Assignment 1

Python Basics: Operators & Mathematics

Department of Mathematics & Computer Science

February 2, 2026

★ General Instructions

- **Total Marks:** 200 (Part A: 100 marks, Part B: 100 marks)
- **Time Allocation:** 180 minutes (3 hours)
- Show **all intermediate steps** in your calculations
- Round final answers to **4 decimal places** where applicable
- Use a calculator or computational tool to **verify your answers**
- Clearly indicate the **order of operations** used
- Write answers in the designated answer spaces
- **Academic Integrity:** This is an individual assignment

⋆ Grading Rubric (Per Question)

Part A - Mathematical Expressions:

- Correct final answer: **5 marks**
- Correct intermediate steps: **3 marks**
- Proper notation: **2 marks**

Part B - Python Operations:

- Correct final answer: **5 marks**
- Binary representations: **3 marks**
- Step-by-step working: **2 marks**

PART A: Advanced Mathematical Expressions (100 marks)

Evaluate the following mathematical expressions using the given values. Show all working and apply proper mathematical notation.

Question 1 (10 marks) - Given $x = 3$

Evaluate:

$$f(x) = \pi x^3 + \log(x) + \sqrt{x^3} + \tan(x) + e^x$$

• Hint

Calculate each term separately: πx^3, natural logarithm $\ln(x)$, cube root component, tangent in radians, and exponential. Then sum all terms.

Answer: _____

Question 2 (10 marks) - Given $x = 2$

Evaluate:

$$g(x) = \frac{e^{2x} + \cosh(x)}{\sinh(x) + \log_{10}(x)} + \sqrt[3]{x^5} - \tan^{-1}(x)$$

• Hint

Remember that $\cosh(x) = \frac{e^x + e^{-x}}{2}$ and $\sinh(x) = \frac{e^x - e^{-x}}{2}$. Use common logarithm (base 10) for $\log_{10}$.

Answer: _____

Question 3 (10 marks) - Given $x = \frac{\pi}{4}$

Evaluate:

$$h(x) = \sin(2x) \cdot \cos(x) + \frac{\tan(x)}{\cot(x)} + e^{\sin(x)} + \ln(\cos(x)) + x^{\pi}$$

• Hint

Note that $\frac{\tan(x)}{\cot(x)} = \tan^2(x)$. Calculate trigonometric functions first, then logarithm and exponential.

Answer: _____

Question 4 (10 marks) - Given $x = 1.5$

Evaluate:

$$p(x) = \sqrt{e^x + \sinh(2x)} + \frac{\ln(x^2 + 1)}{\tanh(x)} + x^x - \cos^{-1}\left(\frac{x}{2}\right) + \cosh^2(x)$$

• Hint

$\tanh(x) = \frac{\sinh(x)}{\cosh(x)}$. Be careful with inverse cosine domain. Calculate x^x as $e^{x\ln(x)}$.

Answer: _____

Question 5 (10 marks) - Given $x = 0.5$

Evaluate:

$$q(x) = \frac{\pi^x \cdot e^{2x}}{\sqrt{\sinh(x) + \cosh(x)}} + \log_2(x+1) \cdot \tan\left(\frac{\pi x}{6}\right) + \sqrt[4]{x^7} - \sin^{-1}(x)$$

• Hint

$\sinh(x) + \cosh(x) = e^x$. Use change of base formula: $\log_2(y) = \frac{\ln(y)}{\ln(2)}$.

Answer: _____

Question 6 (10 marks) - Given $x = 2.5$

Evaluate:

$$r(x) = \left(e^{x} + e^{-x}\right)^{2} - 4\sinh^{2}(x) + \frac{\ln(x!)}{\sqrt{x}} + \cos(x) \cdot \sec(x) + \tanh^{-1}\left(\frac{x-2}{x}\right)$$

• Hint

Use Sterling's approximation: $\ln(n!) \approx n\ln(n) - n$ for non-integer x. Note: $\cos(x) \cdot \sec(x) = 1$. Domain check for $\tanh^{-1}$ is important.

Answer: _____

Question 7 (10 marks) - Given $x = \frac{\pi}{6}$

Evaluate:

$$s(x) = \frac{\sin^{3}(x) + \cos^{3}(x)}{\sin(x) + \cos(x)} + e^{\tan(x)} - \ln\left(\frac{1}{\csc(x)}\right) + \sinh(x) \cdot \operatorname{csch}(x) + \sqrt{\pi x^{2}}$$

• Hint

Use algebraic identity: $\frac{a^3+b^3}{a+b} = a^2 - ab + b^2$. Note: $\operatorname{csch}(x) = \frac{1}{\sinh(x)}$ and $\csc(x) = \frac{1}{\sin(x)}$.

