Vrije Universiteit Amsterdam

Group Assignment

Advanced Econometrics 2024

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Contents

1 Quality Assurance: equipment failure times

Basic Plots and Descriptive Statistics

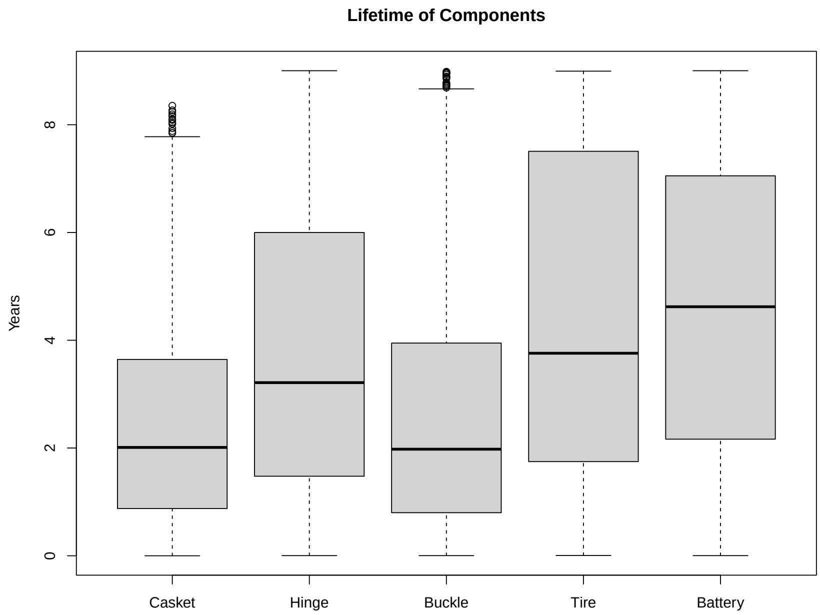
This section shows some basic plots and descriptive statistics of the data.

Table 1: Number of times different components failed

Component	Battery	Buckle	Casket	Hinge	Tire
Number	1381	2402	2598	1712	1497

This table shows the number of times the different components failed and were replaced.

Figure 1: Box-plot of the Lifetime of the different Components



This figure shows a box-plot of the lifetime in years of the different components.

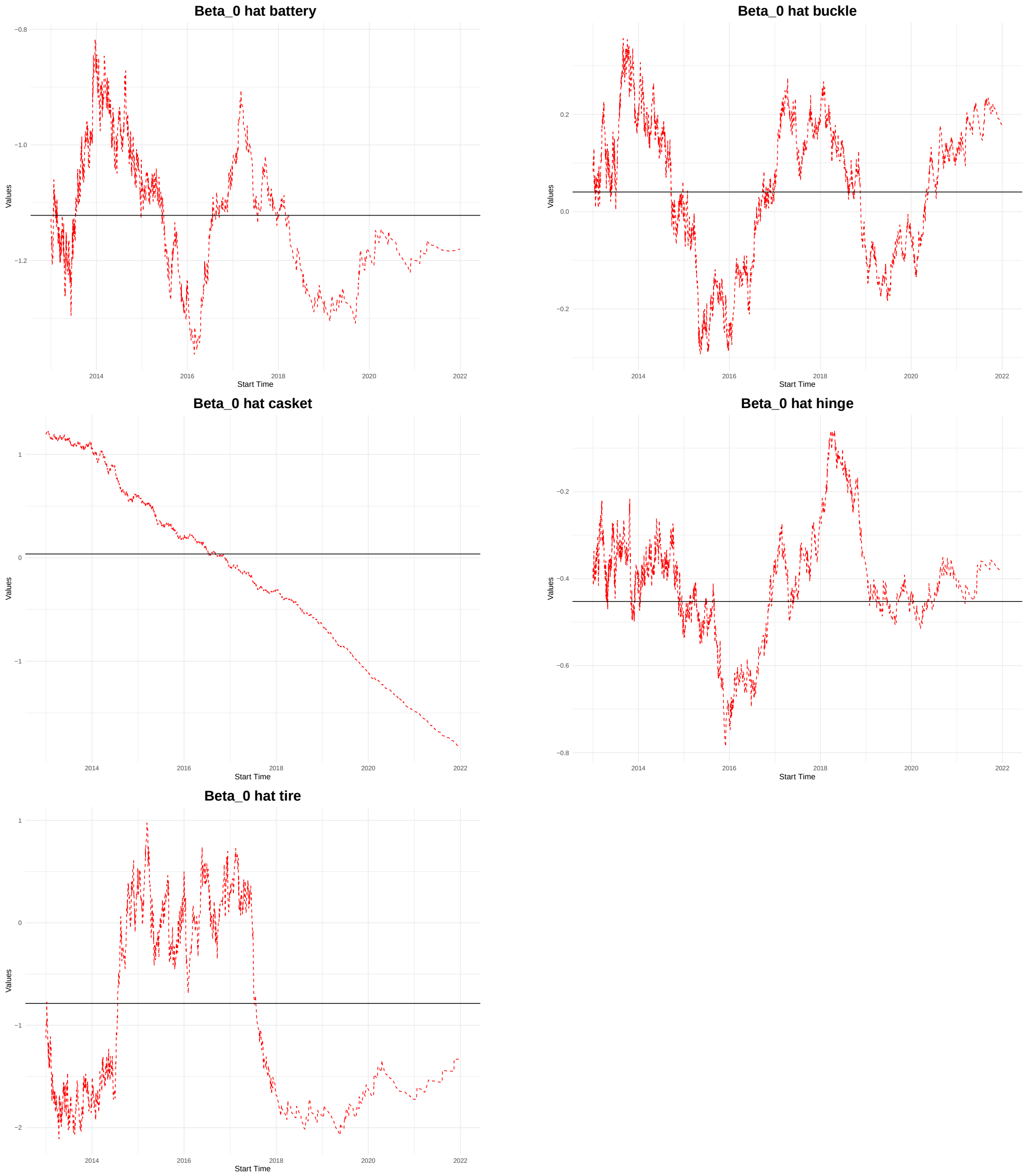
Questions 1.1, 1.2, 1.3 and 1.4

See the attached excel file in Canvas for the solutions of questions 1, 2, 3 and 4.

Question 1.5

Figure 2 shows plots of maximum likelihood estimates for the five different components. To explain what an increase/decrease of the red dashed line means, it is important to know what increase/decrease of $\hat{\beta}_{0,s}$ means. An increase (decrease) in $\hat{\beta}_{0,s}$ for a component indicates a higher (lower) rate of failures. This implies a deteriorating (improving) quality of the component.
When looking at the component "casket", it is observed that this component has suffered a trending pattern in quality. Namely, the ML estimate $\hat{\beta}_{0,s}$ of this component has linearly

Figure 2: Plots of maximum likelihood estimates for the different components



The figures above show plots of the maximum likelihood estimates for the different components. The start time is shown on the horizontal axis, while the vertical axis shows values for the ML estimates. The solid black line shows the ML estimate of $\hat{\beta}_{0,j}$ and the red dashed line shows the ML estimate of $\hat{\beta}_{0,s}$.

decreased over time, as can be seen from Figure 2. As the ML estimate has decreased over time, this means that the quality of the component has increased.
When looking at the component “tire” in Figure 2, we observe that this component has suffered a temporary break in quality, as the ML estimate of this component increased suddenly halfway through 2014 from -1.5 to approximately 1. Then, around halfway through 2017, the ML estimate drops again from 1 to approximately -2. Although the components “battery”, “buckle” and “hinge” also vary in their ML estimates, these do not show a sudden increase and decrease. As the ML estimate suddenly increased and thereafter decreased, it means that the quality of the “tire” temporarily decreased. Thus, the department manager was right in his suspicion that the quality of the different suppliers was inferior.

Question 1.6

Table 2 compares the Log Likelihoods of questions 2 and 4. From the Table it is observed that all components have an increase in Log Likelihood. However, the components “casket” and “tire” have the strongest increase. This indicates a change in supplier quality and coincides with the results found in question 1.5, which results indicate that these two components had a shift in quality.

Table 2: Log Likelihoods Comparison

Component	**Log Likelihood**	
	Q1.2	**Q1.4**
Battery	-1695.241	-1691.946
Buckle	-3565.504	-3547.506
Casket	**-3855.981**	**-3588.343**
Hinge	-2416.535	-2410.431
Tire	**-1971.385**	**-1725.961**

This table shows a comparison of the Log Likelihoods of the different components for questions 2 and 4.

