

Q1 a)

<< Notation`

Symbolize[$\mathbf{x}_{t+\Delta t}$]

Symbolize[$\dot{\mathbf{x}}_{t+\Delta t}$]

Symbolize[$\ddot{\mathbf{x}}_{t+\Delta t}$]

Symbolize[$\mathbf{x}_{t+\gamma\Delta t}$]

Symbolize[$\dot{\mathbf{x}}_{t+\gamma\Delta t}$]

Symbolize[$\ddot{\mathbf{x}}_{t+\gamma\Delta t}$]

Symbolize[\mathbf{x}_t]

Symbolize[$\dot{\mathbf{x}}_t$]

Symbolize[$\ddot{\mathbf{x}}_t$]

Symbolize[$\mathbf{r}_{t+\Delta t}$]

Symbolize[$\mathbf{r}_{t+\gamma\Delta t}$]

Symbolize[Ω_o]

Symbolize[$\overline{\Omega_d}$]

Symbolize[$\overline{\xi}$]

Symbolize[β_1]

Symbolize[β_2]

Symbolize[\mathbf{X}_t]

ClearAll["Global`*"]

(*

Writing in the modal form

*)

$\xi = 0;$

Solve[

$$\ddot{x}_{t+\gamma\Delta t} + 2\xi\omega\dot{x}_{t+\gamma\Delta t} + \omega^2 x_{t+\gamma\Delta t} == r_{t+\gamma\Delta t} \&\&$$

$$x_{t+\gamma\Delta t} == x_t + \frac{\gamma\Delta t}{2} (\dot{x}_t + \dot{x}_{t+\gamma\Delta t}) \&\&$$

$$\dot{x}_{t+\gamma\Delta t} == \dot{x}_t + \frac{\gamma\Delta t}{2} (\ddot{x}_t + \ddot{x}_{t+\gamma\Delta t}),$$

$$\{\ddot{x}_{t+\gamma\Delta t}, \dot{x}_{t+\gamma\Delta t}, x_{t+\gamma\Delta t}\}]$$

$$\begin{aligned} \text{Solve}\left[-\frac{4\pi^2(-4x_t - 4\dot{x}_t\gamma\Delta t - r_{t+\gamma\Delta t}\gamma^2\Delta t^2 - x_t\gamma^2\Delta t^2)}{T^2\left(4 + \frac{4\pi^2\gamma^2\Delta t^2}{T^2}\right)} - \right. \\ \left. -\frac{-4r_{t+\gamma\Delta t} + \frac{16\pi^2x_t}{T^2} + \frac{16\pi^2\dot{x}_t\gamma\Delta t}{T^2} + \frac{4\pi^2x_t\gamma^2\Delta t^2}{T^2}}{4 + \frac{4\pi^2\gamma^2\Delta t^2}{T^2}} == r_{t+\gamma\Delta t} \&\& \right. \\ \left. -\frac{-4x_t - 4\dot{x}_t\gamma\Delta t - r_{t+\gamma\Delta t}\gamma^2\Delta t^2 - x_t\gamma^2\Delta t^2}{4 + \frac{4\pi^2\gamma^2\Delta t^2}{T^2}} == x_t + \frac{1}{2}\gamma\Delta t\left(\dot{x}_t - \right. \right. \\ \left. \left. \left(-4\dot{x}_t - 2r_{t+\gamma\Delta t}\gamma\Delta t - 2x_t\gamma\Delta t + \frac{8\pi^2x_t\gamma\Delta t}{T^2} + \frac{4\pi^2\dot{x}_t\gamma^2\Delta t^2}{T^2}\right) \middle/ \left(4 + \frac{4\pi^2\gamma^2\Delta t^2}{T^2}\right)\right) \&\& \right. \\ \left. -\frac{1}{4 + \frac{4\pi^2\gamma^2\Delta t^2}{T^2}}\left(-4\dot{x}_t - 2r_{t+\gamma\Delta t}\gamma\Delta t - 2x_t\gamma\Delta t + \frac{8\pi^2x_t\gamma\Delta t}{T^2} + \frac{4\pi^2\dot{x}_t\gamma^2\Delta t^2}{T^2}\right) == \right. \\ \left. \dot{x}_t + \frac{1}{2}\gamma\Delta t\left(x_t - \frac{-4r_{t+\gamma\Delta t} + \frac{16\pi^2x_t}{T^2} + \frac{16\pi^2\dot{x}_t\gamma\Delta t}{T^2} + \frac{4\pi^2x_t\gamma^2\Delta t^2}{T^2}}{4 + \frac{4\pi^2\gamma^2\Delta t^2}{T^2}}\right), \right. \\ \left. \left\{-\frac{-4r_{t+\gamma\Delta t} + \frac{16\pi^2x_t}{T^2} + \frac{16\pi^2\dot{x}_t\gamma\Delta t}{T^2} + \frac{4\pi^2x_t\gamma^2\Delta t^2}{T^2}}{4 + \frac{4\pi^2\gamma^2\Delta t^2}{T^2}}, \right. \right. \\ \left. -\frac{1}{4 + \frac{4\pi^2\gamma^2\Delta t^2}{T^2}}\left(-4\dot{x}_t - 2r_{t+\gamma\Delta t}\gamma\Delta t - 2x_t\gamma\Delta t + \frac{8\pi^2x_t\gamma\Delta t}{T^2} + \frac{4\pi^2\dot{x}_t\gamma^2\Delta t^2}{T^2}\right), \right. \\ \left. \left. -\frac{-4x_t - 4\dot{x}_t\gamma\Delta t - r_{t+\gamma\Delta t}\gamma^2\Delta t^2 - x_t\gamma^2\Delta t^2}{4 + \frac{4\pi^2\gamma^2\Delta t^2}{T^2}}\right\}\right] \end{aligned}$$

ClearAll["Global`*"]

$$\begin{aligned} \ddot{x}_{t+\gamma\Delta t} &= -\frac{-4r_{t+\gamma\Delta t} + 4x_t\omega^2 + 4\dot{x}_t\gamma\Delta t\omega^2 + \ddot{x}_t\gamma^2\Delta t^2\omega^2}{4 + \gamma^2\Delta t^2\omega^2} \\ &- \frac{-4r_{t+\gamma\Delta t} + \frac{16\pi^2x_t}{T^2} + \frac{16\pi^2\dot{x}_t\gamma\Delta t}{T^2} + \frac{4\pi^2x_t\gamma^2\Delta t^2}{T^2}}{4 + \frac{4\pi^2\gamma^2\Delta t^2}{T^2}} \\ \dot{x}_{t+\gamma\Delta t} &= -\frac{1}{4 + \gamma^2\Delta t^2\omega^2}\left(-4\dot{x}_t - 2r_{t+\gamma\Delta t}\gamma\Delta t - 2\ddot{x}_t\gamma\Delta t + 2x_t\gamma\Delta t\omega^2 + \dot{x}_t\gamma^2\Delta t^2\omega^2\right) \\ &- \frac{1}{4 + \frac{4\pi^2\gamma^2\Delta t^2}{T^2}}\left(-4\dot{x}_t - 2r_{t+\gamma\Delta t}\gamma\Delta t - 2x_t\gamma\Delta t + \frac{8\pi^2x_t\gamma\Delta t}{T^2} + \frac{4\pi^2\dot{x}_t\gamma^2\Delta t^2}{T^2}\right) \\ x_{t+\gamma\Delta t} &= -\frac{-4x_t - 4\dot{x}_t\gamma\Delta t - r_{t+\gamma\Delta t}\gamma^2\Delta t^2 - \ddot{x}_t\gamma^2\Delta t^2}{4 + \gamma^2\Delta t^2\omega^2} \\ &- \frac{-4x_t - 4\dot{x}_t\gamma\Delta t - r_{t+\gamma\Delta t}\gamma^2\Delta t^2 - x_t\gamma^2\Delta t^2}{4 + \frac{4\pi^2\gamma^2\Delta t^2}{T^2}} \end{aligned}$$

Solve [$\ddot{x}_{t+\Delta t} + 2 \xi \omega \dot{x}_{t+\Delta t} + \omega^2 x_{t+\Delta t} == r_{t+\Delta t}$ &&

$x_{t+\Delta t} == x_t + \gamma \Delta t \left((1 - \beta_1) \dot{x}_t + \beta_1 \dot{x}_{t+\gamma \Delta t} \right) + (1 - \gamma) \Delta t \left((1 - \beta_2) \dot{x}_{t+\gamma \Delta t} + \beta_2 \dot{x}_{t+\Delta t} \right)$ &&

