VE311 Hw4 周维凯 518021911039

1. (a). NMOS

(1). When $V_{DD} > V_x$,

we have $V_D = 5V$, $V_S = V_x$.

$V_{GS} = 5 - V_x$, $V_{DS} = 5 - V_x$,

① When $V_{GS} = 5 - V_x < V_{TH} = 0.7 \Rightarrow V_x > 4.3\,V$,
we know $I_x = 0$.

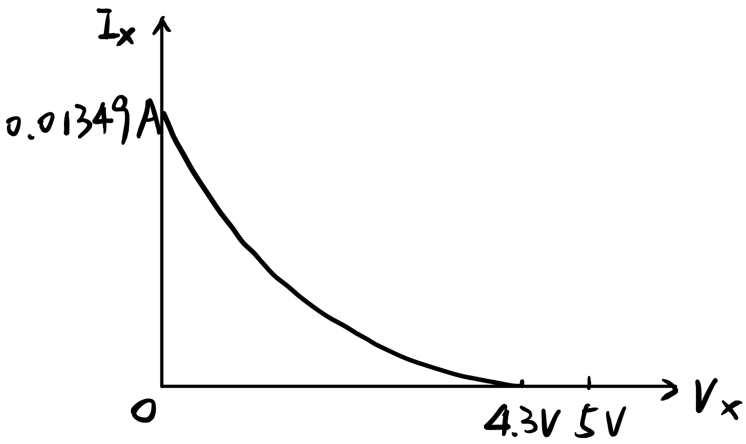
② When $V_{GS} = 5 - V_x > V_{TH} = 0.7. \Rightarrow V_x < 4.3V$,
we know $V_{DS} = 5 - V_x > V_{GS} - V_{TH} = 5 - V_x - 0.7$.
Therefore, it is in saturation region.

$$I_x = I_D = \frac{1}{2} \times 350 \times 10^{-4} \times 3.835 \times 10^{-3} \times \frac{20 \times 10^{-6}}{2 \times 10^{-6} - 2 \times 0.08 \times 10^{-6}} (5 - V_x - 0.7)^2$$

$$= 7.29 \times 10^{-4} (V_x - 4.3)^2$$

(2). When $V_{DD} < V_x$, we have $V_x > 5V$.

According to the directions, we do not need to consider this case.



(b). NMOS

(1) When $V_x < 1V$, we have $V_D = 1V$, $V_S = V_x$

$V_{GS} = 1.9 - V_x$, $V_{DS} = 1 - V_x$

Since $V_x < 1V$, we know $V_{GS} = 1.9 - V_x > 0.9 > V_{TH}$.

Since $V_{DS} = 1 - V_x < V_{GS} - V_{TH} = 1.9 - V_x - 0.7$,

We know it is in triode region.

$$I_x = -I_D = -350\times10^{-4} \times 3.835\times10^{-3} \times \frac{20\times10^{-6}}{2\times10^{-6} - 2\times0.08\times10^{-6}}\left[(1.2-V_x)(1-V_x) - \frac{1}{2}(1-V_x)^2\right]$$

