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<< Notation`;

Symbolize[ $u^t$ ]; Symbolize[ $\dot{u}^t$ ]; Symbolize[ $\ddot{u}^t$ ]; Symbolize[ $u^{t+\Delta t}$ ];
Symbolize[ $\dot{u}^{t+\Delta t}$ ]; Symbolize[ $\ddot{u}^{t+\Delta t}$ ]; Symbolize[ $u^{t+\gamma\Delta t}$ ]; Symbolize[ $\dot{u}^{t+\gamma\Delta t}$ ];
Symbolize[ $\ddot{u}^{t+\gamma\Delta t}$ ]; Symbolize[ $\beta_1$ ]; Symbolize[ $\beta_2$ ]; Symbolize[ $a_k$ ];
Symbolize[ $n_x$ ]; Symbolize[ $n_t$ ]; Symbolize[ $u_1^{t+\Delta t}$ ]; Symbolize[ $u_2^{t+\Delta t}$ ];
Symbolize[ $u_3^{t+\Delta t}$ ]; Symbolize[ $u_4^{t+\Delta t}$ ]; Symbolize[ $u_5^{t+\Delta t}$ ]; Symbolize[ $u_6^{t+\Delta t}$ ];
Symbolize[ $u_1^{t+\gamma\Delta t}$ ]; Symbolize[ $u_2^{t+\gamma\Delta t}$ ]; Symbolize[ $u_3^{t+\gamma\Delta t}$ ]; Symbolize[ $u_4^{t+\gamma\Delta t}$ ];
Symbolize[ $u_5^{t+\gamma\Delta t}$ ]; Symbolize[ $u_6^{t+\gamma\Delta t}$ ]; Symbolize[ $u_1^t$ ]; Symbolize[ $u_2^t$ ];
Symbolize[ $u_3^t$ ]; Symbolize[ $u_4^t$ ]; Symbolize[ $u_5^t$ ]; Symbolize[ $u_0^1$ ];
Symbolize[ $u_1^1$ ]; Symbolize[ $u_2^1$ ]; Symbolize[ $u_0^{1/2}$ ]; Symbolize[ $u_1^{1/2}$ ];
Symbolize[ $u_2^{1/2}$ ]; Symbolize[ $u_0^0$ ]; Symbolize[ $u_1^0$ ]; Symbolize[ $u_2^0$ ];

ClearAll["Global`*"]

 $\gamma = \frac{1}{2}; \beta_2 = 2 \beta_1;$ 

eq1 = m  $\ddot{u}^t$  +  $c_0^2 k u^t$  == 0;
eq2 = m  $\ddot{u}^{t+\gamma\Delta t}$  +  $c_0^2 k u^{t+\gamma\Delta t}$  == 0;
eq3 = m  $\ddot{u}^{t+\Delta t}$  +  $c_0^2 k u^{t+\Delta t}$  == 0;
eq4 =  $u^{t+\gamma\Delta t}$  ==  $u^t + \frac{\gamma \Delta t}{2} (\dot{u}^t + \dot{u}^{t+\gamma\Delta t})$ ;
eq5 =  $\dot{u}^{t+\gamma\Delta t}$  ==  $\dot{u}^t + \frac{\gamma \Delta t}{2} (\ddot{u}^t + \ddot{u}^{t+\gamma\Delta t})$ ;
eq6 =  $u^{t+\Delta t}$  ==  $u^t + \gamma \Delta t ((1 - \beta_1) \dot{u}^t + \beta_1 \dot{u}^{t+\gamma\Delta t}) + (1 - \gamma) \Delta t ((1 - \beta_2) \dot{u}^{t+\gamma\Delta t} + \beta_2 \dot{u}^{t+\Delta t})$ ;
eq7 =  $\dot{u}^{t+\Delta t}$  ==  $\dot{u}^t + \gamma \Delta t ((1 - \beta_1) \ddot{u}^t + \beta_1 \ddot{u}^{t+\gamma\Delta t}) + (1 - \gamma) \Delta t ((1 - \beta_2) \ddot{u}^{t+\gamma\Delta t} + \beta_2 \ddot{u}^{t+\Delta t})$ ;

lmf = Collect[Eliminate[{eq1, eq2, eq3, eq4, eq5, eq6, eq7},
{ $\ddot{u}^t, \ddot{u}^{t+\gamma\Delta t}, \ddot{u}^{t+\Delta t}, \dot{u}^t, \dot{u}^{t+\gamma\Delta t}, \dot{u}^{t+\Delta t}$ }] // FullSimplify, { $u^t, u^{t+\gamma\Delta t}, u^{t+\Delta t}$ }]
 $u^t (8 m + k \beta_1 \Delta t^2 c_0^2 + k \beta_2 \Delta t^2 c_0^2 - 2 k \beta_1 \beta_2 \Delta t^2 c_0^2) +$ 
 $u^{t+\gamma\Delta t} (-16 m + k \beta_1 \Delta t^2 c_0^2 + k \beta_2 \Delta t^2 c_0^2 + 2 k \beta_1 \beta_2 \Delta t^2 c_0^2 - 2 k \beta_2^2 \Delta t^2 c_0^2) +$ 
 $u^{t+\Delta t} (8 m + 2 k \beta_2^2 \Delta t^2 c_0^2) == 0$ 

 $m = \frac{\Delta x}{6} \begin{pmatrix} 1 & 4 & 1 \end{pmatrix};$ 

 $k = \frac{1}{\Delta x} \begin{pmatrix} -1 & 2 & -1 \end{pmatrix};$ 

 $\Delta t = \frac{cfl \Delta x}{c_0};$ 

 $u^{t+\Delta t} = \begin{pmatrix} u_0^1 \\ u_1^1 \\ u_2^1 \end{pmatrix}; u^{t+\gamma\Delta t} = \begin{pmatrix} u_0^{1/2} \\ u_1^{1/2} \\ u_2^{1/2} \end{pmatrix}; u^t = \begin{pmatrix} u_0^0 \\ u_1^0 \\ u_2^0 \end{pmatrix};$ 