$\dot{x}_{t+\Delta t} == \dot{x}_t + \gamma \Delta t \left((1 - \beta_1) \ddot{x}_t + \beta_1 \ddot{x}_{t+\gamma \Delta t} \right) + (1 - \gamma) \Delta t \left((1 - \beta_2) \ddot{x}_{t+\gamma \Delta t} + \beta_2 \ddot{x}_{t+\Delta t} \right),$

$\{ \ddot{x}_{t+\Delta t}, \dot{x}_{t+\Delta t}, x_{t+\Delta t} \}$]

{ { $x_{t+\Delta t} \rightarrow$

$- \left(\left(1. \left(-0.0025665 r_{t+\Delta t} T^4 + 0.101321 T^2 x_t + 0.101321 T^2 \dot{x}_t \Delta t + 0.0506606 r_{t+\gamma \Delta t} T^2 \right. \right. \right.$

$\gamma \Delta t^2 + 0.0506606 T^2 x_t \gamma \Delta t^2 - 2. x_t \gamma \Delta t^2 -$

$0.0253303 r_{t+\gamma \Delta t} T^2 \gamma^2 \Delta t^2 - 0.0253303 r_{t+\Delta t} T^2 \gamma^2 \Delta t^2 -$

$0.0253303 T^2 x_t \gamma^2 \Delta t^2 + 2. x_t \gamma^2 \Delta t^2 - 1. \dot{x}_t \gamma^2 \Delta t^3 + 1. \dot{x}_t \gamma^3 \Delta t^3 \left. \right) \left/ \right.$

$\left(\left(0.101321 T^2 + 1. \gamma^2 \Delta t^2 \right) \left(0.0253303 T^2 + 1. \Delta t^2 - 2. \gamma \Delta t^2 + 1. \gamma^2 \Delta t^2 \right) \right),$

$\dot{x}_{t+\Delta t} \rightarrow \left(0.0759909 \left(0.0337737 T^4 \dot{x}_t + 0.0337737 r_{t+\Delta t} T^4 \Delta t - 1.33333 T^2 x_t \Delta t + \right. \right.$

$0.0168869 r_{t+\gamma \Delta t} T^4 \gamma \Delta t - 0.0337737 r_{t+\Delta t} T^4 \gamma \Delta t + 0.0168869 T^4 x_t \gamma \Delta t +$

$0.666667 T^2 x_t \gamma \Delta t - 1.33333 T^2 \dot{x}_t \gamma \Delta t^2 + 1. T^2 \dot{x}_t \gamma^2 \Delta t^2 -$

$0.333333 r_{t+\gamma \Delta t} T^2 \gamma^2 \Delta t^3 + 0.333333 r_{t+\Delta t} T^2 \gamma^2 \Delta t^3 - 0.333333 T^2 x_t \gamma^2 \Delta t^3 +$

$0.333333 r_{t+\gamma \Delta t} T^2 \gamma^3 \Delta t^3 - 0.333333 r_{t+\Delta t} T^2 \gamma^3 \Delta t^3 + 0.333333 T^2 x_t \gamma^3 \Delta t^3 \left. \right) \left/ \right.$

$\left(\left(0.101321 T^2 + 1. \gamma^2 \Delta t^2 \right) \left(0.0253303 T^2 + 1. \Delta t^2 - 2. \gamma \Delta t^2 + 1. \gamma^2 \Delta t^2 \right) \right),$

$x_{t+\Delta t} \rightarrow \left(1. \left(1. T^4 x_t + 1. T^4 \dot{x}_t \Delta t + 1. r_{t+\Delta t} T^4 \Delta t^2 + 0.5 r_{t+\gamma \Delta t} T^4 \gamma \Delta t^2 - 2. r_{t+\Delta t} T^4 \gamma \Delta t^2 + \right. \right.$

$0.5 T^4 x_t \gamma \Delta t^2 - 19.7392 T^2 x_t \gamma \Delta t^2 - 0.25 r_{t+\gamma \Delta t} T^4 \gamma^2 \Delta t^2 + 1. r_{t+\Delta t} T^4 \gamma^2 \Delta t^2 -$

$0.25 T^4 x_t \gamma^2 \Delta t^2 + 19.7392 T^2 x_t \gamma^2 \Delta t^2 - 9.8696 T^2 \dot{x}_t \gamma^2 \Delta t^3 + 9.8696 T^2 \dot{x}_t \gamma^3 \Delta t^3 +$

$9.8696 r_{t+\Delta t} T^2 \gamma^2 \Delta t^4 - 19.7392 r_{t+\Delta t} T^2 \gamma^3 \Delta t^4 + 9.8696 r_{t+\Delta t} T^2 \gamma^4 \Delta t^4 \left. \right) \left/ \right.$

$\left(\left(1. T^2 + 9.8696 \gamma^2 \Delta t^2 \right) \left(1. T^2 + 39.4784 \Delta t^2 - 78.9568 \gamma \Delta t^2 + 39.4784 \gamma^2 \Delta t^2 \right) \right) \}$ }

