```
ClearAll["Global`*"];
(* This function clears any past definitions of the variables *)



(* THERMOPHYSICAL PROPERTIES OF THE PARAFFIN *)

Tm = 303; (* melting tempreraturе of the paraffin *)
ρs = 880; ρl = 760; ρ = ρl; (* density at solid and liquid states *)
ks = 0.24;
kl = 0.15; (* thermal conductivity at solid and liquid states *)
cps = 2.4 * 10^3;
cpl = 1.8 * 10^3; (* specific heat at solid and liquid states *)
q = 179 * 10^3; (* latent heat of fusion*)
vd = 3.42;
vk = vd/ρ; (* dynamic viscosity (vd)
 and kinematic viscosity (vk) of paraffin *)


(* FORMULAS AND TRANSFORMATIONS*)

αs = ks/(ρs cps);
αl = kl/(ρl cpl); (* thermal diffusivity at solid and liquid states *)
pr = vk/αl; (* Prandtl number*)
gdot = G kl (T0 - Tm)/r^2; (* heat sink parameter *)
ξ = gdot/(ρ q) (r^2 τ/αl); (* mass proportion of liquid in the mixture;
t taken as r^2 τ/αl *)
T0 = Tm + q ste/cpl;(* temperature outside the sphere;
it depends on the stefan number desired for the simulation*)

γ = ks/kl;
Γ = αs/αl;
θi = (Ti - Tm)/(T0 - Tm); (* dimensionless initial temperature *)



keq = 1; (* equivalent thermal conductivity *)
```

```
(* since Rayleigh number = 0 (i.e. ≤ 5×10^4) for figure 3(c),
keq is assumed equal to 1 *)


(* PARAMETER VALUES *)

r = 0.05; (* radius of the sphere *)
Ti = 295; (* initial temperature of the paraffin *)


(* PARAMETER VALUES SPECIFIC TO FIGURE 3(c) OF BECHIRI ET AL. 2020 *)

ste = 0.05; (* Stefan number *)
ra = 0; (* Rayleigh number *)
bi = 10; (* Biot number *)
G = 0; (* dimensionless heat sink parameter *)


(* SOLVING THE EQUATION SYSTEM (6) *)
```

$$\text{lhs} = \left(\frac{\text{keq bi}}{\text{splus}[\tau]}\text{Exp}\left[-\frac{\text{splus}[\tau]^2}{4\,\text{keq}\,\tau}\right]\right)\Big/ \left(\text{keq Exp}\left[-\frac{1}{4\,\text{keq}\,\tau}\right] - \text{bi}\left(\text{Exp}\left[-\frac{1}{4\,\text{keq}\,\tau}\right] - \frac{1}{\text{splus}[\tau]}\text{Exp}\left[-\frac{\text{splus}[\tau]^2}{4\,\text{keq}\,\tau}\right] - \sqrt{\frac{\pi}{4\,\text{keq}\,\tau}}\left(\text{Erfc}\left[\frac{1}{2\sqrt{\text{keq}\,\tau}}\right] - \text{Erfc}\left[\frac{\text{splus}[\tau]}{2\sqrt{\text{keq}\,\tau}}\right]\right)\right)\right) + \frac{G}{6\,\text{splus}[\tau]}\left(\frac{\text{bi}\left(\text{splus}[\tau]^2-1\right) - 2\,\text{keq}}{\text{bi} + \text{splus}[\tau]\,(\text{keq}-\text{bi})}\right) - \frac{2}{\text{splus}[\tau]}\sum_{n=1}^{\infty}\left(\gamma\,\theta\text{i} + \frac{G\,\text{splus}[\tau]^2}{n^2\pi^2}\right)\text{Exp}\left[-\frac{n^2\pi^2\,\Gamma\,\tau}{\text{splus}[\tau]^2}\right]$$

