Introduction to algebra

4.1 - The Use of Letters In Algebra

A) The Use Of Letters

In arithmetic, we manipulate numbers that have definite values. In algebra, we use symbols to represent numbers. Algebra can be considered as an extension of arithmetic. The symbols generally used are the letters of the alphabet *a, b, c, … x, y, z, A, B, C, … X, Y, Z*. A letter may represent any possible numerical values. We call the letter a variable and an expression which consists of numbers, letters and operations, an algebraic expression.

In general, an algebraic expression involves numbers and letters that are connected with operation symbols such as '+', '-', 'x', '÷'.

For example, 9*x* - *y, m* x *n* x *p* and 4 x *t* ÷ *z* are algebraic expressions.

B) Basic Notations in Algebra

In algebra, there are rules for writing algebraic expressions. The operation symbols '+', '-', 'x', '÷' and '='have the same meanings in algebra as in arithmetic. Let us learn how to use algebraic expressions.

Tom is 12 years old now. Find his age after *x* years.
Answer: (12 + *x*) years old.

The breadth of a rectangle is 6cm shorter than half of its length. Find the breadth of the rectangle when the length is *y* cm.
Answer: ½ x *y* - 6 = (*y*/2 - 6) cm

Note:
- $y/2 = y \div 2$
- $y/2 - 6$ is an algebraic expression in variable y.

The following table summarises and compares the arithmetic expressions and algebraic expressions for the four basic operations.

Operation	Statement	Arithmetic Expression	Statement	Algebraic Expression
Addition	• Add 5 to 7 • Sum of 5 and 7 • 5 plus 7	5 + 7 = 7 + 5	• Add a to b • Sum of a and b • a plus b	$a + b$ $= b + a$
Subtraction	• Subtract 10 from 18 • Take away 10 from 18 • 18 minus 10	18 - 11 $\neq$ 11 -18	• Subtract c from d • Take away c from d • d minus c	d - c $\neq c$ - d In general
Multiplication	• Multiply 2 by 6 • 2 times 6 • Twice of 6 • Product of 2 and 6	2 x 6 = 6 x 2	• Multiply g by h • g times h • Product of g and h	g x h $= h$ x g $= gh$
Division	• Divide 24 by 3 • Quotient of 24 divided by 3	24 $\div$ 3 = 24/3 $\neq$ 3 $\div$ 24	• Divide x by y, where $y \neq 0$ • Quotient of x divided by, where $y \neq 0$	$x \div y$ $= x/y$

In arithmetic, we write 7 x 7 x 7 = 7^3. In algebra, the notation is similar.

a x a = a^2, read as a squared;
b x b x b = b^3, read as b cubed;
c x c x c x c = c^4, read as c to the power of 4.

Note: $a^2 \neq 2a = a + a$
$\qquad b^3 \neq 3b = b + b + b$
$\qquad c^4 \neq 4c = c + c + c + c$

4.2 - Evaluation of Algebraic Expressions And Formulae

A) Evaluation of Algebraic Expressions

The total price of of buying 1 watermelon at $5 each and n mangoes at $2 each = $(5 + 2n)$. For buying 1 watermelon and 7 mangoes, the total price can be evaluated by putting n = 7 in the expression. Thus, the total price
= $[5 + 2(7)]$
= $19.
Similarly, for buying 1 watermelon and 10 mangoes, then n = 10 and the total price
= $[5 + 2(10)]$
=$25.

The process of replacing each variable with its value to find the actual value is called substitution.

B) Formulae

We have learnt that the area of a rectangle is given by:

area = length x breadth.

If we let l and b represent the length and breadth of the rectangle respectively in cm, and let A represent the area in cm^2, then the relation above can be expressed as

$A = lb$.

This equality of connecting two or more variables is called a formula. When the values of l and b are known, we can find the value of A in the formula by substitution.

4.3 - Translation of Real-world Situations Into Algebraic Expressions

We may use algebraic expressions and formulae to express the relationship between two or more quantities in our daily life. Let us see an example:

John is 6cm taller than Mary.
   (a)   Let x cm be the height of a John. express the height of Mary in terms of x.
   (b)   Let y cm be the height of Mary. write a formula connecting x and y.

Solution:
   (a)   Mary's height = $(x - 6)$ cm
   (b)   From (a), we have $y = x - 6$.