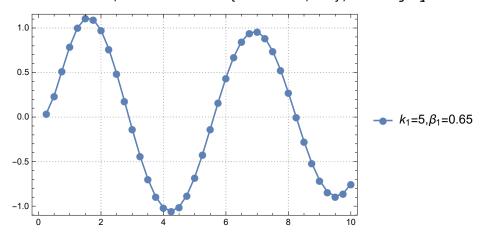
```
<< Notation`;
Symbolize \begin{bmatrix} u_1^t \\ i \end{bmatrix}; Symbolize \begin{bmatrix} u_2^t \\ i \end{bmatrix}; Symbolize \begin{bmatrix} u_2^t \\ i \end{bmatrix}; Symbolize \begin{bmatrix} \dot{u}_1^t \\ i \end{bmatrix};
Symbolize \begin{bmatrix} \dot{u}_2^t \\ \dot{u}_2^t \end{bmatrix}; Symbolize \begin{bmatrix} \dot{u}_3^t \\ \dot{u}_3^t \end{bmatrix}; Symbolize \begin{bmatrix} \dot{u}_2^t \\ \dot{u}_2^t \end{bmatrix}; Symbolize \begin{bmatrix} \dot{u}_2^t \\ \dot{u}_2^t \end{bmatrix};
Symbolize \begin{bmatrix} \ddot{u}_{3}^{t} \end{bmatrix}; Symbolize \begin{bmatrix} u_{1}^{t+\Delta t} \end{bmatrix}; Symbolize \begin{bmatrix} u_{2}^{t+\Delta t} \end{bmatrix}; Symbolize \begin{bmatrix} u_{3}^{t+\Delta t} \end{bmatrix};
Symbolize \left[\begin{array}{c} \dot{\mathbf{u}}_{1}^{t+\Delta t} \end{array}\right]; Symbolize \left[\begin{array}{c} \dot{\mathbf{u}}_{2}^{t+\Delta t} \end{array}\right]; Symbolize \left[\begin{array}{c} \dot{\mathbf{u}}_{3}^{t+\Delta t} \end{array}\right]; Symbolize \left[\begin{array}{c} \dot{\mathbf{u}}_{1}^{t+\Delta t} \end{array}\right];
Symbolize \begin{bmatrix} \ddot{u}_{2}^{t+\Delta t} \end{bmatrix}; Symbolize \begin{bmatrix} \ddot{u}_{3}^{t+\Delta t} \end{bmatrix}; Symbolize \begin{bmatrix} u_{1}^{t+\Delta t} \end{bmatrix};
Symbolize \left[ \begin{array}{c} \mathbf{u}_{2}^{t+\gamma\Delta t} \end{array} \right]; Symbolize \left[ \begin{array}{c} \mathbf{u}_{3}^{t+\gamma\Delta t} \end{array} \right]; Symbolize \left[ \begin{array}{c} \dot{\mathbf{u}}_{1}^{t+\gamma\Delta t} \end{array} \right];
Symbolize \left[\begin{array}{c} \dot{\mathbf{u}}_{2}^{t+\gamma\Delta t} \end{array}\right]; Symbolize \left[\begin{array}{c} \dot{\mathbf{u}}_{3}^{t+\gamma\Delta t} \end{array}\right]; Symbolize \left[\begin{array}{c} \dot{\mathbf{u}}_{1}^{t+\gamma\Delta t} \end{array}\right];
Symbolize \begin{bmatrix} \ddot{u}_2^{t+\gamma\Delta t} \end{bmatrix}; Symbolize \begin{bmatrix} \ddot{u}_3^{t+\gamma\Delta t} \end{bmatrix}; Symbolize \begin{bmatrix} \beta_1 \end{bmatrix}; Symbolize \begin{bmatrix} \beta_2 \end{bmatrix};
        ClearAll["Global`*"];
\gamma = \frac{1}{2}; \Delta t = 0.25; \beta_2 = 2 \beta_1;
m_2 = 1;
       m_3 = 1;
       k_2 = 1;