recursive =

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{
  Collect[ $\ddot{x}_{t+\Delta t} -$ 
     $\left( -\frac{1}{-1 - \beta_2^2 (1 - \gamma)^2 \Delta t^2 \omega^2} \left( r_{t+\Delta t} + \beta_2 (1 - \gamma) \Delta t \omega^2 (-\dot{x}_t + ((1 - \beta_2) (1 - \gamma) \Delta t (-4 r_{t+\gamma \Delta t} + 4 x_t \omega^2 + 4 \dot{x}_t \gamma \Delta t \omega^2 + \ddot{x}_t \gamma^2 \Delta t^2 \omega^2)) \right) / (4 + \gamma^2 \Delta t^2 \omega^2) - \gamma \Delta t (\dot{x}_t (1 - \beta_1) - (\beta_1 (-4 r_{t+\gamma \Delta t} + 4 x_t \omega^2 + 4 \dot{x}_t \gamma \Delta t \omega^2 + \ddot{x}_t \gamma^2 \Delta t^2 \omega^2)) / (4 + \gamma^2 \Delta t^2 \omega^2)) \right) + \omega^2 (-x_t + ((1 - \beta_2) (1 - \gamma) \Delta t (-4 \dot{x}_t - 2 r_{t+\gamma \Delta t} \gamma \Delta t - 2 \ddot{x}_t \gamma \Delta t + 2 x_t \gamma \Delta t \omega^2 + \ddot{x}_t \gamma^2 \Delta t^2 \omega^2)) / (4 + \gamma^2 \Delta t^2 \omega^2) - \gamma \Delta t (\dot{x}_t (1 - \beta_1) - (\beta_1 (-4 \dot{x}_t - 2 r_{t+\gamma \Delta t} \gamma \Delta t - 2 \ddot{x}_t \gamma \Delta t + 2 x_t \gamma \Delta t \omega^2 + \ddot{x}_t \gamma^2 \Delta t^2 \omega^2)) / (4 + \gamma^2 \Delta t^2 \omega^2))) \right), \{ \ddot{x}_t, \dot{x}_t, x_t, r_{t+\Delta t}, r_{t+\gamma \Delta t} \}],$ 
    Collect[ $\ddot{x}_{t+\Delta t} -$ 
       $\left( - \left( (-4 \dot{x}_t - 4 r_{t+\gamma \Delta t} \Delta t + 4 r_{t+\gamma \Delta t} \beta_2 \Delta t - 4 r_{t+\Delta t} \beta_2 \Delta t + 4 r_{t+\gamma \Delta t} \gamma \Delta t - 4 \ddot{x}_t \gamma \Delta t - 4 r_{t+\gamma \Delta t} \beta_1 \gamma \Delta t + 4 \ddot{x}_t \beta_1 \gamma \Delta t - 4 r_{t+\gamma \Delta t} \beta_2 \gamma \Delta t + 4 r_{t+\Delta t} \beta_2 \gamma \Delta t + 4 x_t \Delta t \omega^2 - 4 x_t \gamma \Delta t \omega^2 + 4 x_t \beta_1 \gamma \Delta t \omega^2 + 4 \dot{x}_t \beta_2 \Delta t^2 \omega^2 - 4 \ddot{x}_t \beta_2^2 \Delta t^2 \omega^2 + 4 \dot{x}_t \gamma \Delta t^2 \omega^2 - 8 \dot{x}_t \beta_2 \gamma \Delta t^2 \omega^2 + 8 \dot{x}_t \beta_2^2 \gamma \Delta t^2 \omega^2 - 5 \dot{x}_t \gamma^2 \Delta t^2 \omega^2 + 4 \dot{x}_t \beta_1 \gamma^2 \Delta t^2 \omega^2 + 4 \dot{x}_t \beta_2 \gamma^2 \Delta t^2 \omega^2 - 4 \dot{x}_t \beta_2^2 \gamma^2 \Delta t^2 \omega^2 + 2 r_{t+\gamma \Delta t} \beta_2 \gamma \Delta t^3 \omega^2 + 2 \ddot{x}_t \beta_2 \gamma \Delta t^3 \omega^2 - 2 r_{t+\gamma \Delta t} \beta_2^2 \gamma \Delta t^3 \omega^2 - 2 \ddot{x}_t \beta_2^2 \gamma \Delta t^3 \omega^2 + \ddot{x}_t \gamma^2 \Delta t^3 \omega^2 - 4 r_{t+\gamma \Delta t} \beta_2 \gamma^2 \Delta t^3 \omega^2 - r_{t+\Delta t} \beta_2 \gamma^2 \Delta t^3 \omega^2 - 5 \ddot{x}_t \beta_2 \gamma^2 \Delta t^3 \omega^2 + 2 r_{t+\gamma \Delta t} \beta_1 \beta_2 \gamma^2 \Delta t^3 \omega^2 + 2 \ddot{x}_t \beta_1 \beta_2 \gamma^2 \Delta t^3 \omega^2 + 4 r_{t+\gamma \Delta t} \beta_2^2 \gamma^2 \Delta t^3 \omega^2 + 4 \ddot{x}_t \beta_2^2 \gamma^2 \Delta t^3 \omega^2 - 2 \ddot{x}_t \gamma^3 \Delta t^3 \omega^2 + 2 \ddot{x}_t \beta_1 \gamma^3 \Delta t^3 \omega^2 + 2 r_{t+\gamma \Delta t} \beta_2 \gamma^3 \Delta t^3 \omega^2 + r_{t+\Delta t} \beta_2 \gamma^3 \Delta t^3 \omega^2 + 3 \ddot{x}_t \beta_2 \gamma^3 \Delta t^3 \omega^2 - 2 r_{t+\gamma \Delta t} \beta_1 \beta_2 \gamma^3 \Delta t^3 \omega^2 - 2 \ddot{x}_t \beta_1 \beta_2 \gamma^3 \Delta t^3 \omega^2 - 2 r_{t+\gamma \Delta t} \beta_2^2 \gamma^3 \Delta t^3 \omega^2 - 2 \ddot{x}_t \beta_2^2 \gamma^3 \Delta t^3 \omega^2 - 2 x_t \beta_2 \gamma \Delta t^3 \omega^4 + 2 x_t \beta_2^2 \gamma \Delta t^3 \omega^4 + 5 x_t \beta_2 \gamma^2 \Delta t^3 \omega^4 - 2 x_t \beta_1 \beta_2 \gamma^2 \Delta t^3 \omega^4 - 4 x_t \beta_2^2 \gamma^2 \Delta t^3 \omega^4 - 3 x_t \beta_2 \gamma^3 \Delta t^3 \omega^4 + 2 x_t \beta_1 \beta_2 \gamma^3 \Delta t^3 \omega^4 + 2 x_t \beta_2^2 \gamma^3 \Delta t^3 \omega^4 - \dot{x}_t \beta_2 \gamma^2 \Delta t^4 \omega^4 + \dot{x}_t \beta_2^2 \gamma^2 \Delta t^4 \omega^4 + 3 \dot{x}_t \beta_2 \gamma^3 \Delta t^4 \omega^4 - 2 \ddot{x}_t \beta_1 \beta_2 \gamma^3 \Delta t^4 \omega^4 - 2 \ddot{x}_t \beta_2^2 \gamma^3 \Delta t^4 \omega^4 - 2 \ddot{x}_t \beta_2 \gamma^4 \Delta t^4 \omega^4 + 2 \ddot{x}_t \beta_1 \beta_2 \gamma^4 \Delta t^4 \omega^4 + \ddot{x}_t \beta_2^2 \gamma^4 \Delta t^4 \omega^4 \right) / ((4 + \gamma^2 \Delta t^2 \omega^2) (1 + \beta_2^2 \Delta t^2 \omega^2 - 2 \beta_2^2 \gamma \Delta t^2 \omega^2 + \beta_2^2 \gamma^2 \Delta t^2 \omega^2)) \right), \{ \ddot{x}_t, \dot{x}_t, x_t, r_{t+\Delta t}, r_{t+\gamma \Delta t} \}],$ 
    Collect[ $x_{t+\Delta t} -$ 
       $\left( - \left( (-4 x_t - 4 \dot{x}_t \Delta t - 4 r_{t+\gamma \Delta t} \beta_2 \Delta t^2 + 4 r_{t+\gamma \Delta t} \beta_2^2 \Delta t^2 - 4 r_{t+\Delta t} \beta_2^2 \Delta t^2 - 2 r_{t+\gamma \Delta t} \gamma \Delta t^2 - 2 \ddot{x}_t \gamma \Delta t^2 + 10 r_{t+\gamma \Delta t} \beta_2 \gamma \Delta t^2 - 2 \ddot{x}_t \beta_2 \gamma \Delta t^2 - 4 r_{t+\gamma \Delta t} \beta_1 \beta_2 \gamma \Delta t^2 + 4 \ddot{x}_t \beta_1 \beta_2 \gamma \Delta t^2 - 8 r_{t+\gamma \Delta t} \beta_2^2 \gamma \Delta t^2 + 8 r_{t+\Delta t} \beta_2^2 \gamma \Delta t^2 + 2 r_{t+\gamma \Delta t} \gamma^2 \Delta t^2 + 2 \ddot{x}_t \gamma^2 \Delta t^2 - 2 r_{t+\gamma \Delta t} \beta_1 \gamma^2 \Delta t^2 - 2 \ddot{x}_t \beta_1 \gamma^2 \Delta t^2 - 6 r_{t+\gamma \Delta t} \beta_2 \gamma^2 \Delta t^2 + 2 \ddot{x}_t \beta_2 \gamma^2 \Delta t^2 + 4 r_{t+\gamma \Delta t} \beta_1 \beta_2 \gamma^2 \Delta t^2 - 4 \ddot{x}_t \beta_1 \beta_2 \gamma^2 \Delta t^2 + 4 r_{t+\gamma \Delta t} \beta_2^2 \gamma^2 \Delta t^2 - 4 r_{t+\Delta t} \beta_2^2 \gamma^2 \Delta t^2 + 4 x_t \beta_2 \Delta t^2 \omega^2 - 4 x_t \beta_2^2 \Delta t^2 \omega^2 + 2 x_t \gamma \Delta t^2 \omega^2 - 10 x_t \beta_2 \gamma \Delta t^2 \omega^2 + 4 x_t \beta_1 \beta_2 \gamma \Delta t^2 \omega^2 + 8 x_t \beta_2^2 \gamma \Delta t^2 \omega^2 - 3 x_t \gamma^2 \Delta t^2 \omega^2 + 2 x_t \beta_1 \gamma^2 \Delta t^2 \omega^2 + 6 x_t \beta_2 \gamma^2 \Delta t^2 \omega^2 - 4 x_t \beta_1 \beta_2 \gamma^2 \Delta t^2 \omega^2 - 4 x_t \beta_2^2 \gamma^2 \Delta t^2 \omega^2 + 4 \dot{x}_t \beta_2 \gamma \Delta t^3 \omega^2 - 4 \ddot{x}_t \beta_2^2 \gamma \Delta t^3 \omega^2 + \dot{x}_t \gamma^2 \Delta t^3 \omega^2 - 10 \dot{x}_t \beta_2 \gamma^2 \Delta t^3 \omega^2 + 4 \dot{x}_t \beta_1 \beta_2 \gamma^2 \Delta t^3 \omega^2 + 8 \dot{x}_t \beta_2^2 \gamma^2 \Delta t^3 \omega^2 - 2 \ddot{x}_t \gamma^3 \Delta t^3 \omega^2 + 2 \ddot{x}_t \beta_1 \gamma^3 \Delta t^3 \omega^2 + 6 \ddot{x}_t \beta_2 \gamma^3 \Delta t^3 \omega^2 - 4 \ddot{x}_t \beta_1 \beta_2 \gamma^3 \Delta t^3 \omega^2 - 4 \ddot{x}_t \beta_2^2 \gamma^3 \Delta t^3 \omega^2 + \ddot{x}_t \beta_2 \gamma^2 \Delta t^4 \omega^2 - r_{t+\Delta t} \beta_2^2 \gamma^2 \Delta t^4 \omega^2 - \ddot{x}_t \beta_2^2 \gamma^2 \Delta t^4 \omega^2 - 3 \ddot{x}_t \beta_2 \gamma^3 \Delta t^4 \omega^2 + 2 \ddot{x}_t \beta_1 \beta_2 \gamma^3 \Delta t^4 \omega^2 + 2 r_{t+\Delta t} \beta_2^2 \gamma^3 \Delta t^4 \omega^2 + 2 \ddot{x}_t \beta_2^2 \gamma^3 \Delta t^4 \omega^2 + 2 \ddot{x}_t \beta_2 \gamma^4 \Delta t^4 \omega^2 - 2 \ddot{x}_t \beta_1 \beta_2 \gamma^4 \Delta t^4 \omega^2 - r_{t+\Delta t} \beta_2^2 \gamma^4 \Delta t^4 \omega^2 - \ddot{x}_t \beta_2^2 \gamma^4 \Delta t^4 \omega^2 \right) / ((4 + \gamma^2 \Delta t^2 \omega^2) (1 + \beta_2^2 \Delta t^2 \omega^2 - 2 \beta_2^2 \gamma \Delta t^2 \omega^2 + \beta_2^2 \gamma^2 \Delta t^2 \omega^2)) \right), \{ \ddot{x}_t, \dot{x}_t, x_t, r_{t+\gamma \Delta t}, r_{t+\Delta t} \}]$ 
    (* /. { -1 -  $\beta_2^2 (1 - \gamma)^2 \Delta t^2 \omega^2 \rightarrow \eta_1$ ,  $4 + \gamma^2 \Delta t^2 \omega^2 \rightarrow \eta_2$ ,  $1 + \beta_2^2 \Delta t^2 \omega^2 - 2 \beta_2^2 \gamma \Delta t^2 \omega^2 + \beta_2^2 \gamma^2 \Delta t^2 \omega^2 \rightarrow \eta_3$  } *)