$$= -1.46\times10^{-3}(0.5V_x^2 - 1.2V_x + 0.7)$$

(2). When $V_x > 1V$, we have $V_D = V_x$, $V_S = 1V$

$V_{GS} = 0.9V$, $V_{DS} = V_x - 1$

Since $V_{GS} = 0.9V > 0.7V$, it is turned on

① When $V_{DS} = V_x - 1 < V_{GS} - V_{TH} = 0.2 \Rightarrow 1 < V_x < 1.2$,

We know it is in triode region

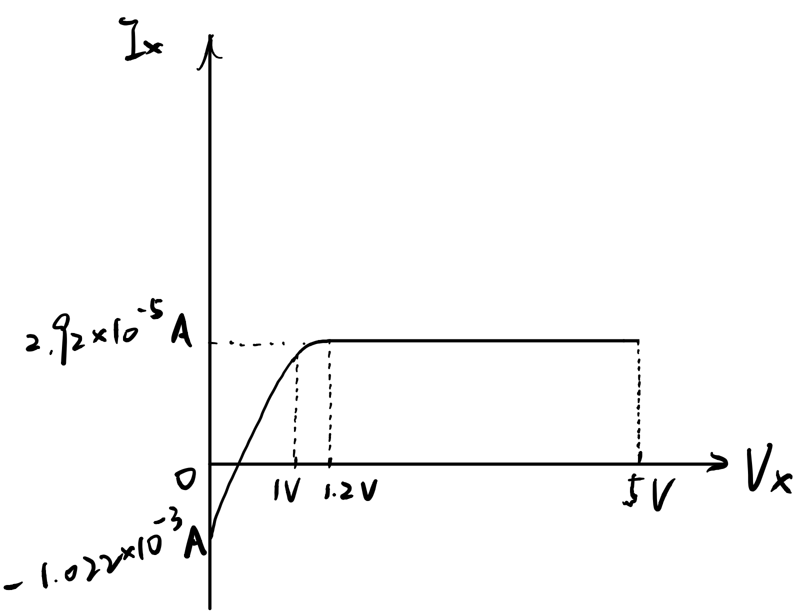
$$I_x = I_D = 350\times10^{-4} \times 3.835\times10^{-3} \times \frac{20\times10^{-6}}{2\times10^{-6} - 2\times0.08\times10^{-6}}\left[(0.9-0.7)(V_x-1) - \frac{1}{2}(V_x-1)^2\right]$$

$$= 1.46\times10^{-3}(-0.5V_x^2 + 1.2V_x - 0.7)$$

② When $V_{DS} = V_x - 1 > V_{GS} - V_{TH} = 0.2 \Rightarrow V_x > 1.2$, in saturation.

$$I_x = I_D = \frac{1}{2}\times350\times10^{-4} \times 3.835\times10^{-3} \times \frac{20\times10^{-6}}{2\times10^{-6} - 2\times0.08\times10^{-6}}[0.2]^2$$

$$= 2.92\times10^{-5}$$



(C). NMOS

(1). When $V_x < 1.9V$, we have $V_D = 1.9V$, $V_S = V_x$

$V_{GS} = 1 - V_x$, $V_{DS} = 1.9 - V_x$

① When $V_{GS} = 1 - V_x < 0.7V \Rightarrow V_x > 0.3V$,

we have $I_x = 0$

② When $V_{GS} = 1 - V_x > 0.7V \Rightarrow V_x < 0.3V$,

we know $V_{DS} = 1.9 - V_x > V_{GS} - V_{TH} = 0.3 - V_x$, in saturation

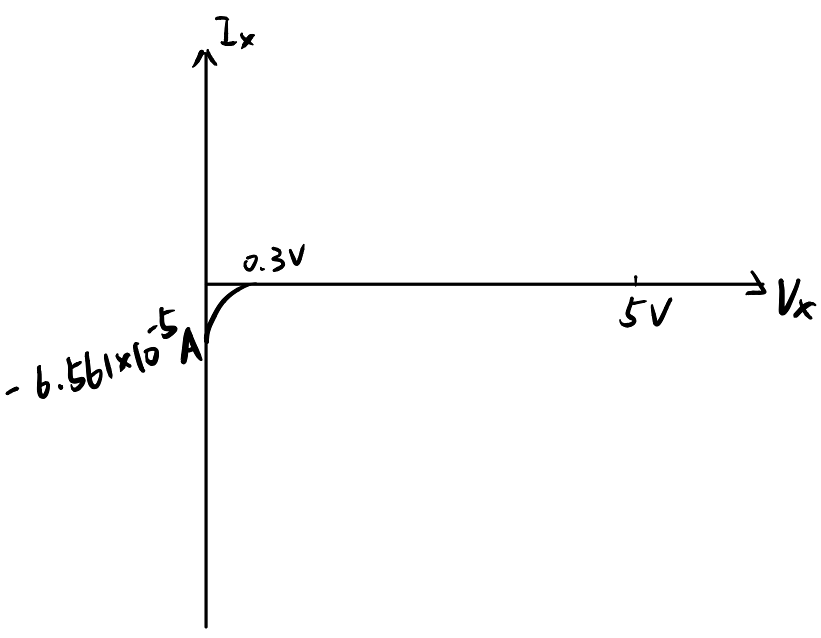
$$I_x = -I_D = \frac{-1}{2} \times 350 \times 10^{-4} \times 3.835 \times 10^{-3} \times \frac{20 \times 10^{-6}}{2 \times 10^{-6} - 2 \times 0.08 \times 10^{-6}} [0.3 - V_x]^2$$

$$= -7.29 \times 10^{-4} (V_x - 0.3)^2$$

(2). When $V_x > 1.9V$, we have $V_D = V_x$, $V_S = 1.9V$,

$V_{GS} = 1 - 1.9 = -0.9V < V_{TH} = 0.7V$.

