Introduction

Let's introduce a geometrical optimization problem, named **cones problem**, with the following characteristics:

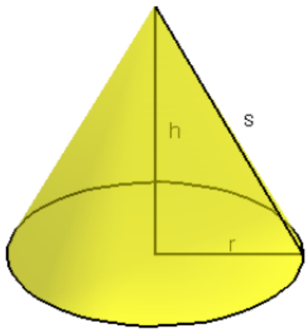
- **multi-objective** problem (two objective functions): the solution is not a single optimum design, but instead it is represented by the set of designs belonging to the *Pareto frontier*
- **constrained** problem: objectives space and designs space present *feasible* and *unfeasible* regions

Problem definition

Right circular cone:

r = base radius
h = height
s = slant height

V = volume
B = base area
S = lateral surface area
T = total area



$$s = \sqrt{r^2 + h^2}$$

$$V = \frac{\pi}{3}\, r^2\, h$$

$$B = \pi\, r^2$$
$$S = \pi\, r\, s$$
$$T = B + S = \pi\, r\,(r + s)$$

Cones problem

- two input variables: r, h

$$r \in [0, 10]\ \text{cm}\ , \quad h \in [0, 20]\ \text{cm}$$

The cone shape (i.e. the design) is defined univocally when both *r* and *h* are given.

- two objectives:

$$\min S$$
$$\min T$$

We want to minimize both the lateral surface area and the total surface area

- one constraint:

$$V > 200\ \text{cm}^3$$

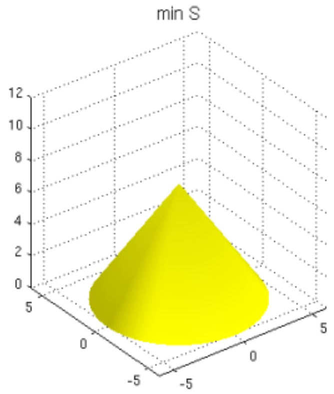
A constraint for the cone volume is given, in order to guarantee a minimum volume.

Final considerations

Let's consider the difference between

- single-objective problem solutions: two different minima
- multi-objective problem solutions: the Pareto frontier

Single-objectives minima



$\min S$
$r = 5.131$ cm
$h = 7.256$ cm
$V = 200$ cm^3
$S = 143.23$ cm^2
$T = 225.92$ cm^2

Each design represents the optimum solution for its corresponding single-objective problem.

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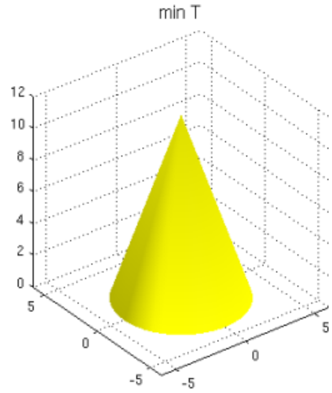
...but what about the in between designs?

...we would like to get a compromise solution. A trade-off of the two objectives...

What we want is the **Pareto frontier**!



$\min T$
$r = 4.072$ cm
$h = 11.518$ cm
$V = 200$ cm^3
$S = 156.28$ cm^2
$T = 208.38$ cm^2

The Pareto frontier