```
<< Notation`;

Symbolize[ $u_1^t$ ]; Symbolize[ $u_2^t$ ]; Symbolize[ $u_3^t$ ]; Symbolize[ $\dot{u}_1^t$ ];

Symbolize[ $\dot{u}_2^t$ ]; Symbolize[ $\dot{u}_3^t$ ]; Symbolize[ $\ddot{u}_1^t$ ]; Symbolize[ $\ddot{u}_2^t$ ];

Symbolize[ $\ddot{u}_3^t$ ]; Symbolize[ $u_1^{t+\Delta t}$ ]; Symbolize[ $u_2^{t+\Delta t}$ ]; Symbolize[ $u_3^{t+\Delta t}$ ];

Symbolize[ $\dot{u}_1^{t+\Delta t}$ ]; Symbolize[ $\dot{u}_2^{t+\Delta t}$ ]; Symbolize[ $\dot{u}_3^{t+\Delta t}$ ]; Symbolize[ $\ddot{u}_1^{t+\Delta t}$ ];

Symbolize[ $\ddot{u}_2^{t+\Delta t}$ ]; Symbolize[ $\ddot{u}_3^{t+\Delta t}$ ]; Symbolize[ $u_1^{t+\gamma\Delta t}$ ];

Symbolize[ $u_2^{t+\gamma\Delta t}$ ]; Symbolize[ $u_3^{t+\gamma\Delta t}$ ] ; Symbolize[ $\dot{u}_1^{t+\gamma\Delta t}$ ];

Symbolize[ $\dot{u}_2^{t+\gamma\Delta t}$ ]; Symbolize[ $\dot{u}_3^{t+\gamma\Delta t}$ ]; Symbolize[ $\ddot{u}_1^{t+\gamma\Delta t}$ ];

Symbolize[ $\ddot{u}_2^{t+\gamma\Delta t}$ ]; Symbolize[ $\ddot{u}_3^{t+\gamma\Delta t}$ ]; Symbolize[ $\beta_1$ ]; Symbolize[ $\beta_2$ ];

For[
  ClearAll["Global`*"];
  $\gamma = \frac{1}{2}$; $\Delta t = 0.25$; $\beta_2 = 2 \beta_1$;
  $m_2 = 1$;
  $m_3 = 1$;
  $k_2 = 1$;
  $\omega = 1.2$;
  $u_1^{t+\Delta t} = Sin[\omega p]$;
  $u_1^{t+\gamma\Delta t} = Sin[\omega (p - \frac{\Delta t}{2})]$;
eq112 = $m_2 \ddot{u}_2^{t+\gamma\Delta t} + (k_1 + k_2) u_2^{t+\gamma\Delta t} + (-k_2) u_3^{t+\gamma\Delta t} == k_1 u_1^{t+\gamma\Delta t}$;