Therefore, $I_x = 0$.



(d). PMOS

(1). When $V_x > 1.9V$, $V_S = V_x$, $V_D = 1.9V$.

$V_{SG} = V_x - 1$, $V_{SD} = V_x - 1.9$

Since $V_{SG} = V_x - 1 > 0.9 > 0.8 = |V_{TH}|$, it is turned on.

Since $V_{SD} = V_x - 1.9 < V_{SG} - |V_{TH}| = V_x - 0.2$, in triode region.

$$I_x = I_D = 100\times10^{-4} \times 3.835\times10^{-3} \times \frac{20\times10^{-6}}{2\times10^{-6} - 2\times0.09\times10^{-6}}\left[(V_x - 1 - 0.8)(V_x - 1.9) - \frac{1}{2}(V_x - 1.9)^2\right]$$

$$= 4.21\times10^{-4}(0.5V_x^2 - 1.8V_x + 1.615)$$

(2). When $V_x < 1.9V$, $V_S = 1.9V$, $V_D = V_x$

$V_{SG} = 1.9 - 1 = 0.9V$, $V_{SD} = 1.9 - V_x$

① When $V_{SD} = 1.9 - V_x < V_{SG} - |V_{TH}| = 0.1 \Rightarrow V_x > 1.8$, in triode region.

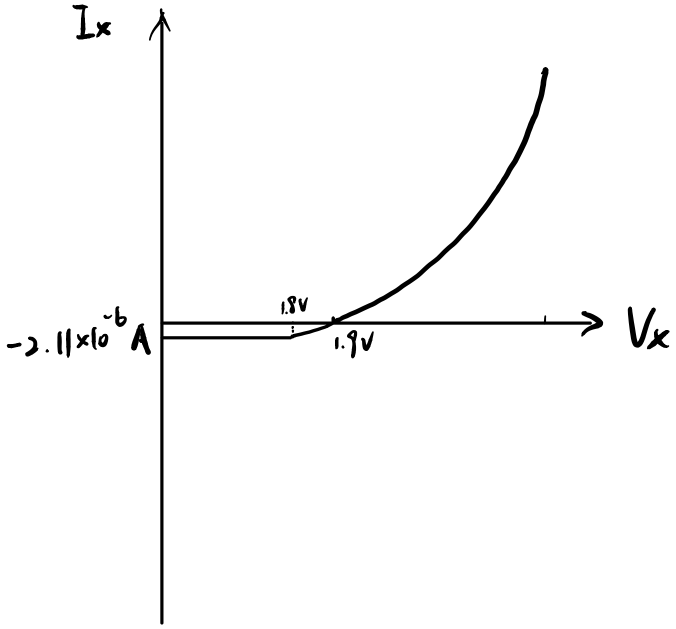
$$I_x = -I_D = -100\times10^{-4} \times 3.835\times10^{-3} \times \frac{20\times10^{-6}}{2\times10^{-6} - 2\times0.09\times10^{-6}}\left[(0.9 - 0.8)(1.9 - V_x) - \frac{1}{2}(1.9 - V_x)^2\right]$$

$$= 4.21\times10^{-4}(0.5V_x^2 - 1.8V_x + 1.615)$$

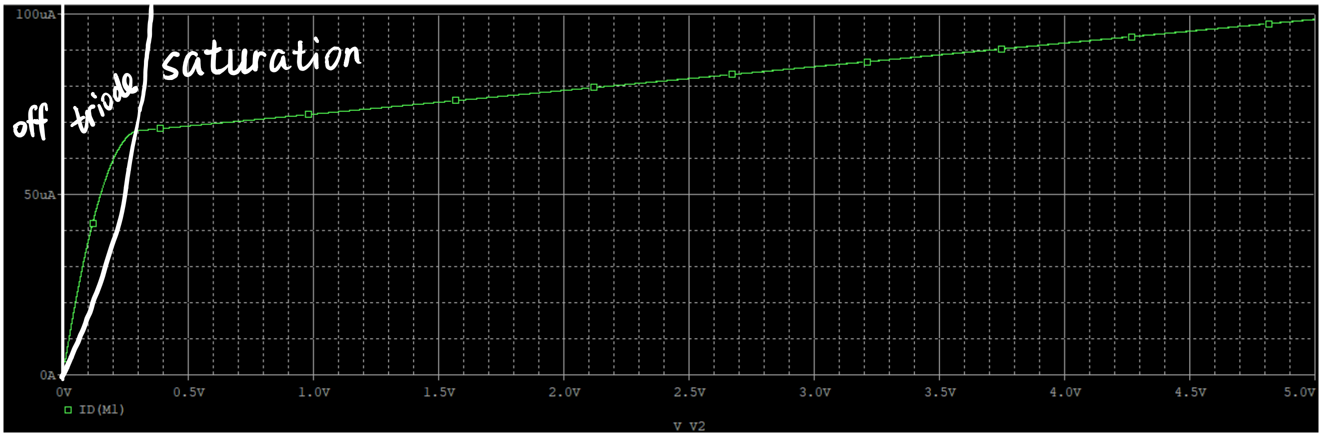
② When $V_{SD} = 1.9 - V_x > V_{SG} - |V_{TH}| = 0.1 \Rightarrow V_x < 1.8$, in saturation region.

