GRE Physics Study Notes

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Contents

Introduction

By no means comprehensive, this list is meant to serve as additional study material to re-reading textbooks, practicing GRE tests, and nagging your physics friends for study help.

1 Waves

1.1 Doppler Effect

$$f = f_0 \left(\frac{v + v_s}{v + v_0} \right) \tag{1}$$

$$v_0 = \begin{cases} + & \text{away} \\ - & \text{towards} \end{cases}$$

$$v_s = \begin{cases} + & \text{towards} \\ - & \text{away} \end{cases}$$

$$\frac{\lambda}{\lambda_0} = \sqrt{\frac{1+\beta}{1-\beta}}\ , \beta = \frac{v}{c} \text{ for relativistic doppler shift} \tag{2}$$

$$= \frac{f_0}{f} \tag{3}$$

2 Optics

2.1 Thin Lenses

$$d_i = \frac{f \cdot d_o}{f - d_o} \tag{4}$$

$$M = \frac{h_i}{h_o} = -\frac{d_i}{d_o} \tag{5}$$

2.2 Ray Diagrams

1. Through f, $\|$ other side
2. Through center, continues along path
3. $\|$, goes through f on other side

2.3 Index of Refraction

$$n = \frac{c}{v} \tag{6}$$

$$v = v_Q = \frac{\omega}{k} = \sqrt{\frac{1}{\epsilon\mu}} \tag{7}$$

$$\lambda = \frac{\lambda_0}{n} \text{ inside a medium} \tag{8}$$

$$n = \frac{c}{v} = \sqrt{\frac{\epsilon\mu}{\epsilon_0\mu_0}} \tag{9}$$

2.4 Telescope and Magnification

2 lenses share a common focal point

$$M = -\frac{f_{\text{o}}}{f_{\text{e}}} = \frac{\theta_{\text{eye}}}{\theta_{\text{object}}} \tag{10}$$

$$d_{\text{o}} + d_{\text{e}} = f_{\text{o}} + f_{\text{e}} \tag{11}$$

2.5 Thin Films

$$\Delta\phi = \begin{cases} 0 & n_2 < n_1 \\ \pi & n_2 > n_1 \end{cases}$$

$$2d = \begin{cases} n\lambda/2 & \Delta\phi_{\text{tot}} = \pi \\ n\lambda & \Delta\phi_{\text{tot}} = 0, 2\pi \end{cases}$$

2.6 Resistance

$$R = \frac{\rho L}{A} = \frac{L}{\sigma A} \tag{12}$$

3 Electricity and Magnetism

3.1 Gauss' Laws

$$\int E \cdot da = \frac{Q_{\text{encl}}}{\epsilon_0} \tag{13}$$

$$\nabla \cdot E = \frac{\rho}{\epsilon_0} \tag{14}$$

$$\int B \cdot dl = \mu_0 I_{\text{encl}} (\text{in Amperes}) \rightarrow \int B \cdot da = 0 \tag{15}$$

$$\nabla \cdot B = 0 \tag{16}$$

$$\int g \cdot da = 4\pi MG \tag{17}$$

$$\nabla \cdot g = -4\pi Gs\rho \tag{18}$$

3.2 Cyclotron

$$\omega = \frac{qB}{m} \tag{19}$$

$$F_c = F_B \rightarrow \frac{mv^2}{r} = qvB \tag{20}$$
$$v = \frac{qBr}{m} = r\omega \tag{21}$$
$$\omega = \frac{qB}{m} \tag{22}$$

3.3 Conductivity / Current Density

$$J = nq\bar{v} \tag{23}$$
$$J = \frac{ne^2\tau}{m}E = \sigma E \ , \quad \sigma = \frac{ne^2\tau}{m} \tag{24}$$

3.4 Potential and Electric Field

$$E = \int \frac{kdQ}{r^2} = k\int \frac{\sigma dA}{r^2} = k\int \frac{\rho dv}{r^2} = k\int \frac{\lambda dl}{r^2} \tag{25}$$
$$V = \int \frac{kdq}{r} \tag{26}$$

3.4.1 Example Ring of Charge

Imagine a ring with radius R and a point P above the ring at a height z making an angle θ above the ring plane.

$$E = k\int \frac{dQ}{r^2} \tag{27}$$
$$r^2 = R^2 + z^2 \tag{28}$$
$$dq = \lambda dl = Q \tag{29}$$
$$\sin\theta = \frac{z}{r} = \frac{E_z}{E} \tag{30}$$
$$E = \frac{kQ}{R^2+z^2} \text{ ,But } E = \hat{E}_z \tag{31}$$
$$E = \frac{kQ}{R^2+z^2}\sin\theta = \frac{kQ}{R^2+z^2}\frac{z}{r} = kQz \tag{32}$$

3.5 Electrostatics

$$F = \frac{kq_1q_2}{r^2} \ , \quad k_{\text{vacuum}} = \frac{1}{4\pi\epsilon_0} \ , \quad k_{\text{medium}} = \frac{1}{4\pi\epsilon} \ , \quad \epsilon = k\epsilon_0 \tag{33}$$
$$\nabla \cdot E = \frac{\rho_{\text{in}}}{\epsilon_0} \tag{34}$$
$$\nabla \times E = 0 \tag{35}$$

3.6 Classic E Examples

1. Sphere $\propto \frac{1}{r^2}$
2. Infinite Line $\propto \frac{1}{r}$
3. Infinite Plane doesn't fall off
   $E_{\text{plane}} = \frac{\sigma}{2\epsilon_0}\hat{n}$
4. Ring of charge:
   $E \propto \frac{x}{d^3} = \frac{x}{(x^2+R^2)^{3/2}}$
5. Disk of charge:
   $E = \frac{\sigma}{2\epsilon_0}\left(1 - \frac{z}{(z^2+R^2)^{1/2}}\right)$, $\sigma =$ area charge density

3.7 Parallel Plate Capacitor

Model as infinite planes:

$$E_{\text{out}} = 0 \tag{36}$$

$$E_{\text{in}} = \frac{\sigma}{\epsilon_0} \tag{37}$$

3.7.1 Limits

1. As $x \to \infty$, all finite objects look like point charges
2. Sometimes must use binomial approximation to get behavior at ∞. Disk of charge $\to 0$ if you don't use it.
3. $(1+X)^n \sim 1 + nX$ for small x

