Factors and Multiples

1.1 - Primes, Prime Factorisation and Index Notation

A)Factors

The number 24 can be expressed as products of 2 numbers as follows:

24 = 1 x 24

24 = 2 x 12

24 = 3 x 8

24 = 4 x 6

1, 2, 3, 4, 6, 8, 12, 24, are called **factors** of 24. Notice that 24 is **divisible** by each of its factors.

B)Multiples

When a number is multiplied by a whole number (not including zero), we get a **multiple** of the number.

The multiples of 3 are: **3 x 1 = 3**, **3 x 2 = 6**, **3 x 3 = 9**, …

C)Primes

A prime number is a whole number greater than 1 that only has 2 factors, 1 and itself.

Notice that the first 10 prime numbers are: 2, 3, 5, 7, 9, 11, 13, 17, 19, 23, 29.

A composite number is a whole number greater than 1 that has more than 2 factors. The number 6 has four factors, 1, 2, 3, 6, thus it's a composite number.

0 and 1 are neither prime nor composite numbers.

D)Prime factorisation

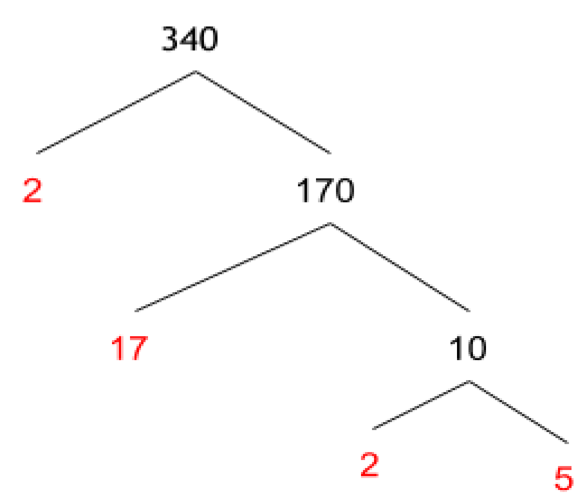
A prime number which is a factor of a composite number is called a **prime factor** of the composite number. For example,

$$30 = 2 \times 3 \times 5$$

where 30 is a composite number and 2, 3 and 5 are its prime factors.

The way to express a composite number as a product of prime factors only is called **prime factorisation.**

We can use a **factor tree** to find the prime factorisation of a composite number. For example, in expressing 340 in prime factors, we can use a factor tree as follows:



Thus, $340 = 2 \times 2 \times 5 \times 17$.

E) Index Notation

When a number is multiplied by itself more than once, we can use **index notation** to represent the product as shown below:

$8 \times 8 = 8^2$ (read as 8 squared)

The number 8 is the **base** and 2 is the **index**. The index shows the number of times the base is multiplied by itself.

1.2 - Highest Common Factor (HCF)

Let us consider the factors of 18 and 24.

18: **1**, **2**, **3**, **6**, 9, 18
24: **1**, **2**, **3**, 4, **6**, 8, 12, 24

Common factors: 1, 2, 3, 6. The largest common factor is 6. We say that 6 is the **highest common factor (HCF)** of 18 and 24.

<u>The highest common factor (HCF) of 2 or more positive whole numbers is the largest positive integer that divides the numbers without a remainder.</u>

There are different methods of finding the HCF of 2 or more numbers. We can use prime factorisation to find their HCF in a more efficient way.

Take the numbers 225 and 750. The prime factorisation of those numbers are:
$225 = 3^2 \times 5^2$
$750 = 2 \times 3 \times 5^3$

We can then visualise the process of taking the common factors:

225 = 3 x 3 x 5 x 5
750 = 2 x 3 x 5 x 5 x 5
HCF = 3 x 5 x 5
= 3 x 5^2(3 is of a lower power than 3^2)
= 75 (5^2 is of a lower power than 5(3))

Note: We see that the HCF is obtained by multiplying the lowest power of each common prime factor (3 and 5^2) of the given numbers.

1.3 - Lowest Common Multiple (LCM)

Let us consider the first 10 multiples of 6 and 8.
Multiples of 6: 6, 12, 18, **24**, 30, 36, 42, **48**, 54, 60, ...
Multiples of 8: 8, 16, 24, 32, 40, **48**, 56, 64, 72, 80, …

24 and 48 are common multiples of 6 and 8. The smallest common multiple is 24. Thus, the **lowest common multiple** (LCM) of 6 and 8 is 24.

The lowest common multiple (LCM) of 2 or more whole numbers is the smallest common multiple of the numbers.

We can use prime factorisation to find the lowest common multiple of 2 or more numbers.

Take the numbers 24 and 90. The prime factorisation of those numbers are:
24 = 2^3 x 3
90 = 2 x 3^2 x 5

We can visualise the process of identifying all the different factors:

$24 \;\; = 2 \times 2 \times 2 \times 3$

$90 \;\; = 2 \qquad\qquad \times 3 \times 3 \times 5$

LCM $= 2 \times 2 \times 2 \times 3 \times 3 \times 5$

$= 2^3 \times 3^2 \times 5$ (2^3 is of a higher power than 2)

$= 360$ (3^2 is of a higher power than 3)

Note: We see that LCM is obtained by multiplying the highest power of each prime factor (2^3, 3^2 and 5) of the given numbers.

1.4 - Square Roots and Cube Roots

A)Square roots

Since $3^2 = 9$, 9 is called the **square** of 3. We also say that 3 is the positive **square root** of 9 and it is denoted by $\sqrt{9} = 3$.

Similarly, we write:

$2^2 = 4$ and $\sqrt{4} = 2$,

$4^2 = 16$ and $\sqrt{16} = 4$.

The numbers 1, 4, 9, 25, … whose square roots are whole numbers are called **perfect squares**. We can find the root of a square using prime factorisation.

Take the value of $\sqrt{144}$. The prime factorization of 144 is:

$144 = 2^4 \times 3^2$

Split the factor into 2 equal groups.

$144 = (2^2 \times 3) \times (2^2 \times 3)$

$= (2^2 \times 3)^2$

$\sqrt{144} = 2^2 \times 3$

$= 12$

B)Cube roots

We can express 8 as a product of 3 identical numbers as follows:

$$2 \times 2 \times 2 = 8.$$

We say that 8 is the **cube** of 2 and 2 is the **cube root** of 8 which is denoted by:

$$\sqrt[3]{8} = 2.$$

Similarly, we write:

$$1^3 = 1 \times 1 \times 1 = 1 \text{ and } \sqrt[3]{1} = 1,$$

$$3^3 = 3 \times 3 \times 3 = 27 \text{ and } \sqrt[3]{27} = 3.$$

The numbers 1, 8, 27, 64, … whose cube roots are whole numbers are called **perfect cubes**. We can find the cube root of a number using prime factorisation.

Take the number 216. The prime factorisation of 216 is:

$216 = 2^3 \times 3^3$

Split the prime factors into 3 different groups.

$216 = (2 \times 3) \times (2 \times 3) \times (2 \times 3)$

$\quad = (2 \times 3)^3$

$\sqrt[3]{216} = 2 \times 3$

$\quad = 6$