Answer: _____

Question 8 (10 marks) - Given $x = 1$

Evaluate:

$$t(x) = \sqrt[3]{e^{3x} + \pi^{x}} + \frac{\cosh(2x) - 1}{\sinh^{2}(x)} + \ln\left(\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right) + x^{\pi} - \cos^{-1}\left(\frac{1}{e}\right)$$

• Hint

Use identity: $\cosh(2x) - 1 = 2\sinh^{2}(x)$. Therefore, the fraction simplifies to 2.

Answer: _____

Question 9 (10 marks) - Given $x = 0.8$

Evaluate:

$$u(x) = e^{x^{2}} \cdot \sin(\pi x) + \frac{\ln(1 + x^{2})}{\tanh(2x)} + \sqrt{\cosh^{2}(x) - \sinh^{2}(x)} + \tan^{-1}(e^{x}) - \frac{\pi^{2}x}{e^{x}}$$

• **Hint**

Important identity: $\cosh^2(x) - \sinh^2(x) = 1$. Calculate exponential and trigonometric terms carefully.

Answer: _____

Question 10 (10 marks) - Given $x = \frac{\pi}{3}$

Evaluate:

$$v(x) = \frac{e^{\sin(x)} + e^{\cos(x)}}{2} + \ln(\sin(x) + \cos(x)) + \sqrt{\tan^2(x) + 1} + \sinh^{-1}(x) - \cosh\left(\frac{x}{2}\right) + \frac{\pi x^2}{e} + \sec^2(x) - \tan^2(x)$$

• Hint

Use identity: $\sec^2(x) - \tan^2(x) = 1$. Also, $\sqrt{\tan^2(x) + 1} = \sec(x)$ for x in first quadrant. $\sinh^{-1}(x) = \ln(x + \sqrt{x^2 + 1})$.

Answer: _____

♣ Formulas Reference - Part A

Hyperbolic Functions:

$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$
$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$
$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)}$$

Hyperbolic Identities:

$$\cosh^2(x) - \sinh^2(x) = 1$$
$$\cosh(2x) - 1 = 2\sinh^2(x)$$
$$\sinh(x) + \cosh(x) = e^x$$

Trigonometric Identities:

$$\sin^2(x) + \cos^2(x) = 1$$
$$\sec^2(x) - \tan^2(x) = 1$$
$$\tan^2(x) + 1 = \sec^2(x)$$
$$\sin(2x) = 2\sin(x)\cos(x)$$

Logarithm Properties:

$$\ln(ab) = \ln(a) + \ln(b)$$
$$\ln(a^n) = n\ln(a)$$
$$\log_b(x) = \frac{\ln(x)}{\ln(b)}$$

PART B: Bitwise, Arithmetic & Logical Expressions (100 marks)

Evaluate the following Python expressions using proper operator precedence. Show all intermediate steps and binary representations where relevant.

▷ Python Operator Precedence (Highest to Lowest)

1. Parentheses `()`
2. Exponentiation `**`
3. Unary `+`, `-`, `~`
4. Multiplication `*`, Division `/`, Floor division `//`, Modulo `%`
5. Addition `+`, Subtraction `-`
6. Bitwise shifts `<<`, `>>`
7. Bitwise AND `&`
8. Bitwise XOR `^`
9. Bitwise OR `|`
10. Comparisons `<`, `<=`, `>`, `>=`, `!=`, `==`
11. Logical NOT `not`
12. Logical AND `and`
13. Logical OR `or`

Question 1 (10 marks)

Evaluate the following expression:

```
(7 ^ 3) * (8 >> 2) + 9 % 4
```

• Hint

Remember that `^` is XOR, `>>` is right shift, and `%` is modulo. Follow operator precedence: bitwise operations before arithmetic.

Answer: _____

Question 2 (10 marks)

Evaluate the following expression:

```
(14 >> 1) + (9 % 5) * (6 ^ 2)
```

• Hint

Right shift by 1 is equivalent to integer division by 2. XOR operates on binary representations.

Answer: _____

Question 3 (10 marks)

Evaluate the following expression:

```
(5**3 // 4 % 7 << 2) + (18 >> 2) * (9 ^ 6 % 5)
```

• Hint

Evaluate exponentiation first, then floor division, modulo, and finally bitwise shifts. Left shift by 2 multiplies by 4.

