Local Antimagic Labelling on Broom Graph

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Contents

Definitions

Local Antimagic Labelling

Let $G = (V, E)$ be a connected graph with $|V| = n$ and $|E| = m$. A bijection $f\colon E \to \{1, 2,.., m\}$ is called a local antimagic labeling if for any two adjacent vertex $w(u) \neq w(u)$, where $w(u) = \sum_{e \in E(u)} f(e)$ and $E(u)$ is the set of edges incident to u. Thus, any local antimagic labeling induces a proper vertex coloring of G where the vertex v is assigned the color $w(v)$. [Hartsfield and Ringel]

Local Antimagic Chromatic Index

For a vertex $x \in V$, define weight $w(x)$ as a sum from the label of the edges that is adjacent with the vertex x.

Local antimagic chromatic index ($\chi'_{la}(G)$) is defined as the minimum number of colors taken over all colorings of G induced by local antimagic labelings of G.

[Alison Marr]

Broom Graph

A Broom Graph $B_{n,d}$ is a graph of n vertices, which have a path P with d vertices and $(n - d)$ pendant, all of these being adjacent to the end of path P.

[M.J. Morgan, S. Mukwembi, H.C.]

General Form of Broom Graph

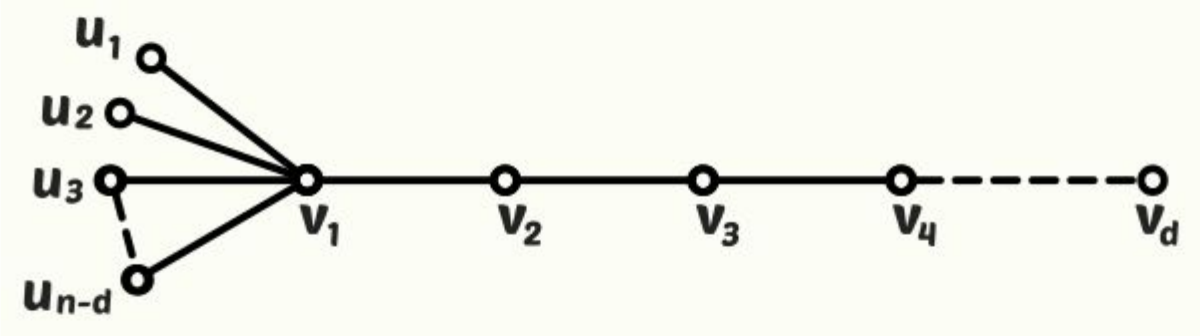


Figure of Broom Graph $B_{n,d}$

Hypothesis

Broom graph $B_{n,d}$ **with** $n \geq 5$ **and** $d \geq 3$ **have** $\chi_{la}(B_{n,d}) = n - d + 2$.

Proofing

Broom Graph $B_{n,d}$ with $n \geq 5$ and $d \geq 3$ where the vertices set of is

$$V = \{v_1\} \cup \{v_i | 2 \leq i \leq d - 1\} \cup \{u_j | 1 \leq j \leq n - d\}$$

and the set of edges is

$$E = \{v_i v_{i+1} | 2 \leq i \leq d - 1\} \cup \{v_1 u_j | 1 \leq j \leq n - d\}$$

Labeling Edges: With Local Antimagic

We will do labeling the edges of the graph (the broom graph) $B_{n,d}$ with using bijection function $f: E \to \{1, 2, n - 1\}$.

f defined as local antimagic labeling, and we get:

$$f(v_i u_j) = n - j$$

$$f(v_i v_{i+1}) = \begin{cases} \dfrac{d+i}{2}, & for\ d\ even\ and\ i\ odd, or\ d\ odd\ and\ i\ even \\ \dfrac{d-i+1}{2}, & for\ d\ even\ and\ i\ even, or\ d\ odd\ and\ i\ odd \end{cases}$$

Labeling Vertices: With Local Antimagic

w **defined as weight of the vertices, and we get:**

with: $i: 1, 2, \dots, d-1$, $j: 1, 2, \dots, n-d$

$$w(v_1) = \begin{cases} \dfrac{n^2 - d^2 - n + 2d + 1}{2}, & for\ d\ odd \\ \dfrac{n^2 - d^2 - n + 2d}{2}, & for\ d\ even \end{cases}$$

$$w(v_i) = \begin{cases} d, & for\ d\ even\ and\ i\ odd, or\ d\ odd\ and\ i\ even \\ \dfrac{d+1}{2}, & for\ d\ even\ and\ i\ even, or\ d\ odd\ and\ i\ odd \end{cases}$$

$$w(v_d) = 1$$

$$w(u_j) = n - j$$

Finding: Upper Bound

For $j = n - d$ **we will get:**

$$w\left(u_{n-d}\right) = (n - d) + d - (n - d) = d$$

For $j = n - d - 1$ **we will get:**

$$w\left(u_{n-d-1}\right) = (n - d) + d - (n - d - 1) = d + 1$$

So, $w\left(v_i\right)$ **will produce same color with** $w\left(u_{n-d}\right)$ **and** $w\left(u_{n-d-1}\right)$

Finding: Upper Bound

$w\left(u_j\right)$ **will produce** $n - d$ **different color.**

$w\left(v_1\right)$ **will produce** 1 **different color.**

$w\left(v_d\right)$ **will produce 1 different color.**

We will have $(n - d) + 1 + 1 = n - d + 2$ **different color, so**
$\chi_{la}(B_{n,d}) \leq n - d + 2$

Finding: Lower Bound

We can use the theorem "For any tree T with l leaves, $\chi_{la}(G) \geq l + 1$".

