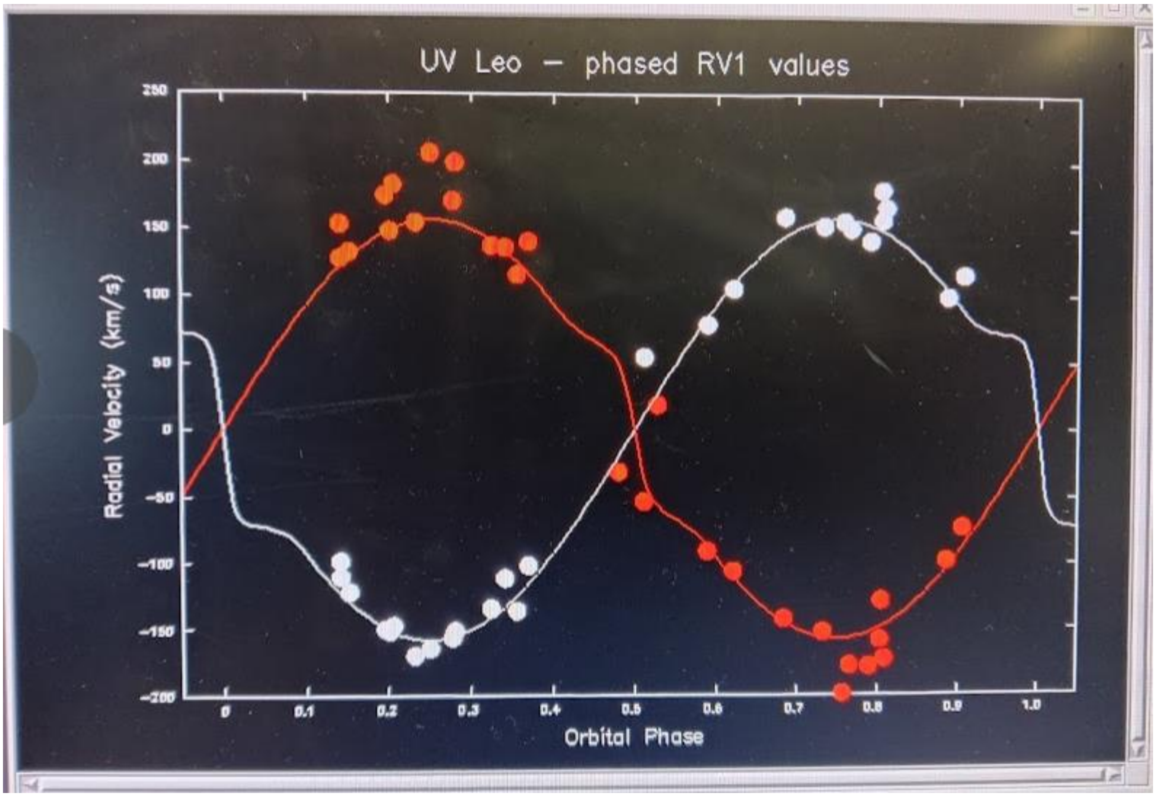
From varying the inclination values, we can observe that the clearer the change in brightness (delta mag) on the light curve, the closer we get to the actual R-V graph. When stars are very far from us, as is usual, looking at them from an angle perpendicular to the orbital plane gives us no idea of their radial velocities, as this is not directly measurable. In this case, the light curve is also trivial and thus of no help.

But when the light-curve becomes more significant, we can now use it to calculate the radial velocity curves. Thus, the properties of the light-curve affect the outcome of the R-V graph.

