Real Numbers

2.1 - Idea of Negative Numbers And The Number Line

A)Negative Numbers and integers

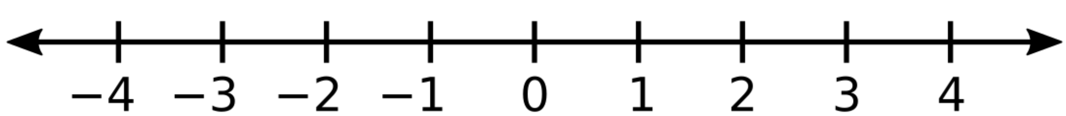
In our daily life, we often come across numbers with the plus sign '+' or the minus sign '-'. It often represents quantities with opposite directions or meanings.

We have learnt that whole numbers are 0, 1, 2, … . The numbers +1, +2, +3, … are called positive integers, and the numbers -1, -2, -3, … are called negative integers. When we write positive integers, we can omit the '+' sign, example, +3 can be simply be written as 3. The collection of numbers.., -3, -2, -1, 0, 1, 2, 3… is called a set of integers. Note that zero is neither positive nor negative.

Similarly, the numbers such as 4/3 and 0.8 are called positive numbers, while -⅔ and -0.69 are called negative numbers.

B)The Number Line

We can use a number line to show the order of numbers. The diagram below shows a number line. An arrow at each end of the line indicates that the line can be extended further from both ends.



Negative numbers Positive numbers

On a horizontal number line,

- All positive numbers are to the right of zero (0),
- All negative numbers are to the left of zero (0),
- Numbers are arranged in ascending order from left to right,
- Every number is less than any number on it’s right and greater than any number on its left.

Thus, -2 is less than 3 and is denoted by ‘-2 < 3’.
-2 is greater than -5 and is denoted by ‘-2 > -5’.

The symbols ‘<’and ‘<’ are called inequality signs. The other inequality signs are ≤ (less than or equal to) and ≥ (greater than or equal to).

2.2 - Addition And Subtraction Of Integers

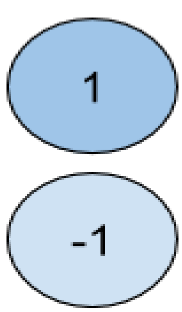
A) Addition

Suppose the temperature of a piece of frozen meat is -3°C. During defrosting, its temperature rises by 4°C. What is the temperature then?

The answer involves addition of integers (-3) + 4.

We can represent the addition of integers using algebra discs. Each disc has 2 sides with labels 1 and -1 respectively. These discs can be flipped to display the front and back. A key concept of these discs is that 1 and -1 form a zero pair.

Thus, 1 + (-1) = 0 and 3 + (-3) = 0.



We can also write 3 + (-3) as 3 - 3 or (-3) + 3.

B) Subtraction

The negative of a number is obtained by changing its sign. Thus, the negative of 8 is -8, (-8) = -8. The negative of -8 is 8, -(-8) = 8.

For subtraction of integers, we can consider the operation as ‘adding the negative’. Let us take a look at how this can be done.

For example, evaluate 3 - 7.

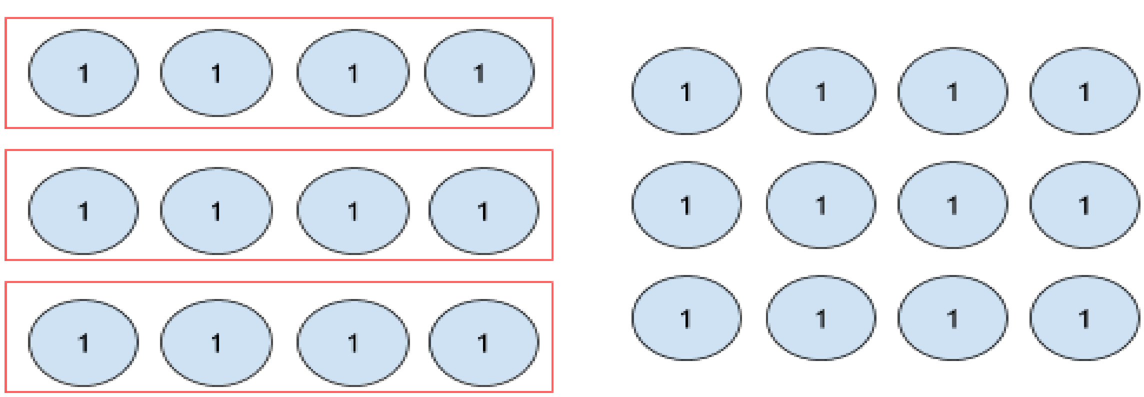
$$\begin{aligned} 3 - 7 &= 3 + (-7) \\ &= -4 \end{aligned}$$

2.3 - Multiplication, Division And Combined Operations Of Integers

A) Multiplication

The multiplication of whole numbers can be regarded as repeated addition.

