$\boxed{5.1, 2.1, 2.2, 5.4}$

1 Main Sequence

Know how to calculate $\epsilon(\rho, T)$ in units of ergs/s/g. Quickly review major points of MS:

$$\boxed{\frac{dP}{dr} = -\rho\frac{GM_r}{r^2} = -\rho g} \quad (1)$$

$$P = P_{gas} + P_{rad}(+P_{degen}) \quad (2)$$

$$= \frac{\rho kT}{\mu m_p} + \frac{1}{3}aT^4 \quad (3)$$

$$\boxed{\frac{dM_r}{dr} = 4\pi r^2\rho} \quad (4)$$

$$E_{tot} = U/2 = -K \text{ , for non-rel} \quad (5)$$

$$E_{tot} \approx 0 \text{ , } K = -U \text{ , for rel} \quad (6)$$

$$(7)$$

1.1 Energy Transport (Radiation):

$$F_r = \frac{L_r}{4\pi r^2} = -\frac{4}{3}\frac{aT^3c}{\kappa\rho}\frac{dT}{dR} \text{ , } \kappa = \text{ opacity} \quad (8)$$

$$l = \frac{1}{\kappa\rho} = \frac{1}{n\sigma} \text{ , } \kappa = \frac{\sigma}{m} \text{ , m is average mass of a particle} \quad (9)$$

$$\kappa_T, \kappa_{ff}, \kappa_{boundfree}, \kappa_{H^-}, ... \quad (10)$$

1.2 Energy Transport (Buoyancy):

Convection sets in if $\frac{ds}{dr} < 0$. Exponentially driven instability driven by buoyancy of matter. Whether or not it's convecting is dependent on the entropy gradient. Another way to put it:

$$\frac{d\ln T}{d\ln P} > \frac{\gamma - 1}{\gamma} \quad (11)$$

We can get away with rough estimates using mixing length to find work done by the buoyancy force.
For convective Flux:

$$F = \frac{1}{2}\rho v_c^3 \quad (12)$$

$$= \frac{1}{2}c_s^3|\frac{H}{c_p}\frac{ds}{dr}|^{3/2} \quad (13)$$

When convection is present, $\frac{ds}{dr} = 0$ and:

$$\boxed{\frac{1}{T}\frac{dT}{dr} = \frac{\gamma - 1}{\rho}\frac{d\rho}{dr}} \ . \tag{14}$$

This is useful for fuly convective objects because then $P \propto \rho^{\gamma} \propto \rho^{5/3}$. This is an exampl of the $n = 3$ polytrope. We can therefore comute $T(r)$ and $\rho(r)$ (relatively) easy.

1.3 Energy Generation in Stars:

Gravity: KH contraction, at a minimum

$$\boxed{L = -\frac{1}{2}\frac{dU}{dt} \approx -\frac{GM^2}{R^2}|\frac{dR}{dt}|} \tag{15}$$

For the sun, $t_{KH} \approx 30$ million years. This contraction drives T_c up and eventually fusion sets in. $\epsilon(\rho, T, \text{composition})$ is the energy generation by fusion. Fusion is a collisional process and you need high densities and temperature for two particle to get close enough to tunnel through the cuolomb barrier.

$$L = \in \epsilon dM_r = \int_0^R 4\pi r^2 \rho \epsilon dr \tag{16}$$

H fusion in the sun lasts about 10^{10} years, which is about 3 orders higher than KH contraction. i.e. fusion is much more important than KH for luminosity. The variables we care about are: P, ρ, T, L_r, M_r and the equations are: HE, Equation of State, $dM_r = 4\pi r^2 \rho$, energy transport, energy generation. While these equations are good, we need boundary conditions/initial conditions. If you specify the mass and initial composition of a star, that determines *everything* $(T_c, T_{off}, \rho, R, L, T(r), P(r), ...)$. For KH contraction, we could calculate $R(M, t)$ and $L(M, t)$.

