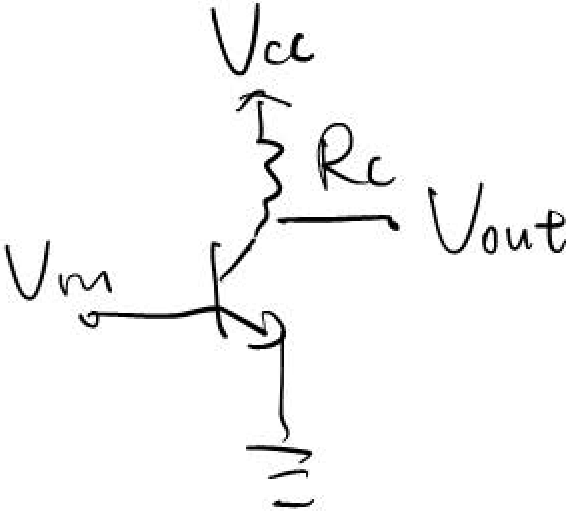
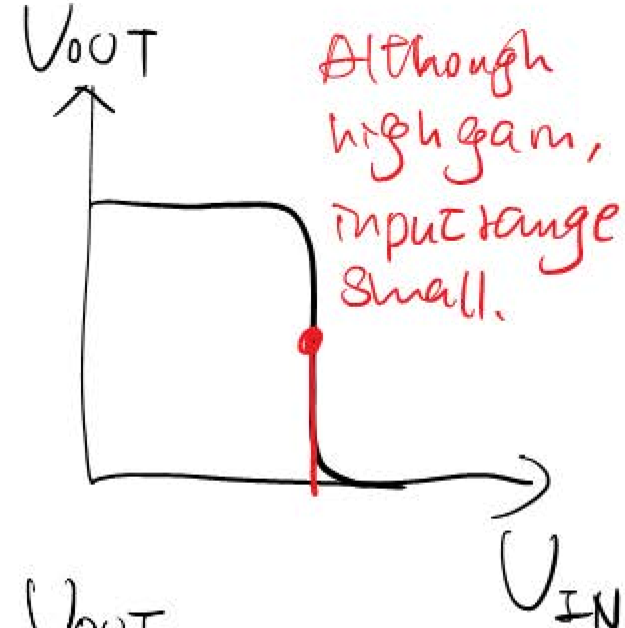


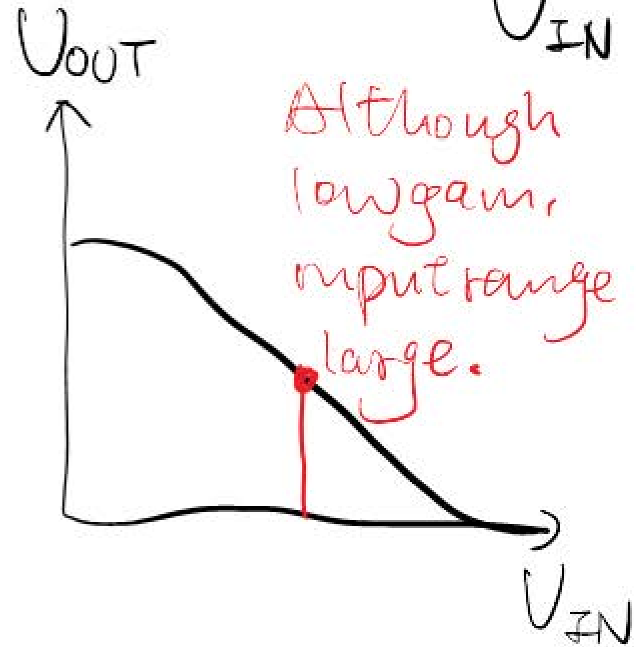
If Rc large

trade-off

If Rc small



Although high gain, input range small.



Although low gain, input range large.

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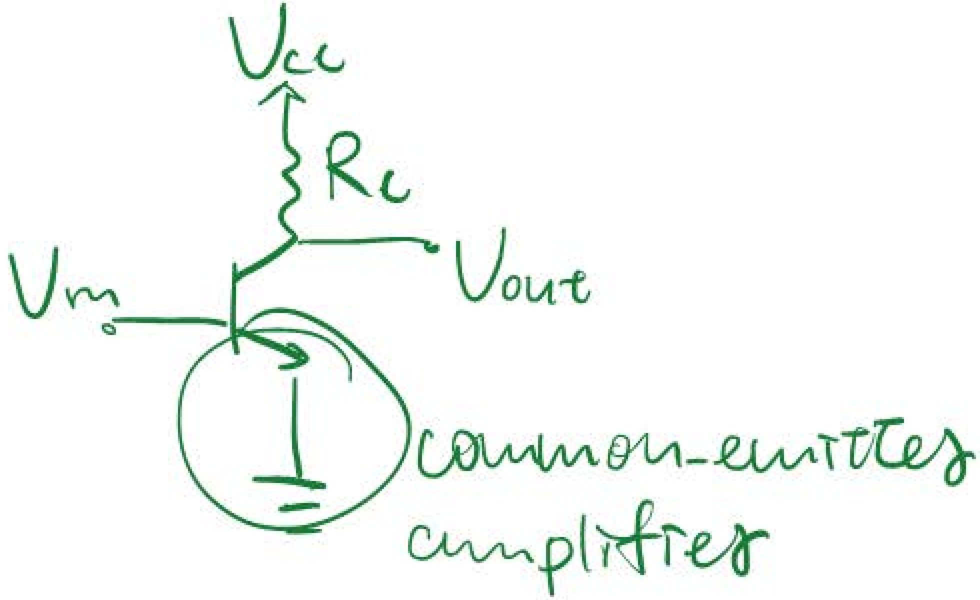
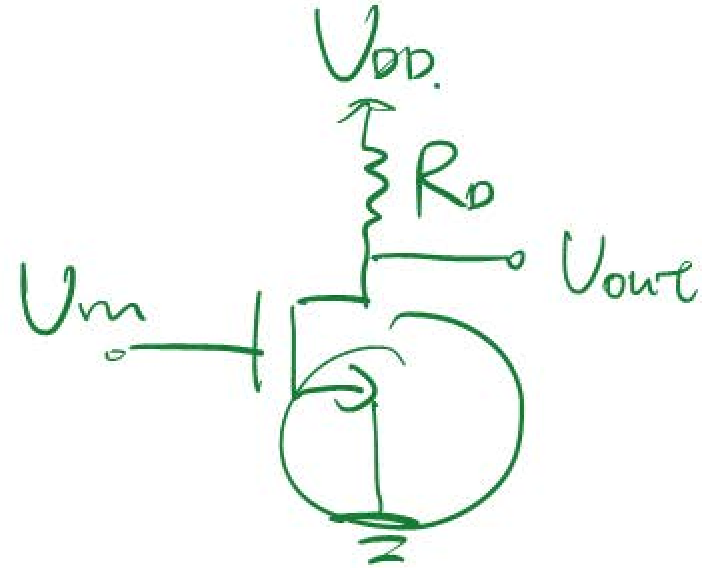
FET Single Stage Amplifier

Ve311 Electronic Circuits (Fall 2020)

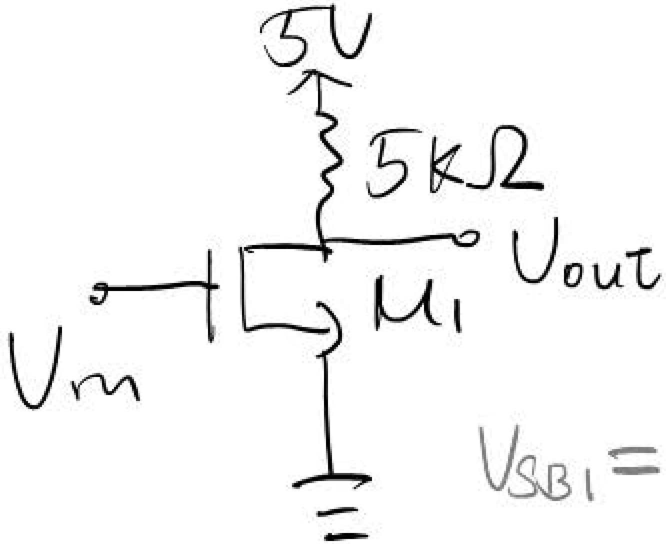
Dr. Chang-Ching Tu

Last Updated on **October 21, 2020**





Common-Source with Resistive Load



$V_{SB1} = 0$

M_1 has size $\frac{10\,\mu m}{2\,\mu m} = \frac{W_{drawn}}{L_{drawn}}$

$\lambda \neq 0$, $\gamma \neq 0$

$V_{in} = 0.8 + 0.001 \sin(2\pi 100 t)$

$V_{out} = V_{OUT} + v_{out} = ?$

1° Find out $V_{OUT} = ?$

then confirm whether

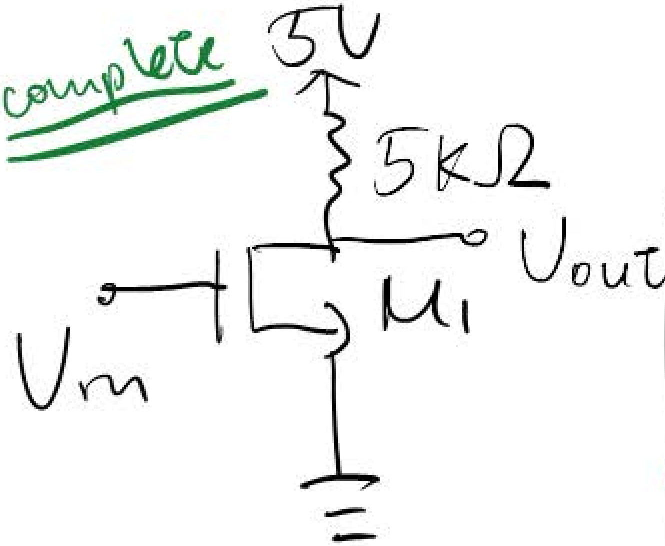
$V_{DS} \geq V_{GS} - V_{TH}$

$V_{OUT} \geq \underset{\parallel \atop 0.8}{V_{IN}} - \underset{\parallel \atop 0.7}{V_{TH}} = 0.1$

$V_{OUT} = 5 - (5K) \cdot I_{D1}$

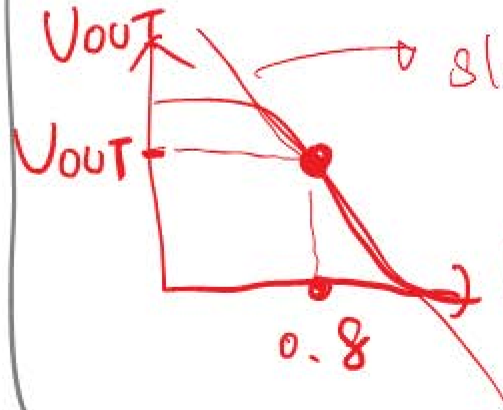
$I_{D1} = \frac{1}{2} \mu_n C_{ox} \left(\frac{10\,\mu m}{2\,\mu m - 2L_D} \right) (0.8 - 0.7)^2 (1 + \lambda V_{OUT})$

complete



2° Find out $v_{out} = ?$

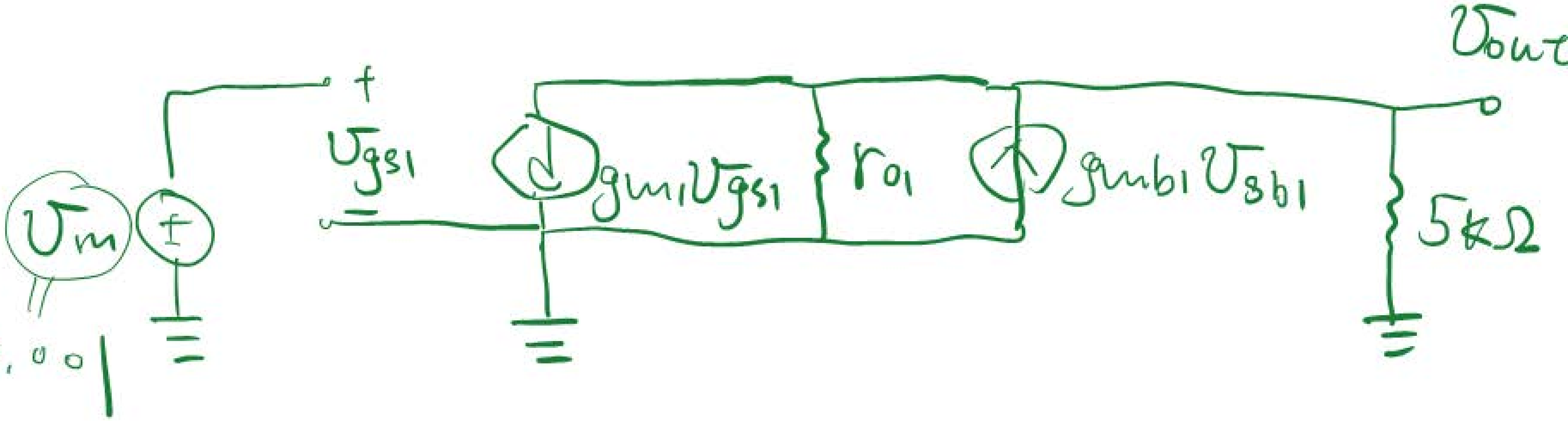
If using Pspice, do DC sweep.



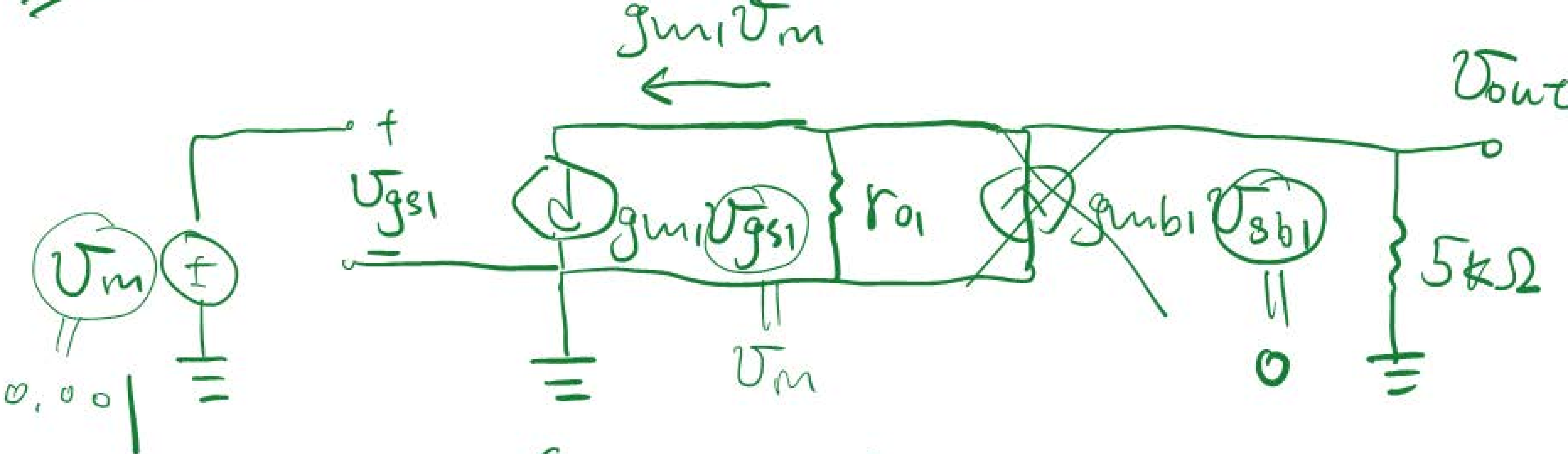
slope $= \boxed{A_v = \left.\dfrac{dV_{OUT}}{dV_{IN}}\right|_{@V_{IN}=0.8}}$

$v_{out} = 0.001 A_v \sin(\omega t)$

small-signal



Small-signal



$$v_{out} = -(g_{m1}v_{in})(r_{o1} \parallel 5k)$$

$$\begin{cases} g_{m1} = \mu_n C_{ox}\left(\frac{W}{L_{eff}}\right)(0.1)^2(1+\lambda V_{OUT}) \\ r_{o1} \cong \frac{1}{I_{D1}\lambda} \end{cases}$$

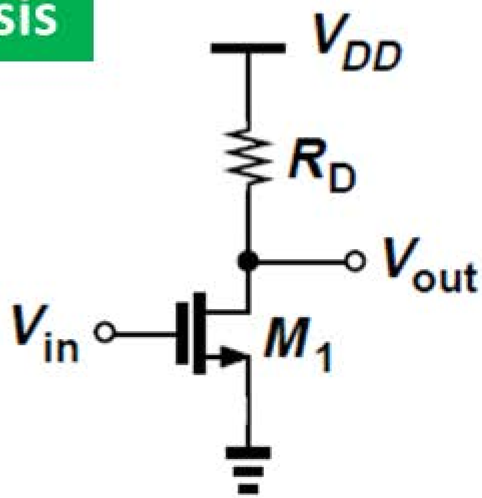
$$\boxed{A_v = \frac{v_{out}}{v_{in}}} = -g_{m1}(r_{o1} \parallel 5k)$$

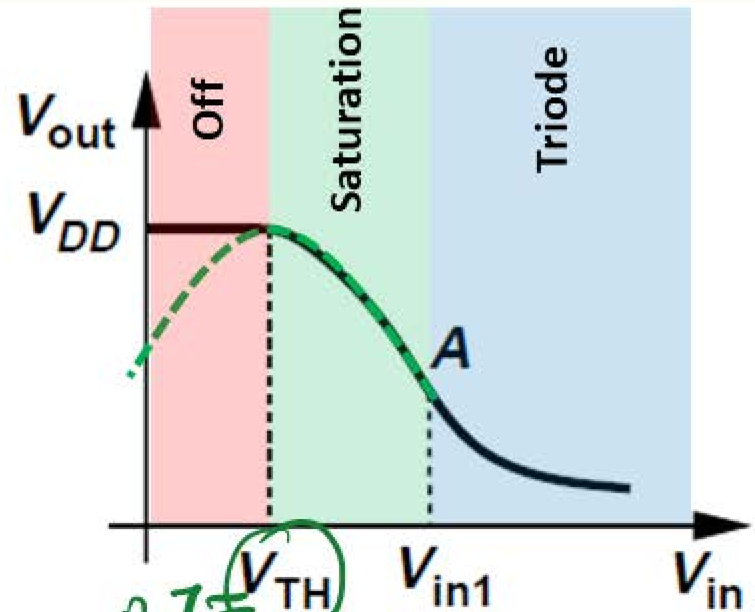
Common-Source with Resistive Load

DC Analysis

$\lambda = 0$

$\gamma = 0$





$0.7 =$

$$V_{out} = V_{in1} - V_{TH}$$
$$= V_{DD} - R_D \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in1} - V_{TH})^2$$

- **$V_{in} < V_{TH} \rightarrow M_1$ Off**

  $$V_{out} = V_{DD}$$

- **$V_{in1} > V_{in} > V_{TH} \rightarrow M_1$ in Saturation**

  $$V_{out} = V_{DD} - R_D \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in} - V_{TH})^2 \quad = I_{D1}$$

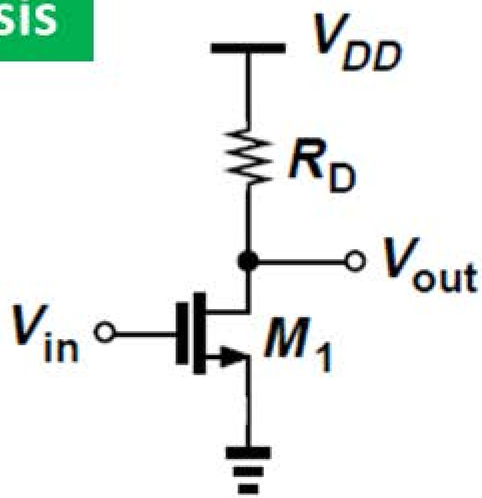
- **$V_{in} > V_{in1} \rightarrow M_1$ in Triode**

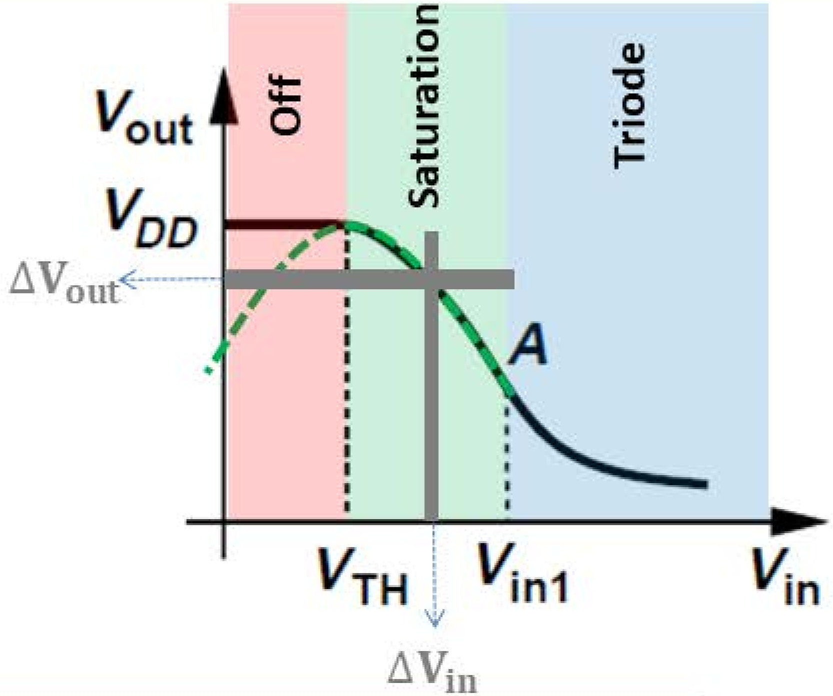
  $$V_{out} = V_{DD} - R_D \mu_n C_{ox}\frac{W}{L}\left[(V_{in} - V_{TH})V_{out} - \frac{1}{2}{V_{out}}^2\right] \quad = I_{D1}$$

Common-Source with Resistive Load

DC Analysis

$\lambda = 0$
$\gamma = 0$





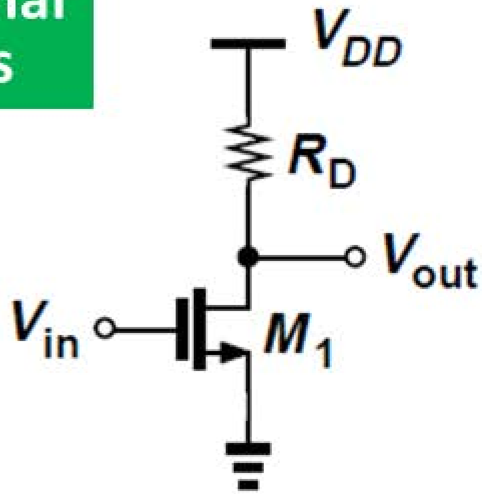
$$A_v = \frac{\partial V_{out}}{\partial V_{in}} = -R_D \mu_n C_{ox} \frac{W}{L}(V_{in} - V_{TH})$$
$$= -gm \cdot R_D$$

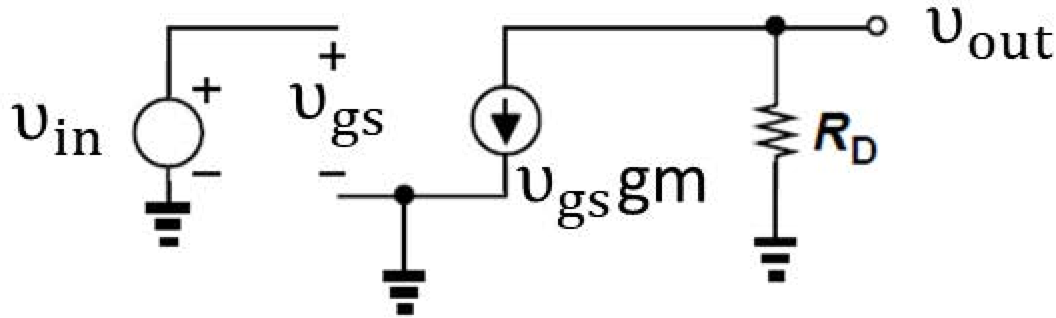
- V_{gs} increases by ΔV_{in} → I_d increases by $\Delta V_{in} \cdot gm$ → V_{out} decreases by $\Delta V_{in} \cdot (gm \cdot R_D)$

Common-Source with Resistive Load

Small-signal Analysis

$\lambda = 0$
$\gamma = 0$





$$\boxed{A_v = \frac{\upsilon_{out}}{\upsilon_{in}} = -gm \cdot R_D}$$

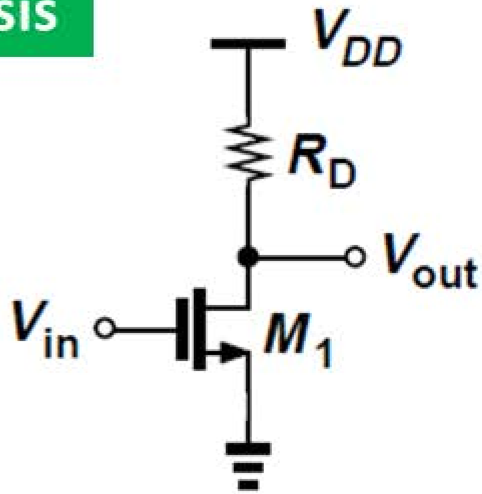
- Small-signal analysis leads to the same result as DC analysis.

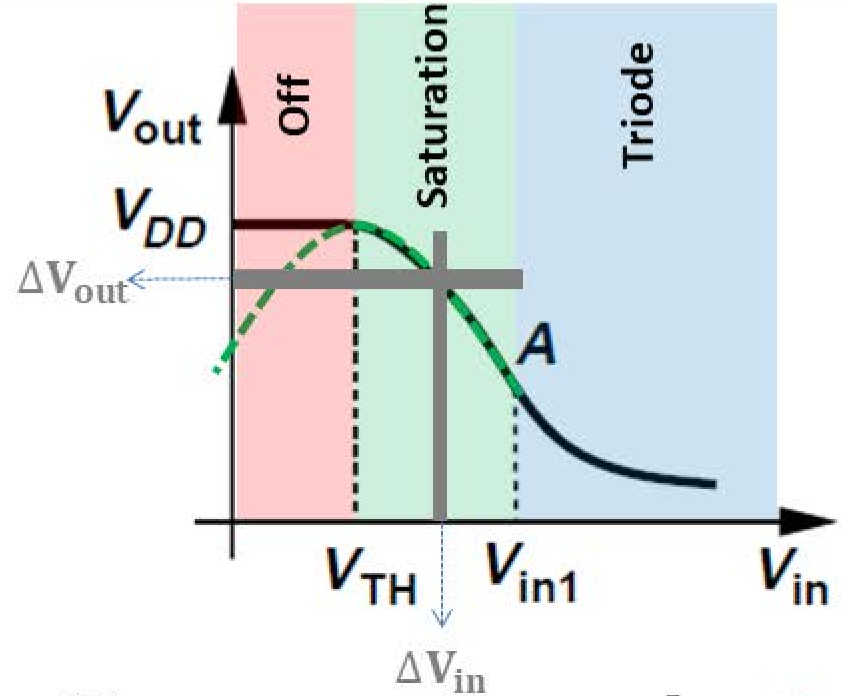
Common-Source with Resistive Load

DC Analysis

$\lambda \neq 0$

$\gamma \neq 0$





$$A_v = \frac{\partial V_{out}}{\partial V_{in}} = \frac{\partial\left[V_{DD} - R_D \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in}-V_{TH})^2(1+\lambda V_{out})\right]}{\partial V_{in}}$$

I_{D1}

$$= -R_D \underbrace{\mu_n C_{ox}\frac{W}{L}(V_{in}-V_{TH})(1+\lambda V_{out})}_{=\,gm} - R_D \underbrace{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in}-V_{TH})^2}_{\approx\, I_D}\lambda \underbrace{\frac{\partial V_{out}}{\partial V_{in}}}_{=\,A_v}$$

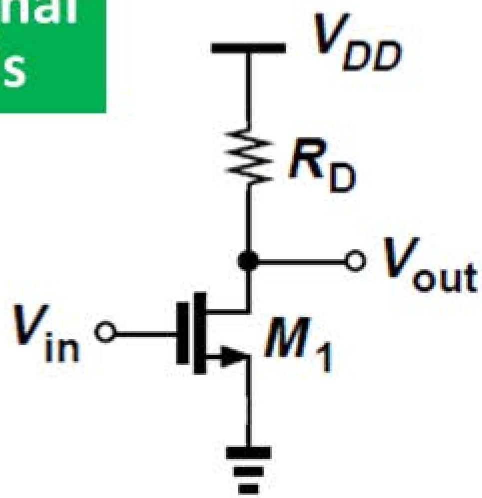
$$A_v = \frac{-gmR_D}{1+R_D\underbrace{I_D\lambda}_{=\,1/r_o}} = -gm\frac{1}{\frac{1}{R_D}+\frac{1}{r_o}} = \boxed{-gm(R_D \parallel r_o)}$$

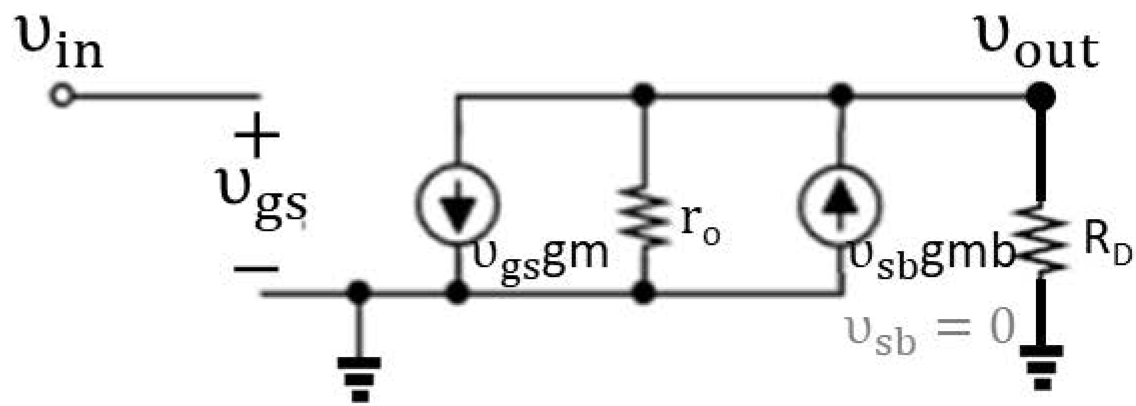
Common-Source with Resistive Load

Small-signal Analysis

$\lambda \neq 0$

$\gamma \neq 0$

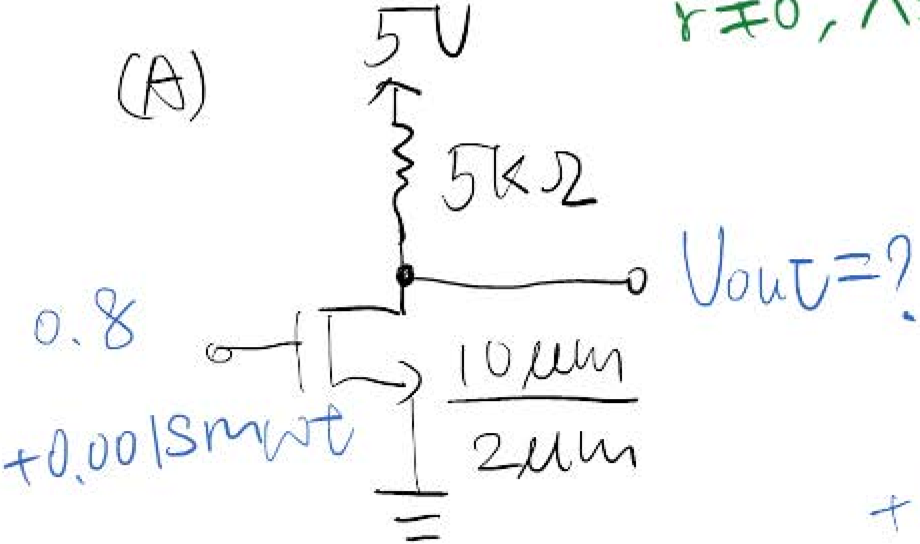


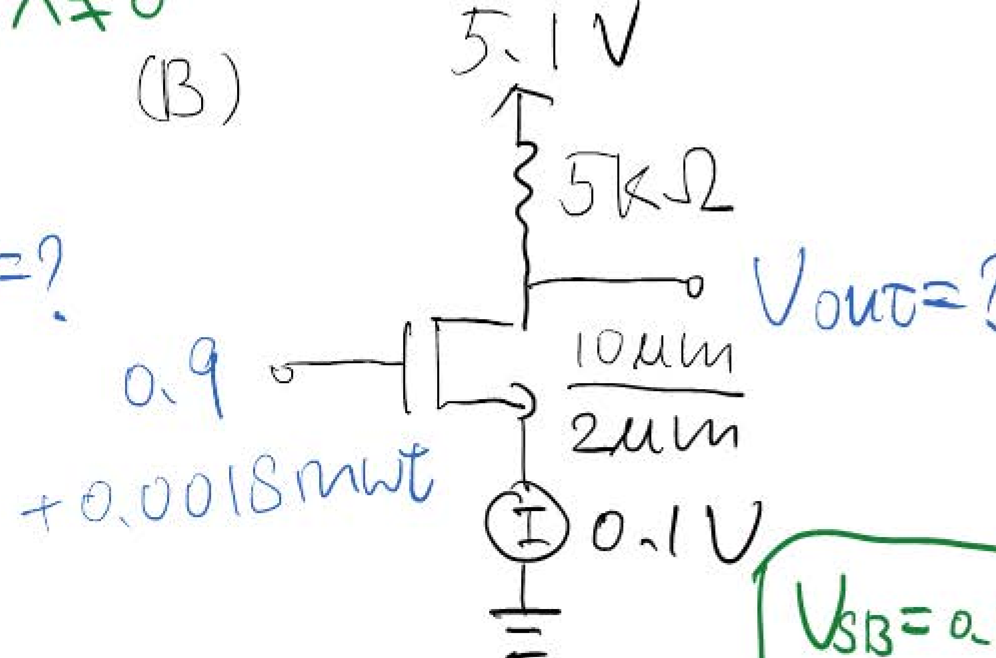


$$A_v = \frac{\upsilon_{out}}{\upsilon_{in}} = -gm \cdot (R_D \parallel r_o)$$

- Small-signal analysis leads to the same result as DC analysis.
- gm is a function of V_{GS} and V_{DS}, while r_o is a function of I_D. → **Nonlinearity**

$r \neq 0,\ \lambda \neq 0$



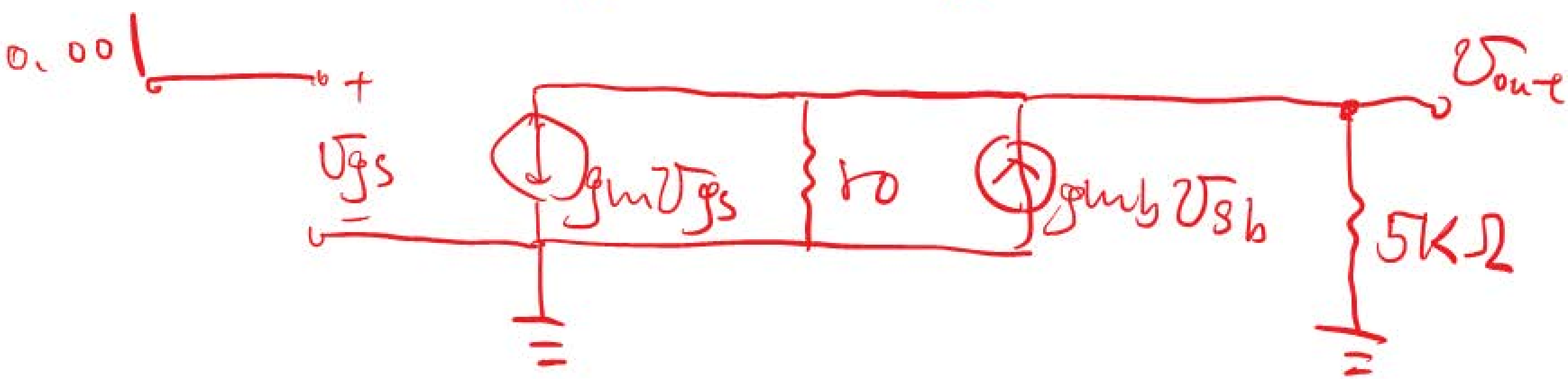


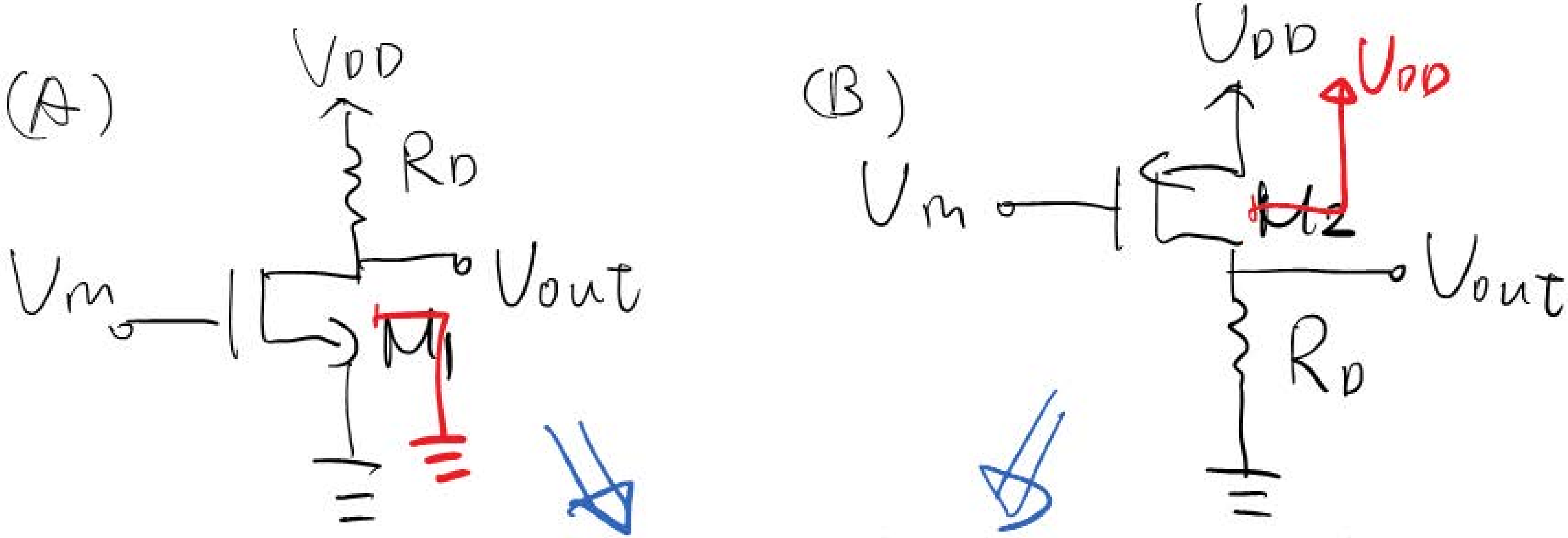
$$\boxed{\begin{aligned} V_{SB} &= 0 \\ v_{sb} &= 0 \end{aligned}}$$

$V_{TH} = V_{TH0} = 0.7$

$$\boxed{\begin{aligned} V_{SB} &= 0.1 \\ v_{sb} &= 0 \end{aligned}}$$

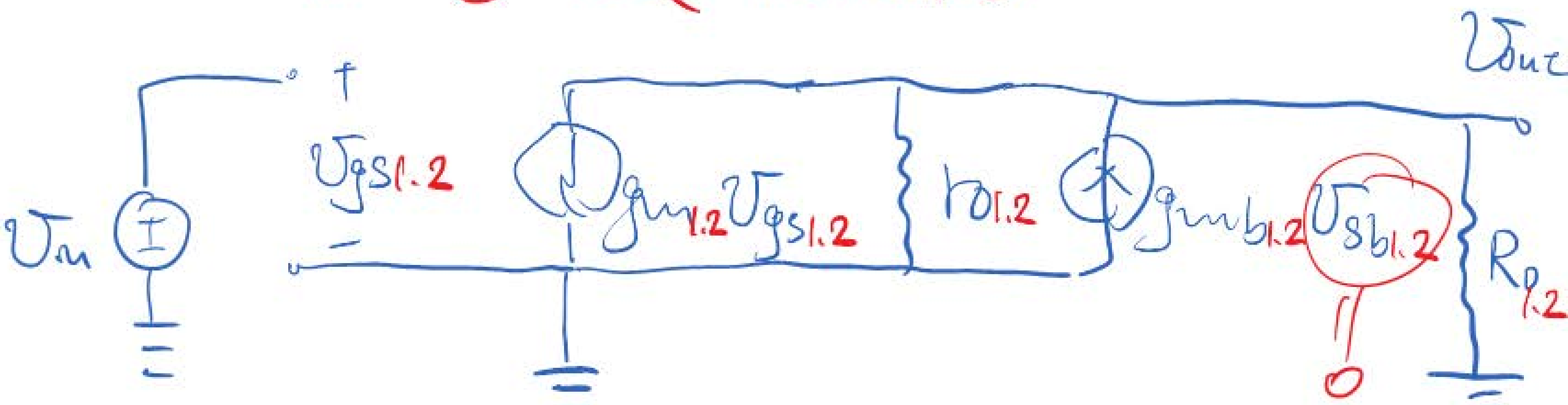
$V_{TH} > V_{TH0} = 0.7$





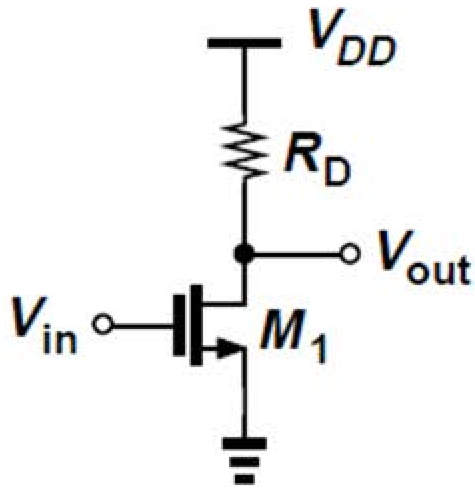
What are the analytical expressions for Av?

$A_v = -g_{m1,2}(r_{o1,2} // R_{D1,2})$

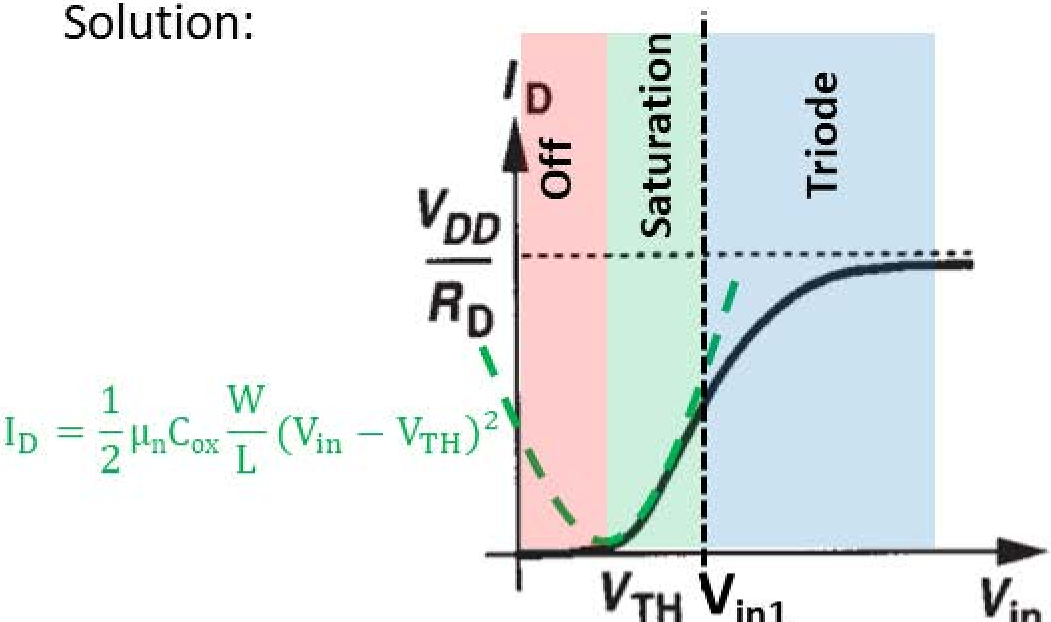


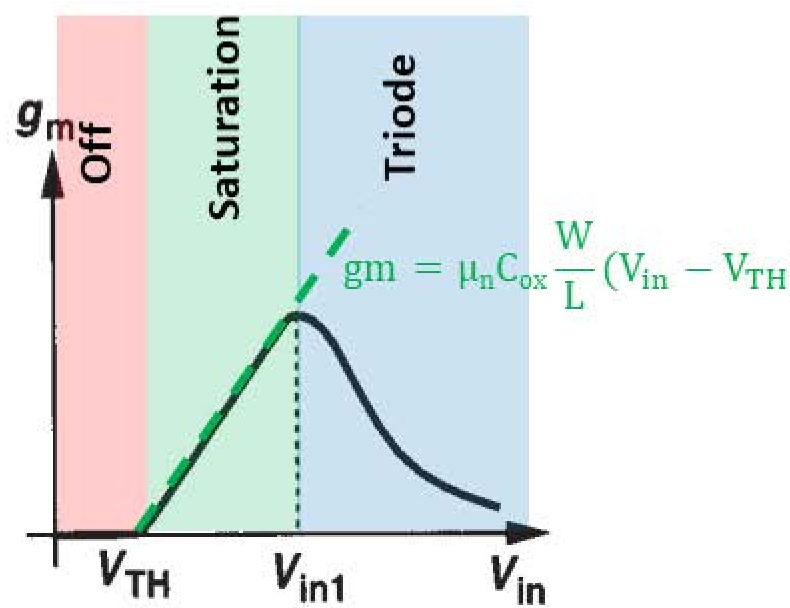
Example

Sketch the drain current and transconductance of M_1 as a function of input voltage. Assume $\lambda = \gamma = 0$.



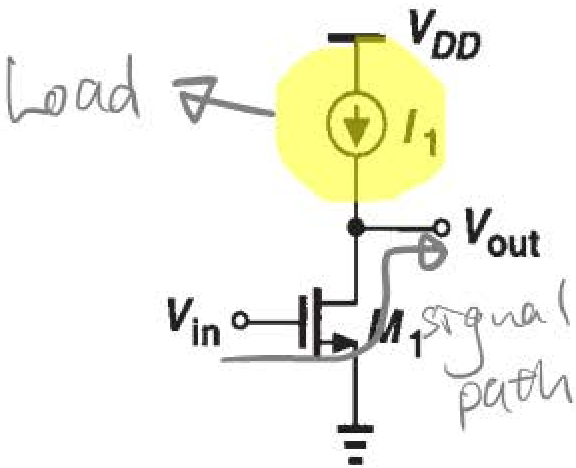
Solution:

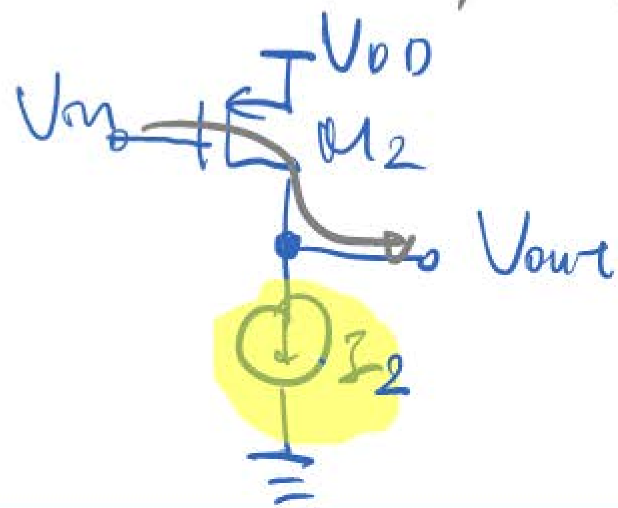




Example

Assuming M_1 in saturation, calculate its small-signal gain. (analytical form)





Solution:

$r \neq 0$

$\lambda \neq 0$

- Small-signal Analysis:

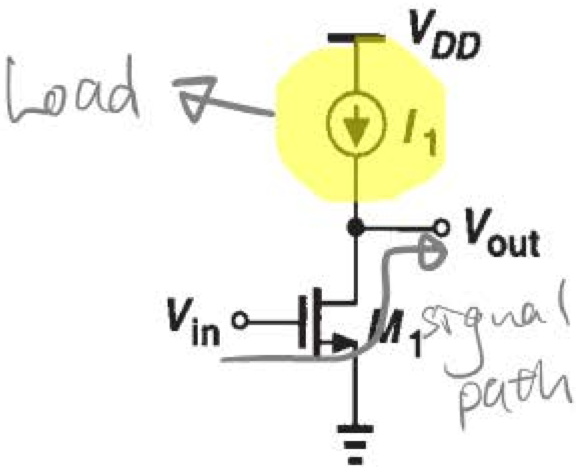
$$A_v = \frac{\upsilon_{out}}{\upsilon_{in}} = -gm_1 r_{o1} \cong \frac{1}{\lambda I_{D1}}$$

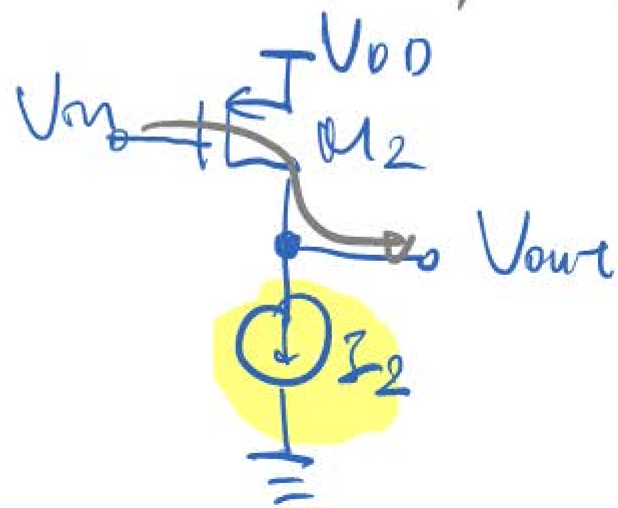
- DC Analysis:

$$I_1 = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in} - V_{TH})^2(1 + \lambda V_{DS})$$

Example

Assuming M_1 in saturation, calculate its small-signal gain. (analytical form)





Solution:

$\lambda = 0$

$r = 0$

- Small-signal Analysis:

$$A_v = \frac{\upsilon_{out}}{\upsilon_{in}} = -g m_1 r_{o1} \rightarrow \infty$$

- DC Analysis:

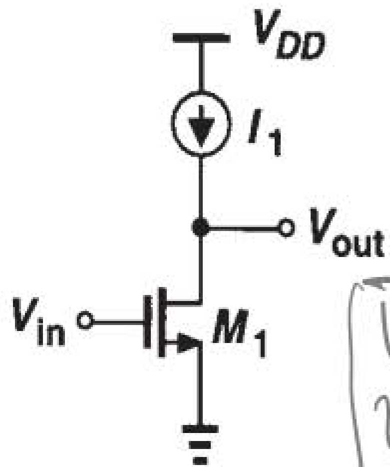
$$I_1 \neq \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in} - V_{TH})^2 \rightarrow V_{OUT} \text{ undefined. (floating)}$$

Example

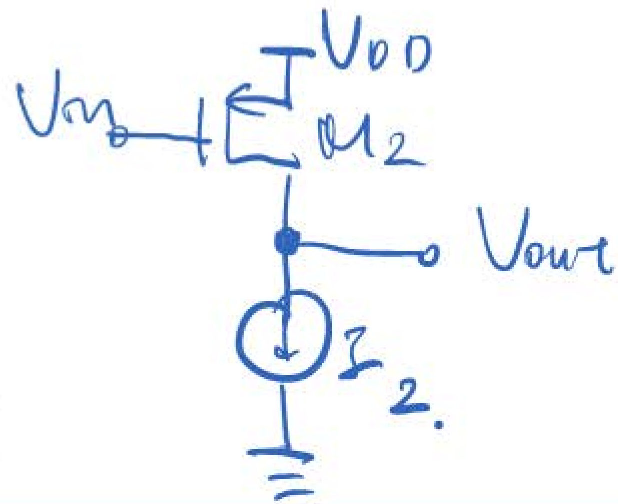
Assuming M_1 in saturation, calculate its small-signal gain.

$\lambda \neq 0$

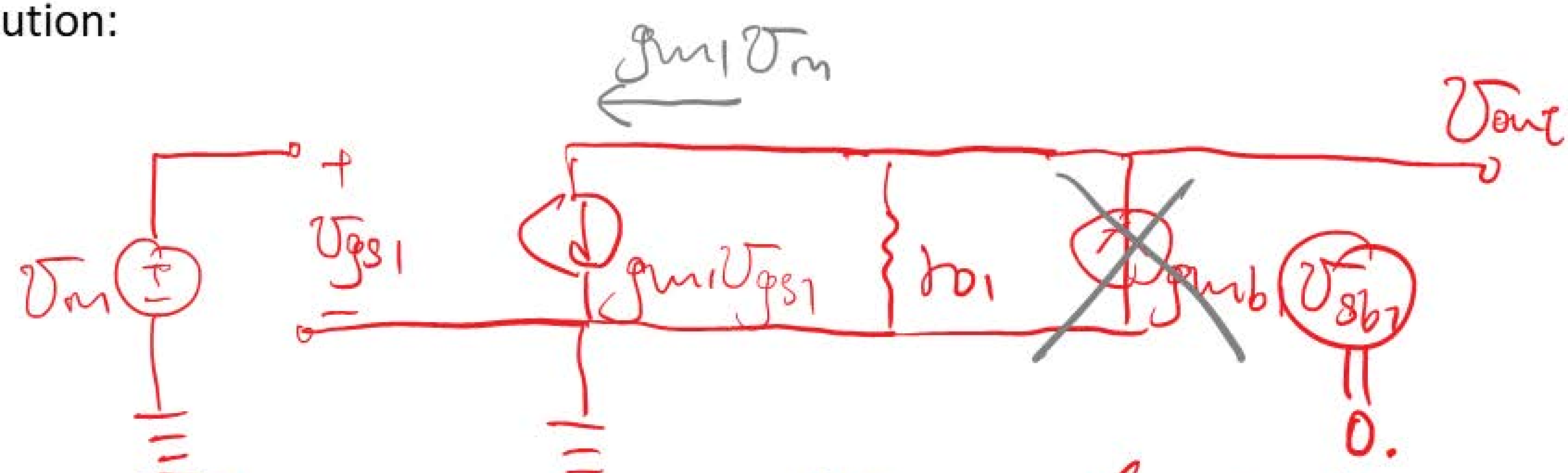
$\gamma \neq 0$



$$\boxed{\begin{aligned} V_{SB1} &= 0 \\ v_{sb1} &= 0 \end{aligned}}$$



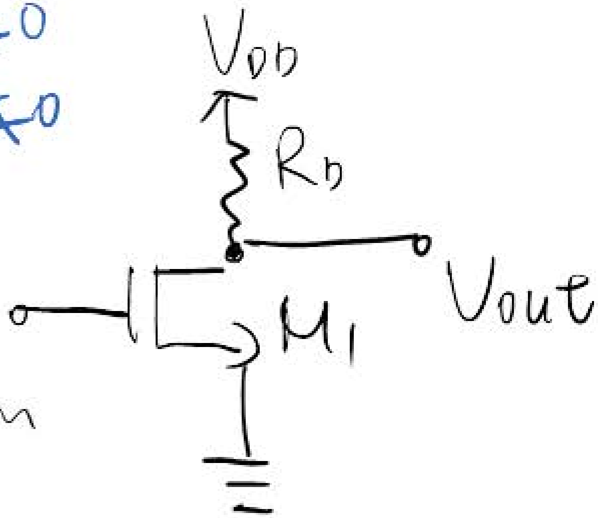
Solution:



$$A_v = \frac{v_{out}}{v_{in}} = -g_{m1} r_{o1}$$

$$v_{out} = -(g_{m1} v_{in}) r_{o1}$$

$\lambda \neq 0$
$r \neq 0$



Find out the analytical expressions for A_v, R_{in} and R_{out}?

1° $A_v = -g_{m1}(r_{o1} \parallel R_D)$

2° $R_{out} = ?$

a. In small-signal circuit, turn v_{in} off.

