Analysis of Daily COVID-19 Cases, Hospitalizations, and Deaths in NYC

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Introduction

Publicly available data for COVID-19 collected by governments, healthcare facilities, and academic institutions has facilitated research on the evolution of the pandemic. In this study, we analyzed COVID-19 trends in each of the five boroughs of New York City (Bronx, Brooklyn, Manhattan, Queens, and Staten Island) using data collected by the New York Department of Health (NYDOH). The NYDOH dataset includes city-wide and borough-specific daily counts of COVID-19 cases, hospitalizations, and deaths since February 29, 2020 (the day of the first laboratory-confirmed case in New York City). A subset of this data up to December 11, 2020 was included in our analyses.

Figure 1 shows the citywide daily counts of COVID-19 cases, hospitalizations, and deaths from February 29 to December 11, 2020. We defined two pandemic waves in this time frame based on observations of daily cases: the first wave starts from February 29, 2020 and ends on September 14, 2020, in which a spike in cases was seen around April and dropped significantly until mid-September; the second wave starts on September 15 and is characterized by a resurgence of cases as winter approached.

The Richard's growth function is a S-shaped function that is commonly used to model the growth of a population. Let $N(t)$ be a population at time t, the function takes the form:

$$N(t) = \frac{a}{\{1 + d\exp\{-k(t - t_0)\}\}^{1/d}}$$

where (a, k, d, t_0) are shape parameters. The parameter a is the upper bound of the function; k is the growth rate, which controls the slope at an inflection (where the curve changes from convex to concave); t_0 is the time at an inflection; and d is another shape parameter that has no clear substantive meaning.

Let t_i be the number of days since the beginning of a pandemic wave. Let $(y_i, t_i)_{i=1,\ldots,n}$ be a sequence of observed daily cases/hospitalizations/deaths at time t_i. Let $Y_i = \sum_{k=1}^{i} y_k$ be the cumulative number of cases/hospitalizations/deaths by time t_i. We assumed that (Y_i, t_i) follows the following non-linear model:

$$Y_i = N(t_i, \boldsymbol{\theta}) + \epsilon_i$$

where $N(t_i, \boldsymbol{\theta})$ is the Richard's growth function with parameters $\boldsymbol{\theta} = (a, k, d, t_0)^T$ and ϵ_i is the random error with mean zero, i.e. $E[\epsilon_i] = 0$.

Our analysis was composed of three tasks. Task 1 developed a Newton-Raphson algorithm to fit a Richard's curve to each borough's cumulative cases. Task 2 then applied the same algorithm to each borough's cumulative hospitalizations and deaths. The fitted curves were compared across the five boroughs as well as the two pandemic waves. Lastly, Task 3 aimed to predict the trends (in cases, hospitalizations, and deaths) after December 11, 2020 for each borough in order to provide suggestions regarding the distribution of vaccination, which was authorized by the FDA on December 11.

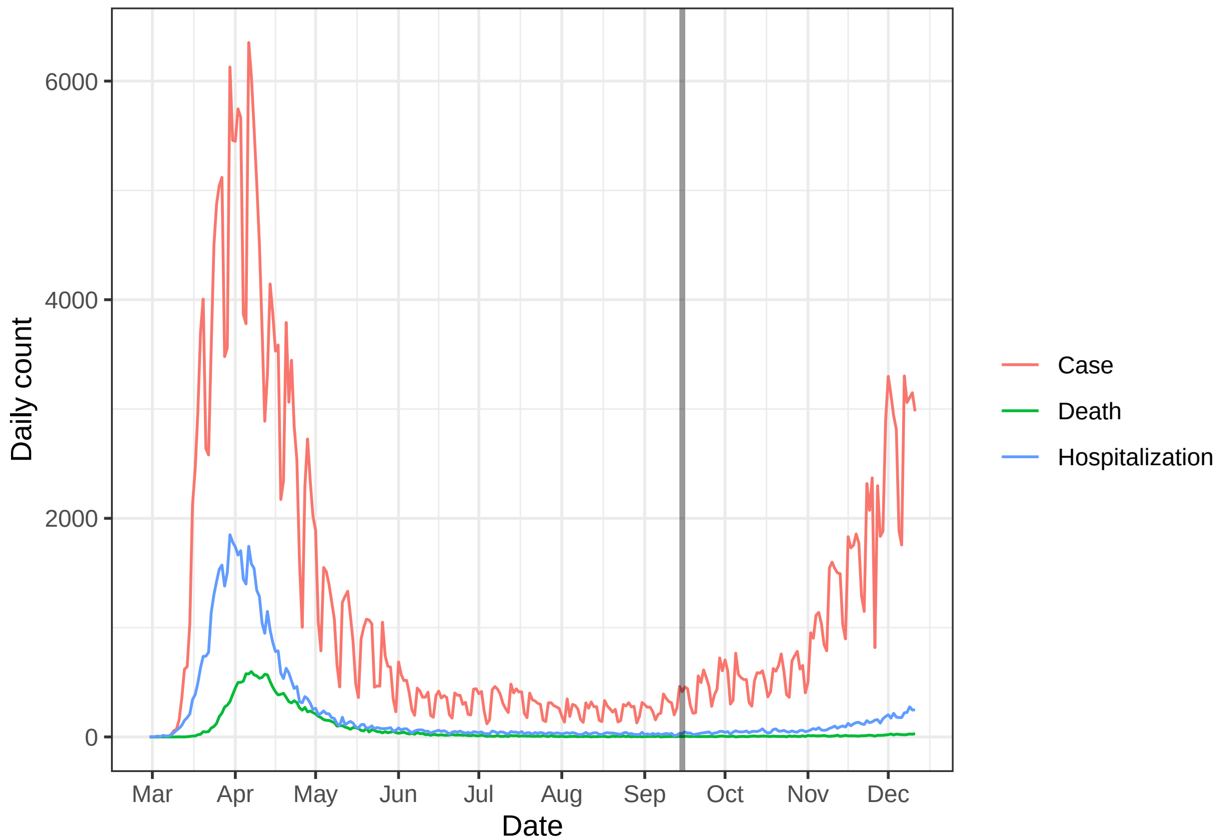


Figure 1: Daily counts of COVID-19 cases, hospitalizations, and deaths in New York City

Task 1

Our goal for Task 1.1 was to develop a Newton-Raphson algorithm to fit Richard curves to each NYC borough's cumulative number of cases for a pandemic wave. We used a simple gradient descent algorithm that aims to minimize the sum of squared errors (SSE), $h(\boldsymbol{\theta})$.

