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USN: IBM18CS083

Course: Advance Algorithm Code:

Date: 06/01/2021

Time: 1:00 - 2:15 pm

Part-C

4b)

```
#include <iostream>
```

```
using namespace std;
```

```
struct point {
```

```
    int x;
```

```
    int y; };
```

```
bool onSegment (Point p, Point q, Point r) {
```

```
    if (q.x <= max (p.x, r.x) &&
```

```
        q.x >= min (p.x, r.x) &&
```

```
        q.y <= max (p.y, r.y) &&
```

```
        q.y >= min (p.y, r.y))
```

```
        return true;
```

```
    return false; }
```

```
int orientation (Point p, Point q, Point r) {
```

```
    int val = (q.y - p.y) * (r.x - q.x) -
```

```
        (q.x - p.x) * (r.y - q.y);
```

```
    if (val == 0) return 0;
```

```
    return (val > 0) ? 1 : 2; }
```

```
bool doInterest (Point p1, point q1, Point p2, point q2)
```

```
{
```

```
    int o1 = orientation (p1, q1, p2);
```

```
    int o2 = orientation (p1, q1, q2);
```

```
    int o3 = orientation (p2, q2, q1);
```

```
    int o4 = orientation (p2, p1, q2);
```

```
    if (o1 != o2 && o3 != o4)
```

```
        return true;
```

01

PTD

Rohit

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4b continued

```
if (O1==0 && onSegment(P1, P2, q1)) return true;
if (O2==0 && onSegment(P1, q2, q1)) return true;
if (O3==0 && onSegment(P2, P1, q2)) return true;
if (O4==0 && onSegment(P2, P1, q2)) return true;
}
```

```
int main () {
```

```
    struct point P1 = {10, 10}, q1 = {10, 30};
```

```
    struct point P2 = {30, 30}, q2 = {40, 10};
```

```
    doIntersect (P1, q1, P2, q2)? cout << "yes" ; cout << "NO" ;
```

```
    P1 = {10, 0}, q1 = {0, 10};
```

```
    P2 = {0, 0}, q2 = {10, 10};
```

```
    doIntersect (P1, q1, P2, q2)? cout << "yes" ; cout << "NO" ;
```

```
    return 0;
```

```
}
```

// algorithm

```
SEGMENT - INTERSECT (P1, P2, P3, P4)
```

```
d1 = DIRECTION (P3, P4, P1)
```

```
d2 = DIRECTION (P3, P4, P2)
```

```
d3 = DIRECTION (P1, P2, P3)
```

```
d4 = DIRECTION (P1, P2, P4)
```

```
if ((d1 > 0 and d2 < 0) or (d1 < 0 and d2 > 0) and
```

```
((d3 > 0 and d4 < 0) or (d3 < 0 and d4 > 0))
```

```
    return TRUE;
```

```
else if d1==0 and ON-SEGMENT (P3, P4, P1)
```

```
    return TRUE;
```


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4b continued

elseif $d_3 == 0$ and ON-SEGMENT(P_1, P_2, P_3)

return TRUE

elseif $d_4 == 0$ and ON-SEGMENT(P_1, P_2, P_4)

return TRUE

else

return FALSE

DIRECTION(P_i, P_j, P_k)

return $(P_k - P_i) \times (P_j - P_i)$

ON-SEGMENT(P_i, P_j, P_k)