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$$\begin{aligned}
& \left\{ 0. + \mathbf{x}_{t+\Delta t} + \frac{\mathbf{r}_{t+\Delta t}}{-1 - \frac{39.4784 (1-\gamma)^2 \Delta t^2}{T^2}} + \right. \\
& \mathbf{x}_t \left(- \frac{4 \pi^2}{T^2 \left(-1 - \frac{39.4784 (1-\gamma)^2 \Delta t^2}{T^2} \right)} + \frac{3117.09 (1-\gamma) \gamma \Delta t^2}{T^4 \left(-1 - \frac{39.4784 (1-\gamma)^2 \Delta t^2}{T^2} \right) \left(4 + \frac{4 \pi^2 \gamma^2 \Delta t^2}{T^2} \right)} + \right. \\
& \left. \left. \frac{1558.55 \gamma^2 \Delta t^2}{T^4 \left(-1 - \frac{39.4784 (1-\gamma)^2 \Delta t^2}{T^2} \right) \left(4 + \frac{4 \pi^2 \gamma^2 \Delta t^2}{T^2} \right)} \right) + \mathbf{r}_{t+\gamma \Delta t} \right. \\
& \left(- \frac{78.9568 (1-\gamma) \gamma \Delta t^2}{T^2 \left(-1 - \frac{39.4784 (1-\gamma)^2 \Delta t^2}{T^2} \right) \left(4 + \frac{4 \pi^2 \gamma^2 \Delta t^2}{T^2} \right)} - \frac{39.4784 \gamma^2 \Delta t^2}{T^2 \left(-1 - \frac{39.4784 (1-\gamma)^2 \Delta t^2}{T^2} \right) \left(4 + \frac{4 \pi^2 \gamma^2 \Delta t^2}{T^2} \right)} \right) + \\
& \dot{\mathbf{x}}_t \left(- \frac{39.4784 (1-\gamma) \Delta t}{T^2 \left(-1 - \frac{39.4784 (1-\gamma)^2 \Delta t^2}{T^2} \right)} - \frac{19.7392 \gamma \Delta t}{T^2 \left(-1 - \frac{39.4784 (1-\gamma)^2 \Delta t^2}{T^2} \right)} - \right. \\
& \frac{78.9568 \gamma \Delta t}{T^2 \left(-1 - \frac{39.4784 (1-\gamma)^2 \Delta t^2}{T^2} \right) \left(4 + \frac{4 \pi^2 \gamma^2 \Delta t^2}{T^2} \right)} + \frac{3117.09 (1-\gamma) \gamma^2 \Delta t^3}{T^4 \left(-1 - \frac{39.4784 (1-\gamma)^2 \Delta t^2}{T^2} \right) \left(4 + \frac{4 \pi^2 \gamma^2 \Delta t^2}{T^2} \right)} + \\
& \left. \frac{779.273 \gamma^3 \Delta t^3}{T^4 \left(-1 - \frac{39.4784 (1-\gamma)^2 \Delta t^2}{T^2} \right) \left(4 + \frac{4 \pi^2 \gamma^2 \Delta t^2}{T^2} \right)} \right) + \\
& \mathbf{x}_t \left(- \frac{19.7392 (1-\gamma) \gamma \Delta t^2}{T^2 \left(-1 - \frac{39.4784 (1-\gamma)^2 \Delta t^2}{T^2} \right)} - \frac{39.4784 \gamma^2 \Delta t^2}{T^2 \left(-1 - \frac{39.4784 (1-\gamma)^2 \Delta t^2}{T^2} \right) \left(4 + \frac{4 \pi^2 \gamma^2 \Delta t^2}{T^2} \right)} + \right. \\
& \left. \frac{779.273 (1-\gamma) \gamma^3 \Delta t^4}{T^4 \left(-1 - \frac{39.4784 (1-\gamma)^2 \Delta t^2}{T^2} \right) \left(4 + \frac{4 \pi^2 \gamma^2 \Delta t^2}{T^2} \right)} \right), 0. + \dot{\mathbf{x}}_{t+\Delta t} + \\
& \dot{\mathbf{x}}_t \left(- \left(4 \left/ \left(\left(1 + \frac{39.4784 \Delta t^2}{T^2} - \frac{78.9568 \gamma \Delta t^2}{T^2} + \frac{39.4784 \gamma^2 \Delta t^2}{T^2} \right) \left(4 + \frac{4 \pi^2 \gamma^2 \Delta t^2}{T^2} \right) \right) \right) \right) + \\
& (157.914 \gamma \Delta t^2) \left/ \left(T^2 \left(1 + \frac{39.4784 \Delta t^2}{T^2} - \frac{78.9568 \gamma \Delta t^2}{T^2} + \frac{39.4784 \gamma^2 \Delta t^2}{T^2} \right) \right. \right. \\
& \left. \left. \left(4 + \frac{4 \pi^2 \gamma^2 \Delta t^2}{T^2} \right) \right) - (118.435 \gamma^2 \Delta t^2) \right/ \\
& \left(T^2 \left(1 + \frac{39.4784 \Delta t^2}{T^2} - \frac{78.9568 \gamma \Delta t^2}{T^2} + \frac{39.4784 \gamma^2 \Delta t^2}{T^2} \right) \left(4 + \frac{4 \pi^2 \gamma^2 \Delta t^2}{T^2} \right) \right) \right) + \mathbf{x}_t \\
& \left((16 \pi^2 \Delta t) \left/ \left(T^2 \left(1 + \frac{39.4784 \Delta t^2}{T^2} - \frac{78.9568 \gamma \Delta t^2}{T^2} + \frac{39.4784 \gamma^2 \Delta t^2}{T^2} \right) \left(4 + \frac{4 \pi^2 \gamma^2 \Delta t^2}{T^2} \right) \right) \right) + \right. \\
& (78.9568 \gamma \Delta t) \left/ \right. \\
& \left(T^2 \left(1 + \frac{39.4784 \Delta t^2}{T^2} - \frac{78.9568 \gamma \Delta t^2}{T^2} + \frac{39.4784 \gamma^2 \Delta t^2}{T^2} \right) \left(4 + \frac{4 \pi^2 \gamma^2 \Delta t^2}{T^2} \right) \right) - \\
& (16 \pi^2 \gamma \Delta t) \left/ \left(T^2 \left(1 + \frac{39.4784 \Delta t^2}{T^2} - \frac{78.9568 \gamma \Delta t^2}{T^2} + \frac{39.4784 \gamma^2 \Delta t^2}{T^2} \right) \right. \right. \\
& \left. \left. \left(4 + \frac{4 \pi^2 \gamma^2 \Delta t^2}{T^2} \right) \right) - (4.54747 \times 10^{-13} \gamma^3 \Delta t^3) \right/ \\
& \left(T^4 \left(1 + \frac{39.4784 \Delta t^2}{T^2} - \frac{78.9568 \gamma \Delta t^2}{T^2} + \frac{39.4784 \gamma^2 \Delta t^2}{T^2} \right) \left(4 + \frac{4 \pi^2 \gamma^2 \Delta t^2}{T^2} \right) \right) \right) + \\
& \mathbf{r}_{t+\gamma \Delta t} \left(- \left((2. \gamma \Delta t) \left/ \left(\left(1 + \frac{39.4784 \Delta t^2}{T^2} - \frac{78.9568 \gamma \Delta t^2}{T^2} + \frac{39.4784 \gamma^2 \Delta t^2}{T^2} \right) \right. \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left(4 + \frac{4 \pi^2 \gamma^2 \Delta t^2}{T^2} \right) \Big) \Big) + (39.4784 \gamma^2 \Delta t^3) / \\
& \left(T^2 \left(1 + \frac{39.4784 \Delta t^2}{T^2} - \frac{78.9568 \gamma \Delta t^2}{T^2} + \frac{39.4784 \gamma^2 \Delta t^2}{T^2} \right) \left(4 + \frac{4 \pi^2 \gamma^2 \Delta t^2}{T^2} \right) \right) - \\
& (39.4784 \gamma^3 \Delta t^3) / \left(T^2 \left(1 + \frac{39.4784 \Delta t^2}{T^2} - \frac{78.9568 \gamma \Delta t^2}{T^2} + \frac{39.4784 \gamma^2 \Delta t^2}{T^2} \right) \right. \\
& \left. \left(4 + \frac{4 \pi^2 \gamma^2 \Delta t^2}{T^2} \right) \right) + x_t \left(- \left((2 \cdot \gamma \Delta t) / \right. \right. \\
& \left. \left(1 + \frac{39.4784 \Delta t^2}{T^2} - \frac{78.9568 \gamma \Delta t^2}{T^2} + \frac{39.4784 \gamma^2 \Delta t^2}{T^2} \right) \left(4 + \frac{4 \pi^2 \gamma^2 \Delta t^2}{T^2} \right) \right) \Big) + \\
& (39.4784 \gamma^2 \Delta t^3) / \left(T^2 \left(1 + \frac{39.4784 \Delta t^2}{T^2} - \frac{78.9568 \gamma \Delta t^2}{T^2} + \frac{39.4784 \gamma^2 \Delta t^2}{T^2} \right) \right. \\
& \left. \left(4 + \frac{4 \pi^2 \gamma^2 \Delta t^2}{T^2} \right) \right) - (39.4784 \gamma^3 \Delta t^3) / \\
