MODULE - IV

* Numerical Methods:-

1. Regular Falsi method -
2. Bisection method
3. Iteration method
4. Newton - Raphson method
5. Secant Method -
6. Ramanujan's method.
7. Muller's method.

1. Regular Falsi method:-

Method of False position:-

$$x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

problem:-

① Find a real root of $x^3 - 4x + 1 = 0$ using false position method.

Sol:- $f(x) = x^3 - 4x + 1 = 0$

put $x = 0$, $f(0) = 0 - 0 + 1$
$= 1$ (positive)

put $x = 1$, $f(1) = 1 - 4 + 1$
$= -2$ (negative)

Root lies between '0' and '1'.

Then the root lies between $\underset{a}{0}$ and $\underset{b}{1}$.

$$x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

$$= \frac{0 - 1}{-2 - 1} = \frac{-1}{-3} = \frac{1}{3} = 0.333$$

now,

$f(x_1) = f(0.33)$

$= (0.33)^3 - 4(0.33) + 1$

$f(x_1) = -0.2962$ (Negative).

$b \to$ replaced by "x_1" ($\because$ we got $f(x_1) = -ve$)

now, root lies between $\underset{a}{0}$ and $\underset{b}{0.333}$

now,

$$x_2 = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

$$= \frac{0 - 0.333(1)}{-0.2962 - 1}$$

$x_2 = 0.2571$

$f(x_2) = (0.2571)^3 - 4(0.2571) + 1$

$f(x_2) = -0.0115$ (negative)

Now 'b' $\to$ replace by "x_2"

Now, Roots lies between 0 and 0.2571

$$\text{now } x_3 = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

$$= \frac{0 - 0.2571(1)}{-0.0115 - 1}$$

$x_3 = 0.2541$

now, $f(x_3) = (0.2541)^3 - 4(0.2541) + 1$

$f(x_3) = +0.000026$ (positive)

$\therefore$ 'a' $\to$ '0' replaced by "x_3"

Now, root lies between

$0.2541 \downarrow a$ and $0.2571 \downarrow b$

now

$$x_4 = \frac{af(b)-bf(a)}{f(b)-f(a)}$$

$$= \frac{0.2541(-0.0115) - 0.2571(0.000006)}{-0.0115-0.000006}$$

$$x_4 = 0.2541$$

↳ Repeated.

∴ Required root is 0.2541 //.

2) Find a real root of $3x^3-4x-9=0$

Ans:- $sol = 1.7467$.

$f(x) = 3x^3-4x-9=0$

$f(0) = 0-0-9 = -9$ (negative)

$f(1) = 3-4-9 = -10$ (negative).

$f(2) = 3(8)-4(2)-9$

$= 24-8-9$

$= 7$ (positive)

∴ Root lies between $\underset{a}{\overset{-ve}{1}}$ and $\underset{b}{\overset{+ve}{2}}$

$$x_1 = \frac{af(b)-bf(a)}{f(b)-f(a)}$$

$$= \frac{1(7)-2(-10)}{7+10} = \frac{7+20}{17}$$

$$= 1.5882$$

$f(x_1) = 3(1.5882)^3 - 4(1.5882) - 9$

$= -3.3340$ (Negative)

$1 \rightarrow$ replaced by 1.5882

Now, Root lies between

1.5882 and 2

$\downarrow \quad\quad \downarrow$

a $\quad\quad$ b

$$x_2 = \frac{af(b) - bf(a)}{f(b)\ f(a)}$$

$$= \frac{1.5882(7) - 2(-3.3340)}{7 - (-3.3340)}$$

$x_2 = 1.7210$

$f(x_2) = 3(1.7210)^3 - 4(1.7210) - 9 = 0$

$= -0.59073$ (Negative).

$\therefore a \rightarrow x_2$

Now, Root lies between

1.7210 and 2

$\downarrow \quad\quad \downarrow$

a $\quad\quad$ b

$$x_3 = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

$$= \frac{1.7210(7) - 2(-0.59073)}{7 - (-0.59073)}$$

$x_3 = 1.8739$ (4)

$f(x_3) = 3(1.8739)^3 - 4(1.8739) - 9 = 0$

$= 0.10093$ (Positive)

-ve

$\therefore b \rightarrow$ replaced by x_3

Now root lies between

1.7210 and 1.8739

wrong

$x_4 = af(b) - bf(a)$

$a \to$ replaced by x_3

root lies between

1.742 and 2
$\downarrow$ a $\downarrow$ b)

$$x_4 = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

$$= \frac{1.742(7) - 2(-0.109)}{7 - (-0.109)}$$

$x_4 = 1.74$

$\therefore 1.74$ is a root //.