3.8 Coulomb

$$E = k\int \frac{dQ}{r^2} \tag{38}$$

$$dQ = \lambda dl \sim \sigma dA \sim \rho dV \text{ , be careful of symmetry when integrating!} \tag{39}$$

For a ring of radius R in the x-y plane, a point is a distance r from the ring, making an angle θ on the z-axis. For ring of charge, must integrate by saying:

$$E = E_{\hat{z}} \tag{40}$$

$$\cos\theta = \frac{z}{r} \tag{41}$$

$$r = \sqrt{z^2 + R^2} \tag{42}$$

$$\lambda = \frac{Q}{2\pi R} \tag{43}$$

$$dl = RdQ \tag{44}$$

$$dE_{\hat{z}} = dE\cos\theta = \frac{k\lambda dl}{r^2}\cos\theta \tag{45}$$

$$E = k\lambda\int \frac{dl}{r^2}\cos\theta = \frac{kQ}{2\pi R}(R)\int_0^{2\pi} \frac{dQ}{r^2}\frac{z}{r} \tag{46}$$

$$R = \frac{kQ}{2\pi}(2\pi)\frac{z}{r^3} = \frac{kQz}{(R^2+z^2)^{3/2}} \tag{47}$$

3.9 Motion Through a Capacitor or Uniform Field

Kinematics equation: $F = ma = eE$. Find v_c, a, t to get θ deflection

3.10 E of a Dipole

$$\bar{p} = qd \tag{48}$$

$$E_{\text{dipole}} = \begin{cases} \frac{k2\bar{p}}{r^3} & \text{on axis of } \hat{d} \\ \frac{k\bar{p}}{r^3} & \text{plane perpendicular to } \hat{d} \end{cases}$$

3.11 Gauss

$$\Phi_E = \int E \cdot dA = \frac{Q_{\text{enclosed}}}{\epsilon_0} \tag{49}$$

3.12 Current Density

Continuity Equation:

$$\frac{\partial \rho}{\partial t} = \overline{\nabla} \cdot \bar{J} \tag{50}$$

$$J = q_\alpha n_\alpha v_\alpha \text{ , } I = JA = \frac{\text{current}}{m^2} \text{ of the cross section } = \frac{A}{m^2} \tag{51}$$

3.13 Drift Speed

$$v_{\text{drift}} = \frac{e\tau E}{m} \tag{52}$$

3.14 Capacitors

C depends upon geometry of electrodes

3.15 Non-ohmic Materials

Do not obey $V = IR$: batteries, semiconductors, capacitors, inductors

3.16 Convention of Battery

Long side of battery is positive.

3.17 RC Circuit

$$Q = Q_0 e^{-t/\tau} \tag{53}$$
$$I = I_0 e^{-t/\tau} \ , \tau = RC \tag{54}$$

$$Q = VC \tag{55}$$
$$V = V_0 e^{-t/\tau} \ , \text{decay} \tag{56}$$
$$= V_0(1 - e^{-t/\tau}) \ , \text{charging up} \tag{57}$$

3.18 Work

$$W = F \cdot d = eE \cdot d = e\Delta V \tag{58}$$

3.19 Magnetostatics

3.19.1 Magnetic Field

Biot-Savart Law:

$$B = \frac{\mu_0}{4\pi} \frac{\bar{I} \times \hat{r}}{r^2} = \frac{\mu_0}{4\pi} I \frac{d\bar{l} \times \hat{r}}{r^2} \ , d\bar{l} = \text{ the actual length of the segment, not just the direction} \tag{59}$$

$$\text{Tesla} = T = \frac{N}{A \cdot m} \tag{60}$$

3.19.2 Current

$$I = \int J da_{\perp} \tag{61}$$

$$\text{if } J = Kr, I = \int_0^{2\pi} \int_0^{r} kr'(r'dr'd\phi) = \frac{2\pi}{3} kr^3 \tag{62}$$

3.19.3 Force

$$\bar{F} = q\bar{v} \times B = I(d\bar{l} \times \bar{B}) \tag{63}$$

3.19.4 Cyclotron Motion

$$v_{\|} B \rightarrow \text{helical} \tag{64}$$
$$\frac{mv^2}{r} = qvB \tag{65}$$

3.19.5 Cycloid

E in $+z$ direction and and B in $+x$ direction with particle traveling in $+y$ direction make a cycloid.

3.19.6 Solenoid

$$B = \begin{cases} \mu_0 n I \hat{z} & \text{inside }, n = \frac{N}{L} \\ 0 & \text{outside} \end{cases}$$

3.19.7 Ring of Current

$$B = \frac{\mu_0 I}{2R} \tag{66}$$

Any displacement along center of ring should reduce to this equation as $x \to 0$, as $x \to \infty$ should be field of dipole.

3.19.8 Infinite Wire

$$B = \frac{\mu_0 I}{2\pi r} \ , \ \ \text{in limits, } \theta_1 = -\frac{\pi}{2} \ , \theta_2 = \frac{\pi}{2} \ , \ \text{ and } r \text{ is the distance from the wire} \tag{67}$$

3.19.9 Surface Current

$$B = \begin{cases} -\frac{\mu_0}{2} & z > 0 \\ \frac{\mu_0}{2} & z < 0 \end{cases} , \ \text{use Amperian square loop}$$

3.19.10 Toroid

$$B = \begin{cases} \frac{\mu_0 I N}{2\pi r} \hat{\phi} & \text{inside} \\ 0 & \text{outside} \end{cases}$$

3.19.11 Dipole

$$B \propto \frac{\mu}{r^3} \tag{68}$$

$$\mu = IA \ , x \to \infty \text{ limit looks like this} \tag{69}$$

Field far away from a current loop = field of a dipole.

Magnetic fields do no work.

3.19.12 Inductance

$$\Phi = LI \tag{70}$$

$$\epsilon = -L\frac{dI}{dt} \tag{71}$$

$$\Phi_B = \int B \cdot dA \tag{72}$$

$$\text{Henry} = H = \frac{Vs}{A} \tag{73}$$

Inductor in serial with resistor: $\tau = \frac{L}{R}$.

$$W = \frac{1}{2}LI^2 = U_{\text{stored}} \tag{74}$$

L is like mass, the greater the L the harder it is to try and change the current.

