18/01/20

MODULE - II

Linear Differential Eq^ns of Second and Higher Orders

* **Definition:-** An eq^n of the form

$$\frac{d^n y}{dx^n} + P_1(x)\frac{d^{n-1}y}{dx^{n-1}} + \cdots + P_n(x)y = Q(x) \longrightarrow (1)$$

Here $P_1(x), P_2(x), \cdots\cdots, P_n(x)$ and $Q(x)$ are all continuous and real valued func^ns of "x" is called differential eq^n of order "n" with Constant coefficients

In eq (1) is the most general sol^n of can be written as $y = y_c + y_p$. Here "y_c" is called Complimentary func^n (C.F) and "y_p" is called Particular Integral (P.I). (y = C.F + P.I)

* **Complimentary Function :-** (C.F) (or) (y_c) :-

Let $f(D)y = 0 \longrightarrow (1)$ $\left(\frac{d}{dx} = D, \frac{d^2}{dx^2} = D^2\right)$

eq^n (1) can be ~~converted into~~ written as algebraic eq^n

It can be converted into auxiliary eq^n (A.E) of $f(m) = 0$ is a polynomial eqn of degree (n). It will have "n" roots $m_1, m_2, \ldots, m_n$

There are totaley four cases:

Case-1 :-

If $m_1, m_2, \ldots, m_n$ are all roots of real & distinct then

$$y_c = c_1 e^{m_1 x} + c_2 e^{m_2 x} + \cdots + c_n e^{m_n x}$$

Case-2 :-

If $m_1, m_2, \ldots, m_n$ are all roots of real & equal then

$$y_c = (c_1 + c_2 x + c_3 x^2 + \cdots + c_n x^{n-1}) e^{mx}$$

case-3 :-

If $m_1, m_2, \ldots, m_n$ are all roots of complex $(\alpha \pm i\beta)$ then
or
(conjugate complex roots)

$$y_c = (c_1 \cos\beta x + c_2 \sin\beta x) e^{\alpha x}$$

case-4 :-

If $m_1, m_2, \ldots, m_n$ are all roots of real & distinct $(\alpha \pm \beta)$ then

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$y_c = (c_1 \cosh \beta x + c_2 \sinh \beta x) e^{\alpha x}$

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(1) Solve $\frac{d^3y}{dx^3} - 9\frac{d^2y}{dx^2} + 23\frac{dy}{dx} - 15y = 0$

Sol:- Given $\frac{d^3y}{dx^3} - 9\frac{d^2y}{dx^2} + 23\frac{dy}{dx} - 15y = 0$

$$D^3y - 9D^2y + 23Dy - 15y = 0$$

$$(D^3 - 9D^2 + 23D - 15)y = 0$$

Now A.E is $\Rightarrow m^3 - 9m^2 + 23m - 15 = 0$

$$\begin{array}{c|cccc}
m=3 & 1 & -9 & 23 & -15 \\
 & 0 & 3 & -18 & 15 \\
\hline
m=5 & 1 & -6 & 5 & \boxed{0} \\
 & 0 & 5 & -5 & \\
\hline
 & 1 & -1 & \boxed{0} &
\end{array}$$

$$m - 1 = 0$$

$\therefore m = 1, 3, 5$

m is real and distinct roots

$$y_c = c_1 e^{x} + c_2 e^{3x} + c_3 e^{5x}$$

(2) Solve $(D^4 - 2D^3 - 3D^2 + 4D + 4)y = 0$

Sol: A.E $\Rightarrow (m^4 - 2m^3 - 3m^2 + 4m + 4) = 0$

$$
\begin{array}{r|rrrrr}
m=2 & 1 & -2 & -3 & 4 & 4 \\
 & 0 & 2 & 0 & -6 & -4 \\
\hline
m=2 & 1 & 0 & -3 & -2 & \boxed{0} \\
 & 0 & 2 & 4 & 2 & \\
\hline
m=-1 & 1 & 2 & 1 & \boxed{0} & \\
 & 0 & -1 & -1 & & \\
\hline
 & 1 & 1 & \boxed{0} & &
\end{array}
$$

$m + 1 = 0 = -1$

$\therefore m = -1, -1, 2, 2$

$y_c = (c_1 + c_2 x)e^{-x} + (c_3 + c_4 x)e^{2x}$

(3) Solve $(D^2 + D + 1)y = 0$.