if $\min(x_i, x_j) \leq x_k \leq \max(x_i, x_j)$ and $\min(y_i, y_j) \leq$

$y_k \leq \max(y_i, y_j)$

return TRUE

else

return FALSE

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Part B

2 a) minimize $x_1 + 2x_2$

Subjected to $x_1 + x_2 \geq 40$

$x_1 - x_2 = 14$

$x_1 - 2x_2 \leq 3$

① maximize $-(x_1 + 2x_2) \Rightarrow -x_1 - 2x_2$

② x_1 & x_2 has no negative constraints

replace x_1 with $(x_1' - x_1'')$ and

x_2 with $(x_2' - x_2'')$

add constraints $(x_1', x_1'', x_2', x_2'') \geq 0$

∴ maximize $-(x_1' - x_1'') - 2(x_2' - x_2'')$

$\Rightarrow -x_1' + x_1'' - 2x_2' + 2x_2''$

Subjected to

$x_1' - x_1'' + x_2' - x_2'' \geq 40$

$x_1' - x_1'' + x_2'' - x_2' = 14$

$x_1' - x_1'' - 2x_2' + 2x_2'' \leq 3$

$x_1', x_1'', x_2', x_2'' \geq 0$

③ Replace equality constraints with pair of inequality constraints

maximize $= x_1'' - x_1' + 2x_2'' - 2x_2'$

Subjected to $x_1' - x_1'' + x_2' - x_2'' \geq 40$

$x_1' - x_1'' + x_2'' - x_2' \geq 14$

$x_2' - x_1'' + x_2'' - x_2' \leq 14$

$x_1' - x_1'' - 2x_2' + 2x_2'' \leq 3$

$x_1', x_2', x_1'', x_2'' \geq 0$

④ convert \geq to \leq by multiplying -1

∴ maximize $= x_1'' - x_1' + 2x_2'' - 2x_2'$

Subjected to $x_1'' - x_1' + x_2'' - x_2' \leq -40$

$x_1'' - x_1' - x_2'' + x_2' \leq -14$

$x_2' - x_1'' + x_2'' - x_2' \leq 14$

$x_1' - x_1'' - 2x_2' + 2x_2'' \leq 3$

$x_1', x_1'', x_2', x_2'' \geq 0$

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Part - B

2b) Let P_1 be $(10, 25)$, P_2 be $(30, 30)$
 $P_1 \times P_2 = \det \begin{pmatrix} 10 & 30 \\ 25 & 30 \end{pmatrix}$
 $= -450$

As $P_1 \times P_2 < 0$
hence point $P_1 (10, 25)$ is to the left of point
 $P_2 (30, 30)$,

2c) Assume $c(u, v) = 0$ if $(u, v) \notin E$ and there are no
antiparallel edges, we can express the max flow
problem as a LP.

$$\text{maximize } \sum_{v \in V} f_{sv} - \sum_{v \in V} f_{vt}$$

$$\text{subjected to } f_{uv} \leq c(u, v) \quad \forall u, v \in V$$

$$\sum_{v \in V} f_{vu} = \sum_{v \in V} f_{uv} \quad \forall u \in V - \{s, t\}$$

$$f_{uv} \geq 0 \quad \forall u, v \in V$$

This LP has $|V|^2$ variables, corresponding to
the flow between each pairs of vertices and it has
 $2|V|^2 + |V| - 2$ constraints, so the more efficient to
rewrite the LP so that, it has $O(V+E)$ constraints.

$$\max Z = F_1 + F_2 + F_3 + \dots + F_n$$

$$\text{subjected to } \sum_{i=1}^k F_i \leq U_e \quad \forall e = E(u, v) \text{ when } e \in P_i,$$

