COMP-2650-01
Assignment #1

1. **(10 marks)** If $(C12B)_b = (3210)_5$, what is the value of base b? What is the decimal value of $(C12B)_b$ given the value you obtained for b?

   Solution:
   $(3210)_5 = (3\text{x}5^3) + (2\text{x}5^2) + (1\text{x}5^1) + (0\text{x}5^0)$
   $= 375 + 50 + 5 + 0$
   $= (430)_{10}$
   $(C12B)_b = (12\text{x}b^3) + (1\text{x}b^2) + (2\text{x}b^1) + (11\text{x}b^0)$ * $(C)_{16} = (12)_{10}$ & $(B)_{16} = (11)_{10}$
   $= (12b^3 + b^2 + 2b + 11)_{10}$
   $(C12B)_b = (3210)_5$
   $12b^3 + b^2 + 2b + 11 = 430$
   $12b^3 + b^2 + 2b - 419 = 0$
   $b \approx 3.22$ (not possible as a b value since it is not an integer)

2. **(20 marks)** Convert the number $(1357.7531)_9$ into base 11. Then convert back from base 11 to base 9. How close is the final base-9 result to the original base-9 number $(1357.7531)_9$? *If necessary, you can limit your precisions to a maximum of 7 fractional digits, in all your calculations*

   Solution:
   - $(1357.7531)_9 \rightarrow N_{11}$
     $(1357.7531)_9 = (1\text{x}9^3) + (3\text{x}9^2) + (5\text{x}9^1) + (7\text{x}9^0) \bullet (7\text{x}9^{-1}) + (5\text{x}9^{-2}) + (3\text{x}9^{-3}) + (1\text{x}9^{-4})$
     $= 729 + 243 + 45 + 7 \bullet 7/9 + 5/81 + 1/243 + 1/6561$
     $= 1024 + 0.8437738$
     $= (1024.8437738)_{10}$
     $(1024.8437738)_{10} \rightarrow N_{11}$

     Integral part:

1024/11	= 93	1 ↑
93/11	= 8	5
8/11	= 0	8

     Fractional part:

     0.8437738(11) = <u>9</u>.2815118 ↓
     0.2815118(11) = <u>3</u>.0966298
     0.0966298(11) = <u>1</u>.0629278
     0.0629278(11) = <u>0</u>.6922058
     0.6922058(11) = <u>7</u>.6142638
     0.6142638(11) = <u>6</u>.7569018
     0.7569018(11) = <u>8</u>.3259198

     $(1024.8437738)_{10} = (851.9310768)_{11}$
     $\therefore (1357.7531)_9 = (851.9310768)_{11}$

- $(851.9310768)_{11} \rightarrow N_9$

  $(851.9310768)_{11} = (8\text{x}11^{2}) + (5\text{x}11^{1}) + (1\text{x}11^{0}) \bullet (9\text{x}11^{-1}) + (3\text{x}11^{-2}) + (1\text{x}11^{-3}) + (0\text{x}11^{-4})$
  $+ (7\text{x}11^{-5}) + (6\text{x}11^{-6}) + (8\text{x}11^{-7})$
  $= (1024.8437738)_{10}$

  $(1024.8437738)_{10} \rightarrow N_9$

  Integral part:

1024/9	= 113	7	↑
113/9	= 12	5	
12/9	= 1	3	
1/9	= 0	1	

  Fractional part:

  $0.8437738(9) = \underline{7}.5939642$ ↓
  $0.5939642(9) = \underline{5}.3456778$
  $0.3456778(9) = \underline{3}.1111002$
  $0.1111002(9) = \underline{0}.9999018$
  $0.9999018(9) = \underline{8}.9991162$
  $0.9991162(9) = \underline{8}.9920458$
  $0.9920458(9) = \underline{8}.9284122$

  $(1024.8437738)_{10} = (1357.7530888)_9$
  $\therefore\ (851.9310768)_{11} = (1357.7530888)_9$
  The final base-9 result is very close to the original base-9 number, since if it is rounded to 4 decimal digits, it is the same as the original.

3. **(10 marks)** Perform the following operations in 1CF and 2CF arithmetics. Also for each operation, give the decimal equivalent of the operands and result.
   (a) 10010110 + 10100101
   (b) 01111000 − 11010010
   make sure to tell whether there is overflow or not for each operation

<u>*Solution:*</u>
(a) 10010110 + 10100101

- In 1CF:

$$
\begin{array}{rl}
10010110 & \rightarrow \text{1CF: } 01101001 = 2^0 + 2^3 + 2^5 + 2^6 = (+105)_{10} \rightarrow (10010110)_2 = (-105)_{10} \\
\underline{+\ 10100101} & \rightarrow \text{1CF: } 01011010 = 2^1 + 2^3 + 2^4 + 2^6 = (+90)_{10} \rightarrow (10100101)_2 = (-90)_{10} \\
100111011 & \\
\underline{+\qquad\quad 1} & \\
00111100 & = 2^2 + 2^3 + 2^4 + 2^5 = (+60)_{10} \rightarrow (00111100)_2 = (+60)_{10}
\end{array}
$$

$\therefore$ **Yes, there is an overflow**, since the operands are the same sign, but the result is a different sign. Also, (-105) + (-90) = (-195) which doesn't equal the result obtained (00111100).