Since $E[Y_i] = N(t_i, \boldsymbol{\theta})$, then $h(\boldsymbol{\theta})$ is defined as:

$$
\begin{aligned}
h(\boldsymbol{\theta}) &= \sum_{i=1}^{n} (Y_i - N(t_i, \boldsymbol{\theta}))^2 \\
&= \sum_{i=1}^{n} \left[Y_i - a \left\{ 1 + d \exp\left\{ -k(t_i - t_0) \right\} \right\}^{-1/d} \right]^2
\end{aligned}
$$

The gradient of $h(\boldsymbol{\theta})$, $\nabla h(\boldsymbol{\theta})$, is calculated as:

$$
\begin{aligned}
\nabla h(\boldsymbol{\theta}) &= -2 \sum_{i=1}^{n} [Y_i - N(t_i, \boldsymbol{\theta})] \cdot \nabla N(t_i, \boldsymbol{\theta}) \\
&= -2 \sum_{i=1}^{n} [Y_i - N(t_i, \boldsymbol{\theta})] \cdot \left(\frac{\partial N(t_i, \boldsymbol{\theta})}{\partial a}, \frac{\partial N(t_i, \boldsymbol{\theta})}{\partial k}, \frac{\partial N(t_i, \boldsymbol{\theta})}{\partial d}, \frac{\partial N(t_i, \boldsymbol{\theta})}{\partial t_0} \right)^T
\end{aligned}
$$

where

$$
\begin{aligned}
\frac{\partial N(t_i, \boldsymbol{\theta})}{\partial a} &= \frac{1}{(1 + de^{-k(t-t_0)})^{1/d}} \\
\frac{\partial N(t_i, \boldsymbol{\theta})}{\partial k} &= \frac{a(t - t_0)e^{-k(t-t_0)}}{(1 + de^{-k(t-t_0)})^{1+1/d}} \\
\frac{\partial N(t_i, \boldsymbol{\theta})}{\partial d} &= -\frac{a(e^{-k(t-t_0)}d - \ln(1 + e^{-k(t-t_0)}d)(1 + e^{-k(t-t_0)}d))}{d^2(1 + de^{-k(t-t_0)})^{1+1/d}} \\
\frac{\partial N(t_i, \boldsymbol{\theta})}{\partial t_0} &= -\frac{ake^{-k(t-t_0)}}{(1 + de^{-k(t-t_0)})^{1+1/d}}
\end{aligned}
$$

Our gradient descent algorithm can be summarized as follows:

1. Set $\boldsymbol{\theta}_0$, the starting values for $\boldsymbol{\theta}$ (see later in this section for more details on how to pick these).
2. Update $\boldsymbol{\theta}$ based on $\boldsymbol{\theta}_j = \boldsymbol{\theta}_{j-1} - \lambda \cdot I_{4\times4} \nabla h(\boldsymbol{\theta}_{j-1})$, where λ is a user-chosen learning rate (setting λ is to a small value, such as 1^{-10}, works well for this data).
3. If $h(\boldsymbol{\theta}_j) \geq h(\boldsymbol{\theta}_{j-1})$, then decrease the learning rate further and recalculate $h(\boldsymbol{\theta}_j)$, replacing λ with $\frac{\lambda}{10}$. Continue repeating this step until $h(\boldsymbol{\theta}_j) < h(\boldsymbol{\theta}_{j-1})$, i.e. until there is a decrease in SSE from the previous iteration.
4. Continue repeating steps 3 and 4 until convergence is reached, i.e. when the absolute difference between $h(\boldsymbol{\theta}_j)$ and $h(\boldsymbol{\theta}_{j-1})$ is smaller than a very small tolerance level.

Although our algorithm is not the most efficient algorithm, it has two main advantages. One, it is simple to compute and does not rely on the calculation of the Hessian, which is complicated. Two, using the symmetric and positive definite matrix $I_{4\times4}$ as a replacement for the Hessian guarantees that we have a descent direction. This means that we will be able to find some $\lambda \in (0, 1)$ that ensures that the updated $\boldsymbol{\theta}_j$ has a smaller SSE than the previous iteration's $\boldsymbol{\theta}_{j-1}$.

