Q1 c)

```
<< Notation`

Symbolize[ x_{t+Δt} ]

Symbolize[ ẋ_{t+Δt} ]

Symbolize[ ẍ_{t+Δt} ]

Symbolize[ x_{t+γΔt} ]

Symbolize[ ẋ_{t+γΔt} ]

Symbolize[ ẍ_{t+γΔt} ]

Symbolize[ x_t ]

Symbolize[ ẋ_t ]

Symbolize[ ẍ_t ]

Symbolize[ r_{t+Δt} ]

Symbolize[ r_{t+γΔt} ]

Symbolize[ Ω_o ]

Symbolize[ Ω̄_d ]

Symbolize[ ξ̄ ]

Symbolize[ β_1 ]

Symbolize[ β_2 ]

Symbolize[ X_t ]

ClearAll["Global`*"]

(*
Writing in the modal form
*)

ξ = 0;
```

```
Solve[
  ẍ_{t+γΔt} + 2 ξ ω ẋ_{t+γΔt} + ω² x_{t+γΔt} == r_{t+γΔt} &&
    x_{t+γΔt} == x_t + (γ Δt)/2 (ẋ_t + ẋ_{t+γΔt}) &&
    ẋ_{t+γΔt} == ẋ_t + (γ Δt)/2 (ẍ_t + ẍ_{t+γΔt}),
  {ẍ_{t+γΔt}, ẋ_{t+γΔt}, x_{t+γΔt}}]
```

$$\{\{x_{t+\gamma\Delta t} \to - ((-4\, r_{t+\gamma\Delta t} + 4\, x_t\, \omega^2 + 4\, \dot{x}_t\, \gamma\, \Delta t\, \omega^2 + x_t\, \gamma^2\, \Delta t^2\, \omega^2) / (4 + \gamma^2\, \Delta t^2\, \omega^2)),$$

$$\dot{x}_{t+\gamma\Delta t} \to - ((-4\, \dot{x}_t - 2\, r_{t+\gamma\Delta t}\, \gamma\, \Delta t - 2\, x_t\, \gamma\, \Delta t + 2\, x_t\, \gamma\, \Delta t\, \omega^2 + \dot{x}_t\, \gamma^2\, \Delta t^2\, \omega^2) / (4 + \gamma^2\, \Delta t^2\, \omega^2)),$$

$$x_{t+\gamma\Delta t} \to - ((-4\, x_t - 4\, \dot{x}_t\, \gamma\, \Delta t - r_{t+\gamma\Delta t}\, \gamma^2\, \Delta t^2 - x_t\, \gamma^2\, \Delta t^2) / (4 + \gamma^2\, \Delta t^2\, \omega^2))\}\}$$

```
ClearAll["Global`*"]
```

$$\ddot{x}_{t+\gamma\Delta t} = - ((-4\, r_{t+\gamma\Delta t} + 4\, x_t\, \omega^2 + 4\, \dot{x}_t\, \gamma\, \Delta t\, \omega^2 + \ddot{x}_t\, \gamma^2\, \Delta t^2\, \omega^2) / (4 + \gamma^2\, \Delta t^2\, \omega^2))$$

$$- ((-4\, r_{t+\gamma\Delta t} + 4\, x_t\, \omega^2 + 4\, \dot{x}_t\, \gamma\, \Delta t\, \omega^2 + x_t\, \gamma^2\, \Delta t^2\, \omega^2) / (4 + \gamma^2\, \Delta t^2\, \omega^2))$$

$$\dot{x}_{t+\gamma\Delta t} = - ((-4\, \dot{x}_t - 2\, r_{t+\gamma\Delta t}\, \gamma\, \Delta t - 2\, \ddot{x}_t\, \gamma\, \Delta t + 2\, x_t\, \gamma\, \Delta t\, \omega^2 + \dot{x}_t\, \gamma^2\, \Delta t^2\, \omega^2) / (4 + \gamma^2\, \Delta t^2\, \omega^2))$$

$$- ((-4\, \dot{x}_t - 2\, r_{t+\gamma\Delta t}\, \gamma\, \Delta t - 2\, x_t\, \gamma\, \Delta t + 2\, x_t\, \gamma\, \Delta t\, \omega^2 + \dot{x}_t\, \gamma^2\, \Delta t^2\, \omega^2) / (4 + \gamma^2\, \Delta t^2\, \omega^2))$$

$$x_{t+\gamma\Delta t} = - \frac{-4\, x_t - 4\, \dot{x}_t\, \gamma\, \Delta t - r_{t+\gamma\Delta t}\, \gamma^2\, \Delta t^2 - \ddot{x}_t\, \gamma^2\, \Delta t^2}{4 + \gamma^2\, \Delta t^2\, \omega^2}$$

$$- \frac{-4\, x_t - 4\, \dot{x}_t\, \gamma\, \Delta t - r_{t+\gamma\Delta t}\, \gamma^2\, \Delta t^2 - x_t\, \gamma^2\, \Delta t^2}{4 + \gamma^2\, \Delta t^2\, \omega^2}$$

```
Solve[ ẍ_{t+Δt} + 2 ξ ω ẋ_{t+Δt} + ω² x_{t+Δt} == r_{t+Δt} &&
   x_{t+Δt} == x_t + γ Δt ((1 - β₁) ẋ_t + β₁ ẋ_{t+γΔt}) + (1 - γ) Δt ((1 - β₂) ẋ_{t+γΔt} + β₂ ẋ_{t+Δt}) &&
   ẋ_{t+Δt} == ẋ_t + γ Δt ((1 - β₁) ẍ_t + β₁ ẍ_{t+γΔt}) + (1 - γ) Δt ((1 - β₂) ẍ_{t+γΔt} + β₂ ẍ_{t+Δt}),
 {ẍ_{t+Δt}, ẋ_{t+Δt}, x_{t+Δt}}]
```

$$
\left\{\left\{\ddot{x}_{t+\Delta t} \to \right.\right.
$$

$$
-\frac{1}{-1 - \beta_2^2 (1-\gamma)^2 \Delta t^2 \omega^2} \left(r_{t+\Delta t} + \beta_2 (1-\gamma) \Delta t \omega^2 \left(-\dot{x}_t + \left((1-\beta_2) (1-\gamma) \Delta t \left(-4 r_{t+\gamma\Delta t} + 4 x_t \right.\right.\right.\right.
$$

$$
\left.\omega^2 + 4 \dot{x}_t \gamma \Delta t \omega^2 + x_t \gamma^2 \Delta t^2 \omega^2 \right)\right) \Big/ \left(4 + \gamma^2 \Delta t^2 \omega^2\right) - \gamma \Delta t \left(x_t (1-\beta_1) - \right.
$$

$$
\left.\beta_1 \left(-4 r_{t+\gamma\Delta t} + 4 x_t \omega^2 + 4 \dot{x}_t \gamma \Delta t \omega^2 + x_t \gamma^2 \Delta t^2 \omega^2\right)\right) \Big/ \left(4 + \gamma^2 \Delta t^2 \omega^2\right)\right) +
$$

$$
\omega^2 \left(-x_t + \left((1-\beta_2) (1-\gamma) \Delta t \left(-4 \dot{x}_t - 2 r_{t+\gamma\Delta t} \gamma \Delta t - 2 x_t \gamma \Delta t + 2 x_t \gamma \Delta t \omega^2 + \dot{x}_t \right.\right.\right.
$$

$$
\left.\gamma^2 \Delta t^2 \omega^2 \right)\right) \Big/ \left(4 + \gamma^2 \Delta t^2 \omega^2\right) - \gamma \Delta t \left(\dot{x}_t (1-\beta_1) - \left(\beta_1 \left(-4 \dot{x}_t - 2 r_{t+\gamma\Delta t} \gamma \Delta t - \right.\right.\right.
$$

$$
\left.\left.\left.\left.\left. 2 x_t \gamma \Delta t + 2 x_t \gamma \Delta t \omega^2 + \dot{x}_t \gamma^2 \Delta t^2 \omega^2\right)\right) \Big/ \left(4 + \gamma^2 \Delta t^2 \omega^2\right)\right)\right)\right),
$$

$$
\dot{x}_{t+\Delta t} \to -\Big(\Big(-4 \dot{x}_t - 4 r_{t+\gamma\Delta t} \Delta t + 4 r_{t+\gamma\Delta t} \beta_2 \Delta t - 4 r_{t+\Delta t} \beta_2 \Delta t + 4 r_{t+\gamma\Delta t} \gamma \Delta t -
$$

$$
4 x_t \gamma \Delta t - 4 r_{t+\gamma\Delta t} \beta_1 \gamma \Delta t + 4 x_t \beta_1 \gamma \Delta t - 4 r_{t+\gamma\Delta t} \beta_2 \gamma \Delta t + 4 r_{t+\Delta t} \beta_2 \gamma \Delta t +
$$

