GRE Physics Study Notes - Lessons from Practice Exams

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Contents

Introduction

By no means comprehensive, this list is meant to serve as additional study material to re-reading textbooks, practicing GRE tests, and nagging your physics friends for study help.

1 Some Stuff

1.1 Cherenkov Radiation

A charged particle which passes through a media with a speed greater than the speed of light in that media will emit E&M radiation (light)

$$n = \frac{c}{v}\ , v = \frac{c}{n} \leftarrow \text{ minimum velocity of the particle} \tag{1}$$

This does not violate relativity because $n > 1$ so $v < c$.

1.2 Bremsstrahlung Radiation:

A continuous spectra of radiation emitted when a charged particle is decelerated in a metal target.

2 Problems

2.1 Hoops Hung on a Nail, Undergoing Small Oscillations

1. Approximate using small θ pendulum, with $\omega = \sqrt{\frac{g}{l}}$
2. Works because it's a hoop, with extended object use a physical pendulum.

2.2 Physical Pendulum

$\tau = I\dot{\omega} = I\alpha \quad \tau = mgL_{\text{cm}} \sin\theta$
$\ddot{\theta} = \frac{mgL_{\text{cm}}\theta}{I} \quad \omega = \sqrt{\frac{mgL_{\text{cm}}}{I}}$

For hoop, $I = MR^2 = ML_{\text{cm}}^2 \to \omega = \sqrt{\frac{mg}{L}} = \sqrt{\frac{g}{\text{radius}}}$
Often dealing with ratios so reduces out mass + constant factor in I.

$$\frac{\omega_1}{\omega_2} = \sqrt{\frac{L_1 \cdot R_2^2}{L_2 \cdot R_1^2}} \tag{2}$$

2.3 Intrinsic Magnetic Moment

$$\bar{\mu}_s = \frac{gq}{2m}\bar{s} \tag{3}$$

Mass is the dominant factor because $g \sim 10$ s, $q \sim 100$ s but m is many orders of magnitude.

2.4 Equations of Motion

Look for boundary values. A given $x(t)$, $y(t)$, and v_0, differentiate and evaluate at $t = 0$ to see which one yields v_0.

2.5 Moments of Inertia

The moment of a plate is $I + \frac{1}{3}Md^2$ where d is the width from the center rotation axis of the plate to the edge of the plate.

2.5.1 Stretch Axis Theorem

If you stretch an object along the dimension it rotates about, then the moment is unchanged.

e.g., Disk and cylinder both have $I = \frac{1}{2}MR^2$

2.6 Moments

1. Hoop: $I = MR^2$
2. Disk: $I = \frac{1}{2}MR^2$
3. Hollow Sphere: $I = \frac{2}{3}MR^2$
4. Solid Sphere: $I = \frac{2}{5}MR^2$
5. Rod Center: $I = \frac{1}{12}ML^2$
6. Rod End: $I = \frac{1}{3}ML^2$
7. Thick Cylinder: $I = \frac{1}{2}m(r_1^2 + r_2^2)$
8. Cuboid: $I_{\hat{y}} = \frac{1}{12}M(x_l^2 + y_l^2)$

2.7 Hermitian Matrix

1. Described observables: The eigenvalues are real
2. Square matrix

$$A = A^T \rightarrow \text{the entries are equal to their conjugate transpose} \tag{4}$$

$$A = \begin{bmatrix} 3 & 2+i \\ 2-i & 1 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 3 & (2-i)^* \\ (2+i)^* & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2+i \\ 2-i & 1 \end{bmatrix} = A$$

Transpose:

1. Row A $\rightarrow$ column A
2. Row B $\rightarrow$ column B
3. The entries on the diagonal are real
4. The sum of any 2 Hermitian matrices is Hermitian
5. The product is Hermitian if and only if $\hat{A}\hat{B} = \hat{B}\hat{A} \Rightarrow [\hat{A}\hat{B}] = 0$ (Commutes)

