<u>COMP-2650-01</u>

<u>Assignment #2</u>

**1. (a) (5 pts) Prove by algebraic method whether $\overline{a} + ab + a\overline{c} + a\overline{b}\overline{c} = \overline{a} + \overline{b} + \overline{c}$.**

**(b) (5 pts) Find the CPOS of $f(x, y, z) = (x + \overline{y})y + \overline{x}z + \overline{x + y} + \overline{y}(\overline{x + z})$.**

*<u>Solution:</u>*

(a) $\overline{a} + ab + a\overline{c} + a\overline{b}\overline{c} = \overline{a} + \overline{b} + \overline{c}$

LHS $= \overline{a} + ab + a\overline{c} + a\overline{b}\overline{c}$

| | |
|---|---|
| $= \overline{a} + a\,(b + \overline{c} + \overline{b}\overline{c})$ | *distributivity* |
| $= \overline{a} + a\,(b + \overline{b}\overline{c} + \overline{c})$ | *commutativity* |
| $= \overline{a} + a\,(b + \overline{c} + \overline{c})$ | *no name* |
| $= \overline{a} + a\,(b + \overline{c})$ | *idempotency* |
| $= \overline{a} + b + \overline{c}$ | *no name* |

$\therefore$ *LHS $\neq$ RHS*

$\therefore \overline{a} + ab + a\overline{c} + a\overline{b}\overline{c} \neq \overline{a} + \overline{b} + \overline{c}$

*(b) CPOS of* $f(x, y, z) = (x + \overline{y})y + \overline{x}z + \overline{x + y} + \overline{y}(\overline{x + z})$

| | |
|---|---|
| $f(x, y, z) = (x + \overline{y})y + \overline{x}z + \overline{x + y} + \overline{y}(\overline{x + z})$ | |
| $= (x + \overline{y})y + \overline{x}z + \overline{x}\,\overline{y} + \overline{y}(\overline{x}\overline{z})$ | *DeMorgan's Law* |
| $= xy + \overline{y}y + \overline{x}z + \overline{x}\,\overline{y} + \overline{y}\overline{x}\overline{z}$ | *Distributivity* |
| $= xy + 0 + \overline{x}z + \overline{x}\,\overline{y} + \overline{yxz}$ | *Complementation* |
| $= xy + \overline{x}z + \overline{x}\,\overline{y} + \overline{yxz}$ | *Identity element* |
| $= xy(z+\overline{z}) + \overline{x}z(y+\overline{y}) + \overline{x}\,\overline{y}(z + \overline{z}) + \overline{yxz}$ | *Complementation* |
| $= xyz + xy\overline{z} + \overline{x}yz + \overline{x}\,\overline{y}z + \overline{x}\,\overline{y}z + \overline{x}\,\overline{y}\,\overline{z} + \overline{x}\,\overline{y}\,\overline{z}$ | *Distributivity/Commutativity* |
| $= m_7 + m_6 + m_3 + m_1 + m_1 + m_0 + m_0$ | |
| $= \Sigma m(0, 1, 3, 6, 7)$ | |
| $= \Pi M(2, 4, 5)$ | |

**2. Let f(w, x, y, z) = $\Sigma m$(1, 3, 8, 11, 12, 13, 15) and $d(w, x, y, z) = \Sigma m$(7, 9).**

**{*d(w, x, y, z)* defines the *don't care conditions* of *f*}.**

**(a) (5 pts) Find the minimal SOP of *f*.**

**(b) (5 pts) Find the minimal POS of *f*.**

**(c) (5+5 pts) Design a circuit from the minimal POS of *f*. The circuit should contain only NOR gates.**

*<u>Solution:</u>*

(a) **K-MAP (SOP)**

| **WX \ YZ** | **00** | **01** | **11** | **10** |
|---|---|---|---|---|
| **00** | | 1 | 1 | |
| **01** | | | X | |
| **11** | 1 | 1 | 1 | |
| **10** | 1 | X | 1 | |