$$
4 x_t \Delta t \omega^2 - 4 x_t \gamma \Delta t \omega^2 + 4 x_t \beta_1 \gamma \Delta t \omega^2 + 4 \dot{x}_t \beta_2 \Delta t^2 \omega^2 - 4 \dot{x}_t \beta_2^2 \Delta t^2 \omega^2 +
$$

$$
4 \dot{x}_t \gamma \Delta t^2 \omega^2 - 8 \dot{x}_t \beta_2 \gamma \Delta t^2 \omega^2 + 8 \dot{x}_t \beta_2^2 \gamma \Delta t^2 \omega^2 - 5 \dot{x}_t \gamma^2 \Delta t^2 \omega^2 +
$$

$$
4 \dot{x}_t \beta_1 \gamma^2 \Delta t^2 \omega^2 + 4 \dot{x}_t \beta_2 \gamma^2 \Delta t^2 \omega^2 - 4 \dot{x}_t \beta_2^2 \gamma^2 \Delta t^2 \omega^2 + 2 r_{t+\gamma\Delta t} \beta_2 \gamma \Delta t^3 \omega^2 +
$$

$$
2 x_t \beta_2 \gamma \Delta t^3 \omega^2 - 2 r_{t+\gamma\Delta t} \beta_2^2 \gamma \Delta t^3 \omega^2 - 2 x_t \beta_2^2 \gamma \Delta t^3 \omega^2 + x_t \gamma^2 \Delta t^3 \omega^2 -
$$

$$
4 r_{t+\gamma\Delta t} \beta_2 \gamma^2 \Delta t^3 \omega^2 - r_{t+\Delta t} \beta_2 \gamma^2 \Delta t^3 \omega^2 - 5 x_t \beta_2 \gamma^2 \Delta t^3 \omega^2 +
$$

$$
2 r_{t+\gamma\Delta t} \beta_1 \beta_2 \gamma^2 \Delta t^3 \omega^2 + 2 x_t \beta_1 \beta_2 \gamma^2 \Delta t^3 \omega^2 + 4 r_{t+\gamma\Delta t} \beta_2^2 \gamma^2 \Delta t^3 \omega^2 +
$$

$$
4 x_t \beta_2^2 \gamma^2 \Delta t^3 \omega^2 - 2 x_t \gamma^3 \Delta t^3 \omega^2 + 2 x_t \beta_1 \gamma^3 \Delta t^3 \omega^2 + 2 r_{t+\gamma\Delta t} \beta_2 \gamma^3 \Delta t^3 \omega^2 +
$$

$$
r_{t+\Delta t} \beta_2 \gamma^3 \Delta t^3 \omega^2 + 3 x_t \beta_2 \gamma^3 \Delta t^3 \omega^2 - 2 r_{t+\gamma\Delta t} \beta_1 \beta_2 \gamma^3 \Delta t^3 \omega^2 -
$$

$$
2 x_t \beta_1 \beta_2 \gamma^3 \Delta t^3 \omega^2 - 2 r_{t+\gamma\Delta t} \beta_2^2 \gamma^3 \Delta t^3 \omega^2 - 2 x_t \beta_2^2 \gamma^3 \Delta t^3 \omega^2 -
$$

$$
2 x_t \beta_2 \gamma \Delta t^3 \omega^4 + 2 x_t \beta_2^2 \gamma \Delta t^3 \omega^4 + 5 x_t \beta_2 \gamma^2 \Delta t^3 \omega^4 - 2 x_t \beta_1 \beta_2 \gamma^2 \Delta t^3 \omega^4 -
$$

$$
4 x_t \beta_2^2 \gamma^2 \Delta t^3 \omega^4 - 3 x_t \beta_2 \gamma^3 \Delta t^3 \omega^4 + 2 x_t \beta_1 \beta_2 \gamma^3 \Delta t^3 \omega^4 + 2 x_t \beta_2^2 \gamma^3 \Delta t^3 \omega^4 -
$$

$$
\dot{x}_t \beta_2 \gamma^2 \Delta t^4 \omega^4 + \dot{x}_t \beta_2^2 \gamma^2 \Delta t^4 \omega^4 + 3 \dot{x}_t \beta_2 \gamma^3 \Delta t^4 \omega^4 - 2 \dot{x}_t \beta_1 \beta_2 \gamma^3 \Delta t^4 \omega^4 -
$$

$$
2 \dot{x}_t \beta_2^2 \gamma^3 \Delta t^4 \omega^4 - 2 \dot{x}_t \beta_2 \gamma^4 \Delta t^4 \omega^4 + 2 \dot{x}_t \beta_1 \beta_2 \gamma^4 \Delta t^4 \omega^4 + \dot{x}_t \beta_2^2 \gamma^4 \Delta t^4 \omega^4\Big) \Big/
$$

$$
\left(\left(4 + \gamma^2 \Delta t^2 \omega^2\right) \left(1 + \beta_2^2 \Delta t^2 \omega^2 - 2 \beta_2^2 \gamma \Delta t^2 \omega^2 + \beta_2^2 \gamma^2 \Delta t^2 \omega^2\right)\right)\Big),
$$

$$
x_{t+\Delta t} \to -\Big(\Big(-4 x_t - 4 \dot{x}_t \Delta t - 4 r_{t+\gamma\Delta t} \beta_2 \Delta t^2 + 4 r_{t+\gamma\Delta t} \beta_2^2 \Delta t^2 - 4 r_{t+\Delta t} \beta_2^2 \Delta t^2 -
$$

$$
2 r_{t+\gamma\Delta t} \gamma \Delta t^2 - 2 x_t \gamma \Delta t^2 + 10 r_{t+\gamma\Delta t} \beta_2 \gamma \Delta t^2 - 2 x_t \beta_2 \gamma \Delta t^2 - 4 r_{t+\gamma\Delta t} \beta_1 \beta_2 \gamma \Delta t^2 +
$$

$$
4 x_t \beta_1 \beta_2 \gamma \Delta t^2 - 8 r_{t+\gamma\Delta t} \beta_2^2 \gamma \Delta t^2 + 8 r_{t+\Delta t} \beta_2^2 \gamma \Delta t^2 + 2 r_{t+\gamma\Delta t} \gamma^2 \Delta t^2 + 2 x_t \gamma^2 \Delta t^2 -
$$

$$
2 r_{t+\gamma\Delta t} \beta_1 \gamma^2 \Delta t^2 - 2 x_t \beta_1 \gamma^2 \Delta t^2 - 6 r_{t+\gamma\Delta t} \beta_2 \gamma^2 \Delta t^2 + 2 x_t \beta_2 \gamma^2 \Delta t^2 +
$$

$$
4 r_{t+\gamma\Delta t} \beta_1 \beta_2 \gamma^2 \Delta t^2 - 4 x_t \beta_1 \beta_2 \gamma^2 \Delta t^2 + 4 r_{t+\gamma\Delta t} \beta_2^2 \gamma^2 \Delta t^2 - 4 r_{t+\Delta t} \beta_2^2 \gamma^2 \Delta t^2 +
$$

$$
4 x_t \beta_2 \Delta t^2 \omega^2 - 4 x_t \beta_2^2 \Delta t^2 \omega^2 + 2 x_t \gamma \Delta t^2 \omega^2 - 10 x_t \beta_2 \gamma \Delta t^2 \omega^2 + 4 x_t \beta_1 \beta_2 \gamma \Delta t^2 \omega^2 +
$$

$$
8 x_t \beta_2^2 \gamma \Delta t^2 \omega^2 - 3 x_t \gamma^2 \Delta t^2 \omega^2 + 2 x_t \beta_1 \gamma^2 \Delta t^2 \omega^2 + 6 x_t \beta_2 \gamma^2 \Delta t^2 \omega^2 -
$$

$$
4 x_t \beta_1 \beta_2 \gamma^2 \Delta t^2 \omega^2 - 4 x_t \beta_2^2 \gamma^2 \Delta t^2 \omega^2 + 4 \dot{x}_t \beta_2 \gamma \Delta t^3 \omega^2 - 4 \dot{x}_t \beta_2^2 \gamma \Delta t^3 \omega^2 +
$$

$$
\dot{x}_t \gamma^2 \Delta t^3 \omega^2 - 10 \dot{x}_t \beta_2 \gamma^2 \Delta t^3 \omega^2 + 4 \dot{x}_t \beta_1 \beta_2 \gamma^2 \Delta t^3 \omega^2 + 8 \dot{x}_t \beta_2^2 \gamma^2 \Delta t^3 \omega^2 -
$$

$$
2 \dot{x}_t \gamma^3 \Delta t^3 \omega^2 + 2 \dot{x}_t \beta_1 \gamma^3 \Delta t^3 \omega^2 + 6 \dot{x}_t \beta_2 \gamma^3 \Delta t^3 \omega^2 - 4 \dot{x}_t \beta_1 \beta_2 \gamma^3 \Delta t^3 \omega^2 -
$$

$$
4 \dot{x}_t \beta_2^2 \gamma^3 \Delta t^3 \omega^2 + x_t \beta_2 \gamma^2 \Delta t^4 \omega^2 - r_{t+\Delta t} \beta_2^2 \gamma^2 \Delta t^4 \omega^2 - x_t \beta_2^2 \gamma^2 \Delta t^4 \omega^2 -
$$

