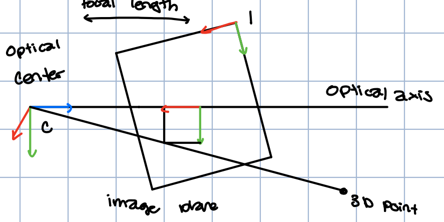
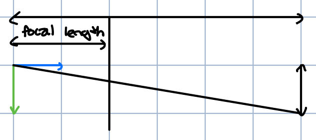
{ Cyrill Stachniss

Geometry: Pinhole camera





side view

Camera intrinsics & extrinsics
↳ parameters of camera model
↳ extrinsics → where is my camera IRL XYZ → 6 dof
↳ point according to pinhole projection ↗
↳ intrinsics → parameters inside camera → assuming camera sits in origin → how point 3D mapped on image
↳ camera constant - c
↳ scale difference x, y (focal length x, y)
↳ principal point → where
↳ Direct Linear Transformation DLT → assumes perf lense

Position - (u^c_m, v^c_m) of projection p^c on camera plane

$$u^c_m = f \frac{P_x^c}{P_z^c} \qquad v^c_m = f \frac{P_y^c}{P_z^c}$$

f - focal length

m - meters

$$P_z^c \begin{bmatrix} u^c_m \\ v^c_m \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} P_x^c \\ P_y^c \\ P_z^c \end{bmatrix}$$

$$P_z^c = \begin{bmatrix} P_x^c \\ P_y^c \\ P_z^c \end{bmatrix} \xrightarrow{\text{homogeneous coordinates}} p^{\sim c} = \begin{bmatrix} P_x^c \\ P_y^c \\ P_z^c \\ 1 \end{bmatrix}$$

$$P_z^c = \begin{bmatrix} u^c_m \\ v^c_m \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} [I_3 \; 0_3] \, \tilde{p}^c$$

conversion to pixels

$$u^I = S_x u^c_m + O_x$$

$$v^I = S_y v^c_m + O_y$$

$$\begin{bmatrix} u^I \\ v^I \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & O_x \\ 0 & S_y & O_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u^c_m \\ v^c_m \\ 1 \end{bmatrix}$$

if pixels skewed:

$$\begin{bmatrix} u^I \\ v^I \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & S_\theta & O_x \\ 0 & S_y & O_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u^c_m \\ v^c_m \\ 1 \end{bmatrix}$$

$$P_z^c \begin{bmatrix} u^I \\ v^I \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & S_\theta & O_x \\ 0 & S_y & O_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} [I_3 \; 0_3] \, \tilde{p}^c$$

$$P_z^c \begin{bmatrix} u^I \\ v^I \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} S_xf & S_\theta f & O_x \\ 0 & S_yf & O_y \\ 0 & 0 & 1 \end{bmatrix}}_{K} \underbrace{[I_3 \; 0_3]}_{\Pi_0} \, p^{\sim c}$$

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$0_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

point coordinates in world frame

$$P_z^c \begin{bmatrix} u^I \\ v^I \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} S_xf & S_\theta f & O_x \\ 0 & S_yf & O_y \\ 0 & 0 & 1 \end{bmatrix}}_{K} \underbrace{[I_3 \; 0_3]}_{\Pi_0} T^c_w \, \tilde{p}^w$$

$[O_x \; O_y]$ - principal point

S_xf - focal length horizontal pixels

S_yf - focal length vertical pixels

S_x, S_y - aspect ratio

$S_\theta f$ - skew of pixel

K - intrinsic calibration matrix

Π_0 - canonical projection

p^w - point in world

Π - projection matrix

$[R_w^c \; t_w^c]$ - extrinsic callibration matrix

1) $P_c = [10, 5, 20]$ (x y z) $\quad f = 0.05$

$$u_m^c = f \cdot \frac{P_x^c}{P_z^c} \qquad v_m^c = f \cdot \frac{P_y^c}{P_z^c}$$

$$u_m^c = 0.05 \cdot \frac{10}{20} \qquad v_m^c = 0.05 \cdot \frac{5}{20}$$

$$= 0.025 \, m \qquad = 0.0125 \, m$$

2) $S_x = 800 \frac{pix}{m}$

$S_y = 800 \frac{pix}{m}$

$O_x = 320 \, pix$

$O_y = 240 \, pix$

$$u^I = S_x u_m + O_x = 800 \cdot 0.025 + 320 = 340$$

$$v^I = S_y v_m^c + O_y = 800 \cdot 0.0125 + 240 = 250$$

3) $S_\theta = 0$

$$K = \begin{bmatrix} S_xf & S_\theta f & O_x \\ 0 & S_yf & O_y \\ 0 & 0 & 1 \end{bmatrix}$$