3. Using false position method find a real root of the equation $x \log x - 1.2 = 0$ in 3 steps

Sol:- $f(x) = x \log x - 1.2$

Ans: 2.7406

$f(0) = 0 - 1.2$
$= -1.2$ (negative)

$f(3) = 3 \log 3 - 1.2$
$= 0.2313$ (+ve)

$f(1) = 1 \log 1 - 1.2$
$= -1.2$ (-ve)

$f(2) = 2 \log 2 - 1.2$
$= -0.5979$ (-ve)

$\therefore$ Root lies b/w 2 & 3
(-ve) $\downarrow$ a $\downarrow$ b (+ve)

$$x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

$$= \frac{2(0.2313) - 3(-0.5979)}{0.2313 - (-0.5979)}.$$

$x_1 = 2.7210$

$f(x_1) = 2.7210 \log(2.7210) - 1.2$

$= -0.0171$ (-ve)

$\therefore$ Roots lies b/w ~~2.72~~ 2.7210 and 3
$\downarrow$ a $\downarrow$ b

$$x_2 = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

$$= \frac{2.7210(0.2313) - 3(0.0171)}{0.2313 - (-0.0171)}.$$

$$x_2 = 2.7402 \text{ (+ve)}$$

Now, root lies between 2.7210 and

$$f(x_2) = 0.6806 \log(0.6806) - 1.2$$
$$= -0.0038 \text{ (-ve)}$$

$\therefore$ Root lies between 2.7402 and 3

$$x_3 = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

$$= \frac{2.7402(0.2313) - 3(-0.0038)}{0.2313 - (-0.0038)}$$

$$x_3 = 2.7443.$$

$\therefore$ 2.740 is a root.

2) Bisection Method:

problem ①: Find the root of the equation $x^3-x-1=0$ using bisection method. Ans: 1.3247

Sol: $f(x)=x^3-x-1=0$

$f(0)=-1$ (-ve)

$f(1)=1-1-1$
$=-1$ (-ve)

$f(2)=8-2-1$
$=5$ (+ve)

∴ Root lies b/w 1 and 2

$(-ve)$ a b $(+ve)$

$x_1=\frac{1+2}{2}=\frac{3}{2}=1.5$

$f(x_1)=(1.5)^3-1.5-1$
$=0.875$ (+ve)

∴ Root lies between 1 and 1.5

$x_2=\frac{1+1.5}{2}=1.25$

$f(x_2)=(1.25)^3-1.25-1$
$=-0.296875$ (-ve)

∴ Root lies between 1.25 and 1.5

$x_3=\frac{1.25+1.5}{2}=1.375$

$f(x_3)=(1.375)^3-1.375-1$
$=0.22$ (+ve)

∴ Root lies between 1.25 and 1.375

$x_4=\frac{1.25+1.375}{2}=1.3125$

$f(x_4)=(1.3125)^3-1.3125-1$
$=-0.05$ (-ve)

∴ Root lies b/w 1.3125 and 1.375

$$x_5 = \frac{1.3125 + 1.375}{2}$$

$x_5 = 1.34375$

$f(x_5) = (1.34375)^3 - 1.34375 - 1$

$= 0.0826$ (+ve)

∴ Root lies b/w 1.3125 and 1.34375

$$x_6 = \frac{1.3125 + 1.34375}{2}$$

$x_6 = 1.3281$

$f(x_6) = (1.3281)^3 - 1.3281 - 1$

$= 0.0144$ (+ve)

∴ Root lies b/w 1.3125 and 1.3281

$$x_7 = \frac{1.3125 + 1.3281}{2}$$

$x_7 = 1.3203$

∴ 1.32 is a root.

