Assignment 5

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1 Problem 3

1.1 Primal

from Sylvester's criterion we get following conditions:

$$(4y_1 + 2) \geq 0,$$
$$-25y_1^2 \geq 0,$$
$$3y_2 \geq 0$$

Second inequality gives us $y_1 = 0$. So, minimization result is 0.

1.2 Dual

Let's write dual problem in a way

$$\min_Z - \langle Z, G \rangle,$$
$$\text{subject to } \langle Z, F_i \rangle = c_i,$$
$$Z \succeq 0$$

$$\min_Z - \langle Z, G \rangle \to \min -2Z_{3,3}$$

$$\langle Z, F_i \rangle = c_i \to$$
$$10Z_{1,2} + 4Z_{3,3} = 2,$$
$$3Z_{2,2} = 0$$

$$Z \succeq 0, Z_{2,2} = 0 \Rightarrow Z_{1,2} = 0 \Rightarrow Z_{3,3} = \frac{1}{2} \Rightarrow$$
$$\Rightarrow \min -2Z_{3,3} = -1$$

This result differs from 0 for primal problem. That proofs that strong duality fails to hold in this problem.

2 Problem 4

2.1 Equivalent LP program and its dual

$$\min_{x \in \mathbb{R}^n} t,$$
$$a_i^\top x + b_i - t \leq 0, i = 1..m$$

Dual problem:

$$
\begin{gathered}
\mathcal{L}\left(x, t, \lambda\right) = \left(1 - \sum_i \lambda_i\right) t + \lambda^\top A x + \lambda^\top b, \\
\min_{x,t} \mathcal{L} \to \\
\sum \lambda_i = 1, \\
\lambda^\top A = 0
\end{gathered}
$$

And dual problem is

$$
\begin{gathered}
\max \lambda^\top b, \\
\text{subject to} \\
\lambda \geq 0, \\
\lambda^\top A = 0, \\
\sum_i \lambda_i = 0
\end{gathered}
$$

2.2 log function

$$
\begin{gathered}
\min_{x \in \mathcal{R}^n} f_2(x) = \frac{1}{\alpha} \log \left(\sum_{i=1}^m \exp^{\alpha y_i}\right), \\
-y_i + a_i^\top x + b_i = 0
\end{gathered}
$$

Dual:

$$
\begin{gathered}
\mathcal{L}\left(x, y, \nu\right) = \frac{1}{\alpha} \log \left(\sum_{i=1}^m \exp^{\alpha y_i}\right) + \sum_i \nu_i \left(-y_i + a_i^\top x + b_i\right), \\
\Rightarrow \frac{\exp^{\alpha y_i}}{\sum_i \exp^{\alpha y_i}} - \nu_i = 0, \nu^\top A = 0
\end{gathered}
$$

Than, we have

$$
\begin{gathered}
\sum_i \nu_i = 1, \\
y_i = \frac{1}{\alpha} \log \nu_i
\end{gathered}
$$

(last was guessed by substitiuon). And dual problem is

$$
\begin{gathered}
\max \nu^\top b - \sum_i \frac{\nu_i}{\alpha} \log \nu_i, \\
\text{subject to} \\
\nu^\top A = 0, \\
\sum_i \nu_i = 1
\end{gathered}
$$