        \omega=1.2;
        \mathbf{u}_{1}^{\mathsf{t}+\Delta\mathsf{t}} = \mathsf{Sin}[\omega\,\mathsf{p}];
        \mathbf{u}_{1}^{\mathsf{t}+\gamma\Delta\mathsf{t}} = \mathrm{Sin}\left[\omega\left(\mathbf{p} - \frac{\Delta\mathsf{t}}{2}\right)\right];
eq112 = m_2 \dot{u}_2^{t+\gamma\Delta t} + (k_1 + k_2) u_2^{t+\gamma\Delta t} + (-k_2) u_3^{t+\gamma\Delta t} == k_1 u_1^{t+\gamma\Delta t};
        eq113 = m_3 \ddot{u}_3^{t+\gamma\Delta t} + (-k_2) u_2^{t+\gamma\Delta t} + k_2 u_3^{t+\gamma\Delta t} == 0;
        eq122 = u_2^{t+\gamma\Delta t} = u_2^t + \frac{\gamma \Delta t}{2} \left( \dot{u}_2^t + \dot{u}_2^{t+\gamma\Delta t} \right);
        eq123 = u_3^{t+\gamma\Delta t} = u_3^t + \frac{\gamma \Delta t}{2} \left( \dot{u}_3^t + \dot{u}_3^{t+\gamma\Delta t} \right);
        eq133 = \dot{\mathbf{u}}_3^{t+\gamma\Delta t} = \dot{\mathbf{u}}_3^t + \frac{\gamma \Delta t}{2} \left( \ddot{\mathbf{u}}_3^t + \ddot{\mathbf{u}}_3^{t+\gamma\Delta t} \right);
        eq212 = m_2 \dot{u}_2^{t+\Delta t} + (k_1 + k_2) u_2^{t+\Delta t} + (-k_2) u_3^{t+\Delta t} == k_1 u_1^{t+\Delta t};
        eq213 = m_3 u_3^{t+\Delta t} + (-k_2) u_2^{t+\Delta t} + k_2 u_3^{t+\Delta t} == 0;
        eq222 = \mathbf{u}_{2}^{\mathsf{t}+\Delta\mathsf{t}} = \mathbf{u}_{2}^{\mathsf{t}} + \gamma \Delta\mathsf{t} \left( \left( 1 - \beta_{1} \right) \dot{\mathbf{u}}_{2}^{\mathsf{t}} + \beta_{1} \dot{\mathbf{u}}_{2}^{\mathsf{t}+\gamma\Delta\mathsf{t}} \right) + \left( 1 - \gamma \right) \Delta\mathsf{t} \left( \left( 1 - \beta_{2} \right) \dot{\mathbf{u}}_{2}^{\mathsf{t}+\gamma\Delta\mathsf{t}} + \beta_{2} \dot{\mathbf{u}}_{2}^{\mathsf{t}+\Delta\mathsf{t}} \right);
        eq223 = \mathbf{u}_3^{t+\Delta t} = \mathbf{u}_3^t + \gamma \Delta t \left( \left( 1 - \beta_1 \right) \dot{\mathbf{u}}_3^t + \beta_1 \dot{\mathbf{u}}_3^{t+\gamma \Delta t} \right) + \left( 1 - \gamma \right) \Delta t \left( \left( 1 - \beta_2 \right) \dot{\mathbf{u}}_3^{t+\gamma \Delta t} + \beta_2 \dot{\mathbf{u}}_3^{t+\Delta t} \right);
