```
<< Notation`;
  Symbolize ut ; Symbolize ut ; Symbolize ut ; Symbolize ut ; Symbolize ut ;
  Symbolize \left[ \begin{array}{c} \dot{\mathbf{u}}^{\mathsf{t}+\Delta\mathsf{t}} \end{array} \right]; Symbolize \left[ \begin{array}{c} \dot{\mathbf{u}}^{\mathsf{t}+\Delta\mathsf{t}} \end{array} \right]; Symbolize \left[ \begin{array}{c} \dot{\mathbf{u}}^{\mathsf{t}+\gamma\Delta\mathsf{t}} \end{array} \right]; Symbolize \left[ \begin{array}{c} \dot{\mathbf{u}}^{\mathsf{t}+\gamma\Delta\mathsf{t}} \end{array} \right];
  Symbolize \left[\begin{array}{c} \mathbf{i}^{\mathbf{i}^{t+\gamma\Delta t}} \end{array}\right]; Symbolize \left[\begin{array}{c} \beta_1 \end{array}\right]; Symbolize \left[\begin{array}{c} \beta_2 \end{array}\right]; Symbolize \left[\begin{array}{c} \mathbf{a}_k \end{array}\right];
  Symbolize \begin{bmatrix} n_x \end{bmatrix}; Symbolize \begin{bmatrix} n_t \end{bmatrix}; Symbolize \begin{bmatrix} u_1^{t+\Delta t} \end{bmatrix}; Symbolize \begin{bmatrix} u_2^{t+\Delta t} \end{bmatrix};
  Symbolize \begin{bmatrix} u_3^{t+\Delta t} \end{bmatrix}; Symbolize \begin{bmatrix} u_4^{t+\Delta t} \end{bmatrix}; Symbolize \begin{bmatrix} u_6^{t+\Delta t} \end{bmatrix}; Symbolize \begin{bmatrix} u_6^{t+\Delta t} \end{bmatrix};
  Symbolize \begin{bmatrix} u_1^{t+\gamma \Delta t} \end{bmatrix}; Symbolize \begin{bmatrix} u_2^{t+\gamma \Delta t} \end{bmatrix}; Symbolize \begin{bmatrix} u_4^{t+\gamma \Delta t} \end{bmatrix}; Symbolize \begin{bmatrix} u_4^{t+\gamma \Delta t} \end{bmatrix};
  Symbolize \left[\begin{array}{c} u_5^{t+\gamma\Delta t} \end{array}\right]; Symbolize \left[\begin{array}{c} u_6^{t+\gamma\Delta t} \end{array}\right]; Symbolize \left[\begin{array}{c} u_1^{t} \end{array}\right]; Symbolize \left[\begin{array}{c} u_2^{t} \end{array}\right];
  Symbolize \begin{bmatrix} u_3^t \end{bmatrix}; Symbolize \begin{bmatrix} u_4^t \end{bmatrix}; Symbolize \begin{bmatrix} u_5^t \end{bmatrix}; Symbolize \begin{bmatrix} u_0^1 \end{bmatrix};
  Symbolize \begin{bmatrix} u_1^1 \end{bmatrix}; Symbolize \begin{bmatrix} u_2^{1/2} \end{bmatrix}; Symbolize \begin{bmatrix} u_1^{1/2} \end{bmatrix}; Symbolize \begin{bmatrix} u_1^{1/2} \end{bmatrix};
  Symbolize \left[\begin{array}{c} u_2^{1/2} \end{array}\right]; Symbolize \left[\begin{array}{c} u_0^0 \end{array}\right]; Symbolize \left[\begin{array}{c} u_1^0 \end{array}\right]; Symbolize \left[\begin{array}{c} u_2^0 \end{array}\right];
 ClearAll["Global`*"]
\gamma = \frac{1}{2}; \ \beta_2 = 2 \ \beta_1;
 eq1 = m \dot{u}^{t} + c_{0}^{2} k u^{t} = 0;
  eq2 = m \dot{u}^{t+\gamma\Delta t} + c_0^2 k u^{t+\gamma\Delta t} == 0;
  eq3 = m \dot{u}^{t+\Delta t} + c_0^2 k u^{t+\Delta t} == 0;
 eq4 = u^{t+\gamma\Delta t} = u^t + \frac{\gamma \Delta t}{2} (\dot{u}^t + \dot{u}^{t+\gamma\Delta t});
 eq5 = \dot{\mathbf{u}}^{t+\gamma\Delta t} = \dot{\mathbf{u}}^t + \frac{\gamma \Delta t}{2} \left( \dot{\mathbf{u}}^t + \dot{\mathbf{u}}^{t+\gamma\Delta t} \right);
  eq6 = u^{t+\Delta t} = u^t + \gamma \Delta t \left( \left( 1 - \beta_1 \right) \dot{u}^t + \beta_1 \dot{u}^{t+\gamma \Delta t} \right) + \left( 1 - \gamma \right) \Delta t \left( \left( 1 - \beta_2 \right) \dot{u}^{t+\gamma \Delta t} + \beta_2 \dot{u}^{t+\Delta t} \right); 
 lmf = Collect[Eliminate[{eq1, eq2, eq3, eq4, eq5, eq6, eq7},
                 \left\{\ddot{u}^{t},~\ddot{u}^{t+\gamma\Delta t},~\ddot{u}^{t+\Delta t},~\dot{u}^{t},~\dot{u}^{t+\gamma\Delta t},~\dot{u}^{t+\Delta t}\right\}\right] \text{ // FullSimplify, } \left\{u^{t},~u^{t+\gamma\Delta t},~u^{t+\Delta t}\right\}\right]
 u^{t+\gamma\Delta t} \left(-16 \text{ m} + 3 \text{ k} \beta_1 \Delta t^2 \text{ c}_0^2 - 4 \text{ k} \beta_1^2 \Delta t^2 \text{ c}_0^2\right) +
         \mathbf{u}^{t} \left( 8 \, \mathbf{m} + 3 \, \mathbf{k} \, \beta_{1} \, \Delta t^{2} \, \mathbf{c}_{0}^{2} - 4 \, \mathbf{k} \, \beta_{1}^{2} \, \Delta t^{2} \, \mathbf{c}_{0}^{2} \right) + \mathbf{u}^{t + \Delta t} \left( 8 \, \mathbf{m} + 8 \, \mathbf{k} \, \beta_{1}^{2} \, \Delta t^{2} \, \mathbf{c}_{0}^{2} \right) = 0
m = \frac{\Delta x}{6} (1 \ 4 \ 1);
k = \frac{1}{\Lambda x} (-1 \ 2 \ -1);
\Delta t = \frac{cfl \, \Delta x}{c_0};
\mathbf{u}^{\mathsf{t}+\Delta\mathsf{t}} = \begin{pmatrix} \mathbf{u}_0^1 \\ \mathbf{u}_1^1 \\ \mathbf{u}_0^1 \end{pmatrix}; \ \mathbf{u}^{\mathsf{t}+\gamma\Delta\mathsf{t}} = \begin{pmatrix} \mathbf{u}_0^{1/2} \\ \mathbf{u}_1^{1/2} \\ \mathbf{u}_0^{1/2} \end{pmatrix}; \ \mathbf{u}^{\mathsf{t}} = \begin{pmatrix} \mathbf{u}_0^0 \\ \mathbf{u}_1^0 \\ \mathbf{u}_0^1 \end{pmatrix};
```

$$\begin{split} & u_0^1 = E^{T \times \Delta x} \left( 0 - (n_t + 1) \left( \frac{c_0 \, \Delta t}{\Delta x} \right) \left( \frac{c}{c_0} \right) \right) \left( \star \text{notice here, } n_x = 3 \text{ and } n_t = 0 \left( \text{koz } t + 0 \Delta t \right) \star \right) \\ & e^{-\frac{1 \cos t_1 \left( 1 + n_t \right) \, \Delta x \times c}{c_0}} \\ & u_1^1 = E^{T \times \Delta x} \left( 1 - (n_t + 1) \left( \frac{c_0 \, \Delta t}{\Delta x} \right) \left( \frac{c}{c_0} \right) \right) \\ & e^{\frac{1}{\alpha} \, \Delta x \times c} \left( 1 - \frac{\cot t \, (1 + n_t)}{c_0} \right) \\ & u_2^1 = E^{T \times \Delta x} \left( 2 - (n_t + 1) \left( \frac{c_0 \, \Delta t}{\Delta x} \right) \left( \frac{c}{c_0} \right) \right) \\ & e^{\frac{1}{\alpha} \, \Delta x \times c} \left( 2 - \frac{\cot t \, (1 + n_t)}{c_0} \right) \\ & u_0^{1/2} = E^{T \times \Delta x} \left( 0 - \left( n_t + \frac{1}{2} \right) \left( \frac{c_0 \, \Delta t}{\Delta x} \right) \left( \frac{c}{c_0} \right) \right) \left( \star \text{notice here, } n_x = 3 \text{ and } n_t = 0 \left( \text{koz } t + 0 \Delta t \right) \star \right) \\ & e^{-\frac{1}{\alpha} \, \cot t \left( \frac{1}{\alpha} \cdot n_t \right) \, \Delta x \times c} \\ & u_1^{1/2} = E^{T \times \Delta x} \left( 1 - \left( n_t + \frac{1}{2} \right) \left( \frac{c_0 \, \Delta t}{\Delta x} \right) \left( \frac{c}{c_0} \right) \right) \\ & \frac{1}{\alpha} \, \Delta x \times \left( 1 - \frac{\cot t \, \left( \frac{1}{\alpha} \cdot n_t \right)}{c_0} \right) \\ & e^{\frac{1}{\alpha} \, \Delta x \times c} \left( 2 - \frac{\cot t \, \left( \frac{1}{\alpha} \cdot n_t \right)}{c_0} \right) \\ & e^{\frac{1}{\alpha} \, \Delta x \times c} \left( 2 - \frac{\cot t \, \left( \frac{1}{\alpha} \cdot n_t \right)}{c_0} \right) \\ & e^{\frac{1}{\alpha} \, \Delta x \times c} \left( 2 - \frac{\cot t \, \left( \frac{1}{\alpha} \cdot n_t \right)}{c_0} \right) \\ & e^{\frac{1}{\alpha} \, \Delta x \times c} \left( 2 - \frac{\cot t \, \left( \frac{1}{\alpha} \cdot n_t \right)}{c_0} \right) \left( \star \text{notice here, } n_x = 3 \text{ and } n_t = 0 \left( \text{koz } t + 0 \Delta t \right) \star \right) \\ & e^{\frac{1}{\alpha} \, \Delta x \times c} \left( 2 - \frac{\cot t \, \left( \frac{1}{\alpha} \cdot n_t \right)}{c_0} \right) \\ & e^{\frac{1}{\alpha} \, \Delta x \times c} \left( 2 - \frac{\cot t \, \left( \frac{1}{\alpha} \cdot n_t \right)}{c_0} \right) \\ & e^{\frac{1}{\alpha} \, \Delta x \times c} \left( 2 - \frac{\cot t \, \left( \frac{1}{\alpha} \cdot n_t \right)}{c_0} \right) \\ & e^{\frac{1}{\alpha} \, \Delta x \times c} \left( 2 - \frac{\cot t \, \left( \frac{1}{\alpha} \cdot n_t \right)}{c_0} \right) \\ & e^{\frac{1}{\alpha} \, \Delta x \times c} \left( 2 - \frac{\cot t \, \left( \frac{1}{\alpha} \cdot n_t \right)}{c_0} \right) \\ & e^{\frac{1}{\alpha} \, \Delta x \times c} \left( 2 - \frac{\cot t \, \left( \frac{1}{\alpha} \cdot n_t \right)}{c_0} \right) \\ & e^{\frac{1}{\alpha} \, \Delta x \times c} \left( 2 - \frac{\cot t \, \left( \frac{1}{\alpha} \cdot n_t \right)}{c_0} \right) \\ & e^{\frac{1}{\alpha} \, \Delta x \times c} \left( 2 - \frac{\cot t \, \left( \frac{1}{\alpha} \cdot n_t \right)}{c_0} \right) \\ & e^{\frac{1}{\alpha} \, \Delta x \times c} \left( 2 - \frac{\cot t \, \left( \frac{1}{\alpha} \cdot n_t \right)}{c_0} \right) \\ & e^{\frac{1}{\alpha} \, \Delta x \times c} \left( 2 - \frac{\cot t \, \left( \frac{1}{\alpha} \cdot n_t \right)}{c_0} \right) \\ & e^{\frac{1}{\alpha} \, \Delta x \times c} \left( 2 - \frac{\cot t \, \left( \frac{1}{\alpha} \cdot n_t \right)}{c_0} \right) \\ & e^{\frac{1}{\alpha} \, \Delta x \times c} \left( 2 -$$

$$\begin{split} & e^{i \, \Delta x \, \kappa \, \left(1 - \frac{c \, \text{cfl} \, n_t}{c_o}\right)} \\ & u_2^0 \, = \, \boldsymbol{E}^{\text{I} \, \kappa \, \Delta x} \left(2 - n_t \, \left(\frac{c_0 \, \Delta t}{\Delta x}\right) \, \left(\frac{c}{c_o}\right)\right) \end{split}$$

$$\mathbf{u_2} = \mathbf{E} \qquad \qquad \mathbf{v}$$

$$\mathbf{e}^{i \, \Delta \mathbf{x} \, \kappa \, \left(2 - \frac{c \, \text{cfl} \, n_t}{c_0}\right)}$$