$$
3 x_t \beta_2 \gamma^3 \Delta t^4 \omega^2 + 2 x_t \beta_1 \beta_2 \gamma^3 \Delta t^4 \omega^2 + 2 r_{t+\Delta t} \beta_2^2 \gamma^3 \Delta t^4 \omega^2 + 2 x_t \beta_2^2 \gamma^3 \Delta t^4 \omega^2 +
$$

$$
2 x_t \beta_2 \gamma^4 \Delta t^4 \omega^2 - 2 x_t \beta_1 \beta_2 \gamma^4 \Delta t^4 \omega^2 - r_{t+\Delta t} \beta_2^2 \gamma^4 \Delta t^4 \omega^2 - x_t \beta_2^2 \gamma^4 \Delta t^4 \omega^2\Big) \Big/
$$

$$
\left.\left.\left(\left(4 + \gamma^2 \Delta t^2 \omega^2\right) \left(1 + \beta_2^2 \Delta t^2 \omega^2 - 2 \beta_2^2 \gamma \Delta t^2 \omega^2 + \beta_2^2 \gamma^2 \Delta t^2 \omega^2\right)\right)\right)\right\}\right\}
$$

```
recursive =
 {
  Collect[ẍ_{t+Δt} -
      (- 1/(-1 - β_2^2 (1-γ)^2 Δt^2 ω^2) (r_{t+Δt} + β_2 (1-γ) Δt ω^2 (-ẋ_t + ((1-β_2) (1-γ) Δt (-4 r_{t+γΔt} +

            4 x_t ω^2 + 4 ẋ_t γ Δt ω^2 + ẍ_t γ^2 Δt^2 ω^2)) / (4 + γ^2 Δt^2 ω^2) - γ Δt (ẍ_t (1-β_1) -
            (β_1 (-4 r_{t+γΔt} + 4 x_t ω^2 + 4 ẋ_t γ Δt ω^2 + ẍ_t γ^2 Δt^2 ω^2)) / (4 + γ^2 Δt^2 ω^2)))) +
         ω^2 (-x_t + ((1-β_2) (1-γ) Δt (-4 ẋ_t - 2 r_{t+γΔt} γ Δt - 2 ẍ_t γ Δt +
               2 x_t γ Δt ω^2 + ẋ_t γ^2 Δt^2 ω^2)) / (4 + γ^2 Δt^2 ω^2) -
            γ Δt (ẋ_t (1-β_1) - (β_1 (-4 ẋ_t - 2 r_{t+γΔt} γ Δt - 2 ẍ_t γ Δt + 2 x_t γ Δt ω^2 +
                  ẋ_t γ^2 Δt^2 ω^2)) / (4 + γ^2 Δt^2 ω^2))))), {ẍ_t, ẋ_t, x_t, r_{t+Δt}, r_{t+γΔt}}],

  Collect[ẋ_{t+Δt} - (- (((-4 ẋ_t - 4 r_{t+γΔt} Δt + 4 r_{t+γΔt} β_2 Δt - 4 r_{t+Δt} β_2 Δt + 4 r_{t+γΔt} γ Δt -
         4 ẍ_t γ Δt - 4 r_{t+γΔt} β_1 γ Δt + 4 ẍ_t β_1 γ Δt - 4 r_{t+γΔt} β_2 γ Δt + 4 r_{t+Δt} β_2 γ Δt +
         4 x_t Δt ω^2 - 4 x_t γ Δt ω^2 + 4 x_t β_1 γ Δt ω^2 + 4 ẋ_t β_2 Δt^2 ω^2 - 4 ẋ_t β_2^2 Δt^2 ω^2 +
         4 ẋ_t γ Δt^2 ω^2 - 8 ẋ_t β_2 γ Δt^2 ω^2 + 8 ẋ_t β_2^2 γ Δt^2 ω^2 - 5 ẋ_t γ^2 Δt^2 ω^2 + 4 ẋ_t β_1 γ^2 Δt^2 ω^2 +
         4 ẋ_t β_2 γ^2 Δt^2 ω^2 - 4 ẋ_t β_2^2 γ^2 Δt^2 ω^2 + 2 r_{t+γΔt} β_2 γ Δt^3 ω^2 + 2 ẍ_t β_2 γ Δt^3 ω^2 -
         2 r_{t+γΔt} β_2^2 γ Δt^3 ω^2 - 2 ẍ_t β_2^2 γ Δt^3 ω^2 + ẍ_t γ^2 Δt^3 ω^2 - 4 r_{t+γΔt} β_2 γ^2 Δt^3 ω^2 -
         r_{t+Δt} β_2 γ^2 Δt^3 ω^2 - 5 ẍ_t β_2 γ^2 Δt^3 ω^2 + 2 r_{t+γΔt} β_1 β_2 γ^2 Δt^3 ω^2 + 2 ẍ_t β_1 β_2 γ^2 Δt^3 ω^2 +
         4 r_{t+γΔt} β_2^2 γ^2 Δt^3 ω^2 + 4 ẍ_t β_2^2 γ^2 Δt^3 ω^2 - 2 ẍ_t γ^3 Δt^3 ω^2 + 2 ẍ_t β_1 γ^3 Δt^3 ω^2 +
         2 r_{t+γΔt} β_2 γ^3 Δt^3 ω^2 + r_{t+Δt} β_2 γ^3 Δt^3 ω^2 + 3 ẍ_t β_2 γ^3 Δt^3 ω^2 - 2 r_{t+γΔt} β_1 β_2 γ^3 Δt^3 ω^2 -
         2 ẍ_t β_1 β_2 γ^3 Δt^3 ω^2 - 2 r_{t+γΔt} β_2^2 γ^3 Δt^3 ω^2 - 2 ẍ_t β_2^2 γ^3 Δt^3 ω^2 -
         2 x_t β_2 γ Δt^3 ω^4 + 2 x_t β_2^2 γ Δt^3 ω^4 + 5 x_t β_2 γ^2 Δt^3 ω^4 - 2 x_t β_1 β_2 γ^2 Δt^3 ω^4 -
         4 x_t β_2^2 γ^2 Δt^3 ω^4 - 3 x_t β_2 γ^3 Δt^3 ω^4 + 2 x_t β_1 β_2 γ^3 Δt^3 ω^4 + 2 x_t β_2^2 γ^3 Δt^3 ω^4 -
         ẋ_t β_2 γ^2 Δt^4 ω^4 + ẋ_t β_2^2 γ^2 Δt^4 ω^4 + 3 ẋ_t β_2 γ^3 Δt^4 ω^4 - 2 ẋ_t β_1 β_2 γ^3 Δt^4 ω^4 -
         2 ẋ_t β_2^2 γ^3 Δt^4 ω^4 - 2 ẋ_t β_2 γ^4 Δt^4 ω^4 + 2 ẋ_t β_1 β_2 γ^4 Δt^4 ω^4 + ẋ_t β_2^2 γ^4 Δt^4 ω^4) /
        ((4 + γ^2 Δt^2 ω^2) (1 + β_2^2 Δt^2 ω^2 - 2 β_2^2 γ Δt^2 ω^2 + β_2^2 γ^2 Δt^2 ω^2)))),
     {ẍ_t, ẋ_t, x_t, r_{t+Δt}, r_{t+γΔt}}],
  Collect[x_{t+Δt} - (- (((-4 x_t - 4 ẋ_t Δt - 4 r_{t+γΔt} β_2 Δt^2 + 4 r_{t+γΔt} β_2^2 Δt^2 - 4 r_{t+Δt} β_2^2 Δt^2 -
         2 r_{t+γΔt} γ Δt^2 - 2 ẍ_t γ Δt^2 + 10 r_{t+γΔt} β_2 γ Δt^2 - 2 ẍ_t β_2 γ Δt^2 - 4 r_{t+γΔt} β_1 β_2 γ Δt^2 +
         4 ẍ_t β_1 β_2 γ Δt^2 - 8 r_{t+γΔt} β_2^2 γ Δt^2 + 8 r_{t+Δt} β_2^2 γ Δt^2 + 2 r_{t+γΔt} γ^2 Δt^2 + 2 ẍ_t γ^2 Δt^2 -
         2 r_{t+γΔt} β_1 γ^2 Δt^2 - 2 ẍ_t β_1 γ^2 Δt^2 - 6 r_{t+γΔt} β_2 γ^2 Δt^2 + 2 ẍ_t β_2 γ^2 Δt^2 +
         4 r_{t+γΔt} β_1 β_2 γ^2 Δt^2 - 4 ẍ_t β_1 β_2 γ^2 Δt^2 + 4 r_{t+γΔt} β_2^2 γ^2 Δt^2 - 4 r_{t+Δt} β_2^2 γ^2 Δt^2 +
         4 x_t β_2 Δt^2 ω^2 - 4 x_t β_2^2 Δt^2 ω^2 + 2 x_t γ Δt^2 ω^2 - 10 x_t β_2 γ Δt^2 ω^2 + 4 x_t β_1 β_2 γ Δt^2 ω^2 +
         8 x_t β_2^2 γ Δt^2 ω^2 - 3 x_t γ^2 Δt^2 ω^2 + 2 x_t β_1 γ^2 Δt^2 ω^2 + 6 x_t β_2 γ^2 Δt^2 ω^2 -
         4 x_t β_1 β_2 γ^2 Δt^2 ω^2 - 4 x_t β_2^2 γ^2 Δt^2 ω^2 + 4 ẋ_t β_2 γ Δt^3 ω^2 - 4 ẋ_t β_2^2 γ Δt^3 ω^2 +
         ẋ_t γ^2 Δt^3 ω^2 - 10 ẋ_t β_2 γ^2 Δt^3 ω^2 + 4 ẋ_t β_1 β_2 γ^2 Δt^3 ω^2 + 8 ẋ_t β_2^2 γ^2 Δt^3 ω^2 -
         2 ẋ_t γ^3 Δt^3 ω^2 + 2 ẋ_t β_1 γ^3 Δt^3 ω^2 + 6 ẋ_t β_2 γ^3 Δt^3 ω^2 - 4 ẋ_t β_1 β_2 γ^3 Δt^3 ω^2 -
         4 ẋ_t β_2^2 γ^3 Δt^3 ω^2 + ẍ_t β_2 γ^2 Δt^4 ω^2 - r_{t+Δt} β_2^2 γ^2 Δt^4 ω^2 - ẍ_t β_2^2 γ^2 Δt^4 ω^2 -
         3 ẍ_t β_2 γ^3 Δt^4 ω^2 + 2 ẍ_t β_1 β_2 γ^3 Δt^4 ω^2 + 2 r_{t+Δt} β_2^2 γ^3 Δt^4 ω^2 + 2 ẍ_t β_2^2 γ^3 Δt^4 ω^2 +
         2 ẍ_t β_2 γ^4 Δt^4 ω^2 - 2 ẍ_t β_1 β_2 γ^4 Δt^4 ω^2 - r_{t+Δt} β_2^2 γ^4 Δt^4 ω^2 - ẍ_t β_2^2 γ^4 Δt^4 ω^2) /
        ((4 + γ^2 Δt^2 ω^2) (1 + β_2^2 Δt^2 ω^2 - 2 β_2^2 γ Δt^2 ω^2 + β_2^2 γ^2 Δt^2 ω^2)))),
     {ẍ_t, ẋ_t, x_t, r_{t+γΔt}, r_{t+Δt}}]
 } (*/. {-1 - β_2^2 (1-γ)^2 Δt^2 ω^2 → η_1   , 4 + γ^2 Δt^2 ω^2 → η_2,
    1 + β_2^2 Δt^2 ω^2 - 2 β_2^2 γ Δt^2 ω^2 + β_2^2 γ^2 Δt^2 ω^2 → η_3}*)
```