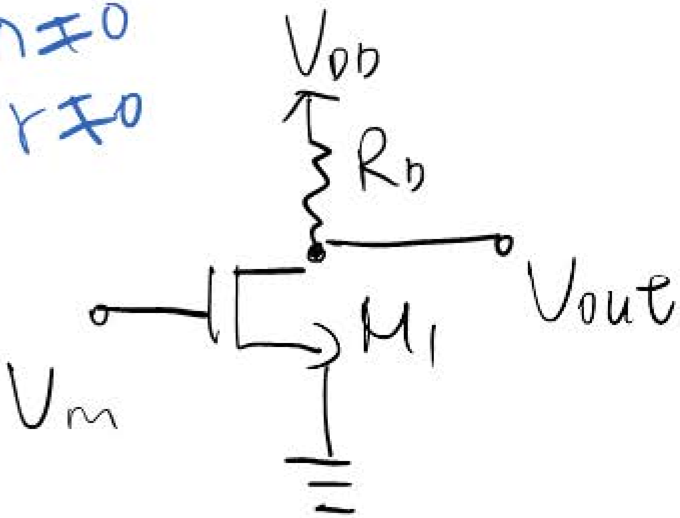
b. Put a test small-signal (v_t) at the output.

c. Calculate small-signal current (i_t) flowing into output.

d. $R_{out} = v_t / i_t$

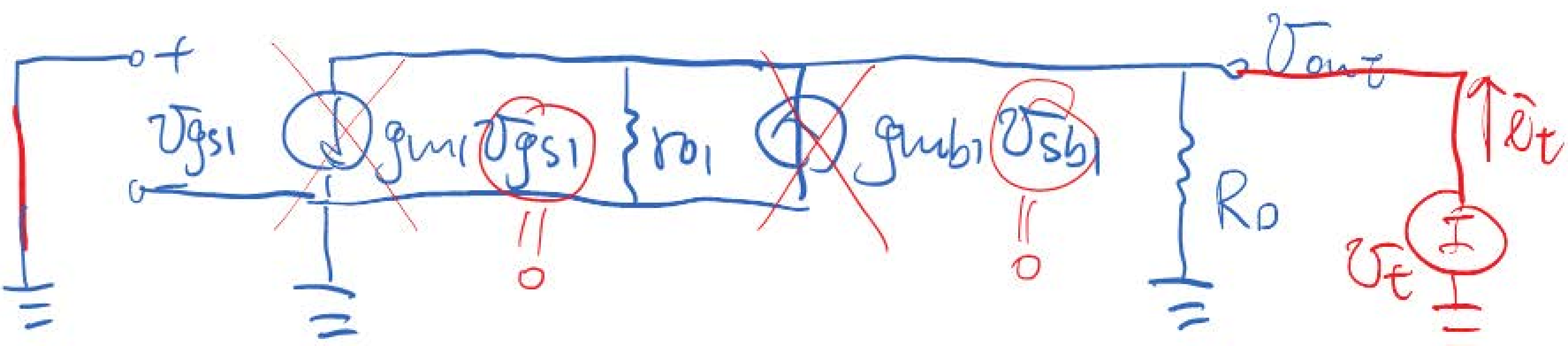
$\lambda \neq 0$

$\gamma \neq 0$

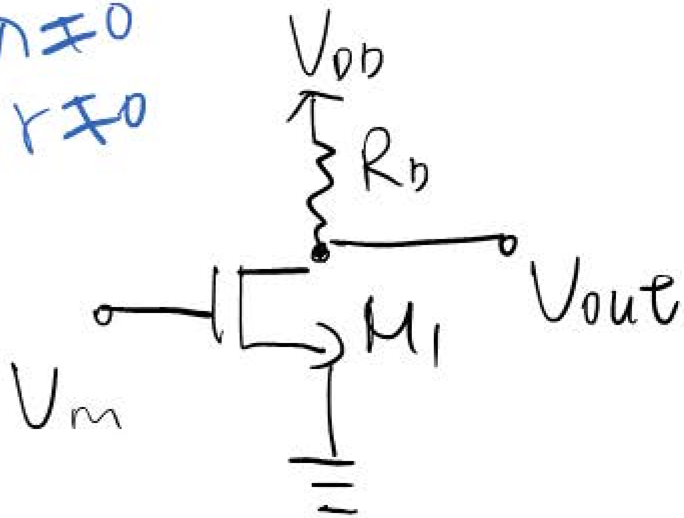


Find out the analytical expressions for A_v, R_{in} and R_{out}?

2° $R_{out} = r_{o1} // R_D$

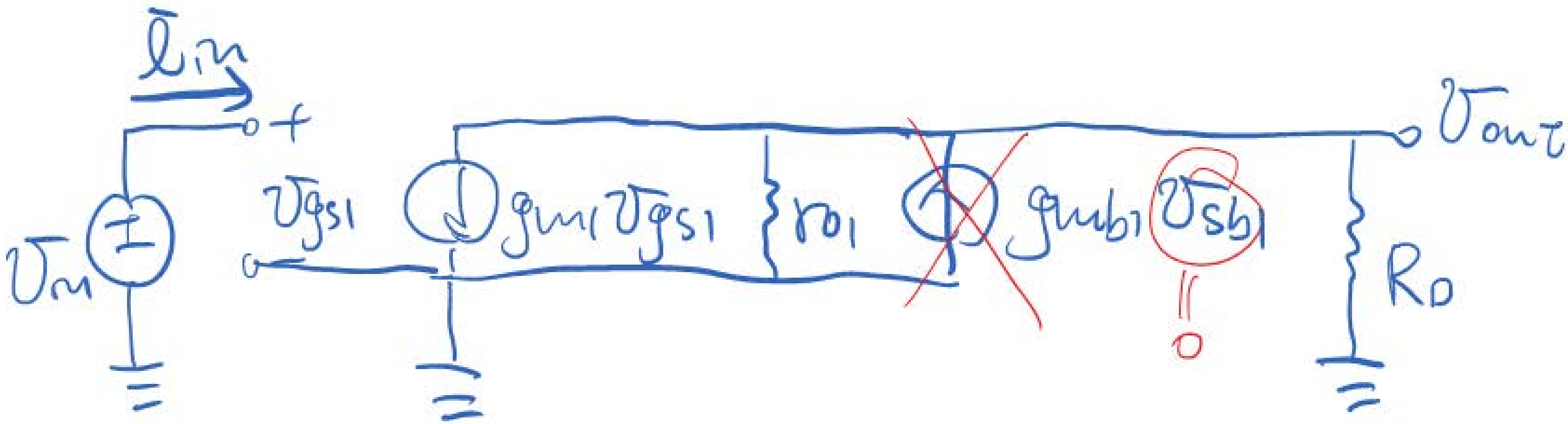


$\lambda \neq 0$
$\gamma \neq 0$

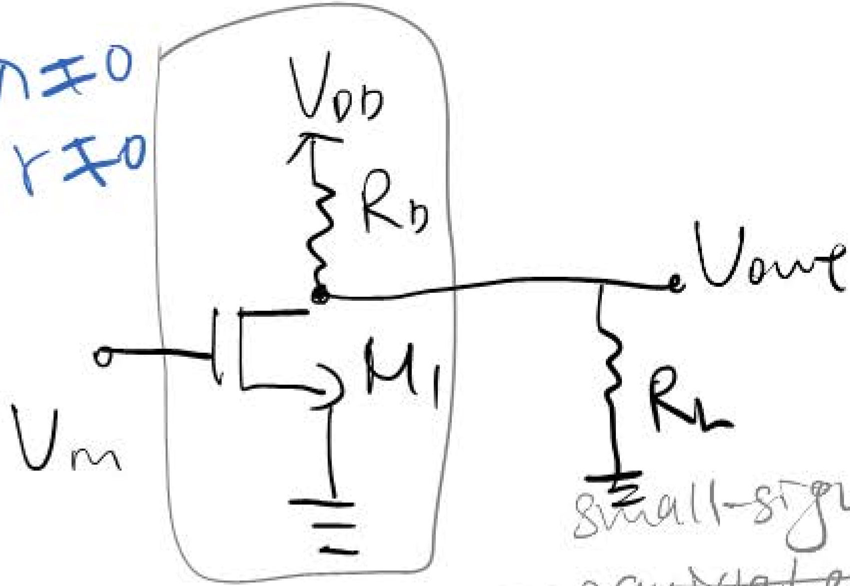


Find out the analytical expressions for A_v, R_{in} and R_{out}?

$$3^\circ \quad R_{in} = \frac{v_{in}}{i_{in}} = \infty$$

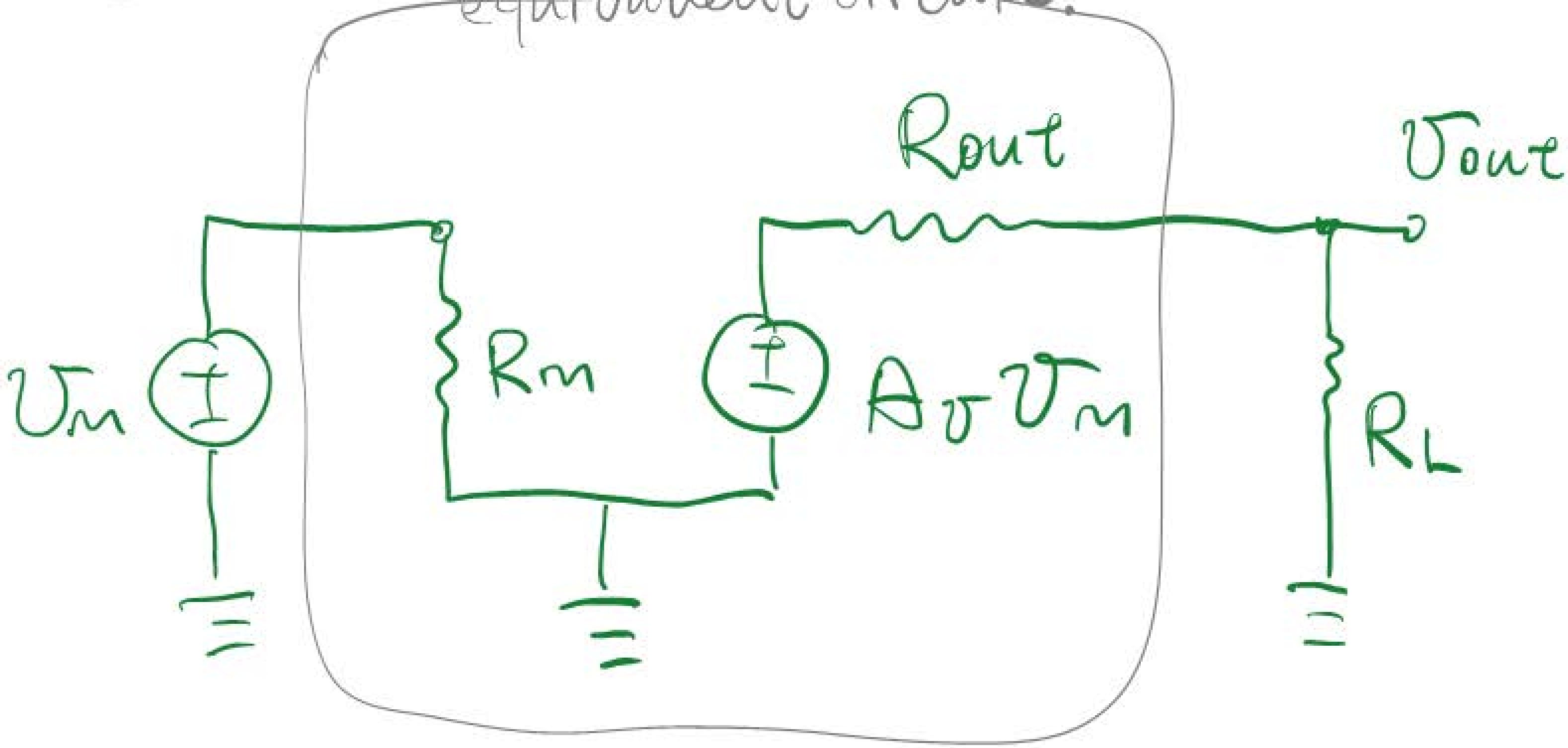


$\lambda \neq 0$
$r \neq 0$

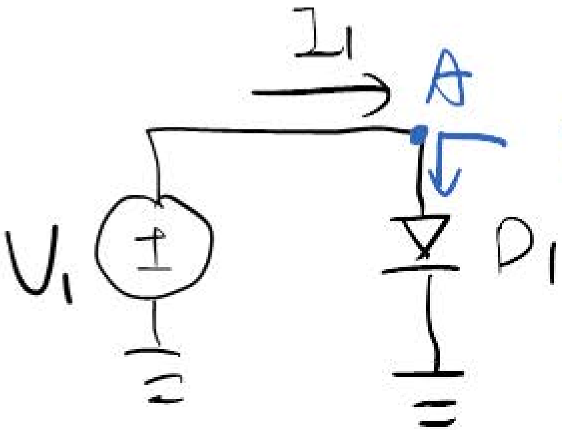


$$v_{out} = v_{in} A_v \frac{R_L}{R_{out} + R_L}$$

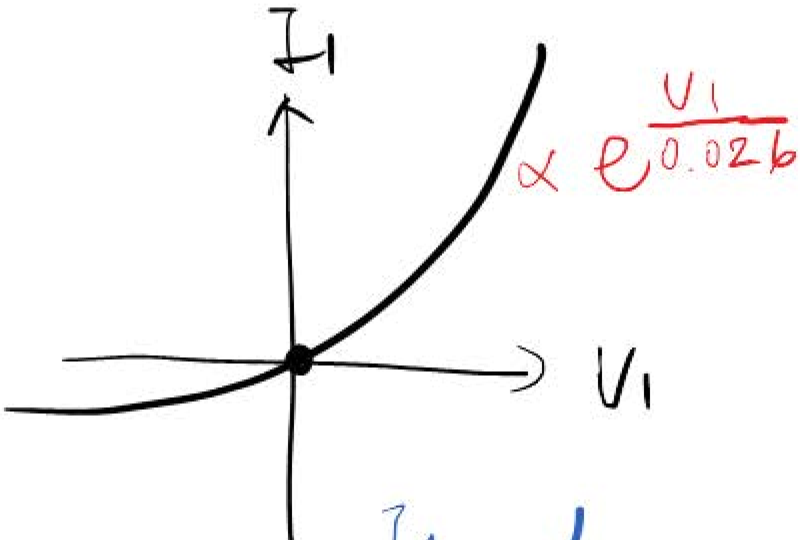
small-signal equivalent circuit.

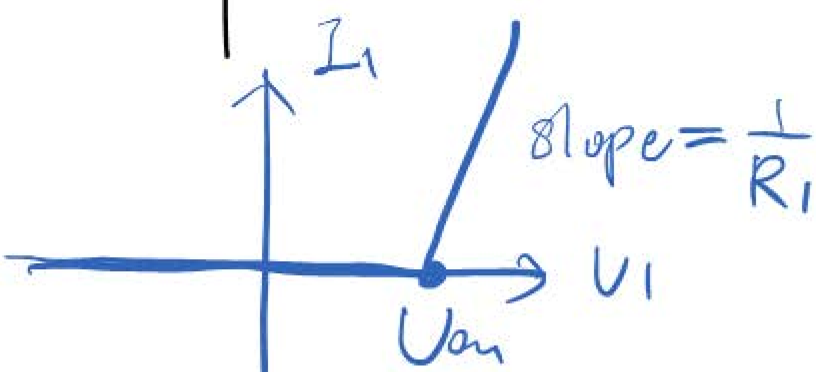


Common-Source with Diode-Connected Load

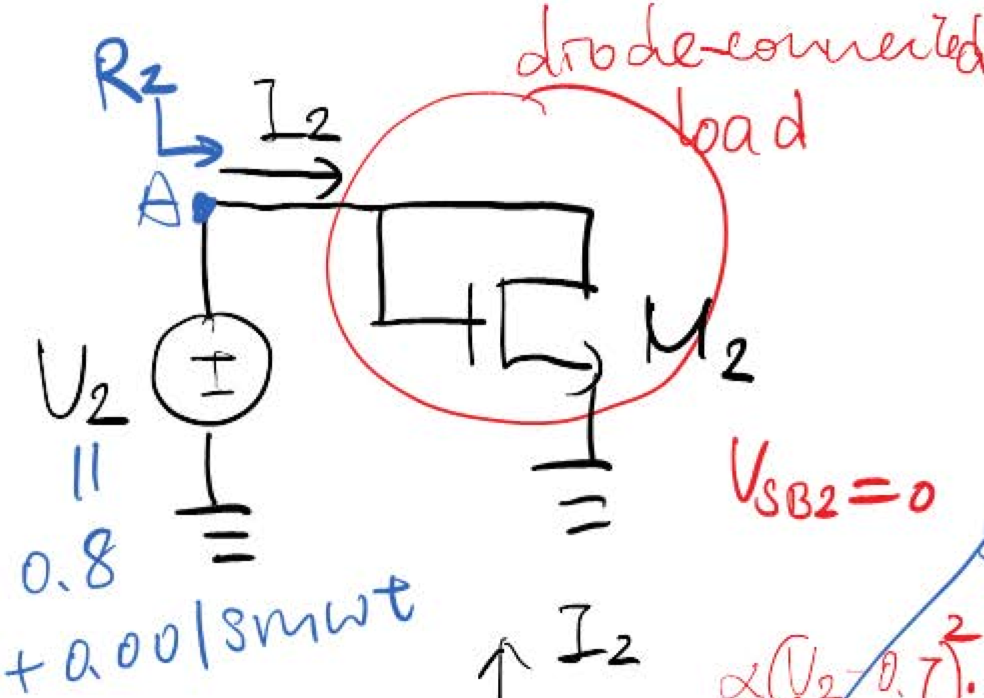


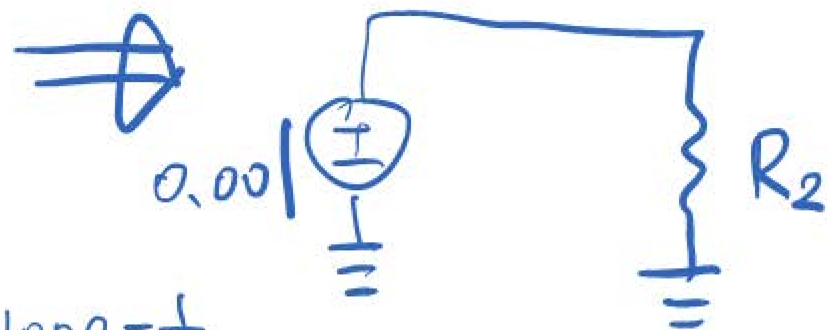
$\frac{1}{R_1}$ (the small signal impedence from node A to small signal ground)

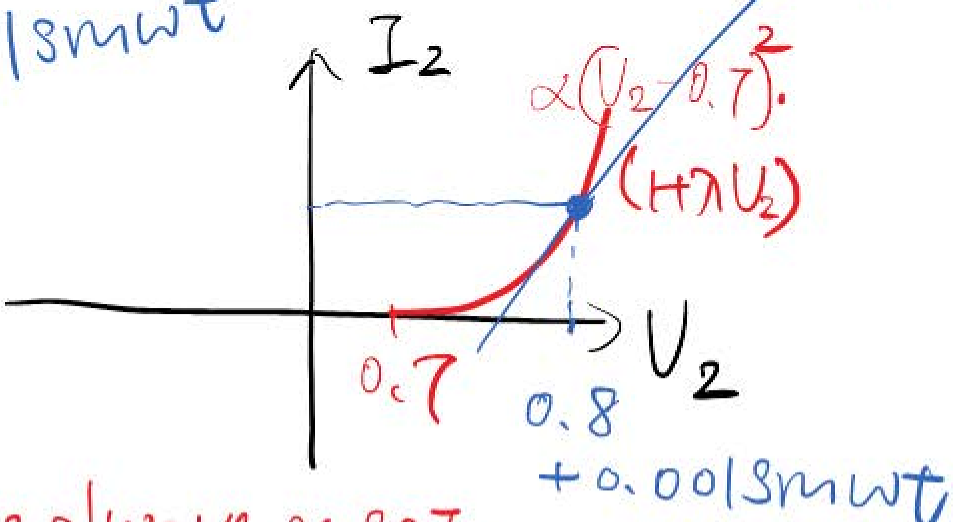




Small signal



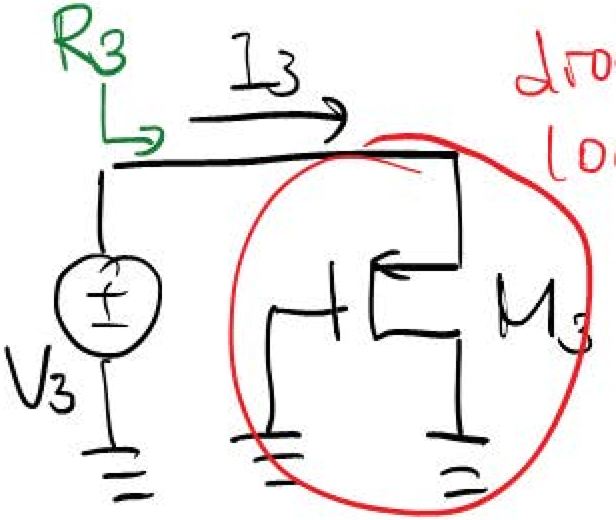




$$R_2 = r_{o2} // \frac{1}{g_{m2}}$$

M_2 always in sat.

$$I_2 = \frac{1}{2} \mu_n C_{ox} \frac{W}{L_{eff}} (V_2 - 0.7)^2 (1+\lambda V_2)$$

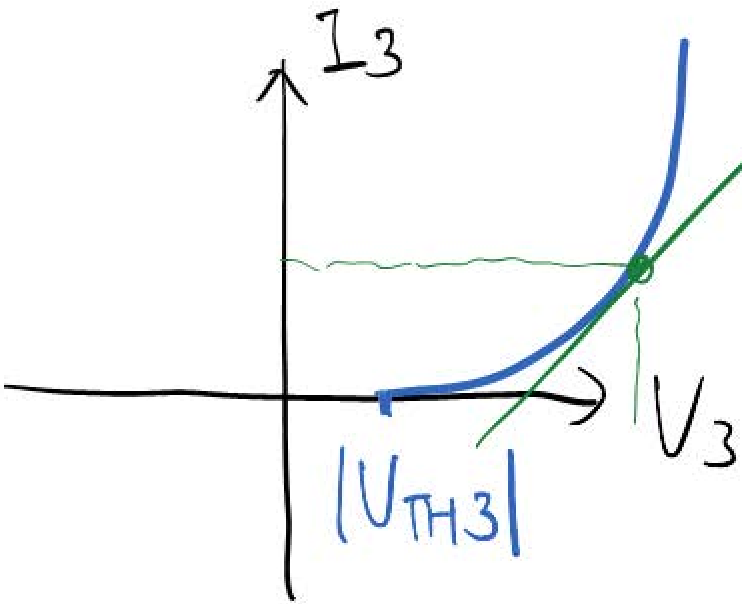


diode connected load

$$R_3 = r_{o3} \,//\, \left(\frac{1}{g_{m3}+g_{mb3}}\right)$$

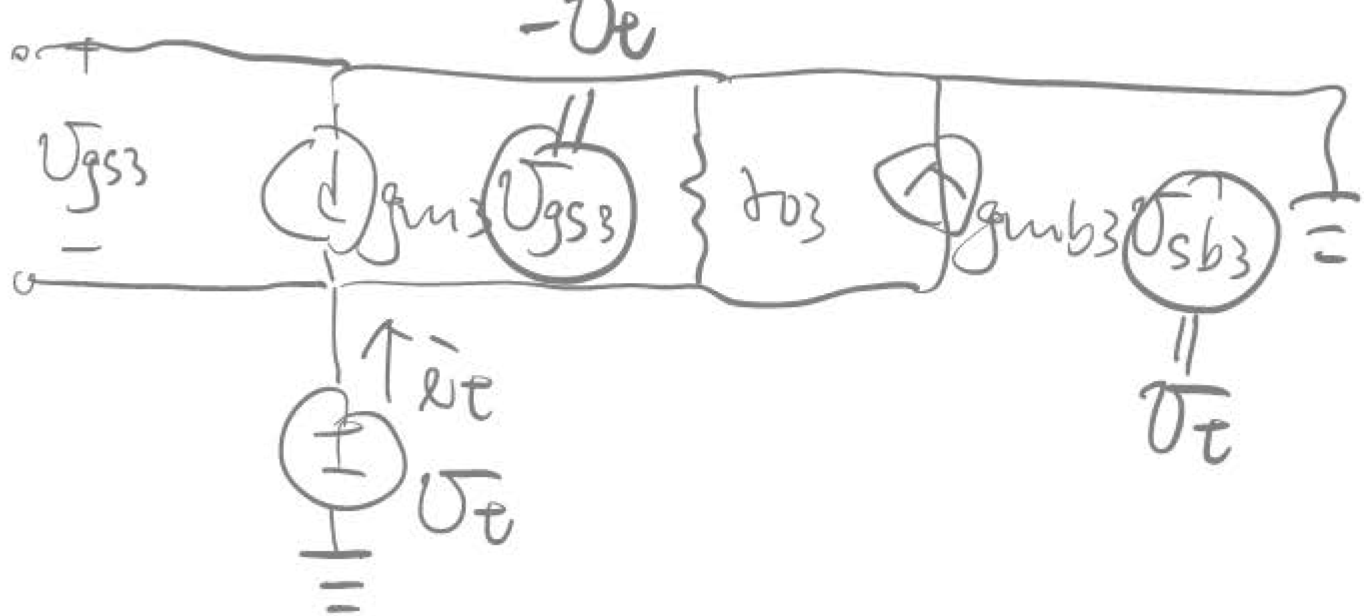
M_3 always in Sat.

$V_{BS3} = V_{DD} - V_3 > 0$, $|V_{TH3}| > 0.8$



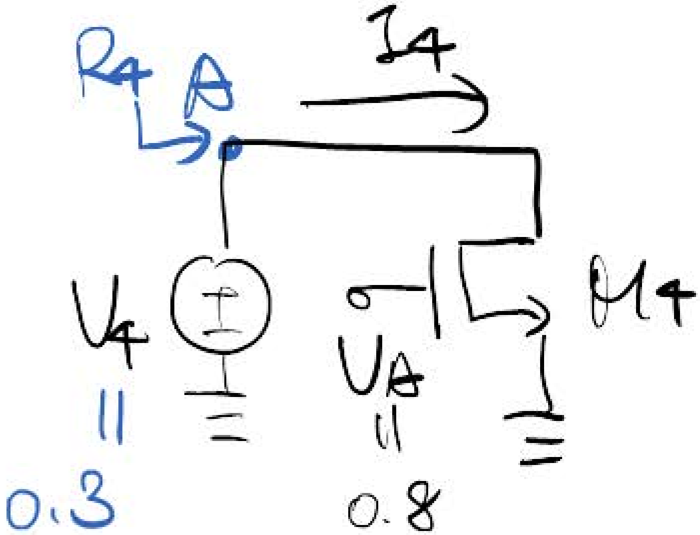
slope $= \frac{1}{R_3}$

$$I_3 = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L_{eff}}\right)_3 \cdot (V_3 - |V_{TH3}|)^2 (1+\lambda V_3)$$



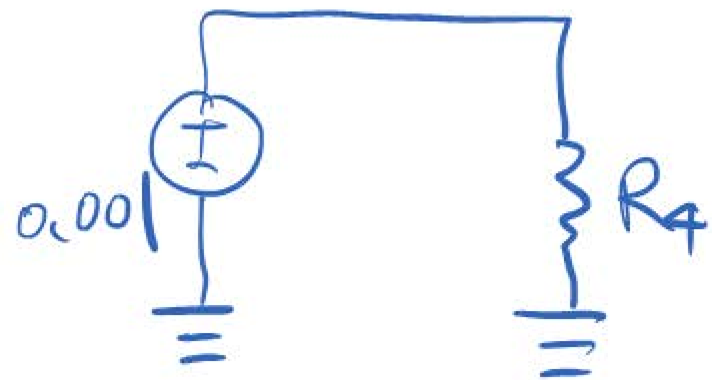
$$i_t = (g_{m3}+g_{mb3})v_t + \frac{v_t}{r_{o3}}$$

$$R_3 = v_t/i_t = r_{o3} \,/\!/\, \left(\frac{1}{g_{m3}+g_{mb3}}\right)$$

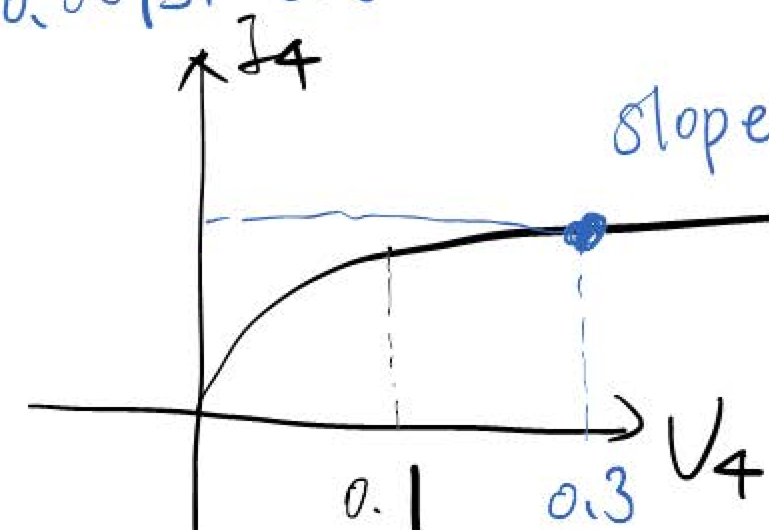


$\Rightarrow$

small signal

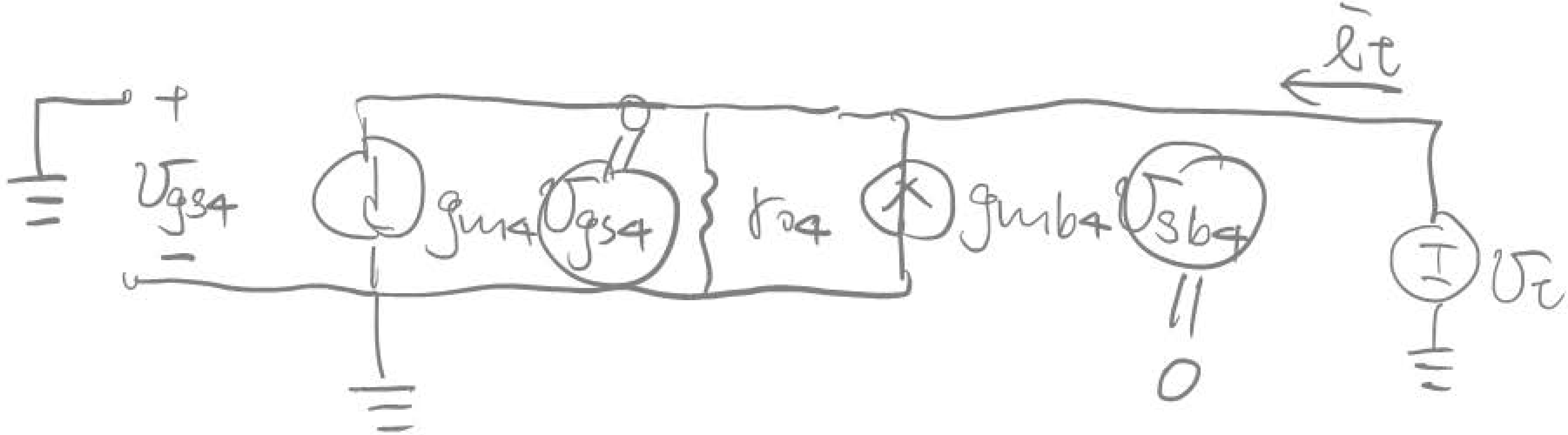


$+0.001 \sin \omega t$

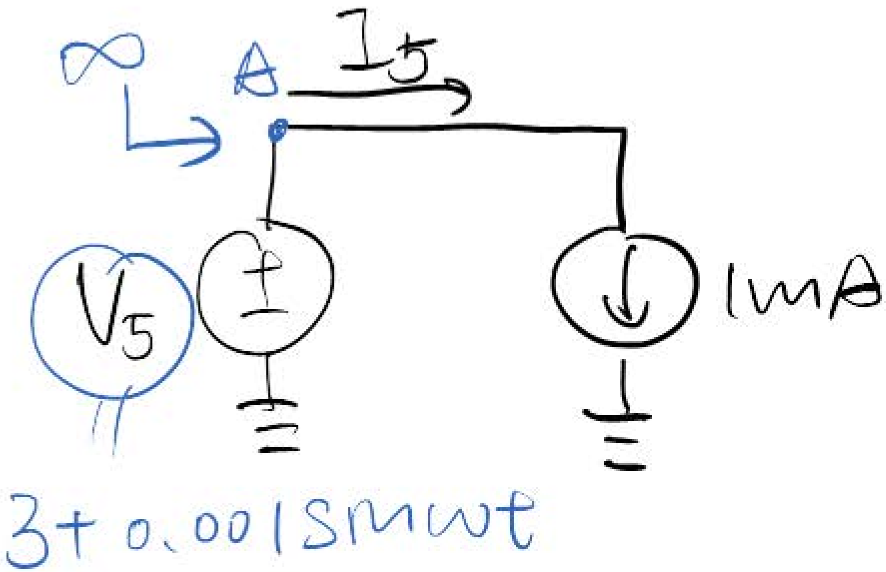


$$\text{slope} = \frac{1}{R_4} = \frac{1}{r_{o4}}$$

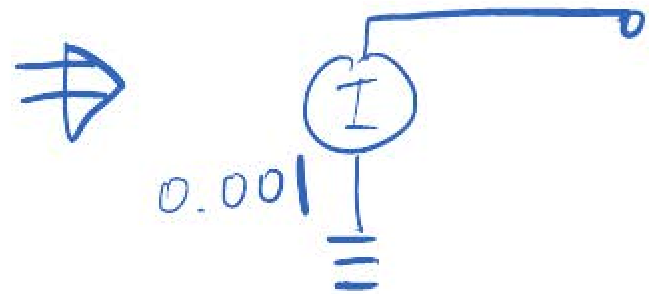
$$= V_{GS4} - V_{TH4}$$
$$= 0.8 - 0.7$$

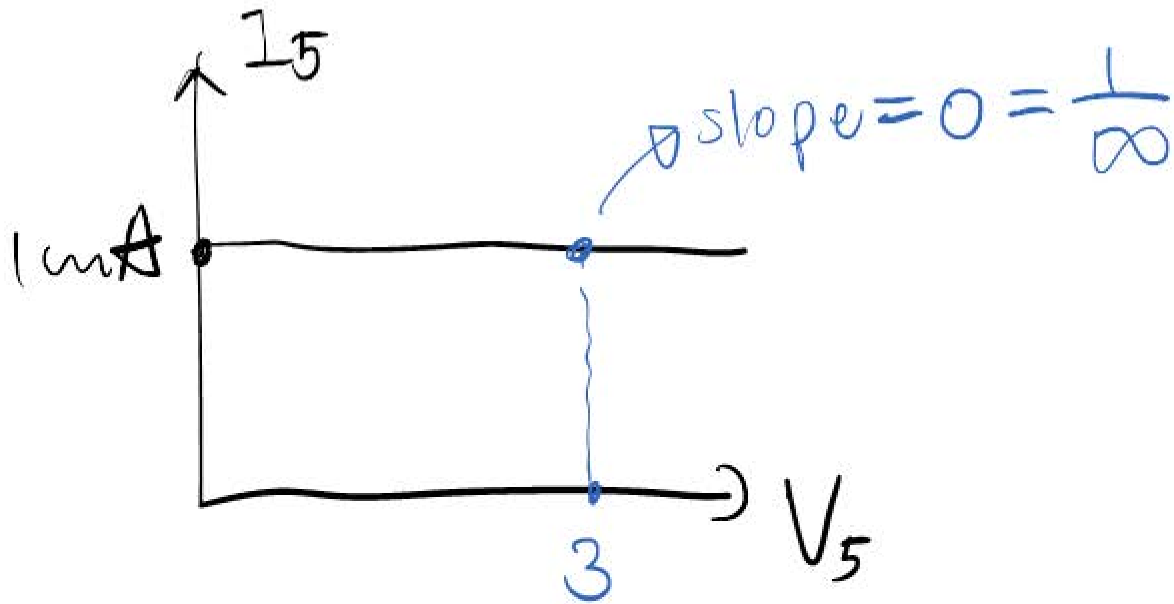


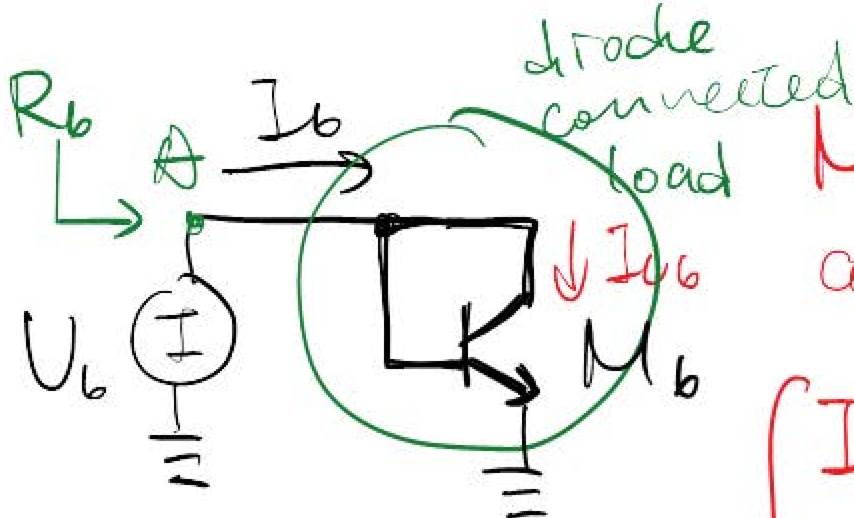
$$R_4 = \frac{v_t}{i_t} = r_{o4}$$



small signal

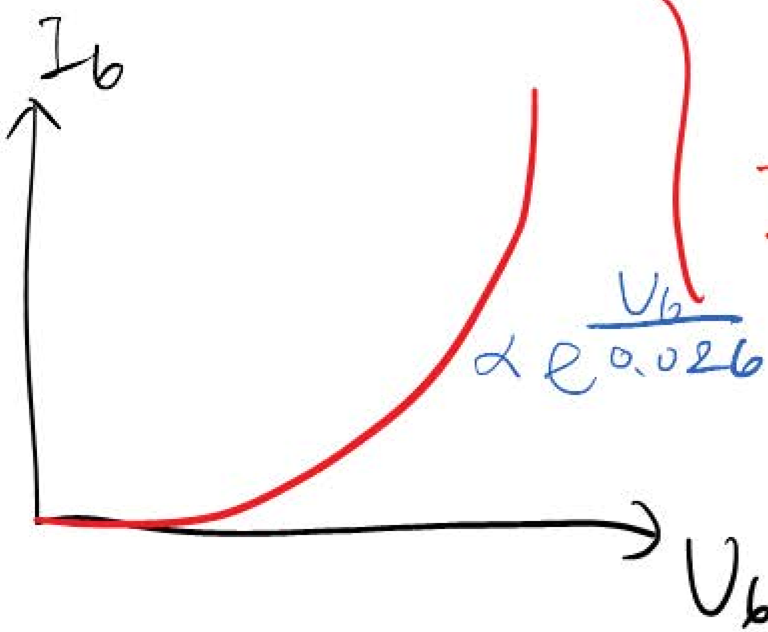




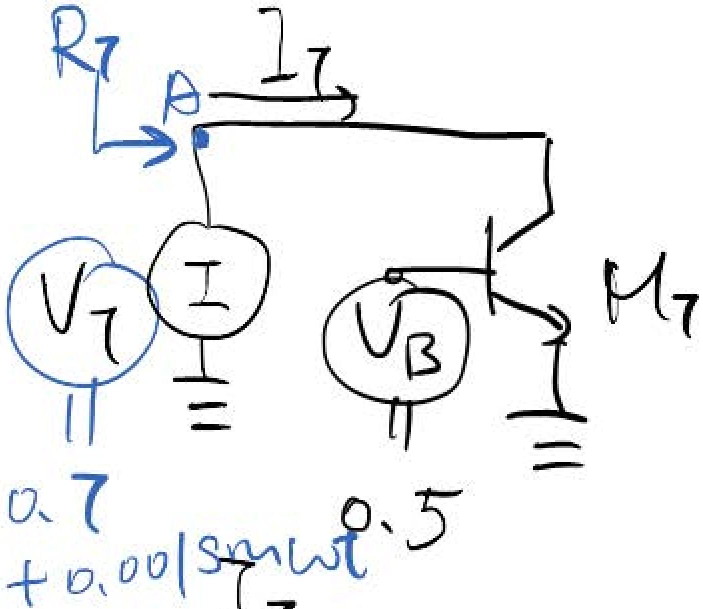


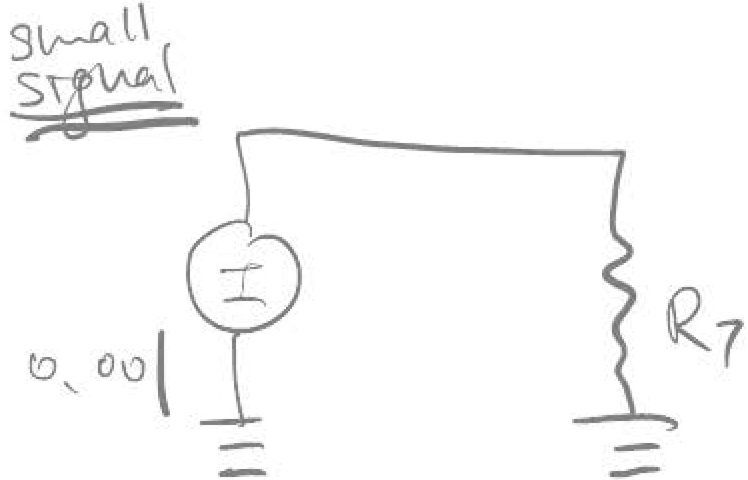
M_b always in forward-active region

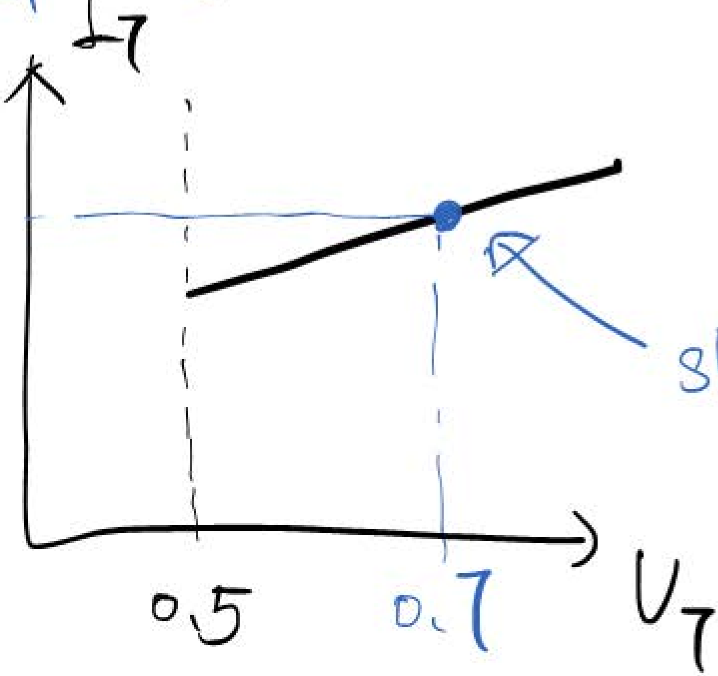
$$\begin{cases} I_{Cb} = I_S\left(e^{\frac{V_b}{0.026}} - 1\right) \cdot \left(1 + \frac{V_b}{V_A}\right) \\ I_b = I_{Cb} + \frac{I_{Cb}}{\beta} \end{cases}$$



$$R_6 = \frac{v_t}{i_t} = r_{\pi 6} \parallel r_{o6} \parallel \frac{1}{g_{m6}}$$

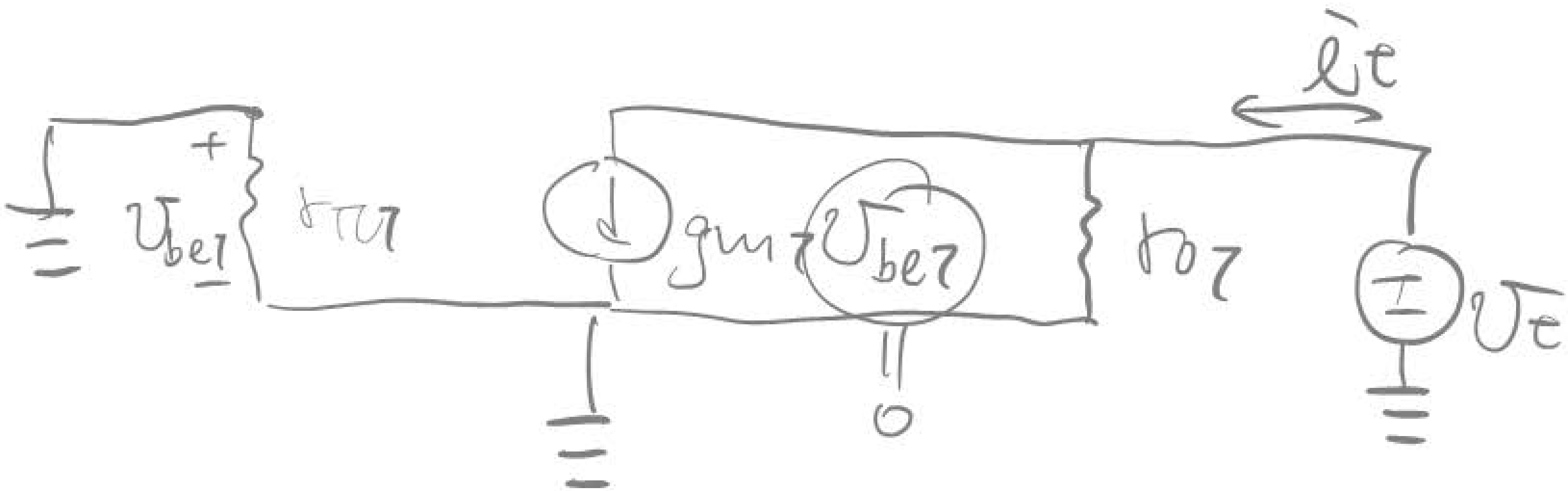




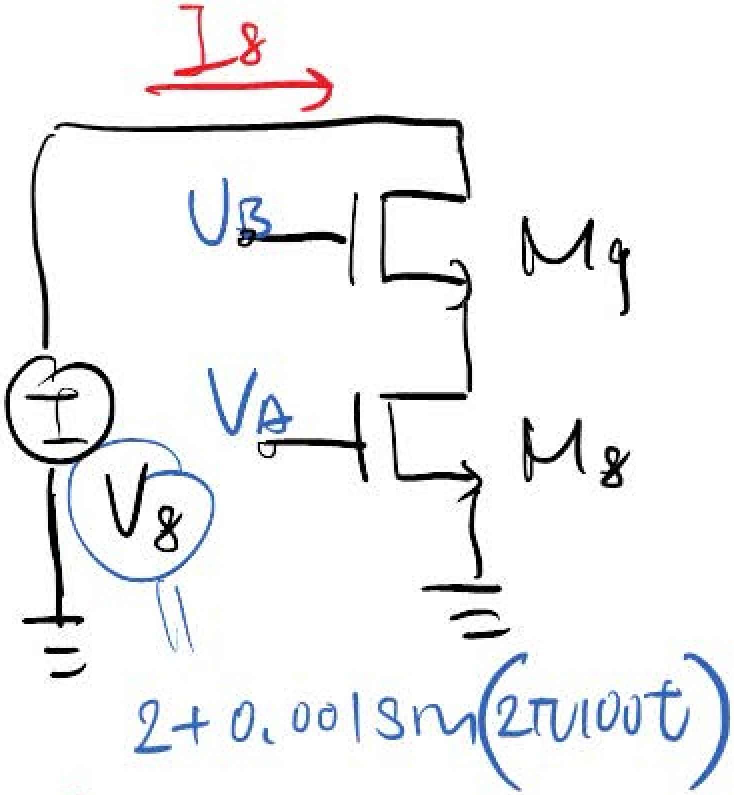


$$I_7 = I_S\left(e^{\frac{0.5}{0.026}} - 1\right)\left(1 + \frac{V_7}{V_A}\right)$$

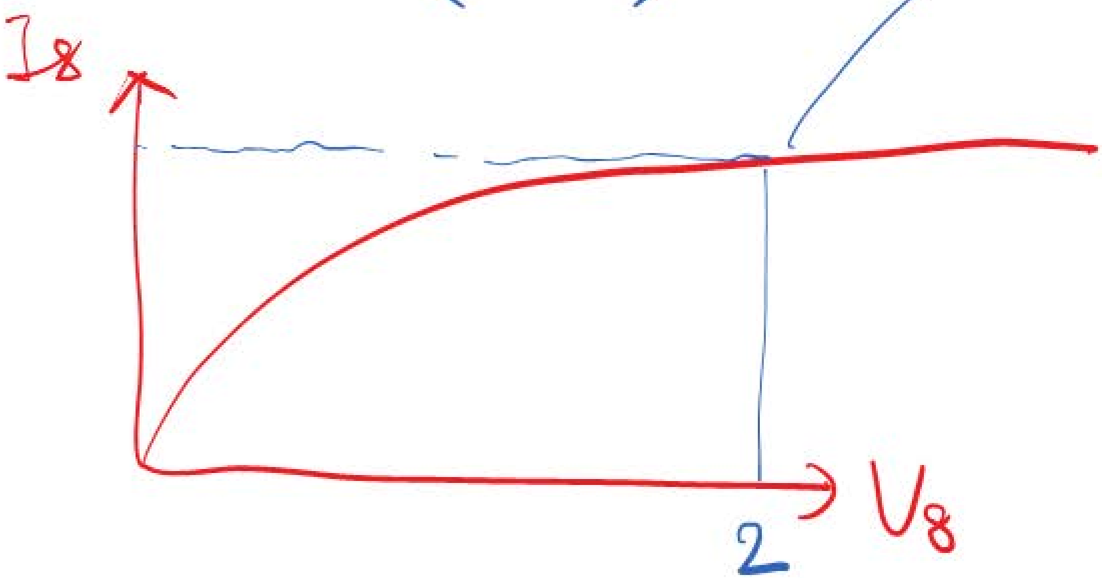
$$\text{slope} = \frac{1}{R_7} = \frac{1}{r_{o7}}$$



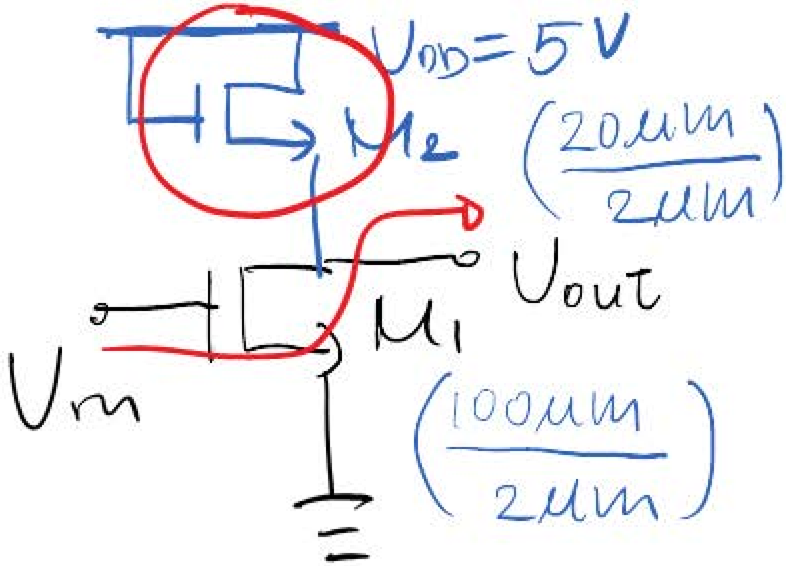
$$R_7 = \frac{v_t}{i_t} = r_{o7}$$



V_A and V_B are DC voltages properly chosen so that $M_{8,9}$ are in the sat.



$$\text{slope} = \frac{1}{\left[r_{o8} + r_{o9} + (g_{m9} + g_{mb9}) r_{o9} r_{o8}\right]}$$



$\lambda \neq 0,\ \gamma \neq 0$

$V_{in} = 0.8 + 0.001 \sin(2\pi 100 t)$

$V_{out} = V_{OUT} + v_{out} = ?$

1° Find out $V_{OUT} = ?$

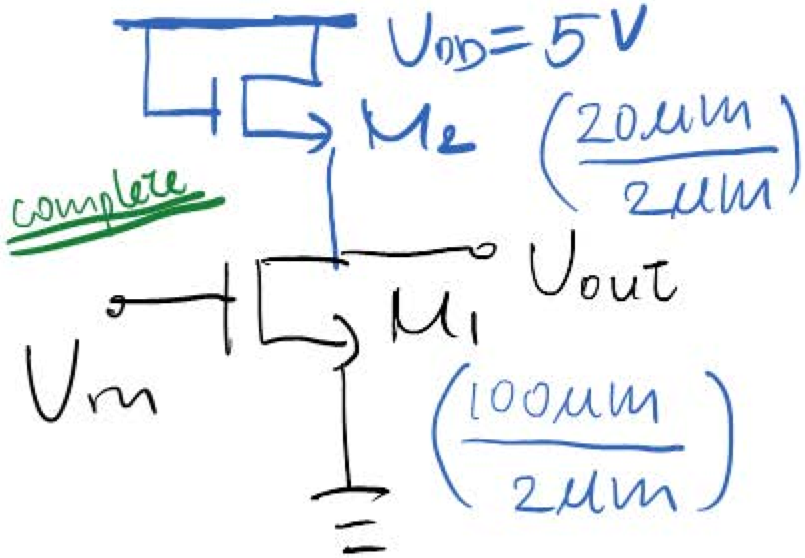
Then we make sure M_1 and M_2 in sat.

$\frac{1}{2}\mu_n C_{ox}\left(\frac{100\mu m}{2\mu m - 2L_D}\right)(0.8-0.7)^2 \cdot (1+\lambda V_{OUT})$

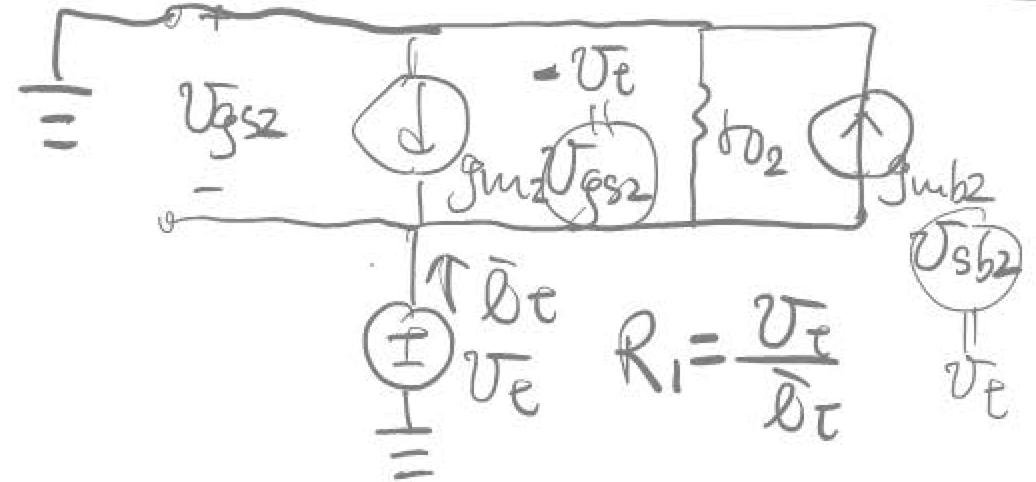
$= \frac{1}{2}\mu_n C_{ox}\left(\frac{20\mu m}{2\mu m - 2L_D}\right) \cdot (5 - V_{OUT} - V_{TH})^2 \cdot [1+\lambda(5-V_{OUT})]$

$V_{SB1} = 0 \Rightarrow V_{TH1} = 0.7$

$V_{SB2} = V_{OUT} \Rightarrow V_{TH2} > 0.7$

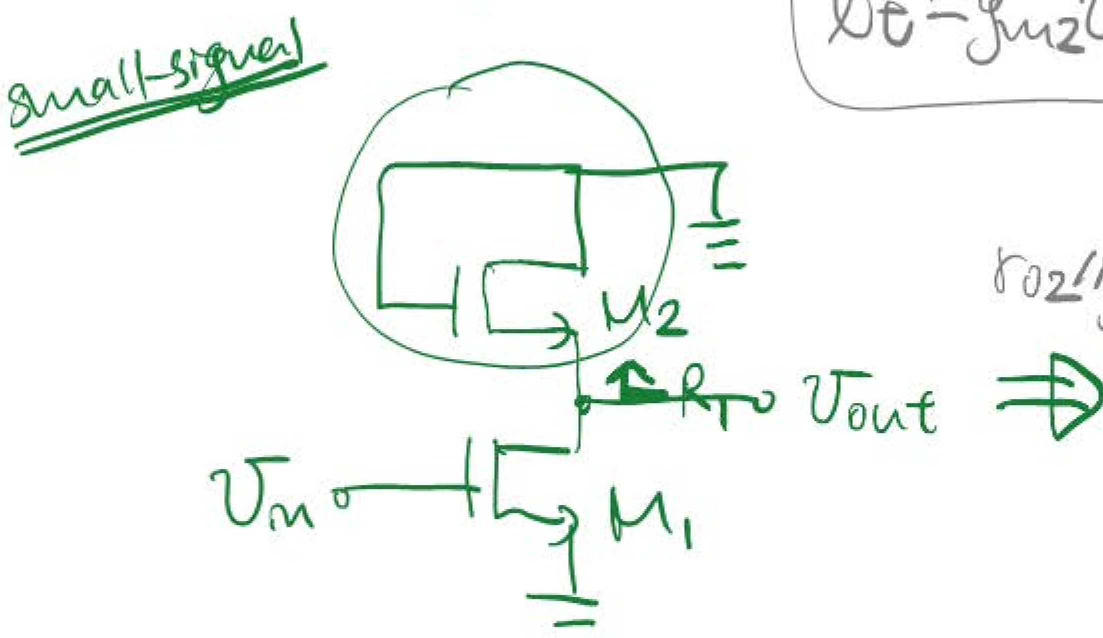


2° Find out v_{out} = ?