$$\begin{split} & \mathbf{lmf} = \ \left( 8 \, \mathbf{m} + \mathbf{k} \, \beta_1 \, \Delta t^2 \, \mathbf{c}_0^2 + \mathbf{k} \, \beta_2 \, \Delta t^2 \, \mathbf{c}_0^2 - 2 \, \mathbf{k} \, \beta_1 \, \beta_2 \, \Delta t^2 \, \mathbf{c}_0^2 \right) \, . \mathbf{u}^t \, + \\ & \left( -16 \, \mathbf{m} + \mathbf{k} \, \beta_1 \, \Delta t^2 \, \mathbf{c}_0^2 + \mathbf{k} \, \beta_2 \, \Delta t^2 \, \mathbf{c}_0^2 + 2 \, \mathbf{k} \, \beta_1 \, \beta_2 \, \Delta t^2 \, \mathbf{c}_0^2 - 2 \, \mathbf{k} \, \beta_2^2 \, \Delta t^2 \, \mathbf{c}_0^2 \right) \, . \mathbf{u}^{t + \gamma \Delta t} \, + \\ & \left( 8 \, \mathbf{m} + 2 \, \mathbf{k} \, \beta_2^2 \, \Delta t^2 \, \mathbf{c}_0^2 \right) \, . \mathbf{u}^{t + \Delta t} \, = 0 \\ & \left\{ \left\{ \mathbf{e}^{i \, \Delta x \, \kappa} \, \left( 1 - \frac{c \, c \, t \, 1 \, n_t}{c_0} \right) \, \left( \frac{16 \, \Delta x}{3} \, + 2 \, c \, f \, 1^2 \, \beta_1 \, \Delta x \, + 2 \, c \, f \, 1^2 \, \beta_2 \, \Delta x \, - 4 \, c \, f \, 1^2 \, \beta_1 \, \beta_2 \, \Delta x \right) \, + \\ & \mathbf{e}^{i \, \Delta x \, \kappa} \, \left( 2 - \frac{c \, c \, t \, 1 \, n_t}{c_0} \right) \, \left( \frac{4 \, \Delta x}{3} \, - \, c \, f \, 1^2 \, \beta_1 \, \Delta x \, - \, c \, f \, 1^2 \, \beta_2 \, \Delta x \, + 2 \, c \, f \, 1^2 \, \beta_1 \, \beta_2 \, \Delta x \right) \, + \\ & \mathbf{e}^{i \, \Delta x \, \kappa} \, \left( 1 - \frac{c \, c \, t \, 1 \, \left( \frac{1}{2} \, n_t \, v \right)}{3} \, - \, c \, f \, 1^2 \, \beta_1 \, \Delta x \, - \, c \, f \, 1^2 \, \beta_2 \, \Delta x \, + 2 \, c \, f \, 1^2 \, \beta_1 \, \beta_2 \, \Delta x \right) \, + \\ & \mathbf{e}^{i \, \Delta x \, \kappa} \, \left( 1 - \frac{c \, c \, t \, 1 \, \left( \frac{1}{2} \, n_t \, v \right)}{3} \, - \, c \, f \, 1^2 \, \beta_1 \, \Delta x \, + 2 \, c \, f \, 1^2 \, \beta_2 \, \Delta x \, + 4 \, c \, f \, 1^2 \, \beta_1 \, \beta_2 \, \Delta x \, - 4 \, c \, f \, 1^2 \, \beta_2^2 \, \Delta x \right) \, + \\ & \mathbf{e}^{i \, \Delta x \, \kappa} \, \left( 2 - \frac{c \, c \, t \, 1 \, \left( \frac{1}{2} \, n_t \, v \right)}{3} \, - \, 2 \, c \, f \, 1^2 \, \beta_2^2 \, \Delta x \right) \, + \, \mathbf{e}^{-i \, c \, c \, f \, 1 \, \left( \frac{1}{2} \, n_t \, v \right)} \, \left( - \frac{8 \, \Delta x}{3} \, - \, c \, f \, 1^2 \, \beta_1 \, \Delta x \, - \, c \, f \, 1^2 \, \beta_2 \, \Delta x \, - \, 2 \, c \, f \, 1^2 \, \beta_1 \, \beta_2 \, \Delta x \, + \, 2 \, c \, f \, 1^2 \, \beta_2^2 \, \Delta x \right) \, + \\ & \mathbf{e}^{i \, \Delta x \, \kappa} \, \left( 2 - \frac{c \, c \, f \, 1 \, \left( \frac{1}{2} \, n_t \, v \right)}{3} \, - \, c \, f \, 1^2 \, \beta_1 \, \Delta x \, - \, c \, f \, 1^2 \, \beta_2 \, \Delta x \, - \, 2 \, c \, f \, 1^2 \, \beta_2^2 \, \Delta x \right) \, + \\ & \mathbf{e}^{i \, \Delta x \, \kappa} \, \left( 2 - \frac{c \, c \, f \, 1 \, \left( \frac{1}{2} \, n_t \, v \right)}{3} \, - \, c \, f \, 1^2 \, \beta_1 \, \Delta x \, - \, c \, f \, 1^2 \, \beta_2 \, \Delta x \, - \, 2 \, c \, f \, 1^2 \, \beta_1 \, \Delta x \, + \, 2 \, c \, f \, 1^2 \, \beta_2^2 \, \Delta x \right) \, + \\ & \mathbf{e}^{i \, \Delta x \, \kappa} \, \left( 2 - \frac{c \, c \, f \, 1 \, \left( \frac{1}{2} \, n_t \, v \right)}{3} \, - \, c \, f \, 1^2 \, \beta$$

$$\begin{split} & \lim f = e^{\frac{i}{\Delta x} \times \left(1 - \frac{\operatorname{cert}(n_{i})}{\sigma_{i}}\right)} \left(\frac{4\Delta x}{3} + 2\operatorname{cfl}^{2}\beta_{1}\Delta x + 2\operatorname{cfl}^{2}\beta_{2}\Delta x - 4\operatorname{cfl}^{2}\beta_{1}\beta_{2}\Delta x\right) + \\ & e^{\frac{i}{\Delta x} \times \left(2 - \frac{\operatorname{cert}(n_{i})}{\sigma_{i}}\right)} \left(\frac{4\Delta x}{3} - \operatorname{cfl}^{2}\beta_{1}\Delta x - \operatorname{cfl}^{2}\beta_{2}\Delta x + 2\operatorname{cfl}^{2}\beta_{1}\beta_{2}\Delta x\right) + \\ & e^{\frac{i}{\Delta x} \times \left(1 - \frac{\operatorname{cert}(\frac{1}{c_{i}} \cdot m_{i})}{\sigma_{i}}\right)} \left(-\frac{32\Delta x}{3} + 2\operatorname{cfl}^{2}\beta_{1}\Delta x + 2\operatorname{cfl}^{2}\beta_{2}\Delta x + 4\operatorname{cfl}^{2}\beta_{1}\beta_{2}\Delta x - 4\operatorname{cfl}^{2}\beta_{2}^{2}\Delta x\right) + \\ & e^{\frac{i}{\Delta x} \times \left(1 - \frac{\operatorname{cert}(\frac{1}{c_{i}} \cdot m_{i})}{\sigma_{i}}\right)} \left(\frac{4\Delta x}{3} - 2\operatorname{cfl}^{2}\beta_{2}^{2}\Delta x\right) + e^{-\frac{i\operatorname{cert}(\frac{1}{c_{i}} \cdot m_{i})}{\sigma_{i}}} \left(\frac{4\Delta x}{3} - 2\operatorname{cfl}^{2}\beta_{2}^{2}\Delta x\right) + \\ & e^{\frac{i}{\Delta x} \times \left(2 - \frac{\operatorname{cert}(\frac{1}{c_{i}} \cdot m_{i})}{\sigma_{i}}\right)} \left(\frac{4\Delta x}{3} - \operatorname{cfl}^{2}\beta_{1}\Delta x - \operatorname{cfl}^{2}\beta_{2}\Delta x - 2\operatorname{cfl}^{2}\beta_{1}\beta_{2}\Delta x + 2\operatorname{cfl}^{2}\beta_{2}^{2}\Delta x\right) + \\ & e^{\frac{i}{\Delta x} \times \left(2 - \frac{\operatorname{cert}(\frac{1}{c_{i}} \cdot m_{i})}{\sigma_{i}}\right)} \left(\frac{-\frac{8\Delta x}{3} - \operatorname{cfl}^{2}\beta_{1}\Delta x - \operatorname{cfl}^{2}\beta_{2}\Delta x - 2\operatorname{cfl}^{2}\beta_{1}\beta_{2}\Delta x + 2\operatorname{cfl}^{2}\beta_{2}^{2}\Delta x\right) + \\ & e^{\frac{i}{\Delta x} \times \left(1 - \frac{\operatorname{cert}(1 \cdot m_{i})}{\sigma_{i}}\right)} \left(\frac{16\Delta x}{3} + 4\operatorname{cfl}^{2}\beta_{2}^{2}\Delta x\right) \\ & e^{\frac{i}{\Delta x} \times \left(1 - \frac{\operatorname{cert}(1 \cdot m_{i})}{\sigma_{i}}\right)} \left(\frac{16\Delta x}{3} - \operatorname{cfl}^{2}\beta_{1}\Delta x - \operatorname{cfl}^{2}\beta_{2}\Delta x + 2\operatorname{cfl}^{2}\beta_{1}\beta_{2}\Delta x\right) + \\ & e^{\frac{i}{\Delta x} \times \left(1 - \frac{\operatorname{cert}(1 \cdot m_{i})}{\sigma_{i}}\right)} \left(\frac{4\Delta x}{3} - \operatorname{cfl}^{2}\beta_{1}\Delta x - \operatorname{cfl}^{2}\beta_{2}\Delta x + 2\operatorname{cfl}^{2}\beta_{1}\beta_{2}\Delta x\right) + \\ & e^{\frac{i}{\Delta x} \times \left(1 - \frac{\operatorname{cert}(1 \cdot m_{i})}{\sigma_{i}}\right)} \left(\frac{4\Delta x}{3} - \operatorname{cfl}^{2}\beta_{1}\Delta x - \operatorname{cfl}^{2}\beta_{2}\Delta x + 2\operatorname{cfl}^{2}\beta_{1}\beta_{2}\Delta x\right) + \\ & e^{\frac{i}{\Delta x} \times \left(2 - \frac{\operatorname{cert}(1 \cdot m_{i})}{\sigma_{i}}\right)} \left(\frac{4\Delta x}{3} - \operatorname{cfl}^{2}\beta_{1}\Delta x - \operatorname{cfl}^{2}\beta_{2}\Delta x + 2\operatorname{cfl}^{2}\beta_{1}\beta_{2}\Delta x\right) + \\ & e^{\frac{i}{\Delta x} \times \left(2 - \frac{\operatorname{cert}(1 \cdot m_{i})}{\sigma_{i}}\right)} \left(-\frac{32\Delta x}{3} + 2\operatorname{cfl}^{2}\beta_{1}\Delta x - \operatorname{cfl}^{2}\beta_{2}\Delta x + 2\operatorname{cfl}^{2}\beta_{2}\Delta x\right) + \\ & e^{\frac{i}{\Delta x} \times \left(2 - \frac{\operatorname{cert}(\frac{1 \cdot m_{i})}{\sigma_{i}}\right)} \left(-\frac{32\Delta x}{3} - \operatorname{cfl}^{2}\beta_{1}\Delta x - \operatorname{cfl}^{2}\beta_{2}\Delta x - 2\operatorname{cfl}^{2}\beta_{2}\Delta x + 2\operatorname{cfl}^{2}\beta_{2}\Delta x\right) + \\ & e^{\frac{i}{\Delta x} \times \left(2 - \frac{\operatorname{cert}(\frac{1$$

## $lmf / \Delta x * 3 // Simplify$

$$\begin{array}{l} & e^{-\frac{i \, \operatorname{ccfl} \, (1+n_{\mathrm{t}}) \, \Delta x \, \kappa}{c_0}} \, \left( \, 4 \, - \, 6 \, \operatorname{cfl}^2 \, \beta_2^2 \right) \, - \, 2 \, e^{i \, \Delta x \, \kappa} \, \left( 2 - \frac{c \, \operatorname{cfl} \, (1+n_{\mathrm{t}})}{c_0} \right) \, \left( - \, 2 \, + \, 3 \, \operatorname{cfl}^2 \, \beta_2^2 \right) \, + \\ & 4 \, e^{i \, \Delta x \, \kappa} \, \left( 1 - \frac{c \, \operatorname{ccfl} \, (1+n_{\mathrm{t}})}{c_0} \right) \, \left( \, 4 \, + \, 3 \, \operatorname{cfl}^2 \, \beta_2^2 \right) \, + \, 2 \, e^{i \, \Delta x \, \kappa} \, \left( 1 - \frac{c \, \operatorname{ccfl} \, \left( \frac{1}{2} + n_{\mathrm{t}} \right)}{c_0} \right) \, \left( - \, 16 \, + \, 3 \, \operatorname{cfl}^2 \, \left( \beta_1 \, + \, \beta_2 \, + \, 2 \, \beta_1 \, \beta_2 \, - \, 2 \, \beta_2^2 \right) \right) \, - \\ & e^{i \, \Delta x \, \kappa} \, \left( 2 - \frac{c \, \operatorname{cfl} \, \left( \frac{1}{2} + n_{\mathrm{t}} \right)}{c_0} \right) \, \left( \, 8 \, + \, 3 \, \operatorname{cfl}^2 \, \left( \, \beta_1 \, + \, \beta_2 \, + \, 2 \, \beta_1 \, \beta_2 \, - \, 2 \, \beta_2^2 \right) \right) \, - \\ & e^{-\frac{i \, c \, \operatorname{ccfl} \, \left( \frac{1}{2} + n_{\mathrm{t}} \right) \, \Delta x \, \kappa}{c_0}} \, \left( \, 8 \, + \, 3 \, \operatorname{cfl}^2 \, \left( \, \beta_1 \, + \, \beta_2 \, + \, 2 \, \beta_1 \, \beta_2 \, - \, 2 \, \beta_2^2 \right) \right) \, - \\ & 2 \, e^{i \, \Delta x \, \kappa} \, \left( 1 - \frac{c \, \operatorname{ccfl} \, n_{\mathrm{t}}}{c_0} \right) \, \left( \, - \, 8 \, + \, \operatorname{cfl}^2 \, \left( \, - \, 3 \, \beta_2 \, + \, \beta_1 \, \left( \, - \, 3 \, + \, 6 \, \beta_2 \right) \, \right) \right) \, + \\ & e^{-\frac{i \, \Delta x \, \kappa}{c_0}} \, \left( 2 - \frac{c \, \operatorname{ccfl} \, n_{\mathrm{t}}}{c_0} \right) \, \left( 4 \, + \, \operatorname{cfl}^2 \, \left( \, - \, 3 \, \beta_2 \, + \, \beta_1 \, \left( \, - \, 3 \, + \, 6 \, \beta_2 \right) \, \right) \right) \right) \, + \\ & e^{-\frac{i \, \operatorname{ccfl} \, n_{\mathrm{t}}}{c_0}} \, \left( \, 4 \, + \, \operatorname{cfl}^2 \, \left( \, - \, 3 \, \beta_2 \, + \, \beta_1 \, \left( \, - \, 3 \, + \, 6 \, \beta_2 \right) \, \right) \right) \right) \, + \\ & e^{-\frac{i \, \operatorname{ccfl} \, n_{\mathrm{t}}}{c_0}} \, \left( \, 4 \, + \, \operatorname{cfl}^2 \, \left( \, - \, 3 \, \beta_2 \, + \, \beta_1 \, \left( \, - \, 3 \, + \, 6 \, \beta_2 \right) \, \right) \right) \, \right) \, + \\ & e^{-\frac{i \, \operatorname{ccfl} \, n_{\mathrm{t}}}{c_0}} \, \left( \, 4 \, + \, \operatorname{cfl}^2 \, \left( \, - \, 3 \, \beta_2 \, + \, \beta_1 \, \left( \, - \, 3 \, + \, 6 \, \beta_2 \right) \, \right) \, \right) \, \right) \, + \\ & e^{-\frac{i \, \Delta x \, \kappa}{c_0}} \, \left( \, 4 \, + \, \operatorname{ccfl}^2 \, \left( \, - \, 3 \, \beta_2 \, + \, \beta_1 \, \left( \, - \, 3 \, + \, 6 \, \beta_2 \right) \, \right) \, \right) \, \right) \, + \\ & e^{-\frac{i \, \Delta x \, \kappa}{c_0}} \, \left( \, 4 \, + \, \operatorname{ccfl}^2 \, \left( \, - \, 3 \, \beta_2 \, + \, \beta_1 \, \left( \, - \, 3 \, + \, 6 \, \beta_2 \right) \, \right) \, \right) \, \right) \, \right) \, + \, \left( \, - \, \frac{i \, \Delta x \, \kappa}{c_0}} \, \left( \, - \, \frac{i \, \Delta x \, \kappa}{c_0} \, \left( \, - \, \frac{i \, \Delta x \, \kappa}{c_0} \, \right) \, \right) \, \right) \, + \, \left( \, - \, \frac{i \, \Delta x \, \kappa}{c_0} \, \left( \, - \, \frac{i \, \Delta x \, \kappa}{c_0} \, \left($$