  eq113 = $m_3 \ddot{u}_3^{t+\gamma\Delta t} + (-k_2) u_2^{t+\gamma\Delta t} + k_2 u_3^{t+\gamma\Delta t} == 0$;
  eq122 = $u_2^{t+\gamma\Delta t} == u_2^t + \frac{\gamma \Delta t}{2} (\dot{u}_2^t + \dot{u}_2^{t+\gamma\Delta t})$;
  eq123 = $u_3^{t+\gamma\Delta t} == u_3^t + \frac{\gamma \Delta t}{2} (\dot{u}_3^t + \dot{u}_3^{t+\gamma\Delta t})$;
  eq132 = $\dot{u}_2^{t+\gamma\Delta t} == \dot{u}_2^t + \frac{\gamma \Delta t}{2} (\ddot{u}_2^t + \ddot{u}_2^{t+\gamma\Delta t})$;
  eq133 = $\dot{u}_3^{t+\gamma\Delta t} == \dot{u}_3^t + \frac{\gamma \Delta t}{2} (\ddot{u}_3^t + \ddot{u}_3^{t+\gamma\Delta t})$;
  eq212 = $m_2 \ddot{u}_2^{t+\Delta t} + (k_1 + k_2) u_2^{t+\Delta t} + (-k_2) u_3^{t+\Delta t} == k_1 u_1^{t+\Delta t}$;
  eq213 = $m_3 \ddot{u}_3^{t+\Delta t} + (-k_2) u_2^{t+\Delta t} + k_2 u_3^{t+\Delta t} == 0$;
  eq222 = $u_2^{t+\Delta t} == u_2^t + \gamma \Delta t ((1 - \beta_1) \dot{u}_2^t + \beta_1 \dot{u}_2^{t+\gamma\Delta t}) + (1 - \gamma) \Delta t ((1 - \beta_2) \dot{u}_2^{t+\gamma\Delta t} + \beta_2 \dot{u}_2^{t+\Delta t})$;
  eq223 = $u_3^{t+\Delta t} == u_3^t + \gamma \Delta t ((1 - \beta_1) \dot{u}_3^t + \beta_1 \dot{u}_3^{t+\gamma\Delta t}) + (1 - \gamma) \Delta t ((1 - \beta_2) \dot{u}_3^{t+\gamma\Delta t} + \beta_2 \dot{u}_3^{t+\Delta t})$;