In step 1 of the algorithm, the user is required to choose the starting values, $\boldsymbol{\theta}_0$. We found that poor choices of $\boldsymbol{\theta}_0$ resulted in non-convergence or incorrect convergence issues. Choices of $\boldsymbol{\theta}_0$ based on the observed cumulative cases for the borough and the epidemiological interpretations of the Richard growth parameters resulted in fast convergence and good final estimates of Y_i. Our guidelines for how to tailor the starting values for each borough and wave are as follows:

- Since a is the upper bound of the Richard growth function, then let its starting value be the maximum number of cumulative cases in the pandemic wave. For the first wave, this is easily calculated as $\max(Y_i)$. For the second wave, since we only have observed data for the first half of the wave (i.e. up until about the inflection point), we must estimate what the maximum number of cumulative cases will be. We recommend using about $2 \times \max(Y_i)$ under the assumption that half of the maximum cumulative cases occurs at the inflection point. We made this assumption because we saw this trend across the boroughs in the first wave and assumed that the second wave would follow a similar pattern.
- Since t_0 is the time of inflection, then let its starting value be the time t_i where the inflection point occurs. This can be chosen as the point where the curve goes from convex to concave based on a plot of Y_i against t_i. Note that for the second wave, since we only have observed data for the first half of the wave, this point will occur towards the end of the observed data.
- Since k is the growth rate that controls the slope at the inflection point, then let its starting value be calculated as the slope at the inflection point t_0 standardized by the cumulative cases at the inflection point: $\frac{(Y_{i,t_0+m} - Y_{i,t_0-m})/[(t_0+m)-(t_0-m)]}{Y_{i,t_0}}$, where Y_{i,t_0-m} and Y_{i,t_0+m} are the cumulative cases corresponding to m days before and after the inflection point, respectively, Y_{i,t_0} is the cumulative cases corresponding to the inflection point, and $(t_0+m)-(t_0-m)$ is a small range around the inflection point. This can easily be computed by using two points around the inflection point to fit a linear regression, regressing Y_i on t_i.
- Since d has no epidemiological interpretation, picking its starting value is tricky. We found that values between 0 and 0.5 worked well. However, we advise investigators to try a few different values for d and pick what works best.

Figure 2 gives an example of how a starting curve for Wave 1 data on NYC would look, compared to the final fitted curve. We can see that the starting curve follows the general trend of the observed cumulative cases, but the final curve obtained from running our algorithm fits the observed data much better. We also note that from about $t_i = 100$ to $t_i = 200$ of the first wave, the observed cumulative cases follow a linear trend, so we would expect that our final Richard growth curve will never be able to fit the observed data very well in this portion of the wave since the Richard growth function assumes a sigmoidal shape.

The algorithm described in this section was also used in Task 1.2 to fit Richard curves to each NYC borough's cumulative number of hospitalizations and deaths for each pandemic wave.

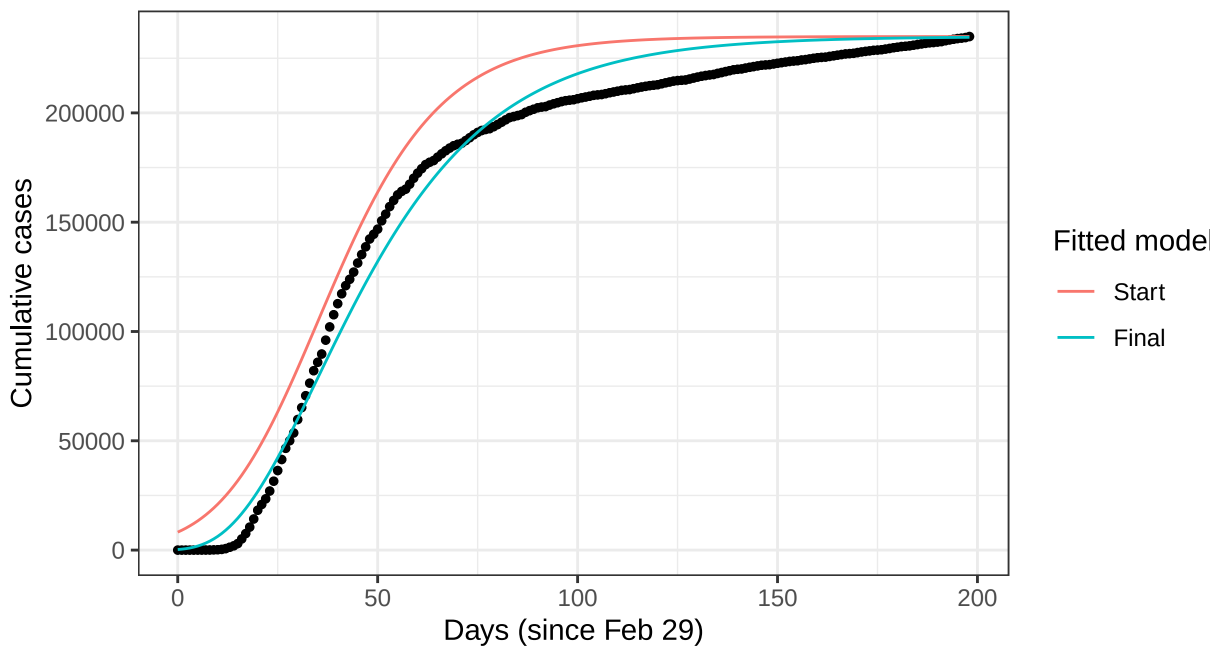
Task 2

Cumulative Cases

We first present the results for cumulative cases in all boroughs to investigate the algorithm performance. The two COVID-19 waves are defined as follows: waves 1: February 29, 2020 to September 14, 2020; wave 2: September 14, 2020 to December 11, 2020.

As discussed in the previous section, the starting values are chosen based on the following guidelines:

- $a = \max(Y_i)$;
- $t_o \approx \text{argmax}(Y_i)$;
- k is the standardized slope for cumulative cases 4 days before and after inflection date.
- d is 0.5



	a	k	d	t0
start	234933	0.062	0.5	35
final	234933	0.04	−0.169	34.976

Figure 2: Example for Wave 1 NYC data comparing starting values and starting curve to the final values and fitted curve obtained from our algorithm.

In the first figure, we present the fitted curves (red) and compare the true cumulative cases (black). From a direct observation, we conclude that the algorithm performed well in approximating the real data.