Q. Find the root of the equation $x^3 - 2x - 5 = 0$ by using bisection method.

Sol: $x_{12} = 2.0946$

$f(x) = x^3 - 2x - 5 = 0$

$f(0) = -5$ (-ve)

$f(1) = 1 - 2 - 5$ (-ve)

$= -6$

$f(2) = 8 - 4 - 5 = -1$ (-ve)

$f(3) = 27 - 6 - 5$

$= 16$ (+ve)

∴ Root lies b/w $\underset{a}{2}$ and $\underset{b}{3}$

(-ve) (+ve)

$$x_1 = \frac{2+3}{2}$$

$$= \frac{5}{2} = 2.5$$

$$f(x_1) = (2.5)^3 - 2(2.5) - 5$$

$$= 5.625 \text{ (+ve)}$$

$\therefore$ Root lies b/w $2\cancel{.3}$ and 2.5

$$x_2 = \frac{2+2.5}{2} = 2.25$$

$$f(x_2) = (2.25)^3 - 2(2.25) - 5$$

$$= 1.89 \text{ (+ve)}$$

$\therefore$ Root lies

3) Iteration Method:

problems:

① Find the root of equation $x^3 - x - 11 = 0$ using iteration method.

Sol:- $x=0, f(0) = -11$ (-ve)

$x=1, f(1) = -11$ (-ve)

$x=2, f(2) = -5$ (-ve)

$x=3, f(3) = 13$ (+ve)

So that root lies between 2 and 3.

let $x_0 = 2.5$ $((2+3)/2)$

from the given equation,

$$x^3 - x - 11 = 0.$$

$$x^3 = x + 11$$

$$x = (x+11)^{1/3}$$

$$\phi(x) = (x+11)^{1/3}$$

$$x_1 = \phi(x_0) = (x_0+11)^{1/3}$$

$$x_1 = (2.5+11)^{1/3} = 2.3811$$

$$x_2 = \phi(x_1) = (x_1+11)^{1/3}$$

$$x_2 = 2.3740$$

$$x_3 = \phi(x_2) = (x_2+11)^{1/3}$$

$$x_3 = 2.3736$$

$$x_4 = \phi(x_3) = (x_3+11)^{1/3}$$

$$x_4 = 2.3736$$

Hence, the root is 2.3736 //.

2) Find the root of equation $x^2 - x - 50 = 0$ using iteration method.

Sol:- $f(x) = x^2 - x - 50 = 0$

$x=0$, $f(0) = -50$ (-ve)

$x=1$, $f(1) = -50$ (-ve)

$x=2$, $f(2) = -48$ (-ve)

$x=3$, $f(3) = -44$ (-ve)

$x=7$, $f(7) = 49 - 7 - 50 = -8$
$= (-ve)$

$x=8$, $f(8) = 64 - 8 - 50$
$= 6$ (+ve)

Root lies between 7 and 8

$$x_0 = \frac{7+8}{2} = 7.5$$

now, $x^2 = x + 50$

$$\phi(x) = x = (x+50)^{1/2}$$

$$x_1 = \phi(x_0) = (x_0 + 50)^{1/2}$$
$$= (7.5 + 50)^{1/2}$$
$$x_1 = 7.5828$$

$$x_2 = \phi(x_1) = (7.5828 + 50)^{1/2}$$
$$= 7.5883$$

$$x_3 = \phi(x_2) = (7.5883 + 50)^{1/2}$$
$$= 7.5886$$

$$x_4 = \phi(x_3) = (7.5886 + 50)^{1/2}$$
$$= 7.5887$$

$$x_5 = \phi(x_4) = (7.5887 + 50)^{1/2}$$
$$= 7.5887$$

$\therefore$ The root is 7.5887 //.

4) NEWTON RAPHSON EQUATION:-

method:-

$$x_{n+1} = x_n - f(x_n)/f'(x_n) \quad \text{for } n = 0, 1, 2, ----$$

problem-①:-

using Newton Raphson method. solve $x^3-3x-5=0$

sol:- $x=0, f(x) = -5$ (-ve)

$x=1, f(x) = -7$ (-ve)

$x=2, f(2) = -3$ (-ve)

$x=3, f(3) = 13$ (+ve)

so the root lies between 2 and 3.

let $x_0 = \frac{2+3}{2} = 2.5$

$f(x) = x^3 - 3x - 5$

$f'(x) = 3x^2 - 3$

$f(x_0) = (2.5)^3 - 3(2.5) - 5$

$= 3.125$

$f'(x_0) = 3(2.5)^2 - 3$

$= 15.75$

Now, $x_1 = x_0 - f(x_0)/f'(x_0)$

$x_1 = 2.3015$

put $n=1$

$x_2 = x_1 - f(x_1)/f'(x_1)$

$x_2 = 2.2792$.

$x_3 = x_2 - f(x_2)/f'(x_2)$

$= 2.2792 - f(x_2)/f'(x_2)$

$= 2.2790$

$x_4 = x_3 - f(x_3)/f'(x_3)$

$x_4 = 2.2790$

Hence the required root is 2.2790//.