  eq232 = $\dot{u}_2^{t+\Delta t} == \dot{u}_2^t + \gamma \Delta t ((1 - \beta_1) \ddot{u}_2^t + \beta_1 \ddot{u}_2^{t+\gamma\Delta t}) + (1 - \gamma) \Delta t ((1 - \beta_2) \ddot{u}_2^{t+\gamma\Delta t} + \beta_2 \ddot{u}_2^{t+\Delta t})$;
  eq233 = $\dot{u}_3^{t+\Delta t} == \dot{u}_3^t + \gamma \Delta t ((1 - \beta_1) \ddot{u}_3^t + \beta_1 \ddot{u}_3^{t+\gamma\Delta t}) + (1 - \gamma) \Delta t ((1 - \beta_2) \ddot{u}_3^{t+\gamma\Delta t} + \beta_2 \ddot{u}_3^{t+\Delta t})$;
s1and2 = Solve[eq112 && eq113 && eq122 && eq123 && eq132 &&
      eq133 && eq212 && eq213 && eq222 && eq223 && eq232 && eq233,
    {$\ddot{u}_2^{t+\gamma\Delta t}$, $\ddot{u}_3^{t+\gamma\Delta t}$, $\dot{u}_2^{t+\gamma\Delta t}$, $\dot{u}_3^{t+\gamma\Delta t}$, $u_2^{t+\gamma\Delta t}$, $u_3^{t+\gamma\Delta t}$, $\ddot{u}_2^{t+\Delta t}$, $\ddot{u}_3^{t+\Delta t}$, $\dot{u}_2^{t+\Delta t}$, $\dot{u}_3^{t+\Delta t}$, $u_2^{t+\Delta t}$, $u_3^{t+\Delta t}$}];


  $\ddot{u}_2^{t+\Delta t}$ = $\ddot{u}_2^{t+\Delta t}$ /. s1and2[[1, 7]];
  $\ddot{u}_3^{t+\Delta t}$ = $\ddot{u}_3^{t+\Delta t}$ /. s1and2[[1, 8]];
```

```
u̇₂ᵗ⁺ᐞᵗ = u̇₂ᵗ⁺ᐞᵗ /. s1and2[[1, 9]];
u̇₃ᵗ⁺ᐞᵗ = u̇₃ᵗ⁺ᐞᵗ /. s1and2[[1, 10]];
u₂ᵗ⁺ᐞᵗ = u₂ᵗ⁺ᐞᵗ /. s1and2[[1, 11]];
u₃ᵗ⁺ᐞᵗ = u₃ᵗ⁺ᐞᵗ /. s1and2[[1, 12]];
k₁ = 5; β₁ = 0.39;
p = 0.25; ü₂ᵗ = 0; ü₃ᵗ = 0; u̇₂ᵗ = 0; u̇₃ᵗ = 0; u₂ᵗ = 0; u₃ᵗ = 0,
p ≤ 30,
p = p + 0.25,
uk005b0392ₚ = u₂ᵗ⁺ᐞᵗ;
uk005b0393ₚ = u₃ᵗ⁺ᐞᵗ;
uk005b039d2ₚ = u̇₂ᵗ⁺ᐞᵗ;
uk005b039d3ₚ = u̇₃ᵗ⁺ᐞᵗ;
uk005b039dd2ₚ = ü₂ᵗ⁺ᐞᵗ;
uk005b039dd3ₚ = ü₃ᵗ⁺ᐞᵗ;
