EECS 442 Computer Vision
Homework 2

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1 Problem 1

(a) Solution

Because $\mathbf{M} = [A, b]$ with rank 3, we can get a matrix $\mathbf{H_1}$ with the form

$$\mathbf{H_1} = \begin{bmatrix} A^{-1} & -A^{-1}b \\ 0 & 1 \end{bmatrix}$$

such that

$$\mathbf{M} \cdot \mathbf{H_1} = [I, 0]$$

Apply $\mathbf{H_1}$ to $\mathbf{M}^{`}$, we can get

$$\mathbf{M}^{`} \cdot \mathbf{H_1} = [A^{`}A^{-1}, -A^{`}A^{-1}b + b^{`}]$$

which is a 3×4 matrix with each element noted as $m^{`}_{ij}$.
Now we can multiply another matrix

$$\mathbf{H_2} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\frac{m^{`}_{31}}{m^{`}_{34}} & -\frac{m^{`}_{32}}{m^{`}_{34}} & -\frac{m^{`}_{33}}{m^{`}_{34}} & \frac{1}{m^{`}_{34}} \end{bmatrix}$$

We multiply both $\mathbf{H_1}$ and $\mathbf{H_2}$ to $\mathbf{M}$ and $\mathbf{M}^{`}$ respectively. We will have

$$MH_1H_2 = \begin{bmatrix} I, 0 \end{bmatrix} \text{ and } M^{`}H_1H_2 = \begin{bmatrix} m^{`}_{11} - \frac{m^{`}_{14}m^{`}_{31}}{m^{`}_{34}} & m^{`}_{12} - \frac{m^{`}_{14}m^{1}_{32}}{m^{`}_{34}} & m^{`}_{13} - \frac{m^{`}_{14}m^{`}_{33}}{m^{`}_{34}} & \frac{m^{`}_{14}}{m^{`}_{34}} \\ m^{`}_{21} - \frac{m^{`}_{24}m^{`}_{31}}{m^{`}_{34}} & m^{`}_{22} - \frac{m^{`}_{24}m^{1}_{32}}{m^{`}_{34}} & m^{`}_{23} - \frac{m^{`}_{24}m^{`}_{33}}{m^{`}_{34}} & \frac{m^{`}_{24}}{m^{`}_{34}} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Therefore, the projection matrix $\mathbf{H} = \mathbf{H_1}\mathbf{H_2}$ is the projection matrix we want.

(b) Proof

Since $\mathbf{MX} = (\mathbf{MH})(\mathbf{H}^{(} - \mathbf{1})\mathbf{X})$. If x and $x^{`}$ are corresponding points with respect to $(\mathbf{M}, \mathbf{M}^{`})$ for the 3D point $\mathbf{X}$, they are also matched with respect to cameras $(MH, M^{`}H)$ for the 3D point $\mathbf{H^{-1}X}$. Therefore, the fundamental matrices are the same for both camera systems.

(c) Solution

From the conclusion in (b), we know that the fundamental matrix for for camera pair $\mathbf{M}, \mathbf{M}^{`}$ is the same as $(\hat{M}, \hat{M}^{`})$, which by definition is $[\hat{b}_{\times}\hat{A}]$.
Thus, F can be calculated as

$$\begin{bmatrix} 0 & -1 & b_2 \\ 1 & 0 & -b_1 \\ -b_2 & b_1 & 0 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} -a_{21} & -a_{22} & -a_{23} \\ a_{11} & a_{12} & a_{13} \\ -a_{11}b_2 + a_{21}b_1 & -a_{12}b_2 + a_{22}b1 & -a_{13}b_2 + a_{23}b_1 \end{bmatrix}$$

We can divide F by a_{12} to get its seven parameter expression.
The seven parameters are $-\frac{a_{21}}{a_{12}}, -\frac{a_{22}}{a_{12}}, -\frac{a_{23}}{a_{12}}, \frac{a_{11}}{a_{12}}, \frac{a_{13}}{a_{12}}, b_1, b_2$

2 Problem 2

Suppose x is the intersection of l and k, we can get $x = k \times l$. Since $l^{`} = Fx = F(k \times l) = F[k]_{\times} l$

3 Problem 3

(a) Program

The matlab program is archived along with this pdf.
Run script *GenerateFMatrix.m*. It will call functions in *FMat.m* and *FMatNorm.m* that calculate fundamental matrices with basic 8-point algorithm and its normalized version. The script will return fundamental matrices, distances for both data set with both methods as well as the images with epipolar lines. See Figure 1
The distance for basic 8-point algorithm is 28.026 and 25.163 for data set 1, 9.7014 and 14.568 for data set 2.
With its normalized version, the distance is 0.88440 and 0.82422 for data set 1, 0.89140 and 0.89353 for data set 2.

(b) Program

Run script *ImgRectification.m*. It will call function in *Rectification.m*, which will use FMatNorm function in (a). It will return the transformation matrix H and H_prime for the image pair. The average error among transformed feature points pairs will also be returned.
The transformation matrix H, $H^{`}$ and error is shown in figure 2.

