

DOOMED DICE PROBLEM:

PART A

1. Each face of Die A can be paired with each face of Die B. So in order to get the total number of combinations, We just need to multiply the number of faces.

Hence,

total combinations = number of faces on Die A * number of faces on Die B = $6 * 6 = 36$

2. In order to represent the distribution of all possible combination, we can use a $6 * 6$ matrix.

Where (i,j)th entry will represent the combination sum obtained by rolling Die A with value i and rolling Die B with value j.

3. To find the probability of a particular combination of two dice values, We can divide the number of favourable cases by the total number of combinations, that is 36.

For example, probability of getting a sum of 3 is $2/36$ since the number of favourable combinations are (1,2) and (2,1) only.

PART B

For making new dices with same probabilities for all possible sums as in the case of normal dice throw, we first create a matrix of probable sums where (i,j)th entry corresponds to the sum when i is shown up in die A and j is shown up in die B. The matrix for the newly crafted Dice would contain same frequency of sums even if their positions may vary.

The original matrix will have one instance of the sum 2. So new matrix should also have a single entry for the sum 2. That causes each of the newly created dice to contain exactly one 1 each.

Same is the case with the sum 12. But here, we have a restriction since the first die can only have a maximum of 4 spots in a face. This translates to the possibility for die B to have a side with 8 spots. Proceeding in this fashion, We can find all the possibilities.

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Frequencies of the sums should be preserved in the new matrix in order to get same probability. That means, new matrix should contain,

Exactly one 2, exactly two 3 and so on.

We can fill in the possibilities using this information:

	1	2	2	3	3	4
1	2					5
3						
4						
5						
6						
8	9					12

Proceeding in this fashion, the final matrix would be,

	1	2	2	3	3	4
1	2	3	3	4	4	5
3	4	5	5	6	6	7
4	5	6	6	7	7	8
5	6	7	7	8	8	9
6	7	8	8	9	9	10
8	9	10	10	11	11	12

Hence the final result would be the dice with the following configuration:

Die 1: [1, 2, 2, 3, 3, 4]

Die 2: [1, 3, 4, 5, 6, 8]