Reasoning with Data Notes

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Contents

Course Summary

- see `lecture1written.pdf`

Exploratory Data Analysis

EDA for 1-variable categorical data

- population: complete set on interest (eg all US workers). can't be measured perfectly
- sample: subset of pop that can actually be obtained
- parameter: summary of population (eg average weight). also can't be measured
- statistic: estimation of parameter using sample.
- inference: specifying estimate of a parameter (ie gving estimate and measure of how far off it is)
- individuals/units/cases: objects described in dataset
- variable: column of spreadsheet, something measured
- quantitative: numerical (eg age)
- categorical/qualitative: not numerical (eg ethnicity)
- frequency table: show how often each *categorical* variable shows up
  - relative freq. table: how percent often they show up, adding up to 1 or 100%: best summary of a categorical variable
- frequency/percent/relative (add to 1) frequency bar graphs: to visualize categorical data. v. similar to their freq table variants.
- pie graph: to visualize a *single* categorical variable, with each slice being a category.

EDA for 1-variable quantitative data

- grouped frequency/grouped relative frequency table: for one quantitative var. boring version of a histogram.
- histogram: for one quantitative variable. visualizes a grouped (rel.) freq table.
- distribution of one quantitative var (anal. doesn't really hold up for qualitative)
  - modality: how many 'clusters'? (eg unimodal/bimodal/etc)
  - symmetric/skewness? (skew right means tail is to right)
  - center: mean ($\overline{x}$)
    - median if heavy outliers. using mode is cursed but you do you
  - spread: standard deviation (S) (usually, $\frac{2}{3}$ of values are 1 stddev away from mean)
    - interquartile range (IQR) if heavy outliers.
  - outliers?
- fun facts: variance: square of std dev. easier in formulas because it removes the square root, but in real world, std dev is preferred because it has same units as the data
- box plot: other way to graph quantitative data (using min, first quartile, median, third quartile, max)
  - interquartile range (IQR): difference between quartiles, a measure of spread. is rough, not as useful as stddev.
  - shows less data than histogram, so best for concise comparison of *multiple* distributions (see section on 2-variable EDA)

EDA for 2-variable data

- relationship/association: one variable can tell you about another
- explanatory variable: “input” variable, the x-axis
- response variable: “output” variable, the y-axis
- if we need analysis from explanatory → response
    - categorical → quantitative: side by side boxplots
        - more difference if boxes are futher apart on “y-axis”
        - summaries: numerical summaries of response for each category
    - categorical → categorical: contingency table
        - summaries: conditional percents of responses, conditioned on explanatory (ie what percent of people from Cali are stat majors?)
    - quantitative → quantitative: scatterplots
        - describe: direction, form, outliers
        - summaries (only if reasonably linear): correlation coefficient (R), least squares regression line ($\hat{y} = b_0 + b_1X$)
    - quantitative → categorical: outside scope of this course

Study Design

- no matter what, we want to consider and get...
  - reliability, statistical significance: low random variation/error
    - use a large sample size
    - can be measured with stddev
  - validity: trustworthy estimates and predictions
    - consider and declare outliers. remove them if needed, but with caution!
    - beware extrapolation outside range of data that produced the model
    - validate model (eg check for linearity if you use linear regression)
  - generalizability: no bias aka systemic error
    - called instrument bias if instrument is set wrong (can be social instrument such as misleading survey)
      - remove via resetting instrument (reset scale, rewrite survey)
    - called sampling bias if sample is systematically not representative
      - remove via random sampling. such as simple random sampling (SRS): every individual has same chance to be chosen as any other; every pair same chance as other pair; etc
- and if we have 2 or more variable relationships...
  - causality: no lurking/confounding variables; if we want to find causation and not just correlation
    - use randomized assignment of explanatory variable
    - experimental study: study where lurking/confounding variables are removed
    - observational study: not experimental. subjects decided which treatments to get (eg survey vitamin c usage vs flu symptoms)
    - placebo control group: 'non-active' treatment to avoid placebo effect
    - 'double blind': neither the researchers nor the subjects know which treatment they are receiving

good luck on exam 1 <3

Elementary Probability

- probablility: measure of variation due to a random phenomenon
- random: produced so that probability applies. satisfies:
  - equal likelihood (A's selection is as likely as B's)
  - independence (A's selection doesn't influence B's)
- coin: P(heads)= 0.5
- relative frequency: if selections are random (eq likelihood and independent),

$$\text{P(outcome)} = \frac{\text{\# of ways outcome occurs}}{\text{\# of possible outcomes}}$$

