- an undirected graph is $G = (V, E)$, with V the vertices, E the edges. $V = \{V_1, V_2, ..., V_n\}$ is a set of objects, E a set of connections between the objects s.t. $\forall e \in E, e = \{u, v\}$ with $u, v \in V$.
- a directed graph differs in its edges E, where $\forall e \in E, e = (u, v)$ with $u, v \in V$.
- a subgraph of G is $H = (V_H, E_H)$ where $V_n \subseteq V, E_H \subseteq E$, and $(\forall e = (u, v) \in E)(u, v \in V_H)$
- connected component: maximal connected subgraph (can't make it more connected) (what???)
- a cycle in $G = (V, E)$ is a sequence of nodes $\{v_k\} \in V$ s.t. $\{v_i, v_{i+1}\} \in E \wedge \{v_1, v_k\} \in E$
- a tree is an acyclic connected graph
- Minimum spanning tree (MST) problem: given an undirected connected graph G and nonnegative weights $d(e) = d(u, v)$, find the subgraph T that connects all vertices and minimizes

$$\text{cost}(T) = \sum_{\{u,v\} \in T} d(u, v)$$

  - think about why T must be a tree. if there are cycles, you can trivially rm an edge to cut costs
  - Prim's alg
    - description
      - given graph G, choose arb node s, the start of T
      - repeating $|V| - 1$ times,
        - add to T, the lowest edge (and nodes) from "in T" to "not in T"
    - proof
      - we prove Prim's produces T, an MST.

        First, we prove the MST Cut Property: let $S \subseteq V$, s.t. $1 \leq |S| < |V|$ i.e. S is not empty but not all nodes. Call a pair $(S, V \setminus S)$ a cut of the graph. Every MST contains an edge on the cut ($e = \{u, v\}, u \in S, v \in V \setminus S$) that has min weight.

        Now on to Prim's. The pair ("in T", "not in T") is a cut of G. By the Cut Property, the MST contains the lowest cost edge crossing this cut, which is by defn the next edge Prim's adds, i.e. Prim's adds edges in the MST.

        At any point in the algorithm, $T = (V_T, E_T)$ is a subgraph and a tree. T grows by 1 vertex and 1 edge at each step, stopping at $|V_G| - 1$ steps, and so results in a spanning tree. Since these edges are in the MST from above, the spanning tree is the MST. □