Group 1
Group 2
Group 3

Group 1

| w | x | y | z |
|---|---|---|---|
| 0 | **0** | 0 | **1** |
| 0 | **0** | 1 | **1** |
| 1 | **0** | 0 | **1** |
| 1 | **0** | 1 | **1** |

$\bar{x}$z

Group 2

| w | x | y | z |
|---|---|---|---|
| **1** | 1 | **0** | 0 |
| **1** | 1 | **0** | 1 |
| **1** | 0 | **0** | 0 |
| **1** | 0 | **0** | 1 |

w$\bar{y}$

Group 3

| w | x | y | z |
|---|---|---|---|
| 0 | 0 | **1** | **1** |
| 0 | 1 | **1** | **1** |
| 1 | 1 | **1** | **1** |
| 1 | 0 | **1** | **1** |

yz

∴ Minimal SOP: $\bar{x}$z + w$\bar{y}$ + yz

(b) **K-MAP (POS)**

| **WX \ YZ** | **00** | **01** | **11** | **10** |
|---|---|---|---|---|
| **00** | 0 | | | 0 |
| **01** | 0 | 0 | X | 0 |
| **11** | | | | 0 |
| **10** | | X | | 0 |

Group 1
Group 2
Group 3

Group 1

| w | x | y | z |
|---|---|---|---|
| **0** | 0 | 0 | **0** |
| **0** | 0 | 1 | **0** |
| **0** | 1 | 0 | **0** |
| **0** | 1 | 1 | **0** |

w+z

Group 2

| w | x | y | z |
|---|---|---|---|
| **0** | **1** | 0 | 0xs |
| **0** | **1** | 0 | 1 |
| **0** | **1** | 1 | 1 |
| **0** | **1** | 1 | 0 |

w+$\bar{x}$

Group 3

| w | x | y | z |
|---|---|---|---|
| 0 | 0 | **1** | **0** |
| 0 | 1 | **1** | **0** |
| 1 | 1 | **1** | **0** |
| 1 | 0 | **1** | **0** |

$\bar{y}$+z

∴ Minimal POS: (w+z)(w+$\bar{x}$)( $\bar{y}$+z)

(c) POS of *f*(*w*, *x*, *y*, *z*) = (w+z)(w+$\bar{x}$)( $\bar{y}$+z)

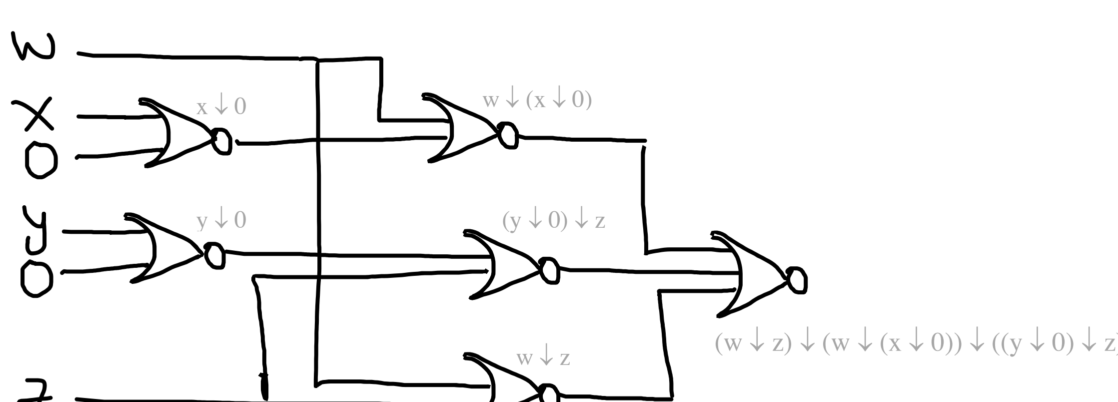
$$= \overline{\overline{(w+z)(w+\bar{x})(\bar{y}+z)}}$$

$$= \overline{\left(\overline{w+z}\right) + \left(\overline{w+\bar{x}}\right) + \left(\overline{\bar{y}+z}\right)}$$

$$= \left(\overline{w+z}\right) \downarrow \left(\overline{w+\bar{x}}\right) \downarrow \left(\overline{\bar{y}+z}\right)$$