the capacity constraint, U_e - capacity of e

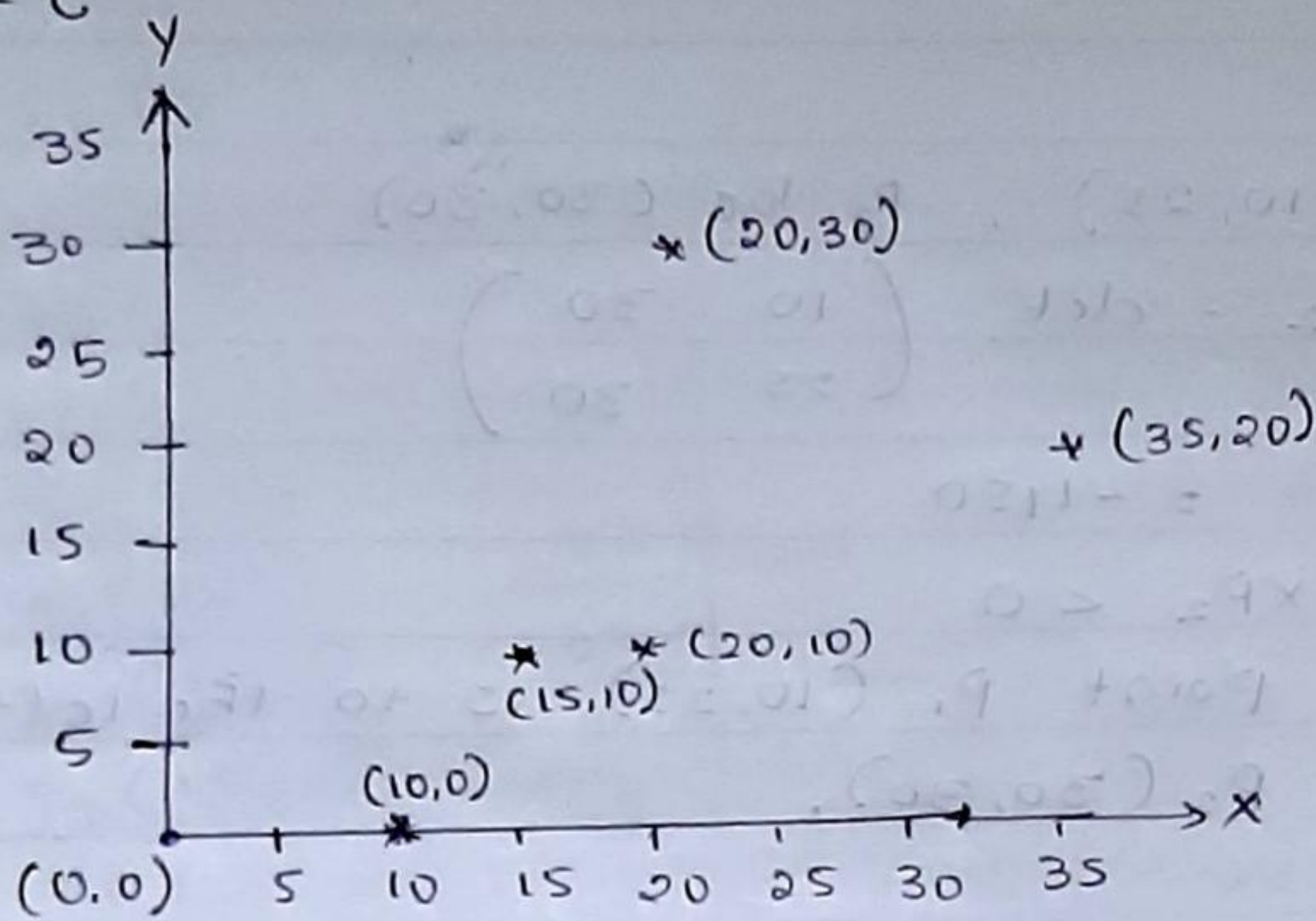
The constraints indicates that the sum of flow on an
edge e must be bounded by U_e (capacity of e).

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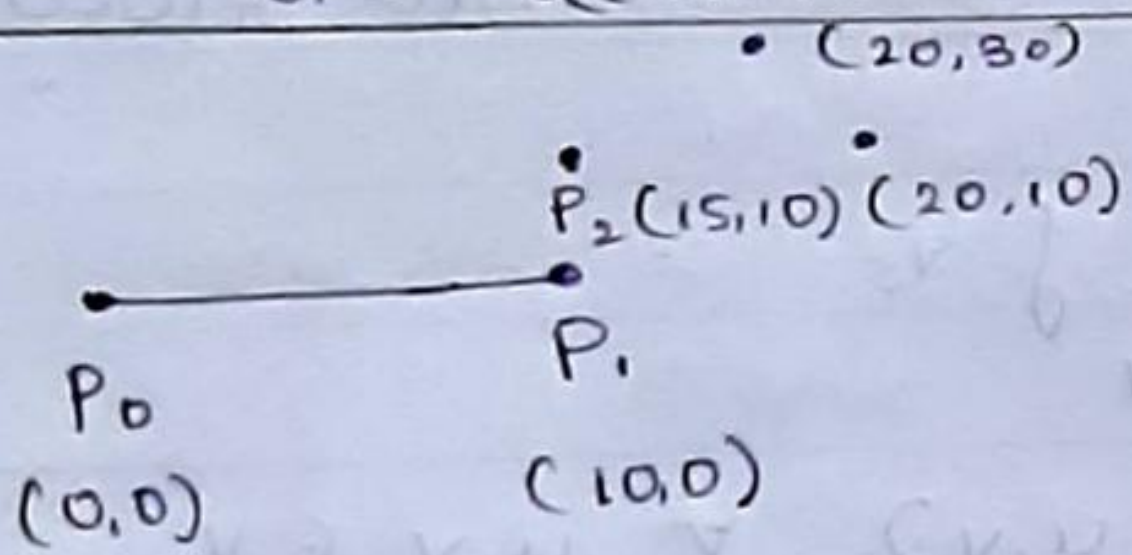
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Part - C