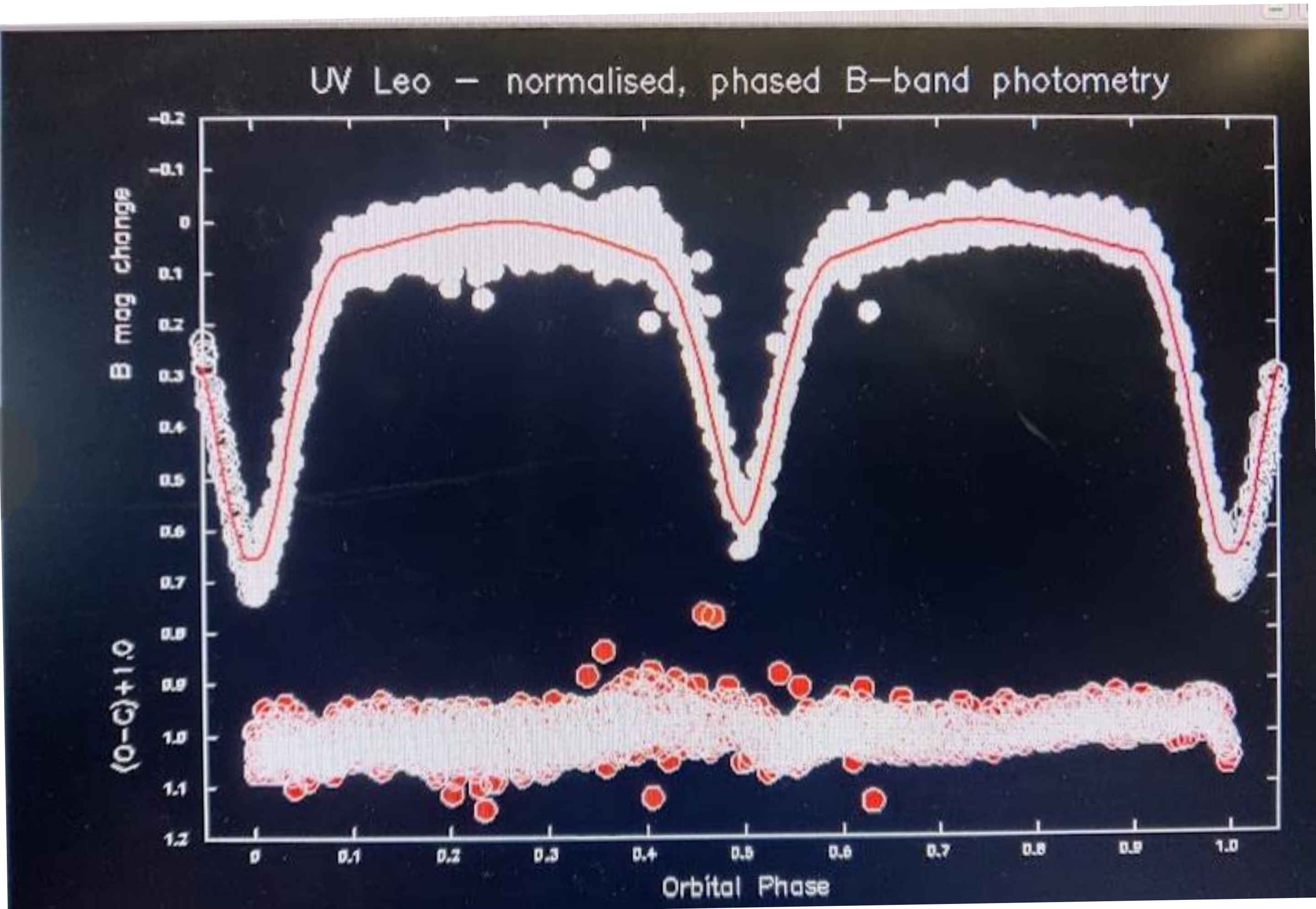
<u>Initial Values for combined mass and Mass ratio:</u>

Combined mass (Solar masses): **2.00**
Mass Ratio: **1.00**

R-V Model after adjustment



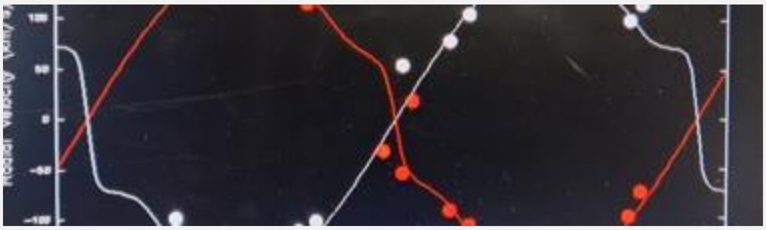
(B-band) Light-curve Model after adjustment



Note that the radial-velocity curves may not be the sine waves you might expect for two stars in circular orbits.

•**Question:** Why do you think this is? (Hint: NIGHTFALL assumes that the stars rotate synchronously; i.e., that each star's rotation period is the same as the orbital period. Do the departures from a sine wave occur in the RV curve of the eclipsing star, or or the eclipsed star?)

In my observation, the general shape of the model for both stars resembled a sine-wave, but there were irregularities in the model for both stars.



These were all found at points where the two curves meet, which suggests that the cause is the eclipse. When looking at which eclipses each curve's individual irregularities coincide with, it can be understood that the star with irregularities is the one that is being eclipsed. This irregularity might be due to insufficient information on the star's movements when it is eclipsed, leading to lesser data that the model makes up for with those irregularities.

Final summary of parameters and uncertainties

Basic tab

MR = 1.00 ± 0.05	**Inc = 86 ± 1.0°**
FFP = 0.87 ± 0.02	FFS = 0.65 ± 0.02
Temp Primary = 5900.0 ± 100K	Temp Secondary = 6100.0 ± 100K

Using B-band

Data fitting

Period (days) = 0.6001
Combined Mass (solar masses) = 2.00 ± 0.05

From the NightfallCurve.dat file, I got the radius of the two stars as follows:

Primary Star Radius = 9.08 x 10^8 m = 1.304$\odot$
Secondary Star Radius = 6.31 x 10^8 m = 0.906$\odot$

I could not compute uncertainty as the mean radius was not a mean of the polar radius and the point radius, but instead closer to the polar radius. Thus I did not compute uncertainties for the radius.

