Unit - 2

* Propositional Logic :-

→ Formal Logic :-

* [Proposition] → Statement

- A Proposition is a declarative Sentence which is either true / false
↳ this is truth value.

Eg:- i) $1+1=2$ ✓ True

(ii) $9<6$ ✗ False

(iii) London is in denmark ✗ False

(iv) Paris is in France ✓ True

(v) where are you going ? → questioning Sentence

(vi) sit down → Command Sentence

∴ this kind of statements doesn't come under proposition only the normal Sentence is proposition.

* Compound Proposition :-

When one or more Propositions are Connected through Various Connectivies (and / OR) it is called Compound Proposition.

Eg:- Roses are Red & violets are blue

Roses are Red → Proposition 1

& → Connectivity

violets are blue → Proposition 2

→ A proposition is Said to be primitive, if it Cannot be broken down into Similar propositions.
↳ a single proposition, Cannot be broken.

Eg:- Roses are Red → Unbreakable.

Basic Logical Operations :-

① Conjunction → AND ($\wedge$)
② Disjunction → OR ($\vee$)
③ Negation → NOT ($\sim$)

} most Used.

④ NAND → NOT + AND
⑤ NOR → NOT + OR
⑥ EXOR →
⑦ EXNOR →

} Not Used in mathematics by used in digital electronics.

* Conjunction (AND "$\wedge$") :-

Any two Propositions Can be Connected by the word "AND" to form a Compound proposition Called Conjunction.

let, p & q ⇒ two Propositions

$(p \wedge q) \rightarrow$ Conjunction

Truth table :- 4, every Proposition is having truth value, so '2' Proposition it is 4 values.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

② **Disjunction:-**

Any two Proposition is Combined by the word "OR" is Called disjunction.

ie (p ∨ q) ⟶ disjunction.

Truth Table:- → if any one value is true then, the o/p is true

p	q	p∨q
T	T	T
T	F	T
F	T	T
F	F	F

③ **Negation (~):-**

A single proposition which value can be changed by using negation.

~p , ~q

p	~p
T	F
F	T

q	~q
T	F
F	T

* **Tautologies & Contradictions:-**

1. Tautology → Truth values are true, for any truth value of variable.
2. Contradiction → Truth values are false, for any truth value of variables.
3. Contigency → Some truth values are true & some are false.

Eg:- (i) p ∨ Q (ii) p ∧ ~p (iii) ~(p ∧ q)

p	Q	p∨Q
T	T	T
T	F	T
F	T	T
F	F	F

Contigency

p	~p	p∧~p
T	F	F
F	T	F

Contradiction

p∨~p

p	~p	p∨~p
T	F	T
F	T	T

Tautology

p	q	p∧q	~(p∧q)
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

Contradiction.

→ Logical Equivalence :-

Two propositions are said to be logically equivalent (or) equal, if they have same truth values

denoted by, $P \equiv Q$ (if P & Q are two propositions)

Eg:- $\sim(p \vee q)$, $\sim p \wedge \sim q$

p	q	$\sim p$	$\sim q$	$(p \vee q)$	$\sim p \wedge \sim q$	$\sim(p \vee q)$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

$\therefore \sim(p \vee q) \equiv \sim p \wedge \sim q$

→ Conditional Statement :-

Statements is in the form "if p then q" i.e., $p \rightarrow q$

Truth table :-

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

→ Biconditional statement :-

Statements is in the form "p if and only if q" i.e., $p \leftrightarrow q$

Truth table

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

* Laws of algebraic Propositions :-

① Idempotent law ⇒ $P \vee P \equiv P$, $P \wedge P \Leftrightarrow P$

② Associative law ⇒ $(p \vee q) \vee r \equiv p \vee (q \vee r)$
$(p \wedge q) \wedge r = p \wedge (q \wedge r)$

③ Commutative law ⇒ $p \vee q \equiv q \vee p$, $p \wedge q = q \wedge p$

④ Distributive law ⇒ $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

⑤ identity / identical law ⇒ $P \vee T \equiv T$, $P \wedge T \equiv P$

⑥ Complement ⇒ $P \vee \sim P \equiv T$, $\sim T = F$

⑦ Involution ⇒ $\sim\sim P \equiv P$

⑧ *** Demorgan's law ⇒ $\sim(p \vee q) \equiv \sim p \wedge \sim q$ / $\sim(p \wedge q) \equiv \sim p \vee \sim q$

p	q	r
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

* Problems

① Propositions & Logical Operations :-

P = It is cold, q = it is raining.

write the verbal Sentence for the given statements.

