I am absolutely starving.

The balance we're interested in is:

$$L_{fision} \approx L_{rad/conv} \tag{1}$$
$$+\text{HE} \rightarrow \text{main sequence} \tag{2}$$

For stars of $M \leq M_{\odot}$, suppoerted by P_{gas}, pp chain fusion, and $\kappa \sim \kappa_{ff}$. And if γ carry energy out $L_{rad} \propto \frac{M^{5.5}}{\sqrt{R}}$. Estimating $L_{fusion} = \epsilon_c M$. In the case of pp fusion, $\epsilon_{pp} \propto \rho T_c^{4/5}$ where $kT_c \sim \frac{GM\mu m_p}{R}$.

$$L_{fusion} \propto \frac{M^{6/5}}{R^{7.5}} \tag{3}$$

If you change the L of the star, T and ρ must change to accommodate. In steady state:

$$L_{fusion} \propto \frac{M^{6/5}}{R^{7.5}} \propto \frac{M^{5.5}}{\sqrt{R}} \tag{4}$$
$$R \propto M^{1/7} \tag{5}$$
$$T_c \propto M^{6/7} \tag{6}$$
$$L \propto M^{5.4} \tag{7}$$
$$L \propto T_{eff}^4 \ \propto 4\pi R^2 \sigma T^4 \text{ ,but R dep is so weak it's estimated to be constant} \tag{8}$$

$$M \uparrow \ \ ,T_c \uparrow \sim M^{6/7} \tag{9}$$
$$\epsilon_{pp} \propto T^{4.5} \ ,\epsilon_{CNO} \propto T^{20} \tag{10}$$

Stars that are more massive than the sun are very T dependent.

For $M \geq M_{\odot}$: $\kappa \sim \kappa_T \sim$ constant. γ still dominate E transport. CNO cycle is dominant mechanism and P_{gas} dominates. $L_{rad} \propto M^3$.

$$L_{fusion} = \int \epsilon_{CNO} dM_r \tag{11}$$
$$\sim \epsilon_{CNO} M \ \ ,\epsilon_{CNO} \propto \rho T_c^{20} \tag{12}$$
$$\epsilon_{CNO} \propto \frac{M^{21}}{R^{23}} \tag{13}$$
$$L_{fusion} \sim L_{rad} \tag{14}$$
$$\frac{M^{22}}{R^{23}} \propto M^3 \tag{15}$$
$$R \propto M^{19/23} \propto M^{.8} \tag{16}$$
$$T_c \propto M^{.2} \tag{17}$$

$$L = 4\pi R^2 \sigma T_{eff}^4 \tag{18}$$
$$L \propto R^2 T_{eff}^4 \tag{19}$$
$$R^2 \propto M^{1.6} \propto L^{1/2} \tag{20}$$
$$L \propto M^3 \tag{21}$$
$$\propto L^{1/2} T_{eff}^4 \tag{22}$$
$$L^{1/2} \propto T_{eff}^4 \tag{23}$$
$$L \propto T_{eff}^8 \tag{24}$$
$$T_{eff} \propto M^{3/8} \tag{25}$$

A huge change in L corresponds to a small change in T. Only for stars with masses slightly more than the sun.

For $M \geq 1.2 M_\odot$, they have convective cores and γ transport energy on outer part of star. Reverse of our sun. Convection sets in if $\frac{ds}{dr} < 0$. $\frac{ds}{dr}$ is implied by γ transport of energy. You can then use Radiative diffusion equation to see if $\frac{ds}{dr} < 0$. i.e. Convection sets in if $\frac{d \ln T}{d \ln P} > \frac{\gamma - 1}{\gamma}$. γ is the one for photons, not of the particles convecting.

$$\frac{d \ln T}{d \ln P} = \frac{1}{4} \frac{P}{P_{rad}} \frac{L}{L_{edd}} \frac{L_r/L}{M_r/M} \; , L_{edd} = \frac{4\pi G M_c}{\kappa} \tag{26}$$

P is the total pressure. This tells us convection sets in if $\frac{1}{4} \frac{P}{P_{rad}} \frac{L}{L_{edd}} \frac{L_r/L}{M_r/M} > \frac{2}{5}$. Recall for CNO, $\epsilon \propto \rho T^\beta$. At almost all r, $L_r \approx L$. For CNO-dominated stars, only 1% of star's mass fuses. TINY! It's this enormous flux that originates so close so the core that it drives convection. $\frac{M_r}{M} < \frac{5}{8} \frac{P}{P_{rad}} \frac{L}{L_{edd}}$, then convection sets in. We're interested where $P \approx P_{rad}$. We want to know $\frac{P}{P_{rad}}$ and $\frac{L}{L_{edd}}$. $L \propto M^3$ and $L \propto M$ so $\frac{L}{L_{edd}} = 4.5 \times 10^{-5} \left(\frac{M}{M_\odot} \right)^2$.

In the sun, $r \sim 0 \to .5 R_\odot$, $\frac{P_{gas}}{P_{rad}} \sim 3000$.

$$P_{gas} \propto \rho T \propto \frac{M^2}{R^4} \tag{27}$$
$$P_{rad} = \frac{1}{3} a T^4 \propto \frac{M^4}{R^4} \tag{28}$$
$$\frac{P_{gas}}{P_{rad}} \propto M^{-2} \tag{29}$$
$$\frac{P_{gas}}{P_{rad}} \approx 3000 \left(\frac{M}{M_\odot} \right)^{-2} \tag{30}$$

Convection sets in if $\frac{M_r}{M} \leq 0.1$.

Lifetime of MS star $\approx \frac{E_{nuc}}{L}$.

$$L = L_\odot \left(\frac{M}{M_\odot} \right)^{3.5} \tag{31}$$
$$E_{nuc} = N_p E \tag{32}$$
$$\approx .1 \frac{M}{m_p} \cdot 7 \text{ MeV} \tag{33}$$
$$\frac{E_{nuc}}{L} \approx 10^{10} \left(\frac{M}{M_\odot} \right)^{-2.5} \tag{34}$$

ANY star with a mass less than $.85M_\odot$ is still fusing after 13.7 billion years. For $M \sim 30M_\odot$, t_{MS} is about 10^6 years. Massive stars live and die where they are born.