2 Finance: Returns and volatilities

2.1 Univariate volatility modeling

Question 2.1

To analyze the impact of shocks on conditional volatility, the news-impact curves of the Robust-GARCH-with-Leverage-Effect model were examined using varying values of λ's and δ's. We used the following predetermined parameter values: $(\sigma^2_{t-1}, \omega\ , \alpha\ , \beta) = (1, 0\ , 0.05, 0.9)$. Figure 3 displays four plots, each corresponding to a specific δ, with all four λ values indicated in the legend. The plot in the upper left corner, where δ equals 0, reveals symmetry for each λ. In contrast, the other news-impact curves, indicating the model with a leverage effect ($\delta \neq 0$), are asymmetric.

The theory suggests that the leverage effect implies that negative returns have a more significant impact on volatility than positive returns. Figure 3 further illustrates that a higher δ value leads to increased volatility. Moreover, as λ rises, the model becomes more sensitive to outliers.

Comparing with a standard GARCH model, the curve in the upper right corner, where δ equals 0 and λ equals 0, demonstrates that larger return values (x) result in higher future volatility compared to other curves. Consequently, for λ equals 0 and an increasing δ, the volatility decreases for positive returns.

For other lambda values (λ equals 0.01, 0.1, and 1), the news-impact curve shows reduced volatilities for positive returns (large values of x), similar to the robust GARCH model. As λ approaches zero, the model resembles the Quadratic GARCH model. If λ tends to zero and δ approaches zero, the model resembles a standard GARCH model.

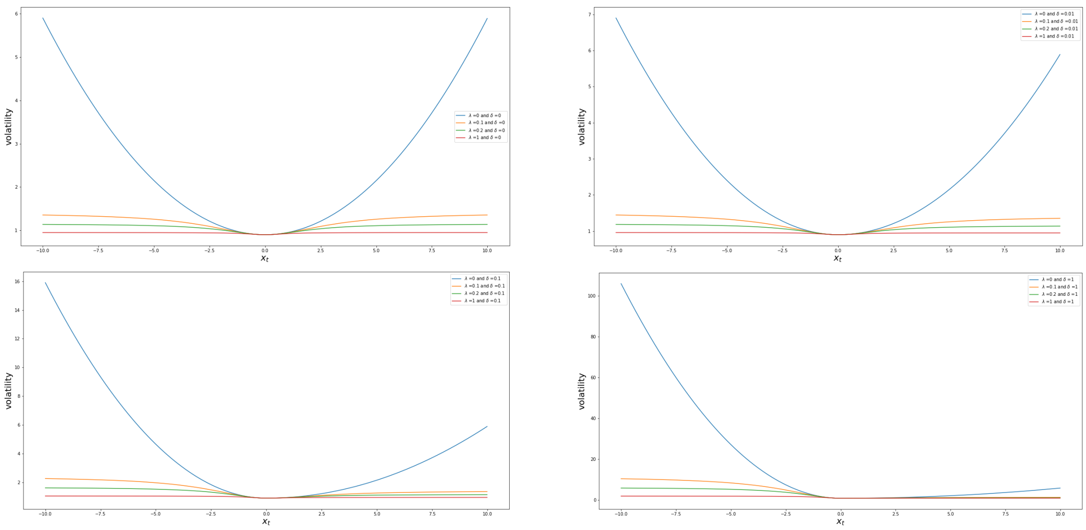


Figure 3: News-impact-curves of Robust-GARCH-with-Leverage-effect model for $\delta = (0, 0.01, 0.1, 1)$ and $\lambda = (0, 0.01, 0.1, 1)$

Question 2.2

Descriptive Statistic	**SM**	**KO**	**GM**	**TSLA**
Mean	0.000	0.000	0.000	0.000
Median	-0.0157	0.008	-0.0008	-0.0834
Standard Deviation	1.3711	1.1149	2.1813	3.5503
Skewness	0.1004	-0.6390	0.1320	0.3641
Excess Kurtosis	3.1912	9.1719	7.1824	5.1761
Minimum Value	-8.6574	-9.7128	-17.3522	-21.2659
Maximum Value	9.5430	6.4392	19.9138	24.1998

Table 3: The descriptive statistics of the data

In Table 3 above, the descriptive statistics for the returns of the four stocks are presented. One striking observation is that all stocks demonstrate mean returns close to zero, a common characteristic in financial time series data. However, there are variations in the median values among the stocks, with Coca Cola being the only stock with a positive median. The standard deviation for all stocks falls within the range of 1.1 to 3.6. Among them, Tesla has the highest volatility, while Coca Cola displays the lowest volatility.
Additionally, the skewness of the returns is slightly positive for all stocks except Coca Cola, suggesting that the returns tend to be slightly more positive than negative in our data. A kurtosis value greater than 3 indicates heavier tails in the data compared to a normal distribution. This implies that investors can expect more extreme returns, either positive or negative.
Notably, KO stands out with a negative skewness, indicating a greater tendency for negative returns, and a kurtosis of 9.1719, which is three times higher than the kurtosis of a normal distribution. This information indicates that outliers are expected to occur more frequently in the data for Coca Cola.

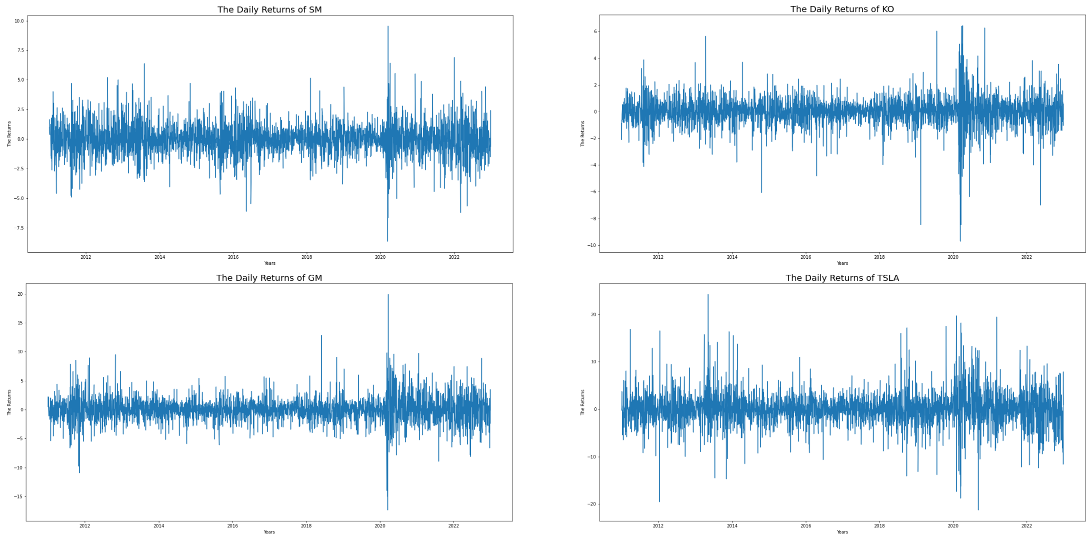


Figure 4: Plots of the demeaned and scaled holding period returns for SM (Spectra Motors), KO (Coca Cola), GM (General Motors) and TSLA (Tesla)

The observation from Figure 4 is evident: substantial returns fluctuations are typically succeeded by significant subsequent changes, while minor returns shifts are succeeded by modest adjustments, indicating a pattern of volatility clustering. In the case of KO, the most significant fluctuations, both positive and negative, occurred in 2020, totalling almost 10%. Similarly, Spectra Motors and General Motors experienced their most substantial gains and losses in 2020. Tesla, on the other hand, saw its highest returns in 2013 and its lowest in 2020, both exceeding 20%.