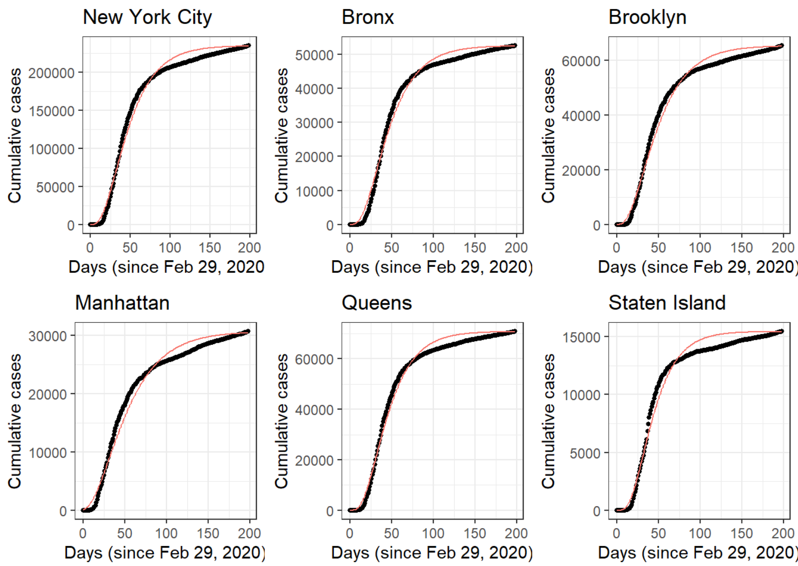


Figure 3: Fitted curves of cumulative cases for first wave

We further present the estimates parameters for the first wave cumulative cases. Since we have access to the first wave data, the estimated parameters can serve as a validation for the algorithm performance. By a direction observation of estimated a, t_0 parameters, we conclude our algorithm performed well since the estimated parameters approached the truth well.

a	k	d	t0	location
65371	0.0347605	-0.3283364	32.03968	Brooklyn
52568	0.0378430	-0.2973221	32.05189	Bronx
30672	0.0295018	-0.3390702	31.53425	Manhattan
70855	0.0411500	-0.2674628	32.04801	Queens
15458	0.0449558	-0.2370432	32.02065	Staten Island

Figure 4: Estimated parameters of cumulative cases for first wave

We then present the fitted curves and estimated parameters for the second wave cumulative cases. Since we only obtained partial data for second wave, several assumptions about starting values are made for the estimation.

- We first assume the inflection point t_0 is at around the end of the observed data
- $a = 2 * y_{t0}$ with the assumption that the maximum number of cumulative cases will be two times the case number at the inflection point
- k is the standardized slope for cumulative cases 4 days before and after inflection date.
- d is 0.5

From the blow figure and estimated parameters, we notice a good recovery of the true cases cruves with our estimated curves in all boroughs.

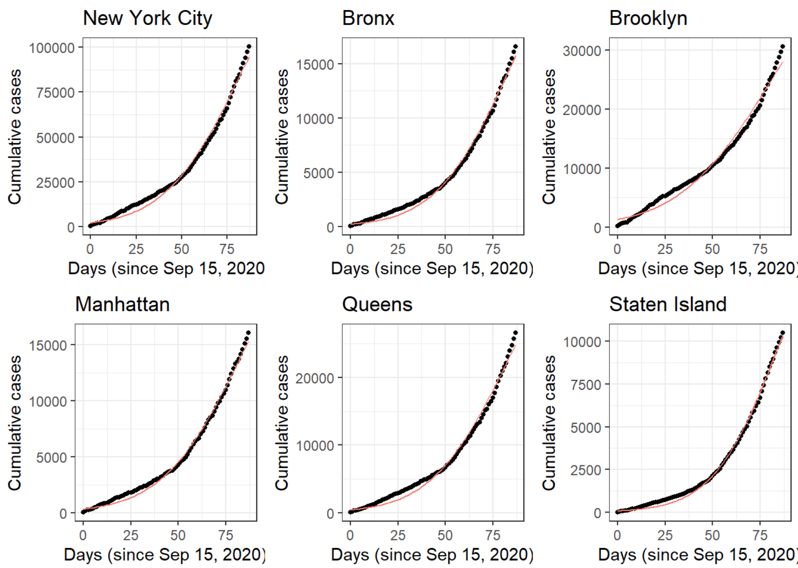


Figure 5: Fitted curves of cumulative cases for second wave

a	k	d	t0	location
200482	0.0359293	0.5644271	85.14072	New York City
61054	0.0287465	0.4861351	85.00972	Brooklyn
33152	0.0394349	0.5673617	85.00639	Bronx
32140	0.0374023	0.5982085	84.99862	Manhattan
53170	0.0364060	0.5210164	85.03427	Queens
20942	0.0488719	0.6912236	85.02036	Staten Island

Figure 6: Estimated parameters of cumulative cases for second wave

Cumulative Hospitalizations

We fit the Richard's growth curve for cumulative hospitalizations in all boroughs in NYC. Similar to case observations, we split hospitalizations into two waves: 1) February 29, 2020 to September 14, 2020; 2) September 14, 2020 to December 11, 2020.

When fitting the first wave of hospitalizations, we use the following assumptions to pick starting values:

- a is $\max(Hospitalizations)$ for all boroughs and NYC, assuming there is not much increase in hospitalization counts or health resources to accommodate patients after the curve levels out.
- Different inflection date t_0 for different locations, as we observe slopes at different dates fit observations better. There are minor differences among inflection dates. Brooklyn and Manhattan have March 31 as starting value for inflection date; Staten Island has April 1; Queens, Bronx, and NYC have April 3, 2020.
- k is the standardized slope for cumulative hospitalization data on 4 days before and after inflection date.
- d is 0.5

Figure 7 are plots for fitted curve for the first hospitalization wave. Figure 8 are the parameters used to fit the first wave.

