HW #12

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Problem 1:

$$P_{degen} \propto \frac{n^{5/3}}{m} \text{ , } P_{gas} = \frac{\rho kT}{\mu m_p} \tag{1}$$

$$\frac{\rho^{5/3}}{(\mu m_p)^{5/3} m} \propto \frac{\rho kT}{\mu m_p} \tag{2}$$

$$\frac{\rho^{5/3}}{(\mu m_p)^{5/3} m} \propto \frac{MkT}{R^3 \mu m_p} \tag{3}$$

$$\frac{\rho^{5/3}}{(\mu m_p)^{5/3} m} \propto \frac{Mk}{R^3 \mu m_p} \frac{M}{R} \tag{4}$$

$$\left(\frac{M}{R^3}\right)^{5/3} \frac{1}{(\mu m_p)^{5/3} m} \propto \frac{M^2}{R^4} \tag{5}$$

$$\frac{M^{5/3}}{R^5 m} \propto \frac{M^2}{R^4} \tag{6}$$

$$\boxed{\frac{M^{-1/3}}{m} \propto R} \tag{7}$$

Problem 2:

$$p_F = \left(\frac{3n}{8\pi}\right)^{1/3} h \tag{8}$$

$$\epsilon_F(n) \approx m_n c^2 + \frac{p_F(n)^2}{2m_n} \text{ , } \epsilon_F(p) \approx m_p c^2 + \frac{p_F(p)^2}{2m_p} \text{ , } \epsilon_F(e) \approx p_F(e) c \tag{9}$$

$$n_e = n_p \rightarrow \text{ from charge neutrality } \rightarrow \tag{10}$$

$$\frac{1}{2m_p}\left(\frac{3n_p}{8\pi}\right)^{2/3} h^2 + m_p c^2 + \left(\frac{3n_p}{8\pi}\right)^{1/3} hc = \frac{1}{2m_n}\left(\frac{3n_n}{8\pi}\right)^{2/3} h^2 + m_n c^2 \tag{11}$$

$$n_p^{2/3}\frac{h^2}{2m_p}\left(\frac{3}{8\pi}\right)^{2/3} + m_p c^2 + n_p^{1/3}\left(\frac{3}{8\pi}\right)^{1/3} hc = n_n^{2/3}\left(\frac{3}{8\pi}\right)^{2/3}\frac{h^2}{2m_n} + m_n c^2 \tag{12}$$

$$n_p^{2/3}\frac{h^2}{2m_p}\left(\frac{3}{8\pi}\right)^{2/3} + n_p^{1/3}\left(\frac{3}{8\pi}\right)^{1/3} hc = n_n^{2/3}\left(\frac{3}{8\pi}\right)^{2/3}\frac{h^2}{2m_n} \tag{13}$$

$$\text{Um.. Wolfram Alpha isn't happy with this equation} \tag{14}$$

Problem 3a:

$$L_\nu = L_\gamma \tag{15}$$

$$10^{12} T_9^3 e^{-11.9/T_9} \left(\frac{R_c}{R_{WD}}\right)^3 L_\odot = 3\times 10^5 L_\odot \tag{16}$$

$$10^{12} T_9^3 e^{-11.9/T_9} \left(\frac{R_c}{R_{WD}}\right)^3 = 3\times 10^5 \tag{17}$$

Solving for T gives us:

$$\boxed{T \approx 115 \text{ keV}} \tag{18}$$

Problem 3b:

$$t = \frac{U}{L} \tag{19}$$

$$U = \text{ total energy released in fusion of } {}^{20}\text{Ne into } {}^{56}\text{Fe} \tag{20}$$

$$= \underbrace{N_{Fe}}_{\#\text{ of Iron}} \cdot \underbrace{56}_{\text{Nucleons per Iron}} \cdot \underbrace{E_{b,Fe}}_{\text{Binding energy per nucleon}} - \underbrace{N_{Ne}}_{\#\text{ of Neon}} \cdot \underbrace{20}_{\text{Nucleons per Neon}} \cdot \underbrace{E_{b,Ne}}_{\text{Binding energy per nucleon}} \tag{21}$$

We assume that the core starts out as all Neon and all of it is converted into Iron, i.e. the number of nucleons is conserved and $N_{Fe}\cdot 56 = N_{Ne}\cdot 20$!

