HW #8

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October 28, 2011

Problem 1a: ρ_{min} =?, if $T \sim 300$ K?

$$
\begin{aligned}
P_{gas} &= P_{degen} \\
\frac{\rho k T}{\mu m_p} &= \frac{h^2}{5 m_e}\left(\frac{3}{8\pi}\right)^{2/3} n^{5/3} \\
\frac{\rho k T}{\mu m_p} &= \frac{h^2}{5 m_e}\left(\frac{3}{8\pi}\right)^{2/3} \left(\frac{\rho}{\mu m_p}\right)^{5/3} \\
\frac{kT}{\mu m_p}(\mu m_p)^{5/3}\frac{5 m_e}{h^2}\left(\frac{8\pi}{3}\right)^{2/3} &= \frac{\rho^{5/3}}{\rho} \\
\frac{kT}{\mu m_p}(\mu m_p)^{5/3}\frac{5 m_e}{h^2}\left(\frac{8\pi}{3}\right)^{2/3} &= \rho^{2/3} \\
\left(\frac{kT}{\mu m_p}(\mu m_p)^{5/3}\frac{5 m_e}{h^2}\left(\frac{8\pi}{3}\right)^{2/3}\right)^{3/2} &= \rho \\
.0146 \text{ g cm}^{-3} &\approx \rho
\end{aligned}
$$

$$
\begin{aligned}
P_{gas} &= P_{degen} \\
\frac{\rho k T}{\mu m_p} &= \frac{h^2}{5 m_e}\left(\frac{3}{8\pi}\right)^{2/3} n^{5/3} \\
\frac{\rho k T}{\mu m_p} &= \frac{h^2}{5 m_e}\left(\frac{3}{8\pi}\right)^{2/3} \left(\frac{\rho}{\mu m_p}\right)^{5/3} \\
T &= \frac{h^2}{5 m_e}\left(\frac{3}{8\pi}\right)^{2/3} \left(\frac{\rho}{\mu m_p}\right)^{5/3} \frac{\mu m_p}{\rho k} \\
T &= \frac{h^2}{5 m_e}\left(\frac{3}{8\pi}\right)^{2/3} \left(\frac{\rho}{\mu m_p}\right)^{2/3} \frac{1}{k} \\
T &\approx 142.57 \text{ K}
\end{aligned}
$$

Problem 1b:

Compare E_E, E_F, and E_{thm}.

$$\begin{aligned} E_E &= \frac{(Z_1 Z_2 q_1 q_2)}{r} \\ &= \frac{(Z_1 Z_2 q_1 q_2)}{n^{-1/3}} \\ &= (Z_1 Z_2 q_1 q_2) n^{1/3} \\ &= (Z_1 Z_2 q_1 q_2) \left(\frac{\rho}{\mu m_p} \right)^{1/3} \end{aligned}$$

$$\boxed{E_E \approx 2.61 \times 10^{-10} \text{ ergs}}$$

$$\begin{aligned} E_F &= \frac{h^2}{2m} \left(\frac{3}{8\pi} \right)^{2/3} \left(\frac{\rho}{\mu m_p} \right)^{2/3} \\ &\approx 8.6 \times 10^{-12} \text{ ergs} \end{aligned}$$

$$\begin{aligned} E_{thm} &= \frac{3}{2} kT \\ E_{thm} &\approx 6.2 \times 10^{-14} \text{ ergs} \end{aligned}$$

E_E is the dominant force.

Problem 2a:

$$\begin{aligned} E_G &= \frac{2\pi^2 m_r e^4 Z_1^2 Z_2^2}{\hbar^2} \\ &\boxed{\approx 4.22 \times 10^{-6} \text{ ergs}} \end{aligned}$$

$$\boxed{< \sigma v >= 2.65 S(E_0) \frac{E_G^{1/6}}{(kT)^{2/3} \sqrt{m_r}} e^{-3 \sqrt[3]{(E_G/4kT)}}}$$

$$< \sigma v > = 4.78 \times 10^{-15} T^{-2/3} e^{-5909 T^{-1/3}}$$

Problem 2b:

As radius decreases, t_c increases because $t \propto R^{-3} \rightarrow t_c \propto R \times R^{-4} = R^{-3}$.

$$
\begin{aligned}
L = \frac{GM^2}{2R^2}\left|\frac{dR}{dt}\right| &= 0.2L_\odot \left(\frac{M}{M_\odot}\right)^{4/7} \left(\frac{R}{R_\odot}\right)^2 \\
\left|\frac{dR}{dt}\right| &= 0.2L_\odot \left(\frac{M}{M_\odot}\right)^{4/7} \left(\frac{R}{R_\odot}\right)^2 2 \left(\frac{R}{R_\odot}\right)^2 R_\odot^2 \frac{1}{GM_\odot^2} \left(\frac{M}{M_\odot}\right)^{-2} \\
\left|\frac{dR}{dt}\right| &= 0.4L_\odot \left(\frac{M}{M_\odot}\right)^{-10/7} \left(\frac{R}{R_\odot}\right)^4 \frac{R_\odot^2}{GM_\odot^2} \\
t_c = \frac{R}{\left|\frac{dR}{dt}\right|} &= \frac{1}{0.4L_\odot \left(\frac{M}{M_\odot}\right)^{-10/7} \left(\frac{R}{R_\odot}\right)^4 \frac{R_\odot^2}{GM_\odot^2}} \\
&\boxed{= \left(\frac{M}{M_\odot}\right)^{10/7} \left(\frac{R}{R_\odot}\right)^{-3} \frac{5GM_\odot^2}{2R_\odot L_\odot}}
\end{aligned}
$$

Problem 2c:

$$
\begin{aligned}
t_D &= \frac{1}{n_p < \sigma v >} \\
n_p &= \frac{\rho}{\mu m_p} \\
t_D(\rho, T) &= \frac{\mu m_p}{\rho < \sigma v >} \ ,
\end{aligned}
$$

where $< \sigma v >$ is defined above.

$$
\begin{aligned}
&\rho_c = \bar{\rho} a_n \ , a_n = 5.99 \\
&T_c = 7.5 \times 10^6 \left(\frac{M}{M_\odot}\right) \left(\frac{R}{R_\odot}\right)^{-1} \ \mathrm{K} \ ,
\end{aligned}
$$

and plug those into $< \sigma v >$ to get it in terms of M and R.

$$t_D(\rho,T) = \frac{\mu m_p}{\rho < \sigma v >}$$
$$= \frac{\mu m_p 4\pi R^3}{3Ma_n}\frac{1}{< \sigma v >}$$
$$< \sigma v > = 1.2\times 10^{-19}\left(\frac{M}{M_\odot}\right)^{-2/3}\left(\frac{R}{R_\odot}\right)^{2/3} e^{-4.4\times 10^{10}\left(\frac{M}{M_\odot}\right)^{-1/3}\left(\frac{R}{R_\odot}\right)^{1/3}}$$
$$\frac{1}{< \sigma v >} = 8.33\times 10^{18}\left(\frac{M}{M_\odot}\right)^{2/3}\left(\frac{R}{R_\odot}\right)^{-2/3} e^{4.4\times 10^{10}\left(\frac{M}{M_\odot}\right)^{-1/3}\left(\frac{R}{R_\odot}\right)^{1/3}}$$
$$t_D(M,R) = \frac{\mu m_p 4\pi}{3a_n}\left(\frac{R}{R_\odot}\right)^{3}\left(\frac{M}{M_\odot}\right)^{-1}\frac{R_\odot^3}{M_\odot}8.33\times 10^{18}\left(\frac{M}{M_\odot}\right)^{2/3}\left(\frac{R}{R_\odot}\right)^{-2/3} e^{4.4\times 10^{10}\left(\frac{M}{M_\odot}\right)^{-1/3}\left(\frac{R}{R_\odot}\right)^{1/3}}$$
$$\boxed{t_D(M,R) = \frac{\mu m_p 4\pi}{3a_n}\left(\frac{M}{M_\odot}\right)^{-1/3}\left(\frac{R}{R_\odot}\right)^{7/3}\frac{R_\odot^3}{M_\odot}8.33\times 10^{18} e^{4.4\times 10^{10}\left(\frac{M}{M_\odot}\right)^{-1/3}\left(\frac{R}{R_\odot}\right)^{1/3}}}$$