Question 2.3

In Figure 5, the news-impact-curves of the three models: non-robust GARCH, Robust GARCH without leverage and Robust GARCH with leverage of Spectra Motors are shown. The estimated parameters are stored in table 4. The non-robust model suggests that volatility reacts symmetrically to positive and negative shocks. The robust GARCH without leverage model lies below the blue line. It is less sensitive to both positive and negative shocks compared to the non-robust GARCH model. Finally, the robust GARCH with leverage clearly shows the asymmetric behavior we often observe in financial markets, where bad news increases volatility more than good news. Based on the plot, a robust GARCH without leverage model seems to be dampening the effects of large shocks, both positive and negative. This might indicate that for Spectra returns, robust filtering might not be crucial for capturing the dynamics of Spectra returns. The fact that the robust GARCH with leverage curve is significantly different from the other two, especially for negative returns, implies that the leverage effect might be present in Spectra returns. Negative shocks appear to have a more pronounced effect on volatility than positive ones.

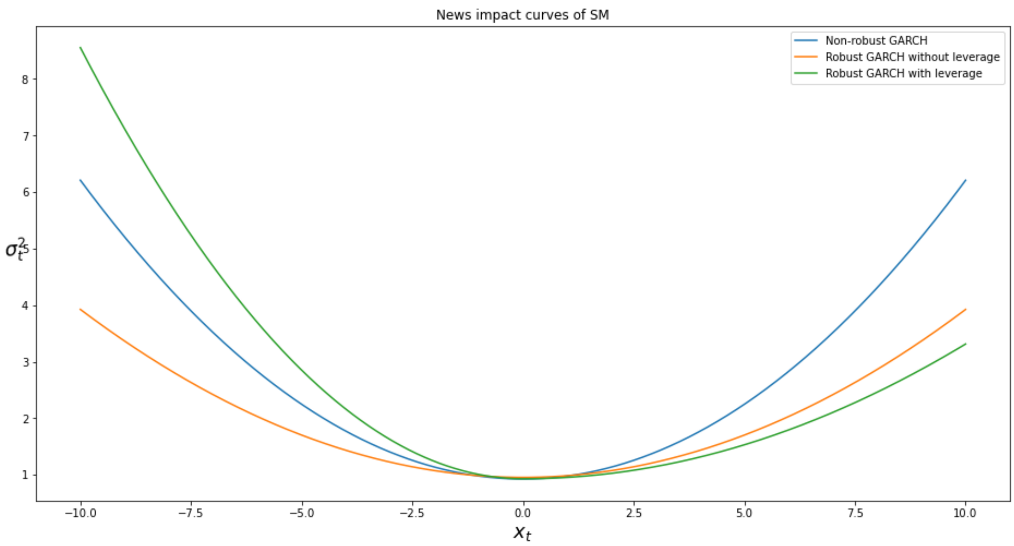


Figure 5: news-implied-curves of Spectra Motors

Question 2.4

In this question, the first 2500 observations are also utilized. Employing maximum likelihood estimation (MLE), we estimate the parameters for the data of all three stocks. Two distinct estimations are made: one where the parameters δ and λ are considered, and another where both δ and λ are set to 0. Additionally, apart from the estimated parameters, their corresponding standard errors are derived. These results, along with the total log-likelihood, AIC, and BIC values, are shown in Table 4 below.

	SM		**GM**		**TSLA**	
	no leverage	with leverage	no leverage	with leverage	no leverage	with leverage
ω	0.026	0.023	0.029	0.032	0.071	0.081
	0.013	0.009	0.009	0.011	nan	0.039
α	0.0528	0.0240	0.0285	0.0497	0.0164	0.0338
	0.013	0.008	0.000	0.024	nan	0.014
β	0.9008	0.9078	0.9377	0.9014	0.9515	0.9272
	0.028	0.022	0.000	0.020	0.002	0.020
ν	5.9079	5.9259	4.4914	4.5543	3.5014	3.4988
	0.692	0.022	0.411	0.424	0.264	0.263
λ	0	0.0001	0	0.2202	0	0.0397
		0.00		0.135		0.029
δ	0	0.0529	0	0.1075	0	0.0093
		0.002		0.050		0.010
AIC	8022.1121	8003.6068	9915.9309	9888.4530	12578.5604	12575.0979
BIC	8076.7044	8085.4954	9970.5233	9970.3415	12633.1528	12656.9865
Log-lik	-4007.0560	-3995.8034	-4953.9655	-4938.2265	-6285.2802	-6281.5489

Table 4: Non-robust GARCH and Robust GARCH with leverage, fitted to daily scaled returns of SM, GM and TSLA

The log-likelihood of each stock is higher in the full model, which includes the leverage effect, compared to the standard model. This is expected because the standard model is a subset of the model with leverage, making the former model more adaptable. However, in both scenarios, the total log-likelihoods obtained are negative, and there isn't a big variation between the models for each stock. This small variation could be attributed to the estimated robust and leverage effects, the values of which are detailed in Table 4.

Ultimately, the AIC and BIC values are calculated to gauge the model fit. Lower values in these measures signify better models. The AIC consistently favors the full model for all stocks. However, in the case of BIC, it favors the standard model only for Spectra Motors and Tesla.

Question 2.5

To assess the necessity of a robust filter and the presence of a leverage effect, we perform hypothesis tests. We use a 5% significance level for the tests. For the robust filter test, we get an estimated degree of freedom of 3015 and for the leverage effect test we get an estimated degree of freedom of 3014.
For the robust filter test we need to test whether λ is equal to zero or not. If it is equal to zero then it means that we do not have a robust filter. If it is not equal to zero then it means that there is a robust filter necessary. We have the following null and alternative hypotheses:

$$H_0 : \text{The use of a robust filter is not necessary}$$
$$H_1 : \text{the use of a robust filter is necessary}$$

We get the following results:

- **SM:** Failed to reject the H_0: the use of a robust filter is not necessary
- **GM:** Reject the H_0: the use of a robust filter is necessary
- **TSLA:** Reject the H_0: the use of a robust filter is necessary
- **KO:** Reject the H_0: the use of a robust filter is necessary

We can see from the results that for all the stock models, except for the SM, that the robust filter is needed. This means that λ should not be equal to zero.
For the leverage effect test we need to test whether δ is equal to zero or not. If it is equal to zero then it means that is no leverage effect present. If it is not equal to zero then it means that there is a leverage effect present. We have the following null and alternative hypotheses:

$$H_0 : \text{The use of a leverage effect is not present}$$
$$H_1 : \text{the use of a leverage effect is present}$$

We get the following results:

- **SM:** Failed to reject the H_0: the use of a leverage effect is not present
- **GM:** Failed to reject the H_0: the use of a leverage effect is not present
- **TSLA:** Reject the H_0: the use of a leverage effect is present
- **KO:** Failed to reject the H_0: the use of a leverage effect is not present

We can see from the results that only for TSLA the leverage effect is present. That means that based on this it best to use a model which uses a robust filter, but where there is no leverage effect present. The results for the competitors do not necessarily surprise us. Stocks from different companies can exhibit vastly different behaviors, and the need for a robust filter or the presence of the leverage effect can vary significantly. We can see since SM is a fictitious company that it is not surprising that no robust filter is needed for this stock.

Question 2.6

In Figure 6, we present the news impact curves and filtered volatilities for both the robust GARCH model with leverage and the model without leverage, individually for each of the stocks. For the news impact curves of the model with leverage, we observe that in the case of negative returns, the associated volatilities are higher compared to the model without leverage. Furthermore, as returns approach zero, the disparity in volatilities between the two model diminishes. Conversely, for positive returns, the GARCH model without leverage effect tends to have a larger volatility than the GARCH model with leverage for the cases of GM and TSLA. For Spectra Motors, the model without leverage has only a bit higher volatility when the returns are between -5 and 5. Outside this range, the model with leverage has much higher volatility compared to the model without leverage.

On the right side of Figure 6, filtered volatilities for each stock are presented, with a clear distinction between the models with and without leverage. These plots reveal a similarity between the two models, both within the sample of 2500 observations and out of this range. This similarity can be attributed to the presence of the auto-regressive term $\beta\sigma^2_{t-1}$, which accounts for the dependency in the volatility time series and thereby maintains the consistency in volatility patterns across the models.

