HW #7

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October 17, 2011

Problem 1a:

$$\epsilon_{pp} \sim \frac{r_{12}Q}{\rho} \tag{1}$$

$$\sim \frac{n_p^2 < \sigma v > Q}{\rho} \tag{2}$$

$$\sigma = \frac{S(E)}{E}P(E) \tag{3}$$

so if we increase S by 12 orders of magnitude, then we increase σ and also ϵ_{pp}. ϵ_{CNO} stays the same because we haven't changed its S value. $\frac{\epsilon'_{pp}}{\epsilon_{CNO}}$ is still much larger than the original $\frac{\epsilon_{pp}}{\epsilon_{CNO}}$.

Problem 1b:

$$L' \approx \epsilon' M' \tag{4}$$

$$L \approx \epsilon M \tag{5}$$

$$\frac{L'}{L} \approx \frac{\epsilon' M}{\epsilon M} \tag{6}$$

$$\frac{L'}{L} \approx \frac{\epsilon'}{\epsilon} \tag{7}$$

$$\frac{L'}{L} \approx \frac{\rho' T'^{\beta}}{\rho T^{\beta}} \tag{8}$$

$$\frac{L'}{L} \approx 1 \tag{9}$$

$$\rho T^{\beta} \approx \rho' T'^{\beta} \tag{10}$$

$$\frac{\rho}{\rho'} \approx \frac{T'^{\beta}}{T^{\beta}} \tag{11}$$

$$\frac{R'^3}{R^3} \approx \frac{T'^{\beta}}{T^{\beta}} \tag{12}$$

$$\tag{13}$$

$$\epsilon \approx 5 \times 10^5 \frac{\rho X^2}{T_7^{2/3}} e^{-15.7 T_7^{-1/3}} = \text{ constant} \tag{14}$$

$$L \approx \epsilon M \tag{15}$$

$$\approx 5 \times 10^5 \frac{\rho X^2}{T_7^{2/3}} e^{-15.7 T_7^{-1/3}} M \tag{16}$$

$$L' \approx \epsilon' M \tag{17}$$

$$\frac{L'}{L} \approx \frac{\epsilon' M}{\epsilon M} \tag{18}$$

$$\frac{L'}{L} \approx \frac{\epsilon'}{\epsilon} \tag{19}$$

$$\frac{L'}{L} \approx 1 \tag{20}$$

$$L_\odot \approx \epsilon' M_\odot \tag{21}$$

$$3.4 \times 10^{33} \approx 10^{12} \cdot 5 \times 10^5 \frac{\rho_c X^2}{T_7'^{2/3}} e^{-15.7 T_7'^{-1/3}} M_\odot \tag{22}$$

$$\boxed{T_c' \approx 3.6 \times 10^5 K} \tag{23}$$

$$T = \frac{GM\mu m_p}{3Rk} \tag{24}$$

$$R = \frac{GM\mu m_p}{3Tk} \tag{25}$$

$$R = \frac{GM_\odot \mu m_p}{3k \cdot 3.6 \times 10^5} \tag{26}$$

$$\boxed{R = 6.4 \times 10^{11} \text{ cm}} \tag{27}$$

$$L_\odot = 4\pi\sigma R^2 T_{eff}^4 \tag{28}$$

$$= 4\pi\sigma (6.4 \times 10^{11})^2 T_{eff}^4 \tag{29}$$

$$\left(\frac{L_\odot}{4\pi\sigma(6.4 \times 10^{11})^2} \right)^{1/4} = T_{eff} \tag{30}$$

$$\boxed{1.84 \times 10^3 \text{ K} \approx T_{eff}} \tag{31}$$

Problem 2a:

Need to find:

* $R(M)$
* $L(M)$
* $T_c(M)$

* $L(T_{eff})$

$$\beta = -\frac{2}{3} + 5.2T_7^{-1/3} \tag{32}$$
$$= 5.88 \tag{33}$$

$$2K = U \tag{34}$$
$$K = \frac{U}{2} \tag{35}$$
$$\frac{3}{2}NkT = \frac{GM^2}{2R} \tag{36}$$
$$\boxed{T \propto \frac{GM}{3Rk}} \tag{37}$$

$$L \approx \epsilon M \tag{38}$$
$$= \rho T^{\beta} M \tag{39}$$
$$0.2\left(\frac{M}{M_\odot}\right)^{4/7}\left(\frac{R}{R_\odot}\right)^2 L_\odot = \rho\left(\frac{GM}{3Rk}\right)^{\beta} M \tag{40}$$
$$\left(\frac{\rho R_\odot^2\left(\frac{GM}{3k}\right)^{\beta}}{0.2L_\odot\left(\frac{M}{M_\odot}\right)^{4/7}}\right)^{1/(2+\beta)} = R \tag{41}$$
$$(M^{5.42})^{1/(2+\beta)} \propto R \tag{42}$$
$$\boxed{M^{.6775} \propto R(M)} \tag{43}$$

$$L(M) \sim 0.2\left(\frac{M}{M_\odot}\right)^{4/7}\left(\frac{R}{R_\odot}\right)^2 L_\odot = L_{\text{conv}} \tag{44}$$
$$\propto M^{4/7}R^2 \tag{45}$$
$$\propto M^{4/7}M^{0.6775*2} \tag{46}$$
$$\boxed{L(M) \propto M^{1.92}} \tag{47}$$

$$T_c = \frac{GM}{3Rk} \tag{48}$$
$$\propto \frac{M}{R} \tag{49}$$
$$\propto \frac{M}{M^{.6775}} \tag{50}$$
$$\boxed{T_c(M) \propto M^{.3225}} \tag{51}$$

$$T_{eff} \approx 4000 \left(\frac{L}{L_\odot}\right)^{1/102} \left(\frac{M}{M_\odot}\right)^{7/51} K \tag{52}$$

$$L = 4\pi R^2 \sigma T_{eff}^4 \tag{53}$$

$$= 4\pi\sigma T_{eff}^4 (M^{.6775})^2 \tag{54}$$

$$\frac{L}{4\pi\sigma T_{eff}^4} = M^{1.355} \tag{55}$$

$$\left(\frac{L}{4\pi\sigma T_{eff}^4}\right)^{1/1.355} = M \tag{56}$$

$$T_{eff} \approx 4000 \left(\frac{L}{L_\odot}\right)^{1/102} \left(\left(\frac{L}{4\pi\sigma T_{eff}^4}\right)^{1/1.355} \frac{1}{M_\odot}\right)^{7/51} K \tag{57}$$

$$\propto L^{1/102} \left(\left(\frac{L}{T_{eff}^4}\right)^{1/1.355}\right)^{7/51} \tag{58}$$

$$\propto L^{1/102} \left(\left(\frac{L}{T_{eff}^4}\right)^{.74}\right)^{7/51} \tag{59}$$

$$\propto L^{1/102} \left(\frac{L}{T_{eff}^4}\right)^{.1} \tag{60}$$

$$T_{eff}^{1+.4} \propto L^{1/102+.1} \tag{61}$$

$$L^{.11} \propto T_{eff}^{1.4} \tag{62}$$

$$L \propto (T_{eff}^{1.4})^{1/.11} \tag{63}$$

$$\boxed{L(T_{eff}) \propto T_{eff}^{12.73}} \tag{64}$$

Problem 2b:

Information from HW #4:

$$\rho_c = \frac{3Ma_n}{4\pi R^3} \tag{65}$$

$$T_c = \frac{0.6 m_p d_n a_n^{1/3} G}{k} \left(\frac{3}{4\pi}\right)^{1/3} \frac{M}{R} \tag{66}$$

$$n = \frac{3}{2} \text{ , } a_n = 5.99 \text{ , } d_n = .478 \tag{67}$$

$$\rho_c = \frac{3M}{4\pi R^3} 5.99 \tag{68}$$

$$= \frac{M_\odot}{R_\odot^3} \frac{3 \cdot 5.99}{4\pi} \frac{M}{M_\odot} \frac{R_\odot^3}{R^3} \frac{\text{cm}^3}{\text{g}} \tag{69}$$