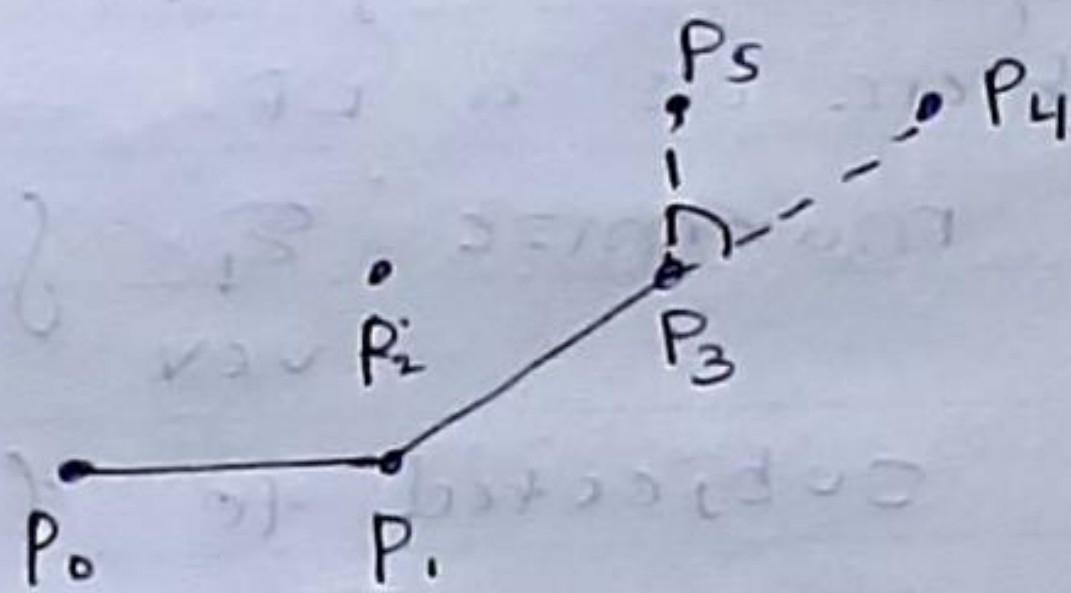
4a)



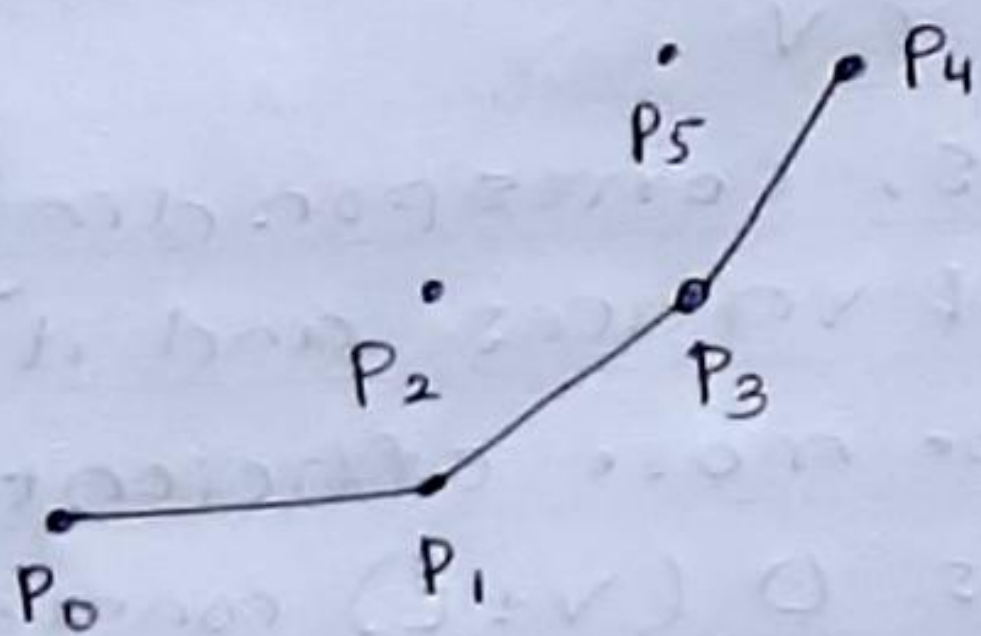
$$Q = \{(0,0), (10,0), (20,10), (15,10), (20,30), (35,20)\}$$