Sol: Given $(D^2 + D + 1)y = 0$

A.E $\Rightarrow m^2 + m + 1 = 0$

$m = -1 \pm \sqrt{1^2 - 4(1)(1)}$

$= \frac{-1 \pm \sqrt{3}}{2}$

$$= \frac{-1 \pm \sqrt{-3}}{2}$$

$$m = \frac{-1}{2} \pm i\frac{\sqrt{3}}{2}$$

$\therefore$ Here 'm' is Complex roots

$$y_c = \left(c_1 \cos\frac{\sqrt{3}}{2}x + c_2 \sin\frac{\sqrt{3}}{2}x\right)e^{-\frac{1}{2}x} \quad //$$

(4) Solve $(D^3 - 14D + 8)y = 0$

Sol:- A.E $\Rightarrow m^3 - 14m + 8 = 0$

$$\begin{array}{r|rrr|r} & 1 & 0 & -14 & 8 \\ \cancel{m=2}\ \ m=-4 & 0 & -4 & 16 & -8 \\ \hline & 1 & -4 & 2 & 0 \end{array}$$

$$m^2 - 4m + 2 = 0$$

$$m = \frac{4 \pm \sqrt{16 - 4(2)(1)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{8}}{2} = \frac{4 \pm 2\sqrt{2}}{2} = 2 \pm \sqrt{2}$$

$$m = -4,\ 2 \pm \sqrt{2}$$

$$y_c = c_1 e^{-4x} + (c_2 \cosh\sqrt{2}x + c_3 \sinh\sqrt{2}x)e^{2x}$$

H/w sums

S ① $y'' - y' - 2y = 0$

② $(D^4 + 8D^2 + 16)y = 0$

③ $D^2(D^2+4)y = 0$

④ $(D^4 - 1)y = 0$

① given $y'' - y' - 2y = 0$.

$D^2y - Dy - 2y = 0$

A.E = ~~$m^2y - my$~~ $m^2 - m - 2 = 0$

$\Rightarrow \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow \cancel{\frac{1 \pm \sqrt{1+4(1)(2)}}{2(1)}}$

$m^2 - 2m + m - 2 = 0$

$m(m-2) + 1(m-2)$ $\cancel{= \frac{1 \pm \sqrt{1+8}}{2} = \frac{1 \pm \sqrt{9}}{2}}$

$m+1, m-2$ ~~$m = \frac{1}{2} \pm \frac{\sqrt{9}}{2}$~~

$m = -1, 2$

~~$y_c = (c_1 \cosh\frac{\sqrt{9}}{2}x + c_2 \sinh\frac{\sqrt{9}}{2}x)e^{\frac{1}{2}x}$~~

$y_c = c_1 e^{-x} + c_2 e^{2x}$

② $(D^4 + 8D^2 + 16)y = 0$

$AE = m^4 + 8m^2 + 16 = 0$

$m=2$	1	0	8	0	16
	0	2	4		
	1	2	12		

(3) $D^2(D^2+4)y = 0$

A.E $= m^4 + 4m^2 = 0$

$m^2(m^2+4) = 0$

$m = 0, 0, \pm i\sqrt{4}$

$m = 0, 0; 0 \pm i\sqrt{4}$

$y_c = (c_1 + c_2 x)e^{0x} + (c_3 \cos\sqrt{4}x + c_4 \sin\sqrt{4}x)e^{0x}$

$y_c = c_1 + c_2 x + c_3 \cos\sqrt{4}x + c_4 \sin\sqrt{4}x$

(4) $(D^4-1)y=0$

A.E is $m^4-1=0$

$$\begin{array}{r|rrrrr}
m=-1 & 1 & 0 & 0 & 0 & -1 \\
 & 0 & -1 & 1 & -1 & 1 \\
\hline
m=1 & 1 & -1 & 1 & -1 & \boxed{0} \\
 & 0 & 1 & 0 & 1 & \\
\hline
 & 1 & 0 & 1 & \boxed{0} &
\end{array}$$

$m^2+1=0$

$$m = \frac{-0 \pm \sqrt{0-4(1)(1)}}{2(1)}$$

$$m = \pm i\frac{\sqrt{4}}{2}, 1, -1$$

$$y_c = c_1e^{x}+c_2e^{-x}+\left(c_3\cos\frac{\sqrt{4}}{2}x + c_4\sin\frac{\sqrt{4}}{2}x\right)e^{0x}$$

$$y_c = c_1e^{x}+c_2e^{-x}+c_3\cos\frac{\sqrt{4}}{2}x + c_4\sin\frac{\sqrt{4}}{2}x$$

21/01/20 * Method - I :-

* Rules for finding particular integrals in some cases:-

* In particular integral of $f(D)y = Q(x)$ ⟶ (1)

where $Q(x) = e^{ax}$. Here "a" is constant

then $y_p = \frac{1}{f(D)} Q(x)$. Now substitute

$D = a$ then we get $e^{ax}\left[\frac{1}{f(a)}\right]$ ⟹

$$y_p = \frac{1}{f(D)} Q(x)$$

Now Substituting ⟹ D = a

$$= \frac{1}{f(D)} e^{ax} \quad [D = a] \quad [\because f(D) \neq 0]$$

$$= e^{ax}\left[\frac{1}{f(a)}\right]$$

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If $f(D) = 0$ in the above case we can use

the particular integral is

$$y_p = \frac{e^{ax}}{\phi(a)} \frac{x^n}{n!}$$

(Here $\phi(a)$ is repeated root of given $f(D)$ and $\phi(a) \neq 0$)