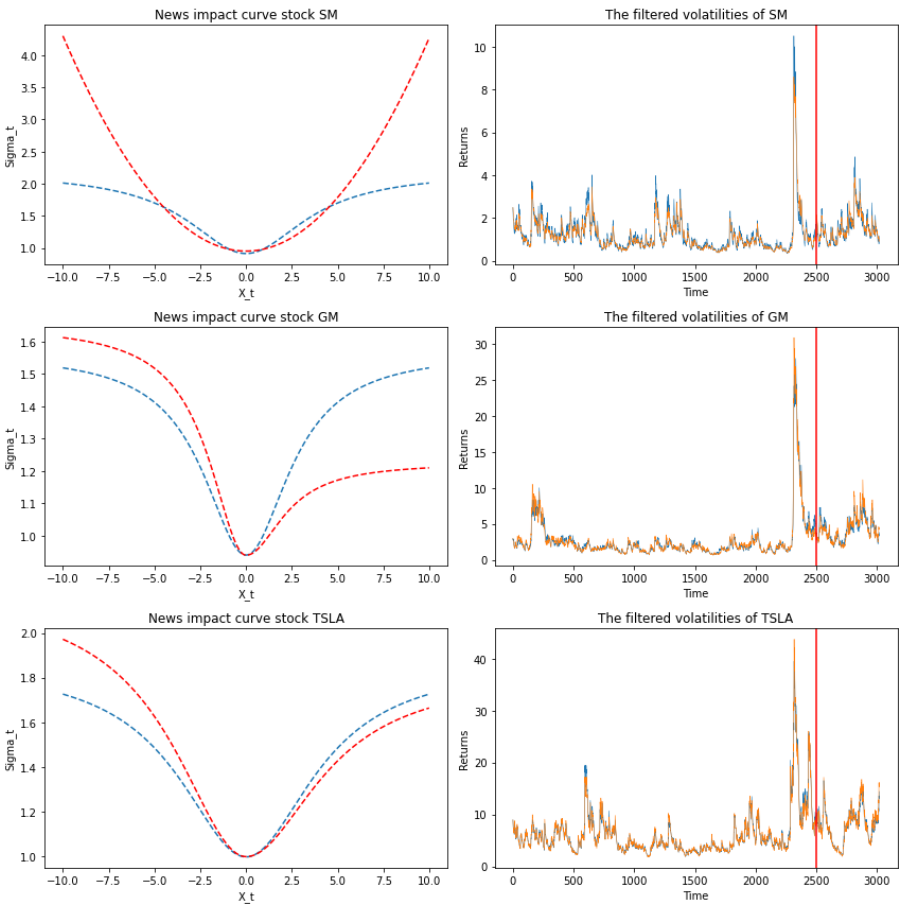


Figure 6: news-implied-curves versus filtered volatilities of SM, GM and TSLA

Question 2.7

In the table presented below, we have estimated the 1-day, 5-day, and 20-day Value at Risk (VaR) figures at the 1%, 5%, and 10% significance levels. We used the estimations of the parameters that we obtained in question 2.3. We can see that the difference between the models without the leverage and with the leverage is not that big. We do see that the difference is bigger when the days increase. We can also see a difference between the significance level. The larger the significance level is, the smaller the VaR is.

	SM		**GM**		**TSLA**	
	no leverage	with leverage	no leverage	with leverage	no leverage	with leverage
1% - 1 day-ahead	-3.280	-2.970	-5.215	-5.529	-10.645	-10.420
1% - 5 days-ahead	-7.209	-6.702	-11.025	-11.803	-21.561	-21.564
1% - 20 days-ahead	-13.835	-14.414	-21.330	-23.990	-39.212	-39.599
5% - 1 day-ahead	-2.021	-1.909	-3.161	-3.354	-5.766	-5.804
5% - 5 days-ahead	-4.702	-4.300	-7.134	-7.864	-13.411	-13.580
5% - 20 days-ahead	-9.248	-8.953	-14.215	-15.680	-26.120	-26.721
10% - 1 day-ahead	-1.505	-1.404	-2.290	-2.475	-3.969	-4.075
10% - 5 days-ahead	-3.486	-3.230	-5.453	-5.799	-9.907	-10.038
10% - 20 days-ahead	-6.918	-6.683	-10.966	-11.762	-20.122	-20.747

Table 5: Estimated Value-at-Risk (VaR) of SM, GM and TSLA with non-robust GARCH and Robust GARCH with leverage

Question 2.8

We compute the VaR again, but this time a little different. We only use a backtest to compute the 1-day-ahead VaR prediction. We also use only the first 2500 observations to estimate the parameters. We still use the significance levels of 1%, 5% and 10%. We calculated the sample hit-rate, standard error and the miss-specification standard error. For the miss-specification standard error, we used the following formula:

$$s_{NW}^2 = \frac{1}{H}\sum_{h=1}^{H} y_{t_0+h}^2 + \frac{2}{H}\sum_{l=1}^{L}\sum_{h=l+1}^{H} w_l y_{t_0+h} y_{t_0+hl}$$

Here H is the number of observations in the forecasting period and L = int($H^{1/5}$). The standard error of the hit-rate is calculated by $s_{NW}/\sqrt{H}$.

The results are presented in the Table 6. We can see that there is almost no difference between the normal standard error and the miss-specification standard error. We can also see that the hit-rate of the model with no leverage effect is higher than the hit-rate of the model with the leverage effect present. The hit-rates of VaR(10%) are the highest, so that means that the VaR is more successfully captured at a higher significance level.

		SM		**GM**		**TSLA**	
		no leverage	with leverage	no leverage	with leverage	no leverage	with leverage
	Hit-rate	0.013	0.012	0.008	0.006	0.010	0.006
VaR(1%)	Standard Error	0.005	0.005	0.004	0.003	0.004	0.003
	$s_N W$	0.005	0.005	0.004	0.003	0.004	0.003
	Hit-rate	0.067	0.065	0.065	0.056	0.079	0.075
VaR(5%)	Standard Error	0.011	0.011	0.011	0.010	0.012	0.012
	$s_N W$	0.011	0.011	0.011	0.010	0.011	0.011
	Hit-rate	0.117	0.112	0.137	0.131	0.148	0.137
VaR(10%)	Standard Error	0.014	0.014	0.015	0.015	0.016	0.015
	$s_N W$	0.015	0.014	0.015	0.015	0.016	0.014

Table 6: Backtesting 1-step (VaR) of SM, GM and TSLA with non-robust GARCH and Robust GARCH with leverage

3 Marketing and Sales: campaign optimization and price analysis

3.1 Marketing campaign effectiveness

Question 3.1

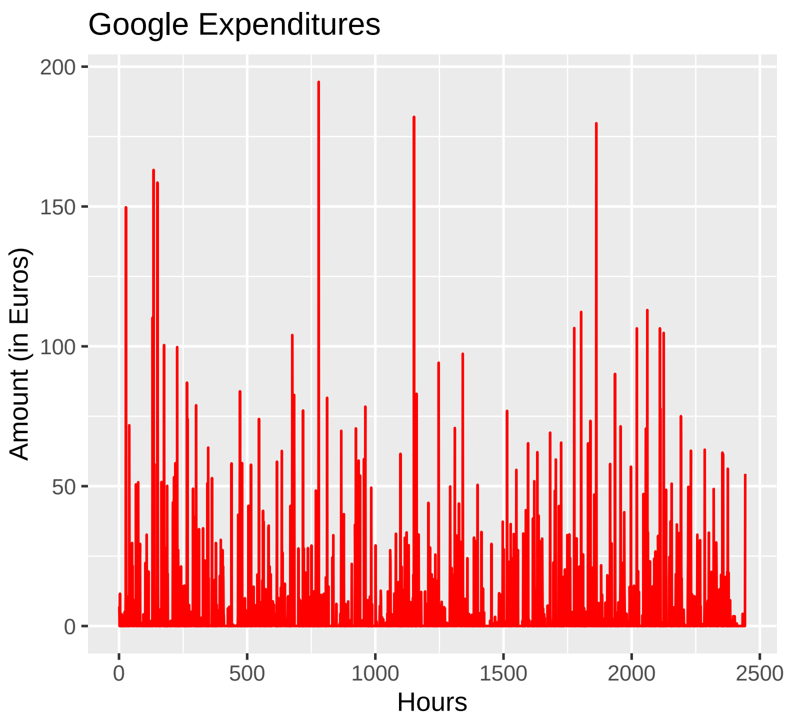
Figure 7 shows plots of the on data sales, Google search expenditures and YouTube expenditures of the self-driving subscription of E-tech X. Sales data oscillates between approximately 25 and 130, while the total sum of sales over the dataset is €162,861.05. Google expenditures are on average €5.64, while the maximum is €194.57. YouTube expenditures are on average €5.22, while the maximum is €181.29. If the hours that don't have any expenditures are not taken into account, the average YouTube expenditures are €16.95, while the average Google expenditures are then €17.73.