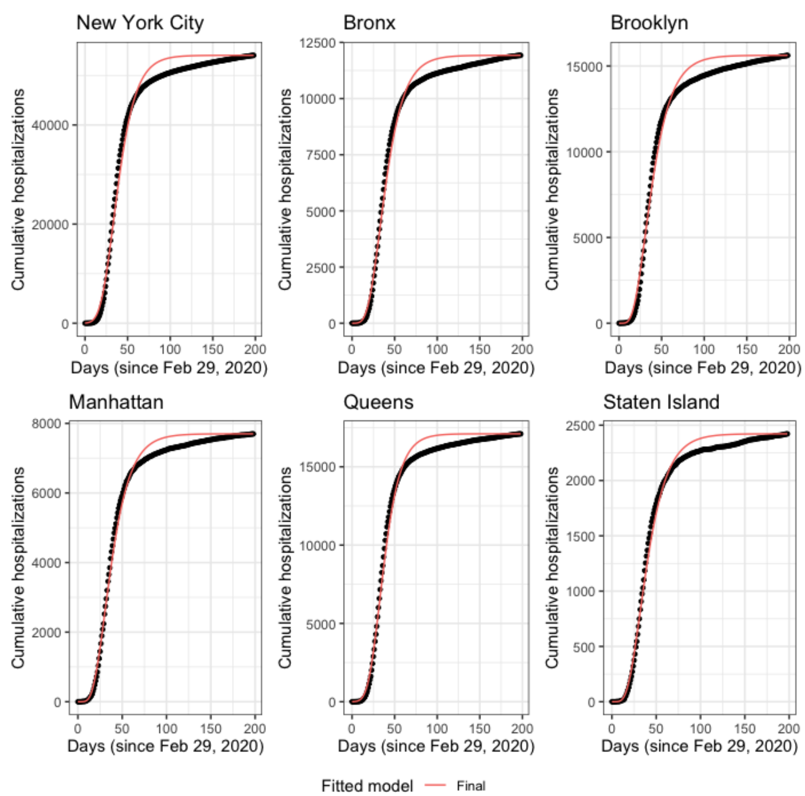


Figure 7: The first hospitalization wave - fitted curves

The fit is very close to the observations in all locations. The fitted curve is overestimating hospitalizations around the dates that the curve starts to level out. However, since healthcare resources were extremely limited during the outbreak of the pandemic, it is safe to overestimate than underestimate hospitalization counts. The final fitted t_0 for all locations are indeed a couple of days apart, corresponding to our initial belief.

a	k	d	t0	location
54041.997	0.0708072	0.0997570	33.59340	New York City
15621.999	0.0593658	-0.1582541	31.05414	Brooklyn
11923.966	0.0648936	-0.0554206	32.94398	Bronx
7699.978	0.0634434	-0.1427894	30.67906	Manhattan
17104.972	0.0771895	0.0816411	32.68012	Queens
2421.995	0.0596605	-0.1477803	32.04848	Staten Island

Figure 8: The first hospitalization wave - parameters

Observed hospitalization in the second wave seems to be the first half of a regular Richard's growth curve. We do see a gradual increase and then more rapid increase in hospitalization during this period of time. The changes in slopes are tricky to model this time. For the second wave of hospitalization, the initial values are decided as the following:

- a is $2.2 \times \max(\mathit{hospitalization})$ for all boroughs and NYC, assuming that half of the maximum hospitalization occurs at the inflection point.
- December 9, 2020 as t_0 for all locations.
- k is the standardized slope for cumulative hospitalization data on 2 days before and after inflection date due to rapid change in hospitalizations around our assumed inflection date.
- d is 0.25 for Staten Island and 0.5 for all other locations.

Figure 9 are plots for fitted curve for the second hospitalization wave. Figure 10 are the parameters used to fit the second wave.

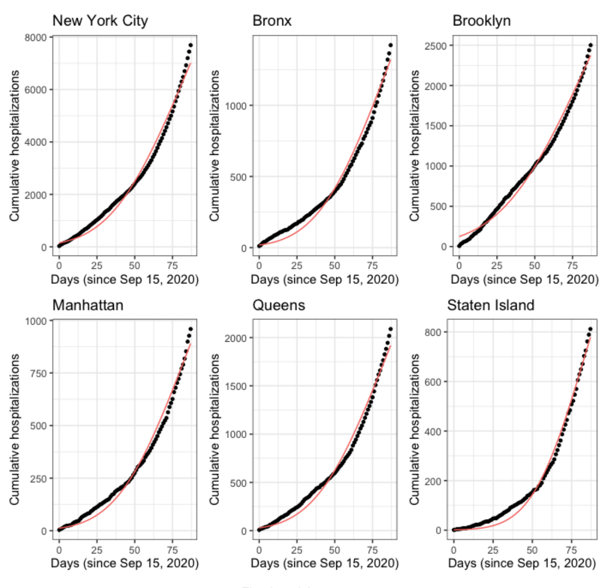


Figure 9: The second hospitalization wave - fitted curves

The fitted curves still look very close to the observation. We tend to underestimate when the observed hospitalization is mildly increasing, and overestimate when the speed of the observed increase is growing. The estimate values for growth rate parameter k reflect how different the slope changes in each borough.

a	k	d	t0	location
16922.400	0.0236201	0.1807922	85.03988	New York City
5502.200	0.0229339	0.2981760	85.01396	Brooklyn
3130.600	0.0265017	0.2164909	85.03464	Bronx
2109.799	0.0265173	0.2180125	85.03329	Manhattan
4595.800	0.0254216	0.1943075	85.05150	Queens
1786.400	0.0365780	0.2671106	85.00761	Staten Island

Figure 10: The second hospitalization wave - parameters

Cumulative Deaths

We also fit growth curves to estimate cumulative deaths in NYC. We split the deaths into two waves, in the same way as mentioned previously.