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$$u_0^1 = E^{I \kappa \Delta x} \left(0 - (n_t + 1) \left(\frac{c_0 \Delta t}{\Delta x} \right) \left(\frac{c}{c_0} \right) \right) (*\text{notice here, } n_x=3 \text{ and } n_t=0 (\text{koz } t+0\Delta t) *)$$

$$e^{-\frac{i c \text{ cfl } (1+n_t) \Delta x \kappa}{c_0}}$$

$$u_1^1 = E^{I \kappa \Delta x} \left(1 - (n_t + 1) \left(\frac{c_0 \Delta t}{\Delta x} \right) \left(\frac{c}{c_0} \right) \right)$$

$$e^{i \Delta x \kappa \left(1 - \frac{c \text{ cfl } (1+n_t)}{c_0} \right)}$$

$$u_2^1 = E^{I \kappa \Delta x} \left(2 - (n_t + 1) \left(\frac{c_0 \Delta t}{\Delta x} \right) \left(\frac{c}{c_0} \right) \right)$$

$$e^{i \Delta x \kappa \left(2 - \frac{c \text{ cfl } (1+n_t)}{c_0} \right)}$$

$$u_0^{1/2} = E^{I \kappa \Delta x} \left(0 - \left(n_t + \frac{1}{2} \right) \left(\frac{c_0 \Delta t}{\Delta x} \right) \left(\frac{c}{c_0} \right) \right) (*\text{notice here, } n_x=3 \text{ and } n_t=0 (\text{koz } t+0\Delta t) *)$$

$$e^{-\frac{i c \text{ cfl } \left(\frac{1}{2} + n_t \right) \Delta x \kappa}{c_0}}$$

$$u_1^{1/2} = E^{I \kappa \Delta x} \left(1 - \left(n_t + \frac{1}{2} \right) \left(\frac{c_0 \Delta t}{\Delta x} \right) \left(\frac{c}{c_0} \right) \right)$$

$$e^{i \Delta x \kappa \left(1 - \frac{c \text{ cfl } \left(\frac{1}{2} + n_t \right)}{c_0} \right)}$$

$$u_2^{1/2} = E^{I \kappa \Delta x} \left(2 - \left(n_t + \frac{1}{2} \right) \left(\frac{c_0 \Delta t}{\Delta x} \right) \left(\frac{c}{c_0} \right) \right)$$

$$e^{i \Delta x \kappa \left(2 - \frac{c \text{ cfl } \left(\frac{1}{2} + n_t \right)}{c_0} \right)}$$

$$u_0^0 = E^{I \kappa \Delta x} \left(0 - n_t \left(\frac{c_0 \Delta t}{\Delta x} \right) \left(\frac{c}{c_0} \right) \right) (*\text{notice here, } n_x=3 \text{ and } n_t=0 (\text{koz } t+0\Delta t) *)$$

$$e^{-\frac{i c \text{ cfl } n_t \Delta x \kappa}{c_0}}$$

$$u_1^0 = E^{I \kappa \Delta x} \left(1 - n_t \left(\frac{c_0 \Delta t}{\Delta x} \right) \left(\frac{c}{c_0} \right) \right)$$

$$e^{i \Delta x \kappa \left(1 - \frac{c \text{ cfl } n_t}{c_0} \right)}$$

$$u_2^0 = E^{I \kappa \Delta x} \left(2 - n_t \left(\frac{c_0 \Delta t}{\Delta x} \right) \left(\frac{c}{c_0} \right) \right)$$

$$e^{i \Delta x \kappa \left(2 - \frac{c \text{ cfl } n_t}{c_0} \right)}$$

$$\begin{aligned} \text{lmf} = & \left(8 m + k \beta_1 \Delta t^2 c_0^2 + k \beta_2 \Delta t^2 c_0^2 - 2 k \beta_1 \beta_2 \Delta t^2 c_0^2 \right) \cdot u^t + \\ & \left(-16 m + k \beta_1 \Delta t^2 c_0^2 + k \beta_2 \Delta t^2 c_0^2 + 2 k \beta_1 \beta_2 \Delta t^2 c_0^2 - 2 k \beta_2^2 \Delta t^2 c_0^2 \right) \cdot u^{t+\gamma \Delta t} + \\ & \left(8 m + 2 k \beta_2^2 \Delta t^2 c_0^2 \right) \cdot u^{t+\Delta t} = 0 \end{aligned}$$

$$\begin{aligned} & \left\{ \left\{ e^{i \Delta x \kappa \left(1 - \frac{c \text{fl } n_t}{c_0} \right)} \left(\frac{16 \Delta x}{3} + 2 \text{cfl}^2 \beta_1 \Delta x + 2 \text{cfl}^2 \beta_2 \Delta x - 4 \text{cfl}^2 \beta_1 \beta_2 \Delta x \right) + \right. \right. \\ & e^{i \Delta x \kappa \left(2 - \frac{c \text{fl } n_t}{c_0} \right)} \left(\frac{4 \Delta x}{3} - \text{cfl}^2 \beta_1 \Delta x - \text{cfl}^2 \beta_2 \Delta x + 2 \text{cfl}^2 \beta_1 \beta_2 \Delta x \right) + \\ & e^{-\frac{i c \text{fl } n_t \Delta x \kappa}{c_0}} \left(\frac{4 \Delta x}{3} - \text{cfl}^2 \beta_1 \Delta x - \text{cfl}^2 \beta_2 \Delta x + 2 \text{cfl}^2 \beta_1 \beta_2 \Delta x \right) + \\ & e^{i \Delta x \kappa \left(1 - \frac{c \text{fl} \left(\frac{1}{2} + n_t \right)}{c_0} \right)} \left(-\frac{32 \Delta x}{3} + 2 \text{cfl}^2 \beta_1 \Delta x + 2 \text{cfl}^2 \beta_2 \Delta x + 4 \text{cfl}^2 \beta_1 \beta_2 \Delta x - 4 \text{cfl}^2 \beta_2^2 \Delta x \right) + \\ & e^{i \Delta x \kappa \left(2 - \frac{c \text{fl} (1+n_t)}{c_0} \right)} \left(\frac{4 \Delta x}{3} - 2 \text{cfl}^2 \beta_2^2 \Delta x \right) + e^{-\frac{i c \text{fl} (1+n_t) \Delta x \kappa}{c_0}} \left(\frac{4 \Delta x}{3} - 2 \text{cfl}^2 \beta_2^2 \Delta x \right) + \\ & e^{i \Delta x \kappa \left(2 - \frac{c \text{fl} \left(\frac{1}{2} + n_t \right)}{c_0} \right)} \left(-\frac{8 \Delta x}{3} - \text{cfl}^2 \beta_1 \Delta x - \text{cfl}^2 \beta_2 \Delta x - 2 \text{cfl}^2 \beta_1 \beta_2 \Delta x + 2 \text{cfl}^2 \beta_2^2 \Delta x \right) + \\ & e^{-\frac{i c \text{fl} \left(\frac{1}{2} + n_t \right) \Delta x \kappa}{c_0}} \left(-\frac{8 \Delta x}{3} - \text{cfl}^2 \beta_1 \Delta x - \text{cfl}^2 \beta_2 \Delta x - 2 \text{cfl}^2 \beta_1 \beta_2 \Delta x + 2 \text{cfl}^2 \beta_2^2 \Delta x \right) + \\ & \left. \left. e^{i \Delta x \kappa \left(1 - \frac{c \text{fl} (1+n_t)}{c_0} \right)} \left(\frac{16 \Delta x}{3} + 4 \text{cfl}^2 \beta_2^2 \Delta x \right) \right\} \right\} = 0 \end{aligned}$$