VLR Circuit: Voltage log's to V.
Ohms':

$$\epsilon_0 - L\frac{dI}{dt} = IR = V \tag{75}$$

Solution to differential equation:

$$I(t) = \frac{\epsilon_0}{R} + ke^{-(R/L)t}\ , \tau = \frac{L}{R} \tag{76}$$

If $t = 0, V = 0$, just plugged in, $k = -\frac{\epsilon_0}{R}$

$$I(t) = \frac{\epsilon_0}{R}\left(1 - e^{-(k/L)t}\right) \tag{77}$$

3.19.13 Maxwell's Equations in Matter

$$\begin{array}{llll} \nabla \cdot D = \rho f & \nabla \times \bar{E} = -\frac{\partial B}{\partial t} & D = \epsilon E & \epsilon = \epsilon_0(1 + \chi_e) \\ \nabla \cdot B = 0 & \nabla \times H = \bar{J}_f + \frac{\partial D}{\partial t} & B = \mu H & \mu = \mu_0(1 + \chi_m) \end{array}$$

3.20 Dielectrics

3.20.1 Dipoles and Bound Charges

$$\begin{array}{lll} \rho_b = -\overline{\nabla} \cdot \bar{p} & \sigma_b = \bar{p} \cdot \hat{n} & \bar{p} = q\bar{d} \\ \tau = \bar{p} \times \bar{E} & \bar{U} = -\bar{p} \cdot \bar{E} & \end{array}$$

3.20.2 Dielectrics

1. Electric Displacement: $\bar{D} = \epsilon_0\bar{E} + \bar{p}$
2. Gauss' Law:

$$\nabla \cdot D = \rho_{\text{free}} \tag{78}$$

$$\int D \cdot da = Q_{\text{enclosed}} \tag{79}$$

3.20.3 Linear Dielectrics

Conduction:

$$\bar{p} = \epsilon_0 \underbrace{\chi_e}_{\text{electric susceptibility}} \bar{E} \tag{80}$$

$$F = \frac{1}{4\pi \underbrace{\epsilon}_{\text{only thing that changes } (\epsilon_0 \to \epsilon)}} \frac{qQ}{r^2} \tag{81}$$

$$= \frac{1}{4\pi\epsilon_0\epsilon_r}\frac{qQ}{r^2} = \frac{F_{\text{vac}}}{\epsilon_r} = F_{\text{medium}} \tag{82}$$

$$E_{\text{medium}} = \frac{E_{\text{vac}}}{\epsilon_r} \to E = \frac{E_0}{k} \tag{83}$$

1. Permittivity = ϵ
2. Dielectric Constant: $\epsilon_r = \frac{\epsilon}{\epsilon_0}$
3. Displacement: $\bar{D} = \epsilon\bar{E}$

3.21 Radiation

3.21.1 Electric Dipole

$$P \propto q^2\omega^4 d^2 \tag{84}$$
$$\langle s\rangle \propto \frac{q^2 d^2\omega^4}{r^2}\sin^2\theta \tag{85}$$

Where the $\sin^2\theta$ component is so we don't see along the direction of motion.

3.21.2 Point Charge

$$P \propto q^2 a^2 \tag{86}$$
$$\langle s\rangle \propto \frac{q^2 a^2\sin^2\theta}{r^2} \tag{87}$$

Once again, no power radiated along motion direction. $\langle s\rangle_{\text{max}}$ @ $\theta = 90$ to motion.

3.21.3 An Oscillating Sphere with Changing Radius

...emits no radiation. Use Gauss' law for symmetry problems, E is constant. an uncharged particle accelerates more than a charged particle because the charged particle emits radiation, $\bar{F}_{\text{in}} - \hat{d}$.

3.21.4 Magnetic Dipole Radiation

1. Model a wire loop with alternating current
2. $P \propto b^4 I_0^2\omega^4$
3. $\langle s\rangle \propto \frac{b^4 I_0^2\omega^4\sin^2\theta}{r^2}$

3.22 Maxwell Equations

$\nabla\cdot E = \frac{\rho}{\epsilon_0}$ $\quad\nabla\times E = -\frac{\partial B}{\partial t}$
$\nabla\cdot B = 0$ $\quad\nabla\times B = \mu_0 J - \mu_0\epsilon_0\frac{\partial E}{\partial t}$
Magnetic monopoles would symmetrize the equations... *wrings hands*

$\oint E\cdot dA = \frac{Q_{\text{in}}}{\epsilon_0}$ $\quad\oint E\cdot dl = -\frac{\partial\Phi_B}{\partial t}$
$\oint B\cdot dA = 0$ $\quad\oint B\cdot dl = \mu_0 I + \mu_0\epsilon_0\frac{\partial\Phi_E}{\partial t}$

3.23 Ampere's Law

$$\oint B\cdot dl = \mu_0 I_{\text{enclosed}} \tag{88}$$

3.24 Current

$$I = \int J \cdot dA \tag{89}$$

3.25 Boundary Conditions E&M Waves

1. $E_{\parallel} = 0 \;\; B_{\perp} = 0 \rightarrow$ reflections
2. For reflection, $E_{\text{tot}} = 0 \; B_{\text{tot}} = 2B_{\text{wave}}$
3. $E_{\perp}$ is always discontinuous by $\frac{\sigma}{\epsilon_0}$ @ boundary
4. $E_{\parallel}$ is always continuous

$$\begin{array}{ll} \epsilon_1 E_1 - \epsilon_2 E_2^{\perp} = \sigma_p & E_1^{\parallel} = E_2^{\parallel} \\ B_1^{\perp} = B_2^{\perp} & \frac{1}{\mu_1} B_1^{\parallel} - \frac{1}{\mu_2} B_2^{\parallel} = \underbrace{k_f}_{\text{free current}} x\hat{n} \end{array}$$

3.26 E&M Fields

E/B are in phase and perpendicular

1. $B_0 = \frac{k}{\omega} E_0 = \frac{1}{c} E_0$
2. Radiation Pressure: $p = \frac{\langle s \rangle}{c}$
3. Energy Density: $\langle U \rangle = \frac{1}{2} \epsilon_0 E^2$
4. $\bar{s} = \frac{1}{\mu_0} (\bar{E} \times \bar{B})$
5. Intensity: $I = \langle s \rangle = \frac{1}{2} c \epsilon_0 E^2$
6. $\hat{s}$ = propagation of E&M field