$$U = N_{\text{Nucleons}}(E_{b,Fe} - E_{b,Ne}) \tag{22}$$

$$= \frac{1M_\odot}{m_p}(8.8 \text{ MeV} - 8.03 \text{ MeV}) \tag{23}$$

$$= 1.47\times 10^{51} \text{ ergs} \tag{24}$$

$$t = \frac{1.47\times 10^{51} \text{ ergs}}{3\times 10^5 L_\odot} \tag{25}$$

$$= 1.29\times 10^{12} \text{ secs} \tag{26}$$

$$\boxed{\approx 1.07\times 10^6 \text{ fortnights}} \tag{27}$$

Problem 4:

$$\gamma +^4\text{He} \rightleftarrows 2n + 2p \tag{28}$$

$$\mu(\gamma) + \mu(^4\text{He}) = 2\mu(n) + 2\mu(p) + \chi \tag{29}$$

$$\mu(\gamma) + m_{\text{He}}c^2 - kT\ln\left(\frac{g_{\text{He}}n_{Q,\text{He}}}{n_{\text{He}}}\right) = m_{2p}c^2 - 2kT\ln\left(\frac{g_p n_{Q,p}}{n_p}\right) + m_{2n}c^2 - 2kT\ln\left(\frac{g_n n_{Q,n}}{n_n}\right) + \chi \tag{30}$$

$$\cancel{\mu(\gamma)} + \cancel{m_{\text{He}}c^2} - kT\ln\left(\frac{g_{\text{He}}n_{Q,\text{He}}}{n_{\text{He}}}\right) = \cancel{m_{2p}c^2} - 2kT\ln\left(\frac{g_p n_{Q,p}}{n_p}\right) + \cancel{m_{2n}c^2} - 2kT\ln\left(\frac{g_n n_{Q,n}}{n_n}\right) + \chi \tag{31}$$

$$-kT\ln\left(\frac{g_{\text{He}}n_{Q,\text{He}}}{n_{\text{He}}}\right) = -2kT\ln\left(\frac{g_p n_{Q,p}}{n_p}\right) - 2kT\ln\left(\frac{g_n n_{Q,n}}{n_n}\right) + \chi \tag{32}$$

$$\ln\left(\frac{g_{\text{He}}n_{Q,\text{He}}}{n_{\text{He}}}\right) = \ln\left(\frac{g_p n_{Q,p}}{n_p}\right)^2 + \ln\left(\frac{g_n n_{Q,n}}{n_n}\right)^2 - \chi/kT \tag{33}$$

$$\left(\frac{g_{\text{He}}n_{Q,\text{He}}}{n_{\text{He}}}\right) = \left(\frac{g_p n_{Q,p}}{n_p}\right)^2 \left(\frac{g_n n_{Q,n}}{n_n}\right)^2 e^{-\chi/kT} \ , n_n = n_p \tag{34}$$

$$\left(\frac{g_{\text{He}}n_{Q,\text{He}}}{n_{\text{He}}}\right) = \left(\frac{g_p n_{Q,p} g_n n_{Q,n}}{n_p^2}\right)^2 e^{-\chi/kT} \tag{35}$$

$$\frac{n_p^4}{n_{\text{He}}} = \left(\frac{g_p n_{Q,p} g_n n_{Q,n}}{g_{\text{He}}n_{Q,\text{He}}}\right)^2 e^{-\chi/kT} \ , \ g\text{'s cancel out} \tag{36}$$

$$\frac{n_p^4}{n_{\text{He}}} = \left(\frac{n_{Q,p} n_{Q,n}}{n_{Q,\text{He}}}\right)^2 e^{-\chi/kT} \tag{37}$$

$$n_{\text{He}} = 2n_n + 2n_p \ , \ \text{since } n_n = n_p, \tag{38}$$

$$= 4n_p \tag{39}$$

$$\frac{n_p^4}{4n_p} = \left(\frac{n_{Q,p}n_{Q,n}}{n_{Q,\mathrm{He}}}\right)^2 e^{-\chi/kT} \tag{40}$$

$$\frac{n_p^3}{4} = \left(\frac{n_{Q,p}n_{Q,n}}{n_{Q,\mathrm{He}}}\right)^2 e^{-\chi/kT} \; , n_Q = \left(\frac{2\pi mkT}{h^2}\right)^{3/2} \tag{41}$$

$$\frac{n_p^3}{4} = \left(\left(\frac{2\pi m_p kT}{h^2}\frac{m_n}{m_{\mathrm{He}}}\right)^{3/2}\right)^2 e^{-\chi/kT} \tag{42}$$

$$\frac{n_p^3}{4} = \left(\left(\frac{2\pi m_p kT}{h^2}\frac{m_p}{4m_p}\right)^{3/2}\right)^2 e^{-\chi/kT} \tag{43}$$

$$\frac{n_p^3}{4} = \left(\left(\frac{\pi m_p kT}{2h^2}\right)^{3/2}\right)^2 e^{-\chi/kT} \tag{44}$$

$$\frac{n_p^3}{4} = \left(\frac{\pi m_p kT}{2h^2}\right)^3 e^{-\chi/kT} \tag{45}$$

$$\frac{n_p^3}{4}\left(\frac{2h^2}{\pi m_p k}\right)^3 = T^3 e^{-\chi/kT} \tag{46}$$

$$\frac{\rho^3}{(\mu m_p)^3 4}\left(\frac{2h^2}{\pi m_p k}\right)^3 = T^3 e^{-\chi/kT} \tag{47}$$

$$\frac{2\rho^3 h^6}{(\mu\pi k)^3 m_p^6} = T^3 e^{-\chi/kT} \; , \text{ solve for } T \text{ with good 'ol Wolfram} \tag{48}$$