$$K = \begin{bmatrix} 40 & 0 & 320 \\ 0 & 40 & 240 \\ 0 & 0 & 1 \end{bmatrix}$$

$S_xf = S_x \cdot f = 800 \cdot 0.05 = 40$

$S_yf = S_y \cdot f = 800 \cdot 0.05 = 40$

$S_\theta f = S_\theta \cdot f = 0 \cdot 0.05 = 0$

4) $P_w = [2, 3, 50]$ (x y z)

$$T^c_w = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$t_w^c = [0, 0, 10]$

$$P_z^c \begin{bmatrix} u^I \\ v^I \\ 1 \end{bmatrix} = K \, [R_w^c \; t_w^c]$$

$$R_w^c = I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad t_w^c = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}$$

$$R_w^c \, t_w^c = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \end{bmatrix}$$

$$= \begin{bmatrix} 40 & 0 & 320 \\ 0 & 40 & 240 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \end{bmatrix}$$

$$p^w + t_w^c = [2, 3, 50] + [0, 0, 10]$$

$$= [2, 3, 60] \quad (x \; y \; z)$$

$$P_z^c \begin{bmatrix} u^I \\ v^I \\ 1 \end{bmatrix} = \begin{bmatrix} S_xf & S_\theta f & O_x \\ 0 & S_yf & O_y \\ 0 & 0 & 1 \end{bmatrix}$$

$f = 1 \quad \rightarrow \quad u_m = \frac{P_x}{P_z} \qquad v_m = \frac{P_y}{P_z}$

$$u_m = S_x \frac{P_x}{P_z} + O_x = 40 \cdot \frac{2}{60} + 320 = 321.\overline{3}$$

$$v_m = S_y \frac{P_y}{P_z} + O_y = 40 \cdot \frac{3}{60} + 240 = 242$$

5) $u^I_{distort}, \; v^I_{distort} = 400, 350$

$a_1 = 0.0001 \quad a_2 = 0.000\,000\,2$

$O_x, \; O_y = 320, 240$

$$r^2 = (u^I_{distort} - O_x)^2 + (v^I_{distort} - O_y)^2$$

$$= (400 - 320)^2 + (350 - 240)^2 = 18{,}500$$

$$r^4 = 342{,}250{,}000$$

$$u^I = (1 + a_1 r^2 + a_2 r^4)(u^I_{distort} - O_x) + O_x = 6024$$

$$v^I = (1 + a_1 r^2 + a_2 r^4)(v^I_{distort} - O_y) + O_y = 8083$$

$$P_z^c \begin{bmatrix} u^I \\ v^I \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} S_x f & S_\theta f & O_x \\ 0 & S_y f & O_y \\ 0 & 0 & 1 \end{bmatrix}}_{K} \underbrace{\begin{bmatrix} R_w^c & t_w^c \end{bmatrix}}_{\text{extrinsic callibration}} \qquad \tilde{p}^w = \Pi \tilde{p}^w$$

Projection in Practice

computing pixel projection of a 3D point if pose of camera T_w^c is known

$$u^I = \frac{[\Pi \tilde{p}^w]_1}{[\Pi \tilde{p}^w]_3} \qquad v^I = \frac{[\Pi \tilde{p}^w]_2}{[\Pi \tilde{p}^w]_3}$$

$[\Pi \tilde{p}^w]_i$ - denotes the i^{th} entry

cameras w/ wide field of view $\rightarrow$ radial distortion

$$u^c = (1 + a_1 r^2 + a_2 r^4)\, u^c_{distort} \qquad r^2 = (u^c_{distort})^2 + (v^c_{distort})^2$$

$$v^c = (1 + a_1 r^2 + a_2 r^4)\, v^c_{distort} \qquad a_i = \text{distortion coefs}$$

model using image frame $\qquad r^2 = (u^I_{distort} - O_x)^2 + (v^I_{distort} - O_y)^2$

$$u^I = (1 + a_1 r^2 + a_2 r^4)(u^I_{distort} - O_x) + O_x$$

$$v^I = (1 + a_1 r^2 + a_2 r^4)(v^I_{distort} - O_y) + O_y$$