        eq232 = \dot{\mathbf{u}}_{2}^{\mathsf{t}+\Delta\mathsf{t}} = \dot{\mathbf{u}}_{2}^{\mathsf{t}} + \gamma \Delta\mathsf{t} \left( \left( 1 - \beta_{1} \right) \ \dot{\mathbf{u}}_{2}^{\mathsf{t}} + \beta_{1} \ \dot{\mathbf{u}}_{2}^{\mathsf{t}+\gamma\Delta\mathsf{t}} \right) + \left( 1 - \gamma \right) \Delta\mathsf{t} \left( \left( 1 - \beta_{2} \right) \ \dot{\mathbf{u}}_{2}^{\mathsf{t}+\gamma\Delta\mathsf{t}} + \beta_{2} \ \dot{\mathbf{u}}_{2}^{\mathsf{t}+\Delta\mathsf{t}} \right);
        eq233 = \dot{\mathbf{u}}_3^{t+\Delta t} == \dot{\mathbf{u}}_3^t + \gamma \Delta t \left( \left( 1 - \beta_1 \right) \dot{\mathbf{u}}_3^t + \beta_1 \dot{\mathbf{u}}_3^{t+\gamma \Delta t} \right) + \left( 1 - \gamma \right) \Delta t \left( \left( 1 - \beta_2 \right) \dot{\mathbf{u}}_3^{t+\gamma \Delta t} + \beta_2 \dot{\mathbf{u}}_3^{t+\Delta t} \right);
sland2 = Solve eq112 && eq113 && eq122 && eq123 && eq132 &&
                    eq133 && eq212 && eq213 && eq222 && eq223 && eq232 && eq233,
                 \left\{\ddot{u}_{2}^{t+\gamma\Delta t},~\ddot{u}_{3}^{t+\gamma\Delta t},~\dot{u}_{2}^{t+\gamma\Delta t},~\dot{u}_{3}^{t+\gamma\Delta t},~u_{2}^{t+\gamma\Delta t},~u_{3}^{t+\gamma\Delta t},~\dot{u}_{3}^{t+\Delta t},~\dot{u}_{3}^{t+\Delta t},~\dot{u}_{3}^{t+\Delta t},~\dot{u}_{3}^{t+\Delta t},~u_{3}^{t+\Delta t}\right\}\right];
        \ddot{u}_{2}^{t+\Delta t} = \ddot{u}_{2}^{t+\Delta t} /. s1and2[[1, 7]];
        \ddot{u}_{3}^{t+\Delta t} = \ddot{u}_{3}^{t+\Delta t} /. sland2[[1, 8]];
```

```
\dot{u}_{2}^{t+\Delta t} = \dot{u}_{2}^{t+\Delta t} /. \text{ s1and2}[[1, 9]];
  \dot{u}_{3}^{t+\Delta t} = \dot{u}_{3}^{t+\Delta t} /. sland2[[1, 10]];
  u_2^{t+\Delta t} = u_2^{t+\Delta t} /. sland2[[1, 11]];
  u_3^{t+\Delta t} = u_3^{t+\Delta t} /. s1and2[[1, 12]];
  k_1 = 5; \beta_1 = 0.65;
  p = 0.25; \; \dot{u}_{2}^{t} = 0; \; \dot{u}_{3}^{t} = 0; \; \dot{u}_{2}^{t} = 0; \; \dot{u}_{3}^{t} = 0; \; u_{2}^{t} = 0; \; u_{3}^{t} = 0,
  p = p + 0.25,
  uk005b0652_p = u_2^{t+\Delta t};
  uk005b0653_p = u_3^{t+\Delta t};
  uk005b065d2_p = \dot{u}_2^{t+\Delta t};
  uk005b065d3_p = \dot{u}_3^{t+\Delta t};
  uk005b065dd2_p = \dot{u}_2^{t+\Delta t};
  uk005b065dd3_p = \dot{u}_3^{t+\Delta t};
  \ddot{u}_{2}^{t} = \ddot{u}_{2}^{t+\Delta t}; \ \ddot{u}_{3}^{t} = \ddot{u}_{3}^{t+\Delta t}; \ \dot{u}_{2}^{t} = \dot{u}_{2}^{t+\Delta t}; \ \dot{u}_{3}^{t} = \dot{u}_{3}^{t+\Delta t}; \ u_{2}^{t} = u_{2}^{t+\Delta t}; \ u_{3}^{t} = u_{3}^{t+\Delta t}
];
```