$$\begin{aligned}
\text{lmf} = & e^{\frac{i \Delta x \kappa}{c_0} \left(1 - \frac{c \text{cfl } n_t}{c_0}\right)} \left(\frac{16 \Delta x}{3} + 2 \text{cfl}^2 \beta_1 \Delta x + 2 \text{cfl}^2 \beta_2 \Delta x - 4 \text{cfl}^2 \beta_1 \beta_2 \Delta x \right) + \\
& e^{\frac{i \Delta x \kappa}{c_0} \left(2 - \frac{c \text{cfl } n_t}{c_0}\right)} \left(\frac{4 \Delta x}{3} - \text{cfl}^2 \beta_1 \Delta x - \text{cfl}^2 \beta_2 \Delta x + 2 \text{cfl}^2 \beta_1 \beta_2 \Delta x \right) + \\
& e^{-\frac{i c \text{cfl } n_t \Delta x \kappa}{c_0}} \left(\frac{4 \Delta x}{3} - \text{cfl}^2 \beta_1 \Delta x - \text{cfl}^2 \beta_2 \Delta x + 2 \text{cfl}^2 \beta_1 \beta_2 \Delta x \right) + \\
& e^{\frac{i \Delta x \kappa}{c_0} \left(1 - \frac{c \text{cfl} \left(\frac{1}{2} + n_t\right)}{c_0}\right)} \left(-\frac{32 \Delta x}{3} + 2 \text{cfl}^2 \beta_1 \Delta x + 2 \text{cfl}^2 \beta_2 \Delta x + 4 \text{cfl}^2 \beta_1 \beta_2 \Delta x - 4 \text{cfl}^2 \beta_2^2 \Delta x \right) + \\
& e^{\frac{i \Delta x \kappa}{c_0} \left(2 - \frac{c \text{cfl} (1+n_t)}{c_0}\right)} \left(\frac{4 \Delta x}{3} - 2 \text{cfl}^2 \beta_2^2 \Delta x \right) + e^{-\frac{i c \text{cfl} (1+n_t) \Delta x \kappa}{c_0}} \left(\frac{4 \Delta x}{3} - 2 \text{cfl}^2 \beta_2^2 \Delta x \right) + \\
& e^{\frac{i \Delta x \kappa}{c_0} \left(2 - \frac{c \text{cfl} \left(\frac{1}{2} + n_t\right)}{c_0}\right)} \left(-\frac{8 \Delta x}{3} - \text{cfl}^2 \beta_1 \Delta x - \text{cfl}^2 \beta_2 \Delta x - 2 \text{cfl}^2 \beta_1 \beta_2 \Delta x + 2 \text{cfl}^2 \beta_2^2 \Delta x \right) + \\
& e^{-\frac{i c \text{cfl} \left(\frac{1}{2} + n_t\right) \Delta x \kappa}{c_0}} \left(-\frac{8 \Delta x}{3} - \text{cfl}^2 \beta_1 \Delta x - \text{cfl}^2 \beta_2 \Delta x - 2 \text{cfl}^2 \beta_1 \beta_2 \Delta x + 2 \text{cfl}^2 \beta_2^2 \Delta x \right) + \\
& e^{\frac{i \Delta x \kappa}{c_0} \left(1 - \frac{c \text{cfl} (1+n_t)}{c_0}\right)} \left(\frac{16 \Delta x}{3} + 4 \text{cfl}^2 \beta_2^2 \Delta x \right) \\
& e^{\frac{i \Delta x \kappa}{c_0} \left(1 - \frac{c \text{cfl } n_t}{c_0}\right)} \left(\frac{16 \Delta x}{3} + 2 \text{cfl}^2 \beta_1 \Delta x + 2 \text{cfl}^2 \beta_2 \Delta x - 4 \text{cfl}^2 \beta_1 \beta_2 \Delta x \right) + \\
& e^{\frac{i \Delta x \kappa}{c_0} \left(2 - \frac{c \text{cfl } n_t}{c_0}\right)} \left(\frac{4 \Delta x}{3} - \text{cfl}^2 \beta_1 \Delta x - \text{cfl}^2 \beta_2 \Delta x + 2 \text{cfl}^2 \beta_1 \beta_2 \Delta x \right) + \\
& e^{-\frac{i c \text{cfl } n_t \Delta x \kappa}{c_0}} \left(\frac{4 \Delta x}{3} - \text{cfl}^2 \beta_1 \Delta x - \text{cfl}^2 \beta_2 \Delta x + 2 \text{cfl}^2 \beta_1 \beta_2 \Delta x \right) + \\
& e^{\frac{i \Delta x \kappa}{c_0} \left(1 - \frac{c \text{cfl} \left(\frac{1}{2} + n_t\right)}{c_0}\right)} \left(-\frac{32 \Delta x}{3} + 2 \text{cfl}^2 \beta_1 \Delta x + 2 \text{cfl}^2 \beta_2 \Delta x + 4 \text{cfl}^2 \beta_1 \beta_2 \Delta x - 4 \text{cfl}^2 \beta_2^2 \Delta x \right) + \\
& e^{\frac{i \Delta x \kappa}{c_0} \left(2 - \frac{c \text{cfl} (1+n_t)}{c_0}\right)} \left(\frac{4 \Delta x}{3} - 2 \text{cfl}^2 \beta_2^2 \Delta x \right) + e^{-\frac{i c \text{cfl} (1+n_t) \Delta x \kappa}{c_0}} \left(\frac{4 \Delta x}{3} - 2 \text{cfl}^2 \beta_2^2 \Delta x \right) + \\
& e^{\frac{i \Delta x \kappa}{c_0} \left(2 - \frac{c \text{cfl} \left(\frac{1}{2} + n_t\right)}{c_0}\right)} \left(-\frac{8 \Delta x}{3} - \text{cfl}^2 \beta_1 \Delta x - \text{cfl}^2 \beta_2 \Delta x - 2 \text{cfl}^2 \beta_1 \beta_2 \Delta x + 2 \text{cfl}^2 \beta_2^2 \Delta x \right) + \\
& e^{-\frac{i c \text{cfl} \left(\frac{1}{2} + n_t\right) \Delta x \kappa}{c_0}} \left(-\frac{8 \Delta x}{3} - \text{cfl}^2 \beta_1 \Delta x - \text{cfl}^2 \beta_2 \Delta x - 2 \text{cfl}^2 \beta_1 \beta_2 \Delta x + 2 \text{cfl}^2 \beta_2^2 \Delta x \right) + \\
& e^{\frac{i \Delta x \kappa}{c_0} \left(1 - \frac{c \text{cfl} (1+n_t)}{c_0}\right)} \left(\frac{16 \Delta x}{3} + 4 \text{cfl}^2 \beta_2^2 \Delta x \right)
\end{aligned}$$

