GRE Physics Study Notes v2

Courtesy of Nicole Duncan, transcribed by Jeren Suzuki

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Contents

1 Hermitian Matrix

1. Square Matrix
2. A = A† $\rightarrow$ the matrix is equal to its conjugate transpose
3. Entries on the diagonal are real
4. Sum of 2 Hermitian matrices is Hermitian
5. Product of 2 Hermitian matrices is Hermitian only if they commute
6. Eigenvalues are orthogonal
7. The determinant is real

2 Doppler Effect

$$f = f_0 \left[\frac{v + v_s}{v + v_0}\right] \tag{1}$$

$$\Delta d \downarrow = \begin{cases} +v_s \\ -v_0 \end{cases}$$

towards e/o

$$\Delta d \uparrow = \begin{cases} -v_s \\ +v_0 \end{cases}$$

away from e/o

2.1 Relativistic

$$\frac{f_0}{f} = \frac{\lambda}{\lambda_0} = \sqrt{\frac{1+\beta}{1-\beta}}\, , \beta = \frac{v}{c} \tag{2}$$

3 Lagrangian and Hamiltonian

$$L = T - U \tag{3}$$

$$\frac{\partial L}{\partial x} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = 0 \tag{4}$$

$$H = T + U \text{ iff } U \neq U(\dot{x})\ , U \neq U(t) \tag{5}$$

$$\rho = \frac{\partial L}{\partial \dot{q}} \qquad \dot{q} = \frac{\partial H}{\partial \rho} \qquad \dot{p} = \frac{\partial H}{\partial q} \tag{6}$$

4 The Structure of Hydrogen

1. Fine: Spin/orbit + relativistic correction
   Breaks l degeneracy , preserves j
   why $E_{2s} < E_{2p}$
2. Hyperfine: spin/spin coupling of e^-/nucleus
   Responsible for 21 cm line

$$\mu_p = \frac{ge}{2m_p}\langle \bar{s}_p \rangle \ \mu_e = -\frac{e}{m_e}\langle \bar{s}_e \rangle \tag{7}$$

$$E_{n'f} = \frac{\mu_0 g_p e^2}{3\pi m_p m_e a^3}\langle \bar{s}_p \cdot \bar{s}_e \rangle \tag{8}$$

3. Stark Effect: Atom in external E
   Not spin dependent
   $H' = eE_z$ if $E = \hat{E}_z$
   Hydrogen:

$$E_1' = \langle H' \rangle = eE\int_0^\infty d^3rz|\Psi_{100}|^2 = 0 \tag{9}$$

4. Zeeman Effect: Atom in external B
   Spin/orbital angular momentum(l) + B coupling

$$H_z' = (\bar{\mu}_e + \bar{\mu}_s) \cdot B_{\text{ext}} \tag{10}$$

   Weak: $B_{\text{ext}} \ll B_{\text{int}}$
   $E' \propto mj \to$ breaks into $2j+1$ levels
   Strong: $B_{\text{ext}} \gg B_{\text{int}}$
   $E' = \mu_B B_{\text{ext}}(m_e + 2m_s)$

5 Particle in a Box

$$E_n = n^2 E_0 \tag{11}$$

$$E_0 = \frac{\hbar^2 k^2}{2m} \tag{12}$$

$$k = \frac{n\pi}{a} \tag{13}$$

$$\Psi = \sqrt{\frac{2}{a}}\sin(kx)\ , p = \hbar k \tag{14}$$

$$\text{3D: } E = \frac{\hbar^2}{2m}[k_x^2 + k_y^2 + k_z^2] \tag{15}$$

6 Free Particle

$$\Psi = Ae^{i(kx-\omega t)} \tag{16}$$

$$\Delta p \Delta x = \frac{\hbar}{2} \tag{17}$$

$$\Delta x \Delta k \sim 1 \tag{18}$$

Packet moves with group velocity... $v_g = \frac{\partial \omega}{\partial k}$

$$\Psi = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \Phi(k) e^{ikx} dk \qquad \Phi = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \Psi(x) e^{-ikx} dx \tag{19}$$

7 Schrödinger's Equation

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\Psi = i\hbar\frac{\partial \Psi}{\partial t} \tag{20}$$

Separable $\Psi(x,t) = \Psi(x)\phi(t)$

$$\phi = e^{-iE_n t/\hbar} \tag{21}$$

8 Index of Refraction

$$n = \frac{c}{v} = \sqrt{\frac{\epsilon\mu}{\epsilon_0\mu_0}} \qquad v = v_\phi = \sqrt{\frac{1}{\epsilon\mu}} \tag{22}$$

$\lambda = \frac{\lambda_0}{n}$ inside a medium

8.1 Cherenkov Radiation

A charged particle passing through a medium which travels faster than the speed of light in that medium will emit light

$$n = \frac{c}{v} \longrightarrow v_{\text{min}} = \frac{c}{n} \tag{23}$$

8.2 Bremsstrahlung Radiation

Continuous spectrum of radiation emitted when a charged particle is decelerated in a metal target

9 Gauss' Laws

$$\int E \cdot da = \frac{Q_{\text{in}}}{\epsilon_0} \to \nabla \cdot E = \frac{\rho_{\text{in}}}{\epsilon_0} \tag{24}$$

$$\int B \cdot da = 0 \to \nabla \cdot B = 0 \tag{25}$$

$$\int g \cdot da = 4\pi MG \to \nabla \cdot g = -4\pi G\rho_m \tag{26}$$

10 Damped Driven Oscillator

$$F = -\underbrace{kx}_{\text{spring}} - \underbrace{b\dot{x}}_{\text{damp}} + \underbrace{A\cos\theta}_{\text{driving}}, \omega = \sqrt{\frac{k}{m}}, \beta = \frac{b}{2m} \tag{27}$$