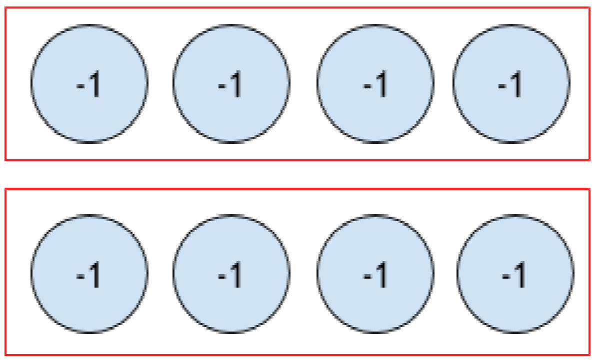
For example, 3 x 4 = 4 + 4 + 4 = 12.
Using algebra discs, 3 x 4 can be considered as ‘3 groups of 4’. Thus,



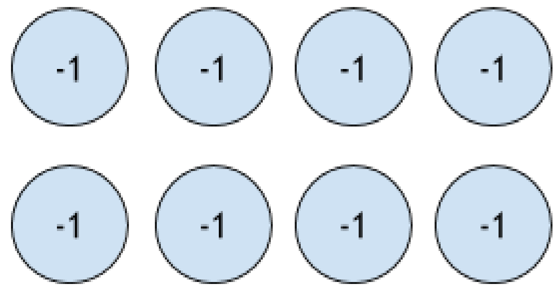
Take 3 groups of 4 Ungroup the discs

We can extend this idea to find the products of integers by using the algebra discs.

For example, evaluate 2 x (-4).



Take 2 groups of -5



Ungroup the discs

2 x (-4) = -8 (Observe that the answer is negative. Here, we multiply a positive number by a negative number.)

B) Division

Division is related to multiplication as illustrated below.

4 x 3 = 12, 12 ÷ 3 = 12/3 = 12 x ⅓ = 4
4 x (-3) = -12, (-12) ÷ (-3) = -12/-3 = -12 x 1/-3 = 4
(-4) x 3 = -12, (-12) ÷ 3 = -12/3 = -12 x ⅓ = -4
(-4) x (-3) = 12, 12 ÷ (-3) = 12/-3 = 12 x 1/-3 = -4.

Hence, the rules of the division of integers can be derived from the rules of multiplication of integers.

Note: 4 x 0 = 0 but 4 ÷ 0 ≠ 0. This is because 'division by zero' is invalid. In fact, a ÷ 0 is undefined for all numbers a.

C) Combined Operations of Integers

The order of operations on whole numbers can be extended to operations on integers as follows:

- Evaluate the expressions within the innermost pair of brackets first if there is more than 1 pair of brackets.
- Evaluate the powers.
- Multiply and divide from left to right.
- Add and subtract from left and right.

2.4 Rational numbers

A)Definition

The division of 2 integers may not always result in integers. For instance:

3 ÷ 8 = ⅜ and (-11) ÷ 7 = -11/7 are fractions.

Hence, the integer number system has to be extended to include fractions of form *a*/*b*. We define rational numbers as follows:

Rational numbers are numbers that can be expressed in the form of *a*/*b*, where *a* and *b* are integers and b ≠ 0.

From this definition, we can see that 3/1 (=3), -4/1 (= -4), 32/9 and -⅞ are rational numbers. Therefore rational numbers include all integers and fractions.

B)Addition and Subtraction

The rules of operations on integers hold for rational numbers. We express rational numbers as equivalent fractions with the same denominator before we add or subtract them.

C) Multiplication and Division

The steps of multiplying two rational numbers are:

1. If there are mixed numbers, change them to improper fractions first.
2. Multiply the numerator of the rational numbers to get the new numerator.
3. Multiply the denominator of the rational number to get the new denominator.
4. Simplify the resulting fraction of possible.

When a rational number is divided by another rational number, it is equal to multiplying the first number by the reciprocal of the second number.

D) Division

Decimals which have a finite number of digits are rational numbers. This is because they can be expressed as fractions.

Addition and subtraction of decimals are like adding and subtracting whole numbers. The thing we have to take note is to line up the decimal points.

When multiplying two decimals together:
1. Multiply them as if there were no decimal points in them.
2. Count the number of decimal places in each given number and add them together. This will be the number of decimals in the answer.

3. Insert the decimal accordingly to obtain the answer.

When dividing a decimal with another decimal:

1. Shift the decimal point of the divisor to make it a whole number.
2. Shift the decimal point of the dividend an equal number of spaces as step 1.
3. Use long division to get the answer, making sure that the decimal points of the quotient and the dividend are in alignment.

2.5 Real Numbers

Every rational number can be expressed as a decimal by dividing the numerator of the rational number by the denominator.

A number is called a real number if it can be represented by a point on the number line. By this argument, we can conclude that all rational numbers are real numbers. All rational numbers are either terminating decimals or recurring decimals. However, on the number line, there are some decimals which are non-recurring and non-terminating, we say that these are real numbers which are not rational. It's called an irrational number.