2) Find the root of the equation

$x^3 - x - 1 = 0$

Sol:- Ans:- $X_4 = 1.32475$

$x = 0,\ f(0) = -1\ (-ve)$

$x = 1,\ (f(1)) = -1\ (-ve)$

$x = 2,\ f(2) = 5\ (+ve)$

$\therefore$ Root lies between 1 and 2

$x_0 = \frac{1+2}{2} = 1.5$

Now

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$ $\quad f(x) = x^3 - x - 1$

$f'(x) = 3x^2 - 1$

$x_1 = 1.3478$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_2)}$

$x_2 = 1.3251$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$x_3 = 1.32471$

$x_4 = x_3 - \frac{f(x_2)}{f'(x_2)}$

$x_4 = 1.32471$

Hence, the required root is 1.32471 //.

5) Secant Method:-

Method:-

$$x_{n+1} = x_{n-1} f(x_n) - x_n f(x_{n-1}) / (f(x_n) - f(x_{n-1}))$$

problem - ①:-

Find root of $x^3 - 5x + 1 = 0$ in interval $(0, 1)$ using secant method.

Sol:- $x = 0, f(0) = 1$ (+ve) $\rightarrow x_0$

$x = 1, f(1) = -3$ (−ve) $\rightarrow x_1$

$x_{n+1} = x_{n-1} f(x_n) - x_n f(x_{n-1}) / f(x_n) - f(x_{n-1})$

put $n = 1$

$x_2 = x_0 f(x_1) - x_1 f(x_0) / f(x_1) - f(x_0)$

$x_2 = 0.25$

$f(x_2) = -0.234$

put $n = 2$

$x_3 = x_1 f(x_2) - x_2 f(x_1) / f(x_2) - f(x_1)$

$x_3 = 0.1864$

$f(x_3) = 0.07428$

put $n = 3$

$x_4 = x_2 f(x_3) - x_3 f(x_2) / f(x_3) - f(x_2)$

$$= 0.25 \times 0.07428 - \frac{0.1864 (0.234)}{0.07428 - (-0.234)}$$

$= 0.01857 + 0.5872 + 0.234$

$= 0.20164$

$f(x_4) = -0.00048$

put $n = 4$

$x_5 = x_3 f(x_4) - x_4 f(x_3) / f(x_4) - f(x_3)$

$x_5 = 0.20081$

$f(x_5) = 0$

Hence the required root is 0.20081 //.

② Find root of $x^3 - 7x + 5 = 0$ using secent method.