10.1 Critically Damped $\to \omega = \beta$

$$X_e = Ae^{-\omega_0 t} + A_2 te^{-\omega_0 t} \tag{28}$$

10.2 Overdamped $\to \omega < \beta$

$$X_o = Ae^{-\beta t}e^{-\omega'' t} \ , \omega'' = \sqrt{\beta^2 - \omega_0^2} \tag{29}$$

10.3 Underdamped $\to \omega > \beta$

$$X_u = Ae^{-\beta t}\cos(\omega' t + \phi) \ , \omega' = \sqrt{\omega_0^2 - \beta^2} \tag{30}$$

11 Traveling Wave Formalism

$$v_\phi = \frac{\omega}{k} \quad \psi = A\cos k(vt - x) = A\cos(\omega t - kx) \tag{31}$$

In one period, $x - vt = 2\pi$

12 Maxwell Velocity Distribution

$$D(v) \propto v^2 e^{-E/kT} \tag{32}$$

13 Mean Free Path

$$l = \frac{1}{n\sigma} \tag{33}$$

14 Cross Section

$$N_s = N_i \frac{N_t}{A}\sigma \tag{34}$$

15 Particle Diffusion (Fick's Law)

$$J = -D\nabla n \tag{35}$$

16 Thermal Diffusion (Fourier's Law)

$$\phi = -\sigma\nabla T \tag{36}$$

Interaction	Quantity	Variable	Formula
Mech	vol	P	$P = -\frac{\partial U}{\partial V} = T\left(\frac{\partial S}{\partial V}\right)$
Thermal	temp/energy	T	$T = \frac{\partial}{\partial S}$
Diffuse	particles	μ	$\mu = -\frac{\partial U}{\partial N} = T\frac{\partial S}{\partial N}$

17 Thermodynamic Identity

$$TdS - PdV + \mu dN = dU \tag{37}$$

18 Heat Capacity

$$C \equiv \frac{dQ}{dT} \tag{38}$$

$$C_p = \frac{dQ}{dT} = T\frac{dS}{dT} \qquad C_v = \frac{dQ}{dT} = \frac{dU}{dT} \tag{39}$$

At constant P, E lost to work $\Rightarrow T_p < T_V \Rightarrow C_p > C_v$

19 Water

$\rho = 1$ g/cm^3, 1 L = 1 kg

20 Decay

$^{0}_{-1}\beta + \bar{\nu}$ $\quad ^{0}_{1}\beta + \nu$ $\quad ^{2}_{1}D$

$^{4}_{2}\alpha$ $\quad ^{0}_{0}\gamma$ $\quad ^{A}_{Z}X$

21 Beats

$$f_0 = f_1 - f_2 \ , \qquad T_b = \frac{1}{f_1 - f_2} \ , \qquad \text{occur when } f_1 + f_2 \text{ are close} \tag{40}$$

The tuned frequency:

$$f = \underbrace{n}_{\text{harmonic}} \overbrace{f_f}^{\text{fundamental}} \tag{41}$$

22 First Law of Thermodynamics

$$U = Q + W \tag{42}$$

22.1 Second Law of Thermodynamics

S increases or stays the same for any cyclic process.

22.2 Third Law of Thermodynamics

$$S(T=0) = 1 \quad C_v \to 0 \text{ as } T \to 0 \tag{43}$$

23 Fundamental Assumption of Statistical Mechanics

All accessible microscopic states are equally likely

24 Isothermal

Slow so T can equalize.

$$P_1V_1 = P_2V_2 \quad W = Nk\ln\left(\frac{V_i}{V_f}\right) \quad ,W = -\int_{V_i}^{V_f} PdV \tag{44}$$

$$U = 0 \ \text{ since } \Delta T = 0 \ ,U = \frac{f}{2}Nk\Delta T \tag{45}$$

25 Adiabatic Compression

Fast, so no ΔQ lost, like opening a soda can.

$$\Delta Q = 0 \to U = W \tag{46}$$
$$\Delta U = Nk\Delta T = W \tag{47}$$

$$\gamma = \frac{f+2}{f} \quad ,W = \frac{P_fV_f - P_iV_i}{1-\gamma} \quad ,V_f^{\gamma}P_f = V_i^{\gamma}P_i \tag{48}$$

26 Heat

$$Q = TdS \tag{49}$$
$$Q = mc\Delta T \tag{50}$$
$$Q = Pt \tag{51}$$
$$U = Q + W \tag{52}$$

27 Cyclotron

$$\omega = \frac{qB}{m} \qquad F_c = F_B \to \frac{mv^2}{r} = qvB \to v = \frac{qBr}{m} = r\omega \tag{53}$$

28 Fermi

$$T_f = \frac{E}{k_B} \qquad E_f = \frac{\hbar^2k^2}{2m} \qquad p = \hbar k \qquad v_f = \frac{p_f}{m} \qquad k_F = \left(\frac{3\pi^2 N}{\text{vol}}\right)^{1/3} \tag{54}$$

$$p_f = \frac{2}{3}\frac{E_f}{v} \tag{55}$$

Degenerate Fermi gas: so cold that all states below E_F are occupied

29 Telescope

$$M = -\frac{f_{\text{object}}}{f_{\text{eye}}} = \frac{\theta_{\text{eye}}}{\theta_{\text{object}}} \tag{56}$$

30 Multiplicity/States

Probability $(\Omega_n) = \frac{\Omega_n}{\Omega_{all}}$ where Ω is the multiplicity # of things.