$$\boxed{\rho_c \approx 8.48 \frac{M}{M_\odot} \left(\frac{R_\odot}{R}\right)^3 \frac{\text{cm}^3}{\text{g}}} \tag{70}$$

$$T_c = \frac{0.6 m_p d_n a_n^{1/3} G}{k} \left(\frac{3}{4\pi}\right)^{1/3} \frac{M}{R} \tag{71}$$

$$= \frac{M_\odot}{R_\odot} \frac{0.6 m_p d_n a_n^{1/3} G}{k} \left(\frac{3}{4\pi}\right)^{1/3} \frac{M}{M_\odot} \frac{R_\odot}{R} \tag{72}$$

$$\boxed{T_c \approx 3.53 \times 10^6 \frac{M}{M_\odot} \frac{R_\odot}{R} \mathrm{K}} \tag{73}$$

$$\tag{74}$$

$$\epsilon_{pp} \approx 5.05 \times 10^5 \rho_c X^2 T_7^\beta \ , \tag{75}$$

and we're going to evaluate it at the center of the star.

$$\epsilon_{pp_c} \approx 5.05 \times 10^5 \cdot 8.48 \frac{M}{M_\odot} \left(\frac{R_\odot}{R}\right)^3 \frac{\mathrm{cm}^3}{\mathrm{g}} X^2 \left(\frac{3.53 \times 10^6}{1 \times 10^7} \frac{M}{M_\odot} \frac{R_\odot}{R}\right)^\beta \tag{76}$$

$$\boxed{\epsilon_{pp_c} \approx 4.6 \times 10^3 \left(\frac{M}{M_\odot}\right)^{1+\beta} \left(\frac{R_\odot}{R}\right)^{3+\beta}} \tag{77}$$

Problem 2c:

$$
\begin{aligned}
L_{fus} = L_{conv} &= 0.2\left(\frac{M}{M_\odot}\right)^{4/7}\left(\frac{R}{R_\odot}\right)^{2} L_\odot && (78)\\
L_{fus} &\simeq= \frac{2.4\epsilon_c M}{(3+\beta)^{3/2}} && (79)\\
&\simeq \frac{2.4\epsilon_c M}{8.88^{3/2}} && (80)\\
&\simeq 9.07\times 10^{-2}\epsilon_c M && (81)\\
0.2\left(\frac{M}{M_\odot}\right)^{4/7}\left(\frac{R}{R_\odot}\right)^{2} L_\odot &\simeq 9.07\times 10^{-2}\epsilon_c M && (82)\\
&\simeq 9.07\times 10^{-2} M_\odot \frac{M}{M_\odot}\cdot 4.6\times 10^{3}\left(\frac{M}{M_\odot}\right)^{1+\beta}\left(\frac{R_\odot}{R}\right)^{3+\beta} && (83)\\
&\simeq 8.34\times 10^{35}\left(\frac{M}{M_\odot}\right)^{2+\beta}\left(\frac{R_\odot}{R}\right)^{3+\beta} && (84)\\
\left(\frac{M}{M_\odot}\right)^{4/7}\left(\frac{R}{R_\odot}\right)^{2} L_\odot &\simeq 4.17\times 10^{36}\left(\frac{M}{M_\odot}\right)^{2+\beta}\left(\frac{R_\odot}{R}\right)^{3+\beta} && (85)\\
\left(\frac{M}{M_\odot}\right)^{4/7}\left(\frac{R}{R_\odot}\right)^{2} &\simeq 1.1\times 10^{3}\left(\frac{M}{M_\odot}\right)^{2+\beta}\left(\frac{R_\odot}{R}\right)^{3+\beta} && (86)\\
\left(\frac{R}{R_\odot}\right)^{2} &\simeq 1.1\times 10^{3}\left(\frac{M}{M_\odot}\right)^{2+\beta-4/7}\left(\frac{R}{R_\odot}\right)^{-(3+\beta)} && (87)\\
\left(\frac{R}{R_\odot}\right)^{2+3+\beta} &\simeq 1.1\times 10^{3}\left(\frac{M}{M_\odot}\right)^{2+\beta-4/7} && (88)\\
&\boxed{\frac{R}{R_\odot} \simeq 1.903\left(\frac{M}{M_\odot}\right)^{.6717}} && (89)
\end{aligned}
$$

