Computing for Mathematical Physics 2022/23

Homework4

Mark for homework4: 38/42
(to be competed by your marker)

Feedback from marker:
(to be competed by your marker)

Q1 16/20: Marks lost here were for not labelling axes in plots, which you can easily do (and you may well need these in e.g. an exam). Please see my comments under the various bits of this question for more info and help. In particular, make sure you are also comfortable ending Which[...] functions with a if-all-the-above-fails condition + return value, of the form "..., True, BlahBlah] " (See my alternative to your hitQ in Q1b for an example of this.)

Q2 8/8 Good. Solid. Please see my comments under Q2a though, in relation to Which[...] again.

Q3 10/10 Good effort on a question that tripped a lot of people up. Very few people I marked got this right. See my comment under Q3c for a slightly simpler version of the code there (your code is nevertheless perfectly correct though).

Q4 4/4 Nice to see this one come out confidently.

Give your answers in the code cells marked (* Enter your solution here *)

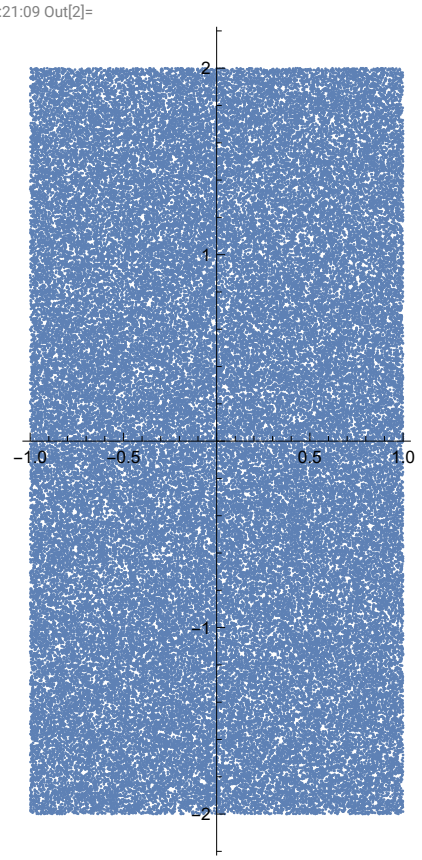
Double click the vertical braces on the RHS of each of the question headings to open and view them.

- 1. Monte Carlo integration procedure to estimate π. [20 marks]
  - a) Use `Table` and `RandomReal` to make a list of 100,000 points, called, `shots`, each one in the form of a two-element list `{x, y}`, with `x` values in the range `-1 ≤ x ≤ 1` and `y` values in `-2 ≤ y ≤ 2`. *You **must** use a semi-colon to terminate the `Table` command, to avoid filling the screen with 200,000 random numbers.* Use `ListPlot` with option `AspectRatio→Automatic`, together with any other due formatting, to display the points.
    [6 marks]

2/23/23 22:21:08 In[1]:=

```
(*Create large list containing two inner lists*)
shots = Table[{RandomReal[{-1, 1}], RandomReal[{-2, 2}]}, 100 000];
ListPlot[shots, AspectRatio → Automatic]
```

2/23/23 22:21:09 Out[2]=



Q1a 4/6

- *Got the basics right.*
- *Forgot to label axes though — question asks for 'any other due formatting'.*

■ b) Write a function `hitQ`, using `Which`, that *specifically* takes a two element list, `{x, y}`, as input. If $x^2 + y^2 \leq 1$, the function should return `True`, and `False` otherwise.
[4 marks]

2/23/23 22:21:09 In[3]:=

```
(*Create delayed function whicb checks if point lies in the circle *)
hitQ[x_ ] := Which[x[[1]]^2 + x[[2]]^2 ≤ 1, True, x[[1]]^2 + x[[2]]^2 > 1, False ]
```

