```
ClearAll["Global`*"];
(* This function clears any past definitions of the variables *)


(* THERMOPHYSICAL PROPERTIES OF THE PARAFFIN *)

Tm = 303; (* melting tempreratue of the paraffin *)
ρs = 880; ρl = 760; ρ = ρl; (* density at solid and liquid states *)
ks = 0.24;
kl = 0.15; (* thermal conductivity at solid and liquid states *)
cps = 2.4 * 10^3;
cpl = 1.8 * 10^3; (* specific heat at solid and liquid states *)
q = 179 * 10^3; (* latent heat of fusion*)
vd = 3.42;
vk = vd/ρ; (* dynamic viscosity (vd)
 and kinematic viscosity (vk) of paraffin *)


(* FORMULAS AND TRANSFORMATIONS*)

αs = ks/(ρs cps);
αl = kl/(ρl cpl); (* thermal diffusivity at solid and liquid states *)
pr = vk/αl; (* Prandtl number*)
gdot = (G kl (T0 - Tm))/R^2; (* heat sink parameter *)
ξ = gdot/(ρ q) (R^2 τ/αl); (* mass proportion of liquid in the mixture;
t taken as r^2τ/αl *)
T0 = Tm + (q ste)/cpl;(* temperature outside the sphere;
it depends on the stefan number desired for the simulation *)

γ = ks/kl;
Γ = αs/αl;
θi = (Ti - Tm)/(T0 - Tm); (* dimensionless initial temperature *)

λn = (n π)/splus;
```

```
keq = 1; (* equivalent thermal conductivity *)

(* since Rayleigh number = 0 (i.e. ≤ 5×10^4) for figure 3(c),
keq is assumed equal to 1 *)


(* PARAMETER VALUES *)

R = 5 * 10^-2; (* radius of the sphere *)
Ti = 295; (* initial temperature of the paraffin *)


(* PARAMETER VALUES SPECIFIC TO FIGURE 3(c) OF BECHIRI ET AL. 2020 *)

ste = 0.05; (* Stefan number *)
ra = 0; (* Rayleigh number *)
bi = 10; (* Biot number *)
G = 0; (* dimensionless heat sink parameter *)
```

$$\theta l = \left(-bi\left(\sqrt{keq\,\tau}\left(\frac{Exp\left[\frac{-\eta^2}{4\,keq\,\tau}\right]}{\eta} - \frac{Exp\left[\frac{-splus^2}{4\,keq\,\tau}\right]}{splus}\right) + \frac{\sqrt{\pi}}{2}\left(Erfc\left[\frac{splus}{2\sqrt{keq\,\tau}}\right] - Erfc\left[\frac{\eta}{2\sqrt{keq\,\tau}}\right]\right)\right)\right) \Big/ \left(keq\sqrt{keq\,\tau}\;Exp\left[\frac{-1}{4\,keq\,\tau}\right] - bi\left(\sqrt{keq\,\tau}\left(Exp\left[\frac{-1}{4\,keq\,\tau}\right] - \frac{1}{splus}\,Exp\left[\frac{-splus^2}{4\,keq\,\tau}\right]\right) + \frac{\sqrt{\pi}}{2}\left(Erfc\left[\frac{splus}{2\sqrt{keq\,\tau}}\right] - Erfc\left[\frac{1}{2\sqrt{keq\,\tau}}\right]\right)\right)\right) - \frac{G}{6\,keq}\left(splus^2 - \eta^2 + \left(\frac{splus}{\eta} - 1\right)\frac{bi\left(splus^2 - 1\right) - 2\,keq}{bi + splus\,(keq - bi)}\right)$$