$$\begin{array}{l} \mathbf{e}^{-\frac{i \cdot \operatorname{crf1}\left(1 + \operatorname{in_2}\right) \operatorname{ax}}{c_0}} \left( \mathbf{4} - 6 \operatorname{cfl}^2 \beta_2^2 \right) - \\ 2 \operatorname{e}^{\frac{i \cdot \operatorname{dx} \times \left(2 - \frac{\operatorname{cerf1}\left(1 + \operatorname{in_2}\right)}{c_0}\right)}{c_0}} \left( -2 + 3 \operatorname{cfl}^2 \beta_2^2 \right) + 4 \operatorname{e}^{\frac{i \cdot \operatorname{dx} \times \left(1 - \frac{\operatorname{cerf1}\left(1 + \operatorname{in_2}\right)}{c_0}\right)}{c_0}} \right) \left( 4 + 3 \operatorname{cfl}^2 \beta_2^2 \right) + \\ 2 \operatorname{e}^{-\frac{i \cdot \operatorname{dx} \times \left(2 - \frac{\operatorname{cerf1}\left(\frac{1}{2} + \operatorname{in_2}\right)}{c_0}\right)}{c_0}} \left( 8 + 3 \operatorname{cfl}^2 \left( \beta_1 + \beta_2 + 2 \beta_1 \beta_2 - 2 \beta_2^2 \right) \right) - \\ \mathbf{e}^{-\frac{i \cdot \operatorname{cerf1}\left(\frac{1}{2} + \operatorname{in_2}\right) \operatorname{dax}}{c_0}} \left( 8 + 3 \operatorname{cfl}^2 \left( \beta_1 + \beta_2 + 2 \beta_1 \beta_2 - 2 \beta_2^2 \right) \right) - \\ 2 \operatorname{e}^{\frac{i \cdot \operatorname{dx} \times \left(1 - \frac{\operatorname{cerf1} \operatorname{in_2}}{c_0}\right)}{c_0}} \left( 8 + 3 \operatorname{cfl}^2 \left( \beta_1 + \beta_2 + 2 \beta_1 \beta_2 - 2 \beta_2^2 \right) \right) - \\ 2 \operatorname{e}^{\frac{i \cdot \operatorname{dx} \times \left(1 - \frac{\operatorname{cerf1} \operatorname{in_2}}{c_0}\right)}{c_0}} \left( 4 + \operatorname{cfl}^2 \left( -3 \beta_2 + \beta_1 \left( -3 + 6 \beta_2 \right) \right) \right) + \\ \mathbf{e}^{\frac{i \cdot \operatorname{dx} \times \left(2 - \frac{\operatorname{cerf1} \operatorname{in_2}}{c_0}\right)}{c_0}} \left( 4 + \operatorname{cfl}^2 \left( -3 \beta_2 + \beta_1 \left( -3 + 6 \beta_2 \right) \right) \right) / \cdot \operatorname{\Deltax} \times \rightarrow \operatorname{\Deltax} \times \operatorname{By} \pi \operatorname{Pi} \right) \\ \mathbf{e}^{-\frac{i \cdot \operatorname{cerf1} \left(1 + \operatorname{in_2}\right) \operatorname{Ta} \operatorname{Ax} \times \operatorname{By} \pi}{c_0}} \left( 4 - 6 \operatorname{cfl}^2 \beta_2^2 \right) - \\ 2 \operatorname{e}^{\frac{i \cdot \operatorname{TA} \times \times \operatorname{By} \pi}{c_0}} \left( 2 - \frac{\operatorname{cerf1} \left(\frac{1}{2} \cdot \operatorname{in_2}\right)}{c_0} \right) \left( -2 + 3 \operatorname{cfl}^2 \beta_2^2 \right) + 4 \operatorname{e}^{\frac{i \cdot \operatorname{TA} \times \times \operatorname{By} \pi}{c_0}} \left( 1 - \frac{\operatorname{cerf1} \left(\frac{1}{2} \cdot \operatorname{in_2}\right)}{c_0} \right) \left( 4 + 3 \operatorname{cfl}^2 \left( \beta_1 + \beta_2 + 2 \beta_1 \beta_2 - 2 \beta_2^2 \right) \right) - \\ \mathbf{e}^{\frac{i \cdot \operatorname{TA} \times \times \operatorname{By} \pi}{c_0}} \left( 2 - \frac{\operatorname{cerf1} \left(\frac{1}{2} \cdot \operatorname{in_2}\right)}{c_0} \right) \left( 8 + 3 \operatorname{cfl}^2 \left( \beta_1 + \beta_2 + 2 \beta_1 \beta_2 - 2 \beta_2^2 \right) \right) - \\ 2 \operatorname{e}^{\frac{i \cdot \operatorname{TA} \times \times \operatorname{By} \pi}{c_0}} \left( 8 + 3 \operatorname{cfl}^2 \left( \beta_1 + \beta_2 + 2 \beta_1 \beta_2 - 2 \beta_2^2 \right) \right) - \\ 2 \operatorname{e}^{\frac{i \cdot \operatorname{TA} \times \times \operatorname{By} \pi}{c_0}} \left( 8 + 3 \operatorname{cfl}^2 \left( \beta_1 + \beta_2 + 2 \beta_1 \beta_2 - 2 \beta_2^2 \right) \right) - \\ 2 \operatorname{e}^{\frac{i \cdot \operatorname{TA} \times \times \operatorname{By} \pi}{c_0}} \left( 8 + 3 \operatorname{cfl}^2 \left( -3 \beta_2 + \beta_1 \left( -3 + 6 \beta_2 \right) \right) \right) + \\ \mathbf{e}^{\frac{i \cdot \operatorname{TA} \times \times \operatorname{By} \pi}{c_0}} \left( 4 + \operatorname{cfl}^2 \left( -3 \beta_2 + \beta_1 \left( -3 + 6 \beta_2 \right) \right) \right) + \\ \mathbf{e}^{\frac{i \cdot \operatorname{TA} \times \times \operatorname{By} \pi}{c_0}} \left( 4 + \operatorname{cfl}^2 \left( -3 \beta_2 + \beta_1 \left( -3 + 6 \beta_2 \right) \right) \right) \right) + \\ \mathbf{e}^{\frac{i \cdot \operatorname{TA} \times \times \operatorname{By} \pi}{c_0}} \left( 4 + \operatorname{cfl}^2 \left( -3 \beta_2 + \beta_1 \left( -3 + 6$$

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e^{-\frac{i\,\text{cofl}\,\left(1+n_{\text{t}}\right)\,\pi\,\Delta x \kappa B y \pi}{c_0}}\,\left(4-6\,\text{cfl}^2\,\beta_2^2\right)\,-
                         2 e^{i \pi \Delta x \kappa B y \pi \left(2 - \frac{\cot (1 + n_1)}{c_0}\right)} \left(-2 + 3 \text{ cfl}^2 \beta_2^2\right) + 4 e^{i \pi \Delta x \kappa B y \pi \left(1 - \frac{\cot (1 + n_1)}{c_0}\right)} \left(4 + 3 \text{ cfl}^2 \beta_2^2\right) + \frac{2}{3} e^{i \pi \Delta x \kappa B y \pi \left(1 - \frac{\cot (1 + n_1)}{c_0}\right)} \left(4 + 3 \text{ cfl}^2 \beta_2^2\right) + \frac{2}{3} e^{i \pi \Delta x \kappa B y \pi \left(1 - \frac{\cot (1 + n_1)}{c_0}\right)} \left(4 + 3 \text{ cfl}^2 \beta_2^2\right) + \frac{2}{3} e^{i \pi \Delta x \kappa B y \pi \left(1 - \frac{\cot (1 + n_1)}{c_0}\right)} \left(4 + 3 \text{ cfl}^2 \beta_2^2\right) + \frac{2}{3} e^{i \pi \Delta x \kappa B y \pi \left(1 - \frac{\cot (1 + n_1)}{c_0}\right)} \left(4 + 3 \text{ cfl}^2 \beta_2^2\right) + \frac{2}{3} e^{i \pi \Delta x \kappa B y \pi \left(1 - \frac{\cot (1 + n_1)}{c_0}\right)} \left(4 + 3 \text{ cfl}^2 \beta_2^2\right) + \frac{2}{3} e^{i \pi \Delta x \kappa B y \pi \left(1 - \frac{\cot (1 + n_1)}{c_0}\right)} \left(4 + 3 \text{ cfl}^2 \beta_2^2\right) + \frac{2}{3} e^{i \pi \Delta x \kappa B y \pi \left(1 - \frac{\cot (1 + n_1)}{c_0}\right)} \left(4 + 3 \text{ cfl}^2 \beta_2^2\right) + \frac{2}{3} e^{i \pi \Delta x \kappa B y \pi \left(1 - \frac{\cot (1 + n_1)}{c_0}\right)} \left(4 + 3 \text{ cfl}^2 \beta_2^2\right) + \frac{2}{3} e^{i \pi \Delta x \kappa B y \pi \left(1 - \frac{\cot (1 + n_1)}{c_0}\right)} \left(4 + 3 \text{ cfl}^2 \beta_2^2\right) + \frac{2}{3} e^{i \pi \Delta x \kappa B y \pi \left(1 - \frac{\cot (1 + n_1)}{c_0}\right)} \left(4 + 3 \text{ cfl}^2 \beta_2^2\right) + \frac{2}{3} e^{i \pi \Delta x \kappa B y \pi \left(1 - \frac{\cot (1 + n_1)}{c_0}\right)} \left(4 + 3 \text{ cfl}^2 \beta_2^2\right) + \frac{2}{3} e^{i \pi \Delta x \kappa B y \pi \left(1 - \frac{\cot (1 + n_1)}{c_0}\right)} \left(4 + 3 \text{ cfl}^2 \beta_2^2\right) + \frac{2}{3} e^{i \pi \Delta x \kappa B y \pi \left(1 - \frac{\cot (1 + n_1)}{c_0}\right)} \left(4 + 3 \text{ cfl}^2 \beta_2^2\right) + \frac{2}{3} e^{i \pi \Delta x \kappa B y \pi \left(1 - \frac{\cot (1 + n_1)}{c_0}\right)} \left(4 + 3 \text{ cfl}^2 \beta_2^2\right) + \frac{2}{3} e^{i \pi \Delta x \kappa B y \pi \left(1 - \frac{\cot (1 + n_1)}{c_0}\right)} \left(4 + 3 \text{ cfl}^2 \beta_2^2\right) + \frac{2}{3} e^{i \pi \Delta x \kappa B y \pi \left(1 - \frac{\cot (1 + n_1)}{c_0}\right)} \left(4 + 3 \text{ cfl}^2 \beta_2^2\right) + \frac{2}{3} e^{i \pi \Delta x \kappa B y \pi \left(1 - \frac{\cot (1 + n_1)}{c_0}\right)} \left(4 + 3 \text{ cfl}^2 \beta_2^2\right) + \frac{2}{3} e^{i \pi \Delta x \kappa B y \pi \left(1 - \frac{\cot (1 + n_1)}{c_0}\right)} \left(4 + 3 \text{ cfl}^2 \beta_2^2\right) + \frac{2}{3} e^{i \pi \Delta x \kappa B y \pi \left(1 - \frac{\cot (1 + n_1)}{c_0}\right)} \left(4 + 3 \text{ cfl}^2 \beta_2^2\right) + \frac{2}{3} e^{i \pi \Delta x \kappa B y \pi \left(1 - \frac{\cot (1 + n_1)}{c_0}\right)} \left(4 + 3 \text{ cfl}^2 \beta_2^2\right) + \frac{2}{3} e^{i \pi \Delta x \kappa B y \pi \left(1 - \frac{\cot (1 + n_1)}{c_0}\right)} \left(4 + 3 \text{ cfl}^2 \beta_2^2\right) + \frac{2}{3} e^{i \pi \Delta x \kappa B y \pi \left(1 - \frac{\cot (1 + n_1)}{c_0}\right)} \left(4 + 3 \text{ cfl}^2 \beta_2^2\right) + \frac{2}{3} e^{i \pi \Delta x \kappa B y \pi \left(1 - \frac{\cot (1
                        2 e^{\frac{i \pi \Delta x \kappa B y \pi \left(1 - \frac{c \text{ cfl}\left(\frac{1}{2} + n_{\epsilon}\right)}{c_{0}}\right)} \left(-16 + 3 \text{ cfl}^{2} \left(\beta_{1} + \beta_{2} + 2 \beta_{1} \beta_{2} - 2 \beta_{2}^{2}\right)\right) - \frac{c \pi \Delta x \kappa B y \pi \left(1 - \frac{c \text{ cfl}\left(\frac{1}{2} + n_{\epsilon}\right)}{c_{0}}\right)}{c_{0}}
