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1 Picking Up Where We Left Off

$$F = -\frac{4}{3}\frac{caT^3}{\kappa\rho}\frac{dT}{dr} \leftarrow \text{ for radiative diffusion}$$
$$= -\frac{1}{3}lc\frac{dU}{dr}$$

L is strictly NOT dependent on fusion, it is more dependent on $\frac{dU}{dT}$ of the photons. Fusion creates E and the photons but doesn't determine the rate of energy leaving.
χ is constant, but dependent on the composition of the star.

$$L \propto \mu^4\mu_e M^3$$
$$H \to HE$$
$$\mu \to \mu \uparrow$$
$$L \to L \uparrow$$

L was lower in the past. L when the Earth forced was only 20% of the current $L_\odot . This brings about the problem that the Earth$

$$\frac{3}{2}nkT > aT^4$$
$$F = -\frac{1}{3}lv\frac{dU}{dx}$$
$$= -\chi\frac{dT}{dx}, \chi = \frac{1}{3}lv\frac{dU}{dT}$$
$$\frac{F_{rad}}{F_{e^-}} = \frac{-\chi_{rad}}{-\chi_{e^-}}, \frac{dT}{dx} \text{ is the same for both}$$
$$\sim \frac{l_\gamma}{l_{e^-}}\frac{c}{v_{e^-}}\frac{aT^4}{\frac{3}{2}nkT}$$

$\frac{l_\gamma}{l_{e^-}} \ggg 1$, $\frac{c}{v_{e^-}} \gg 1$, and $\frac{aT^4}{\frac{3}{2}nkT} \ll 1$.

$$F = -\frac{4}{3}\frac{caT^4}{n\sigma}\frac{dT}{dr}$$

We want to find the time for thermal energy to leak out by photon diffusion.

$$
\begin{aligned}
t_{KH} &= \frac{E}{L} \\
L &\sim 4\pi R^2 \frac{4}{3} a T^3 \frac{1}{n\sigma} \frac{T}{R} \\
&= \frac{\frac{3}{2} nkT \cdot \frac{4}{3}\pi R^3}{\left(\frac{4 \cdot 16\pi a T^4 R}{R 3 n\sigma}\right)} \\
&= \frac{\frac{3}{2} nkT R^2 n\sigma}{4aT^4 c} \\
&\sim \frac{nkT}{aT^4} \frac{R^2}{lc} \\
&\sim \frac{nkT}{aT^4} t_{RW} \text{ , where } t_{RW} = \frac{R^2}{lc} = \text{random walk time}
\end{aligned}
$$

$$
\begin{aligned}
< |D|^2 > &= Nl^2 \text{ , where } N = \text{number of steps} \\
< |D|^2 >^{1/2} &= \text{ RMS Distance = typical distance a photon will find itself after } N \text{ scatterings} \\
&= \sqrt{N} l
\end{aligned}
$$

What is N so that the photon leaves the star?

$$
< |D|^2 >^{1/2} = R
$$

$$
N \sim \left(\frac{R}{l}\right)^2
$$

$$
\frac{R}{l} \sim 10^{11}, N \sim 10^{22}
$$

How long does it take to get out?

$$
\begin{aligned}
N t_{step} = t_{esc} = N \frac{l}{c} \sim \left(\frac{R}{l}\right)^2 \cdot \frac{l}{c} \sim \frac{R^2}{lc} \\
\sim 10^4 \text{ yeards for our sun}
\end{aligned}
$$

If photons didn't bounce around, it would escape in 2 seconds. (ν_e can get out in about 2 seconds, l_ν must be greater than $R_\odot$) Also, the time it takes for heat to diffuse throughout a room is: $\frac{R^2}{l v_{thm}}$.

$$
\begin{aligned}
F &= -\frac{4}{3}\frac{caT^4}{n\sigma}\frac{dT}{dr} \\
&= -lc\frac{d}{dr}\frac{1}{3}aT^4 \\
F_r &= -lc\frac{d}{dr}P_{rad}\ , \text{ we must be careful, } F \text{ and } L \text{ depend on } r \\
\frac{L_r}{4\pi r^2} &= -lc\frac{d}{dr}P_{rad} \\
-\frac{L_r}{4\pi lcr^2} &= \frac{d}{dr}P_{rad} \\
-\frac{L_r\kappa\rho}{4\pi cr^2} &= \frac{d}{dr}P_{rad}
\end{aligned}
$$

We're interested in how P_{rad} changes with P_{tot}.

$$\frac{dP_{rad}}{dP} = \frac{L_r\kappa}{4\pi GM_r c} \equiv \frac{L_r}{L_{EDD}}$$

Roughly, if $P_{rad} \sim P_{tot}$, then $L_r \sim L_{EDD}$.

1.1 Eddington Luminosity

$F_g = -\frac{GMm}{r^2}$

$F_{rad} = \frac{dp}{dt}$

$p_{proton} = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda}$

What is the total p per unit time produced by the star? $\sum_i^\infty p_i$ is too hard...

$$
\begin{aligned}
p_{photon} &= \frac{E}{c} = \frac{L}{c} \\
F_{Rad} &= \frac{dp}{dt} = \frac{L}{c4\pi r^2}\sigma
\end{aligned}
$$

The Eddington Luminosity is where the radiation force equals the forge of gravity. The Eddington "Limit" is:

$$L_{EDD} = \frac{4\pi GMc}{\sigma/m} = \frac{4\pi GMc}{\kappa}$$

If $L > L_{EDD}$, $F_{rad} > F_{grav}$, and material is "blown" out.

1.2 Fully Ionized H

$L_{EDD} = \frac{4\pi GMc}{\kappa}$

$\kappa = \frac{\sigma_T}{m}$

$l = \frac{1}{n\sigma} = \frac{1}{n_e\sigma_T} = \frac{1}{\kappa\sigma}$

For fully ionized H, $\mu_e = 1$

since photons are only interacting with e^- and not p, $\mu = 1 = \mu_e$

$$\begin{aligned}\kappa &= \frac{\sigma_T}{m} \\ &= 0.4 \text{ cm}^2 \text{ g}^{-1}\end{aligned}$$

$$L_{EDD} = 1.3 \times 10^{38} \left(\frac{M}{M_\odot}\right) \text{ ergs s}^{-1}$$

If $L \sim L_{EDD}$, $F_{rad} \sim F_{grav}$ which results in the radiation force not being important in the sun. It's dominated by gas pressure.

As $M \uparrow, L_{EDD} \uparrow$ so higher M means P_{rad} becomes more important $\rightarrow$ it becomes the dominant force