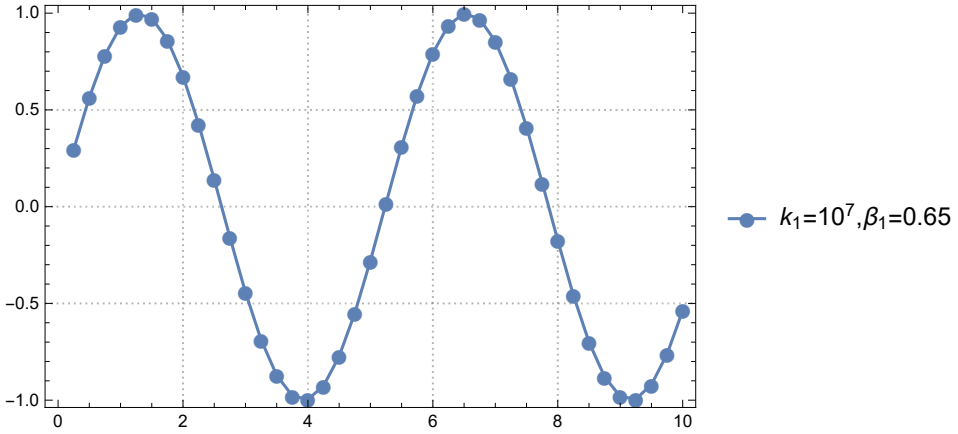
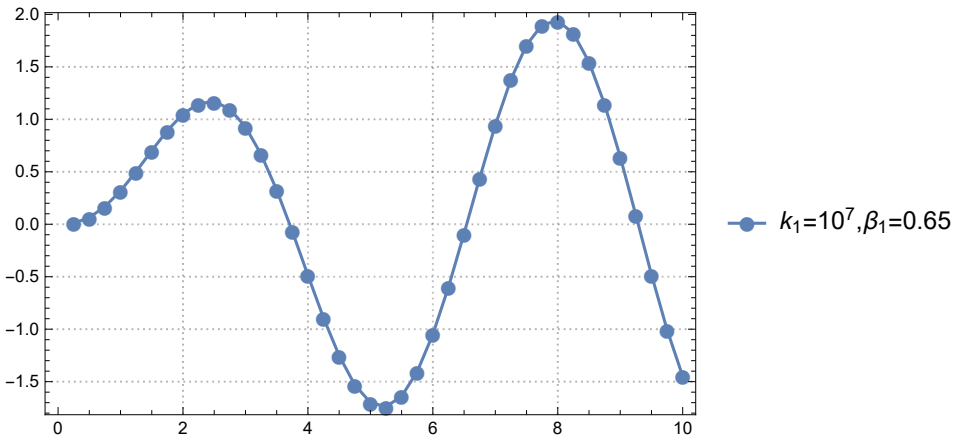
```
<< Notation`;

Symbolize[ u_1^t ]; Symbolize[ u_2^t ]; Symbolize[ u_3^t ]; Symbolize[ u̇_1^t ];

Symbolize[ u̇_2^t ]; Symbolize[ u̇_3^t ]; Symbolize[ ü_1^t ]; Symbolize[ ü_2^t ];

Symbolize[ ü_3^t ]; Symbolize[ u_1^{t+Δt} ]; Symbolize[ u_2^{t+Δt} ]; Symbolize[ u_3^{t+Δt} ];

Symbolize[ u̇_1^{t+Δt} ]; Symbolize[ u̇_2^{t+Δt} ]; Symbolize[ u̇_3^{t+Δt} ]; Symbolize[ ü_1^{t+Δt} ];

Symbolize[ ü_2^{t+Δt} ]; Symbolize[ ü_3^{t+Δt} ]; Symbolize[ u_1^{t+γΔt} ];

Symbolize[ u_2^{t+γΔt} ]; Symbolize[ u_3^{t+γΔt} ] ; Symbolize[ u̇_1^{t+γΔt} ];

Symbolize[ u̇_2^{t+γΔt} ]; Symbolize[ u̇_3^{t+γΔt} ]; Symbolize[ ü_1^{t+γΔt} ];

Symbolize[ ü_2^{t+γΔt} ]; Symbolize[ ü_3^{t+γΔt} ]; Symbolize[ β_1 ]; Symbolize[ β_2 ];

For[
  ClearAll["Global`*"];
γ = 1/2; Δt = 0.25; β_2 = 2 β_1;
m_2 = 1;
  m_3 = 1;
  k_2 = 1;
  ω = 1.2;
  u_1^{t+Δt} = Sin[ω p];
  u_1^{t+γΔt} = Sin[ω (p - Δt/2)];
eq112 = m_2 ü_2^{t+γΔt} + (k_1 + k_2) u_2^{t+γΔt} + (-k_2) u_3^{t+γΔt} == k_1 u_1^{t+γΔt};
  eq113 = m_3 ü_3^{t+γΔt} + (-k_2) u_2^{t+γΔt} + k_2 u_3^{t+γΔt} == 0;
  eq122 = u_2^{t+γΔt} == u_2^t + γ Δt/2 (u̇_2^t + u̇_2^{t+γΔt});
  eq123 = u_3^{t+γΔt} == u_3^t + γ Δt/2 (u̇_3^t + u̇_3^{t+γΔt});
  eq132 = u̇_2^{t+γΔt} == u̇_2^t + γ Δt/2 (ü_2^t + ü_2^{t+γΔt});
  eq133 = u̇_3^{t+γΔt} == u̇_3^t + γ Δt/2 (ü_3^t + ü_3^{t+γΔt});
  eq212 = m_2 ü_2^{t+Δt} + (k_1 + k_2) u_2^{t+Δt} + (-k_2) u_3^{t+Δt} == k_1 u_1^{t+Δt};