1. Total # microstates: (# of states can be in)$^{\text{\# of things}}$
2. Ways to choose n things from N

$$\Omega\binom{N}{n} = \frac{N!}{(N-n)!n!} \tag{57}$$

31 Rocket Motion

$$u\frac{dm}{dt} + M\frac{dv}{dt} = 0 \tag{58}$$

$$v_f = v_0 + u\ln\left(\frac{M_i}{M_f}\right) \tag{59}$$

32 Collisions

1. Momentum is always conserved $p_i = p_f$
   Don't forget to use $(+)$ and $(-)$ for before and after velocity collisions
2. KE and U conserved before or after collision only
3.

$$\epsilon = \frac{\overbrace{|v_1| + |v_2|}^{\text{after}}}{\underbrace{|U_1| + |U_2|}_{\text{before}}} \tag{60}$$

4. Impulse $J = F\Delta t = \Delta p = \Delta L$
5. Cross section $N_s = N_I \frac{N_t}{A}\sigma$

33 Springs/Single Harmonic Oscillator

$$F = -kx \quad U = \frac{1}{2}kx^2 \quad \omega = \sqrt{\frac{k}{m}} \tag{61}$$

$$ma = -kx = m\ddot{x} \tag{62}$$

$$E_{\text{tot}} = \frac{1}{2}kA^2\ ,\ A = \text{max amplitude} \tag{63}$$

34 Thin Films

$$\Delta\phi = \begin{cases} 0 & n_2 < n_1 \\ \pi & n_2 > n_1 \end{cases} \qquad 2d = \begin{cases} n\lambda/2 & \Delta\phi_{\text{tot}} = \pi \\ n\lambda & \Delta\phi_{\text{tot}} = 0, 2\pi \end{cases}, \; n = \text{odd \#'s only}$$

35 Conductivity/Current Density

$$\bar{J} = ne\bar{v} = \sigma E \tag{64}$$

$$\sigma = \frac{ne^2\tau}{m} \tag{65}$$

36 Resistance

$$R = \frac{\rho L}{A} \tag{66}$$

37 Boltzmann Statistics

$$Z = \sum_i g_i e^{-E_i/kT} \;, g_i = \text{degeneracy of state } i \tag{67}$$

$$p_s = \frac{g_s e^{-E_s/k_B t}}{Z} \qquad \frac{p_A}{p_B} = \frac{g_A}{g_B}\frac{e^{-A/kT}}{e^{-B/kT}} = \frac{g_A}{g_B} e^{(-A+B)/kT} \tag{68}$$

$$\langle \bar{X} \rangle = \frac{\sum_i e^{-E_i/kT}}{Z} \rightarrow \langle \bar{E} \rangle = \frac{1}{Z}\sum_i E_i e^{-E_i/kT} \tag{69}$$

$$U = N\bar{E} \rightarrow \text{total Energy of system} \tag{70}$$

38 Density of State Distribution

Fermions:

$$N_i = \frac{g_i}{e^{(E_i-\mu)/kT} + 1} \tag{71}$$

Bosons:

$$N_i = \frac{g_i}{e^{(E_i-\mu)/kT} - 1} \tag{72}$$

39 Band Pass Filter

$$j\omega C + \frac{1}{j\omega L} = Z = \frac{-\omega^2 CL + 1}{j\omega L} \qquad \omega_o = \frac{1}{\sqrt{LC}} \tag{73}$$

40 Resonant Frequency

Inductor and capacitor in series:

$$j\omega L = \frac{1}{j\omega C} \to \omega_0^2 LC = 1 \to \omega = \frac{1}{\sqrt{LC}} \tag{74}$$

Inductor and capacitor in parallel:

$$j\omega C + \frac{1}{j\omega L} = 0 \to j\omega C = \frac{1}{j\omega L} \to \omega = \frac{1}{\sqrt{LC}} \tag{75}$$

41 Central Force Motion

$$\begin{array}{lll} \mu = \frac{m_1 m_2}{m_1+m_2} & R_{\text{CM}} = \frac{\sum m_i r_i}{\sum m_i} & T = \frac{1}{2}\mu|\dot{r}|^2 \\ r_1 = \frac{m_2}{m_1+m_2} r & r_2 = \frac{m_1}{m_1+m_2} r & \bar{r} = \bar{r}_1 - \bar{r}_2 \end{array}$$

42 Moments of Inertia

1. Hoop: MR^2
2. Disk: $\frac{1}{2}MR^2$
3. Solid Sphere: $\frac{2}{3}MR^2$
4. Hollow Sphere: $\frac{2}{5}MR^2$
5. Rod End: $\frac{1}{3}ML^2$
6. Rod Middle: $\frac{1}{12}ML^2$

43 Blackbody Radiation

$$T\lambda = 3 \text{ mm}K \qquad P \propto T^4 \qquad \rho \propto AT^4 \text{ (for photons)} \tag{76}$$

44 Heat Engine

$$e \leq 1 - \frac{T_c}{T_h} \qquad e = \frac{\text{benefit}}{\text{cost}} = \frac{W}{Q_h} \qquad W = Q_h - Q_c \tag{77}$$

45 Refrigerator

$$e \leq \frac{T_c}{T_h - T_c} \qquad e = \frac{\text{benefit}}{\text{cost}} = \frac{Q_c}{W} \tag{78}$$

46 Space-Time Diagram

$$s^2 = x^2 - (ct)^2 \tag{79}$$

1. $s > 0$ Spacelike
   Δt can equal 0, simultaneous events occur
2. $s < 0$ Timelike
   Events can occur at same point in space, $\Delta x = 0$, but not simultaneously $\Delta t \neq 0$
3. $\Delta s = 0$ Lightlike

47 Low-Pass

RC or LR perpendicular to each other. For RC:

$$\frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{j\omega CR + 1}\ , \omega \to 0, \to 1 \tag{80}$$

For LR:

$$\frac{R}{R + j\omega L}\ , \omega \to 0, \to 1 \tag{81}$$

Square signals turn into wave-like signals with crests.

$$V_{\text{out}} = V_{\text{in}} \left(\frac{Z_2}{Z_1 + Z_2} \right) \tag{82}$$

Where Z_1 and Z_2 are on either side of a perpendicular V_{out} with a V_{in} leading into Z_1.

48 High Pass

CR or RL, turns square signals into signals where the flat tops of square turns into decaying to 0.

