Master Thesis Financial Econometrics

AI vs. Wall Street: A Volatility Forecasting Study Using GARCH and GAS Models

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Abstract

This thesis conducts research to evaluate which model is the best to use for forecasting the volatility of the stock market from two different sectors: banking and technology. Multiple models are applied, such as ARCH, GARCH, GJR-GARCH, EGARCH, and GAS. These models are examined under three distributions: the Normal distribution, the Student-t distribution, and the Skewed Student-t distribution. The approaches used to estimate the models' parameters are the Frequentist approach and the Bayesian approach. For the evaluation of these models, multiple performance tests are conducted, which will show which model performs the best and has the highest accuracy for forecasting volatility. From the findings, it can be concluded that overall the EGARCH model performs the best, especially under the Skewed Student-t distribution and using the Bayesian approach.

Keywords: ARCH; GARCH; GJR-GARCH; EGARCH; GAS; Frequentist Approach; Bayesian Approach; Maximum Likelihood Estimation; Random Walk Metropolis-Hastings Method; Normal Distribution; Student-t Distribution; Skewed Student-t Distribution; JPMorgan Chase & Co.; NVIDIA Corporation; Mean Squared Error; Mean Absolute Error; Mean Absolute Percentage Error; Root Mean Squared Error; Jarque-Bera Test; Akaike Information Criterion; Bayesian Information Criterion; Value-at-Risk Backtesting; Unconditional Coverage Test; Independence Test; Conditional Coverage Test; Logarithmic Scoring Rule; Diebold-Mariano Test

Contents

1 Introduction

Financial markets play an important role in helping businesses and in moving the economy forward. There is also a lot of uncertainty in financial markets, which is something that many investors and portfolio managers face by managing the risk from volatile asset returns. Studying the volatility is essential for risk management, portfolio optimization, and Value-at-Risk (VaR) estimation. That is why volatility modeling became important to the empirical finance.

There are various models that help analyze the volatility of assets. One of the most commonly used in time series analysis is the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) family of models. These models use time-varying parameters and the likelihood function to estimate the conditional variance, which represents volatility. For this study, multiple volatility models from the GARCH family are used, including ARCH, GJR-GARCH, and EGARCH. The Generalized Autoregressive Score (GAS) model is also used, which is a more recent alternative for modeling volatility. To estimate the parameters for the modeling, we use two different approaches, the Frequentist and Bayesian approaches.

This research focuses on two stocks from distinct sectors of the market: banking and technology. The selected companies are JPMorgan Chase & Co. (JPM) and NVIDIA Corporation (NVDA), representing the financial and technology sectors, respectively. The technology sector, and particularly companies involved in artificial intelligence, has experienced significant growth in recent years. As a result, AI-driven stocks like NVDA tend to exhibit higher volatility and attract increased investor attention. In comparison, financial stocks such as JPM typically demonstrate more stable and predictable return behavior.

This contrast provides a valuable opportunity to explore how different volatility models perform under varying market environments. One environment is shaped by rapid innovation and speculative activity, while the other reflects more traditional and regulated financial operations. The objective of this study is to examine how sector-specific characteristics influence return dynamics and volatility patterns. Additionally, it evaluates the suitability of various volatility models, such as GARCH-type and GAS models, in capturing the behavior of both AI-focused technology stocks and more established financial assets.

To estimate the parameters for the modeling, two different approaches are used: the Frequentist and the Bayesian approaches. For the Frequentist approach, Maximum Likelihood Estimation (MLE) is used, while for the Bayesian approach, the Random Walk Metropolis-Hastings (RWMH) algorithm within the Markov Chain Monte Carlo (MCMC) framework is used. The three distributions applied for parameter estimation are the normal distribution, the Student-t distribution, and the skewed Student-t distribution, which allow for the modeling of fat tails and skewness in the dataset.

To evaluate the volatility forecasts, various performance tests are used. In this paper, we apply Value-at-Risk (VaR) backtesting, as well as in-sample and out-of-sample performance tests. The VaR backtesting includes the Unconditional Coverage (UC), Independence (IND), and Conditional Coverage (CC) tests. For in-sample performance tests, model selection criteria such as the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC) are used. The out-of-sample performance tests involve the use of loss functions (MSE, MAE, MAPE, RMSE), logarithmic scoring, and the Diebold-Mariano test.

The aim of this thesis is to provide an evaluation of volatility forecasting techniques, focusing on how different modeling choices influence the ability to forecast and manage financial risk. Based on the obtained findings, it can be concluded which models are best suited to different types of market dynamics. In this case, the analysis is applied to the banking and technology sectors.

This thesis is structured as follows. Section 2 provides an analysis of the data. In Section 3 the GARCH models are considered. Section 4 discusses the frequentist and Bayesian estimation methods. Section 5 considers the normal, Student-t and skewed Student-t distributions. Section 6 discusses the Value-at-Risk backtests, information criteria and loss functions. In Section 7 the results are discussed and Section 8 provides the conclusions. The Appendices A-H provide graphs and tables of the results.

2 Data Analysis

As introduced earlier, this research paper is based on two companies, each from a different sector. JPMorgan Chase & Co. (JPM) represents the banking sector and is a major global financial institution with broad exposure to commercial banking, investment services, and asset management. NVIDIA Corporation (NVDA), selected from the technology sector, is widely recognized for its leadership in artificial intelligence and high-performance computing. Its business model makes it a prime example of a highly AI-driven stock.

The datasets of both stocks consist of the daily closing prices and are retrieved from Yahoo Finance. The sample period is from January 1, 2015, to December 31, 2024, which gives a 10-year window with 2514 observations. This window ensures that it captures various market conditions like, policy shifts, and periods of economic uncertainty. This means that the dataset is suitable for volatility modeling, since it involves both regular market conditions and periods of significant turbulence. In Figure 1 in Appendix A, the plots of the adjusted closing prices can be seen.

The datasets were transformed to log-returns, so that they can be used for volatility modeling. Figure 2 shows the plots of the log-returns. Log-returns are widely used in financial econometrics due to their beneficial properties. They help stabilize the variance and achieve stationarity, which is important to use for time series models like the GARCH models. The log-returns were calculated using the following formula:

$$r_t = (\log(p_t) - \log(p_{t-1})) \times 100$$

Table 1 shows the descriptive statistics of the log-returns of JPM and NVDA. From this it can be seen that both stocks have a sample mean close to 0, with the mean of NVDA being higher than that of JPM. It also shows that the standard deviation of NVDA is higher than of JPM. This is because NVDA returns are more volatile than JPM returns. A high standard deviation indicates that the stock has a higher volatility. NVDA displays a wider interquartile range and both higher maximum and lower minimum log-returns than JPM, indicating that it has more outliers in its return. This is common for stocks that are more volatile.

Table 1: Descriptive statistics of the log-returns of JPM and NVDA

	JPM	NVDA
Number of Observations	2514	2514
Mean	0.0641	0.2248
Standard Deviation	1.7213	3.0378
Lowest Log Return	-16.2106	-20.7711
25% Quantile	-0.7175	-1.2514
50% Quantile	0.0586	0.2648
75% Quantile	0.8705	1.7473
Highest Log Return	16.5620	26.0876

The Jarque-Bera (JB) test (Jarque and Bera, 1987) is also performed on the log-returns, the results are presented in Table 2. The JB test is a statistical test that is used to analyze whether the dataset has a normal distribution or not. The sample skewness (S) and sample kurtosis (K) of the data are used for the JB test. The following formulas are used for the calculations:

- $JB = \frac{n}{6}(S^2 + \frac{1}{4}(K-3)^2) \sim \chi^2_2$
- $S = \frac{1}{n}\sum_{t=1}^{n} \frac{(x_t - \mu)^3}{s^3}$
- $K = \frac{1}{n}\sum_{t=1}^{n} \frac{(x_t - \mu)^4}{s^4}$

A hypothesis test is conducted for the JB test, where the null and alternative hypotheses are stated as follows:

$$H_0 : \text{The data have a normal distribution}$$
$$H_1 : \text{The data do not have a normal distribution}$$

From the results of Table 2 it can be seen that the p-values are lower than the chosen significance level, hence we reject the null hypothesis that the data of both stocks have a normal distribution. It can also be seen that the sample skewness of JPM is negative and the sample skewness of NVDA is positive, which indicate that there is a fatter tail on the left and right side of the distribution, respectively. The sample kurtosis of both stocks are higher than 3, hence both stocks have heavy tails.

Table 2: Jarque-Bera test of the log-returns of JPM and NVDA

	JPM	NVDA
Skewness	-0.0205	0.2087
Kurtosis	13.5088	6.8299
Jarque-Bera Test	19032.9641	4881.7198
p-value	0.0000	0.0000
Reject/Accept	Reject	Reject

In Figure 3, the QQ-plots of the stocks are shown. These plots compare the quantiles of the normal distribution with the quantiles of our log-returns. It can be observed that, for both stocks, the ends of the plots do not follow a straight line. This indicates that the returns are not normally distributed and that alternative distributions are required for the analysis. Two possibly suitable alternatives are the Student-t distribution and the skewed Student-t distribution, both of which will be discussed further in Section 5.

3 GARCH Models

One of the most used models to help with forecasting the volatility of stock returns is the GARCH model. Various types of GARCH models are used on the two stocks that are analyzed. The main purpose of the GARCH models is that they are used to calculate the conditional variance. This is done by using past information from the dataset. In order to calculate the conditional variance the parameters of the GARCH models need to be estimated. The methods used to estimate the parameters are described in Section 5 and 4.

3.1 ARCH

The first and the simplest model, introduced by Engle (1982), that captures the time-varying conditional variances is the Autoregressive Conditional Heteroskedasticity (ARCH) model. For this study, the ARCH(1) model is used, which indicates that the model considers only the lagged squared error term at time $t-1$ to capture the time-varying volatility. The formula used to calculate the conditional variance with the ARCH(1) model is as follows:

$$\sigma_t^2 = \omega + \alpha_1 \epsilon_{t-1}^2$$

Here, the error term is ϵ_{t-1}, which represents the return minus its conditional mean, typically set to 0 or a constant parameter μ. The parameters ω and α_1 need to be estimated. Since the conditional variance must be positive, the parameters need to satisfy the conditions $\omega > 0$ and $\alpha_1 \geq 0$. However, this model is not very efficient at capturing the time-varying volatility, as it often requires a large number of lags to properly show how volatility changes over time.

3.2 GARCH

The extension of the ARCH model is the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model, which was introduced by Bollerslev (1986). This model is widely applied within the financial econometrics to forecast the volatility of stock returns. The GARCH(1,1) is the extension of ARCH(1), which will be used. The formula used for GARCH(1,1) is denoted as follows:

$$\sigma_t^2 = \omega + \alpha_1 \epsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$

The formula shows that the model captures both lagged squared residuals and lagged conditional variances compared to the ARCH(1). The β_1 captures the impact of past volatility, while α_1 captures the influence of past squared shocks. The $\sigma_t^2 > 0$ is obtained with the following restrictions: $\omega > 0$, $\alpha_1 \geq 0$ and $\beta_1 \geq 0$. Compared with an ARCH model with a high order, then the GARCH(1,1) model would be more efficient because it also captures both the short-term shocks and long-term volatility.

3.3 GJR-GARCH

Within the GARCH model family is also the Glosten-Jagannathan-Runkle GARCH (GJR-GARCH) model (Glosten et al., 1993). It builds on the GARCH model by incorporating an extra parameter that captures the asymmetric response to positive and negative shocks. The formula of GJR-GARCH(1,1) is given as:

$$\sigma_t^2 = \omega + \alpha_1 \epsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \gamma_1 \epsilon_{t-1}^2 I_{\{\epsilon_{t-1} \, < \, 0\}}$$

Here, γ_1 is the asymmetry parameter and $I_{\{\epsilon_{t-1} \, < \, 0\}}$ is the indicator function where if the previous shock is positive it gives $I = 0$ and if the previous shock is negative then it gives $I = 1$. The same restrictions of the GARCH model apply for this model to obtain a positive conditional variance, but there is an extra restriction of $\gamma_1 \geq 0$. There is a leverage effect in this model when $\gamma_1 > 0$, because the negative returns increase volatility more than positive returns of the same magnitude. The GJR-GARCH(1,1) model reduces to a GARCH(1,1) model when $I = 0$ or if $\gamma_1 = 0$.

3.4 EGARCH

The Exponential GARCH (EGARCH) model, introduced by Nelson (1991), also builds on the GARCH model as an extension. Similar to the GJR-GARCH model, EGARCH also captures the asymmetry in the volatility. However, there is no indicator function used for the EGARCH model. The EGARCH also uses the logarithm of the conditional variance, so that the conditional variance will always be positive. This indicates that there is no need for any restrictions anymore on the parameters. The formula used for the EGARCH(1,1) is given as:

$$\log(\sigma_t^2) = \omega + \alpha_1 \frac{|\epsilon_{t-1}|}{\sigma_{t-1}} + \beta_1 \log(\sigma_{t-1}^2) + \gamma_1 \frac{\epsilon_{t-1}}{\sigma_{t-1}}$$

Here, $\alpha_1 \frac{|\epsilon_{t-1}|}{\sigma_{t-1}}$ captures the magnitude effect, because of the absolute value of the shocks, reflecting the size of the shock. The term $\gamma_1 \frac{\epsilon_{t-1}}{\sigma_{t-1}}$ captures the asymmetry, which allows negative shocks to have a different impact on volatility compared to positive shocks. For the leverage effect it can be that γ_1 is negative. This model provides more realistic results of the volatility, because it captures the asymmetries and volatility clustering. If there are extreme movements in stock returns, then this model can accommodate it more naturally than the GARCH model.

3.5 GAS

The Generalized Autoregressive Score (GAS) model, introduced by Creal et al. (2013), is a framework for modeling time-varying parameters in time series models, just like the GARCH models. However, what sets the GAS model apart from the other GARCH models is that it is based on the score of the log-likelihood function. This score is used to update the parameter, which allows the dynamics of the system to respond directly to new information. The update formula used for the GAS(1,1) model is as follows:

$$\theta_{t+1} = \omega + \alpha s_t + \beta \theta_t$$

Here, θ_t is the time-varying parameter and s_t is the scaled score function, with the formula $s_t = S_t \cdot \nabla_t$. In this research the logarithm of the conditional variance is used, $\theta_t = \log(\sigma_t^2)$, which implies that we can use the unscaled score, so the scalar matrix is set as $S_t = 1$, which gives $s_t = \nabla_t$ as the score function. The formula for ∇_t is given as:

$$\nabla_t = \frac{\partial \log p(r_t \mid \theta_t)}{\partial \theta_t}$$

4 Estimation of the Models

In Section 3, it was discussed that, in order to forecast the volatility of stock returns, the parameters of the GARCH models need to be estimated. For this research, two approaches are used to estimate these parameters: the Frequentist approach and the Bayesian approach. These approaches will be evaluated based on their performance, which will be explained in Section 6. This evaluation will indicate which of the two approaches is more appropriate for modeling volatility.

