September 6, 2011

Jeren Suzuki

1 Energy Transport by Radiation (& Conduction)

So in the beginning...

2 Energy Transport by Conduction

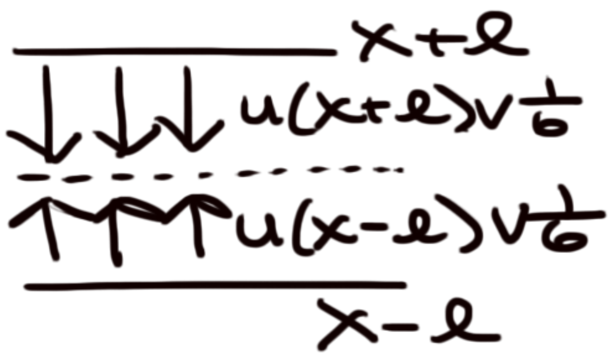

Figure 2 shows the stratification levels of the inside of a star at a given height x with a length l either up or down.

$T =$ temp

$u =$ Thermal Energy Density

$v =$ velocity

$l=$ mean free path

The factor of $\frac{1}{6}$ is used because we multiply $\frac{1}{3}$ (from 3 degrees of translational freedom) with $\frac{1}{2}$ (from going up and down).

$$F \equiv \text{ net flux of energy} = \frac{1}{6}u(x-l)v - \frac{1}{6}u(x+l)v$$

Taylor expanding $u(x-l) = u(x) - \frac{du}{dx}l$, we get:

$$F = -\frac{1}{3}vl\frac{du}{dx} \text{ , or more generally, } F = -\chi \nabla u$$

For a gas of charged particles,

$$U = n\frac{3}{2}kT$$

$$F = -\frac{1}{3}vl\frac{dU}{dT}\frac{dT}{dx} \text{ , where } \frac{dU}{dT} \text{ is the specific heat.}$$

$$\boxed{F = -\frac{1}{2}vlnk\frac{dT}{dx}}$$

In the case of a charged particle moving past another charged particle, there is a characterized distance (b) where the change in trajectory is significant.

$$\frac{q^2}{b} \sim kT$$

$$b \sim \frac{q^2}{kT}$$

$$\text{effective area } = \sigma = \pi b^2$$

$$\boxed{\sigma = \frac{\pi q^4}{(kT)^2}}$$

$$F = -\frac{1}{2}nklv\frac{dT}{dx} \text{ , } v \sim v_{thm} = \sqrt{\frac{kT}{m}}$$

$$\sim -\frac{1}{2}\frac{kv}{\sigma}\frac{dT}{dx}$$

$$= -\chi\frac{dT}{dx} \text{ , where } \chi = \frac{1}{2}\frac{kv}{\sigma} \propto \frac{T^{5/2}}{\sqrt{m}}$$

Electrons move faster than protons so they transfer energy much more effectively. Electrons and protons have the same σ, but the difference in velocities is huge.

2.1 How important is this energy transport in the sun?

$$F = -\chi\frac{dT}{dx}$$

$$L = 4\pi r^2 F$$

$$\sim 4\pi R^2\chi\frac{T_c}{R} \text{ (We're doing a poor man's derivative where } \frac{dT}{dx} = \frac{T_c}{R}\text{)}$$

$$L \sim \frac{k^{7/2}T_c^{7/2}R}{q^4\ln(\Lambda)\sqrt{m_e}}$$

$$\sim 10^{-4}L_\odot\left(\frac{R}{R_\odot}\right)\left(\frac{T_c}{10^7\text{ K}}\right)^{7/2}$$

Doing this, we get that $L_{conduction} \ll L_\odot$ and therefore electron conduction is unimportant to energy transport.

3 Radiation Transport of Energy

$$U = aT^4$$

$l = \frac{1}{n\sigma}$

σ = cross section for photons to interact with matter, not with other photons

$$
\begin{aligned}
F &= -\frac{1}{3} cl \frac{d}{dr} aT^4 \\
&= -\frac{4}{3} claT^3 \frac{dT}{dr} \\
&\boxed{= -\frac{4}{3} \frac{caT^3}{\kappa\rho} \frac{dT}{dr}}
\end{aligned}
$$

For an Ionized plasma with dominant photon-matter interactions through electron scattering (Thomson Scattering),

$$
\begin{aligned}
m_e \bar{a} &= -e(\bar{E} + \frac{\bar{v}}{c} \times \bar{B}) \\
|E| &= |B| \\
m_e \bar{a} &= -e\bar{E} \text{ since } \frac{\bar{v}}{c} \times \bar{B} \text{ is small} \\
\bar{a} &= -\frac{e\bar{E}}{m_e}
\end{aligned}
$$

$$
\begin{aligned}
P &= \frac{2}{3} \frac{e^4}{c^3 m_e^2} |\bar{E}|^2 \\
\sigma F &= P \\
\sigma \frac{c}{4\pi} |\bar{E}| &= \frac{2}{3} \frac{e^4}{c^3 m_e^2} |\bar{E}|^2 \\
&\boxed{\sigma_T = \frac{8\pi}{3} \frac{e^4}{m_e^2 c^4} = \text{ Thomson cross-section or } e^-\text{-scattering cross-section}}
\end{aligned}
$$

$$
\begin{aligned}
\sigma_T &= \frac{8\pi}{3} r_c^2, \quad \text{where } r_c \text{ is the classical radius of the } e^- \\
r_c &= \frac{e^2}{m_e c^2} \approx 2.8 \times 10^{-13} \text{ cm},
\end{aligned}
$$

which should strike as odd since e^- has no observed structure but still has an ”effective radius”.