49 Special Relativity

v/c	γ
.1	1.005
.25	1.033
.5	1.151
.75	1.55
.9	2.3

Motion in $\hat{x}$:

$$x = \gamma(x' + vt') \tag{83}$$

$$t = \gamma \left(t' + \frac{vx}{c^2} \right) \tag{84}$$

$$u'_x = \frac{u_x + v}{1 + \frac{u_x v}{c^2}} \tag{85}$$

$$u'_z = \frac{u_z}{\gamma \left(1 + \frac{u_x v}{c^2} \right)} \tag{86}$$

49.1 Time Dilation

$$t' = \gamma t_0 \tag{87}$$

49.2 Length Contraction

$$x' = \frac{x}{\gamma} \tag{88}$$

49.3 Invariant

$$\Delta s^2 = \Delta x^2 - (ct)^2 \tag{89}$$

49.4 Energy

$$E_r^2 = E_0^2 + (pc)^2 \quad E_r \neq \frac{p_r^2}{2m} \quad E_r = \gamma E_0 \quad p_r = \gamma m v = \gamma p \tag{90}$$

50 Finite Potential Well

$$E \propto n^2 \quad d \propto \frac{1}{\sqrt{V - E_n}} \to d \propto n \tag{91}$$

51 Fundamental Particles

1. Bosons:
   Gauge Boson - Gluon - Strong
   Photon - E&M
   W,Z Bosons - a.k.a weak bosons
   Higgs
   Graviton
   Pion
2. Fermions:
   Quarks (Up, down, top, bottom, strange, charm)
   Leptons (Electron, Muon, Tauon) and neutrino variants of each
3. Composite Fermions: Protons and Neutrons, etc.

52 Single Slit Diffraction

$$d \sin\theta = n\lambda \quad \theta = \text{angle between central max and first minimum} \tag{92}$$

$$\tan\theta = \frac{y}{L} \text{ central max, width } \Delta y_{\max} = \frac{2L\lambda}{d} \tag{93}$$

53 Double Slit

$$d \sin\theta = n\lambda \quad \Delta y = L \tan\theta \tag{94}$$

54 Diffraction Grating

$$d \sin\theta = n\lambda \qquad y = L\tan\theta = L\frac{\sin\theta}{\cos\theta} = \frac{Ln\lambda}{d\cos\theta} \tag{95}$$

55 Bragg

$$2d\sin\theta = n\lambda \qquad d = \frac{a}{\sqrt{l^2 + h^2 + k^2}}\ ,\ \text{letters are miller indices} \tag{96}$$

56 Aperture Limited: Airy Disk: Diffraction Limit: Angular Resolution

$$\sin\theta = \frac{1.22\lambda}{D}\ , D = \text{diameter of aperture}\ , \theta = \text{angular separation} \tag{97}$$

57 Electrostatics

$$F = \frac{kq_1q_2}{r^2} \quad \epsilon = k\epsilon_0 \quad k = 9\times10^9\frac{\text{Nm}}{c^2} \tag{98}$$

1. Sphere: $\propto \frac{1}{r^2}$
2. Infinite Line: $\propto \frac{1}{r}$
3. Infinite Plane doesn't fall off: $E = \frac{\sigma}{2\epsilon_0}$
4. Ring: $\propto \frac{x}{d^3}$, $d = \sqrt{x^2 + R^2}$
5. Capacitor doesn't fall off: $E_{\text{out}} = 0$, $E_{\text{in}} = \frac{\sigma}{\epsilon_0}$

57.1 Limits

As $x \to \infty$ all objects look like point objects. Sometimes use binomial approximation to get behavior at ∞, $(1+x)^n \sim 1 + nx$ for $x \ll 1$.

57.2 Motion Through a Capacitor, etc.

Use kinematics equations $F = ma = qE$ find V, a, t to get θ deflection.

57.3 Dipole

$$\bar{p} = q\bar{d} \tag{99}$$

$$\bar{E}_{\text{dipole}} = \begin{cases} \frac{2k\bar{p}}{r^3} & \text{axis of } \bar{d} \\ -\frac{k\bar{p}}{r^3} & \perp \text{ to } \bar{d} \end{cases}$$

57.4 Current Density

$$J = nev_d\ , I = JA = \frac{\text{current of cross section}}{m^2} \tag{100}$$

57.5 Drift Speed

$$J = \sigma E = \frac{ne^2\tau}{m}E \to v_d = \frac{\sigma E}{ne} = \frac{e\tau E}{m} \tag{101}$$

57.6 Conductivity

$$\sigma = \frac{ne^2\tau}{m} \tag{102}$$

58 Magnetic Field

$$B = \frac{\mu_0 I}{4\pi}\frac{d\bar{l} \times \hat{r}}{r^2} \text{ ,} d\bar{l} = \text{length and direction} \tag{103}$$

58.1 Tesla

$$T = \frac{N}{A \cdot m} \text{ ,current } I = \int J \cdot da_{\perp} \tag{104}$$

58.2 Force

$$F = q\bar{v} \times \bar{B} = I(d\bar{l} \times \bar{B}) \tag{105}$$

58.3 Cyclotron

$$E \parallel B \to \text{ Helical motion} \tag{106}$$

$$v_{\parallel} B \to \text{ Helical} \tag{107}$$

$$\frac{mv^2}{r} = qvB \to F_c = F_m \tag{108}$$

58.4 Cycloid

$$E \perp B \tag{109}$$

58.5 Examples

58.5.1 Solenoid

$$B = \begin{cases} 0 & \text{outside} \\ \frac{\mu_0 IN}{L} & \text{inside} \end{cases}$$

58.5.2 Ring

$$B = \frac{\mu_0 I}{2R} \tag{110}$$

Any displacement along center of ring should reduce tho this equation as $x \to 0$.