① solve $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = e^{2x}$

② solve $(D^3 - 5D^2 + 8D - 4)y = e^{2x}$

③ Solve $(4D^2 - 4D + 1)y = 100$

④ Solve $(D^2 - 3D + 2)y = \cosh x$

Sol:- Given $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = e^{2x}$

G.S is $(D^2 + 4D + 3)y = e^{2x}$

A.E is $m^2 + 4m + 3 = 0$

$m^2 + 3m + m + 3 = 0$

$m(m+3) + 1(m+3) = 0$

$(m+1)(m+3)$

$m = -1, -3$

$\therefore y_c = c_1 e^{-3x} + c_2 e^{-x}$

$P.I = y_p = \frac{1}{f(D)} e^{ax}$ $\left[\because D = a,\ D = 2\right]$

$= \frac{1}{(D^2 + 4D + 3)} e^{2x}$

$= e^{2x}\left(\frac{1}{(2^2 + 4(2) + 3)}\right)$

$= \frac{e^{2x}}{15}$

$\therefore y = y_c + y_p$

$= c_1 e^{-3x} + c_2 e^{-x} + \frac{e^{2x}}{15}$

2 sol:- Given $(D^3 - 5D^2 + 8D - 4)y = e^{2x}$

A.E is $m^3 - 5m^2 + 8m - 4 = e^{2x}$

$m - 1 = 0$

$m = 1, 2, 2$

$$\begin{array}{c|cccc} & 1 & -5 & 8 & -4 \\ 1 & 0 & 1 & -4 & 4 \\ \hline & 1 & -4 & 4 & 0 \\ 2 & 0 & 2 & -4 & \\ \hline & 1 & -2 & 0 & \end{array}$$

$\therefore y_c = c_1 e^{x} + (c_2 + c_3 x) e^{2x}$

$P.I = y_p = \frac{1}{f(D)} e^{ax}$ $\left[\because D = a \\ D = 2\right]$

$= \frac{1}{D^3 - 5D^2 + 8D - 4} e^{2x}$

$= \frac{1}{2^3 - 5(2)^2 + 8(2) - 4} e^{2x}$

$= \left[\frac{1}{8 - 20 + 16 - 4}\right] e^{2x}$

$= \frac{1}{8 + 16 - 24} e^{2x}$

$= \frac{1}{24 - 24} e^{2x}$ $\Rightarrow \frac{1}{0} e^{x}$

Denominator should not become zero

So, $y_p = \frac{1}{f(D)} e^{ax} \quad [\because D = a]$

$= \frac{1}{(D^3 - 5D^2 + 8D - 4)} e^{2x}$

$= \frac{1}{(D-1)(D-2)^2} e^{2x}$

$= \frac{e^{2x}}{(2-1)} \frac{x^2}{2!} \quad \left(\frac{e^{ax}}{\phi(a)} \frac{x^n}{n!} \right)$

$= \frac{e^{2x}}{1} \frac{x^2}{2} = \frac{x^2 e^{2x}}{2}$

$y = c_1 e^x + (c_2 + c_3 x) e^{2x} + \frac{x^2 e^{2x}}{2}$

(3) Given $(4D^2 - 4D + 1)y = 100$

A.E: $4m^2 - 4m + 1 = 0$

$m = \frac{4 \pm \sqrt{16 - 4(4)(1)}}{2(4)}$

$= \frac{4 \pm \sqrt{16 - 4(4)}}{8}$

$= \frac{4 \pm 0}{8}$

$= \frac{1}{2}, \frac{1}{2}$

$y_c = (c_1 + c_2 x) e^{x/2}$

$$P.I = y_p = \frac{1}{f(D)} e^{ax} \quad \left[\because D = a,\ D = 2\right]$$

$$= \frac{1}{(4D^2 - 4D + 1)} \, 100\, e^{0x}$$

$$= \frac{e^{0x}\, 100}{0 - 0 + 1} = 100$$

$$y = y_c + y_p = (c_1 + c_2 x)e^{x/2} + 100$$

(4) Given $(D^2 - 3D + 2)y = \cosh x$

$$A.E = m^2 - 3m + 2 = 0$$

$$= m^2 - 2m - m + 2 = 0$$

$$= m(m-2) - 1(m-2) = 0$$

$$= (m-1)(m-2)$$

$$= m = 1, 2$$

$$y_c = c_1 e^{x} + c_2 e^{2x}.$$

~~P.I = $y_p = \frac{1}{f(D)} e^{ax}$ $[\because D = a]$~~

$$P.I = y_p = \frac{1}{D^2 - 3D + 2} \cosh x$$

$$= \frac{1}{D^2 - 3D + 2}\left[\frac{e^{x} + e^{-x}}{2}\right]$$

$$= \frac{1}{2}\left[\frac{1}{D^2 - 3D + 2} e^{x} + \frac{1}{D^2 - 3D + 2} e^{-x}\right]$$