4.1 Frequentist Approach

The Frequentist approach is a widely used approach, because the implementation is relatively straightforward and simple to understand. For this approach the Maximum Likelihood Estimation (MLE) method is used to estimate the parameters (Rossi, 2010). This method uses the log-likelihood function, which is obtained by taking the logarithm of a specific PDF function. The log-likelihood depends on specific distributions, which will be given in Section 5. MLE maximizes the log-likelihood function with respect to the parameter vector θ. For the maximization of the log-likelihood, the numerical optimization technique of Nelder-Mead is used. The formula for MLE is given as:

$$\hat{\theta}_{\text{MLE}} = \arg\max_{\theta} \; l(r;\theta)$$

Here, the joint likelihood is decomposed using the conditional density of each observation, given past information. The general log-likelihood function is given as:

$$l(r;\theta) = \log p(r_1, \ldots, r_n \mid \theta) = \sum_{t=1}^{n} \log p(r_t \mid r_{t-1}, r_{t-2}, \ldots, \theta)$$

4.2 Bayesian Approach

Unlike the Frequentist approach, the Bayesian approach treats the parameters as random variables. The Bayesian approach is based on Bayes' theorem and uses probability distributions to represent uncertainty about the model parameters (Hoogerheide, 2022). The posterior distribution is obtained by using the prior density and the observed data, which gives the following formula:

$$\Pr(\theta \mid Y) = \frac{\Pr(\theta)\Pr(Y \mid \theta)}{\Pr(Y)}$$

Here, $\Pr(\theta \mid Y)$ is the posterior probability, which updates the beliefs about θ. $\Pr(\theta)$ is the prior probability of θ, which represents the initial belief about the parameter values before observing data. $\Pr(Y \mid \theta)$ is the likelihood function, which represents the distributions of the observed data given the parameters. $\Pr(Y)$ is the marginal likelihood of the observed data, which is difficult to compute analytically. In case of continuous distributions (instead of discrete distributions), the probability density functions is used instead of the probabilities above. To avoid computing the marginal likelihood, Markov Chain Monte Carlo (MCMC) is used to approximate the posterior distributions.

The MCMC generates samples by constructing a Markov chain that converges to the posterior distribution as its stationary distribution. The Monte Carlo concept uses the Law of Large Numbers, so the formula that is used is as follows:

$$\mathbb{E}(\theta \mid Y) \approx \frac{1}{N}\sum_{i=1}^{N}\theta^{(i)}, \quad \theta^{(i)} \sim Pr(\theta \mid Y)$$

One of the widely used methods within the MCMC is the Metropolis-Hastings (MH) algorithm, which was proposed by Metropolis et al (1953) and generalized by Hastings (1970). It approximates the unknown posterior distribution by drawing from a time-reversible Markov chain. The Markov chain converges to the stationary distribution. For this research, the Random Walk Metropolis-Hastings Method (RWMHM) is used, which is a form of the MH algorithm. In RWMHM the proposal is a symmetric random walk centered at the current value.

For both methods a total number of draws, a burn-in period, and an initial θ_0 are chosen. For each iteration there is a $\tilde{\theta}$ simulated from the proposal distribution. Then the acceptance probability is calculated to see whether $\tilde{\theta}$ should be accepted or rejected as the next state in the chain. Algorithm 1 gives the pseudo-code of the general MH algorithm. The pseudo-code for RWMHM is given in Algorithm 2.

Algorithm 1 Metropolis-Hastings

Choose n_{draws} and $n_{burn-in}$
Choose initial θ_0
for $i = 1, 2, \ldots, n_{draws}$ **do**
  Simulate proposal: $\tilde{\theta} \sim Q(\cdot \mid \theta_{i-1})$ ▷ $Q(\cdot \mid \theta_{i-1})$ denotes the candidate distribution
  Compute acceptance probability:

$$\alpha = \min\left\{\frac{\Pr(\tilde{\theta})Q(\theta_{i-1} \mid \tilde{\theta})}{\Pr(\theta_{i-1})Q(\tilde{\theta} \mid \theta_{i-1})}, 1\right\}$$

  Simulate $U \sim \text{Uniform}(0, 1)$
  if $U \leq \alpha$ **then**
    Accept proposal: $\theta_i = \tilde{\theta}$
  else
    Reject proposal: $\theta_i = \theta_{i-1}$
  end if
end for

Algorithm 2 Random Walk Metropolis-Hastings Method

Choose n_{draws} and $n_{burn-in}$
Choose initial θ_0
for $i = 1, 2, \ldots, n_{draws}$ **do**
  Simulate candidate draw: $\tilde{\theta} \sim Q(\cdot \mid \theta_{i-1})$ ▷ $Q(\cdot \mid \theta_{i-1})$ denotes the candidate distribution
  Compute acceptance probability: $\alpha = \min\left\{\frac{\Pr(\tilde{\theta})}{\Pr(\theta_{i-1})}, 1\right\}$
  Simulate $U \sim \text{Uniform}(0, 1)$
  if $U \leq \alpha$ **then**
    Accept candidate draw: $\theta_i = \tilde{\theta}$
  else if $U > \alpha$ **then**
    Reject candidate draw: $\theta_i = \theta_{i-1}$
  end if
end for

5 Distributions

The estimation of the GARCH model parameters is needed to obtain the conditional variances. Section 4 discussed which approaches are used to estimate the parameters. The log-likelihood functions are used for these estimations and those are based on distributions. In Section 2 it was concluded from the Jarqua-Bera test and the QQ-plots that the log-returns are not normally distributed. It was also observed that the JPM stock has a fatter tail on the left side while the NVDA stock has a fatter tail on the right side due to the skewness. In order to capture the presence of heavy tails, the Student-t distribution was chosen as an alternative to the normal distribution. To capture the skewness the Skewed Student-t distribution was chosen. The Normal distribution is also used for comparison with the alternative distributions.

5.1 Normal Distribution

The Normal distribution is also known as the Gaussian distribution. It is one of the most commonly used distributions for statistical modeling. It is a very desirable distribution because of its symmetry and finite moments. It uses the parameters μ and σ representing the mean and variance, respectively. This distribution does not always properly capture the characteristics of financial return data. Financial data often show skewness and high kurtosis, just like the dataset used for this research. The Normal distribution is symmetric around its mean and features thin tails, which means it cannot properly capture financial datasets with extreme market movements. That is why other alternatives are considered to be better for datasets with asymmetry and heavy tails. The PDF for the Normal distribution (Hoogerheide, 2023) is as follows:

$$f(r_t \mid \theta) = \frac{1}{\sqrt{2\pi\sigma_t^2}} e^{\dfrac{-(r_t - \mu_t)^2}{2\sigma_t^2}}$$

Since the log-likelihood functions are used for these estimations, the log-likelihood function for the Normal distribution (Hoogerheide, 2023) is:

$$l(r_t; \theta) = -\frac{n}{2}\log(2\pi) - \frac{1}{2}\sum_{t=1}^{n}\left(\log(\sigma_t^2) + \frac{(r_t - \mu_t)^2}{\sigma_t^2}\right)$$

5.2 Student-t Distribution

The Student-t distribution is a good alternative to the Normal distribution and it is also very widely used. This distribution is more suitable for datasets with high kurtosis, hence having a heavier tail. The PDF used for Student-t distribution (Hoogerheide, 2023) is:

$$f(r_t \mid \theta) = \frac{\Gamma\left(\dfrac{\nu+1}{2}\right)}{\Gamma\left(\dfrac{\nu}{2}\right)\sqrt{(\nu-2)\pi\sigma_t^2}} \left(1 + \frac{(r_t-\mu_t)^2}{(\nu-2)\sigma_t^2}\right)^{\dfrac{-(\nu+1)}{2}}$$

The corresponding log-likelihood function of the student-t distribution (Hoogerheide, 2023) is given as:

$$\begin{aligned} l(r_t;\theta) = n &\left[\log\Gamma\left(\frac{\nu+1}{2}\right) - \log\Gamma\left(\frac{\nu}{2}\right) - \frac{1}{2}\log\left(\pi(\nu-2)\right)\right] \\ &- \frac{1}{2}\sum_{t=1}^{n}\left[\log(\sigma_t^2) + (\nu+1)\log\left(1 + \frac{(r_t-\mu)^2}{(\nu-2)\sigma_t^2}\right)\right] \end{aligned}$$

Here, the distribution incorporates an extra parameter ν compared to the Normal distribution. This parameter represents the degrees of freedom, which controls the heaviness of the tails. Just like the Normal distribution, the Student-t distribution is symmetric around the mean and does not account for skewness in the data.

5.3 Skewed Student-t Distribution

A financial dataset with a kurtosis value greater than 3 indicates that it has heavy tails, which makes the Student-t distribution a suitable distribution to use for modeling. However, it does not capture the skewness of the datasets, because it stays symmetric around the mean. The limitation of the Student-t distribution shows that there is an extension needed which captures the skewness, namely the Skewed Student-t distribution. The PDF for the Skewed Student-t distribution (Hoogerheide, 2023) is denoted as:

$$f(r_t \mid \theta) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)\sqrt{(\nu-2)\pi\sigma_t^2}} \times s \times \left(\frac{2}{\xi+\frac{1}{\xi}}\right) \times \left(1 + \frac{(s\frac{r_t-\mu_t}{\sigma_t}+m)^2}{\nu-2}\xi^{-2I_t}\right)^{\frac{-(\nu+1)}{2}}$$

Here, the parameters s, m and I_t have the following formulations:

- $s = \sqrt{\left(\xi^2 + \frac{1}{\xi^2} - 1\right) - m^2}$
- $m = \frac{\Gamma\left(\frac{\nu-1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \times \sqrt{\frac{\nu-2}{\pi}} \times \left(\xi - \frac{1}{\xi}\right)$
- $I_t = \begin{cases} 1 & \text{if } s\frac{r_t-\mu_t}{\sigma_t} + m \geq 0 \\ -1 & \text{if } s\frac{r_t-\mu_t}{\sigma_t} + m < 0 \end{cases}$

The log-likelihood function of the Skewed Student-t distribution (Hoogerheide, 2023) is given as follows:

$$\begin{aligned} l(r_t;\theta) = \; & \log\Gamma\left(\frac{\nu+1}{2}\right) - \log\Gamma\left(\frac{\nu}{2}\right) - \frac{1}{2}\log\left((\nu-2)\pi\sigma_t^2\right) \\ & + \log(s) + \log\left(\frac{2}{\xi+\frac{1}{\xi}}\right) - \frac{\nu+1}{2}\log\left(1 + \frac{\left(s\frac{r_t-\mu_t}{\sigma_t}+m\right)^2}{\nu-2}\xi^{-2I_t}\right) \end{aligned}$$

The parameter ξ captures the skewness making it useful for modeling with data that has heavy tails and skewness. If $\xi > 1$, then distribution skews to the right and if $\xi < 1$ then it skews to the left. The distribution reduces to the symmetric Student-t distribution when $\xi = 1$, $s = 1$, and $m = 0$. Financial datasets usually show excess kurtosis but also significant asymmetries. Hence, the Skewed Student-t distribution is suitable for such datasets, unlike the Student-t and Normal distribution.

6 Performance Tests

Volatility forecasts for financial returns can sometimes be inaccurate, because of misspecification or non-linear dynamics. To find the best model for capturing volatility, it is important to evaluate and compare model performance. This is done by using different diagnostic tests and evaluation metrics, which assess in-sample fit and out-of-sample accuracy. The evaluations will show how accurate the model fit is to the observed data. For this paper, both in-sample and out-of-sample tests are used on the models. VaR backtesting is used to evaluate the accuracy of the forecasted VaR and to determine whether the model underestimates or overestimates risk. Information Criteria will help with selecting the best model and Loss Functions give an indication of which model has the best accuracy with the forecasting. The Diebold-Mariano test uses the results of the Loss Functions to compare two models with each other to see which one is more accurate.

6.1 Value-at-Risk Backtesting

The Value-at-Risk (VaR) is used to estimate the maximum potential loss of the stock returns. VaR is widely used in risk management, especially when modeling volatility. The VaR is computed using the inverse CDF, which depends on the distributional assumption. The formula to obtain the VaR is given as:

$$\text{VaR}_{\alpha,t} = -\hat{\sigma}_t \cdot \Phi^{-1}(\alpha)$$

Here, Φ^{-1} is the inverse CDF of the standardized distribution and the confidence level α that is used in this paper to forecast the VaR is 5%. The estimated VaR is used for backtesting, which evaluates the accuracy of these forecasts by comparing them with realized returns. The results will indicate how much the model will capture the actual risk. For backtesting there is an indicator variable I_t used, which indicates if a violation occurred. A violation occurs when $r_t < \text{VaR}_t$, which is denoted as as $I_t = 1$, and if there are no violation then $I_t = 0$. Here, r_t is denoted as the stock returns. The violations are used for the calculations of the violation ratio, which indicates whether the model underestimates risk or overestimates risk. If the violation ratio is approximately 1 then the model predicts VaR violations very accurately. If the violation ratio is bigger than 1 then the model has too many violations, which means it underestimates risk. For a violation rate smaller than 1 it means that the model does not have a lot of violations and therefore the model overestimates risk.

6.1.1 Unconditional Coverage Test

The Unconditional Coverage (UC) test (Kupiec, 1995) is used to check if the observed number of VaR violations matches the expected number under the model. The likelihood ratio of a binomial model is used for this test and a hypothesis test is conducted which is stated as follows:

$$\begin{aligned} H_0 &: \text{The violation rate} = \alpha \\ H_1 &: \text{The violation rate} \neq \alpha \end{aligned}$$

A good model does not reject the null hypothesis, which indicates that the observed violation rate is equal to the nominal VaR level α. Under the null hypothesis the UC test statistic asymptotically follows a chi-squared distribution. For the likelihood ratio of UC the numbers of non-violations, n_0, and the number of violations, n_1, are used, which can be seen below (where $\hat{\pi}_1 = n_1/(n_0 + n_1)$):

$$LR_{\text{UC}} = -2 \log \frac{(1-\alpha)^{n_0}(\alpha)^{n_1}}{(1-\hat{\pi}_1)^{n_0}(\hat{\pi}_1)^{n_1}} \sim \chi^2(1)$$

6.1.2 Independence Test

The Independence (IND) test (Christoffersen, 1998) is used to evaluate whether violations occur independently over time. The hypothesis test that is conducted for IND is stated as follows:

$$\begin{aligned} H_0 &: \text{VaR violations are independently distributed} \\ H_1 &: \text{VaR violations are serially dependent} \end{aligned}$$

A good model will have randomly scattered violations instead of clusters, hence the null hypothesis should not be rejected for a good model. The likelihood ratio of IND also asymptotically follows a chi-squared distributed under the null hypothesis and is as follows:

$$\pi_{01} = \frac{n_{01}}{n_{00} + n_{01}}, \quad \pi_{11} = \frac{n_{11}}{n_{10} + n_{11}}, \quad \pi_1 = \frac{n_{01} + n_{11}}{n_{00} + n_{01} + n_{10} + n_{11}}$$

$$LR_{\text{IND}} = -2 \log \left(\frac{(1 - \pi_1)^{n_{00}+n_{10}} \pi_1^{n_{01}+n_{11}}}{(1 - \pi_{01})^{n_{00}} \pi_{01}^{n_{01}} (1 - \pi_{11})^{n_{10}} \pi_{11}^{n_{11}}} \right) \sim \chi^2(1)$$

Here, the likelihood ratio of IND uses $n_{i,j}$ to give the number of times a violation transition occurs from state i to state j. So, n_{00} is defined as the total number of observations where no violations occur in both state j and the previous state i. For n_{11} it is the total number of observations where violations occur in both state j and the previous state i. Lastly, n_{01} is the number of observations a violation occurred following a period without violation, while n_{10} is the number of observations a period without violation followed a violation.

6.1.3 Conditional Coverage Test

The Conditional Coverage (CC) test (Christoffersen, 1998) evaluates if the model has both the right number of violations and the independence of violations. Hence, it is a joint test that combines the UC test and the IND test. The hypothesis test for this is given below:

H_0 : Model satisfies both unconditional coverage and independence

H_1 : Model does not satisfy both unconditional coverage and independence

Again, the best model would be the model where the null hypothesis does not get rejected. The formula for the likelihood ratio of CC is the sum of the likelihood ratio of UC and the likelihood ratio of IND. Since the UC and IND test statistics are asymptotically independent under the null hypothesis, the CC test statistic asymptotically follows a chi-squared distribution with 2 degrees of freedom under the null hypothesis. The likelihood ratio of CC is given with the following formula:

$$LR_{\text{CC}} = LR_{\text{UC}} + LR_{\text{IND}} \sim \chi^2(2)$$

6.2 Information Criteria

A quantitative method that is used to evaluate competing models based on their goodness-of-fit is the Information Criteria. There are two commonly used Information Criteria, which are the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC). These criteria are used in an in-sample context and both are used for this research.

