HW #6

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Who are you.

Problem 1a

For 2 protons:

$$r_c = \frac{Z_1 Z_2 e^2}{E} \tag{1}$$
$$= \frac{1\ e^2}{2\ \text{keV}} \tag{2}$$
$$= 7.2 \times 10^{-11}\ \text{cm} \tag{3}$$
$$\tag{4}$$
$$P \approx e^{-(E_g/E)^{1/2}} \tag{5}$$
$$m_r = \frac{1}{2} m_p \tag{6}$$
$$E_g = \frac{2\pi^2 m_r e^4 Z_1^2 Z_2^2}{\hbar^2} \tag{7}$$
$$P \approx 1.46 \times 10^{-7} \tag{8}$$

For 2 ^{4}He:

$$r_c = \frac{Z_1 Z_2 e^2}{E} \tag{9}$$
$$= \frac{4\ e^2}{2\ \text{keV}} \tag{10}$$
$$= 2.88 \times 10^{-10}\ \text{cm} \tag{11}$$
$$\tag{12}$$
$$P \approx e^{-(E_g/E)^{1/2}} \tag{13}$$
$$m_r = 2m_p \tag{14}$$
$$P \approx 2.2 \times 10^{-54} \tag{15}$$

For ^{4}He and p:

$$r_c = \frac{Z_1 Z_2 e^2}{E} \tag{16}$$
$$= \frac{2\ e^2}{2\ \text{keV}} \tag{17}$$
$$= 1.44 \times 10^{-10}\ \text{cm} \tag{18}$$
$$\tag{19}$$
$$P \approx e^{-(E_g/E)^{1/2}} \tag{20}$$
$$m_r = \frac{4}{5} m_p \tag{21}$$
$$P \approx 1.08 \times 10^{-17} \tag{22}$$

Problem 1b:

For 2 ^{4}He:

$$1.46 \times 10^{-7} \approx e^{-(E_g/E)^{1/2}} \tag{23}$$
$$[-\ln(1.46 \times 10^{-7})]^2 = \frac{E_g}{E} \tag{24}$$
$$E = \frac{E_g}{[-\ln(1.46 \times 10^{-7})]^2} \tag{25}$$
$$E = \frac{1}{[-\ln(1.46 \times 10^{-7})]^2} \frac{2\pi^2 2m_p e^4 4 \cdot 4}{\hbar^2} \tag{26}$$
$$E = 1.9 \times 10^{-7} \text{ ergs} \tag{27}$$
$$\tag{28}$$
$$\frac{3}{2}kT = E \tag{29}$$
$$T = 9.2 \times 10^8 \text{ K} \tag{30}$$

For p + ^{4}He:

$$E = \frac{1}{[-\ln(1.46 \times 10^{-7})]^2} \frac{2\pi^2 4m_p e^4 4}{\hbar^2} \tag{31}$$
$$E = 1.9 \times 10^{-8} \text{ ergs} \tag{32}$$
$$\tag{33}$$
$$T = 9.18 \times 10^7 \text{ K} \tag{34}$$

For ^{12}C + ^{12}C:

$$E = \frac{1}{[-\ln(1.46 \times 10^{-7})]^2} \frac{2\pi^2 6m_p e^4 36 \cdot 36}{\hbar^2} \tag{35}$$
$$E = 7.67 \times 10^{-6} \text{ ergs} \tag{36}$$
$$T = 3.7 \times 10^{10} \text{ K} \tag{37}$$

Problem 2a:

$$E_0 = \left(\frac{1}{2}E_g^{1/2}kT\right)^{2/3} \tag{38}$$
$$MB = \frac{2}{kT}\left(\frac{E}{\pi kT}\right)^{1/2} e^{\left(-\frac{E}{kT}\right)} dE \ , \tag{39}$$

where MB is just the shorthand for the Maxwell-Boltzmann Distribution.
Plugin E_0 for E and dE:

$$MB = \frac{2}{kT}\left(\frac{E_0}{\pi kT}\right)^{1/2} e^{\left(-\frac{E_0}{kT}\right)} E_0 \tag{40}$$
$$= .113 \tag{41}$$
$$= 11.3 \ \% \tag{42}$$

Problem 2b:

$$S = 3.78 \times 10^{-22} \text{ keV barn} \tag{43}$$

$$\sigma = \frac{S}{E} e^{(E_g/E)^{1/2}} \tag{44}$$

$$\tag{45}$$

We have to take into account that only 10% of the mass in the sun is fusing and therefore must multiply our l by 0.1.

$$l = \frac{.1}{n\sigma} \ , n = \frac{\rho\mu}{m_p} \tag{46}$$

$$= \frac{0.1 m_p}{\rho\mu\sigma} \tag{47}$$

$$= \frac{0.1 m_p E}{\rho\mu S e^{(E_g/E)^{1/2}}} \tag{48}$$

$$= 3.29 \times 10^{24} \text{ cm} \tag{49}$$

$$l = 4.72 \times 10^{13} R_\odot \tag{50}$$

$$\tag{51}$$

$$t = \frac{l}{v} \tag{52}$$

$$\frac{1}{2} m v^2 = \frac{3}{2} kT \tag{53}$$

$$v = \sqrt{\frac{3kT}{m_p}} \tag{54}$$

$$= 6.23 \times 10^7 \text{ cm/s} \tag{55}$$

$$t = \frac{3.29 \times 10^{24} \text{ cm}}{6.23 \times 10^7 \text{ cm/s}} \tag{56}$$

$$t \approx 1.67 \text{ billion years} \tag{57}$$

Problem 3:

$$I = \int_0^\infty e^{-f(E)} dE \tag{58}$$

$$\text{Taylor expand } f(E) \text{ around } E_0 \tag{59}$$

$$f(E) = f(E_0) + \frac{f'(E_0)}{1!}[E - E_0] + \frac{f''(E_0)}{2!}[E - E_0]^2 \ , \tag{60}$$

and the first derivative is zero, so we can just get rid of that middle term.