Figure 8 shows an overlapped plot of the data on sales, Google search expenditures and YouTube expenditures of the self-driving subscription of E-tech X. From this plot it becomes clear that a spike in Google or YouTube expenditures does not always coincide with a spike in sales. This is due to the nature of the sales, which is calculated with the ad-stock. The ad-stock in turn depends on past ad-stock, thus creating a delay in the response of sales to expenditures.

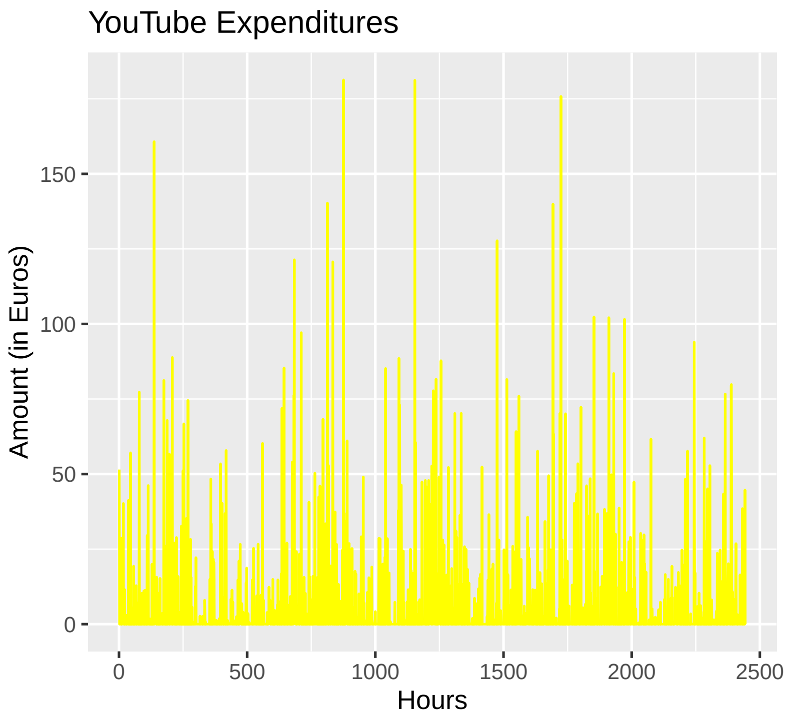
Figure 7: Plots of the data on sales, Google search expenditures and YouTube expenditures of the self-driving subscription of E-tech X

(a) Sales

Figure 7: Plots of the data on sales, Google search expenditures and YouTube expenditures of the self-driving subscription of E-tech X (continued)



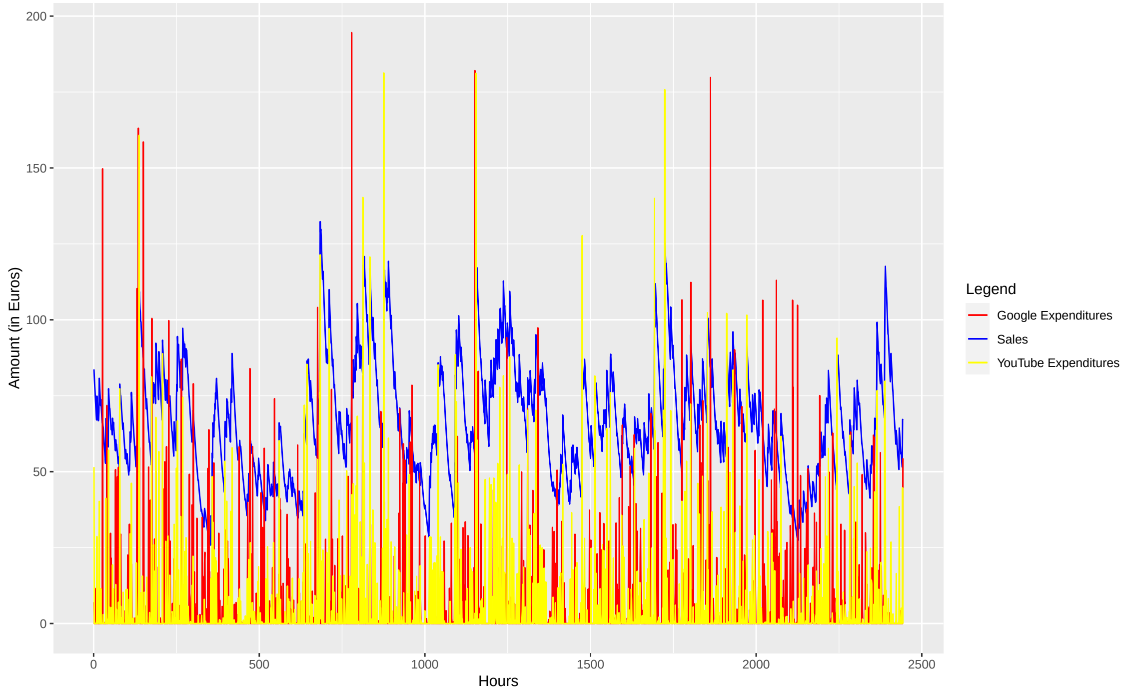
(b) Google Expenditures



(c) YouTube Expenditures

The three plots above show de-trended and de-seasonalized data on sales, Google expenditures and YouTube expenditures for the time period of the dataset. The y-axis shows the amount in euros and the x-axis shows the time (in hours). The dataset consists of 2443 observations.

Figure 8: Overlapped plot of the data on sales, Google search expenditures and YouTube expenditures of the self-driving subscription of E-tech X



Question 3.2

As already mentioned in Question 3.1, the sales are calculated with the adstock. For this, the following nonlinear dynamic parameter model is used:

$$s_t = \mu + \phi_1 \text{gads}_t{}^{\delta_1} + \phi_2 \text{yads}_t{}^{\delta_2} + \varepsilon_t, \tag{1}$$

$$\text{gads}_t = \beta_1 \text{gads}_{t-1} + \alpha_1 \text{g}_t, \tag{2}$$

$$\text{yads}_t = \beta_2 \text{yads}_{t-1} + \alpha_2 \text{y}_t, \tag{3}$$

where s_t is the sales at time t, gads_t is the Google adstock at time t and yads_t is the YouTube adstock at time t. g_t is the Google expenditures at time t and y_t is the YouTube expenditures at time t.
The parameter vector $\boldsymbol{\theta} = (\mu, \phi_1, \phi_2, \delta_1, \delta_2, \alpha_1, \alpha_2, \beta_1, \beta_2)$ is estimated by minimizing the following least squares function for predicted sales:

$$\hat{\theta}_t = \underset{\boldsymbol{\theta} \in \Theta}{\text{argmin}} \frac{1}{T} \sum_{t=2}^{T} (s_t - \hat{s}_t(\boldsymbol{\theta}))^2 \tag{4}$$

where $s_t = \mu + \phi_1 \text{gads}_t{}^{\delta_1} + \phi_2 \text{yads}_t{}^{\delta_2}$, using $\boldsymbol{\theta} = (\mu, \phi_1, \phi_2, \delta_1, \delta_2, \alpha_1, \alpha_2, \beta_1, \beta_2) = (1, 1, 1, 0.5, 0.5, 5, 5, 0.9, 0.9)$ as initial values. Additionally, the parameters $\mu, \phi_1, \phi_2, \alpha_1, \alpha_2$ are restricted to be positive and the parameters $\delta_1, \delta_2, \beta_1, \beta_2$ are restricted to be between 0 and 1. To initialise the adstock of Google and YouTube at time $t = 0$, the unconditional mean is used. The unconditional means are equal to:

$$\mathbb{E}[\text{gads}] = \frac{\alpha_1 * \mu_\text{g}}{1 - \beta_1}, \tag{5}$$

$$\mathbb{E}[\text{yads}] = \frac{\alpha_2 * \mu_\text{y}}{1 - \beta_2}, \tag{6}$$

where μ_g is the mean Google expenditures and μ_y is the mean YouTube expenditures.

The parameter estimates obtained by this estimator are shown in Table 7. The value of the minimised objective function is 1.008.