6.2.1 Akaike Information Criterion

The Akaike Information Criterion (AIC), introduced by Akaike (1969), measures the quality of statistical models based on their maximum likelihood estimation. AIC helps with determining which model is the best to use for volatility modeling. It estimates the Kullback-Leibler divergence between the true model and the estimated model. AIC does not capture the sample size, but applies a fixed penalty per parameter which can lead to overfitting in the dataset. The formula used to obtain the AIC is as follows:

$$AIC = 2K - 2L$$

Here, L is the maximized log-likelihood and K is the numbers of parameters in the model. The lower the AIC value of a model is, the better the model is to use for volatility modeling. So it is important to see which model obtains the lowest AIC value, which will then indicate that it is the best model to capture volatility.

6.2.2 Bayesian Information Criterion

The Bayesian Information Criterion (BIC), introduced by Schwarz (1978), is also an Information Criterion but derived from the Bayesian framework. It is similar to the AIC, but the BIC does capture the sample size of the dataset. The formula of BIC is at follows:

$$BIC = \log(n)K - 2L$$

Here, n is for the sample size. The BIC favors models that are less complex, because it has a stronger penalty for models that have more numbers of parameters. Just like the AIC, the model which has the lowest BIC value is preferred.

6.3 Loss Functions

Loss functions are quantitative tools that are used to evaluate the forecasting accuracy of the models and they are used in out-of-sample context. The idea involves the difference between the predicted values and the actual observations, which will provide insight into the model's predictive performance. For this paper, four loss functions are used, Mean Squared Error (MSE), Mean Absolute Error (MAE), Mean Absolute Percentage Error (MAPE), and Root Mean Squared Error (RMSE).

6.3.1 Mean Squared Error

The Mean Squared Error (MSE) (Chicco et al., 2021) computes the average of the squared difference between the estimated value $\hat{\sigma_t}^2$ and the actual observed value σ_t^2. This result gives what the distance is between the two. The MSE gets squared which ensures that all errors are positive. The lower the MSE is the shorter the distance is between the predicted value and the observed value, which indicates that the predicted value is close to the actual value. That means that if the MSE value is small that the prediction of the model is accurate. Hence, the model has a good performance for forecasting the volatility. The formula for the MSE is as follows:

$$MSE = \frac{1}{n} \sum_{t=1}^{n} (\sigma_t^2 - \hat{\sigma_t}^2)^2$$

6.3.2 Mean Absolute Error

The Mean Absolute Error (MAE) (Chicco et al., 2021) looks similar to the MSE. The difference is that the MAE looks at the average of the absolute differences between the estimated value $\hat{\sigma_t}^2$ and the actual observed value σ_t^2. The absolute value treats all deviations equally and is therefore more robust to outliers compared to the MSE. Just like with MSE, the model with the lowest MAE value is preferred, as it indicates better performance in forecasting volatility. The formula used to obtain MAE is as follows:

$$MAE = \frac{1}{n} \sum_{t=1}^{n} |\sigma_t^2 - \hat{\sigma_t}^2|$$

6.3.3 Mean Absolute Percentage Error

The Mean Absolute Percentage Error (MAPE) (Chicco et al., 2021) gives the average absolute percentage of the distance between the actual value $\hat{\sigma}^2$ and the predicted value σ^2. The result will be presented in percentages of how accurate the models are. If the result is a small percentage then it indicates that the deviation is small, hence that the model has a good performance.

$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{\sigma_t^2 - \hat{\sigma_t}^2}{\sigma_t^2}\right| \times 100\%$$

6.3.4 Root Mean Squared Error

The Root Mean Squared Error (RMSE) (Chicco et al., 2021) is the square root of MSE. The difference is that it does penalize large errors more than the MAE, however it is still not robust to outliers like MSE. The formula of RMSE is given as:

$$RMSE = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(\sigma_t^2 - \hat{\sigma_t}^2)^2}$$

6.4 Logarithmic Scoring Rule

The Logarithmic Scoring Rule (LSR) is like the Loss Functions also an out-of-sample evaluation metric. It is a scoring rule that evaluates not just one predicted value but the entire predicted probability distribution and in return gives a score for this. A higher LSR score indicates that the model has a better performance. The formula to obtain the LSR score is as follows:

$$LS = \frac{1}{n}\sum_{t=1}^{n}\log p(r_t|\theta, r_{t-1}, r_{t-1}, \ldots)$$

Here, the average of the score function is taken, which uses the PDF of the distribution that the model uses. The $\log p(r_t|\theta, r_{t-1}, r_{t-2}, \ldots)$ gives the log-density of the observed value. The higher the value of the LSR, the better the model performs. A more negative value indicates that the model has a worse predictive performance.

6.5 Diebold-Mariano Test

The Diebold-Mariano (DM) test, introduced by Diebold and Mariano (1995), is used to compare the accuracy of the forecast between two models. This test is based on the Loss Functions, where it uses the difference in forecasting performance of the models. In this paper, the GARCH models are getting compared with each other separately for each distribution. The test is conducted separately under Maximum Likelihood Estimation (MLE) and Random Walk Metropolis-Hastings method (RWMHM). The formula for the DM Test is given as:

$$DM = \frac{\bar{d}}{\hat{\sigma}_{\bar{d}}},$$

The difference between the Loss Functions of the two models is given by $d_t = g(e_{1,t}) - g(e_{2,t})$. Here, $g(\cdot)$ is a specific Loss Function, $e_{1,t}$ and $e_{2,t}$ are the forecast error from model 1 and model 2, respectively. If $DM < 0$ then that indicates that model 1 is better than model 2, because the average loss of model 1 is lower. If $DM > 0$ then the average loss of model 1 is higher than model 2, which indicated that model 1 gives a worse forecast. The $\bar{d}$ is the sample mean of the loss differential which is given by $\bar{d} = \frac{1}{n}\sum_{t=1}^{n} d_t$. The $\hat{\sigma}_{\bar{d}}$ gives the standard error of $\bar{d}$, which was introduced by Newey and West (1994). The DM test statistic is compared to the critical values of the standard normal distribution, and a 5% significance level is used to assess statistical significance. The hypotheses for the DM test are stated as follows:

H_0 : The forecasting accuracy of the two models is equal

H_1 : The forecasting accuracy of one model is better than that of the other

If the null hypothesis is not rejected and the DM value is approximately 0 then there is no significant difference in accuracy of the forecast between the two models. If the null hypothesis is rejected and the DM value is negative then model 1 is more accurate than model 2. If the DM value is positive then model 2 is more accurate than model 1.

7 Results

In this section, the results of the models are going to be discussed. The purpose of this thesis is to compare the forecasting performance of the volatility models applied to stocks from different sectors. The stocks that were chosen are JPMorgan Chase & Co. (JPM) from the banking sector and NVIDIA Corporation (NVDA) from the technology sector. The reason for this is to explore how the volatility models perform for two different sectors.

7.1 GARCH Model Results

The GARCH models are used to estimate the conditional variance, which represents the time-varying volatility of a stock. The estimation of the parameters is presented in the tables in Appendix B. Tables 3 and 4 show the Random Walk Metropolis-Hastings Method acceptance percentages. The rates are between 20% and 55%, which shows that the proposal distribution works well. The graphs of the conditional variances can be seen in Appendix C. These graphs show that the ARCH(1) model overreacts the most to large shocks, while the other models have smoother estimates of the conditional variance. The GAS model also produces more frequent spikes in conditional variance, which can be a reaction to changes in the stock return. The GJR-GARCH(1,1) model is similar to the GARCH(1,1) model, but it is better at capturing asymmetric shocks. The EGARCH(1,1) model has smooth and persistent variance paths, especially with the Student-t and Skewed Student-t distributions.

It can be seen that the Bayesian approach captures the conditional variance more smoothly and stably than the Frequentist approach. The Frequentist approach shows more extreme spikes and clustering; hence, the Bayesian approach is potentially better at capturing the model's volatility. Comparing the three distributions shows that the Normal distribution is a poorer fit with all models. This was expected because of the outcome obtained with the JB test in Section 2. The Student-t and Skewed Student-t distributions show that they capture heavy tails better and that they provide more accurate estimates of the conditional variance. Thus, the EGARCH and the GJR-GARCH models with the Student-t and Skewed Student-t distributions appear to be the best models for capturing the volatility.

7.2 AIC and BIC Results

The AIC and BIC were computed, and the results are presented in Appendix D. Table 9 shows the results for stock JPM and Table 10 shows the results for stock NVDA. For both stocks it can be seen that the EGARCH model has the highest log-likelihood and the lowest AIC and BIC values. This indicates that EGARCH is the best model to capture the volatility. It can also be seen that the Skewed Student-t distribution is the most suitable distribution to use. However, for JPM the Student-t distribution has the lowest BIC value, indicating that it may be preferred when penalizing model complexity more heavily.

7.3 Loss Function Results

The loss function is important for determining how well a volatility model can predict and manage financial risk. In Appendix E, the results of MSE, MAE, MAPE, and RMSE are presented. Table 11 shows that the models EGARCH and GJR-GARCH perform the best with the Frequentist approach, especially under the Normal distribution. With the Bayesian approach, it can be seen in Table 12 that the GJR-GARCH model is the best model with the use of the Student-t distribution. Looking at the NVDA stock next, it can be seen in Table 13 that EGARCH is the best performing model with the Normal distribution for the Frequentist approach. For the Bayesian approach, in Table 14, it can be seen that the GJR-GARCH model performs the best with the Skewed Student-t distribution. Hence, under the Frequentist approach, EGARCH with the Normal distribution is the best performing model, while the GJR-GARCH model with the Student-t or Skewed Student-t distribution is the best performing model under the Bayesian approach.

7.4 Value-at-Risk Backtesting Results

In Appendix F, the results of the Value-at-Risk backtesting are presented in Table 15 and 16. For the JPM stock, the lowest UC value is obtained with the EGARCH model with the Normal distribution for both the Frequentist and Bayesian approach. However, the lowest IND and CC values are obtained for the GJR-GARCH model with the Skewed Student-t distribution for the Frequentist approach. For the NVDA stock, the lowest IND value (0.0000) is obtained for the GJR-GARCH model with the Student-t distribution for the Frequentist approach. The null hypothesis also does not get rejected, so this indicates that there is perfect independence. However, here the lowest UC value is obtained for the GARCH model with the Normal distribution for both the Frequentist and Bayesian approach. For both stocks, it can be seen that for all CC tests the null hypothesis gets rejected, which indicates that either the violation count or the independence forms a problem. In this case, all the UC tests also show that the null hypothesis gets rejected, so none of the models has a violation rate that is equal to the nominal VaR level α. So, the EGARCH and GARCH models with the Normal distribution are the best models when it comes to the violation count. However, the GJR-GARCH model with the Student-t and Skewed Student-t distributions performs the best on independence.

7.5 Logarithmic Scoring Rule Results

The results of the Logarithmic scoring rule can be found in Appendix G. From both Tables 17 and 18, it can be seen that the lowest score is obtained for the EGARCH model with the Normal distribution and for the Bayesian approach. This indicates that this model gives the most accurate density forecast. The other two models that come close to the lowest score are the GARCH model and the GJR-GARCH model, both also with the Normal distribution. The worst-performing models are the ARCH and GAS models with the Student-t and Skewed Student-t distributions. The reason why the Normal distribution scores better than the Student-t and Skewed Student-t distributions is likely due to the fact that the log-score does not improve with those distributions, even if they capture fat tails better.

7.6 Diebold-Mariano Test Results

The results for the DM test are represented in the tables in Appendix H. For stock JPM, in Table 19, it can be seen that the EGARCH model with the Skewed Student-t distribution and for the Bayesian approach outperforms all the other models. It can also be seen that every model outperforms the ARCH model and that the GJR-GARCH model outperforms the GARCH model. In Table 20, it can also be seen that the EGARCH model for the Bayesian approach again outperforms the other models, however, this time with the Student-t distribution. Also, the GAS model only outperforms the GJR-GARCH model and the GARCH model under the Normal distribution. In Table 21, the p-values indicate that no pair of models shows a statistically significant difference in MAPE at the 5% level. However, the GARCH model appears to outperform most models. The best distribution here is the Skewed Student-t distribution for the Bayesian approach. In Table 22, it shows again that the EGARCH model under the Student-t distribution for the Bayesian approach is the best performing model compared to the other models.

For stock NVDA, in Table 23, it can be seen that the GJR-GARCH model gets outperformed by both the ARCH and GARCH model. The ARCH model also outperforms the GAS model. However, the EGARCH model with the Skewed Student-t for the Bayesian approach outperforms all the other models. In Table 24, the GARCH and GJR-GARCH models outperform the GAS model. Overall, the EGARCH model under the Skewed Student-t distribution for the Bayesian approach outperforms the other models. In Table 25, EGARCH with the Skewed Student-t distribution for the Bayesian approach is again the best performing model compared to the other models. However, this time the GAS model under the Normal distribution for the Bayesian approach is also a very good performing model. In the last Table 26, the EGARCH with the Skewed Student-t distribution for the Bayesian approach again outperforms the other models. Here, the GARCH and GJR-GARCH models do outperform the GAS model.

Hence, looking at the results of the DM test, it can be seen that overall the EGARCH model under the Skewed Student-t distribution for the Bayesian approach outperforms the other models. This is most likely due to the fact that the EGARCH model captures large shocks and asymmetry better, and the Skewed Student-t distribution captures datasets with fat tails and skewness better. The Bayesian approach also performed better than the Frequentist approach, because it often produces smoother, more robust parameter estimates, especially when combined with fat-tailed or skewed distributions. The ARCH model is the most outperformed model, which was expected since it is the simplest model and reacts strongly to large shocks.

8 Conclusion

The objective of this thesis was to evaluate and compare different models that forecast the volatility of stock returns from two different stock sectors. This thesis used the JPMorgan Chase & Co. (JPM) stock from the banking sector and the NVIDIA Corporation (NVDA) stock from the technology sector. From the results, it can be concluded that overall the model that performed the best was the EGARCH model under the Skewed Student-t distribution using the Bayesian approach. This was expected since this model captures fat tails, asymmetry, and clustering in stock returns. The EGARCH model is especially useful in capturing the volatility of NVDA, which is a stock in a volatile market. The ARCH model was overall the weakest model, giving poor statistical performance. The VaR backtesting showed that none of the models passed all the tests, with the Unconditional Coverage (UC) or Conditional Coverage (CC) tests not being achieved; however, EGARCH and GJR-GARCH performed the best.

The Bayesian approach outperformed the Frequentist approach in most cases because it gives more stable and smoother estimates of the conditional variance. It also performed statistically stronger in the DM test compared to the Frequentist approach. The Normal distribution was outperformed by the other two distributions. This is because it does not capture the fat tails and skewness of a dataset well, like the Student-t and Skewed Student-t distributions. However, the Normal distribution did score the best for the Logarithmic Scoring Rule. This indicates that there is better density forecasting accuracy when there are no extreme conditions.