The T_{eff} values were:

Primary star: 5900.0 ± 100K
Secondary Star: 6100.0 ± 100K

Thus, using

$$L = 4\pi R^2 \sigma T^4$$

$$L = luminosity$$
$$R = stellar\ radius$$
$$T = surface\ temperature$$
$$\sigma = Stefan - Boltzmann\ constant$$

I got the luminosities:

Primary Star Lum = 7.118 x 10^{26} W*K^3
Secondary Star Lum = 3.93 x 10^{26} W*K^3

Then I divided their lums by the solar luminosity and took the log of that:

$$\log_{10}(L/L_\odot)$$

Primary star log lum = 0.269
Secondary star log lum = 0.0112

7.1

The log of the effective temperatures for the stars are:

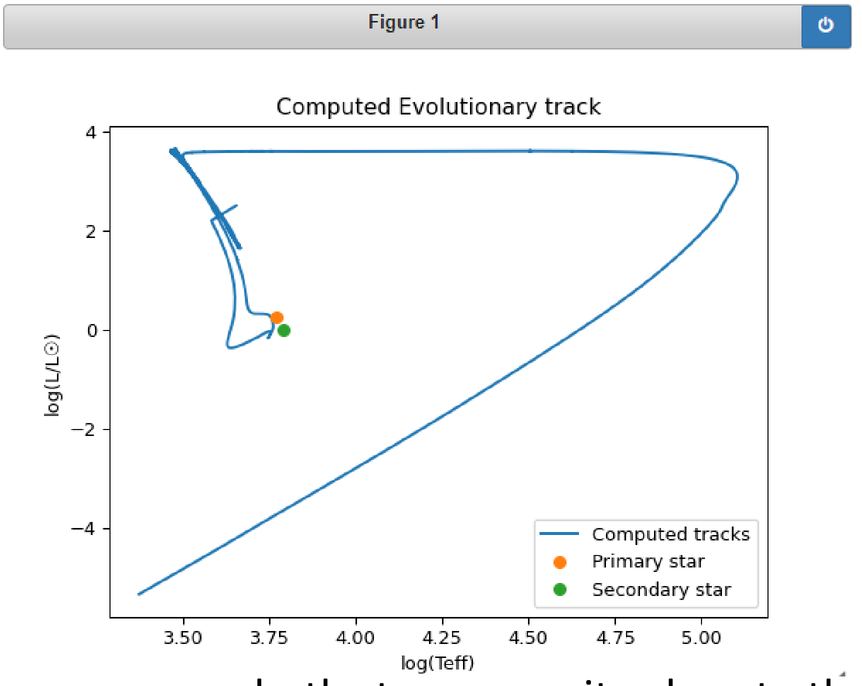
Primary star Log T_{eff} = 3.77
Secondary star Log T_{eff} = 3.79

Since both stars have same mass, I plotted only one evolutionary track plot but plotted the data from above on it for both stars.

In [26]:
```
%matplotlib notebook
y , x = np.loadtxt('./trimmed_history.data', skiprows = 6, usecols = (3,5), unpack = True)
PLL = 0.269
SLL = 0.0112
PLT = 3.77
SLT = 3.79

plt.figure()
plt.title('Computed Evolutionary track')
plt.legend()
plt.plot(x,y, label='Computed tracks')
plt.plot(PLT,PLL, 'o', label='Primary star')
plt.plot(SLT,SLL, 'o', label='Secondary star')
plt.xlabel('log(Teff)')
plt.ylabel('log(L/L☉)')

plt.legend()
```



As you can see, both stars are quite close to the evolutionary tracks. The point which they are closest to in the graph suggests they are both about 10^{10} years old, with the primary slightly further along its tracks. Both are pretty early in their evolutionary lives, only just having started their cosmic lives. They haven't even started burning helium yet according to the data.

