

Calc in 3d Notes

saffron_

Contents

Chapter 2: Vectors in Space	2
Appendix	4

Chapter 2: Vectors in Space

Graphing

TEMPLATE

TEMPLATE

The Vector

ex image here

- a quantity with a magnitude and direction

- unit vector $\hat{u} = \frac{\vec{v}}{\|\vec{v}\|}$

Vector Operations

- Addition
 - $\vec{a} = \langle 1, 2 \rangle, \vec{b} = \langle 3, 4 \rangle$
 - $\vec{a} + \vec{b} = \langle 4, 6 \rangle$
- Dot Product (also 2d!)
 - geom.:
 - img
 - $\vec{u} \cdot \vec{v} = \|\vec{u}\| \|\vec{v}\| \cos \theta$
 - alg.:
 - $\vec{u} = \langle u_1, u_2, u_3 \rangle, \vec{v} = \langle v_1, v_2, v_3 \rangle$
 - $\vec{u} \cdot \vec{v} = \sum u_i v_i$
 - $\vec{v} \cdot \vec{v} = \|\vec{v}\|^2$
 - two vectors are orthogonal aka \perp iff their dot product is 0
 - $\cos \theta = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \|\vec{v}\|}$ wtf is equation 2.5
"unique over this range" on abt
 - work = $\vec{F} \cdot \vec{D}$
 - comp (scalar projection)

$$\begin{aligned} \text{comp}_{\vec{v}} \vec{u} &= \|\vec{v}\| \cos \theta \\ &= \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\|} \end{aligned}$$
 - proj (vector projection)

$$\begin{aligned} \text{proj}_{\vec{u}} \vec{v} &= \text{comp}_{\vec{u}} \vec{v} \cdot \frac{\vec{u}}{\|\vec{u}\|} \\ &= \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\|^2} \vec{u} \end{aligned}$$
- Scalar Multiplication
 - $\vec{v} = \langle 1, 3 \rangle, c = 2$
 - $c\vec{v} = \langle 2, 6 \rangle$
- Cross Product (3d)
 - geom.:
 - img
 - $\|\vec{u} \times \vec{v}\| = \|\vec{u}\| \|\vec{v}\| \sin \theta$
 - alg.:
 - $\vec{u} \times \vec{v} = \det \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$
 - result is \perp to both input vectors, direction by right hand rule
 - triple scalar product:

$$\begin{aligned} \vec{u} \cdot (\vec{v} \times \vec{w}) &= (\vec{u} \times \vec{v}) \cdot \vec{w} \\ &= \det \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix} \end{aligned}$$
 - areas and volumes (see appendix)
 - if \vec{u} and \vec{v} are the sides of a parallelogram, then its area is $\|\vec{u} \times \vec{v}\|$
 - if a parallelepiped has edges $\vec{u}, \vec{v}, \vec{w}$, its volume is the absolute value of its triple scalar product
 - torque = $\vec{\tau} = \vec{r} \times \vec{F}$

simple inverses for subtraction and scalar division exist.

Appendix

matrices, 3x3 determinants

Recall that the elements of a matrix are enumerated a_{ij} where i is column and j is row, both 1-indexed.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

Recall that the minor of the matrix element here is the 2 by 2 determinant when you take away the row and column of the element in a 3 by 3 matrix. Picking an arbitrary row (or even column), with a_{ij} being an element and M_{ij} being a minor, a 3 by 3 determinant is calculated by

$$\sum_{j=1}^3 (-1)^{i+j} a_{ij} M_{ij}$$

In the figure `media/3x3determinant.png`, $(-1)^{i+j}$ is in green, a_{ij} in orange, and M_{ij} in blue.

cross/dot products

- see:
 - `media/cross_prod_area_ex.png`
 - `media/scalar_projection_composition_ex.png`
 - `media/scalar_triple_prod_parallelepiped_ex.png`