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A Appendix: Graphs of Prices and Log-returns

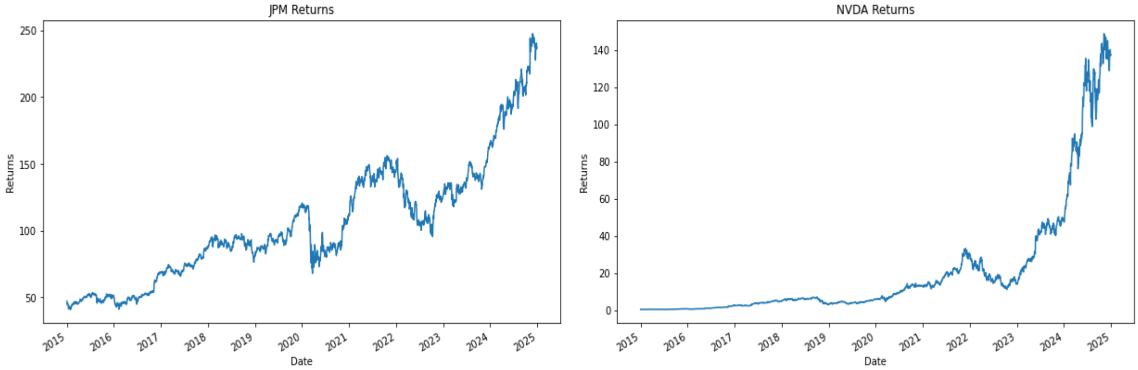

Figure 1: Daily adjusted closing prices of the chosen stock markets

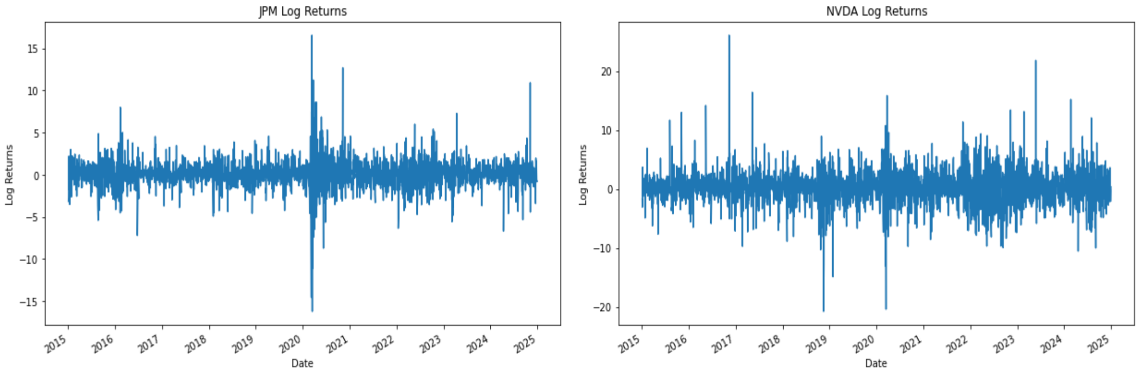

Figure 2: Log-returns of the chosen stock markets

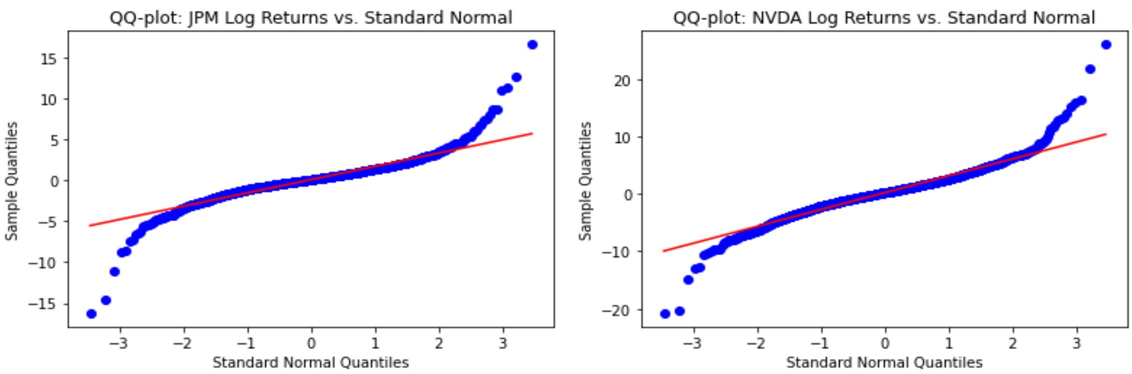

Figure 3: QQ-plots of the chosen stock markets

B Appendix: Results of Parameter Estimation

Table 3: Acceptance Percentages of JPM

	Normal	Student-t	Skewed t
ARCH	54.18%	44.36%	38.18%
GARCH	47.45%	37.64%	28.91%
GJR-GARCH	39.91%	33.18%	26.73%
EGARCH	39.18%	33.18%	27.00%
GAS	46.18%	39.73%	33.91%

Table 4: Acceptance Percentages of NVDA

	Normal	Student-t	Skewed t
ARCH	53.18%	42.36%	36.64%
GARCH	45.36%	40.36%	33.00%
GJR-GARCH	42.55%	32.36%	28.55%
EGARCH	38.91%	34.18%	26.64%
GAS	49.09%	43.18%	36.82%

Table 5: Posterior Means and Posterior Standard Deviations of JPM

	Parameter	Normal	Student-t	Skewed t
ARCH	$\hat{\omega}$	1.6829	1.7647	1.8159
		(0.0647)	(0.1249)	(0.1413)
	$\hat{\alpha_1}$	0.4069	0.4289	0.4371
		(0.0384)	(0.0544)	(0.0587)
	$\hat{\nu}$		4.0094	3.8504
			(0.3426)	(0.3140)
	$\hat{\xi}$			0.9686
				(0.0215)
GARCH	$\hat{\omega}$	0.2320	0.1589	0.1549
		(0.0638)	(0.0536)	(0.0423)
	$\hat{\alpha_1}$	0.1404	0.1483	0.1467
		(0.0262)	(0.0320)	(0.0263)
	$\hat{\beta_1}$	0.7678	0.7988	0.8014
		(0.0488)	(0.0438)	(0.0384)
	$\hat{\nu}$		4.6230	4.7132
			(0.4040)	(0.4176)
	$\hat{\xi}$			0.9634
				(0.0192)
GJR-GARCH	$\hat{\omega}$	0.1699	0.1540	0.1414
		(0.0390)	(0.0410)	(0.0358)
	$\hat{\alpha_1}$	0.0351	0.0414	0.0371
		(0.0116)	(0.0158)	(0.0155)
	$\hat{\beta_1}$	0.8167	0.8071	0.8185
		(0.0306)	(0.0351)	(0.0359)
	$\hat{\gamma_1}$	0.1815	0.2257	0.2177
		(0.0307)	(0.0521)	(0.0522)
	$\hat{\nu}$		4.8551	4.8554
			(0.4297)	(0.4082)
	$\hat{\xi}$			0.9639
				(0.0236)
EGARCH	$\hat{\omega}$	-0.0851	-0.1088	-0.0961
		(0.0133)	(0.0171)	(0.0173)
	$\hat{\alpha_1}$	0.1704	0.1961	0.1804
		(0.0215)	(0.0275)	(0.0294)
	$\hat{\beta_1}$	0.9530	0.9591	0.9602
		(0.0080)	(0.0086)	(0.0086)
	$\hat{\gamma_1}$	-0.1167	-0.1272	-0.1233
		(0.0143)	(0.0173)	(0.0156)
	$\hat{\nu}$		5.0209	4.9170
			(0.4495)	(0.4703)
	$\hat{\xi}$			0.9603
				(0.0251)
GAS	$\hat{\omega}$	0.0315	0.0351	0.0334
		(0.0075)	(0.0125)	(0.0104)
	$\hat{\alpha_1}$	0.0501	0.1551	0.1529
		(0.0064)	(0.0233)	(0.0220)
	$\hat{\beta_1}$	0.9601	0.9553	0.9586
		(0.0093)	(0.0133)	(0.0122)
	$\hat{\nu}$		4.6731	4.6682
			(0.3948)	(0.4418)
	$\hat{\xi}$			0.9580
				(0.0265)

Table 6: Posterior Means and Posterior Standard Deviations of NVDA

	Parameter	Normal	Student-t	Skewed t
ARCH	$\hat{\omega}$	7.5955	7.6847	8.0256
		(0.2884)	(0.5447)	(0.6882)
	$\hat{\alpha_1}$	0.2002	0.2457	0.2622
		(0.0327)	(0.0484)	(0.0515)
	$\hat{\nu}$		3.9176	3.7143
			(0.3208)	(0.3185)
	$\hat{\xi}$			0.9232
				(0.0211)
GARCH	$\hat{\omega}$	0.8803	0.2690	0.2577
		(0.1779)	(0.0927)	(0.0843)
	$\hat{\alpha_1}$	0.1112	0.0857	0.0879
		(0.0217)	(0.0164)	(0.0159)
	$\hat{\beta_1}$	0.7992	0.8917	0.8961
		(0.0329)	(0.0226)	(0.0202)
	$\hat{\nu}$		4.3556	4.1053
			(0.3672)	(0.3654)
	$\hat{\xi}$			0.9180
				(0.0238)
GJR-GARCH	$\hat{\omega}$	1.2776	0.4189	0.4304
		(0.2033)	(0.1230)	(0.1483)
	$\hat{\alpha_1}$	0.0240	0.0333	0.0351
		(0.0114)	(0.0100)	(0.0125)
	$\hat{\beta_1}$	0.7346	0.8572	0.8586
		(0.0366)	(0.0285)	(0.0303)
	$\hat{\gamma_1}$	0.2560	0.1730	0.1735
		(0.0445)	(0.0437)	(0.0447)
	$\hat{\nu}$		4.5163	4.3697
			(0.3864)	(0.4819)
	$\hat{\xi}$			0.9224
				(0.0222)
EGARCH	$\hat{\omega}$	0.0780	-0.0487	-0.0505
		(0.0272)	(0.0231)	(0.0208)
	$\hat{\alpha_1}$	0.2330	0.2036	0.2057
		(0.0237)	(0.0298)	(0.0263)
	$\hat{\beta_1}$	0.8884	0.9559	0.9573
		(0.0155)	(0.0114)	(0.0115)
	$\hat{\gamma_1}$	-0.1396	-0.0984	-0.0940
		(0.0196)	(0.0206)	(0.0182)
	$\hat{\nu}$		4.6859	4.5047
			(0.3905)	(0.4692)
	$\hat{\xi}$			0.9234
				(0.0215)
GAS	$\hat{\omega}$	0.2103	0.0524	0.0702
		(0.0471)	(0.0127)	(0.0259)
	$\hat{\alpha_1}$	0.0436	0.1110	0.1246
		(0.0079)	(0.0149)	(0.0217)
	$\hat{\beta_1}$	0.9026	0.9747	0.9673
		(0.0214)	(0.0061)	(0.0114)
	$\hat{\nu}$		4.5332	4.3491
			(0.3656)	(0.3586)
	$\hat{\xi}$			0.9243
				(0.0224)

Table 7: Maximum Likelihood Estimators and Standard Errors of JPM

	Parameter	Normal	Student-t	Skewed Student-t
ARCH	$\hat{\omega}$	1.6762	1.7522	1.7750
		(0.0635)	(0.1290)	(0.1337)
	$\hat{\alpha_1}$	0.4075	0.4265	0.4249
		(0.0379)	(0.0584)	(0.0588)
	$\hat{\nu}$		3.9359	3.8982
			(0.3381)	(0.3347)
	$\hat{\xi}$			0.9704
				(0.0220)
GARCH	$\hat{\omega}$	0.1882	0.1295	0.1258
		(0.0690)	(0.0420)	(0.0412)
	$\hat{\alpha_1}$	0.1218	0.1310	0.1288
		(0.0280)	(0.0280)	(0.0276)
	$\hat{\beta_1}$	0.8028	0.8249	0.8296
		(0.0533)	(0.0377)	(0.0368)
	$\hat{\nu}$		4.6279	4.5895
			(0.4296)	(0.4263)
	$\hat{\xi}$			0.9582
				(0.0236)
GJR-GARCH	$\hat{\omega}$	0.1417	0.1232	0.1184
		(0.0367)	(0.0360)	(0.0355)
	$\hat{\alpha_1}$	0.0275	0.0309	0.0297
		(0.0099)	(0.0143)	(0.0141)
	$\hat{\beta_1}$	0.8400	0.8359	0.8423
		(0.0296)	(0.0340)	(0.0337)
	$\hat{\gamma_1}$	0.1687	0.1999	0.1939
		(0.0309)	(0.0434)	(0.0426)
	$\hat{\nu}$		4.8499	4.7957
			(0.4605)	(0.4551)
	$\hat{\xi}$			0.9615
				(0.0245)
EGARCH	$\hat{\omega}$	-0.0813	-0.0964	-0.0939
		(0.0119)	(0.0185)	(0.0182)
	$\hat{\alpha_1}$	0.1605	0.1729	0.1698
		(0.0193)	(0.0296)	(0.0289)
	$\hat{\beta_1}$	0.9576	0.9650	0.9658
		(0.0072)	(0.0083)	(0.0081)
	$\hat{\gamma_1}$	-0.1131	-0.1198	-0.1191
		(0.0129)	(0.0166)	(0.0164)
	$\hat{\nu}$		4.9596	4.9048
			(0.4743)	(0.4688)
	$\hat{\xi}$			0.9552
				(0.0245)
GAS	$\hat{\omega}$	0.0281	0.0278	0.0281
		(0.0063)	(0.0098)	(0.0098)
	$\hat{\alpha_1}$	0.0478	0.1441	0.1431
		(0.0055)	(0.0218)	(0.0216)
	$\hat{\beta_1}$	0.9645	0.9635	0.9641
		(0.0074)	(0.0113)	(0.0111)
	$\hat{\nu}$		4.6593	4.6292
			(0.4236)	(0.4219)
	$\hat{\xi}$			0.9594
				(0.0238)

Table 8: Maximum Likelihood Estimators and Standard Errors of NVDA

	Parameter	Normal	Student-t	Skewed Student-t
ARCH	$\hat{\omega}$	7.5788	7.6010	7.9101
		(0.2834)	(0.5516)	(0.6215)
	$\hat{\alpha_1}$	0.1978	0.2342	0.2431
		(0.0337)	(0.0482)	(0.0504)
	$\hat{\nu}$		3.9183	3.7557
			(0.3323)	(0.3175)
	$\hat{\xi}$			0.9265
				(0.0205)
GARCH	$\hat{\omega}$	0.8167	0.1975	0.1985
		(0.1594)	(0.0776)	(0.0769)
	$\hat{\alpha_1}$	0.1029	0.0730	0.0751
		(0.0183)	(0.0160)	(0.0160)
	$\hat{\beta}_1$	0.8126	0.9103	0.9118
		(0.0300)	(0.0203)	(0.0193)
	$\hat{\nu}$		4.3157	4.1006
			(0.3754)	(0.3542)
	$\hat{\xi}$			0.9175
				(0.0219)
GJR-GARCH	$\hat{\omega}$	1.0000	0.3241	0.3154
		(0.1697)	(0.1104)	(0.1083)
	$\hat{\alpha_1}$	0.0206	0.0280	0.0295
		(0.0104)	(0.0104)	(0.0106)
	$\hat{\beta}_1$	0.7804	0.8802	0.8849
		(0.0312)	(0.0257)	(0.0244)
	$\hat{\gamma}_1$	0.2209	0.1461	0.1433
		(0.0406)	(0.0363)	(0.0357)
	$\hat{\nu}$		4.4937	4.2722
			(0.4005)	(0.3786)
	$\hat{\xi}$			0.9223
				(0.0223)
EGARCH	$\hat{\omega}$	0.0688	-0.0555	-0.0553
		(0.0306)	(0.0195)	(0.0194)
	$\hat{\alpha_1}$	0.2213	0.1893	0.1902
		(0.0283)	(0.0276)	(0.0276)
	$\hat{\beta}_1$	0.8965	0.9638	0.9650
		(0.0182)	(0.0106)	(0.0103)
	$\hat{\gamma}_1$	-0.1348	-0.0909	-0.0893
		(0.0194)	(0.0190)	(0.0190)
	$\hat{\nu}$		4.6066	4.3880
			(0.4160)	(0.3943)
	$\hat{\xi}$			0.9235
				(0.0225)
GAS	$\hat{\omega}$	0.1785	0.0002	0.0010
		(0.0447)	(0.0070)	(0.0071)
	$\hat{\alpha_1}$	0.0393	0.1029	0.1045
		(0.0075)	(0.0152)	(0.0153)
	$\hat{\beta}_1$	0.9174	1.0000	1.0000
		(0.0207)	(0.0032)	(0.0032)
	$\hat{\nu}$		4.3148	4.1378
			(0.3631)	(0.3471)
	$\hat{\xi}$			0.9220
				(0.0221)