3.27 Energy Stored in E&M

$$U = \epsilon_0 E^2 = \frac{1}{\mu_0} B^2 \;, U_E = U_B \tag{90}$$

3.28 Poynting Vector

$$\bar{S} = \frac{1}{\mu_0} \bar{E} \times \bar{B} \tag{91}$$

3.29 Irradiance

$$I = \langle s \rangle \tag{92}$$
$$= c\epsilon_0 \langle E^2 \rangle \tag{93}$$
$$= \frac{c}{\mu_0} \langle B^2 \rangle \tag{94}$$

3.30 Relativistic E&M

1. E&M consistent with relativity
2. Between reference frames the E&M processes change but particle motion and outcome is always the same
3. Charge is invariant

3.30.1 Example: Parallel Plate Capacitor

$$S : E^{\perp} = \frac{\sigma_0}{\epsilon_0}\hat{y} \tag{95}$$

$$S' : E^{\perp} = \frac{\sigma}{\epsilon_0}\hat{y} \text{ ,only } \sigma \text{ changes} \tag{96}$$

Charge on each plate is invariant, width is unchanged, but the length (along direction of motion) is contracted.

$$l = \frac{l_0}{\gamma} \rightarrow \sigma = \frac{\sigma_0}{\gamma} \tag{97}$$

For motion in $\hat{x}$, $E_{\hat{y}}$ is changed while $E_{\hat{x}}$ is unchanged since $E = E_{\hat{y}}$.

$$E_{\perp} = \gamma E_{\perp} \tag{98}$$

$$E_{\|} = E_{\|} \tag{99}$$

3.30.2 Special Cases

If $B = 0$ in any one reference frame,

$$\bar{B} = -\frac{1}{c^2}(\bar{V} \times \bar{E}) \tag{100}$$

If $E = 0$ in any one reference frame,

$$\bar{E} = \bar{V} \times \bar{B} \tag{101}$$

3.31 Coordinate Systems

1. Cartesian: $dl = dx\hat{x} + dy\hat{y} + dz\hat{z}$,$dV = dxdydz$
2. Spherical: $dl = dr\hat{r} + rd\theta\hat{\theta} + r\sin\theta d\phi\hat{\phi}$,$dV = r^2 \sin\theta dr d\phi d\theta$
3. Cylindrical: $dl = ds\hat{s} + sd\phi\hat{\phi} + dz\hat{z}$,$dV = sdsd\phi dz$

3.32 Vectors

$$\nabla \times (\nabla \times A) = \nabla(\nabla \cdot A) - \nabla^2 A \tag{102}$$

3.33 Diamagnetism

Caused by change in orbital moment (μ) induced by B. Acts to negate B, anti-parallel to B.

3.34 Paramagnetism

In a magnetic field, breaking of energy levels by spin/spin or spin/orbit coupling induced along B.

3.35 Ferromagnetism

Any material that exhibits a spontaneous B. (A net magnetic moment in the absence of an external B)

3.36 Radiation Pressure

Energy Density of the wave

$$P = U = V_e + U_B \tag{103}$$

$$\langle p \rangle = \frac{\langle s \rangle}{c} \tag{104}$$

$$\tag{105}$$

1. Perfect reflection: light enters with $+c$ and exits with $-c$
   so $\Delta v = 2c \rightarrow \langle p \rangle = \frac{2\langle s \rangle}{c}$

Curl-less Fields: $\bar{E}$	Div-less Fields: $\bar{B}$
$\nabla \times F = 0$ everywhere	$\nabla \cdot F = 0$
$\int_a^b F \cdot dl =$ pattern independent	$\int F \cdot dA =$ independent of any bound line
$\oint F \cdot dl = 0$ closed loop	$\oint F \cdot dA = 0$ for all surfaces
$F = -\nabla V$	$\bar{F} = \bar{\nabla} \times \bar{A}$

4 Circuits

4.1 Resistivity

$$\rho(T_2) = \rho(T_1)(1 + \alpha \Delta T) \tag{106}$$

For metals:

1. $\alpha = (+)$
2. $\rho \uparrow \; T \uparrow$
3. Doping increases ρ

while for semiconductors:

1. $\alpha = (-)$
2. $\rho \downarrow \; T \uparrow$
3. Doping decreases ρ

4.2 Types of Cells

1. Conventional Cell: Contains more than 1 lattice point.
2. Primitive Cell: Contains 1 lattice point. $V_{cc}/N_{\text{cc lattice points}} = V_{pc}$

4.3 Band Pass

$$\omega_0 \rightarrow \omega_0 L = \frac{1}{\omega_0 C} \rightarrow \omega_0 = \frac{1}{\sqrt{LC}} \tag{107}$$

4.4 Low Pass

Either looks like a RC or LR circuit, but perpendicular to each other. Purpose is to cut out high frequencies, essentially letting low frequencies pass through.

$$T_1 = \frac{\frac{1}{jwc}}{R + \frac{1}{jwc}} \tag{108}$$

$$= \frac{\frac{1}{jwc}}{\frac{R_{jwc}+1}{jwc}} \tag{109}$$

$$= \frac{1}{1 + jwcR} \tag{110}$$

1. As $\omega \to \infty$, $T_1 \to 0$
2. As $\omega \to 0$, $T_1 \to 1$

$$T_2 = \frac{R}{jwc + R} \tag{111}$$

1. As $\omega \to \infty$, $T_2 \to 0$
2. As $\omega \to 0$, $T_2 \to 1$

4.5 High Pass

Either looks like a CR or RL circuit, like a low pass filter configuration but with elements reversed. Cuts out low frequencies.

$$T_1 = \frac{R}{R + \frac{1}{jwc}} \tag{112}$$

$$= \frac{R}{\frac{jwcR+1}{jwc}} \tag{113}$$

$$= \frac{jwcR}{jwcR + 1} \tag{114}$$

1. As $\omega \to \infty$, $T_1 \to 1$
2. As $\omega \to 0$, $T_1 \to 0$

$$T_2 = \frac{jwL}{R + jwL} \tag{115}$$

1. As $\omega \to \infty$, $T_2 \to 1$
2. As $\omega \to 0$, $T_2 \to 0$

5 Quantum Mechanics

5.1 Operators

$$\hat{x} = x \tag{116}$$

$$\hat{H} = -\frac{\hbar^2}{2m}\nabla^2 + V \tag{117}$$

$$\hat{p} = -ih\frac{\partial}{\partial x} \tag{118}$$

5.2 Hermitian Operators

1. Represent observables
2. $\langle \hat{Q} \rangle = \langle \Psi | \hat{Q} | \Psi \rangle$ = real #
3. Conditions: $\langle f | \hat{Q} g \rangle = \langle \hat{Q} f | g \rangle \Rightarrow \hat{a}^\dagger = \hat{a}^* = \hat{a}$
4. Determinant states are eigenfunctions of $\hat{Q}$
5. $\left(\frac{\partial}{\partial x}\right)^\dagger = -\frac{\partial}{\partial x}$, note