$$
\Big\{ \mathtt{x}_{t+\Delta t} + \frac{\mathtt{r}_{t+\Delta t}}{-1 - \beta_2^2 \ (1-\gamma)^2 \ \Delta t^2 \ \omega^2} +
$$

$$
\mathtt{r}_{t+\gamma\Delta t} \left(- \left(\left(4 \ (1-\beta_2) \ \beta_2 \ (1-\gamma)^2 \ \Delta t^2 \ \omega^2 \right) \Big/ \left(\left(-1 - \beta_2^2 \ (1-\gamma)^2 \ \Delta t^2 \ \omega^2 \right) \left(4 + \gamma^2 \ \Delta t^2 \ \omega^2 \right) \right) \right) -
$$

$$
\frac{2 \ (1-\beta_2) \ (1-\gamma) \ \gamma \ \Delta t^2 \ \omega^2}{\left(-1 - \beta_2^2 \ (1-\gamma)^2 \ \Delta t^2 \ \omega^2 \right) \left(4 + \gamma^2 \ \Delta t^2 \ \omega^2 \right)} - \frac{4 \ \beta_1 \ \beta_2 \ (1-\gamma) \ \gamma \ \Delta t^2 \ \omega^2}{\left(-1 - \beta_2^2 \ (1-\gamma)^2 \ \Delta t^2 \ \omega^2 \right) \left(4 + \gamma^2 \ \Delta t^2 \ \omega^2 \right)} -
$$

$$
\frac{2 \ \beta_1 \ \gamma^2 \ \Delta t^2 \ \omega^2}{\left(-1 - \beta_2^2 \ (1-\gamma)^2 \ \Delta t^2 \ \omega^2 \right) \left(4 + \gamma^2 \ \Delta t^2 \ \omega^2 \right)} \right) +
$$

$$
\mathtt{x}_t \left(- \frac{\omega^2}{-1 - \beta_2^2 \ (1-\gamma)^2 \ \Delta t^2 \ \omega^2} + \frac{4 \ (1-\beta_2) \ \beta_2 \ (1-\gamma)^2 \ \Delta t^2 \ \omega^4}{\left(-1 - \beta_2^2 \ (1-\gamma)^2 \ \Delta t^2 \ \omega^2 \right) \left(4 + \gamma^2 \ \Delta t^2 \ \omega^2 \right)} +
$$

$$
\frac{2 \ (1-\beta_2) \ (1-\gamma) \ \gamma \ \Delta t^2 \ \omega^4}{\left(-1 - \beta_2^2 \ (1-\gamma)^2 \ \Delta t^2 \ \omega^2 \right) \left(4 + \gamma^2 \ \Delta t^2 \ \omega^2 \right)} + \frac{4 \ \beta_1 \ \beta_2 \ (1-\gamma) \ \gamma \ \Delta t^2 \ \omega^4}{\left(-1 - \beta_2^2 \ (1-\gamma)^2 \ \Delta t^2 \ \omega^2 \right) \left(4 + \gamma^2 \ \Delta t^2 \ \omega^2 \right)} +
$$

$$
\frac{2 \ \beta_1 \ \gamma^2 \ \Delta t^2 \ \omega^4}{\left(-1 - \beta_2^2 \ (1-\gamma)^2 \ \Delta t^2 \ \omega^2 \right) \left(4 + \gamma^2 \ \Delta t^2 \ \omega^2 \right)} \right) +
$$

$$
\dot{\mathtt{x}}_t \left(- \frac{\beta_2 \ (1-\gamma) \ \Delta t \ \omega^2}{-1 - \beta_2^2 \ (1-\gamma)^2 \ \Delta t^2 \ \omega^2} + \frac{(-1 + \beta_1) \ \gamma \ \Delta t \ \omega^2}{-1 - \beta_2^2 \ (1-\gamma)^2 \ \Delta t^2 \ \omega^2} -
$$

$$
\frac{4 \ (1-\beta_2) \ (1-\gamma) \ \Delta t \ \omega^2}{\left(-1 - \beta_2^2 \ (1-\gamma)^2 \ \Delta t^2 \ \omega^2 \right) \left(4 + \gamma^2 \ \Delta t^2 \ \omega^2 \right)} - \frac{4 \ \beta_1 \ \gamma \ \Delta t \ \omega^2}{\left(-1 - \beta_2^2 \ (1-\gamma)^2 \ \Delta t^2 \ \omega^2 \right) \left(4 + \gamma^2 \ \Delta t^2 \ \omega^2 \right)} +
$$

$$
\frac{4 \ (1-\beta_2) \ \beta_2 \ (1-\gamma)^2 \ \gamma \ \Delta t^3 \ \omega^4}{\left(-1 - \beta_2^2 \ (1-\gamma)^2 \ \Delta t^2 \ \omega^2 \right) \left(4 + \gamma^2 \ \Delta t^2 \ \omega^2 \right)} + \frac{(1-\beta_2) \ (1-\gamma) \ \gamma^2 \ \Delta t^3 \ \omega^4}{\left(-1 - \beta_2^2 \ (1-\gamma)^2 \ \Delta t^2 \ \omega^2 \right) \left(4 + \gamma^2 \ \Delta t^2 \ \omega^2 \right)} +
$$

$$
\frac{4 \ \beta_1 \ \beta_2 \ (1-\gamma) \ \gamma^2 \ \Delta t^3 \ \omega^4}{\left(-1 - \beta_2^2 \ (1-\gamma)^2 \ \Delta t^2 \ \omega^2 \right) \left(4 + \gamma^2 \ \Delta t^2 \ \omega^2 \right)} + \frac{\beta_1 \ \gamma^3 \ \Delta t^3 \ \omega^4}{\left(-1 - \beta_2^2 \ (1-\gamma)^2 \ \Delta t^2 \ \omega^2 \right) \left(4 + \gamma^2 \ \Delta t^2 \ \omega^2 \right)} \right) +
$$