C Appendix: Graphs of Conditional Variances

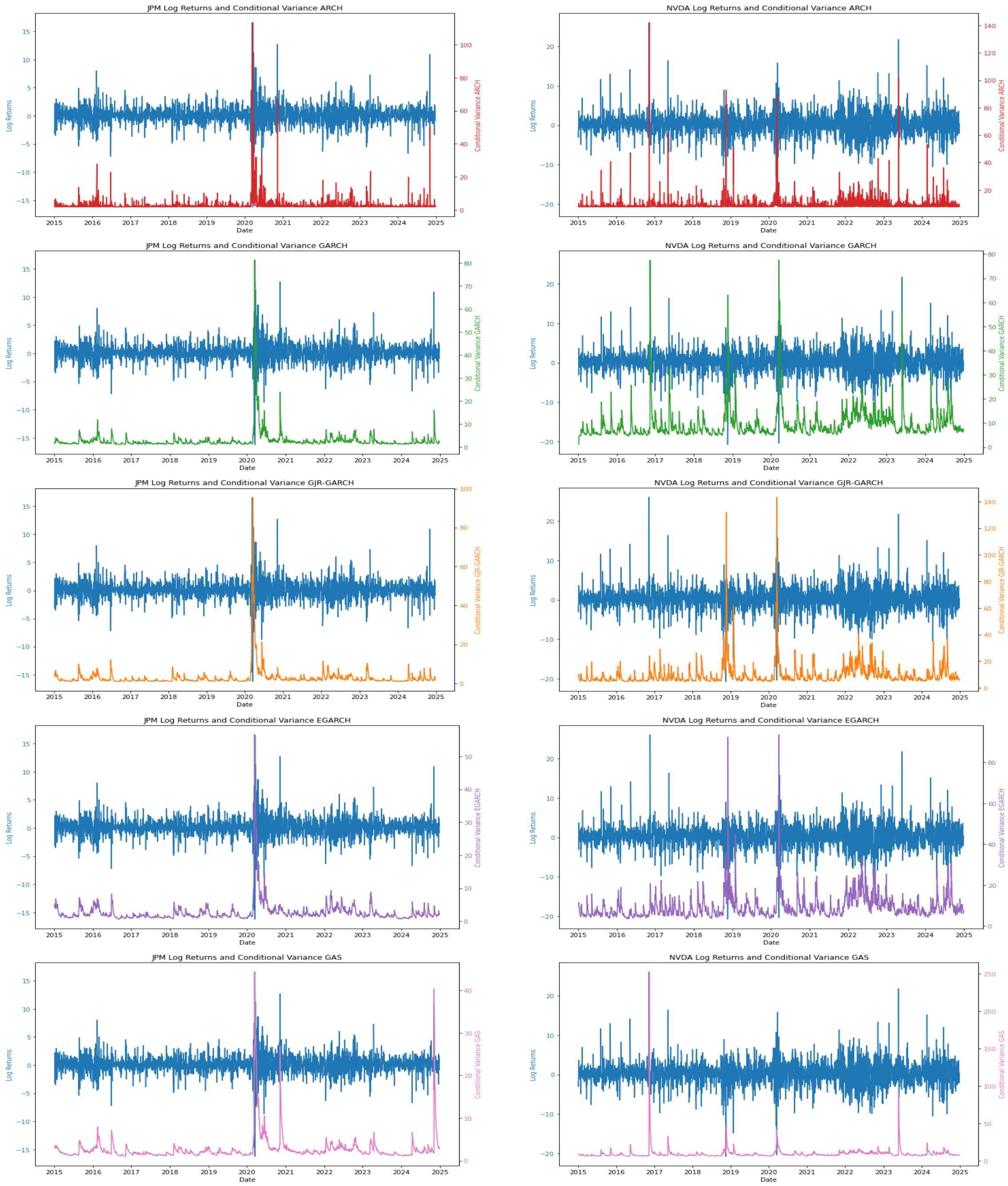

Figure 4: Conditional variances with the Normal Distribution obtained by the Frequentist Approach

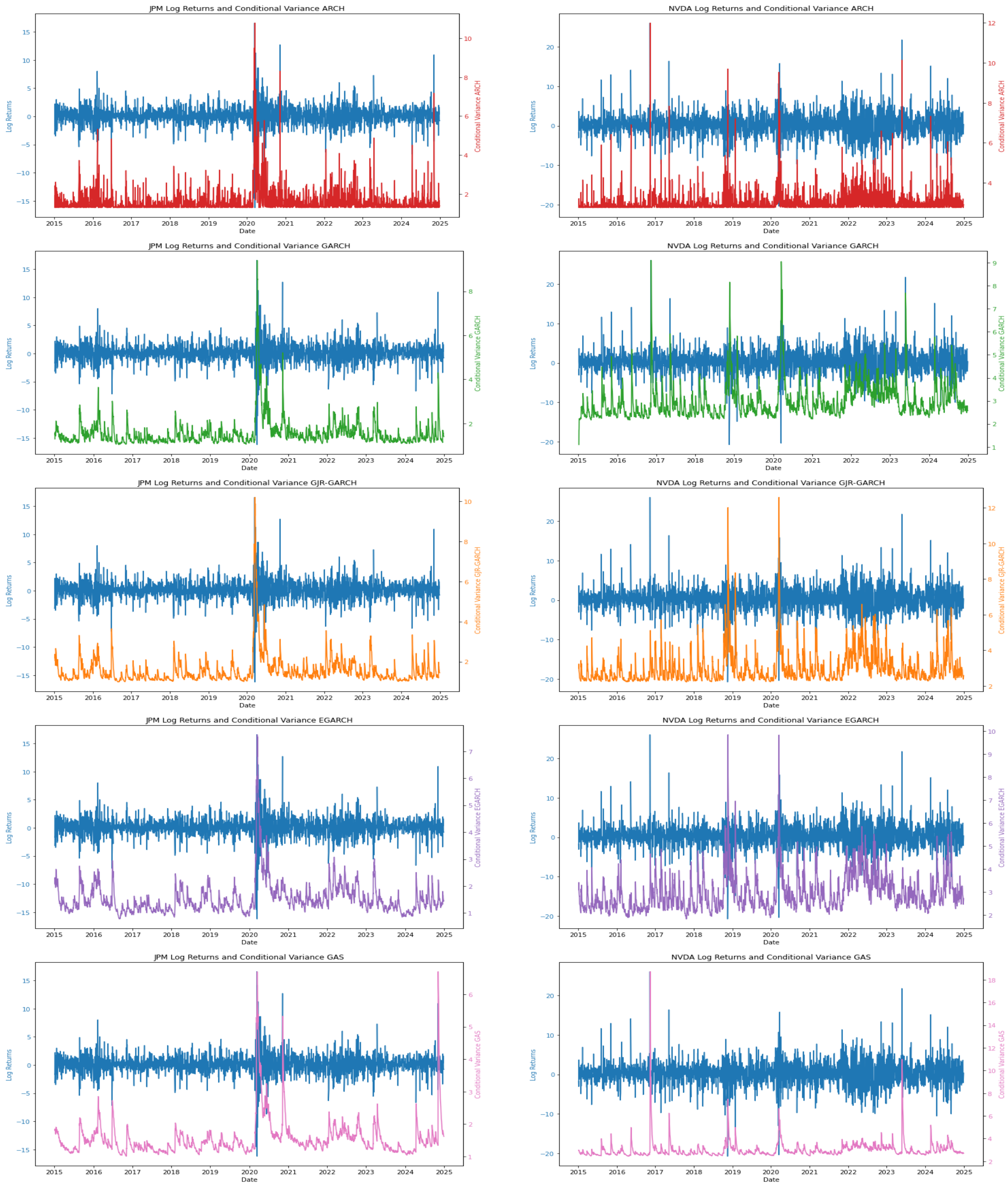

Figure 5: Conditional variances with the Normal Distribution obtained by the Bayesian Approach

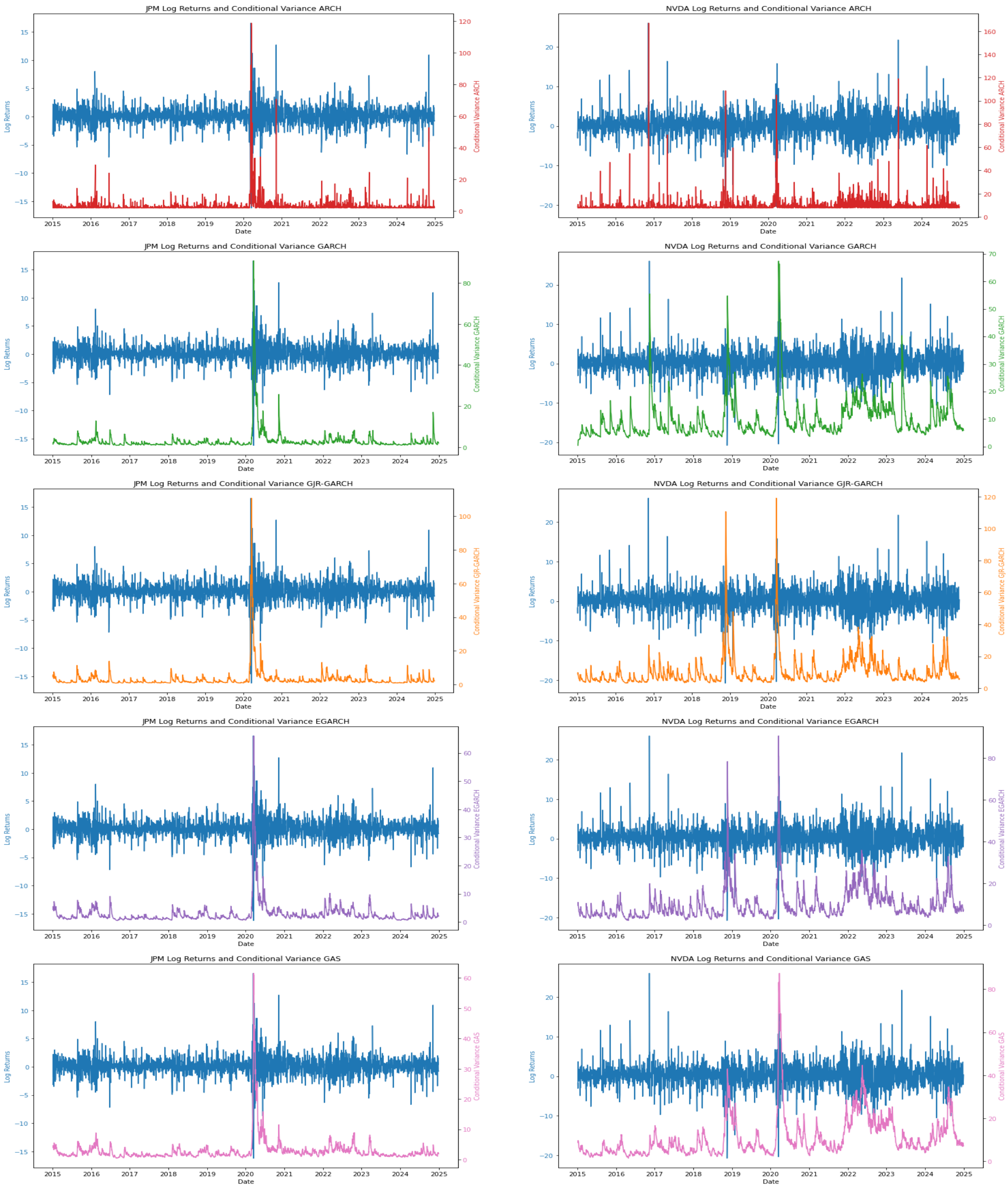

Figure 6: Conditional variances with the Student-t Distribution obtained by the Frequentist Approach

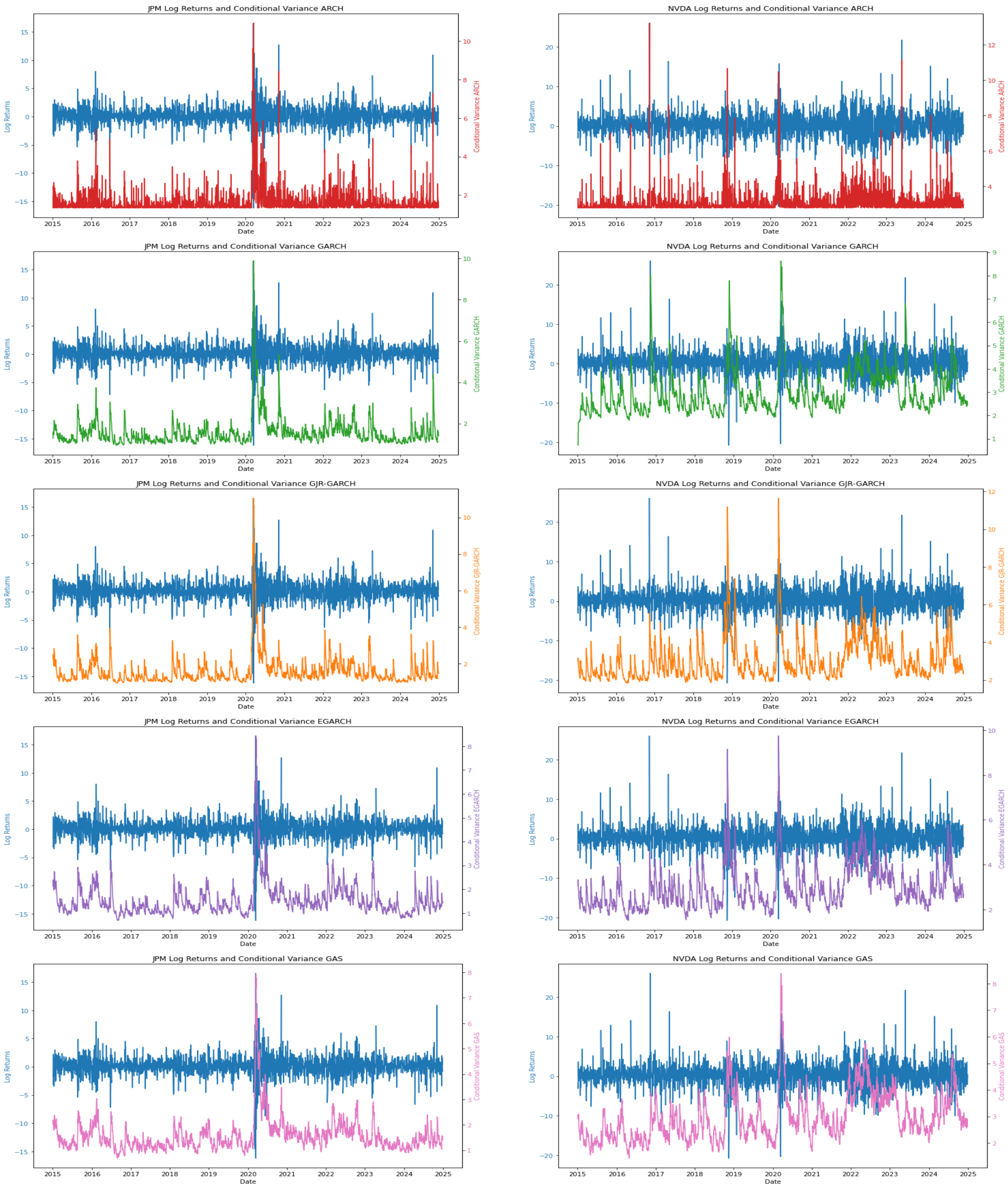

Figure 7: Conditional variances with the Student-t Distribution obtained by the Bayesian Approach