5.3 Transmission / Reflection / Tunneling Through Barrier

1. Incident: Ae^{ikx}
2. Reflection: Re^{-ikx}
3. Transmission: Te^{-ikx}
4. Limits:
   $v_0 \to 0\ ,\quad R \to 0$
   $v_0 \to \infty\ ,\quad T \to 0$
5. Probability(Transmission) = $|T/A|^2$
6. Probability(Reflection) = $|R/A|^2$
7. Probability(Transmission) + Probability(Reflection) = 1
   $T^2 + R^2 = A^2$
8. Tunneling Depth $d \propto \frac{1}{\sqrt{V-E}}$

5.4 Hyperfine Splitting

1. Spin/spin of e^- nucleus
2. Responsible for 21 cm line

$$\mu_p = \frac{ge}{Zm_p}\overline{s_p} \tag{119}$$

$$\mu_e = \frac{-e}{m_e}\overline{s_e} \tag{120}$$

$$E_{n'_f} = \frac{\mu_0 g_p e^2}{3\pi m_p m_e a^3}\langle \overline{s_p} \cdot \overline{s_e} \rangle \tag{121}$$

$$E_{n'_f} \propto \frac{e^2}{m_p m_e a^3}\langle \overline{s_p} \cdot \overline{s_e} \rangle \tag{122}$$

5.5 Fine Structure

1. Spin/orbit coupling + relativistic correction
2. Breaks l degeneracy, retains j degeneracy
3. Why $E_{2s} < E_{2p}$

5.6 Zeeman Effect

1. Atom in external $\bar{B}$
2. Spin+orbital angular momentum/B coupling

3. $H_{z'} = (-\bar{\mu}_e + \bar{\mu}_s) \cdot \bar{B}_{\text{ext}}$
4. Weak $B_{\text{ext}} \ll B_{\text{int}} \rightarrow E' = \mu_b g_j \underbrace{m_j}_{\text{breaks } m_i \text{ degeneracy into } 2j+1 \text{ levels}} B_{\text{ext}}$
5. Strong $B_{\text{ext}} \gg B_{\text{int}} \rightarrow E' = \mu_b B_{\text{ext}}(m_l + 2m_s)$

5.7 Stark Effect

1. External $\bar{E}$
2. not spin dependent
3. $H' = eE_z$ if $E = \hat{E}_z$
4. Hydrogen, $E_1' = \langle H' \rangle = eE \int_0^\infty d^3r \underbrace{z}_{\text{odd}} |\underbrace{\Psi_{100}}_{\text{even}}|^2 = 0$

5.8 Degenerate Perturbation Theory

1. A state when n degenerate states breaks into n distinct E levels
2. Tensor, $w_{aa}, w_{bb}, w_{cc} = E_a, E_b, E_c$ of unperturbed states
3. $$\begin{pmatrix} w_{aa} & w_{ab} \\ w_{ba} & w_{bb} \end{pmatrix} \Rightarrow w_{ab} = w_{ba}^*$$

5.9 Non degenerate Perturbation Theory

1. $H = H' + H^0$
2. First order: $E_n' = \langle \Psi_n | H' | \Psi_n \rangle = \langle H' \rangle$

$$\Psi_{n'} \sum_{m,n} \frac{\langle \Psi_m^0 | H' | \Psi_n^0 \rangle}{E_n^0 - E_m^0} \Psi_m^0 \tag{123}$$

1. If E introduced

$$H' = eE \rightarrow E' = 0 \tag{124}$$

1. Potential raised by constant

$$H' = v_0 \rightarrow E' = v_0 \tag{125}$$

5.10 Particle in a Box - Infinite Square Well

$$E_n = n^2 E_0 \tag{126}$$

$$E_0 = \frac{\hbar^2 k_0^2}{2m} = \frac{p_0^2}{2m} \tag{127}$$

$$k_n = \frac{n\pi}{a} \tag{128}$$

$$p_n = \hbar k_n \tag{129}$$

$$\psi = \sqrt{\frac{2}{a}} \sin(k_n x) \tag{130}$$

$$\text{3D: } E = \frac{\hbar^2}{2m}[k_x^2 + k_y^2 + k_z^2] \tag{131}$$

5.11 Schrödinger's Equation

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\Psi = ih\frac{\partial \Psi}{\partial t} \tag{132}$$

Separable Solutions:

$$\Psi = \Phi(t)\Psi(x) \tag{133}$$

$$\Phi(t) = e^{-iE_n t/\hbar} \tag{134}$$

5.12 Free Particle

$$\Psi = Ae^{i(kx-\omega t)} \tag{135}$$

5.12.1 Wave Packet Solutions

$$\Psi = \frac{1}{\sqrt{2\pi}} \int\limits_{-\infty}^{\infty} \Phi(k) e^{ikx} dk \tag{136}$$

$$\Phi = \frac{1}{\sqrt{2\pi}} \int\limits_{-\infty}^{\infty} \Psi(x) e^{-ikx} dx \tag{137}$$

1. Packet moves at group velocity, $v_g = \frac{\partial \omega}{\partial k}$
2. $\Delta x \Delta k \sim 1$
3. $\Delta x \Delta p \sim \hbar$, $p = \hbar k$

5.13 Traveling Wave Formalism

$$\frac{\partial^2 \Psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \Psi}{\partial t^2} \text{ , } v = \sqrt{\frac{\text{restoring force}}{\text{density}}} \tag{138}$$

$$v_\phi = \frac{\omega}{k} \tag{139}$$

$$\Psi = A\cos(k(vt - X)) = A\cos(\omega t - kx) \tag{140}$$

In one period, $x - vT = 2\pi$

5.14 Finite Potential Well

$$E \propto n^2 \tag{141}$$
$$d \propto \frac{1}{\sqrt{V - E_n}}\ , d = \frac{\hbar}{\sqrt{2m(V - E_N)}} \tag{142}$$
$$d \propto n \tag{143}$$

5.15 Fundamental Particles

1. Bosons: Force carriers
   Gauge Boson: Gluon-strong
   W,Z Boson - a.k.a Weak Boson
   photons - E&M
   other: Higgs, graviton, pion
2. Fermions: Associated with matter
   Quarks: up, down, top, bottom, strange, charm
   Leptons: electron, muon, tauon, neutrino flavors of each
3. Composite Fermions: Protons and Neutrons, etc.