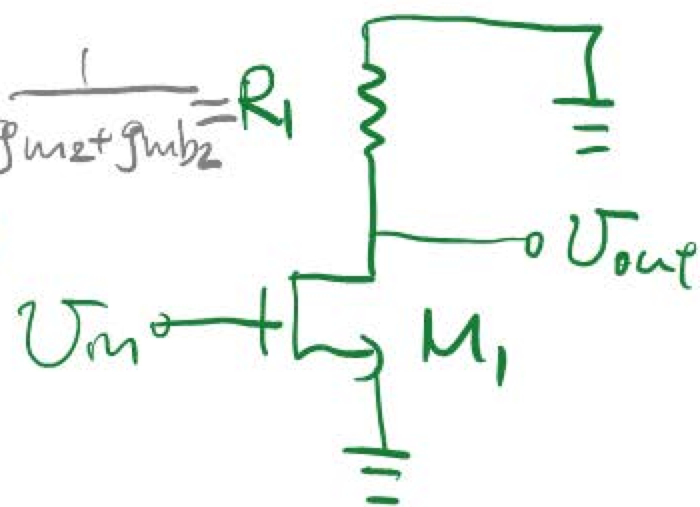


$$R_1 = \frac{v_t}{i_t}$$

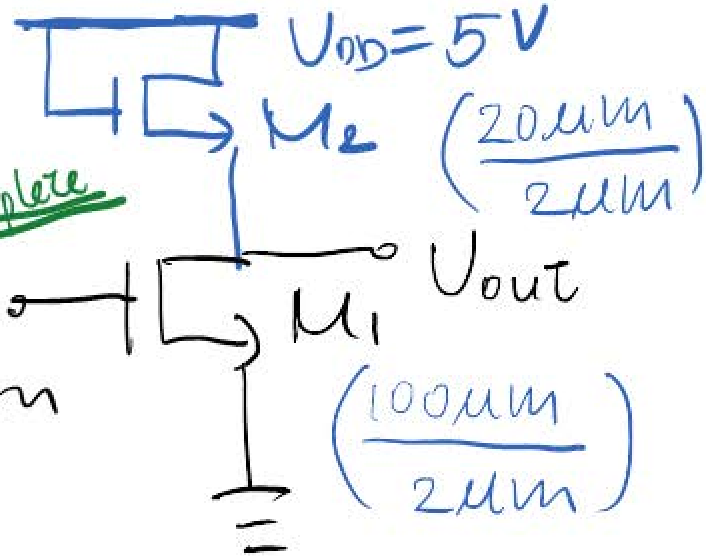
$$i_t = g_{m2}v_t + \frac{v_t}{r_{o2}} + g_{mb2}v_t$$



$$r_{o2} \,\|\, \frac{1}{g_{m2}+g_{mb2}} = R_1$$



complete



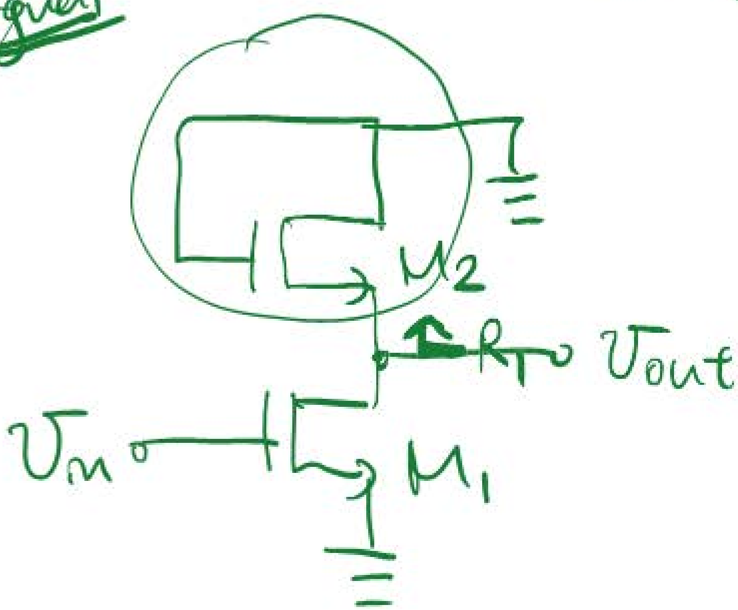
2° Find out $v_{out} = ?$

$$v_{out} = -g_{m1} v_{in} (r_{o1} // R_1)$$

$$= -g_{m1} v_{in} \left(r_{o1} // r_{o2} // \frac{1}{g_{m2}+g_{mb2}}\right)$$

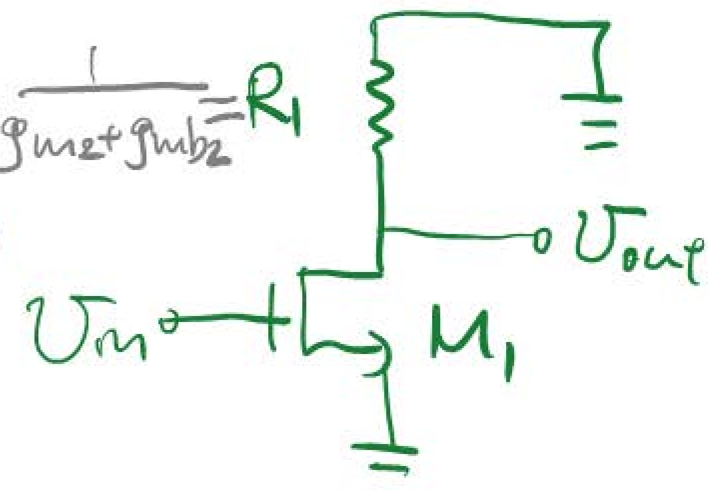
$$\cong -g_{m1} v_{in} \frac{1}{g_{m2}+g_{mb2}} \cong -v_{in} \frac{g_{m1}}{g_{m2}}$$

small-signal

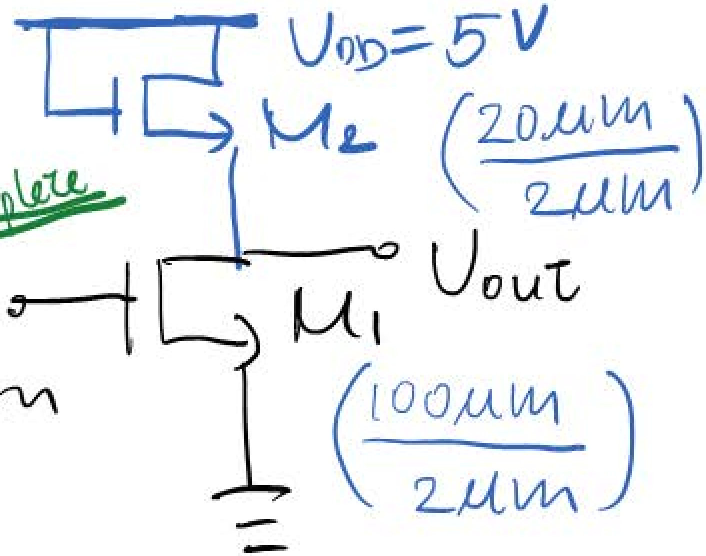


$$\Rightarrow$$

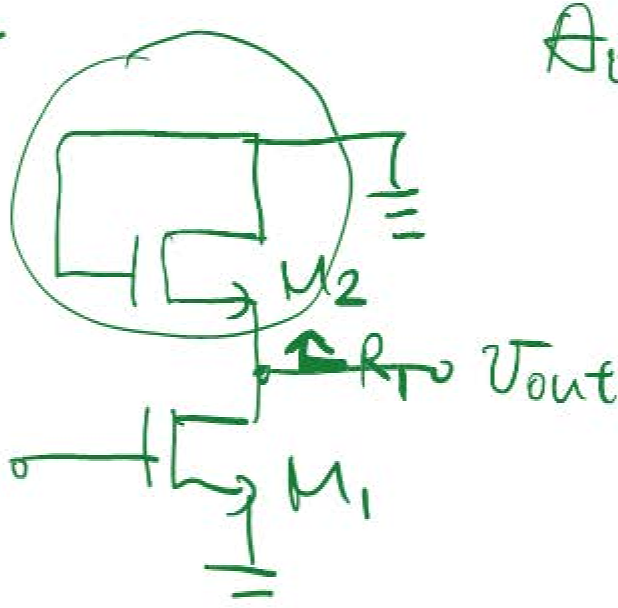
$$r_{o2} // \frac{1}{g_{m2}+g_{mb2}} = R_1$$



complete



small-signal



2° Find out v_{out} = ?

$$v_{out} = -g_{m1} v_{in} (r_{o1} // R_1)$$

$$= -g_{m1} v_{in} \left(r_{o1} // r_{o2} // \frac{1}{g_{m2}+g_{mb2}}\right)$$

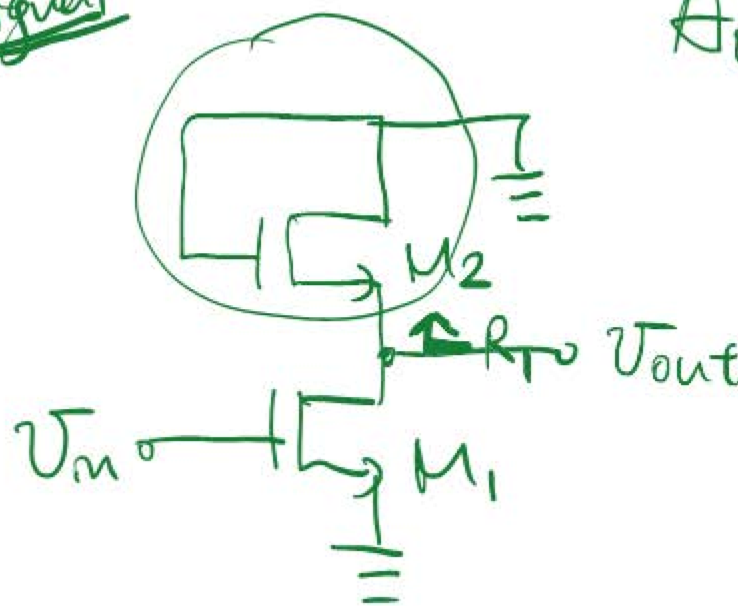
$$A_v = \frac{v_{out}}{v_{in}}$$

$$= -g_{m1} \left(r_{o1} // r_{o2} // \frac{1}{g_{m2}+g_{mb2}}\right)$$

(设 $\lambda = 0$, $r \neq 0$)

$$= -\frac{g_{m1}}{g_{m2}+g_{m2}\eta} = -\frac{g_{m1}}{g_{m2}}\left(\frac{1}{1+\eta}\right)$$

small-signal



$$A_v = \frac{v_{out}}{v_{in}}$$

$$= g_{m1}\left(r_{o1} /\!/ r_{o2} /\!/ \frac{1}{g_{m2}+g_{mb2}}\right)$$

(if $\lambda = 0$, $\gamma \neq 0$)

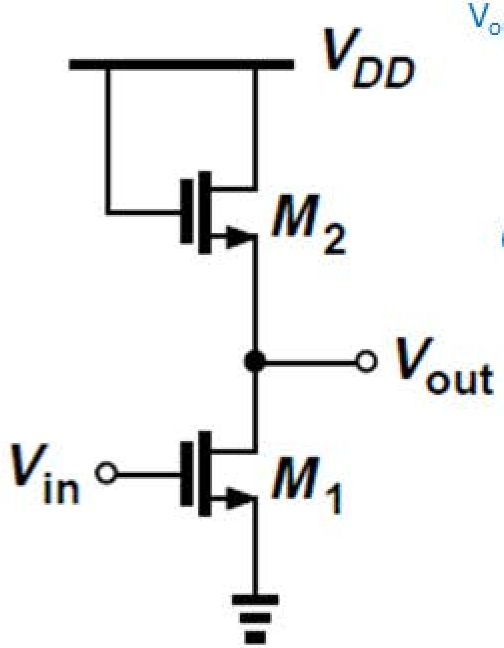
$$= -\frac{g_{m1}}{g_{m2}+g_{m2}\eta} = -\frac{g_{m1}}{g_{m2}}\left(\frac{1}{1+\eta}\right)$$

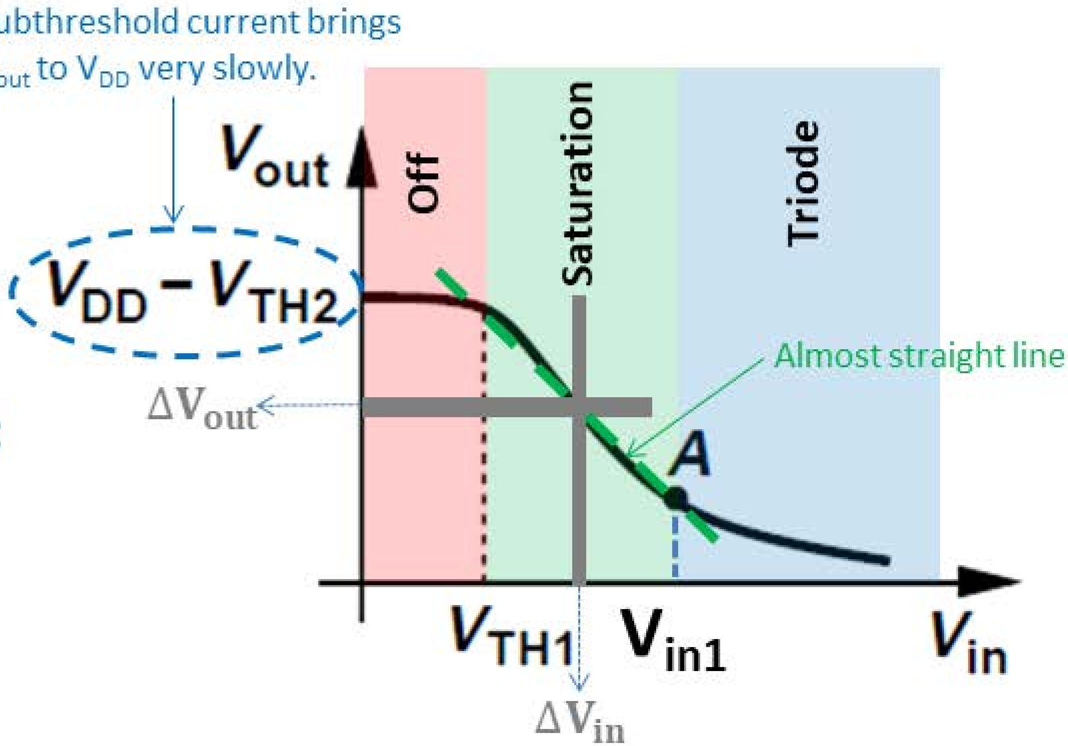
$$= -\frac{\sqrt{\cancel{2}\cancel{\mu_n}\cancel{C_{ox}}\left(W/L_{eff}\right)_1 \cancel{I_{D1}}}}{\sqrt{\cancel{2}\cancel{\mu_n}\cancel{C_{ox}}\left(W/L_{eff}\right)_2 \cancel{I_{D2}}}}\left(\frac{1}{1+\eta}\right)$$

$$= -\sqrt{\frac{\left(W/L_{eff}\right)_1}{\left(W/L_{eff}\right)_2}}\left(\frac{1}{1+\eta}\right)$$

Common-Source with Diode-Connected Load

DC Analysis $\lambda = 0 \quad \gamma \neq 0$





$$V_{out} = V_{in1} - V_{TH1}$$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in1} - V_{TH1})^2 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 [V_{DD} - (V_{in1} - V_{TH1}) - V_{TH2}]^2$$

- V_{gs} increases by ΔV_{in} → I_d increases by $\Delta V_{in} \cdot gm$ → V_{out} decreases

Common-Source with Diode-Connected Load

DC Analysis $\lambda = 0 \quad \gamma \neq 0$

- **$V_{in1} > V_{in} > V_{TH} \rightarrow M_1$ in Saturation**

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - V_{TH1})^2 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 [V_{DD} - V_{out} - V_{TH2}]^2$$

$$\sqrt{\left(\frac{W}{L}\right)_1} (V_{in} - V_{TH1}) = \sqrt{\left(\frac{W}{L}\right)_2} (V_{DD} - V_{out} - V_{TH2})$$

$$\sqrt{\left(\frac{W}{L}\right)_1} = \sqrt{\left(\frac{W}{L}\right)_2} \left(-\frac{\partial V_{out}}{\partial V_{in}} - \frac{\partial V_{TH2}}{\partial V_{in}}\right)$$

$V_{out} = V_{SB2}$

$$\sqrt{\left(\frac{W}{L}\right)_1} = \sqrt{\left(\frac{W}{L}\right)_2} \left(-\frac{\partial V_{out}}{\partial V_{in}} - \frac{\partial V_{TH2}}{\partial V_{out}}\frac{\partial V_{out}}{\partial V_{in}}\right)$$

$$= \eta = \frac{\gamma}{2\sqrt{2\Phi_F + V_{SB}}}$$

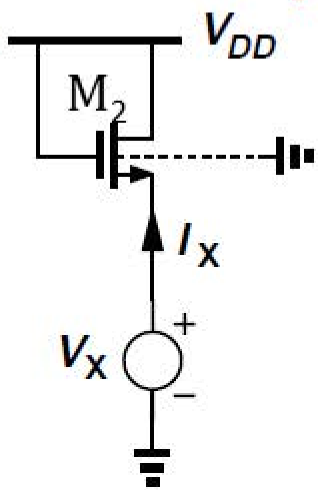
$$\boxed{A_v = \frac{\partial V_{out}}{\partial V_{in}} = -\sqrt{\frac{(W/L)_1}{(W/L)_2}}\frac{1}{1+\eta}}$$

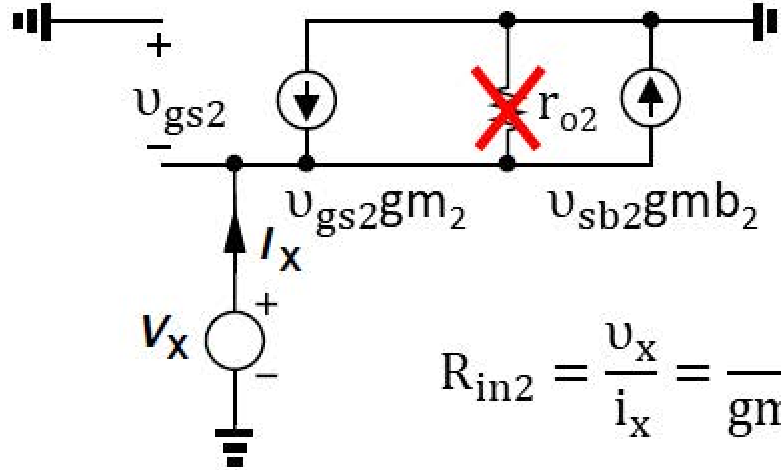
- η is a function of V_{SB}.
- A_v is almost linear for M_1 in saturation.

Common-Source with Diode-Connected Load

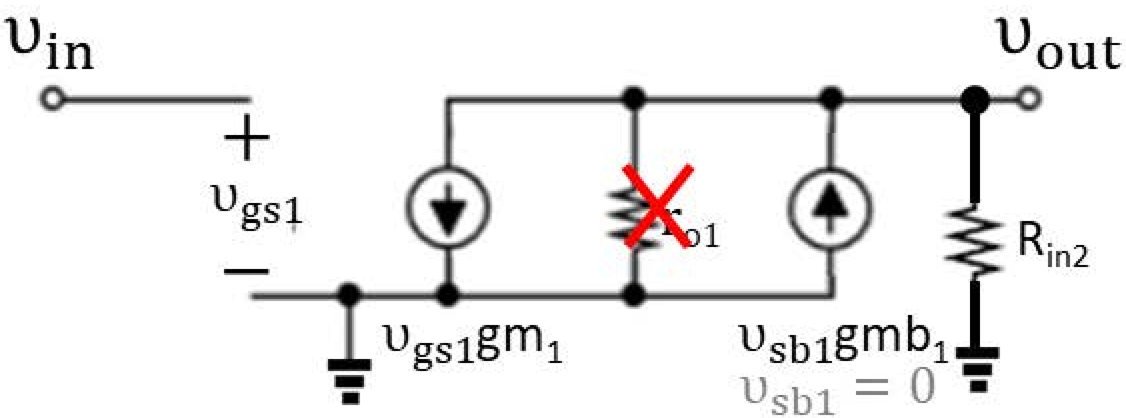
Small-signal Analysis

$\lambda = 0 \quad \gamma \neq 0$





$$R_{in2} = \frac{\upsilon_x}{i_x} = \frac{1}{gm_2 + gmb_2}$$



$$A_v = \frac{\upsilon_{out}}{\upsilon_{in}} = \frac{-gm_1}{gm_2 + gmb_2}$$

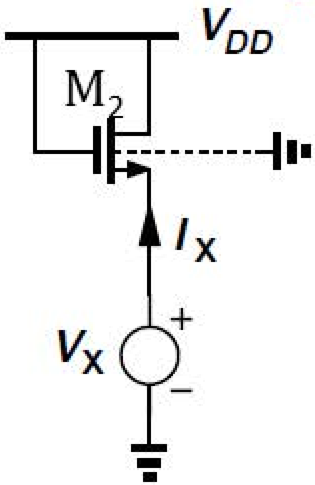
$$= -\sqrt{\frac{(W/L)_1}{(W/L)_2}} \frac{1}{1 + \eta}$$

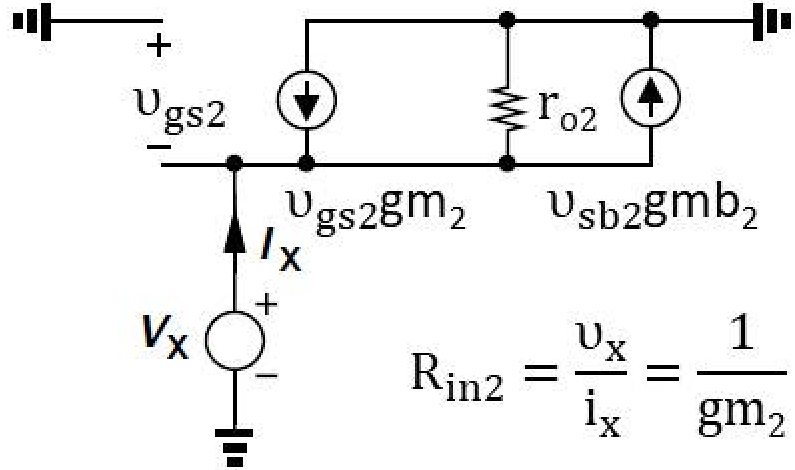
- Small-signal analysis leads to the same result as DC analysis.

Common-Source with Diode-Connected Load

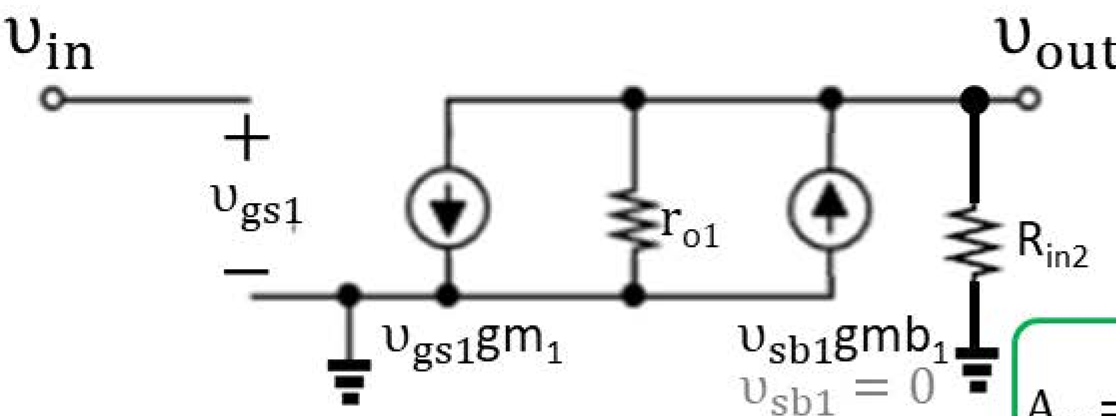
Small-signal Analysis

$\lambda \neq 0 \quad \gamma \neq 0$





$$R_{in2} = \frac{\upsilon_x}{i_x} = \frac{1}{gm_2} \parallel \frac{1}{gmb_2} \parallel r_{o2}$$

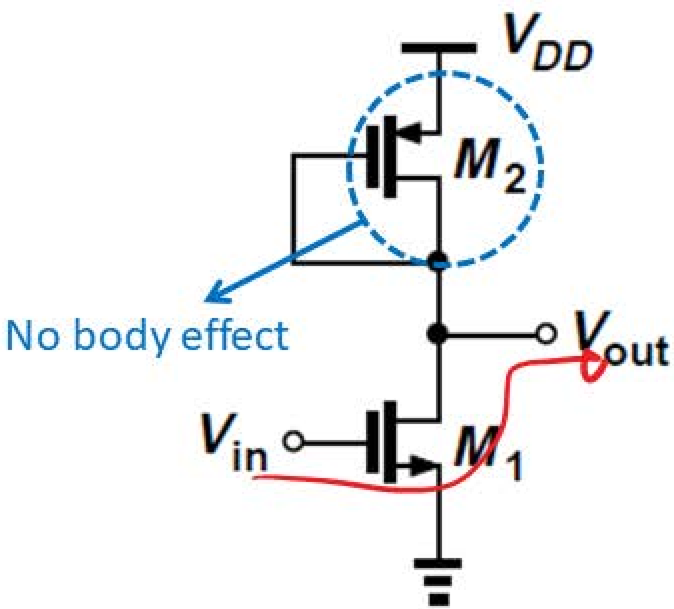


$r_o >> 1/gm$

$$A_v = \frac{\upsilon_{out}}{\upsilon_{in}}$$
$$= -gm_1 \left(\frac{1}{gm_2} \parallel \frac{1}{gmb_2} \parallel r_{o2} \parallel r_{o1} \right)$$

Common-Source with Diode-Connected Load

Small-signal Analysis $\lambda \neq 0 \quad \gamma \neq 0$



$r_o >> 1/gm$

$$A_v = \frac{\upsilon_{out}}{\upsilon_{in}}$$

$$= -gm_1\left(\frac{1}{gm_2} \parallel r_{o2} \parallel r_{o1}\right)$$

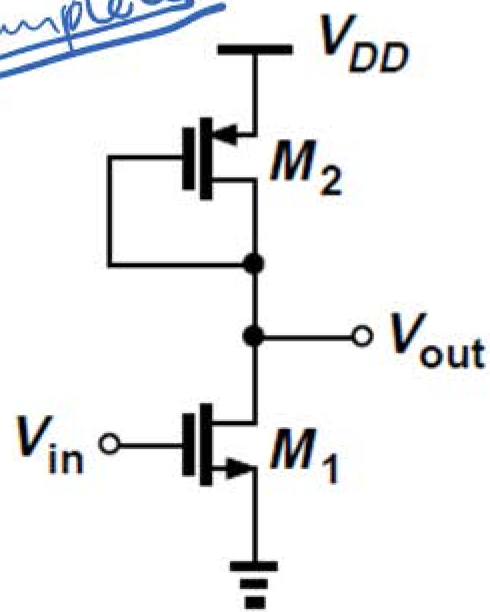
$$\approx -\frac{gm_1}{gm_2} = -\frac{\sqrt{\cancel{2}\mu_n C_{ox}(W/L_{eff})_1 I_{D1}}}{\sqrt{\cancel{2}\mu_p C_{ox}(W/L_{eff})_2 I_{D2}}}$$

$$= -\sqrt{\frac{\mu_n (W/L_{eff})_1}{\mu_p (W/L_{eff})_2}} = -\frac{\frac{2I_{D1}}{V_{GS1}-V_{TH1}}}{\frac{2I_{D2}}{V_{SG2}-|V_{TH2}|}}$$

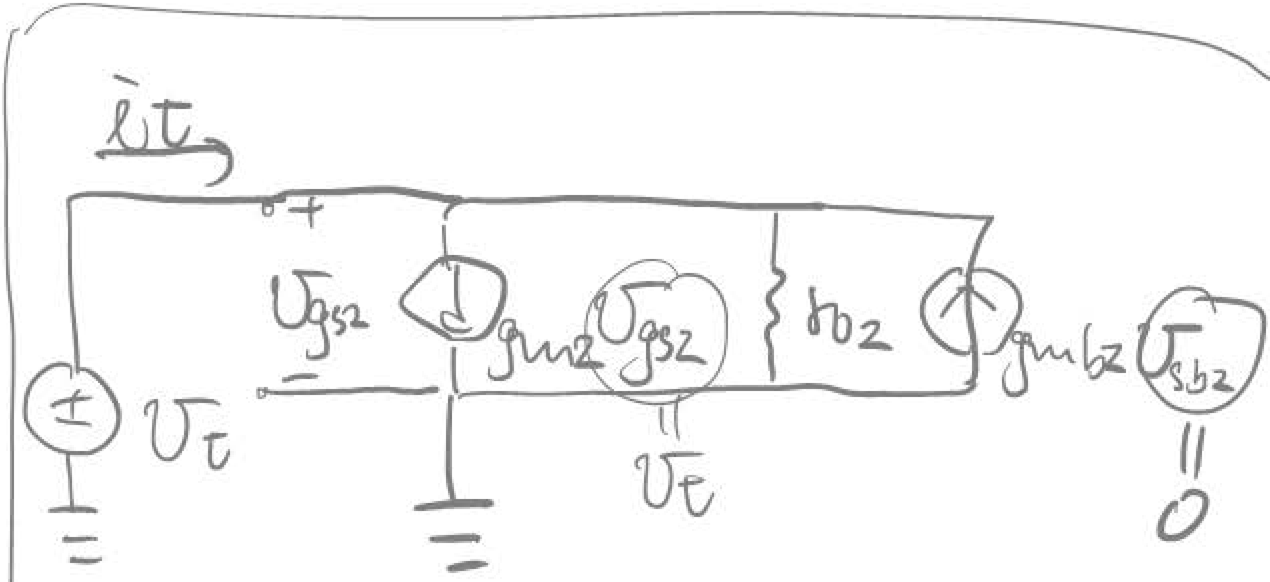
$$= -\frac{V_{SG2} - |V_{TH2}|}{V_{GS1} - V_{TH1}}$$

- For $A_v = 10$, $(W/L)_1 >> (W/L)_2$ → **Disproportionally large transistor**
- For $A_v = 10$, $(V_{SG2} - V_{TH2}) = 10 \times (V_{GS1} - V_{TH1})$ → **Limited output swing**

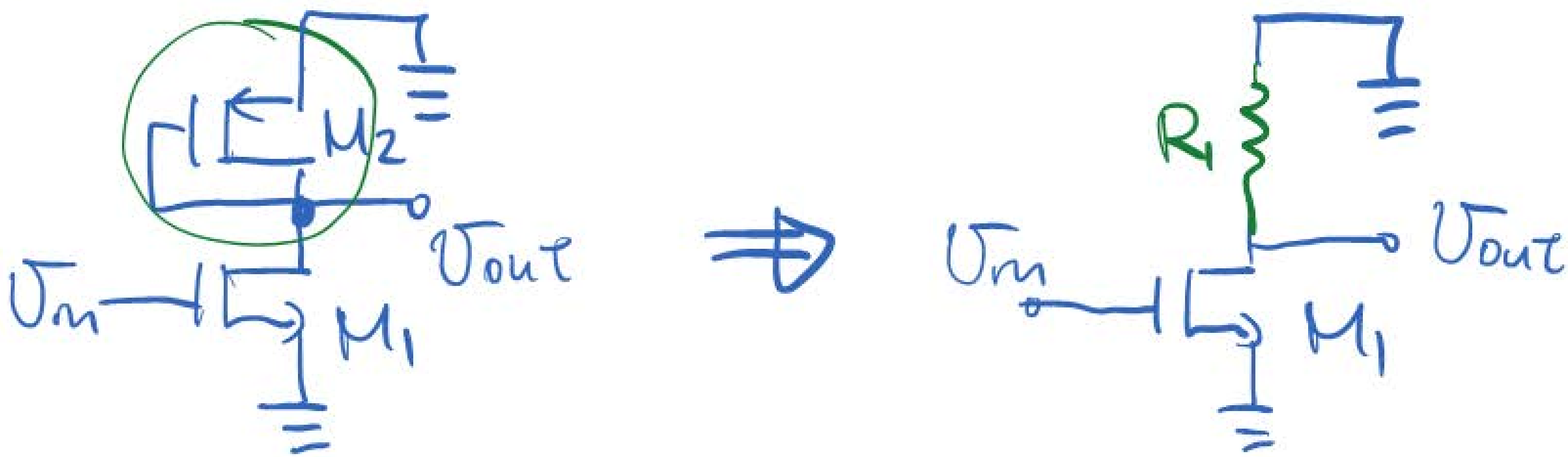
complete



small-signal



$$R_1 = \frac{v_t}{i_t} = \frac{1}{g_{m2}} \,//\, r_{o2}$$

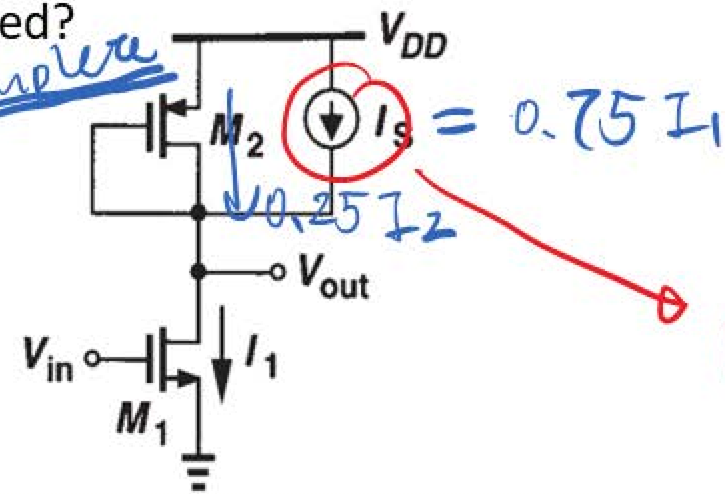


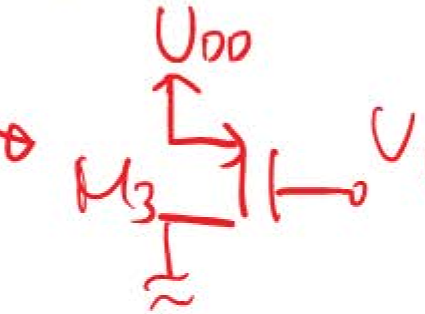
Example

$\lambda = 0$
$r \neq 0$

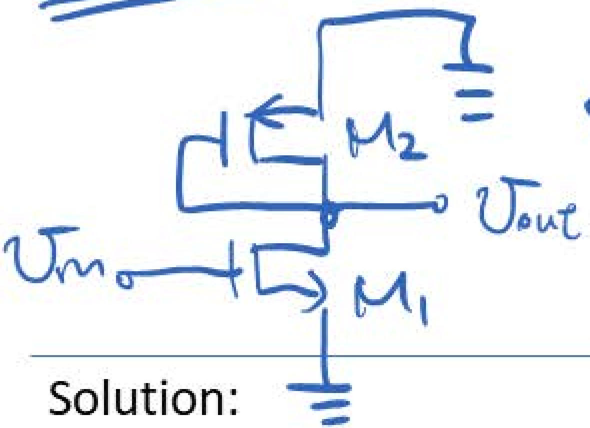
M_1 in saturation and $I_S = 0.75 \times I_1$. How do the disadvantages of CS stage with diode-connected load get improved?

complete





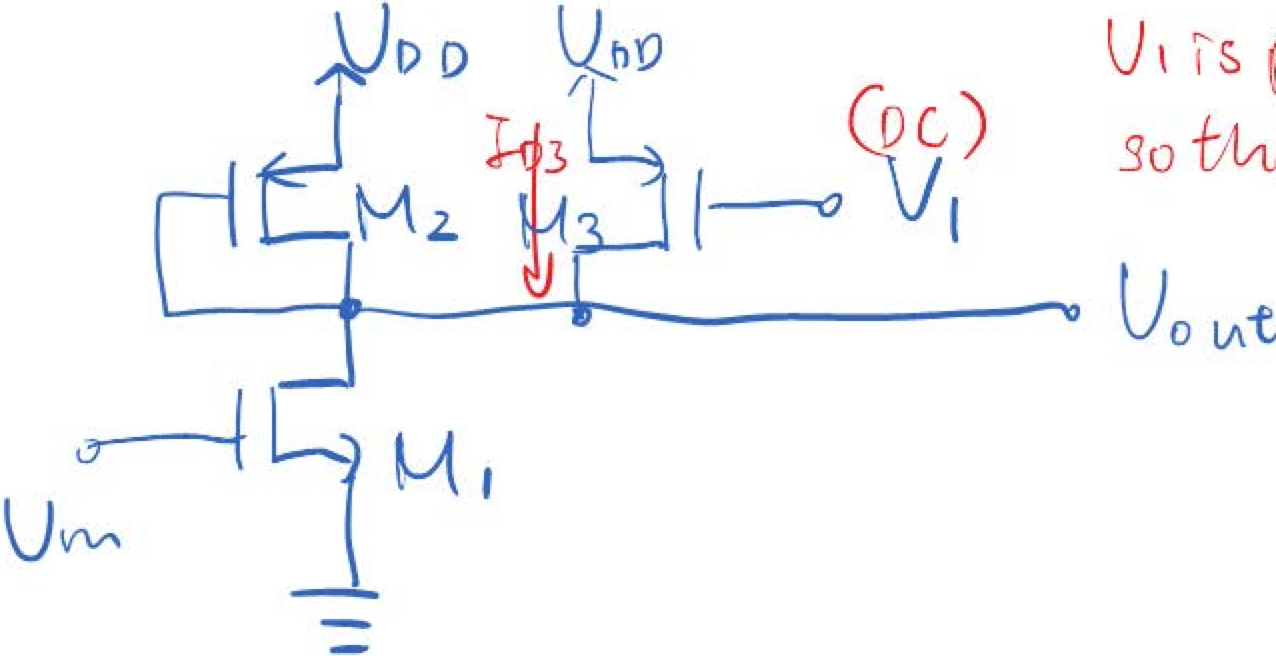
small-signal



Solution:

- Small-signal Analysis ($\lambda = 0$):

$$A_v = \frac{\upsilon_{out}}{\upsilon_{in}} = -\frac{gm_1}{gm_2} = -\frac{\sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_1 I_{D1}}}{\sqrt{2\mu_p C_{ox}\left(\frac{W}{L}\right)_2 I_{D2}}} = -\frac{\sqrt{4\mu_n\left(\frac{W}{L}\right)_1}}{\sqrt{\mu_p\left(\frac{W}{L}\right)_2}} = -\frac{4(V_{SG2} - V_{TH2})}{(V_{GS1} - V_{TH1})}$$

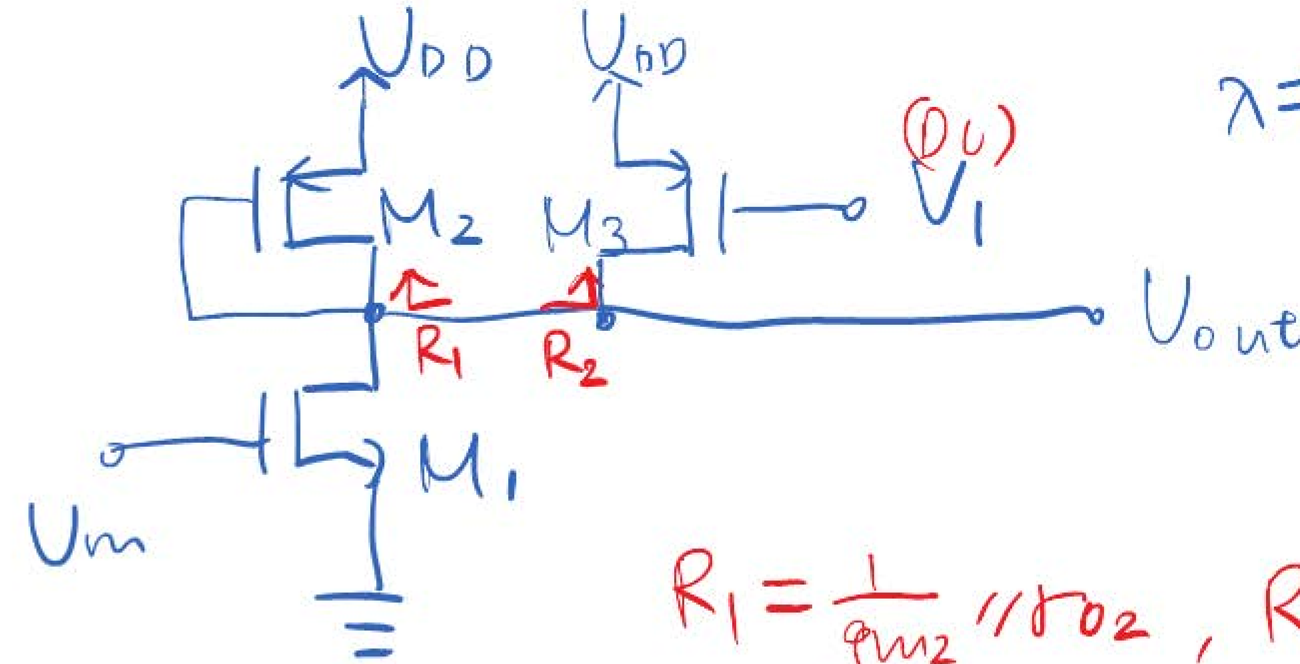


V1 is properly chosen so that M3 in sat.

$$I_{D3} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L_{eff}}\right)_3 (V_{DD}-V_1-0.8)^2\left[1+\lambda(V_{DD}-V_{OUT})\right]$$

$\Downarrow$ if $\lambda=0$, $\gamma\neq 0$

$$I_{D3} = \boxed{\text{constant}}\,(V_{DD}-0.8-V_1)^2 = \boxed{\text{constant}}$$



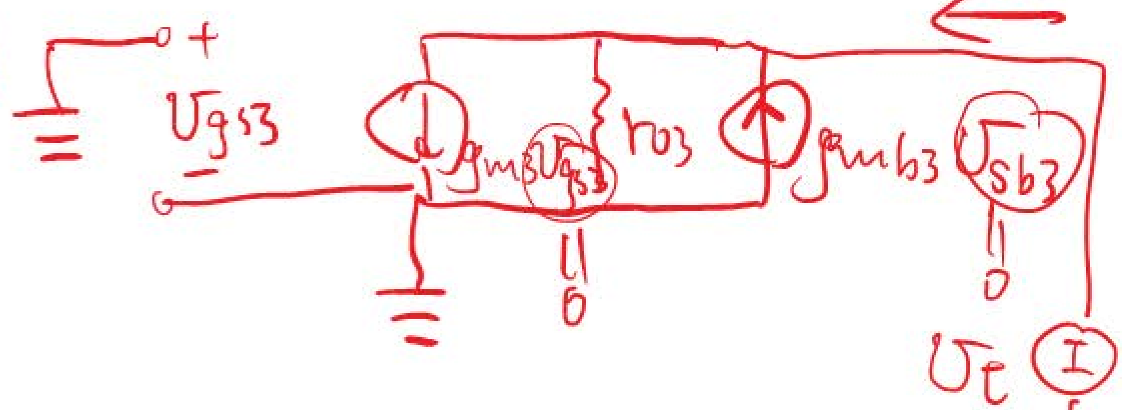
$\lambda \neq 0,\ \gamma \neq 0$

$A_v = ?$

$R_1 = \frac{1}{g_{m2}} // r_{o2}$, $R_2 = r_{o3}$

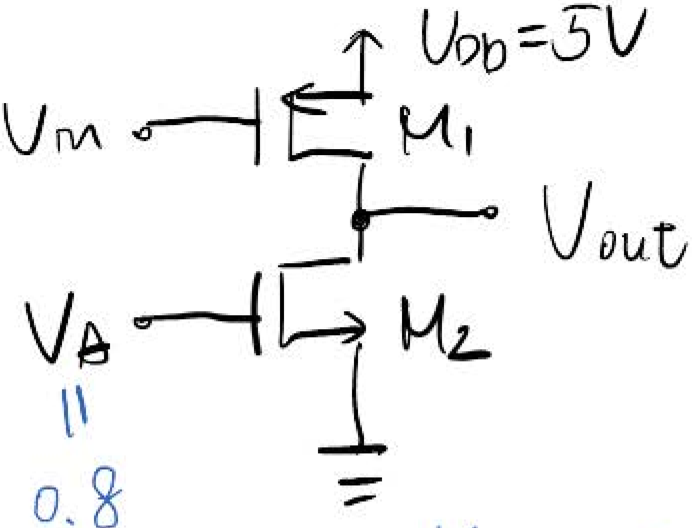
$A_v = -g_{m1}(r_{o1} // R_1 // R_2)$

$= -g_{m1} \cdot (r_{o1} // \frac{1}{g_{m2}} // r_{o2} // r_{o3})$



$R_2 = \frac{v_t}{i_t} = r_{o3}$

Common-Source with Current-Source Load



1° Find out $V_{OUT} = ?$

Then we make sure M_1 and M_2 in sat.

$V_{BS1} = 0 \Rightarrow |V_{TH1}| = 0.8$

$V_{SB2} = 0 \Rightarrow V_{TH2} = 0.7$

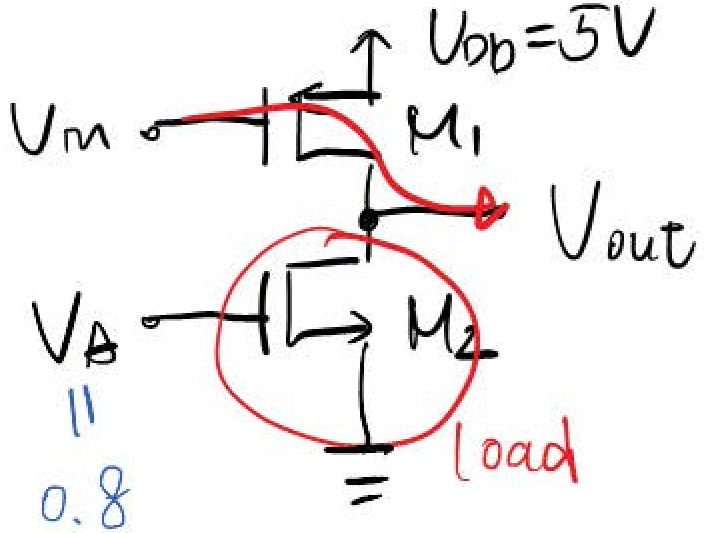
$\lambda \neq 0, \ \gamma \neq 0$

$V_{in} = 4.1 + 0.001 \sin(2\pi 100 t)$

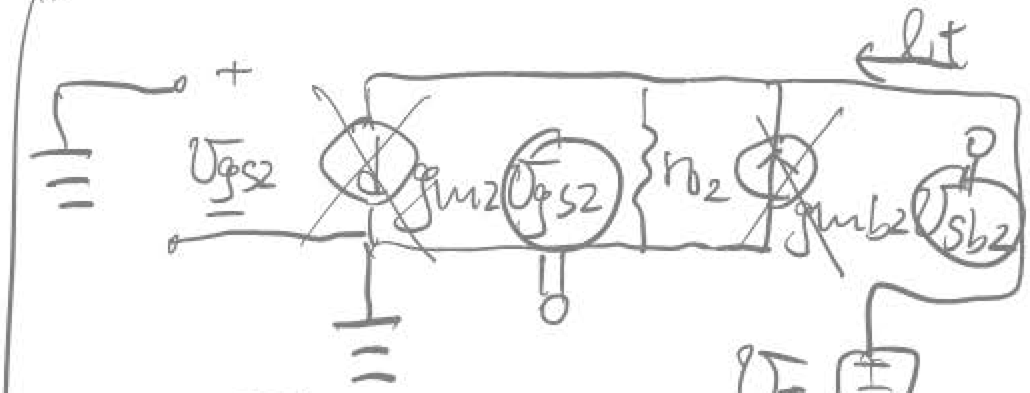
$V_{out} = V_{OUT} + v_{out} = ?$

$$\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L_{eff}}\right)_1 (5-4.1-0.8)^2 \cdot [1+\lambda(5-V_{OUT})]$$

$$= \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L_{eff}}\right)_2 (0.8-0.7)^2 [1+\lambda V_{OUT}]$$

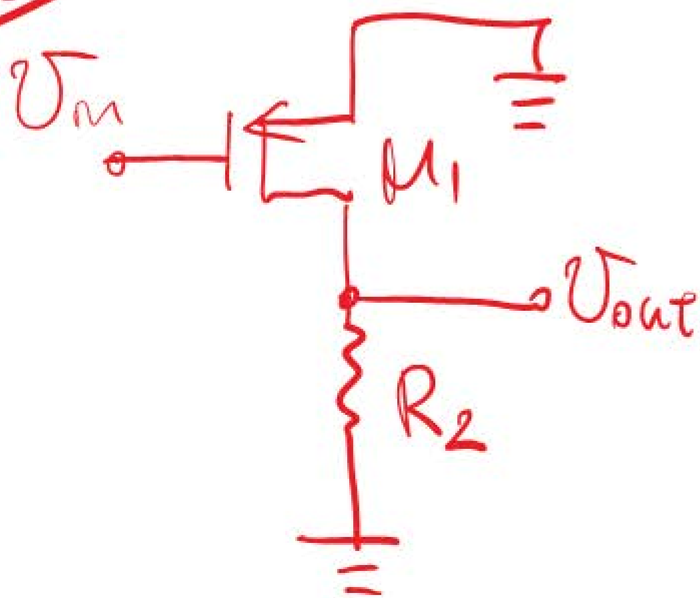


2° Find out $v_{out} = ?$



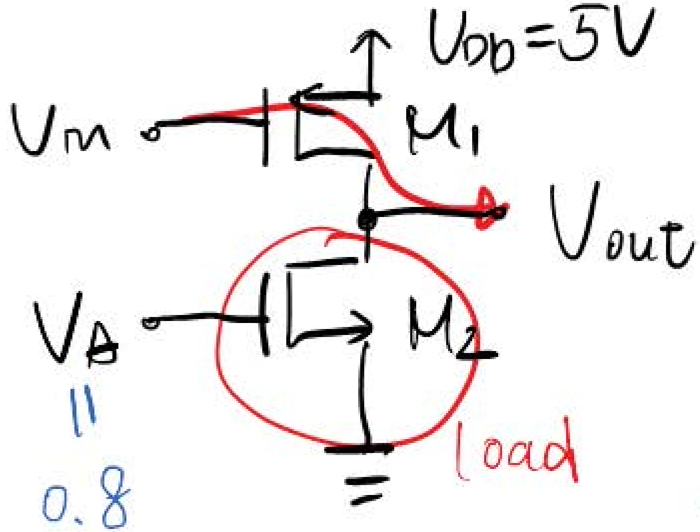
$$R_2 = v_t / i_t = r_{o2}$$

small signal



$$A_v = \frac{v_{out}}{v_{in}} = -g_{m1}(r_{o1} // R_2)$$

$$= -g_{m1}(r_{o1} // r_{o2})$$



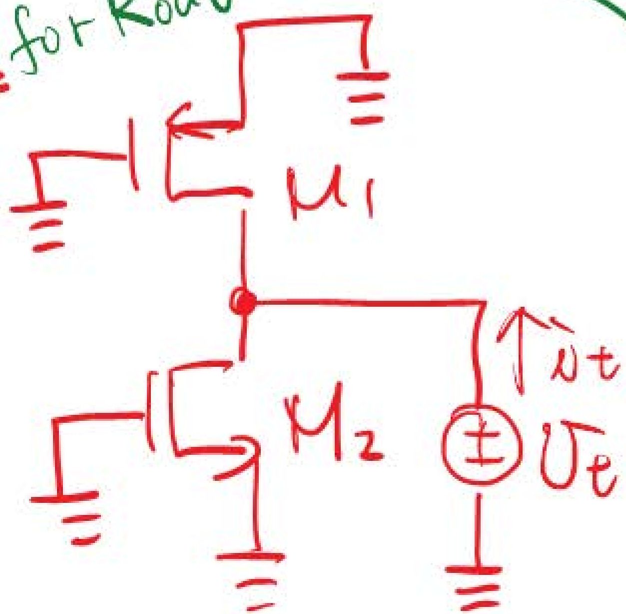
3° Find out $R_{in} = ?$

$R_{in} = \infty$

4° Find out $R_{out} = ?$

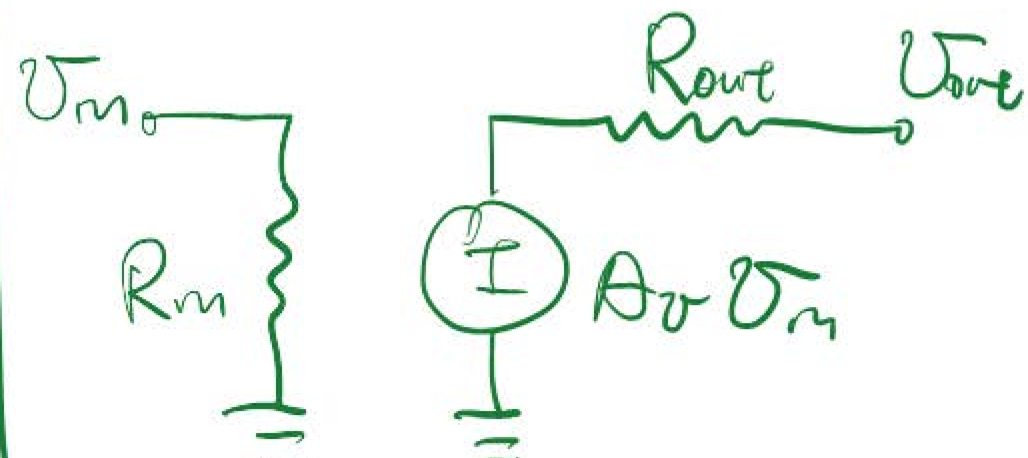
$R_{out} = r_{o1} // r_{o2}$

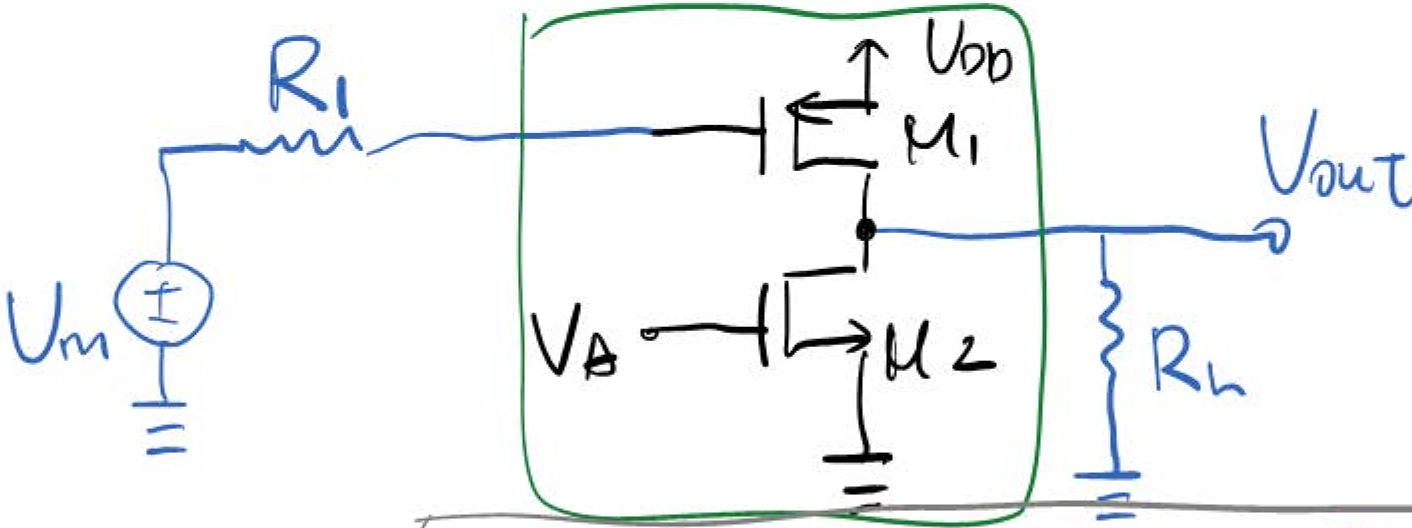
small signal for Rout



$R_{out} = v_t / i_t$

Equivalent Small Signal Circuit

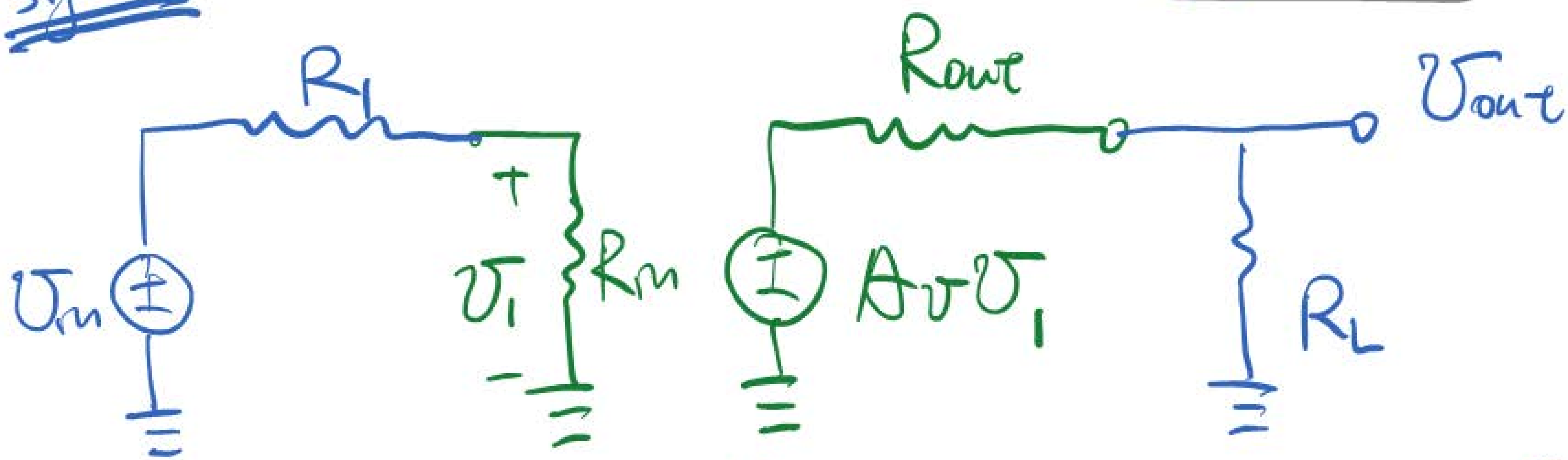




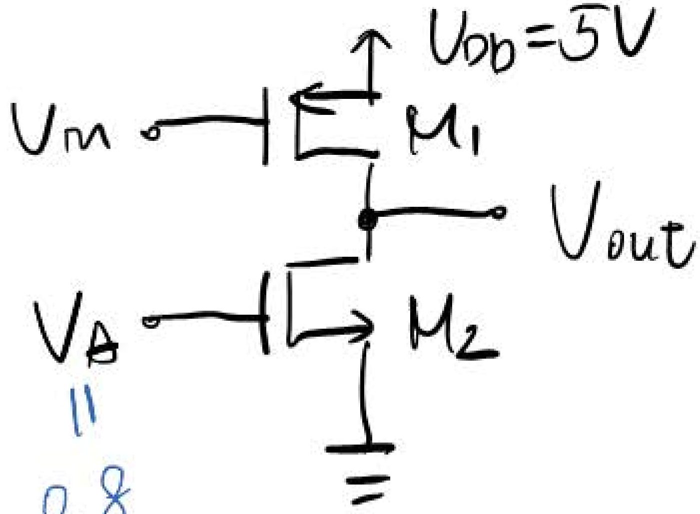
$A_v = ?$

$$\boxed{A_v = -g_{m1}(r_{o1} // r_{o2}) \frac{R_L}{R_{out}+R_L}}$$

small signal



$R_{in}=\infty$, $R_{out}=r_{o1}//r_{o2}$, $A_v=-g_{m1}(r_{o1}//r_{o2})$



$V_{BS1} = 0 \Rightarrow |V_{TH1}| = 0.8$

$V_{SB2} = 0 \Rightarrow V_{TH2} = 0.7$

$\lambda \neq 0,\ \gamma \neq 0$

$V_{in} = 4.1 + 0.001 \sin(2\pi 100 t)$

$V_{out} = V_{OUT} + v_{out} = ?$

5° Find out available output swing range.