$$\boxed{T \approx 4.6 \times 10^{25} K} \tag{49}$$

Problem 5a:

$$L = -\frac{GM^2}{2R^2}\frac{dR}{dt} \tag{50}$$

$$Ldt = -\frac{GM^2}{2R^2}dR \tag{51}$$

$$U = -\int \frac{GM^2}{2R^2}dR \tag{52}$$

$$= -\frac{GM^2}{2}\int \frac{1}{R^2}dR \tag{53}$$

$$= \frac{GM^2}{2}\left(\frac{1}{R}\right) \; , R = 11R_\odot \tag{54}$$

$$\boxed{= 1.04 \times 10^{40} \text{ BTU}_{\mathrm{IT}}} \tag{55}$$

Problem 5b:

$$E_{nuc} = N_{\text{reactions}} \cdot E_{\text{per reaction}} \tag{56}$$

$$= 0.1\frac{M}{\mu 4 m_p}E_{\text{per reaction}} \; , E_{\text{per reaction}} = 26.7 \text{ MeV} \tag{57}$$

$$\boxed{= 1.46 \times 10^{63} \text{ hartrees}} \tag{58}$$

Problem 5c:

$$\begin{aligned} E_{nuc} &= N_{\text{nucleons}} \cdot \Delta E_b && (59) \\ &= 0.1 \frac{M}{\mu m_p}(8.8 \text{ MeV} - 6.7 \text{ MeV}) && (60) \\ &\boxed{= 7.6 \text{ foes}} && (61) \end{aligned}$$

Problem 5d:

$$\begin{aligned} \Delta E &= E_f - E_i && (62) \\ &= \frac{GM_{core}^2}{R_{NS}} - \frac{GM_{core}^2}{R_{core}} && (63) \\ &= \frac{GM_{core}^2}{R_{NS}} \text{ , since } R_{core} \gg R_{NS} && (64) \end{aligned}$$

We took a leap of faith here and are using the educated guesses that $M_{core} \approx M_{NS} \approx 1.4 M_\odot$ and $R_{NS} \approx 10$ km. The former is used because it's the most common NS radius and the latter is used because once again, most NSs are observed at this radii.

$$\boxed{\Delta E = 7.01 \times 10^{43} \text{ hp in 1 second}} \quad (65)$$

Problem 6a:

$$\begin{aligned} v_\nu &= \frac{D}{t_\nu} && (66) \\ &= \frac{D}{t_\gamma - 3 \text{ hours}} && (67) \\ &= \frac{D}{\frac{D}{c} - 3 \text{ hours}} && (68) \\ &\boxed{\approx 2.08 \times 10^8 \text{ overall width of stock 2011 Prius microfortnight}^{-1}} && (69) \end{aligned}$$

Problem 6b:
If by some miracle $\Delta v/c \simeq 2.37 \pm 0.32 \times 10^{-5}$, then:

$$\Delta v \simeq 2.37 \pm 0.32 \times 10^{-5} c \tag{70}$$
$$v_{\nu,\text{``nu''}} - c \simeq 2.37 \pm 0.32 \times 10^{-5} c \tag{71}$$
$$v_{\nu,\text{``nu''}} \simeq 2.37 \pm 0.32 \times 10^{-5} c + c \tag{72}$$
$$v_{\nu,\text{``nu''}} \simeq (2.37 \pm 0.32 \times 10^{-5} + 1)c \tag{73}$$
$$v_{\nu,\text{``nu''}} \approx 3.000071100 \times 10^{10} \text{ cm s}^{-1} \gg v_\nu \tag{74}$$
$$t_{diff} = \frac{D}{c} - \frac{D}{v_{\nu,\text{``nu''}}} \tag{75}$$
$$\boxed{\approx 27.02 \text{ dog years}} \tag{76}$$

Problem 6c:

$$E_1 = mc^2\gamma_1 \tag{77}$$
$$E_1 = \frac{mc^2}{\sqrt{1 - \left(\frac{v_1}{c}\right)^2}} \tag{78}$$
$$v_1 = c\sqrt{1 - \left(\frac{mc^2}{E_1}\right)^2} \tag{79}$$

Repeat for E_2 to get:

$$v_2 = c\sqrt{1 - \left(\frac{mc^2}{E_2}\right)^2} \tag{80}$$

Because the neutrinos arrived within a time difference of 10 seconds,

$$v = \frac{D}{t} \tag{81}$$
$$t = \frac{D}{v} \tag{82}$$
$$\Delta t = \frac{D}{v_2} - \frac{D}{v_1} \tag{83}$$
$$10 \text{ secs} = D\left(\frac{1}{v_2} - \frac{1}{v_1}\right) \tag{84}$$

Solving for m, we get:

$$\boxed{3.67 \times 10^{-11} \text{ Benzoylmethyl ecogine (molecular cocaine)}} \tag{85}$$

Problem 6d:
It's very realistic! It's also very small, like 10 times less massive than an electron.