Answer: _____

Question 4 (10 marks)

Evaluate the following expression:

```
(27 % 5 * 3**2 // 4) ^ (48 >> 3) + (14 & 9 << 1)
```

• Hint

`&` is bitwise AND. Be careful with the precedence of shift operators versus arithmetic operators.

Answer: _____

Question 5 (10 marks)

Evaluate the following expression:

```
((19 | 6) ^ (28 & 14)) * (3 << 2) % (5**2 // 3)
```

• Hint

`|` is bitwise OR. Evaluate parentheses first, then shifts, then arithmetic operations.

Answer: _____

Question 6 (10 marks)

Evaluate the following expression:

```
(45 & 27) << 2 // (9 % 4 + 2**3) + (16 ^ (5 << 1))
```

• Hint

Floor division has the same precedence as multiplication. Work through nested parentheses carefully.

Answer: _____

Question 7 (10 marks)

Evaluate the following logical expression:

```
((15**2 % 7 + 48 >> 2) ^ (19 & 13 | 6)) * (7 << 3 // 5)
+ ((42 % 5 * 3**3 // 4) > (18 >> 1))
and not ((9 | 4) ^ (12 & 7) < (3**2 + 5))
or (27 // 4 % 3 == 2)
```

• Hint

Evaluate all arithmetic and bitwise operations first, then comparisons, then logical operators (`not`, `and`, `or`).

Answer: _____

Question 8 (10 marks)

Evaluate the following expression:

```
((63 & 45) << 1 | (72 >> 3)) ^ ((11 % 3 * 2**4) // (6 + 1))
+ (((8 | 3) ^ 5) * (16 >> 2) % 7) - ((25 % 6) << 2)
```

• Hint

Break down into major sub-expressions. Work from innermost parentheses outward. Remember that `<<` and `>>` have lower precedence than arithmetic operators.

Answer: _____

Question 9 (10 marks)

Evaluate the following expression:

```
(((128 >> 3) & (45 | 18)) ^ (7**2 % 10)) * ((36 // 5) << 1)
- ((91 % 7 * 2**3) ^ (54 & 23)) + ((15 | 6) >> 1) % 5
```

• Hint

Pay attention to the order: exponentiation → multiplication/division/modulo → addition/subtraction → shifts → bitwise AND → bitwise XOR → bitwise OR.

Answer: _____

Question 10 (10 marks)

Evaluate the following complex logical expression:

```
(((17**2 % 13) << 2) ^ ((64 >> 2) | (9 & 5))) > ((45 % 7 * 3**2) //
    6)
and not (((28 | 12) ^ (35 & 19)) < (2**5 + 7 % 3))
or ((((81 // 9) << 1) % 5) == 3 and ((56 >> 3) & (15 | 4)) != 7)
and not ((99 % 8 * 2**3 // 5) ^ (42 & 27) > 25)
```

• Hint

This is a compound logical expression. First evaluate all arithmetic and bitwise operations, then all comparison operators (`>`, `<`, `==`, `!=`), and finally logical operators in order: `not`, `and`, `or` (left to right for same precedence).

Answer: _____

♠ Bitwise Operations Quick Reference

Bitwise Operators:

- `&` - AND: both bits must be 1
- `|` - OR: at least one bit is 1
- `^` - XOR: bits are different
- `~` - NOT: flip all bits
- `<<` - Left shift: multiply by 2^n
- `>>` - Right shift: divide by 2^n

Common Conversions:

- $7_{10} = 0111_2$
- $8_{10} = 1000_2$
- $14_{10} = 1110_2$
- $9_{10} = 1001_2$
- $15_{10} = 1111_2$
- $16_{10} = 10000_2$

Verification Using Python:

```
# Example verification
result = (7 ^ 3) * (8 >> 2) + 9 % 4
print(f"Result: {result}")
print(f"Binary: {bin(result)}")
```

♢ Submission Information

- Submit your answers in a **clear, organized format**
- Show your **step-by-step working** for each question
- Include **binary representations** where relevant (Part B)
- Double-check your **operator precedence**
- Use Python to **verify your final answers**
- Ensure all answers are written in the designated spaces

Student Name: _____ **Student ID:** _____

Submission Date: _____ **Instructor Signature:** _____

✓ Good Luck!

Remember to manage your time wisely and verify all calculations.