& \left(T^2 \left(1 + \frac{39.4784 \Delta t^2}{T^2} - \frac{78.9568 \gamma \Delta t^2}{T^2} + \frac{39.4784 \gamma^2 \Delta t^2}{T^2} \right) \left(4 + \frac{4 \pi^2 \gamma^2 \Delta t^2}{T^2} \right) \right) + r_{t+\Delta t} \\
& \left(- \left((4 \cdot \Delta t) / \left(\left(1 + \frac{39.4784 \Delta t^2}{T^2} - \frac{78.9568 \gamma \Delta t^2}{T^2} + \frac{39.4784 \gamma^2 \Delta t^2}{T^2} \right) \left(4 + \frac{4 \pi^2 \gamma^2 \Delta t^2}{T^2} \right) \right) \right) + \right. \\
& (4 \cdot \gamma \Delta t) / \left(\left(1 + \frac{39.4784 \Delta t^2}{T^2} - \frac{78.9568 \gamma \Delta t^2}{T^2} + \frac{39.4784 \gamma^2 \Delta t^2}{T^2} \right) \left(4 + \frac{4 \pi^2 \gamma^2 \Delta t^2}{T^2} \right) \right) - \\
& (39.4784 \gamma^2 \Delta t^3) / \left(T^2 \left(1 + \frac{39.4784 \Delta t^2}{T^2} - \frac{78.9568 \gamma \Delta t^2}{T^2} + \frac{39.4784 \gamma^2 \Delta t^2}{T^2} \right) \right. \\
& \left. \left(4 + \frac{4 \pi^2 \gamma^2 \Delta t^2}{T^2} \right) \right) + (39.4784 \gamma^3 \Delta t^3) / \\
& \left(T^2 \left(1 + \frac{39.4784 \Delta t^2}{T^2} - \frac{78.9568 \gamma \Delta t^2}{T^2} + \frac{39.4784 \gamma^2 \Delta t^2}{T^2} \right) \left(4 + \frac{4 \pi^2 \gamma^2 \Delta t^2}{T^2} \right) \right) \Big), \\
& 0. + x_{t+\Delta t} + r_{t+\gamma \Delta t} \left(- \left((2 \cdot \gamma \Delta t^2) / \left(\left(1 + \frac{39.4784 \Delta t^2}{T^2} - \frac{78.9568 \gamma \Delta t^2}{T^2} + \frac{39.4784 \gamma^2 \Delta t^2}{T^2} \right) \right. \right. \right. \\
& \left. \left. \left(4 + \frac{4 \pi^2 \gamma^2 \Delta t^2}{T^2} \right) \right) \right) + (1 \cdot \gamma^2 \Delta t^2) / \\
& \left(\left(1 + \frac{39.4784 \Delta t^2}{T^2} - \frac{78.9568 \gamma \Delta t^2}{T^2} + \frac{39.4784 \gamma^2 \Delta t^2}{T^2} \right) \left(4 + \frac{4 \pi^2 \gamma^2 \Delta t^2}{T^2} \right) \right) \Big) + \\
& x_t \left(- \left((2 \cdot \gamma \Delta t^2) / \left(\left(1 + \frac{39.4784 \Delta t^2}{T^2} - \frac{78.9568 \gamma \Delta t^2}{T^2} + \frac{39.4784 \gamma^2 \Delta t^2}{T^2} \right) \right. \right. \right. \\
& \left. \left. \left(4 + \frac{4 \pi^2 \gamma^2 \Delta t^2}{T^2} \right) \right) \right) + (1 \cdot \gamma^2 \Delta t^2) / \\
& \left(\left(1 + \frac{39.4784 \Delta t^2}{T^2} - \frac{78.9568 \gamma \Delta t^2}{T^2} + \frac{39.4784 \gamma^2 \Delta t^2}{T^2} \right) \left(4 + \frac{4 \pi^2 \gamma^2 \Delta t^2}{T^2} \right) \right) \Big) + \\
& x_t \left(- \left(4 / \left(\left(1 + \frac{39.4784 \Delta t^2}{T^2} - \frac{78.9568 \gamma \Delta t^2}{T^2} + \frac{39.4784 \gamma^2 \Delta t^2}{T^2} \right) \left(4 + \frac{4 \pi^2 \gamma^2 \Delta t^2}{T^2} \right) \right) \right) + \right. \\
& (78.9568 \gamma \Delta t^2) / \left(T^2 \left(1 + \frac{39.4784 \Delta t^2}{T^2} - \frac{78.9568 \gamma \Delta t^2}{T^2} + \frac{39.4784 \gamma^2 \Delta t^2}{T^2} \right) \right. \\
& \left. \left(4 + \frac{4 \pi^2 \gamma^2 \Delta t^2}{T^2} \right) \right) - (78.9568 \gamma^2 \Delta t^2) / \\
& \left(T^2 \left(1 + \frac{39.4784 \Delta t^2}{T^2} - \frac{78.9568 \gamma \Delta t^2}{T^2} + \frac{39.4784 \gamma^2 \Delta t^2}{T^2} \right) \left(4 + \frac{4 \pi^2 \gamma^2 \Delta t^2}{T^2} \right) \right) \Big) + \\
& \dot{x}_t \left(- \left((4 \Delta t) / \left(\left(1 + \frac{39.4784 \Delta t^2}{T^2} - \frac{78.9568 \gamma \Delta t^2}{T^2} + \frac{39.4784 \gamma^2 \Delta t^2}{T^2} \right) \left(4 + \frac{4 \pi^2 \gamma^2 \Delta t^2}{T^2} \right) \right) \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& \left(39.4784 \gamma^2 \Delta t^3 \right) / \left(T^2 \left(1 + \frac{39.4784 \Delta t^2}{T^2} - \frac{78.9568 \gamma \Delta t^2}{T^2} + \frac{39.4784 \gamma^2 \Delta t^2}{T^2} \right) \right. \\
& \quad \left. \left(4 + \frac{4 \pi^2 \gamma^2 \Delta t^2}{T^2} \right) \right) - \left(39.4784 \gamma^3 \Delta t^3 \right) / \\
& \quad \left(T^2 \left(1 + \frac{39.4784 \Delta t^2}{T^2} - \frac{78.9568 \gamma \Delta t^2}{T^2} + \frac{39.4784 \gamma^2 \Delta t^2}{T^2} \right) \left(4 + \frac{4 \pi^2 \gamma^2 \Delta t^2}{T^2} \right) \right) + r_{t+\Delta t} \\
& \left(- \left(4 \Delta t^2 \right) / \left(\left(1 + \frac{39.4784 \Delta t^2}{T^2} - \frac{78.9568 \gamma \Delta t^2}{T^2} + \frac{39.4784 \gamma^2 \Delta t^2}{T^2} \right) \left(4 + \frac{4 \pi^2 \gamma^2 \Delta t^2}{T^2} \right) \right) \right) + \\
& \quad \left(8 \gamma \Delta t^2 \right) / \left(\left(1 + \frac{39.4784 \Delta t^2}{T^2} - \frac{78.9568 \gamma \Delta t^2}{T^2} + \frac{39.4784 \gamma^2 \Delta t^2}{T^2} \right) \left(4 + \frac{4 \pi^2 \gamma^2 \Delta t^2}{T^2} \right) \right) - \\
& \quad \left(4 \gamma^2 \Delta t^2 \right) / \left(\left(1 + \frac{39.4784 \Delta t^2}{T^2} - \frac{78.9568 \gamma \Delta t^2}{T^2} + \frac{39.4784 \gamma^2 \Delta t^2}{T^2} \right) \left(4 + \frac{4 \pi^2 \gamma^2 \Delta t^2}{T^2} \right) \right) - \\
& \quad \left(39.4784 \gamma^2 \Delta t^4 \right) / \\
& \quad \left(T^2 \left(1 + \frac{39.4784 \Delta t^2}{T^2} - \frac{78.9568 \gamma \Delta t^2}{T^2} + \frac{39.4784 \gamma^2 \Delta t^2}{T^2} \right) \left(4 + \frac{4 \pi^2 \gamma^2 \Delta t^2}{T^2} \right) \right) + \\
& \quad \left(78.9568 \gamma^3 \Delta t^4 \right) / \left(T^2 \left(1 + \frac{39.4784 \Delta t^2}{T^2} - \frac{78.9568 \gamma \Delta t^2}{T^2} + \frac{39.4784 \gamma^2 \Delta t^2}{T^2} \right) \right. \\
& \quad \left. \left(4 + \frac{4 \pi^2 \gamma^2 \Delta t^2}{T^2} \right) \right) - \left(39.4784 \gamma^4 \Delta t^4 \right) / \\
& \quad \left(T^2 \left(1 + \frac{39.4784 \Delta t^2}{T^2} - \frac{78.9568 \gamma \Delta t^2}{T^2} + \frac{39.4784 \gamma^2 \Delta t^2}{T^2} \right) \left(4 + \frac{4 \pi^2 \gamma^2 \Delta t^2}{T^2} \right) \right) \}
\end{aligned}$$