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$$= \frac{1}{2}\left[\frac{1}{(D-1)(D-2)} e^{x}\right] + \frac{1}{2}\left[\frac{e^{-x}}{1+3+2}\right]$$

$$= \frac{1}{2}\left[\frac{e^{x}}{(1-2)} \cdot \frac{x^{1}}{1!}\right] + \frac{1}{2}\frac{e^{-x}}{6}$$

$$y_p = \frac{-xe^{x}}{2} + \frac{e^{-x}}{12}$$

$$y = y_c + y_p = c_1e^{x} + c_2e^{2x} + \left[\frac{-xe^{x}}{2} + \frac{e^{-x}}{12}\right]$$

(5) Solve $(D^2+5D+6)y = e^{x}$

Sol:- Given $(D^2+5D+6)y = e^{x}$

$A.E = m^2+5m+6 = 0$

$= \cancel{m^2+5m+m+6=0}$

$\cancel{= m(m+5)+1(m}$

$= m^2+3m+2m+6 = 0$

$= m(m+3)+2(m+3) = 0$

$= (m+2)(m+3)$

$\Rightarrow m = -2, -3$

$$y_c = c_1e^{-2x} + c_2e^{-3x}$$

$$y_p = \frac{1}{D^2+5D+6} e^{x}$$

$$= \frac{1}{1+5(1)+6} e^{x} = \frac{e^{x}}{12}$$

$$y = c_1 e^{-2x} + c_2 e^{-3x} + \frac{e^x}{12}$$

(6) Solve $(D^2+6D+9)y = 2e^{-3x}$

Sol: Given $(D^2+6D+9)y = 2e^{-3x}$

$$A.E = m^2+6m+9 = 0$$

$$= m^2+3m+3m+9 = 0$$

$$= m(m+3)+3(m+3) = 0$$

$$= (m+3)(m+3) = 0$$

$$= m = -3, -3$$

$$y_c = (c_1+c_2x)e^{-3x}$$

$$y_p = \frac{1}{f(D)} 2e^{-3x}$$

$$= \frac{1}{(D+3)(D+3)} 2e^{-3x}$$

$$= \frac{1}{(D+3)^2} 2e^{-3x}$$

$$= \cancel{2} e^{-3x} \cdot \frac{x^{\cancel{-3}\,2}}{(\cancel{-2})!}$$

$$y = y_c + y_p$$

$$= (c_1+c_2x)e^{-3x} + \cancel{2e^{-3x}\cdot\frac{x^{-3}}{(-3)!}}\; x^2 e^{-3x}$$

H/w $(D^3-3D^2+4)y = (1+e^{-x})^3$

$(D^3-3D^2+4)y = 1+3e^{-x}+3e^{-2x}+e^{-3x}$

$m^3-3m^2+4=0$

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* Method - 2 :-

Let $f(D)y = \sin bx$ (or) $\cos bx \longrightarrow$ (1)

then $y_p = \frac{1}{f(D)} \sin bx$ (or) $\cos bx$. $(f(D) \neq 0)$

Let $f(D) = \phi(D^2)$

then $\phi(D^2) \sin bx = \phi(-b^2)$

$\therefore y_p = \frac{1}{\phi(-b^2)} \sin bx$ (or) $\cos bx$

If 'D' should be occured then rationalize with $(-b^2)$ term then we get D^2 term again substitute $-b^2$, continuiting this process we get constant term, Here numerical "D" act as derivating. The Denominator "D" of integrating part.

(1) $\frac{1}{D^2+b^2} \sin bx = \frac{-x \cos bx}{2b}$

(2) $\frac{1}{D^2+b^2} \cos bx = \frac{x \sin bx}{2b}$

1) Solve $(D^2+3D+2)y = \sin 3x$

Sol: A.E = $m^2+3m+2 = 0$

$m^2+2m+m+2 = 0$

$\Rightarrow m(m+2)+1(m+2) = 0$ $m = -1, -2$

$$y_c = c_1 e^{-x} + c_2 e^{-2x}$$

$$y_p = \frac{1}{D^2+3D+2} \sin 3x \quad \left[\begin{array}{l} \because D^2 = -b^2 \\ b = 3 \\ D^2 = -9 \end{array}\right]$$

$$= \sin 3x \left[\frac{1}{-9+3D+2}\right]$$

$$= \frac{\sin 3x}{3D-7}$$

$$= \frac{(3D+7)\sin 3x}{(3D-7)(3D+7)}$$

$$= \frac{3D\sin 3x + 7\sin 3x}{9D^2-49}$$

$$= \frac{3D\sin 3x + 7\sin 3x}{9(-9)-49}$$

$$= \frac{3(3\cos 3x) + 7\sin 3x}{-130}$$

$$= \frac{9\cos 3x + 7\sin 3x}{-130}$$

$$y = y_c + y_p = c_1 e^{-x} + c_2 e^{-2x} + \frac{9\cos 3x + 7\sin 3x}{-130}$$

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(2) Solve $(D^2+4)y = \sin 2x + \cos 2x$

