

<< Notation`;

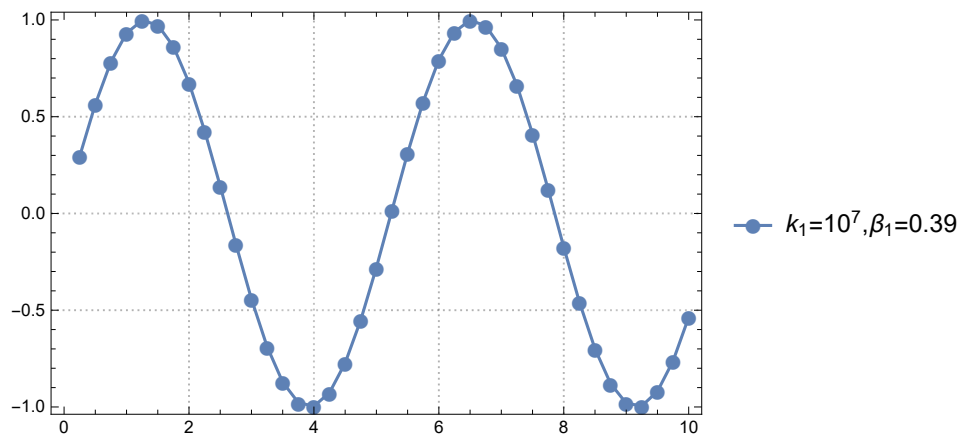
```
Symbolize[ $u_1^t$ ]; Symbolize[ $u_2^t$ ]; Symbolize[ $u_3^t$ ]; Symbolize[ $\dot{u}_1^t$ ];  
Symbolize[ $\dot{u}_2^t$ ]; Symbolize[ $\dot{u}_3^t$ ]; Symbolize[ $\ddot{u}_1^t$ ]; Symbolize[ $\ddot{u}_2^t$ ];  
Symbolize[ $\ddot{u}_3^t$ ]; Symbolize[ $u_1^{t+\Delta t}$ ]; Symbolize[ $u_2^{t+\Delta t}$ ]; Symbolize[ $u_3^{t+\Delta t}$ ];  
Symbolize[ $\dot{u}_1^{t+\Delta t}$ ]; Symbolize[ $\dot{u}_2^{t+\Delta t}$ ]; Symbolize[ $\dot{u}_3^{t+\Delta t}$ ]; Symbolize[ $\ddot{u}_1^{t+\Delta t}$ ];  
Symbolize[ $\ddot{u}_2^{t+\Delta t}$ ]; Symbolize[ $\ddot{u}_3^{t+\Delta t}$ ]; Symbolize[ $u_1^{t+\gamma\Delta t}$ ];  
Symbolize[ $u_2^{t+\gamma\Delta t}$ ]; Symbolize[ $u_3^{t+\gamma\Delta t}$ ]; Symbolize[ $\dot{u}_1^{t+\gamma\Delta t}$ ];  
Symbolize[ $\dot{u}_2^{t+\gamma\Delta t}$ ]; Symbolize[ $\dot{u}_3^{t+\gamma\Delta t}$ ]; Symbolize[ $\ddot{u}_1^{t+\gamma\Delta t}$ ];  
Symbolize[ $\ddot{u}_2^{t+\gamma\Delta t}$ ]; Symbolize[ $\ddot{u}_3^{t+\gamma\Delta t}$ ]; Symbolize[ $\beta_1$ ]; Symbolize[ $\beta_2$ ];
```

```

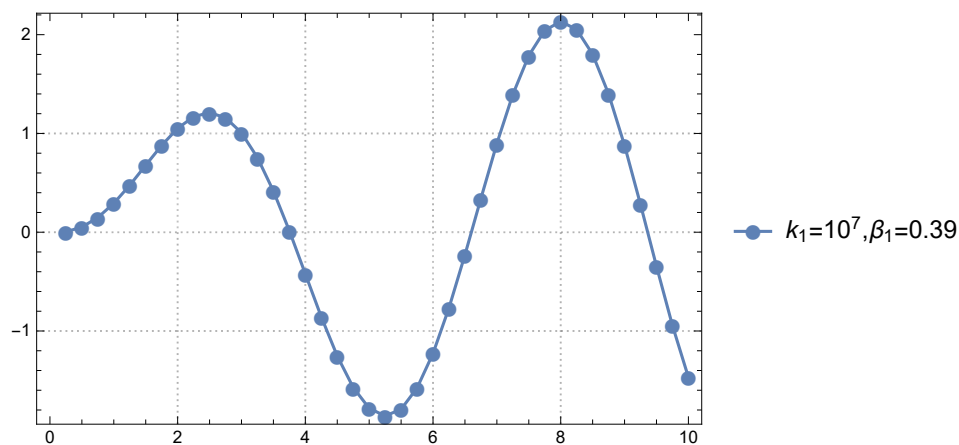
For[
  ClearAll["Global`*"];
   $\gamma = \frac{1}{2}; \Delta t = 0.25; \beta_2 = 2 \beta_1;$ 
  m2 = 1;
  m3 = 1;
  k2 = 1;
   $\omega = 1.2;$ 
   $u_1^{t+\Delta t} = \text{Sin}[\omega p];$ 
   $u_1^{t+\gamma\Delta t} = \text{Sin}\left[\omega\left(p - \frac{\Delta t}{2}\right)\right];$ 
  eq112 = m2  $\ddot{u}_2^{t+\gamma\Delta t} + (k_1 + k_2) u_2^{t+\gamma\Delta t} + (-k_2) u_3^{t+\gamma\Delta t} = k_1 u_1^{t+\gamma\Delta t};$ 