a =

$$\begin{aligned}
& \left(\begin{array}{lll} \text{Coefficient}[\text{Part}[\text{recursive}, 1], \dot{x}_t] & \text{Coefficient}[\text{Part}[\text{recursive}, 1], \dot{x}_t] & \text{Coeff} \\ \text{Coefficient}[\text{Part}[\text{recursive}, 2], \dot{x}_t] & \text{Coefficient}[\text{Part}[\text{recursive}, 2], \dot{x}_t] & \text{Coeff} \\ \text{Coefficient}[\text{Part}[\text{recursive}, 3], \dot{x}_t] & \text{Coefficient}[\text{Part}[\text{recursive}, 3], \dot{x}_t] & \text{Coeff} \end{array} \right. \\
& \left. \right);
\end{aligned}$$

a // MatrixForm // Simplify

$$\begin{pmatrix} \frac{0. + T^2 \gamma (-19.7392 + 9.8696 \gamma) \Delta t^2}{(T^2 + 39.4784 (1. - 1. \gamma)^2 \Delta t^2) (T^2 + 9.8696 \gamma^2 \Delta t^2)} & \frac{-39.4784 T^2 \Delta t + (389.636 - 389.636 \gamma) \gamma^2 \Delta t^3}{(T^2 + 39.4784 (1. - 1. \gamma)^2 \Delta t^2) (T^2 + 9.8696 \gamma^2 \Delta t^2)} & \frac{-39.4784 T^2 + (779.27}{(T^2 + 39.4784 (1. - 1. \gamma)^2 \Delta t^2) (T^2 + 9.8696 \gamma^2 \Delta t^2)} \\ \frac{T^2 \gamma \Delta t (0.5 T^2 + \gamma (-9.8696 + 9.8696 \gamma) \Delta t^2)}{(1. T^2 + 39.4784 (1. - 1. \gamma)^2 \Delta t^2) (T^2 + \pi^2 \gamma^2 \Delta t^2)} & \frac{T^2 (T^2 + \gamma (-39.4784 + 29.6088 \gamma) \Delta t^2)}{(T^2 + 39.4784 (1. - 1. \gamma)^2 \Delta t^2) (T^2 + 9.8696 \gamma^2 \Delta t^2)} & \frac{T^2 (-39.4784 + 19.7392 \gamma)}{(T^2 + 39.4784 (1. - 1. \gamma)^2 \Delta t^2) (T^2 + 9.8696 \gamma^2 \Delta t^2)} \\ - \frac{0.25 T^4 \gamma (-2. + 1. \gamma) \Delta t^2}{(1. T^2 + 39.4784 (1. - 1. \gamma)^2 \Delta t^2) (T^2 + \pi^2 \gamma^2 \Delta t^2)} & \frac{T^2 \Delta t (1. T^2 + \gamma^2 (-9.8696 + 9.8696 \gamma) \Delta t^2)}{(1. T^2 + 39.4784 (1. - 1. \gamma)^2 \Delta t^2) (T^2 + \pi^2 \gamma^2 \Delta t^2)} & \frac{T^2 (T^2 + \gamma (-19.7392}{(1. T^2 + 39.4784 (1. - 1. \gamma)^2 \Delta t^2) (T^2 + \pi^2 \gamma^2 \Delta t^2)} \end{pmatrix}$$