0.1 ~ 4.9

$$V_{out,max} = 5 - (5 - 4.1 - 0.8)$$

$$= 4.9$$

$$= V_{DD} - \boxed{(V_{SG1} - |V_{TH1}|)}$$

overdrive of M_1

$$V_{out,min} = 0.8 - 0.7$$

$$= \boxed{V_{GS2} - V_{TH2}}$$

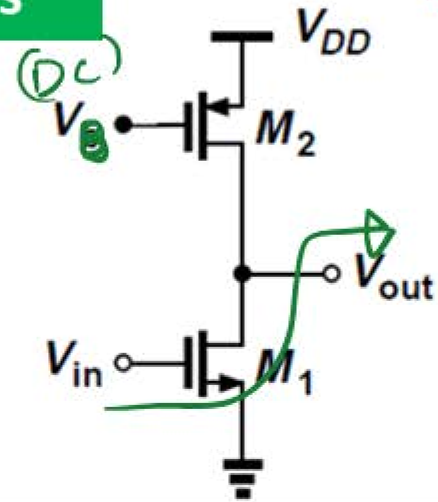
$$= 0.1$$

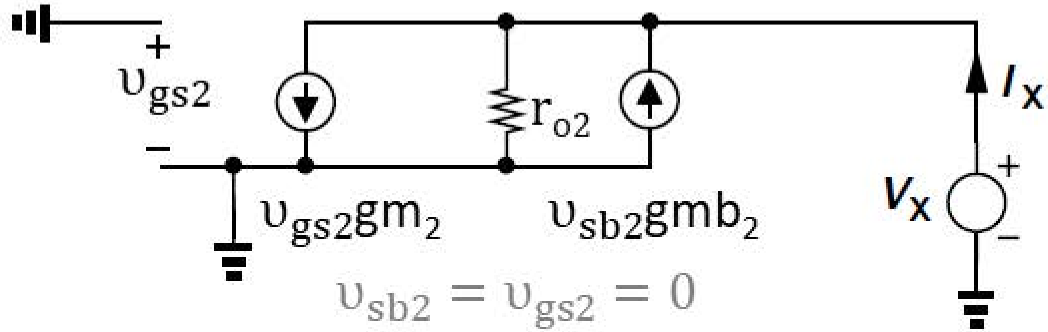
overdrive of M_2.

Common-Source with Current-Source Load

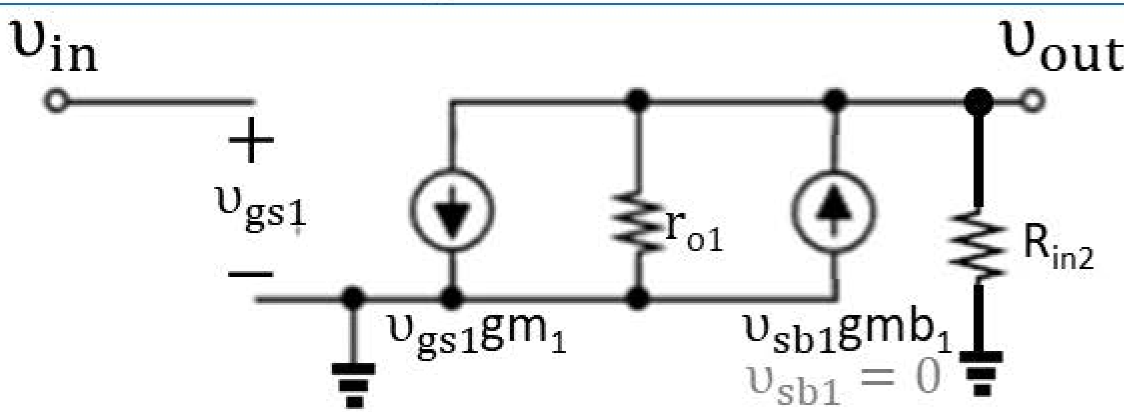
Small-signal Analysis

$\lambda \neq 0 \quad \gamma \neq 0$



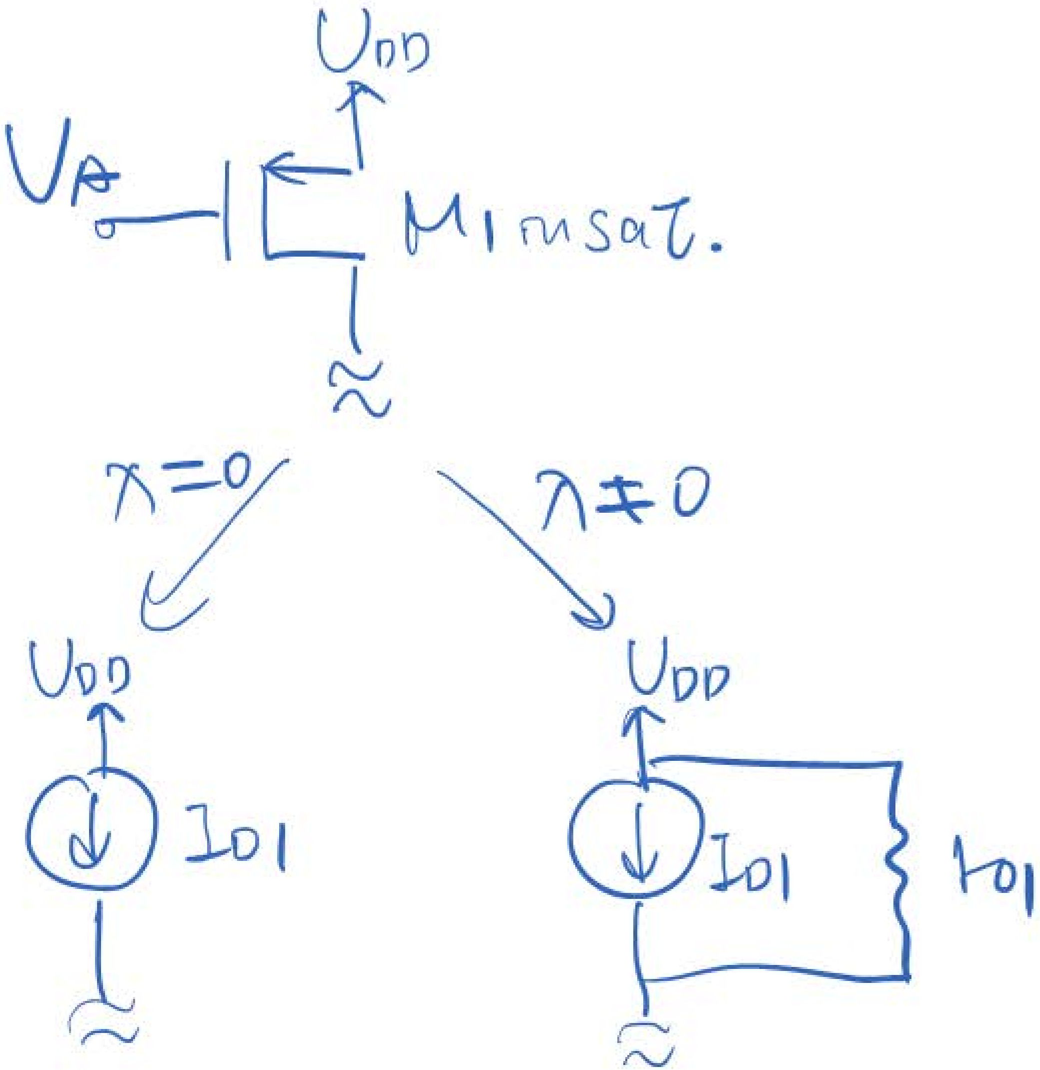


$$R_{in2} = \frac{\upsilon_x}{i_x} = r_{o2}$$



$$A_v = \frac{\upsilon_{out}}{\upsilon_{in}} = -gm_1(r_{o2} \parallel r_{o1})$$

- To achieve high A_v, the output swing is severely limited in the CS stages with resistive load and diode-connected load.
- Here $V_{out,\,max} = V_{DD} - (V_{SG2} - |V_{TH2}|)$, which can be quite close to V_{DD}.

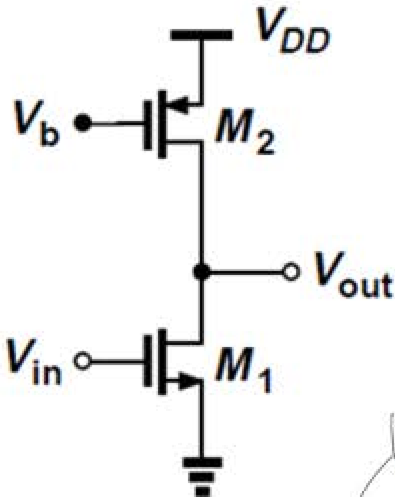


V_{DD}
V_B
M_1 in sat.
$\lambda = 0$
$\lambda \neq 0$
V_{DD}
I_{D1}
V_{DD}
I_{D1}
r_{o1}

Common-Source with Current-Source Load

Small-signal Analysis

$\lambda \neq 0 \quad \gamma \neq 0$



$$A_v = \frac{\upsilon_{out}}{\upsilon_{in}}$$
$$= -gm_1(r_{o2} \parallel r_{o1})$$

$$r_o \approx \frac{1}{\lambda I_D} \quad \lambda \propto \frac{1}{L}$$

high

$$gm_1 = \mu_n C_{ox} \left(\frac{W}{L_{eff}}\right) \boxed{(V_{GS} - V_{TH})}$$

small

large

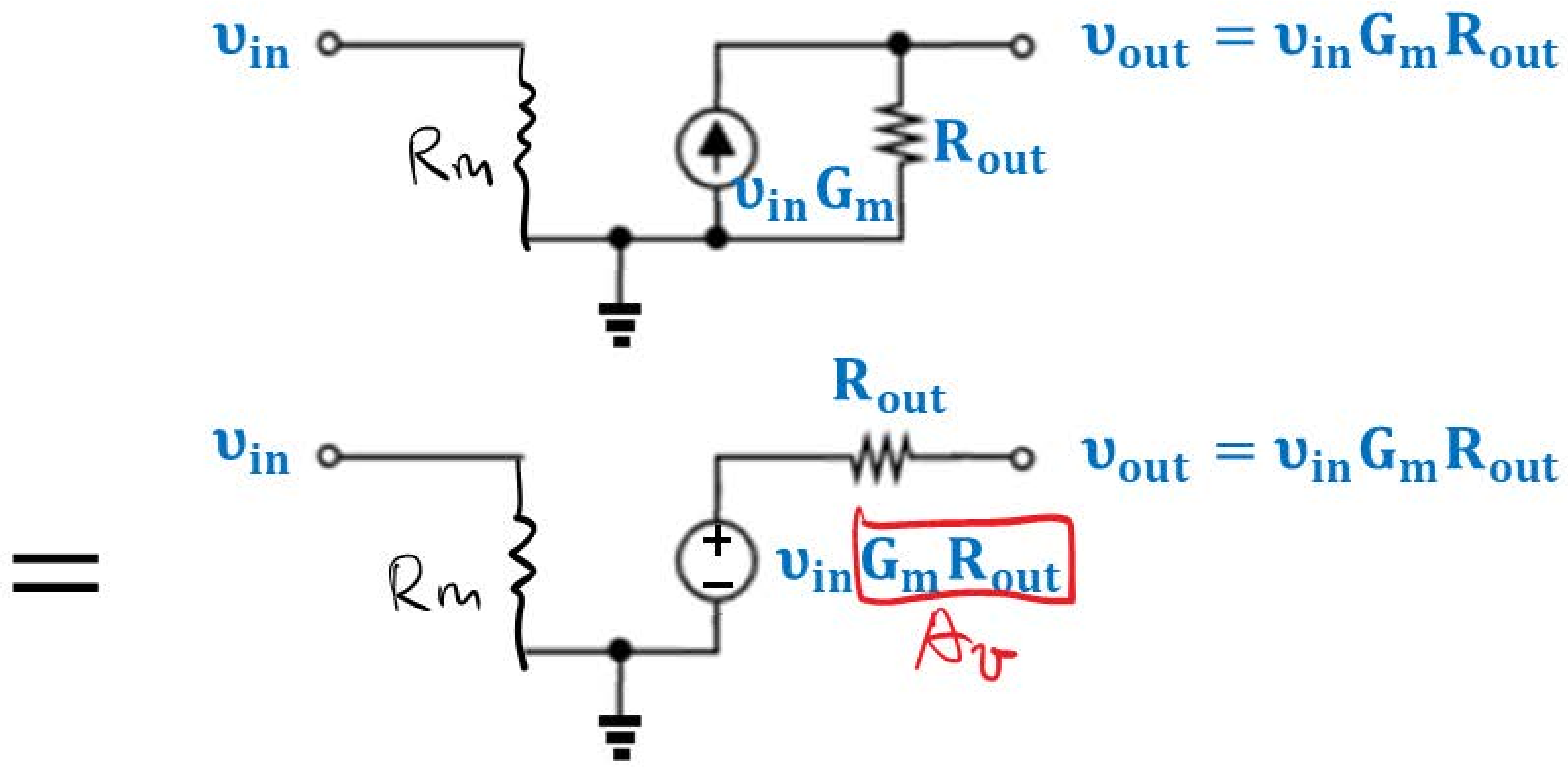
$$(1 + \lambda V_{DS})$$

- $V_{out,\,max} = V_{DD} - (V_{SG2} - |V_{TH2}|)$
- $V_{out,\,min} = (V_{GS1} - V_{TH1})$
- For high gm_1 and small $(V_{GS1} - V_{TH1})$, W of M_1 needs to be large.
- For high r_{o1} and r_{o2}, L of M_1 and M_2 need to be large and L of M_1 and M_2 needs to be increased proportionally. The cost is the **large parasitic drain junction capacitance** at the output.

Degeneration

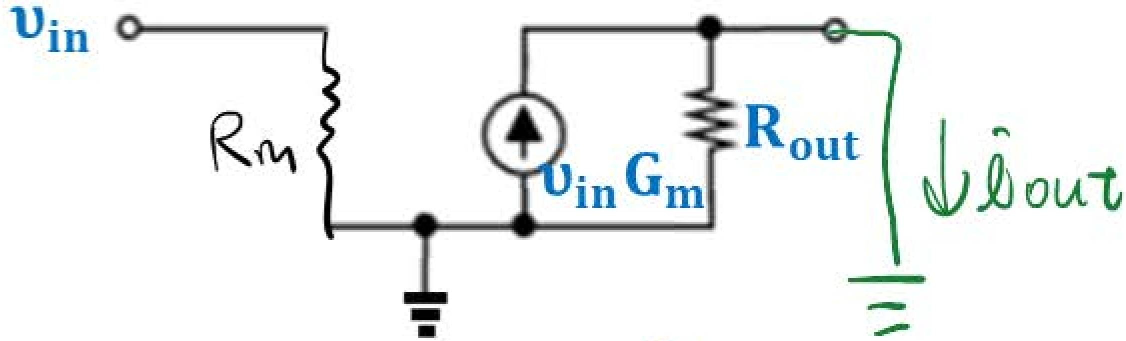
Common-Source with Source Degradation

Amplifier Equivalent Circuit



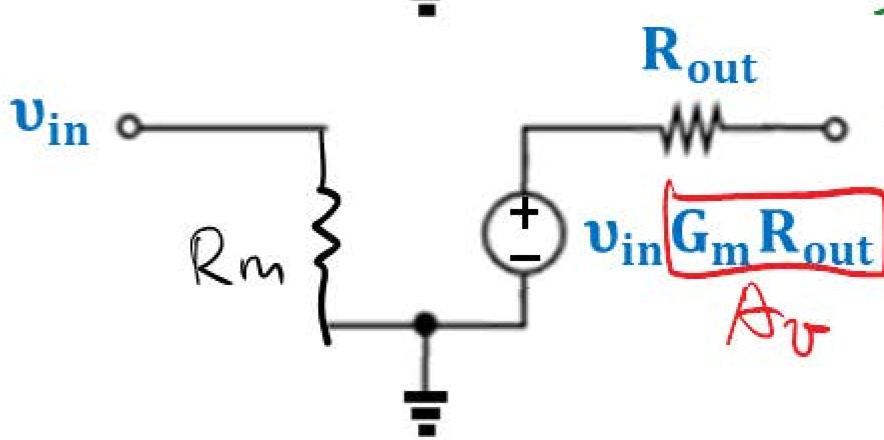
- How to calculate G_m? υ_{out} shorted to ground. $\mathbf{G_m = i_{out}/\upsilon_{in}}$
- How to calculate R_{out}? υ_{in} shorted to ground and υ_{out} connected to υ_{test}. $\mathbf{R_{out} = \upsilon_{test}/i_{test}}$

Amplifier Equivalent Circuit



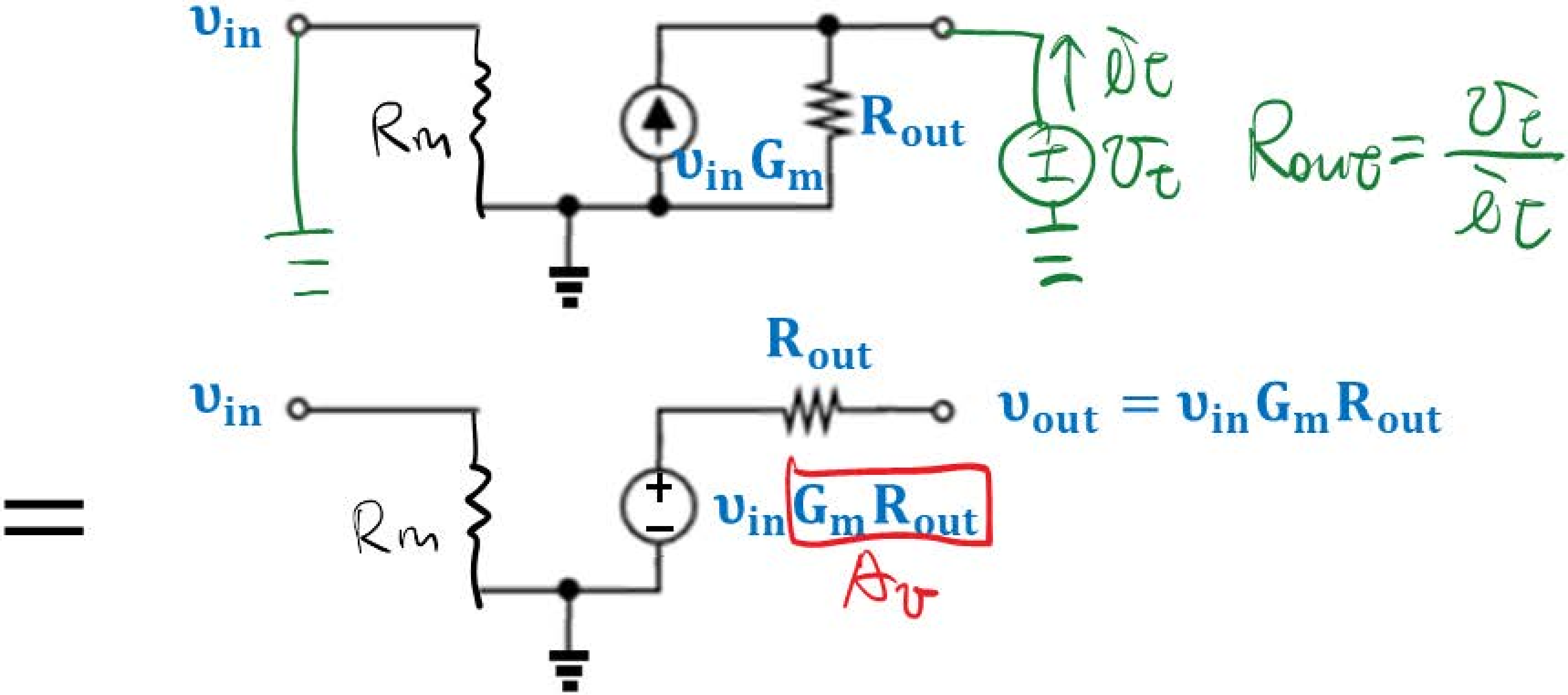
$G_m = \frac{i_{out}}{v_{in}}$

$=$

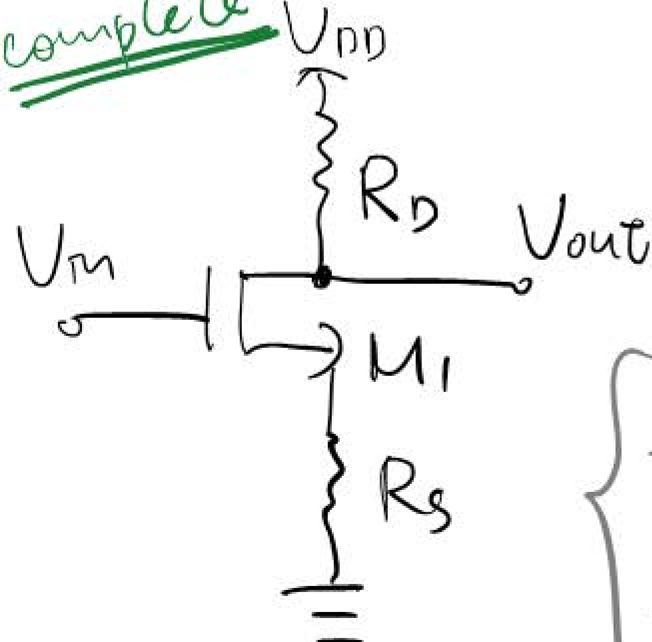


$v_{out} = v_{in} G_m R_{out}$

Amplifier Equivalent Circuit



complete



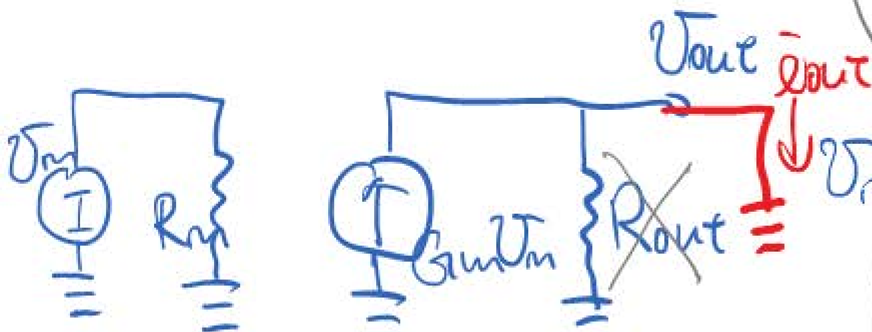
$\lambda \neq 0,\ \gamma \neq 0$

$A_v = ?$

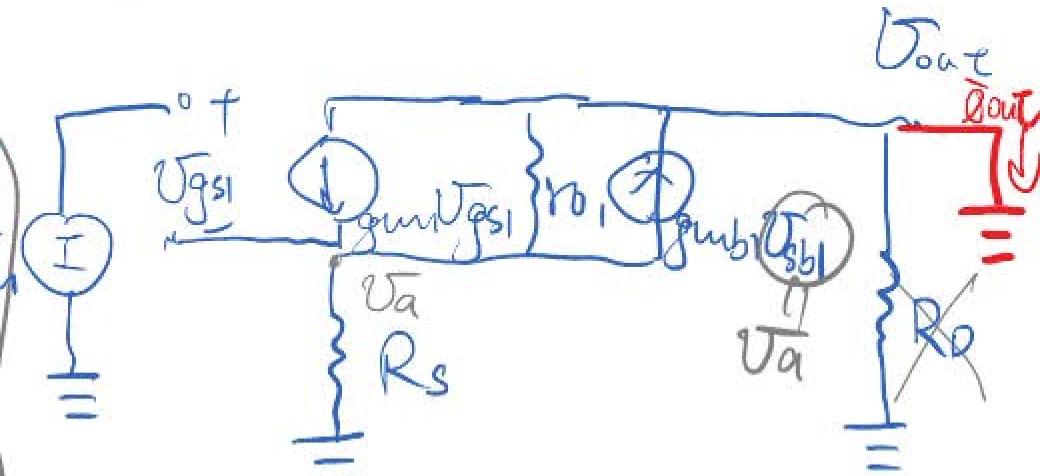
$G_m = i_{out} / v_{in}$

$$\begin{cases} \dfrac{v_a}{R_S} + (v_a - v_{in}) g_{m1} + \dfrac{v_a}{r_{o1}} + g_{mb1} v_a = 0 \\ -\dfrac{v_a}{R_S} = i_{out} \end{cases}$$

Equivalent small signal



small signal

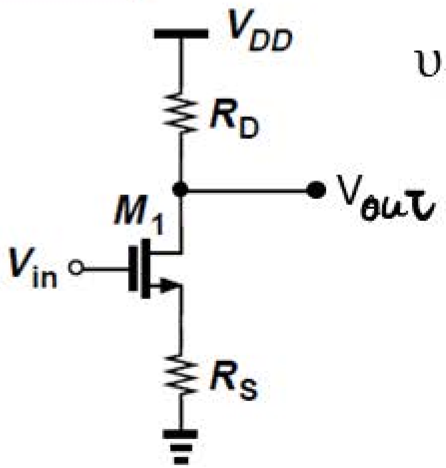


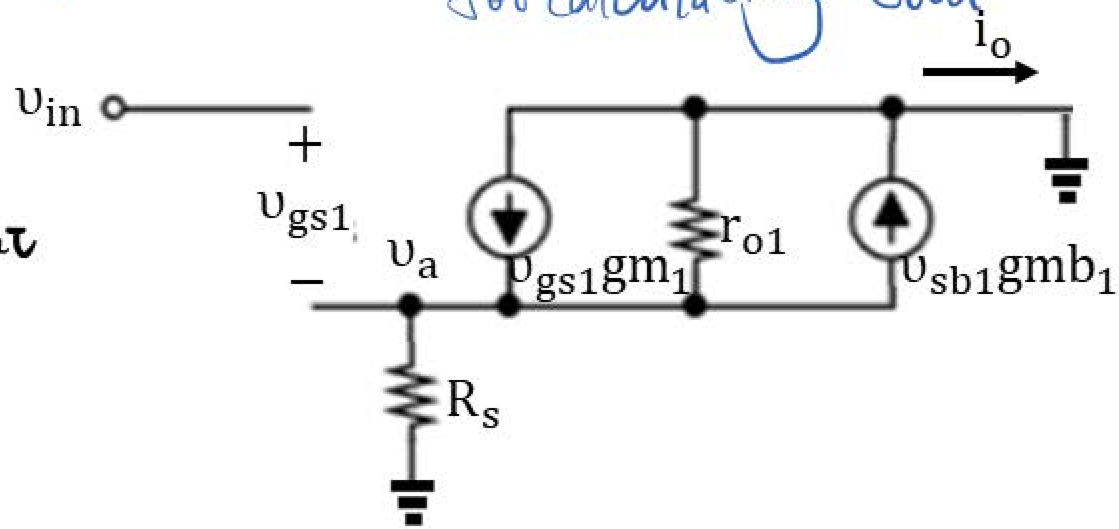
Common-Source with Source Degradation

Small-signal Analysis

$\lambda \neq 0 \quad \gamma \neq 0$

small signal circuit for calculating Gm





$$\begin{cases} i_o = \dfrac{-\upsilon_a}{R_S} \\ (\upsilon_{in} - \upsilon_a)gm_1 + i_o = \dfrac{\upsilon_a}{r_{o1}} + \upsilon_a gmb_1 \end{cases}$$

$$G_m = \frac{i_o}{\upsilon_{in}} = \frac{-gm_1 r_{o1}}{R_S + r_{o1} + \boxed{(gm_1 + gmb_1)r_{o1}}R_S} \approx -\frac{1}{R_S}$$

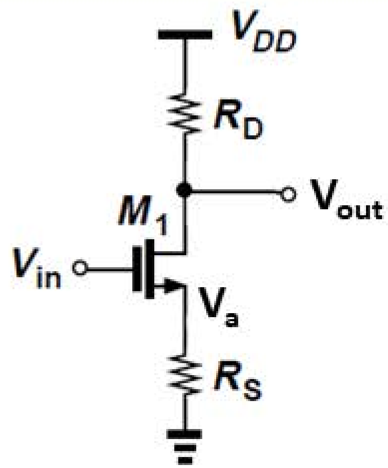
intrinsic gain of M1

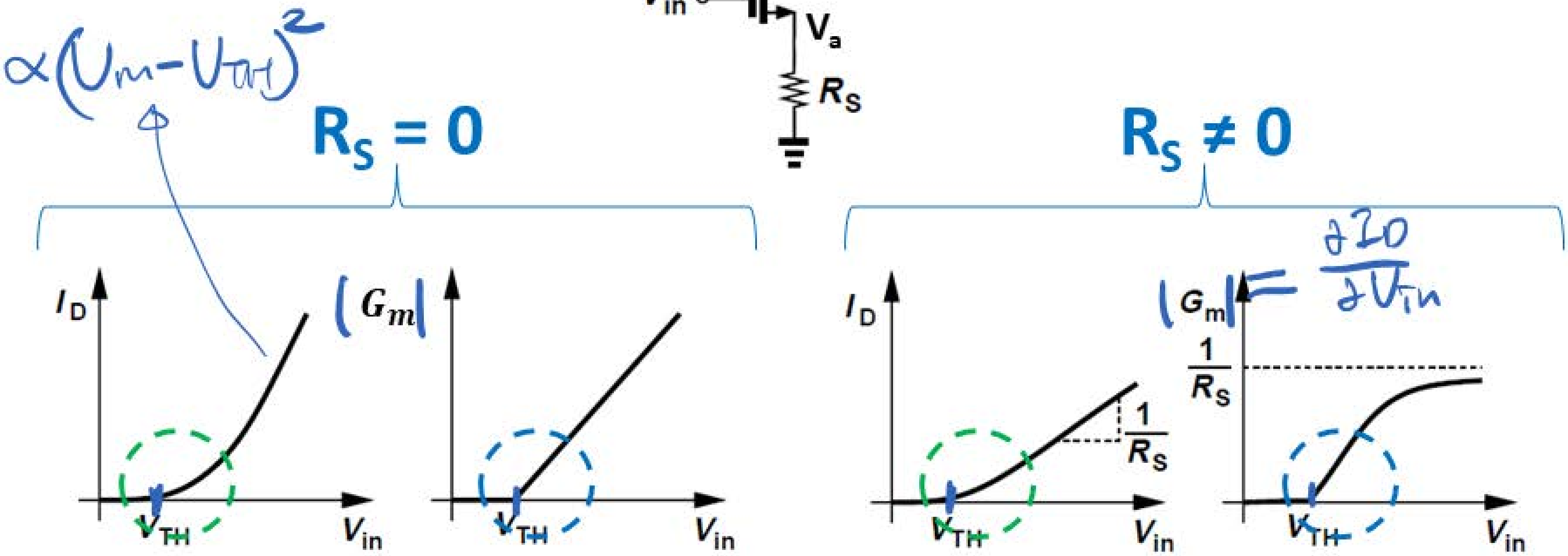
if gmb1 << gm1

If $(gm_1 + gmb_1)r_{o1}R_S$ >> r_{o1} and R_S

Common-Source with Source Degradation

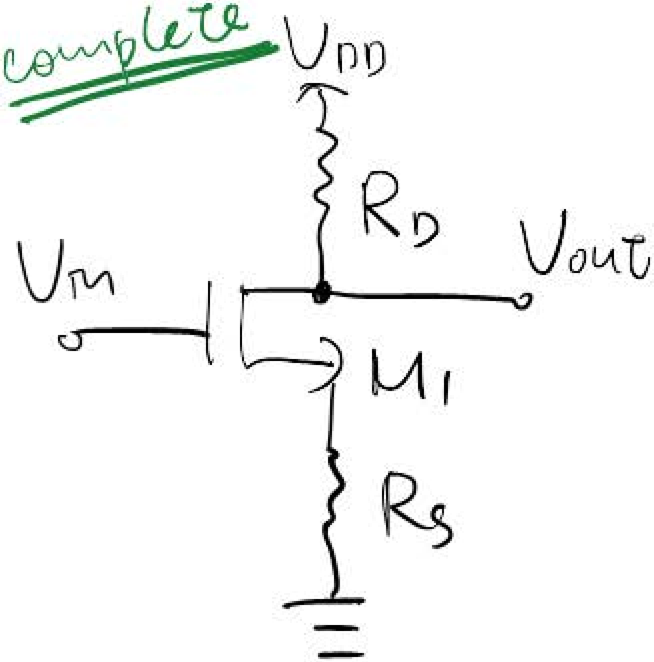
DC Analysis





- At low V_{in} (gm small), turn-on behavior of $R_S \neq 0$ is similar to that of $R_S = 0$.
- At large V_{in} (gm large), the effect of R_s, i.e. degradation, becomes more significant.
- V_{in} = 0 V → M_1 off, no current flowing → V_a = 0 V and $V_{out} = V_{DD}$

complete

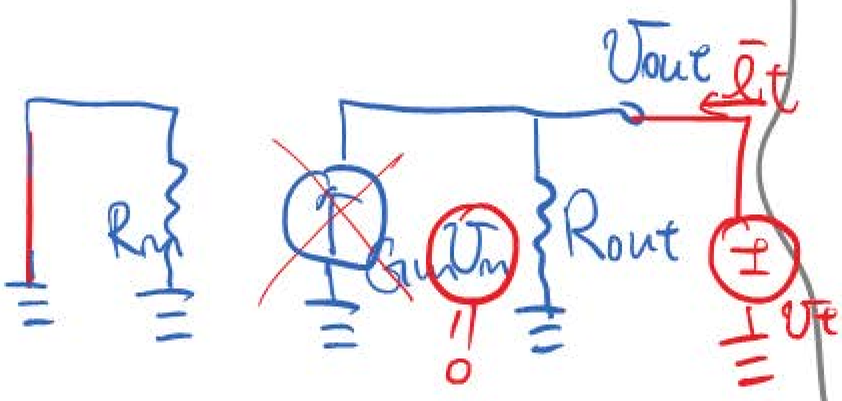


$\lambda \neq 0, \ \gamma \neq 0$

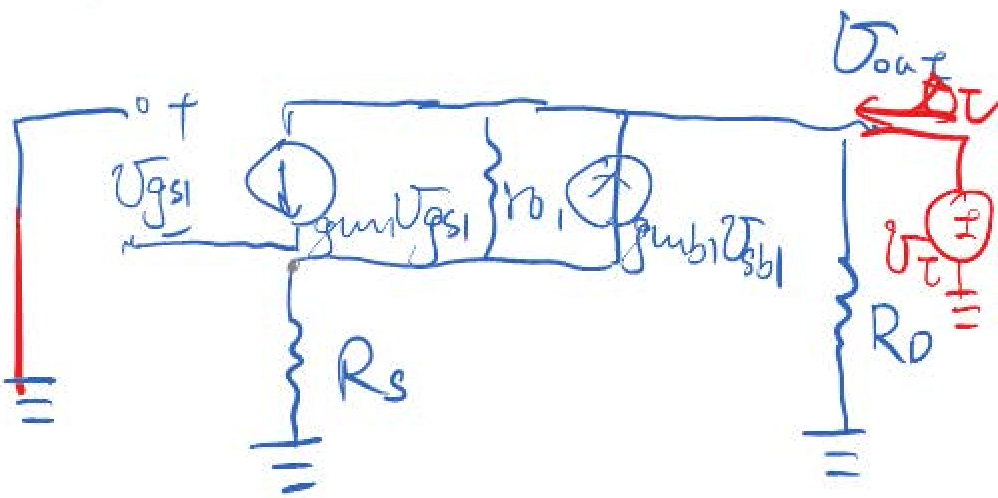
$A_v = ?$

$R_{out} = v_t / i_t$

Equivalent small signal



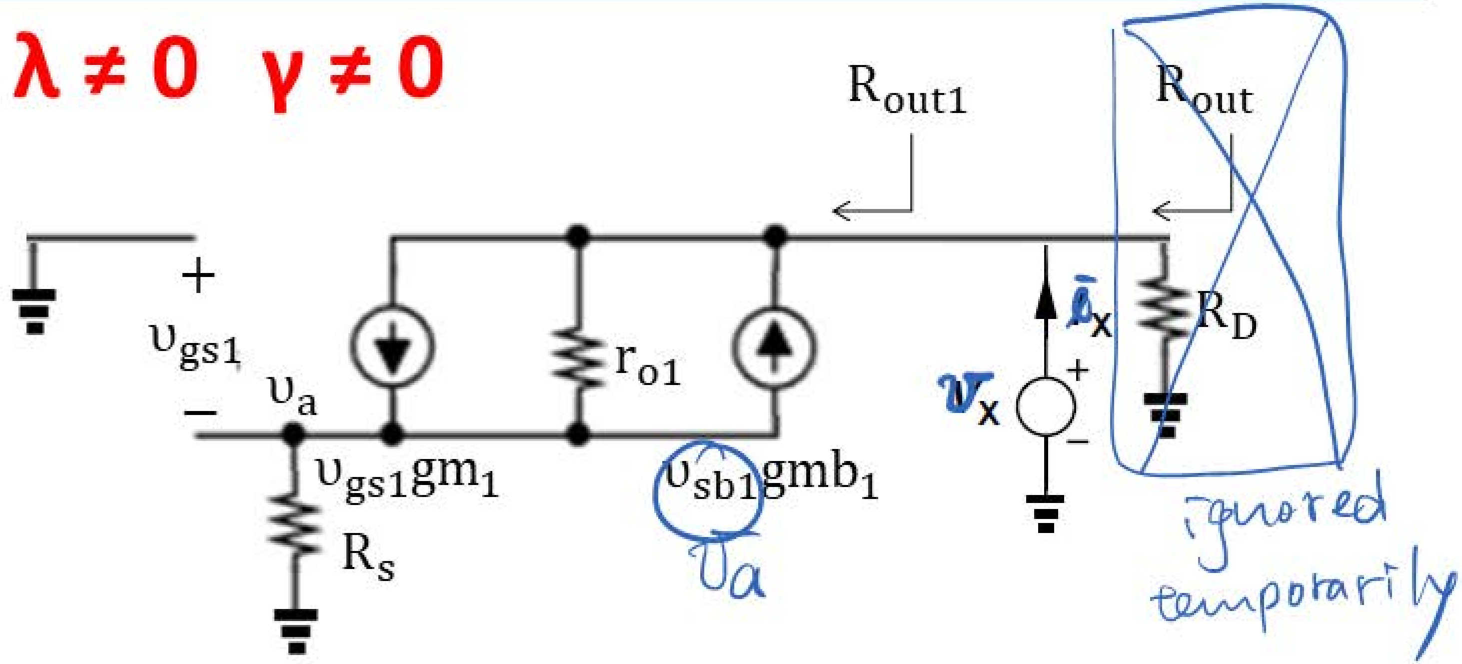
Small signal



Common-Source with Source Degradation

Small-signal Analysis

$\lambda \neq 0 \quad \gamma \neq 0$



$$
\begin{cases}
i_x = \dfrac{\upsilon_a}{R_S} \\
\upsilon_a gm_1 + \upsilon_a gmb_1 + \dfrac{\upsilon_a - \upsilon_x}{r_o} + i_x = 0
\end{cases}
$$

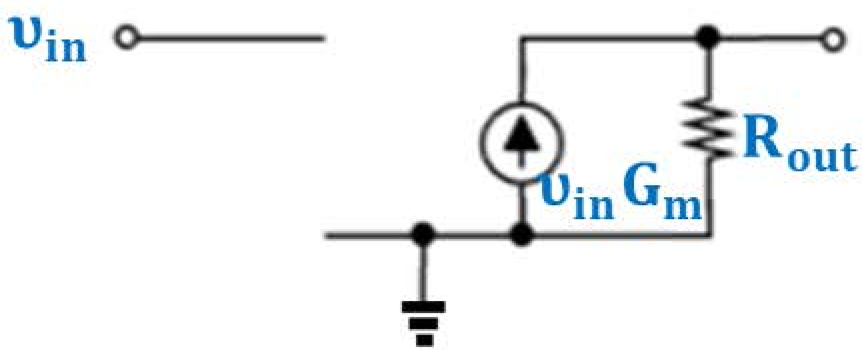
$$R_{out} = R_{out1} \parallel R_D = \underbrace{[R_S + r_{o1} + (gm_1 + gmb_1)r_{o1}R_S]}_{= \upsilon_x / i_x = R_{out1}} \parallel R_D \approx R_D$$

If $(gm_1 + gmb_1)r_{o1}R_S >> R_D$

Common-Source with Source Degradation

Small-signal Analysis

$\lambda \neq 0 \quad \gamma \neq 0$



$\upsilon_{out} = \upsilon_{in} G_m R_{out}$

$$A_v = \frac{\upsilon_{out}}{\upsilon_{in}} = G_m R_{out}$$

$$= \frac{-gm_1 r_{o1}}{\cancel{R_S + r_{o1} + (gm_1 + gmb_1) r_{o1} R_S}} \cdot \frac{\cancel{[R_S + r_{o1} + (gm_1 + gmb_1) r_{o1} R_S]} R_D}{[R_S + r_{o1} + (gm_1 + gmb_1) r_{o1} R_S] + R_D}$$

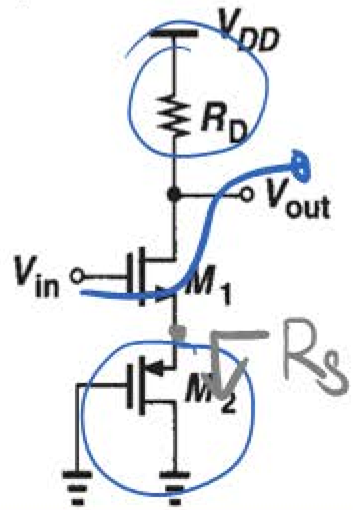
$$\approx -\frac{R_D}{R_S}$$

If $(gm_1 + gmb_1) r_{o1}$, the intrinsic gain, is large.

if gmb<<gm

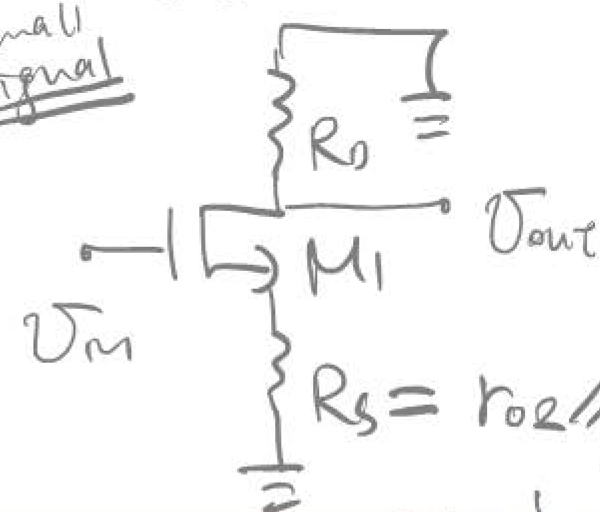
Example

Assuming λ = γ = 0, calculate the small signal voltage gain of the circuit below.



⇒

small signal



$R_S = r_{o2} \,\|\, \left(\frac{1}{g_{m2}+g_{mb2}}\right) = \frac{1}{g_{m2}}$

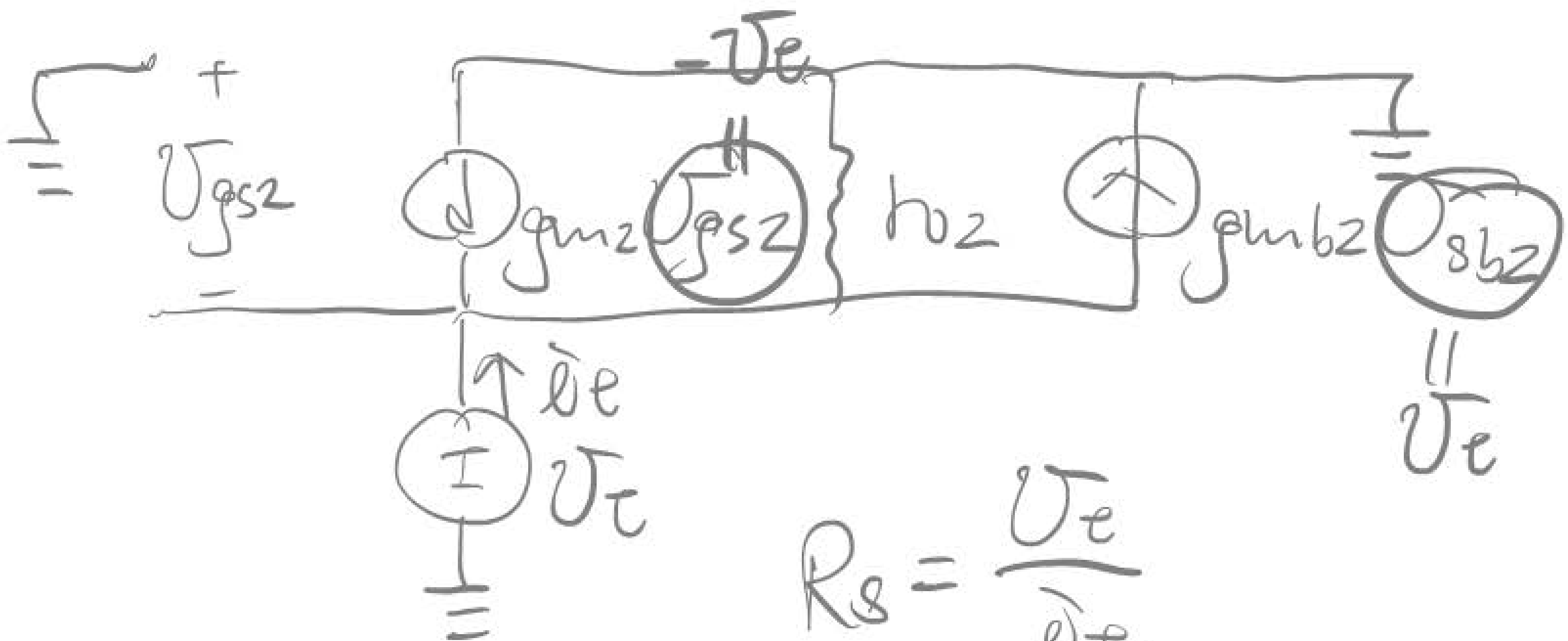
Solution:

$$G_m = -\frac{1}{\frac{1}{gm_1}+\frac{1}{gm_2}} = \frac{-g_{m1} r_{o1}}{\cancel{R_S} + r_{o1} + (g_{m1}+g_{mb1}) r_{o1} R_S}$$

$$R_{out} = R_D$$

$$= \frac{-g_{m1} \cancel{r_{o1}}}{\cancel{r_{o1}} + g_{m1} \cancel{r_{o1}} R_S}$$

$$A_v = G_m R_{out}$$

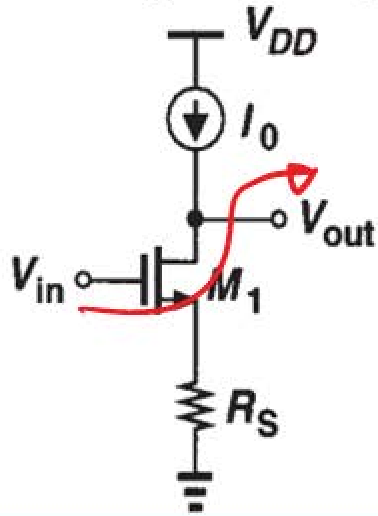


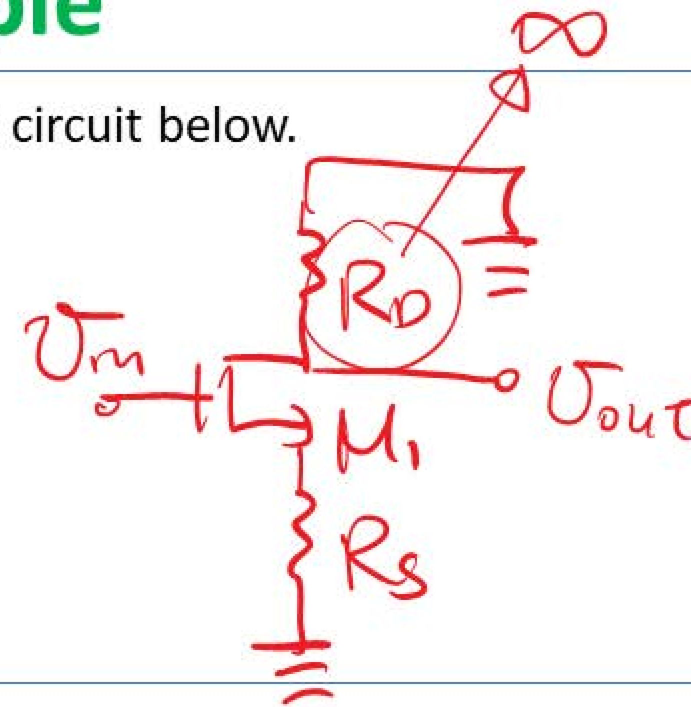
$$R_s = \frac{v_e}{i_t}$$

$$= r_{o2} \parallel \left(\frac{1}{g_{m2} + g_{mb2}} \right)$$

Example

Calculate the small signal voltage gain of the circuit below.