We use the following assumptions to pick starting values:

- a is $\max(Deaths)$ for all boroughs and NYC.
- All locations have the same starting value t_0, which is April 10, 2020. This inflection point is later than the ones assumed for cases and hospitalizations, as we assume that a rise in deaths will trail several days behind rises in cases and hospitalizations.
- k is the standardized slope for cumulative deaths data on 4 days before and after inflection date.
- d is 0.5.

Figure 11 shows plots for fitted death curves for the first wave. Figure 12 shows the final parameters used to fit the first wave curves.

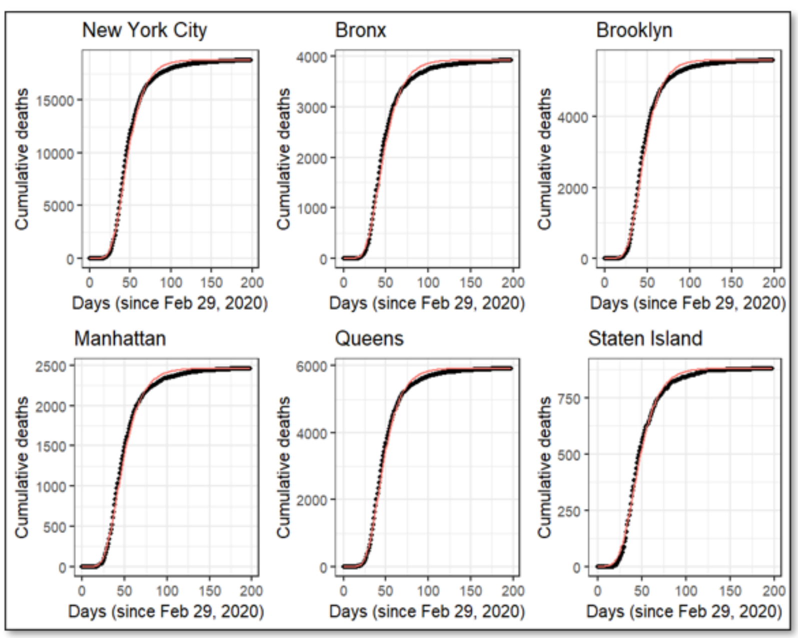


Figure 11: The first death wave - fitted curves

The estimated Richard's curve fits the observed deaths curve very well for each borough. is very close to the observations in all locations.

a	k	d	t0	location
18799.9975	0.0705570	0.0167689	40.85405	New York City
5602.9895	0.0747471	0.0592430	40.78483	Brooklyn
3931.9844	0.0680085	-0.0037896	40.80752	Bronx
2461.9921	0.0653303	-0.0179362	40.98036	Manhattan
5922.9928	0.0711746	0.0033253	40.88255	Queens
879.9963	0.0690191	0.1212710	41.03465	Staten Island

Figure 12: The first death wave - parameters

For the second wave of deaths, the initial values are decided as the following:

- a is $2 \times \max(Deaths)$ for all boroughs and NYC, assuming that half of the maximum deaths occur at the inflection point.
- December 9, 2020 as t_0 for all locations. We would assume that the inflection point would be later for deaths than it is for cases and hospitalizations, but since we only have data until December 11, it did not make sense to try to push the starting inflection date further.
- k is the standardized slope for cumulative deaths data on 2 days before and after the inflection date.
- d is 0.25 for Staten Island and 0.5 for all other locations.

Figure 13 are plots for fitted curve for the second death wave. Figure 14 shows the final parameters used to fit the second wave.

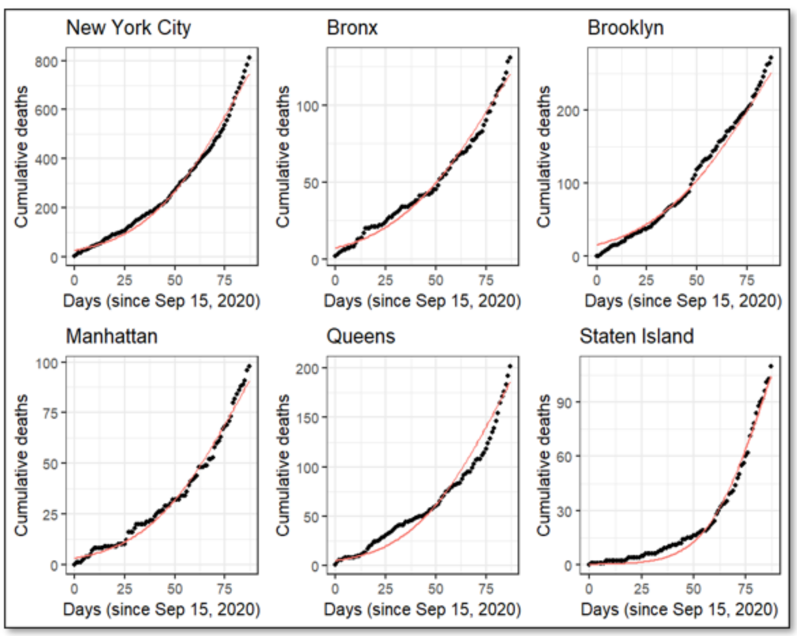


Figure 13: The second death wave - fitted curves

The fitted curves look relatively close to the observed death trends. The shapes of the curves across boroughs seem very different, showing that the predicted trends in deaths by the end of the second wave will vary across boroughs.

a	k	d	t0	location
1625.9999	0.0300735	0.4806397	85.00461	New York City
544.0002	0.0275453	0.5243478	84.99782	Brooklyn
262.0000	0.0271101	0.4989538	85.00016	Bronx
196.0000	0.0308474	0.5006601	85.00000	Manhattan
403.9998	0.0319881	0.4858160	85.00171	Queens
220.0000	0.0524564	0.5004613	85.00010	Staten Island

Figure 14: The second death wave - parameters

Task 3

Cumulative Cases

From the previous task, we successfully estimated and obtained parameters that approximated the first half of the second wave data well. A natural extrapolation would be extending the curves so that we may predict the cumulative cases based on th observed the data. Hence, we utilize the estimated final parameters and use that for cumulative cases prediction.