Table 7: Parameter Estimates for Question 3.2

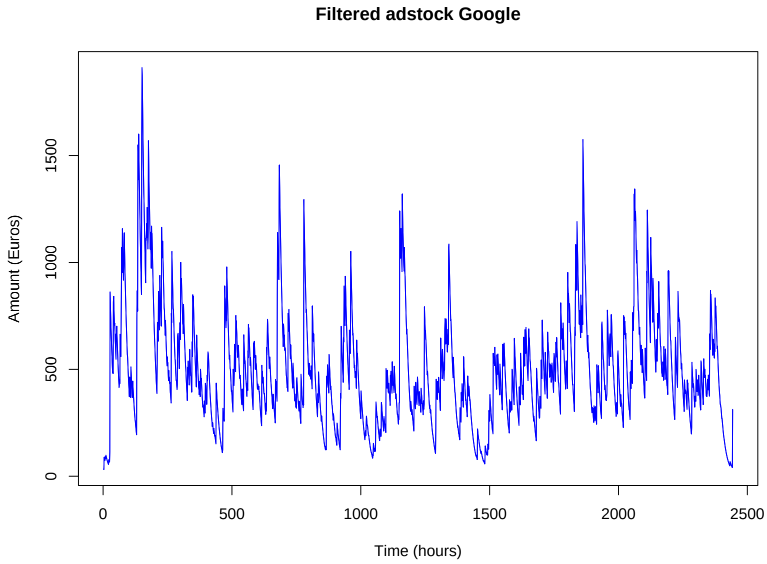
Parameter	Estimate
$\hat{\mu}$	1.027
$\hat{\phi}_1$	1.358
$\hat{\phi}_2$	1.476
$\hat{\delta}_1$	0.250
$\hat{\delta}_2$	0.596
$\hat{\alpha}_1$	5.036
$\hat{\alpha}_2$	5.049
$\hat{\beta}_1$	0.943
$\hat{\beta}_2$	0.950

This table shows the parameter estimates using the least squares optimizer in Equation 4.

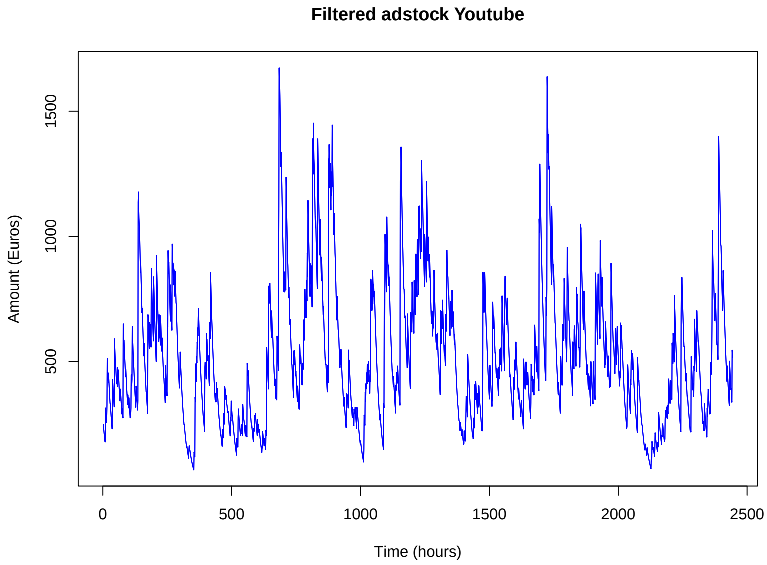
Question 3.3

Figure 9 shows plots of the filtered adstock generated by marketing on Google search and YouTube. For the initialisation of gads_t at time $t = 0$ we assumed the previous adstock to be equal to 0 and we used: $\text{gads}_0 = \hat{\alpha}_1 * \text{g}_0$ and for yads_t we used: $\text{yads}_0 = \hat{\alpha}_2 * \text{y}_0$. These plots show that the filtered adstock for Google was the highest in the beginning of the sample, at approximately hour 200. For YouTube the filtered adstock has two peaks, at around hour 750 and 1700. For the Google adstock this is quite surprising, as the Google expenditures are not the highest at this time, but rather later in the sample. For the YouTube adstock it is within expectations, as the YouTube expenditures are highest around the same time that the filtered adstock is the highest. Also the YouTube adstock shows a peak at the end of the sample, while the YouTube expenditures do not show a peak at that time. This has to do with the fact that the adstock is scaled up with a factor α_2, which is approximately equal to 5.

Figure 9: Plots of the filtered adstock generated by marketing on Google search and YouTube



(a) Google adstock



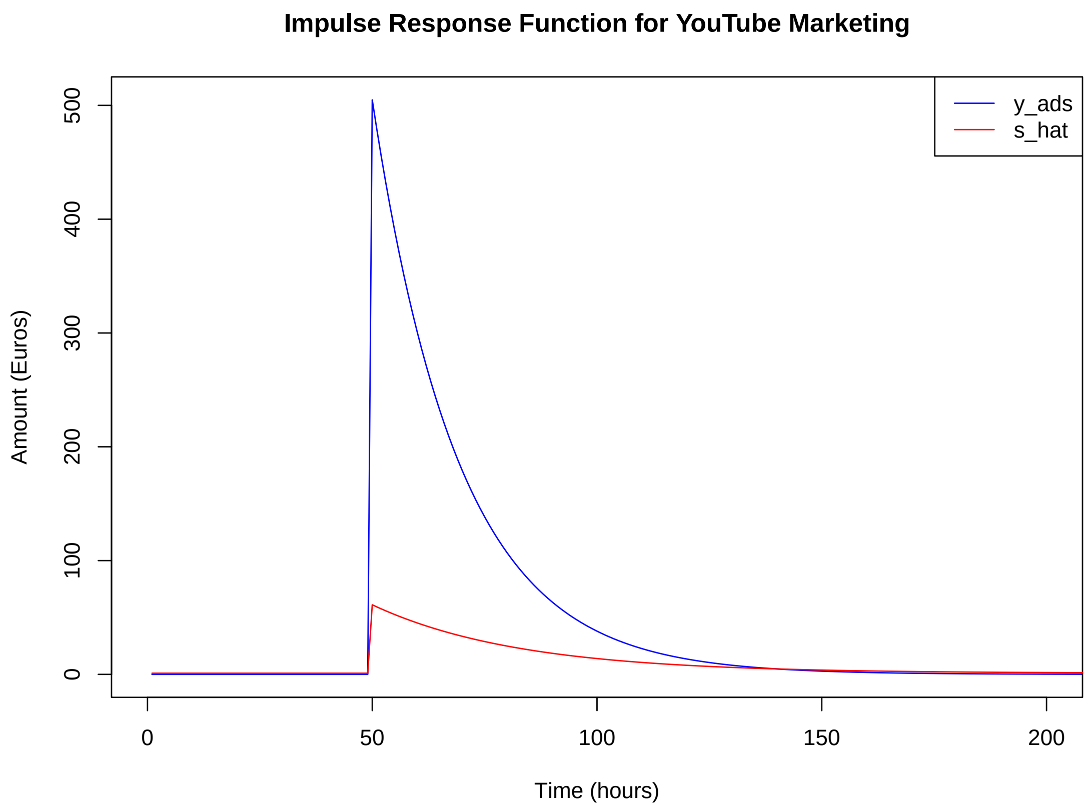
(b) YouTube adstock

The two plots above show the filtered adstock generated by marketing on Google search and YouTube. The top plot shows the filtered adstock for Google while the plot below shows the filtered adstock for YouTube. The data consists of 2442 datapoints.

Question 3.4 and 3.5

Figure 10 shows the Impulse Response Function (IRF) for sales and YouTube adstock generated by spending 100 euros with marketing on YouTube on $t = 50$. Impulse response functions show how a system responds dynamically to a shock. As observed from the figure, the spending of 100 euros results in a direct peak for both YouTube adstock and sales at time $t = 50$. Thereafter, both YouTube adstock and sales rapidly decrease back to their origin (0 for YouTube adstock and μ for sales, where μ is the average for sales). In approximately 100 hours, both IRFs are back to their origin. This is due to the fact that β_2, which determines the YouTube adstock in the next period, is smaller than 1 and exponentially decreases. Thus, an increase in marketing expenditures has a fairly long impact on the generated adstock.