$$\frac{2.5743\left(-1. + \text{EllipticTheta}\left[3, 0., 2.71828^{-\frac{10.2285\,\tau}{\text{splus}[\tau]^2}}\right]\right)}{\text{splus}[\tau]} + \frac{10\,e^{-\frac{\text{splus}[\tau]^2}{4\tau}}}{\left(e^{-\frac{1}{4}/\tau} - 10\left(e^{-\frac{1}{4}/\tau} - \frac{1}{2}\sqrt{\pi}\sqrt{\frac{1}{\tau}}\left(\text{Erfc}\left[\frac{1}{2\sqrt{\tau}}\right] - \text{Erfc}\left[\frac{\text{splus}[\tau]}{2\sqrt{\tau}}\right]\right) - \frac{e^{-\frac{\text{splus}[\tau]^2}{4\tau}}}{\text{splus}[\tau]}\right)\right)\text{splus}[\tau]}$$

$$\text{rhs} = \left(\frac{1-\xi}{\text{ste}}\right)\text{splus}'[\tau]$$

20. splus′[τ]

```
sol = NDSolve[{lhs == rhs, splus[0.001] == 1}, splus[τ], {τ, 0.01, 4}]
 (* this line gives solution of equation system (6) with initial
   condition S⁺(0.001)=1; the solution is stored in the variable 'sol'*)
```

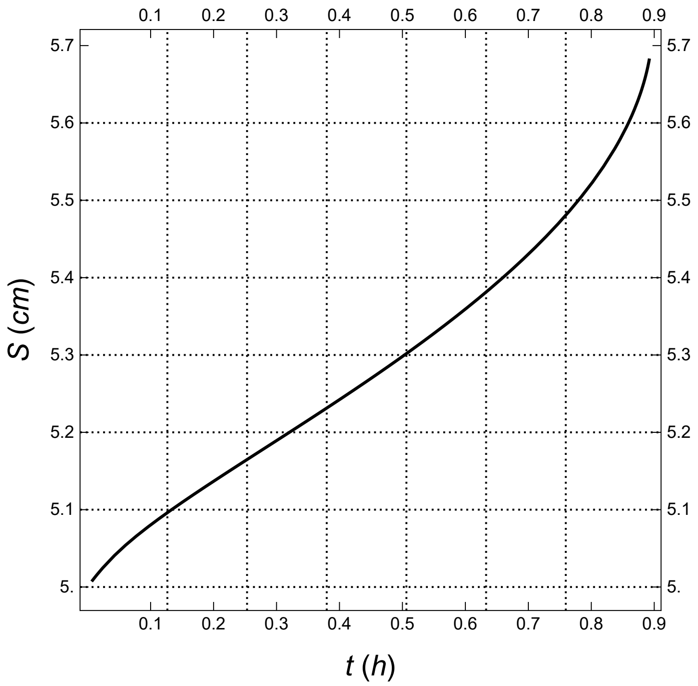
NDSolve::ndsz : At τ == 0.1418614794948729`, step size is effectively zero; singularity or stiff system suspected. >>

```
{{splus[τ] → InterpolatingFunction[ Domain: {{0.01, 0.142}} Output: scalar ][τ]}}
```

```
Plot[Evaluate[splus[τ] /. sol], {τ, 0.001, 0.141},
 PlotTheme → {"Detailed", "Monochrome"}, AspectRatio → 1,
 PlotLegends → None, FrameLabel → {Style["t (h)", 15], Style["S (cm)", 15]},
 FrameTicks →
  {Table[{0.0158(* τ = 0.158 <=> t = 1 h *) i, 0.1 i}, {i, 1, 200}],
   Table[{1 + 0.02 i, r * 100 * (1 + 0.02 i)}, {i, 0, 200}]}]
(* plotting the solution of the differential equation *)
```

InterpolatingFunction::dmval :

Input value {0.00100286} lies outside the range of data in the interpolating function. Extrapolation will be used. >>



```
(*

NOTES:

    - since Rayleigh number = 0 (i.e. Ra ≤ 5×10^4), keq is assumed equal to 1

    - the equation is solved with initial condition S+(0.001)=1,
i.e. the initial condition is set at τ=0.001.
    This is done because having τ=
  0 leads to the indeterminate form 1/0 appearing inside equation 6.1.

     - τ = 0.158 units corresponds to t = 1h,
τ = 0.316 units corresponds to t = 2h,
and so on. τ = 3.78947 units corresponds to t = 24 h.

*)
```