We incorporate borough population to infer cases per 100,000 capita so that based on the y axis we are able to make a direct comparison of the severity of the second wave. Based on the interpretation of the estimated parameters, we obtained the following observations:

- a can be interpreted as maximum number of cumulative cases in each borough. Staten Island is predicted to have the largest cases per 100,000 capita and Manhattan is predicted to have the smallest
- k can be interpreted as standardized slope at the inflection point. It Provides intuition of the rate of cases increase at inflection point. Staten Island has biggest k while Brooklyn has smallest k.

Therefore, based on the previous prediction and observation, we would suggestion that distributing vaccination to the boroughs with higher predicted cumulative cases per capita and large rate of cases increase. In this case, Staten Island should be prioritized for vaccination rollout.

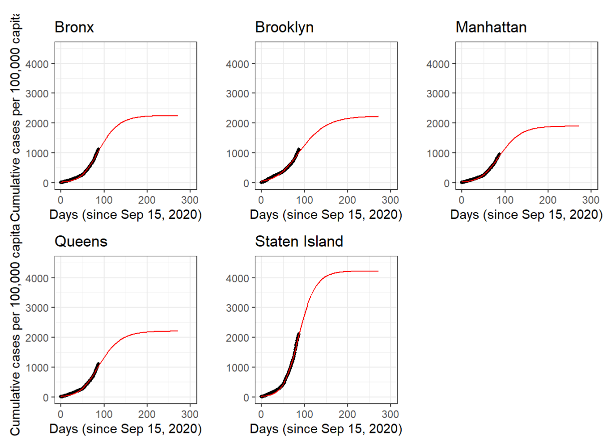


Figure 15: Predicted cases for each borough

a	k	d	t0	location
200482	0.0359293	0.5644271	85.14072	New York City
61054	0.0287465	0.4861351	85.00972	Brooklyn
33152	0.0394349	0.5673617	85.00639	Bronx
32140	0.0374023	0.5982085	84.99862	Manhattan
53170	0.0364060	0.5210164	85.03427	Queens
20942	0.0488719	0.6912236	85.02036	Staten Island

Figure 16: Estimated parameters of cumulative cases for second wave

Cumulative Hospitalizations

We used the parameters in the fit for second wave of cumulative hospitalizations to make long-term predictions and hope to draw insight on vaccine distribution from the predicted curve.

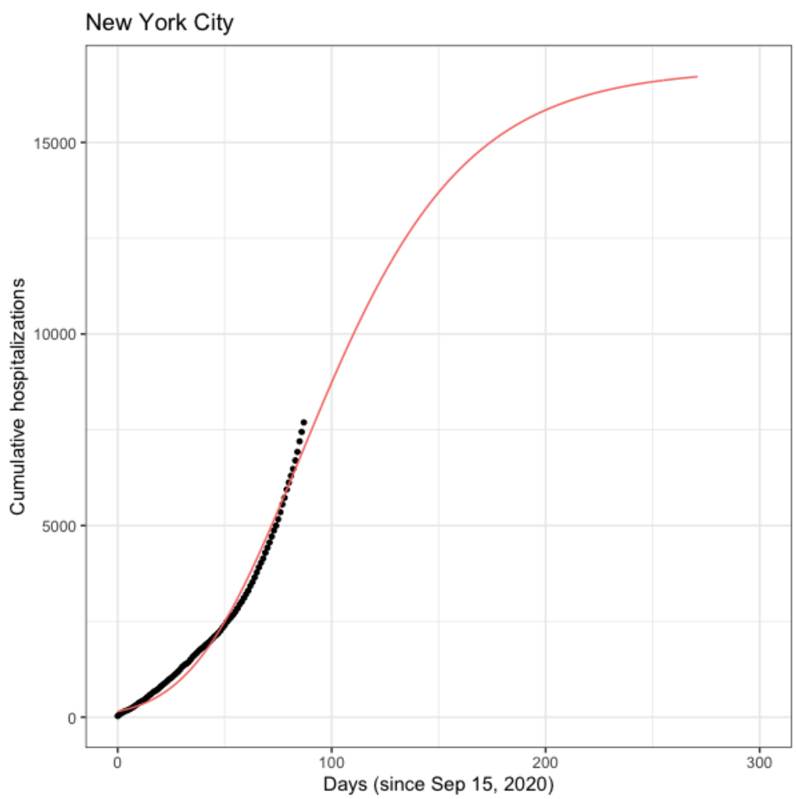


Figure 17: Predicted hospitalization for NYC

To make our predictions more comparable, we plotted predicted cumulative hospitalizations per 100,000 capita for each borough.

Unfortunately, Staten Island has the highest predicted cumulative hospitalizations per 100,000 capita and its predicted growth rate is the highest.

Bronx and Manhattan both have the second highest predicted growth rate. But Bronx has the higher predicted hospitalizations per 100,000 capita.

Even though growth rate and predicted hospitalization per 100,000 capita are different for each borough, the predicted inflection date is almost the same for all boroughs. $t_0 = 85$ corresponds to December 9, 2020.