Q1b 4/4

- *Good. Correct.*
- *The following would also work.*

```
hitQ[{x_, y_}] :=
 Which[
  (* If x^2+y^2≤1 we are expected to return True.     *)
  x^2 + y^2 ≤ 1, True,
  (* If none of the other conditions above return    *)
  (* True, i.e. if x^2+y^2>1 we return False.        *)
  True, False
 ]
```

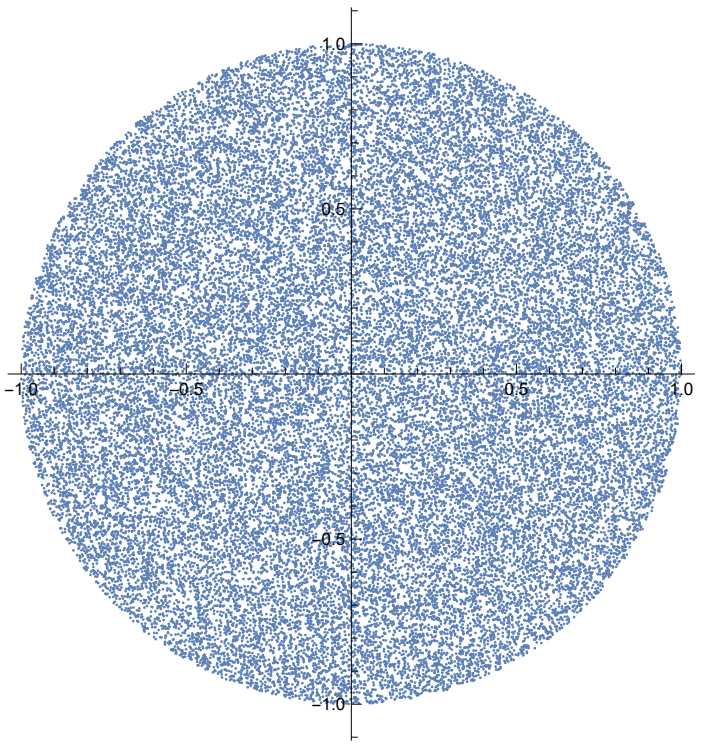
• *Basically the Which[...] function checks each conditional inside it, in order, one at a time, until it finds one that evaluates to True, and when it does, it returns whatever is on the RHS of* ***that*** *conditional as the answer. So, by having a "True" sitting right at the end of each Which[...] command we are basically saying, "this last thing at the very end is what we should return if* ***all*** *conditionals above it have failed". I.e. the last " Which[..., ..., True, Blah] " bit of any Which[...] command is like a final "if all else fails return Blah" catch. Ask me in class if you'd like a face-to-face explanation of this. It's important that you are comfortable with how Which[...] works.*

- c) Use `Select` with `hitQ` to generate a new list called `hitList`, comprised of those points inside `shots` with $x^2 + y^2 \leq 1$. *You* ***must*** *use a semi-colon to terminate the `Table` command, to avoid filling the screen with a few 10,000 random numbers.* Use `ListPlot` with option `AspectRatio→Automatic`, together with any other due formatting, to view the `hitList` points.
  [5 marks]

2/23/23 22:21:09 In[4]:=

```
hitList = Select[shots, hitQ];
ListPlot[hitList, AspectRatio → Automatic]
```

2/23/23 22:21:11 Out[5]=



Q1c 3/5

- *Lost marks for not labelling any axes.*

- d) Compute the *fraction* of points inside `shots` with $x^2 + y^2 \leq 1$, `hitFraction`. [1 mark]

2/23/23 22:21:11 In[6]:=

```
hitFraction = Length[hitList] / Length[shots]
```

2/23/23 22:21:11 Out[6]=

$$\frac{39\,109}{100\,000}$$

Q1d 1/1

- *Good.*

- e) By considering `hitFraction` together with the initial area covered with the points in `shots`, determine a numerical estimate for π. Display your estimate to 5 s.f. For full marks in this part, you will need to list your logic, clearly, in 2-3 short sentences, in a text cell. [4 marks]

Area of a circle is πr^2 and area covered by the whole graph is 2r * 4r = 8 r^2. So fraction of area of circle to area of whole graph is $\pi r^2 / 8 r^2 = \pi / 8$. So we can approximate the value of π by multiplying the ration of the area of the circle to the area of the whole graph by 8.