  - long-run approx. can be as precise as desired: Law of Large Numbers
  - not replacing violates independence
    - consider a bag with a blue and red ball. selecting blue and not putting it back makes P(red) $= 1$.
    - but this is common—a pollster doesn't survey two houses twice
  - not replacing is fine if population ≥ 10 or 20 times the sample
    - consider 2000 blue, 2000 red. selecting blue makes P(red) ≈ 0.5
- probablility experiment: an outcome's success/failure varies
  - eg flipping a coin: outcome (like get heads) varies
  - trials: different "runs" of a probability experiment
- disjoint/mutually exclusive: $A \cap B$ is empty
- elementary outcomes: irreducible for assigning probabilities. mutually disjoint.
- sample space: S, the set of all elementary outcomes in a probability experiment
- complement: A^C, "opposite" of a probability. $A + A^C = 1$.
- intersection: $\cap$, "and" of two probabilities. middle of venn diagram.
- union: $\cup$, "or" of two probabilities. all circles' overlap area.
  - $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$
- joint probability tables
  - each inner box is an elementary outcome (and a joint probability like $A \cap B$)
  - rows sum to row marginal probabilities, same with columns
  - 2x2 tables can represent venn diagrams

- conditional probability (A, if we know that B):
    - $\mathrm{P}(A \text{ given } B) = \mathrm{P}(A \mid B) = \mathrm{P}(A \cap B) \div \mathrm{P}(B)$
    - for a d20, $\mathrm{P}(4 \mid \text{even}) = 0.1$
    - take from box and margin of a joint probability table
- statistically independent: unassociated events. if you know the outcome of one event, it doesn't change the probability of the other.
    - with variables: association b/t A and B is nearly what is expected from chance
    - with sampling: any person similarly likely to be selected, regardless of previous selection
    - A and B are independent
        - iff $\mathrm{P}(A \mid B) = \mathrm{P}(A)$
        - iff $\mathrm{P}(A \mid B) = \mathrm{P}(A \mid B^C)$
        - iff $\mathrm{P}(A \cap B) = \mathrm{P}(A) \cdot \mathrm{P}(B)$

Probability Models of Data

- random variable: outcome of randomness that takes on numerical values
- discrete rv: takes on "separated" outcomes. modeled with table or rel-freq histogram
- continuous rv: takes on an interval of outcomes. modeled with density curve
- rv example: cars occupancy
    1. define prob. experiment: randomly select car on road
    2. define rv: let X = number of people in car
    3. assign probabilites to X. denote prob. of $X = 1$ as $\mathrm{P}(X = 1)$
- Binomial Counts "Experiment"/Study/Trial
    - conditions:
        (a) fixed sample size n for every run of study
        (b) two categories (success/failure) for each observation
        (c) observations must succeed or fail independantly
        (d) fixed prob p of success for each observation
    - conditions c and d are guaranteed if selections are random
    - the count of successes in n trials is a discrete rv X, and follows the Binomial Distribution (ie, X is binomial. see by graphing the histogram)
    - $\mathrm{P}(X = x) = {}_nC_x \cdot p^x \cdot (1 - p)^{n-x}$
    - binomial coefficient ${}_nC_k = C^n_k = \binom{n}{k} = \frac{n!}{x!(n-x)!}$ (recall $0! = 1$)
    - eg: 8 rand selected lab animals get vaccine w/ prevention rate of 40% and then are infected. Then X = surviving animals out of $n = 8$ with $p = 0.4$.