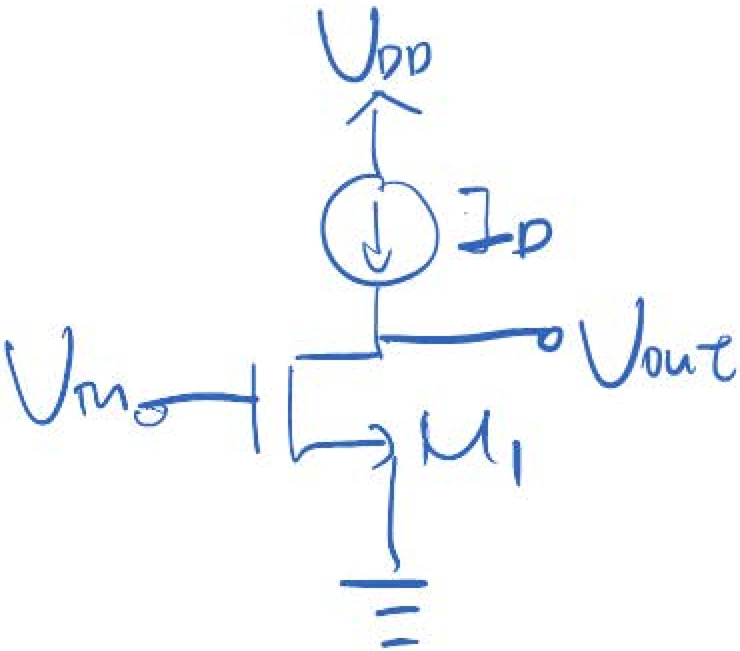
Solution:

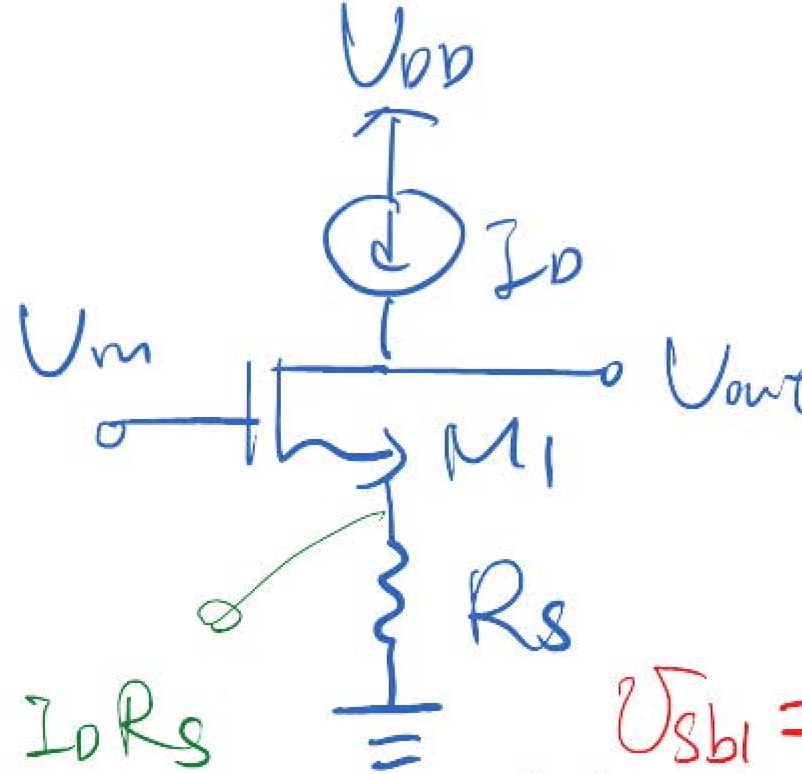
$$G_m = \frac{-gm_1 r_{o1}}{r_{o1} + R_S + (gm_1 + gmb_1) r_{o1} R_S}$$

$$R_{out} = r_{o1} + R_S + (gm_1 + gmb_1) r_{o1} R_S$$

$$A_v = G_m R_{out} = -gm_1 r_{o1}$$

- I_o is ideal current source → Voltage across R_s is constant → M_1 source shorted to ground
- R_D replaced by current source → Nonlinearity issue arises again





$V_{SB1} = 0$

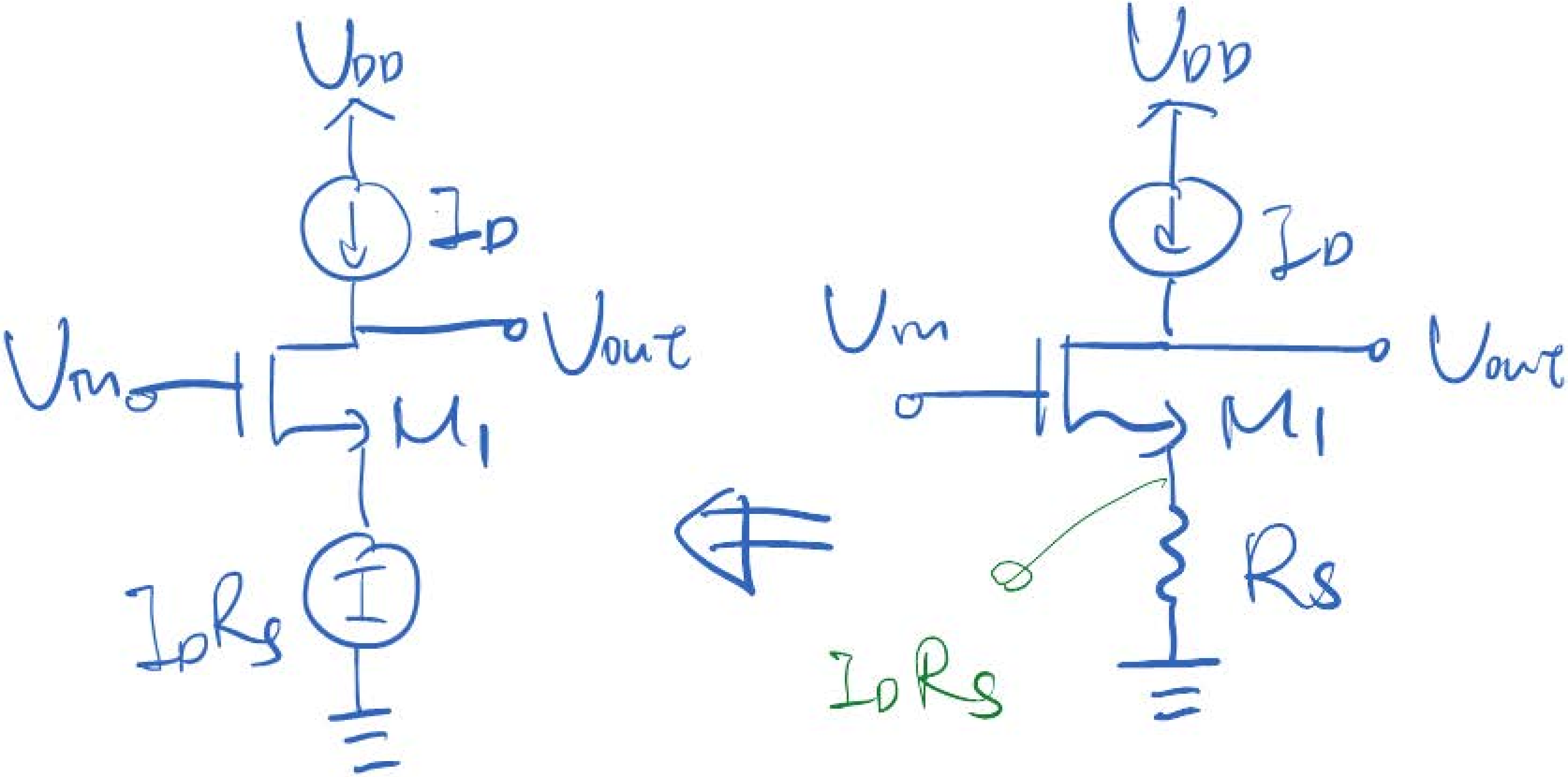
$v_{sb1} = 0$

$A_v = -g_{m1} r_{o1}$

$v_{sb1} = 0$

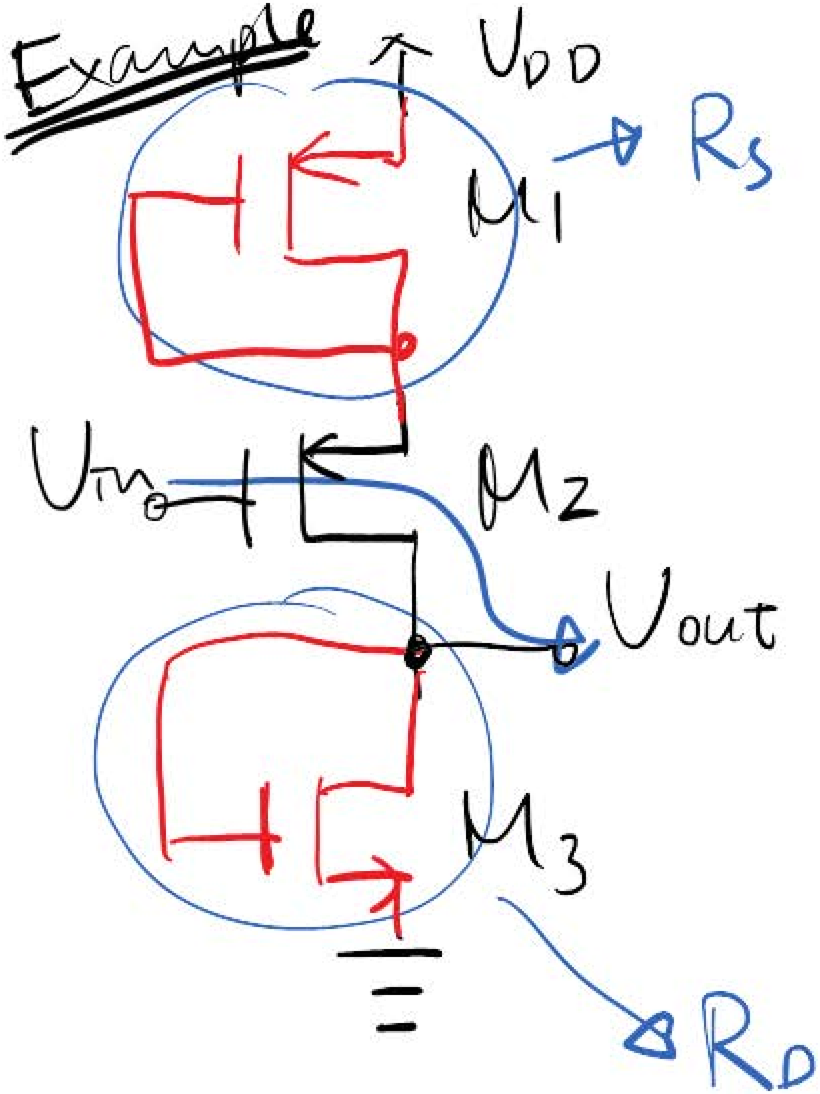
$V_{SB1} = I_D R_S$

$A_v = -g_{m1} r_{o1}$



$$A_v = -g_{m1} r_{o1}$$

Example



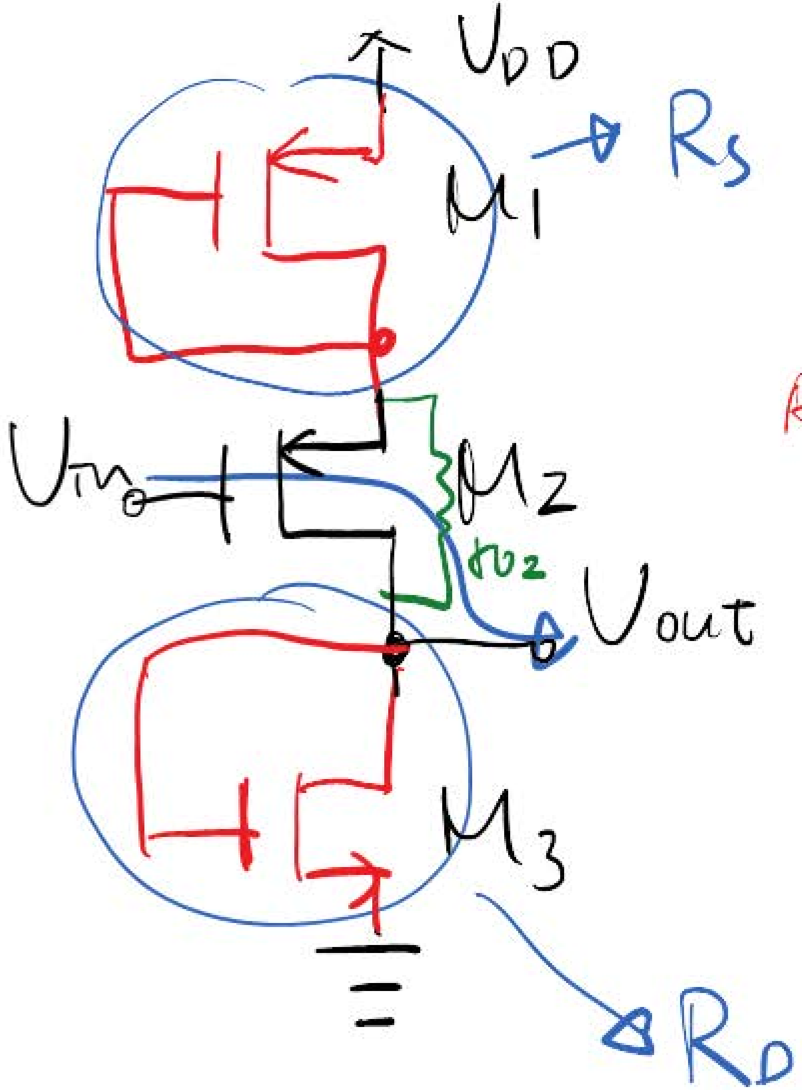
$G_m = ?$ $R_{out} = ?$

$\lambda = 0, \gamma \neq 0$

$$R_S = r_{o1} \,//\, \frac{1}{g_{m1}} = \frac{1}{g_{m1}}$$

$$R_D = \frac{1}{g_{m3}}$$

$$G_m = \frac{-g_{m2} r_{o2}}{r_{o2} + R_S + (g_{m2} + g_{mb2}) r_{o2} R_S} = \frac{-g_{m2}}{1 + (g_{m2} + g_{mb2}) R_S}$$



$G_m = ?$ $\quad$ $R_{out} = ?$

$\lambda = 0, \ \gamma \neq 0$

Assume all transistors in sat.

$$R_S = r_{o1} \,//\, \frac{1}{g_{m1}} = \frac{1}{g_{m1}}$$

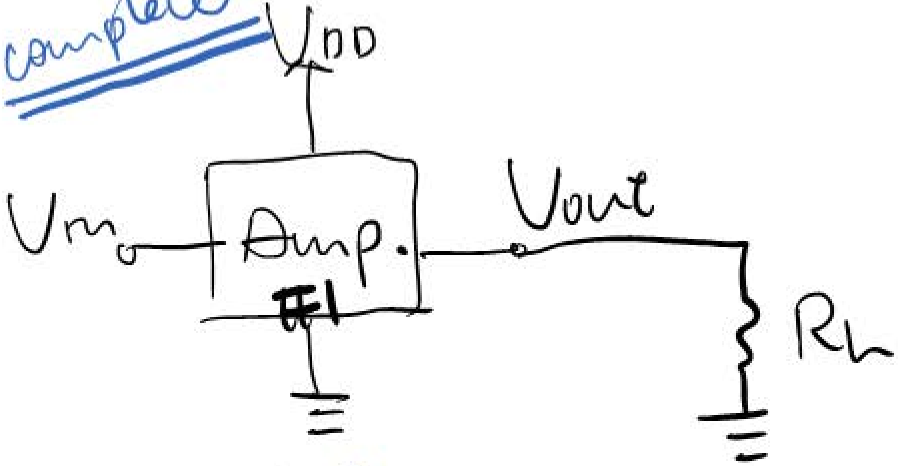
$$R_D = \frac{1}{g_{m3}}$$

$$R_{out} = R_D \,//\, \left[r_{o2} + R_S + \overbrace{\boxed{(g_{m2} + g_{mb2}) r_{o2}}}^{\text{intrinsic gain of } M_2} R_S \right] = \frac{1}{g_{m3}}$$

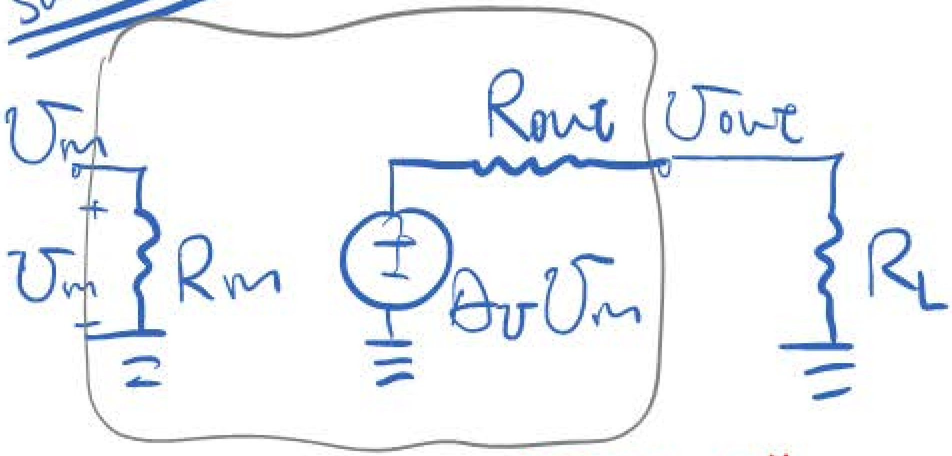
Common-Drain

Source Follower (Buffer)

complete



small-signal



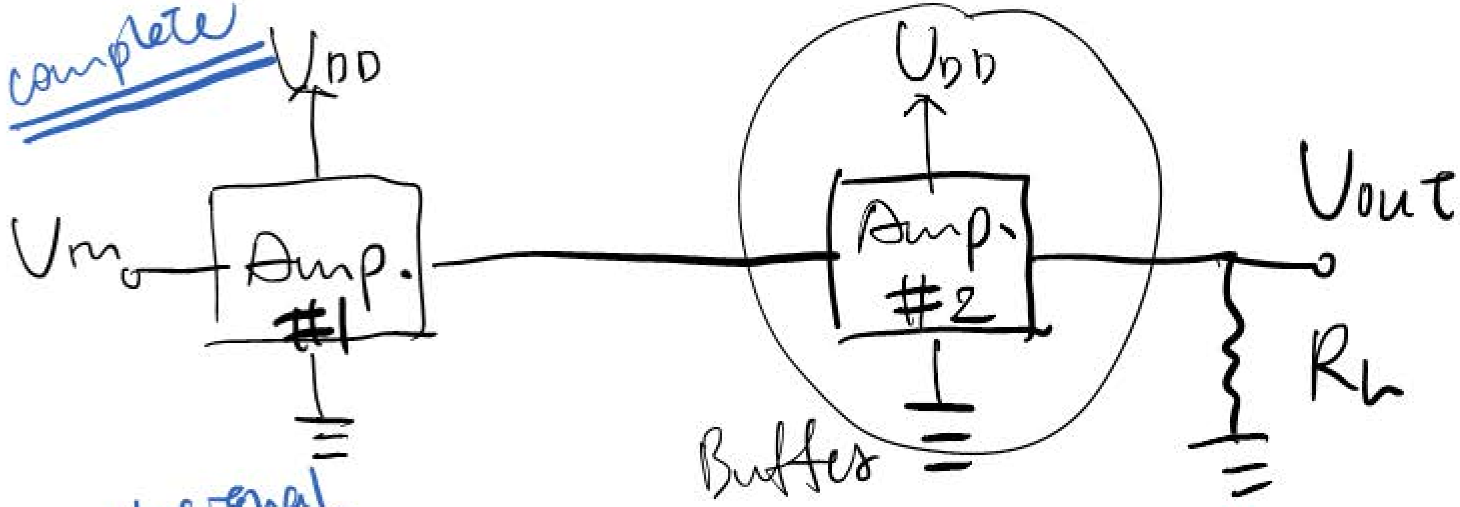
small

$$\frac{v_{out}}{v_{in}} = A_v \cdot \frac{R_L}{R_{out}+R_L} \Rightarrow$$

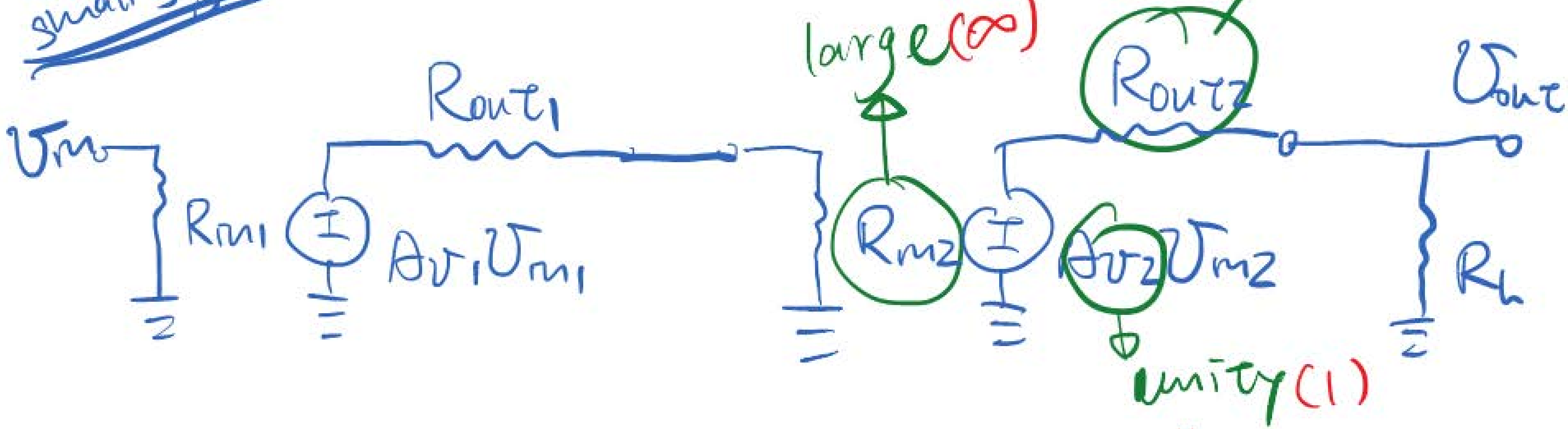
large

A_v consumed by the factor $\frac{R_L}{R_{out}+R_L}$

complete

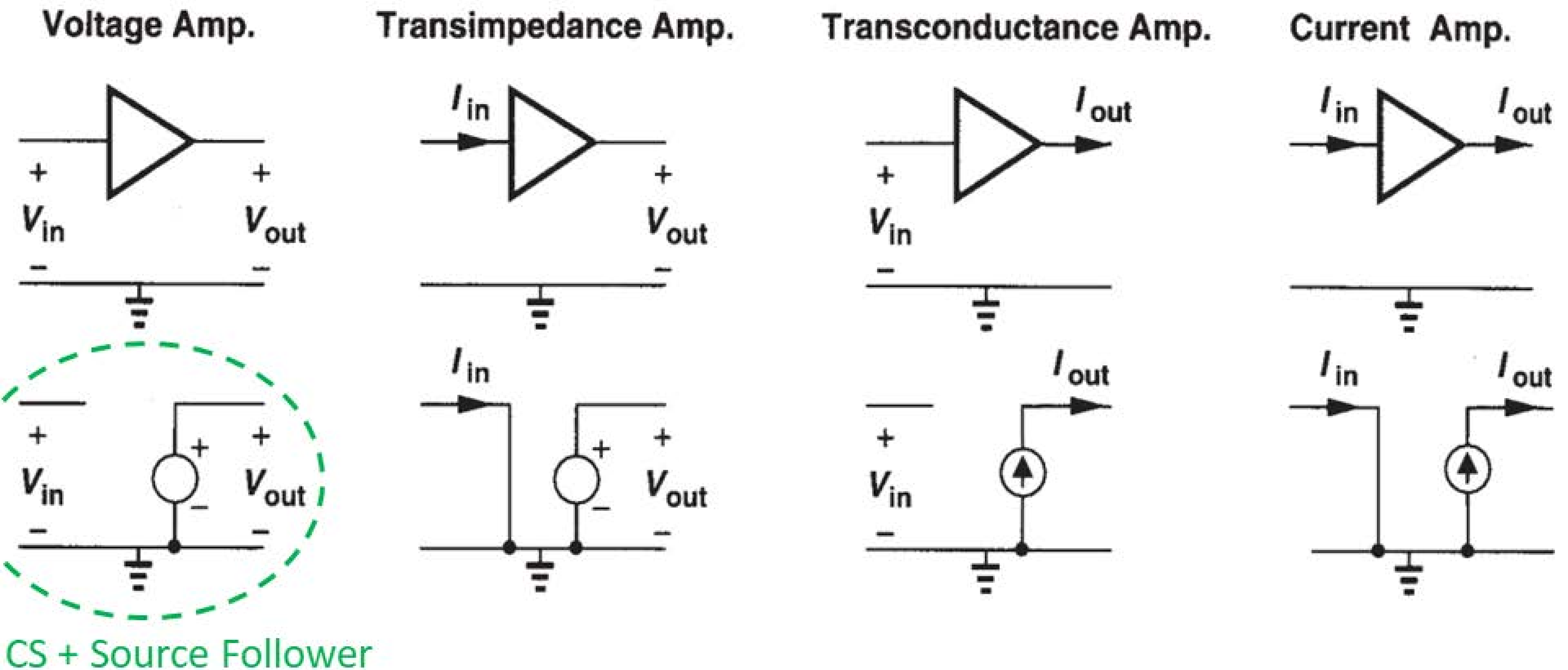


small-signal

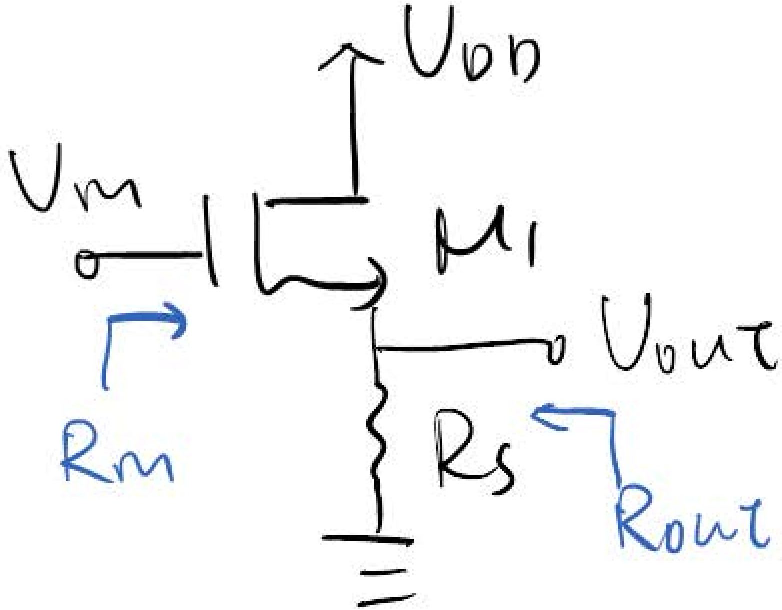


$$\frac{v_{out}}{v_{in}} = A_{v1}\frac{R_{in2}}{\underset{\text{large}}{R_{out1}}+R_{in2}}\,A_{v2}\frac{\overset{\text{small}}{R_L}}{R_{out2}+R_L}$$

Ideal Amplifier



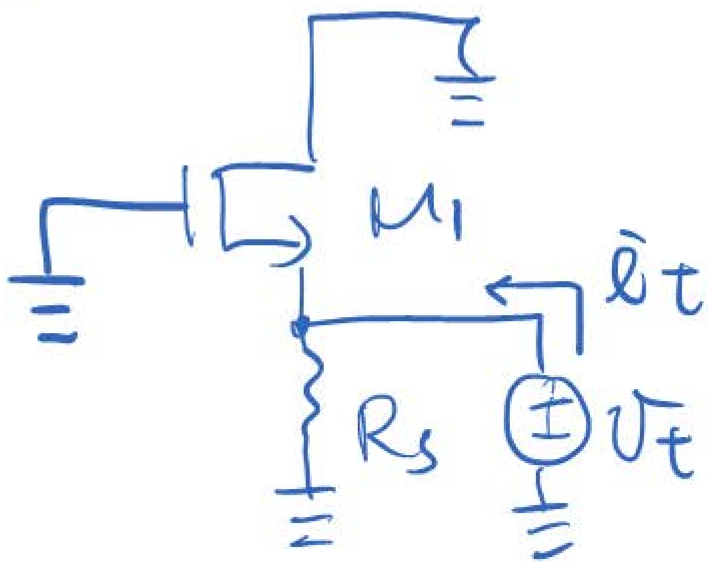
- For driving a low impedance load, source follower, as a buffer, provides **no gain** but **large input impedance** and **low output impedance**.



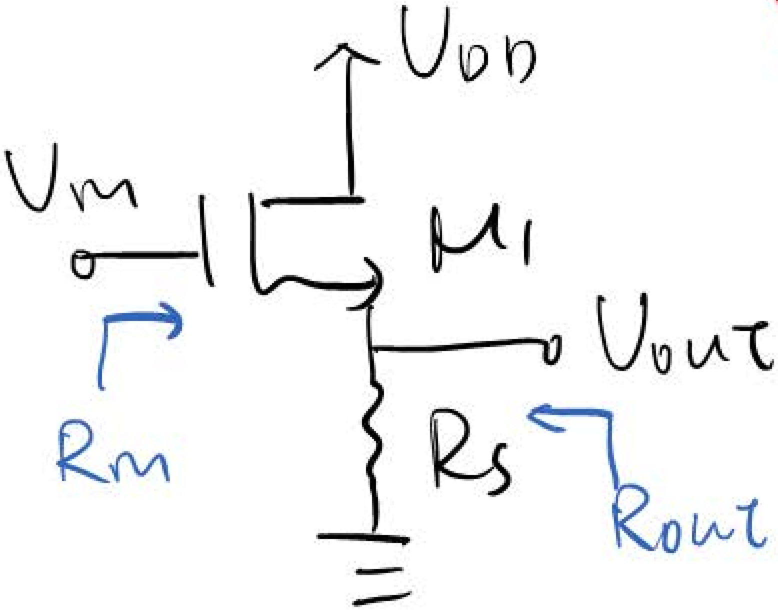
$$R_{in} = \infty$$

$$R_{out} = R_S \parallel r_{o1} \parallel \left(\frac{1}{g_{m1} + g_{mb1}}\right)$$

small-signal



$$R_{out} = \frac{v_t}{i_t}$$



$\lambda \neq 0,\ \gamma \neq 0$

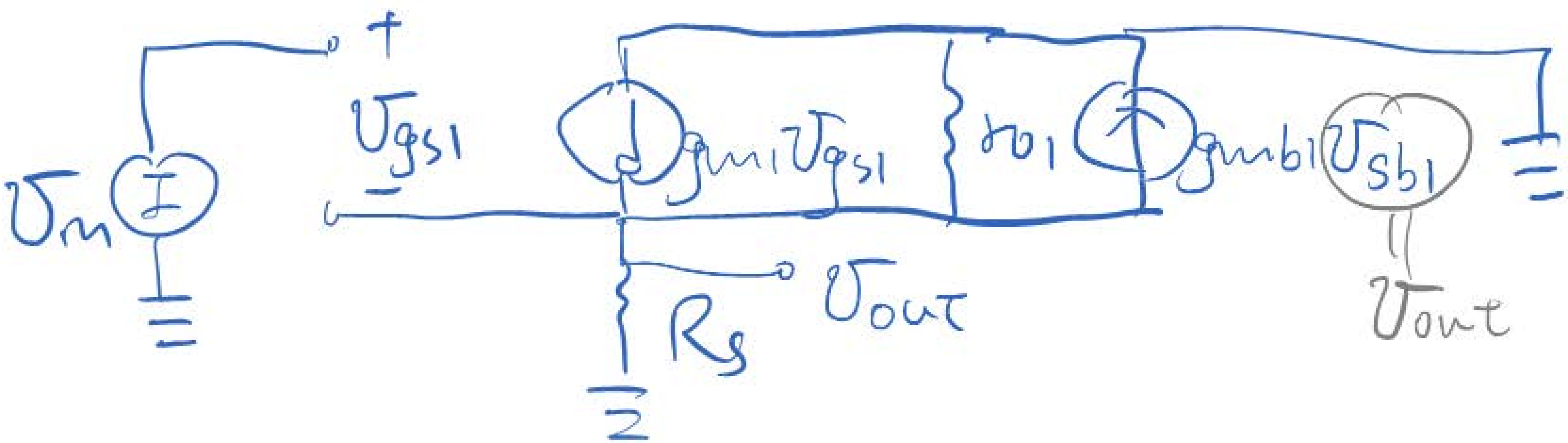
$$R_{in} = \infty$$

$$R_{out} = R_S \,//\, r_{o1} \,//\, \left(\frac{1}{g_{m1}+g_{mb1}}\right)$$

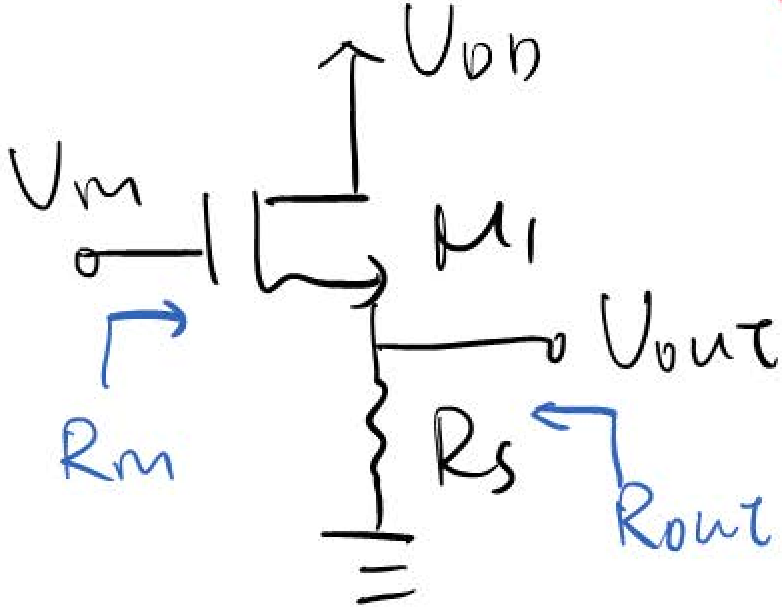
$$A_v = \frac{v_{out}}{v_{in}}$$

small-signal

$$\frac{v_{out}}{R_S} + (v_{out} - v_{in})g_{m1} + \frac{v_{out}}{r_{o1}} + g_{mb1}v_{out} = 0$$



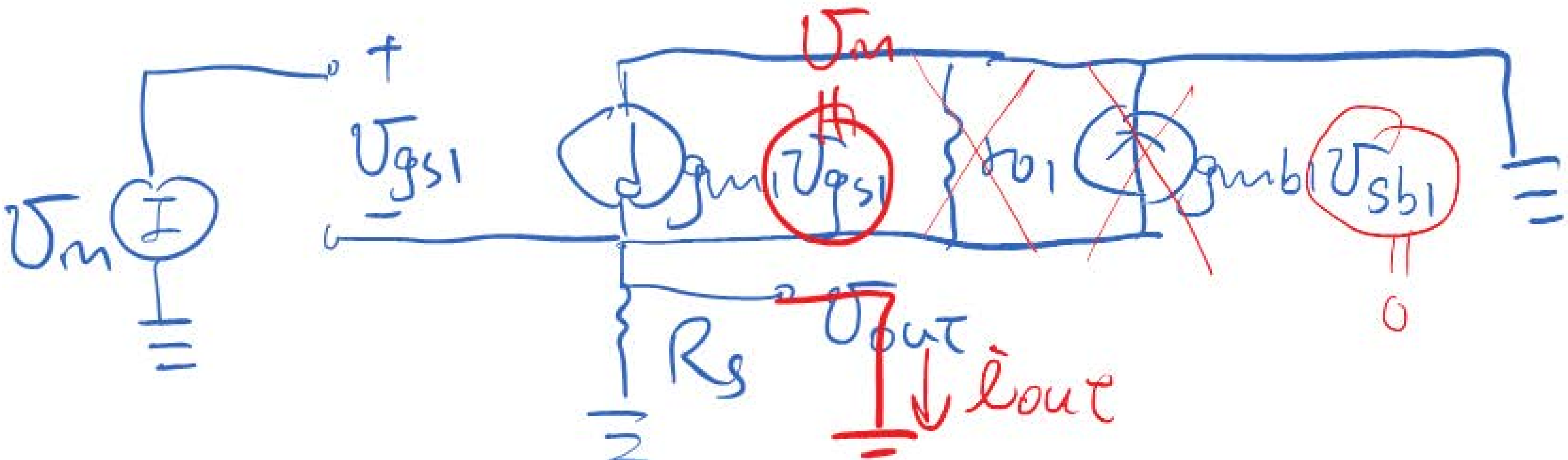
$\lambda \neq 0, \gamma \neq 0$

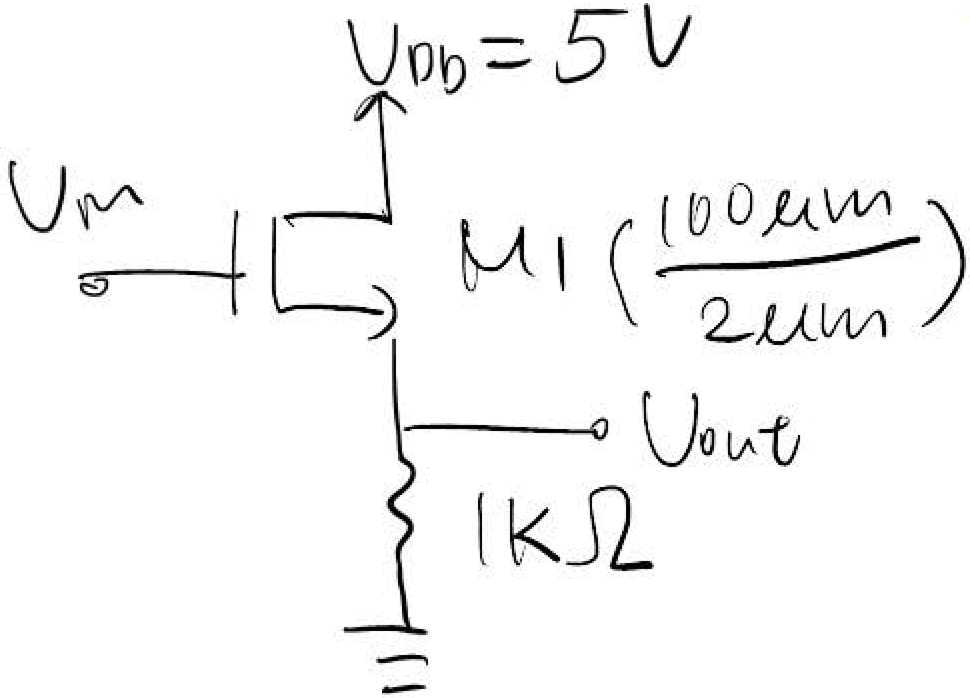


$$G_m = ? = \frac{i_{out}}{v_{in}} = g_{m1}$$

$$i_{out} = g_{m1} v_{in}$$

small-signal





$\lambda \neq 0, \gamma \neq 0$

1° $V_{OUT} = ?$

DC biasing analysis.

M_1 must be in sat.

$$\frac{V_{OUT}}{1k} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L_{eff}}\right) \cdot (3 - V_{OUT} - V_{TH})^2 \cdot [1 + \lambda(5 - V_{out})]$$

$V_{in} = 3 + 0.001\sin(2\pi 100t)$

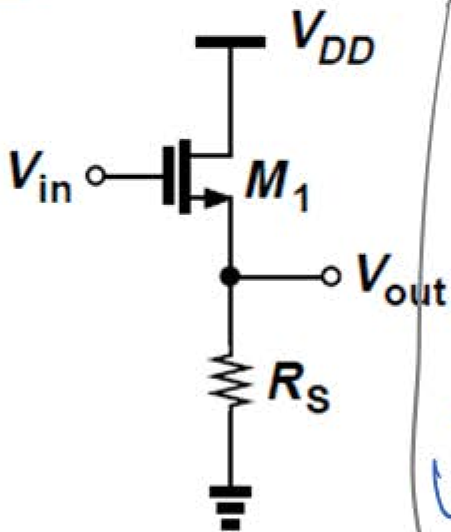
$V_{out} = ?$

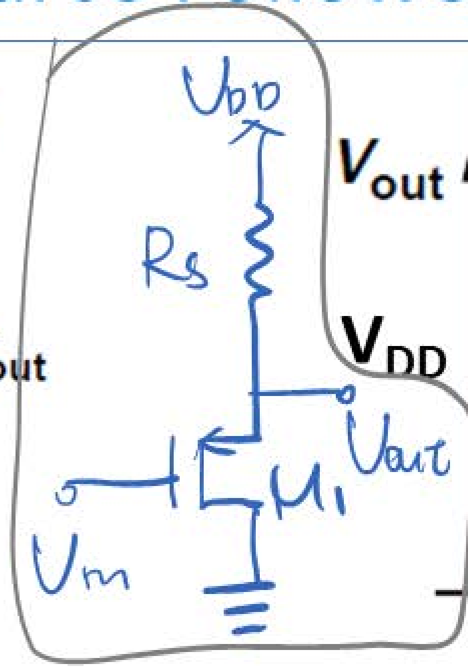
Source Follower

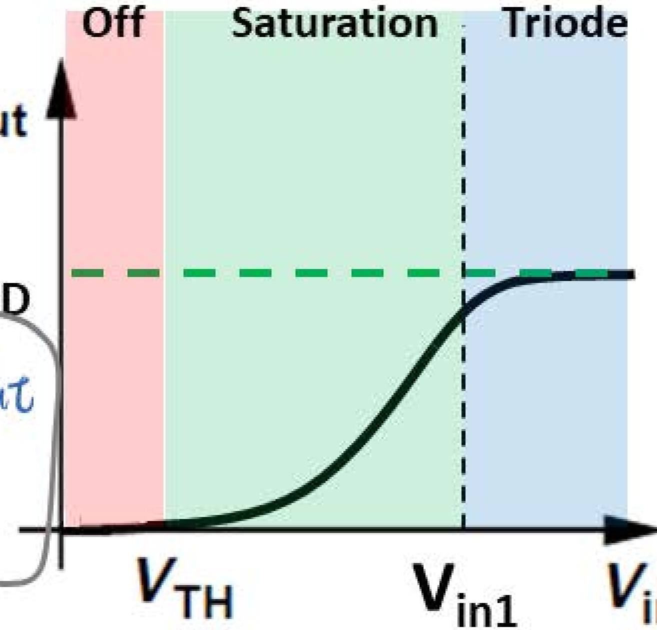
DC Analysis

$\lambda = 0$

$\gamma \neq 0$







- $V_{in} < V_{TH}$ → M_1 Off

$$V_{out} = 0$$

$$V_{DD} - V_{out} = V_{in1} - V_{out} - V_{TH}$$

$$\rightarrow V_{in1} = V_{DD} + V_{TH}$$

- $V_{in1} > V_{in} > V_{TH}$ → M_1 in Saturation

$$R_S \frac{1}{2} \mu_n C_{ox} \frac{W}{L_{eff}} (V_{in} - V_{out} - V_{TH})^2 = V_{out}$$

- $V_{in} > V_{in1}$ → M_1 in Triode

$$R_S \mu_n C_{ox} \frac{W}{L_{eff}} \left[(V_{in} - V_{out} - V_{TH})(V_{DD} - V_{out}) - \frac{1}{2}(V_{DD} - V_{out})^2 \right] = V_{out}$$

Source Follower

DC Analysis

$V_{out} = V_{SB}$.

$\lambda = 0$

$\gamma \neq 0$

- **$V_{in1} > V_{in} > V_{TH} \rightarrow M_1$ in Saturation**

$$R_S \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in} - V_{out} - V_{TH})^2 = V_{out}$$

$$R_S \frac{1}{2}\mu_n C_{ox}\frac{W}{L}2(V_{in} - V_{out} - V_{TH})\left(1 - \frac{\partial V_{out}}{\partial V_{in}} - \frac{\partial V_{TH}}{\partial V_{in}}\right) = \frac{\partial V_{out}}{\partial V_{in}}$$

$$R_S \underbrace{\boxed{\mu_n C_{ox}\frac{W}{L}(V_{in} - V_{out} - V_{TH})}}_{= gm}\left(1 - \frac{\partial V_{out}}{\partial V_{in}} - \underbrace{\frac{\partial V_{TH}}{\partial V_{out}}}_{= \frac{\partial V_{TH}}{\partial V_{SB}} = \eta = \frac{\gamma}{2\sqrt{2\Phi_F + V_{SB}}}}\frac{\partial V_{out}}{\partial V_{in}}\right) = \frac{\partial V_{out}}{\partial V_{in}}$$

$$\boxed{A_v = \frac{gmR_S}{1 + gmR_S(1+\eta)} = \frac{gmR_S}{1 + (gm + gmb)R_S} \approx \frac{1}{1+\eta}}$$

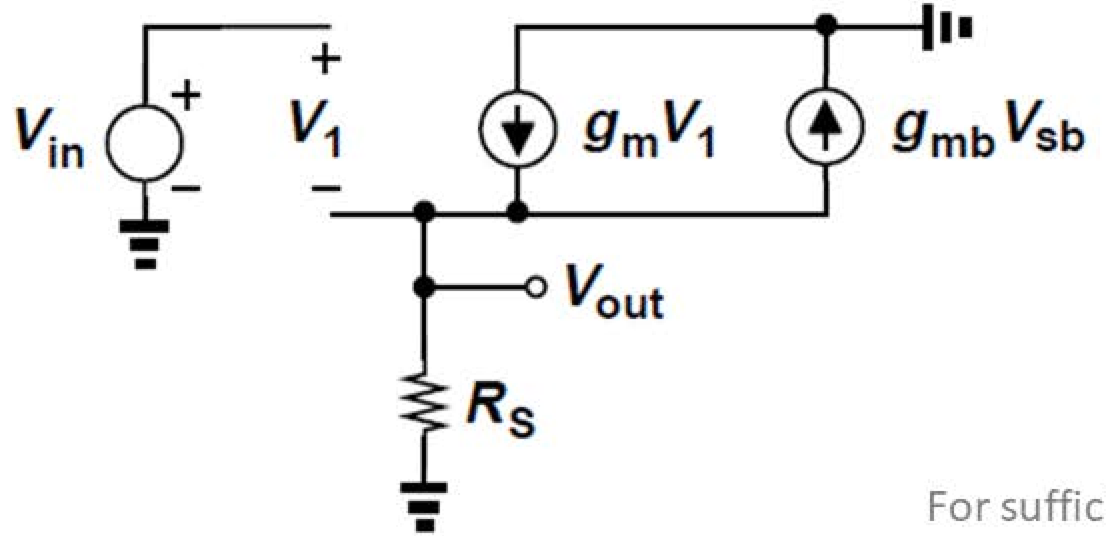
If $(gm + gmb)R_S >> 1$

Source Follower

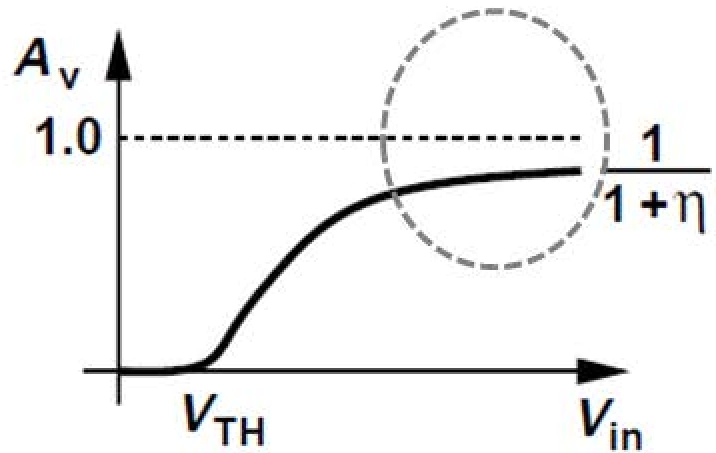
Small-signal Analysis

λ = 0

γ ≠ 0



For sufficiently large V_{in}, I_D and thus gm.



$$G_m = gm$$

$$R_{out} = R_S \parallel \left(\frac{1}{gm + gmb}\right)$$

$$A_v = \frac{gmR_S}{1 + (gm + gmb)R_S} \approx \frac{1}{1 + \eta}$$

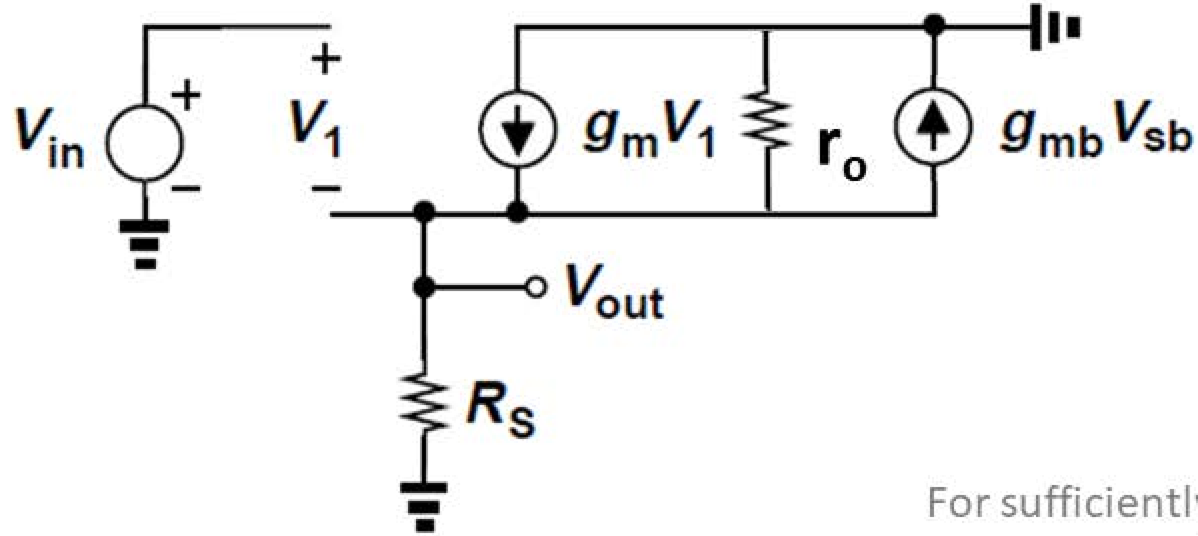
If $(gm + gmb)R_S >> 1$

Source Follower

Small-signal Analysis

$\lambda \neq 0$

$\gamma \neq 0$

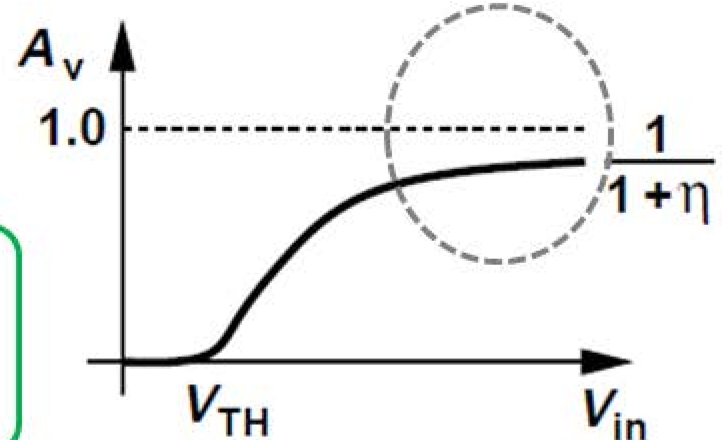


$$G_m = gm$$

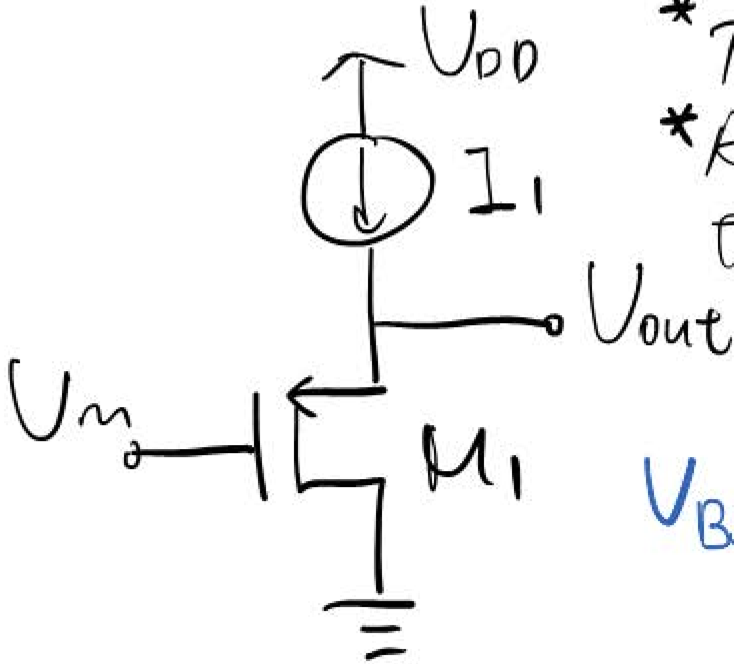
$$R_{out} = r_o \parallel R_S \parallel \left(\frac{1}{gm + gmb}\right)$$

$$\boxed{A_v = \frac{gmr_oR_S}{r_o + R_S + (gm + gmb)r_oR_S} \approx \frac{1}{1 + \eta}}$$

For sufficiently large V_{in}, I_D and thus gm.



If $(gm + gmb)r_oR_S >> r_o$ and R_S

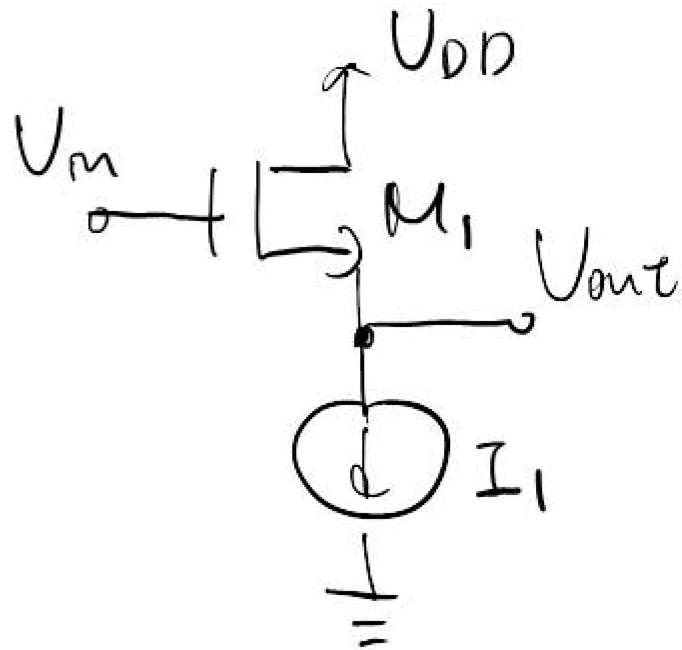


* $\lambda = \gamma = 0$
* Assume all transistors in sat.

$V_{BS1} > 0 \not\Rightarrow |V_{THP}| > 0.8$ if $\gamma \neq 0$

$$\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L_{eff}}\right)_1\left(V_{out} - V_{in} - |V_{THP}|\right)^2 = I_1$$

constants



$\lambda \neq 0, \gamma \neq 0$

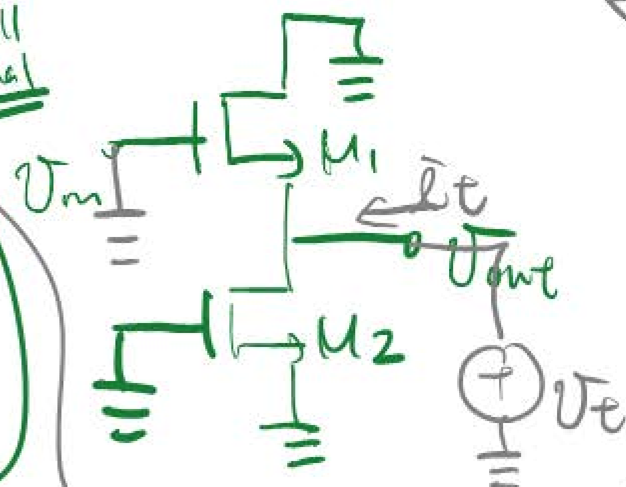
All transistors operate in sat.

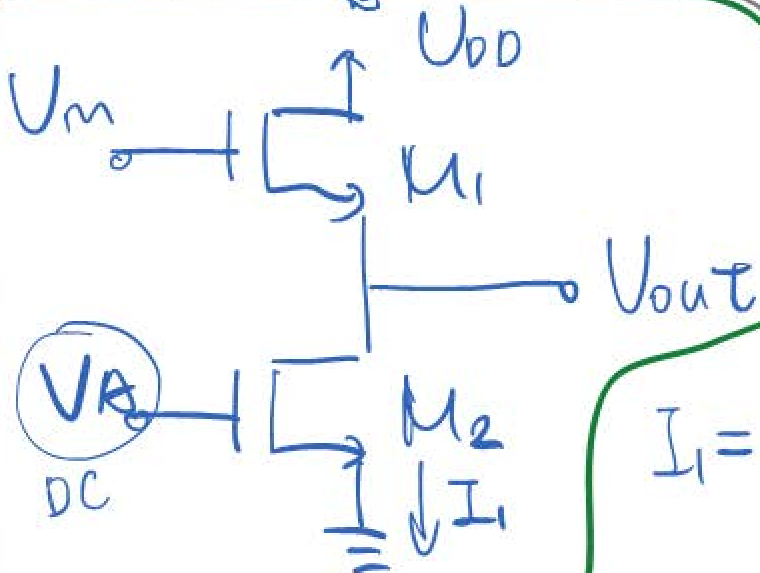
$R_{in} = ?$ $R_{out} = ?$ $A_v = ?$

$R_{in} = \infty$

$R_{out} = r_{o2} \,//\, r_{o1} \,//\, \left(\frac{1}{g_{m1} + g_{mb1}}\right)$

small signal

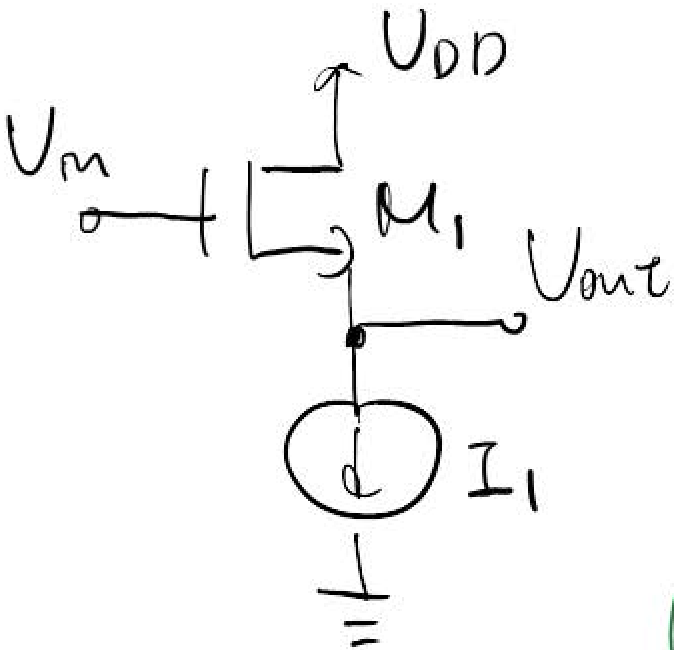




$$I_1 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L_{eff}}\right)(V_A - 0.7)^2(1 + \lambda V_{OUT})$$

$\lambda \neq 0,\ \gamma \neq 0$

All transistors operate in sat.