                         e^{\frac{i \pi \Delta x \kappa B y \pi \left(2 - \frac{c \text{ of } 1 \left(\frac{1}{2} + n_t\right)}{c_0}\right)}} \left(8 + 3 \text{ cfl}^2 \left(\beta_1 + \beta_2 + 2 \beta_1 \beta_2 - 2 \beta_2^2\right)\right) - \frac{c \pi \Delta x \kappa B y \pi \left(2 - \frac{c \text{ of } 1 \left(\frac{1}{2} + n_t\right)}{c_0}\right)}{c_0}\right)
                        e^{-\frac{i \cot \left(\frac{1}{2} + n_{t}\right) \pi \Delta x \kappa B y \pi}{c_{0}}} \left(8 + 3 \text{ cfl}^{2} \left(\beta_{1} + \beta_{2} + 2 \beta_{1} \beta_{2} - 2 \beta_{2}^{2}\right)\right) - \frac{c_{0}}{c_{0}}}
                         2 e^{i \pi \Delta x \kappa B y \pi \left(1 - \frac{c \operatorname{cfl} n_t}{c_0}\right)} \left(-8 + \operatorname{cfl}^2 \left(-3 \beta_2 + \beta_1 \left(-3 + 6 \beta_2\right)\right)\right) + 
                         e^{i \pi \Delta x \times By \pi \left(2 - \frac{c \text{ ofl } n_t}{c_0}\right)} \left(4 + \text{ cfl}^2 \left(-3 \beta_2 + \beta_1 \left(-3 + 6 \beta_2\right)\right)\right) + C \left(-3 \beta_2 + \beta_1 \left(-3 + 6 \beta_2\right)\right)
                         e^{-\frac{i \circ \text{cfl} \, n_c \, \pi \, \Delta x \times By \pi}{c_0}} \, \left( 4 + \text{cfl}^2 \, \left( -3 \, \beta_2 + \beta_1 \, \left( -3 + 6 \, \beta_2 \right) \right) \right) \, / \, . \, \, \frac{c}{c_0} \rightarrow \text{cByc}
   e^{-i \text{ cByc cfl } (1+n_t) \pi \Delta x \kappa B y \pi} \left(4-6 \text{ cfl}^2 \beta_2^2\right) -
               2 e^{i (2-cByc\,cfl\,(1+n_t))\,\pi\,\Delta x \kappa By\pi} \left(-2+3\,cfl^2\,\beta_2^2\right) + 4 e^{i \,(1-cByc\,cfl\,(1+n_t))\,\pi\,\Delta x \kappa By\pi} \left(4+3\,cfl^2\,\beta_2^2\right) + 4 e^{i \,(1-cByc\,cfl\,(1+n_t))\,\pi\,\Delta x \kappa By\pi} \left(4+3\,cfl^2\,\beta_2^2\right
             2\ e^{i\ \left(1-\text{cByc ofl}\ \left(\frac{1}{2}+n_{t}\right)\right)\ \pi\ \Delta x \kappa By\pi}\ \left(-16+3\ \text{cfl}^{2}\ \left(\beta_{1}+\beta_{2}+2\ \beta_{1}\ \beta_{2}-2\ \beta_{2}^{2}\right)\right)\ -
               e^{-i\;\text{cByc cfl}\;\left(\frac{1}{2}+n_t\right)\;\pi\;\Delta x \kappa B y \pi}\;\left(8+3\;\text{cfl}^2\;\left(\beta_1+\beta_2+2\;\beta_1\;\beta_2-2\;\beta_2^2\right)\right)\;-
             e^{i\left(2-\text{cByc cfl}\left(\frac{1}{2}+n_{t}\right)\right)\,\pi\,\Delta x \kappa B y \pi}\,\left(8+3\,\text{cfl}^{2}\,\left(\beta_{1}+\beta_{2}+2\,\beta_{1}\,\beta_{2}-2\,\beta_{2}^{2}\right)\right)\,-\,2\,\beta_{2}^{2}}
               2 e^{i (1-cByc\,cfl\,n_t)\,\pi\,\Delta x \kappa By\pi} \left(-8+cfl^2 \left(-3\,\beta_2+\beta_1\,\left(-3+6\,\beta_2\right)\right)\right) +
               e^{-i\;\text{cByc cfl}\;n_t\;\pi\;\Delta x \kappa \text{By}\pi}\;\left(\,4\,+\,\text{cfl}^2\;\left(\,-\,3\;\beta_2\,+\,\beta_1\;\left(\,-\,3\,+\,6\;\beta_2\,\right)\,\right)\,\right)\,+\,
               e^{i (2-cByc\,cfl\,n_t)\,\pi\,\Delta x \kappa By\pi}\,\left(4+cfl^2\,\left(-3\,eta_2+eta_1\,\left(-3+6\,eta_2
ight)
ight)
ight)
 nd = e^{-i cByc cfl (1+n_t) \pi \Delta x \kappa By\pi} (4 - 6 cfl^2 \beta_2^2) -
                         2\;e^{i\;(2-cByc\;cfl\;(1+n_t))\;\pi\;\Delta x \kappa By\pi}\;\left(-2+3\;cfl^2\;\beta_2^2\right) + 4\;e^{i\;(1-cByc\;cfl\;(1+n_t))\;\pi\;\Delta x \kappa By\pi}\;\left(4+3\;cfl^2\;\beta_2^2\right) + 4\;e^{i(1-cByc\;cfl\;(1+n_t))\;\pi\;\Delta x \kappa By\pi}\;\left(4+3\;cfl^2\;\beta_2^2\right) + 4\;e^{i(1-cByc\;cfl\;(1+n_t))}\;\pi\;\Delta x \kappa By\pi}\;\left(4+3\;cfl^2\;\beta
                         2~e^{i\left(1-\text{cByc cfl}\left(\frac{1}{2}+n_{t}\right)\right)~\pi~\Delta x \kappa B y \pi}~\left(-16+3~\text{cfl}^{2}~\left(\beta_{1}+\beta_{2}+2~\beta_{1}~\beta_{2}-2~\beta_{2}^{2}\right)\right)~-4.5
                         \mathrm{e}^{-\mathrm{i}\,\mathrm{cByc}\,\mathrm{cfl}\,\left(\frac{1}{2}+n_{\mathrm{t}}\right)\,\pi\,\Delta x \kappa B y \pi}\,\left(8+3\,\mathrm{cfl}^2\,\left(\beta_1+\beta_2+2\,\beta_1\,\beta_2-2\,\beta_2^2\right)\right)\,-\,\frac{1}{2}\,\mathrm{cfl}^2\left(\beta_1+\beta_2+2\,\beta_1\,\beta_2-2\,\beta_2^2\right)\right)\,-\,\frac{1}{2}\,\mathrm{cfl}^2\left(\beta_1+\beta_2+2\,\beta_1\,\beta_2-2\,\beta_2^2\right)
                         \mathrm{e}^{\frac{i}{\hbar}\left(2-\mathrm{cByc\;cfl}\left(\frac{1}{2}+n_{\mathrm{t}}\right)\right)\;\pi\;\Delta\kappa\kappa\mathrm{By}\pi}\;\left(8+3\;\mathrm{cfl^2}\;\left(\beta_{1}+\beta_{2}+2\;\beta_{1}\;\beta_{2}-2\;\beta_{2}^{2}\right)\right)\;-1}
                           2 e^{i (1-cByc cfl n_t) \pi \Delta x \kappa By\pi} \left(-8 + cfl^2 \left(-3 \beta_2 + \beta_1 \left(-3 + 6 \beta_2\right)\right)\right) +
                           e^{-i cByc cfl n_t \pi \Delta x \kappa By\pi} \left(4 + cfl^2 \left(-3 \beta_2 + \beta_1 \left(-3 + 6 \beta_2\right)\right)\right) +
                           e^{i(2-cByc\ cfl\ n_t)\ \pi\ \Delta x \kappa By\pi} \left(4+cfl^2\left(-3\ \beta_2+\beta_1\ \left(-3+6\ \beta_2\right)\right)\right)
   e^{-i \text{ cByc cfl } (1+n_t) \pi \Delta x \kappa By \pi} \left(4-6 \text{ cfl}^2 \beta_2^2\right) -
               2 e^{i (2-cByc\,cfl\,\,(1+n_t))\,\,\pi\,\Delta x \kappa By\pi} \left(-2+3\,cfl^2\,\beta_2^2\right) + 4 e^{i \,\,(1-cByc\,cfl\,\,(1+n_t))\,\,\pi\,\Delta x \kappa By\pi} \left(4+3\,cfl^2\,\beta_2^2\right) + 2 e^{i \,\,(1-cByc\,cfl\,\,(1+n_t))\,\,\pi\,\Delta
             2~e^{i~\left(1-\text{cByc cfl}~\left(\frac{1}{2}+n_{t}\right)\right)~\pi~\Delta x \kappa B y \pi}~\left(-16+3~\text{cfl}^{2}~\left(\beta_{1}+\beta_{2}+2~\beta_{1}~\beta_{2}-2~\beta_{2}^{2}\right)\right)~-
             e^{-i\;\text{cByc cfl}\;\left(\frac{1}{2}+n_t\right)\;\pi\;\Delta x \kappa B y \pi}\;\left(8+3\;\text{cfl}^2\;\left(\beta_1+\beta_2+2\;\beta_1\;\beta_2-2\;\beta_2^2\right)\right)\;-1
             e^{i\left(2-\text{cByc cfl}\left(\frac{1}{2}+n_{t}\right)\right)\,\pi\,\Delta x \kappa B y \pi}\,\left(8+3\,\text{cfl}^{2}\,\left(\beta_{1}+\beta_{2}+2\,\beta_{1}\,\beta_{2}-2\,\beta_{2}^{2}\right)\right)\,-\,2\,\beta_{2}^{2}}
               2 e^{i (1-cByc\,cfl\,n_t)\,\pi\,\Delta x \kappa By\pi} \left(-8+cfl^2 \left(-3\,\beta_2+\beta_1 \left(-3+6\,\beta_2\right)\right)\right) +
             e^{-i\;\text{cByc cfl}\;n_t\;\pi\;\Delta x\kappa By\pi}\;\left(\,4\,+\,\text{cfl}^2\;\left(\,-\,3\;\beta_2\,+\,\beta_1\;\left(\,-\,3\,+\,6\;\beta_2\,\right)\,\right)\,\right)\,+\,
               e^{i (2-cByc\,cfl\,n_t)\,\pi\,\Delta x \kappa By\pi}\,\left(4+cfl^2\,\left(-3\,eta_2+eta_1\,\left(-3+6\,eta_2
ight)
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eq = nd // Cancel // ExpandAll // Simplify
  e^{-i\,\text{cByc cfl }(1+n_t)\,\,\pi\,\Delta x \kappa \text{By}\pi\,\,\left(4-6\,\,\text{cfl}^2\,\,\beta_2^2+e^{2\,\,i\,\,\pi\,\Delta x \kappa \text{By}\pi}\,\,\left(4-6\,\,\text{cfl}^2\,\,\beta_2^2\right)\,+
                                                       4\ e^{i\ \pi\ \Delta x \kappa B y \pi}\ \left(4+3\ cfl^{2}\ \beta_{2}^{2}\right) + e^{i\ (1+cByc\ cfl)\ \pi\ \Delta x \kappa B y \pi}\ \left(16+6\ cfl^{2}\ (\beta_{1}+\beta_{2}-2\ \beta_{1}\ \beta_{2})\right) + e^{i\ (1+cByc\ cfl)\ \pi\ \Delta x \kappa B y \pi}
                                                    2 \; e^{\frac{1}{2} \; i \; (2 + c B y c \; c f 1) \; \pi \; \Delta x \kappa B y \pi} \; \left( -16 + 3 \; c f 1^2 \; \left( \beta_1 + \beta_2 + 2 \; \beta_1 \; \beta_2 - 2 \; \beta_2^2 \right) \right) \; - \; C \; d^2 
                                                    e^{\frac{1}{2}i \operatorname{cByc} \operatorname{cfl} \pi \operatorname{\Delta} x \times \operatorname{By} \pi} \left( 8 + 3 \operatorname{cfl}^2 \left( \beta_1 + \beta_2 + 2 \beta_1 \beta_2 - 2 \beta_2^2 \right) \right) - C \operatorname{Col} \pi \operatorname{C} x \times \operatorname{Col} \pi \operatorname{Col} \pi \operatorname{C} x \times \operatorname{Col} \pi \operatorname{Col} \pi
                                                    e^{\frac{1}{2}\,i\,\left(4+cByc\,cfl\right)\,\,\pi\,\Delta x \kappa By\pi}\,\left(8+3\,cfl^2\,\left(\beta_1+\beta_2+2\,\,\beta_1\,\,\beta_2-2\,\,\beta_2^2\right)\right)\,+
                                                       e^{i \text{ cByc cfl } \pi \text{ } \Delta x \times By\pi} \left(4 + \text{cfl}^2 \left(-3 \beta_2 + \beta_1 \left(-3 + 6 \beta_2\right)\right)\right) +
                                                       \mathrm{e}^{\mathrm{i}\;\left(2+\mathrm{cByc\;cfl}\right)\;\pi\;\Delta\mathrm{x}\mathrm{x}\mathrm{By}\pi\;\left(\,4+\mathrm{cfl}^{\,2}\;\left(\,-\,3\;\beta_{2}\,+\,\beta_{1}\;\left(\,-\,3\,+\,6\;\beta_{2}\,\right)\,\right)\,\right)\,}
nd2 = 4 - 6 cfl^{2} \beta_{2}^{2} + e^{2 i \pi \Delta x \kappa B y \pi} (4 - 6 cfl^{2} \beta_{2}^{2}) +