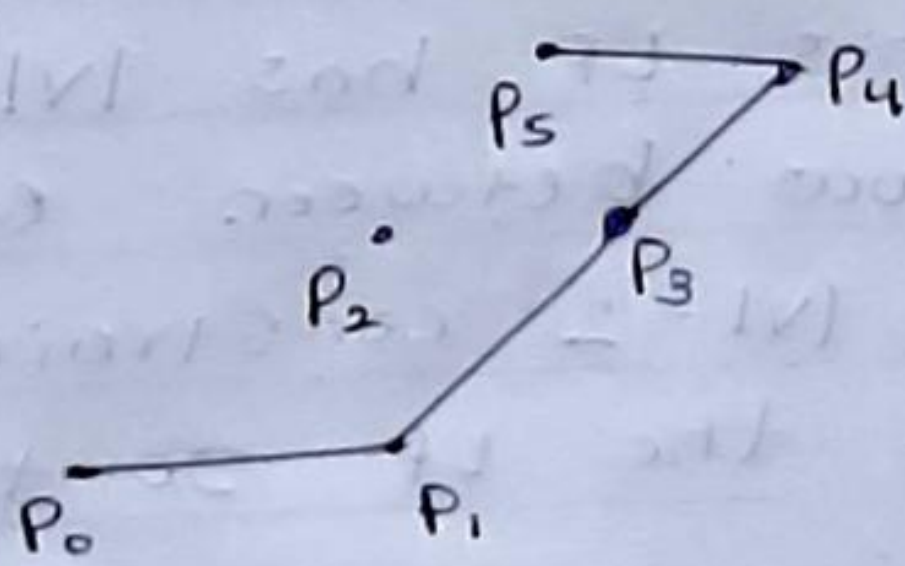
(a)



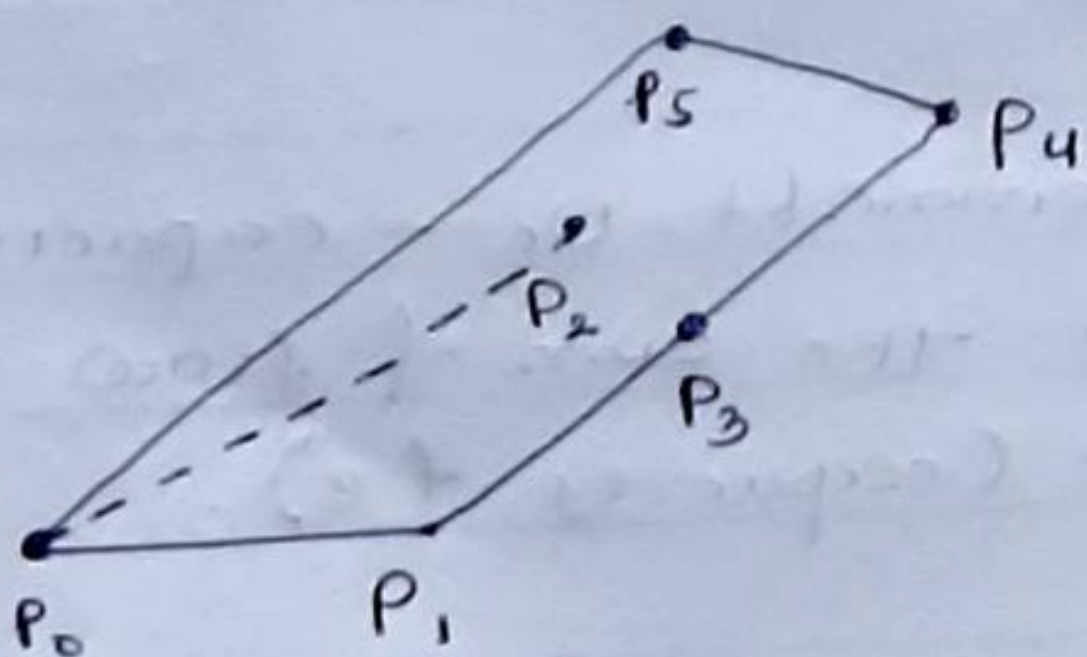
(b)



