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1 Convection

Second Law of Thermodynamics: $TdS = dE + PdV$, which isn't all that useful for stars, really.

$U = \frac{E}{\mu}$: Energy per unit mass

$s = \frac{S}{U}$: Entropy

$M =$ conserved, l is small

$\rho = \frac{M}{V}, V = \frac{M}{\rho} \rightarrow \boxed{dV = -d\rho\frac{M}{\rho^2}}$: second law, for astrophysicists

1.1 Review of the Adiabatic Process

$$\epsilon = E/\text{unit volume}, NR : P = \frac{2}{3}\epsilon$$

$$R : P = \frac{1}{3}\epsilon$$

$$U = \frac{\epsilon}{\rho} = \frac{P}{\rho} = \phi U \text{ , where } \phi \text{ is either } 1/3 \text{ or } 2/3$$

$$dU = \frac{P}{\rho^2}d\rho = \phi U\frac{d\rho}{\rho}$$

$$\frac{dU}{U} = \phi\frac{d\rho}{\rho}$$

$$U \propto \rho^{\phi} \text{ , for an adiabatic process}$$

$$P \propto \rho U \propto \rho^{\phi+1} \propto \rho^{\gamma} \text{ ,} \phi + 1 \text{ is the adiabatic index}$$

$$\text{For a NR gas: } \phi = \frac{2}{3}, \gamma = \frac{5}{3} \text{ ,} P \propto \rho^{5/3}, T \propto \rho^{2/3} \text{ for an adiabatic process}$$

$$\text{For a R gas: } \phi = \frac{1}{3}, \gamma = \frac{4}{3} \text{ ,} P \propto \rho^{4/3}, T \propto \rho^{1/3} \text{ for an adiabatic process}$$

1.2 What is the Entropy of an Ideal Gas?

$$
\begin{aligned}
TdS &= dU - \frac{P}{\rho^2}d\rho \\
\frac{TdS}{U} &= \frac{dU}{U} - (\gamma-1)\frac{U\frac{d\rho}{\rho}}{U} \\
U &= \frac{P}{\rho}\frac{1}{\gamma-1}\frac{kT}{m} \\
\frac{m(\gamma-1)}{k}dS &= \frac{dU}{U} - (\gamma-1)\frac{d\rho}{\rho} \\
\frac{m(\gamma-1)}{k}s &= \ln U - (\gamma-1)\ln\rho + c \\
&\boxed{s = \frac{k}{m}\frac{1}{\gamma-1}\ln\left(\frac{U}{\rho^{\gamma-1}}\right) + c} \\
s &= \frac{k}{m}\frac{1}{\gamma-1}\ln\left(\frac{P}{\rho^{\gamma}}\right) + c
\end{aligned}
$$

For an adiabatic process, $s = 0$.

$$
\begin{aligned}
\frac{Tds}{dt} &= \frac{dU}{dt} - \frac{P}{\rho^2}\frac{d\rho}{dt} \\
&= E_{fusion} - \frac{1}{\rho}(\bar{\nabla}\cdot\bar{F})
\end{aligned}
$$

Say a blob is gaining/losing heat. E_{fusion} is the heating per mass per time and $\bar{F}$ is the flux of E. In general:

$$
\begin{aligned}
\text{total cooling} &= \int \bar{F}\cdot d\bar{A} \\
&= \int \bar{\nabla}\cdot\bar{F}d\bar{V} \\
\text{cooling per unit } V &= \bar{\nabla}\cdot\bar{F} \\
\text{cooling per unit mass} &= \frac{1}{\rho}(\bar{\nabla}\cdot F)
\end{aligned}
$$

If a blob moves up a distance dr, given $T(r), P(r)$, and $\rho(r)$, is the fluid buoyantly stable? i.e. $\rho_{blob} \gtrless \rho_*$? We'll be making 2 assumptions which we will then confirm *post-facto*.

Motion is adiabatic $\leftarrow$ valid is the time scale to move (~ 1 month) is sufficiently smaller than the time to exchange heat with the surroundings ($\sim 10^7$ years)

$P_{blob} = P_*$ at all times; in pressure equilibrium with surroundings

The time scale to establish HE: $\sim \frac{1}{\sqrt{G\rho}} \sim 1$ hr $\ll$ time to move dr, which is about a month. If it's adiabatic, $s_{blob} = s \neq s_*$ in general, where s_{blob} is the blob at the new position, s is the initial entropy, and s_* uis the background entropy of the star at the new position.

$\frac{ds}{dr} < 0$	$\frac{ds}{dr} > 0$
$s > s_*$	$s < s_*$
$s_{blob} > s_*$	$s_{blob} < s_*$
$P_{blob} = P_*$	$P_{blob} = P_*$
$\rho_{blob} < \rho_*$	$\rho_{blob} > \rho_*$
buoyancy unstable, rises	sinks back down (stable)