lmf / Δx * 3 // Simplify

$$\begin{aligned}
& e^{-\frac{i c \text{cfl} (1+n_t) \Delta x \kappa}{c_0}} \left(4 - 6 \text{cfl}^2 \beta_2^2 \right) - 2 e^{\frac{i \Delta x \kappa}{c_0} \left(2 - \frac{c \text{cfl} (1+n_t)}{c_0}\right)} \left(-2 + 3 \text{cfl}^2 \beta_2^2 \right) + \\
& 4 e^{\frac{i \Delta x \kappa}{c_0} \left(1 - \frac{c \text{cfl} (1+n_t)}{c_0}\right)} \left(4 + 3 \text{cfl}^2 \beta_2^2 \right) + 2 e^{\frac{i \Delta x \kappa}{c_0} \left(1 - \frac{c \text{cfl} \left(\frac{1}{2} + n_t\right)}{c_0}\right)} \left(-16 + 3 \text{cfl}^2 \left(\beta_1 + \beta_2 + 2 \beta_1 \beta_2 - 2 \beta_2^2 \right) \right) - \\
& e^{\frac{i \Delta x \kappa}{c_0} \left(2 - \frac{c \text{cfl} \left(\frac{1}{2} + n_t\right)}{c_0}\right)} \left(8 + 3 \text{cfl}^2 \left(\beta_1 + \beta_2 + 2 \beta_1 \beta_2 - 2 \beta_2^2 \right) \right) - \\
& e^{-\frac{i c \text{cfl} \left(\frac{1}{2} + n_t\right) \Delta x \kappa}{c_0}} \left(8 + 3 \text{cfl}^2 \left(\beta_1 + \beta_2 + 2 \beta_1 \beta_2 - 2 \beta_2^2 \right) \right) - \\
& 2 e^{\frac{i \Delta x \kappa}{c_0} \left(1 - \frac{c \text{cfl } n_t}{c_0}\right)} \left(-8 + \text{cfl}^2 \left(-3 \beta_2 + \beta_1 \left(-3 + 6 \beta_2 \right) \right) \right) + \\
& e^{\frac{i \Delta x \kappa}{c_0} \left(2 - \frac{c \text{cfl } n_t}{c_0}\right)} \left(4 + \text{cfl}^2 \left(-3 \beta_2 + \beta_1 \left(-3 + 6 \beta_2 \right) \right) \right) + \\
& e^{-\frac{i c \text{cfl } n_t \Delta x \kappa}{c_0}} \left(4 + \text{cfl}^2 \left(-3 \beta_2 + \beta_1 \left(-3 + 6 \beta_2 \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& e^{-\frac{i \text{ c f l } (1+n_t) \Delta x \kappa}{c_0}} \left(4 - 6 \text{ c f l}^2 \beta_2^2 \right) - \\
& 2 e^{i \Delta x \kappa \left(2 - \frac{\text{c f l } (1+n_t)}{c_0} \right)} \left(-2 + 3 \text{ c f l}^2 \beta_2^2 \right) + 4 e^{i \Delta x \kappa \left(1 - \frac{\text{c f l } (1+n_t)}{c_0} \right)} \left(4 + 3 \text{ c f l}^2 \beta_2^2 \right) + \\
& 2 e^{i \Delta x \kappa \left(1 - \frac{\text{c f l } \left(\frac{1}{2} + n_t \right)}{c_0} \right)} \left(-16 + 3 \text{ c f l}^2 \left(\beta_1 + \beta_2 + 2 \beta_1 \beta_2 - 2 \beta_2^2 \right) \right) - \\
& e^{i \Delta x \kappa \left(2 - \frac{\text{c f l } \left(\frac{1}{2} + n_t \right)}{c_0} \right)} \left(8 + 3 \text{ c f l}^2 \left(\beta_1 + \beta_2 + 2 \beta_1 \beta_2 - 2 \beta_2^2 \right) \right) - \\
& e^{-\frac{i \text{ c f l } \left(\frac{1}{2} + n_t \right) \Delta x \kappa}{c_0}} \left(8 + 3 \text{ c f l}^2 \left(\beta_1 + \beta_2 + 2 \beta_1 \beta_2 - 2 \beta_2^2 \right) \right) - \\
& 2 e^{i \Delta x \kappa \left(1 - \frac{\text{c f l } n_t}{c_0} \right)} \left(-8 + \text{c f l}^2 \left(-3 \beta_2 + \beta_1 \left(-3 + 6 \beta_2 \right) \right) \right) + \\
& e^{i \Delta x \kappa \left(2 - \frac{\text{c f l } n_t}{c_0} \right)} \left(4 + \text{c f l}^2 \left(-3 \beta_2 + \beta_1 \left(-3 + 6 \beta_2 \right) \right) \right) + \\
& e^{-\frac{i \text{ c f l } n_t \Delta x \kappa}{c_0}} \left(4 + \text{c f l}^2 \left(-3 \beta_2 + \beta_1 \left(-3 + 6 \beta_2 \right) \right) \right) / . \Delta x \kappa \rightarrow \Delta x \kappa \text{By} \pi \text{Pi} \\
& e^{-\frac{i \text{ c f l } (1+n_t) \pi \Delta x \kappa \text{By} \pi}{c_0}} \left(4 - 6 \text{ c f l}^2 \beta_2^2 \right) - \\
& 2 e^{i \pi \Delta x \kappa \text{By} \pi \left(2 - \frac{\text{c f l } (1+n_t)}{c_0} \right)} \left(-2 + 3 \text{ c f l}^2 \beta_2^2 \right) + 4 e^{i \pi \Delta x \kappa \text{By} \pi \left(1 - \frac{\text{c f l } (1+n_t)}{c_0} \right)} \left(4 + 3 \text{ c f l}^2 \beta_2^2 \right) + \\
& 2 e^{i \pi \Delta x \kappa \text{By} \pi \left(1 - \frac{\text{c f l } \left(\frac{1}{2} + n_t \right)}{c_0} \right)} \left(-16 + 3 \text{ c f l}^2 \left(\beta_1 + \beta_2 + 2 \beta_1 \beta_2 - 2 \beta_2^2 \right) \right) - \\
& e^{i \pi \Delta x \kappa \text{By} \pi \left(2 - \frac{\text{c f l } \left(\frac{1}{2} + n_t \right)}{c_0} \right)} \left(8 + 3 \text{ c f l}^2 \left(\beta_1 + \beta_2 + 2 \beta_1 \beta_2 - 2 \beta_2^2 \right) \right) - \\
& e^{-\frac{i \text{ c f l } \left(\frac{1}{2} + n_t \right) \pi \Delta x \kappa \text{By} \pi}{c_0}} \left(8 + 3 \text{ c f l}^2 \left(\beta_1 + \beta_2 + 2 \beta_1 \beta_2 - 2 \beta_2^2 \right) \right) - \\
& 2 e^{i \pi \Delta x \kappa \text{By} \pi \left(1 - \frac{\text{c f l } n_t}{c_0} \right)} \left(-8 + \text{c f l}^2 \left(-3 \beta_2 + \beta_1 \left(-3 + 6 \beta_2 \right) \right) \right) + \\
& e^{i \pi \Delta x \kappa \text{By} \pi \left(2 - \frac{\text{c f l } n_t}{c_0} \right)} \left(4 + \text{c f l}^2 \left(-3 \beta_2 + \beta_1 \left(-3 + 6 \beta_2 \right) \right) \right) + \\
& e^{-\frac{i \text{ c f l } n_t \pi \Delta x \kappa \text{By} \pi}{c_0}} \left(4 + \text{c f l}^2 \left(-3 \beta_2 + \beta_1 \left(-3 + 6 \beta_2 \right) \right) \right)
\end{aligned}$$