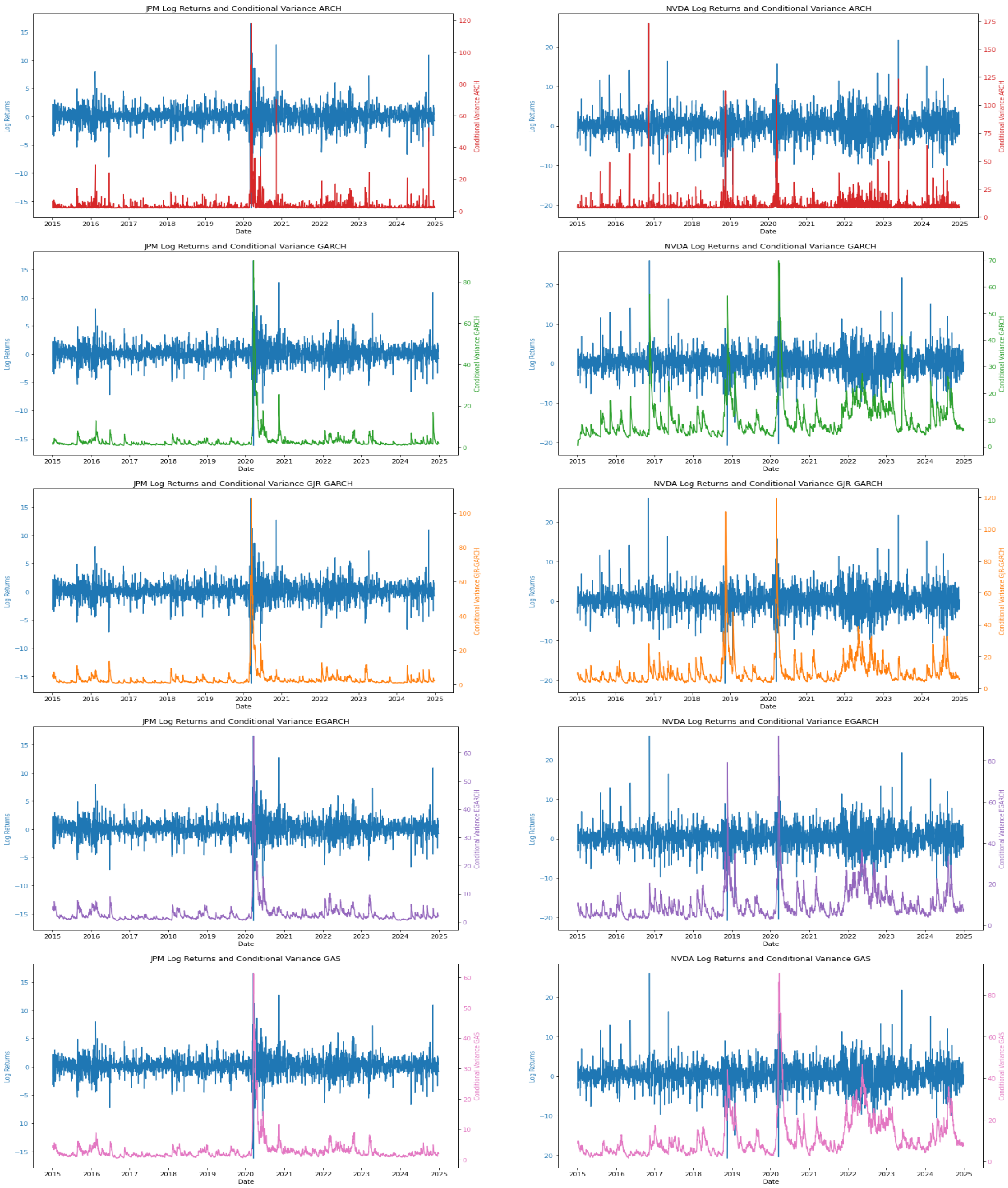

Figure 8: Conditional variances with the Skewed Student-t Distribution obtained by the Frequentist Approach

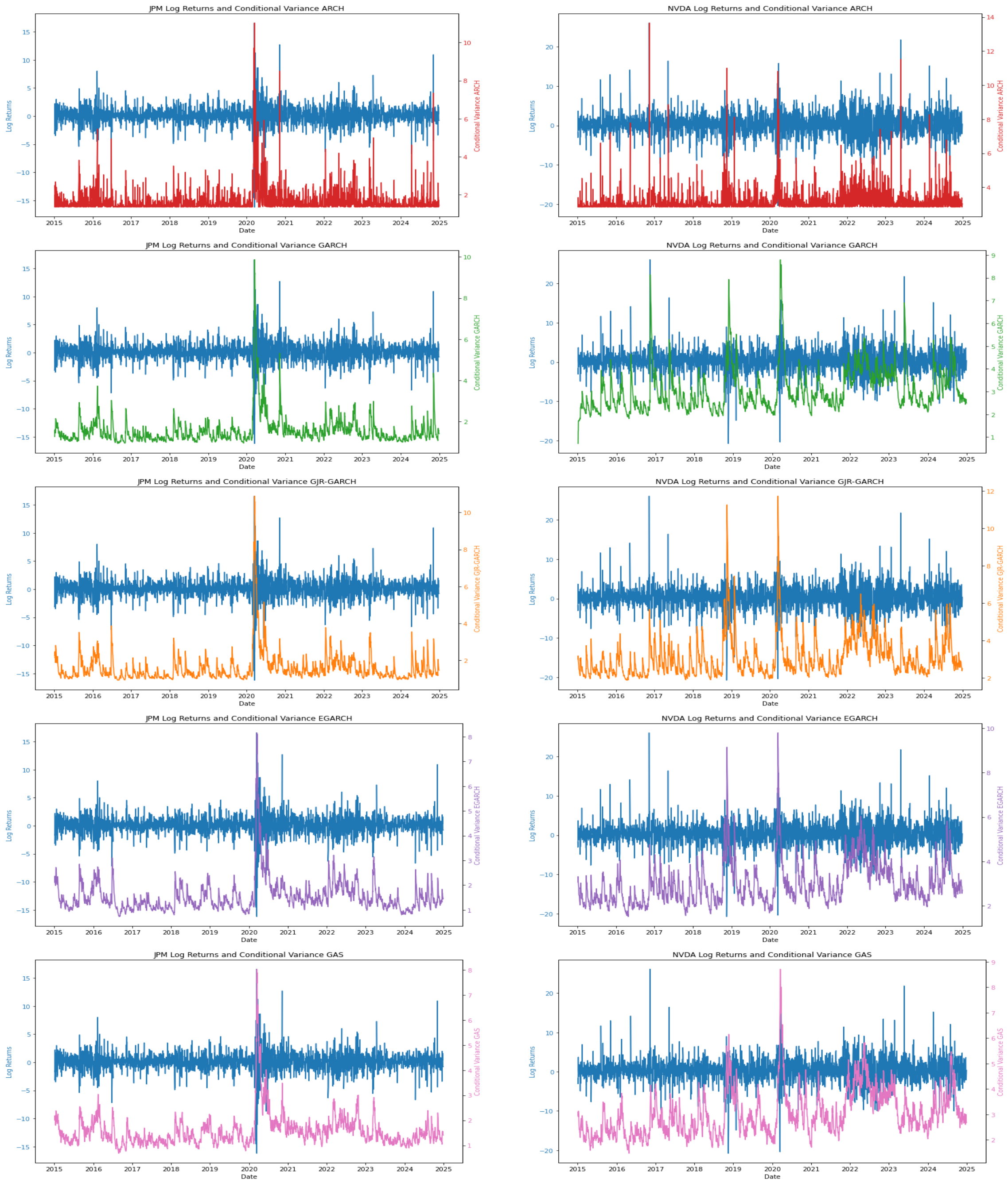

Figure 9: Conditional variances with the Skewed Student-t Distribution obtained by the Bayesian Approach

D Appendix: Results of AIC and BIC

Table 9: Log-likelihood, AIC and BIC of JPM

	Distribution	Log-likelihood	AIC	BIC
ARCH	Normal	-4634.5836	9273.1671	9284.8264
	Student-t	-4472.1636	8950.3273	8967.8162
	Skewed t	-4471.2856	8950.5711	8973.8896
GARCH	Normal	-4539.3143	9084.6286	9102.1175
	Student-t	-4401.3361	8810.6723	8833.9908
	Skewed t	-4399.8312	8809.6624	8838.8105
GJR-GARCH	Normal	-4506.0018	9020.0037	9043.3222
	Student-t	-4372.6857	8755.3715	8784.5196
	Skewed t	-4371.4966	8754.9932	8789.9709
EGARCH	Normal	-4495.2674	8998.5347	9021.8532
	Student-t	-4364.9163	8739.8325	8768.9807
	Skewed t	-4363.3167	8738.6335	8773.6113
GAS	Normal	-4568.8070	9143.6140	9161.1029
	Student-t	-4396.9411	8801.8822	8825.2007
	Skewed t	-4395.5357	8801.0713	8830.2195

Table 10: Log-likelihood, AIC and BIC of NVDA

	Distribution	Log-likelihood	AIC	BIC
ARCH	Normal	-6318.3090	12640.6179	12652.2772
	Student-t	-6127.3404	12260.6807	12278.1696
	Skewed t	-6121.4271	12250.8541	12274.1727
GARCH	Normal	-6249.9688	12505.9375	12523.4264
	Student-t	-6051.5942	12111.1885	12134.5070
	Skewed t	-6045.1616	12100.3232	12129.4714
GJR-GARCH	Normal	-6217.5387	12443.0775	12466.3960
	Student-t	-6030.7126	12071.4253	12100.5734
	Skewed t	-6025.2377	12062.4754	12097.4532
EGARCH	Normal	-6202.1653	12412.3306	12435.6491
	Student-t	-6019.9524	12049.9049	12079.0530
	Skewed t	-6014.7588	12041.5176	12076.4954
GAS	Normal	-6281.5196	12569.0392	12586.5281
	Student-t	-6044.9569	12097.9138	12121.2323
	Skewed t	-6039.3025	12088.6050	12117.7531

E Appendix: Results of Loss Functions

Table 11: MSE, MAE, MAPE, and RMSE of the Frequentist Approach (JPM)

	Distribution	MSE	MAE	MAPE	RMSE
ARCH	Normal	110.6538	3.3681	60.23%	10.5192
	Student-t	110.5771	3.4392	60.60%	10.5156
	Skewed t	110.5821	3.4490	60.68%	10.5158
GARCH	Normal	105.9970	3.1179	58.41%	10.2955
	Student-t	105.2838	3.1838	58.38%	10.2608
	Skewed t	105.4300	3.1958	58.45%	10.2679
GJR-GARCH	Normal	104.4600	3.0828	57.76%	10.2206
	Student-t	104.6803	3.1416	57.70%	10.2313
	Skewed t	104.7984	3.1486	57.75%	10.2371
EGARCH	Normal	108.1973	3.0275	57.57%	10.4018
	Student-t	105.8246	3.0571	57.48%	10.2871
	Skewed t	105.8734	3.0670	57.54%	10.2895
GAS	Normal	115.4379	3.2040	58.69%	10.7442
	Student-t	107.9845	3.1452	58.32%	10.3916
	Skewed t	108.0215	3.1551	58.38%	10.3933

Table 12: MSE, MAE, MAPE, and RMSE of the Bayesian Approach (JPM)

	Distribution	MSE	MAE	MAPE	RMSE
ARCH	Normal	132.3945	2.9066	57.74%	11.5063
	Student-t	132.1522	2.9184	57.84%	11.4957
	Skewed t	132.0543	2.9253	57.89%	11.4915
GARCH	Normal	131.4744	2.8526	57.03%	11.4662
	Student-t	130.7750	2.8457	56.89%	11.4357
	Skewed t	130.7835	2.8448	56.88%	11.4361
GJR-GARCH	Normal	130.5377	2.8220	56.62%	11.4253
	Student-t	129.7456	2.8147	56.48%	11.3906
	Skewed t	129.7820	2.8133	56.46%	11.3922
EGARCH	Normal	132.0751	2.8246	56.50%	11.4924
	Student-t	131.3372	2.8147	56.35%	11.4602
	Skewed t	131.4770	2.8206	56.42%	11.4663
GAS	Normal	133.1458	2.8818	57.21%	11.5389
	Student-t	131.8068	2.8431	56.76%	11.4807
	Skewed t	131.7050	2.8433	56.75%	11.4763

Table 13: MSE, MAE, MAPE, and RMSE of the Frequentist Approach (NVDA)

	Distribution	MSE	MAE	MAPE	RMSE
ARCH	Normal	750.4778	10.7814	60.41%	27.3948
	Student-t	755.8963	10.9691	60.60%	27.4936
	Skewed t	758.0087	11.1814	60.92%	27.5320
GARCH	Normal	735.9778	10.4435	59.05%	27.1289
	Student-t	740.3648	10.5128	58.53%	27.2096
	Skewed t	742.3663	10.7219	58.86%	27.2464
GJR-GARCH	Normal	731.3751	10.3233	58.64%	27.0439
	Student-t	737.7527	10.4563	58.26%	27.1616
	Skewed t	740.1152	10.6323	58.54%	27.2051
EGARCH	Normal	719.7117	10.0941	58.43%	26.8274
	Student-t	721.5387	10.1803	57.99%	26.8615
	Skewed t	722.4923	10.3383	58.26%	26.8792
GAS	Normal	791.8116	10.7168	59.57%	28.1391
	Student-t	746.8080	10.9468	58.46%	27.3278
	Skewed t	750.9972	11.1639	58.76%	27.4043

Table 14: MSE, MAE, MAPE, and RMSE of the Bayesian Approach (NVDA)

	Distribution	MSE	MAE	MAPE	RMSE
ARCH	Normal	792.8981	8.5728	56.50%	28.1584
	Student-t	791.5550	8.5742	56.46%	28.1346
	Skewed t	790.5038	8.5804	56.44%	28.1159
GARCH	Normal	790.0343	8.4891	56.02%	28.1075
	Student-t	788.9930	8.4601	55.82%	28.0890
	Skewed t	787.8809	8.4619	55.78%	28.0692
GJR-GARCH	Normal	787.4058	8.4570	55.94%	28.0608
	Student-t	785.9313	8.4265	55.74%	28.0345
	Skewed t	785.2542	8.4287	55.71%	28.0224
EGARCH	Normal	788.6690	8.4545	55.90%	28.0833
	Student-t	787.4707	8.4265	55.76%	28.0619
	Skewed t	786.9089	8.4278	55.73%	28.0519
GAS	Normal	792.8919	8.5333	56.21%	28.1583
	Student-t	788.6929	8.4478	55.84%	28.0837
	Skewed t	787.9144	8.4526	55.84%	28.0698

F Appendix: Results of Value-at-Risk Backtesting

Table 15: VaR Backtesting Results of JPM

Model	Distribution	Method	Violations	Ratio	UC	p-value	IND	p-value	CC	p-value
ARCH	Normal	MLE	113	0.90	12.2695	0.0005	0.7161	0.3974	12.9856	0.0015
		RWMHM	113	0.90	12.2695	0.0005	0.7161	0.3974	12.9856	0.0015
	Student-t	MLE	128	1.02	12.4522	0.0004	0.9450	0.3310	13.3973	0.0012
		RWMHM	128	1.02	12.4522	0.0004	0.9450	0.3310	13.3973	0.0012
	Skewed t	MLE	140	1.11	12.5873	0.0004	2.2133	0.1368	14.8006	0.0006
		RWMHM	135	1.07	12.5321	0.0004	1.8943	0.1687	14.4264	0.0007
GARCH	Normal	MLE	103	0.82	12.1377	0.0005	8.6276	0.0033	20.7652	0.0000
		RWMHM	107	0.85	12.1914	0.0005	7.4814	0.0062	19.6729	0.0001
	Student-t	MLE	127	1.01	12.4406	0.0004	11.8388	0.0006	24.2793	0.0000
		RWMHM	125	0.99	12.4170	0.0004	10.3756	0.0013	22.7926	0.0000
	Skewed t	MLE	138	1.10	12.5654	0.0004	8.3483	0.0039	20.9137	0.0000
		RWMHM	142	1.13	12.6089	0.0004	10.8581	0.0010	23.4671	0.0000
GJR-GARCH	Normal	MLE	107	0.85	12.1914	0.0005	1.2426	0.2650	13.4341	0.0012
		RWMHM	106	0.84	12.1781	0.0005	1.3456	0.2460	13.5238	0.0012
	Student-t	MLE	116	0.92	12.3074	0.0005	1.2613	0.2614	13.5687	0.0011
		RWMHM	113	0.90	12.2695	0.0005	1.5852	0.2080	13.8547	0.0010
	Skewed t	MLE	128	1.02	12.4522	0.0004	0.3491	0.5546	12.8013	0.0017
		RWMHM	124	0.99	12.4051	0.0004	0.5872	0.4435	12.9923	0.0015
EGARCH	Normal	MLE	100	0.80	12.0964	0.0005	3.4770	0.0622	15.5733	0.0004
		RWMHM	100	0.80	12.0964	0.0005	3.4770	0.0622	15.5733	0.0004
	Student-t	MLE	114	0.91	12.2822	0.0005	5.5926	0.0180	17.8748	0.0001
		RWMHM	113	0.90	12.2695	0.0005	5.8257	0.0158	18.0952	0.0001
	Skewed t	MLE	131	1.04	12.4869	0.0004	3.6317	0.0567	16.1186	0.0003
		RWMHM	128	1.02	12.4522	0.0004	2.8864	0.0893	15.3386	0.0005
GAS	Normal	MLE	109	0.87	12.2178	0.0005	11.1255	0.0009	23.3433	0.0000
		RWMHM	109	0.87	12.2178	0.0005	11.1255	0.0009	23.3433	0.0000
	Student-t	MLE	116	0.92	12.3074	0.0005	11.0330	0.0009	23.3404	0.0000
		RWMHM	113	0.90	12.2695	0.0005	5.8257	0.0158	18.0952	0.0001
	Skewed t	MLE	133	1.06	12.5096	0.0004	11.7112	0.0006	24.2209	0.0000
		RWMHM	133	1.06	12.5096	0.0004	11.7112	0.0006	24.2209	0.0000