ⓐ $\sim p$, ⓑ $p \wedge q$ ⓒ $p \vee q$ ⓓ $q \vee \sim p$.

② Truth Values & Truth tables.

ⓐ $\underset{F}{4+2=5}$ and ($\wedge$) $\underset{T}{6+3=9}$ = F

ⓑ $3+2=5$ and $4+7=11$
$T \wedge T = T$

ⓒ $3+2=5$ or $6+1=7$
$T \vee T = T$

③ Find the truth table of $\sim p \wedge q$. ⇒ F, F, T, F

④ Verify the Proposition $p \vee \sim(p \wedge q)$ is tautology → yes.

⑤ Show that $\sim(p \wedge q)$ & $\sim p \vee q$ are logically equivalent. → not equal
↳ $\sim(p \wedge q)$ = F, T, T, T
$\sim p \vee q$ = F, F, T, F

⑥ $p \wedge (\sim q \vee r)$ is tautology / contradiction / Contigency? yes.
8 Combination

$p, q \rightarrow 2^2 = 4$, $p, q, r = 2^3 = 8$

* Consider a proposition having, $P \to Q$, convert them into inverse, converse, contrapositive.

Formulas:- Inverse $\Rightarrow \sim p \to \sim q$

Converse $\to Q \to P$

Contrapositive $\Rightarrow \sim Q \to \sim P$

Eg:- ① If I study hard, I shall succeed.

p = I study hard, q = I shall succeed.

$\therefore p \to q$ (given)

Inverse:- $\sim p \to \sim q \Rightarrow$ if I donot study hard then I shall not succeed.

Converse:- $Q \to P \Rightarrow$ if I shall succeed then I study hard.

Contrapositive:- $\sim Q \to \sim P \Rightarrow$ if I shall not succeed the I don't study hard.

② if $\underbrace{\text{it rains}}_{p}$ then $\underbrace{\text{I get wet}}_{q}$

Inverse:- if it doesn't rain then I don't get wet

Converse:- if i get wet then it rains.

C.P:- if i don't get wet then it doesn't rain.

③ if $\underbrace{\text{I tell the truth}}_{p}$, then $\underbrace{\text{gold exists}}_{q}$?

Inverse: $\sim p \to \sim q$

Converse: $q \to p$

C.P:- $\sim q \to \sim p$

* Predicates [Property that the subject of statement have]

→ Two/more statements have some features common then we take the common feature as Predicate.

Eg:- $B(x)$ = x is a bechlor } common ∴ it is predicate

x = John → John is a bechlor.

→ Here x is a place holder until it is replaced by name of the object, it will be a statement after that,

→ Predicate can have n- no. of place holder.

$T(x,y) = x$ is taller than y.

$S(x,y,z) = x$ sits in b/w y & z.

Eg:- $a=4, b=6$

let, $\underset{a}{4}$ is less than $\underset{c}{5}$; $a<c$ → this relationship is taken as $Q(a,c)$

$\underset{b}{6}$ is greater than $\underset{a}{4}$; $b>a$ → $Q(b,a)$

} this two are called predicates i.e., 2 place holder predicate

* Rules of Inference:-

① $P \wedge Q \Rightarrow P$ (or) $P \wedge Q \Rightarrow Q$ [Simplification]

② $P \Rightarrow P \vee Q$ (or) $Q \Rightarrow P \vee Q$ [Addition]

③ $\sim P \Rightarrow P \rightarrow Q$

④ $Q \Rightarrow P \rightarrow Q$

⑤ $\sim(P \rightarrow Q) \Rightarrow P$

⑥ $\sim(P \rightarrow Q) \Rightarrow \sim Q$

⑦ $P, Q \Rightarrow P \wedge Q$ [Conjunction]

⑧ $P \vee Q, \sim P \Rightarrow Q$ [Disjunctive Syllogism]

⑨ $\sim(P \wedge Q), P \Rightarrow \sim Q$ [Conjunctive Syllogism]

