PHAS0007 Computing Final Assignment 2020-21: Simulating Planetary Orbits

Revision date: December 16, 2020
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1. Introduction

You will take the programming skills you have developed throughout the course and use them, combined with your knowledge of classical mechanics, to create a VPython animation of planets orbiting a star.

The assignment is divided into two parts:

- You can ask questions and get advice from us on part A.

- Part B must be completed independently. You must work on this entirely on your own, without the assistance of other students, course staff or anyone else.

Everything in your submission must be either completely your own work, or have the source explicitly acknowledged and suitably referenced. We will be using several methods to check for any plagiarised or copied assignments

2. The physics

2.1   Newton's law of universal gravitation

Newton's law of universal gravitation states that the gravitational force between any two objects is proportional to their masses, and inversely proportional to the square of the distance between them:

$$F = \frac{G m_1 m_2}{r^2}$$

In SI units, the gravitational constant $G = 6.674 \times 10^{-11}$ N m^2 kg^{-2}. (This is very small: gravity is a very weak force!)

The image below illustrates Newton's law of universal gravitation. A point mass m_1 attracts another point mass m_2 by a force F_2 pointing along the line intersecting both points. In the same way, m_2 attracts m_1 with an equal force F_1 pointing in the opposite direction.
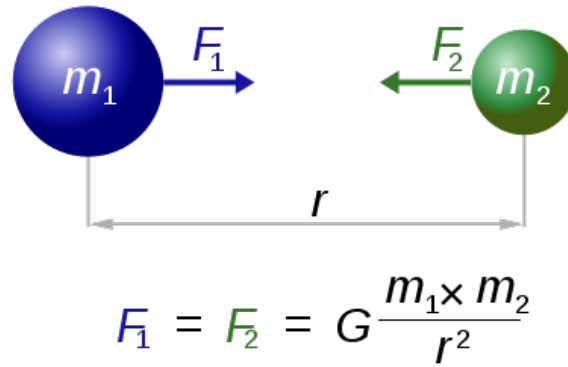
Figure 1: *Illustration of Newton's law of gravitation. Image created by Dennis Nilsson, CC BY 3.0, via Wikimedia Commons.*

Consider two masses m_1 and m_2, as shown in the figure. The gravitational force exerted on m_1 due to m_2 will be

$$\mathbf{F}_{12} = -G\frac{m_1 m_2}{|\mathbf{r}_{21}|^2}\hat{\mathbf{r}}_{21},$$

where $\mathbf{r}_{21}$ is the position vector from m_2 to m_1, i.e. $\mathbf{r}_1 - \mathbf{r}_2$

Similarly, the force exerted on m_2 due to m_1 will be

$$\mathbf{F}_{21} = -G\frac{m_1 m_2}{|\mathbf{r}_{12}|^2}\hat{\mathbf{r}}_{12}$$

and clearly $\mathbf{F}_{12} = -\mathbf{F}_{21}$: the forces are equal and opposite, as they should be according to Newton's third law.

From Newton's second law, we also have $\mathbf{F} = m\mathbf{a}$, so therefore we can see that the accelerations experienced by m_1 due to m_2 and vice versa will be

$$\mathbf{a}_1 = -\frac{Gm_2}{r_{12}^2}\hat{\mathbf{r}}_{12}$$

$$\mathbf{a}_2 = -\frac{Gm_1}{r_{21}^2}\hat{\mathbf{r}}_{21}$$

These accelerations will not be equal in magnitude unless the objects have equal masses.