5.16 Single Slit Diffraction

$$w \sin\theta = n\lambda\ , \quad \tan\theta = \frac{y}{L} \tag{144}$$

Central maximum width:

$$\frac{2L\lambda}{d} = \Delta y_{\text{max}} \tag{145}$$

5.17 Diffraction Grating

$$d \sin\theta = n\lambda \tag{146}$$
$$y = L\tan\theta = L\frac{\sin\theta}{\cos\theta} = \frac{Ln\lambda}{d\cos\theta} \tag{147}$$

5.18 Double Slit Interference

$$d \sin\theta = n\lambda\ , d\sin\theta = n\left(\lambda + \frac{\lambda}{2}\right) \tag{148}$$

5.19 Bragg Diffraction

$$2d \sin\theta = n\lambda \tag{149}$$
$$d = \frac{\overbrace{a}^{\text{lattice spacing}}}{\underbrace{\sqrt{h^2 + k^2 + l^2}}_{\text{miller indices}}} \tag{150}$$

6 Harmonics

6.1 Harmonic Oscillator Potential

$$E_n = \hbar\omega\left(n + \frac{1}{2}\right) \text{ , lowest n} = 0 \tag{151}$$

$$\langle v \rangle = \langle T \rangle = \frac{1}{2}\hbar\omega\left(n + \frac{1}{2}\right) \tag{152}$$

$$\omega = \sqrt{\frac{k}{m}} \tag{153}$$

$$X = A\sin\omega t + B\cos\omega t \tag{154}$$

$$\Psi_n \propto e^{-\frac{m\omega x^2}{2\hbar}} H_n(x) \tag{155}$$

6.2 Damped-Driven Oscillator

$$F = -k\underbrace{x}_{\text{Hooke's}} - b\overbrace{\dot{x}}^{\text{dampening} \propto v} + A\underbrace{\cos\theta}_{\text{driver}} \tag{156}$$

$$\omega_0 = \sqrt{\frac{k}{m}} \tag{157}$$

$$\beta = \frac{b}{2m} \tag{158}$$

6.2.1 Underdamped $\omega_0 > \beta$

$$X_u = Ae^{-\beta t}\cos(\omega' t + \phi) \text{ ,} \omega' = \sqrt{\omega_0^2 - \beta^2} \tag{159}$$

6.2.2 Overdamped $\omega_0 < \beta$

$$X_o = Ae^{-\beta t}e^{-\omega'' t} \text{ ,} \omega'' = \sqrt{\beta^2 - \omega_0^2} \tag{160}$$

6.2.3 Critically Damped $\omega_0 = \beta$

$$X_c = A_1 e^{-\omega_0 t} + A_2 t e^{-\omega_0 t} \tag{161}$$

6.3 Springs and Simple Harmonic Oscillators

$$F = -kx \Rightarrow U = \frac{1}{2}kx^2 \text{ ,} \omega = \sqrt{\frac{k}{m}} \tag{162}$$

$$ma = -kx \tag{163}$$

$$\ddot{x} = -\omega_0^2 x = -\frac{k}{m}x \tag{164}$$

Solutions: sines and cosines

1. A = max amplitude

2. $E_{tot} = \frac{1}{2}KA^2$
3. $KE = \frac{1}{2}KA^2\cos^2(\omega_0 t)$
4. $PE = \frac{1}{2}KA^2\sin^2(\omega_0 t)$

To find oscillations about the minimum of E in an arbitrary u:

1. Find equilibrium value: $\frac{\partial u}{\partial x} = 0 \to x_0 = ?$
2. 2nd derivative of taylor series gives $\omega_a \to \frac{1}{2}v''(x_0) = \frac{1}{2}m\omega^2$

6.4 Beats

$$f_b = f_1 - f_2 \tag{165}$$

$$T_b = \frac{1}{f_1 - f_2} \tag{166}$$

7 Kinematics

7.1 Linear $\rightarrow$ Rotational Kinematics

$$\begin{array}{rll}
x \to \theta & s_{\text{arc}} = r\theta & \Delta x = v_0 t + \frac{1}{2}at^2 \to \Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2 \\
v \to \omega & v_\perp = r \times \omega & v = v_0 + at \\
a \to \alpha & a_\perp = r \times \alpha & v^2 = v_0^2 + 2a\Delta x \\
p \to L & L = r \times p & L = I\omega \ (p = mv) \\
F \to \tau & \tau = r \times F & \tau = \frac{\partial L}{\partial t} \ (F = \frac{\partial p}{\partial t}) \\
m \to I & I \propto mr^2 &
\end{array}$$

7.2 Lagrangian

$$\begin{array}{rl}
L = T - U & H = T + U \text{ if } U \neq U(v) \neq U(t) \\
\frac{\partial L}{\partial q} - \underbrace{\frac{d}{dt}}_{\text{EOMS}}\left(\frac{\partial L}{\partial \dot{q}}\right) = 0 & p = \frac{\partial L}{\partial \dot{q}}
\end{array}$$

7.2.1 EOMS:

$$\dot{q}_k = \frac{\partial H}{\partial p_k} \tag{167}$$

$$-\dot{p}_k = \frac{\partial H}{\partial q_k} \tag{168}$$

7.3 Rocket Motion

$$U\frac{dm}{dt} + M\frac{dv}{dt} = 0 \tag{169}$$

$$v_f = v_0 + u\ln\left(\frac{M_i}{M_f}\right) \tag{170}$$

7.4 Collisions

1. Momentum + mass are always conserved classically
2. Use p equalities for before/after collisions even if elastic
3. Elastic $\rightarrow$ conservation of kinetic energy

$$\epsilon = 1 = \frac{\overbrace{|v_1| + |v_2|}^{\text{final}}}{\underbrace{|u_1| + |u_2|}_{\text{before}}} \tag{171}$$

Don't forget to include $(-)$ and $(+)$ for direction of velocity in momentum equations!