Figure 10: Impulse Response Function for sales and YouTube adstock generated by spending 100 euros with marketing on YouTube on $t = 50$



This figure shows the Impulse Response Function for sales and YouTube adstock generated by spending 100 euros with marketing on YouTube on $t = 50$. The blue line shows the IRF for the YouTube adstock while the red line shows the IRF for predicted sales $\hat{s}_t$. The data for the IRF consists of 2443 observations.

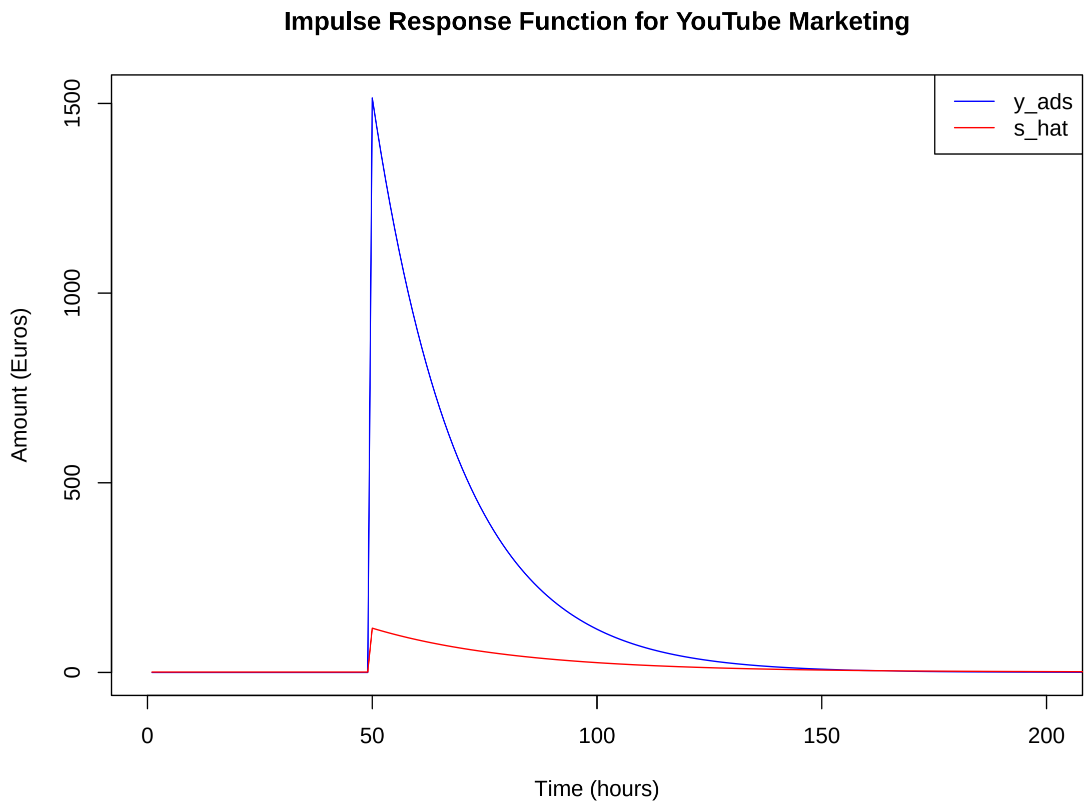
Question 3.6

The IRF from the previous question is used to calculate the expected accumulated additional sales produced by the impulse of 100 euro expenditure on YouTube. It is calculated as the sum of the predicted sales in the IRF minus the sum of sales at the origin (which is equal to the average). The expected accumulated additional sales are: €1,981.37.

Question 3.7

A similar IRF as in the previous questions is used to calculate the expected accumulated additional sales produced by the impulse of 300 euro expenditure on YouTube on $t = 50$. The IRF is shown in Figure 11. The expected accumulated additional sales are: €3,811.98. The ratio of the two IRFs is $\frac{3{,}811.98}{1{,}981.37} = 1.92$. Thus, when spending 300 euros rather than 100 we do not obtain three times as much additional sales. This is due to the fact that the expected accumulated additional sales are calculated using the YouTube adstock. In turn, this YouTube adstock is calculated with the previous adstock as: $\text{yads}_{s+1} = \beta_2^n + \text{yads}_s$, where s is the shock time ($t = 50$) and $n = 1, 2, 3, ...$ As the β_2 has an exponent, future adstock decreases (as $\beta_2 < 1$) exponentially to the origin. As the shock is three times as high in the second IRF, it also decreases faster to the origin. Due to the faster (exponentially) decrease to the origin, the expected accumulated additional sales are not three times as high with a shock three times as high.

Figure 11: Impulse Response Function for sales and YouTube adstock generated by spending 300 euros with marketing on YouTube on $t = 50$



This figure shows the Impulse Response Function for sales and YouTube adstock generated by spending 300 euros with marketing on YouTube on $t = 50$. The blue line shows the IRF for the YouTube adstock while the red line shows the IRF for predicted sales $\hat{s}_t$. The data for the IRF consists of 2443 observations.

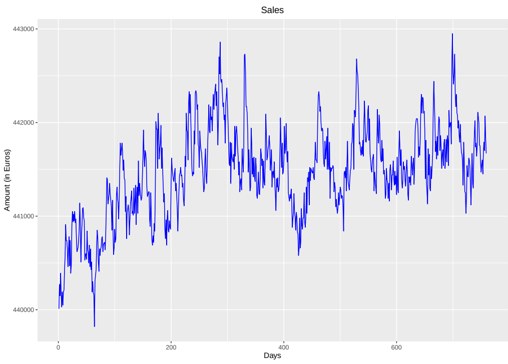
Question 3.8

The AB-testing procedure is important to understand the causal relationships between variables. To search for the impact that an exogenous, ceteris-paribus shift in one variable has on another variable. That will be in our case, what effects marketing expenditures, YouTube or Google, have on sales. In AB-testing, you will randomly assign two groups: a control group and one or more treatment groups. The control group remains unchanged, while the treatment groups are exposed to the variable being tested. AB testing relies on running experiments and thus it is only useful in situations where you can control and manipulate the regressor. As this is possible in our case, the AB-testing procedure is feasible. In this case, they divided the groups, into the control group that is not exposed to any marketing expenditure. Furthermore, you will have a treatment group that is exposed to Google AdWords advertising and a treatment group exposed to YouTube Advertising. Both the treatment groups are compared with the sales of the control group and then with statistical tests it is determined what the effect is of these advertisements. With this procedure, you will recover the true effects of each advertisement strategy on sales. Without AB testing, there could be simultaneity bias. There can be simultaneity because the marketing expenditures can affect the sales, and also the sales can affect the marketing expenditures. Without the AB-testing you can observe correlations, however, it is hard to establish the direction of causality. Furthermore, it is also hard to isolate the impact of each marketing strategy. As other factors can influence sales simultaneously, such as marketing conditions or seasonality. These factors can make it unclear which strategy, Google Adwords or Youtube Advertisement, is responsible for the changes in sales. In conclusion, AB-testing will give us unbiased and consistent estimates of the parameters by disentangling the causal relationships and thus mitigating the simultaneity bias.

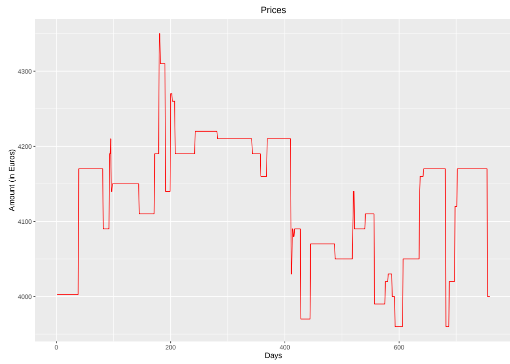
3.2 Dynamic Pricing

Question 3.9

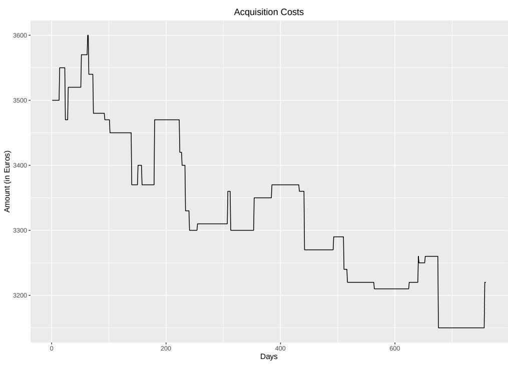
Figure 12: Plots of data on sales, prices, acquisition costs and marketing expenditures



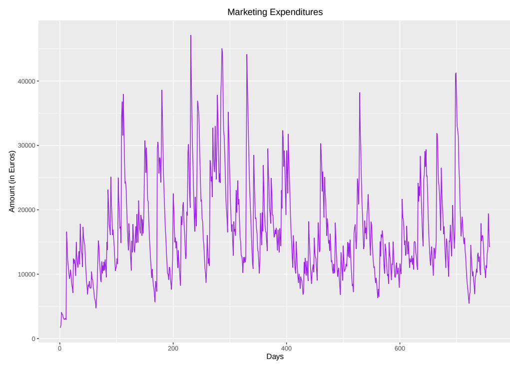
(a) Sales



(b) Prices



(c) Acquisition Costs



(d) Marketing Expenditures

The figures above show plots of daily de-seasonalised data on sales, prices, acquisition costs and marketing expenditures. The y-axis shows the amount in euros while the x-axis shows the number of days.