Table 16: VaR Backtesting Results of NVDA

Model	Distribution	Method	Violations	Ratio	UC	p-value	IND	p-value	CC	p-value
ARCH	Normal	MLE	101	0.80	12.1102	0.0005	0.3287	0.5664	12.4389	0.0020
		RWMHM	101	0.80	12.1102	0.0005	0.3287	0.5664	12.4389	0.0020
	Student-t	MLE	124	0.99	12.4051	0.0004	0.1341	0.7143	12.5392	0.0019
		RWMHM	122	0.97	12.3811	0.0004	0.0010	0.9743	12.3822	0.0020
	Skewed t	MLE	145	1.15	12.6410	0.0004	0.0186	0.8914	12.6596	0.0018
		RWMHM	141	1.12	12.5981	0.0004	0.1229	0.7259	12.7210	0.0017
GARCH	Normal	MLE	96	0.76	12.0399	0.0005	0.5035	0.4780	12.5434	0.0019
		RWMHM	96	0.76	12.0399	0.0005	0.5035	0.4780	12.5434	0.0019
	Student-t	MLE	117	0.93	12.3199	0.0004	1.2113	0.2711	13.5311	0.0012
		RWMHM	113	0.90	12.2695	0.0005	0.1906	0.6624	12.4601	0.0020
	Skewed t	MLE	136	1.08	12.5433	0.0004	1.0046	0.3162	13.5479	0.0011
		RWMHM	137	1.09	12.5544	0.0004	0.9171	0.3382	13.4715	0.0012
GJR-GARCH	Normal	MLE	99	0.79	12.0824	0.0005	0.2440	0.6213	12.3264	0.0021
		RWMHM	103	0.82	12.1377	0.0005	0.4254	0.5142	12.5631	0.0019
	Student-t	MLE	112	0.89	12.2567	0.0005	0.0000	0.9976	12.2567	0.0022
		RWMHM	111	0.88	12.2438	0.0005	0.0020	0.9643	12.2458	0.0022
	Skewed t	MLE	131	1.04	12.4869	0.0004	0.0046	0.9460	12.4915	0.0019
		RWMHM	130	1.03	12.4754	0.0004	0.0905	0.7635	12.5659	0.0019
EGARCH	Normal	MLE	99	0.79	12.0824	0.0005	0.2440	0.6213	12.3264	0.0021
		RWMHM	101	0.80	12.1102	0.0005	0.3287	0.5664	12.4389	0.0020
	Student-t	MLE	111	0.88	12.2438	0.0005	0.1940	0.6596	12.4378	0.0020
		RWMHM	114	0.91	12.2822	0.0005	0.0065	0.9360	12.2887	0.0021
	Skewed t	MLE	132	1.05	12.4983	0.0004	0.1740	0.6766	12.6723	0.0018
		RWMHM	132	1.05	12.4983	0.0004	0.1740	0.6766	12.6723	0.0018
GAS	Normal	MLE	103	0.82	12.1377	0.0005	0.1473	0.7011	12.2850	0.0021
		RWMHM	102	0.81	12.1240	0.0005	0.1821	0.6696	12.3061	0.0021
	Student-t	MLE	112	0.89	12.2567	0.0005	4.3626	0.0367	16.6192	0.0002
		RWMHM	115	0.92	12.2948	0.0005	0.5743	0.4486	12.8691	0.0016
	Skewed t	MLE	131	1.04	12.4869	0.0004	2.4333	0.1188	14.9202	0.0006
		RWMHM	140	1.11	12.5873	0.0004	1.3214	0.2503	13.9087	0.0010

G Appendix: Results of Logarithmic Scoring Rule

Table 17: Logarithmic Scoring Rules of JPM

Model	Distribution	MLE	RWMHM
ARCH	Normal	-1.6310	-1.6319
	Student-t	-1.7782	-1.7783
	Skewed t	-1.7778	-1.7778
GARCH	Normal	-1.6131	-1.6088
	Student-t	-1.7506	-1.7506
	Skewed t	-1.7500	-1.7501
GJR-GARCH	Normal	-1.6054	-1.6050
	Student-t	-1.7386	-1.7387
	Skewed t	-1.7382	-1.7383
EGARCH	Normal	-1.6032	-1.6013
	Student-t	-1.7355	-1.7357
	Skewed t	-1.7349	-1.7350
GAS	Normal	-1.6180	-1.6165
	Student-t	-1.7483	-1.7484
	Skewed t	-1.7477	-1.7477

Table 18: Logarithmic Scoring Rules of NVDA

Model	Distribution	MLE	RWMHM
ARCH	Normal	-1.9662	-1.9649
	Student-t	-2.4374	-2.4375
	Skewed t	-2.4350	-2.4350
GARCH	Normal	-1.9538	-1.9521
	Student-t	-2.4085	-2.4085
	Skewed t	-2.4059	-2.4058
GJR-GARCH	Normal	-1.9471	-1.9436
	Student-t	-2.3990	-2.3992
	Skewed t	-2.3968	-2.3969
EGARCH	Normal	-1.9428	-1.9423
	Student-t	-2.3947	-2.3948
	Skewed t	-2.3926	-2.3926
GAS	Normal	-1.9584	-1.9585
	Student-t	-2.4046	-2.4006
	Skewed t	-2.4024	-2.3981

H Appendix: Results of Diebold-Mariano Test

Table 19: Diebold-Mariano Test Results of JPM (MSE)

Model 1	Model 2	Distribution	Method	DM Statistic	p-value
ARCH	GARCH	Normal	MLE	0.9216	0.3568
			RWMHM	0.8562	0.3919
		Student-t	MLE	0.5444	0.5862
			RWMHM	0.5113	0.6092
		Skewed t	MLE	0.5068	0.6123
			RWMHM	1.0950	0.2735
ARCH	GJR-GARCH	Normal	MLE	0.1706	0.8646
			RWMHM	0.1147	0.9087
		Student-t	MLE	0.0392	0.9687
			RWMHM	-0.2219	0.8244
		Skewed t	MLE	0.0463	0.9631
			RWMHM	0.2938	0.7689
ARCH	EGARCH	Normal	MLE	1.9190	0.0550
			RWMHM	2.0323	0.0421
		Student-t	MLE	1.9697	0.0489
			RWMHM	2.0100	0.0444
		Skewed t	MLE	1.9445	0.0518
			RWMHM	2.4467	0.0144
ARCH	GAS	Normal	MLE	1.5303	0.1259
			RWMHM	1.5780	0.1146
		Student-t	MLE	1.5111	0.1308
			RWMHM	1.7585	0.0787
		Skewed t	MLE	1.4897	0.1363
			RWMHM	2.0171	0.0437
GARCH	GJR-GARCH	Normal	MLE	-1.9305	0.0535
			RWMHM	-1.7992	0.0720
		Student-t	MLE	-1.3800	0.1676
			RWMHM	-2.0093	0.0445
		Skewed t	MLE	-1.2862	0.1984
			RWMHM	-1.9021	0.0572
GARCH	EGARCH	Normal	MLE	2.4857	0.0129
			RWMHM	2.6578	0.0079
		Student-t	MLE	3.2596	0.0011
			RWMHM	3.2539	0.0011
		Skewed t	MLE	3.2842	0.0010
			RWMHM	2.9147	0.0036
GARCH	GAS	Normal	MLE	1.1584	0.2467
			RWMHM	1.2735	0.2028
		Student-t	MLE	2.4178	0.0156
			RWMHM	2.7137	0.0067
		Skewed t	MLE	2.4688	0.0136
			RWMHM	2.1386	0.0325
GJR-GARCH	EGARCH	Normal	MLE	3.6083	0.0003
			RWMHM	3.7757	0.0002
		Student-t	MLE	3.9632	0.0001
			RWMHM	4.4003	0.0000
		Skewed t	MLE	3.9419	0.0001
			RWMHM	3.9393	0.0001
GJR-GARCH	GAS	Normal	MLE	1.7411	0.0817
			RWMHM	1.7890	0.0736
		Student-t	MLE	2.5276	0.0115
			RWMHM	3.1011	0.0019
		Skewed t	MLE	2.5188	0.0118
			RWMHM	2.6849	0.0073
EGARCH	GAS	Normal	MLE	-1.2963	0.1949
			RWMHM	-1.3516	0.1765
		Student-t	MLE	-2.4393	0.0147
			RWMHM	-1.6428	0.1004
		Skewed t	MLE	-2.4246	0.0153
			RWMHM	-2.4492	0.0143

Table 20: Diebold-Mariano Test Results of JPM (MAE)

Model 1	Model 2	Distribution	Method	DM Statistic	p-value
ARCH	GARCH	Normal	MLE	6.4506	0.0000
			RWMHM	6.6005	0.0000
		Student-t	MLE	7.1030	0.0000
			RWMHM	7.2825	0.0000
		Skewed t	MLE	7.0845	0.0000
			RWMHM	8.9844	0.0000
ARCH	GJR-GARCH	Normal	MLE	6.3787	0.0000
			RWMHM	6.2853	0.0000
		Student-t	MLE	7.4021	0.0000
			RWMHM	6.8593	0.0000
		Skewed t	MLE	7.4621	0.0000
			RWMHM	8.4193	0.0000
ARCH	EGARCH	Normal	MLE	8.2585	0.0000
			RWMHM	8.5873	0.0000
		Student-t	MLE	9.7281	0.0000
			RWMHM	10.0517	0.0000
		Skewed t	MLE	9.7262	0.0000
			RWMHM	10.8762	0.0000
ARCH	GAS	Normal	MLE	4.5145	0.0000
			RWMHM	4.7207	0.0000
		Student-t	MLE	7.7398	0.0000
			RWMHM	8.2055	0.0000
		Skewed t	MLE	7.7846	0.0000
			RWMHM	9.1866	0.0000
GARCH	GJR-GARCH	Normal	MLE	2.1227	0.0338
			RWMHM	1.8333	0.0668
		Student-t	MLE	2.5806	0.0099
			RWMHM	1.1915	0.2334
		Skewed t	MLE	2.7386	0.0062
			RWMHM	1.4483	0.1475
GARCH	EGARCH	Normal	MLE	4.6390	0.0000
			RWMHM	5.0918	0.0000
		Student-t	MLE	5.7935	0.0000
			RWMHM	6.1407	0.0000
		Skewed t	MLE	5.7732	0.0000
			RWMHM	4.5444	0.0000
GARCH	GAS	Normal	MLE	-2.7428	0.0061
			RWMHM	-1.9833	0.0473
		Student-t	MLE	1.4149	0.1571
			RWMHM	1.8556	0.0635
		Skewed t	MLE	1.5075	0.1317
			RWMHM	0.4439	0.6571
GJR-GARCH	EGARCH	Normal	MLE	3.8132	0.0001
			RWMHM	4.9108	0.0000
		Student-t	MLE	4.9020	0.0000
			RWMHM	7.1708	0.0000
		Skewed t	MLE	4.7479	0.0000
			RWMHM	4.4396	0.0000
GJR-GARCH	GAS	Normal	MLE	-3.0584	0.0022
			RWMHM	-2.4356	0.0149
		Student-t	MLE	-1.2662	0.2054
			RWMHM	0.2581	0.7964
		Skewed t	MLE	-1.3337	0.1823
			RWMHM	-0.9351	0.3497
EGARCH	GAS	Normal	MLE	-5.4577	0.0000
			RWMHM	-5.3281	0.0000
		Student-t	MLE	-6.2068	0.0000
			RWMHM	-6.1071	0.0000
		Skewed t	MLE	-6.1121	0.0000
			RWMHM	-5.5898	0.0000

Table 21: Diebold-Mariano Test Results of JPM (MAPE)

Model 1	Model 2	Distribution	Method	DM Statistic	p-value
ARCH	GARCH	Normal	MLE	1.2939	0.1957
			RWMHM	1.3602	0.1738
		Student-t	MLE	1.5838	0.1132
			RWMHM	1.6440	0.1002
		Skewed t	MLE	1.5896	0.1119
			RWMHM	1.7960	0.0725
ARCH	GJR-GARCH	Normal	MLE	0.8970	0.3697
			RWMHM	0.9735	0.3303
		Student-t	MLE	1.2275	0.2197
			RWMHM	1.2771	0.2016
		Skewed t	MLE	1.2251	0.2205
			RWMHM	1.4147	0.1571
ARCH	EGARCH	Normal	MLE	0.3927	0.6945
			RWMHM	0.4770	0.6333
		Student-t	MLE	0.7335	0.4632
			RWMHM	0.8764	0.3808
		Skewed t	MLE	0.7301	0.4654
			RWMHM	0.8558	0.3921
ARCH	GAS	Normal	MLE	0.6325	0.5271
			RWMHM	0.7156	0.4743
		Student-t	MLE	0.9010	0.3676
			RWMHM	0.9834	0.3254
		Skewed t	MLE	0.9117	0.3619
			RWMHM	1.0791	0.2805
GARCH	GJR-GARCH	Normal	MLE	-0.9886	0.3229
			RWMHM	-0.8689	0.3849
		Student-t	MLE	-1.1893	0.2343
			RWMHM	-1.0996	0.2715
		Skewed t	MLE	-1.2694	0.2043
			RWMHM	-1.2161	0.2239
GARCH	EGARCH	Normal	MLE	-1.1788	0.2385
			RWMHM	-1.0520	0.2928
		Student-t	MLE	-1.3260	0.1849
			RWMHM	-1.0602	0.2890
		Skewed t	MLE	-1.3620	0.1732
			RWMHM	-1.4860	0.1373
GARCH	GAS	Normal	MLE	-1.8826	0.0598
			RWMHM	-1.7561	0.0791
		Student-t	MLE	-1.3536	0.1759
			RWMHM	-1.4098	0.1586
		Skewed t	MLE	-1.3580	0.1745
			RWMHM	-1.4098	0.1586
GJR-GARCH	EGARCH	Normal	MLE	-1.1815	0.2374
			RWMHM	-1.0497	0.2939
		Student-t	MLE	-1.2117	0.2256
			RWMHM	-0.8400	0.4009
		Skewed t	MLE	-1.2371	0.2161
			RWMHM	-1.4121	0.1579
GJR-GARCH	GAS	Normal	MLE	-1.0497	0.2939
			RWMHM	-1.0349	0.3007
		Student-t	MLE	-1.0573	0.2904
			RWMHM	-0.8517	0.3944
		Skewed t	MLE	-1.0316	0.3022
			RWMHM	-1.0493	0.2941
EGARCH	GAS	Normal	MLE	0.0861	0.9314
			RWMHM	0.0215	0.9828
		Student-t	MLE	0.6506	0.5153
			RWMHM	0.3351	0.7376
		Skewed t	MLE	0.7073	0.4794
			RWMHM	0.8741	0.3821