$$
\mathtt{x}_t \left(\frac{(-1 + \beta_1) \ \beta_2 \ (1-\gamma) \ \gamma \ \Delta t^2 \ \omega^2}{-1 - \beta_2^2 \ (1-\gamma)^2 \ \Delta t^2 \ \omega^2} - \frac{2 \ (1-\beta_2) \ (1-\gamma) \ \gamma \ \Delta t^2 \ \omega^2}{\left(-1 - \beta_2^2 \ (1-\gamma)^2 \ \Delta t^2 \ \omega^2 \right) \left(4 + \gamma^2 \ \Delta t^2 \ \omega^2 \right)} -
$$

$$
\frac{2 \ \beta_1 \ \gamma^2 \ \Delta t^2 \ \omega^2}{\left(-1 - \beta_2^2 \ (1-\gamma)^2 \ \Delta t^2 \ \omega^2 \right) \left(4 + \gamma^2 \ \Delta t^2 \ \omega^2 \right)} + \frac{(1-\beta_2) \ \beta_2 \ (1-\gamma)^2 \ \gamma^2 \ \Delta t^4 \ \omega^4}{\left(-1 - \beta_2^2 \ (1-\gamma)^2 \ \Delta t^2 \ \omega^2 \right) \left(4 + \gamma^2 \ \Delta t^2 \ \omega^2 \right)} +
$$

$$
\frac{\beta_1 \ \beta_2 \ (1-\gamma) \ \gamma^3 \ \Delta t^4 \ \omega^4}{\left(-1 - \beta_2^2 \ (1-\gamma)^2 \ \Delta t^2 \ \omega^2 \right) \left(4 + \gamma^2 \ \Delta t^2 \ \omega^2 \right)} \right),
$$

$$
\dot{\mathtt{x}}_{t+\Delta t} + \mathtt{r}_{t+\Delta t} \left(- \left((4 \ \beta_2 \ \Delta t) \Big/ \left(\left(4 + \gamma^2 \ \Delta t^2 \ \omega^2 \right) \left(1 + \beta_2^2 \ \Delta t^2 \ \omega^2 - 2 \ \beta_2^2 \ \gamma \ \Delta t^2 \ \omega^2 + \beta_2^2 \ \gamma^2 \ \Delta t^2 \ \omega^2 \right) \right) \right) \right) +
$$

$$
(4 \ \beta_2 \ \gamma \ \Delta t) \Big/ \left(\left(4 + \gamma^2 \ \Delta t^2 \ \omega^2 \right) \left(1 + \beta_2^2 \ \Delta t^2 \ \omega^2 - 2 \ \beta_2^2 \ \gamma \ \Delta t^2 \ \omega^2 + \beta_2^2 \ \gamma^2 \ \Delta t^2 \ \omega^2 \right) \right) -
$$

$$
\left(\beta_2 \ \gamma^2 \ \Delta t^3 \ \omega^2 \right) \Big/ \left(\left(4 + \gamma^2 \ \Delta t^2 \ \omega^2 \right) \left(1 + \beta_2^2 \ \Delta t^2 \ \omega^2 - 2 \ \beta_2^2 \ \gamma \ \Delta t^2 \ \omega^2 + \beta_2^2 \ \gamma^2 \ \Delta t^2 \ \omega^2 \right) \right) +
$$

$$
\left(\beta_2 \ \gamma^3 \ \Delta t^3 \ \omega^2 \right) \Big/ \left(\left(4 + \gamma^2 \ \Delta t^2 \ \omega^2 \right) \left(1 + \beta_2^2 \ \Delta t^2 \ \omega^2 - 2 \ \beta_2^2 \ \gamma \ \Delta t^2 \ \omega^2 + \beta_2^2 \ \gamma^2 \ \Delta t^2 \ \omega^2 \right) \right) \right) +
$$

$$
\mathtt{r}_{t+\gamma\Delta t} \left(- \left((4 \ \Delta t) \Big/ \left(\left(4 + \gamma^2 \ \Delta t^2 \ \omega^2 \right) \left(1 + \beta_2^2 \ \Delta t^2 \ \omega^2 - 2 \ \beta_2^2 \ \gamma \ \Delta t^2 \ \omega^2 + \beta_2^2 \ \gamma^2 \ \Delta t^2 \ \omega^2 \right) \right) \right) \right) +
$$

$$
(4 \ \beta_2 \ \Delta t) \Big/ \left(\left(4 + \gamma^2 \ \Delta t^2 \ \omega^2 \right) \left(1 + \beta_2^2 \ \Delta t^2 \ \omega^2 - 2 \ \beta_2^2 \ \gamma \ \Delta t^2 \ \omega^2 + \beta_2^2 \ \gamma^2 \ \Delta t^2 \ \omega^2 \right) \right) +
$$

$$
(4 \ \gamma \ \Delta t) \Big/ \left(\left(4 + \gamma^2 \ \Delta t^2 \ \omega^2 \right) \left(1 + \beta_2^2 \ \Delta t^2 \ \omega^2 - 2 \ \beta_2^2 \ \gamma \ \Delta t^2 \ \omega^2 + \beta_2^2 \ \gamma^2 \ \Delta t^2 \ \omega^2 \right) \right) -
$$

$$
(4 \ \beta_1 \ \gamma \ \Delta t) \Big/ \left(\left(4 + \gamma^2 \ \Delta t^2 \ \omega^2 \right) \left(1 + \beta_2^2 \ \Delta t^2 \ \omega^2 - 2 \ \beta_2^2 \ \gamma \ \Delta t^2 \ \omega^2 + \beta_2^2 \ \gamma^2 \ \Delta t^2 \ \omega^2 \right) \right) -
$$

$$
(4 \ \beta_2 \ \gamma \ \Delta t) \Big/ \left(\left(4 + \gamma^2 \ \Delta t^2 \ \omega^2 \right) \left(1 + \beta_2^2 \ \Delta t^2 \ \omega^2 - 2 \ \beta_2^2 \ \gamma \ \Delta t^2 \ \omega^2 + \beta_2^2 \ \gamma^2 \ \Delta t^2 \ \omega^2 \right) \right) +
$$

$$
\left(2 \ \beta_2 \ \gamma \ \Delta t^3 \ \omega^2 \right) \Big/ \left(\left(4 + \gamma^2 \ \Delta t^2 \ \omega^2 \right) \left(1 + \beta_2^2 \ \Delta t^2 \ \omega^2 - 2 \ \beta_2^2 \ \gamma \ \Delta t^2 \ \omega^2 + \beta_2^2 \ \gamma^2 \ \Delta t^2 \ \omega^2 \right) \right) -
$$

$$
\left(2 \ \beta_2^2 \ \gamma \ \Delta t^3 \ \omega^2 \right) \Big/ \left(\left(4 + \gamma^2 \ \Delta t^2 \ \omega^2 \right) \left(1 + \beta_2^2 \ \Delta t^2 \ \omega^2 - 2 \ \beta_2^2 \ \gamma \ \Delta t^2 \ \omega^2 + \beta_2^2 \ \gamma^2 \ \Delta t^2 \ \omega^2 \right) \right) -
$$

$$
\left(4 \ \beta_2 \ \gamma^2 \ \Delta t^3 \ \omega^2 \right) \Big/ \left(\left(4 + \gamma^2 \ \Delta t^2 \ \omega^2 \right) \left(1 + \beta_2^2 \ \Delta t^2 \ \omega^2 - 2 \ \beta_2^2 \ \gamma \ \Delta t^2 \ \omega^2 + \beta_2^2 \ \gamma^2 \ \Delta t^2 \ \omega^2 \right) \right) +
$$

$$
\left(2 \ \beta_1 \ \beta_2 \ \gamma^2 \ \Delta t^3 \ \omega^2 \right) \Big/ \left(\left(4 + \gamma^2 \ \Delta t^2 \ \omega^2 \right) \left(1 + \beta_2^2 \ \Delta t^2 \ \omega^2 - 2 \ \beta_2^2 \ \gamma \ \Delta t^2 \ \omega^2 + \beta_2^2 \ \gamma^2 \ \Delta t^2 \ \omega^2 \right) \right) +
$$

$$
\left(4 \ \beta_2^2 \ \gamma^2 \ \Delta t^3 \ \omega^2 \right) \Big/ \left(\left(4 + \gamma^2 \ \Delta t^2 \ \omega^2 \right) \left(1 + \beta_2^2 \ \Delta t^2 \ \omega^2 - 2 \ \beta_2^2 \ \gamma \ \Delta t^2 \ \omega^2 + \beta_2^2 \ \gamma^2 \ \Delta t^2 \ \omega^2 \right) \right) +
$$

$$
\left(2 \ \beta_2 \ \gamma^3 \ \Delta t^3 \ \omega^2 \right) \Big/ \left(\left(4 + \gamma^2 \ \Delta t^2 \ \omega^2 \right) \left(1 + \beta_2^2 \ \Delta t^2 \ \omega^2 - 2 \ \beta_2^2 \ \gamma \ \Delta t^2 \ \omega^2 + \beta_2^2 \ \gamma^2 \ \Delta t^2 \ \omega^2 \right) \right) -
$$