Sol:- $x=0, f(0) = 5$ (+ve) $\rightarrow x_0$

$x=1, f(1) = -1$ (-ve) $\rightarrow x_1$

put n=1

$$x_{n+1} = x_{n-1} f(x_n) - x_n f(x_{n-1}) / (f(x_n) - f(x_{n-1}))$$

$$x_2 = x_0 f(x_1) - x_1 f(x_0) / (f(x_1) - f(x_0))$$

$$x_2 = 0 - 1\left(\frac{5}{(-1-5)}\right)$$

$$= -1\left(\frac{5}{-6}\right)$$

$x_2 = 0.8333$

$f(x_2) = 0.2546$

put n=2

$$x_3 = x_1 f(x_2) - x_2 f(x_1) / (f(x_2) - f(x_1))$$

$$= 1(5) - 0 - (-1)$$

$$= 5+1$$

$x_3 = 6$

$f(x_3) =$

$$x_3 = 1(-0.2546) - 0.8333 / (-0.2546+1)$$

$x_3 = -1.3725$

Sol:- $x=0; f(0) = 5$ (+ve) $\rightarrow x_0$

$x=1, f(1) = -1$ (-ve) $\rightarrow x_1$

$$x_{n+1} = x_{n-1} f(x_n) - x_n f(x_{n-1}) / (f(x_n) - f(x_{n-1}))$$

put n=1

$$x_2 = x_0 f(x_1) - x_1 f(x_0) / (f(x_1) - f(x_0))$$

$$x_2 = 0 - (1)(5) / (-1-5)$$

$$= \frac{-5}{-6} = \frac{5}{6} = 0.8333$$

$$f(x_2) = (0.8333)^3 - 7(0.8333) + 5$$

$$= -0.2546$$

put $n=2$

$$x_3 = x_1 f(x_2) - x_2 f(x_1) / (f(x_2) - f(x_1))$$

$$= 1(-0.2546) - 0.8333(-1) / (-0.2546 - (-1))$$

$$= -0.2546 + 0.8333 / (-0.2546 + 1)$$

$$x_3 = 0.8633$$

$$f(x_3) = -0.3998$$

put $n=3$

$$x_4 = x_2 f(x_3) - x_3 f(x_2) / (f(x_3) - f(x_2))$$

$$= 0.8333(-0.3998) - 0.8633(-0.2546) / (-0.3998 + 0.2546)$$

$$= 0.3331 - 1.5137$$

$$x_4 = -1.1805$$

$$f(x_4) = 11.61$$

put $n=4$

$$x_5 = x_3 f(x_4) - x_4 f(x_3) / (f(x_4) - f(x_3))$$

$$= 0.8633(11.61) - \frac{(-1.1805)(-0.3998)}{(11.61 + 0.3998)}$$

$$= (0.8633)(11.61) - 0.0392$$

$$= 9.9836$$

$$f(x_5) = 930$$

6) RAMANUJAN'S METHOD:

Iteration method used to determine smallest root of the equation $f(x)=0$

$$f(x) = 1-(a_1x+a_2x^2+a_3x^3+----+a_nx^n).$$

for smallest value of x we can write

$$[1-(a_1x+a_2x^2+a_3x^3+----+a_nx^n]^{-1}$$

$$= b_1+b_2x+b_3x^2+----$$

expanding LHS equation by using binomial theorem.

$$1+(a_1x+a_2x^2+---)+(a_1x+a_2x^2+---)^2+----$$

$$= b_1+b_2x+----$$

By comparing the co-efficient of x which has like powers on both sides we get

$$b_1=1$$

$$b_2=a_1b_1$$

$$b_3=a_1b_2+a_2b_1$$

$$b_4=a_1b_3+a_2b_2+a_3b_1$$

$$b_5=a_1b_4+a_2b_3+a_3b_2+a_4b_1$$

roots:

$$\frac{b_n}{b_{n+1}} \quad \text{i.e.}, \frac{b_1}{b_2}, \frac{b_2}{b_3}, \frac{b_3}{b_4}, ---$$

called the convergent approach in the limit the smallest root of $f(x)=0$.

problems:

① Find the smallest root of the equation $x^3-9x^2+26x-24=0$.

sol: $f(x) = x^3-9x^2+26x-24=0$

$$f(x) = \frac{x^3-9x^2+26x}{24}-1=0$$

$$= 1-\frac{x^3}{24}+\frac{9x^2}{24}-\frac{26x}{24}$$

$$= 1-\frac{x^3}{24}+\frac{3x^2}{8}-\frac{26x}{24}$$

$$f(x) = 1-\left(\frac{13}{12}x-\frac{3}{8}x^2+\frac{1}{24}x^3\right)$$

$a_1=\frac{13}{12}$ $\quad a_2=\frac{-3}{8}$ $\quad a_3=\frac{1}{24}$ $\quad a_4=a_5=a_6=0$

$b_1=1$

$b_2=a_1b_1$

$= \frac{13}{12}(1) = 1.0833$

$b_3 = a_1b_2+a_2b_1$

$= \frac{13}{12}(1.0833)+\left(\frac{-3}{8}\right)(1)$

$= 1.173575$

$=0.7985$

$b_4=a_1b_3+a_2b_2+a_3b_1$

$=0.5007$

$b_5=a_1b_4+a_2b_3+a_3b_2+a_4b_1$

$= 0.2880$

$b_6=a_1b_5+a_2b_4+a_3b_3+a_4b_2+a_5b_1 = 0.1575$

$b_7=a_1b_6+a_2b_5+a_3b_4+a_4b_3+a_5b_2+a_6b_1 = 1.8862$

$b_8=a_1b_7+a_2b_6+a_3b_5+a_4b_4+a_5b_3+a_6b_2+a_7b_1$

$=1.8862$

$b_9=a_1b_8+a_2b_7+a_3b_6+a_4b_5+a_5b_4+a_6b_3+a_7b_2+a_8b_1$

$=0.223$

Roots:-

$\frac{b_1}{b_2} = 0.923$

$\frac{b_2}{b_3} = 1.356$

$\frac{b_3}{b_4} = 1.595$

$\frac{b_4}{b_5} = 1.7382$

$\frac{b_5}{b_6} = 1.8286$

$\frac{b_6}{b_7} = 1.8862$

$\frac{b_7}{b_8} = 1.9240$

$\frac{b_8}{b_9} = 1.9462.$

∴ The smallest root is "2".