7.2

Now, I will do the calculations required for part 7.2

$$F = \pi B_\lambda = \frac{2\pi h c^2}{\lambda^5} \cdot \frac{1}{e^{hc/kT\lambda} - 1}$$

Spectral irradiance

$h = 6.626 \times 10^{-34} \frac{m^2 \cdot kg}{s}$

$c = 3 \times 10^8 \ m/s$

$\lambda = 550 \times 10^{-9} m = 5.5 \times 10^{-7} \ m$

$k_B = 1.38 \times 10^{-23} \ J/K$ or $\frac{m^2 kg}{s^2 K}$

$T_1 = 5900K$

$T_2 = 6100 \ K$

From .dat file

$R_{*1} = 9.08 \times 10^8 m$

$R_{*2} = 6.31 \times 10^8 m$

$$\cancel{F_1 = 2 \cdot \pi \cdot 6.626 \times 10^{-34} \times (3 \times 10^8)^2 \cdot}$$

$$F_1 = 7.445 \times 10^{15} \cdot \frac{1}{e^{hc/kT_1\lambda} - 1} = 8.896 \times 10^{13} \ J/s^3 \cdot m^3 \text{ or } kg/s^3 \cdot m$$
$$= 8.896 \times 10^4 \ J/s^3 \cdot m^2 \cdot nm$$

$$F_2 = 7.445 \times 10^{15} \cdot \frac{1}{e^{hc/kT_2\lambda} - 1} = 1.031 \times 10^{14} \ J/s^3 \cdot m^3 \text{ or } kg/s^3 \cdot m$$
$$= 1.031 \times 10^5 \ J/s^3 m^2 nm$$

in e^x -> $\frac{m^2 \cdot kg}{s} \cdot \frac{m}{s} \cdot \frac{s^2 K}{m^2 kg} \cdot \frac{1}{K} \cdot \frac{1}{m}$

in frac -> $\frac{m^2 \cdot kg}{s} \cdot \frac{m^2}{s^2} \cdot \frac{1}{m^5} = kg/s^3 \cdot m$

$J/s \cdot \frac{m^2}{s^2} \cdot \frac{1}{m^5} = J/s^3 \cdot m^3$

-> Some unit calculations

Total emitted V-band flux: (for full surface of star)

① -> $F_1 \cdot 4\pi R_{1*}^2 = 8.896 \times 10^{13} \times 4\pi \times (9.08 \times 10^8)^2 = 9.2167 \times 10^{32} \ J/s^3 \cdot m$

② -> $F_2 \cdot 4\pi R_{2*}^2 = 1.031 \times 10^{14} \times 4\pi \times (6.31 \times 10^8)^2 = 5.1585 \times 10^{32} \ J/s^3 \cdot m$

$J/s^3 \cdot m^3 \cdot m^2 = J/s^3 \cdot m$

$J/(s^3 \cdot m^2) \ ; \ \frac{m^3 \cdot s}{J} \cdot \frac{J}{s^3 \cdot m^2} = \frac{m^2}{s^2}$

$J/(s^3 \cdot m^2) \ ; \ \frac{m^3 \cdot s}{J} \cdot \frac{J}{s^3 \cdot m^2} = \frac{m}{s^2}$

-> More unit calculations

$V = -2.5 \log_{10}(f_V) - 26.1 \quad ; \quad V = 8.9$

$8.9 = -2.5 \log_{10}(f_V) - 26.1$

$\log_{10}(f_V) = -14$

$f_V = 10^{-14} \; J/m^2 \cdot nm \cdot s$

$$\left.\begin{array}{l} ① = 9.2167 \times 10^{32} \; J/s^3 \cdot m \\ ② = 5.1585 \times 10^{32} \; J/s^3 \cdot m \end{array}\right\} \text{Total Emitted flux}$$

$$f_V = 10^{-14} \; J/m^2 \cdot nm \cdot s = 10^{-5} \; J/m^3 \cdot s \;\Big\}\; \text{Apparent flux (from V=8.9)}$$

d = distance from earth to the star.

Distances from earth

$① = 4\pi \cdot d_1^2 \cdot f_V$

$d_1 = \sqrt{\dfrac{①}{4\pi \cdot f_V}} = 2.71 \times 10^{18} \, m = \boxed{87.825 \text{ parsec}}$

$d_2 = \sqrt{\dfrac{②}{4\pi f_V}} = 2.026 \times 10^{18} \, m = \boxed{65.658 \text{ parsec}}$

$$\left.\begin{array}{l} M = -5 \log\left(\frac{d}{10}\right) + m \\ \quad = -5 \log\left(\frac{87.825}{10}\right) + 8.91 \\ \quad = 4.19 \end{array}\right\} \text{Sanity check via magnitudes}$$

Makes some sense

As you can see from the sanity check, the AbsMag is pretty close to sun's, but since I have used the brighter star there, which has a bigger radius than the sun, it is justified that the AbsMag is lower and the star is more luminous.

Note: The two distances d1 and d2 were too different from each other. I know that the differences arise when calculating the total emitted V-band flux using the radii on the .dat file. Since the values were too far apart (~22 parsecs, about the size of Orion nebula), I consulted stellarium and found that 87.825 parsec was closer to the real-life value than 65.658 parsecs, and thus I used that value for the sanity check.