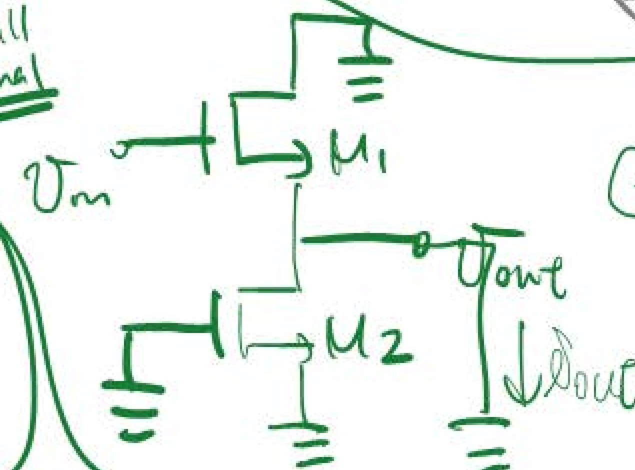
$R_{in} = ?$ $\quad R_{out} = ?$ $\quad A_v = ?$

$= G_m R_{out}$

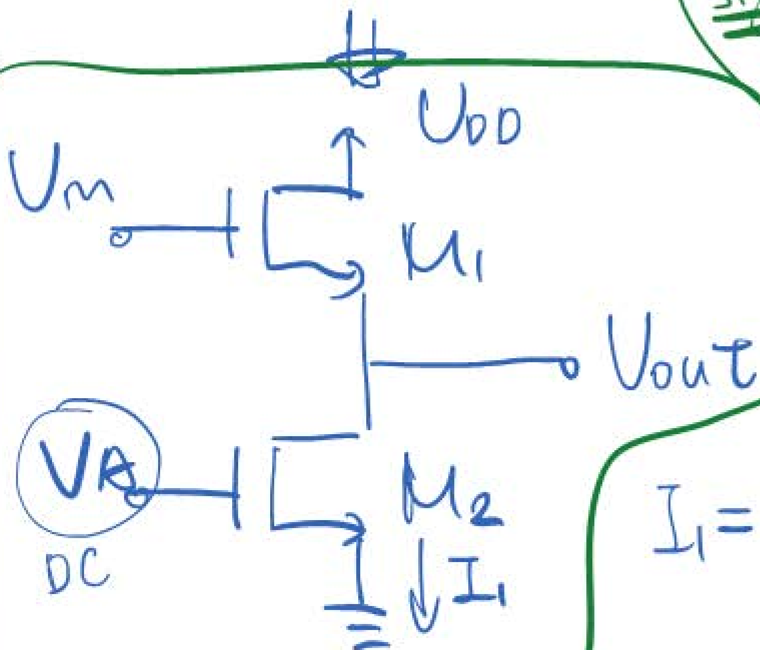
$R_{in} = \infty$

$R_{out} = r_{o2} \,//\, r_{o1} \,//\, \left(\dfrac{1}{g_{m1} + g_{mb1}}\right)$

Small signal



$G_m = g_{m1}$



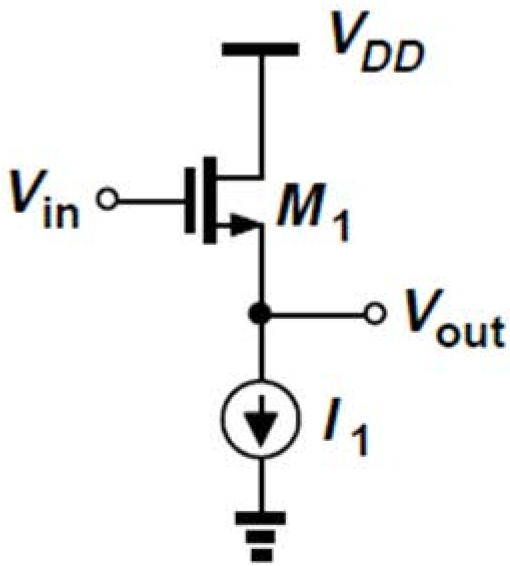
$I_1 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L_{eff}}\right)(V_A - 0.7)^2(1+\lambda V_{OUT})$

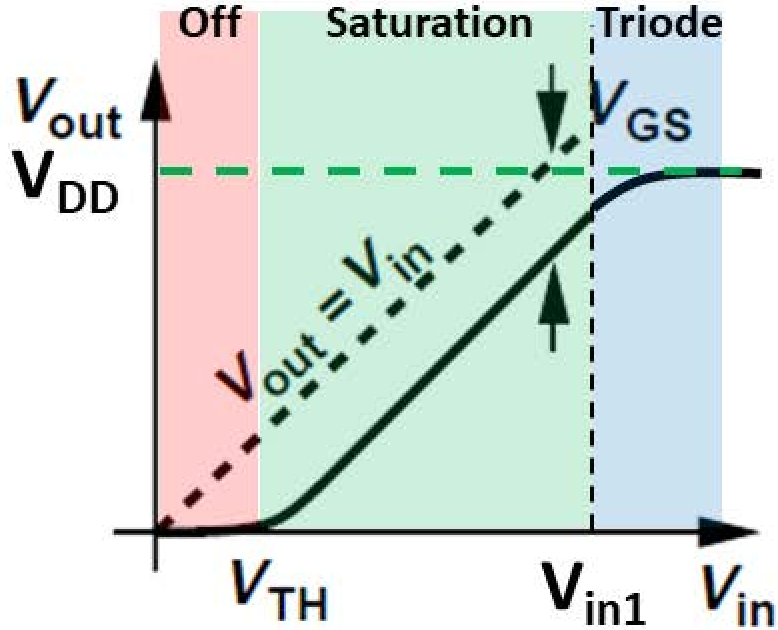
Source Follower with Current Source

DC Analysis

$\lambda = 0$

$\gamma \neq 0$





$$\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in} - V_{out} - V_{TH})^2 = I_1$$

$$\frac{1}{2}\mu_n C_{ox}\frac{W}{L}2(V_{in} - V_{out} - V_{TH})\left(1 - \frac{\partial V_{out}}{\partial V_{in}} - \frac{\partial V_{TH}}{\partial V_{in}}\right) = 0$$

$$\underbrace{\boxed{\mu_n C_{ox}\frac{W}{L}(V_{in} - V_{out} - V_{TH})}}_{= gm}\left(1 - \frac{\partial V_{out}}{\partial V_{in}} - \underbrace{\boxed{\frac{\partial V_{TH}}{\partial V_{out}}}}_{= \eta}\frac{\partial V_{out}}{\partial V_{in}}\right) = 0$$

$$A_v = \frac{1}{1 + \eta}$$

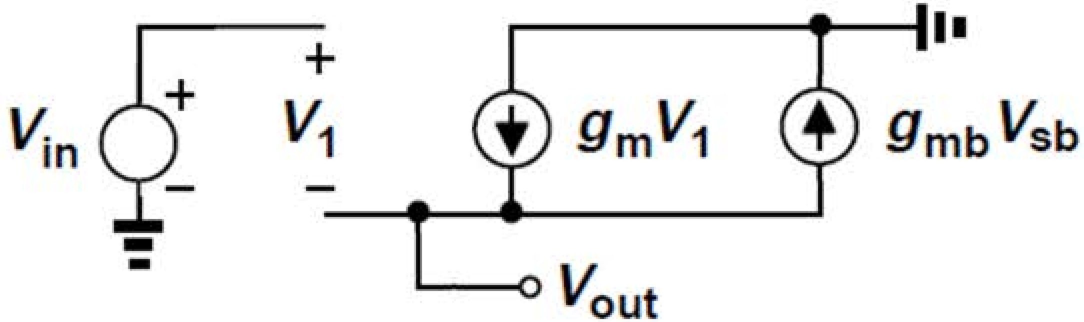
If $\gamma = 0$, $A_v = 1$.

Source Follower with Current Source

Small-signal Analysis

$\lambda = 0$

$\gamma \neq 0$

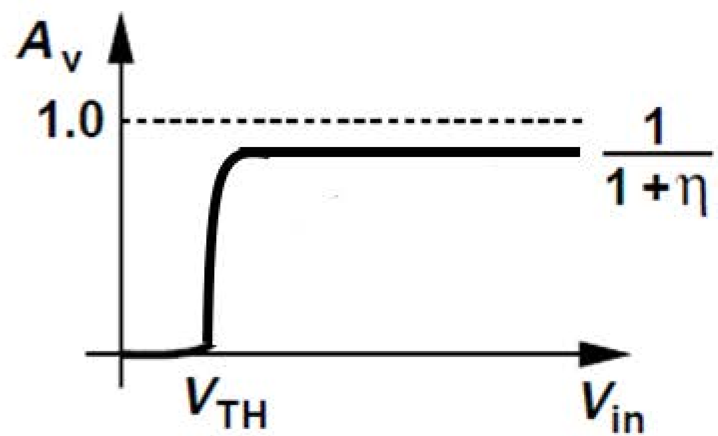


$$G_m = gm$$

$$R_{out} = \frac{1}{gm + gmb}$$

$$A_v = \frac{1}{1+\eta}$$

If $\gamma = 0$, $A_v = 1$.

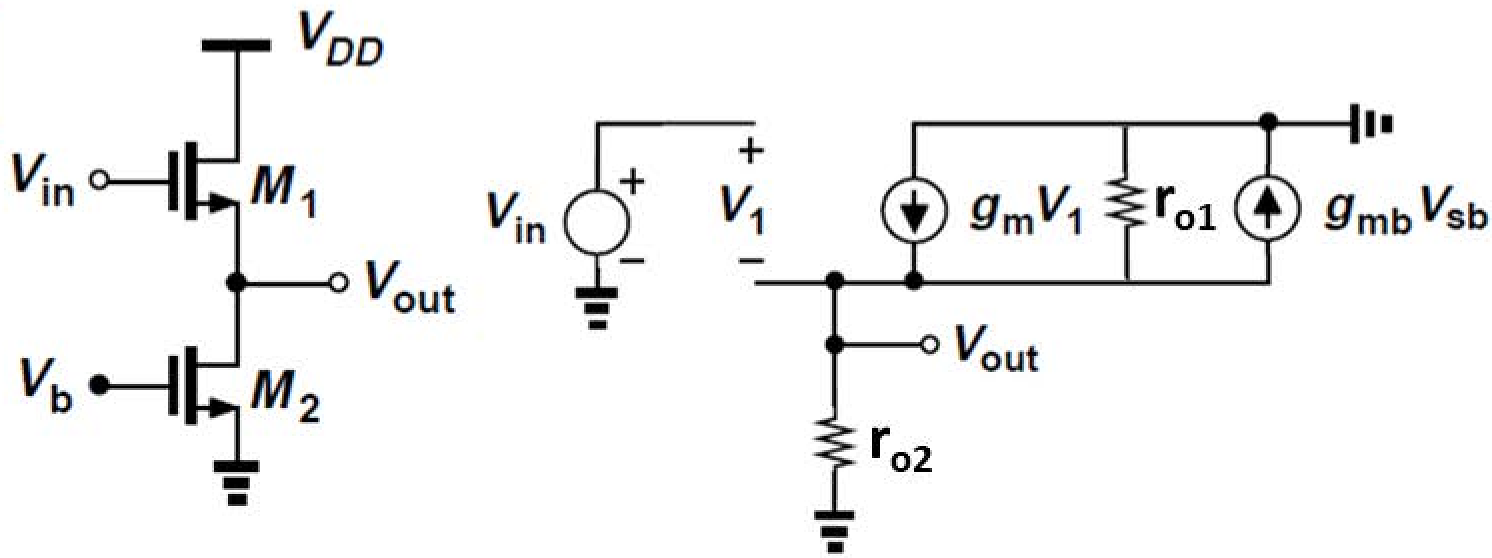


Source Follower with Current Source

Small-signal Analysis

$\lambda \neq 0$

$\gamma \neq 0$



$$G_m = gm_1$$

$$R_{out} = r_{o1} \parallel r_{o2} \parallel \left(\frac{1}{gm_1 + gmb_1}\right)$$

$$\boxed{A_v = \frac{gmr_{o1}r_{o2}}{r_{o1} + r_{o2} + (gm + gmb)r_{o1}r_{o2}}}$$

If r_{o1} and r_{o2} large, A_v is linear.

Source Follower with Current Source ($V_{BS} = 0$)

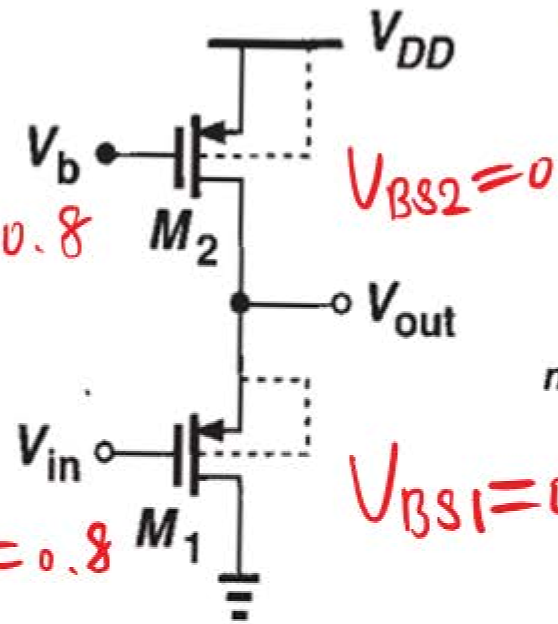
Small-signal Analysis

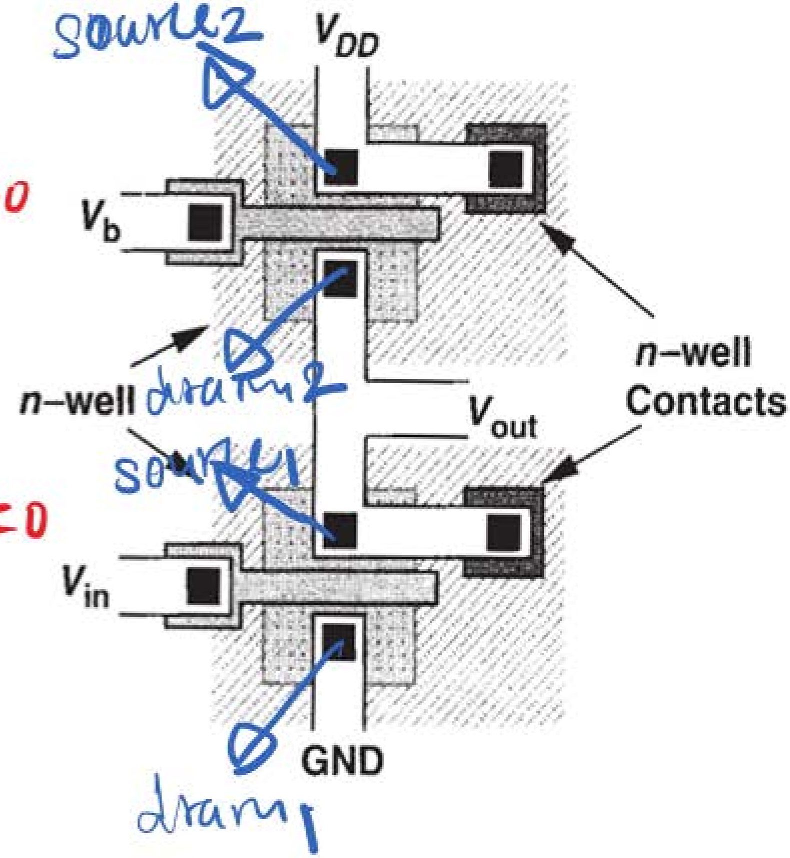
$\lambda \neq 0$

$\gamma \neq 0$

$|V_{TH2}| = 0.8$

$|V_{TH1}| = 0.8$





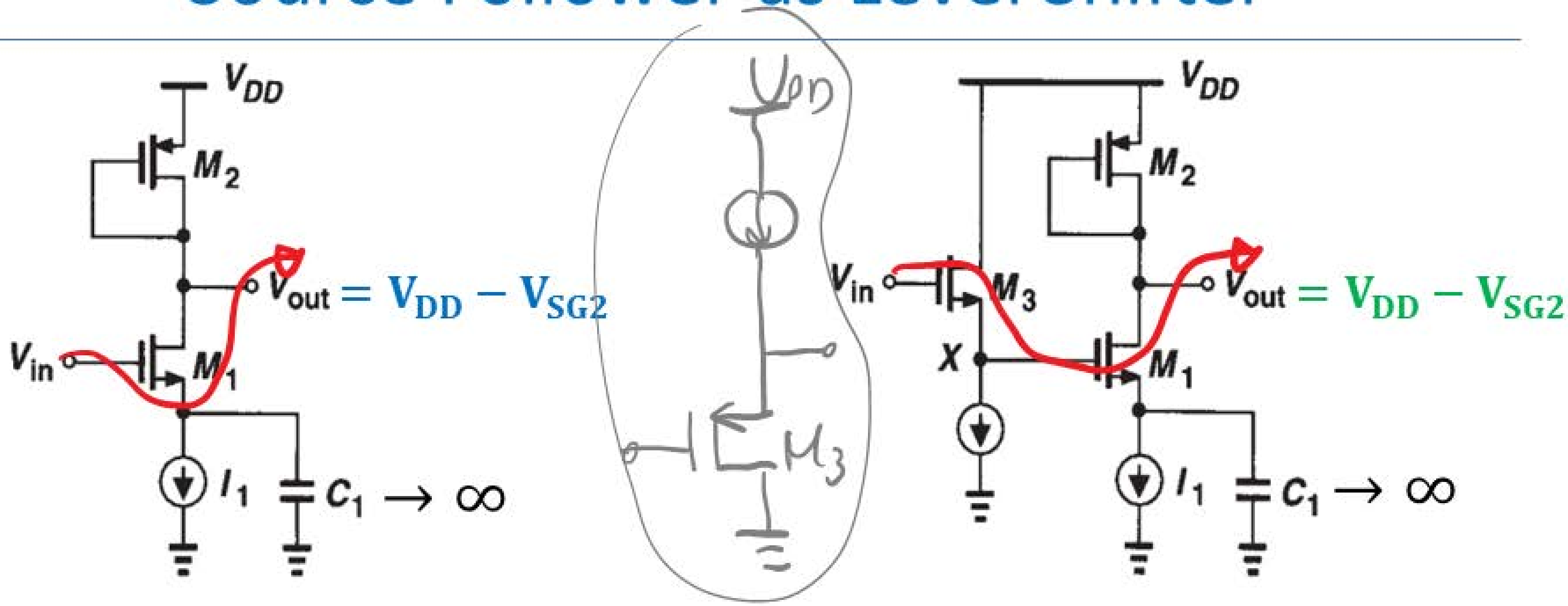
$$G_m = gm_1$$

$$R_{out} = r_{o1} \parallel r_{o2} \parallel \frac{1}{gm_1}$$

$$\boxed{A_v = \frac{gm_1 r_{o1} r_{o2}}{r_{o1} + r_{o2} + gm_1 r_{o1} r_{o2}}}$$

- The sacrifice here is the higher output impedance due to smaller mobility of holes relative to electrons.

Source Follower as Level Shifter



$$V_{in} \le V_{DD} - V_{SG2} + V_{TH1}$$

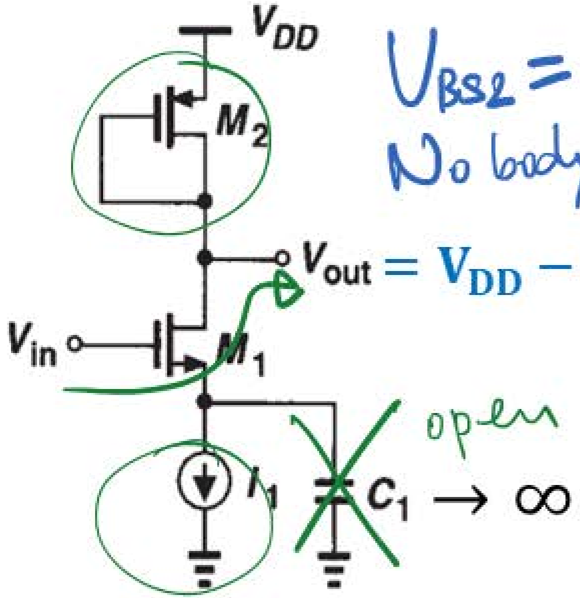
$$V_{in} - V_{GS3} \le V_{DD} - V_{SG2} + V_{TH1}$$

$$\begin{cases} G_m = -gm_1 \\ R_{out} = r_{o1} \parallel r_{o2} \parallel \dfrac{1}{gm_2} \end{cases}$$

$$\begin{cases} G_{m(left)} = gm_3 \\ R_{out(left)} = r_{o3} \parallel \dfrac{1}{gm_3 + gmb_3} \end{cases}$$

$$\begin{cases} R_{in(right)} = \infty \\ G_{m(right)} = -gm_1 \\ R_{out(right)} = r_{o1} \parallel r_{o2} \parallel \dfrac{1}{gm_2} \end{cases}$$

Source Follower as Level Shifter



$V_{BS2} = 0$
No body effect

1° DC biasing analysis

$$I_1 = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L_{eff}}\right)_2 \cdot \left(V_{DD} - V_{OUT} - |V_{TH2}|\right)^2 \cdot \left[1 + \lambda(V_{DD} - V_{OUT})\right]$$

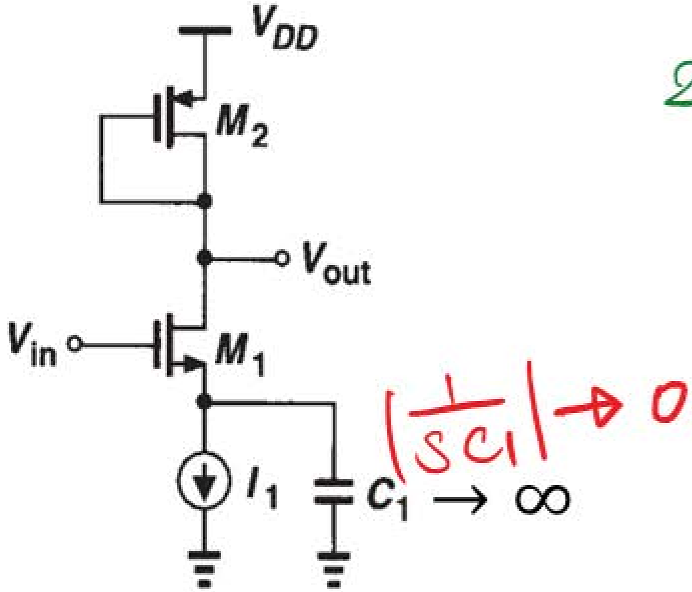
0.8

$V_{OUT} = ?$

$$\mathbf{V_{in} \leq \underbrace{\boxed{V_{DD} - V_{SG2}}}_{V_{OUT}} + V_{TH1}}$$

When V_{in} higher than $V_{OUT} + V_{TH1}$, M_1 will be driven into triode.

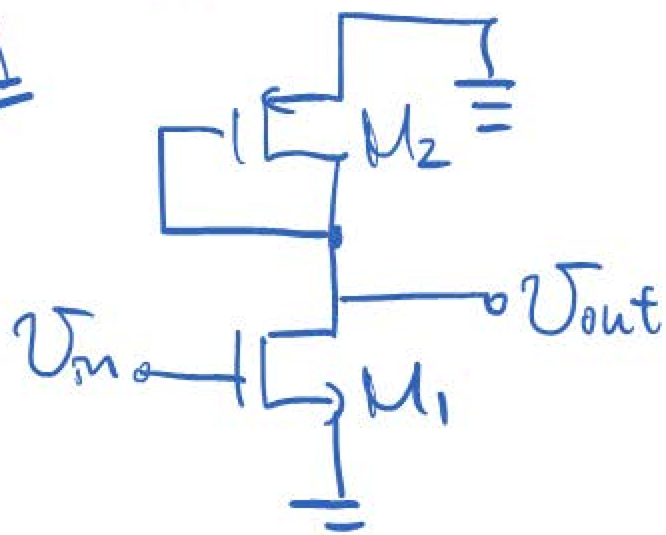
Source Follower as Level Shifter



2° small signal analysis

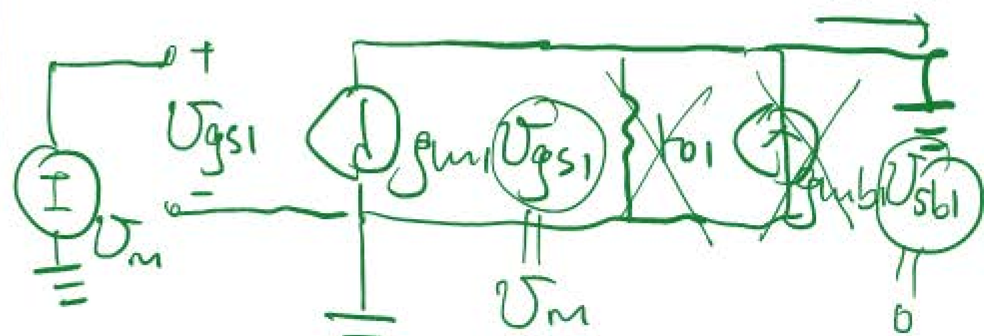
$$\begin{cases} G_m = -g_{m1} \\ R_{out} = r_{o1} \parallel r_{o2} \parallel \dfrac{1}{g_{m2}} \\ R_{in} = \infty \end{cases}$$

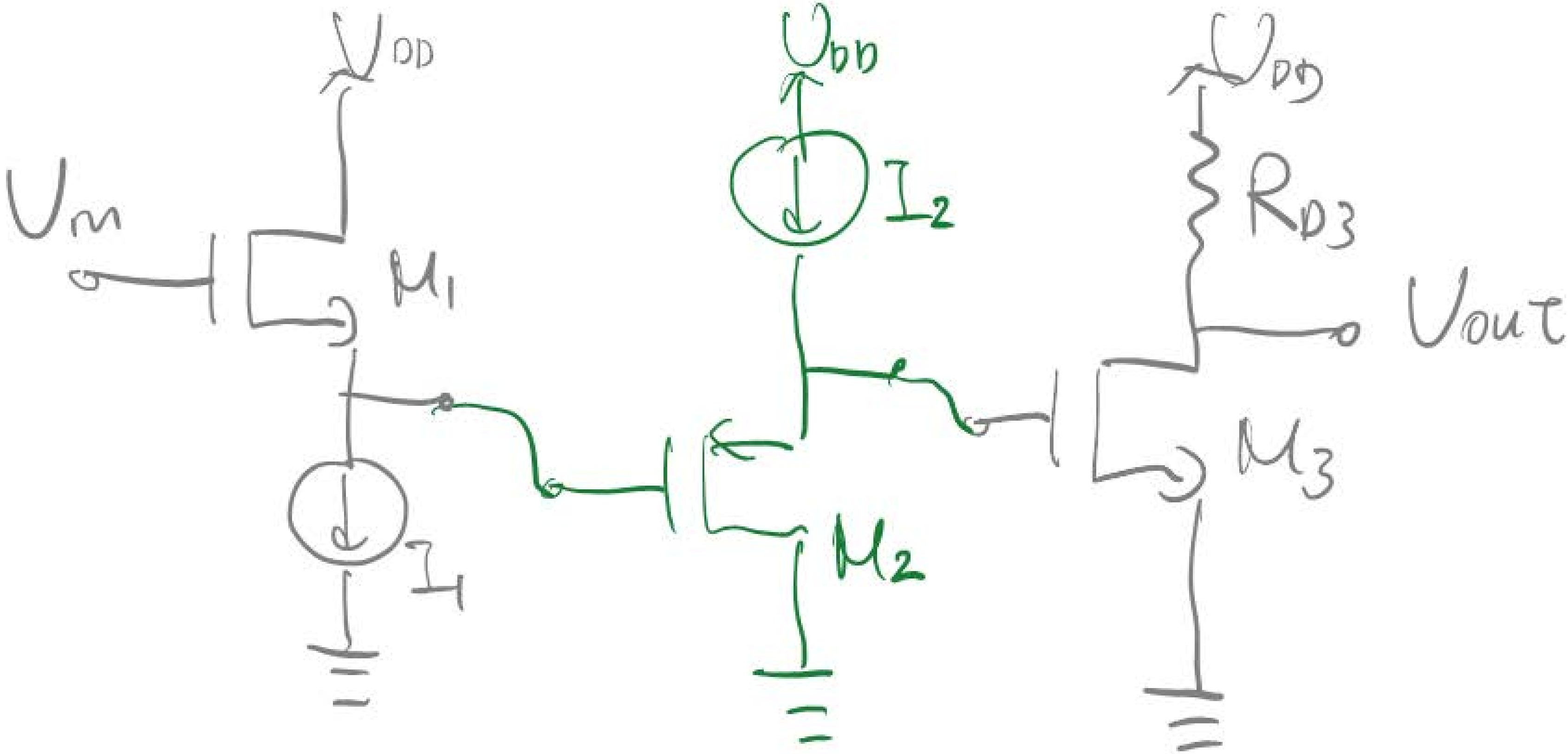
small signal



$$G_m = \frac{i_{out}}{v_{in}}$$

$$-g_{m1}v_{in} = i_{out}$$

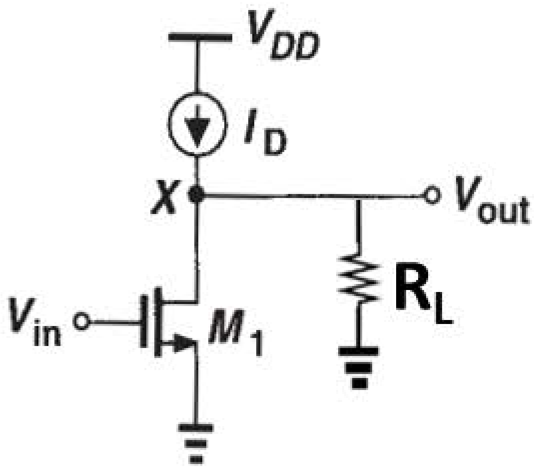




Assume $\lambda = \gamma = 0$, $A_v = ?$

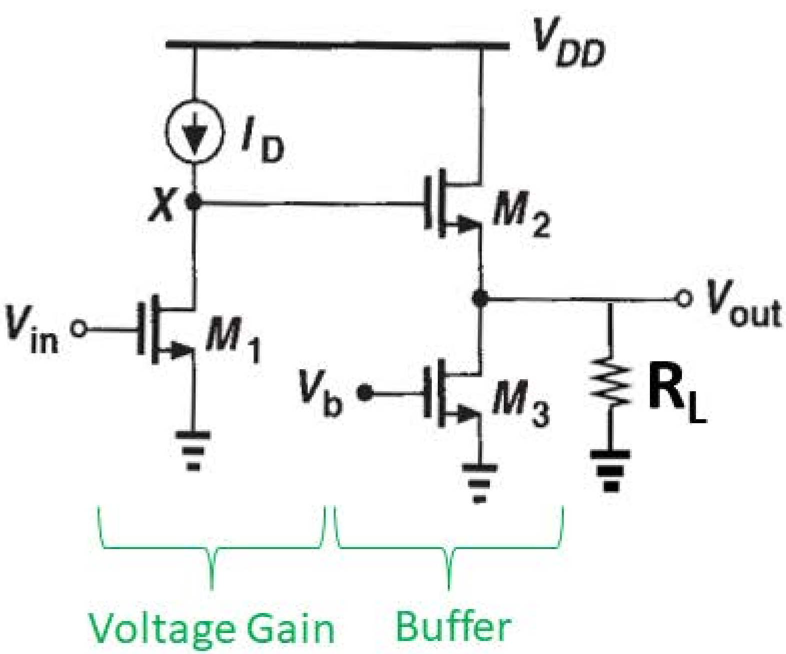
$$A_v = 1 \times 1 \times (-g_{m3} R_{D3})$$

CS + Source Follower



$$A_v = -gm_1(r_{o1} \parallel R_L)$$

- Voltage gain severely reduced when R_L very small

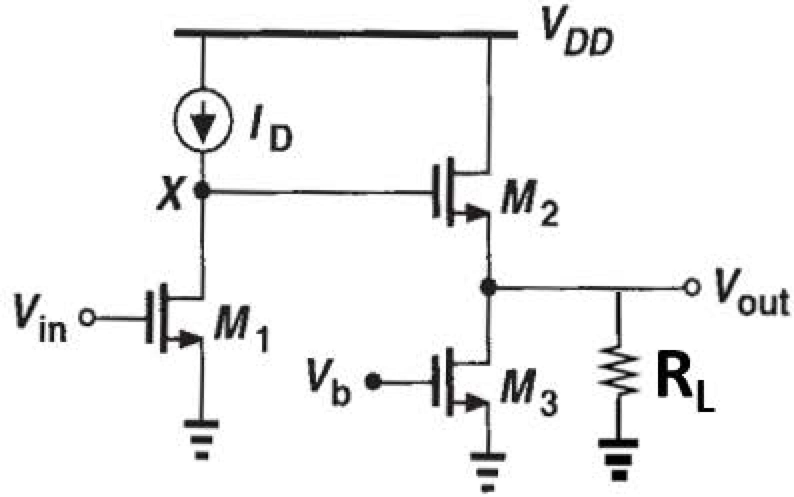


$$A_v = -gm_1 r_{o1} \times$$

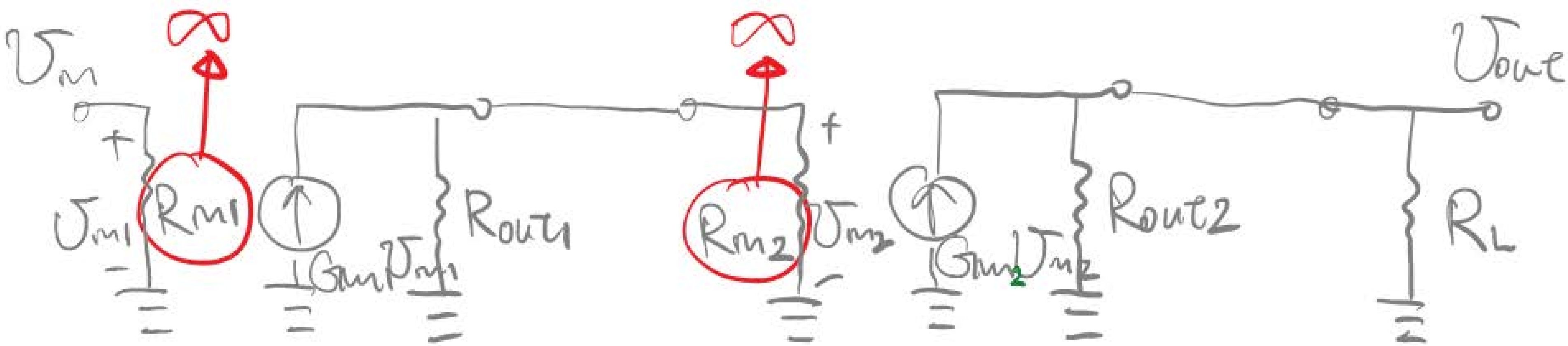
small

$$gm_2\left(r_{o2} \parallel \frac{1}{gm_2 + gmb_2} \parallel r_{o3} \parallel R_L\right)$$

- Voltage gain maintained when R_L very small



$$A_v = -gm_1 r_{o1} \times gm_2\left(r_{o2} \parallel \frac{1}{gm_2 + gmb_2} \parallel r_{o3} \parallel R_L\right)$$



$G_{m1} = -g_{m1}$

$R_{out1} = r_{o1}$

$G_{m2} = g_{m2}$

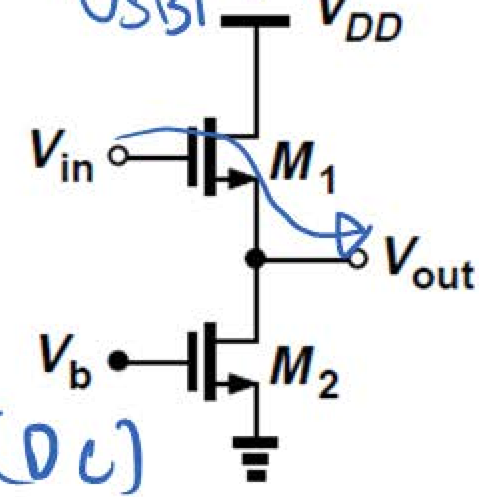
$R_{out2} = r_{o2} // \frac{1}{g_{m2} + g_{mb2}} // r_{o3}$

Example

$V_{SB1} \neq 0$

$V_{SB1} \neq 0$

$(W/L)_1 = 20/0.5$, $I_D = 0.2$ mA, $V_{THO} = 0.6$ V, $2\phi_F = 0.7$ V, $\mu_n C_{ox} = 50\ \mu A/V^2$, $\gamma = 0.4\ V^{1/2}$ and $\lambda = 0$. (a) Calculate V_{out} for $V_{in} = 1.2$ V. (b) Minimum $(W/L)_2$ for which M_2 remains saturated.



Solution:

(a)

V_{TH1}

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L_{eff,1}}(V_{in} - V_{out} - V_{THO})^2 \rightarrow V_{out} = 0.153\ \text{V} = V_{SB1}$$

$$V_{TH1} = V_{THO} + \gamma\left(\sqrt{2\Phi_F + V_{out}} - \sqrt{2\Phi_F}\right) = 0.635\ \text{V} \rightarrow V_{out} \approx 0.118\ \text{V}$$

(b)

$$V_{out} = 0.118\ \text{V} \geq V_{GS2} - V_{TH2}$$ for M_2 to stay in sat.

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS2} - V_{TH2})^2 \rightarrow \left(\frac{W}{L}\right)_2 \geq \frac{283}{0.5}$$

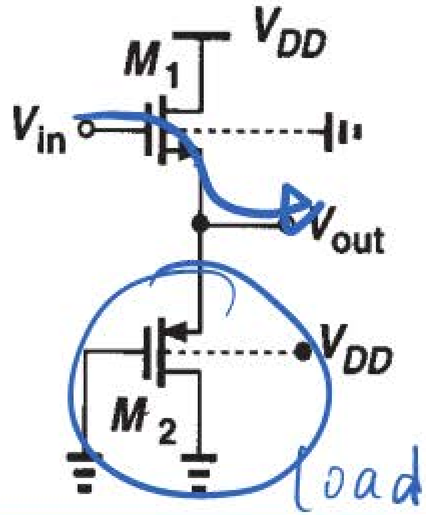
Example

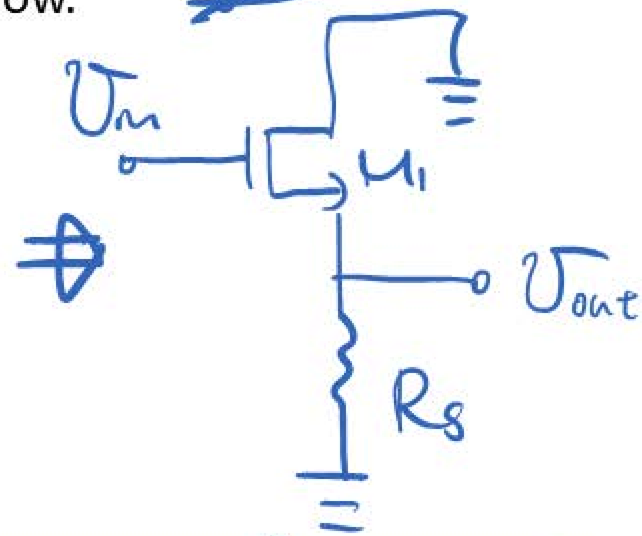
small signal

Calculate the small signal voltage gain of the circuit below.

Assume all in sat.

Assume $\lambda \neq 0, \gamma \neq 0$





Solution:

$R_S = r_{o2} \parallel \left(\frac{1}{g_{m2}+g_{mb2}}\right)$

$$G_m = gm_1$$

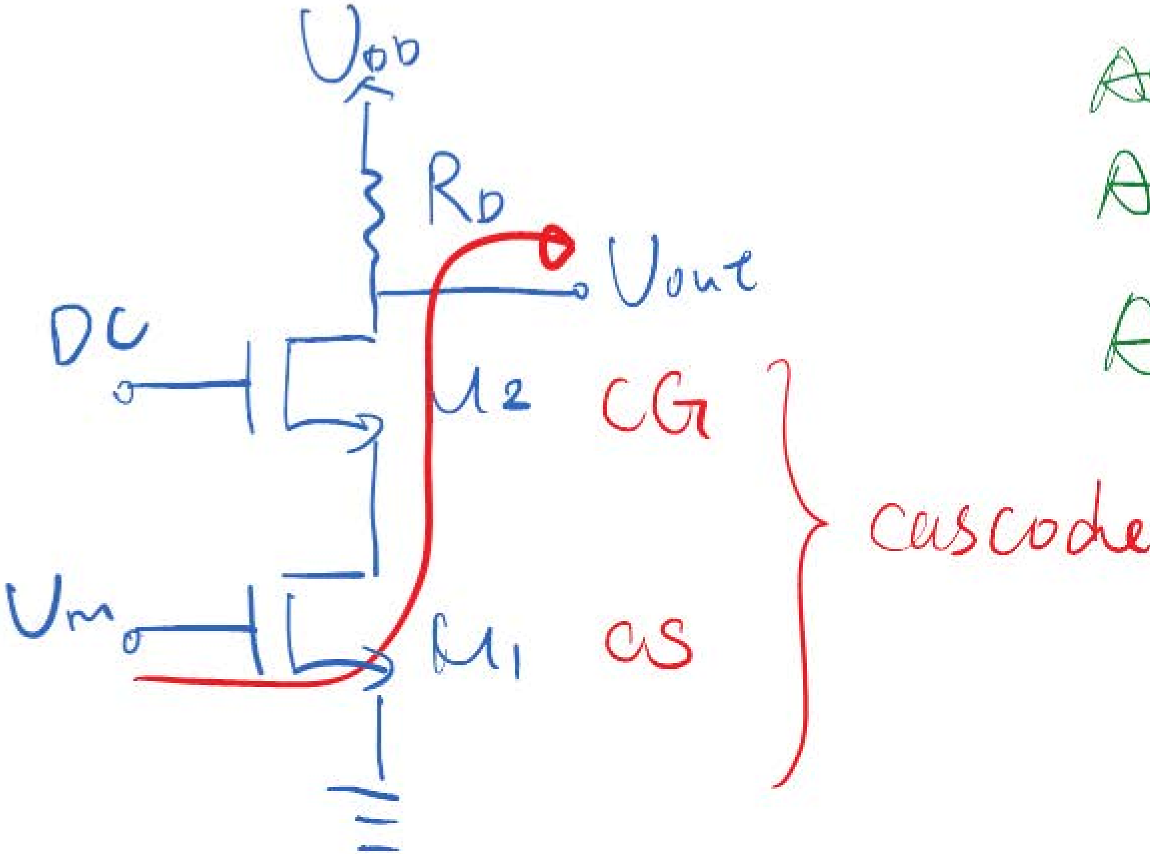
$$R_{out} = \frac{1}{gm_1 + gmb_1} \parallel r_{o1} \parallel \frac{1}{gm_2 + gmb_2} \parallel r_{o2}$$

$$A_v = G_m R_{out}$$

CS
SF } both are voltage-in-voltage-out amplifiers.

Common-Gate (CG)

Current-in-voltage-out

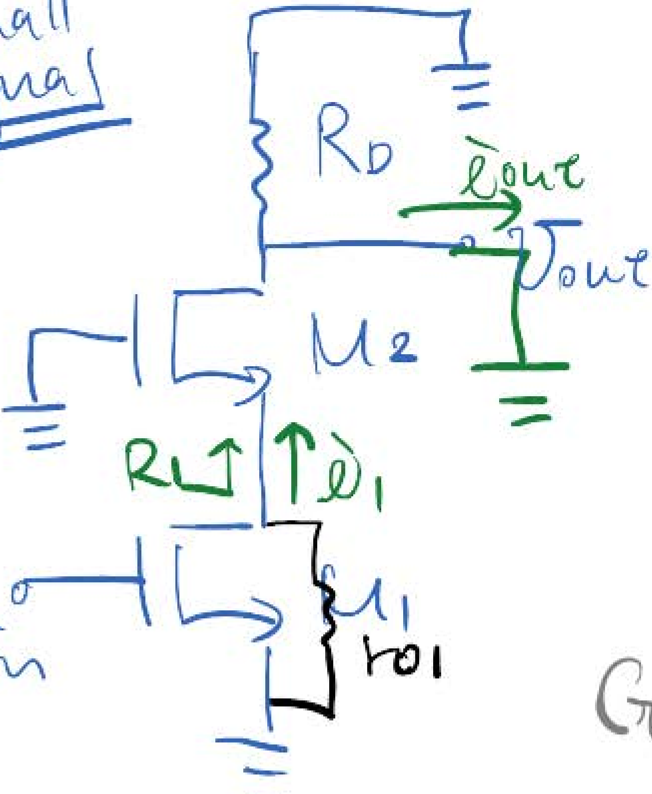


Assume M_1 and M_2 in sat.

Assume $\lambda \neq 0$, $\gamma \neq 0$

$A_v = ?$

small signal



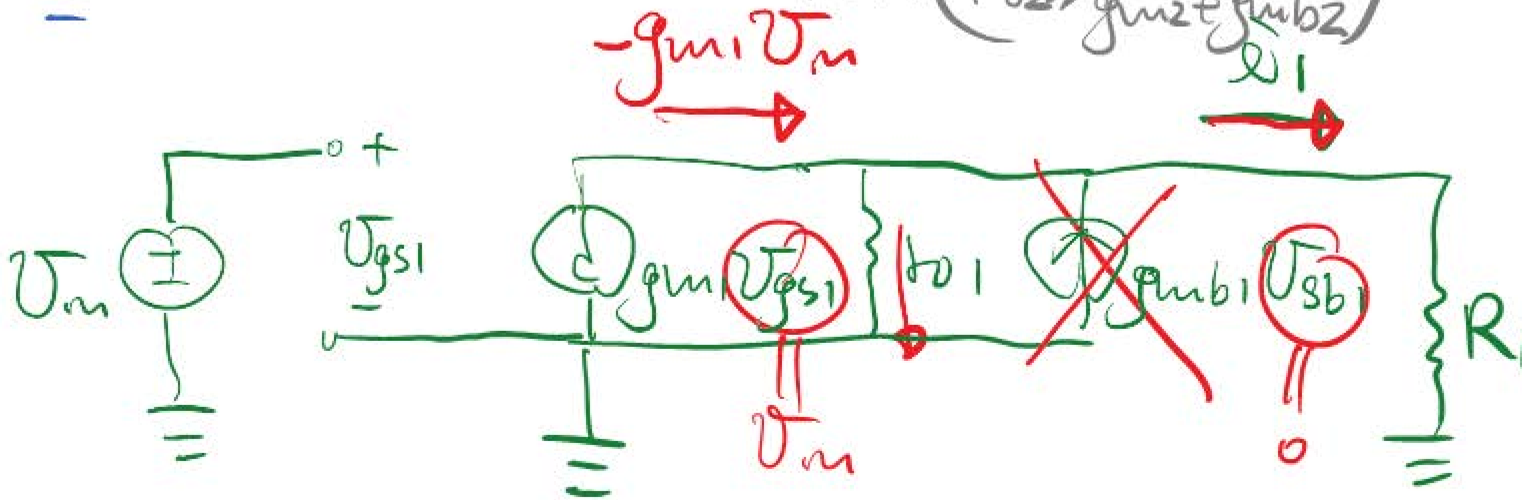
$$G_m = i_{out} / v_{in}$$

$$i_1 = i_{out}$$

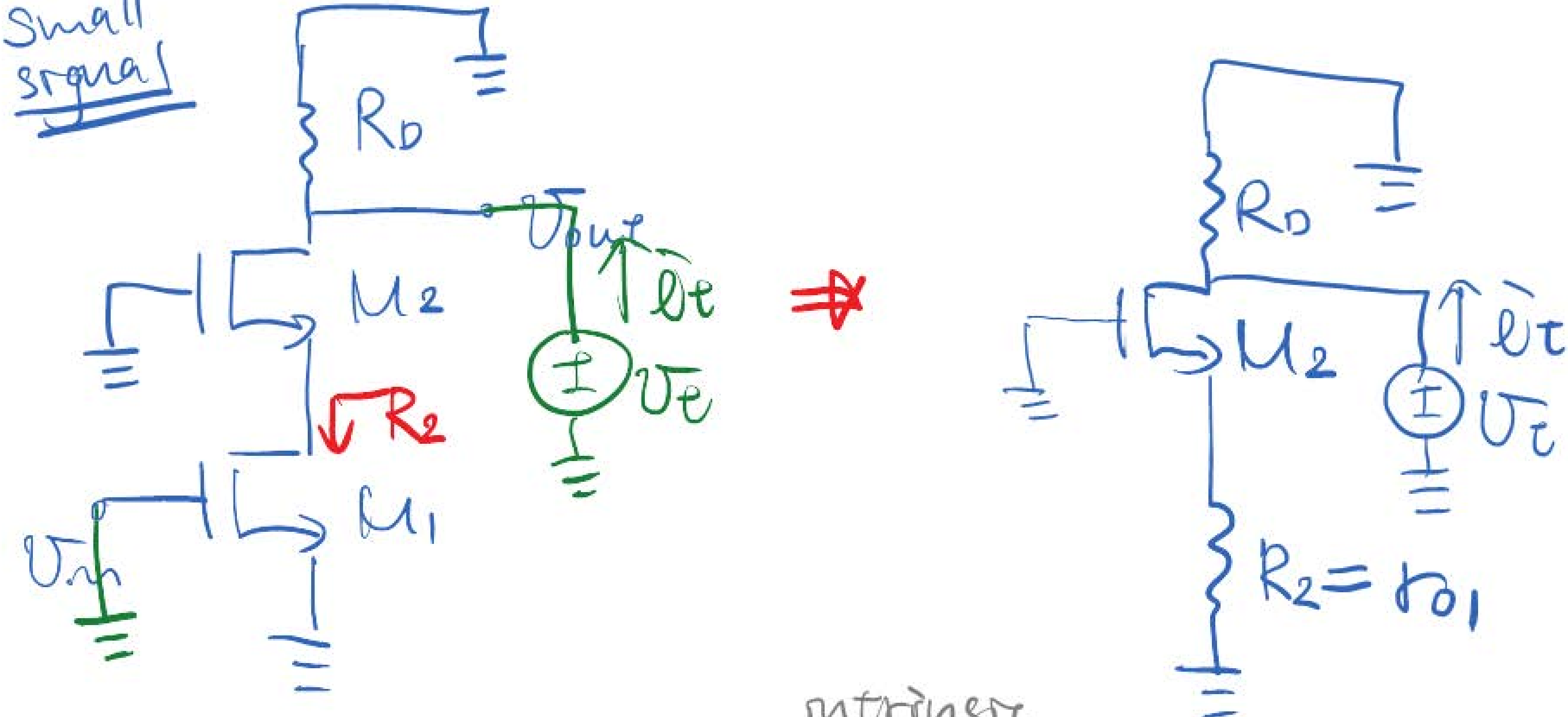
$$R_1 = r_{o2} \,//\, \frac{1}{g_{m2}+g_{mb2}}$$ small

$$i_1 = (-g_{m1} v_{in}) \frac{r_{o1}}{r_{o1} + R_1}$$

$$G_m = -g_{m1} \frac{r_{o1}}{r_{o1} + \left(r_{o2} \,//\, \frac{1}{g_{m2}+g_{mb2}}\right)}$$



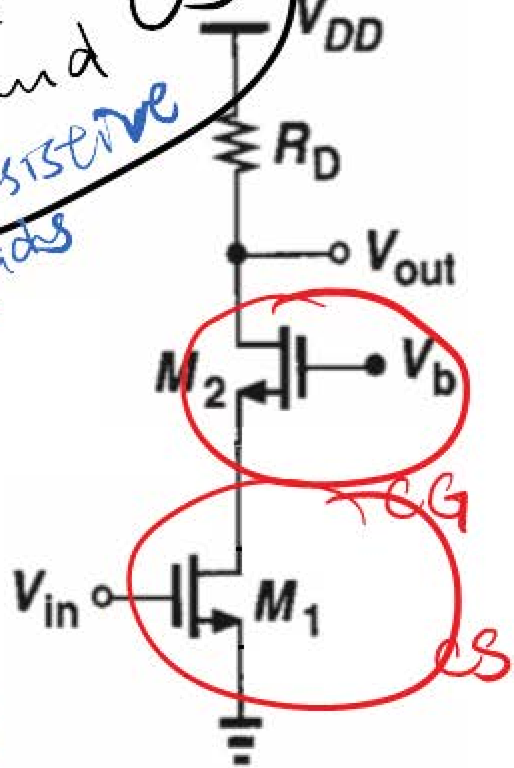
Small signal

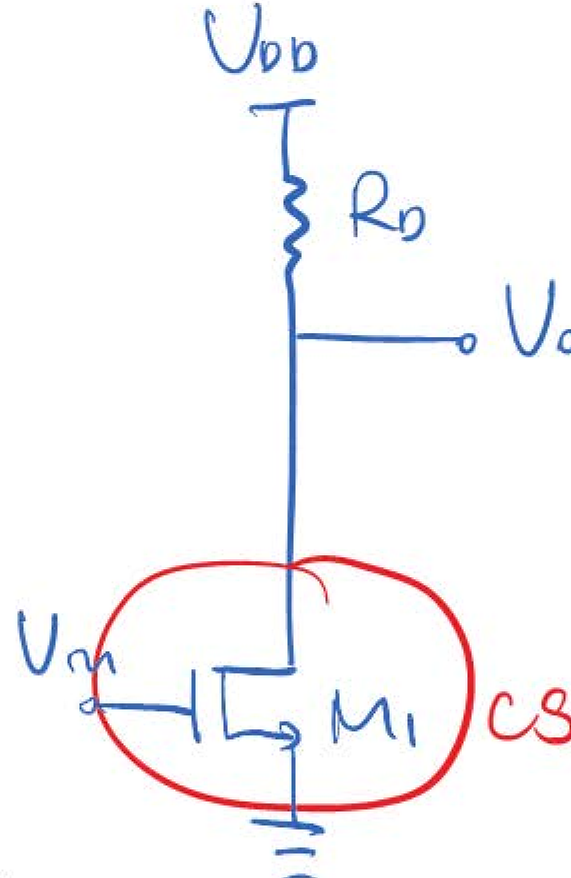


$$R_{out} = R_D // \left[r_{o2} + r_{o1} + \underbrace{(g_{m2} + g_{mb2}) r_{o2}}_{\text{intrinsic gain of } M_2} r_{o1} \right]$$

Comparison between CG+CS and CS with resistive loads

cascode



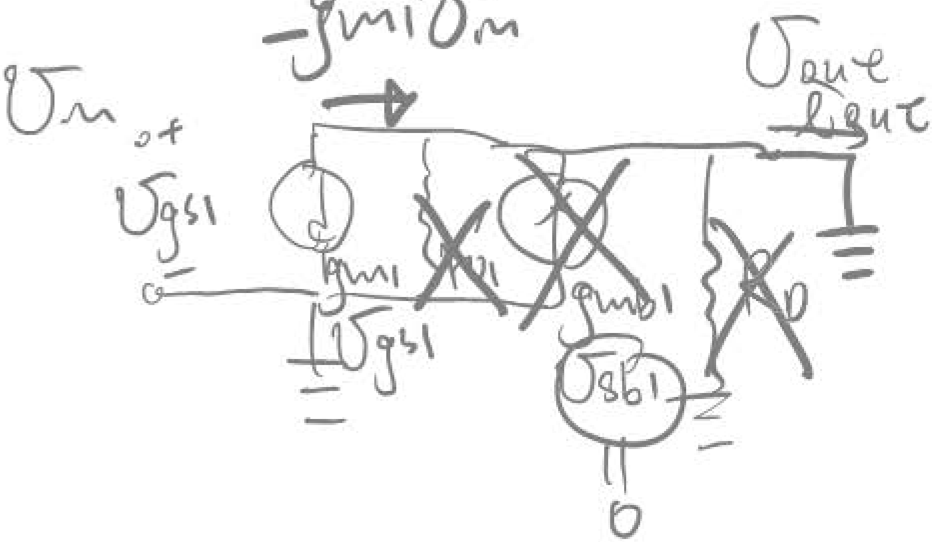


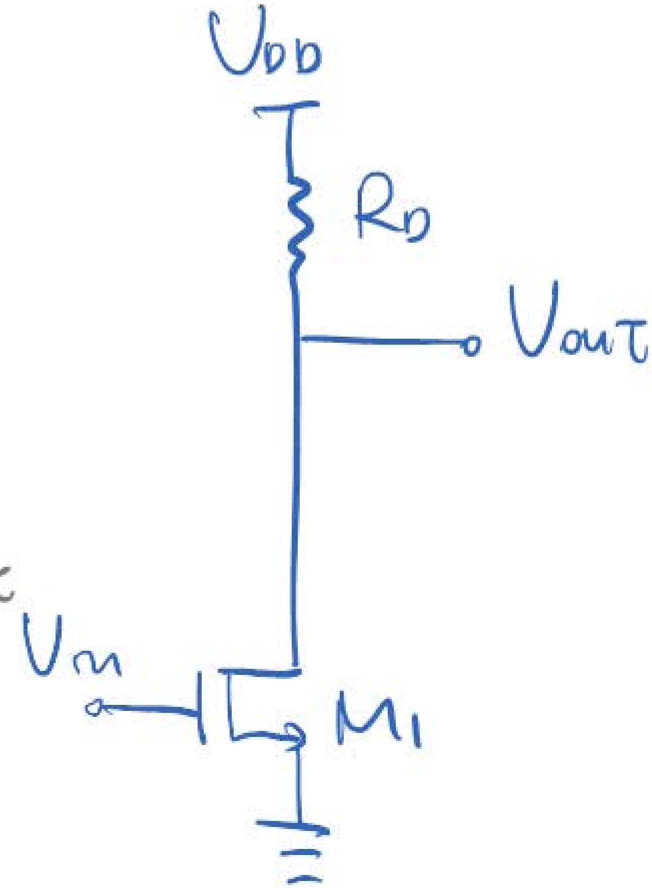
$$G_m = -g_{m1}\frac{r_{o1}}{r_{o1}+\left(r_{o2} \parallel \frac{1}{g_{m2}+g_{mb2}}\right)}$$

$$G_m = -g_{m1}$$

$$R_{out} = R_D \parallel \left[r_{o2}+r_{o1}+(g_{m2}+g_{mb2})r_{o2}r_{o1}\right]$$

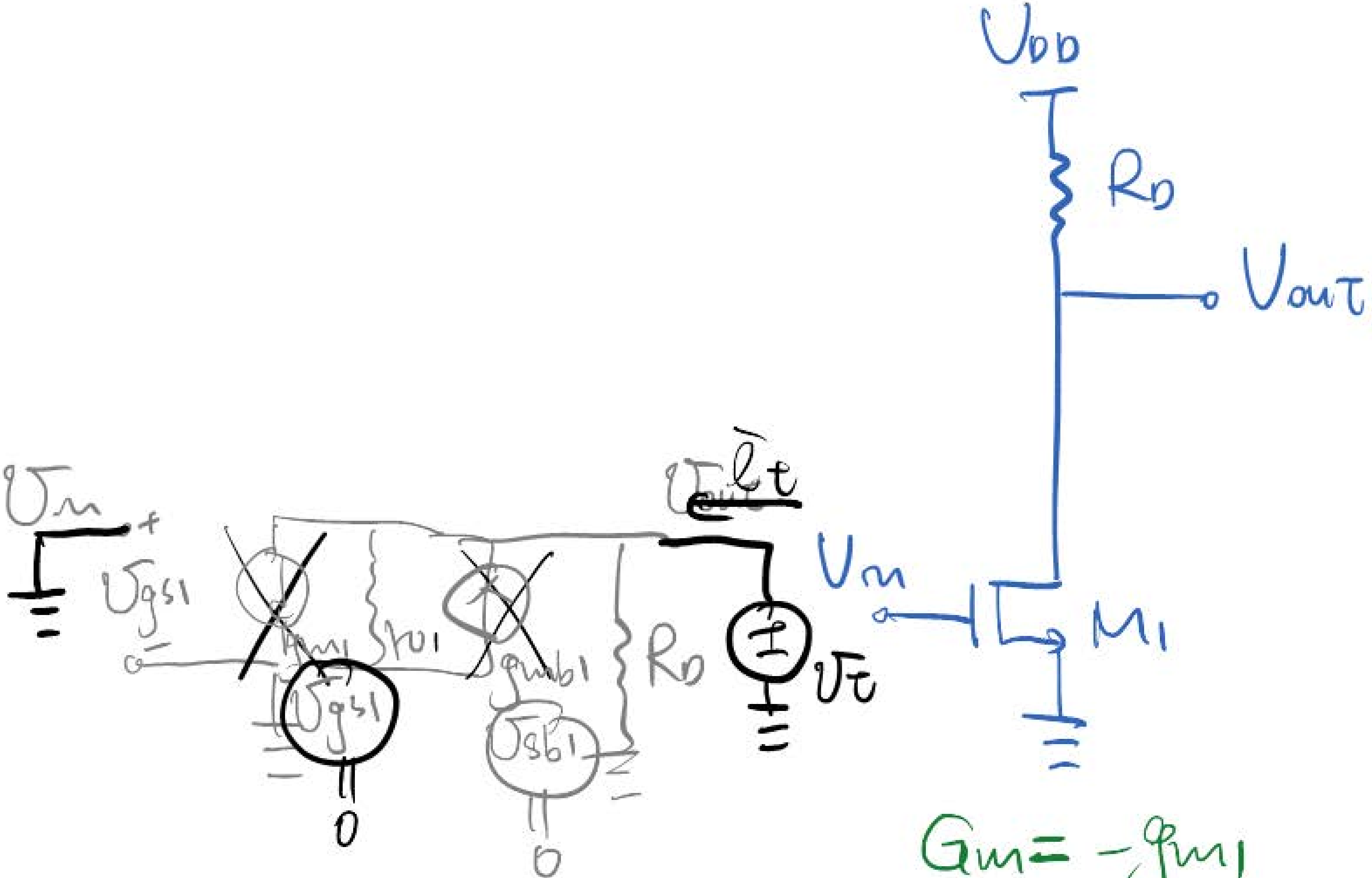
$$R_{out} = r_{o1} \parallel R_D$$





$G_m = -g_{m1}$

$i_{out} = -g_{m1} v_{in}$

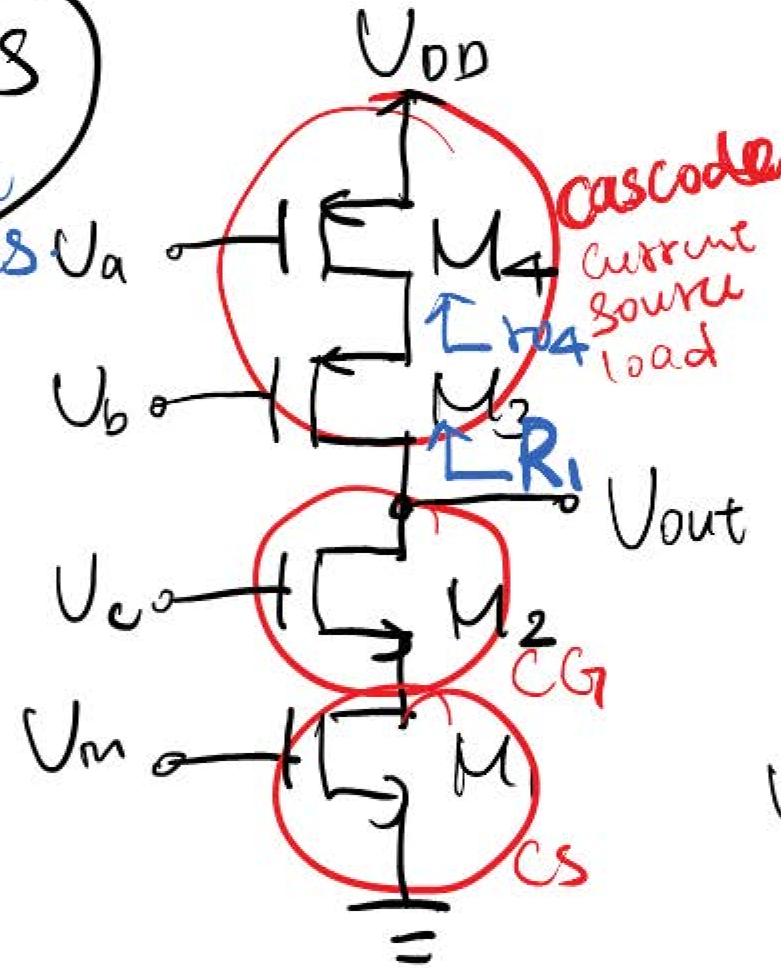


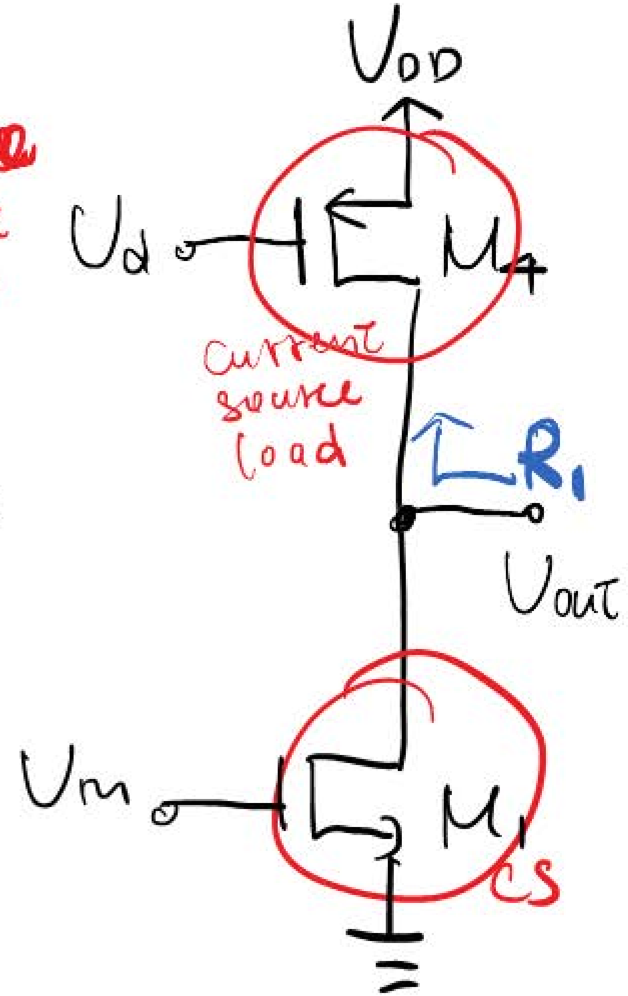
$G_m = -g_{m1}$

$$R_{out} = \frac{v_t}{i_t} = R_D \parallel r_{o1}$$

Comparison between CG+CS and CS with current source loads.