Sol:- Given $(D^2+4)y = \sin 2x + \cos 2x$

$A.E = m^2+4 = 0$

$\Rightarrow m^2 = -4$

$\Rightarrow m = \pm\sqrt{-4}$

$\therefore m = \cancel{\pm 2} \ \pm 2i$

$m = 0 \pm i2$

~~$y_c =$~~

$y_c = (c_1 \cos 2x + c_2 \sin 2x) e^{0x}$

$$y_p = \frac{1}{f(D)} Q(x)$$

$$= \frac{1}{D^2+4} (\sin 2x + \cos 2x)$$

$$= \frac{1}{D^2+4} \sin 2x + \frac{1}{D^2+4} \cos 2x$$

$$y_p = \frac{-x\cos 2x}{2(2)} + \frac{x \sin 2x}{2(2)} \quad \left[\begin{array}{l} \because \frac{1}{D^2+b^2} \sin bx = \frac{-x\cos bx}{2b} \\ \frac{1}{D^2+b^2} \cos bx = \frac{x \sin bx}{2b} \end{array} \right]$$

$$= \frac{-x\cos 2x}{4} + \frac{x \sin 2x}{4}$$

** (3) Solve $y'' + 4y' + 4y = 4\cos x + 3\sin x$,

$y(0) = 0, \quad y'(0) = 0$

Solve : $(D^2-4D+3)y = \sin 3x \cos 2x$

(4)

(5) solve $y'' - 2y' + 2y = e^x + \cos x + 10$

Sol:- Given $y'' + 4y' + 4y$

$$m^2 + 4m + 4 = 0$$

$$m^2 + 2m + 2m + 4 = 0$$

$$m(m+2) + 2(m+2) = 0$$

$$(m+2)(m+2) = 0$$

$$m = -2, -2$$

$$y_c = (c_1 + c_2 x)e^{-2x}$$

$$y_p = \frac{1}{(D^2+4D+4)}(4\cos x) + \frac{1}{D^2+4D+4}(3\sin x)$$

[Here $D^2 = -b^2$, $b = 1$, $D^2 = -1$] [Here $D^2 = -b^2$, $b = 1$, $D^2 = -1$]

$$= \frac{4\cos x}{(-1+4D+4)} + \frac{3\sin x}{-1+4D+4}$$

$$= \frac{4\cos x}{4D+3} + \frac{3\sin x}{4D+3}$$

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$$= \frac{4\cos x(4D-3)}{(4D+3)(4D-3)} + \frac{3\sin x(4D-3)}{(4D+3)(4D-3)}$$

$$= \frac{4\cos x(4D-3)}{16D^2-9} + \frac{3\sin x(4D-3)}{16D^2-9} \quad [\because D^2=-1]$$

$$= \frac{16D\cos x - 12\cos x}{-16-9} + \frac{12D\sin x - 9\sin x}{-16-9}$$

$$= \frac{16(-\sin x) - 12\cos x}{-25} + \frac{12\cos x - 9\sin x}{-25}$$

$$= \frac{16\sin x + \cancel{12\cos x} - \cancel{12\cos x} + 9\sin x}{25}$$

$$= \frac{\cancel{25}\sin x}{\cancel{25}}$$

$$y_p = \sin x$$

$$y = y_c + y_p = (c_1 + c_2 x)e^{-2x} + \sin x \longrightarrow (1)$$

$\Rightarrow x=0, y=0$

$$0 = (c_1 + c_2(0))e^0 + 0$$

$$\boxed{\Rightarrow c_1 = 0}$$

$$y = (c_1 e^{-2x} + c_2 x e^{-2x}) + \sin x$$

$$\frac{dy}{dx} = y' = c_1 e^{-2x}(-2) + c_2\left(x e^{-2x}(-2) + e^{-2x}\right) + \cos x$$

$y'(0) = 0 \Rightarrow y' = 0,\ x = 0,\ c_1 = 0$

$$0 = 0 + c_2(0) + c_2 e^{-0} + \cos 0$$

$c_2 + 1 = 0$

$\boxed{c_2 = -1}$

Now sub c_1 & c_2 values in ① we get

$$y = (0 + (-1)x)e^{-2x} + \sin x$$

$$y = -xe^{-2x} + \sin x$$

(4) Given $(D^2 - 4D + 3)y = \sin 3x \cos 3x$.