2.2 Converting Newton's laws to a numerical model

Let's consider a system with two objects: a star with mass M and a planet with mass m. Under normal circumstances clearly $M \gg m$ and so the acceleration experienced by the star due to the planet will be small compared to the acceleration experienced by the planet due to the star. The gravitational force on the planet due to the star is then

$$\mathbf{F}_{mM} = -GMm\frac{\hat{\mathbf{r}}_{Mm}}{|\mathbf{r}_{Mm}|^2}$$

To keep things simple, we will assume the acceleration on the star due to the planet is small enough that we can neglect it, and put the star in a fixed position at the origin of our co-ordinate system. Thus $\mathbf{r}_{Mm} = \mathbf{r}_m - \mathbf{r}_M$ becomes $\mathbf{r}_m$. We'll drop the now unnecessary subscript on $\mathbf{r}$, and replace the unit vector $\hat{\mathbf{r}}$ with $\mathbf{r}/|r|$, to give a simpler-looking expression:

$$\mathbf{F}_{mM} = -GMm\frac{\mathbf{r}}{|\mathbf{r}|^3}.$$

We can work out the motion of the planet straight from this, just by using the laws of motion:

$$\mathbf{F}_{mM} = m\mathbf{a} = m\frac{d\mathbf{v}}{dt} = m\frac{d^2\mathbf{r}}{dt^2}$$

In order to calculate the motion of the planet around the star, we need to take the initial position and velocity of the planet as the starting positions, and integrate in order to calculate the force. We have to convert the differentials to small finite differences, and here we will use Euler's method, as in module 9. This is the simplest, most naive method and definitely not the most accurate, as you will learn in later computing courses.

We therefore rewrite the force as

$$\mathbf{F}_{mM} = m\frac{d\mathbf{v}}{dt} \approx m\frac{\delta\mathbf{v}}{\delta t}$$

and then rearrange this to give the change in velocity of the planet in the time δt as

$$\delta\mathbf{v} = \frac{\mathbf{F}_{Mm}}{m}\delta t = -GM\frac{\mathbf{r}}{|\mathbf{r}|^3}\delta t$$

where $\delta\mathbf{v}$ and δt are very small finite quantities. We can define δt (usually known as the timestep) in our code as a parameter.

Similarly, we can write the change in the planet's position over the same δt as

$$\delta\mathbf{r} = \mathbf{v}\delta t$$

Given our starting point of the initial position and velocity of the planet, and a fixed timestep δt, we can calculate how $\mathbf{r}$ and $\mathbf{v}$ change:

$$\mathbf{r}(t + \delta t) = \mathbf{r}(t) + \delta\mathbf{r}$$

$$= \mathbf{r}(t) + \mathbf{v}\delta t$$

$$\mathbf{v}(t + \delta t) = \mathbf{v}(t) + \delta \mathbf{v}$$

$$= \mathbf{v}(t) - \frac{GM\mathbf{r}}{|\mathbf{r}|^3}\delta t = \mathbf{v}(t) - \frac{GM\hat{\mathbf{r}}}{|\mathbf{r}|^2}\delta t$$

We can use these equations to calculate the path of the planet, and create an animation of this, just as we simulated the path of a projectile with air resistance in module 9.

3. Task instructions

3.1 Part A (40% of marks)

- Download the template notebook provided.

- Complete and edit this template to create your own self-contained notebook, completing the tasks described.

- The template includes outline functions for you to complete to

    1) calculate the gravitational force between two objects;

    2) update the position of the star; in each time step;

    3) display an animation of a planet orbiting a star.

- You will also need to complete the introduction text cell, and add further code and text cells in the "investigation" section.

- The template also includes some additional code to help you ensure each function is working correctly. You may remove this once you have fully tested your code, or leave it in the notebook, or even add your own tests if you wish.

3.2 Part B

1) Two planets (20% of marks)

a) Copy the `animate_planet` function from part A into a new code cell, and adapt it so that instead of a single planet it shows the paths of two separate planets in the gravitational field of the same star. You should not take into account the gravitational attraction between the planets in this section, only the force between each planet and the star.

Instead of a single position and velocity, your function will need to take as arguments **two** positions and velocities (a position and velocity for each planet) and return **two** positions and velocities.

Your function can call `move_planet` from part A, and hence make use of the existing `force` function. You do not need to repeat or change these functions.

b) Test your code using the following initial conditions:

- first planet: position $(1.85, 0, 0)$, velocity $(0, 23.25, 0)$

- second planet: position $(1.96, 0.4, 0)$, velocity $(-4.44, 21.9, 0)$

Describe and comment on the behaviour in a text cell.