2.8 Hermitian Operators

Condition:

$$\langle f|\hat{A}f\rangle = \langle \hat{A}f|f\rangle \tag{5}$$

is equivalent to saying $A = A^*$

2.9 Balancing Problem

Torque is the long and hard way to find where the pivot should be. Classic problem, where to put the fulcrum. Want to set up $\sum \tau = 0$ problem. Instead, put the origin somewhere convenient (center of rod if it has mass) and calculate the center of mass.

$$CM = \frac{\sum\limits_i m_i r_i}{\sum\limits_i m_i} \tag{6}$$

2.10 Decay Rates

$$\frac{dA}{dt} = -kA \rightarrow A = A_0 e^{-kt} \rightarrow \frac{A}{A_0} = \frac{1}{2}e^{-kt} \rightarrow \frac{\ln(2)}{k} = t_{1/2} \tag{7}$$

for a substance with multiple decay modes,

$$k_{\text{tot}} A = (k_1 + k_2)A \tag{8}$$

2.11 Conservation of Energy

Total energy must be conserved.

$$U + KE_{\text{roll}} + KE_{\text{trans}} = E_{\text{tot}} \tag{9}$$

Example: Roll down a hill at a given v_{transl}

$$U = KE_{\text{roll}} + KE_{\text{transl}} \tag{10}$$

$$mgh = \frac{1}{2}I\omega^2 + \frac{1}{2}mv_{\text{transl}}^2 \tag{11}$$

2.12 Interferometer

Fringe shifts occur for changing distances of mirrors or changing wavelengths. A fringe shift occurs for $2d = \lambda$. When using a gas to change wavelength, it changes with n.

$$2d = m(\lambda_{\text{gas}} - \lambda_{\text{vac}}) = m\lambda_{\text{vac}} \left(\frac{1}{n} - 1\right) , \lambda_{\text{gas}} = \frac{\lambda_{\text{vac}}}{n} \tag{12}$$

2.13 Vector Calculus

1. The div of a curl is 0 $\rightarrow \nabla \cdot (\nabla \times F) = 0$
2. The curl of a gradient is 0 $\rightarrow \nabla \times (\nabla \cdot F) = 0$

2.14 Thermodynamic Work

The area under the PV graph, or in a closed cycle. If clockwise, $+W$, if counter clockwise, $-W$.

2.15 Special Relativity, Momentum

$$E_{\text{rel}} \neq \frac{p_{\text{rel}}^2}{2m} \tag{13}$$

Instead use

$$E_{\text{rel}}^2 = (pc)^2 + E_0^2 \tag{14}$$

Only photons move at c, duh. However, particles can move faster than the speed of light in a medium, $v_\phi = \frac{c}{n}$ and it will emit Cherenkov radiation.

2.16 Decay

1. Write out coefficients!
2. e^-/e^+ always accompanied by $\bar{\nu}/\nu$ by conservation of lepton #. Any combo of capture and emission.

$$^A_Z X =^{p^+ + n^0}_{p^+} X \tag{15}$$

2.17 Springs

Add like capacitors.
For series:

$$\frac{1}{k_{\text{tot}}} = \frac{1}{k_1} + \frac{1}{k_2} \tag{16}$$

For parallel:

$$k_{\text{tot}} = k_1 + k_2 \tag{17}$$

2.18 Speed of Sound in an Ideal Gas

$$v \propto T^{1/2} \tag{18}$$

2.19 Conservative Field

$$\nabla \times F = 0 \rightarrow F = -\nabla V \tag{19}$$

2.20 Orbit Problems

First, think Kepler ($T^2 \propto R^3$). Minimum E is a circular orbit.

2.21 Intensity / Radiation Problems

Radiation spreads like a spherical wavefront.

$$\text{\# particles detected / counts} = \frac{\text{Area detector}}{\text{Area sphere @ detector}} \tag{20}$$

2.22 Partition Function

$$\frac{f}{2}NkT \text{ , } f = \# \text{ squared terms in Hamiltonian} \tag{21}$$

$f =$ Degrees of freedom; is a corollary.