- In 2CF:

$$
\begin{array}{rl}
10010110 & \rightarrow \text{2CF: } 01101010 = 2^1 + 2^3 + 2^5 + 2^6 = (+106)_{10} \rightarrow (10010110)_2 = (-106)_{10} \\
\underline{+\ 10100101} & \rightarrow \text{2CF: } 01011011 = 2^0 + 2^1 + 2^3 + 2^4 + 2^6 = (+91)_{10} \rightarrow (10100101)_2 = (-91)_{10} \\
\cancel{1}00111011 & = 2^0 + 2^1 + 2^3 + 2^4 + 2^5 = (+59)_{10} \rightarrow (00111100)_2 = (+59)_{10}
\end{array}
$$

$\therefore$ **Yes, there is an overflow**, since the operands are the same sign, but the result is a different sign. Also, (-106) + (-91) = (-197) which doesn't equal the result obtained (00111011).

(b) 01111000 − 11010010

- In 1CF:

$$\begin{array}{rl} 01111000 & 01111000 = 2^3 + 2^4 + 2^5 + 2^6 = (+120)_{10} \rightarrow (01111000)_2 = (+120)_{10} \\ \underline{-\ 11010010} \rightarrow \text{1CF: } \underline{+\ 00101101} & = 2^0 + 2^2 + 2^3 + 2^5 = (+45)_{10} \rightarrow (11010010)_2 = (-45)_{10} \\ 10100101 & \rightarrow \text{1CF: } 01011010 = 2^1 + 2^3 + 2^4 + 2^6 = (+90)_{10} \\ & (10100101)_2 = (-90)_{10} \end{array}$$

∴ **Yes, there is an overflow**, since the operands are the same sign, but the result is a different sign. Also, (+120) - (-45) = (+165) which doesn't equal the result obtained (10100101).

- In 2CF:

$$\begin{array}{rl} 01111000 & 01111000 = 2^3 + 2^4 + 2^5 + 2^6 = (+120)_{10} \rightarrow (01111000)_2 = (+120)_{10} \\ \underline{-\ 11010010} \rightarrow \text{2CF: } \underline{+\ 00101110} & = 2^1 + 2^2 + 2^3 + 2^5 = (+46)_{10} \rightarrow (11010010)_2 = (-46)_{10} \\ 10100110 & \rightarrow \text{2CF: } 01011001+1 = 01011010 \\ & 01011010 = 2^1 + 2^3 + 2^4 + 2^6 = (+90)_{10} \\ & (10100110)_2 = (-90)_{10} \end{array}$$

∴ **Yes, there is an overflow**, since the operands are the same sign, but the result is a different sign. Also, (+120) - (-46) = (+166) which doesn't equal the result obtained (10100110).

4. **(10 marks)** Perform the following operations in 1CF and 2CF arithmetics; in binary.
   (a) Addition of (+125) + (−37) in 1CF and 2CF arithmetics
   (b) Subtraction of (−125) − (−37) in 1CF and 2CF arithmetics

show the operations, the operands, and the results in binary, and be sure to tell whether there is overflow or not for each operation

Solution

Let n=8 be the number of bits used to represent an integer.

$(125)_{10} \rightarrow N_2$

125/2	= 62	1
62/2	= 31	0
31/2	= 15	1
15/2	= 7	1
7/2	= 3	1
3/2	= 1	1
1/2	= 0	1

$(37)_{10} \rightarrow N_2$

37/2	= 18	1
18/2	= 9	0
9/2	= 4	1
4/2	= 2	0
2/2	= 1	0
1/2	= 0	1

$(125)_{10} = (1111101)_2$
$(+125)_{10} = (01111101)_2$
$(-125)_{10} =$ 1CF: $(10000010)_2$
$(-125)_{10} =$ 2CF: $(10000011)_2$

$(37)_{10} = (100101)_2$
$(+37)_{10} = (00100101)_2$
$(-37)_{10} =$ 1CF: $(11011010)_2$
$(-37)_{10} =$ 2CF: $(11011011)_2$

(a) $(+125) + (-37) = \mathbf{(+88)}$

- In 1CF:

$$
\begin{array}{r}
01111101 \\
\underline{+\ 11011010} \\
101010111 \\
\underline{+ \qquad\qquad 1} \\
01011000
\end{array}
= 2^3 + 2^4 + 2^6 = (+88)_{10}
$$

$\therefore$ **There is no overflow.**

- In 2CF:

$$
\begin{array}{r}
01111101 \\
\underline{+\ 11011011} \\
\cancel{1}01011000
\end{array}
= 2^3 + 2^4 + 2^6 = (+88)_{10}
$$

$\therefore$ **There is no overflow.**

(b) $(-125) - (-37) = \mathbf{(-88)}$

- In 1CF:

$$
\begin{array}{r}
10000010 \\
\underline{-\ 11011010}
\end{array}
\rightarrow \text{1CF: }
\begin{array}{r}
10000010 \\
\underline{+\ 00100101} \\
10100111
\end{array}
\rightarrow \text{1CF: } 01011000 = 2^3 + 2^4 + 2^6 = (+88)_{10}
$$

$(1010011)_2 = (-88)_{10}$

$\therefore$ **There is no overflow.**

- In 2CF:

$$
\begin{array}{r}
10000011 \\
\underline{-\ 11011011}
\end{array}
\rightarrow \text{2CF: }
\begin{array}{r}
10000011 \\
\underline{+\ 00100101} \\
10101000
\end{array}
\rightarrow \text{2CF: } 01010111+1 = 01011000
$$

$= 2^3 + 2^4 + 2^6 = (+88)_{10}$

$(10101000)_2 = (-88)_{10}$

$\therefore$ **There is no overflow.**