ü₂ᵗ = ü₂ᵗ⁺ᐞᵗ; ü₃ᵗ = ü₃ᵗ⁺ᐞᵗ; u̇₂ᵗ = u̇₂ᵗ⁺ᐞᵗ; u̇₃ᵗ = u̇₃ᵗ⁺ᐞᵗ; u₂ᵗ = u₂ᵗ⁺ᐞᵗ; u₃ᵗ = u₃ᵗ⁺ᐞᵗ
];
```
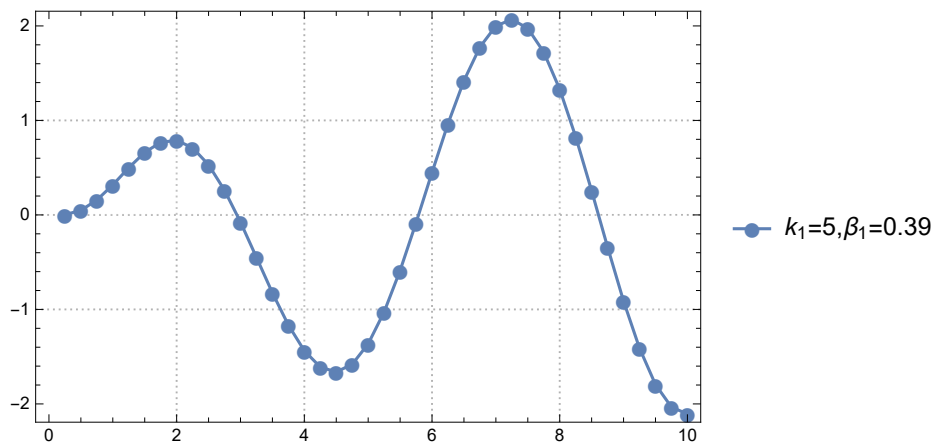
```
DiscretePlot[{uk005b0392ₚ}, {p, 0, 10, Δt},
 PlotLegends -> {"k₁=5,β₁=0.39"}, PlotTheme -> "Detailed",
 Joined → True, PlotMarkers -> {Automatic, 12}, FillingStyle → White]
```
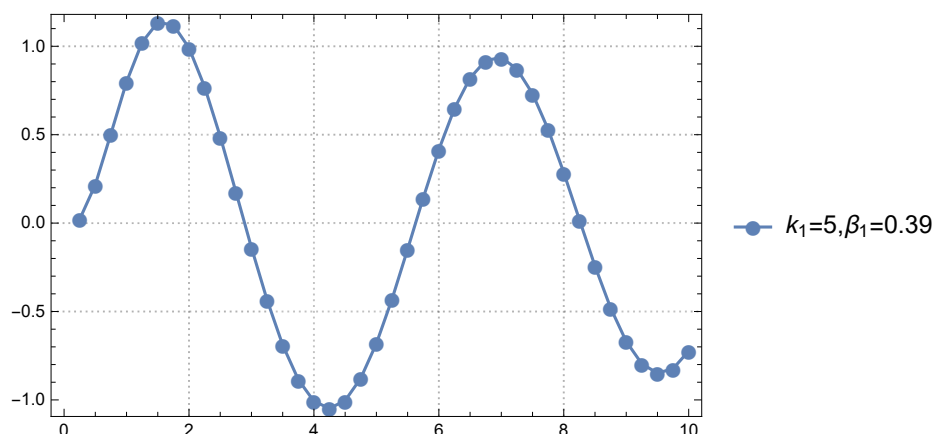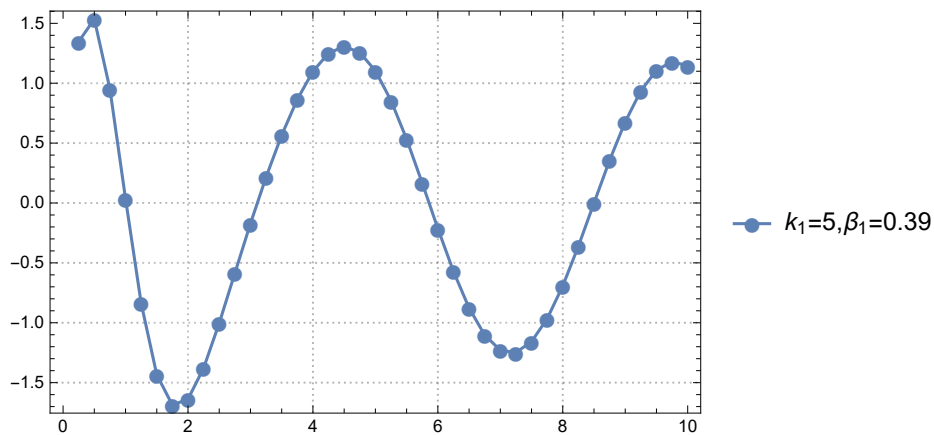


```
DiscretePlot[{uk005b0393ₚ}, {p, 0, 10, Δt},
 PlotLegends -> {"k₁=5,β₁=0.39"}, PlotTheme -> "Detailed",
 Joined → True, PlotMarkers -> {Automatic, 12}, FillingStyle → White]
```
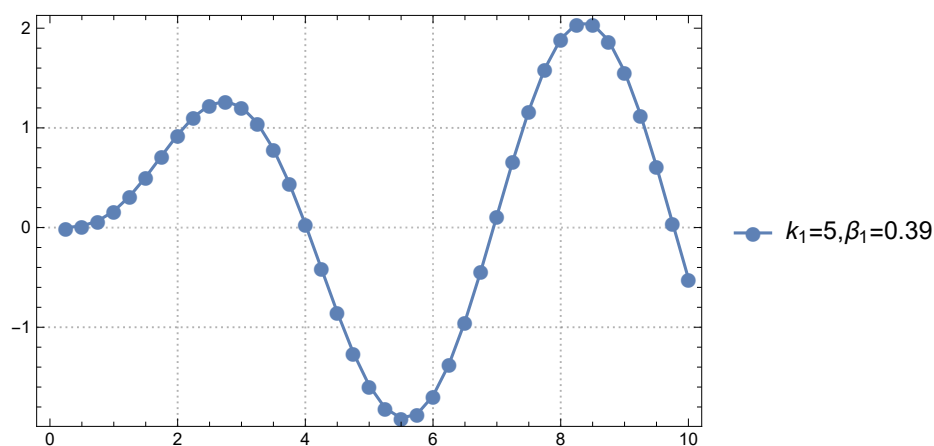
```
DiscretePlot[{uk005b039d2ₚ}, {p, 0, 10, Δt},
 PlotLegends -> {"k₁=5,β₁=0.39"}, PlotTheme -> "Detailed",
 Joined → True, PlotMarkers -> {Automatic, 12}, FillingStyle → White]
```
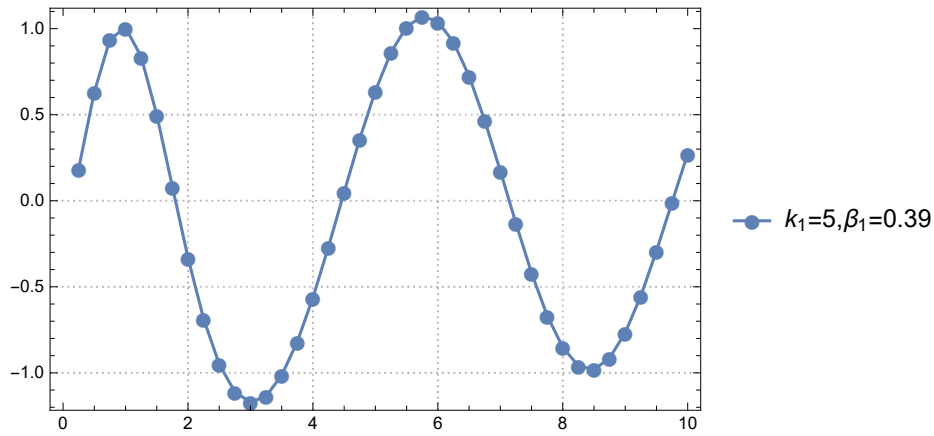


```
DiscretePlot[{uk005b039d3ₚ}, {p, 0, 10, Δt},
 PlotLegends -> {"k₁=5,β₁=0.39"}, PlotTheme -> "Detailed",
 Joined → True, PlotMarkers -> {Automatic, 12}, FillingStyle → White]
```



```
DiscretePlot[{uk005b039dd2ₚ}, {p, 0, 10, Δt},
 PlotLegends -> {"k₁=5,β₁=0.39"}, PlotTheme -> "Detailed",
 Joined → True, PlotMarkers -> {Automatic, 12}, FillingStyle → White]
```

```
DiscretePlot[{uk005b039dd3_p}, {p, 0, 10, Δt},
 PlotLegends -> {"k_1=5,β_1=0.39"}, PlotTheme -> "Detailed",
 Joined → True, PlotMarkers -> {Automatic, 12}, FillingStyle → White]
```


$k_1=5,\beta_1=0.39$