$$
\begin{aligned}
L(M) &\sim 0.2\left(\frac{M}{M_\odot}\right)^{4/7}\left(\frac{R}{R_\odot}\right)^{2} L_\odot && (90)\\
&\sim 7.6\times 10^{32}\left(\frac{M}{M_\odot}\right)^{4/7}\left(\frac{R}{R_\odot}\right)^{2} && (91)\\
&\sim 7.6\times 10^{32}\left(\frac{M}{M_\odot}\right)^{4/7}\left(1.903\left(\frac{M}{M_\odot}\right)^{.6717}\right)^{2} && (92)\\
&\sim 2.75\times 10^{33}\left(\frac{M}{M_\odot}\right)^{4/7+1.34} && (93)\\
&\boxed{L(M) \sim 2.75\times 10^{33}\left(\frac{M}{M_\odot}\right)^{1.915}} && (94)
\end{aligned}
$$

$$T_c = 3.53 \times 10^6 \frac{M}{M_\odot} \frac{R_\odot}{R} \mathrm{K} \tag{95}$$

$$= 3.53 \times 10^6 \frac{M}{M_\odot} \left(1.903 \left(\frac{M}{M_\odot} \right)^{.6717} \right)^{-1} \tag{96}$$

$$= 1.85 \times 10^6 \frac{M}{M_\odot} \left(\frac{M}{M_\odot} \right)^{-.6717} \tag{97}$$

$$\boxed{T_c = 1.85 \times 10^6 \left(\frac{M}{M_\odot} \right)^{.3283}} \tag{98}$$

$$L = 4\pi R^2 \sigma T_{eff}^4 \tag{99}$$

$$= 4\pi R_\odot^2 \left(\frac{R}{R_\odot}\right)^2 \sigma T_{eff}^4 \tag{100}$$

$$= 4\pi R_\odot^2 \left(1.903 \left(\frac{M}{M_\odot}\right)^{.6717}\right)^2 \sigma T_{eff}^4 \tag{101}$$

$$= 1.25 \times 10^{19} \left(\frac{M}{M_\odot}\right)^{1.343} T_{eff}^4 \tag{102}$$

$$\frac{L}{1.25 \times 10^{19} T_{eff}^4} = \left(\frac{M}{M_\odot}\right)^{1.343} \tag{103}$$

$$\left(\frac{L}{1.25 \times 10^{19} T_{eff}^4}\right)^{1/1.343} = \frac{M}{M_\odot} \tag{104}$$

$$4000 \left(\frac{L}{L_\odot}\right)^{1/102} \left(\frac{M}{M_\odot}\right)^{7/51} \approx T_{eff} \tag{105}$$

$$4000 \left(\frac{L}{L_\odot}\right)^{1/102} \left(\left(\frac{L}{1.25 \times 10^{19} T_{eff}^4}\right)^{1/1.343}\right)^{7/51} \approx T_{eff} \tag{106}$$

$$44.7 \left(\frac{L}{L_\odot}\right)^{1/102} \left(\left(\frac{L}{T_{eff}^4}\right)^{1/1.343}\right)^{7/51} \approx T_{eff} \tag{107}$$

$$44.7 \left(\frac{L}{L_\odot}\right)^{1/102} \left(\frac{L}{T_{eff}^4}\right)^{.102} \approx T_{eff} \tag{108}$$

$$44.7 \left(\frac{L}{L_\odot}\right)^{1/102} L^{.102} \approx T_{eff}^{1.408} \tag{109}$$

$$44.7 \left(\frac{L}{L_\odot}\right)^{1/102} L_\odot^{.102} \left(\frac{L}{L_\odot}\right)^{.102} \approx T_{eff}^{1.408} \tag{110}$$

$$1.19 \times 10^5 \left(\frac{L}{L_\odot}\right)^{.112} \approx T_{eff}^{1.408} \tag{111}$$

$$\left(\frac{L}{L_\odot}\right)^{.112} \approx 8.4 \times 10^{-6} T_{eff}^{1.408} \tag{112}$$

$$\frac{L}{L_\odot} \approx \left(8.4 \times 10^{-6} T_{eff}^{1.408}\right)^{1/.112} \tag{113}$$

$$\boxed{\frac{L}{L_\odot} \approx 4.82 \times 10^{-46} T_{eff}^{12.57}} \tag{114}$$