eq = nd // Cancel // ExpandAll // Simplify

$$\begin{aligned} & e^{-i \text{cBuc cfl} (1+n_t)} \pi \Delta x \kappa \text{Buc} \pi \left(4 - 6 \text{cfl}^2 \beta_2^2 + e^{2i \pi \Delta x \kappa \text{Buc} \pi} \left(4 - 6 \text{cfl}^2 \beta_2^2 \right) + \right. \\ & 4 e^{i \pi \Delta x \kappa \text{Buc} \pi} \left(4 + 3 \text{cfl}^2 \beta_2^2 \right) + e^{i (1+\text{cBuc cfl}) \pi \Delta x \kappa \text{Buc} \pi} \left(16 + 6 \text{cfl}^2 (\beta_1 + \beta_2 - 2 \beta_1 \beta_2) \right) + \\ & 2 e^{\frac{1}{2} i (2+\text{cBuc cfl}) \pi \Delta x \kappa \text{Buc} \pi} \left(-16 + 3 \text{cfl}^2 (\beta_1 + \beta_2 + 2 \beta_1 \beta_2 - 2 \beta_2^2) \right) - \\ & e^{\frac{1}{2} i \text{cBuc cfl} \pi \Delta x \kappa \text{Buc} \pi} \left(8 + 3 \text{cfl}^2 (\beta_1 + \beta_2 + 2 \beta_1 \beta_2 - 2 \beta_2^2) \right) - \\ & e^{\frac{1}{2} i (4+\text{cBuc cfl}) \pi \Delta x \kappa \text{Buc} \pi} \left(8 + 3 \text{cfl}^2 (\beta_1 + \beta_2 + 2 \beta_1 \beta_2 - 2 \beta_2^2) \right) + \\ & e^{i \text{cBuc cfl} \pi \Delta x \kappa \text{Buc} \pi} \left(4 + \text{cfl}^2 (-3 \beta_2 + \beta_1 (-3 + 6 \beta_2)) \right) + \\ & \left. e^{i (2+\text{cBuc cfl}) \pi \Delta x \kappa \text{Buc} \pi} \left(4 + \text{cfl}^2 (-3 \beta_2 + \beta_1 (-3 + 6 \beta_2)) \right) \right) \end{aligned}$$

$$\begin{aligned} \text{nd2} = & 4 - 6 \text{cfl}^2 \beta_2^2 + e^{2i \pi \Delta x \kappa \text{Buc} \pi} \left(4 - 6 \text{cfl}^2 \beta_2^2 \right) + \\ & 4 e^{i \pi \Delta x \kappa \text{Buc} \pi} \left(4 + 3 \text{cfl}^2 \beta_2^2 \right) + e^{i (1+\text{cBuc cfl}) \pi \Delta x \kappa \text{Buc} \pi} \left(16 + 6 \text{cfl}^2 (\beta_1 + \beta_2 - 2 \beta_1 \beta_2) \right) + \\ & 2 e^{\frac{1}{2} i (2+\text{cBuc cfl}) \pi \Delta x \kappa \text{Buc} \pi} \left(-16 + 3 \text{cfl}^2 (\beta_1 + \beta_2 + 2 \beta_1 \beta_2 - 2 \beta_2^2) \right) - \\ & e^{\frac{1}{2} i \text{cBuc cfl} \pi \Delta x \kappa \text{Buc} \pi} \left(8 + 3 \text{cfl}^2 (\beta_1 + \beta_2 + 2 \beta_1 \beta_2 - 2 \beta_2^2) \right) - \\ & e^{\frac{1}{2} i (4+\text{cBuc cfl}) \pi \Delta x \kappa \text{Buc} \pi} \left(8 + 3 \text{cfl}^2 (\beta_1 + \beta_2 + 2 \beta_1 \beta_2 - 2 \beta_2^2) \right) + \\ & e^{i \text{cBuc cfl} \pi \Delta x \kappa \text{Buc} \pi} \left(4 + \text{cfl}^2 (-3 \beta_2 + \beta_1 (-3 + 6 \beta_2)) \right) + e^{i (2+\text{cBuc cfl}) \pi \Delta x \kappa \text{Buc} \pi} \\ & \left(4 + \text{cfl}^2 (-3 \beta_2 + \beta_1 (-3 + 6 \beta_2)) \right) // \text{Cancel // ExpandAll // Simplify} \end{aligned}$$

$$\begin{aligned} & 4 - 6 \text{cfl}^2 \beta_2^2 + e^{2i \pi \Delta x \kappa \text{Buc} \pi} \left(4 - 6 \text{cfl}^2 \beta_2^2 \right) + 4 e^{i \pi \Delta x \kappa \text{Buc} \pi} \left(4 + 3 \text{cfl}^2 \beta_2^2 \right) + \\ & e^{i (1+\text{cBuc cfl}) \pi \Delta x \kappa \text{Buc} \pi} \left(16 + 6 \text{cfl}^2 (\beta_1 + \beta_2 - 2 \beta_1 \beta_2) \right) + \\ & 2 e^{\frac{1}{2} i (2+\text{cBuc cfl}) \pi \Delta x \kappa \text{Buc} \pi} \left(-16 + 3 \text{cfl}^2 (\beta_1 + \beta_2 + 2 \beta_1 \beta_2 - 2 \beta_2^2) \right) - \\ & e^{\frac{1}{2} i \text{cBuc cfl} \pi \Delta x \kappa \text{Buc} \pi} \left(8 + 3 \text{cfl}^2 (\beta_1 + \beta_2 + 2 \beta_1 \beta_2 - 2 \beta_2^2) \right) - \\ & e^{\frac{1}{2} i (4+\text{cBuc cfl}) \pi \Delta x \kappa \text{Buc} \pi} \left(8 + 3 \text{cfl}^2 (\beta_1 + \beta_2 + 2 \beta_1 \beta_2 - 2 \beta_2^2) \right) + \\ & e^{i \text{cBuc cfl} \pi \Delta x \kappa \text{Buc} \pi} \left(4 + \text{cfl}^2 (-3 \beta_2 + \beta_1 (-3 + 6 \beta_2)) \right) + \\ & e^{i (2+\text{cBuc cfl}) \pi \Delta x \kappa \text{Buc} \pi} \left(4 + \text{cfl}^2 (-3 \beta_2 + \beta_1 (-3 + 6 \beta_2)) \right) \end{aligned}$$