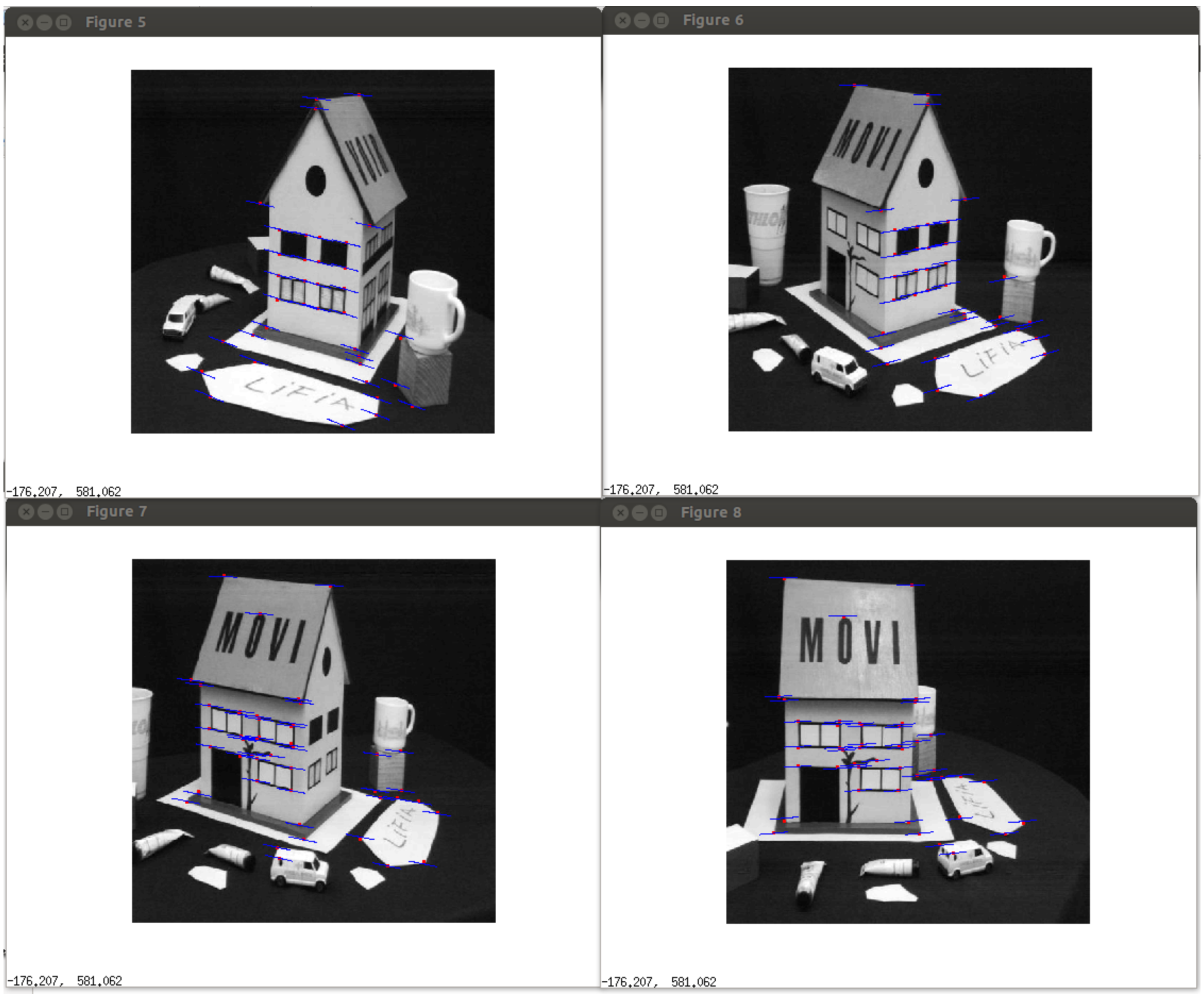

Figure 1: Epipolar lines with normalized 8-point algorithm

```
>>>H1 =

   2.1065e-06  -1.6470e-06  -3.3540e-03
   1.2335e-06  -5.9127e-06  -1.7170e-03
  -6.2663e-09   2.0130e-10   1.0212e-05

H1_prime =

  -9.9489e-01  -1.0100e-01  -2.8055e+02
   1.0100e-01  -9.9489e-01  -2.2884e+02
   9.3144e-04   9.4559e-05   1.2627e+00

err1 =  74.279
H2 =

  -7.3225e-07   2.7156e-06   2.2629e-03
  -5.4272e-07   5.2278e-06   1.4334e-03
   2.2287e-09  -9.5275e-11  -7.5068e-06

H2_prime =

  -9.9279e-01  -1.1988e-01  -2.8484e+02
   1.1988e-01  -9.9279e-01  -2.2347e+02
   5.8878e-04   7.1092e-05   1.1689e+00

err2 =  58.188
```

Figure 2: Transformation matrices and errors