$$
\left(2 \ \beta_1 \ \beta_2 \ \gamma^3 \ \Delta t^3 \ \omega^2 \right) \Big/ \left(\left(4 + \gamma^2 \ \Delta t^2 \ \omega^2 \right) \left(1 + \beta_2^2 \ \Delta t^2 \ \omega^2 - 2 \ \beta_2^2 \ \gamma \ \Delta t^2 \ \omega^2 + \beta_2^2 \ \gamma^2 \ \Delta t^2 \ \omega^2 \right) \right) -
$$

$$\left(2\,\beta_2^2\,\gamma^3\,\Delta t^3\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)\right)+$$

$$x_t\left(-\left(\left(4\,\gamma\,\Delta t\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)\right)\right)+$$

$$\left(4\,\beta_1\,\gamma\,\Delta t\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)+$$

$$\left(2\,\beta_2\,\gamma\,\Delta t^3\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)-$$

$$\left(2\,\beta_2^2\,\gamma\,\Delta t^3\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)+$$

$$\left(\gamma^2\,\Delta t^3\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)-$$

$$\left(5\,\beta_2\,\gamma^2\,\Delta t^3\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)+$$

$$\left(2\,\beta_1\,\beta_2\,\gamma^2\,\Delta t^3\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)+$$

$$\left(4\,\beta_2^2\,\gamma^2\,\Delta t^3\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)-$$

$$\left(2\,\gamma^3\,\Delta t^3\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)+$$

$$\left(2\,\beta_1\,\gamma^3\,\Delta t^3\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)+$$

$$\left(3\,\beta_2\,\gamma^3\,\Delta t^3\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)-$$

$$\left(2\,\beta_1\,\beta_2\,\gamma^3\,\Delta t^3\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)-$$

$$\left(2\,\beta_2^2\,\gamma^3\,\Delta t^3\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)\right)+$$

$$x_t\left(\left(4\,\Delta t\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)-$$

$$\left(4\,\gamma\,\Delta t\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)+$$

$$\left(4\,\beta_1\,\gamma\,\Delta t\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)-$$

$$\left(2\,\beta_2\,\gamma\,\Delta t^3\,\omega^4\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)+$$

$$\left(2\,\beta_2^2\,\gamma\,\Delta t^3\,\omega^4\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)+$$

$$\left(5\,\beta_2\,\gamma^2\,\Delta t^3\,\omega^4\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)-$$

$$\left(2\,\beta_1\,\beta_2\,\gamma^2\,\Delta t^3\,\omega^4\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)-$$

$$\left(4\,\beta_2^2\,\gamma^2\,\Delta t^3\,\omega^4\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)-$$

$$\left(3\,\beta_2\,\gamma^3\,\Delta t^3\,\omega^4\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)+$$

$$\left(2\,\beta_1\,\beta_2\,\gamma^3\,\Delta t^3\,\omega^4\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)+$$

$$\left(2\,\beta_2^2\,\gamma^3\,\Delta t^3\,\omega^4\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)\right)+$$

$$\dot{x}_t\left(-\left(4\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)\right)+$$

$$\left(4\,\beta_2\,\Delta t^2\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)-$$

$$\left(4\,\beta_2^2\,\Delta t^2\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)+$$

$$\left(4\,\gamma\,\Delta t^2\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)-$$

$$\left(8\,\beta_2\,\gamma\,\Delta t^2\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)+$$

$$\left(8\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)-$$

$$\left(5\,\gamma^2\,\Delta t^2\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)+$$

$$\left(4\,\beta_1\,\gamma^2\,\Delta t^2\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)+$$

$$\left(4\,\beta_2\,\gamma^2\,\Delta t^2\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)-$$

$$\left(4\,\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)-$$

$$\left(\beta_2\,\gamma^2\,\Delta t^4\,\omega^4\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)+$$

$$\left(\beta_2^2\,\gamma^2\,\Delta t^4\,\omega^4\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)+$$

$$\left(3\,\beta_2\,\gamma^3\,\Delta t^4\,\omega^4\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)-$$

$$\left(2\,\beta_1\,\beta_2\,\gamma^3\,\Delta t^4\,\omega^4\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)-$$

$$\left(2\,\beta_2^2\,\gamma^3\,\Delta t^4\,\omega^4\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)-$$

$$\left(2\,\beta_2\,\gamma^4\,\Delta t^4\,\omega^4\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)+$$

$$\left(2\,\beta_1\,\beta_2\,\gamma^4\,\Delta t^4\,\omega^4\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)+$$

$$\left(\beta_2^2\,\gamma^4\,\Delta t^4\,\omega^4\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)\right),$$

$$x_{t+\Delta t}+r_{t+\gamma\Delta t}\left(-\left(\left(4\,\beta_2\,\Delta t^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)\right)\right)+$$

$$\left(4\,\beta_2^2\,\Delta t^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)-$$

$$\left(2\,\gamma\,\Delta t^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)+$$

$$\left(10\,\beta_2\,\gamma\,\Delta t^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)-$$
$$\left(4\,\beta_1\,\beta_2\,\gamma\,\Delta t^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)-$$
$$\left(8\,\beta_2^2\,\gamma\,\Delta t^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)+$$
$$\left(2\,\gamma^2\,\Delta t^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)-$$
$$\left(2\,\beta_1\,\gamma^2\,\Delta t^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)-$$
$$\left(6\,\beta_2\,\gamma^2\,\Delta t^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)+$$
$$\left(4\,\beta_1\,\beta_2\,\gamma^2\,\Delta t^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)+$$
$$\left(4\,\beta_2^2\,\gamma^2\,\Delta t^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)\right)+$$
$$x_t\left(-\left(4\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)\right)\right)+$$
$$\left(4\,\beta_2\,\Delta t^2\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)-$$
$$\left(4\,\beta_2^2\,\Delta t^2\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)+$$
$$\left(2\,\gamma\,\Delta t^2\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)-$$
$$\left(10\,\beta_2\,\gamma\,\Delta t^2\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)+$$
$$\left(4\,\beta_1\,\beta_2\,\gamma\,\Delta t^2\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)+$$
$$\left(8\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)-$$
$$\left(3\,\gamma^2\,\Delta t^2\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)+$$
$$\left(2\,\beta_1\,\gamma^2\,\Delta t^2\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)+$$
$$\left(6\,\beta_2\,\gamma^2\,\Delta t^2\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)-$$
$$\left(4\,\beta_1\,\beta_2\,\gamma^2\,\Delta t^2\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)-$$
$$\left(4\,\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)\right)+$$
$$\dot{x}_t\left(-\left(\left(4\,\Delta t\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)\right)\right)+$$
$$\left(4\,\beta_2\,\gamma\,\Delta t^3\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)-$$
$$\left(4\,\beta_2^2\,\gamma\,\Delta t^3\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)+$$
$$\left(\gamma^2\,\Delta t^3\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)-$$
$$\left(10\,\beta_2\,\gamma^2\,\Delta t^3\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)+$$
$$\left(4\,\beta_1\,\beta_2\,\gamma^2\,\Delta t^3\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)+$$
$$\left(8\,\beta_2^2\,\gamma^2\,\Delta t^3\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)-$$
$$\left(2\,\gamma^3\,\Delta t^3\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)+$$
$$\left(2\,\beta_1\,\gamma^3\,\Delta t^3\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)+$$
$$\left(6\,\beta_2\,\gamma^3\,\Delta t^3\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)-$$
$$\left(4\,\beta_1\,\beta_2\,\gamma^3\,\Delta t^3\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)-$$
$$\left(4\,\beta_2^2\,\gamma^3\,\Delta t^3\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)\right)+$$
$$r_{t+\Delta t}\left(-\left(\left(4\,\beta_2^2\,\Delta t^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)\right)\right)+$$
$$\left(8\,\beta_2^2\,\gamma\,\Delta t^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)-$$
$$\left(4\,\beta_2^2\,\gamma^2\,\Delta t^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)-$$
$$\left(\beta_2^2\,\gamma^2\,\Delta t^4\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)+$$
$$\left(2\,\beta_2^2\,\gamma^3\,\Delta t^4\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)-$$
$$\left(\beta_2^2\,\gamma^4\,\Delta t^4\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)\right)+$$
$$x_t\left(-\left(\left(2\,\gamma\,\Delta t^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)\right)\right)-$$
$$\left(2\,\beta_2\,\gamma\,\Delta t^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)+$$
$$\left(4\,\beta_1\,\beta_2\,\gamma\,\Delta t^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)+$$
$$\left(2\,\gamma^2\,\Delta t^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)-$$
$$\left(2\,\beta_1\,\gamma^2\,\Delta t^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)+$$
$$\left(2\,\beta_2\,\gamma^2\,\Delta t^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)-$$
$$\left(4\,\beta_1\,\beta_2\,\gamma^2\,\Delta t^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)+$$
$$\left(\beta_2\,\gamma^2\,\Delta t^4\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)-$$