Reduce = Solve[nd2 == 0, cByc]

Solve::ifun : Inverse functions are being used by Solve, so

some solutions may not be found; use Reduce for complete solution information. >>

Set::wrsym : Symbol Reduce is Protected. >>

$$\left\{ \left\{ \text{cByc} \rightarrow -\frac{1}{\text{cfl} \pi \Delta \text{хкВул}} 2 \operatorname{Log} \left[\left(8 + 32 e^{i \pi \Delta \text{хкВул}} + 8 e^{2 i \pi \Delta \text{хкВул}} + 3 \text{cfl}^2 \beta_1 - 6 \text{cfl}^2 e^{i \pi \Delta \text{хкВул}} \beta_1 + 3 \text{cfl}^2 e^{2 i \pi \Delta \text{хкВул}} \beta_1 + 3 \text{cfl}^2 \beta_2 - 6 \text{cfl}^2 e^{i \pi \Delta \text{хкВул}} \beta_2 + 3 \text{cfl}^2 e^{2 i \pi \Delta \text{хкВул}} \beta_2 + 6 \text{cfl}^2 \beta_1 \beta_2 - 12 \text{cfl}^2 e^{i \pi \Delta \text{хкВул}} \beta_1 \beta_2 + 6 \text{cfl}^2 e^{2 i \pi \Delta \text{хкВул}} \beta_1 \beta_2 - 6 \text{cfl}^2 \beta_2^2 + 12 \text{cfl}^2 e^{i \pi \Delta \text{хкВул}} \beta_2^2 - 6 \text{cfl}^2 e^{2 i \pi \Delta \text{хкВул}} \beta_2^2 - \sqrt{\left(-4 \left(4 + 16 e^{i \pi \Delta \text{хкВул}} + 4 e^{2 i \pi \Delta \text{хкВул}} - 3 \text{cfl}^2 \beta_1 + 6 \text{cfl}^2 e^{i \pi \Delta \text{хкВул}} \beta_1 - 3 \text{cfl}^2 e^{2 i \pi \Delta \text{хкВул}} \beta_1 - 3 \text{cfl}^2 \beta_2 + 6 \text{cfl}^2 e^{i \pi \Delta \text{хкВул}} \beta_2 - 3 \text{cfl}^2 e^{2 i \pi \Delta \text{хкВул}} \beta_2 + 6 \text{cfl}^2 \beta_1 \beta_2 - 12 \text{cfl}^2 e^{i \pi \Delta \text{хкВул}} \beta_1 \beta_2 + 6 \text{cfl}^2 e^{2 i \pi \Delta \text{хкВул}} \beta_1 \beta_2 \right)} \left(4 + 16 e^{i \pi \Delta \text{хкВул}} + 4 e^{2 i \pi \Delta \text{хкВул}} - 6 \text{cfl}^2 \beta_2^2 + 12 \text{cfl}^2 e^{i \pi \Delta \text{хкВул}} \beta_2^2 - 6 \text{cfl}^2 e^{2 i \pi \Delta \text{хкВул}} \beta_2^2 \right) + \left(-8 - 32 e^{i \pi \Delta \text{хкВул}} - 8 e^{2 i \pi \Delta \text{хкВул}} - 3 \text{cfl}^2 \beta_1 + 6 \text{cfl}^2 e^{i \pi \Delta \text{хкВул}} \beta_1 - 3 \text{cfl}^2 e^{2 i \pi \Delta \text{хкВул}} \beta_1 - 3 \text{cfl}^2 \beta_2 + 6 \text{cfl}^2 e^{i \pi \Delta \text{хкВул}} \beta_2 - 3 \text{cfl}^2 e^{2 i \pi \Delta \text{хкВул}} \beta_2 - 6 \text{cfl}^2 \beta_1 \beta_2 + 12 \text{cfl}^2 e^{i \pi \Delta \text{хкВул}} \beta_1 \beta_2 - 6 \text{cfl}^2 e^{2 i \pi \Delta \text{хкВул}} \beta_1 \beta_2 + 6 \text{cfl}^2 \beta_2^2 - 12 \text{cfl}^2 e^{i \pi \Delta \text{хкВул}} \beta_2^2 + 6 \text{cfl}^2 e^{2 i \pi \Delta \text{хкВул}} \beta_2^2 \right)^2 \right) \right] \right\}, \left\{ \text{cByc} \rightarrow -\frac{1}{\text{cfl} \pi \Delta \text{хкВул}} 2 \operatorname{Log} \left[\left(8 + 32 e^{i \pi \Delta \text{хкВул}} + 8 e^{2 i \pi \Delta \text{хкВул}} + 3 \text{cfl}^2 \beta_1 - 6 \text{cfl}^2 e^{i \pi \Delta \text{хкВул}} \beta_1 + 3 \text{cfl}^2 e^{2 i \pi \Delta \text{хкВул}} \beta_1 + 3 \text{cfl}^2 \beta_2 - 6 \text{cfl}^2 e^{i \pi \Delta \text{хкВул}} \beta_2 + 3 \text{cfl}^2 e^{2 i \pi \Delta \text{хкВул}} \beta_2 + 6 \text{cfl}^2 \beta_1 \beta_2 - 12 \text{cfl}^2 e^{i \pi \Delta \text{хкВул}} \beta_1 \beta_2 + 6 \text{cfl}^2 e^{2 i \pi \Delta \text{хкВул}} \beta_1 \beta_2 + 6 \text{cfl}^2 e^{2 i \pi \Delta \text{хкВул}} \beta_1 \beta_2 - 6 \text{cfl}^2 \beta_2^2 + 12 \text{cfl}^2 e^{i \pi \Delta \text{хкВул}} \beta_2^2 - 6 \text{cfl}^2 e^{2 i \pi \Delta \text{хкВул}} \beta_2^2 - \sqrt{\left(-4 \left(4 + 16 e^{i \pi \Delta \text{хкВул}} + 4 e^{2 i \pi \Delta \text{хкВул}} - 3 \text{cfl}^2 \beta_1 + 6 \text{cfl}^2 e^{i \pi \Delta \text{хкВул}} \beta_1 - 3 \text{cfl}^2 e^{2 i \pi \Delta \text{хкВул}} \beta_1 - 3 \text{cfl}^2 \beta_2 + 6 \text{cfl}^2 e^{i \pi \Delta \text{хкВул}} \beta_2 - 3 \text{cfl}^2 e^{2 i \pi \Delta \text{хкВул}} \beta_2 - 3 \text{cfl}^2 \beta_1 \beta_2 + 6 \text{cfl}^2 e^{i \pi \Delta \text{хкВул}} \beta_1 \beta_2 - 12 \text{cfl}^2 e^{i \pi \Delta \text{хкВул}} \beta_1 \beta_2 + 6 \text{cfl}^2 e^{2 i \pi \Delta \text{хкВул}} \beta_1 \beta_2 + 6 \text{cfl}^2 \beta_2^2 + 12 \text{cfl}^2 e^{i \pi \Delta \text{хкВул}} \beta_2^2 - 6 \text{cfl}^2 e^{2 i \pi \Delta \text{хкВул}} \beta_2^2 \right)} \left(4 + 16 e^{i \pi \Delta \text{хкВул}} + 4 e^{2 i \pi \Delta \text{хкВул}} - 3 \text{cfl}^2 \beta_1 + 6 \text{cfl}^2 e^{i \pi \Delta \text{хкВул}} \beta_1 - 3 \text{cfl}^2 e^{2 i \pi \Delta \text{хкВул}} \beta_1 - 3 \text{cfl}^2 \beta_2 + 6 \text{cfl}^2 e^{i \pi \Delta \text{хкВул}} \beta_2 - 3 \text{cfl}^2 e^{2 i \pi \Delta \text{хкВул}} \beta_2 - 6 \text{cfl}^2 \beta_1 \beta_2 + 12 \text{cfl}^2 e^{i \pi \Delta \text{хкВул}} \beta_1 \beta_2 - 6 \text{cfl}^2 e^{2 i \pi \Delta \text{хкВул}} \beta_1 \beta_2 + 6 \text{cfl}^2 \beta_2^2 - 12 \text{cfl}^2 e^{i \pi \Delta \text{хкВул}} \beta_2^2 + 6 \text{cfl}^2 e^{2 i \pi \Delta \text{хкВул}} \beta_2^2 \right)^2 \right) \right] \right\} \right\}$$

