Final: Finite Element Method 2 (Dec. 10, 2018 – Dec. 17, 2018) - Prof. Gunwoo Noh

This is a take-home, open-book test. You may use any materials (including software), but no collaborations are allowed for the answers. You may get help from other people only for the indirect subjects (e.g. usage of the software, etc.). Submit your answers including the related codes **to E6-319 by 5:00 pm Dec. 17, 2018.** Describe the procedures in detail.

Consider the following composite method:

<First substep>

$$\mathbf{M}\,{}^{t+\gamma\Delta t}\ddot{\mathbf{U}} + \mathbf{C}\,{}^{t+\gamma\Delta t}\dot{\mathbf{U}} + \mathbf{K}\,{}^{t+\gamma\Delta t}\mathbf{U} = {}^{t+\gamma\Delta t}\mathbf{R}$$

$${}^{t+\gamma\Delta t}\mathbf{U} = {}^{t}\mathbf{U} + \frac{\gamma\Delta t}{2}({}^{t}\dot{\mathbf{U}} + {}^{t+\gamma\Delta t}\dot{\mathbf{U}})$$

$${}^{t+\gamma\Delta t}\dot{\mathbf{U}} = {}^{t}\dot{\mathbf{U}} + \frac{\gamma\Delta t}{2}({}^{t}\ddot{\mathbf{U}} + {}^{t+\gamma\Delta t}\ddot{\mathbf{U}})$$

<Second substep>

$$\mathbf{M}\,{}^{t+\Delta t}\ddot{\mathbf{U}} + \mathbf{C}\,{}^{t+\Delta t}\dot{\mathbf{U}} + \mathbf{K}\,{}^{t+\Delta t}\mathbf{U} = {}^{t+\Delta t}\mathbf{R}$$

$${}^{t+\Delta t}\mathbf{U} = {}^{t}\mathbf{U} + \gamma\Delta t\,((1-\beta_1)\,{}^{t}\dot{\mathbf{U}} + \beta_1\,{}^{t+\gamma\Delta t}\dot{\mathbf{U}}) + (1-\gamma)\Delta t((1-\beta_2)\,{}^{t+\gamma\Delta t}\dot{\mathbf{U}} + \beta_2\,{}^{t+\Delta t}\dot{\mathbf{U}})$$

$${}^{t+\Delta t}\dot{\mathbf{U}} = {}^{t}\dot{\mathbf{U}} + \gamma\Delta t\,((1-\beta_1)\,{}^{t}\ddot{\mathbf{U}} + \beta_1\,{}^{t+\gamma\Delta t}\ddot{\mathbf{U}}) + (1-\gamma)\Delta t((1-\beta_2)\,{}^{t+\gamma\Delta t}\ddot{\mathbf{U}} + \beta_2\,{}^{t+\Delta t}\ddot{\mathbf{U}})$$

Consider the case of no physical damping. Consider $\gamma = 1/2$.

1. (25 points) Dispersion analysis in 1D case

a) **(5 points)** Construct the linear multistep form of the method

b) **(10 points)** Draw the numerical dispersion curves in 1D case for $\beta_2 = 2\beta_1$ when $\beta_1 = 0.39$ and 0.65: i.e. $\frac{c - c_0}{c_0}$ with respect to $\frac{k\Delta x}{\pi}$ where c = numerical wave speed, c_0 = exact wave speed, k= numerical wave number and Δx =element length.

c) **(5 points)** Identify the values of $k\Delta x$ where $\Delta t / T$ =0.3 for various CFL numbers and β_1 ($\beta_2 = 2\beta_1$).

d) **(5 points)** Identify the optimal CFL values for various values of β_1 when $\beta_2 = 2\beta_1$. Discuss its dispersion properties.

2. (25 points) 3-DOF-spring system

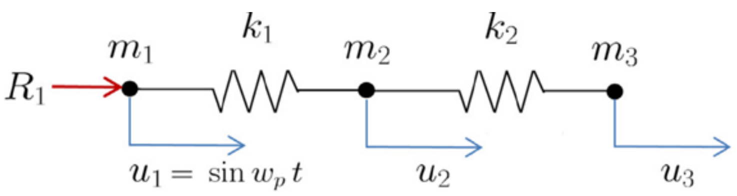

Fig. Model problem of three degrees of freedom spring system, $k_2 = 1,\ m_1 = 0,\ m_2 = 1,\ m_3 = 1,\ \omega_p = 1.2,\ k_1 = 10^7$ or 5.

Calculate the displacements, velocities and accelerations at nodes 2 and 3 for two cases, $k_1 = 5$ and 10^7 (all with $\omega_p = 1.2$), using the above time integration method, from time t = 0 to 30 with $\Delta t = 0.25$. In the calculations, consider $\beta_1 = 0.39$ and 0.65 when $\beta_2 = 2\beta_1$.

You may use the following equation in your calculations:

$$\begin{bmatrix} m_2 & 0 \\ 0 & m_3 \end{bmatrix} \begin{bmatrix} \ddot{u}_2 \\ \ddot{u}_3 \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} k_1 u_1 \\ 0 \end{bmatrix}$$