* Assume all in sat.
* Assume $\lambda \neq 0, \gamma \neq 0$
* $V_{a,b,c,d}$ are DC



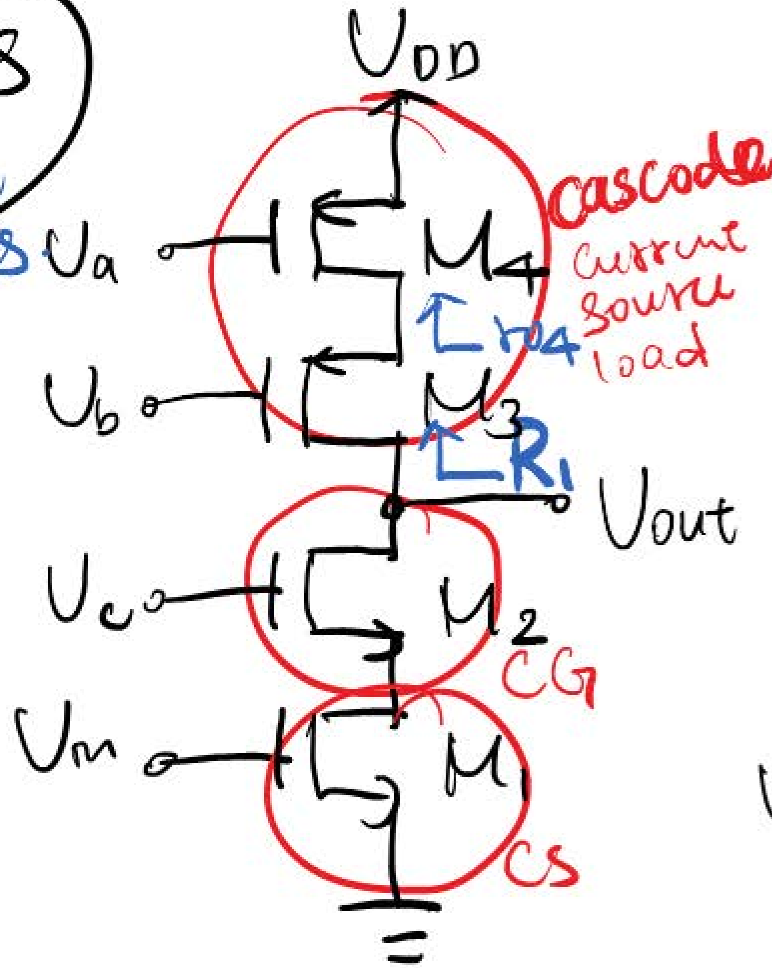


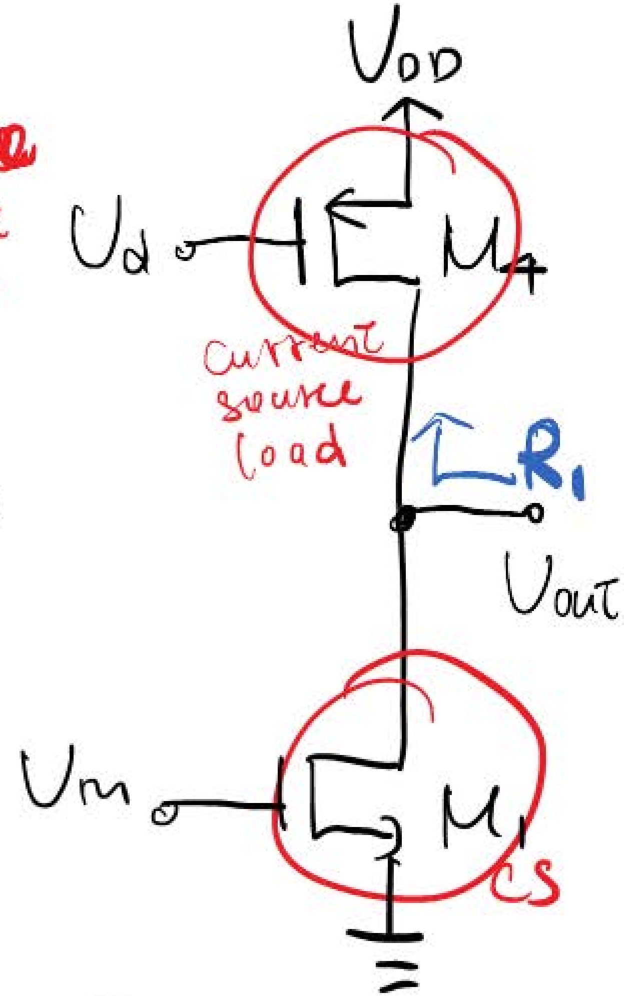
$$R_1 = r_{o3} + r_{o4} + (g_{m3} + g_{mb3}) r_{o3} r_{o4}$$

intrinsic gain of M_3

$$R_1 = r_{o4}$$

Comparison between CG+CS and CS with current source loads.



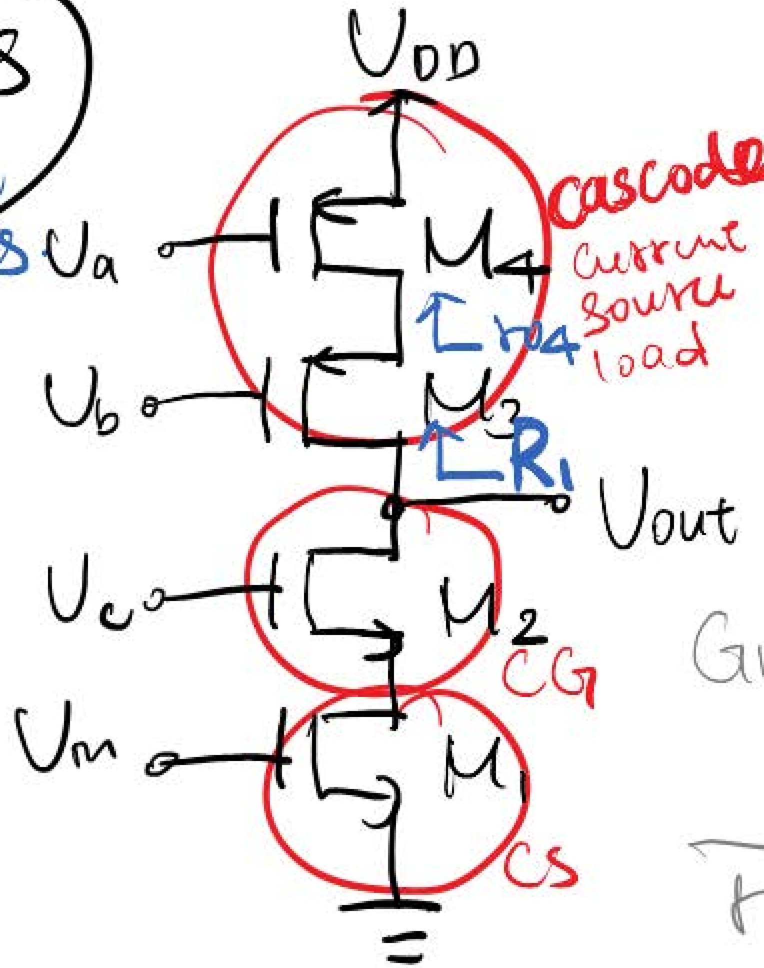


* Assume all in sat.
* Assume $\lambda \neq 0, \gamma \neq 0$
* $V_{a,b,c,d}$ are DC

$G_m = -g_{m1}$

$R_{out} = r_{o1} // r_{o4}$

Comparison between CG+CS and CS with current source loads.



* Assume all in sat.

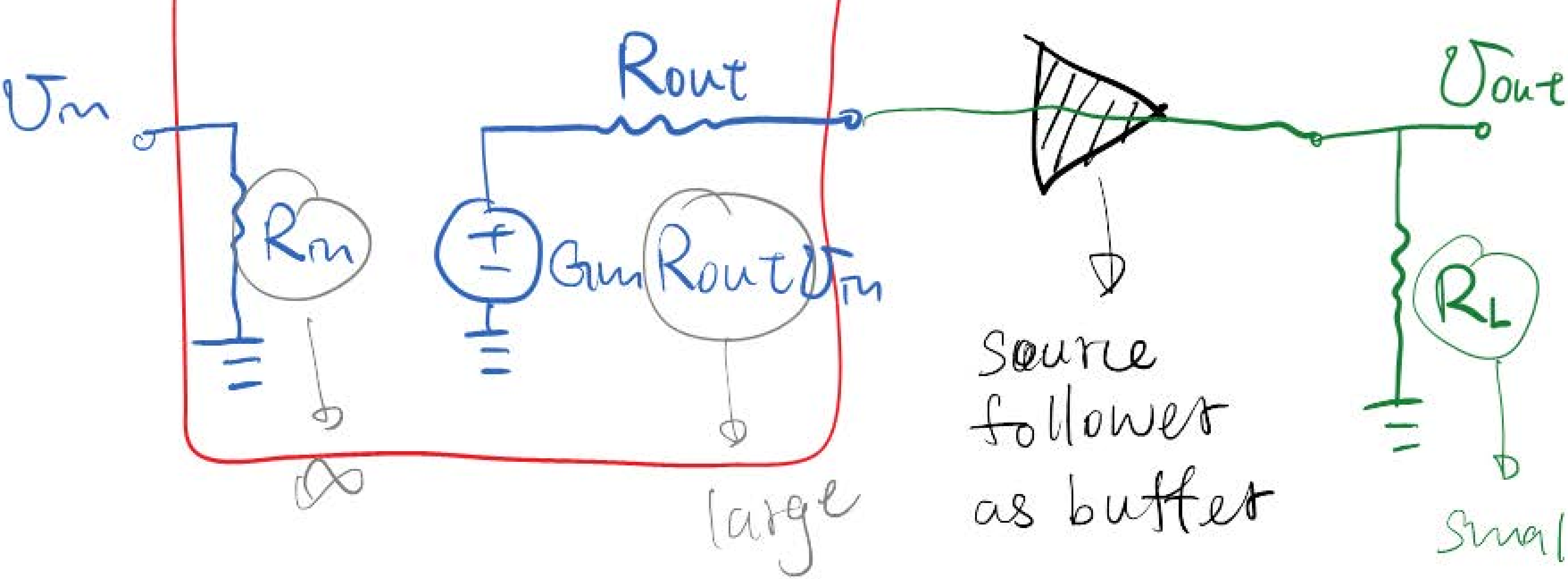
* Assume $\lambda \neq 0, \gamma \neq 0$

* $V_{a,b,c,d}$ are DC

$$G_m = -g_{m1} \cdot \frac{r_{o1}}{r_{o1} + \left(r_{o2} \parallel \frac{1}{g_{m2}+g_{mb2}} \right)}$$

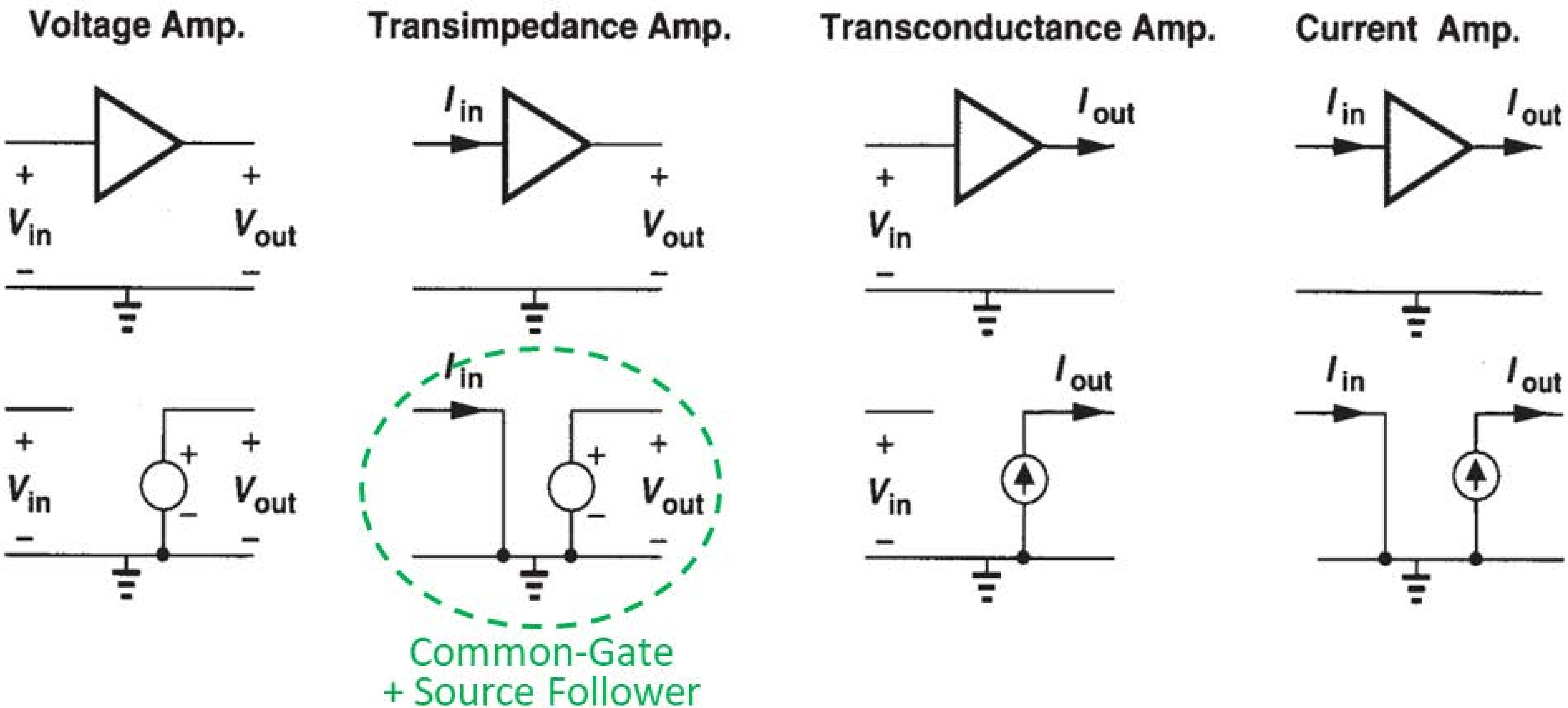
$$R_{out} = \left[r_{o3} + r_{o4} + (g_{m3}+g_{mb3}) r_{o3} r_{o4} \right] \parallel \left[r_{o2} + r_{o1} + (g_{m2}+g_{mb2}) r_{o2} r_{o1} \right]$$

Equivalent small signal circuit for cascode with cascode current source.



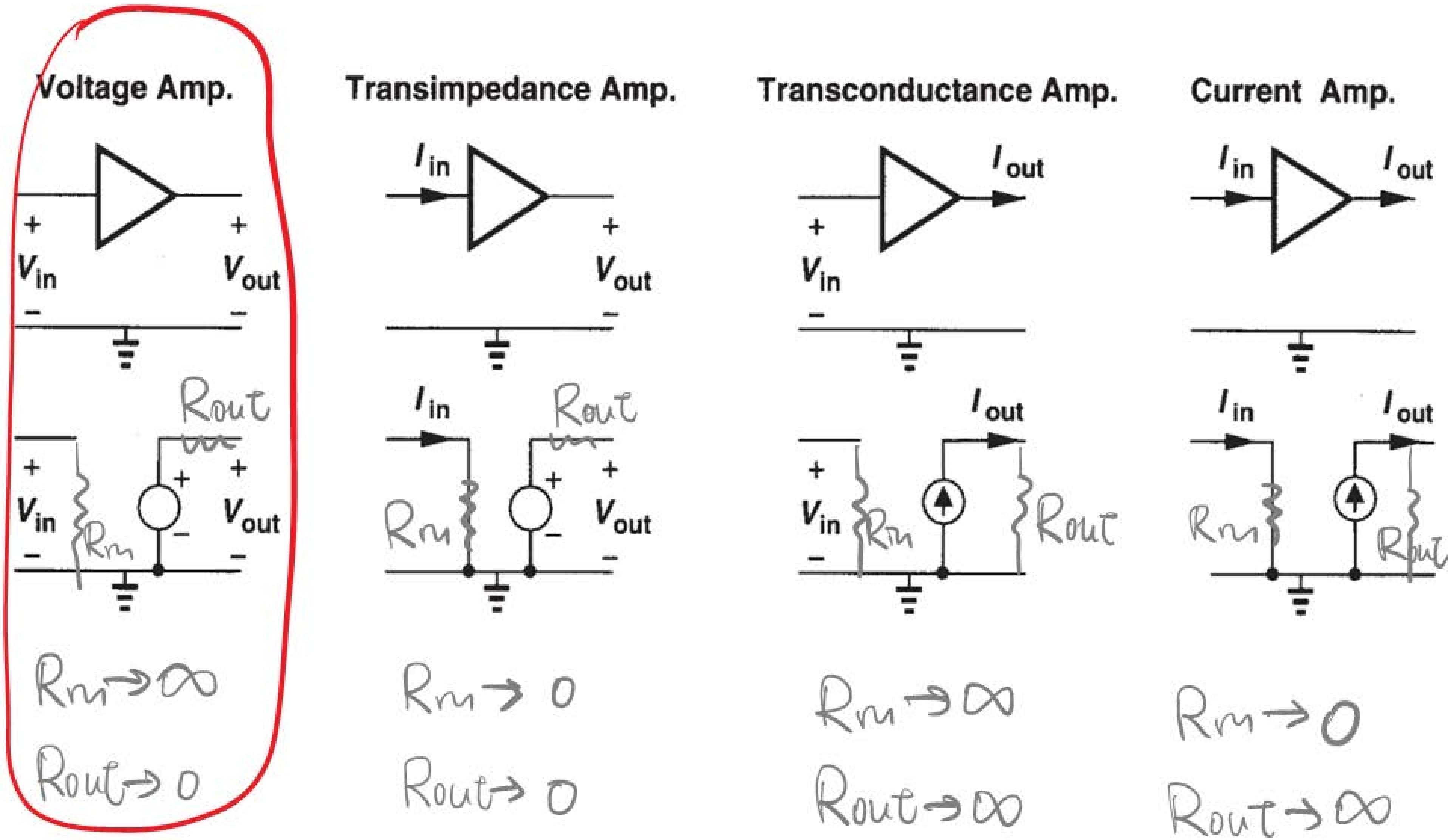
$$A_v = \frac{v_{out}}{v_{in}} = G_m R_{out} \frac{R_L}{R_{out} + R_L}$$ (without buffer)

Ideal Amplifier

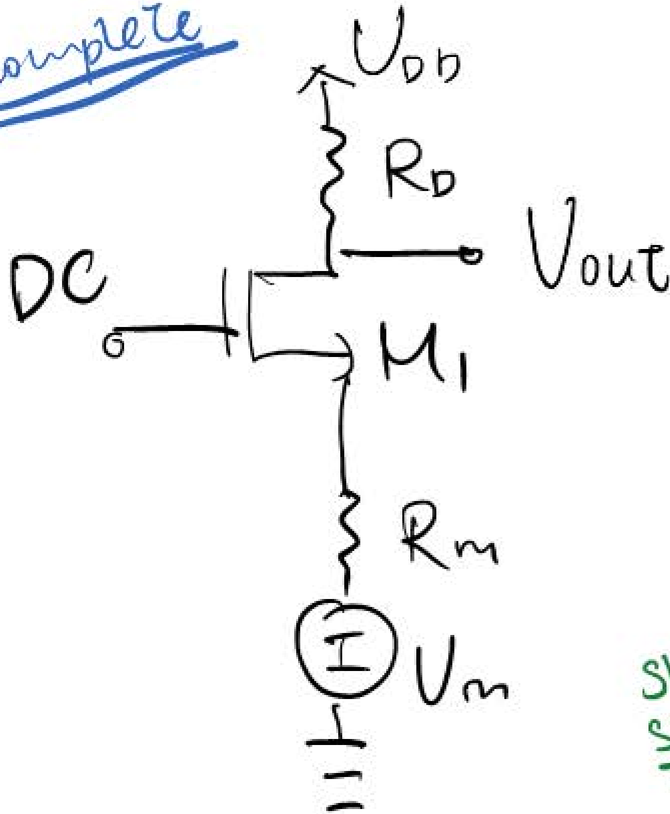


- For converting and amplifying small-signal current to voltages, common-gate provides **low input impedance** and **moderate gain**, but relatively **large output impedance**.

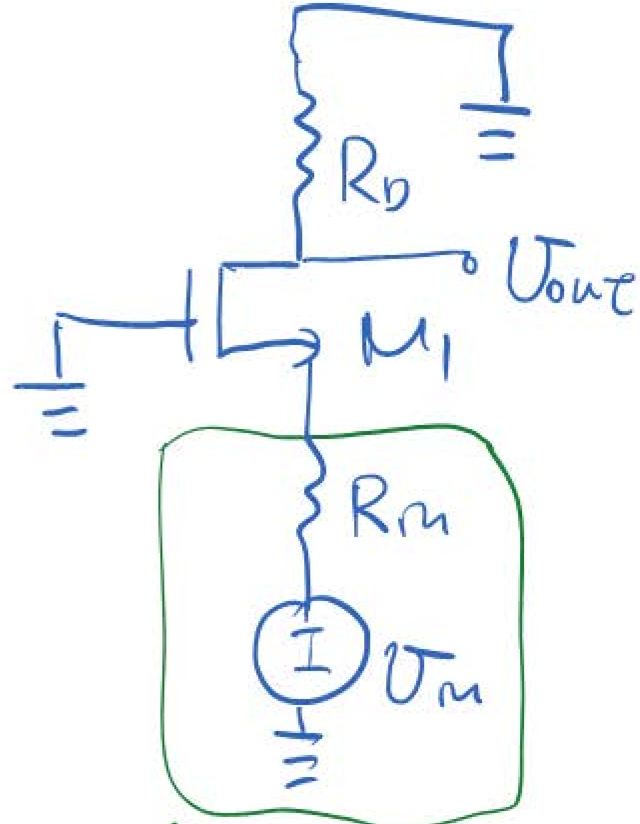
NOT Ideal Amplifier



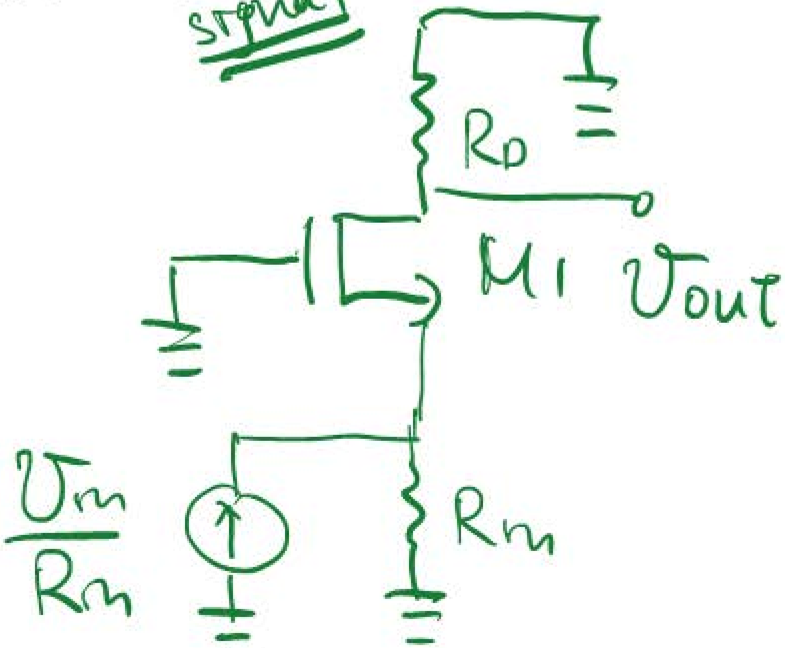
complete



small signal

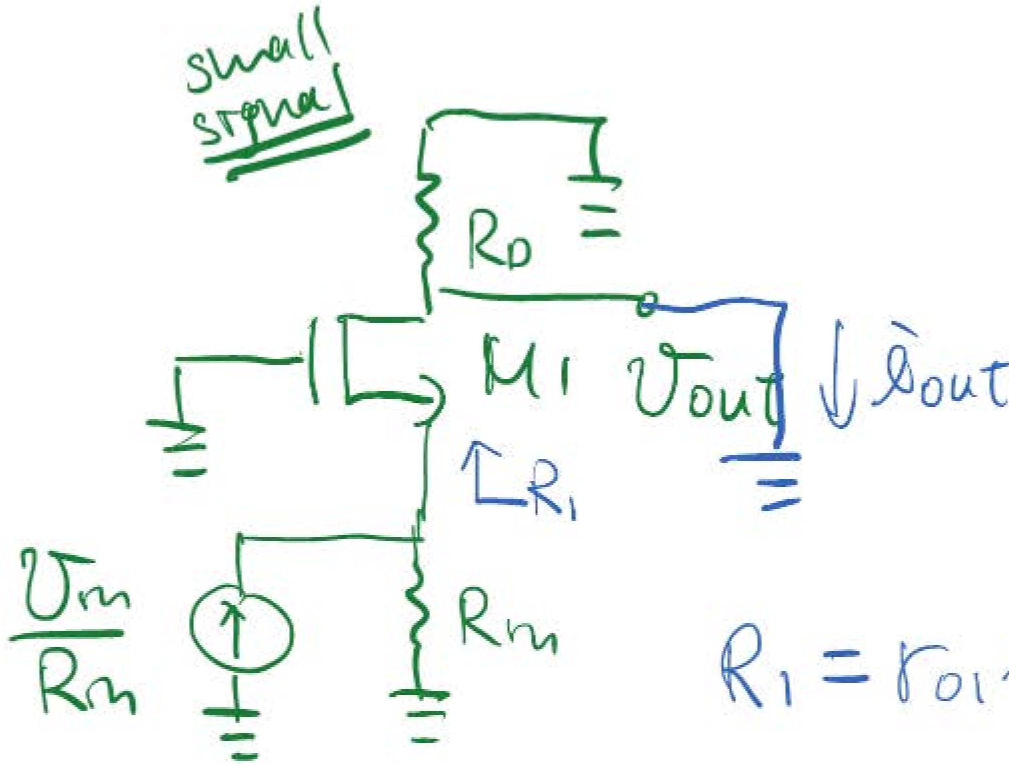


small signal



When calculating $G_m = ?$

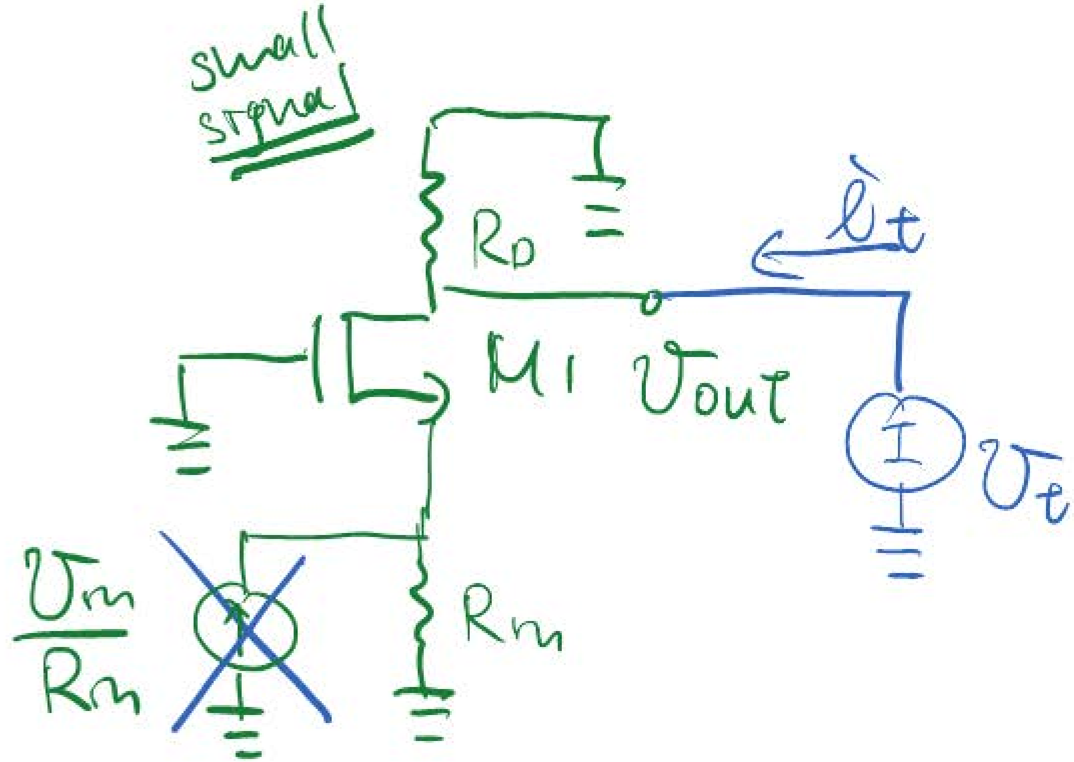
$$i_{out} = \frac{v_{in}}{\cancel{R_{in}}} \cdot \frac{\cancel{R_{in}}}{R_1 + R_{in}} = v_{in} \frac{1}{\left(r_{o1} \parallel \frac{1}{g_{m1}+g_{mb1}}\right) + R_{in}}$$



$$R_1 = r_{o1} \parallel \frac{1}{g_{m1}+g_{mb1}}$$

When calculating R_{out} = ?

$$R_{out} = \frac{v_t}{i_t} = R_D \parallel [r_{o1} + R_{in} + (g_{m1} + g_{mb1}) r_{o1} R_{in}]$$



Common-Gate

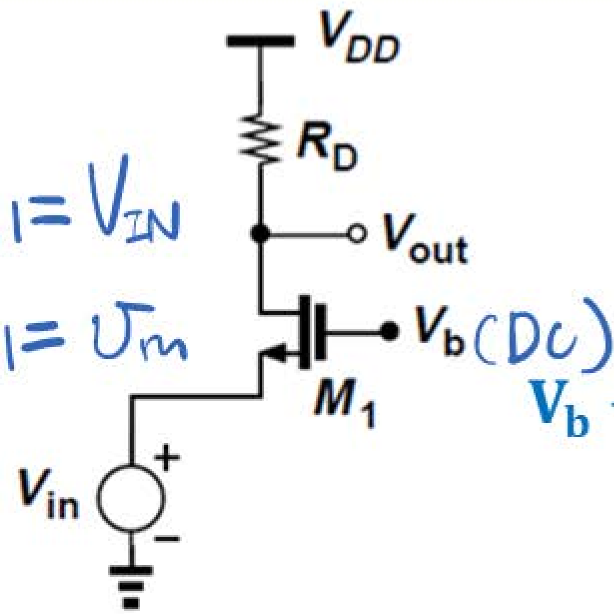
DC Analysis

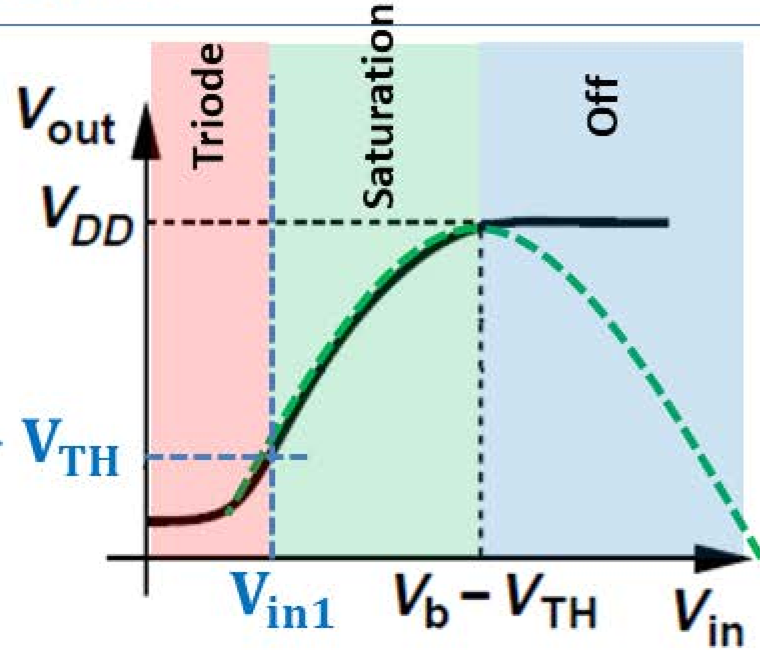
$\lambda = 0$

$\gamma \neq 0$

$V_{SB1} = V_{IN}$

$v_{sb1} = v_{in}$





$$V_{out} = V_b - V_{TH}$$
$$= V_{DD} - R_D \frac{1}{2}\mu_n C_{ox}\frac{W}{L_{eff}}(V_b - V_{in1} - V_{TH})^2$$

- **$V_{in} > V_b - V_{TH} \rightarrow M_1$ Off**

  $V_{out} = V_{DD}$

- **$V_b - V_{TH} > V_{in} > V_{in1} \rightarrow M_1$ in Saturation**

  $$V_{out} = V_{DD} - R_D \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_b - V_{in} - V_{TH})^2$$

- **$V_{in} < V_{in1} \rightarrow M_1$ in Triode**

  $$V_{out} = V_{DD} - R_D \mu_n C_{ox}\frac{W}{L}\left[(V_b - V_{in} - V_{TH})(V_{out} - V_{in}) - \frac{1}{2}(V_{out} - V_{in})^2\right]$$

Common-Gate

DC Analysis

$\lambda = 0$

$\gamma \neq 0$

- **$V_b - V_{TH} > V_{in} > V_{in1} \rightarrow M_1$ in Saturation**

$$V_{out} = V_{DD} - R_D \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_b - V_{in} - V_{TH})^2$$

$$\frac{\partial V_{out}}{\partial V_{in}} = -R_D \frac{1}{2} \mu_n C_{ox} \frac{W}{L} 2(V_b - V_{in} - V_{TH}) \left(-1 - \frac{\partial V_{TH}}{\partial V_{in}}\right)$$

$$= R_D \underbrace{\boxed{\mu_n C_{ox} \frac{W}{L} (V_b - V_{in} - V_{TH})}}_{= gm} \left(1 + \underbrace{\frac{\partial V_{TH}}{\partial V_{in}}}_{= \eta = \frac{\gamma}{2\sqrt{2\Phi_F + V_{SB}}}}\right)$$

$V_{in} = V_{SB}$

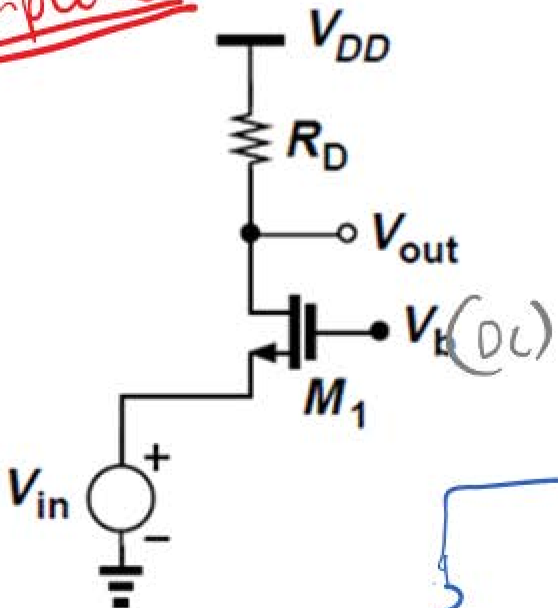
$$\boxed{A_v = \frac{\partial V_{out}}{\partial V_{in}} = R_D gm(1 + \eta)} = R_D(g_m + g_{mb})$$

- gm is a function of I_D and η is a function of V_{SB}.
- A_v is not quite linear.

complete

Assume $\lambda = 0$, $\gamma \neq 0$

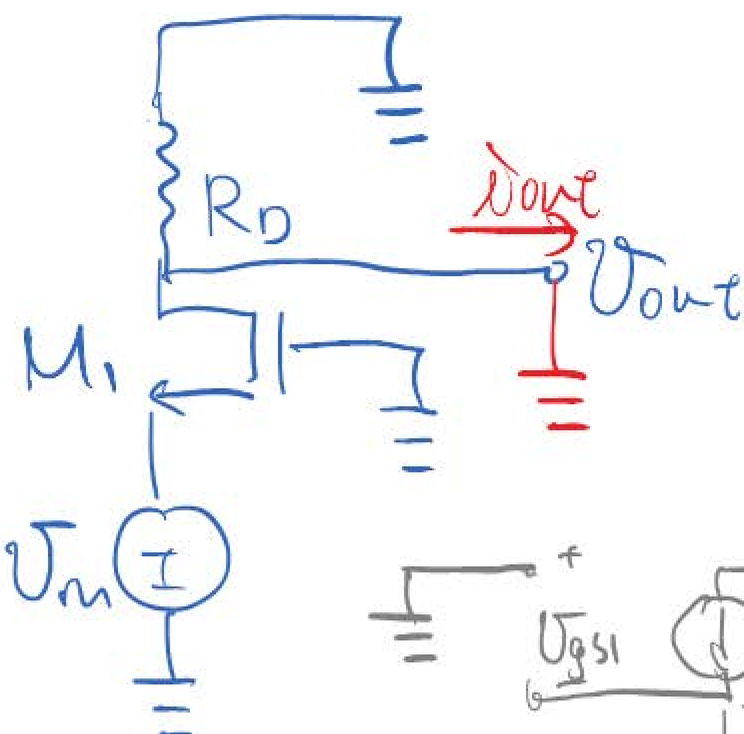
Assume M_1 in sat.



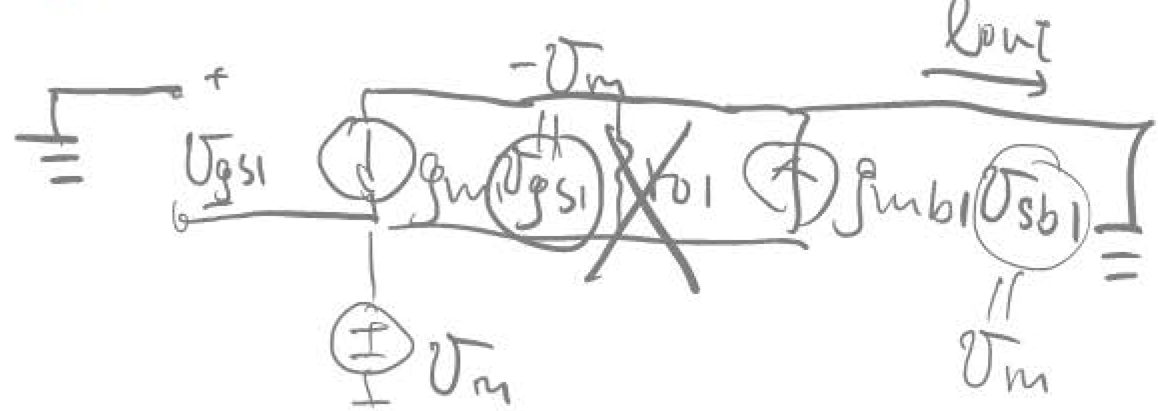
1° $G_m = ?$

$$G_m = \frac{i_{out}}{v_{in}}$$

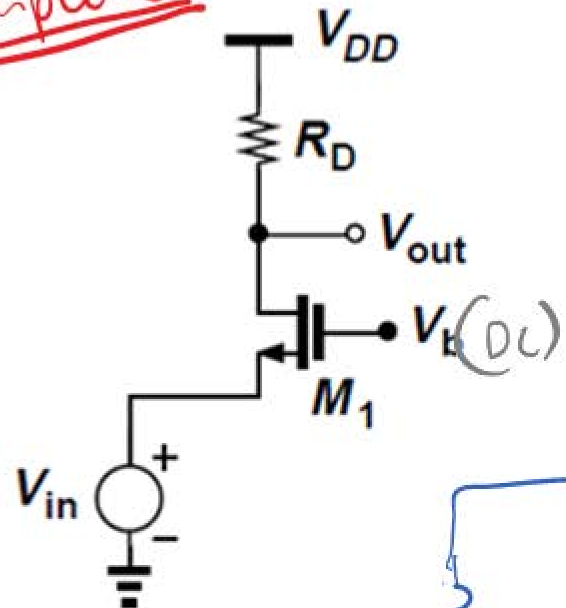
small signal



$$= g_{m1} + g_{mb1}$$



complete



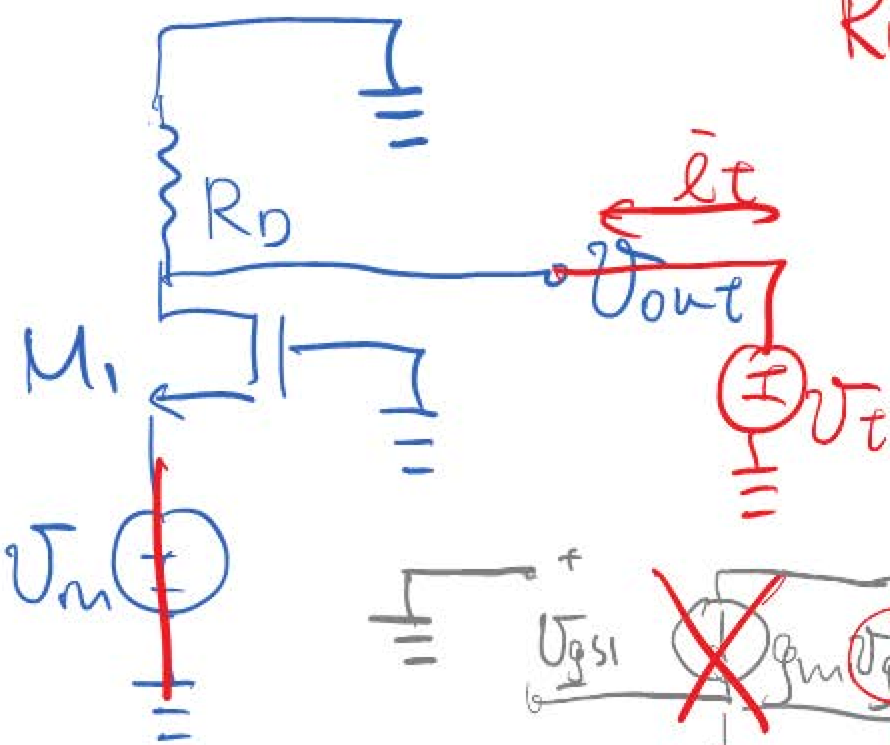
Assume $\lambda = 0$, $\gamma \neq 0$

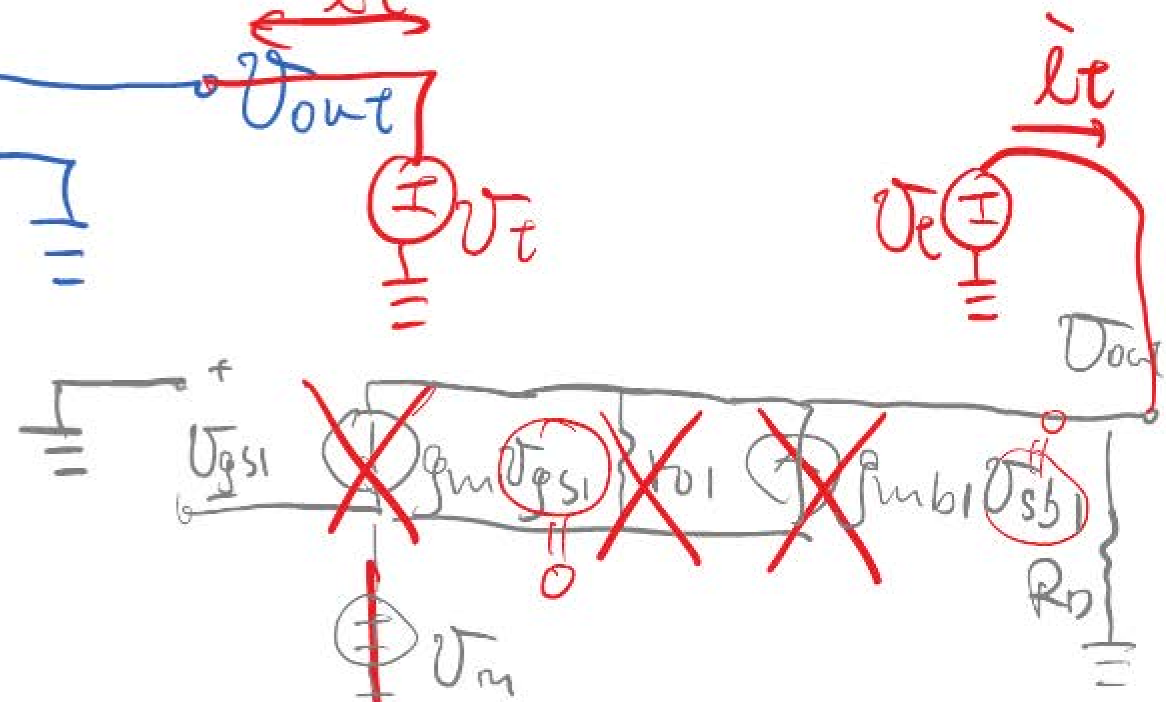
Assume M_1 in sat.

