In[214]:=

⋮

Computing for Mathematical Physics 2022/23

Homework1

Mark for homework1: /40

(to be competed by your marker)

Feedback from marker:

(to be competed by your marker)

Give your answers in the code cells marked (* Enter your solution here *)

- 1. Integrate Sin[t] × Exp[−a t] with respect to t, from 0 to ∞. Use `Assumptions` to specify that the parameter a should be assumed to be greater than 0 when performing the integral.
  [2 marks]

In[215]:=

```
Integrate[ⅇ^(-a t) Sin[t], {t, 0, Infinity}, Assumptions → a > 0]
```

- 2. Plot a single graph showing the following four functions, superimposed, each with a different colour:
  a) 1-Exp[-x],
  b) $x - \frac{x^2}{2}$,
  c) $x - \frac{x^2}{2} + \frac{x^3}{6}$,
  d) $x - \frac{x^2}{2} + \frac{x^3}{6} - \frac{x^4}{24}$
  Label the axes with x and y, and add a plot legend to your graph, which labels each line according to its corresponding expression. Consult the local or online Wolfram documentation on `Plot`, `AxesLabel` and `PlotLegends` as need be.

[6 marks]

In[216]:=

```
Plot[{1 - e^-x, x - x^2/2, x - x^2/2 + x^3/6, x - x^2/2 + x^3/6 + x^4/24},
 {x, 0, 3}, AxesLabel → {x, y}, PlotLegends → Automatic]
```

In[217]:=

- 3. Solve the following set of two simultaneous equations for x and y
  - $3x + 2y - 20 + a == 0$
  - $4x - y + 8 + a^2 == 0$

  Your solutions will be in terms of a.
  [4 marks]

In[218]:=

```
eq2 = 3 x + 2 y - 20 + a == 0
eq2 = 4 x - y + 8 + a^2 == 0
sol = Solve[{eq2, eq2}, {a}]
```

- 4. Solve the equation $a x^5 + b x^3 + c x == 0$.
  [2 marks]

In[221]:=

```
eq = a x^5 + b x ^3 + c == 0
Solve[eq, x]
```

- 5. Series expand $\frac{1}{1-\text{Cos}[x]}$ in powers of x up to and including terms of order x^2, about $x = 0$.
  [2 marks]

In[223]:=

```
Series[1 / 1 - Cos[x], {x, 0, 2}]
```

- 6. Find the limit of the function, $\frac{x}{\text{Abs}[x]}$ $\text{Exp}[-x^2]$, as $x \to 0$, from above and from below.
  [2 marks]

In[224]:=

```
Limit[x Exp[-x^2] / Abs[x], x → 0, Direction → 1]
Limit[x Exp[-x^2] / Abs[x], x → 0, Direction → -1]
```

- 7. Integrate $(x-2)/(x^3 - 11x^2 + 38x - 40)$ with respect to x. Then return to the integrand and partial fraction it using `Apart`, into terms whose denominators are linear in x. Integrate the latter new expression for the integrand with respect to x. Test that the two integrals, so obtained, are identical using == .
  [6 marks]

In[226]:=

```
int1 = Integrate[(x - 2) / (x^3 - 11 x^2 + 38 x - 40), x]
ap1 = Apart[(x - 2) / (x^3 - 11 x^2 + 38 x - 40)]
int2 = Integrate[ap1, x]
int1 == int2
```

In[230]:=

- 8. Solving a differential equation.

a) Solve the differential equation $\frac{d^2}{dx^2}y[x] - 2x\frac{d}{dx}y[x] + \lambda y[x] = 0$ for $y[x]$. Your solution for $y[x]$ should be of the form $y[x] \to C[1] \times f[x] + C[2] \times g[x]$, wherein $f[x]$ and $g[x]$ comprise of combinations of special functions which you may not have met before, and two constant coefficients, $C[1]$ and $C[2]$.

- Note 1: λ can be entered by typing ESC l ESC, but writing `lambda` instead of λ is acceptable here.
- Note 2: the constant coefficients, $C[1]$ and $C[2]$, may be displayed by *Mathematica* as c_1 and c_2.

[4 marks]

In[231]:=

```
DSolve[D[y[x], {x, 2}] - 2 x D[y[x], x] + λ y[x] == 0, y[x], x]
```

- b) Extract the $f[x]$ and $g[x]$ functions from your $y[x]$ solution, manually or otherwise, calling them `f` and `g` respectively. Set λ=4 and hence plot `f` and `g` over the range $-1 \leq x \leq 1$. Label the axes of your plots, giving each one a clear and useful title (see https://reference.wolfram.com/language/ref/PlotLabel.html).

  [8 marks]

In[232]:=

```
f = HermiteH[λ / 2, x];
g = Hypergeometric1F1[-λ / 4, 1 / 2, x^2];
λ = 4;
Plot[f, {x, -1, 1}, AxesLabel → {x, y}, PlotLabel → hermite polynomial ]
Plot[g, {x, -1, 1}, AxesLabel → {x, y}, PlotLabel → hypergeometric ]
```

In[237]:=

- c) Find the limits of the $f[x]$ and $g[x]$ functions as $x \to 0$.

  [2 marks]

In[238]:=

```
Limit[f, x → 0]
Limit[g, x → 0]
```

- d) If $C[1] = 1$ what value must $C[2]$ take in order that the solution to the differential equation for λ=4 satisfies $y[0] = 0$? Explain your reasoning, *clearly*, in one sentence, in a text cell.
  - Note: to start a text cell, click the space after a cell, or between two cells, and press ALT+7 on Windows, ⌘+7 on MacOS, or select Format→Style→Text from the drop-down menus.

  [2 marks]

For the equation $C[1]$ x f[x] + C[2] x g[x] = 0, when x tends towards 0, the equation becomes 1 x -2 + C2 x 1 = 0, which simplifies to -2 + C2 = 0. Therefore C2 = 2

In[240]:=

In[241]:=

- **Total marks available: 40**

- **Solutions are due *by* 1200 noon on Thursday January 19th *here*: allow time for uploading on moodle.**
- A 10% mark deduction will be made (4 marks) if the template isn’t used.
- Name your solution notebook file in the format **WK1_HMWK_<Initials>_<Family Name>.nb**, e.g. **WK1_HMWK_K_Hamilton.nb**
- Make a *backup copy* of your solutions.
- Delete all cell evaluation output by selecting **Cell → Delete All Output** from the drop-down menus at the top of the screen, then save and upload *that* file to Moodle.
- The first thing your marker will do when they receive your notebook is to evaluate all of it, to regenerate the output, by clicking **Evaluation → Evaluate Notebook** from the drop-down menus at the top of the screen. *It is your responsibility to check that carrying out this process will produce the output you intend it to, before you upload your work.*

K. Hamilton

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