Table 22: Diebold-Mariano Test Results of JPM (RMSE)

Model 1	Model 2	Distribution	Method	DM Statistic	p-value
ARCH	GARCH	Normal	MLE	6.4506	0.0000
			RWMHM	6.6005	0.0000
		Student-t	MLE	7.1030	0.0000
			RWMHM	7.2825	0.0000
		Skewed t	MLE	7.0845	0.0000
			RWMHM	8.9844	0.0000
ARCH	GJR-GARCH	Normal	MLE	6.3787	0.0000
			RWMHM	6.2853	0.0000
		Student-t	MLE	7.4021	0.0000
			RWMHM	6.8593	0.0000
		Skewed t	MLE	7.4621	0.0000
			RWMHM	8.4193	0.0000
ARCH	EGARCH	Normal	MLE	8.2585	0.0000
			RWMHM	8.5873	0.0000
		Student-t	MLE	9.7281	0.0000
			RWMHM	10.0517	0.0000
		Skewed t	MLE	9.7262	0.0000
			RWMHM	10.8762	0.0000
ARCH	GAS	Normal	MLE	4.5145	0.0000
			RWMHM	4.7207	0.0000
		Student-t	MLE	7.7398	0.0000
			RWMHM	8.2055	0.0000
		Skewed t	MLE	7.7846	0.0000
			RWMHM	9.1866	0.0000
GARCH	GJR-GARCH	Normal	MLE	2.1227	0.0338
			RWMHM	1.8333	0.0668
		Student-t	MLE	2.5806	0.0099
			RWMHM	1.1915	0.2334
		Skewed t	MLE	2.7386	0.0062
			RWMHM	1.4483	0.1475
GARCH	EGARCH	Normal	MLE	4.6390	0.0000
			RWMHM	5.0918	0.0000
		Student-t	MLE	5.7935	0.0000
			RWMHM	6.1407	0.0000
		Skewed t	MLE	5.7732	0.0000
			RWMHM	4.5444	0.0000
GARCH	GAS	Normal	MLE	-2.7428	0.0061
			RWMHM	-1.9833	0.0473
		Student-t	MLE	1.4149	0.1571
			RWMHM	1.8556	0.0635
		Skewed t	MLE	1.5075	0.1317
			RWMHM	0.4439	0.6571
GJR-GARCH	EGARCH	Normal	MLE	3.8132	0.0001
			RWMHM	4.9108	0.0000
		Student-t	MLE	4.9020	0.0000
			RWMHM	7.1708	0.0000
		Skewed t	MLE	4.7479	0.0000
			RWMHM	4.4396	0.0000
GJR-GARCH	GAS	Normal	MLE	-3.0584	0.0022
			RWMHM	-2.4356	0.0149
		Student-t	MLE	-1.2662	0.2054
			RWMHM	0.2581	0.7964
		Skewed t	MLE	-1.3337	0.1823
			RWMHM	-0.9351	0.3497
EGARCH	GAS	Normal	MLE	-5.4577	0.0000
			RWMHM	-5.3281	0.0000
		Student-t	MLE	-6.2068	0.0000
			RWMHM	-6.1071	0.0000
		Skewed t	MLE	-6.1121	0.0000
			RWMHM	-5.5898	0.0000

Table 23: Diebold-Mariano Test Results of NVDA (MSE)

Model 1	Model 2	Distribution	Method	DM Statistic	p-value
ARCH	GARCH	Normal	MLE	-0.2288	0.8190
			RWMHM	-0.4793	0.6317
		Student-t	MLE	0.0772	0.9384
			RWMHM	0.6650	0.5060
		Skewed t	MLE	0.0471	0.9624
			RWMHM	0.5845	0.5589
ARCH	GJR-GARCH	Normal	MLE	-0.7428	0.4576
			RWMHM	-0.7750	0.4383
		Student-t	MLE	-0.7121	0.4764
			RWMHM	-0.6530	0.5138
		Skewed t	MLE	-0.4921	0.6226
			RWMHM	0.1067	0.9150
ARCH	EGARCH	Normal	MLE	0.7703	0.4411
			RWMHM	0.8800	0.3789
		Student-t	MLE	0.7912	0.4288
			RWMHM	1.9185	0.0550
		Skewed t	MLE	1.1046	0.2693
			RWMHM	2.9608	0.0031
ARCH	GAS	Normal	MLE	0.0248	0.9802
			RWMHM	-0.2453	0.8062
		Student-t	MLE	-4.0417	0.0001
			RWMHM	1.7653	0.0775
		Skewed t	MLE	-3.9438	0.0001
			RWMHM	2.5311	0.0114
GARCH	GJR-GARCH	Normal	MLE	-0.8680	0.3854
			RWMHM	-0.6073	0.5436
		Student-t	MLE	-1.4860	0.1373
			RWMHM	-2.1786	0.0294
		Skewed t	MLE	-1.0546	0.2916
			RWMHM	-0.7346	0.4626
GARCH	EGARCH	Normal	MLE	1.4766	0.1398
			RWMHM	1.9093	0.0562
		Student-t	MLE	1.4996	0.1337
			RWMHM	2.6272	0.0086
		Skewed t	MLE	2.2512	0.0244
			RWMHM	5.0223	0.0000
GARCH	GAS	Normal	MLE	0.2575	0.7968
			RWMHM	0.1215	0.9033
		Student-t	MLE	-8.7101	0.0000
			RWMHM	2.8270	0.0047
		Skewed t	MLE	-8.4786	0.0000
			RWMHM	4.7281	0.0000
GJR-GARCH	EGARCH	Normal	MLE	3.6075	0.0003
			RWMHM	3.6880	0.0002
		Student-t	MLE	4.6799	0.0000
			RWMHM	6.7961	0.0000
		Skewed t	MLE	4.9845	0.0000
			RWMHM	7.3502	0.0000
GJR-GARCH	GAS	Normal	MLE	0.6336	0.5263
			RWMHM	0.3879	0.6981
		Student-t	MLE	-6.5102	0.0000
			RWMHM	3.7128	0.0002
		Skewed t	MLE	-6.8379	0.0000
			RWMHM	3.7837	0.0002
EGARCH	GAS	Normal	MLE	-0.5798	0.5621
			RWMHM	-0.7362	0.4616
		Student-t	MLE	-10.7231	0.0000
			RWMHM	-0.0698	0.9444
		Skewed t	MLE	-11.1879	0.0000
			RWMHM	-0.7427	0.4577

Table 24: Diebold-Mariano Test Results of NVDA (MAE)

Model 1	Model 2	Distribution	Method	DM Statistic	p-value
ARCH	GARCH	Normal	MLE	5.2795	0.0000
			RWMHM	5.0420	0.0000
		Student-t	MLE	6.1689	0.0000
			RWMHM	7.0203	0.0000
		Skewed t	MLE	6.1094	0.0000
			RWMHM	6.8064	0.0000
ARCH	GJR-GARCH	Normal	MLE	5.9172	0.0000
			RWMHM	5.7936	0.0000
		Student-t	MLE	6.5493	0.0000
			RWMHM	6.6635	0.0000
		Skewed t	MLE	6.8478	0.0000
			RWMHM	7.7014	0.0000
ARCH	EGARCH	Normal	MLE	7.8595	0.0000
			RWMHM	8.0580	0.0000
		Student-t	MLE	8.6671	0.0000
			RWMHM	10.2027	0.0000
		Skewed t	MLE	9.1323	0.0000
			RWMHM	11.7089	0.0000
ARCH	GAS	Normal	MLE	3.0518	0.0023
			RWMHM	3.1235	0.0018
		Student-t	MLE	1.9849	0.0472
			RWMHM	9.6424	0.0000
		Skewed t	MLE	2.1616	0.0307
			RWMHM	10.8041	0.0000
GARCH	GJR-GARCH	Normal	MLE	2.3306	0.0198
			RWMHM	2.1983	0.0279
		Student-t	MLE	1.6792	0.0931
			RWMHM	0.6652	0.5059
		Skewed t	MLE	2.3340	0.0196
			RWMHM	2.5906	0.0096
GARCH	EGARCH	Normal	MLE	4.3809	0.0000
			RWMHM	4.8527	0.0000
		Student-t	MLE	4.6508	0.0000
			RWMHM	5.6805	0.0000
		Skewed t	MLE	5.5840	0.0000
			RWMHM	8.5206	0.0000
GARCH	GAS	Normal	MLE	-3.3021	0.0010
			RWMHM	-2.3581	0.0184
		Student-t	MLE	-5.0153	0.0000
			RWMHM	5.4064	0.0000
		Skewed t	MLE	-4.6536	0.0000
			RWMHM	7.4330	0.0000
GJR-GARCH	EGARCH	Normal	MLE	4.0680	0.0000
			RWMHM	5.1601	0.0000
		Student-t	MLE	5.8944	0.0000
			RWMHM	9.3251	0.0000
		Skewed t	MLE	6.4443	0.0000
			RWMHM	10.5893	0.0000
GJR-GARCH	GAS	Normal	MLE	-3.3497	0.0008
			RWMHM	-2.8016	0.0051
		Student-t	MLE	-6.1174	0.0000
			RWMHM	3.7015	0.0002
		Skewed t	MLE	-6.3435	0.0000
			RWMHM	3.5695	0.0004
EGARCH	GAS	Normal	MLE	-4.8966	0.0000
			RWMHM	-4.5981	0.0000
		Student-t	MLE	-10.0075	0.0000
			RWMHM	-1.4143	0.1573
		Skewed t	MLE	-10.5107	0.0000
			RWMHM	-2.9645	0.0030

Table 25: Diebold-Mariano Test Results of NVDA (MAPE)

Model 1	Model 2	Distribution	Method	DM Statistic	p-value
ARCH	GARCH	Normal	MLE	0.2272	0.8203
			RWMHM	0.2574	0.7969
		Student-t	MLE	0.6639	0.5067
			RWMHM	0.5449	0.5858
		Skewed t	MLE	0.6807	0.4961
			RWMHM	0.5622	0.5740
ARCH	GJR-GARCH	Normal	MLE	1.9239	0.0544
			RWMHM	1.9531	0.0508
		Student-t	MLE	1.8483	0.0646
			RWMHM	1.8306	0.0672
		Skewed t	MLE	1.8617	0.0626
			RWMHM	1.7971	0.0723
ARCH	EGARCH	Normal	MLE	1.9298	0.0536
			RWMHM	1.8942	0.0582
		Student-t	MLE	2.1188	0.0341
			RWMHM	2.1027	0.0355
		Skewed t	MLE	2.1234	0.0337
			RWMHM	2.0961	0.0361
ARCH	GAS	Normal	MLE	-0.4394	0.6604
			RWMHM	-0.4670	0.6405
		Student-t	MLE	1.8844	0.0595
			RWMHM	1.8355	0.0664
		Skewed t	MLE	1.8924	0.0584
			RWMHM	1.8635	0.0624
GARCH	GJR-GARCH	Normal	MLE	1.0741	0.2828
			RWMHM	1.1155	0.2646
		Student-t	MLE	0.8044	0.4212
			RWMHM	0.8312	0.4059
		Skewed t	MLE	0.8292	0.4070
			RWMHM	0.8636	0.3878
GARCH	EGARCH	Normal	MLE	1.4391	0.1501
			RWMHM	1.4594	0.1444
		Student-t	MLE	1.3473	0.1779
			RWMHM	1.3905	0.1644
		Skewed t	MLE	1.3723	0.1700
			RWMHM	1.4546	0.1458
GARCH	GAS	Normal	MLE	-1.1526	0.2491
			RWMHM	-1.2137	0.2249
		Student-t	MLE	1.7406	0.0818
			RWMHM	1.6702	0.0949
		Skewed t	MLE	1.7422	0.0815
			RWMHM	1.6609	0.0967
GJR-GARCH	EGARCH	Normal	MLE	0.5771	0.5639
			RWMHM	0.6366	0.5244
		Student-t	MLE	1.2502	0.2112
			RWMHM	1.4349	0.1513
		Skewed t	MLE	1.2706	0.2039
			RWMHM	1.4809	0.1386
GJR-GARCH	GAS	Normal	MLE	-1.6795	0.0931
			RWMHM	-1.7499	0.0801
		Student-t	MLE	1.1627	0.2449
			RWMHM	1.0543	0.2918
		Skewed t	MLE	1.1680	0.2428
			RWMHM	0.9635	0.3353
EGARCH	GAS	Normal	MLE	-2.0175	0.0436
			RWMHM	-2.0429	0.0411
		Student-t	MLE	1.0114	0.3118
			RWMHM	0.6211	0.5346
		Skewed t	MLE	1.0128	0.3111
			RWMHM	0.4093	0.6823

Table 26: Diebold-Mariano Test Results of NVDA (RMSE)

Model 1	Model 2	Distribution	Method	DM Statistic	p-value
ARCH	GARCH	Normal	MLE	5.2795	0.0000
			RWMHM	5.0420	0.0000
		Student-t	MLE	6.1689	0.0000
			RWMHM	7.0203	0.0000
		Skewed t	MLE	6.1094	0.0000
			RWMHM	6.8064	0.0000
ARCH	GJR-GARCH	Normal	MLE	5.9172	0.0000
			RWMHM	5.7936	0.0000
		Student-t	MLE	6.5493	0.0000
			RWMHM	6.6635	0.0000
		Skewed t	MLE	6.8478	0.0000
			RWMHM	7.7014	0.0000
ARCH	EGARCH	Normal	MLE	7.8595	0.0000
			RWMHM	8.0580	0.0000
		Student-t	MLE	8.6671	0.0000
			RWMHM	10.2027	0.0000
		Skewed t	MLE	9.1323	0.0000
			RWMHM	11.7089	0.0000
ARCH	GAS	Normal	MLE	3.0518	0.0023
			RWMHM	3.1235	0.0018
		Student-t	MLE	1.9849	0.0472
			RWMHM	9.6424	0.0000
		Skewed t	MLE	2.1616	0.0307
			RWMHM	10.8041	0.0000
GARCH	GJR-GARCH	Normal	MLE	2.3306	0.0198
			RWMHM	2.1983	0.0279
		Student-t	MLE	1.6792	0.0931
			RWMHM	0.6652	0.5059
		Skewed t	MLE	2.3340	0.0196
			RWMHM	2.5906	0.0096
GARCH	EGARCH	Normal	MLE	4.3809	0.0000
			RWMHM	4.8527	0.0000
		Student-t	MLE	4.6508	0.0000
			RWMHM	5.6805	0.0000
		Skewed t	MLE	5.5840	0.0000
			RWMHM	8.5206	0.0000
GARCH	GAS	Normal	MLE	-3.3021	0.0010
			RWMHM	-2.3581	0.0184
		Student-t	MLE	-5.0153	0.0000
			RWMHM	5.4064	0.0000
		Skewed t	MLE	-4.6536	0.0000
			RWMHM	7.4330	0.0000
GJR-GARCH	EGARCH	Normal	MLE	4.0680	0.0000
			RWMHM	5.1601	0.0000
		Student-t	MLE	5.8944	0.0000
			RWMHM	9.3251	0.0000
		Skewed t	MLE	6.4443	0.0000
			RWMHM	10.5893	0.0000
GJR-GARCH	GAS	Normal	MLE	-3.3497	0.0008
			RWMHM	-2.8016	0.0051
		Student-t	MLE	-6.1174	0.0000
			RWMHM	3.7015	0.0002
		Skewed t	MLE	-6.3435	0.0000
			RWMHM	3.5695	0.0004
EGARCH	GAS	Normal	MLE	-4.8966	0.0000
			RWMHM	-4.5981	0.0000
		Student-t	MLE	-10.0075	0.0000
			RWMHM	-1.4143	0.1573
		Skewed t	MLE	-10.5107	0.0000
			RWMHM	-2.9645	0.0030