58.5.3 Sheet of Current

$$B = \begin{cases} -\frac{\mu_0}{2} & z > 0 \\ \frac{\mu_0}{2} & z < 0 \end{cases}$$

58.5.4 Toroid

$$B = \begin{cases} 0 & \text{out} \\ \frac{\mu_0 IN}{2\pi R} & \text{in} \end{cases}$$

58.5.5 Dipole

$$B \propto \frac{\mu}{r^3} \text{ ,} \mu = IA = \text{dipole moment} \tag{111}$$

$$B \propto \frac{IA}{r^3} \text{ , as } x \to \infty, \text{ this is twice the field of a current loop} \tag{112}$$

59 Inductance

$$\Phi = LI \quad \epsilon = -\frac{d\Phi}{dt} \qquad \Phi_B = \int B \cdot dA \tag{113}$$

$$W = \frac{1}{2}LI^2 \left(\text{corollary: cap } W = \frac{t}{2}CV^2\right) \tag{114}$$

$$I(t) = \frac{\epsilon_0}{R}\left[1 - e^{-(R/L)t}\right] \quad \tau = \frac{L}{R} \tag{115}$$

60 Radiation

Electric Dipole	Magnetic Dipole	Point Charge
$P \propto q^2 d^2 \omega^4$	$P \propto I^2 \omega^4$	$P \propto q^2 a^2$

$P_{\max} \perp$ to $\hat{a}$.

61 Maxwell's Equations

$$\begin{array}{ll} \nabla \cdot E = \frac{\rho_{\text{in}}}{\epsilon_0} & \nabla \times E = -\frac{\partial B}{\partial t} \\ \nabla \cdot B = 0 & \nabla \times B = \mu_0 J - \mu_0 \epsilon_0 \frac{\partial E}{\partial t} \end{array}$$

$$\begin{array}{ll} \oint E \cdot dA = \frac{Q_{\text{in}}}{\epsilon_0} & \oint E \cdot dl = -\frac{\partial \Phi_B}{\partial t} \\ \oint B \cdot dA = 0 & \oint B \cdot dl = \mu_) I + \mu_0 \epsilon_0 \frac{\partial \Phi_E}{\partial t} \end{array}$$

62 Boundary Conditions

$$E_{\parallel} = 0 \ , \quad B_{\perp} = 0 \ \text{ for reflected waves } \ E_{\text{tot}} = 0 \ \ B_{\text{tot}} = 2B_{\text{wave}} \tag{116}$$

$$\epsilon_1 E_1^{\perp} - \epsilon_2 E_2^{\perp} = \sigma_F \quad E_1^{\parallel} = E_2^{\parallel} \tag{117}$$

$$B_1^{\perp} = B_2^{\perp} \qquad \frac{B_1^{\parallel}}{\mu_1} - \frac{B_2^{\parallel}}{\mu_2} = k_f \times \hat{n} \tag{118}$$

63 E&M Fields

B/E are in phase and perpendicular

$$B_+0 = \frac{k}{\omega}E_0 = \frac{1}{c}E_0 \tag{119}$$

$$\text{Energy Density } \langle U \rangle = \frac{\epsilon_0}{2}E^2 \quad \text{Intensity } \langle I \rangle = \frac{1}{2}c\epsilon E^2 \tag{120}$$

$$\text{Radiation Pressure } p = \frac{\langle s \rangle}{c} \quad \bar{s} = \frac{\bar{E} \times \bar{B}}{\mu_0} \ , \quad \hat{s} = \text{propagation of } \frac{E}{\mu} \tag{121}$$

64 Relativistic E&M

Processes change between frames, but outcome is same.
Example: Parallel Plate Capacitor:

$$\sigma_0 = \frac{Q}{A} \quad \sigma' = \frac{Q}{A'} \ , \text{ length contracts, } A' < A \rightarrow \sigma' > \sigma_0 \quad \sigma' = \gamma\sigma_0 \tag{122}$$

$$\text{Also, } A = lw \qquad a' = \frac{l}{\gamma}w = \frac{lw}{\gamma} \tag{123}$$

$$\text{so, } E_{\perp} = \gamma E_0 \qquad E_{\parallel} = E_{\parallel} \tag{124}$$

65 Coordinate Systems

1. Cartesian: $dl = \hat{x}dx + \hat{y}dy + \hat{z}dz$, $dV = dxdydz$
2. Spherical: $dl = \hat{r}dr + rd\theta\hat{\theta} + r\sin\theta d\phi\hat{\phi}$, $dV = r^2 \sin\theta dr d\phi d\theta$
3. Cylindrical: $dl = \hat{s}ds + sd\phi\hat{\phi} + \hat{z}dz$, $dV = sdsd\phi dz$

66 Positronium

$$\mu = \frac{m_e}{2} \rightarrow E_p = \frac{E_H}{2} \qquad E_{pos} = \frac{-13.6 \text{ eV}}{2} \qquad E_p = \frac{-6.8}{n^2} \tag{125}$$

67 Free Expansion

$$W = 0 \ , \quad Q = 0 \ , \quad \Delta S > 0 \ , \quad \Delta S = Nk\ln\left(\frac{V_i}{V_f}\right) \tag{126}$$

68 Entropy

$$S = k\ln(\Omega) \quad , Q = TdS \quad S_{\text{tot}} = S_A + S_B \tag{127}$$

69 P/N Junctions

1. n donate e^- to CB
2. p donate holes to VB

70 Wave Velocity

$$\text{Group Velocity } v_g = \frac{\partial\omega}{\partial k} \quad \text{phase } v_\phi = \frac{\omega}{k} = \sqrt{\frac{1}{\epsilon\mu}} = \frac{\lambda}{T} \tag{128}$$

71 Polarizers

1. $I = I_0\cos^2\theta$ for plane polarized
2. $I = \frac{I_0}{2}$ for natural light

72 Heisenberg

$$\sigma_p\sigma_x \geq \frac{\hbar}{2} \qquad \sigma_A\sigma_B \geq \frac{1}{2i}\langle[\hat{A},\hat{B}]\rangle \tag{129}$$