2) **Two planets with gravitational interactions (30% of marks)**

a) Create a new function, similar to `move_planet`, but this time dealing with two planets and taking into account the gravitational attraction between the planets as well as between each planet and the star.

In addition to the positions and velocities of the two planets, this function will need to take two more arguments to represent the masses of the planets.

Your function can call the `force` function from part A: you do not need to repeat or change this function.

b) Create a new function, similar to `animate_planet`, but this time dealing with two planets and using your new `move_planet` function to take into account the gravitational attraction between the planets as well as between each planet and the star.

c) Test your code using the same initial conditions as for the previous case, and with each planet having a mass of 2.0 in the units we are using for this assignment.

Describe and comment on the behaviour in a text cell.

3) **Further investigation (10% of marks)**

Try varying some of the parameters of your simulation: the masses of the star and the planets, and the starting positions and velocities of the planets. There are many aspects you could investigate, but you should pick one or two. Create a few animations to illustrate some of the effects you observe, and write a conclusion describing the behaviour and attempting to explain it.

For example, you could

- calculate the required velocity to achieve a stable circular orbit with a given radius, and see how close or how massive the planets have to be to have a significant effect on each other's orbit;

- turn one of the planets into a moon orbiting the other, by careful choice of masses and initial conditions. (This is **hard**.)

4. Strategies and hints

You are recommended to break each problem down into small steps. It is easier to get a small, simple piece of code working and then build on it than to write a larger section in one go and then have to debug it. In part A we have helped by splitting the overall task into three functions for you to complete one at a time. In part B it is up to you how you manage the process.

Make sure you explain all choices you make in your implementation, especially in the comments. If you use an unusual coding approach, you **must** explain it clearly in the comments, and state why you have chosen this approach, otherwise you may not receive full credit for it. You must also ensure that if you reuse or repurpose any code from another source (including your own coursework submissions) you include a reference to this in both the comments and the bibliography.

As usual, you are recommended to comment your code as you go along, as this forms an essential part of the debugging procedure. You may also find it useful to enable line numbering in your code cells (press ESC-L in a code cell).

You are strongly encouraged to make full use of the Jupyter VPython documentation available at http://www.glowscript.org/docs/VPythonDocs/index.html.

Before you submit your code, make sure that you have tested it fully.

You are **strongly advised** not to attempt to complete this assignment in a single coding session. Plan your working time sensibly to allow enough time for completion before the published deadline.

5. Assessment

Everything in your submission must be either **completely your own work**, or have the source **explicitly acknowledged and suitably referenced**. We will be using several methods to check for any plagiarised or copied assignments.

5.1 Anonymity

As for the Reading Week assignment, this assignment will be graded anonymously. Please ensure that

- your name does not appear anywhere in your submission;

- you include your student number in the name of your submitted file;

- you include your student number at the start of your submitted Jupyter notebook;

- if you need to include a reference to one of your own submissions in your notebook, you should replace your name with your student number in the reference.

We will not use the student number to identify you while marking, but it is helpful in case there are any technical problems that require someone to check the status of your file.

It is your responsibility to ensure that you submit the correct file. After uploading your notebook, you should download it again and run the downloaded version right through to ensure it works as expected.

5.2 Grading criteria

Your work will be graded using a mark scheme based on the following assessment criteria:

- whether your calculations are correct (40% of available marks);

- the quality of your code, including structure, layout, style, and comments (25% of available marks);

- the quality and presentation of your text commentary including referencing, discussion, and any conclusions you have drawn from your results (25% of available marks);

- the appearance of your animations (5% of available marks);

The remaining 5% of the marks are reserved for any improvements or extensions that are relevant to the task but go beyond what is required.

Remember that this is a formal assignment: your document should be clearly laid out in sections, with complete, grammatically correct sentences and paragraphs rather than bullet point lists.

You must a bibliography, using the Harvard referencing style, as for the Reading Week assignment. As a minimum, this should include a reference to this script, but you should also include any and all external sources you referred to in order to complete this assignment.