1. Only use kinetic energy for conservation of total energy either before or after the collision
2. Impulse $J = F\Delta t = \Delta p = \Delta L$
3. Cross section:
   $N_{\text{scat}} = \frac{N_{\text{target}}}{\text{area}} N_{\text{incident}}\sigma$

7.5 Central Force Motion

$$\mu = \frac{m_1 \cdot m_2}{m_1 + m_2} \tag{172}$$

$$r_{\text{CM}} = \frac{\sum\limits_i m_i r_i}{\sum\limits_i m_i} \tag{173}$$

$$T = \frac{1}{2}\mu|\dot{r}|^2 \tag{174}$$

$$\overline{r}_1 = \frac{m_2}{m_1 + m_2}\overline{r} \tag{175}$$

$$\overline{r}_2 = \frac{m_1}{m_1 + m_2}\overline{r} \tag{176}$$

$$\overline{r} = \overline{r}_1 - \overline{r}_2 \tag{177}$$

7.6 Moments of Inertia

1. $I = CMR^2$, where C is a constant
2. $I_{\text{hoop}} = MR^2$
3. $I_{\text{disk}} = \frac{1}{2}MR^2$
4. $I_{\text{hollow sphere}} = \frac{2}{3}MR^2$
5. $I_{\text{solid sphere}} = \frac{2}{5}MR^2$
6. $I_{\text{point mass}} = MR^2$
7. $I_{\text{rod end}} = \frac{1}{3}ML^2$
8. $I_{\text{rod center}} = \frac{1}{12}ML^2$
9. $L_{\text{rot}} = I\omega$
10. $T_{\text{rot}} = \frac{1}{2}I\omega^2$

11. $\tau = I dv = \frac{dL}{dt}$
12. $I_{\text{parallel axis}} = I_{\text{CM}} + MR^2_{\text{displaced}}$

8 Statistical Thermodynamics

8.1 Laws of Thermodynamics

8.1.1 1st Law

$$\Delta U = Q + W \tag{178}$$

8.1.2 2nd Law

E flows spontaneously until the system is at the most likely microstate $\Rightarrow$ entropy tends to increase

8.1.3 3rd Law

$$S(T = 0) = 1 \text{ , so } C_v \to 0 \text{ as } T \to 0 \tag{179}$$

8.2 Maxwell Velocity Distribution

Speed of molecules in ideal gas:

$$D(v) \propto v^2 e^{-E/k_b T} \tag{180}$$

8.3 Mean Free Path

$$l = \frac{1}{n\sigma} \tag{181}$$

$$n = \frac{\text{particles}}{\text{volume}} \tag{182}$$

$$\sigma = \text{scattering cross section} \tag{183}$$

8.4 Particle Diffusion

Fick's Law:

$$J_p = -\underbrace{D}_{\text{constant}} \nabla n \tag{184}$$

8.5 Thermal Diffusion

Fourier's Law:

$$J_q = \Phi_q = -\underbrace{\sigma}_{\text{conductivity (Thermal)}} \nabla T \tag{185}$$

$$\underbrace{\Phi_q}_{\text{Flow of energy/time} \cdot \text{area, units of } \frac{\text{W}}{\text{m}^2}} = -k\nabla T \text{ , } k = \text{thermal conductivity with units} = \frac{\text{W}}{\text{m degrees K}} \tag{186}$$

Type of Interaction	Quantity	Variable	Formula
Mechanical	volume	P	$P = -\left(\frac{\partial U}{\partial V}\right)_{U,N} = T\left(\frac{\partial S}{\partial V}\right)_{U,N}$
Thermal	Temperature/Energy	T	$T = \left(\frac{\partial U}{\partial S}\right)_{V,N}$
Diffusive	Particles	μ	$\mu = -\left(\frac{\partial U}{\partial N}\right) = T\left(\frac{\partial S}{\partial N}\right)$

8.5.1 Thermodynamic Identity

$$dU = TdS - PdV + \mu dN \tag{187}$$

8.6 Heat Capacity

$$c = \frac{dQ}{dt} \tag{188}$$

$$c_p = \left(\frac{\partial Q}{\partial T}\right)_P = T\left(\frac{\partial S}{\partial T}\right) \tag{189}$$

$$c_v = \left(\frac{\partial Q}{\partial T}\right)_V = \left(\frac{\partial U}{\partial T}\right)_V \text{ , } U = \text{ total } E \tag{190}$$

$c_P > c_V$ since at constant P the system loses E in the form of work $\Rightarrow$ for the same Q, $dT_P < dT_V$, thus $c_P > c_V$.

8.7 Isothermal Compression (Slow)

$$P_1V_1 = P_2V_2 \tag{191}$$

$$W = Nk\ln(V_i/V_f) \text{ , } W = -\int_{V_i}^{V_f} PdV \tag{192}$$

$$\Delta U = 0 \text{ since } \Delta T = 0 \text{ , } \Delta U = \frac{f}{2}Nk\Delta T \tag{193}$$

8.8 Adiabatic Compression (Fast)

If no heat flows,

$$\Delta Q = 0 \rightarrow \Delta U = W \tag{194}$$

Equipartion Theorem,

$$\Delta U = Nk\Delta T = W \tag{195}$$

$$V_f T_f^{f/2} = V_i T_i^{f/2} \text{ , } f = \text{Degrees of Freedom} \tag{196}$$

$$V_f^{\gamma} P_f = V_i^{\gamma} P_f \text{ , } \gamma = \frac{f+2}{f} \tag{197}$$

$$W = \frac{P_f V_f - P_i V_i}{1-\gamma} \tag{198}$$

$$PV^{\gamma} = C \rightarrow P = \frac{C}{V^{\gamma}} \tag{199}$$

$$W = \int PdV = C\frac{dV}{V^{\gamma}} = \frac{1}{\gamma}\frac{C}{V^{\gamma-1}}\Big|_{V_1}^{V_2} \tag{200}$$

8.9 Heat

$$Q = TdS \tag{201}$$
$$= mc\Delta T \tag{202}$$
$$= \text{Power} \cdot t \tag{203}$$

8.10 Multiplicity/States

$$\text{Probability}(\Omega_n) = \Omega(n)/\Omega(\text{all}) \tag{204}$$

1. Ω = multiplicity = how many different microstates yield a macrostate
2. Total number of macrostates = (# states thing can be in)$^{(\text{\# of things})}$
3. e.g., 3 coins $\rightarrow 2^3 = 8 = \Omega$
4. # of ways to choose n things from N: $\Omega\binom{N}{n} = \frac{N!}{(N-n)!n!}$