2/23/23 22:21:11 In[7]:=

```
N[8 * hitFraction, 5]
```

2/23/23 22:21:11 Out[7]=

3.1287

Q1e 4/4

• Right answer.

2/23/23 22:21:11 In[8]:=

■ 2. Recursive function. [8 marks]

■ Let the function $P_n(x)$ be defined by the recursion relation:

$$n\,P_n(x) \;=\; (2n-1)\,x\,P_{n-1}(x) \;-\; (n-1)\,P_{n-2}(x)$$

with $P_0(x) = 1,\; P_1(x) = x$.

■ a) Implement the function $P_n(x)$ as a recursive function called `PA[n_,x_]` using `Which`.

2/23/23 22:21:11 In[9]:=

```
(*Create function which has conditions
 for the first two numbers of the series*)
PA[n_, x_] := Which [ n == 0, 1, n == 1, x,
  n > 1, ((2 n - 1) x PA[n - 1, x] - (n - 1) PA[n - 2, x]) / n]
```

■ b) Implement the function $P_n(x)$ again, now as `PB[n_,x_]`, *without* using `If` or `Which`. Instead, carry out the implementation by providing three separate (overloaded) definitions for `PB[n_,x_]`:
i) a definition specific to the `n=0` case, `PB[0,x_]`,
ii) a definition specific to the `n=1` case, `PB[1,x_]`,
iii) a definition for all other general `n` values, `PB[n_,x_]`.

2/23/23 22:21:11 In[10]:=

```
(*Create normal function, then overload it*)
PB[n_, x_] := ((2 n - 1) x PB[n - 1, x] - (n - 1) PB[n - 2, x]) / n
PB[0, x_] = 1;
PB[1, x_] := x
```

■ c) Use `Simplify` together with the equal operator, ==, to test whether `PA[15,x]` and `PB[15,x]` equal *Mathematica's* internal `LegendreP[15,x]` function. [8 marks]

2/23/23 22:21:11 In[13]:=

```
Simplify[PA[15, x ]] == Simplify[LegendreP[15, x]]
(*Simplify both sides so that tehy are in the exact same format*)
Simplify[PB[15, x]] == Simplify[LegendreP[15, x]]
```

2/23/23 22:21:11 Out[13]=

True

2/23/23 22:21:11 Out[14]=

True

Nice.

Q2a 3/3

• Good. Perfectly correct.

• Again note, however, that instead of the last " ... n>1, ((2n-1) x PA[n-1, x]-(n-1)PA[n-2, x])/n] " in the Which[...] you could have had instead " ... True, ((2n-1) x PA[n-1, x]-(n-1)PA[n-2, x])/n] " . It would be good if you could convince yourself, and get comfortable, with the latter way of doing things too, as it effectively amounts to saying "if none of the above were True, do this" , which is a very useful construct, that can come up in a lot of situations. In this particular case of Q2a, this means, make sure you are also comfortable with this minor modification of your answer:

PA[n_, x_]:= Which [n== 0, 1, n ==1, x, True, ((2n-1) x PA[n-1, x]-(n-1)PA[n-2, x])/n]

Q2b 3/3

• Nice. Solid.

Q2c 2/2

• Good.

• Would be slightly safer, in future questions, to wrap the whole expression inside Simplify[...] as in

Simplify[PA[15,x]==LegendreP[15,x]]

Simplify [PB[15,x]==LegendreP[15,x]]

instead of what you have above.

3. List manipulation with user defined functions. [10 marks]

- For this question you will need to evaluate the following code cell to store the list `marks` in memory, corresponding to fictitious student exam marks.