(c)



(d)



(e)

Revised

Part C

4a)

$P_0 (0,0)$

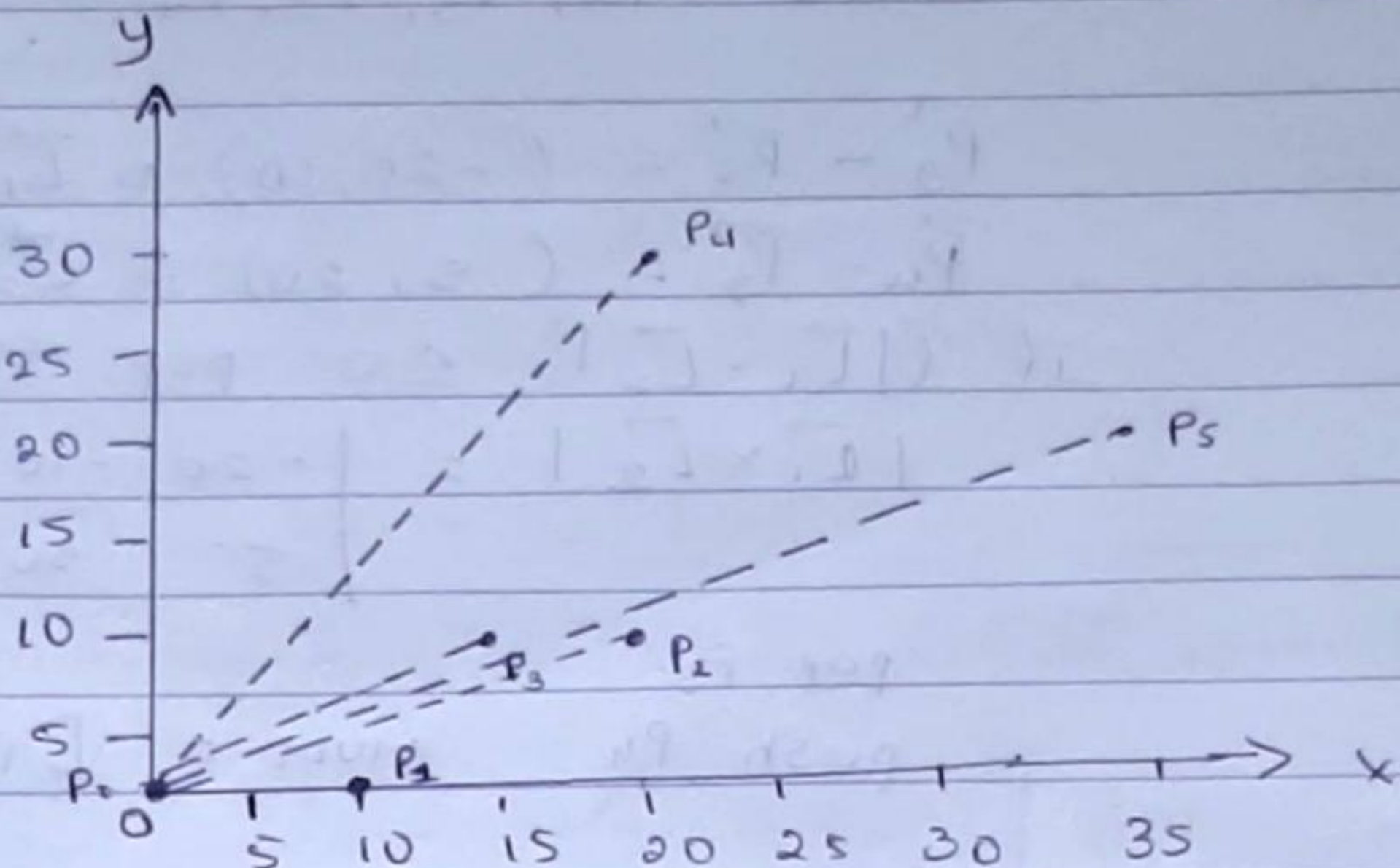
$P_1 (10,0)$

$P_2 (20,10)$

$P_3 (15,10)$

$P_4 (20,30)$

$P_5 (35,20)$



Sorted according to polar angles
points $[P_0, P_1, P_2, P_3, P_5, P_4]$

Step 1: push P_0, P_1, P_2 to stack (Null)