                                                                                         4 \, e^{i \, \pi \, \Delta x \kappa B y \pi} \, \left( 4 + 3 \, \text{cfl}^2 \, \beta_2^2 \right) + e^{i \, (1 + c B y c \, c f 1) \, \pi \, \Delta x \kappa B y \pi} \, \left( 16 + 6 \, c f 1^2 \, \left( \beta_1 + \beta_2 - 2 \, \beta_1 \, \beta_2 \right) \right) + e^{i \, \pi \, \Delta x \kappa B y \pi} \, \left( 16 + 6 \, c f 1^2 \, \left( \beta_1 + \beta_2 - 2 \, \beta_1 \, \beta_2 \right) \right) + e^{i \, \pi \, \Delta x \kappa B y \pi} \, \left( 16 + 6 \, c f 1^2 \, \left( \beta_1 + \beta_2 - 2 \, \beta_1 \, \beta_2 \right) \right) + e^{i \, \pi \, \Delta x \kappa B y \pi} \, \left( 16 + 6 \, c f 1^2 \, \left( \beta_1 + \beta_2 - 2 \, \beta_1 \, \beta_2 \right) \right) + e^{i \, \pi \, \Delta x \kappa B y \pi} \, \left( 16 + 6 \, c f 1^2 \, \left( \beta_1 + \beta_2 - 2 \, \beta_1 \, \beta_2 \right) \right) + e^{i \, \pi \, \Delta x \kappa B y \pi} \, \left( 16 + 6 \, c f 1^2 \, \left( \beta_1 + \beta_2 - 2 \, \beta_1 \, \beta_2 \right) \right) + e^{i \, \pi \, \Delta x \kappa B y \pi} \, \left( 16 + 6 \, c f 1^2 \, \left( \beta_1 + \beta_2 - 2 \, \beta_1 \, \beta_2 \right) \right) + e^{i \, \pi \, \Delta x \kappa B y \pi} \, \left( 16 + 6 \, c f 1^2 \, \left( \beta_1 + \beta_2 - 2 \, \beta_1 \, \beta_2 \right) \right) + e^{i \, \pi \, \Delta x \kappa B y \pi} \, \left( 16 + 6 \, c f 1^2 \, \left( \beta_1 + \beta_2 - 2 \, \beta_1 \, \beta_2 \right) \right) + e^{i \, \pi \, \Delta x \kappa B y \pi} \, \left( 16 + 6 \, c f 1^2 \, \left( \beta_1 + \beta_2 - 2 \, \beta_1 \, \beta_2 \right) \right) + e^{i \, \pi \, \Delta x \kappa B y \pi} \, \left( 16 + 6 \, c f 1^2 \, \left( \beta_1 + \beta_2 - 2 \, \beta_1 \, \beta_2 \right) \right) + e^{i \, \pi \, \Delta x \kappa B y \pi} \, \left( 16 + 6 \, c f 1^2 \, \left( \beta_1 + \beta_2 - 2 \, \beta_1 \, \beta_2 \right) \right) + e^{i \, \pi \, \Delta x \kappa B y \pi} \, \left( 16 + 6 \, c f 1^2 \, \left( \beta_1 + \beta_2 - 2 \, \beta_1 \, \beta_2 \right) \right) + e^{i \, \pi \, \Delta x \kappa B y \pi} \, \left( 16 + 6 \, c f 1^2 \, \left( \beta_1 + \beta_2 - 2 \, \beta_1 \, \beta_2 \right) \right) + e^{i \, \pi \, \Delta x \kappa B y \pi} \, \left( 16 + 6 \, c f 1^2 \, \left( \beta_1 + \beta_2 - 2 \, \beta_1 \, \beta_2 \right) \right) + e^{i \, \pi \, \Delta x \kappa B y \pi} \, \left( 16 + 6 \, c f 1^2 \, \left( \beta_1 + \beta_2 - 2 \, \beta_1 \, \beta_2 \right) \right) + e^{i \, \pi \, \Delta x \kappa B y \pi} \, \left( 16 + 6 \, c f 1^2 \, \left( \beta_1 + \beta_2 - 2 \, \beta_1 \, \beta_2 \right) \right) + e^{i \, \pi \, \Delta x \kappa B y \pi} \, \left( 16 + 6 \, c f 1^2 \, \left( \beta_1 + \beta_2 - 2 \, \beta_1 \, \beta_2 \right) \right) + e^{i \, \pi \, \Delta x \kappa B y \pi} \, \left( 16 + 6 \, c f 1^2 \, \left( \beta_1 + \beta_2 - 2 \, \beta_1 \, \beta_2 \right) \right) + e^{i \, \pi \, \Delta x \kappa B y \pi} \, \left( 16 + 6 \, c f 1^2 \, \left( \beta_1 + \beta_2 - 2 \, \beta_1 \, \beta_2 \right) \right) + e^{i \, \pi \, \Delta x \kappa B y \pi} \, \left( 16 + 6 \, c f 1^2 \, \left( \beta_1 + \beta_2 - 2 \, \beta_1 \, \beta_2 \right) \right) + e^{i \, \pi \, \Delta x \kappa B y \pi} \, \left( 16 + 6 \, c f 1^2 \, \left( \beta_1 + \beta_2 - 2 \, \beta_1 \, \beta_2 \right) \right) + e^{i \, \pi \, \Delta x \kappa B y \pi} \, \left( 16 + 6 \, c f 1 \, \beta_2 + 2 \, c \beta_1 \, \beta_2 \right) \right) + e^{i \, \pi \, 
                                                                                      2\ e^{\frac{1}{2}\,\dot{a}\ (2+cByc\ cfl)\ \pi\,\Delta x \kappa By\pi}\ \left(-\,16+3\ cfl^2\ \left(\beta_1+\beta_2+2\ \beta_1\ \beta_2-2\ \beta_2^2\right)\right)\ -
                                                                                      \mathrm{e}^{\frac{1}{2}\,\mathrm{ii}\,\,(4+\mathrm{cByc\,cfl})\,\,\pi\,\Delta x \kappa B y \pi}\,\left(8+3\,\mathrm{cfl}^2\,\left(\beta_1+\beta_2+2\,\beta_1\,\beta_2-2\,\beta_2^2\right)\right)\,+
                                                                                       e^{i \operatorname{cByc} \operatorname{cfl} \pi \operatorname{\Delta} x \ltimes \operatorname{By} \pi} \left( 4 + \operatorname{cfl}^{2} \left( -3 \beta_{2} + \beta_{1} \left( -3 + 6 \beta_{2} \right) \right) \right) + e^{i \left( 2 + \operatorname{cByc} \operatorname{cfl} \right) \pi \operatorname{\Delta} x \ltimes \operatorname{By} \pi} 
                                                                                                              (4 + cfl^2 (-3 \beta_2 + \beta_1 (-3 + 6 \beta_2))) // Cancel // ExpandAll // Simplify
    4 - 6 \, \mathrm{cfl^2} \, \beta_2^2 + \mathrm{e}^{2 \, \mathrm{i} \, \pi \, \Delta x \kappa \mathrm{By} \pi} \, \left( 4 - 6 \, \mathrm{cfl^2} \, \beta_2^2 \right) + 4 \, \mathrm{e}^{\mathrm{i} \, \pi \, \Delta x \kappa \mathrm{By} \pi} \, \left( 4 + 3 \, \mathrm{cfl^2} \, \beta_2^2 \right) + 2 \, \mathrm{e}^{\mathrm{i} \, \pi \, \Delta x \kappa \mathrm{By} \pi} \, \left( 4 + 3 \, \mathrm{cfl^2} \, \beta_2^2 \right) + 2 \, \mathrm{e}^{\mathrm{i} \, \pi \, \Delta x \kappa \mathrm{By} \pi} \, \left( 4 + 3 \, \mathrm{cfl^2} \, \beta_2^2 \right) + 2 \, \mathrm{e}^{\mathrm{i} \, \pi \, \Delta x \kappa \mathrm{By} \pi} \, \left( 4 + 3 \, \mathrm{cfl^2} \, \beta_2^2 \right) + 2 \, \mathrm{e}^{\mathrm{i} \, \pi \, \Delta x \kappa \mathrm{By} \pi} \, \left( 4 + 3 \, \mathrm{cfl^2} \, \beta_2^2 \right) + 2 \, \mathrm{e}^{\mathrm{i} \, \pi \, \Delta x \kappa \mathrm{By} \pi} \, \left( 4 + 3 \, \mathrm{cfl^2} \, \beta_2^2 \right) + 2 \, \mathrm{e}^{\mathrm{i} \, \pi \, \Delta x \kappa \mathrm{By} \pi} \, \left( 4 + 3 \, \mathrm{cfl^2} \, \beta_2^2 \right) + 2 \, \mathrm{e}^{\mathrm{i} \, \pi \, \Delta x \kappa \mathrm{By} \pi} \, \left( 4 + 3 \, \mathrm{cfl^2} \, \beta_2^2 \right) + 2 \, \mathrm{e}^{\mathrm{i} \, \pi \, \Delta x \kappa \mathrm{By} \pi} \, \left( 4 + 3 \, \mathrm{cfl^2} \, \beta_2^2 \right) + 2 \, \mathrm{e}^{\mathrm{i} \, \pi \, \Delta x \kappa \mathrm{By} \pi} \, \left( 4 + 3 \, \mathrm{cfl^2} \, \beta_2^2 \right) + 2 \, \mathrm{e}^{\mathrm{i} \, \pi \, \Delta x \kappa \mathrm{By} \pi} \, \left( 4 + 3 \, \mathrm{cfl^2} \, \beta_2^2 \right) + 2 \, \mathrm{e}^{\mathrm{i} \, \pi \, \Delta x \kappa \mathrm{By} \pi} \, \left( 4 + 3 \, \mathrm{cfl^2} \, \beta_2^2 \right) + 2 \, \mathrm{e}^{\mathrm{i} \, \pi \, \Delta x \kappa \mathrm{By} \pi} \, \left( 4 + 3 \, \mathrm{cfl^2} \, \beta_2^2 \right) + 2 \, \mathrm{e}^{\mathrm{i} \, \pi \, \Delta x \kappa \mathrm{By} \pi} \, \left( 4 + 3 \, \mathrm{cfl^2} \, \beta_2^2 \right) + 2 \, \mathrm{e}^{\mathrm{i} \, \pi \, \Delta x \kappa \mathrm{By} \pi} \, \left( 4 + 3 \, \mathrm{cfl^2} \, \beta_2^2 \right) + 2 \, \mathrm{e}^{\mathrm{i} \, \pi \, \Delta x \kappa \mathrm{By} \pi} \, \left( 4 + 3 \, \mathrm{cfl^2} \, \beta_2^2 \right) + 2 \, \mathrm{e}^{\mathrm{i} \, \pi \, \Delta x \kappa \mathrm{By} \pi} \, \left( 4 + 3 \, \mathrm{cfl^2} \, \beta_2^2 \right) + 2 \, \mathrm{e}^{\mathrm{i} \, \pi \, \Delta x \kappa \mathrm{By} \pi} \, \left( 4 + 3 \, \mathrm{cfl^2} \, \beta_2^2 \right) + 2 \, \mathrm{e}^{\mathrm{i} \, \pi \, \Delta x \kappa \mathrm{By} \pi} \, \left( 4 + 3 \, \mathrm{cfl^2} \, \beta_2^2 \right) + 2 \, \mathrm{e}^{\mathrm{i} \, \pi \, \Delta x \kappa \mathrm{By} \pi} \, \left( 4 + 3 \, \mathrm{cfl^2} \, \beta_2^2 \right) + 2 \, \mathrm{e}^{\mathrm{i} \, \pi \, \Delta x \kappa \mathrm{By} \pi} \, \left( 4 + 3 \, \mathrm{cfl^2} \, \beta_2^2 \right) + 2 \, \mathrm{e}^{\mathrm{i} \, \pi \, \Delta x \kappa \mathrm{By} \pi} \, \left( 4 + 3 \, \mathrm{cfl^2} \, \beta_2^2 \right) + 2 \, \mathrm{e}^{\mathrm{i} \, \pi \, \Delta x \kappa \mathrm{By} \pi} \, \left( 4 + 3 \, \mathrm{cfl^2} \, \beta_2^2 \right) + 2 \, \mathrm{e}^{\mathrm{i} \, \pi \, \Delta x \kappa \mathrm{By} \pi} \, \left( 4 + 3 \, \mathrm{cfl^2} \, \beta_2^2 \right) + 2 \, \mathrm{e}^{\mathrm{i} \, \pi \, \Delta x \kappa \mathrm{By} \pi} \, \left( 4 + 3 \, \mathrm{cfl^2} \, \beta_2^2 \right) + 2 \, \mathrm{e}^{\mathrm{i} \, \pi \, \Delta x \kappa \mathrm{By} \pi} \, \left( 4 + 3 \, \mathrm{cfl^2} \, \beta_2^2 \right) + 2 \, \mathrm{e}^{\mathrm{i} \, \pi \, \Delta x \kappa \mathrm{By} \pi} \, \left( 4 + 3 \, \mathrm{cfl^2} \, \beta_2 \right) + 2 \, \mathrm{e}^{\mathrm{i} \, \pi \, \Delta x \kappa \mathrm{By
                     e^{i (1+cByccfl) \pi \Delta x \kappa B y \pi} (16+6cfl^2 (\beta_1+\beta_2-2\beta_1\beta_2)) +
                     2 e^{\frac{1}{2}i} (2+cByc cfl) \pi \Delta x \kappa B y \pi \left(-16+3 \text{ cfl}^2 \left(\beta_1+\beta_2+2 \beta_1 \beta_2-2 \beta_2^2\right)\right)
                   e^{\frac{1}{2}i \operatorname{cByc} \operatorname{cfl} \pi \operatorname{\Delta} \operatorname{x} \times \operatorname{By} \pi} \left( 8 + 3 \operatorname{cfl}^2 \left( \beta_1 + \beta_2 + 2 \beta_1 \beta_2 - 2 \beta_2^2 \right) \right) - C
                   e^{\frac{1}{2}\,i\,\left(4+\text{cByc cfl}\right)\,\,\pi\,\Delta x \kappa \text{By}\pi}\,\left(8+3\,\text{cfl}^2\,\left(\beta_1+\beta_2+2\,\beta_1\,\beta_2-2\,\beta_2^2\right)\right)\,+
                     e^{i \text{ cByc cfl } \pi \Delta x \kappa B y \pi} \left( 4 + \text{cfl}^2 \left( -3 \beta_2 + \beta_1 \left( -3 + 6 \beta_2 \right) \right) \right) +
                     e^{i(2+cByccfl)\pi\Delta x \kappa By\pi} \left(4+cfl^2(-3\beta_2+\beta_1(-3+6\beta_2))\right)
```

## Reduce = Solve[nd2 == 0, cByc]

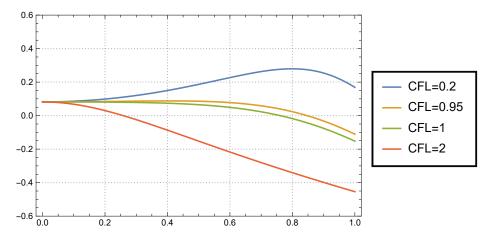