$$\left(\beta_2^2\,\gamma^2\,\Delta t^4\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)-$$
$$\left(3\,\beta_2\,\gamma^3\,\Delta t^4\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)+$$
$$\left(2\,\beta_1\,\beta_2\,\gamma^3\,\Delta t^4\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)+$$
$$\left(2\,\beta_2^2\,\gamma^3\,\Delta t^4\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)+$$
$$\left(2\,\beta_2\,\gamma^4\,\Delta t^4\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)-$$
$$\left(2\,\beta_1\,\beta_2\,\gamma^4\,\Delta t^4\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)-$$
$$\left(\beta_2^2\,\gamma^4\,\Delta t^4\,\omega^2\right)\Big/\left(\left(4+\gamma^2\,\Delta t^2\,\omega^2\right)\left(1+\beta_2^2\,\Delta t^2\,\omega^2-2\,\beta_2^2\,\gamma\,\Delta t^2\,\omega^2+\beta_2^2\,\gamma^2\,\Delta t^2\,\omega^2\right)\right)\Big)\Big\}$$

```
a =
   - ⎛ Coefficient[Part[recursive, 1], ẍₜ]  Coefficient[Part[recursive, 1], ẋₜ]  Coeff
     ⎜ Coefficient[Part[recursive, 2], ẍₜ]  Coefficient[Part[recursive, 2], ẋₜ]  Coeff
     ⎝ Coefficient[Part[recursive, 3], ẍₜ]  Coefficient[Part[recursive, 3], ẋₜ]  Coeff

     ⎞
     ⎟ ;
     ⎠
```

```
a // MatrixForm // Simplify
```

$$\begin{pmatrix}
\frac{2\,(-1-3\,\beta_1+4\,\beta_1^2)\,\Delta t^2\,\omega^2}{(16+\Delta t^2\,\omega^2)\,(1+\beta_1^2\,\Delta t^2\,\omega^2)} & \frac{-16\,\Delta t\,\omega^2+(3-4\,\beta_1)\,\beta_1\,\Delta t^3\,\omega^4}{(16+\Delta t^2\,\omega^2)\,(1+\beta_1^2\,\Delta t^2\,\omega^2)} & \frac{-16\,\omega^2+(1+6\,\beta_1-8\,\beta_1^2)\,\Delta t^2\,\omega^4}{(16+\Delta t^2\,\omega^2)\,(1+\beta_1^2\,\Delta t^2\,\omega^2)} \\[6pt]
\frac{2\,\Delta t\,(4+\beta_1^2\,\Delta t^2\,\omega^2-\beta_1\,(4+\Delta t^2\,\omega^2))}{(16+\Delta t^2\,\omega^2)\,(1+\beta_1^2\,\Delta t^2\,\omega^2)} & \frac{16+(-3-12\,\beta_1+16\,\beta_1^2)\,\Delta t^2\,\omega^2}{(16+\Delta t^2\,\omega^2)\,(1+\beta_1^2\,\Delta t^2\,\omega^2)} & \frac{\Delta t\,\omega^2\,(-8-2\,\beta_1^2\,\Delta t^2\,\omega^2+\beta_1\,(-8+\Delta t^2\,\omega^2))}{(16+\Delta t^2\,\omega^2)\,(1+\beta_1^2\,\Delta t^2\,\omega^2)} \\[6pt]
\frac{2\,(1+3\,\beta_1-4\,\beta_1^2)\,\Delta t^2}{(16+\Delta t^2\,\omega^2)\,(1+\beta_1^2\,\Delta t^2\,\omega^2)} & \frac{\Delta t\,(16-3\,\beta_1\,\Delta t^2\,\omega^2+4\,\beta_1^2\,\Delta t^2\,\omega^2)}{(16+\Delta t^2\,\omega^2)\,(1+\beta_1^2\,\Delta t^2\,\omega^2)} & \frac{16+(-1-6\,\beta_1+8\,\beta_1^2)\,\Delta t^2\,\omega^2}{(16+\Delta t^2\,\omega^2)\,(1+\beta_1^2\,\Delta t^2\,\omega^2)}
\end{pmatrix}$$

```
ClearAll["Global`*"]
```

```
la = - ⎛ Coefficient[Part[recursive, 1], r_{t+γΔt}] ⎞
       ⎜ Coefficient[Part[recursive, 2], r_{t+γΔt}] ⎟ ;
       ⎝ Coefficient[Part[recursive, 3], r_{t+γΔt}] ⎠
```

```
la // MatrixForm // Simplify
```

$$\begin{pmatrix}
\frac{2\,(-1-3\,\beta_1+4\,\beta_1^2)\,\Delta t^2\,\omega^2}{(16+\Delta t^2\,\omega^2)\,(1+\beta_1^2\,\Delta t^2\,\omega^2)} \\[6pt]
\frac{2\,\Delta t\,(4+\beta_1^2\,\Delta t^2\,\omega^2-\beta_1\,(4+\Delta t^2\,\omega^2))}{(16+\Delta t^2\,\omega^2)\,(1+\beta_1^2\,\Delta t^2\,\omega^2)} \\[6pt]
\frac{2\,(1+3\,\beta_1-4\,\beta_1^2)\,\Delta t^2}{(16+\Delta t^2\,\omega^2)\,(1+\beta_1^2\,\Delta t^2\,\omega^2)}
\end{pmatrix}$$

```
lb = - ⎛ Coefficient[Part[recursive, 1], r_{t+Δt}] ⎞
       ⎜ Coefficient[Part[recursive, 2], r_{t+Δt}] ⎟ ;
       ⎝ Coefficient[Part[recursive, 3], r_{t+Δt}] ⎠
```

```
lb // MatrixForm // Simplify
```

$$\begin{pmatrix}
\frac{1}{1+\beta_1^2\,\Delta t^2\,\omega^2} \\[6pt]
\frac{\beta_1\,\Delta t}{1+\beta_1^2\,\Delta t^2\,\omega^2} \\[6pt]
\frac{\beta_1^2\,\Delta t^2}{1+\beta_1^2\,\Delta t^2\,\omega^2}
\end{pmatrix}$$

```
eigA = Eigenvalues[a] /. Δt → p T // Expand // Simplify // Cancel;
```

```
eigA // MatrixForm
```

$$\begin{pmatrix}
0 \\[6pt]
\frac{4\,T^4-3\,p^2\,\pi^2\,T^4-12\,p^2\,\pi^2\,T^4\,\beta_1+16\,p^2\,\pi^2\,T^4\,\beta_1^2-2\,\pi\,\sqrt{-p^2\,T^8\,(4-3\,p^2\,\pi^2\,\beta_1+4\,p^2\,\pi^2\,\beta_1^2)^2}}{(4+p^2\,\pi^2)\,T^4\,(1+4\,p^2\,\pi^2\,\beta_1^2)} \\[6pt]
\frac{4\,T^4-3\,p^2\,\pi^2\,T^4-12\,p^2\,\pi^2\,T^4\,\beta_1+16\,p^2\,\pi^2\,T^4\,\beta_1^2+2\,\pi\,\sqrt{-p^2\,T^8\,(4-3\,p^2\,\pi^2\,\beta_1+4\,p^2\,\pi^2\,\beta_1^2)^2}}{(4+p^2\,\pi^2)\,T^4\,(1+4\,p^2\,\pi^2\,\beta_1^2)}
\end{pmatrix}$$

$\gamma = \dfrac{1}{2}; \beta_2 = 2 \beta_1;$

$\xi = 0;$

$\omega = 2 \text{ Pi} / T;$

$\lambda 1 = \text{Part[eigA, 2]} /. T \to 1 \text{ // Simplify // Expand}$

$$\frac{4}{\left(4 + p^2 \pi^2\right)\left(1 + 4 p^2 \pi^2 \beta_1^2\right)} - \frac{3 p^2 \pi^2}{\left(4 + p^2 \pi^2\right)\left(1 + 4 p^2 \pi^2 \beta_1^2\right)} - \frac{12 p^2 \pi^2 \beta_1}{\left(4 + p^2 \pi^2\right)\left(1 + 4 p^2 \pi^2 \beta_1^2\right)} +$$

$$\frac{16 p^2 \pi^2 \beta_1^2}{\left(4 + p^2 \pi^2\right)\left(1 + 4 p^2 \pi^2 \beta_1^2\right)} - \frac{2 \pi \sqrt{-p^2 \left(4 + p^2 \pi^2 \beta_1 \left(-3 + 4 \beta_1\right)\right)^2}}{\left(4 + p^2 \pi^2\right)\left(1 + 4 p^2 \pi^2 \beta_1^2\right)}$$

$\text{pe} = \dfrac{\Omega_o}{\overline{\Omega}_d} - 1$

$-1 + \dfrac{\Omega_o}{\overline{\Omega}_d}$

$\Omega_o = \omega \, \Delta t \, /. \, \dfrac{\Delta t}{T} \to p$

$2 p \pi$

$\overline{\Omega}_d = \text{ArcTan}\left[\dfrac{2 \pi \sqrt{p^2 \left(-36 + 5 p^2 \pi^2\right)^2}}{36 - 47 p^2 \pi^2}\right]$ **(*simply copy paste this from above*)**