Step 2: $\vec{P_2} - \vec{P_1} = (10,10) \equiv \vec{L_1}$
 $\vec{P_5} - \vec{P_2} = (15,10) \equiv \vec{L_2}$

if $|\vec{L_1} \times \vec{L_2}| \leq 0$ pop P_2

$$|\vec{L_1} \times \vec{L_2}| = \begin{vmatrix} 10 & 10 \\ 15 & 10 \end{vmatrix} = 100 - 150 = -50$$

$\vec{L_2}$ is on right of $\vec{L_1}$

so pop P_2

push P_5

Step 3: Check P_1, P_5, P_3

$$\vec{P_5} - \vec{P_1} = (25,20) \equiv \vec{L_1}$$

$$\vec{P_3} - \vec{P_5} = (-20,-10) \equiv \vec{L_2}$$

if $|\vec{L_1} \times \vec{L_2}| \leq 0$, pop P_5

$$|\vec{L_1} \times \vec{L_2}| = \begin{vmatrix} 25 & 20 \\ -20 & -10 \end{vmatrix} = -250 + 400 = 150$$

HULL = $[P_0, P_1, P_5, P_3]$

4a continued

Step 4 : check P_5, P_3, P_3, P_4

$$\vec{P_3} - \vec{P_5} = (-20, 10) \equiv \vec{L_1}$$

$$\vec{P_4} - \vec{P_3} = (5, 20) \equiv \vec{L_2}$$

if $(|\vec{L_1} \cdot \vec{L_2}|) \leq 0$ pop P_3

$$|\vec{L_1} \times \vec{L_2}| = \begin{vmatrix} -20 & 10 \\ 5 & 20 \end{vmatrix} = -400 + 50 = -350$$

pop P_3

push P_4

$$\text{HULL} = [P_0, P_1, P_5, P_4]$$

Step 5 : check P_1, P_5, P_5, P_4

$$\vec{P_5} - \vec{P_1} = (25, 20) \equiv \vec{L_1}$$

$$\vec{P_4} - \vec{P_5} = (-15, 10) \equiv \vec{L_2}$$

if $(|\vec{L_1} \cdot \vec{L_2}|) \leq 0$ pop P_4

$$|\vec{L_1} \cdot \vec{L_2}| = \begin{vmatrix} 25 & 20 \\ -15 & 10 \end{vmatrix} = 250 + 300 = 550$$

$$\text{HULL} = [P_0, P_1, P_5, P_4]$$

Step 6 : check P_5, P_4, P_4, P_0

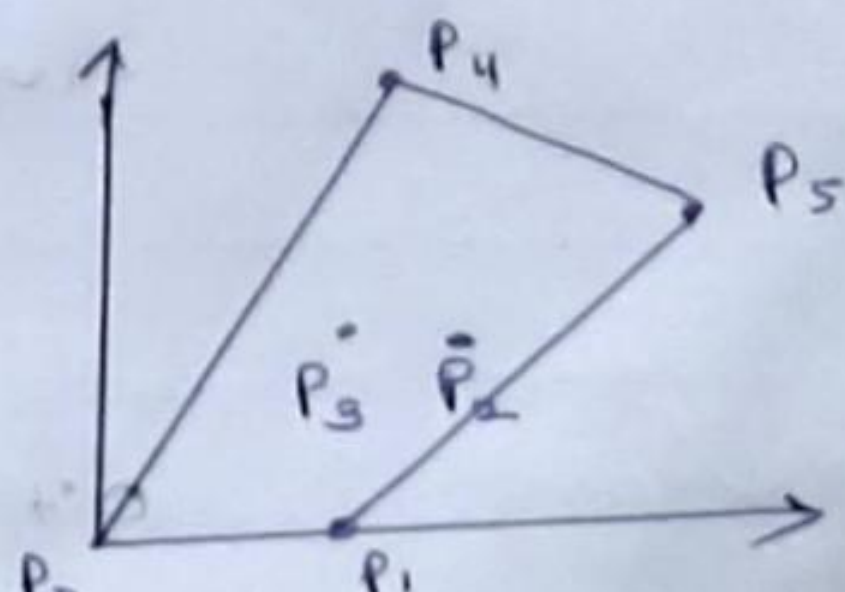
$$\vec{P_0} - \vec{P_4} = (-20, -30) \equiv \vec{L_2}$$

$$\vec{P_4} - \vec{P_5} = (-15, +10) \equiv \vec{L_1}$$

if $(|\vec{L_1} \cdot \vec{L_2}|) \geq 0$ HULL is connected & Final

$$|\vec{L_1} \times \vec{L_2}| = \begin{vmatrix} -15 & +10 \\ -20 & -30 \end{vmatrix} = 650$$