$$I_x = -I_D = \frac{-1}{2}\times100\times10^{-4} \times 3.835\times10^{-3} \times \frac{20\times10^{-6}}{2\times10^{-6} - 2\times0.09\times10^{-6}}(0.9 - 0.8)^2$$

$$= -2.11\times10^{-6}$$



2.(a).



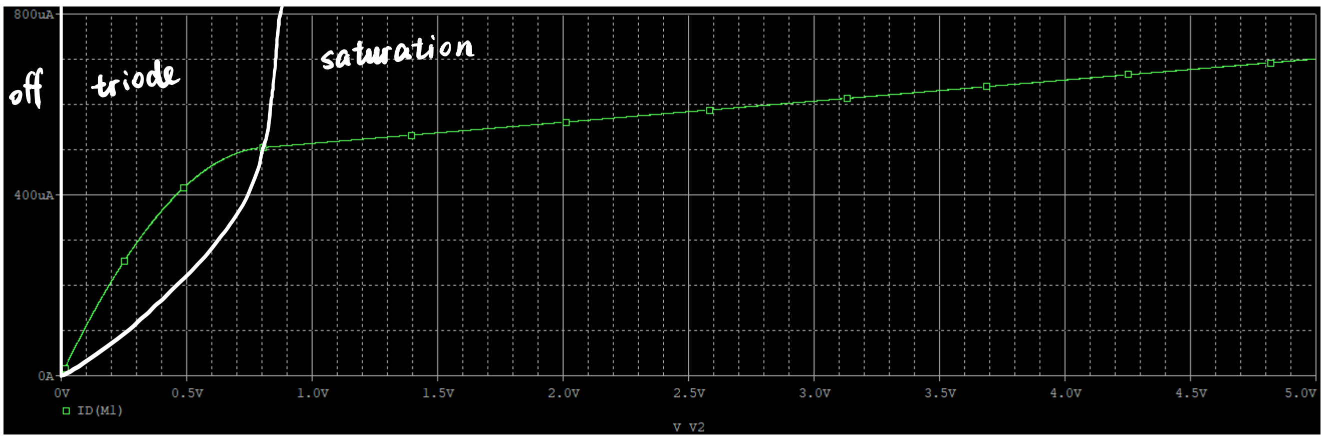
$V_{GS} = 1V$

From the plot, we have $(3, 85.39\times10^{-6}), (3.1, 86.038\times10^{-6})$, then

$$r_o = \frac{1}{\text{slope}} = \frac{3.1-3}{(86.038-85.39)\times10^{-6}} = 1.54\times10^5.$$

Theoretically, $r_o = \dfrac{1}{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH})^2\cdot\lambda} = 1.52\times10^5$

They are close to each other.