Problem 2d:

$$L(M = .1M_\odot) \sim 2.75 \times 10^{33} \left(\frac{.1M_\odot}{M_\odot}\right)^{1.915} \tag{115}$$
$$L(M = .1M_\odot) \sim 2.75 \times 10^{33} \left(.1\right)^{1.915} \tag{116}$$
$$L(M = .1M_\odot) \sim 3.34 \times 10^{31} \text{ ergs/s} \tag{117}$$
$$L(M = .1M_\odot) \sim 8.8 \times 10^{-3} L_\odot \tag{118}$$

$$L(M = .3M_\odot) \sim 2.75 \times 10^{33} \left(\frac{.3M_\odot}{M_\odot}\right)^{1.915} \tag{119}$$
$$L(M = .3M_\odot) \sim 2.75 \times 10^{33} \left(.3\right)^{1.915} \tag{120}$$
$$L(M = .3M_\odot) \sim 2.74 \times 10^{32} \text{ ergs/s} \tag{121}$$
$$L(M = .3M_\odot) \sim .072 L_\odot \tag{122}$$

$$R(M = .1M_\odot) = R_\odot \cdot 1.903 \left(\frac{.1M_\odot}{M_\odot}\right)^{.6717} \tag{123}$$
$$R(M = .1M_\odot) = R_\odot \cdot 1.903 \left(.1\right)^{.6717} \tag{124}$$
$$R(M = .1M_\odot) = .4 R_\odot \tag{125}$$

$$R(M = .3M_\odot) = R_\odot \cdot 1.903 \left(\frac{.3M_\odot}{M_\odot}\right)^{.6717} \tag{126}$$
$$R(M = .3M_\odot) = R_\odot \cdot 1.903 \left(.3\right)^{.6717} \tag{127}$$
$$R(M = .3M_\odot) = .85 R_\odot \tag{128}$$

$$T_{eff}(M = .1M_\odot) = \left(\frac{L(M = .1M_\odot)}{L_\odot} \frac{1}{4.82 \times 10^{-46}}\right)^{1/12.57} \tag{129}$$
$$T_{eff}(M = .1M_\odot) = \left(\frac{8.8 \times 10^{-3} L_\odot}{L_\odot} \frac{1}{4.82 \times 10^{-46}}\right)^{1/12.57} \tag{130}$$
$$T_{eff}(M = .1M_\odot) \approx 2765 \text{ K} \tag{131}$$

$$T_{eff}(M = .3M_\odot) = \left(\frac{L(M = .3M_\odot)}{L_\odot} \frac{1}{4.82 \times 10^{-46}}\right)^{1/12.57} \tag{132}$$
$$T_{eff}(M = .3M_\odot) = \left(\frac{.072 L_\odot}{L_\odot} \frac{1}{4.82 \times 10^{-46}}\right)^{1/12.57} \tag{133}$$
$$T_{eff}(M = .3M_\odot) \approx 3268 \text{ K} \tag{134}$$

Problem 3a:

* $R(M)$

* $L(M)$

* $T_c(M)$
* $L(T_{eff})$ ...again

$$\beta = -2/3 + 23.6T_7^{-1/3} \tag{135}$$
$$\approx 20 \tag{136}$$

$$L_{fusion} \approx \epsilon M \tag{137}$$
$$\approx \rho T^{\beta} M \ , \rho \propto \frac{M}{R^3} \ , T \propto \frac{M}{R} \text{ from VT} \tag{138}$$
$$\approx \frac{M}{R^3}\left(\frac{M}{R}\right)^{20} M \tag{139}$$
$$\approx \frac{M^{22}}{R^{23}} \tag{140}$$
$$\sim L_{rad} \propto M^3 \tag{141}$$
$$\frac{M^{22}}{R^{23}} \propto M^3 \tag{142}$$
$$\frac{M^{22}}{M^3} \propto R^{23} \tag{143}$$
$$\boxed{R \propto M^{19/23} \propto M^{.8}} \tag{144}$$
$$\boxed{T_c \propto M^{.2}} \tag{145}$$