(10) $P \rightarrow Q,\ P \Rightarrow Q$ [modus pones]

(11) $P \rightarrow Q,\ \sim Q \Rightarrow \sim P$ [modus tollens/contrapositive]

(12) $P \rightarrow Q,\ Q \rightarrow R \Rightarrow P \rightarrow R$ [hypothetical Syllogism/transitive law]

(13) $(P \rightarrow Q) \wedge (R \rightarrow S),\ P \vee R \Rightarrow Q \vee S$

(14) $p \vee q \Rightarrow \sim p \rightarrow q$

Proof:-

① $(r \rightarrow s) \wedge (p \rightarrow q) \wedge (r \vee p) \longrightarrow s \vee q$

$r \rightarrow s$

$\left.\begin{array}{l} p \rightarrow q \\ r \vee p \Rightarrow \sim r \rightarrow p \end{array}\right] \Rightarrow \left.\begin{array}{l} \sim r \rightarrow q \\ r \rightarrow s \end{array}\right] \Longrightarrow \begin{array}{l} r \rightarrow \sim q \\ r \rightarrow s \\ \hline \sim q \rightarrow s \end{array} \Rightarrow q \vee s \equiv s \vee q$

Disjunctive syllogism.

② $[(\underbrace{\sim t \rightarrow \sim r}_{1}) \wedge \underbrace{\sim s}_{4} \wedge (\underbrace{t \rightarrow w}_{2}) \wedge (\underbrace{r \vee s}_{3})] \rightarrow w$

$\sim t \rightarrow \sim r \Rightarrow r \rightarrow t$ [∵ Contrapositive $\sim Q \rightarrow \sim p = P \rightarrow q$]

$\begin{array}{l} t \rightarrow w \\ \hline r \rightarrow w \end{array}$ [∵ hypothetical syllogism/transitive law]

$r \vee s \Rightarrow \cancel{\sim s \rightarrow r}$

$\Downarrow$

$s \vee r \Rightarrow \sim s \rightarrow r$

$\begin{array}{l} \sim s \rightarrow r \\ \hline \sim \cancel{s} \rightarrow w \\ \cancel{\sim s} \\ \hline w \end{array}$ → conclusion

③ $(p \rightarrow q) \wedge (q \rightarrow r) \wedge p \rightarrow r$

$\begin{array}{l} p \rightarrow q \\ q \rightarrow r \\ \hline p \rightarrow r \\ p \rightarrow r \\ \hline p \rightarrow r \end{array}$

* Method of Proof and Disproof :-

Given $p \to q$, where p and q are Simple/Compound Propositions, the Process of establishing that the Conditional is true by using laws of logic/ using rules etc. Constitutes a Proof of the Conditional.

→ The Process of establishing that a proposition is false constitute a disproof.

① Direct Method :-

The direct method of proving a Conditional $p \to q$ has the following lines of argument.

a) Hypothesis : First assume that p is true.

b) Analysis : Starting with hypothesis & employing the rules/laws of logic and other known facts infer that q is true.

c) Conclusion : $p \to q$ is true.

② Indirect Proof :-

We know that the Conditional $p \to q$ and its Contra-positive $\sim q \to \sim p$ are logically equivalent.

In Some Situations given a Conditional $p \to q$, a proof of $\sim q \to \sim p$ is easier.

On the basis of this proof. we Conclude that $p \to q$ is true.

$p \to q$ & $\sim q \to \sim p$ are logically equivalent.

5) Proof by Existence :-

We know that a proposition of the form, $\exists x \in S, p(x)$ is true, if any one element a in S is such that p(a) is true is exhibited.

for some

The best way to prove a proposition of the form, $\exists x \in S, p(x)$ is to find one $a \in S$, such that p(a) is true.

This method of proof is called <u>Proof by existence</u>.

---

6) Disproof by Contradiction :-

Suppose we ~~wish~~ like to disprove a proposition $p \to q$.

For this let us assume that p is true and q is true and end up with a contradiction.

Hence, we conclude that $p \to q$ is false.

This method of disproving $p \to q$ is called <u>disproof by Contradiction</u>.

7) Disproof by Counter Example :-

We can disprove a proposition involving the universal quantifier is to find one value for x for which the proposition is false.