= (w ↓ z) ↓ (w ↓ $\bar{x}$) ↓ ($\bar{y}$ ↓ z)

= (w ↓ z) ↓ (w ↓ (x ↓ 0)) ↓ ((y ↓ 0) ↓ z)



**3. Let *f(w, x, y, z)* = Π *M*(4, 9, 12, 13, 14) and *d(w, x, y, z)* = *Σm*(5, 6, 11, 15).**
**{[*d(w, x, y, z)* defines the *don't care conditions* of f}.**

**(a) (5 pts) Find the minimal SOP of *f*.**

**(b) (5 pts) Find the minimal POS of *f*.**

**(c) (5+5 pts) Design a circuit from the minimal SOP of *f*. The circuit should contain only NAND gates.**

*Solution:*

(a) **K-MAP (SOP)**

| **WX \ YZ** | **00** | **01** | **11** | **10** |
|---|---|---|---|---|
| **00** | 1 | 1 | 1 | 1 |
| **01** | | X | 1 | X |
| **11** | | | X | |
| **10** | 1 | | X | 1 |

Group 1
Group 2

Group 1

| w | x | y | z |
|---|---|---|---|
| 0 | **0** | 0 | **0** |
| 0 | **0** | 1 | **0** |
| 1 | **0** | 0 | **0** |
| 1 | **0** | 1 | **0** |

$\overline{x}\,\overline{z}$

Group 2

| w | x | y | z |
|---|---|---|---|
| **0** | 0 | 0 | **1** |
| **0** | 0 | 1 | **1** |
| **0** | 1 | 0 | **1** |
| **0** | 1 | 1 | **1** |

$\overline{w}z$

$\therefore$ Minimal SOP: $\overline{x}\,\overline{z} + \overline{w}z$

(b) **K-MAP (POS)**

| **WX \ YZ** | **00** | **01** | **11** | **10** |
|---|---|---|---|---|
| **00** | | | | |
| **01** | 0 | X | | X |
| **11** | 0 | 0 | X | 0 |
| **10** | | 0 | X | |

Group 1
Group 2

Group 1

| w | x | y | z |
|---|---|---|---|
| 0 | **1** | 0 | **0** |
| 0 | **1** | 1 | **0** |
| 1 | **1** | 0 | **0** |
| 1 | **1** | 1 | **0** |

$\overline{x} + z$

Group 2

| w | x | y | z |
|---|---|---|---|
| **1** | 1 | 0 | **1** |
| **1** | 1 | 1 | **1** |
| **1** | 0 | 0 | **1** |
| **1** | 0 | 1 | **1** |

$\overline{w} + \overline{z}$

$\therefore$ Minimal POS: $(\overline{x} + z)(\overline{w} + \overline{z})$

(c) SOP of *f(w, x, y, z)* $= \overline{x}\,\overline{z} + \overline{w}z$

$$= \overline{\overline{\overline{x}\,\overline{z} + \overline{w}z}}$$

$$= \overline{\overline{\overline{x}\,\overline{z}} \bullet \overline{\overline{w}z}}$$

$$= \overline{\overline{x}\,\overline{z}} \uparrow \overline{\overline{w}z}$$

$$= (\overline{x} \uparrow \overline{z}) \uparrow (\overline{w} \uparrow z)$$

$$= ((x \uparrow 1) \uparrow (z \uparrow 1)) \uparrow ((w \uparrow 1) \uparrow z)$$