$\Rightarrow m^2 - 4m + 3 = 0$

$\Rightarrow \cancel{m^2 - 4m - m + 3} = 0$

$= m^2 - 3m - m + 3 = 0$

$= m(m-3) - 1(m-3) = 0$

$= (m-1)(m-3) = 0$

$= m = 1, 3$

$$y_c = c_1 e^{x} + c_2 e^{3x}$$

$$y_p = \frac{1}{D^2 - 4D + 3} \sin 3x \cos 2x$$

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$$\left[\because 2\sin A\cos B = \sin(A+B)+\sin(A-B) \quad \sin 3x\cos 2x = \frac{\sin(5x)+\sin x}{2}\right]$$

$$y_p = \frac{1}{D^2-4D+3}\left[\frac{\sin 5x+\sin x}{2}\right]$$

$$y_p = \frac{1}{2}\left[\frac{1}{D^2-4D+3}(\sin 5x) + \frac{1}{D^2-4D+3}\sin x\right]$$

$$\left[\because D^2 = -b^2 \quad D^2 = -25\right] \quad \left[\because D^2 = -b^2 \quad D^2 = -1\right]$$

$$= \frac{1}{2}\left[\frac{\sin 5x}{-25-4D+3} + \frac{\sin x}{-1-4D+3}\right]$$

$$= \frac{1}{2}\left[\frac{\sin 5x}{-4D-22} + \frac{\sin x}{-4D+2}\right]$$

$$= \frac{1}{2}\left[\frac{-\sin 5x(4D-22)}{(4D+22)(4D-22)} + \frac{\sin x(2+4D)}{(2-4D)(2+4D)}\right]$$

$$= \frac{1}{2}\left[\frac{-4D\sin 5x+22\sin 5x}{16D^2-484} + \frac{2\sin x+4D\sin x}{4-16D^2}\right]$$

$$[\text{Here } D^2 = -25] \quad [\because D^2 = -1]$$

$$= \frac{1}{2}\left[\frac{-20\cos 5x+22\sin 5x}{-884} + \frac{2\sin x+4\cos x}{20}\right]$$

$$= \frac{10\cos 5x - 11\sin 5x}{884} + \frac{\sin x + 2\cos x}{20}$$

$y = y_c + y_p$

$$= c_1 e^{x} + c_2 e^{3x} + \frac{10\cos 5x - 11\sin 5x}{884} + \frac{\sin x + 2\cos x}{20}$$

(5) Given $y'' - 2y' + 2y = e^{x} + \cos x + 10$

$A.E = m^2 - 2m + 2 = 0$

$$= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{2 \pm \sqrt{4 - 4(1)(2)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{4 - 8}}{2} = \frac{2 \pm \sqrt{-4}}{2}$$

$$= \frac{2 \pm 2i}{2}$$

$= 1 \pm i$

Roots are complex

$y_c = (c_1\cos x + c_2(\sin x))e^{x}$

$y_p = \frac{1}{D}\left[e^{x} + \cos x + 10\right]$

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$$= \frac{1}{D^2-2D+2}\left[e^x + \cos x + 10e^0\right]$$

$$= \frac{e^x}{D^2-2D+2} + \frac{\cos x}{D^2-2D+2} + \frac{10e^0}{D^2-2D+2}$$

$\left[\begin{array}{l} D=a \\ =1 \end{array}\right]$ $\left[\because D^2=-b^2,\ b=1,\ -b^2=-1\right]$ $\left[\because D=a,\ a=0\right]$

$$= \frac{e^x}{\cancel{1-2(1)+2}\ D^2-2(1)+2} + \frac{\cos x}{-1-2D+2} + \frac{10e^0}{-2(0)+2}$$

$$= \frac{e^x}{1} + \frac{\cos x}{1-2D} + \frac{10e^0}{2}$$

$$= \frac{e^x}{1} + \frac{\cos x}{1-2D} + 5e^0$$

$$= e^x + \frac{\cos x(1+2D)}{(1-2D)(1+2D)} + 5e^0$$

$$= e^x + \frac{\cos x - 2\sin x}{1-4D^2} + 5 \quad [\because D^2=-1]$$

$$y = y_c + y_p$$

$$= (c_1\cos x + c_2\sin x)e^x + e^x + \frac{\cos x - 2\sin x}{\cancel{1-4D^2}\ 5} + 5$$

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* Method-3:-

Let $f(D)y = B\, x^k$

Now, par $y_p = \frac{1}{f(D)} x^k$

To ~~exp~~ evaluate y_p (P.I) reduce ~~the~~ $\frac{1}{f(D)}$ to the form $\frac{1}{[1 \pm \phi(D)]}$ by taking out the lowest degree term from $f(D) \Rightarrow [1 \pm \phi(D)]^{-1}$ and expand it in ascending powers of "D" using binomial theorem upto the term containing "D^k".

