

Advanced Algorithms

2015 SPEAAG

Name: Anurag Hanj K

USN: 1BM19C017

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Part -A

1a) Naive string matching

Pseudocode:

Naive_String_Matcher (T, P)

$n \leftarrow \text{length}[T]$

$m \leftarrow \text{length}[P]$

for $s \leftarrow 0$ to $n-m$

do if $P[1 \dots m] = T[s+1 \dots s+m]$

then print "Pattern occurs with shift" s

c++ code:

```
#include <iostream>
```

```
using namespace std;
```

```
void search(char* txt, char* pat)
```

```
{
```

```
    int m = strlen(pat);
```

```
    int n = strlen(txt);
```

```
    for (int i = 0; i <= n-m; i++)
```

```
    {
```

```
for(int j = 0; j < m; j++)  
{  
    if (txt[i+j] != pat[j])  
        break;  
}  
if (j == m)  
    cout << "Pattern occurs with shift " << i << endl;  
}  
}  
  
int main ()  
{  
    char txt[] = "ABAA BAC AAB"  
    char pat[] = "BACA"  
    search(pat, txt);  
    return 0;  
}
```

Output:

Pattern occurs with shift 4

Part-B

2a) Multithreaded ~~fib~~ fibonacci number generation ($n=4$)

~~fib~~ Multithreaded algorithm:

P-Fib(n)

if $n \leq 1$
return n

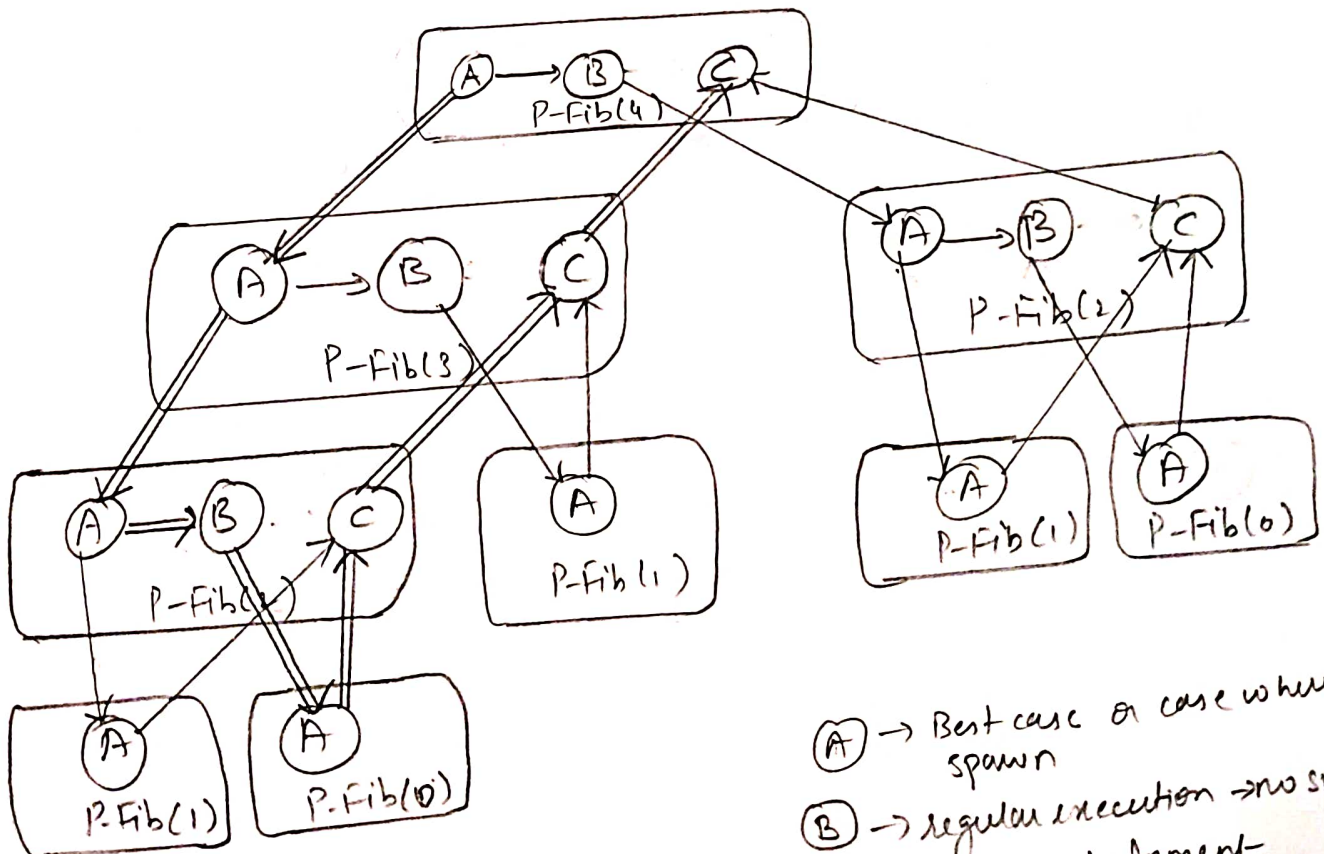
else $x = \text{spawn } \text{P-Fib}(n-1)$