  eq213 = m_3 ü_3^{t+Δt} + (-k_2) u_2^{t+Δt} + k_2 u_3^{t+Δt} == 0;
  eq222 = u_2^{t+Δt} == u_2^t + γ Δt ((1 - β_1) u̇_2^t + β_1 u̇_2^{t+γΔt}) + (1 - γ) Δt ((1 - β_2) u̇_2^{t+γΔt} + β_2 u̇_2^{t+Δt});
  eq223 = u_3^{t+Δt} == u_3^t + γ Δt ((1 - β_1) u̇_3^t + β_1 u̇_3^{t+γΔt}) + (1 - γ) Δt ((1 - β_2) u̇_3^{t+γΔt} + β_2 u̇_3^{t+Δt});
  eq232 = u̇_2^{t+Δt} == u̇_2^t + γ Δt ((1 - β_1) ü_2^t + β_1 ü_2^{t+γΔt}) + (1 - γ) Δt ((1 - β_2) ü_2^{t+γΔt} + β_2 ü_2^{t+Δt});
  eq233 = u̇_3^{t+Δt} == u̇_3^t + γ Δt ((1 - β_1) ü_3^t + β_1 ü_3^{t+γΔt}) + (1 - γ) Δt ((1 - β_2) ü_3^{t+γΔt} + β_2 ü_3^{t+Δt});
s1and2 = Solve[eq112 && eq113 && eq122 && eq123 && eq132 &&
      eq133 && eq212 && eq213 && eq222 && eq223 && eq232 && eq233,
    {ü_2^{t+γΔt}, ü_3^{t+γΔt}, u̇_2^{t+γΔt}, u̇_3^{t+γΔt}, u_2^{t+γΔt}, u_3^{t+γΔt}, ü_2^{t+Δt}, ü_3^{t+Δt}, u̇_2^{t+Δt}, u̇_3^{t+Δt}, u_2^{t+Δt}, u_3^{t+Δt}}];

  ü_2^{t+Δt} = ü_2^{t+Δt} /. s1and2[[1, 7]];
  ü_3^{t+Δt} = ü_3^{t+Δt} /. s1and2[[1, 8]];
```

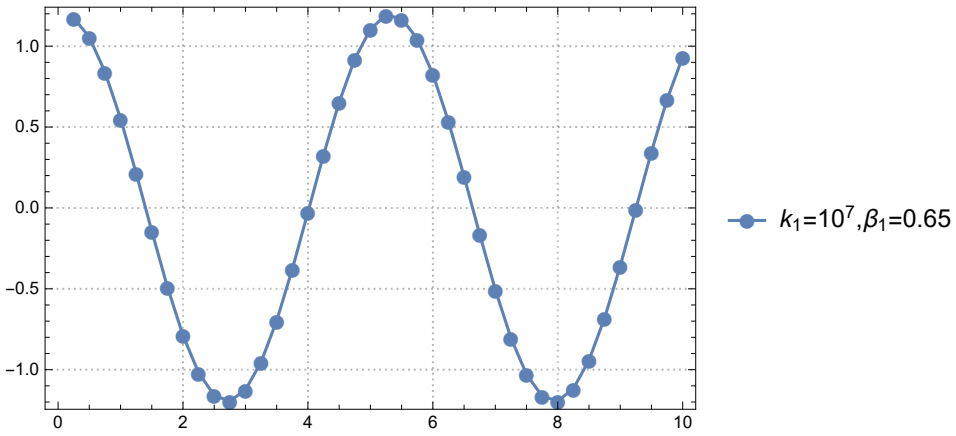
```
  u̇2^(t+Δt) = u̇2^(t+Δt) /. s1and2[[1, 9]];
  u̇3^(t+Δt) = u̇3^(t+Δt) /. s1and2[[1, 10]];
  u2^(t+Δt) = u2^(t+Δt) /. s1and2[[1, 11]];
  u3^(t+Δt) = u3^(t+Δt) /. s1and2[[1, 12]];
  k1 = 10^7; β1 = 0.65;
  p = 0.25; ü2^t = 0; ü3^t = 0; u̇2^t = 0; u̇3^t = 0; u2^t = 0; u3^t = 0,
  p ≤ 30,
  p = p + 0.25,
  uk107b0652p = u2^(t+Δt);
  uk107b0653p = u3^(t+Δt);