This method of disproof is called <u>disproof by Counter example</u>.

③ Proof by the method of Contradiction :-

The indirect method of proof is equivalent to the proof by contradiction

→ The lines of argument in this method of proof of the statement $p \to q$ are as follows.

1) <u>Hypothesis</u> :- First assume that $\overset{T}{p} \to \overset{F}{q} = F$ is false. i.e., p is true and q is false.

2) <u>Analysis</u> :- Assuming q is false and employing the rules / laws of logic and other known facts infer that p is false. This contradicts the assumption that p is true.

3) <u>Conclusion</u> :- Because of the contradiction we infer that $p \to q$ is true.

④ Proof by exhaustion :-

We know that a proposition of the form, $\forall x \in S$, $p(x)$ is true for every x in S.

for every

universe

If S consists of only a limited number of elements we can prove that the statement $\forall x \in S$, $p(x)$ is true by considering $p(a)$ for each a in S and verifying that $p(a)$ is true in each case.

Such a method of proof is called <u>method of exhaustion</u>.

Problems :-

① Give a direct Proof for each of the following :-

(i) For all integers k and l, if k and l are both even, then k + l is even.

(ii) For all integers k and l, if k and l are both even, then k * l is even.

Sol:- Take any 2 integers k and l. Direct method.

Assume both k and l are even.

then, $k = 2m$, $l = 2n$ for $m, n \in \mathbb{Z}$.

(i) $k + l = 2m + 2n = 2(m+n) \longrightarrow$ even.

(ii) $k * l = 2m * 2n = 4mn \longrightarrow$ even.

$\therefore$ k + l and k * l are both even.

Since, k and l are arbitary integers, in view of rule of Universal ~~quantifier~~ generalisation, the proof is Complete.

② Provide a Proof by Contradiction of the following Statements.

"For every integer 'n', if n^2 is odd, then 'n' is odd."

Sol:- let p : n^2 is odd

q : n is odd

Given, $p \rightarrow q$.

Assume $p \rightarrow q$ is false then, p is true and q is false.

Consider q is false

$\Rightarrow$ n is even

$\Rightarrow n = 2k \quad , k \in Z$

$\therefore n^2 = (2k)^2 = 4k^2 \longrightarrow$ even.

$\Rightarrow$ p is false

the contradicts the assumption that p is true.

$\therefore$ By the method of proof by contradiction, $p \rightarrow q$ is true.

i.e., If n^2 is odd, then n is odd.

(3) Give (i) direct proof (ii) an indirect proof and (iii) proof of contradiction for the following statement:

"If n is an odd integer, then n+9 is an even integer".

Sol:- (i) Direct Proof:-

Assume that n is an odd integer.

$\Rightarrow n = 2k+1 \quad , k \in Z$

$\therefore n+9 = (2k+1)+9 = 2k+10 = \underbrace{2(k+5)}_{\text{even}}$

thus n+9 is even.

(ii) Indirect Proof:- (proof by CP).

$2(1+5)$
$2(6) = \underbrace{12}_{\text{even}}$

let p: n is odd ; q: n+9 is even

given:- $p \rightarrow q$

its CP:- $\sim q \rightarrow \sim p$ (to be proved)

Suppose $\sim q$ is ~~take~~ true $\Rightarrow$ n+9 is odd.

(4) then $n+9 = 2k+1$, $k \in Z$

$\Rightarrow n = 2k-8$

$\Rightarrow n = 2(k-4) \rightarrow$ even.

i.e., n is even i.e., $\sim p$ is true.

$\therefore$ this proves the Contrapositive.

(iii) Proof by Contradiction :-

let p: n is odd , q: $n+9$ is even.

Given :- $p \rightarrow q$

Assume that, $p \rightarrow q$ is false

i.e., p is true and q is false.

Consider q is false

$\Rightarrow n+9$ is odd.

$\Rightarrow n+9 = 2k+1$, $k \in Z$

$\Rightarrow n = 2k+1-9$

$\Rightarrow n = 2k-8 = 2(k-4) \rightarrow$ even.

$\therefore$ n is even $\Rightarrow p$ is false

this ~~contradiction~~ contradicts the assumption that n is odd.

$\therefore p \rightarrow q$ is true

④ Prove that every even integer n within $2 \le n \le 26$ can be written as a sum of atmost 3 perfect square.