2/23/23 22:22:29 In[15]:=

```
marks = {
    {1, "Rishi", "Sunak", 13},
    {2, "Donald", "Trump", 1},
    {3, "Donald", "Trump", 0},
    {3, "Rishi", "Sunak", 17},
    {1, "Liz", "Truss", 4},
    {1, "Joe", "Biden", 16},
    {2, "Joe", "Biden", 14},
    {1, "Barak", "Obama", 22},
    {1, "Theresa", "May", 17},
    {2, "Liz", "Truss", 1},
    {1, "Boris", "Johnson"},
    {2, "Theresa", "May", 11},
    {2, "Boris", "Johnson"},
    {2, "Barak", "Obama", 25}
   };
```

- Each entry in the `marks` list has the form

  `{test number, first name, family name, mark}`

  or, if a mark is missing,

  `{test number, first name, family name}` .

- a) Write a function `consolidate[data_]`, using `Map` and `Rest`, capable of taking in the list `marks` and removing the test number from the start of each entry. Apply your `consolidate` function to the list `marks`.
  [2 marks]

2/23/23 22:22:32 In[16]:=

```
(*Rest function removes first part of a list so apply to each part of
 the smaller lists. We don't need to specify level here as the map
 function already chooses the lowest level which is what we want*)
consolidate[data_] := Map[Rest, data]
consolidate[ marks]
```

2/23/23 22:22:32 Out[17]=

{{Rishi, Sunak, 13}, {Donald, Trump, 1}, {Donald, Trump, 0}, {Rishi, Sunak, 17},
{Liz, Truss, 4}, {Joe, Biden, 16}, {Joe, Biden, 14}, {Barak, Obama, 22}, {Theresa, May, 17},
{Liz, Truss, 1}, {Boris, Johnson}, {Theresa, May, 11}, {Boris, Johnson}, {Barak, Obama, 25}}

Q3a 2/2

- *Good!*

- b) Extend your `consolidate[data_]` function so as to additionally apply a *replacement rule* to its output before it is returned. The replacement rule should act to *consolidate* the marks for each student, so that each student entry in the final

output list is of the form

```
{first name, family name, test1_mark, test2_mark,...}
```

I.e., the final output list should have only one member per student.

Each member of the final output list should only contain test marks for tests attempted by the student named in that member, i.e., each member may have a different length.

Apply this modified `consolidate` function to the list `marks`, and store the output as `consolidatedMarks`.
[4 marks]

2/23/23 22:23:07 In[19]:=

```
consolidateRule = {a___, {b_, c_, d___}, e___, {b_, c_, f___}, g___} →
   {a, {b, c, d, f}, e, g};
consolidatedMarks = consolidate[marks] //. consolidateRule
```

2/23/23 22:23:07 Out[20]=

{{Rishi, Sunak, 13, 17}, {Donald, Trump, 1, 0}, {Liz, Truss, 4, 1},
 {Joe, Biden, 16, 14}, {Barak, Obama, 22, 25}, {Theresa, May, 17, 11}, {Boris, Johnson}}

Q3b 4/4

- *Nice to see you get this.*

■ c) Define a function `orderedFirstNamesQ[member1_,member2_]` designed to take two members from the `consolidatedMarks` list as input. This function should, itself, use `OrderedQ` to determine whether `member1` and `member2` are input in alphabetical order, according to the first name contained within each one. If `member1` and `member2` have been input to the function in alphabetical order, as determined by examining the first name in each input member, `orderedFirstNamesQ` should return `True`, otherwise return `False`.