  uk107b065d2p = u̇2^(t+Δt);
  uk107b065d3p = u̇3^(t+Δt);
  uk107b065dd2p = ü2^(t+Δt);
  uk107b065dd3p = ü3^(t+Δt);
  ü2^t = ü2^(t+Δt); ü3^t = ü3^(t+Δt); u̇2^t = u̇2^(t+Δt); u̇3^t = u̇3^(t+Δt); u2^t = u2^(t+Δt); u3^t = u3^(t+Δt)
 ];
DiscretePlot[uk107b0652p, {p, 0, 10, Δt},
 PlotLegends -> {"k1=10^7,β1=0.65"}, PlotTheme -> "Detailed",
 Joined → True, PlotMarkers -> {Automatic, 12}, FillingStyle → White]
```



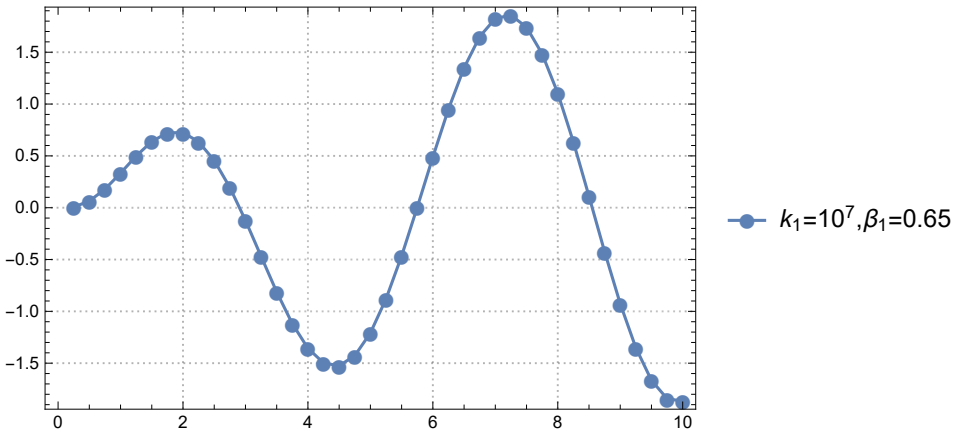

```
DiscretePlot[uk107b0653p, {p, 0, 10, Δt},
 PlotLegends -> {"k1=10^7,β1=0.65"}, PlotTheme -> "Detailed",
 Joined → True, PlotMarkers -> {Automatic, 12}, FillingStyle → White]
```



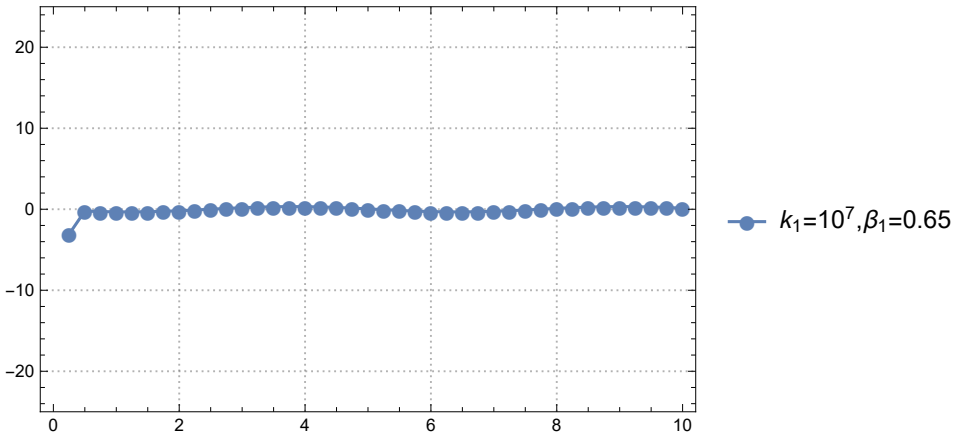
```
DiscretePlot[uk107b065d2p, {p, 0, 10, Δt},
 PlotLegends -> {"k1=10^7,β1=0.65"}, PlotTheme -> "Detailed",
 Joined → True, PlotMarkers -> {Automatic, 12}, FillingStyle → White]
```




```
DiscretePlot[uk107b065d3p, {p, 0, 10, Δt},
 PlotLegends -> {"k1=10^7,β1=0.65"}, PlotTheme -> "Detailed",
 Joined → True, PlotMarkers -> {Automatic, 12}, FillingStyle → White]
```




```
DiscretePlot[uk107b065dd2p, {p, 0, 10, Δt},
 PlotLegends -> {"k1=10^7,β1=0.65"}, PlotTheme -> "Detailed", Joined → True,
 PlotMarkers -> {Automatic, 12}, FillingStyle → White, PlotRange → 25]
```



```
DiscretePlot[uk107b065dd3p, {p, 0, 10, Δt},
 PlotLegends -> {"k1=10^7,β1=0.65"}, PlotTheme -> "Detailed",
 Joined → True, PlotMarkers -> {Automatic, 12}, FillingStyle → White]
```