  eq113 = m3  $\ddot{u}_3^{t+\gamma\Delta t} + (-k_2) u_2^{t+\gamma\Delta t} + k_2 u_3^{t+\gamma\Delta t} = 0;$ 
  eq122 =  $u_2^{t+\gamma\Delta t} = u_2^t + \frac{\gamma \Delta t}{2} (\dot{u}_2^t + \dot{u}_2^{t+\gamma\Delta t});$ 
  eq123 =  $u_3^{t+\gamma\Delta t} = u_3^t + \frac{\gamma \Delta t}{2} (\dot{u}_3^t + \dot{u}_3^{t+\gamma\Delta t});$ 
  eq132 =  $\dot{u}_2^{t+\gamma\Delta t} = \dot{u}_2^t + \frac{\gamma \Delta t}{2} (\ddot{u}_2^t + \ddot{u}_2^{t+\gamma\Delta t});$ 
  eq133 =  $\dot{u}_3^{t+\gamma\Delta t} = \dot{u}_3^t + \frac{\gamma \Delta t}{2} (\ddot{u}_3^t + \ddot{u}_3^{t+\gamma\Delta t});$ 
  eq212 = m2  $\ddot{u}_2^{t+\Delta t} + (k_1 + k_2) u_2^{t+\Delta t} + (-k_2) u_3^{t+\Delta t} = k_1 u_1^{t+\Delta t};$ 
  eq213 = m3  $\ddot{u}_3^{t+\Delta t} + (-k_2) u_2^{t+\Delta t} + k_2 u_3^{t+\Delta t} = 0;$ 
  eq222 =  $u_2^{t+\Delta t} = u_2^t + \gamma \Delta t ((1 - \beta_1) \dot{u}_2^t + \beta_1 \dot{u}_2^{t+\gamma\Delta t}) + (1 - \gamma) \Delta t ((1 - \beta_2) \dot{u}_2^{t+\gamma\Delta t} + \beta_2 \dot{u}_2^{t+\Delta t});$ 
  eq223 =  $u_3^{t+\Delta t} = u_3^t + \gamma \Delta t ((1 - \beta_1) \dot{u}_3^t + \beta_1 \dot{u}_3^{t+\gamma\Delta t}) + (1 - \gamma) \Delta t ((1 - \beta_2) \dot{u}_3^{t+\gamma\Delta t} + \beta_2 \dot{u}_3^{t+\Delta t});$ 
  eq232 =  $\dot{u}_2^{t+\Delta t} = \dot{u}_2^t + \gamma \Delta t ((1 - \beta_1) \ddot{u}_2^t + \beta_1 \ddot{u}_2^{t+\gamma\Delta t}) + (1 - \gamma) \Delta t ((1 - \beta_2) \ddot{u}_2^{t+\gamma\Delta t} + \beta_2 \ddot{u}_2^{t+\Delta t});$ 