73 Compton Effect

Elastic scattering of photons - shows particle nature of light

$$\Delta\lambda = \lambda_c(1-\cos\theta) \qquad \lambda_c = \frac{hc}{E_0} = \frac{h}{mc} \tag{130}$$

74 Photoelectric Effect

$$KE_{\max} = E_p - \Phi = \hbar\omega - \Phi = \frac{hc}{\lambda} - \Phi = hf - \Phi \tag{131}$$

$$\text{Energy of photon } = \frac{hc}{\lambda} = hf = \frac{h}{2\pi}\omega = \hbar\omega \tag{132}$$

Einstein's Equation: $eV = hf - \Phi$, $V = (-)$ value which e^- can be stopped from hitting the cathode, $I \to 0$ (133)

75 Phonon

Displacement from equilibrium values of plane spacing

$$E_{\text{phonon}} = \hbar\omega \tag{134}$$

76 Superconductor

1. Meissner: $B = 0$ inside S_c
2. $\rho \to 0$ at critical temp
3. λ_c penetration depth measures how far B penetrates before $\to 0$
   $B = B_0 e^{-x/\lambda_c}$

77 Mirrors

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \to d_i = \frac{d_o f}{f - d_o} \tag{135}$$

$$M = \frac{h_i}{h_o} = -\frac{d_i}{d_o} \qquad f = \frac{R}{2} \tag{136}$$

Rays go through center and continues in a straight line and goes parallel then through focal point.

77.1 Concave

Converging Mirror, what most diagrams are of

77.2 Convex

Diverging Mirror. Images always smaller, virtual, and upright. They cover a wide field of view.

78 Reflection/Refraction

Snell's Law:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \tag{137}$$

$$\theta_i = \theta_r \text{ },\theta \text{ measured relative to normal} \tag{138}$$

For total internal reflection:

$$n_1 \sin\theta_1 = n_2 \sin 90 \tag{139}$$

$$\sin\theta_1 = \frac{n_2}{n_1} \tag{140}$$

$$n = \frac{c}{v} = \sqrt{\frac{\epsilon\mu}{\epsilon_0\mu_0}} \qquad \epsilon = k\epsilon_0 \qquad \frac{n_1}{n_2} = \frac{v_2}{v_1} \tag{141}$$

79 Bloch's Theorem

Ψ solutions to Schrödinger's Equation are plane waves modulated by a function with the periodicity of the lattice

80 Stern-Gerlach

Expected $2s + 1$ states, saw $2s + 1 = 2$ states. Implied that $H = -\gamma\bar{S} \cdot \bar{B}$

81 Mixing Gases

$$\begin{cases} \text{if } A \neq B & \Delta s = \Delta s_A + \Delta s_B \\ \text{if } A = B & \Delta s = 0 \text{ since } \Delta \ln(\Omega) \sim 0 \text{ since } \Omega = \text{ large \#} \end{cases}$$

82 Equipartition Theorem

$$U = \frac{f}{2} NkT \tag{142}$$

$f =$ # quadratic terms in Hamiltonian (not degrees of freedom, but in general they're equal).

$$kT \sim \frac{1}{40} \text{ eV @ room temperature} \tag{143}$$

83 Degrees of Freedom

1. A quadratic term in PE or KE
2. Translational ($\frac{mv^2}{r}$)
3. Rotational ($I\omega^2$)
4. Vibrational (Counts as 2) (kx^2, mv^2)

84 Simple Pendulum

$$\omega = \sqrt{\frac{g}{l}} \qquad \theta = \theta_{\max} \sin(\omega t) \tag{144}$$

85 Gravity

$$F_g = \frac{Gm_1 m_2}{r^2} = mg \quad \text{Kepler: } T^2 \propto a^3 \tag{145}$$

Escape velocity:

$$F_c = F_g \rightarrow mg = \frac{mv^2}{r} \rightarrow v = \sqrt{gr} \tag{146}$$

Gauss:

$$\int g \cdot dA = -4\pi G \int dM_{\text{in}} \rightarrow \int g \cdot dA = -4\pi G M_{\text{in}} \tag{147}$$

86 Drag Force

$$F_D \propto v^n \quad \text{air: } F_D \propto v^2 \tag{148}$$

$$\text{Terminal velocity } F_D = F_g \rightarrow v = \sqrt{\frac{g}{k}} \tag{149}$$

$$F_D = mkv^2 \tag{150}$$

87 Selection Rules

$$\Delta l = \pm 1 \qquad \Delta m = \pm 1, 0 \qquad \Delta s \text{ no rule} \qquad \Delta n \geq 1 \tag{151}$$

88 Stationary States

$$\langle \Psi | \Psi \rangle \text{ is not a function of time} \tag{152}$$

89 Spectroscopic Notation

$$^{2s+1}L_j \tag{153}$$

1. Spin isn't always $\frac{1}{2}$
2. $L =$ orbital angular momentum
3. $j = s + L =$ total angular momentum

$$L = \begin{cases} s & 0 \\ p & 1 \\ d & 2 \\ f & 3 \end{cases}$$

90 Matter Waves

$$p = \hbar k \qquad k = \frac{2\pi}{\lambda} \qquad p = \frac{h}{\lambda} \to \lambda = \frac{h}{p} \tag{154}$$

91 Pipes/Tubes/Fixed and Open Ends

1. Closed on both ends:
   Ends are nodes, $\sin kx$ dependence, $\Psi = A \sin kx \cos \omega t$
   $\lambda = \frac{2L}{n}$ longest λ is $2L$
2. Open on both ends:
   Ends are antinodes, $\cos kx$ dependence
   $\lambda = \frac{2L}{n}$ $\quad \Psi = A \cos kx \cos \omega t$
3. Closed/Open ends:
   Node/Antinode Longest $\lambda = 4L$ (so $\frac{1}{4}\lambda$ can fit end to end)
   $\lambda = \frac{4L}{2n-1}$ (-1 if n starts at 1)
   $\Psi = A \sin kx \cos \omega t$