Problem 2d:

$$t_D(M,R) = t_c$$
$$\frac{\mu m_p 4\pi}{3a_n}\left(\frac{M}{M_\odot}\right)^{-1/3}\left(\frac{R}{R_\odot}\right)^{7/3}\frac{R_\odot^3}{M_\odot}8.33\times 10^{18} e^{4.4\times 10^{10}\left(\frac{M}{M_\odot}\right)^{-1/3}\left(\frac{R}{R_\odot}\right)^{1/3}} = \left(\frac{M}{M_\odot}\right)^{10/7}\left(\frac{R}{R_\odot}\right)^{-3}\frac{5GM_\odot^2}{2R_\odot L_\odot}$$
$$\frac{\mu m_p 4\pi}{3a_n}\left(\frac{R}{R_\odot}\right)^{16/3}\frac{R_\odot^3}{M_\odot}8.33\times 10^{18} e^{4.4\times 10^{10}\left(\frac{M}{M_\odot}\right)^{-1/3}\left(\frac{R}{R_\odot}\right)^{1/3}} = \left(\frac{M}{M_\odot}\right)^{10/7+1/3}\frac{5GM_\odot^2}{2R_\odot L_\odot}$$
$$\left(\frac{R}{R_\odot}\right)^{16/3} e^{4.4\times 10^{10}\left(\frac{M}{M_\odot}\right)^{-1/3}\left(\frac{R}{R_\odot}\right)^{1/3}} = 1.2\times 10^{-19}\left(\frac{M}{M_\odot}\right)^{37/21}\frac{15a_n GM_\odot^3}{8\mu m_p \pi R_\odot^4 L_\odot}$$
$$\boxed{\left(\frac{R}{R_\odot}\right)^{16/3} e^{4.4\times 10^{10}\left(\frac{M}{M_\odot}\right)^{-1/3}\left(\frac{R}{R_\odot}\right)^{1/3}} = 2.25\times 10^{21}\left(\frac{M}{M_\odot}\right)^{37/21}}$$

$$\left(\frac{R}{R_\odot}\right)^{16/3} e^{4.4\times 10^{10}\left(\frac{M}{M_\odot}\right)^{-1/3}\left(\frac{R}{R_\odot}\right)^{1/3}} = 2.25\times 10^{21}\left(\frac{M}{M_\odot}\right)^{37/21}$$
$$\text{For } M = .03M_\odot, \boxed{\left(\frac{R}{R_\odot}\right) = .685}$$
$$\text{For } M = .1M_\odot, \boxed{\left(\frac{R}{R_\odot}\right) = 1.82}$$

$$
\begin{aligned}
T_c &= 7.5\times 10^6 \left(\frac{M}{M_\odot}\right)\left(\frac{R}{R_\odot}\right)^{-1} \text{ K} \\
T_c(M = .03M_\odot) &= 7.5\times 10^6 \times .03 \times .685^{-1} \text{ K} \\
&\boxed{= 3.28\times 10^5 \text{ K}} \\
T_c(M = .1M_\odot) &= 7.5\times 10^6 \times .03 \times 1.82^{-1} \text{ K} \\
&\boxed{= 4.12\times 10^5 \text{ K}}
\end{aligned}
$$

$$
\begin{aligned}
t_D(M = .03M_\odot) &= 5.34\times 10^{13} \text{ s} \\
t_D(M = .1M_\odot) &= 1.56\times 10^{13} \text{ s}
\end{aligned}
$$

Deuterium fusing happens before the MS because there is some radius R_D where $t_c = t_D$ since we defined t_D as the lifetime of a Deuterium nucleus. As a result, there exists some time where in contraction, Deuterium fuses, which since it's contracting, is therefore before the main sequence.