DiscretePlot[ $\{uk005b0652_p\}$ ,  $\{p, 0, 10, \Delta t\}$ ,

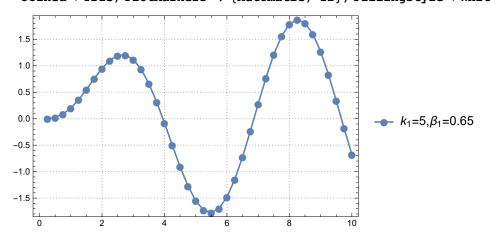
PlotLegends ->  $\{"k_1=5, \beta_1=0.65"\}$ , PlotTheme -> "Detailed",

Joined → True, PlotMarkers -> {Automatic, 12}, FillingStyle → White]



 $\texttt{DiscretePlot}[\left\{uk005b0653_p\right\}, \left\{p, \, 0, \, 10, \, \Delta t\right\},$ 

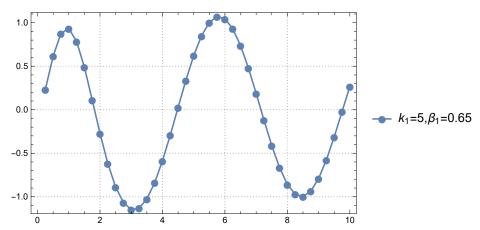
PlotLegends -> {" $k_1=5$ ,  $\beta_1=0.65$ "}, PlotTheme -> "Detailed", Joined → True, PlotMarkers -> {Automatic, 12}, FillingStyle → White]



 $DiscretePlot[\{uk005b065d2_p\}, \{p, 0, 10, \Delta t\},$ 

PlotLegends ->  $\{"k_1=5, \beta_1=0.65"\}$ , PlotTheme -> "Detailed",

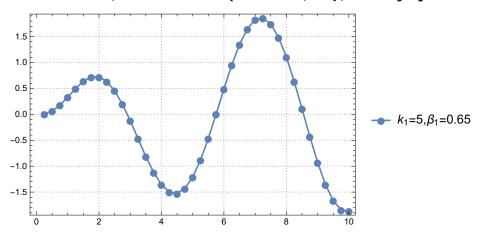
 $\texttt{Joined} \rightarrow \texttt{True}, \; \texttt{PlotMarkers} \; \texttt{->} \; \{\texttt{Automatic}, \; \; \texttt{12}\} \,, \; \texttt{FillingStyle} \rightarrow \texttt{White}]$ 



 $\label{eq:decomposition} \texttt{DiscretePlot}[\{uk005b065d3_p\}\,,\,\{p\,,\,0\,,\,10\,,\,\Delta t\}\,,$ 

PlotLegends ->  $\{"k_1=5, \beta_1=0.65"\}$ , PlotTheme -> "Detailed",

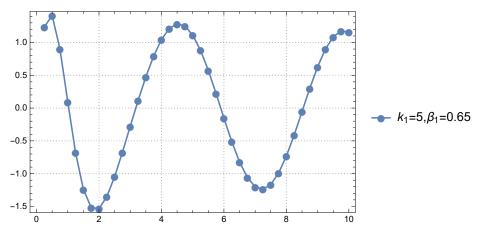
Joined → True, PlotMarkers -> {Automatic, 12}, FillingStyle → White]



 $\label{eq:decomposition} DiscretePlot[\left\{uk005b065dd2_p\right\}, \left\{p, \, 0 \,, \, 10 \,, \, \Delta t\right\},$ 

PlotLegends ->  $\{"k_1=5, \beta_1=0.65"\}$ , PlotTheme -> "Detailed",

Joined → True, PlotMarkers -> {Automatic, 12}, FillingStyle → White]



$$\label{eq:decomposition} \begin{split} & \text{DiscretePlot}[\,\{uk005b065dd3_p\}\,,\,\,\{p,\,\,0\,,\,\,10\,,\,\,\Delta t\,\}\,,\\ & \text{PlotLegends} \,\rightarrow\, \{\,"k_1=5\,,\beta_1=0\,.65\,"\,\}\,,\,\, \text{PlotTheme} \,\rightarrow\, "\text{Detailed"}\,,\\ & \text{Joined} \,\rightarrow\, \text{True}\,,\,\, \text{PlotMarkers} \,\rightarrow\, \{\,\text{Automatic}\,,\,\,\,12\,\}\,,\,\, \text{FillingStyle} \,\rightarrow\, \text{White}\,] \end{split}$$