Solve::ifun: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. >> Set::wrsym: Symbol Reduce is Protected. >>>  $-\frac{1}{\text{cfl }\pi\,\Delta\text{x}\kappa\text{By}\pi}\,2\,\,\text{ii}\,\,\text{Log}\Big[\,\Big(8+32\,\,\text{e}^{\text{i}\,\pi\,\Delta\text{x}\kappa\text{By}\pi}+8\,\,\text{e}^{2\,\,\text{ii}\,\pi\,\Delta\text{x}\kappa\text{By}\pi}+3\,\,\text{cfl}^2\,\,\beta_1-6\,\,\text{cfl}^2\,\,\text{e}^{\text{ii}\,\pi\,\Delta\text{x}\kappa\text{By}\pi}\,\beta_1+3\,\,\text{cfl}^2\,\,\beta_2-6\,\,\text{cfl}^2\,\,\text{e}^{\text{ii}\,\pi\,\Delta\text{x}\kappa\text{By}\pi}\,\beta_2+3\,\,\text{cfl}^2\,\,\beta_3-6\,\,\text{cfl}^2\,\,\text{e}^{\text{ii}\,\pi\,\Delta\text{x}\kappa\text{By}\pi}\,\beta_3+3\,\,\text{cfl}^2\,\,\beta_3-6\,\,\text{cfl}^2\,\,\text{e}^{\text{ii}\,\pi\,\Delta\text{x}\kappa\text{By}\pi}\,\beta_3+3\,\,\text{cfl}^2\,\,\beta_3-6\,\,\text{cfl}^2\,\,\text{e}^{\text{ii}\,\pi\,\Delta\text{x}\kappa\text{By}\pi}\,\beta_3+3\,\,\text{cfl}^2\,\,\beta_3-6\,\,\text{cfl}^2\,\,\text{e}^{\text{ii}\,\pi\,\Delta\text{x}\kappa\text{By}\pi}\,\beta_3+3\,\,\text{cfl}^2\,\,\beta_3-6\,\,\text{cfl}^2\,\,\text{e}^{\text{ii}\,\pi\,\Delta\text{x}\kappa\text{By}\pi}\,\beta_3+3\,\,\text{cfl}^2\,\,\beta_3-6\,\,\text{cfl}^2\,\,\text{e}^{\text{ii}\,\pi\,\Delta\text{x}\kappa\text{By}\pi}\,\beta_3+3\,\,\text{cfl}^2\,\,\beta_3-6\,\,\text{cfl}^2\,\,\text{e}^{\text{ii}\,\pi\,\Delta\text{x}\kappa\text{By}\pi}\,\beta_3+3\,\,\text{cfl}^2\,\,\beta_3-6\,\,\text{cfl}^2\,\,\text{e}^{\text{ii}\,\pi\,\Delta\text{x}\kappa\text{By}\pi}\,\beta_3+3\,\,\text{cfl}^2\,\,\beta_3-6\,\,\text{cfl}^2\,\,\text{e}^{\text{ii}\,\pi\,\Delta\text{x}\kappa\text{By}\pi}\,\beta_3+3\,\,\text{cfl}^2\,\,\beta_3-6\,\,\text{cfl}^2\,\,\text{cfl}^2\,\,\beta_3-6\,\,\text{cfl}^2$  $3\,\mathrm{cfl^2}\,\,\mathrm{e}^{2\,\mathrm{i}\,\pi\,\Delta\mathrm{x}\kappa\mathrm{By}\pi}\,\beta_1 + 3\,\mathrm{cfl^2}\,\beta_2 - 6\,\mathrm{cfl^2}\,\,\mathrm{e}^{\mathrm{i}\,\pi\,\Delta\mathrm{x}\kappa\mathrm{By}\pi}\,\beta_2 + 3\,\mathrm{cfl^2}\,\,\mathrm{e}^{2\,\mathrm{i}\,\pi\,\Delta\mathrm{x}\kappa\mathrm{By}\pi}\,\beta_2 +$ 6 cfl<sup>2</sup>  $\beta_1$   $\beta_2$  - 12 cfl<sup>2</sup>  $e^{i \pi \Delta x \kappa B y \pi}$   $\beta_1$   $\beta_2$  + 6 cfl<sup>2</sup>  $e^{2 i \pi \Delta x \kappa B y \pi}$   $\beta_1$   $\beta_2$  -6 cfl²  $\beta_2^2$  + 12 cfl²  $e^{i\,\pi\,\Delta x\kappa By\pi}\,\beta_2^2$  - 6 cfl²  $e^{2\,i\,\pi\,\Delta x\kappa By\pi}\,\beta_2^2$  - $\sqrt{\left(-4\left(4+16\,\mathrm{e}^{\mathrm{i}\,\pi\,\Delta\mathrm{x}\kappa\mathrm{By}\pi}+4\,\mathrm{e}^{2\,\mathrm{i}\,\pi\,\Delta\mathrm{x}\kappa\mathrm{By}\pi}-3\,\mathrm{cfl}^{2}\,eta_{1}+6\,\mathrm{cfl}^{2}\,\mathrm{e}^{\mathrm{i}\,\pi\,\Delta\mathrm{x}\kappa\mathrm{By}\pi}\,eta_{1}ight.}$  $3\,\mathrm{cfl^2}\,\,\mathrm{e}^{2\,\mathrm{i}\,\pi\,\Delta\mathrm{x}\kappa\mathrm{By}\pi}\,\beta_1 - 3\,\mathrm{cfl^2}\,\beta_2 + 6\,\mathrm{cfl^2}\,\,\mathrm{e}^{\mathrm{i}\,\pi\,\Delta\mathrm{x}\kappa\mathrm{By}\pi}\,\beta_2 - 3\,\mathrm{cfl^2}\,\,\mathrm{e}^{2\,\mathrm{i}\,\pi\,\Delta\mathrm{x}\kappa\mathrm{By}\pi}\,\beta_2 + 3\,\mathrm{cfl^2}\,\mathrm{e}^{2\,\mathrm{i}\,\pi\,\Delta\mathrm{x}\kappa\mathrm{By}\pi}\,\beta_2 + 3\,\mathrm{cfl^2}\,\mathrm{e}^{2\,$ 6 cfl<sup>2</sup>  $\beta_1$   $\beta_2$  - 12 cfl<sup>2</sup>  $e^{i \pi \Delta x \kappa B y \pi}$   $\beta_1$   $\beta_2$  + 6 cfl<sup>2</sup>  $e^{2 i \pi \Delta x \kappa B y \pi}$   $\beta_1$   $\beta_2$ )  $(4+16~\text{e}^{\text{i}~\pi~\Delta\text{x}\kappa\text{By}\pi}+4~\text{e}^{\text{2}~\text{i}~\pi~\Delta\text{x}\kappa\text{By}\pi}-6~\text{cfl}^2~\beta_2^2+12~\text{cfl}^2~\text{e}^{\text{i}~\pi~\Delta\text{x}\kappa\text{By}\pi}~\beta_2^2-12~\text{cfl}^2~\text{e}^{\text{i}~\pi~\Delta\text{x}\kappa\text{By}\pi}$ 6 cfl<sup>2</sup>  $e^{2i\pi\Delta x \kappa By\pi}$   $\beta_2^2$ ) +  $(-8-32 e^{i\pi\Delta x \kappa By\pi}-8 e^{2i\pi\Delta x \kappa By\pi}-8 e^{2i\pi\Delta x \kappa By\pi}-8 e^{2i\pi\Delta x \kappa By\pi}$  $3 \text{ cfl}^2 \beta_1 + 6 \text{ cfl}^2 e^{i \pi \Delta x \kappa B y \pi} \beta_1 - 3 \text{ cfl}^2 e^{2 i \pi \Delta x \kappa B y \pi} \beta_1 - 3 \text{ cfl}^2 \beta_2 +$ 6 cfl²  $e^{i\pi\Delta x\kappa By\pi}\beta_2$  - 3 cfl²  $e^{2i\pi\Delta x\kappa By\pi}\beta_2$  - 6 cfl²  $\beta_1\beta_2$  + 12 cfl²  $e^{i \pi \Delta x \kappa B y \pi} \beta_1 \beta_2$  – 6 cfl²  $e^{2 i \pi \Delta x \kappa B y \pi} \beta_1 \beta_2$  + 6 cfl²  $\beta_2^2$  – 12 cfl<sup>2</sup>  $e^{i \pi \Delta x \kappa B y \pi} \beta_2^2 + 6 \text{ cfl}^2 e^{2 i \pi \Delta x \kappa B y \pi} \beta_2^2$ )  $\Big) \Big/$ (2 (4 + 16  $e^{i\,\pi\,\Delta x \kappa By\pi}$  + 4  $e^{2\,i\,\pi\,\Delta x \kappa By\pi}$  – 3 cfl<sup>2</sup>  $\beta_1$  + 6 cfl<sup>2</sup>  $e^{i\,\pi\,\Delta x \kappa By\pi}\,\beta_1$  – 3 cfl<sup>2</sup>  $e^{2i\pi\Delta x \kappa By\pi}\beta_1$  - 3 cfl<sup>2</sup>  $\beta_2$  + 6 cfl<sup>2</sup>  $e^{i\pi\Delta x \kappa By\pi}\beta_2$  - 3 cfl<sup>2</sup>  $e^{2i\pi\Delta x \kappa By\pi}\beta_2$  +  $\texttt{6cfl}^2 \; \beta_1 \; \beta_2 \; - \; \texttt{12cfl}^2 \; \texttt{e}^{\texttt{i} \; \pi \; \Delta \texttt{x} \kappa \texttt{B} \texttt{y} \pi} \; \beta_1 \; \beta_2 \; + \; \texttt{6cfl}^2 \; \texttt{e}^{\texttt{2} \; \texttt{i} \; \pi \; \Delta \texttt{x} \kappa \texttt{B} \texttt{y} \pi} \; \beta_1 \; \beta_2 \big) \, \big) \, \big] \, \big\} \, \textbf{,}$  $\left\{ \texttt{cByc} \rightarrow -\frac{\texttt{i}}{\texttt{cfl} \; \pi \; \Delta \texttt{x} \kappa \texttt{By} \pi} \; 2 \; \texttt{ii} \; \texttt{Log} \left[ \; \left( 8 + 32 \; \texttt{e}^{\texttt{i} \; \pi \; \Delta \texttt{x} \kappa \texttt{By} \pi} + 8 \; \texttt{e}^{2 \; \texttt{i} \; \pi \; \Delta \texttt{x} \kappa \texttt{By} \pi} + 3 \; \texttt{cfl}^2 \; \beta_1 \; - \right] \right\} \; + \; \texttt{cfl} \; \pi \; \Delta \texttt{x} \kappa \texttt{By} \pi \; + \; \texttt{cfl}^2 \; \beta_1 \; - \; \texttt{cfl}^2 \; \beta_2 \; + \; \texttt{cfl}^2 \; \beta_3 \; + \; \texttt{cfl}^2 \; \beta_4 \; - \; \texttt{cfl}^2 \; \beta_4 \; + \; \texttt{cfl}^2 \; + \; \texttt{cfl}^2 \; \beta_4 \; + \; \texttt{cfl}^2 \; + \; \texttt{cfl}^2$  $6\,\mathrm{cfl^2}\,\,\mathrm{e}^{\mathrm{i}\,\pi\,\Delta\mathrm{x}\kappa\mathrm{By}\pi}\,\beta_1 + 3\,\mathrm{cfl^2}\,\,\mathrm{e}^{2\,\mathrm{i}\,\pi\,\Delta\mathrm{x}\kappa\mathrm{By}\pi}\,\beta_1 + 3\,\mathrm{cfl^2}\,\beta_2 - 6\,\mathrm{cfl^2}\,\,\mathrm{e}^{\mathrm{i}\,\pi\,\Delta\mathrm{x}\kappa\mathrm{By}\pi}\,\beta_2 +$  $3\,\mathtt{cfl^2}\,\, \mathrm{e}^{2\,\mathtt{i}\,\pi\,\Delta\mathtt{x}\kappa\mathtt{B}\mathtt{y}\pi}\,\beta_2 + 6\,\mathtt{cfl^2}\,\beta_1\,\beta_2 - 12\,\mathtt{cfl^2}\,\, \mathrm{e}^{\mathrm{i}\,\pi\,\Delta\mathtt{x}\kappa\mathtt{B}\mathtt{y}\pi}\,\beta_1\,\beta_2 +$  $6\,\mathrm{cfl^2}\,\,\mathrm{e}^{2\,\mathrm{i}\,\pi\,\Delta\mathrm{x}\kappa\mathrm{By}\pi}\,\beta_1\,\beta_2 - 6\,\mathrm{cfl^2}\,\beta_2^2 + 12\,\mathrm{cfl^2}\,\,\mathrm{e}^{\mathrm{i}\,\pi\,\Delta\mathrm{x}\kappa\mathrm{By}\pi}\,\beta_2^2 - 6\,\mathrm{cfl^2}\,\,\mathrm{e}^{2\,\mathrm{i}\,\pi\,\Delta\mathrm{x}\kappa\mathrm{By}\pi}\,\beta_2^2 + 12\,\mathrm{cfl^2}\,\mathrm{e}^{\mathrm{i}\,\pi\,\Delta\mathrm{x}\kappa\mathrm{By}\pi}\,\beta_2^2 + 12\,\mathrm{cfl^2}\,\mathrm{e}^{\mathrm{i}\,\pi\,\Delta\mathrm{x}\kappa\mathrm{By}\pi}\,\beta_2^2 - 6\,\mathrm{cfl^2}\,\mathrm{e}^{2\,\mathrm{i}\,\pi\,\Delta\mathrm{x}\kappa\mathrm{By}\pi}\,\beta_2^2 + 12\,\mathrm{cfl^2}\,\mathrm{e}^{\mathrm{i}\,\pi\,\Delta\mathrm{x}\kappa\mathrm{By}\pi}\,\beta_2^2 - 6\,\mathrm{cfl^2}\,\mathrm{e}^{2\,\mathrm{i}\,\pi\,\Delta\mathrm{x}\kappa\mathrm{By}\pi}\,\beta_2^2 + 12\,\mathrm{cfl^2}\,\mathrm{e}^{\mathrm{i}\,\pi\,\Delta\mathrm{x}\kappa\mathrm{By}\pi}\,\beta_2^2 - 6\,\mathrm{cfl^2}\,\mathrm{e}^{2\,\mathrm{i}\,\pi\,\Delta\mathrm{x}\kappa\mathrm{By}\pi}\,\beta_2^2 + 12\,\mathrm{cfl^2}\,\mathrm{e}^{\mathrm{i}\,\pi\,\Delta\mathrm{x}\kappa\mathrm{By}\pi}\,\beta_2^2 - 6\,\mathrm{cfl^2}\,\mathrm{e}^{2\,\mathrm{i}\,\pi\,\Delta\mathrm{x}\kappa\mathrm{By}\pi}\,\beta_2^2 + 12\,\mathrm{cfl^2}\,\mathrm{e}^{2\,\mathrm{i}\,\pi\,\Delta\mathrm{x}\kappa\mathrm{By}\pi}\,\beta_2^2 - 6\,\mathrm{cfl^2}\,\mathrm{e}^{2\,\mathrm{i}\,\pi\,\Delta\mathrm{x}\kappa\mathrm{By}\pi}\,\beta_2^2 - 6\,\mathrm{cfl^2}\,\mathrm{e}^{2\,\mathrm{i}\,\pi\,\Delta\mathrm{x}\kappa\mathrm{x}\mathrm{By}\pi}\,\beta_2^2 - 6\,\mathrm{cfl^2}\,\mathrm{e}^{2\,\mathrm{i}\,\pi\,\Delta\mathrm{x}\mathrm{x}\mathrm{x}\mathrm{x}\mathrm{x$  $\sqrt{\left(-4 \left(4+16 e^{i \pi \Delta x \kappa B y \pi}+4 e^{2 i \pi \Delta x \kappa B y \pi}-3 cfl^{2} \beta_{1}+6 cfl^{2} e^{i \pi \Delta x \kappa B y \pi} \beta_{1}-6 cfl^{2} \right)}$ 3 cfl $^2$  e $^2$  i  $\pi$   $\Delta$ x $\times$ By $\pi$   $\beta_1$  - 3 cfl $^2$   $\beta_2$  + 6 cfl $^2$  e $^i$   $\pi$   $\Delta$ x $\times$ By $\pi$   $\beta_2$  -6 cfl<sup>2</sup>  $e^{2 i \pi \Delta x \kappa B y \pi} \beta_1 \beta_2$ ) (4 + 16  $e^{i \pi \Delta x \kappa B y \pi}$  + 4  $e^{2 i \pi \Delta x \kappa B y \pi}$  -6 cfl<sup>2</sup>  $\beta_2^2$  + 12 cfl<sup>2</sup>  $e^{i\pi\Delta x \kappa By\pi}$   $\beta_2^2$  - 6 cfl<sup>2</sup>  $e^{2i\pi\Delta x \kappa By\pi}$   $\beta_2^2$ ) +  $(-8-32~\text{e}^{\text{i}~\pi~\Delta x \kappa B y \pi}-8~\text{e}^{\text{2}~\text{i}~\pi~\Delta x \kappa B y \pi}-3~\text{cfl}^{\text{2}}~\beta_{\text{1}}+6~\text{cfl}^{\text{2}}~\text{e}^{\text{i}~\pi~\Delta x \kappa B y \pi}~\beta_{\text{1}} 3\,\mathrm{cfl^2}\,\,\mathrm{e}^{2\,\mathrm{i}\,\pi\,\Delta\mathrm{x}\kappa\mathrm{By}\pi}\,\beta_1 - 3\,\mathrm{cfl^2}\,\beta_2 + 6\,\mathrm{cfl^2}\,\,\mathrm{e}^{\mathrm{i}\,\pi\,\Delta\mathrm{x}\kappa\mathrm{By}\pi}\,\beta_2 - 3\,\mathrm{cfl^2}\,\,\mathrm{e}^{2\,\mathrm{i}\,\pi\,\Delta\mathrm{x}\kappa\mathrm{By}\pi}$  $\beta_2 - 6 \ \text{cfl}^2 \ \beta_1 \ \beta_2 + 12 \ \text{cfl}^2 \ \text{e}^{\text{i} \ \pi \ \Delta x \kappa \text{By} \pi} \ \beta_1 \ \beta_2 - 6 \ \text{cfl}^2 \ \text{e}^{2 \ \text{i} \ \pi \ \Delta x \kappa \text{By} \pi} \ \beta_1 \ \beta_2 + 12 \ \text{cfl}^2 \ \text{e}^{2 \ \text{i} \ \pi \ \Delta x \kappa \text{By} \pi} \ \beta_2 \ \text{cfl}^2 \ \text{e}^{2 \ \text{i} \ \pi \ \Delta x \kappa \text{By} \pi} \ \beta_1 \ \beta_2 + 12 \ \text{cfl}^2 \ \text{e}^{2 \ \text{i} \ \pi \ \Delta x \kappa \text{By} \pi} \ \beta_2 \ \text{cfl}^2 \ \text{e}^{2 \ \text{i} \ \pi \ \Delta x \kappa \text{By} \pi} \ \beta_2 \ \text{cfl}^2 \ \text{e}^{2 \ \text{i} \ \pi \ \Delta x \kappa \text{By} \pi} \ \beta_2 \ \text{cfl}^2 \ \text{e}^{2 \ \text{i} \ \pi \ \Delta x \kappa \text{By} \pi} \ \beta_2 \ \text{cfl}^2 \ \text{e}^{2 \ \text{i} \ \pi \ \Delta x \kappa \text{By} \pi} \ \beta_2 \ \text{cfl}^2 \ \text{cfl}^2 \ \text{e}^{2 \ \text{i} \ \pi \ \Delta x \kappa \text{By} \pi} \ \beta_2 \ \text{cfl}^2 \ \text{e}^{2 \ \text{i} \ \pi \ \Delta x \kappa \text{By} \pi} \ \beta_2 \ \text{cfl}^2 \ \text{cfl}^2$ 6 cfl<sup>2</sup>  $\beta_2^2$  - 12 cfl<sup>2</sup>  $e^{i \pi \Delta x \kappa B y \pi} \beta_2^2$  + 6 cfl<sup>2</sup>  $e^{2 i \pi \Delta x \kappa B y \pi} \beta_2^2$ ) /  $(2 (4+16 e^{i \pi \Delta x \kappa B y \pi}+4 e^{2 i \pi \Delta x \kappa B y \pi}-3 cfl^2 \beta_1+6 cfl^2 e^{i \pi \Delta x \kappa B y \pi} \beta_1-3 cfl^2 \beta_1+6 cfl^2 e^{i \pi \Delta x \kappa B y \pi} \beta_1-3 cfl^2 \beta_1+6 cfl^2 e^{i \pi \Delta x \kappa B y \pi} \beta_1-3 cfl^2 \beta_1+6 cfl^2 e^{i \pi \Delta x \kappa B y \pi} \beta_1-3 cfl^2 \beta_1+6 cfl^2 e^{i \pi \Delta x \kappa B y \pi} \beta_1-3 cfl^2 \beta_1+6 cfl^2 e^{i \pi \Delta x \kappa B y \pi} \beta_1-3 cfl^2 \beta_1+6 cfl^2 e^{i \pi \Delta x \kappa B y \pi} \beta_1-3 cfl^2 \beta_1+6 cfl^2 e^{i \pi \Delta x \kappa B y \pi} \beta_1-3 cfl^2 \beta_1+6 cfl^2 e^{i \pi \Delta x \kappa B y \pi} \beta_1-3 cfl^2 \beta_1+6 cfl^2 e^{i \pi \Delta x \kappa B y \pi} \beta_1-3 cfl^2 \beta_1+6 cfl^2 e^{i \pi \Delta x \kappa B y \pi} \beta_1-3 cfl^2 \beta_1+6 cfl^2 e^{i \pi \Delta x \kappa B y \pi} \beta_1-3 cfl^2 \beta_1+6 cfl^2 e^{i \pi \Delta x \kappa B y \pi} \beta_1-3 cfl^2 \beta_1+6 cfl^2 e^{i \pi \Delta x \kappa B y \pi} \beta_1-3 cfl^2 \beta_1+6 cfl^2 e^{i \pi \Delta x \kappa B y \pi} \beta_1-3 cfl^2 \beta_1+6 cfl^2 e^{i \pi \Delta x \kappa B y \pi} \beta_1-3 cfl^2 \beta_1+6 cfl^2 e^{i \pi \Delta x \kappa B y \pi} \beta_1-3 cfl^2 \beta_1+6 cfl^2 e^{i \pi \Delta x \kappa B y \pi} \beta_1-3 cfl^2 \beta_1+6 cfl^2 e^{i \pi \Delta x \kappa B y \pi} \beta_1-3 cfl^2 \beta_1+6 cfl^2 e^{i \pi \Delta x \kappa B y \pi} \beta_1-3 cfl^2 \beta_1+6 cfl^2 e^{i \pi \Delta x \kappa B y \pi} \beta_1-3 cfl^2 \beta_1+6 cfl^2 e^{i \pi \Delta x \kappa B y \pi} \beta_1$  $3\,\mathrm{cfl^2}\,\,\mathrm{e}^{2\,\mathrm{i}\,\pi\,\Delta\mathrm{x}\kappa\mathrm{By}\pi}\,\beta_1 - 3\,\mathrm{cfl^2}\,\beta_2 + 6\,\mathrm{cfl^2}\,\,\mathrm{e}^{\mathrm{i}\,\pi\,\Delta\mathrm{x}\kappa\mathrm{By}\pi}\,\beta_2 - 3\,\mathrm{cfl^2}\,\,\mathrm{e}^{2\,\mathrm{i}\,\pi\,\Delta\mathrm{x}\kappa\mathrm{By}\pi}\,\beta_2 + 3\,\mathrm{cfl^2}\,\mathrm{e}^{2\,\mathrm{i}\,\pi\,\Delta\mathrm{x}\kappa\mathrm{By}\pi}\,\beta_2 + 3\,\mathrm{cfl^2}\,\mathrm{e}$ 6 cfl<sup>2</sup>  $\beta_1 \beta_2$  - 12 cfl<sup>2</sup>  $e^{i \pi \Delta x \kappa B y \pi} \beta_1 \beta_2$  + 6 cfl<sup>2</sup>  $e^{2 i \pi \Delta x \kappa B y \pi} \beta_1 \beta_2$ ) ] } }  $\beta_1 = 0.39;$ 

cfl = .90;

```
cfl π ΔxκByπ
                          Log \left[ \left( 8 + 32 e^{i \pi \Delta x \kappa B y \pi} + 8 e^{2 i \pi \Delta x \kappa B y \pi} + 3 cfl^{2} \beta_{1} - 6 cfl^{2} e^{i \pi \Delta x \kappa B y \pi} \beta_{1} + 3 cfl^{2} e^{2 i \pi \Delta x \kappa B y \pi} \beta_{1} + 3 cfl^{2} e^{2 i \pi \Delta x \kappa B y \pi} \beta_{1} + 3 cfl^{2} e^{2 i \pi \Delta x \kappa B y \pi} \beta_{1} + 3 cfl^{2} e^{2 i \pi \Delta x \kappa B y \pi} \beta_{1} + 3 cfl^{2} e^{2 i \pi \Delta x \kappa B y \pi} \beta_{1} + 3 cfl^{2} e^{2 i \pi \Delta x \kappa B y \pi} \beta_{1} + 3 cfl^{2} e^{2 i \pi \Delta x \kappa B y \pi} \beta_{1} + 3 cfl^{2} e^{2 i \pi \Delta x \kappa B y \pi} \beta_{1} + 3 cfl^{2} e^{2 i \pi \Delta x \kappa B y \pi} \beta_{1} + 3 cfl^{2} e^{2 i \pi \Delta x \kappa B y \pi} \beta_{1} + 3 cfl^{2} e^{2 i \pi \Delta x \kappa B y \pi} \beta_{1} + 3 cfl^{2} e^{2 i \pi \Delta x \kappa B y \pi} \beta_{1} + 3 cfl^{2} e^{2 i \pi \Delta x \kappa B y \pi} \beta_{1} + 3 cfl^{2} e^{2 i \pi \Delta x \kappa B y \pi} \beta_{1} + 3 cfl^{2} e^{2 i \pi \Delta x \kappa B y \pi} \beta_{1} + 3 cfl^{2} e^{2 i \pi \Delta x \kappa B y \pi} \beta_{1} + 3 cfl^{2} e^{2 i \pi \Delta x \kappa B y \pi} \beta_{1} + 3 cfl^{2} e^{2 i \pi \Delta x \kappa B y \pi} \beta_{1} + 3 cfl^{2} e^{2 i \pi \Delta x \kappa B y \pi} \beta_{1} + 3 cfl^{2} e^{2 i \pi \Delta x \kappa B y \pi} \beta_{1} + 3 cfl^{2} e^{2 i \pi \Delta x \kappa B y \pi} \beta_{1} + 3 cfl^{2} e^{2 i \pi \Delta x \kappa B y \pi} \beta_{1} + 3 cfl^{2} e^{2 i \pi \Delta x \kappa B y \pi} \beta_{1} + 3 cfl^{2} e^{2 i \pi \Delta x \kappa B y \pi} \beta_{1} + 3 cfl^{2} e^{2 i \pi \Delta x \kappa B y \pi} \beta_{1} + 3 cfl^{2} e^{2 i \pi \Delta x \kappa B y \pi} \beta_{1} + 3 cfl^{2} e^{2 i \pi \Delta x \kappa B y \pi} \beta_{1} + 3 cfl^{2} e^{2 i \pi \Delta x \kappa B y \pi} \beta_{1} + 3 cfl^{2} e^{2 i \pi \Delta x \kappa B y \pi} \beta_{1} + 3 cfl^{2} e^{2 i \pi \Delta x \kappa B y \pi} \beta_{1} + 3 cfl^{2} e^{2 i \pi \Delta x \kappa B y \pi} \beta_{1} + 3 cfl^{2} e^{2 i \pi \Delta x \kappa B y \pi} \beta_{1} + 3 cfl^{2} e^{2 i \pi \Delta x \kappa B y \pi} \beta_{1} + 3 cfl^{2} e^{2 i \pi \Delta x \kappa B y \pi} \beta_{1} + 3 cfl^{2} e^{2 i \pi \Delta x \kappa B y \pi} \beta_{1} + 3 cfl^{2} e^{2 i \pi \Delta x \kappa B y \pi} \beta_{1} + 3 cfl^{2} e^{2 i \pi \Delta x \kappa B y \pi} \beta_{1} + 3 cfl^{2} e^{2 i \pi \Delta x \kappa B y \pi} \beta_{1} + 3 cfl^{2} e^{2 i \pi \Delta x \kappa B y \pi} \beta_{1} + 3 cfl^{2} e^{2 i \pi \Delta x \kappa B y \pi} \beta_{1} + 3 cfl^{2} e^{2 i \pi \Delta x \kappa B y \pi} \beta_{1} + 3 cfl^{2} e^{2 i \pi \Delta x \kappa B y \pi} \beta_{1} + 3 cfl^{2} e^{2 i \pi \Delta x \kappa B y \pi} \beta_{1} + 3 cfl^{2} e^{2 i \pi \Delta x \kappa B y \pi} \beta_{1} + 3 cfl^{2} e^{2 i \pi \Delta x \kappa B y \pi} \beta_{1} + 3 cfl^{2} e^{2 i \pi \Delta x \kappa B y \pi} \beta_{1} + 3 cfl^{2} e^{2 i \pi \Delta x \kappa B y \pi} \beta_{1} + 3 cfl^{2} e^{2 i \pi \Delta x \kappa B y \pi} \beta_{1} + 3 c
                                                                                   3\,\mathtt{cfl^2}\,\beta_2 - 6\,\mathtt{cfl^2}\,\,\mathtt{e}^{\mathtt{i}\,\pi\,\Delta\mathtt{x}\mathsf{x}\mathtt{B}\mathtt{y}\pi}\,\beta_2 + 3\,\mathtt{cfl^2}\,\,\mathtt{e}^{2\,\mathtt{i}\,\pi\,\Delta\mathtt{x}\mathsf{x}\mathtt{B}\mathtt{y}\pi}\,\beta_2 + 6\,\mathtt{cfl^2}\,\beta_1\,\beta_2 - 12\,\mathtt{cfl^2}\,\,\mathtt{e}^{\mathtt{i}\,\pi\,\Delta\mathtt{x}\mathsf{x}\mathtt{B}\mathtt{y}\pi}\,\beta_1
                                                                                               \beta_2 + 6 \text{ cfl}^2 \text{ e}^{2 \text{ i} \pi \Delta x \times By \pi} \beta_1 \beta_2 - 6 \text{ cfl}^2 \beta_2^2 + 12 \text{ cfl}^2 \text{ e}^{\text{i} \pi \Delta x \times By \pi} \beta_2^2 - 6 \text{ cfl}^2 \text{ e}^{2 \text{ i} \pi \Delta x \times By \pi} \beta_2^2 - 6 \text{ cfl}^2 \beta_2^2 + 6 \text{ cfl}^2
                                                                                 \sqrt{\left(-4 \left(4+16 e^{i \pi \Delta x \times By \pi}+4 e^{2 i \pi \Delta x \times By \pi}-3 cfl^{2} \beta_{1}+6 cfl^{2} e^{i \pi \Delta x \times By \pi} \beta_{1}-6 cfl^{2} \right)}
                                                                                                                                                                   3\,\mathtt{cfl^2}\,\,\mathtt{e}^{2\,\mathtt{i}\,\pi\,\Delta\mathtt{x}\kappa\mathtt{B}\mathtt{y}\pi}\,\beta_1 - 3\,\mathtt{cfl^2}\,\beta_2 + 6\,\mathtt{cfl^2}\,\,\mathtt{e}^{\mathtt{i}\,\pi\,\Delta\mathtt{x}\kappa\mathtt{B}\mathtt{y}\pi}\,\beta_2 - 3\,\mathtt{cfl^2}\,\,\mathtt{e}^{2\,\mathtt{i}\,\pi\,\Delta\mathtt{x}\kappa\mathtt{B}\mathtt{y}\pi}\,\beta_2 + 6\,\mathtt{cfl^2}\,\,\mathtt{e}^{2\,\mathtt{i}\,\pi\,\Delta\mathtt{x}\kappa\mathtt{B}\mathtt{y}\pi}\,\beta_2 + 6\,\mathtt{cfl^2}\,\,\mathtt{e}^{2\,\mathtt{i}\,\pi\,\Delta\mathtt{x}\kappa\,\Delta\mathtt{x}\kappa\mathtt{B}\mathtt{y}\pi}\,\beta_2 + 6\,\mathtt{cfl^2}\,\,\mathtt{e}^{2\,\mathtt{i}\,\pi\,\Delta\mathtt{x}\kappa\,\Delta\mathtt{x}\kappa\,\Delta\mathtt{x}\kappa\,\Delta\mathtt{x}\kappa\,\Delta\mathtt{x}\kappa}\,\beta_2 + 6\,\mathtt{cfl^2}\,\mathtt{e}^{2\,\mathtt{i}\,\pi\,\Delta\mathtt{x}\kappa\,\Delta\mathtt{x}\kappa\,\Delta\mathtt{x}\kappa\,\Delta\mathtt{x}\kappa\,\Delta\mathtt{x}\,\Delta\mathtt{x}\kappa\,\Delta\mathtt{x}\kappa\,\Delta\mathtt{x}\,\Delta\mathtt{x}\kappa\,\Delta\mathtt{x}\,\Delta\mathtt{x}\,\Delta\mathtt{x}\,\Delta\mathtt{x}\,\Delta\mathtt{x}\,\Delta\mathtt{x}\,\Delta\mathtt{x}\,\Delta\mathtt{x}\,\Delta\mathtt{x}\,\Delta\mathtt{x}\,\Delta\mathtt{x}\,\Delta\mathtt{x}\,\Delta\mathtt{x}\,\Delta\mathtt{x}\,\Delta\mathtt{x}\,\Delta\mathtt{x}\,\Delta\mathtt{x}\,\Delta\mathtt{x}\,\Delta\mathtt{x}\,\Delta\mathtt{x}\,\Delta\mathtt{x}\,\Delta\mathtt{x}\,\Delta\mathtt{x}\,\Delta\mathtt{x}\,\Delta\mathtt{x}\,\Delta\mathtt{x}\,\Delta\mathtt{x}\,\Delta\mathtt{x}\,\Delta\mathtt{x}\,\Delta\mathtt{x}\,\Delta\mathtt{x}\,\Delta\mathtt{x}\,\Delta\mathtt{x}\,\Delta\mathtt{x}\,\Delta\mathtt{x}\,\Delta\mathtt{x}\,\Delta\mathtt{x}\,\Delta\mathtt{x}\,\Delta\mathtt{x}\,\Delta\mathtt{x}\,\Delta\mathtt{x}\,\Delta\mathtt{x}\,\Delta\mathtt{x}\,\Delta\mathtt{x}\,\Delta\mathtt{x}\,\Delta\mathtt{x}\,\Delta\mathtt{x}\,\Delta\mathtt{x}\,\Delta\mathtt{x}\,\Delta\mathtt{x}\,\Delta\mathtt{x}\,\Delta\mathtt{x}\,\Delta\mathtt{x