2.23 Expectation Value

$$\langle \hat{Q} \rangle = \langle \Psi | \hat{Q} | \Psi \rangle = q \langle \Psi | \Psi \rangle \text{ or } q_1 \langle \Psi_1 | \Psi_1 \rangle + q_2 \langle \Psi_2 | \Psi_2 \rangle + \cdots \tag{22}$$

2.24 Commutator Identities

$$[A, B] = -[B, A] \tag{23}$$
$$[A, BC] = B[A, C] + [A, B]C \tag{24}$$
$$[AB, C] = A[B, C] + [A, C]AB \tag{25}$$

2.25 Motion in a Circle

Always a_{radial} component, only $a_{\|}$ if $v_{\tan}$ is changing.

$$F = \frac{mv^2}{r} = ma_r \to a_r = \frac{v^2}{r} \tag{26}$$

Compare to:

$$v = r \times \omega \tag{27}$$
$$a_r = r \times \alpha \tag{28}$$
$$a = a_{\|}^2 + a_r^2 \tag{29}$$

2.26 Particle Decay

$$\frac{dN}{dt} - -kN \to N = N_0 e^{-kt} \text{ , decay is exponential} \tag{30}$$

$k =$ decay constant, $\tau =$ average, lifetime $= \frac{1}{k}$

2.26.1 Half Life

$$\frac{N}{N_0} = \frac{1}{2} = e^{-kt} \to t_{hl} = \frac{\ln(2)}{k} \tag{31}$$

2.26.2 Multiple Decay Channels

$$(K_{\text{tot}})N = K_1 N + K_2 N + \cdots \tag{32}$$

so, $k = \frac{\ln(2)}{t_{hl}}$, $\frac{1}{t_{hl}} = \frac{1}{t_1} + \frac{1}{t_2} + \cdots$

2.27 Specific Heat in a Solid

Best fit for when accounting for e^- specific heat with FD.

1. Einstein Model: Treat atoms as 3N Harmonic Oscillators. They all have same E, (frequency).
2. Debye: Also 3N Harmonic Oscillators. Assigns a range of energies and treats lattice vibrations as phonons in box.
   Correctly predicts low temperature $C_V \propto T^3 Nk$
3. Dulong-Petit: High temps, uses equipartition with harmonic oscillator $f = 6, c = 3Nk$
   Debye and Einstein reduce to this in high T limit.

2.28 Relativistic Doppler Shift

$$\frac{f_0}{f} = \frac{\lambda}{\lambda_0} = \sqrt{\frac{1+\beta}{1-\beta}}\ , \beta = \frac{v}{c} \tag{33}$$

The "redshift": $z = \frac{\lambda_0 - \lambda}{\lambda} = \frac{f - f_0}{f}$

2.29 Fission

1. Conservation of Energy
2. Binding Energies of nucleus is always $(-)$, like BE e^-'s

$$-BE_i + KE_i = -BE_f + KE_f \tag{34}$$

2.30 Wire Resistance

$$R = \frac{\rho L}{A} \tag{35}$$

2.31 Inside a Non-conducting Sphere of Uniform Charge Density

With constant surface potential, like a conductor. $\nabla V = 0 = E$

2.32 Spin Matrices

$$S_i \Psi = \frac{\hbar}{2} \sigma_i \Psi \tag{36}$$

$$S_z |\uparrow\rangle = \frac{\hbar}{2} |\uparrow\rangle = \frac{\hbar}{2}\begin{pmatrix}1\\0\end{pmatrix} \tag{37}$$

$$S_z |\downarrow\rangle = -\frac{\hbar}{2} |\downarrow\rangle = -\frac{\hbar}{2}\begin{pmatrix}0\\1\end{pmatrix} \tag{38}$$

For example, eigenstate of S_x with $-\frac{\hbar}{2}$,

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\frac{1}{\sqrt{2}}(|\uparrow\rangle - |\downarrow\rangle) = \frac{1}{\sqrt{2}}\left[\begin{pmatrix}1\\0\end{pmatrix} - \begin{pmatrix}0\\1\end{pmatrix}\right] = \frac{1}{\sqrt{2}}\begin{pmatrix}\Psi\\-1\end{pmatrix} \tag{39}$$

$$\begin{pmatrix}1\\-1\end{pmatrix}\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix} = \begin{pmatrix}-1\\1\end{pmatrix} \rightarrow S_x\Psi = \frac{\hbar}{2}\left(\frac{1}{\sqrt{2}}\right)\begin{pmatrix}-1\\1\end{pmatrix} = -\frac{\hbar}{2}\Psi \tag{40}$$