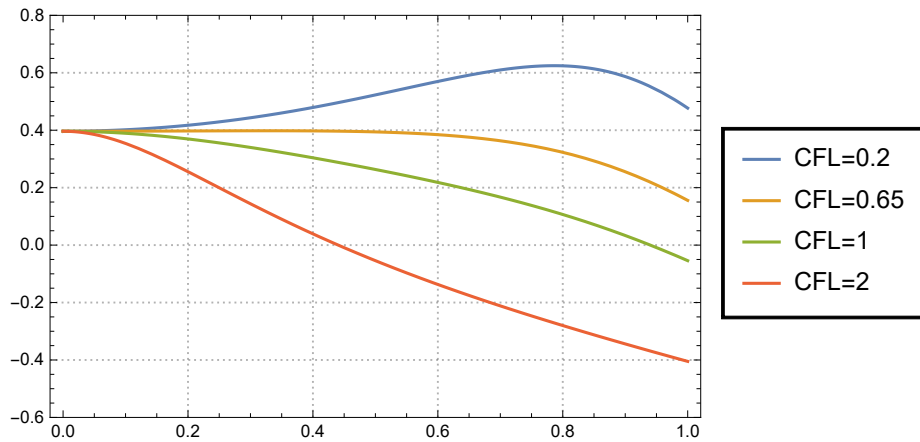
$$\beta_1 = 0.65;$$

$$\text{cfl} = 2;$$

$$\text{solu} = -\frac{1}{\text{cfl} \pi \Delta \kappa \text{By} \pi} 2 \, i \, \text{Log} \left[\left(8 + 32 e^{i \pi \Delta \kappa \text{By} \pi} + 8 e^{2 i \pi \Delta \kappa \text{By} \pi} + 3 \text{cfl}^2 \beta_1 - 6 \text{cfl}^2 e^{i \pi \Delta \kappa \text{By} \pi} \beta_1 + 3 \text{cfl}^2 e^{2 i \pi \Delta \kappa \text{By} \pi} \beta_1 + 3 \text{cfl}^2 \beta_2 - 6 \text{cfl}^2 e^{i \pi \Delta \kappa \text{By} \pi} \beta_2 + 3 \text{cfl}^2 e^{2 i \pi \Delta \kappa \text{By} \pi} \beta_2 + 6 \text{cfl}^2 \beta_1 \beta_2 - 12 \text{cfl}^2 e^{i \pi \Delta \kappa \text{By} \pi} \beta_1 \beta_2 + 6 \text{cfl}^2 e^{2 i \pi \Delta \kappa \text{By} \pi} \beta_1 \beta_2 - 6 \text{cfl}^2 \beta_2^2 + 12 \text{cfl}^2 e^{i \pi \Delta \kappa \text{By} \pi} \beta_2^2 - 6 \text{cfl}^2 e^{2 i \pi \Delta \kappa \text{By} \pi} \beta_2^2 - \sqrt{-4 \left(4 + 16 e^{i \pi \Delta \kappa \text{By} \pi} + 4 e^{2 i \pi \Delta \kappa \text{By} \pi} - 3 \text{cfl}^2 \beta_1 + 6 \text{cfl}^2 e^{i \pi \Delta \kappa \text{By} \pi} \beta_1 - 3 \text{cfl}^2 e^{2 i \pi \Delta \kappa \text{By} \pi} \beta_1 - 3 \text{cfl}^2 \beta_2 + 6 \text{cfl}^2 e^{i \pi \Delta \kappa \text{By} \pi} \beta_2 - 3 \text{cfl}^2 e^{2 i \pi \Delta \kappa \text{By} \pi} \beta_2 + 6 \text{cfl}^2 \beta_1 \beta_2 - 12 \text{cfl}^2 e^{i \pi \Delta \kappa \text{By} \pi} \beta_1 \beta_2 + 6 \text{cfl}^2 e^{2 i \pi \Delta \kappa \text{By} \pi} \beta_1 \beta_2 \right) \left(4 + 16 e^{i \pi \Delta \kappa \text{By} \pi} + 4 e^{2 i \pi \Delta \kappa \text{By} \pi} - 6 \text{cfl}^2 \beta_2^2 + 12 \text{cfl}^2 e^{i \pi \Delta \kappa \text{By} \pi} \beta_2^2 - 6 \text{cfl}^2 e^{2 i \pi \Delta \kappa \text{By} \pi} \beta_2^2 \right) + \left(-8 - 32 e^{i \pi \Delta \kappa \text{By} \pi} - 8 e^{2 i \pi \Delta \kappa \text{By} \pi} - 3 \text{cfl}^2 \beta_1 + 6 \text{cfl}^2 e^{i \pi \Delta \kappa \text{By} \pi} \beta_1 - 3 \text{cfl}^2 e^{2 i \pi \Delta \kappa \text{By} \pi} \beta_1 - 3 \text{cfl}^2 \beta_2 + 6 \text{cfl}^2 e^{i \pi \Delta \kappa \text{By} \pi} \beta_2 - 3 \text{cfl}^2 e^{2 i \pi \Delta \kappa \text{By} \pi} \beta_2 - 6 \text{cfl}^2 \beta_1 \beta_2 + 12 \text{cfl}^2 e^{i \pi \Delta \kappa \text{By} \pi} \beta_1 \beta_2 - 6 \text{cfl}^2 e^{2 i \pi \Delta \kappa \text{By} \pi} \beta_1 \beta_2 + 6 \text{cfl}^2 \beta_2^2 - 12 \text{cfl}^2 e^{i \pi \Delta \kappa \text{By} \pi} \beta_2^2 + 6 \text{cfl}^2 e^{2 i \pi \Delta \kappa \text{By} \pi} \beta_2^2 \right)^2 \right) \right] / \left(2 \left(4 + 16 e^{i \pi \Delta \kappa \text{By} \pi} + 