Sol:- let $S = \{2, 4, 6, 8, \ldots\ldots, 26\}$

let $p(x)$: x is a sum of almost 3 perfect squares.

To prove:- the statement $\forall x \in S$, $p(x)$ is true (Proof by exhaustion)

we have,

$2 = 1^2 + 1^2$	$14 = 3^2 + 2^2 + 1^2$
$4 = 2^2$	$16 = 4^2$
$6 = 2^2 + 1^2 + 1^2$	$18 = 4^2 + 1^2 + 1^2$
$8 = 2^2 + 2^2$	$20 = 4^2 + 2^2$
$10 = 3^2 + 1^2$	$22 = 3^2 + 3^2 + 2^2$
$12 = 2^2 + 2^2 + 2^2$	$24 = 4^2 + 2^2 + 2^2$
	$26 = 5^2 + 1^2$

$\therefore$ each x in S is a sum of atmost 3 perfect squares.

⑤ Disprove the statements :-

" the sum of 2 odd integers is an odd integer."

Sol:- let p: a and b are odd integers.

q: $a+b$ is an odd integer.

then the proposition to be disproved is $p \to q$

Assume p is true & q is true

then $a = 2k_1 + 1$; $b = 2k_2 + 1 \longrightarrow$ ①

$a + b = 2k_3 + 1 \longrightarrow$ ② where, $k_1, k_2, k_3 \in \mathbb{Z}$.

from ①, $a+b = 2k_1 + 1 + 2k_2 + 1$

$= 2k_1 + 2k_2 + 2$

$= 2(k_1 + k_2 + 1) \rightarrow$ even

$\therefore a+b$ is even.

this contradicts the assumption made in (2).

$\therefore p \to q$ is false, which disproves the given statement.

* Consistency of Premise :-

→ **Rule P** :- A Premise may be introduced at any point in the derivation

→ **Rule T** :- A formula S can be introduced in a direction if S is tautologically implied by any one/more of the preceding formulas in the derivation.

* Demonstrate that R is a valid inference from the Premises

$$P \to Q, Q \to R, \text{ and } P.$$

Sol :-

{1}	①	$P \to Q$	Rule P
{2}	②	P	Rule P
{1,2}	③	Q	Rule T, (1), (2) and $(P \to Q) \wedge P \Rightarrow Q$
{3}	④	$Q \to R$	Rule P
{1,2,3}	⑤	R	Rule T, (3), (3) and $(Q \to R) \wedge Q \Rightarrow R$.

→ **Rule CP** :- if we can derive S from R and a set of premises, then we can derive $R \to S$ from the set of Premise alone.

Note :- Rule CP is also called as deduction theorem.

Show that $R \to S$ can be derived from the premises

$$P \to (Q \to S), \sim R \vee P \text{ and } Q.$$

Proof :- Instead of showing $R \to S$, we shall include the premise R as an additional premise and show S first.

means, we have that,

$$P \to (Q \to S), \sim R \vee P, Q \text{ and } R.$$

{2}	(1)	$\sim R \vee P$	Rule P
{4}	(2)	R	Rule P
{2,4}	(3)	P	Rule T
{1}	(4)	$P \to (Q \to S)$	Rule P
{1,2,4}	(5)	$Q \to S$	Rule T (3),(4) and $(P \to Q), P \to Q$
{3}	(6)	Q	Rule P
{1,2,3,4}	(7)	S	Rule T (5),(6) & $(P \to Q), P \to Q$.
{1,2,3,4}	(8)	$R \to S$	Rule CP

$\sim R \vee P \Rightarrow R \to P$
$(\sim R \vee P) \wedge R \Rightarrow P$
$(\sim R \vee P), R \Rightarrow P$ (2) & (3)

* Checking Consistency of premises:- → just verify all the statements are logically equal / not.

"If there was a ball game, then travelling was difficult. If they arrived on time, then travelling was not difficult. they arrived on time. therefore, there was no ball game."

conclusion.

without using truth table, show that these statements constitute a valid argument.

this are mostly used in Neural N/W's.

P: There was a ball game.

Q: Travelling was difficult,

R: they arrived on time.

