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1 Convection Continued

$$
\begin{aligned}
a &= |N|^2 \delta r \\
|N|^2 &= \frac{g}{cp}\left|\frac{ds}{dr}\right| \\
&= \frac{g}{H}\left|\frac{H}{c_p}\frac{ds}{dr}\right| \\
v_c^2 &= a\delta r = |N|^2 \delta r^2 \\
\delta r &\equiv \alpha H \\
v_c &= \alpha c_s \left|\frac{H}{c_p}\frac{ds}{dr}\right|^{1/2} \\
F = \frac{1}{2}\rho v_c^3 &= \frac{1}{2}\rho\alpha^3 c_s^3 \left|\frac{H}{c_p}\frac{ds}{dr}\right|^{3/2}
\end{aligned}
$$

We wanted to find the $F_r = -\frac{4}{3}\frac{caT^3}{\kappa\rho}\frac{dT}{dr}$ equivalent for convection. $F = \frac{1}{2}\rho v_c^3 = \frac{1}{2}\rho\alpha^3 c_s^3 \left|\frac{H}{c_p}\frac{ds}{dr}\right|^{3/2}$ gives the v_c and $\frac{ds}{dr}$ given the flux.

$\left|\frac{H}{c_p}\frac{ds}{dr}\right| \sim 10^{-6}$, $s \sim c_p$, so $\frac{\Delta s}{s} \sim 10^{-6}$ on a length scale H. Ergo, $s =$ constant in the convection zone. This replaces $F_r = -\frac{4}{3}\frac{caT^3}{\kappa\rho}\frac{dT}{dr}$. Let's assume $P \propto \rho^\gamma$ & $\frac{dP}{dr} = -\rho\frac{GM_r}{r^2}$.

$$
\begin{aligned}
\frac{dM_r}{dr} &= 4\pi r^2 \rho \\
\frac{d}{dr}\left(\frac{r^2}{\rho}\frac{dP}{dr} = -\rho G M_r\right) \\
\frac{d}{dr}\left(\frac{r^2}{\rho}\frac{dP}{dr}\right) &= -4\pi r^2 G\rho
\end{aligned}
$$

But... if $P = K\rho^\gamma$, $\frac{dP}{dr} = \gamma K \rho^{\gamma-1}\frac{d\rho}{dr}$! These kinds of models are called:

1.1 Polytropic Models

$$
\begin{aligned}
P &= K\rho^\gamma \\
&= K\rho^{1+1/n}, \gamma = 1 + \frac{1}{n}, \text{ where n is the polytropic index}
\end{aligned}
$$

$$
\begin{aligned}
\theta &= \left(\frac{\rho}{\rho_c}\right)^{1/n}, \rho_c = \rho(r=0) \\
\zeta = \frac{r}{a}, a &= \sqrt{\frac{(n+1)K\rho_c^{\frac{1}{n}-1}}{4\pi G}}, [a] = \text{ cm}
\end{aligned}
$$

$$\boxed{\frac{1}{\zeta} \cdot \frac{d}{d\zeta}\left(\zeta^2 \frac{d\theta}{d\zeta}\right) = -\theta^n}$$

Let's look at the properties of a fully convective star of low mass. Low mass $\rightarrow$ low T $\rightarrow$ high κ.

1.2 $M_* < \frac{1}{3}M_\odot$ on MS

For stars with photons carrying the energy out, $L \propto M^3$ if $\sigma = \sigma_T$ for fully convective stars, $L = 4\pi R^2 F_c$, where $F_c = \rho v_c^3 \propto \left|\frac{ds}{dr}\right|^{3/2}$. Let's look at the surface where photons are carrying the energy out.

$$\text{s} = \text{constant}$$

$$P \propto \rho^{5/3} \propto T^{5/2}$$

$$\rho T \propto \rho^{5/3}, T \propto \rho^{2/3}$$

$$\frac{P_c}{P_{photons}} = \left(\frac{T_c}{T_{eff}}\right)^{5/2}, \text{ now use V.T. to relate } T_c \text{ and } M \text{ \& } R.$$

BUT

We know that this is a $n = 3/2$ polytrope so