  eq233 =  $\dot{u}_3^{t+\Delta t} = \dot{u}_3^t + \gamma \Delta t ((1 - \beta_1) \ddot{u}_3^t + \beta_1 \ddot{u}_3^{t+\gamma\Delta t}) + (1 - \gamma) \Delta t ((1 - \beta_2) \ddot{u}_3^{t+\gamma\Delta t} + \beta_2 \ddot{u}_3^{t+\Delta t});$ 
  s1and2 = Solve[eq112 && eq113 && eq122 && eq123 && eq132 &&
    eq133 && eq212 && eq213 && eq222 && eq223 && eq232 && eq233,
    { $\ddot{u}_2^{t+\gamma\Delta t}, \ddot{u}_3^{t+\gamma\Delta t}, \ddot{u}_2^{t+\Delta t}, \ddot{u}_3^{t+\Delta t}, \dot{u}_2^{t+\gamma\Delta t}, \dot{u}_3^{t+\gamma\Delta t}, \dot{u}_2^{t+\Delta t}, \dot{u}_3^{t+\Delta t}, \ddot{u}_2^{t+\Delta t}, \ddot{u}_3^{t+\Delta t}, \dot{u}_2^{t+\Delta t}, \dot{u}_3^{t+\Delta t}$ }];
   $\ddot{u}_2^{t+\Delta t} = \ddot{u}_2^{t+\Delta t} /. \text{s1and2}[[1, 7]];$ 
   $\ddot{u}_3^{t+\Delta t} = \ddot{u}_3^{t+\Delta t} /. \text{s1and2}[[1, 8]];$ 
   $\dot{u}_2^{t+\Delta t} = \dot{u}_2^{t+\Delta t} /. \text{s1and2}[[1, 9]];$ 
   $\dot{u}_3^{t+\Delta t} = \dot{u}_3^{t+\Delta t} /. \text{s1and2}[[1, 10]];$ 
   $u_2^{t+\Delta t} = u_2^{t+\Delta t} /. \text{s1and2}[[1, 11]]; u_3^{t+\Delta t} = u_3^{t+\Delta t} /. \text{s1and2}[[1, 12]];$ 
  k1 = 107;  $\beta_1 = 0.39;$ 
  p = 0.25;  $\dot{u}_2^t = 0; \dot{u}_3^t = 0; \ddot{u}_2^t = 0; \ddot{u}_3^t = 0; u_2^t = 0; u_3^t = 0,$ 
  p ≤ 30,
  p = p + 0.25,
  uk107b0392p =  $u_2^{t+\Delta t};$ 
  uk107b0393p =  $u_3^{t+\Delta t};$ 
  uk107b039d2p =  $\dot{u}_2^{t+\Delta t};$ 
  uk107b039d3p =  $\dot{u}_3^{t+\Delta t};$ 
  uk107b039dd2p =  $\ddot{u}_2^{t+\Delta t};$ 
