intro

- foreshadowing/context: under all prob computations are *sample spaces*
- rarely work with sample spaces directly, unless they're simple (heads/tails)
  - (so, we start here)

what is prob?

- objective prob: long run freq of occurrence (eg heads in coin flip)
  - often called frequentist/classical methods.
  - used more often in undergrad CMU
- subjective prob: a possibly informed belief in rate of occurrence of event
  - can called bayesian

set notation

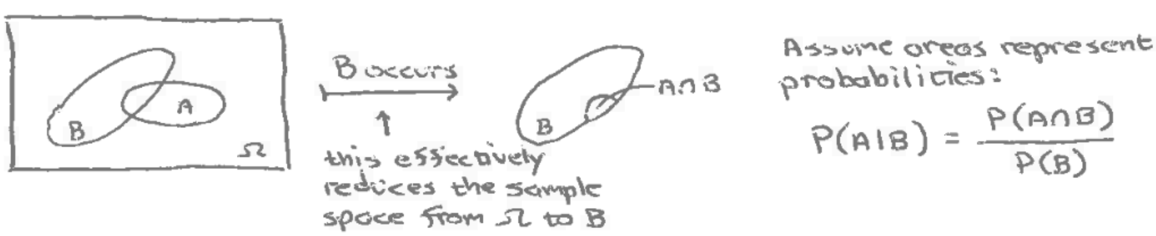
- $A \supset B, A \subset B, A \cup B, A \cap B, \overline{A}$ aka A^C
- let the set of all experimental outcomes $\Omega = A \cup \overline{A}$
- $A \cap B = \emptyset \implies A$ and B are mutually exclusive aka disjoin
- distributive/associative laws
- de morgan's ($\overline{A \cup B} = \overline{A} \cap \overline{B}$, etc)

what are experiments?

- make observations
- passive: just collect data
- active: control setting
- sample space (Ω)
  - two coins tossed? $\Omega = \{\mathrm{HH}, \mathrm{HT}, \mathrm{TH}, \mathrm{TT}\}$
    - HH is simple event
    - TH is compound event ("at least one tail")
  - free throws until miss? Ω = {M, HM, HHM, ...}
  - relative freqs of above? don't know! need more info

probability

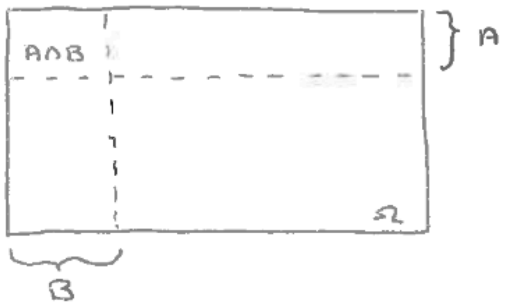
- conditional probability: A, when we know B



- A and B are independent
  - iff $P(A \mid B) = P(A)$
  - iff $P(B \mid A) = P(B)$
  - iff $P(A \cap B) = P(A)P(B)$

- if there are three events, A and B are *conditionally* independent
  - if $P(A \cap B \mid C) = P(A \mid C)P(B \mid C)$

The following is an example of independence as rendered on a Venn diagram:



$$P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{P(A)}{P(\Omega)} = P(A)$$

area ratios are equal

Let A, B, and C be three separate events in Ω, all of which have non-zero probability of occurring. A and B are *conditionally independent* if

$$P(A\cap B \mid C) = P(A\mid C)P(B\mid C)$$

The following shows conditional independence and dependence as rendered on a Venn diagram:

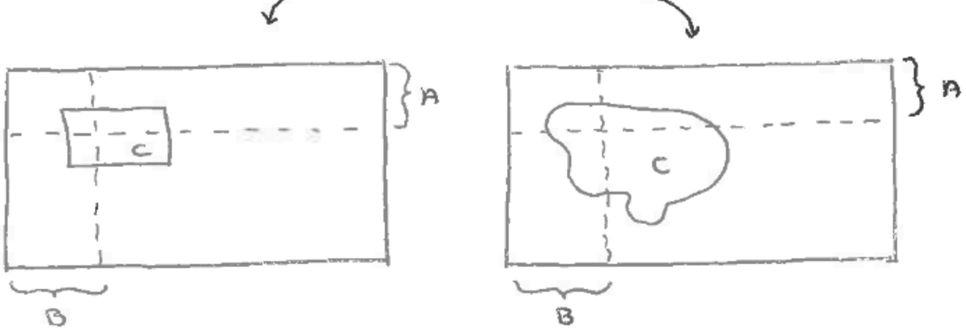


Figure 1: independence visualization