$\text{ArcTan}\left[\dfrac{2 \pi \sqrt{p^2 \left(-36 + 5 p^2 \pi^2\right)^2}}{36 - 47 p^2 \pi^2}\right]$

$\text{Plot[(pe)} * 100, \{p, 0, .2\}, \text{PlotRange} \to \{0, 20\}]$
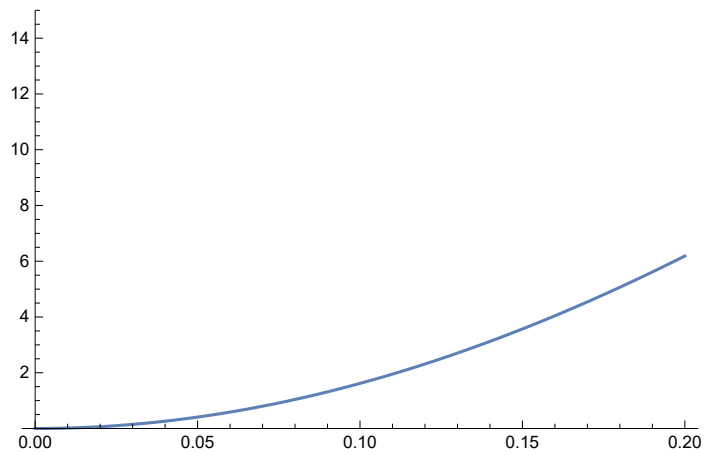
$\text{AD} = 1 - \text{Exp}\left[-2 \text{ Pi } \overline{\xi} \, \dfrac{\Omega_o}{\overline{\Omega}_d}\right]$

$1 - \text{Abs}\left[\dfrac{36 - 47 p^2 \pi^2 - 2 \pi \sqrt{-p^2 \left(-36 + 5 p^2 \pi^2\right)^2}}{\left(4 + p^2 \pi^2\right)\left(9 + 4 p^2 \pi^2\right)}\right]^{\frac{2 \pi}{\text{ArcTan}\left[\frac{2 \pi \sqrt{p^2 \left(-36 + 5 p^2 \pi^2\right)^2}}{36 - 47 p^2 \pi^2}\right]}}$

$$\overline{\xi} = -\frac{1}{\Omega_o} \, \text{Log[Abs[}\lambda 1]]$$

$$-\frac{\text{Log}\Big[\text{Abs}\Big[\frac{36-47\,p^2\,\pi^2-2\,\pi\,\sqrt{-p^2\,(-36+5\,p^2\,\pi^2)^2}}{(4+p^2\,\pi^2)\,(9+4\,p^2\,\pi^2)}\Big]\Big]}{2\,p\,\pi}$$

Plot[(AD) * 100, {p, 0, .2}, PlotRange → {0, 20}]
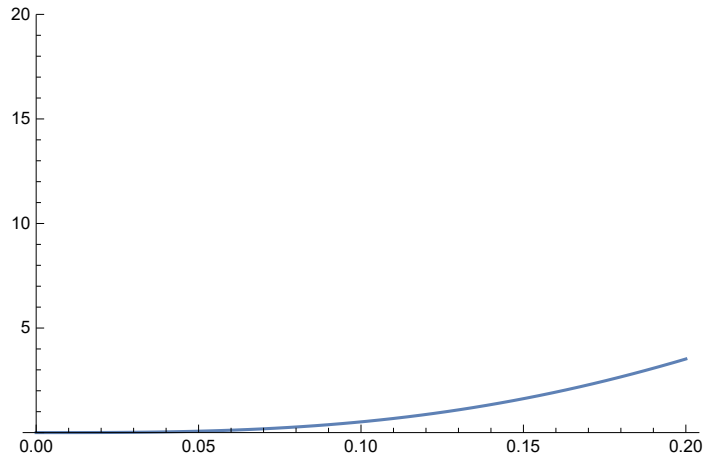
aNot = $-\begin{pmatrix} \text{Coefficient}\big[\text{Part[recursive, 2]}, \dot{x}_t\big] & \text{Coefficient}\big[\text{Part[recursive, 2]}, x_t\big] \\ \text{Coefficient}\big[\text{Part[recursive, 3]}, \dot{x}_t\big] & \text{Coefficient}\big[\text{Part[recursive, 3]}, x_t\big] \end{pmatrix}$

$$\Big\{\Big\{ \frac{144}{\Big(9+\frac{4\,\pi^2\,\Delta t^2}{T^2}\Big)\Big(16+\frac{4\,\pi^2\,\Delta t^2}{T^2}\Big)} - \frac{188\,\pi^2\,\Delta t^2}{T^2\Big(9+\frac{4\,\pi^2\,\Delta t^2}{T^2}\Big)\Big(16+\frac{4\,\pi^2\,\Delta t^2}{T^2}\Big)},$$

$$-\frac{384\,\pi^2\,\Delta t}{T^2\Big(9+\frac{4\,\pi^2\,\Delta t^2}{T^2}\Big)\Big(16+\frac{4\,\pi^2\,\Delta t^2}{T^2}\Big)} + \frac{16\,\pi^4\,\Delta t^3}{T^4\Big(9+\frac{4\,\pi^2\,\Delta t^2}{T^2}\Big)\Big(16+\frac{4\,\pi^2\,\Delta t^2}{T^2}\Big)}\Big\},$$

$$\Big\{ \frac{144\,\Delta t}{\Big(9+\frac{4\,\pi^2\,\Delta t^2}{T^2}\Big)\Big(16+\frac{4\,\pi^2\,\Delta t^2}{T^2}\Big)} - \frac{20\,\pi^2\,\Delta t^3}{T^2\Big(9+\frac{4\,\pi^2\,\Delta t^2}{T^2}\Big)\Big(16+\frac{4\,\pi^2\,\Delta t^2}{T^2}\Big)},$$

$$\frac{144}{\Big(9+\frac{4\,\pi^2\,\Delta t^2}{T^2}\Big)\Big(16+\frac{4\,\pi^2\,\Delta t^2}{T^2}\Big)} - \frac{76\,\pi^2\,\Delta t^2}{T^2\Big(9+\frac{4\,\pi^2\,\Delta t^2}{T^2}\Big)\Big(16+\frac{4\,\pi^2\,\Delta t^2}{T^2}\Big)}\Big\}\Big\}$$

eigaNot = Eigenvalues[aNot] /. Δt → p T // Expand // Simplify // Cancel;

ppp = {0, Part[eigaNot, 1], Part[eigaNot, 2]}

$$\Big\{0,$$

$$\Big(36\,T^4 - 33\,p^2\,\pi^2\,T^4 - 2\,\pi\,\sqrt{-p^2\,(864 - 205\,p^2\,\pi^2 + 5\,p^4\,\pi^4)\,T^8}\Big)\Big/\Big((4+p^2\,\pi^2)\,(9+4\,p^2\,\pi^2)\,T^4\Big),$$

$$\Big(36\,T^4 - 33\,p^2\,\pi^2\,T^4 + 2\,\pi\,\sqrt{-p^2\,(864 - 205\,p^2\,\pi^2 + 5\,p^4\,\pi^4)\,T^8}\Big)\Big/\Big((4+p^2\,\pi^2)\,(9+4\,p^2\,\pi^2)\,T^4\Big)\Big\}$$

qqq = eigA

$$\Big\{0, \Big(36\,T^4 - 47\,p^2\,\pi^2\,T^4 - 2\,\pi\,\sqrt{-p^2\,(-36+5\,p^2\,\pi^2)^2\,T^8}\Big)\Big/\Big((4+p^2\,\pi^2)\,(9+4\,p^2\,\pi^2)\,T^4\Big),$$

$$\Big(36\,T^4 - 47\,p^2\,\pi^2\,T^4 + 2\,\pi\,\sqrt{-p^2\,(-36+5\,p^2\,\pi^2)^2\,T^8}\Big)\Big/\Big((4+p^2\,\pi^2)\,(9+4\,p^2\,\pi^2)\,T^4\Big)\Big\}$$

ppp - qqq // Simplify

$$\left\{ 0, \ \left(2 \pi \left(7 p^2 \pi T^4 + \sqrt{-p^2 \left(-36 + 5 p^2 \pi^2 \right)^2 T^8} - \sqrt{-p^2 \left(864 - 205 p^2 \pi^2 + 5 p^4 \pi^4 \right) T^8} \right) \right) \right/$$

$$\left(\left(4 + p^2 \pi^2 \right) \left(9 + 4 p^2 \pi^2 \right) T^4 \right),$$

$$\left(2 \pi \left(7 p^2 \pi T^4 - \sqrt{-p^2 \left(-36 + 5 p^2 \pi^2 \right)^2 T^8} + \sqrt{-p^2 \left(864 - 205 p^2 \pi^2 + 5 p^4 \pi^4 \right) T^8} \right) \right) \right/$$

$$\left. \left(\left(4 + p^2 \pi^2 \right) \left(9 + 4 p^2 \pi^2 \right) T^4 \right) \right\}$$