$$-\left(\left(10\left(\left(-\frac{e^{-\frac{splus^2}{4\tau}}}{splus} + \frac{e^{-\frac{\eta^2}{4\tau}}}{\eta}\right)\sqrt{\tau} + \frac{1}{2}\sqrt{\pi}\left(Erfc\left[\frac{splus}{2\sqrt{\tau}}\right] - Erfc\left[\frac{\eta}{2\sqrt{\tau}}\right]\right)\right)\right) \Big/ \left(e^{-\frac{1}{4}/\tau}\sqrt{\tau} - 10\left(\left(e^{-\frac{1}{4}/\tau} - \frac{e^{-\frac{splus^2}{4\tau}}}{splus}\right)\sqrt{\tau} + \frac{1}{2}\sqrt{\pi}\left(-Erfc\left[\frac{1}{2\sqrt{\tau}}\right] + Erfc\left[\frac{splus}{2\sqrt{\tau}}\right]\right)\right)\right)\right)$$

$$\theta s = -\frac{G}{6\,\gamma}\left(splus^2 - \eta^2\right) + 2\,Sum\left[\frac{(-1)^{n+1}}{\lambda n^2}\left(\theta i + \frac{G}{\lambda n^2\,\gamma}\right)\frac{Sin[\lambda n\,\eta]}{\eta}\,Exp\left[-\lambda n^2\,\Gamma\,\tau\right],\ \{n, 1, 20\}\right];$$

```
τ = αl t / R^2; splus = s / R; η = r / R;

data3c = {{303.712036, 0.049948025}, {6782.902137, 0.04464657},
   {13 667.04162, 0.04043659}, {20 449.94376, 0.037006237},
   {27 334.08324, 0.033835759}, {34 116.98538, 0.030925156},
   {41 001.12486, 0.028066528}, {47 784.027, 0.025259875},
   {54 668.16648, 0.022401247}, {61 552.30596, 0.019386694},
   {68 335.2081, 0.016216216}, {75 219.34758, 0.012577963},
   {82 002.24972, 0.008056133}, {87 165.35433, 0}};
s = Fit[data3c, {1, t, t^2, t^3, t^4, t^5, t^6, t^7, t^8, t^9, t^10}, t];
sFunc[t_] = s;

Tl[r_, t_] = θl (T0 - Tm) + Tm; Ts[r_, t_] = θs (T0 - Tm) + Tm;
(* transforming back *)
```

```
Manipulate[Plot[
  {
   Piecewise[{{Ts[r, t], 0 < r < sFunc[t]}, {Tl[r, t], r > sFunc[t]}}],
   100 sFunc[t] Sqrt[1 - (r/sFunc[t])^2] + Tm, -100 sFunc[t] Sqrt[1 - (r/sFunc[t])^2] + Tm
  },
  {r, -R, R}, PlotTheme → {"Detailed"}, AspectRatio → 1,
  PlotRange → {298, 308}, PlotLegends → None, PlotStyle → {Red, Blue, Blue},
  FillingStyle → {White}, Filling → {2 → {3}},
  Prolog → {LightBlue, Scaled /@ Rectangle[{0, 0}, {1, 1}]}

 ], {t, 0, 24.2 * 60 * 60}]
```

```
t

Plot[{ { Ts[r, FE`t$$1]  0 < r < sFunc[FE`t$$1]
         Tl[r, FE`t$$1]  r > sFunc[FE`t$$1]     ,
  (100 sFunc[FE`t$$1]) Sqrt[1 - (r/sFunc[FE`t$$1])^2] + Tm,
  (-100 sFunc[FE`t$$1]) Sqrt[1 - (r/sFunc[FE`t$$1])^2] + Tm}, {r, -R, R},
 PlotTheme → {Detailed}, AspectRatio → 1, PlotRange → {298, 308},
 PlotLegends → None, PlotStyle → {Red, Blue, Blue},
 FillingStyle → {White}, Filling → {2 → {3}},
 Prolog → {LightBlue, Scaled /@ Rectangle[{0, 0}, {1, 1}]}]
```

Plot::plln : Limiting value −R in {r, −R, R} is not a machine−sized real number. >>