Problem 2e:

$$
\begin{aligned}
L = -\frac{1}{2}\frac{GM^2}{R}\left|\frac{dR}{dt}\right| &= 5.5 \text{ MeV} \times \frac{\text{reactions}}{\text{sec}} \\
-\frac{1}{2}\frac{GM^2}{t_c} &= 5.5 \text{ MeV} \times \frac{\text{reactions}}{\text{sec}} \\
-\frac{1}{2}\frac{GM^2}{t_c} &= 8.8\times 10^{-6} \text{ ergs} \times \frac{\text{reactions}}{\text{sec}} \\
10^{34} \text{ ergs s}^{-1} &\simeq 8.8\times 10^{-6} \text{ ergs} \times n_1 n_2 < \sigma v > \\
10^{34} \text{ ergs s}^{-1} &\simeq 10^{-21} \text{ ergs s}^{-1}
\end{aligned}
$$

We see that the luminosity of Deuterium is much smaller than the fusion luminosity and it will never halt contraction of the star.

Problem 3a:

$$\rho_c = \bar{\rho} a_n$$
$$P_c = GM^{2/3}\rho^{4/3} d_n$$

$$
\begin{aligned}
P_c &= K\rho_c^{5/3} \\
GM^{2/3}\rho_c^{4/3} d_n &= K(\bar{\rho}a_n)^{5/3} \\
GM^{2/3}\left(\frac{3M}{4\pi R^3}a_n\right)^{4/3} d_n &= K\left(\frac{3M}{4\pi R^3}a_n\right)^{5/3} \\
R &= \left(\frac{3Ma_n}{4\pi}\right)^{5/3} \frac{K(4\pi)^{4/3}}{G(3a_n)^{4/3}M^2 d_n} \\
R(M) &= M^{-1/3}\left(\frac{3a_n}{4\pi}\right)^{5/3} \frac{K(4\pi)^{4/3}}{G(3a_n)^{4/3} d_n}
\end{aligned}
$$

To find K,

$$
\begin{aligned}
\frac{h^2}{5m_e}\left(\frac{3}{8\pi}\right)^{2/3} n^{5/3} &= K\rho^{5/3} \\
\frac{h^2}{5m_e}\left(\frac{3}{8\pi}\right)^{2/3}\left(\frac{\rho}{\mu_e m_p}\right)^{5/3} &= K\rho^{5/3} \\
\frac{h^2}{5m_e}\left(\frac{3}{8\pi}\right)^{2/3}\left(\frac{1}{\mu_e m_p}\right)^{5/3} &= K
\end{aligned}
$$

$$
\begin{aligned}
R(M) &= M^{-1/3}\left(\frac{3a_n}{4\pi}\right)^{5/3} \frac{(4\pi)^{4/3}}{G(3a_n)^{4/3} d_n}\frac{h^2}{5m_e}\left(\frac{3}{8\pi}\right)^{2/3}\left(\frac{1}{\mu_e m_p}\right)^{5/3} \\
\left(\frac{R}{R_\odot}\right) &= \frac{1}{R_\odot}\left(\frac{M}{M_\odot}\right)^{-1/3} M_\odot^{-1/3}\left(\frac{3a_n}{4\pi}\right)^{5/3} \frac{(4\pi)^{4/3}}{G(3a_n)^{4/3} d_n}\frac{h^2}{5m_e}\left(\frac{3}{8\pi}\right)^{2/3}\left(\frac{1}{\mu_e m_p}\right)^{5/3} \\
&\boxed{\left(\frac{R}{R_\odot}\right) = \frac{1}{R_\odot}\left(\frac{M}{M_\odot}\right)^{-1/3} M_\odot^{-1/3}\left(\frac{3a_n}{4\pi}\right)^{1/3} \frac{1}{Gd_n}\frac{h^2}{5m_e}\left(\frac{3}{8\pi}\right)^{2/3}\left(\frac{1}{\mu_e m_p}\right)^{5/3}}
\end{aligned}
$$

Problem 3b:

$$\left(\frac{R}{R_\odot}\right) \text{ where } M = M_J = 1.9 \times 10^{30} \text{ g},$$

$$\boxed{\left(\frac{R}{R_\odot}\right) \approx 0.04}\text{, in reality, } \frac{R_J}{R_\odot} \approx 0.1$$