2.33 Collisions

1. For $i \rightarrow f$ conditions, use conservation of momentum only!
2. For converting between U and KE, use KE only.
3. $\epsilon = 1$ if elastic, $\epsilon = 0$ if inelastic

$$\frac{\overbrace{|V_2| + |V_1|}^{\text{final}}}{\underbrace{|U_2| + |U_1|}_{\text{before}}} \tag{41}$$

4. KE is conserved in elastic collisions only

2.34 For what v will car stay on hill?

$$F_c = F_g = mg \tag{42}$$

$$\frac{mv^2}{r} = mg \tag{43}$$

2.35 Böhr Model

1. e^- have classical motions
2. $\Delta E = hf$
3. Quantization of angular momentum, $L = n\hbar$
4. $E_n = -\frac{Z^2E_0}{n^2}$, $E_n \propto \mu$
5. $\Delta E = E_0\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \rightarrow \frac{1}{\lambda} = R_y\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$, $R_y = 1 \times 10^7\ \text{m}^{-1}$
6. Positronium: $\mu = \frac{m_e}{2} \rightarrow E_p = \frac{E_0}{2n^2}$

2.36 Hydrogen Spectral Series

1. Lyman: $n_f = 1 \rightarrow$ UV
2. Balmer: $n_f = 2 \rightarrow$ Visible
3. Paschen: $n_f = 3 \rightarrow$ IR

2.37 Fluids

Equilibrium when $F_a = F_b$

2.38 Gauss with non-uniform densities

Must integrate. For example:

$$\rho = Ar^2 \ , \rho \propto r^2 \ , dV = 4\pi r^2 dr \tag{44}$$

$$\int E \cdot dA = \int \frac{\rho dV}{\epsilon_0} = \int_0^R \frac{\rho(4\pi r^2 dr)}{\epsilon_0} = 4\pi \int \frac{r^4 dr}{\epsilon_0} \tag{45}$$

2.39 Capacitors

If capacitors in series, $Q_1 = Q_2$. If parallel, $V_1 = V_2$.

2.40 Diffraction Limit

Airy Disk: circular aperture diffraction

$$\theta = \frac{1.22\lambda}{d} \ , \Delta l = \frac{1.22 f\lambda}{d} \tag{46}$$

2.41 Time Dilation and Mass Contraction

$$t = \gamma t_0 \ , X = \frac{X}{\gamma} \tag{47}$$

Used to relate a moving frame t, x to a rest frame's x_0, t_0. Cannot use these equations to relate two moving frames.

2.42 Expectation Value Problems

Look for even/odd functions

$$\int_0^T \sin x \cos x = 0 \text{ because orthogonal, } T = \text{period} \tag{48}$$

If $\frac{\partial}{\partial x}$ for $\langle p \rangle$ doesn't bring out an i, then $\langle p \rangle = 0$

2.43 Normal Modes

1. Highest normal mode frequency when out of phase
2. Use limits if possible, if $M \to \infty$, etc.
3. # Frequencies = # masses
4. If odd # masses, one ω will be $\omega_0 9$, others will be above/below.