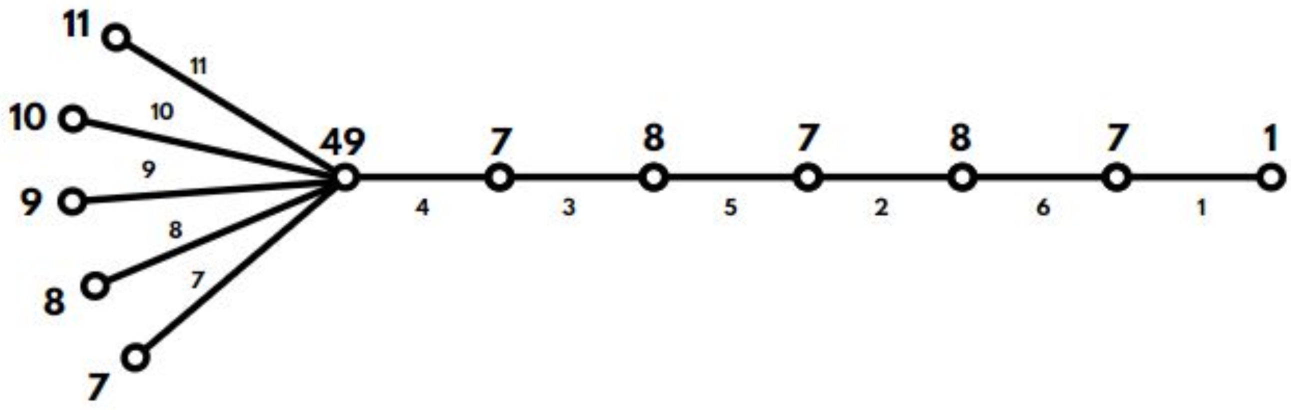
We know that graph $B_{n,d}$ has $n - d + 1$ leaves, so $l = n - d + 1$.

According to the theorem, we get $\chi_{la}(B_{n,d}) \geq n - d + 2$.

By finding upper and lower bound its proving Broom graph $B_{n,d}$ **with** $n \geq 5$ **and** $d \geq 3$ **have** $\chi_{la}(B_{n,d}) = n - d + 2$

Example

Local antimagic labeling on $B_{12,7}$ with $\chi_{la}(B_{12,7}) = 7$



Edge Labelling

$$f(v_1u_1) = 11 = 12 - 1 = n - j$$

$$f(v_1u_2) = 10 = 12 - 2 = n - j$$

$$f(v_1u_3) = 9 = 12 - 3 = n - j$$

$$f(v_1u_4) = 8 = 12 - 4 = n - j$$

$$f(v_1u_5) = 7 = 12 - 5 = n - j$$

$$f(v_1v_2) = \frac{d+i}{2} = \frac{7+1}{2} = 4$$

$$f(v_2v_3) = \frac{d-i+1}{2} = \frac{7-2+1}{2} = 3$$

$$f(v_3v_4) = \frac{d+i}{2} = \frac{7+3}{2} = 5$$

$$f(v_4v_5) = \frac{d-i+1}{2} = \frac{7-4+1}{2} = 2$$

$$f(v_5v_6) = \frac{d+i}{2} = \frac{7+5}{2} = 6$$

$$f(v_6v_7) = \frac{d-i+1}{2} = \frac{7-6+1}{2} = 1$$

Vertex Labeling

$$w(u_1) = n - j = 12 - 1 = 11$$

$$w(u_2) = n - j = 12 - 2 = 10$$

$$w(u_3) = n - j = 12 - 3 = 9$$

$$w(u_4) = n - j = 12 - 4 = 8$$

$$w(u_5) = n - j = 12 - 5 = 7$$

$$w(v_1) = \frac{n^2 - d^2 - n + 2d + 1}{2} = \frac{12^2 - 7^2 - 12 + 2(7) + 1}{2} = 49$$

$$w(v_i) = \begin{cases} d = 7 & i = 3,5 \\ d + 1 = 8 & i = 2,4,6 \end{cases}$$

$$w(v_7) = 1$$

Lower Bound

There are 7 colors needed which are 1,7,8,9,10,11, and 49. So $\chi_{la}(B_{12,7}) \leq 7$

Upper Bound

There are 6 leaf node which makes the upper bound $\chi_{la}(B_{12,7}) \geq 7$

Conclusuion

By $\chi_{la}(B_{12,7}) \leq 7$ **and** $\chi_{la}(B_{12,7}) \geq 7$**, or in other way** $7 \leq \chi_{la}(B_{12,7}) \leq 7$

we get $\chi_{la}(B_{12,7}) = 7$

Thank

You!

Bibliography

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Arumugam, S, K. Premalatha, Martin Bača, Andrea Semaničová-Fenovčíková. 2017. Local Antimagic Vertex Coloring of a Graph. Japan: Springer