COMP-2650-01
Assignment #2

1. (a) (5 pts) Prove by algebraic method whether $\overline{a} + ab + a\overline{c} + a\overline{b}\,\overline{c} = \overline{a} + \overline{b} + \overline{c}$.
(b) (5 pts) Find the CPOS of $f(x, y, z) = (x + \overline{y})y + \overline{x}z + \overline{x + y} + \overline{y}(\overline{x + z})$.

Solution:

(a) $\overline{a} + ab + a\overline{c} + a\overline{b}\,\overline{c} = \overline{a} + \overline{b} + \overline{c}$

LHS $= \overline{a} + ab + a\overline{c} + a\overline{b}\,\overline{c}$

$= \overline{a} + a\,(b + \overline{c} + \overline{b}\,\overline{c})$ *distributivity*

$= \overline{a} + a\,(b + \overline{b}\,\overline{c} + \overline{c})$ *commutativity*

$= \overline{a} + a\,(b + \overline{c} + \overline{c})$ *no name*

$= \overline{a} + a\,(b + \overline{c})$ *idempotency*

$= \overline{a} + b + \overline{c}$ *no name*

$\therefore$ *LHS ≠ RHS*

$\therefore \overline{a} + ab + a\overline{c} + a\overline{b}\,\overline{c} \neq \overline{a} + \overline{b} + \overline{c}$

(b) *CPOS of* $f(x, y, z) = (x + \overline{y})y + \overline{x}z + \overline{x + y} + \overline{y}(\overline{x + z})$

$f(x, y, z) = (x + \overline{y})y + \overline{x}z + \overline{x + y} + \overline{y}(\overline{x + z})$

$= (x + \overline{y})y + \overline{x}z + \overline{x}\,\overline{y} + \overline{y}(\overline{x}\,\overline{z})$ *DeMorgan's Law*

$= xy + \overline{y}y + \overline{x}z + \overline{x}\,\overline{y} + \overline{y}\,\overline{x}\,\overline{z}$ *Distributivity*

$= xy + 0 + \overline{x}z + \overline{x}\,\overline{y} + \overline{y}\,\overline{x}\,\overline{z}$ *Complementation*

$= xy + \overline{x}z + \overline{x}\,\overline{y} + \overline{y}\,\overline{x}\,\overline{z}$ *Identity element*

$= xy(z+\overline{z}) + \overline{x}z(y+\overline{y}) + \overline{x}\,\overline{y}(z + \overline{z}) + \overline{y}\,\overline{x}\,\overline{z}$ *Complementation*

$= xyz + xy\overline{z} + \overline{x}yz + \overline{x}\,\overline{y}z + \overline{x}\,\overline{y}z + \overline{x}\,\overline{y}\,\overline{z} + \overline{x}\,\overline{y}\,\overline{z}$ *Distributivity/Commutativity*

$= m_7 + m_6 + m_3 + m_1 + m_1 + m_0 + m_0$

$= \Sigma m(0, 1, 3, 6, 7)$

$= \Pi M(2, 4, 5)$

2. Let f(w, x, y, z) = Σm(1, 3, 8, 11, 12, 13, 15) and $d(w, x, y, z) = \Sigma m$(7, 9).
{$d(w, x, y, z)$ defines the *don't care conditions* of f}.

(a) (5 pts) Find the minimal SOP of f.
(b) (5 pts) Find the minimal POS of f.
(c) (5+5 pts) Design a circuit from the minimal POS of f. The circuit should contain only NOR gates.

Solution:

(a) **K-MAP (SOP)**

WX \ YZ	**00**	**01**	**11**	**10**
00		1	1	
01			X	
11	1	1	1	
10	1	X	1	

Group 1
Group 2
Group 3

Group 1

w	x	y	z
0	**0**	0	**1**
0	**0**	1	**1**
1	**0**	0	**1**
1	**0**	1	**1**

$\bar{x}z$

Group 2

w	x	**y**	z
1	1	**0**	0
1	1	**0**	1
1	0	**0**	0
1	0	**0**	1

$w\bar{y}$

Group 3

w	x	**y**	**z**
0	0	**1**	**1**
0	1	**1**	**1**
1	1	**1**	**1**
1	0	**1**	**1**

yz

$\therefore$ Minimal SOP: $\bar{x}z + w\bar{y} + yz$

(b) **K-MAP (POS)**

WX \ YZ	**00**	**01**	**11**	**10**
00	0			0
01	0	0	X	0
11				0
10		X		0

Group 1
Group 2
Group 3

Group 1

w	x	y	**z**
0	0	0	**0**
0	0	1	**0**
0	1	0	**0**
0	1	1	**0**

w+z

Group 2

w	**x**	y	z
0	**1**	0	0xs
0	**1**	0	1
0	**1**	1	1
0	**1**	1	0

$w+\bar{x}$

Group 3

w	x	**y**	**z**
0	0	**1**	**0**
0	1	**1**	**0**
1	1	**1**	**0**
1	0	**1**	**0**

$\bar{y}+z$

$\therefore$ Minimal POS: $(w+z)(w+\bar{x})(\bar{y}+z)$

(c) POS of $f(w, x, y, z) = (w+z)(w+\bar{x})(\bar{y}+z)$

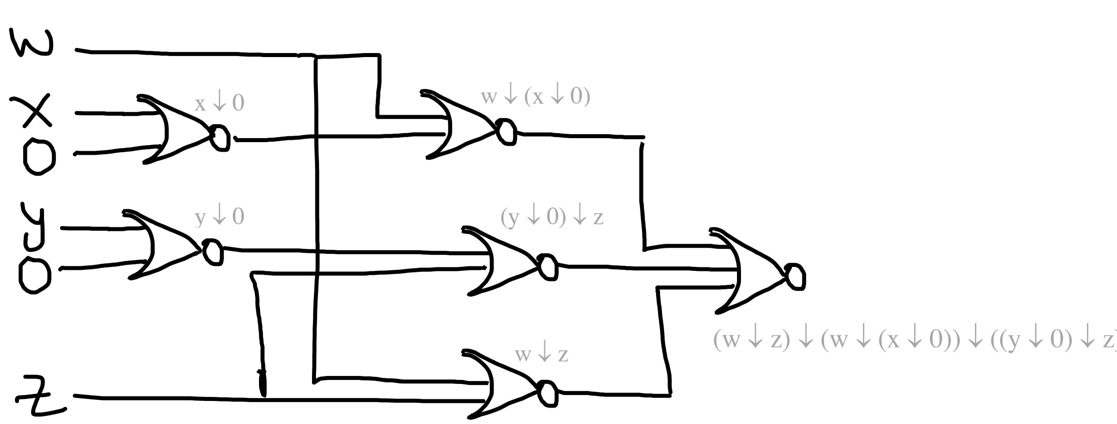
$$= \overline{\overline{(w+z)(w+\bar{x})(\bar{y}+z)}}$$

$$= \overline{\overline{(w+z)} + \overline{(w+\bar{x})} + \overline{(\bar{y}+z)}}$$

$$= \overline{(w+z)} \downarrow \overline{(w+\bar{x})} \downarrow \overline{(\bar{y}+z)}$$

$$= (w \downarrow z) \downarrow (w \downarrow \bar{x}) \downarrow (\bar{y} \downarrow z)$$

$$= (w \downarrow z) \downarrow (w \downarrow (x \downarrow 0)) \downarrow ((y \downarrow 0) \downarrow z)$$



3. Let *f(w, x, y, z)* = ∏ *M*(4, 9, 12, 13, 14) and *d(w, x, y, z)* = *Σm*(5, 6, 11, 15).
{[*d(w, x, y, z)* defines the *don't care conditions* of f}.

(a) (5 pts) Find the minimal SOP of *f*.

(b) (5 pts) Find the minimal POS of *f*.

(c) (5+5 pts) Design a circuit from the minimal SOP of *f*. The circuit should contain only NAND gates.

Solution:

(a) **K-MAP (SOP)**

WX \ YZ	**00**	**01**	**11**	**10**
00	1	1	1	1
01		X	1	X
11			X	
10	1		X	1

Group 1
Group 2

Group 1

w	x	y	z
0	**0**	0	**0**
0	**0**	1	**0**
1	**0**	0	**0**
1	**0**	1	**0**

$\overline{x}\,\overline{z}$

Group 2

w	x	y	z
0	0	0	**1**
0	0	1	**1**
0	1	0	**1**
0	1	1	**1**

$\overline{w}z$

∴ Minimal SOP: $\overline{x}\,\overline{z} + \overline{w}z$

(b) **K-MAP (POS)**

WX \ YZ	**00**	**01**	**11**	**10**
00				
01	0	X		X
11	0	0	X	0
10		0	X	

Group 1
Group 2

Group 1

w	x	y	z
0	**1**	0	**0**
0	**1**	1	**0**
1	**1**	0	**0**
1	**1**	1	**0**

$\overline{x} + z$

Group 2

w	x	y	z
1	1	0	**1**
1	1	1	**1**
1	0	0	**1**
1	0	1	**1**

$\overline{w} + \overline{z}$

∴ Minimal POS: $(\overline{x} + z)(\overline{w} + \overline{z})$

(c) SOP of *f(w, x, y, z)* $= \bar{x}\,\bar{z} + \bar{w}z$

$$= \overline{\overline{\bar{x}\,\bar{z} + \bar{w}z}}$$

$$= \overline{\bar{x}\,\bar{z} \bullet \bar{w}z}$$

$$= \overline{\bar{x}\,\bar{z}} \uparrow \overline{\bar{w}z}$$

$$= (\bar{x} \uparrow \bar{z}) \uparrow (\bar{w} \uparrow z)$$

$$= ((x \uparrow 1) \uparrow (z \uparrow 1)) \uparrow ((w \uparrow 1) \uparrow z)$$