Use `Sort` together with the `orderedFirstNamesQ` function to rearrange the members of `consolidatedMarks` such that their first names appear in alphabetical order.
[4 marks]

2/23/23 22:23:58 In[22]:=

```
(*function takes in two names and determines
 whether the first names are in alphabetical order*)
orderedFirstNamesQ[x_, y_] := If[OrderedQ[{x[[1]], y[[1]]}], True, False]
Sort[consolidatedMarks, orderedFirstNamesQ]
```

2/23/23 22:23:58 Out[23]=

{{Barak, Obama, 22, 25}, {Boris, Johnson}, {Donald, Trump, 1, 0},
 {Joe, Biden, 16, 14}, {Liz, Truss, 4, 1}, {Rishi, Sunak, 13, 17}, {Theresa, May, 17, 11}}

Q3c 4/4

- *Very well done.*

• *Note that since OrderedQ is a function that by itself returns just True or False, you could have just had*
OrderedQ[{x[[1]],y[[1]]}]
instead of
If[OrderedQ[{x[[1]],y[[1]]}], True,False]

- 4. Pure functions. [4 marks]
  - In this question we seek to compute a very basic numerical ***approximation*** to the integral of the function x^2 Cos[x]^2 over the range $0 \le x \le 3.5\,\pi$, based on approximating the area under the function in this range by a number of thin rectangles, each of the same width.
  - *Set* `rectangleWidths=0.00001;`
  - *Set* `rectangleUpperEdges=Range[rectangleWidths,3.5π,rectangleWidths];`
  - a) Create a list of values, called `rectangleHeights`, by passing `rectangleUpperEdges` to `Map`, together with a ***pure function*** representation of x^2 Cos[x]^2. [I.e. if the `i'th` element in `rectangleUpperEdges` has a value, say, x_i, then the `i'th` element of `rectangleHeights` should have a value x_i^2 `Cos[`x_i`]`^{2}.]
    [2 marks]

2/23/23 22:26:54 In[26]:=
```
rectangleWidths = 0.00001;
rectangleUpperEdges = Range[rectangleWidths, 3.5 π, rectangleWidths];
```

2/23/23 22:26:56 In[28]:=
```
rectangleHeights = Map[#^2 Cos[#]^2 &, rectangleUpperEdges];
```

  - b) Create a list called `rectangleAreas`, by passing `rectangleHeights` to `Map`, together with a suitably defined ***pure function***, to multiply each element in `rectangleHeights` by the constant `rectangleWidths`.
    [1 mark]

2/23/23 22:27:00 In[29]:=
```
rectangleAreas = Map[rectangleWidths # &, rectangleHeights];
```

  - c) Compute the sum of the elements in `rectangleAreas`, and compare it to what one obtains when one integrates x^2 Cos[x]^2 over $0 \le x \le 3.5\,\pi$ using *Mathematica*'s native `Integrate` function.
    [1 mark]

2/23/23 22:27:02 In[30]:=
```
N[Total[rectangleAreas], 10]
N[Integrate[x^2 Cos[x]^2, {x, 0, 3.5 π}], 10]
```

2/23/23 22:27:02 Out[30]=
218.817

2/23/23 22:27:03 Out[31]=
218.817

Answers are exactly the same, no further precision past 6 significant figures.

Q4 4/4

• *Excellent. Well done!*

- **Total marks available: 42**

- **Solutions are due *by* 1200 noon on Thursday February 9th *here*: allow time for uploading on moodle.**
- A 10% mark deduction will be made (4 marks) if the template isn't used.
- Name your solution notebook file in the format **WK4_HMWK_<Initials>_<Family Name>.nb**, e.g. **WK4_HMWK_K_Hamilton.nb**
- Make a *backup copy* of your solutions.
- Delete all cell evaluation output by selecting **Cell → Delete All Output** from the drop-down menus at the top of the screen, then save and upload *that* file to Moodle.
- The first thing your marker will do when they receive your notebook is to evaluate all of it, to regenerate the output, by clicking **Evaluation → Evaluate Notebook** from the drop-down menus at the top of the screen. *It is your responsibility to check that carrying out this process will produce the output you intend it to, before you upload your work.*

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