2° $R_{out} = ?$

$$R_{out} = \frac{v_t}{i_t} = R_D$$

small signal



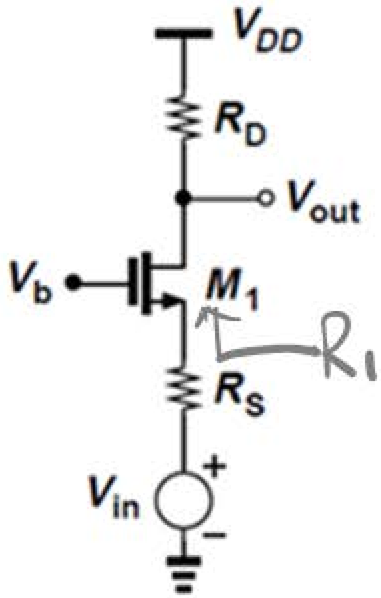


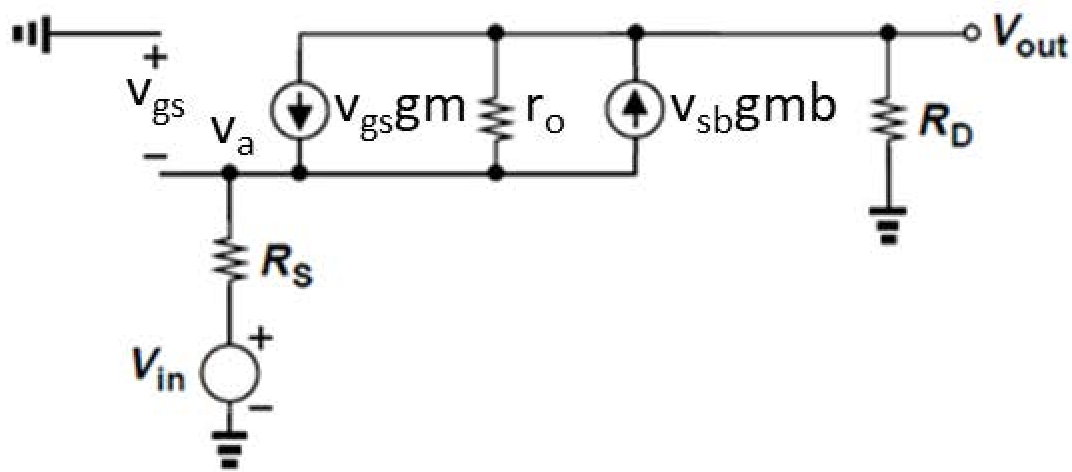
Common-Gate

Small-signal Analysis

$\lambda \neq 0$

$\gamma \neq 0$





$$G_m = \frac{(gm + gmb)r_o + 1}{r_o + R_S + (gm + gmb)r_o R_S}$$

$$R_{out} = R_D \parallel [r_o + R_S + (gm + gmb)r_o R_S]$$

$$\boxed{A_v = \frac{(gm + gmb)r_o + 1}{r_o + R_S + (gm + gmb)r_o R_S + R_D} R_D} \approx R_D gm(1 + \eta)$$

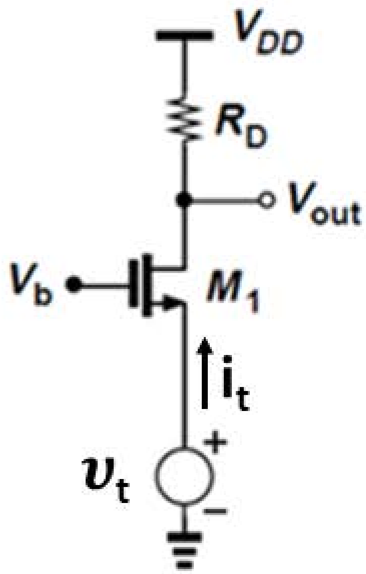
If $R_S = 0$ and $r_o = \infty$

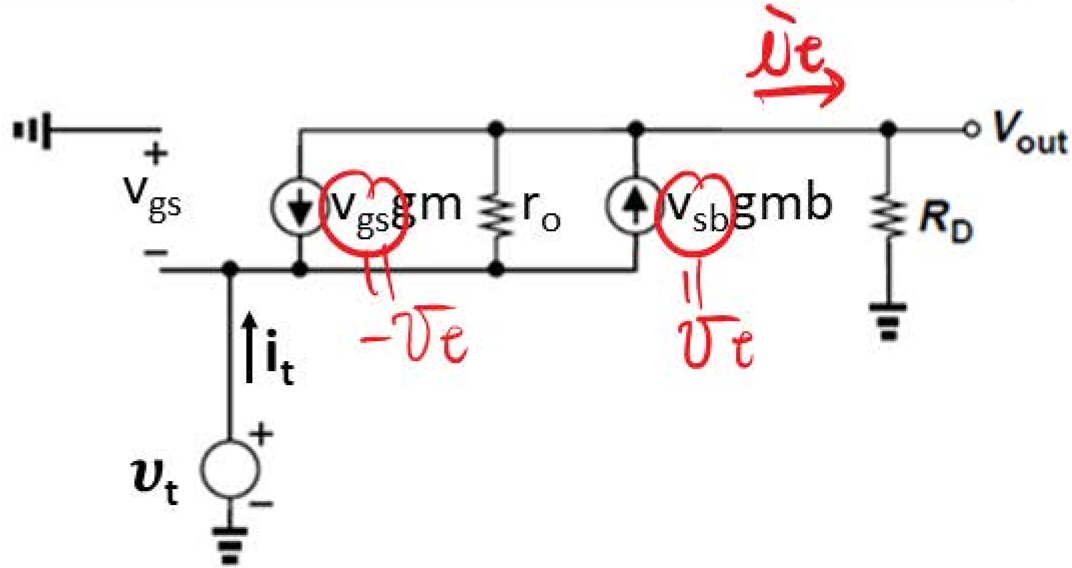
Common-Gate (Input Impedance)

Small-signal Analysis

$\lambda \neq 0$

$\gamma \neq 0$



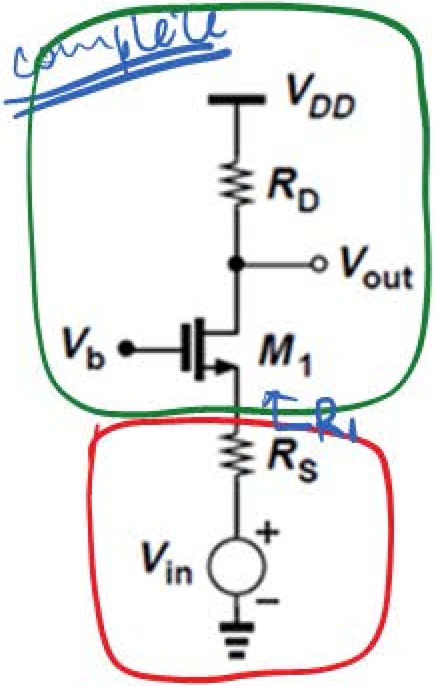


$$\begin{cases} i_t = \upsilon_t(gm + gmb) + \dfrac{\upsilon_t - \upsilon_{out}}{r_o} \\ \upsilon_{out} = R_D i_t \end{cases}$$

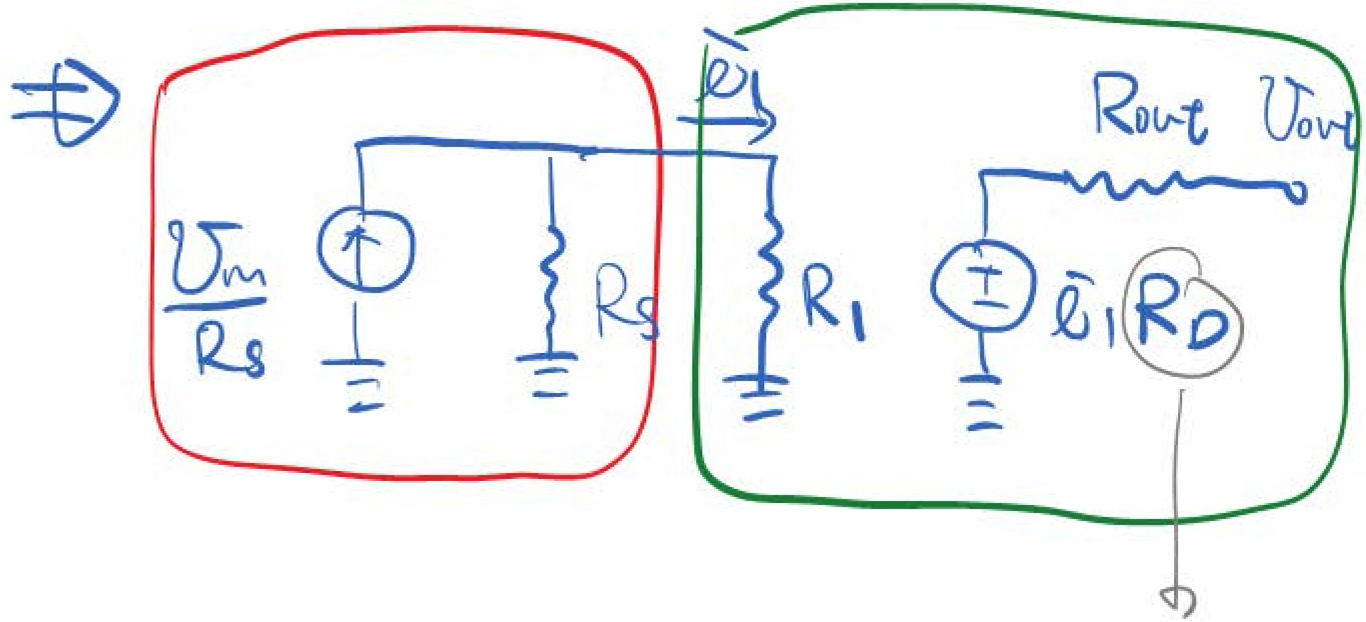
$$\boxed{R_{in} = \frac{R_D + r_o}{1 + (gm + gmb)r_o}}$$

If $R_D = 0$ → $R_{in} = r_o \parallel \dfrac{1}{gm} \parallel \dfrac{1}{gmb}$

If $R_D = \infty$ → $R_{in} = \infty$



Equivalent small-signal



$$R_1 = \frac{R_D + r_{o1}}{1 + (g_{m1} + g_{mb1}) r_{o1}}$$

trans-impedance gain.

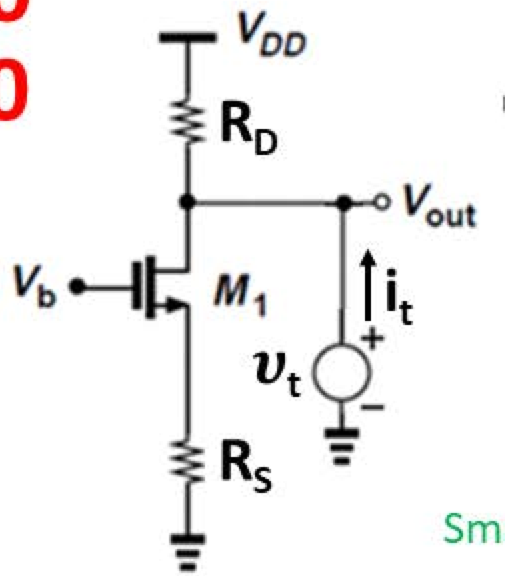
$$i_1 = \frac{R_S}{R_S + R_1}$$

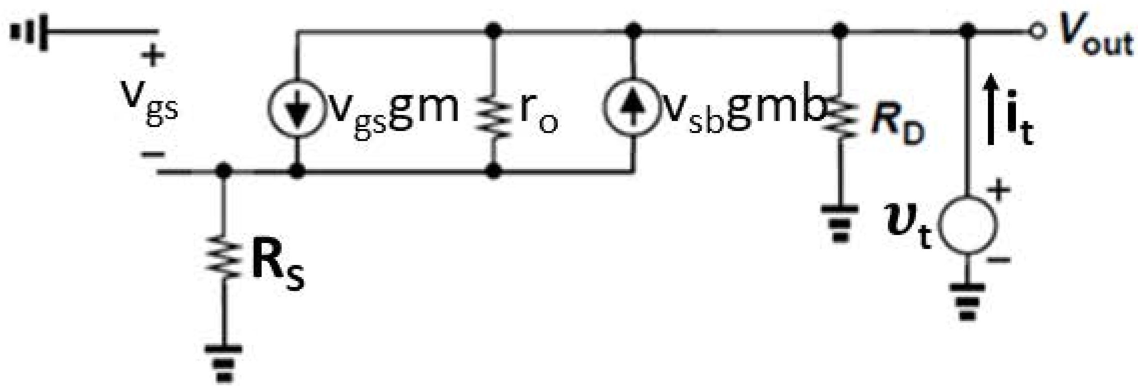
Common-Gate (Output Impedance)

Small-signal Analysis

$\lambda \neq 0$

$\gamma \neq 0$





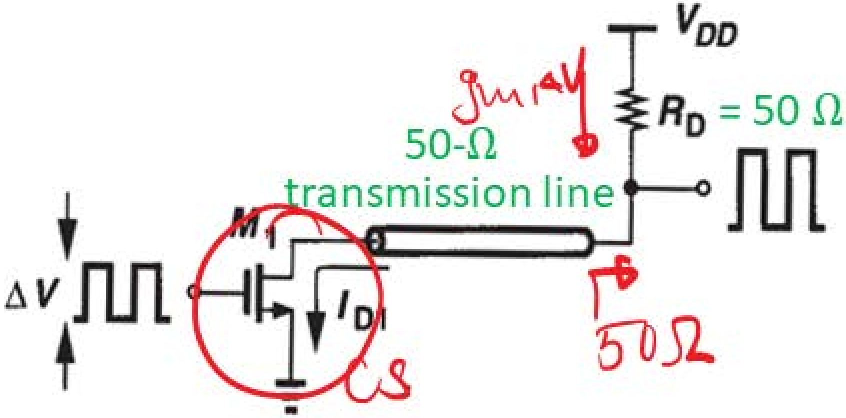
Small-signal model the same as CS with source degradation

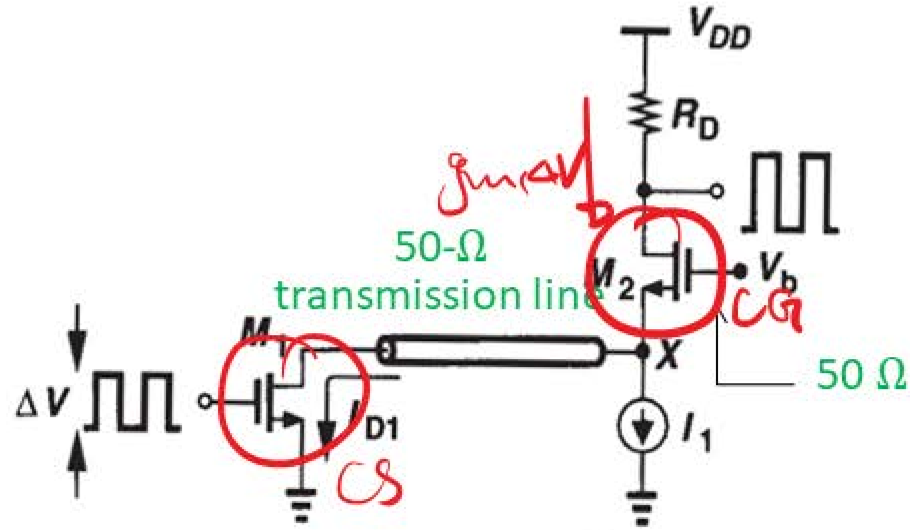
$$R_{out} = [R_S + r_{o1} + (gm_1 + gmb_1) r_{o1} R_S] \parallel R_D$$

Example

Calculate the small-signal voltage gain at low frequencies of the circuits below. To minimize wave reflection at point X, the input impedance must be equal to 50 Ω.

Assume $\lambda = 0$



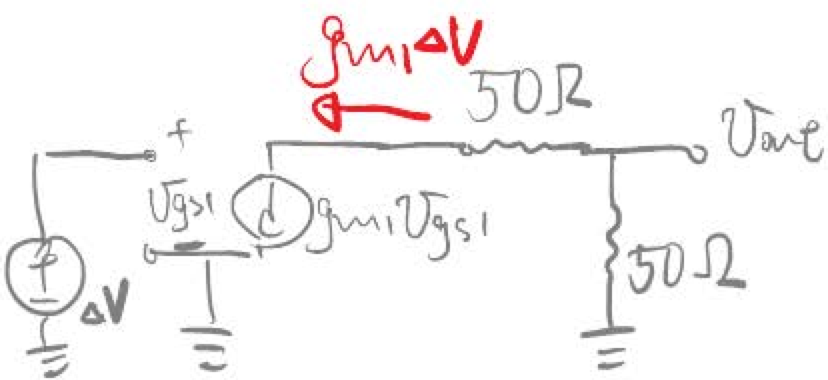


$$A_v = -gm_1 R_D$$

= 50 Ω

$$A_v = -gm_1 R_D$$

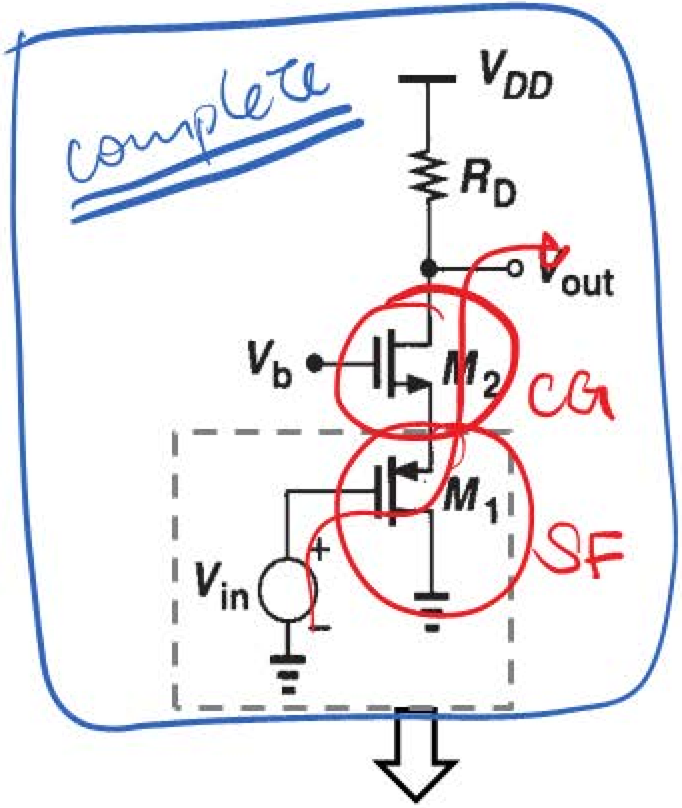
can be much larger than 50 Ω, so as to achieve a much higher gain.



$$R_{in} = \frac{R_D + r_{o2}}{1 + (gm_2 + gmb_2)r_{o2}} = 50\ \Omega$$

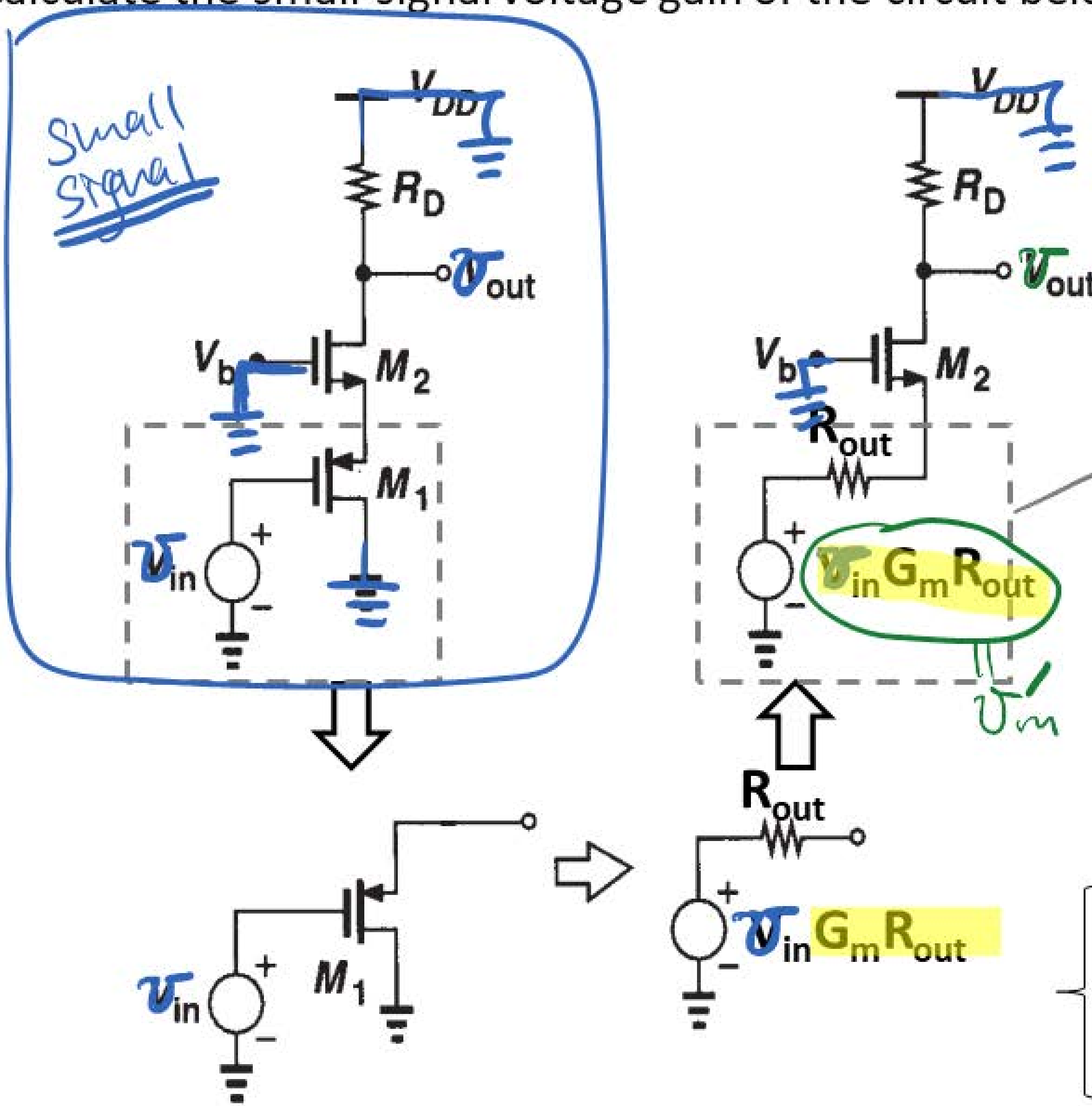
Example

Calculate the small-signal voltage gain of the circuit below. ($\lambda \neq 0$, $\gamma \neq 0$)



Example

Calculate the small-signal voltage gain of the circuit below. (λ ≠ 0, γ ≠ 0)



$$\frac{v_{out}}{v_{in}} = ?$$

Equivalent small signal circuit.

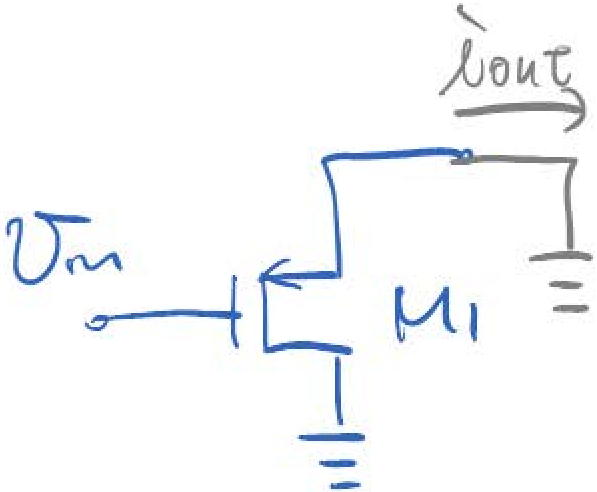
$$\begin{cases} G_m = gm_1 \\ R_{out} = r_{o1} \parallel \dfrac{1}{gm_1} \parallel \dfrac{1}{gmb_1} \end{cases}$$

$$\frac{v_{out}}{v_{in}} = \frac{(g_{m2} + g_{mb2})r_{o2} + 1}{r_{o2} + R_{out} + (g_{m2} + g_{mb2})r_{o2}R_{out} + R_D} R_D$$

$$v_{in} = v_{in}\left[g_{m1}\left(r_{o1} \,//\, \frac{1}{g_{m1}+g_{mb1}}\right)\right]$$

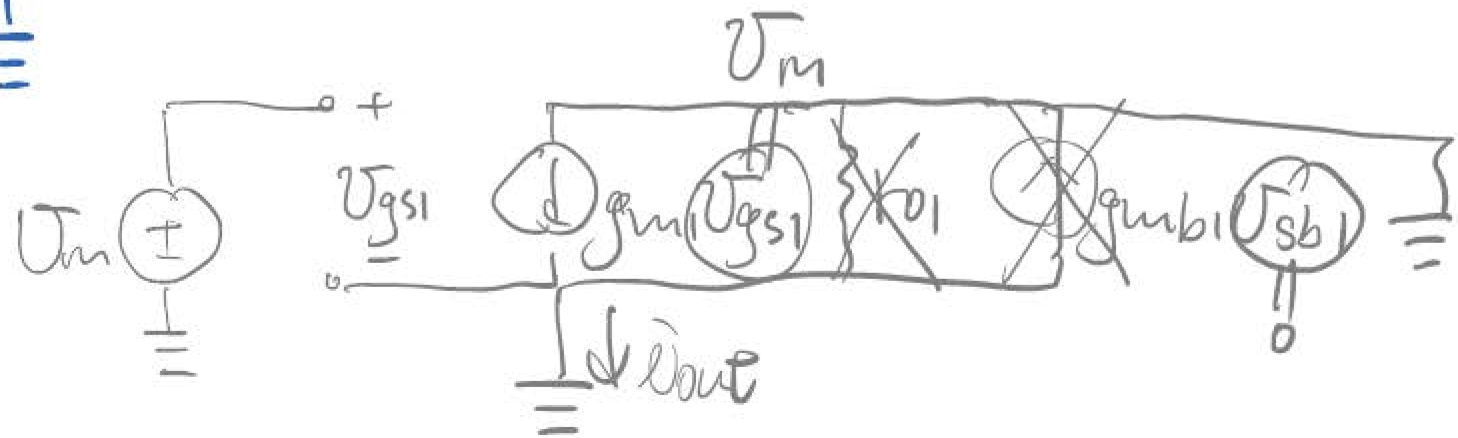
$$R_{out} = r_{o1} \,//\, \frac{1}{g_{m1}+g_{mb1}}$$

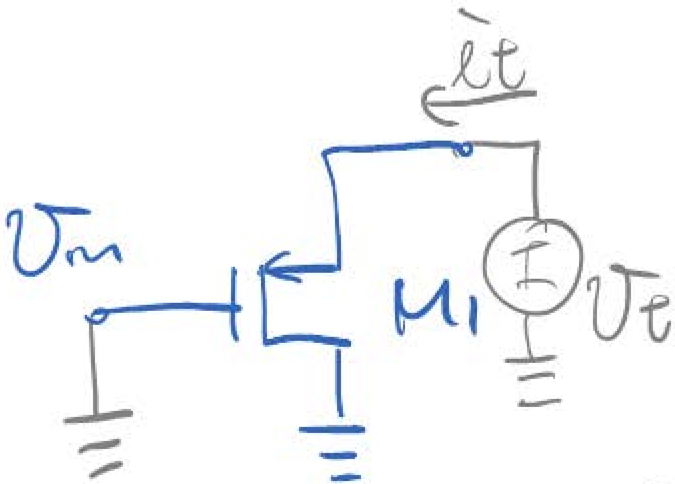
$$A_v = \frac{v_{out}}{v_{in}} = \left[g_{m1}\left(r_{o1} \,//\, \frac{1}{g_{m1}+g_{mb1}}\right)\right] \frac{\quad}{\quad} R_D$$



1° $G_m = ?$

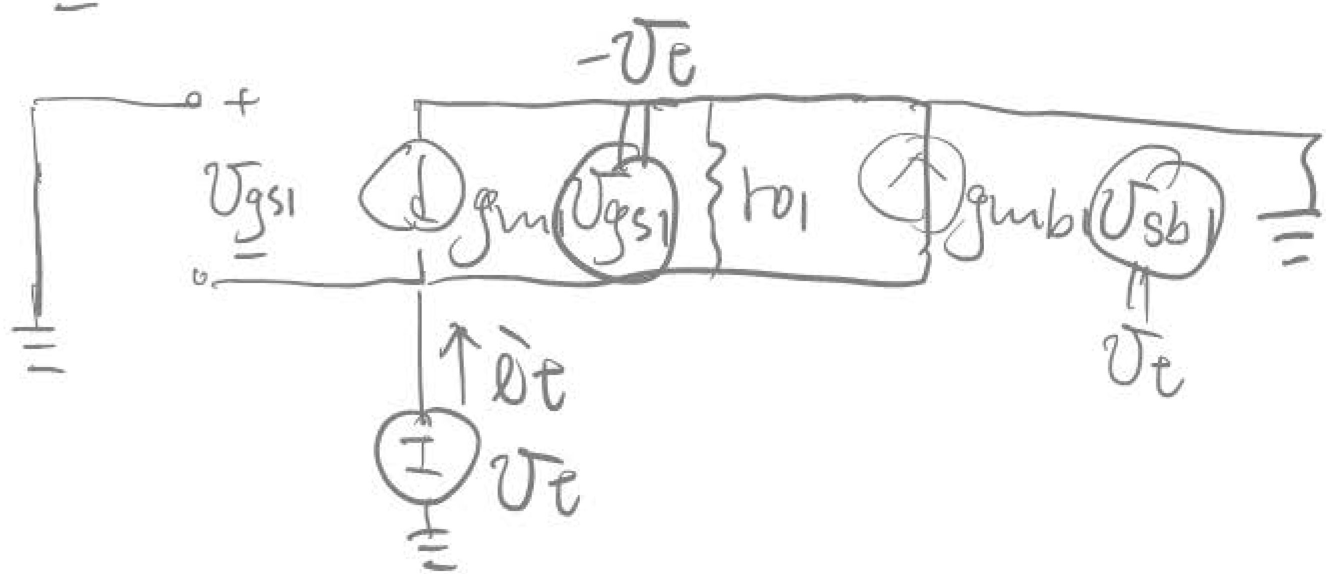
$$G_m = i_{out}/v_{in} = g_{m1}$$





2° $R_{out} = ?$

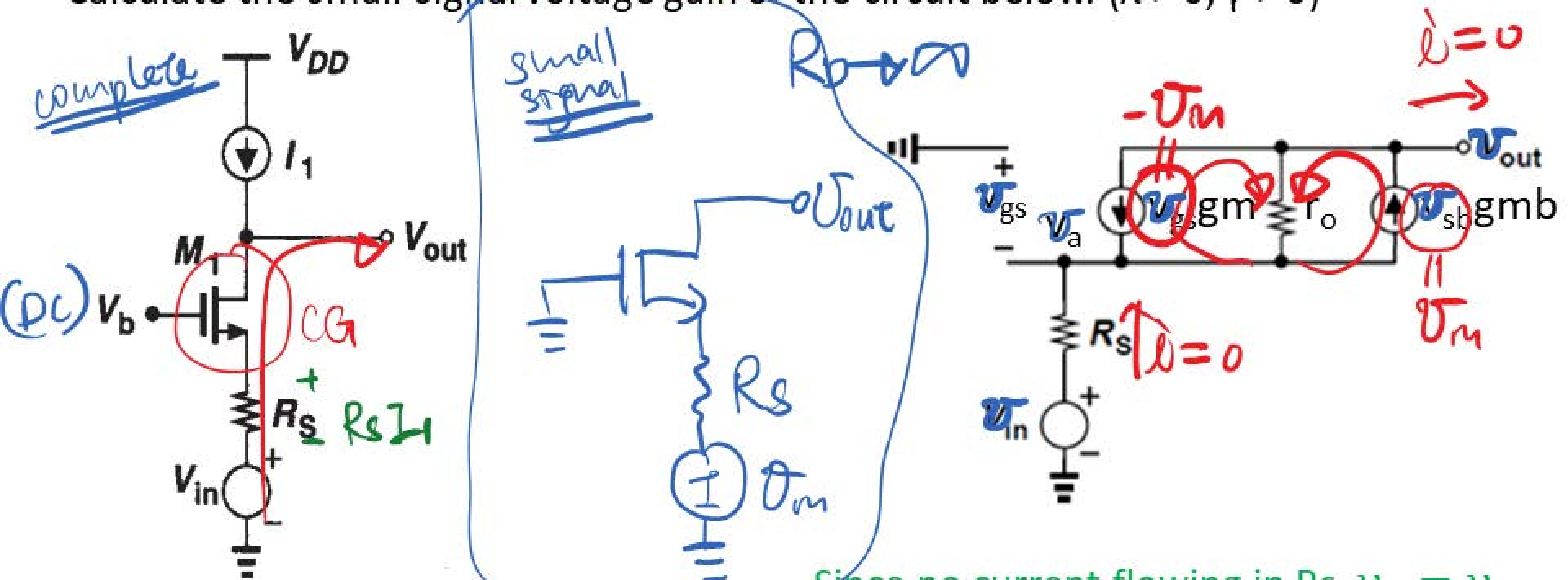
$$R_{out} = \frac{v_t}{i_t} = r_{o1} \,//\, \frac{1}{g_{m1}+g_{mb1}}$$



3° $$A_v = G_m R_{out} = \underbrace{g_{m1}\left(r_{o1} \,//\, \frac{1}{g_{m1}+g_{mb1}}\right)}_{SF}.$$

Example

Calculate the small-signal voltage gain of the circuit below. ($\lambda \neq 0, \gamma \neq 0$)



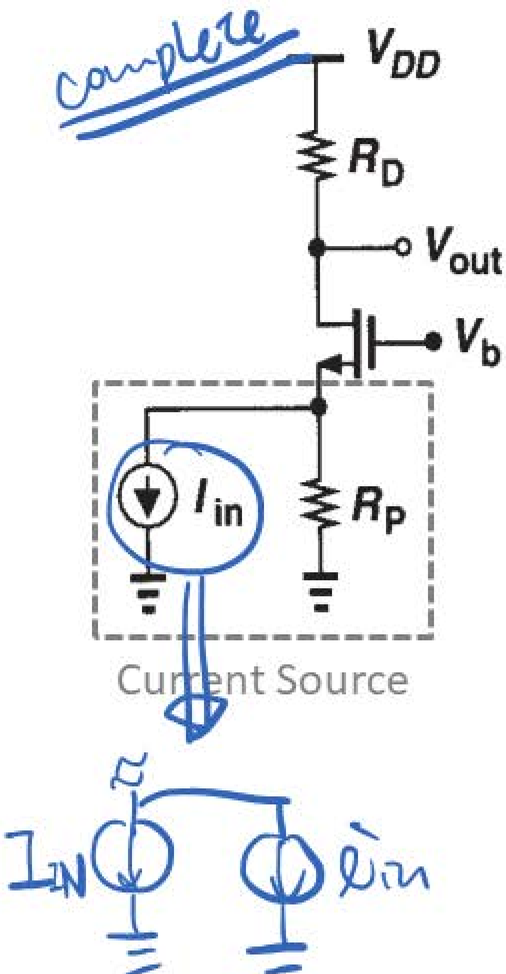
Since no current flowing in Rs, $\upsilon_a = \upsilon_{in}$.

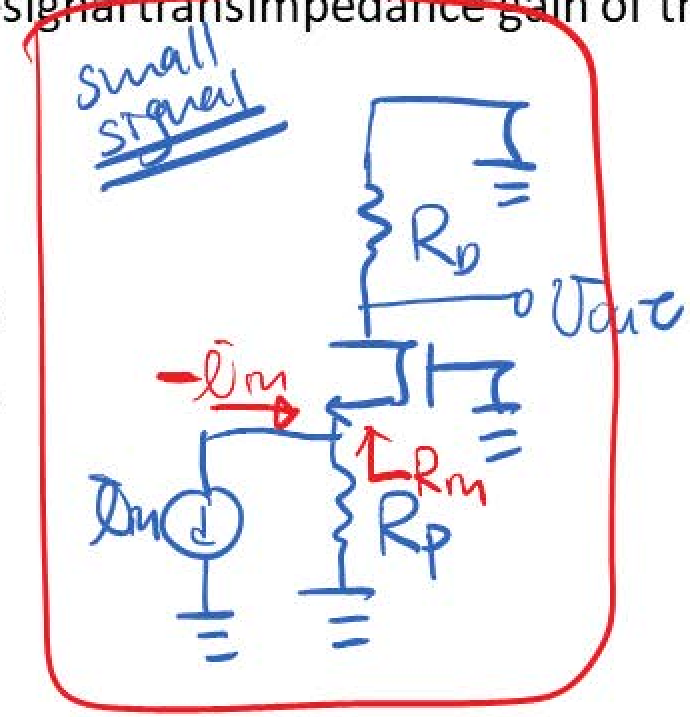
$$\upsilon_{out} - \upsilon_{in}(gm + gmb)r_o = \upsilon_{in}$$

$$A_v = \frac{\upsilon_{out}}{\upsilon_{in}} = 1 + (gm + gmb)r_o$$

Example

Calculate the small-signal transimpedance gain of the circuit below. ($\lambda \neq 0, \gamma \neq 0$)

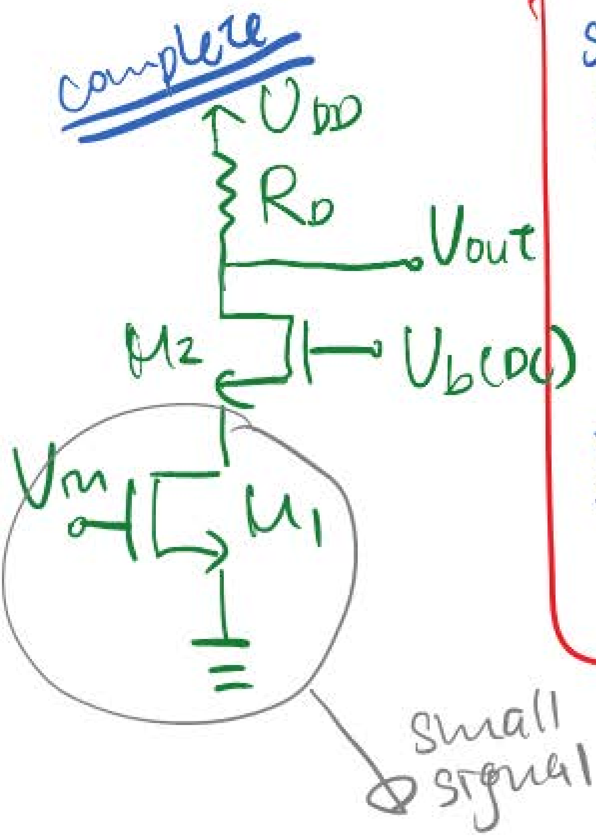


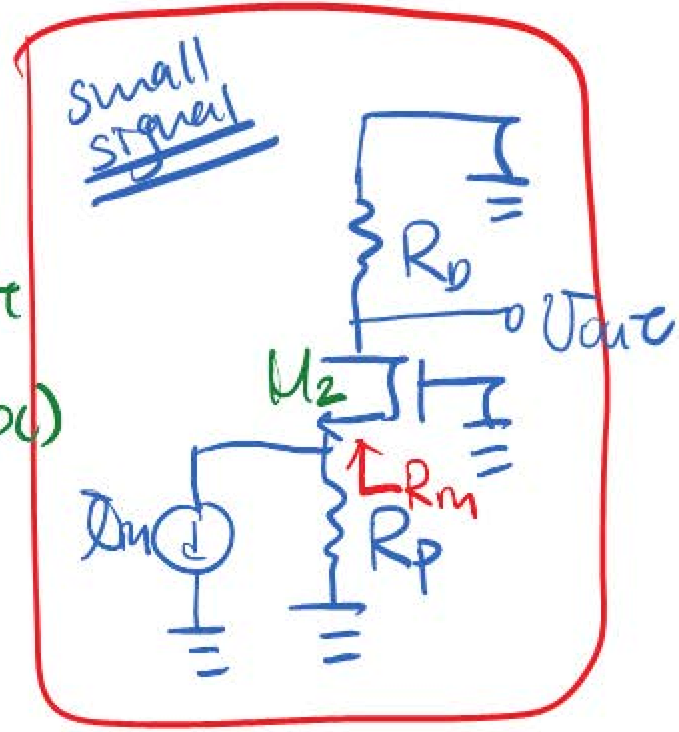


$$R_{in} = \frac{R_D + r_o}{1 + (gm + gmb)r_o}$$

$$\left(-i_{in}\frac{R_P}{R_{in} + R_P}\right)R_D = \upsilon_{out}$$

$$\frac{\upsilon_{out}}{i_{in}} = -\frac{R_P}{R_{in} + R_P}R_D = \frac{-R_P R_D[1 + (gm + gmb)r_o]}{R_D + r_o + R_P + (gm + gmb)r_o R_P} (\Omega)$$

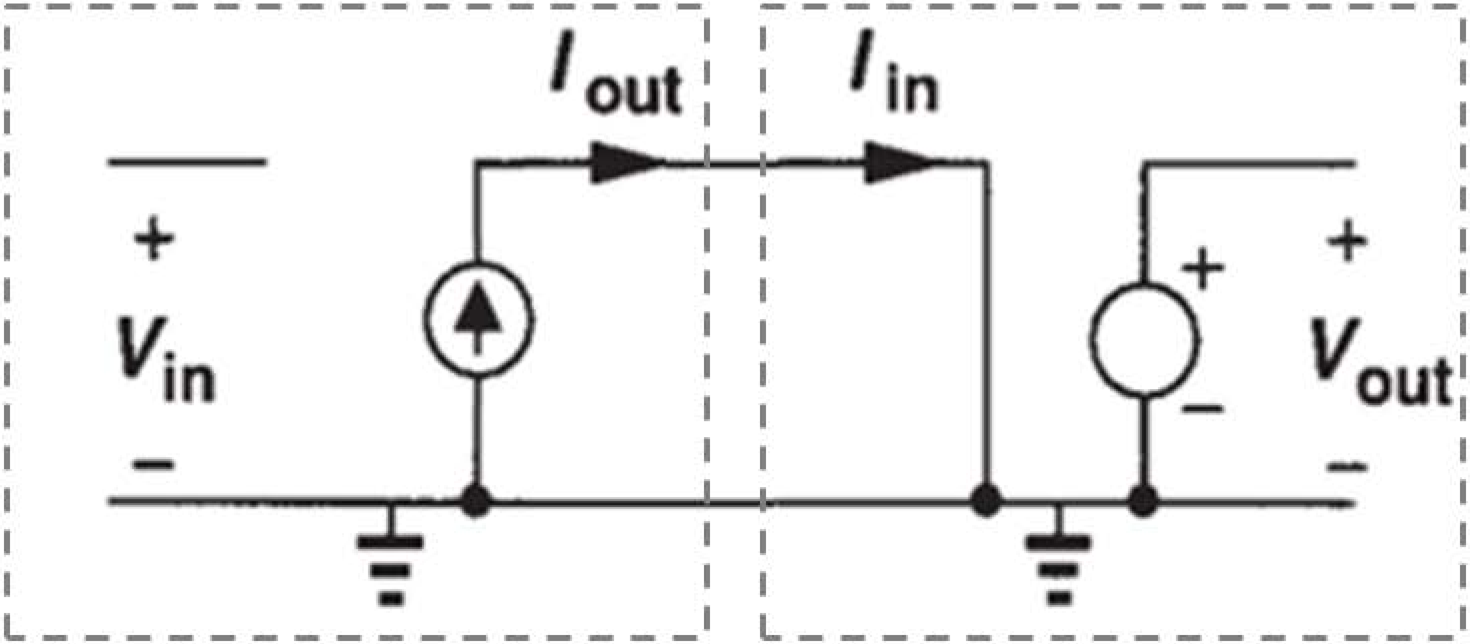




$$i_{in} = g_{m1} v_{in}$$
$$R_P = r_{o1}$$

Cascode

Ideal Amplifier



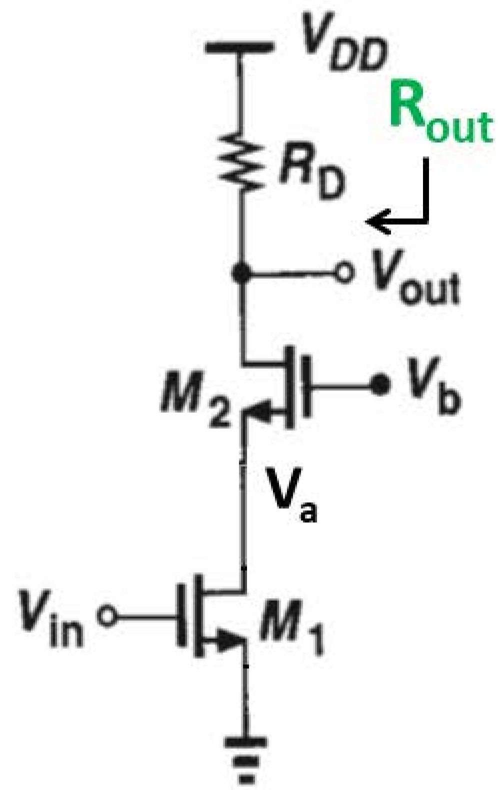
Common-Source

Common-Gate
+ Source Follower

CS + CG with Resistive Load

Small-signal Analysis

$\lambda \neq 0 \quad \gamma \neq 0$



$$\begin{cases} G_m = -gm_1 \dfrac{r_{o1}}{r_{o1} + \left(r_{o2} \parallel \dfrac{1}{gm_2 + gmb_2}\right)} \\ R_{out} = [r_{o1} + r_{o2} + (gm_2 + gmb_2) r_{o2} r_{o1}] \parallel R_D \end{cases}$$

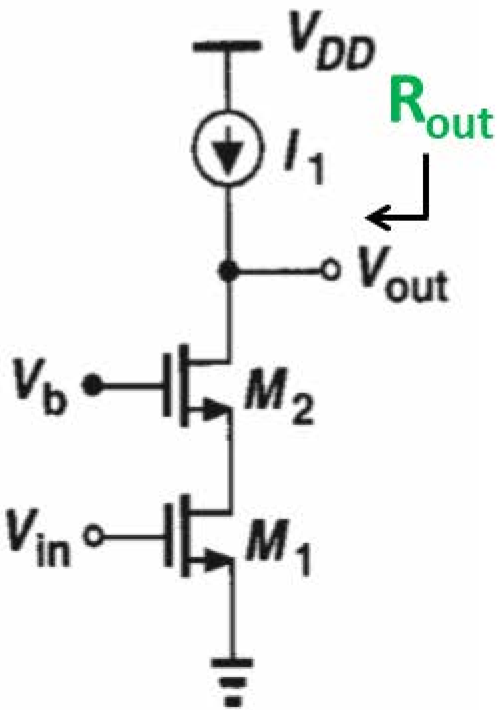
$$A_v = G_m R_{out}$$

$$V_a \geqq V_{in} - V_{TH1}$$
$$V_b - V_{GS2} \geqq V_{in} - V_{TH1}$$
$$V_b \geqq V_{in} - V_{TH1} + V_{GS2}$$
$$V_{out} \geqq V_b - V_{TH2} \geqq (V_{in} - V_{TH1}) + (V_{GS2} - V_{TH2})$$
$$\mathbf{V_{DD} \geqq V_{out} \geqq V_{ov1} + V_{ov2}}$$

CS + CG with Ideal Current Source Load

Small-signal Analysis

$\lambda \neq 0 \quad \gamma \neq 0$



$$\begin{cases} G_m = -gm_1 \dfrac{r_{o1}}{r_{o1} + \left(r_{o2} \parallel \dfrac{1}{gm_2 + gmb_2}\right)} \\ R_{out} = r_{o1} + r_{o2} + (gm_2 + gmb_2) r_{o2} r_{o1} \end{cases}$$

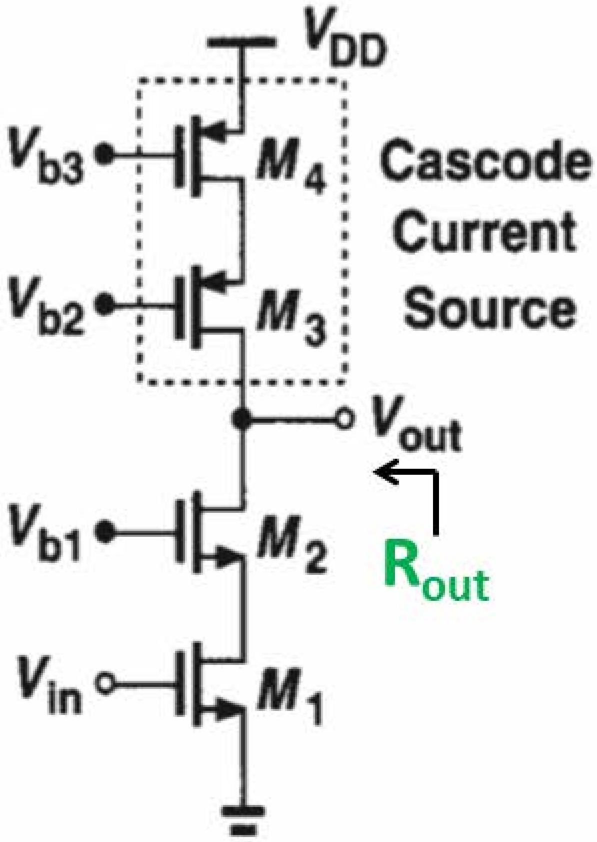
$$A_v = G_m R_{out}$$

CS + CG with Cascode Current Source Load

Small-signal Analysis

Assume $\lambda \neq 0 \quad \gamma \neq 0$

Assume all in sat.



$$
\begin{cases}
G_m = -gm_1 \dfrac{r_{o1}}{r_{o1} + \left(r_{o2} \parallel \dfrac{1}{gm_2 + gmb_2}\right)} \\
R_{out} = [r_{o1} + r_{o2} + (gm_2 + gmb_2) r_{o2} r_{o1}] \\
\quad \parallel [r_{o3} + r_{o4} + (gm_3 + gmb_3) r_{o3} r_{o4}]
\end{cases}
$$

$$A_v = G_m R_{out}$$

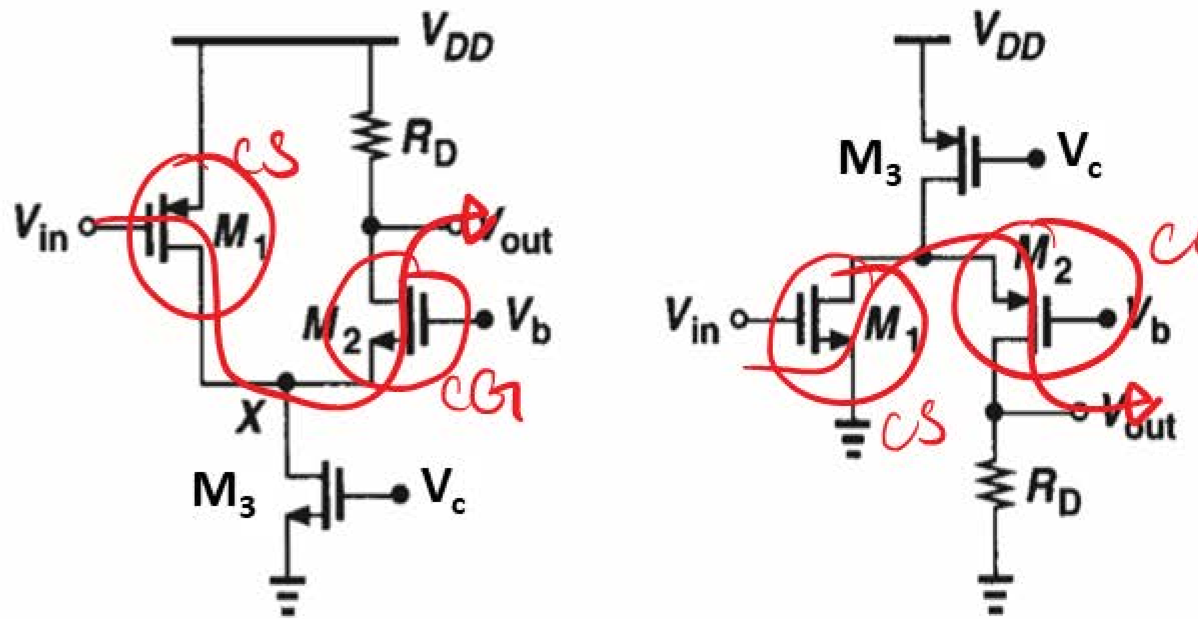
$$\mathbf{V_{DD} - V_{ov3} - V_{ov4} \geqq V_{out} \geqq V_{ov1} + V_{ov2}}$$

Vout headroom

Folded Cascode

Small-signal Analysis

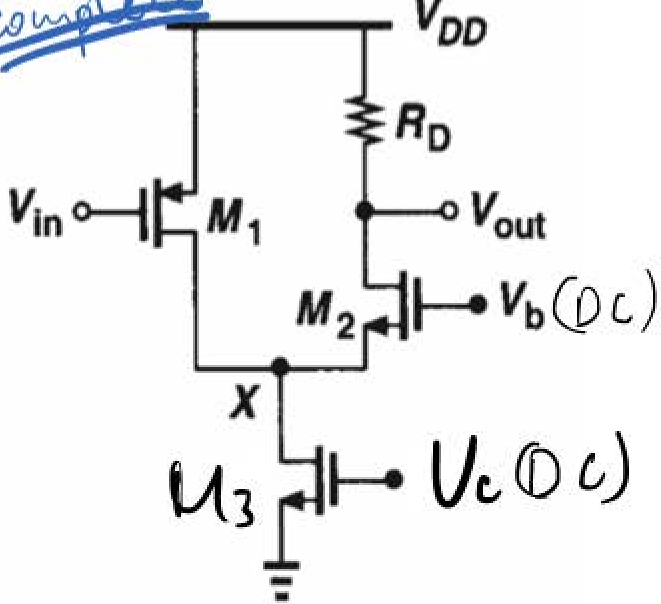
$\lambda \neq 0 \quad \gamma \neq 0$



$$\begin{cases} G_m = -gm_1 \dfrac{(r_{o1} \parallel r_{o3})}{(r_{o1} \parallel r_{o3}) + \left(r_{o2} \parallel \dfrac{1}{gm_2 + gmb_2}\right)} \\ R_{out} = [(r_{o1} \parallel r_{o3}) + r_{o2} + (gm_2 + gmb_2)r_{o2}(r_{o1} \parallel r_{o3})] \parallel R_D \end{cases}$$

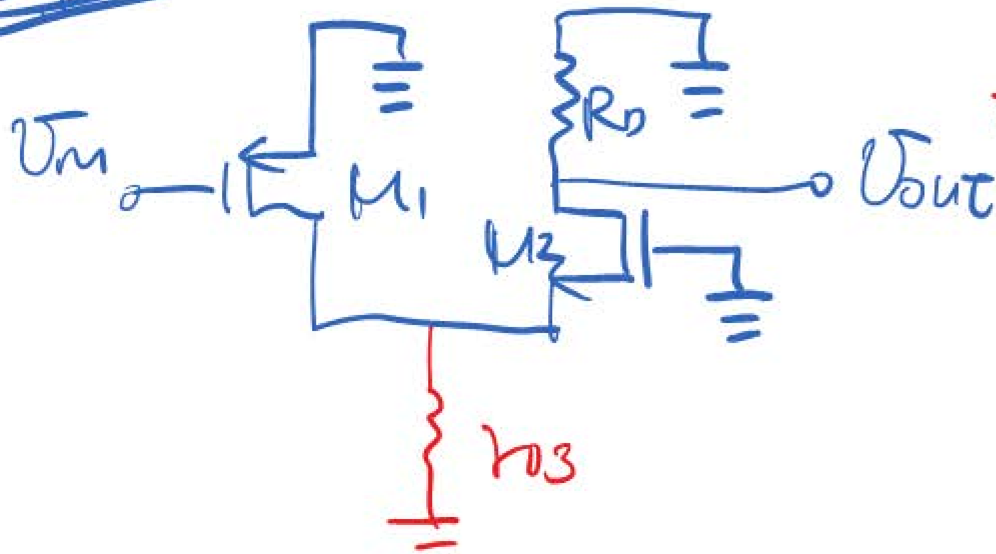
$$A_v = G_m R_{out}$$

complete

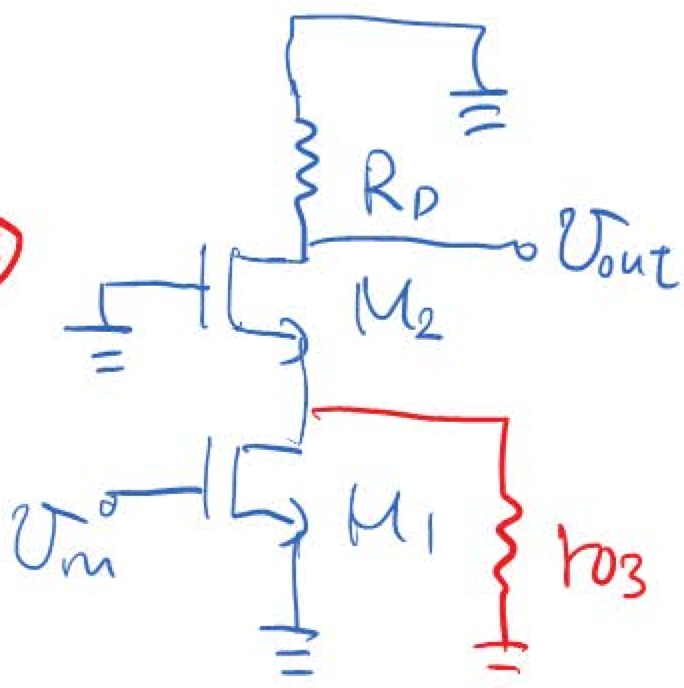


$\lambda \neq 0, \gamma \neq 0$

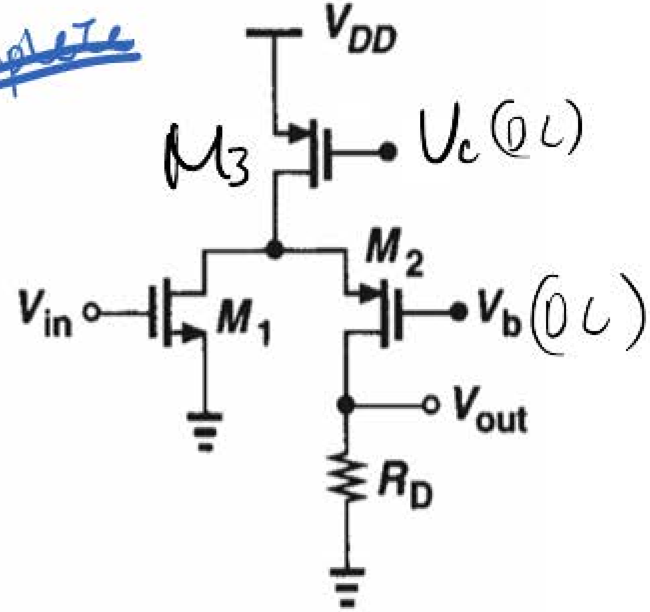
small signal



$\Rightarrow$

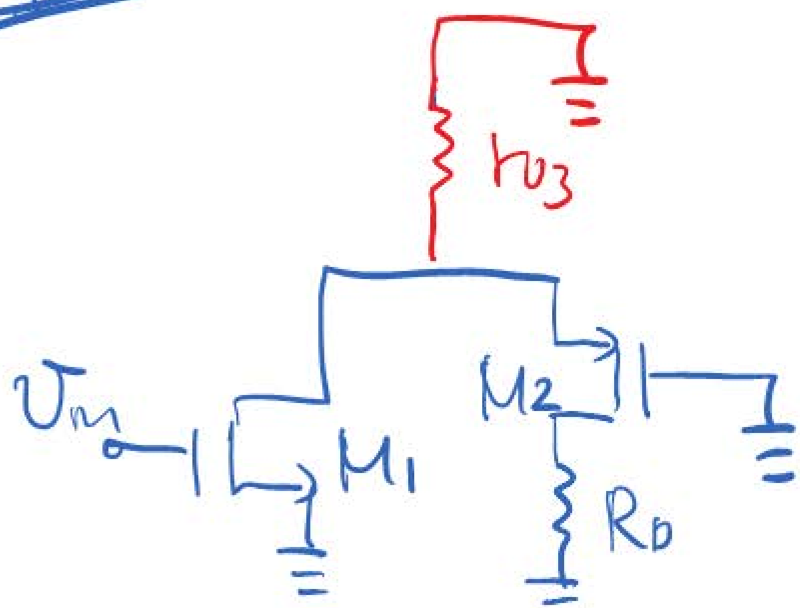


complete



$\lambda \neq 0, \gamma \neq 0$

small signal



$\Rightarrow$