So, the given paragraph can be written as

$H_1: P \to Q$, $H_2: R \to \sim Q$, $H_3: R$, $C: \sim P$

{2}	(1)	$R \to \sim Q$	Rule P
{3}	(2)	R	Rule P
{2,3}	(3)	$\sim Q$	Rule T
{1}	(4)	$P \to Q$	Rule P
{1,2,3}	(5)	$\sim P$	Rule T

* Predicate Logic / Predicate Calculus :-

→ **Predicate** :- Let us have a statement,

① $\underbrace{\text{Rama}}_{\text{subject}}$ $\underbrace{\text{is a good boy}}_{\text{Predicate}}$

② $\underbrace{x}_{\text{Sub}} \underbrace{> 3}_{\text{predicate}}$

③ x is a student

S(x) → predicate
→ variable.

Proposition function (or) statement function

④ x is a man

M(x) → predicate
→ variable

statement function

⑤ x is a women.

w(x)

So, this is about what is a statement function and ~~predicate~~ variable

Here, we can Perform various Operations, we can Construct Compound Statements functions, we know what is a compound proposition.

↳ 2/more statems using connectivities

Likewise we have compound statement functions

↳ if we use connectivites in the statement functions

Eg :-

$M(x) \wedge w(x)$

$M(x) \vee w(x)$

$\sim M(x)$

$M(x) \rightarrow w(x)$

* Quantifiers :-
  - → Universal Quantifiers
  - → Existential Quantifiers.

① Universal Quantifier :-

→ Some statement telling about all individual (or) object belonging to a Certain set.

→ These statements are begining with forall, forevery, everything.

→ It is denoted by "$\forall$"

→ $\forall x\ P(x) \Rightarrow$ for all x, $P(x)$

→ A proposition preceded by '$\forall$' always have a truth value ie, T/F

Eg:- $(\forall n \in N)\ (n+4 > 3)$

$n = 1, 2, 3 \dots n$

ie, $n = 1$, $5+4 > 3$ ✓ T

eg:- $(n+2) < 6$

$n = 5 = 5+2 < 6$ ✓ F

Eg:- 1. All men are mortal.

$\underbrace{\text{All men}}_{x}$

$\forall x \{M(x) \rightarrow H(x)\}$

2. Every apple is red.

$\underbrace{\text{apple}}_{x}$

$\forall x \{A(x) \rightarrow R(x)\}$

③ Any integer is either +ve / -ve.

$\underbrace{\text{integer}}_{x}$

$\forall x \left[I(x) \rightarrow \{P(x) \vee N(x)\} \right]$

Negation :-

$\forall x \Rightarrow \sim\forall(x) \Rightarrow \exists(x)$ → ~~Universal~~ existential.

④ Not every graph is Connected

Graph $(x) \Rightarrow \forall x \{G(x) \rightarrow c(x)\} \Rightarrow \sim\forall(x) [G(x) \rightarrow c(x)]$

not all graph connected.

② Existential Quantifier:-

The statement begin with for Some, there exists (or) there is atleast one. denoted as "$\exists$"

Eg:- ① there exists a man

$$\exists(x)\, M(x)$$

② Some men are clever

$$\exists(x)\left[M(x) \wedge C(x)\right]$$

③ Some real numbers are rational.

$$\exists(x)\left[N(x) \wedge R(x)\right]$$

Negation:-

$$\sim\exists(x) \Rightarrow \forall(x)$$

④ Not all graphs are Connected $\Rightarrow \sim\forall(x)\left[G(x) \rightarrow C(x)\right]$

⑤ Some graphs are not connected $\Rightarrow \exists(x)\left[G(x) \wedge \sim C(x)\right]$

⑥ All graphs are Connected

$$\forall(x)\left[G(x) \rightarrow C(x)\right]$$

⑦ Some cats are black. $\Rightarrow \exists(x)\left[C(x) \rightarrow B(x)\right]$

⑧ $P(x,y) \rightarrow x$ likes y

Somebody likes Someone $\Rightarrow \exists x\ \exists y\ P(x,y)$

Everybody likes everybody $\Rightarrow \forall x\ \forall y\ P(x,y)$

Everybody likes Someone $\Rightarrow \forall x\ \exists y\ P(x,y)$

there is Someone liked by everybody. $\Rightarrow \exists y\ \forall x\ P(x,y)$