92 Solutions to Time-Independent Schrödinger's Equation

$$\langle H \rangle = \sum_n |C_n|^2 E_n = C_1 E_1 + C_2 E_2 + \cdots \quad , \sum_n |C_N|^2 = 1 \tag{155}$$

$$\text{Prob}(a < x < b) = \int_a^b |\Psi(x)|^2 dx \to \text{ area under the } |\Psi|^2 \text{ vs } x \text{ graph} \tag{156}$$

93 Physical Pendulum

$$\tau = I\alpha = I\dot{\omega} = mg\sin\theta L_{\text{cm}} = \bar{r} \times \bar{F} \tag{157}$$

$$\ddot{\theta} = \frac{mgL_{\text{CM}}}{I}\theta \rightarrow \omega = \sqrt{\frac{mgL_{\text{CM}}}{I}}\ ,\quad L_{\text{CM}} = \text{the distance from the pivot point to the center of mass} \tag{158}$$

94 Intrinsic Magnetic Moment

$$\bar{\mu}_s = \frac{gq}{2m}\bar{s} \quad m \text{ is dominant factor} \tag{159}$$

95 Equations of Motions

Look for boundary values $I(s)$ given $x(t)$ and $y(t)$. Differentiate and see which one yields v_0

96 Moments of Inertia

The moment of an object stretched along the axis of rotation doesn't change

$$I_{\text{disk}} = I_{\text{cylinder}} \tag{160}$$

The moment of a cuboid:

$$I = \frac{M}{12}(x^2 + y^2) \tag{161}$$

"Twin Plate", $y = 0$ so $I_z = \frac{M}{12}x^2$. Then, $I_z = \frac{1}{12}(2d)^2 = \frac{M}{3}d^2$

97 Hermitian Matrix

Real eigenvalues, square

$$A = A^T = A^{*T} \text{ the entries are equal to their conjugate tranpose} \tag{162}$$

All diagonals must be real. The sum of two Hermitian matrices is also Hermitian.

$$\langle f|\hat{A}f\rangle = \langle \hat{A}f|f\rangle \Rightarrow A = A^* \tag{163}$$

98 Balancing Problem

Easiest to use center of mass.

$$\text{Center of Mass} = \frac{\sum m_i r_i}{\sum m_i} \text{ one mass is at } -r \tag{164}$$

99 Decay Rates

$$\frac{dA}{dt} = -kA \rightarrow A = A_0 e^{-kt};\ \frac{A}{A_0} = \frac{1}{2} = e^{-kt} \rightarrow t_{1/2} = \frac{\ln(2)}{k} \tag{165}$$

100 Interferometer

Fringe shifts occur for changing distance or λ.

$$2d = m\lambda \quad d = \text{change in distance} \qquad \lambda = \Delta\lambda \qquad \lambda_{\text{gas}} = \frac{\lambda_{\text{vac}}}{n} \tag{166}$$

In a tube where gas $\rightarrow$ vacuum:

$$2d = m(\lambda_{\text{gas}} - \lambda_{\text{vac}}) = m\lambda_{\text{vac}}\left(\frac{1}{n} - 1\right) \tag{167}$$

101 Springs

Add like capacitors. Makes sense because in series they can stretch more so $F = kx$ must be decreased, in parallel, they stretch less so $k \uparrow$ for the same force.

$$\text{Springs in series: } \frac{1}{k_{\text{tot}}} = \frac{1}{k_1} + \frac{1}{k_2} \tag{168}$$

$$\text{Springs in parallel: } k = k_1 + k_2 \tag{169}$$

102 Speed of Sound

In an ideal gas: $v \propto \sqrt{T}$

103 Commutator Identities

$$[A, B] = -[B, A] \tag{170}$$
$$[AB, C] = A[B, C] + [A, C]B \tag{171}$$
$$[A, BC] = B[A, C] + [A, B]C \tag{172}$$

104 Motion in a Circle

Always a_{radial} component. Only $a_{\tan}$ if $v_{\tan}$ changes.

$$F = \frac{mv^2}{r} = ma_r \rightarrow a_r = \frac{v^2}{r} \tag{173}$$

$$a_r = r \times \alpha \qquad v = r \times \omega \tag{174}$$

105 Specific Heat in a Solid

1. Einstein Model:
   Treats atoms as harmonic oscillators, $3N$ total harmonic oscillators and they all have the same energy (frequency) using Bose-Einstein statistics

2. Debye:
   Also $3N$ harmonic oscillators. Assigns a range of energies (frequencies) and treats the lattice vibrations as phonons in a box

3. Dulong-Petit:
   High temperature, uses equipartition theorem with harmonic oscillators ($f = 6$, $c = 3Nk$). Debye and Einstein models reduce to this in the high T limit.

106 Doppler Shift

$$f = f_0 \left(\frac{1 + v_s}{1 + v_0} \right) \qquad \frac{\lambda}{\lambda_0} = \frac{f_0}{f} = \sqrt{\frac{1+\beta}{1-\beta}}\ , \beta = \frac{v}{c} \tag{175}$$

The "redshift": $z = \frac{\lambda_0 - \lambda}{\lambda} = \frac{f - f_0}{f}$

107 Fission

Conservation of energy, binding energy of nucleus is always $(-)$, like e^- binding energy.