7) Muller's Method:

→ Formula to find iterations:

$$x_{i+1} = x_i - \frac{2y_i}{B \pm \sqrt{B^2 - 4Ay_i}}$$

$$A = \frac{1}{(h_{i-1} + h_i)} \left[\frac{\Delta_i}{h_i} - \frac{\Delta_{i-1}}{h_{i-1}} \right]$$

$$B = \frac{\Delta_i}{h_i} + Ah_i$$

problem

① Using Muller's method, find the roots of the equation $f(x) = x^3 - x - 1 = 0$ with the initial approximations 0, 1 and 2.

Sol: $f(x) = x^3 - x - 1$

Initial approximations $x_{i-2} = 0$, $x_{i-1} = 1$, $x_i = 2$

$x_{i-2} = 0$ then $y_{i-2} = f(0) = -1$

$x_{i-1} = 1$ then $y_{i-1} = f(1) = -1$

$x_i = 2$ then $y_i = f(2) = 5$

$\Delta_i = y_i - y_{i-1} = 5 + 1 = 6$

$\Delta_{i-1} = y_{i-1} - y_{i-2} = 0$

$h_i = x_i - x_{i-1} = 2 - 1 = 1$

$h_{i-1} = x_{i-1} - x_{i-2} = 1 - 0 = 1$

$$A = \frac{1}{(h_{i-1} + h_i)} \left[\frac{\Delta_i}{h_i} - \frac{\Delta_{i-1}}{h_{i-1}} \right]$$

$A = 3$

$B = \frac{\Delta_i}{h_i} + Ah_i$

$B = 9$

and $\sqrt{B^2 - 4Ay_i} = \sqrt{21}$

first iteration :

$$x_{i+1} = x_i - \frac{2y_i}{B \pm \sqrt{B^2 - 4Ay_i}}$$

$$x_{i+1} = 1.26376$$

let $x_{i-2} = 1$, $x_{i-1} = 2$, $x_i = 1.26376$

$x_{i-2} = 1$ then $y_{i-2} = f(1) = -1$

$x_{i-1} = 2$ then $y_{i-1} = f(2) = 5$

$x_i = 1.26376$ then $y_i = f(1.26376) = -0.24542$

$\Delta_i = y_i - y_{i-1} = -5.24542$

$\Delta_{i-1} = y_{i-1} - y_{i-2} = 6$

$h_i = x_i - x_{i-1} = 0.73624$

$h_{i-1} = x_{i-1} - x_{i-2} = 1$

$$A = \frac{1}{(h_{i-1} + h_i)} \left[\frac{\Delta_i}{h_i} - \frac{\Delta_{i-1}}{h_{i-1}} \right]$$

$A = 4.26375$

$B = \frac{\Delta_i}{h_i} + Ah_i$

$B = 3.98546$

$\sqrt{B^2 - 4Ay_i} = 4.47990$

Second iteration :

$$x_{i+1} = x_i - \frac{2y_i}{B \pm \sqrt{B^2 - 4Ay_i}}$$

$$x_{i+1} = 1.32174$$

let $x_{i-2} = 2$; $x_{i-1} = 1.26376$, $x_i = 1.32174$

$x_{i-2} = 2$ then $y_{i-2} = f(2) = 5$

$x_{i-1} = 1.26376$ then $y_{i-1} = f(1.26376) = -0.24542$

$x_i = 1.32174$ then $y_i = f(1.132174) = -0.01266$

$\Delta_i = y_i - y_{i-1} = 0.23276$

$\Delta_{i-1} = y_{i-1} - y_{i-2} = -5.24542$

$h_i = x_i - x_{i-1} = 0.05798$

$h_{i-1} = x_{i-1} - x_{i-2} = -0.73624$

$$A = \frac{1}{(h_{i-1} + h_i)} \left[\frac{\Delta_i}{h_i} - \frac{\Delta_{i-1}}{h_{i-1}} \right]$$

$A = 4.58549$

$$B = \frac{\Delta_i}{h_i} + A h_i$$

$B = 4.28035$ $\qquad \sqrt{B^2 - 4Ay_i} = 4.30739$

Third iteration:-

$$x_{i+1} = x_i - \frac{2y_i}{B \pm \sqrt{B^2 - 4Ay_i}}$$

$x_{i+1} = 1.32469$

$\therefore$ 1.32 is the required root.