$$L \propto R^2 T_{eff}^4 \tag{146}$$
$$R^2 \propto (M^{.8})2 \tag{147}$$
$$R^2 = M^{1.6} \propto L^{1/2} \tag{148}$$
$$L_{rad} \propto M^3 \tag{149}$$
$$L \propto L^{1/2} T_{eff}^4 \tag{150}$$
$$L^{1/2} \propto T_{eff}^4 \tag{151}$$
$$L \propto T_{eff}^8 \tag{152}$$
$$M^3 \propto T_{eff}^8 \tag{153}$$
$$\boxed{M^{3/8} \propto T_{eff}} \tag{154}$$

$$L \propto R^2 T^4 \tag{155}$$
$$\propto M^{1.6}(M^{3.8})^4 \tag{156}$$
$$\boxed{L \propto M^{3.1}} \tag{157}$$

$$L \propto M^{3.1} \tag{158}$$
$$L \propto (R^{1/.8})^{3.1} \tag{159}$$
$$L \propto R^{3.1/.8} \tag{160}$$
$$L^{.8/3.1} \propto R \tag{161}$$
$$L^{1.6/3.1} \propto R^2 \tag{162}$$
$$L \propto R^2 T_{eff}^4 \tag{163}$$
$$L \propto L^{1.6/3.1} T_{eff}^4 \tag{164}$$
$$L^{1.5/3.1} \propto T_{eff}^4 \tag{165}$$
$$L \propto T_{eff}^{4 \cdot 3.1/1.5} \tag{166}$$
$$\boxed{L \propto T_{eff}^{8.36}} \tag{167}$$

Problem 3b:

$$\frac{M_r}{M} < \frac{5}{8} \frac{P}{P_{rad}} \frac{L}{L_{edd}} \text{ , taken from lecture} \tag{168}$$

$$\frac{L}{L_{edd}} \approx 1 \text{ for very massive stars} \tag{169}$$
$$\frac{P}{P_{rad}} \approx 1 \text{ for very massive stars} \tag{170}$$
$$\frac{M_r}{M} < \frac{5}{8} \frac{P}{P_{rad}} \frac{L}{L_{edd}} \lesssim \frac{5}{8} \cdot 1 \cdot 1 \tag{171}$$
$$\frac{M_r}{M} \lesssim .62 \tag{172}$$

Problem 3c:

$$t \approx \frac{E_{nuc}}{L} \tag{173}$$

$$\frac{L}{L_\odot} = \left(\frac{M}{M_\odot}\right)^{3.5} \tag{174}$$

$$E_{nuc} = N_p E \tag{175}$$

$$E_{nuc} \approx .1 \frac{M}{m_p} \cdot 7 \text{ MeV} \tag{176}$$

$$t \approx \frac{.1 \frac{M}{m_p} \cdot 7 \text{ MeV}}{L_\odot \left(\frac{M}{M_\odot}\right)^{3.5}} \tag{177}$$

$$t \approx \frac{.1 M_\odot \frac{M}{M_\odot} \cdot 7 \text{ MeV}}{L_\odot m_p \left(\frac{M}{M_\odot}\right)^{3.5}} \tag{178}$$

$$t \approx \frac{.1 M_\odot \cdot 7 \text{ MeV}}{L_\odot m_p} \left(\frac{M}{M_\odot}\right)^{-2.5} \tag{179}$$

$$t \approx 2.5 \times 10^{18} \left(\frac{M}{M_\odot}\right)^{-2.5} \text{ s} \tag{180}$$

$$t \approx 7.8 \times 10^{10} \left(\frac{M}{M_\odot}\right)^{-2.5} \text{ years} \tag{181}$$

$$t(M = 50 M_\odot) \approx 4.43 \times 10^{6} \text{ years} \tag{182}$$

$$t(M = 100 M_\odot) \approx 7.84 \times 10^{5} \text{ years} \tag{183}$$