$$f(E) = f(E_0) + \frac{1}{2} f''(E_0)[E - E_0]^2 \tag{61}$$

$$I = \int_0^\infty e^{-f(E_0) - \frac{1}{2} f''(E_0)[E - E_0]^2} dE \tag{62}$$

$$I = e^{-f(E_0)} \int_0^\infty e^{-\frac{1}{2} f''(E_0)[E - E_0]^2} dE \tag{63}$$

We want to make line 63 look like a Gaussian integral,

$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi} \ , \tag{64}$$

so that we can easily plug in $\sqrt{\pi}$.

$$-\frac{1}{2}f''(E_0)[E-E_0]^2 = -x^2 \tag{65}$$

$$\sqrt{\frac{f''(E_0)}{2}}(E-E_0) = x \tag{66}$$

$$\sqrt{\frac{f''(E_0)}{2}}dE = dx \tag{67}$$

$$\tag{68}$$

$$I = e^{-f(E_0)}\int_0^\infty e^{-x^2}dx \cdot \sqrt{\frac{2}{f''(E_0)}} \tag{69}$$

$$I = e^{-f(E_0)}\sqrt{\frac{2}{f''(E_0)}}\sqrt{\pi} \tag{70}$$

$$I \approx \frac{\sqrt{2\pi}e^{-f(E_0)}}{\sqrt{f''(E_0)}} \tag{71}$$

We can ignore the $\int_0^\infty$ difference in the integral given in the homework problem with $\int_{-\infty}^\infty$ of the actual Gaussian equation because the Gaussian integral from $\int_{-\infty}^\infty$ assumes it's symmetrical around 0. In our case, the Gaussian is symmetrical around E_0, which is far enough away from 0 that there is no contributing factor to the Gaussian at 0. Therefore, integrating from $-\infty \to \infty$ will be the same as $0 \to \infty$.

Problem 4:

$$E \ll E_g \tag{72}$$

$$E \ll \frac{2\pi^2 e^4 Z_1^2 Z_2^2 m_r}{\hbar^2} \text{ , and } e^4 Z_1^2 Z_2^2 \text{ looks like } E = \frac{e^2 Z_1 Z_2}{r} \tag{73}$$

$$E \ll \frac{2\pi^2 m_r}{\hbar^2}(Er)^2 \tag{74}$$

$$\frac{1}{E} \ll \frac{2\pi^2 m_r}{\hbar^2}r^2 \text{ , } E = \frac{p^2}{2m} \tag{75}$$

$$\frac{2m}{p^2} \ll \frac{2\pi^2 m_r}{\hbar^2}r^2 \text{ , } \lambda = \frac{h}{p} \tag{76}$$

$$\frac{\lambda^2 2m}{h^2} \ll \frac{2\pi^2 m_r}{\hbar^2}r^2 \tag{77}$$

$$\frac{\lambda^2}{h^2} \ll \frac{\pi^2}{\hbar^2}r^2 \tag{78}$$

$$\lambda^2 \ll (2\pi)^2\pi^2 r^2 \tag{79}$$

$$\lambda \ll 2\pi^2 r \tag{80}$$

$$\lambda \ll r \tag{81}$$

Problem 5a:

$$\beta = -\frac{2}{3} + 23.6 \cdot T_7^{-1/3} \tag{82}$$

$$\beta(1.5 \times 10^7 \text{ K}) \approx 19.95 \tag{83}$$

$$\beta(3 \times 10^7 \text{ K}) \approx 15.7 \tag{84}$$

Problem 5b:

$$L_{CNO} = .016 L_\odot \tag{85}$$
$$L = \int \epsilon dM \tag{86}$$
$$\sim \epsilon M \tag{87}$$
$$\epsilon \propto 4.4 \times 10^{27} \frac{\rho X Z}{T_7^{2/3}} e^{-70.7 T_7^{-1/3}} \tag{88}$$

We divide luminosities at different Ts to cancel out ρ, X, and anything that isn't T-dependent.

$$\frac{L'_{CNO}}{L_{CNO}} \sim \left(\frac{T_7}{T'_7}\right)^{2/3} \frac{e^{-70.7 T_7'^{-1/3}}}{e^{-70.7 T_7^{-1/3}}} \text{, where } L'_{CNO} \text{ is the Luminosity of the CNO chain at the new } T' \tag{89}$$
$$\sim \left(\frac{T_7}{T'_7}\right)^{2/3} e^{-70.7 (T_7'^{-1/3} - T_7^{-1/3})} \tag{90}$$
$$\sim \left(\frac{1.5}{1.65}\right)^{2/3} e^{-70.7 (1.65^{-1/3} - 1.5^{-1/3})} \tag{91}$$
$$L'_{CNO} \sim 8.5 L_{CNO} \tag{92}$$
$$\sim 8.5 \cdot .016 L_\odot \tag{93}$$
$$\sim .1360 L_\odot \tag{94}$$
$$\sim 13.6\% \ L_\odot \tag{95}$$

Problem 5c:

Since $\epsilon \propto T^\beta$, we can rearrange the proportionality to find T in terms of ϵ and get $T \propto \epsilon^{1/\beta}$. At higher T, β essentially becomes constant at around 20. Looking at the Line 87, we see that we can change the ρ, X, and Z of a star, but that change will be suppressed by any $^{1/20}$ dependence.