$\vec{L_1} \times \vec{L_2} > 0 \Rightarrow$ HULL is obtained
HULL = $[P_0, P_1, P_5, P_4]$



OB

Sheet

Part c

3a) maximise $z = 7x_1 + 5x_2$

subject to $2x_1 + x_2 \leq 100$

$4x_1 + 3x_2 \leq 240$

Standard form $2x_1 + x_2 + s_1 = 100$ — (i)

$4x_1 + 3x_2 + s_2 = 240$ — (ii)

$\max z = 7x_1 + 5x_2 + 0s_1 + 0s_2$

$s_1, s_2 \geq 0$

optimizer

condition for max

$\forall C_j - z_j \leq 0$

* initialize simplex table

CB _i	C _j Basic variable	7 x ₁	5 x ₂	0 s ₁	0 s ₂	Solution	Ratio
0	s ₁	2	1	1	0	100	100/2 = 50
0	s ₂	4	3	0	1	240	240/4 = 60
(Z _j = Σ C _B a _{ij}) Z _j		0	0	0	0	0	

$C_j - z_j$

7

5

0

0

↑

$R_1 \Rightarrow R_1/2$

$R_2 \rightarrow R_2 - R_1$

iteration 1

CB _i	C _j	7	5	0	0	Solution	Ratio
7	x ₁	1	0.5	0.5	0		
0	s ₂	2.5	-0.5	1			

PTO

3a continued

iteration 1

C_B	C_j	7	5	0	0	Solution	Ratio
7	X_1	1	0.5	0.5	0	50	100
0	S_2	3	2.5	-0.5	1	190	76 \leftarrow Key Row
	Z_j	7	3.5	3.5	0		
	$C_j - Z_j$	0	2.5	0.5	1		

↑ key column

$R_1 \rightarrow R_1/2$

iteration 1

C_B	C_j	7	5	0	0	Sol ⁿ	Ratio
7	X_1	1	0.5	0.5	0	50	100
0	S_2	0	1	-2	1	40	40 \leftarrow Key row
	Z_j	7	3.5	3.5	0		
	$C_j - Z_j$	0	1.5	-3.5	0		

↑ key row

iteration 2

C_B	C_j	7	5	0	0	Sol ⁿ	Ratio
7	X_1	1	0	1.5	-0.5	30	-
5	X_2	1	0	-2	1	40	-
	Z_j	7	5	0.5	1.5	410	
	$C_j - Z_j$	0	0	-0.5	-1.5		

3a continued

$$a_{ij} \quad C_j - Z_j \leq 0$$

Z is optimal at

$$x_1 = 7, x_2 = 5, Z = 410 //$$

Part A

① Linear programming

Solving the problem finding real values for the values, satisfying the constraints & that gives the maximum possible value for object function.

① feasible solution:

$$\text{Ex: } \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

Satisfies all the constraints & is called feasible solution.

⇒ it's objective function value, obtained by evaluating the object at $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ is $0 + 1 - 2(1) = -1$

② optimal solution:

max possible objects function value in the case of maximization problem is called optimal solⁿ.

$$\text{Ex: } \max x + y$$

$$\text{st } 2x + 2y \leq 1$$

⇒ x and y can't exceeds $\frac{1}{2}$

$$\text{Now, both } \begin{bmatrix} x/y \end{bmatrix} = \begin{bmatrix} 1/2 \\ 0 \end{bmatrix} \text{ \& } \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 1/2 \end{bmatrix}$$

are feasible solution

1 continued.

3) Infeasible : Not all LP problem have optimal solution

$$\begin{aligned} \text{Ex : } \min x \\ \text{st } x \leq 1 \\ x \geq 2 \end{aligned}$$

\Rightarrow There is no value for x i.e., at the same time at most 1 and atleast 2

L2

Correct