$y = \text{P-Fib}(n-2)$

sync

return $x+y$

Fibonacci(4) = P-Fib(4)



(A) → Best case or case where we spawn
(B) → regular execution → no spawning
(C) → return statement

SPAN

The span is the ~~largest~~ longest time to execute the threads along any path of the computational DAG.

Span = The number of vertices on a longest or critical path (path shown as double-arrows in the figure)

span = 8 time units (Assuming unit time for each thread)

WORK

The work of a multithreaded computation is the total time to execute the entire computation on one processor

work = sum of the times taken by each thread
= No. of vertices in DAG [\because No. of vertices = No. of threads]

work = 17 time units (Assuming unit time for each thread and 17 vertices = 17 threads)

2b) Multithreaded Matrix Multiplication algorithm

Matrix_Multiply (C, A, B, n):

// Multiplies matrices A and B, storing the result in C
// n is power of 2 (for simplicity)

if $n == 1$

$$C[1, 1] = A[1, 1] \cdot B[1, 1]$$

else

allocate a temporary matrix $T[1 \dots n, 1 \dots n]$

partition A, B, C and T into $(n/2) \times (n/2)$ sub-matrices

spawn Matrix_Multiply ($C_{11}, A_{11}, B_{11}, n/2$)

spawn Matrix_Multiply ($C_{12}, A_{11}, B_{12}, n/2$)

spawn Matrix_Multiply ($C_{21}, A_{21}, B_{11}, n/2$)

spawn Matrix_Multiply ($C_{22}, A_{21}, B_{12}, n/2$)

spawn Matrix_Multiply ($T_{11}, A_{12}, B_{21}, n/2$)

spawn Matrix_Multiply ($T_{12}, A_{12}, B_{22}, n/2$)

spawn Matrix_Multiply ($T_{21}, A_{22}, B_{21}, n/2$)

Matrix_Multiply ($T_{22}, A_{22}, B_{22}, n/2$)

sync

Matrix_Add (C, T, n)

Matrix-Add (C, T, n) :

// Adds matrices C and T in-place, producing $C = C + T$

// n is power of 2 (for simplicity)

if $n == 1$:

$$C[1, 1] = C[1, 1] + T[1, 1]$$

else :

partition C and T into $(n/2) \times (n/2)$ sub-matrices

spawn Matrix-Add (C_{11} , T_{11} , $n/2$)

spawn Matrix-Add (C_{12} , T_{12} , $n/2$)

spawn Matrix-Add (C_{21} , T_{21} , $n/2$)

