Unit-4

Linear Programming

Que. Convert the below LPP to standard form

Minimize: $x_1 + 2x_2$

subjected to :— $x_1 + x_2 > 40$

$x_1 - x_2 = 14$

$x_1 - 2x_2 < 3$

Ans. Minimize $x_1 + 2x_2$

① Multiply by -1,

maximize: $-x_1 - 2x_2$

② change equality sign: $x_1 - x_2 = 14$

$-x_1 + x_2 > 14$

$x_1 - x_2 < 14$

Standard form: Maximize $-x_1 - 2x_2$

subject to: $x_1 + x_2 > 40$

$x_1 - 2x_2 > 3$

$-x_1 + x_2 > 14$

$x_1 - x_2 > 14$

$x_1, x_2 > 0$

Que. Convert the below LPP to standard form

Minimize: $8x + 7y$

subject to :— $4x + 2y >= 20$

$-6x + 4y <= 6$

$x + y >= 4$

$2x - 4 = 2$

Ans. subject to :— $4x + 2y \geq 20$

$-6x + 4y \leq 6$

$x + y \geq 4$

$2x - 4 = 2$

① Change minimize to maximize

$\Rightarrow$ maximize: $-8x - 7y$

Charge equality $\Rightarrow 2x-4=2$

$2x-4\leq 2$

$2x-4\geq 2$

standard form $\Rightarrow$ maximize: $-8x-7y$

subject to: $-4x-2y\leq -20$

$-6x+4y\leq 6$

$-x-y\leq -4$

$2x-4\leq 2$

$-2x+4\leq 2$

$x,y\geq 0$

Que. A baker bakes two types of cakes: cake A & cake B.
He requires for baking:

Cake A - 1 unit butter & 3 unit flour

Cake B - 1 unit butter & 3 unit flour

Totally he has 5 unit of butter & 12 unit flour in sto
He makes a profit of ₹ 6 for each cake A sold & ₹ 5 for cake B sold.
Maximize.

Ans.

	cake A	cake B
Butter	1	1
Flour	3	2
Profit	6	5

Profit $= 6x+5y$

Maximize $= 6x+5y$

subject to: $x+y\leq 5$

$3x+2y\leq 12$

$A,B\geq 0$ [Non negative as it is real world problem]

Unit-5

Que. Solve the below LPP using simplex method.

Maximize: $z=3x_1+x_2+2x_3$

subject to: $x_1+x_2+3x_3\leq 30$

$2x_1+2x_2+5x_3\leq 24$

$4x_1+x_2+2x_3\leq 36$

$x_1,x_2,x_3\geq 0$

Ans. Convert to slack form,

$z=3x_1+x_2+2x_3$

$x_4=30-x_1-x_2-3x_3$

$x_5=24-2x_1-2x_2-5x_3$

$x_6=36-4x_1-x_2-2x_3$

$[x_1,x_2,x_3,x_4,x_5,x_6]=[0,0,0,30,24,36]$

pivot ① with x_1

If we increase $x_1 \Rightarrow$ constraints $= x_1\leq 30$

$2x_1\leq 24$

$4x_1\leq 36$

So equation becomes

$$z=27+\frac{x_2}{4}+\frac{x_3}{2}-\frac{3x_6}{4}$$

$$x_1=9-\frac{x_2}{4}-\frac{x_3}{2}-\frac{x_6}{4}$$

$$x_4=21-\frac{3x_2}{4}-\frac{5x_3}{2}+\frac{x_6}{4}$$

$$x_5=6-\frac{3x_2}{2}-\frac{4x_3}{1}+\frac{x_6}{2}$$

So basic soln is

$(9,0,0,21,6,0)$

$\boxed{z=27}$

Pivot x_3 constraints $\Rightarrow$ $x_3 \le 18$

$x_3 \le 8.4$

$x_3 \le 1.5$

$x_3 \le \frac{3}{2}$ $\rightarrow$ substitute with x

$$x_3 = \frac{3}{2} - \frac{3x_2}{8} - \frac{x_5}{4} + \frac{x_6}{8}$$

$$z = \frac{111}{4} + \frac{x_2}{16} - \frac{x_5}{8} - \frac{11x_6}{16}$$

$$x_1 = \frac{33}{4} - \frac{x_2}{16} + \frac{x_5}{8} - \frac{5x_6}{16}$$

$$x_3 = \frac{3}{2} - \frac{3x_2}{8} - \frac{x_5}{4} + \frac{x_6}{8}$$

$$x_4 = \frac{69}{4} + \frac{3x_2}{16} + \frac{5x_5}{8} - \frac{x_6}{16}$$

so basic solⁿ $\Rightarrow$ $\left(\frac{33}{4}, 0, \frac{3}{2}, \frac{69}{4}, 0, 0\right)$