Problem 3c:

$$
\begin{aligned}
E_E &= E_F \\
\frac{Z_1 Z_2 q_1 q_2}{r} &\sim \frac{1}{2m}\left(\frac{3h^3}{8\pi}\right)^{2/3} n^{2/3}\ , r \sim n^{-1/3} \\
Z_1 Z_2 q_1 q_2 n^{1/3} &\sim \frac{1}{2m}\left(\frac{3h^3}{8\pi}\right)^{2/3} n^{2/3} \\
Z_1 Z_2 q_1 q_2 \left(\frac{\rho}{\mu m_p}\right)^{1/3} &\sim \frac{1}{2m}\left(\frac{3h^3}{8\pi}\right)^{2/3}\left(\frac{\rho}{\mu m_p}\right)^{2/3} \\
\left(\left(\frac{8\pi}{3h^3}\right)^2 2m_e Z_1 Z_2 q_1 q_2\right)^3 \mu m_p &\sim \rho \\
\boxed{0.0984 \text{ g cm}^{-3} \sim \rho}
\end{aligned}
$$

$$
\begin{aligned}
.1 = \rho_c &= \frac{3Ma_n}{4\pi R^3} \\
\frac{.4\pi R^3}{3a_n} &= M \\
\frac{.4\pi}{3a_n} R^3 &= M \\
\frac{.4\pi R_\odot^3}{3a_n M_\odot}\left(\frac{R}{R_\odot}\right)^3 &= \left(\frac{M}{M_\odot}\right) \\
\frac{.4\pi R_\odot^3}{3a_n M_\odot}\left(\frac{1}{R_\odot}\left(\frac{M}{M_\odot}\right)^{-1/3} M_\odot^{-1/3}\left(\frac{3a_n}{4\pi}\right)^{1/3}\frac{1}{Gd_n}\frac{h^2}{5m_e}\left(\frac{3}{8\pi}\right)^{2/3}\left(\frac{1}{\mu_e m_p}\right)^{5/3}\right)^3 &= \left(\frac{M}{M_\odot}\right) \\
\frac{.4\pi R_\odot^3}{3a_n M_\odot}\left(\frac{1}{R_\odot} M_\odot^{-1/3}\left(\frac{3a_n}{4\pi}\right)^{1/3}\frac{1}{Gd_n}\frac{h^2}{5m_e}\left(\frac{3}{8\pi}\right)^{2/3}\left(\frac{1}{\mu_e m_p}\right)^{5/3}\right)^3 &= \left(\frac{M}{M_\odot}\right)^2 \\
\frac{.4\pi R_\odot^3}{3a_n M_\odot}\left(\frac{1}{R_\odot} M_\odot^{-1/3}\left(\frac{3a_n}{4\pi}\right)^{1/3}\frac{1}{Gd_n}\frac{h^2}{5m_e}\left(\frac{3}{8\pi}\right)^{2/3}\left(\frac{1}{\mu_e m_p}\right)^{5/3}\right)^{3/2} &= \left(\frac{M}{M_\odot}\right) \\
9.6 \times 10^{-5} &= \left(\frac{M}{M_\odot}\right) \\
\boxed{1.92 \times 10^{29} \text{ g} = M}
\end{aligned}
$$

$$
\begin{aligned}
R &= \left(\frac{3Ma_n}{.4\pi}\right)^{1/3} \\
&= 1.4 \times 10^{10} \text{ cm} \\
&\boxed{\approx .201 R_\odot}
\end{aligned}
$$

$$E_F > E_C \ , \text{ then degenerate}$$
$$n^{2/3} \gtrsim n^{1/3}$$

Since n_Q is proportional to T, and since $E_F \propto n^{2/3}$, we are at high high n for the degenerate object that we don't care about the T dependence. E_F will be significantly larger than E_C which is in turn larger than E_{thm}. If you get to a regime that $E_F \sim E_C$, then that will be the maximum radius since you can get smaller radii and still be degenerate.