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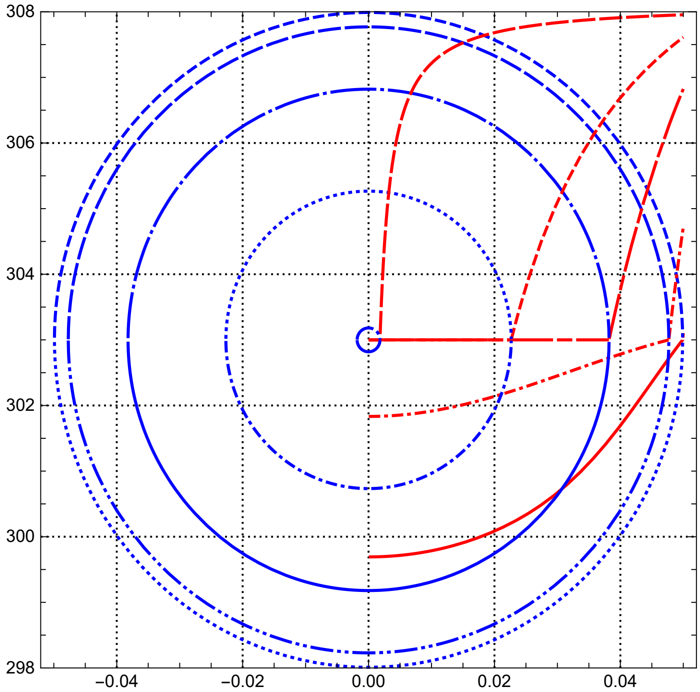
```
t15 = 15 * 60 * 60;
t05 = 5 * 60 * 60;
t24 = 24 * 60 * 60;
tp8 = 0.8 * 60 * 60;
tp1 = 0.1 * 60 * 60;
Plot[
 {
  Piecewise[{{Ts[r, tp1], 0 < r < sFunc[tp1]}, {Tl[r, tp1], r > sFunc[tp1]}}],
  100 sFunc[tp1] Sqrt[1 - (r/sFunc[tp1])^2] + Tm,
  -100 sFunc[tp1] Sqrt[1 - (r/sFunc[tp1])^2] + Tm,
  Piecewise[{{Ts[r, tp8], 0 < r < sFunc[tp8]}, {Tl[r, tp8], r > sFunc[tp8]}}],
  100 sFunc[tp8] Sqrt[1 - (r/sFunc[tp8])^2] + Tm,
  -100 sFunc[tp8] Sqrt[1 - (r/sFunc[tp8])^2] + Tm,
  Piecewise[{{Ts[r, t05], 0 < r < sFunc[t05]}, {Tl[r, t05], r > sFunc[t05]}}],
  100 sFunc[t05] Sqrt[1 - (r/sFunc[t05])^2] + Tm,
  -100 sFunc[t05] Sqrt[1 - (r/sFunc[t05])^2] + Tm,
  Piecewise[{{Ts[r, t15], 0 < r < sFunc[t15]}, {Tl[r, t15], r > sFunc[t15]}}],
  100 sFunc[t15] Sqrt[1 - (r/sFunc[t15])^2] + Tm,
  -100 sFunc[t15] Sqrt[1 - (r/sFunc[t15])^2] + Tm,
  Piecewise[{{Ts[r, t24], 0 < r < sFunc[t24]}, {Tl[r, t24], r > sFunc[t24]}}],
  100 sFunc[t24] Sqrt[1 - (r/sFunc[t24])^2] + Tm, -100 sFunc[t24] Sqrt[1 - (r/sFunc[t24])^2] + Tm
 },
 {r, -R, R}, PlotTheme → {"Detailed", "Monochrome", "CoolColor"}, AspectRatio → 1,
 PlotLegends → None, PlotRange → {298, 308}, PlotStyle → {Red, Blue, Blue}]
```



308
306
304
302
300
298
−0.04
−0.02
0.00
0.02
0.04