4 e^{2 i \pi \Delta \kappa \text{By} \pi} - 3 \text{cfl}^2 \beta_1 + 6 \text{cfl}^2 e^{i \pi \Delta \kappa \text{By} \pi} \beta_1 - 3 \text{cfl}^2 e^{2 i \pi \Delta \kappa \text{By} \pi} \beta_1 - 3 \text{cfl}^2 \beta_2 + 6 \text{cfl}^2 e^{i \pi \Delta \kappa \text{By} \pi} \beta_2 - 3 \text{cfl}^2 e^{2 i \pi \Delta \kappa \text{By} \pi} \beta_2 + 6 \text{cfl}^2 \beta_1 \beta_2 - 12 \text{cfl}^2 e^{i \pi \Delta \kappa \text{By} \pi} \beta_1 \beta_2 + 6 \text{cfl}^2 e^{2 i \pi \Delta \kappa \text{By} \pi} \beta_1 \beta_2 \right) \right) \right];$$

```
plotb0p65CFL2p0 = solu;
```

```
Plot[{Abs[plotb0p65CFL0p2] - 1, Abs[plotb0p65CFL0p65] - 1, Abs[plotb0p65CFL1p0] - 1,
  Abs[plotb0p65CFL2p0] - 1}, {ΔκByπ, 0.000001, 1}, PlotRange → {- .6, .8},
  PlotLegends → LineLegend[{"CFL=0.2", "CFL=0.65", "CFL=1", "CFL=2"},
  LegendFunction → (Framed[#] &)], PlotTheme → "Detailed"]
```



```

q1c = -  $\frac{1}{\text{cfl } \pi \Delta \kappa \text{By} \pi} 2 \, i$ 
Log[ (8 + 32 ei π ΔκByπ + 8 e2 i π ΔκByπ + 3 cfl2 β1 - 6 cfl2 ei π ΔκByπ β1 + 3 cfl2 e2 i π ΔκByπ β1 +
3 cfl2 β2 - 6 cfl2 ei π ΔκByπ β2 + 3 cfl2 e2 i π ΔκByπ β2 + 6 cfl2 β1 β2 - 12 cfl2 ei π ΔκByπ β1
β2 + 6 cfl2 e2 i π ΔκByπ β1 β2 - 6 cfl2 β22 + 12 cfl2 ei π ΔκByπ β22 - 6 cfl2 e2 i π ΔκByπ β22 -
 $\sqrt{-4 (4 + 16 e^{i \pi \Delta \kappa \text{By} \pi} + 4 e^{2 i \pi \Delta \kappa \text{By} \pi} - 3 \text{cfl}^2 \beta_1 + 6 \text{cfl}^2 e^{i \pi \Delta \kappa \text{By} \pi} \beta_1 -$ 
3 cfl2 e2 i π ΔκByπ β1 - 3 cfl2 β2 + 6 cfl2 ei π ΔκByπ β2 - 3 cfl2 e2 i π ΔκByπ β2 + 6
cfl2 β1 β2 - 12 cfl2 ei π ΔκByπ β1 β2 + 6 cfl2 e2 i π ΔκByπ β1 β2) (4 + 16 ei π ΔκByπ +
4 e2 i π ΔκByπ - 6 cfl2 β22 + 12 cfl2 ei π ΔκByπ β22 - 6 cfl2 e2 i π ΔκByπ β22) +
(-8 - 32 ei π ΔκByπ - 8 e2 i π ΔκByπ - 3 cfl2 β1 + 6 cfl2 ei π ΔκByπ β1 -
3 cfl2 e2 i π ΔκByπ β1 - 3 cfl2 β2 + 6 cfl2 ei π ΔκByπ β2 - 3 cfl2 e2 i π ΔκByπ β2 -
6 cfl2 β1 β2 + 12 cfl2 ei π ΔκByπ β1 β2 - 6 cfl2 e2 i π ΔκByπ β1 β2 +
6 cfl2 β22 - 12 cfl2 ei π ΔκByπ β22 + 6 cfl2 e2 i π ΔκByπ β22)2) ) /
(2 (4 + 16 ei π ΔκByπ + 4 e2 i π ΔκByπ - 3 cfl2 β1 + 6 cfl2 ei π ΔκByπ β1 -
3 cfl2 e2 i π ΔκByπ β1 - 3 cfl2 β2 + 6 cfl2 ei π ΔκByπ β2 - 3 cfl2 e2 i π ΔκByπ β2 +
6 cfl2 β1 β2 - 12 cfl2 ei π ΔκByπ β1 β2 + 6 cfl2 e2 i π ΔκByπ β1 β2) ) ] ; cfl // Clear;

cfl = .65; qc = FindRoot[ΔκByπ π -  $\frac{0.6 \pi}{\text{cfl Abs}[q1c]}$ , {ΔκByπ, 0.1}]; qc = qc[[1, 2]]
0.879327

cfl = 1; qc = FindRoot[ΔκByπ π -  $\frac{0.6 \pi}{\text{cfl Abs}[q1c]}$ , {ΔκByπ, 0.1}]; qc = qc[[1, 2]]
0.470257

cfl = 2; qc = FindRoot[ΔκByπ π -  $\frac{0.6 \pi}{\text{cfl Abs}[q1c]}$ , {ΔκByπ, 0.1}]; qc = qc[[1, 2]]
0.219873

```