$$-BE_i + KE_i = -BE_f + KE_f \tag{176}$$

108 Wire Resistance

$$R = \frac{\rho L}{A} \tag{177}$$

109 Spin Matrices

$$S_i \psi = \frac{\hbar}{2} \sigma_i \psi \tag{178}$$

For example, eigenstate of S_x with $-\frac{\hbar}{2}$ and $\sigma_x = \left(\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right)$

$$\frac{1}{\sqrt{2}}(|\uparrow\rangle - |\downarrow\rangle) = \frac{1}{\sqrt{2}}\left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right] = \frac{1}{\sqrt{2}}\left[\begin{pmatrix} 1 \\ -1 \end{pmatrix}\right] \tag{179}$$

$$\begin{pmatrix} 1 \\ -1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \to S_x \psi = \frac{\hbar}{2}\left(\frac{1}{\sqrt{2}}\right)\begin{pmatrix} -1 \\ 1 \end{pmatrix} = -\frac{\hbar}{2}\left(\frac{1}{\sqrt{2}}\right)\begin{pmatrix} -1 \\ 1 \end{pmatrix} = -\frac{\hbar}{2}\psi \tag{180}$$

$$|\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{181}$$

110 For What v Will a Car Stay on a Hill?

$$F_c = F_g \qquad \frac{mv^2}{r} = mg \tag{182}$$

111 Böhr Model

1. e^- have classical motions
2. $\Delta E = hf$
3. Angular momentum quantized $L = n\hbar$
4. $E_n = -\frac{Z^2 E_0}{n^2} \qquad E_n \propto \mu$
5. $\Delta E = E_0 \left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \rightarrow \frac{1}{\lambda} = R_y \left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$, $R_y = 1 \times 10^7\ \mathrm{m}^{-1}$
6. Positronium: $\mu = \frac{m_e}{2} \rightarrow E_p = \frac{E_0}{2n^2}$

112 Fluids

Equilibrium when $F_A = F_B$, where $F = mg$.

113 Gauss' with non-uniform

Must integrate.

$$\rho = Ar^2 \quad dV = 4\pi r^2 dr \quad \int E \cdot dA = \frac{Q_{\text{in}}}{\epsilon_0} = \frac{\int \rho dV}{\epsilon_0} = \frac{\int Ar^2 4\ pir^2 dr}{\epsilon_0} = \frac{A4\pi r^5}{5\epsilon_0} \tag{183}$$

114 Capacitors

In series, $Q_1 = Q_2$ while in parallel, $V_1 = V_2$

115 Diffraction Limit

$$\sin\theta = \frac{1.22\lambda}{d} \quad d = \text{diameter of lens} \tag{184}$$

116 Normal Modes

1. Highest normal mode frequency when out of phase
2. Use limits if possible, with $M \rightarrow \infty$
3. # frequency = # masses
4. If odd # masses, one ω will be ω_0, others above and below

For two hanging masses connected by a spring,

$$\text{In phase: } \omega = \sqrt{\frac{g}{l}} \quad \text{spring's } \Delta x = 0 \tag{185}$$

$$\text{Out of phase: } \omega = \sqrt{\frac{2k}{m} + \frac{g}{l}} \tag{186}$$

For three masses connected by two springs with the mass in the middle larger than the equal masses on the sides:

$$\omega = \sqrt{\frac{k}{m}}\ ,\ \text{like attached to a wall} \tag{187}$$

Side masses are in phase, middle mass is out of phase, $\omega = \sqrt{\frac{2k}{m}}$.

For 2 masses connected by 3 springs with the side masses connected to a wall:

$$\text{In phase: } \Delta x_1 = \Delta x_2 \text{ and } k' \text{ isn't expanded }, \omega = \sqrt{\frac{k}{m}} \tag{188}$$

$$\text{Out of phase: } \Delta x_1 = -\Delta x_2 \text{ and center of mass } k' \text{ stays in place } \ \omega = \sqrt{\frac{k+2k'}{m}} \tag{189}$$

117 Radiation in Atoms

$n_f = \begin{matrix} \text{K} & \text{L} & \text{M} & \text{N} \\ 1 & 2 & 3 & 4 \end{matrix}$ series specify the final states coming from infinity, $n_i = \infty$ when being bombarded.

118 Ionization Energy

E required to liberate the outermost e^-. On the periodic table, increases in the $+y, +x$ direction.

119 Binding Energy

How tightly bound nucleons are

1. Peak at Fe/N $\rightarrow$ elements $Z < Z_{\text{FE}}$ undergo Fusion, $Z > Z_{\text{Fe}}$ undergo fission to release energy
2. When BE/nucleon increases, in reaction, energy is released
3. The mass of a nucleus is always less than the $\sum$ particle's masses
4. "More tightly bound" = less mass/nucleon, more BE/nucleon
5. Created by the strong force
6. Energy given off in fusion/fission is the ΔE between fuel and products

120 Hierarchy of Forces

1. Strong
2. E&M
3. Weak
4. Gravity

121 Pair Production

1. Creation of elementary particle and anti-particle from photon
2. Cannot occur in free space, usually near a nucleus or other photon
3. For e^-, the photon E must exceed twice the rest energy of the e^-, ≈ 1 MeV
4. If 2 photons, 500 keV
5. Dominates at high E

122 Spectral Lines

1. Less dense gas $\rightarrow$ more sharp and precise lines - don't lose E due to collisions
2. Sodium doublet - created by spin/orbit coupling, more pronounced in an external B

123 Photon Interactions with Matter

1. Compton Effect: low E, elastic scattering $< 10^6$ MeV
2. Photoelectric Effect: mid E, $< 10^7$ MeV
3. Pair Production: high E, $> 10^6$ MeV

124 Neutron

A fermion with:

$$ {}^1_0n \tag{190} $$

$$ \text{Decay: } {}^1_0n \rightarrow^1_1 p^+ +^1_{-1} e + \bar{\nu} \tag{191} $$

$$ \text{Capture: } {}^1_1p^+ +^1_{-1} e \rightarrow^1_0 n + \bar{\nu} \tag{192} $$

125 Deuteron

"Heavy Hydrogen", ${}^2_1H$. Also, a boson.

126 Protium

A proton, ${}^1_1H$, Hydrogen nucleus, a fermion