Homework #1

1. In class, I derived the plane-parallel isothermal hydrostatic atmosphere. The Earth's atmosphere is made primarily of molecular oxygen and nitrogen, so that the average mass of a particle is approximately 30 $m_p$. At room temperature calculate

   a) the scale-height of the Earth's atmosphere. How does it compare to the Earth's radius? How does it compare to the height of the tallest mountains on Earth?

   b) the mass of the Earth's atmosphere. How does it compare to the mass of the Earth?

2. The density of the interior of the sun is significantly larger than that of water. Why, then, do we treat stellar interiors as an ideal gas! The important physics here is as follows: in a gas, interparticle forces are typically unimportant (unless particles happen to get very close to each other) while in a liquid, interparticle forces are important even for when particles are at a "typical" separation from each other. For an ionized plasma, the relevant interparticle force is the Coulomb electric force. Note that you do not need any quantum mechanics for this problem. It's purely classical.

   a) Provide a quantitative relation between the temperature and density of a star which indicates when we can treat it as a gas (rather than a liquid) throughout its interior, in spite of the very high densities. Is our assumption valid at the center of the sun?

   b) If all stars have roughly the same central temperature (that of the sun), use a scaling argument to estimate the stellar mass at which the simple non-interacting ideal gas assumption breaks down.

3. Consider a star composed solely of ionized hydrogen. Charge neutrality requires that $n_e = n_p$ and thermal equilibrium implies that $T_e = T_p$. Show that for both the electrons and protons to be separately in force balance (no net force), there must be an electric field in the star. Calculate the magnitude and direction of the electric field.

4. Consider a star of mass $M$ and radius $R$ with a density profile given by

   $$\rho(r) = \rho_c(1 - r/R) \tag{1}$$

   where $\rho_c$ is the central density of the star.

   a) Calculate the gravitational potential energy of the star. What is the total thermal energy of the star required for hydrostatic equilibrium?

   b) Using hydrostatic equilibrium, calculate the pressure as a function of radius in the star.

   c) Find the temperature of the star as a function of radius assuming an ideal gas of pure, ionized hydrogen.

   d) What is the relation between the central temperature, central density, and the mass $M$ of the star? What is the predicted central temperature of the sun?

   e) Use hydrostatic equilibrium and a scaling argument to show that the *proportionality* derived in d) between $T_c$, $\rho_c$, and $M$ is true for any star supported by gas pressure, independent of the specific density profile in equation (1).