$$\boxed{z = \frac{111}{4}}$$

Pivot with x_2, constraints $x_2 \le 4$

$x_2 \le 132$

$x_2 \le 8$

$x_2 \le 4$ $\rightarrow$ substituting with x_3

so $x_2 = \frac{3}{2} - \frac{8x_3}{3} - \frac{x_5}{4} + \frac{x_6}{8}$

$$z = 28 - \frac{x_3}{6} - \frac{x_5}{6} - \frac{2x_6}{4}$$

$$x_1 = 8 + \frac{x_3}{6} + \frac{x_5}{6} - \frac{x_6}{3}$$

$$x_2 = 4 - \frac{8x_3}{3} - \frac{2x_5}{3} + \frac{x_6}{3}$$

$$x_4 = 18 - \frac{x_2}{2} + \frac{x_5}{2}$$

basic solⁿ $[8, 4, 0, 18, 0, 0]$

$$\boxed{z = 28}$$

Unit-5
Geometry

Que. Check whether the point (10,25) is to the left of (30,30)

Ans. Let p_1 be (10,25)
p_2 be (30,30)

$$p_1 \times p_2 = \begin{vmatrix} 10 & 30 \\ 25 & 30 \end{vmatrix} = -45$$

As $p_1 \times p_2$ is -ve or <0
p_1 is left of p_2 (counterclockwise)

Que. How does using the cross product help in determining if a point is in clockwise/counterclockwise from another point? Show with an example.

Ans. Consider points p_1 and p_2

$p_1 = (p_{11}, p_{12})$
$p_2 = (p_{21}, p_{22})$

If $p_1 \times p_2 = \begin{vmatrix} p_{11} & p_{21} \\ p_{12} & p_{22} \end{vmatrix} < 0$

i.e -ve
it is anticlockwise
i.e p_1 is left of p_2

If $p_1 \times p_2 = \begin{vmatrix} p_{11} & p_{21} \\ p_{12} & p_{22} \end{vmatrix} > 0$

i.e +ve
it is clockwise
i.e p_1 is to right of p_2.

Que 3. Explain how sweep line algo. can be used for finding intersection of line segments.

Ans. Using $O(n \log n)$ time

→ The algorithm first sorts the end points along the x axis from left to right, then it passes a vertical line through all points from left to right & checks for intersections.

Que 4. Check whether the points (10, 25) & (10, 55) are co-linear or not.

Ans.

$AB = \sqrt{(10-10)^2 + (55-25)^2}$
$= \sqrt{(30)^2}$
$= 30$

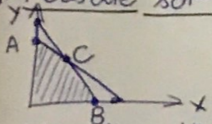
$BA = \sqrt{(10-10)^2 + (25-55)^2}$
$= \sqrt{(30)^2}$
$= 30$

$\boxed{AB = BA}$

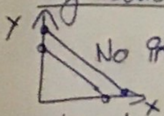
∴ they are collinear

Que. Explain what is :— ① Feasible solⁿ
② Infeasible solⁿ
③ optimal solⁿ

Ans. Feasible solⁿ : In linear program is a solution that satisfies all constraints.



Infeasible solⁿ : If no. solⁿ exists to satisfy all constraints



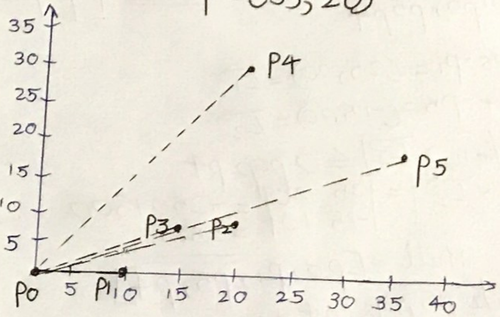
optimal solⁿ : It is a feasible solⁿ with the largest objective function (maximized funcⁿ).