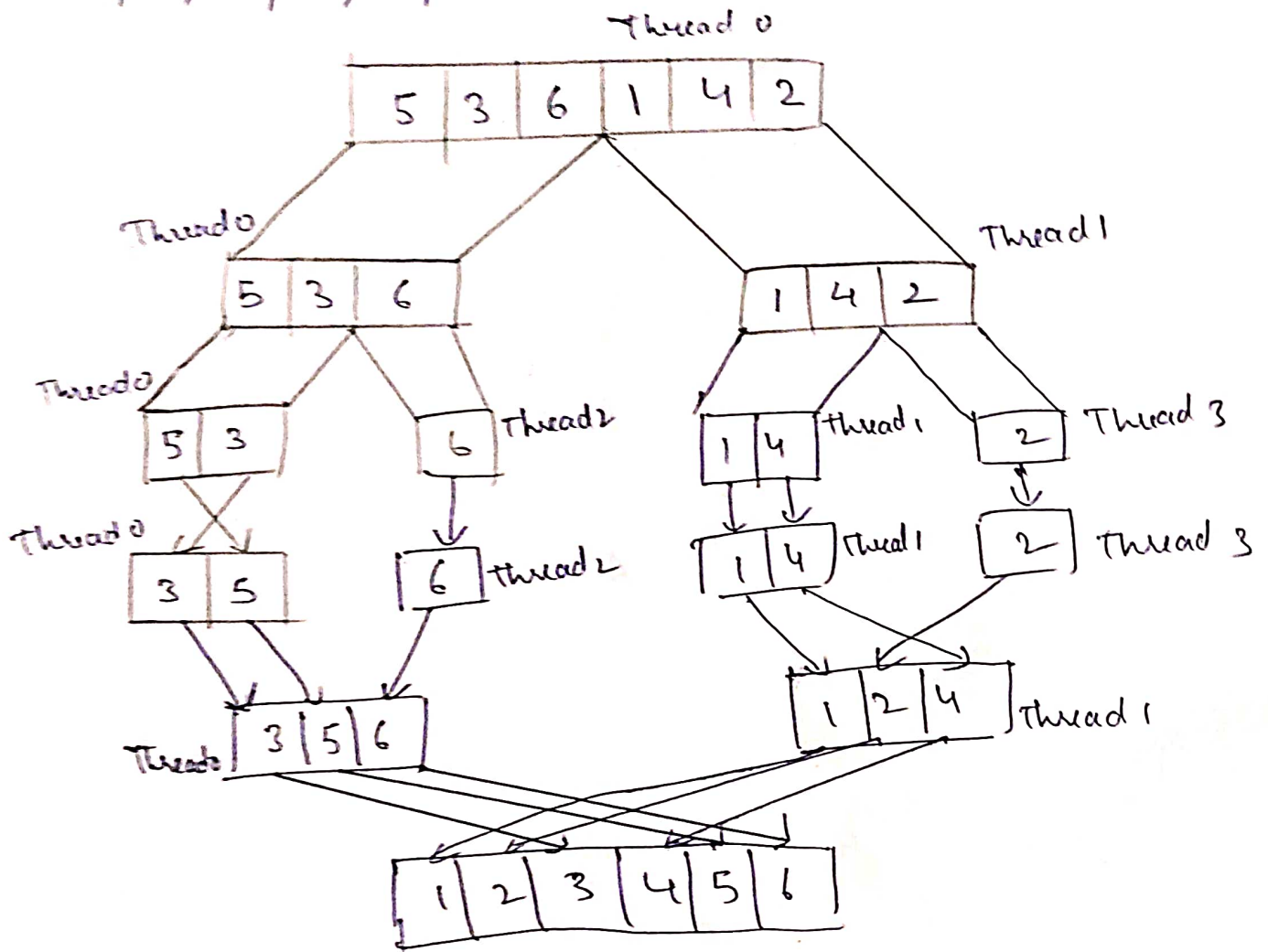
spawn Matrix-Add (C_{22} , T_{22} , $n/2$)

sync

Representation :

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = \begin{bmatrix} \textcircled{C_{11}} & \textcircled{C_{12}} \\ A_{11}B_{11} & A_{11}B_{12} \\ A_{21}B_{11} & A_{21}B_{12} \\ \textcircled{C_{21}} & \textcircled{C_{22}} \end{bmatrix} + \begin{bmatrix} \textcircled{T_{11}} & \textcircled{T_{12}} \\ A_{12}B_{21} & A_{12}B_{22} \\ A_{22}B_{21} & A_{22}B_{22} \\ \textcircled{T_{21}} & \textcircled{T_{22}} \end{bmatrix}$$