  uk107b039dd3p =  $\ddot{u}_3^{t+\Delta t};$ 
   $\ddot{u}_2^t = \ddot{u}_2^{t+\Delta t}; \ddot{u}_3^t = \ddot{u}_3^{t+\Delta t}; \dot{u}_2^t = \dot{u}_2^{t+\Delta t}; \dot{u}_3^t = \dot{u}_3^{t+\Delta t}; u_2^t = u_2^{t+\Delta t}; u_3^t = u_3^{t+\Delta t}$ 
];

```

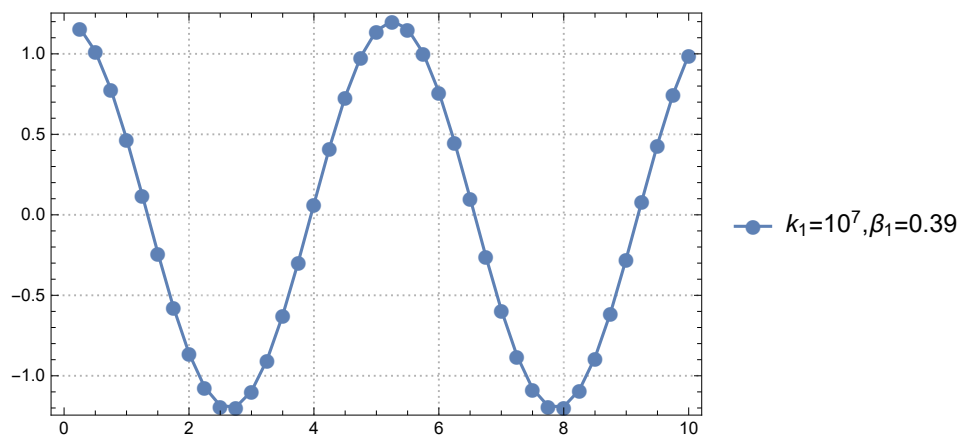
```
DiscretePlot[{uk107b0392p}, {p, 0, 10, Δt},
  PlotLegends -> {"k1=107, β1=0.39"}, PlotTheme -> "Detailed",
  Joined -> True, PlotMarkers -> {Automatic, 12}, FillingStyle -> White]
```



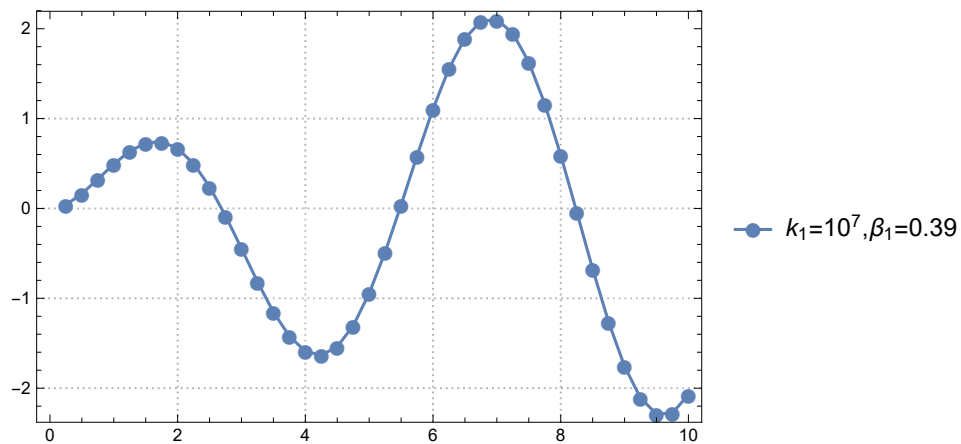
```
DiscretePlot[{uk107b0393p}, {p, 0, 10, Δt},
  PlotLegends -> {"k1=107, β1=0.39"}, PlotTheme -> "Detailed",
  Joined -> True, PlotMarkers -> {Automatic, 12}, FillingStyle -> White]
```



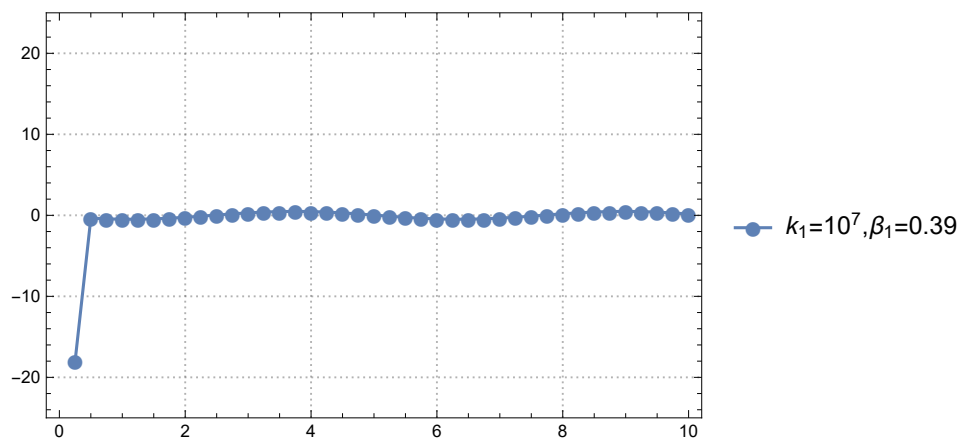
```
DiscretePlot[{uk107b039d2p}, {p, 0, 10, Δt},
  PlotLegends -> {"k1=107, β1=0.39"}, PlotTheme -> "Detailed",
  Joined -> True, PlotMarkers -> {Automatic, 12}, FillingStyle -> White]
```



```
DiscretePlot[{uk107b039d3p}, {p, 0, 10, Δt},
  PlotLegends -> {"k1=107, β1=0.39"}, PlotTheme -> "Detailed",
  Joined -> True, PlotMarkers -> {Automatic, 12}, FillingStyle -> White]
```



```
DiscretePlot[{uk107b039dd2p}, {p, 0, 10, Δt},
  PlotLegends -> {"k1=107, β1=0.39"}, PlotTheme -> "Detailed", Joined -> True,
  PlotMarkers -> {Automatic, 12}, FillingStyle -> White, PlotRange -> 25]
```



```
DiscretePlot[{uk107b039dd3p}, {p, 0, 10, Δt},
  PlotLegends -> {"k1=107, β1=0.39"}, PlotTheme -> "Detailed",
  Joined -> True, PlotMarkers -> {Automatic, 12}, FillingStyle -> White]
```