For $4x+5y$
(8, 12) yields maximized value, hence it is the optimal solⁿ.

Que. Apply Graham scan algo. to find convex hull for the below points :—

p0 (0,0)
p1 (10,0)
p2 (20,10)
p3 (15,10)
p4 (20,30)
p5 (35,20)

Ans.



sorted according to polar angles,
points = [p0, p1, p5, p3, p4]

Step1 : Push p0, p1 and p2 to stack (Hull)

Step 2 : Check p_1p_2 and p_2p_5 angle,

$p_2 - p_1 = (10,10) = \vec{L_1}$

$p_5 - p_2 = (15,10) = \vec{L_2}$

if $|\vec{L_1} \times \vec{L_2}| \le 0$ pop p2

$|\vec{L_1} \times \vec{L_2}| = \begin{vmatrix} 10 & 10 \\ 15 & 10 \end{vmatrix} = 100 - 150 = \underline{-50}$

∴ $\vec{L_2}$ is on right of $\vec{L_1}$
so pop p_2
push p5

Step 3 : Check p_1p_5, p_5p_3

$p_5 - p_1 = (25,20) = \vec{L_1}$

$p_3 - p_5 = (-20,-10) = \vec{L_2}$

if $|\vec{L_3} \times \vec{L_2}|$ so pop p5

$|\vec{L_1} \times \vec{L_2}| = \begin{vmatrix} 25 & 20 \\ -20 & -10 \end{vmatrix} = -250 + 40$
$= \underline{150}$

Hull = [p0, p1, p5, p3]

Step 4: Check $p5p3$, $p3p4$

$p3 - p5 = (-20,-10) = \vec{L_1}$

$p4 - p3 = (5,20) = \vec{L_2}$

If $|\vec{L_1} \times \vec{L_2}| \leq 0$ pop $p3$

$|\vec{L_1} \times \vec{L_2}| = \begin{vmatrix} -20 & -10 \\ 5 & 20 \end{vmatrix} = -400 + 50 = \underline{\underline{-350}}$ pop $p3$

Step 5: Check $p1p5$, $p5p4$

$p5 - p1 = (25,20) = \vec{L_1}$

$p4 - p5 = (-15,10) = \vec{L_2}$

If $|\vec{L_1} \times \vec{L_2}| \leq 0$ pop $p4$

$|\vec{L_1} \times \vec{L_2}| = \begin{vmatrix} 25 & 20 \\ -15 & 10 \end{vmatrix} = +250 + 300 = \underline{\underline{550}}$

Hull $= [p0, p1, p5, p4]$

Step 6: Check $p5p4$, $p4p0$

$p4 - p5 = (-15,10) = \vec{L_1}$

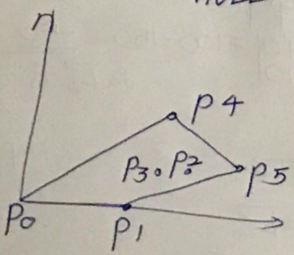
$p0 - p4 = (-20,-30) = \vec{L_2}$

If $|L_1 \times L_2| > 0$ Hull is correct

$|L_1 \times L_2| = \begin{vmatrix} -15 & 10 \\ -20 & -30 \end{vmatrix} = 450 + 200 = \underline{650}$

$\vec{L_1} \times \vec{L_2} > 0 \Rightarrow$ Hull is obtained

HULL $= [p0, p1, p5, p4]$



Que. Describe the Jarvis's march technique of computing the convex hull.

polygon that encloses all points

Ans. Jarvis's march computes the convex hull of a set Q of points by a technique known as package wrapping.

→ We start with the leftmost point (or point with minimum x coordinate value) to we keep wrapping points in counter clockwise direction.

→ To find the next point in the output we use orientation where the next point is selected as the point that beats all other points at a counter clockwise orientation.

Que. Check whether pair of line segments intersects or not.

Ans. $(p1, p2)$ with $(p3, p4)$

$p1 = (15,10)$
$p2 = (45,25)$
$p3 = (20,35)$
$p4 = (30,10)$