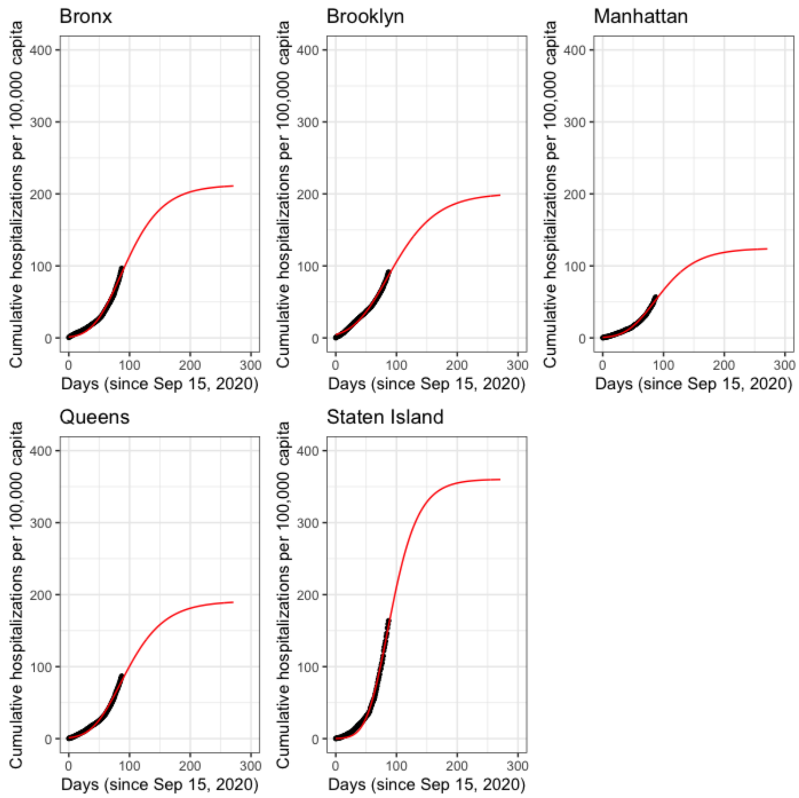


Figure 18: Predicted hospitalization for each borough

This date is also very similar to the predicted inflection date for cumulative cases. This could be a result of rapid disease progression, as some people who were contracted need to be hospitalized immediately.

COVID-19 vaccines first became available in the US on December 14, 2020 according to U.S. Department of Health & Human Services. Therefore, while prioritizing hot spots such as Staten Island and Bronx, the government should also prepare enough inventory to meet needs city-wide to provide maximum protection against COVID-19 due to its rapid disease progression.

Cumulative Deaths

The predicted plots for cumulative deaths in the second wave, scaled for population size, for each borough are shown below in Figure 19.

Based on the final parameters shown in Figure 14, we see that Brooklyn and Manhattan have the highest estimated k values (slope at the inflection point). Brooklyn and Queens have the highest number of estimated cumulative deaths. Staten Island is consistently predicted to increase quickly and have a high rate of deaths overall.

When we scale for population in the plots in the figure above, we see that Brooklyn and the Bronx actually have the highest number of predicted cumulative deaths per 100,000 capita.

Interestingly, in Brooklyn and the Bronx, cases aren't expected to increase at a quick rate, but hospitalizations and deaths are. This would give us motivation us to roll out vaccinations quickly to Brooklyn and the Bronx, to prevent severe cases and therefore hospitalizations and deaths.

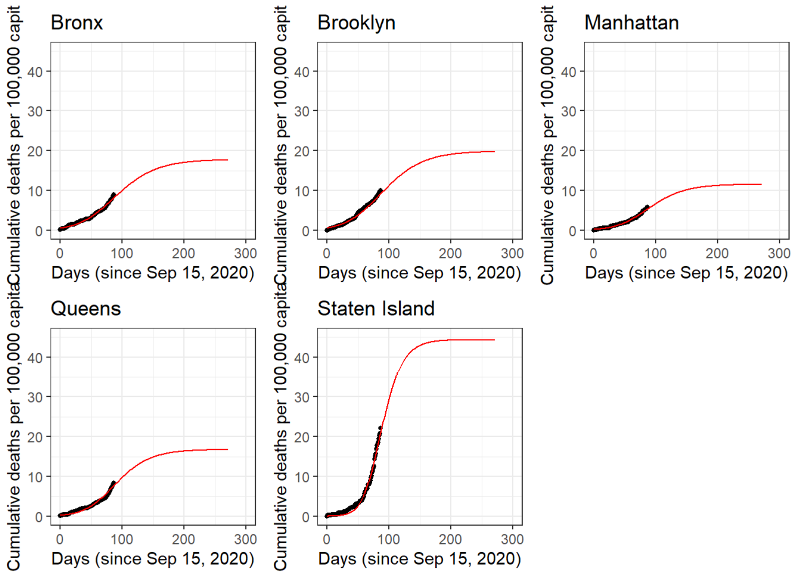


Figure 19: Predicted hospitalization for NYC

Conclusions

Task 1. Richard's growth curves fit relatively well to COVID-19 case, hospitalization, and death data when starting values were chosen well and waves were specified appropriately.

Task 2. From fitting the curves, we learned that Brooklyn and Queens had experienced the greatest number of cases, hospitalizations, and deaths by the end of each wave. Empirically, curves varied more between boroughs during the second wave than during the first.

Task 3. Extending our wave 2 fitted curves and accounting for each borough's population size allowed us to make recommendations for vaccine rollout. We recommend that, if the number of vaccines is limited, that we target Staten Island, which is expected to see the largest number of cases, deaths, and hospitalizations per capita. We also suggest rolling out vaccines quickly to Brooklyn and the Bronx, where the number of hospitalizations and deaths are expected to be large and to increase at a quick rate around the inflection point. It is important to target these areas because we noticed that even though the number of cases may be lower, the large amount of hospitalizations and deaths tells us that COVID-19 may take a large toll on healthcare facilities in these areas.