* Formulae:-

$$(1-x)^{-1} = 1+x+x^2+x^3+ \cdots$$

$$(1+x)^{-1} = 1-x+x^2-x^3+ \cdots$$

$$(1-x)^{-2} = 1+2x+3x^2+4x^3+ \cdots$$

$$(1+x)^{-2} = 1-2x+3x^2-4x^3+ \cdots$$

$$(1-x)^{-3} = 1+3x+6x^2+10x^3+ \cdots$$

$$(1+x)^{-3} = 1-3x+6x^2-10x^3+ \cdots$$

① Solve $(D^2+D+1)y = x^3$

Sol:- A.E $= m^2+m+1$

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$$= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)}$$

$$= \frac{-1 \pm \sqrt{1-4}}{2(1)} = \frac{-1 \pm \sqrt{-3}}{2}$$

$$= \frac{-1 \pm \sqrt{3}}{2} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}$$

$$y_c = \left(c_1 \cos \frac{\sqrt{3}}{2} x + c_2 \sin \frac{\sqrt{3}}{2} x\right) e^{-1/2 x}$$

$$y_p = (D^2 + D + 1) y = x^3$$

$$y_p = \frac{1}{f(D)} x^k$$

$$y_p = \frac{1}{(D^2 + D + 1)} x^3$$

$$= \frac{1}{[1 + (D + D^2)]} x^3$$

$$= [1 + (D + D^2)]^{-1} x^3 \quad [\because (1+x)^{-1} = 1 - x + x^2 - x^3 + \dots]$$

$$= [1 - (D + D^2) + (D + D^2)^2 - (D + D^2)^3] \, x^3$$

$$= [1 - D - \cancel{D^2} + \cancel{D^2} + 2D^3 + D^4 - D^3 - 3D^4 - 3D^5 - D^6] \, x^3$$

$$= [1 - D + D^3] x^3$$

$$= [x^3 - 3x^2 + 6]$$

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$y = y_c + y_p$

$= \left(c_1 \cos\frac{\sqrt{3}}{2}x + c_2 \sin\frac{\sqrt{3}}{2}x\right)e^{-x/2} + (x^3 - 3x^2 + 6)$

(2) Solve $(D^2(D^2+4))y = 320(x^3+2x^2)$

Sol: Given $(D^2(D^2+4))y$

$A \cdot E = \cancel{D^4}\ m^2(m^2+4) = 0$

$\cancel{m^4+4m^2}$

$= m^2 = -4$

$= m = \pm\sqrt{4}$

$= m = \pm 2$

$= 0, 0, \pm 2i$

$y_c = (C_1 + C_2 x)e^{0x} + (C_1\cos 2x + C_2 \sin 2x)e^{0x}$

$y_p = \frac{1}{f(D)} x^k$

$= \frac{1}{D^2(D^2+4)} 320(x^3+2x^2)$

$= 320\left[\frac{1}{4D^2\left(1+\frac{D^2}{4}\right)}\right](x^3+2x^2)$

$= \frac{80}{D^2}\left[\left(1+\frac{D^2}{4}\right)\right]^{-1}(x^3+2x^2)$

$= \frac{80}{D^2}\left[1 - \frac{D^2}{4} + \left(\frac{D^2}{4}\right)^2 - \left(\frac{D^2}{4}\right)^3\right](x^3+2x^2)$

$= \frac{80}{D^2}\left[1 - \frac{D^2}{4} + \frac{D^4}{16} - \frac{D^6}{64}\right](x^3+2x^2)$

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$$= 80\left[\frac{1}{D^2} - \frac{1}{4} + \frac{D^2}{16} - \frac{D^4}{64}\right](x^3+2x^2)$$

$$= 80\left[\frac{(x^3+2x^2)}{D^2} - \frac{(x^3+2x^2)}{4} + \frac{D^2(x^3+2x^2)}{16} - \frac{D^4(x^3+2x^2)}{64}\right]$$

($\because$ $1/D^2$ means two times integral)

$$= 80\left[\frac{x^5}{20} + \frac{2x^4}{12} - \frac{x^3}{4} - \frac{x^2}{2} + \frac{6x}{16} + \frac{4}{16} - \frac{0}{64} - \frac{2(0)}{64}\right]$$

$$= 80\left[\frac{x^5}{20} + \frac{x^4}{6} - \frac{x^3}{4} - \frac{x^2}{2} + \frac{3x}{8} + \frac{1}{4}\right]$$

$$y = y_c + y_p$$

$$= (c_1+c_2x) + (c_1\cos 2x + c_2\sin 2x) + 80\left[\frac{x^5}{20} + \frac{x^4}{6} - \frac{x^3}{4} - \frac{x^2}{2} + \frac{3x}{8} + \frac{1}{4}\right]$$