Figure 12 shows plots of daily de-seasonalised data on sales, prices, acquisition costs and marketing expenditures of Spectra's premium non-electric model. From Figure 12a it is observed that the sales increase over time. Prices stay relatively similar throughout the sample period, as can be seen from Figure 12b. Figure 12c shows that acquisition costs decrease over time. This can possibly be explained by the fact that production costs of the car decrease over time, as the production process is optimised and producing cars in larger amounts decreases the costs per produced car. Therefore, also the acquisition costs decrease. Marketing expenditures swing the most, from approximately €1,600 to €47,000. However, Figure 12d does show that the amount of high peaks of marketing expenditures does decrease over time (in the sense that early in the sample the peaks were more frequent). This indicates that Spectra is less reliant on marketing expenditures for the sales, probably due to the increased popularity and brand awareness by customers (as sales are increasing).

Question 3.10

The parameters of the linear regression in (7) are estimated with the use of Ordinary Least Squares. Where s_t represents the sales at time t and p_t is the price at time t. The estimates of the linear regression are reported in Table 8.

$$s_t = \alpha + \beta * p_t + \epsilon_t \tag{7}$$

Table 8: Parameter Estimates for Question 3.10

Parameter	Estimate
α	4.355 x 10^5 (892.010)
β	1.443 (0.216)

This table shows parameter estimates for Equation 7. The used data consists of 759 observations. Standard errors are given in parentheses.

Question 3.11

The regression, described in Equation 7 analyzes the influence of price on sales. In the context of a predictive model, it aims to forecast sales based on the given parameters α and β. In Table 8, we observe that β is positive, namely 1.443. This positive estimate implies that as price increases, the sales are forecasted to increase as well. This observation corresponds with our intuition for forecasting sales. As there is commonly a co-movement between sales and prices. Consumers tend to associate higher prices with a higher quality or demand. This aligns with the objective of a predictive model, however, in a dynamic pricing context, it would imply that a higher price will lead to an increased demand.

Suppose that the estimator is consistent, this will mean that as the sample increases, the estimated values $\hat{\alpha}$ and $\hat{\beta}$ will converge to their true value of the parameter. As the estimator's properties in the limit are unbiasedness, efficiency and asymptotic normality. This means that in the long run, as the data-set grows, the estimator will have unbiased estimates for α and β, with smaller variances, and the distribution of the estimates will approach a normal distribution.

Thus, consistency ensures that the estimates become more accurate with larger sample sizes. However, it does not guarantee causal relationships. The aim of the predictive model is to forecast the sales based on he observed price sales relationships.

Question 3.12

The estimate for β is positive, and as mentioned in Question 3.11 this aligns with the intuition for a predictive model. However, in a causal context, such as a structural model, this interpretation can be misleading. A positive β in a structural model would imply that increasing the price directly causes an increase in sales. This is counter intuitive, as general economic theory suggests that demand increases with lower prices and demand decreases with higher prices. Thus, very high prices can make a product unattractive and can lead to a reduction of sales. This issue can arise, because the prices are simultaneously determined, as the sales depend on the prices, but observed prices can also depend on the sales. Furthermore, the regression equation can have missing explanatory variables, such as marketing expenditures. Lastly, the realized sales are a noisy measurement of potential sales, that can be influenced by factors like inventory shortages. All this leads to endogeneity in the regression. Thus, in a dynamic pricing model, it is more appropriate to use a causal parameter (β) that is negative. This shows that a decrease in sales is due to an increase in price. This can be obtained with a IV regression model.

Question 3.13

In this question, the sales are calculated using an instrumental variable approach. First, a regression is conducted to predict the price, as shown in Equation **??**. The predicted price p_t is then used, to find the structural-causal relationship between sales and the prices, shown in Equation 9.

$$p_t = \delta + \gamma c_t + \nu_t, \tag{8}$$
$$s_t = \alpha + \beta \hat{p}_t + \epsilon_t, \tag{9}$$

Table 9: Parameter estimates for the instrumental variables regression

Parameter	Estimate
δ	3.654 x 10^3 (84.139)
γ	0.142 (0.025)

This table shows parameter estimates for the instrumental variables regression (Equation 8). The used data consists of 759 observations. Standard errors are given in parentheses.

Parameter	Estimate
α	5.200 x 10^5 (3578.181)
β	-19.029 (0.867)

Table 10: Parameter estimates for the sales regression

This table shows parameter estimates for the sales regression (Equation 9). The used data consists of 759 observations. Standard errors are given in parentheses.

The estimates for the instrumental regression are reported in Table 9 and the estimates for the sales regression are reported in Table 10. The causal impact of an increase in price (p_t), on expected sales (s_t) is shown by the parameter β with an estimate of -19.029. It is a negative value, and thus a decrease in price would result in an increase in sales, which is in line with the general economic theory as described in Question 3.12.

Question 3.14

To test for endogeneity, we can use the Hausman-Durbin-Wu test, with the null hypothesis being that prices are exogenous (in the original regression of Equation 7. After conducting this test, we obtained a test statistic of 596.079, which corresponds to a p-value of 3.656

x 10^{-130}. Thus, we can reject the null hypothesis of exogeneity of the prices. This means that indeed the price variable in Equation 7 is endogenous and therefore this Equation 7 is not suitable for structural analysis.

Question 3.15

Now, we will improve the model by including marketing expenditures, this will result in the following regression:

$$s_t = \alpha + \beta \hat{p}_t + \psi m_t + \epsilon_t \tag{10}$$

The parameter estimates are reported in Table 11. The regression results show a negative β coefficient, which is within expectations. As predicted sales $\hat{p}_t$ is used, the endogeneity is removed. Furthermore, the ψ coefficient is positive, meaning that marketing expenditures has a positive effect on sales. This is also within expectations, as we would increase higher sales as more people are aware of the product due to marketing.

Table 11: Parameter estimates for Question 3.15

Parameter	Estimate
α	5.127 x 10^5 (1.795 x 10^3)
β	-17.429 (0.435)
ψ	4.489 x 10^{-2} (9.415 x 10^{-4})

This table shows parameter estimates for the regression in Equation 10. The used data consists of 759 observations. Standard errors are given in parentheses.

Question 3.16

In this question, the model in Equation (10) is used to calculate the causal impact on expected profit π_t of a unit increase in price p_t. Expected profit is given by:

$$\mathbb{E}[\pi_t|c_t] = (\alpha + \beta p_t + \psi m_t)(p_t - c_t). \tag{11}$$

Using the parameter values from Question 3.15 and using the last observed values in the sample (p_{759} = €4,000, c_{759} = €3,220, m_{759} = €14,250), the causal impact on expected profit π_{759} of a unit increase in price p_{759} to €4,001 is equal to €429,967.10 (using Equation 11).

This means that Spectra should increase the price of their luxury car, as a price increase leads to a profit increase. This is also within expectations, as the optimal price is much higher than the €4,000 which is used at the end of the sample. Namely, setting the derivative of Equation 11 with respect to p_t equal to 0 we get the optimal price (at time t):

$$p_t^* = \frac{c_t\beta - \alpha - \psi m_t}{2\beta} \tag{12}$$

For $t = 759$, the optimal price is €16,335.16. Therefore, the used price is far below the optimal price. Thus, increasing the price increases the profit, as profit is maximised at the optimal price p_t^*.