For 2 masses hung by strings and connected by a spring,

1. In phase: $\omega = \sqrt{\frac{g}{l}}$
2. Out of phase: $\omega = \sqrt{\frac{2k}{m} + \frac{g}{l}} \to F = ma = K_{\text{eff}} x - mg\cos\theta = -2Kx - mg\cos\theta$

For 3 masses held together by strings in an m-M-m configuration:

1. 2 moving in opposite directions, M @ rest, $\omega = \sqrt{\frac{k}{m}}$, like they're attached to a wall.
2. Side masses are in phase, mid mass is out, $\omega_2 = \sqrt{\frac{2k}{m}}$

For 2 masses connected to each other and then connected to 2 walls, all on springs with k-k'-k spring coefficient configuration:

1. In phase $\Delta x_1 = \Delta x_2$ and k' isn't expanded $\omega = \sqrt{\frac{k}{m}}$
2. Out of phase $\Delta x_1 = -\Delta x_2$ CM k' stays in place so k' is split between $m_1/m_2 \rightarrow$ force on each mass $2k'$ since only 1/2 moves $\rightarrow F = k + 2k'$,$\omega = \sqrt{\frac{k+2k'}{m}}$

2.44 Radiation in Atoms

series	K	L	M	N
$n_f =$	1	2	3	4

Specify what the final states are when coming from infinity.

2.45 Ionization Energy

1. E required to liberate outermost e^-

2.46 Binding Energy

1. How tightly bound nucleons are
2. Reaches peak at Iron-56
   Elements below iron release E by fusion
   Elements above iron release E by fission

3. The mass of a nucleus is always <u>less than</u> Σ particle's mass
   The Δ energy is the binding energy
   $c^2(M_{\text{nucleus}} - \Sigma_{\text{nucleons}}) = BE$
4. More tightly bound means less mass/nucleon, more BE/nucleon
5. Created by the strong force
6. The energy given off during fusion/fission is the ΔE between binding energies of fuel and products.

2.47 Hierarchy of Forces

1. Strong (100x E&M, 10^5x weak, 10^{39}x gravity)
2. E&M
3. Weak
4. Gravity

2.48 Pair Production

1. Creation of an elementary particle and it's anti-particle usually from a photon.

2. Cannot occur in free-space since the original momentum of the photon must be absorbed by something
   Usually near a nucleus or other photon
3. For e^- production, the E_{photon} must exceed 2x the rest energy of $e^- = 1$ MeV or if 2 photons involved, $E/\text{photon} = 500$ keV
4. Dominates at high energy ($>$ MeV)
5. Strangeness, momentum, electric charge, must be conserved

2.49 Spectral Lines

1. Less Dense $\rightarrow$ more sharp/precise lines - don't lost E due to collisions
2. Sodium, famous yellow doublet
   created by spin/orbit coupling
   coupling becomes more pronounced in an external B

2.50 Photon Interactions with Matter

1. Low E, elastically scatter $\rightarrow$ Compton $< 10^6$ MeV
2. Med/Low $E \rightarrow$ photoelectric $< 10^7$ MeV
3. High $E \rightarrow$ pair production $> 10^6$ MeV

2.51 Neutron

1. Fermion, spin=$\frac{1}{2}$
2. ${}^1_0n$
3. Decay: ${}^1_0n \rightarrow {}^1_1 p^+ + {}^0_{-1} e + {}^0_0 \bar{\nu}$
4. Capture: ${}^1_1p^+ + {}^0_{-1} e \rightarrow {}^1_0 n + {}^0_0 \bar{\nu}$

2.52 Deuteron

1. Deuterium nucleus: ${}^2_1H \leftrightarrow$ heavy hydrogen
2. Boson

2.53 Protium

1. Hydrogen nucleus ${}^1_1H$
2. Fermion, $s = \frac{1}{2}$

2.54 Davisson-Germer

1. Found diffraction pattern of e^- scattering off N_i
2. Confirmed wave nature of matter
3. Plane spacing $d = D\sin\theta$, $D =$ interatomic spacing
4. Bragg: $2d\sin\theta = n\lambda$

2.55 Kepler

1. $T^2 \propto R^3$
2. $\frac{dA}{dt} \propto L \rightarrow \frac{\text{area swept}}{\text{time}} \propto$ angular momentum

2.56 Beats

1. Beats occur when 2 frequencies are similar
2. The # of beats $\Rightarrow f_b = f_1 - f_2$
3. The harmonic is the index of n or whatever...

$$f_n = \underbrace{n}_{\text{this is the harmonic}} \overbrace{f_0}^{\text{fundamental}} \tag{49}$$