8.11 Boltzmann Statistics

$$P(s) = \frac{g_s e^{E_s/kT}}{Z}\ ,\ Z = \sum_i g_i e^{-E_i/kT}\ ,\ g_i = \text{ degeneracy of I} \tag{205}$$
$$P(A)/P(B) = \frac{g_a}{g_b} e^{(-A+B)/kT} \tag{206}$$
$$\langle \bar{x} \rangle = \frac{1}{Z} \sum_s x_s e^{-E_s/kT} \text{ average of any value} \tag{207}$$
$$\langle \bar{E} \rangle = \frac{1}{Z} \sum_i E_i e^{-E_i/kT} \tag{208}$$
$$U = N\overline{E}: \text{ total energy of the system} \tag{209}$$

8.12 Density of State Distributions

Fermions:

$$N_i = \frac{g_i}{e^{(E_i-\mu)/kT} + 1} \tag{210}$$

Mesons/Bosons:

$$N_i = \frac{g_i}{e^{[E_i-\mu]/kT} - 1} \tag{211}$$

Boltzmann:

$$N_i = g_i e^{(E_i-\mu)/kT} - 1 \tag{212}$$

8.13 Blackbody Radiation

8.13.1 Wein's Law

$$T \cdot \lambda_{\text{max}} = 3 \text{ mm} \cdot K \tag{213}$$

8.13.2 Stephan-Boltzmann

$$P \propto aT^4 \tag{214}$$

8.14 Heat Engines

$$e \leq 1 - \frac{T_c}{T_h} \tag{215}$$

$$e = \frac{\text{benefit}}{\text{cost}} = \frac{W}{Q_h} \text{ }, W = Q_h - Q_c \tag{216}$$

8.15 Refrigerators

$$e \leq \frac{T_c}{T_h - T_c} \tag{217}$$

$$W = Q_h - Q_c \tag{218}$$

$$\Delta S = 0 \text{ , independednt of working substances} \tag{219}$$

8.16 Big People

1. Onnes: Superconductivity in Hg
2. Anderson: Positron
3. Yukawa: Strong Nuclear
4. Fermi: First nuclear reactor
5. Mann + Zweig: Quarks
6. Rontengen: X-rays
7. Penzias & Wilson: Background Radiation
8. Huygens: Wavefronts
9. Cavendish: G
10. Oersted: Connection between E&M
11. Ampere: B force law
12. Hertz: Showed E&M waves existed

9 Relativity

9.1 Space-Time Diagram

$\Delta S > 0$ Spacelike

1. Ordering of events depends on reference frame
2. There exists a reference frame where 2 events occur simultaneously, but they can't occur at the same location in space

$\Delta S < 0$ Timelike

1. Ordering of events is absolute
2. Casual relationships are timelike
3. Two events can occur at same point in space

9.2 Special Relativity

v/c	γ
.1	1.005
.25	1.033
.5	1.151
.75	1.55
.9	2.29

$$x = \gamma(x' + vt') \tag{220}$$
$$t = \gamma\left(t' + \frac{vx}{c^2}\right) \tag{221}$$
$$u'_x = \frac{u_x + v}{1 + \frac{u_x v}{c^2}} \tag{222}$$
$$u'_z = \frac{u_z}{\gamma\left(1 + \frac{u_x v}{c^2}\right)} \tag{223}$$

9.2.1 Time Dilation

$$t' = \gamma t_0 \ , t_0 = \text{ rest time} \tag{224}$$

9.2.2 Length Contraction

$$x' = \frac{x_0}{\gamma} \ , x_0 = \text{ rest length} \tag{225}$$

9.2.3 Invariant Interval

$$\Delta s^2 = \Delta x^2 - (ct)^2 \leftarrow \text{ transform between 2 moving frames} \tag{226}$$

9.2.4 Energy

$$E_{\text{rel}} = \gamma E_0 \tag{227}$$
$$p = \gamma p = \gamma m v \tag{228}$$
$$E_{\text{rel}}^2 = E_0^2 + (pc)^2 \tag{229}$$
$$E_{\text{rel}} \neq \frac{p_{\text{rel}}^2}{2m} \tag{230}$$
$$p_x = \gamma\left(p_{x'} + \frac{v}{c^2}E'\right) \tag{231}$$
$$E = \gamma\left(E' + v p_{x'}\right) \tag{232}$$

Last 2 lines employ the invariant 4-vector, where $p_{y'} = p_y$.

10 Atomic Physics

10.1 Hydrogen Spectral Series

$$\frac{1}{\lambda} = R_y\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \ , \quad R_y \approx 1 \times 10^7 \text{m}^{-1} \tag{233}$$

1. Lyman: $n_f = 1$
2. Balmer: $n_f = 2$
3. Paschen: $n_f = 3$

$$\Delta E = E_0 \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \tag{234}$$

10.2 Atomic Notation

$$^A_Z X \tag{235}$$

1. A = mass number = $p^+ + n^0$
2. Z = number of protons = chemical number

11 Particle Physics

11.1 Fermi

$$E_F = k_b T_F \tag{236}$$

$$p_F = \hbar k_F \rightarrow E_F = \frac{p^2}{2mn} = \frac{\hbar^2 k^2}{2m} \;, \quad v_F = \frac{p_F}{m} \tag{237}$$

$$k_F \left(\frac{3\pi^2 N}{\text{volume}} \right)^{1/3} \;, \quad p_F = \frac{2}{3} \frac{E_F}{v} \tag{238}$$

Degenerate Fermi gas, so cold that nearly all states below E_F are occupied and above states are unoccupied.

11.2 Degeneracy Pressure of a Solid

$P = \frac{3}{2} \frac{E}{V}$: The stabilizing internal pressure that comes from the anti-symmetrization requirement for the wave functions of identical fermions.

12 Misc

12.1 Water Density

$$1 \text{ liter} = 1 \text{ kg} \;, \rho = 1 \text{ g/cm}^3 \tag{239}$$

Beats occur when f_1 and f_2 are close together

12.2 Fundamental Law of Statistical Mechanics

All accessible microstates are equally likely

12.3 Irreversible Process

Creates new entropy

12.4 Reversible Process

Creates no new entropy