If only Io was even smaller... she'd be the only person who is both a moon and a white dwarf. Lol.

$$L = \int \epsilon dM_r \ , \tag{17}$$

so energy transport is determined by L. In the case of photons,

$$\sigma = \sigma_T \ , L \propto M^3 \tag{18}$$

$$\sigma = \sigma_{ff} \ , L \propto M^{5.5}/R \tag{19}$$

$$\text{convection} \ , L \propto M^{4/7} R^2 \tag{20}$$

For the Main Sequence:

$$L_{fusion} \approx L_{rad/conv} \tag{21}$$

$$L_{fusion} = 0 \ , \tag{22}$$

then it contracts from KH.

$$L_{rad/conv} \neq 0 \ , \tag{23}$$

then

$$T \uparrow \ , L_{fusion} \uparrow \ . \tag{24}$$

Then, $L_{fusion} \approx L_{rad/conv}$, and then $E_{tot} \approx$ constant $\rightarrow$ no KH contraction.

2 The Case of the Sun

Due to radiative diffusion, $L \sim L_\odot (L \propto M^3)$.

$$L_\odot = \int 4\pi r^2 dr \rho \epsilon_{pp} \rightarrow T_c \approx 10^7 \text{ K} \tag{25}$$

Fusion generates the energy in a star, but energy transport ultimately determines L of star. Then we can use HE and the VT:

$$kT \approx \frac{GM\mu m_p}{R} \rightarrow R \approx \frac{GM\mu m_p}{kT} \ , \tag{26}$$

and using T is T_c, we get $R \approx 10^{11}$ cm. Then, $L = 4\pi R^2 \sigma T_{eff}^4 = L_{eff}$.

We want to use these same set o arguments, but for stars of other masses. What makes this tricky is opacity, energy generation, energy transport, and pressure. These all depend on ρ, T and hence on the mass of the star.

Conveniently, for the sun, it is at the transition point (in mass) where $\kappa_T \propto \kappa_{ff}$ dominating and $\epsilon \propto \epsilon_{CNO}$ dominating. Specifically,

$$\kappa_{ff} \approx \kappa_T \text{ at center of sun} \tag{27}$$

$$\epsilon_{CNO} \sim 10^{-2} \epsilon_{pp} \text{ at center of sun} \tag{28}$$

$M \uparrow$, $T \uparrow$. $\kappa \sim \rho T^{-7/2} \downarrow$ as $M \uparrow$.

$$\epsilon_{CNO} \propto T^{20} \ , \kappa_T > \kappa_{ff} \ M > M_\odot \tag{29}$$

$$\epsilon_{pp} \propto T^{4.5} \ , \kappa_{ff} > \kappa_T \ M < M_\odot \tag{30}$$

2.1 Little Less Massive than Sun

$M \leq M_\odot$, dominated by pp chain and $\kappa \approx \kappa_{ff}$. $P_{gas} \gg P_{rad}$ and $L_{rad} \propto M^{5.5}/\sqrt{R}$. For MS: $L_{rad} \approx L_{fusion}$, where $L_{fusion} = \int \epsilon dM \sim \epsilon_c M$. $\epsilon_{pp} \propto \rho T^{4/5}$, and $T_c \approx 10^7$K and more specifically, $T_c \propto M/R$. Then, $\epsilon \propto \frac{M}{R^3} T_c^{4.5}$, and $\epsilon_{pp} \propto \frac{M^{5.5}}{R^{7.5}}$. Since $L_{rad} = L_{fusion}$, $\frac{M^{5.5}}{\sqrt{R}} \propto \frac{M^{6.5}}{R^{2.5}}$, then finally you get $R \propto M^{1/7}$. GOING ON

$$T_c \propto M/R \propto M^{6/7} \tag{31}$$

$$L \propto M^{5.5} \ , R \propto M^{1/7} \ , R \propto L^{1/40} \tag{32}$$

$$L = 4\pi R^2 \sigma T_{eff}^4 \rightarrow L \propto T_{eff}^4 \tag{33}$$

$$R \approx R_\odot \left(\frac{M}{M_\odot}\right)^{1/7} \tag{34}$$

$$L \approx L_\odot \left(\frac{M}{M_\odot}\right)^{5.5} \tag{35}$$

$$T_{eff} \approx \ \mathrm{K} \left(\frac{L}{L_\odot}\right)^{1/4} \tag{36}$$

$$T_c \approx 10^7 \ \mathrm{K} \left(\frac{M}{M_\odot}\right)^{6/7} \tag{37}$$