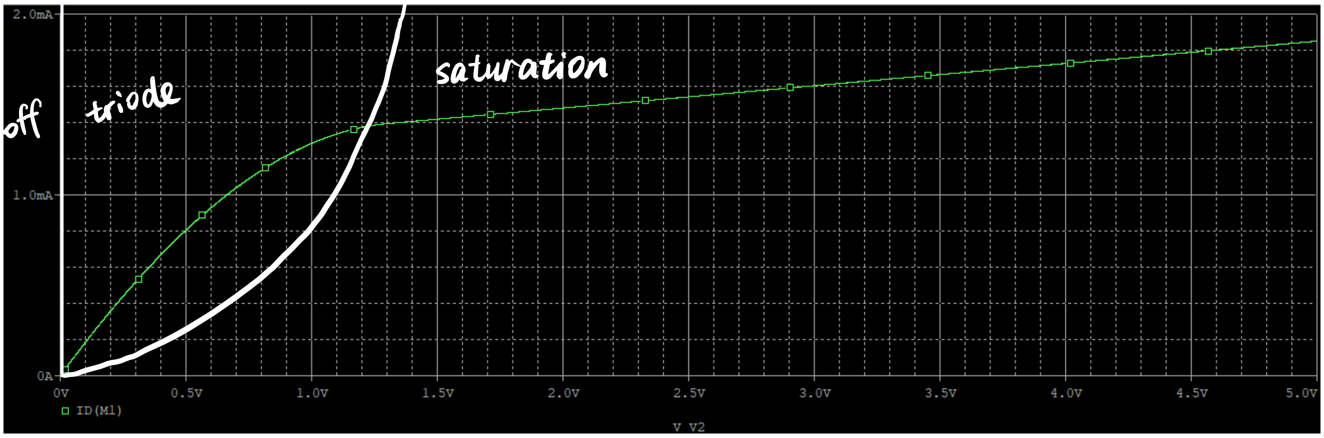
$V_{GS} = 1.5V$

From the plot, we have $(3, 607.22\times10^{-6}), (3.1, 611.83\times10^{-6})$, then

$$r_o = \frac{1}{\text{slope}} = \frac{3.1-3}{(611.83-607.22)\times10^{-6}} = 2.17\times10^4$$

Theoretically, $r_o = \dfrac{1}{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH})^2\cdot\lambda} = 2.14\times10^4$

They are close to each other.



$V_{GS} = 2V$

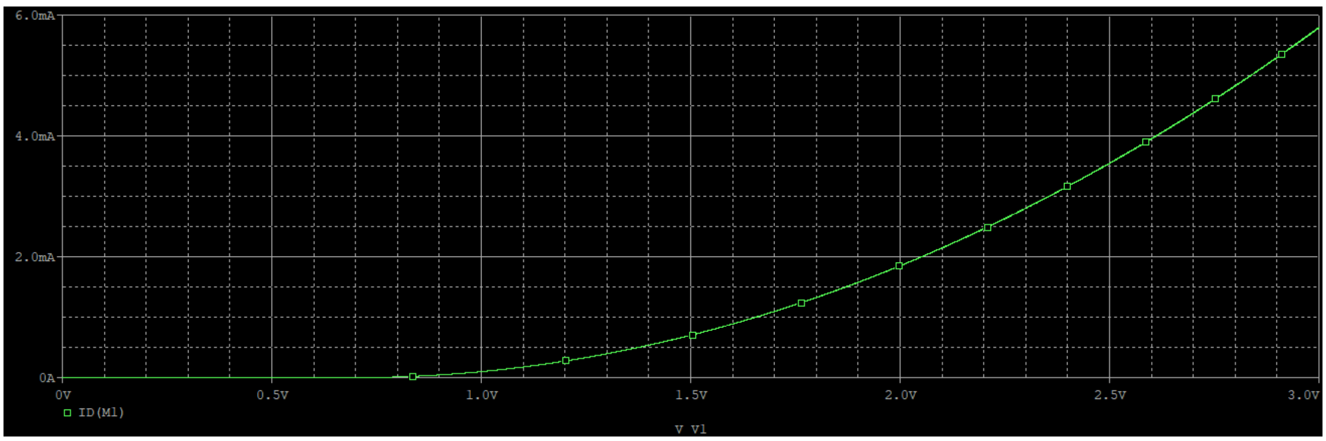
From the plot, we have $(3, 1.6034\times10^{-3})$, $(3.1, 1.6158\times10^{-3})$, then

$$r_o = \frac{1}{slope} = \frac{3.1-3}{(1.6158-1.6034)\times10^{-3}} = 7.81\times10^{3}$$

Theoretically, $r_o = \dfrac{1}{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH})^2\cdot\lambda} = 8.11\times10^{3}$

They are close to each other.

(b).



From the plot, we have $(1.95, 1.7105\times10^{-3})$ $(2.05, 1.9936\times10^{-3})$. Then

$$g_m = slope = \frac{(1.9936-1.7105)\times10^{-3}}{2.05-1.95} = 2.831\times10^{-3}$$

Theoretically, we have $g_m = \mu_n C_{ox}\frac{W}{L_{eff}}(V_{GS}-V_{TH})(1+\lambda V_{DS})$

$= 2.845\times10^{-3}$

They are close to each other