ClearAll["Global`*"]

$$\text{la} = - \left(\begin{array}{l} \text{Coefficient}[\text{Part}[\text{recursive}, 1], r_{t+\gamma \Delta t}] \\ \text{Coefficient}[\text{Part}[\text{recursive}, 2], r_{t+\gamma \Delta t}] \\ \text{Coefficient}[\text{Part}[\text{recursive}, 3], r_{t+\gamma \Delta t}] \end{array} \right);$$

la // MatrixForm // Simplify

$$\begin{pmatrix} \frac{9.8696 T^2 \gamma (-2. + 1. \gamma) \Delta t^2}{(T^2 + 39.4784 (1. - 1. \gamma)^2 \Delta t^2) (T^2 + 9.8696 \gamma^2 \Delta t^2)} \\ \frac{T^2 \gamma \Delta t (0.5 T^2 + \gamma (-9.8696 + 9.8696 \gamma) \Delta t^2)}{(1. T^2 + 39.4784 (1. - 1. \gamma)^2 \Delta t^2) (T^2 + \pi^2 \gamma^2 \Delta t^2)} \\ - \frac{0.25 T^4 \gamma (-2. + 1. \gamma) \Delta t^2}{(1. T^2 + 39.4784 (1. - 1. \gamma)^2 \Delta t^2) (T^2 + \pi^2 \gamma^2 \Delta t^2)} \end{pmatrix}$$

$$\mathbf{lb} = - \begin{pmatrix} \text{Coefficient}[\text{Part}[\text{recursive}, 1], \mathbf{r}_{t+\Delta t}] \\ \text{Coefficient}[\text{Part}[\text{recursive}, 2], \mathbf{r}_{t+\Delta t}] \\ \text{Coefficient}[\text{Part}[\text{recursive}, 3], \mathbf{r}_{t+\Delta t}] \end{pmatrix};$$

lb // MatrixForm // Simplify

$$\begin{pmatrix} \frac{T^2}{T^2 + 39.4784 (1. - 1. \gamma)^2 \Delta t^2} \\ \frac{T^2 \Delta t (T^2 (1. - 1. \gamma) + (9.8696 - 9.8696 \gamma) \gamma^2 \Delta t^2)}{(1. T^2 + 39.4784 (1. - 1. \gamma)^2 \Delta t^2) (T^2 + \pi^2 \gamma^2 \Delta t^2)} \\ \frac{T^2 (1. - 1. \gamma)^2 \Delta t^2 (1. T^2 + 9.8696 \gamma^2 \Delta t^2)}{(1. T^2 + 39.4784 (1. - 1. \gamma)^2 \Delta t^2) (T^2 + \pi^2 \gamma^2 \Delta t^2)} \end{pmatrix}$$

eigA = Eigenvalues[a] /. Δt → p T // Expand // Simplify // Cancel;

eigA // MatrixForm // Simplify

$$\begin{pmatrix} 5.6751 \times 10^{-57} \text{Root}[-6.54678 \times 10^{150} p^4 T^{12} \gamma^2 - 1.30936 \times 10^{151} p^4 T^{12} \gamma^3 - 1.30936 \times 10^{151} p^4 T^{12} \gamma^4 - 4.18994 \times 10^{152} p^6 T^{12} \gamma^4 + 4.18994 \times 10^{152} p^6 T^{12} \gamma^5 - 4.18994 \times 10^{152} p^6 T^{12} \gamma^6 - 8.37988 \times 10^{152} p^8 \gamma^8 + (1.9922 \times 10^{109} + 7.8649 \times 10^{110} p^2 - 1.57298 \times 10^{111} p^2 \gamma + 1.17974 \times 10^{111} p^2 \gamma^2 + 1.55247 \times 10^{112} p^4 \gamma^2 - 3.10494 \times 10^{112} p^4 \gamma^3 + 1.74653 \times 10^{112} p^4 \gamma^4 + 7.66113 \times 10^{112} p^6 \gamma^4 - 1.53223 \times 10^{113} p^6 \gamma^5 + 7.66113 \times 10^{112} p^6 \gamma^6) \#1 + (-8.92682 \times 10^{54} + 3.52417 \times 10^{56} p^2 \gamma - 2.64313 \times 10^{56} p^2 \gamma^2) \#1^2 + 1. \#1^3 \&, 2] \\ 5.6751 \times 10^{-57} \text{Root}[-6.54678 \times 10^{150} p^4 T^{12} \gamma^2 - 1.30936 \times 10^{151} p^4 T^{12} \gamma^3 - 1.30936 \times 10^{151} p^4 T^{12} \gamma^4 - 4.18994 \times 10^{152} p^6 T^{12} \gamma^4 + 4.18994 \times 10^{152} p^6 T^{12} \gamma^5 - 4.18994 \times 10^{152} p^6 T^{12} \gamma^6 - 8.37988 \times 10^{152} p^8 \gamma^8 + (1.9922 \times 10^{109} + 7.8649 \times 10^{110} p^2 - 1.57298 \times 10^{111} p^2 \gamma + 1.17974 \times 10^{111} p^2 \gamma^2 + 1.55247 \times 10^{112} p^4 \gamma^2 - 3.10494 \times 10^{112} p^4 \gamma^3 + 1.74653 \times 10^{112} p^4 \gamma^4 + 7.66113 \times 10^{112} p^6 \gamma^4 - 1.53223 \times 10^{113} p^6 \gamma^5 + 7.66113 \times 10^{112} p^6 \gamma^6) \#1 + (-8.92682 \times 10^{54} + 3.52417 \times 10^{56} p^2 \gamma - 2.64313 \times 10^{56} p^2 \gamma^2) \#1^2 + 1. \#1^3 \&, 2] \\ 5.6751 \times 10^{-57} \text{Root}[-6.54678 \times 10^{150} p^4 T^{12} \gamma^2 - 1.30936 \times 10^{151} p^4 T^{12} \gamma^3 - 1.30936 \times 10^{151} p^4 T^{12} \gamma^4 - 4.18994 \times 10^{152} p^6 T^{12} \gamma^4 + 4.18994 \times 10^{152} p^6 T^{12} \gamma^5 - 4.18994 \times 10^{152} p^6 T^{12} \gamma^6 - 8.37988 \times 10^{152} p^8 \gamma^8 + (1.9922 \times 10^{109} + 7.8649 \times 10^{110} p^2 - 1.57298 \times 10^{111} p^2 \gamma + 1.17974 \times 10^{111} p^2 \gamma^2 + 1.55247 \times 10^{112} p^4 \gamma^2 - 3.10494 \times 10^{112} p^4 \gamma^3 + 1.74653 \times 10^{112} p^4 \gamma^4 + 7.66113 \times 10^{112} p^6 \gamma^4 - 1.53223 \times 10^{113} p^6 \gamma^5 + 7.66113 \times 10^{112} p^6 \gamma^6) \#1 + (-8.92682 \times 10^{54} + 3.52417 \times 10^{56} p^2 \gamma - 2.64313 \times 10^{56} p^2 \gamma^2) \#1^2 + 1. \#1^3 \&, 2] \end{pmatrix}$$

$$\xi = 0;$$

$$\omega = 2 \text{Pi} / T;$$

λ1 = Part[eigA, 2] /. T → 1 // Simplify // Apart

$$\begin{pmatrix} 5.6751 \times 10^{-57} \text{Root}[-6.54678 \times 10^{150} p^4 \gamma^2 - 1.30936 \times 10^{151} p^4 \gamma^3 - 1.30936 \times 10^{151} p^4 \gamma^4 - 4.18994 \times 10^{152} p^6 \gamma^4 + 4.18994 \times 10^{152} p^6 \gamma^5 - 4.18994 \times 10^{152} p^6 \gamma^6 - 8.37988 \times 10^{152} p^8 \gamma^8 + (1.9922 \times 10^{109} + 7.8649 \times 10^{110} p^2 - 1.57298 \times 10^{111} p^2 \gamma + 1.17974 \times 10^{111} p^2 \gamma^2 + 1.55247 \times 10^{112} p^4 \gamma^2 - 3.10494 \times 10^{112} p^4 \gamma^3 + 1.74653 \times 10^{112} p^4 \gamma^4 + 7.66113 \times 10^{112} p^6 \gamma^4 - 1.53223 \times 10^{113} p^6 \gamma^5 + 7.66113 \times 10^{112} p^6 \gamma^6) \#1 + (-8.92682 \times 10^{54} + 3.52417 \times 10^{56} p^2 \gamma - 2.64313 \times 10^{56} p^2 \gamma^2) \#1^2 + 1. \#1^3 \&, 2] \\ (0.02533303 + 1. p^2 - 2. p^2 \gamma + 1. p^2 \gamma^2) (1. + 9.8696 p^2 \gamma^2) \end{pmatrix}$$

$$\beta_1 = 0.5$$

$$0.5$$

$$\beta_2 = 2 \beta_1$$

$$1.$$

$$\mathbf{pe} = \frac{\Omega_o}{\Omega_d} - 1$$

$$-1 + \frac{\Omega_o}{\Omega_d}$$

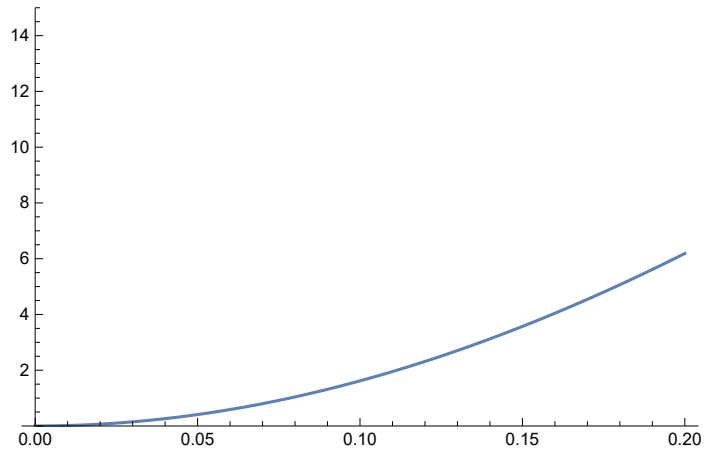
$$\Omega_o = \omega \Delta t /. \frac{\Delta t}{T} \rightarrow p$$

$$2 p \pi$$

$$\bar{\Omega}_d = \text{ArcTan}\left[\frac{2 \pi \sqrt{p^2 (-36 + 5 p^2 \pi^2)^2}}{36 - 47 p^2 \pi^2}\right] \text{ (*simply copy paste this from above*)}$$

$$\text{ArcTan}\left[\frac{2 \pi \sqrt{p^2 (-36 + 5 p^2 \pi^2)^2}}{36 - 47 p^2 \pi^2}\right]$$