- distributions (density curves)
  - graph a histogram, imagine a idealized curve fitting it
    - idealized curve means true populations, so variables are greek letters
    - mean = μ (mu)
    - stddev = σ (sigma)
  - area = proportion of observations = relative freq = probability
  - total area = 1
  - exponential distribution
    - let X be the time until the next event, with events occuring randomly with a mean waiting time λ.
    - denoted $X \sim \text{Exp}(\lambda)$
    - fun fact: this is graphed $f(x) = \lambda e^{-\lambda x}$
    - eg: time until next earthquake, time until a phone call ends
  - uniform distribution
    - a process produces values $\in [a, b]$ with equal likelihood.
    - denoted $X \sim \text{Uniform}(a, b)$
    - fun fact: this is graphed $f(x) = \frac{1}{b-a}$ for $x \in [a, b]$, $f(x) = 0$ elsewhere
    - eg: result of dice roll
  - normal/gaussian distribution
    - models result of mixing many independant factors and Central Lim Thrm
    - denoted $X \sim \text{N}(\mu = u, \sigma = s)$
    - fun fact: these are variations on the graph $f(x) = e^{-x^2}$
    - eg: human height
    - points of inflection are 1 stddev away from mean
    - 68/95/99.7 percent of population falls within 1/2/3 stddev of mean
  - standard normal distribution
    - for discussing probability on normal distributions: on all normal curves, portions with the same Z score have the same area.
    - standardized/Z score: stddevs from mean for a number
      - $Z = \frac{\text{observation} - \mu}{\sigma}$
    - $\text{N}(\mu = 0, \sigma = 1)$ is already standardized. we may denote $Z \sim \text{N}(0, 1)$
    - eg: consider adult male weight X.
      - we know $X \sim \text{N}(\mu = 165, \sigma = 30)$
      - if P weighs 202 pounds, how unusual are they?
      - $Z = \frac{202-165}{30} = 1.23$ and $\text{P}(X < 202) = \text{P}(Z < 1.23) = 0.8907$.
    - Z score tables and such exist for these calculations
    - be able to calculate: Z score to prob, prob to Z score, stdizing to find prob, unstdizing to find score (see Lecture 14 notes)

Sampling Distribution of a Statistic

- population distribution: eg how long it takes for literally every lightbulb to burn out
- (sample) data distribution: eg how long it takes for this random sample of $n = 1000$ lightbulbs to burn out
- sampling distribution of a statistic: the probability distribution of all possible values of a statistic (from taking many samples of same sample size from same population)
- sampling distribution of $\overline{X}$, the mean
  - center: if sampling is random, then mean of sample is the population mean. ie,

    $$\mu_{\overline{x}} = \mu \text{ if sampling is random}$$

  - spread: if sampling is random and outcomes are statistically independant (eg sampling with replacement or infinitely large pop), then

    $$\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}}$$

  - shape: if sample is large ($n \geq 30$), then by the Central Limit Theorem, the sampling distribution of $\overline{X}$ approaches the normal distribution. (resembles population otherwise)
  - lightbulb example problem in Lecture 16. Note AFSOC for inference.
- sampling distribution of $\hat{P}$, proportion
  - consider a binomial study (in particular, random and independant) with sample size n and success probability p. then,
  - center:

    $$\mu_{\hat{p}} = p$$

  - spread:

    $$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

  - shape: if np and $n(1-p)$ both ≥ 10, then by the Central Limit Theorem, $\hat{P}$ approaches the normal distribution. (resembles population otherwise?? maybe?)
  - coinflip example problem in Lecture 18.

good luck on exam 2 <3

Formal Inference (Confidence Intervals, Hypo. Testing)

- we study two branches of formal inference:
  - confidence intervals: what are most likely values of a parameter?
  - hypothesis/significance testing: what is the likelihood of a parameter to be a value?
- Confidence Intervals
  - Theory: $\overline{x}$ is "probably" somewhat "close" to μ.
    - ∗ how probable? confidence level
    - ∗ how close? margin of error
  - if random sampling, stat. independant, normal, then the confidence interval is

$$\overline{X} \pm Z_{\text{critical}} \frac{\sigma}{\sqrt{n}}$$

    - ∗ where n = sample size, $\overline{X}$ = sample mean, σ = assumed population stdddev, $Z_{\text{c}} = 1.645$ for $C = 90\%$, $Z_{\text{c}} = 2$ for $C = 95\%$, $Z_{\text{c}} = 3$ for $C = 99.7\%$, etc
    - ∗ eg: $\approx 95\%$ of $\overline{x}$'s are within $\pm 2\sigma_{\overline{x}}$
  - similarly, if binomial (in particular, random sampling and independent) and normal, the confidence interval is

$$\hat{p} \pm Z_{\text{critical}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

- Hypothesis Testing
  1. State Hypotheses
     - null (H_0): μ = value
     - alt (H_a): $\mu <, >,$ or $\neq$ value
  2. choose α (usually 0.01 or 0.05)
  3. perform study, get sample statistic, get p value of that sample statistic
  4. conclusion: if $p < \alpha$, this is very unlikely. we reject the null hypothesis
  5. state in context: what does parameter represent? what is the likely value/why?
- Note: '$\neq$' confidence test with significance α is equiv to making a confidence interval with level $1 - \alpha$ ie α^C
  - ie: Hypothesis test says do not reject $H_0 \Leftrightarrow H_0$ value is inside confidence interval
- TODO formal template for writing this. an example on lec24 p5 says "since the p value is (not) less than 0.05, we do (not) reject the null hypo. [explain what this means]." do NOT say "accept the null hypo."