                                                                                                                                                                                  \texttt{cfl}^2 \ \beta_1 \ \beta_2 - 12 \ \texttt{cfl}^2 \ \texttt{e}^{\texttt{i} \ \pi \ \Delta \texttt{x} \times \texttt{B} \texttt{y} \pi} \ \beta_1 \ \beta_2 + 6 \ \texttt{cfl}^2 \ \texttt{e}^{2 \ \texttt{i} \ \pi \ \Delta \texttt{x} \times \texttt{B} \texttt{y} \pi} \ \beta_1 \ \beta_2 \Big) \ \left( 4 + 16 \ \texttt{e}^{\texttt{i} \ \pi \ \Delta \texttt{x} \times \texttt{B} \texttt{y} \pi} + 16 \ \texttt{e}^{\texttt{i} \ \pi \ \Delta \texttt{x} \times \texttt{B} \texttt{y} \pi} \right) 
                                                                                                                                                                     4 e^{2 i \pi \Delta x \kappa B y \pi} - 6 cfl^2 \beta_2^2 + 12 cfl^2 e^{i \pi \Delta x \kappa B y \pi} \beta_2^2 - 6 cfl^2 e^{2 i \pi \Delta x \kappa B y \pi} \beta_2^2 +
                                                                                                                             \left(-8-32\ e^{i\pi\Delta x \kappa By\pi}-8\ e^{2\ i\pi\Delta x \kappa By\pi}-3\ cfl^{2}\ eta_{1}+6\ cfl^{2}\ e^{i\pi\Delta x \kappa By\pi}\ eta_{1}-
ight.
                                                                                                                                                                     3 \text{ cfl}^2 \text{ e}^{2 \text{ i} \pi \Delta x \times By \pi} \beta_1 - 3 \text{ cfl}^2 \beta_2 + 6 \text{ cfl}^2 \text{ e}^{\text{i} \pi \Delta x \times By \pi} \beta_2 - 3 \text{ cfl}^2 \text{ e}^{2 \text{ i} \pi \Delta x \times By \pi} \beta_2 - 3 \text{ cfl}^2 
                                                                                                                                                                     6 \text{ cfl}^2 \beta_1 \beta_2 + 12 \text{ cfl}^2 \text{ e}^{\text{i} \pi \Delta x \kappa B y \pi} \beta_1 \beta_2 - 6 \text{ cfl}^2 \text{ e}^{2 \text{ i} \pi \Delta x \kappa B y \pi} \beta_1 \beta_2 + \\
                                                                                                                                                                   6 cfl<sup>2</sup> \beta_2^2 - 12 cfl<sup>2</sup> e<sup>i \pi \Delta x \times By \pi</sup> \beta_2^2 + 6 cfl<sup>2</sup> e<sup>2 i \pi \Delta x \times By \pi</sup> \beta_2^2)<sup>2</sup>) /
                                                            (2 (4+16 e^{i \pi \Delta x \times By\pi} + 4 e^{2 i \pi \Delta x \times By\pi} - 3 cfl^{2} \beta_{1} + 6 cfl^{2} e^{i \pi \Delta x \times By\pi} \beta_{1} -
                                                                                                               3 \text{ cfl}^2 \text{ e}^{2 \text{ i} \pi \Delta x \times By \pi} \beta_1 - 3 \text{ cfl}^2 \beta_2 + 6 \text{ cfl}^2 \text{ e}^{\text{i} \pi \Delta x \times By \pi} \beta_2 - 3 \text{ cfl}^2 \text{ e}^{2 \text{ i} \pi \Delta x \times By \pi} \beta_2 +
                                                                                                               6 cfl<sup>2</sup> \beta_1 \beta_2 - 12 cfl<sup>2</sup> e^{i \pi \Delta x \times By \pi} \beta_1 \beta_2 + 6 cfl<sup>2</sup> e^{2 i \pi \Delta x \times By \pi} \beta_1 \beta_2)
-\frac{1}{\Delta x \kappa B y \pi} \left(0. + 0.707355 i\right) Log \left[\left(9.36469 + 29.2706 e^{i \pi \Delta x \kappa B y \pi} + 6.86866 e^{i \pi \Delta x \kappa B y \pi}\right)\right]
                                                                    4 (1.04318 + 21.9136 e^{i \pi \Delta x \kappa B y \pi} + 1.04318 e^{2 i \pi \Delta x \kappa B y \pi})
                                                                                                                              (2.63531 + 18.7294 e^{i \pi \Delta x \kappa B y \pi} + 2.63531 e^{2 i \pi \Delta x \kappa B y \pi})))
                                             (2(2.63531 + 18.7294 e^{i \pi \Delta x \kappa B y \pi} + 2.63531 e^{2 i \pi \Delta x \kappa B y \pi}))
```

plotb0p39CFL0p90 = solu;

Plot[{Abs[plotb0p39CFL0p2] - 1, Abs[plotb0p39CFL0p90] - 1, Abs[plotb0p39CFL1p0] - 1, Abs[plotb0p39CFL2p0] - 1},  $\{\Delta x \kappa B y \pi, 0.000001, 1\}$ , PlotRange  $\rightarrow \{-.6, .6\}$ ,  ${\tt PlotLegends} \rightarrow {\tt LineLegend[\{"CFL=0.2", "CFL=0.95", "CFL=1", "CFL=2"\}, and all of the content of the cont$ LegendFunction  $\rightarrow$  (Framed[#] &)], PlotTheme  $\rightarrow$  "Detailed"]



$$\begin{aligned} &\operatorname{qlc} = -\frac{1}{\operatorname{cfl} \pi \Delta x \operatorname{KBy} \pi} + 8 \, \operatorname{e}^{2 \, \operatorname{i} \pi \Delta x \operatorname{KBy} \pi} + 3 \, \operatorname{cfl}^{2} \, \beta_{1} - 6 \, \operatorname{cfl}^{2} \, \operatorname{e}^{\operatorname{i} \pi \Delta x \operatorname{KBy} \pi} \, \beta_{1} + 3 \, \operatorname{cfl}^{2} \, \operatorname{e}^{2 \, \operatorname{i} \pi \Delta x \operatorname{KBy} \pi} \, \beta_{1} + 3 \, \operatorname{cfl}^{2} \, \operatorname{e}^{2 \, \operatorname{i} \pi \Delta x \operatorname{KBy} \pi} \, \beta_{1} + 3 \, \operatorname{cfl}^{2} \, \operatorname{e}^{2 \, \operatorname{i} \pi \Delta x \operatorname{KBy} \pi} \, \beta_{1} + 3 \, \operatorname{cfl}^{2} \, \operatorname{e}^{2 \, \operatorname{i} \pi \Delta x \operatorname{KBy} \pi} \, \beta_{1} + 3 \, \operatorname{cfl}^{2} \, \operatorname{e}^{2 \, \operatorname{i} \pi \Delta x \operatorname{KBy} \pi} \, \beta_{1} + 3 \, \operatorname{cfl}^{2} \, \operatorname{e}^{2 \, \operatorname{i} \pi \Delta x \operatorname{KBy} \pi} \, \beta_{1} + 3 \, \operatorname{cfl}^{2} \, \operatorname{e}^{2 \, \operatorname{i} \pi \Delta x \operatorname{KBy} \pi} \, \beta_{2} + 3 \, \operatorname{cfl}^{2} \, \operatorname{e}^{2 \, \operatorname{i} \pi \Delta x \operatorname{KBy} \pi} \, \beta_{2} + 6 \, \operatorname{cfl}^{2} \, \operatorname{e}^{2 \, \operatorname{i} \pi \Delta x \operatorname{KBy} \pi} \, \beta_{1} + 3 \, \operatorname{cfl}^{2} \, \operatorname{e}^{2 \, \operatorname{i} \pi \Delta x \operatorname{KBy} \pi} \, \beta_{2} + 6 \, \operatorname{cfl}^{2} \, \operatorname{e}^{2 \, \operatorname{i} \pi \Delta x \operatorname{KBy} \pi} \, \beta_{2} - 1 \, \operatorname{cfl}^{2} \, \operatorname{e}^{2 \, \operatorname{i} \pi \Delta x \operatorname{KBy} \pi} \, \beta_{2} - 6 \, \operatorname{cfl}^{2} \, \operatorname{e}^{2 \, \operatorname{i} \pi \Delta x \operatorname{KBy} \pi} \, \beta_{2} - 3 \, \operatorname{cfl}^{2} \, \operatorname{e}^{2 \, \operatorname{i} \pi \Delta x \operatorname{KBy} \pi} \, \beta_{2} - 3 \, \operatorname{cfl}^{2} \, \operatorname{e}^{2 \, \operatorname{i} \pi \Delta x \operatorname{KBy} \pi} \, \beta_{2} - 3 \, \operatorname{cfl}^{2} \, \operatorname{e}^{2 \, \operatorname{i} \pi \Delta x \operatorname{KBy} \pi} \, \beta_{2} + 6 \, \operatorname{cfl}^{2} \, \operatorname{e}^{2 \, \operatorname{i} \pi \Delta x \operatorname{KBy} \pi} \, \beta_{2} - 3 \, \operatorname{cfl}^{2} \, \operatorname{e}^{2 \, \operatorname{i} \pi \Delta x \operatorname{KBy} \pi} \, \beta_{2} + 6 \, \operatorname{cfl}^{2} \, \operatorname{e}^{2 \, \operatorname{i} \pi \Delta x \operatorname{KBy} \pi} \, \beta_{2} + 6 \, \operatorname{cfl}^{2} \, \operatorname{e}^{2 \, \operatorname{i} \pi \Delta x \operatorname{KBy} \pi} \, \beta_{2} + 6 \, \operatorname{cfl}^{2} \, \operatorname{e}^{2 \, \operatorname{i} \pi \Delta x \operatorname{KBy} \pi} \, \beta_{2} + 6 \, \operatorname{cfl}^{2} \, \operatorname{e}^{2 \, \operatorname{i} \pi \Delta x \operatorname{KBy} \pi} \, \beta_{2} + 6 \, \operatorname{cfl}^{2} \, \operatorname{e}^{2 \, \operatorname{i} \pi \Delta x \operatorname{KBy} \pi} \, \beta_{2} + 6 \, \operatorname{cfl}^{2} \, \operatorname{e}^{2 \, \operatorname{i} \pi \Delta x \operatorname{KBy} \pi} \, \beta_{2} + 6 \, \operatorname{cfl}^{2} \, \operatorname{e}^{2 \, \operatorname{i} \pi \Delta x \operatorname{KBy} \pi} \, \beta_{2} + 6 \, \operatorname{cfl}^{2} \, \operatorname{e}^{2 \, \operatorname{i} \pi \Delta x \operatorname{KBy} \pi} \, \beta_{2} + 6 \, \operatorname{cfl}^{2} \, \operatorname{e}^{2 \, \operatorname{i} \pi \Delta x \operatorname{KBy} \pi} \, \beta_{2} + 6 \, \operatorname{cfl}^{2} \, \operatorname{e}^{2 \, \operatorname{i} \pi \Delta x \operatorname{KBy} \pi} \, \beta_{2} - 3 \, \operatorname{cfl}^{2} \, \operatorname{e}^{2 \, \operatorname{i} \pi \Delta x \operatorname{KBy} \pi} \, \beta_{2} + 6 \, \operatorname{cfl}^{2} \, \operatorname{e}^{2 \, \operatorname{i} \pi \Delta x \operatorname{KBy} \pi} \, \beta_{2} + 6 \, \operatorname{cfl}^{2} \, \operatorname{e}^{2 \, \operatorname{i} \pi \Delta x \operatorname{KBy} \pi} \, \beta_{2} + 6 \, \operatorname{cfl}^{2} \, \operatorname{e}^{2 \, \operatorname{i} \pi \Delta x \operatorname{KBy} \pi} \, \beta_{2} + 6 \, \operatorname{cfl}^{2} \, \operatorname{e}^{2 \, \operatorname$$