`Plot[(pe) * 100, {p, 0, .2}, PlotRange -> {0, 20}]`



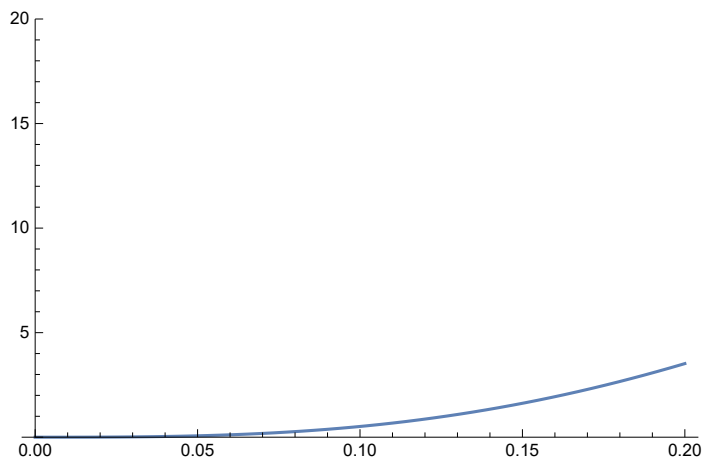
$$AD = 1 - \text{Exp}\left[-2 \pi \bar{\xi} \frac{\Omega_o}{\bar{\Omega}_d}\right]$$

$$1 - \text{Abs}\left[\frac{36 - 47 p^2 \pi^2 - 2 \pi \sqrt{-p^2 (-36 + 5 p^2 \pi^2)^2}}{(4 + p^2 \pi^2) (9 + 4 p^2 \pi^2)}\right] \frac{2 \pi}{\text{ArcTan}\left[\frac{2 \pi \sqrt{-p^2 (-36 + 5 p^2 \pi^2)^2}}{36 - 47 p^2 \pi^2}\right]}$$

$$\bar{\xi} = -\frac{1}{\Omega_o} \text{Log}[\text{Abs}[\lambda_1]]$$

$$-\frac{\text{Log}\left[\text{Abs}\left[\frac{36 - 47 p^2 \pi^2 - 2 \pi \sqrt{-p^2 (-36 + 5 p^2 \pi^2)^2}}{(4 + p^2 \pi^2) (9 + 4 p^2 \pi^2)}\right]\right]}{2 p \pi}$$

`Plot[(AD) * 100, {p, 0, .2}, PlotRange -> {0, 20}]`



$$\mathbf{aNot} = - \begin{pmatrix} \text{Coefficient}[\text{Part}[\text{recursive}, 2], \dot{\mathbf{x}}_t] & \text{Coefficient}[\text{Part}[\text{recursive}, 2], \mathbf{x}_t] \\ \text{Coefficient}[\text{Part}[\text{recursive}, 3], \dot{\mathbf{x}}_t] & \text{Coefficient}[\text{Part}[\text{recursive}, 3], \mathbf{x}_t] \end{pmatrix}$$

$$\left\{ \left\{ \frac{144}{\left(9 + \frac{4 \pi^2 \Delta t^2}{T^2}\right) \left(16 + \frac{4 \pi^2 \Delta t^2}{T^2}\right)} - \frac{188 \pi^2 \Delta t^2}{T^2 \left(9 + \frac{4 \pi^2 \Delta t^2}{T^2}\right) \left(16 + \frac{4 \pi^2 \Delta t^2}{T^2}\right)}, \right. \right.$$

$$- \frac{384 \pi^2 \Delta t}{T^2 \left(9 + \frac{4 \pi^2 \Delta t^2}{T^2}\right) \left(16 + \frac{4 \pi^2 \Delta t^2}{T^2}\right)} + \frac{16 \pi^4 \Delta t^3}{T^4 \left(9 + \frac{4 \pi^2 \Delta t^2}{T^2}\right) \left(16 + \frac{4 \pi^2 \Delta t^2}{T^2}\right)} \left. \right\},$$

$$\left\{ \frac{144 \Delta t}{\left(9 + \frac{4 \pi^2 \Delta t^2}{T^2}\right) \left(16 + \frac{4 \pi^2 \Delta t^2}{T^2}\right)} - \frac{20 \pi^2 \Delta t^3}{T^2 \left(9 + \frac{4 \pi^2 \Delta t^2}{T^2}\right) \left(16 + \frac{4 \pi^2 \Delta t^2}{T^2}\right)}, \right.$$

$$\left. \left. \frac{144}{\left(9 + \frac{4 \pi^2 \Delta t^2}{T^2}\right) \left(16 + \frac{4 \pi^2 \Delta t^2}{T^2}\right)} - \frac{76 \pi^2 \Delta t^2}{T^2 \left(9 + \frac{4 \pi^2 \Delta t^2}{T^2}\right) \left(16 + \frac{4 \pi^2 \Delta t^2}{T^2}\right)} \right\} \right\}$$

eigaNot = Eigenvalues[aNot] /. Δt → p T // Expand // Simplify // Cancel;

ppp = {0, Part[eigaNot, 1], Part[eigaNot, 2]}

$$\{0,$$

$$\left(36 T^4 - 33 p^2 \pi^2 T^4 - 2 \pi \sqrt{-p^2 (864 - 205 p^2 \pi^2 + 5 p^4 \pi^4) T^8}\right) / \left((4 + p^2 \pi^2) (9 + 4 p^2 \pi^2) T^4\right),$$

$$\left(36 T^4 - 33 p^2 \pi^2 T^4 + 2 \pi \sqrt{-p^2 (864 - 205 p^2 \pi^2 + 5 p^4 \pi^4) T^8}\right) / \left((4 + p^2 \pi^2) (9 + 4 p^2 \pi^2) T^4\right) \}$$

qqq = eiga

$$\{0, \frac{36 T^4 - 47 p^2 \pi^2 T^4 - 2 \pi \sqrt{-p^2 (-36 + 5 p^2 \pi^2)^2 T^8}}{(4 + p^2 \pi^2) (9 + 4 p^2 \pi^2) T^4},$$

$$\frac{36 T^4 - 47 p^2 \pi^2 T^4 + 2 \pi \sqrt{-p^2 (-36 + 5 p^2 \pi^2)^2 T^8}}{(4 + p^2 \pi^2) (9 + 4 p^2 \pi^2) T^4} \}$$

ppp - qqq // Simplify

$$\{0, \left(2 \pi \left(7 p^2 \pi T^4 + \sqrt{-p^2 (-36 + 5 p^2 \pi^2)^2 T^8} - \sqrt{-p^2 (864 - 205 p^2 \pi^2 + 5 p^4 \pi^4) T^8}\right)\right) /$$

$$\left((4 + p^2 \pi^2) (9 + 4 p^2 \pi^2) T^4\right),$$

$$\left(2 \pi \left(7 p^2 \pi T^4 - \sqrt{-p^2 (-36 + 5 p^2 \pi^2)^2 T^8} + \sqrt{-p^2 (864 - 205 p^2 \pi^2 + 5 p^4 \pi^4) T^8}\right)\right) /$$

$$\left((4 + p^2 \pi^2) (9 + 4 p^2 \pi^2) T^4\right) \}$$