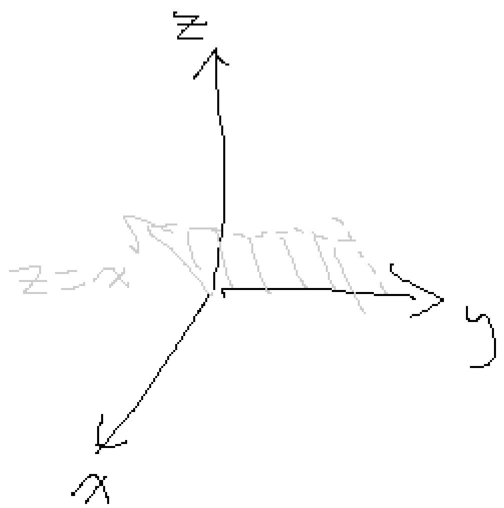
Calc in 3d Notes

saffron_

Contents

Chapter 2: Vectors in Space

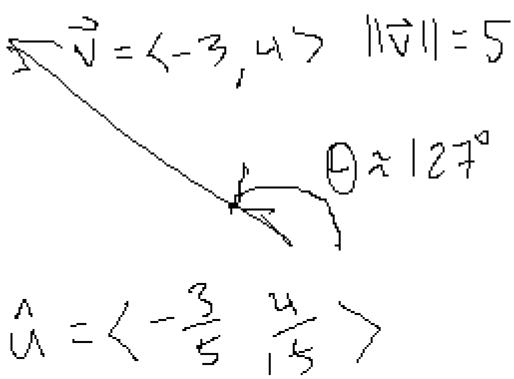
Graphing



convention

sphere: $(x-1)^2 + (y-2)^2 + (z+3)^2 = 9$

The Vector



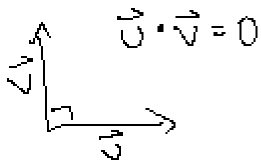
- a quantity with a magnitude and direction
- unit vector $\hat{u} = \dfrac{\vec{v}}{\|\vec{v}\|}$

Vector Operations

- Addition
  - $\vec{a} = \langle 1, 2 \rangle$, $\vec{b} = \langle 3, 4 \rangle$
  - $\vec{a} + \vec{b} = \langle 4, 6 \rangle$
- Scalar Multiplication
  - $\vec{v} = \langle 1, 3 \rangle$, $c = 2$
  - $c\vec{v} = \langle 2, 6 \rangle$

simple inverses for subtraction and scalar division exist.

- Dot Product (also 2d!)
  - geom.:



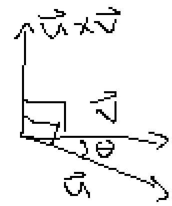
    $\vec{u} \cdot \vec{v} = \|\vec{u}\| \, \|\vec{v}\| \cos\theta$
  - alg.:
    $\vec{u} = \langle u_1, u_2, u_3 \rangle$, $\vec{v} = \langle v_1, v_2, v_3 \rangle$
    $\vec{u} \cdot \vec{v} = \sum u_i v_i$
  - $\vec{v} \cdot \vec{v} = \|\vec{v}\|^2$
  - two vectors are orthogonal aka $\perp$ iff their dot product is 0
  - $\cos\theta = \dfrac{\vec{u} \cdot \vec{v}}{\|u\| \, \|v\|}$ wtf is equation 2.5 "unique over this range" on abt
  - work $= \vec{F} \cdot \vec{D}$
  - comp (scalar projection)

$$\begin{aligned}\operatorname{comp}_{\vec{u}} \vec{v} &= \|\vec{v}\| \cos\theta \\ &= \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\|}\end{aligned}$$

  - proj (vector projection)

$$\begin{aligned}\operatorname{proj}_{\vec{u}} \vec{v} &= \operatorname{comp}_{\vec{u}} \vec{v} \cdot \frac{\vec{u}}{\|\vec{v}\|} \\ &= \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\|^2} \vec{u}\end{aligned}$$

- Cross Product (3d)
  - geom.:



    $\|\vec{u} \times \vec{v}\| = \|\vec{u}\| \, \|\vec{v}\| \sin\theta$
  - alg.:
  - $\vec{u} \times \vec{v} = \det \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$
  - result is $\perp$ to both input vectors, direction by right hand rule
  - triple scalar product:
    $\vec{u} \cdot (\vec{v} \times \vec{w})$
    $= (\vec{u} \times \vec{v}) \cdot \vec{w}$
    $= \det \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$
  - if $\vec{u}$ and $\vec{v}$ are the sides of a parallelogram, then its area is $\|\vec{u} \times \vec{v}\|$
  - if a parallelepiped has edges $\vec{u}, \vec{v}, \vec{w}$, its volume is the absolute value of its triple scalar product
  - torque $= \vec{\tau} = \vec{r} \times \vec{F}$

Appendix

matrices, 3x3 determinants

In depth lesson:

- `https://www.mathcentre.ac.uk/resources/uploaded/sigma-matrices9-2009-1.pdf`

Recall that a elements of a matrix are enumerated a_{ij} where i is column and j is row, both 1-indexed.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

Recall that the minor of the matrix element here is the 2 by 2 determinant when you take away the row and column of the element in a 3 by 3 matrix. Picking an arbitrary row (or even column), with a_{ij} being an element and M_{ij} being a minor, a 3 by 3 determinant is calculated by

$$\sum_{j=1}^{3} (-1)^{i+j} a_{ij} M_{ij}$$

In the figure `media/3x3determinant.png`, $(-1)^{i+j}$ is in green, a_{ij} in orange, and M_{ij} in blue.

cross/dot products

- see:
  - `media/cross_prod_area_ex.png`
  - `media/scalar_projection_composition_ex.png`
  - `media/scalar_triple_prod_parallelepiped_ex.png`