- T-Test for mean when sigma unknown
  - we instead use sample stddev, S.
  - like how we call $\frac{\sigma}{\sqrt{n}}$ $\sigma_{\overline{x}}$, we call $\frac{S}{\sqrt{n}}$ standard error
  - notice how similar our equations are
  - hypothesis test for a mean
    - test stat $= t = \frac{\overline{X} - \mu_{\text{null}}}{S/\sqrt{n}}$ ("stderrs away from mean" instead of stddevs away from mean)
  - confidence interval for a mean
    - $\overline{X} \pm t_{\text{critical}} \cdot \frac{S}{\sqrt{n}}$
    - note: t_{critical} is a bit over 2 for 95% because the t distribution is fatter at tails than the normal
  - instead of $n \geq 30$ for normality, you may inspect the distribution. since t-distribution is robust, vauge normality is even fine as samples sizes increase (as long as no severe outliers). ie, if $n \geq 40$ or so, t dist is valid regardless of shape
  - t approaches norm distribution as n increases.

- Use test for two independent means when wts relationship between *two categorical* explantories and *one quantitative* response.
  - $H_0 : \mu_1 - \mu_2 = 0$ and $H_A : \mu_1 - \mu_2 < / > / \neq 0$
  - estimator of difference: $\overline{x}_1 - \overline{x}_2$
  - we can analyze z or t score for the sampling dist of the estimator (depending on if σ known/t dist valid)

- inference for two proportions when wts relationship between *two categorical* explantories and *two categorical response*
  - $H_0 : p_1 - p_2 = 0$ and $H_A : p_1 - p_2 < / > / \neq 0$
  - estimator of difference: $\hat{p}_1 - \hat{p}_2$
  - must analyze z score. rmbr to check validity ($np, n(1-p) \geq 10$ for both explanatories)

- one-way (one explanatory) ANOVA (ANalysis Of VAriance) test of means
  - TODO group others accordingly like this, descr down here
  - at least *3 categorical* explanatory → *one quantitative* response (like better test of two means)
  - $H_0 : \mu_1 = \mu_2 = \cdots = \mu_k$ and H_A : at least one μ is different
  - we test F score. reqs: random selection. independence of each population's mean (large pop for each). shape ($n \geq 30$ each). each population σ is similar, all within factor of 2 is fine.

- chi square (χ^2) test for contingency tables
  - *numerous* categorical → *numerous* response (like better test of two proportions)
  - H_0 : no association between cat. and resp. and H_A : there is an association between cat. and resp.
  - analyze χ^2 score (fun fact: χ^2 dist. is unimodal right skewed)
    - expected cell = $\frac{\text{row total}\times\text{column total}}{\text{grand total}}$ (for the actual values, not percentages)
    - $\chi^2 = \sum \frac{(\text{observed}-\text{expected})^2}{\text{expected}}$ for all cells
  - reqs: independent cells ie each individual must be in only one cell; random selection; independence (large pop); shape (expected count ≥ 5, use observed count if expected is not reported by software)
  - (what if shape not satisfied? in 36-200, group categories or remove one (be careful!). in later courses, use a distribution-free test)

TODO: independence condition for tests, if we alr know random sampling, is fulfilled by replacement or large pop compared to n. this is all that is needed.

- test of slope (linear regression)
  - *quantitative* categorial → *quantitative* response
  - population line: $Y = \beta_0 + \beta_1 X$, sample line: $\hat{y} = b_0 + b_1 X$
  - no relationship $\iff$ $\beta_1 = 0$. thus,
  - $H_0 : \beta_1 = 0$ and $H_\text{A} : \beta_1 < / > / \neq 0$
  - fun fact: test score is $t = \frac{\text{slope estimate}-\text{null for slope}}{\text{stderr of slope}} = \frac{b_1 - 0}{SE_\text{slope}}$
  - reqs: quant → quant, random, independence (no systemic deviation from line. check via look at scatterplot, or other tools), shape (large n), spread (a req that ANOVA also has) (check if vertical spread reasonably same at all X values)
  - nonlinear?
    - right skewed? try $x^{1/2}, x^{1/3}, \ldots, \ln(x)$. ln should be of strictly greater than 1.
    - left skewed? try $x^2, x^3, \ldots, e^x$