2c) Multithreaded merge sort
5, 3, 6, 1, 4, 2



Part c

3a) Algorithm for string matching using Rabin Karp approach

Rabin-Karp-Matcher (T, P, d, q)

// T is text string, P is pattern, d is radix, q is modulus (prime)

$n \leftarrow \text{length}[T]$

$m \leftarrow \text{length}[P]$

$h \leftarrow d^{m-1} \bmod q$ // higher order digit position for m -digit window

$p \leftarrow 0$

$t_0 \leftarrow 0$

for $i \leftarrow 1$ to m // Preprocessing

do $p \leftarrow (dp + P[i]) \bmod q$

$t_0 \leftarrow (dt_0 + T[i]) \bmod q$

for $s \leftarrow 0$ to $n-m$

do if $p = t_s$

then if $P[1..m] = T[s+1..s+m]$

then "Pattern occurs with shift" s

if $s < n-m$

then $t_{s+1} \leftarrow (d(t_s - T[s+1]h) + T[s+m+1]) \bmod q$

Given $T = "ababaaabbaab"$

$$P = "abba"$$

$$d = 26, \quad q = 13$$

$$\begin{aligned} h(abba) &= h(a, b, b, a) & a - 1 \\ &= (1 \times 10^3 + 2 \times 10^2 + 2 \times 10^1 + 1 \times 10^0) \bmod 13 & b - 2 \\ &= (1000 + 200 + 20 + 1) \div 13 & c - 3 \\ &= (1221) \div 13 = 12 & d - 4 \\ & & e - 5 \end{aligned}$$

~~first substring from $T = abab$~~

$$\begin{aligned} h(abab) &= \cancel{1000} (1 \times 10^3 + 2 \times 10^2 + 1 \times 10^1 + 2 \times 10^0) \div 13 \\ &= (1000 + 200 + 10 + 2) \div 13 \\ &= 1212 \div 13 = 3 \neq 12 \end{aligned}$$

$$\begin{aligned} h(baba) &= (2 \times 10^3 + 1 \times 10^2 + 2 \times 10^1 + 1 \times 10^0) \\ &= (2121) \div 13 = 2 \neq 12 \end{aligned}$$

$$\begin{aligned} h(abaa) &= (1 \times 10^3 + 2 \times 10^2 + 1 \times 10^1 + 1 \times 10^0) \\ &= 1211 \div 13 = 2 \neq 12 \end{aligned}$$

$$\begin{aligned} h(baaa) &= (2 \times 10^3 + 1 \times 10^2 + 1 \times 10^1 + 2 \times 10^0) \\ &= 2112 \div 13 = 6 \neq 12 \end{aligned}$$

$$\begin{aligned} h(aabb) &= (1 \times 10^3 + 1 \times 10^2 + 2 \times 10^1 + 2 \times 10^0) \\ &= 1122 \div 13 = 4 \neq 12 \end{aligned}$$

$$\begin{aligned} h(abba) &= (1 \times 10^3 + 2 \times 10^2 + 2 \times 10^1 + 1 \times 10^0) \\ &= (1221) \div 13 = 12 = 12 // \end{aligned}$$

\therefore Pattern occurs at shift 5

⊕

4a) String matching using Horspool's technique

Horspool-Matching ($P[0 \dots m-1]$, $T[0 \dots n-1]$)

// Input: Pattern $P[0 \dots m-1]$ and text $T[0 \dots n-1]$

// Output: index of first matching substring, -1 if no match

Shift Table ($P[0 \dots m-1]$) // generate Table of shifts

$i \leftarrow m-1$

while $i \leq n-1$ do

$k \leftarrow 0$

while $k \leq m-1$ and $P[m-1-k] = T[i-k]$ do

$k \leftarrow k+1$

if $k = m$

return $i-m+1$

else $i \leftarrow i + \text{Table}[T[i]]$

return -1

$P = "$ prada"

$P = p, r, a, d, a$
0 1 2 3 4

@ shift 5

hr shift 3

h=height

pattern matched
at index 17