H/w solve

③ $(D^3 - 3D - 2)y = x^2$

④ solve $(D^3 - D^2 - 6D)y = 1 + x^2$

***

3 sol:- Given $(D^3 - 3D - 2)y = x^2$

$$A.E = m^3 - 3m - 2$$

$$\begin{array}{r|rrrr} -1 & 1 & 0 & -3 & -2 \\ & 0 & -1 & 1 & 2 \\ \hline 2 & 1 & -1 & -2 & \lfloor 0 \\ & 0 & 2 & 2 & \\ \hline & 1 & 1 & \lfloor 0 & \end{array}$$

$\Rightarrow$ $m = -1,\ m = 2$

$m+1 = 0,\ m = -1$

$m = -1, -1, 2$

$y_c = (c_1 + c_2 x) e^{-1x} + c_3 e^{2x}$

$y_p = (D^3 - 3D - 2)y = x^2$

$y_p = \frac{1}{f(D)} x^k$

$= \frac{1}{D^3 - 3D - 2} x^2$

$= \frac{1}{2\left(\frac{D^3}{2} - \frac{3}{2}D - 1\right)} x^2$

$= \frac{1}{2} \left[\frac{1}{\left(-1 - \frac{3}{2}D + \frac{D^3}{2}\right)}\right] x^2$

$= \frac{1}{2} \left[\frac{1}{-\left(1 + \frac{3}{2}D - \frac{D^3}{2}\right)}\right] x^2$

$= \frac{-1}{2} \left[\frac{1}{\left(1 + \left(\frac{3}{2}D - \frac{D^3}{2}\right)\right)}\right] x^2$

$= \frac{-1}{2} \left[\left(1 + \left(\frac{3}{2}D - \frac{D^3}{2}\right)\right)\right]^{-1} x^2$

$$= -\frac{1}{2}\left[1 - \left(\frac{3D}{2} - \frac{D^3}{2}\right) + \left(\frac{3D}{2} - \frac{D^3}{2}\right)^2\right]x^2$$

$$= -\frac{1}{2}\left[1 - \frac{3D}{2} + \frac{D^3}{2} + \frac{9D^2}{4} + \frac{D^6}{4} - \frac{6D^4}{4}\right]x^2$$

$$= -\frac{1}{2}\left[1 - \frac{3D}{2} + \frac{9D^2}{4}\right]x^2$$

$$= -\frac{1}{2}\left[x^2 - \frac{\overset{3}{\cancel{6}}x}{\underset{1}{\cancel{2}}} + \frac{\overset{9}{\cancel{18}}}{\underset{2}{\cancel{4}}}\right]$$

$$= -\frac{1}{2}\left[x^2 - 3x + \frac{9}{2}\right]$$

$y = y_c + y_p$

$$= (c_1 + c_2x)e^{-1x} + c_3e^{2x} - \frac{1}{2}\left[x^2 - 3x + \frac{9}{2}\right].$$

$$= (c_1 + c_2x)e^{-1x} + c_3e^{2x} - \frac{x^2}{2} + \frac{3x}{2} - \frac{9}{4}$$

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(4) Given $(D^3 - D^2 - 6D)y = 1 + x^2$

A.E $= m^3 - m^2 - 6m$

$= 3, -2, 0$

$$\begin{array}{r|rrrr} 0 & 1 & -1 & -6 & 0 \\ & 0 & 0 & 0 & 0 \\ \hline -2 & 1 & -1 & -6 & 0 \\ & 0 & 4 & & \\ \hline 3 & & -2 & & \end{array}$$

$$y_c = c_1e^{3x} + c_2e^{-2x} + c_3e^{0x}$$

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$$y_p = \frac{1}{(D^3 - D^2 - 6D)}(1+x^2)$$

$$= \frac{1}{-6D\left(1-\left(\frac{D^2}{6}-\frac{D}{6}\right)\right)}(1+x^2)$$

$$= -\frac{1}{6D}\left[1-\left(\frac{D^2}{6}-\frac{D}{6}\right)\right]^{-1}(1+x^2)$$

$$= \frac{-1}{6D}\left[1+\left(\frac{D^2}{6}-\frac{D}{6}\right)+\left(\frac{D^2}{6}-\frac{D}{6}\right)^2\right](1+x^2)$$

$$= \frac{-1}{6D}\left[1+\frac{D^2}{6}-\frac{D}{6}+\frac{D^4}{36}+\frac{D^2}{36}-\frac{D^3}{18}\right](1+x^2)$$

$$= \left[\frac{-1}{6D}-\frac{7D^2}{(6)(36)D}+\frac{D}{36D}-\frac{D^4}{(6)(36)D}+\frac{D^3}{(6)(18)D}\right](1+$$

$$= \left[\frac{-1}{6D}-\frac{7D}{216}+\frac{1}{36}+\frac{D^2}{108}\right](1+x^2)$$

$$= \left[-\frac{(1+x^2)}{6D}-\frac{7D(1+x^2)}{216}+\frac{(1+x^2)}{36}+\frac{D^2(1+x^2)}{108}\right]$$

$$= \left[\frac{-1}{6}\left(x+\frac{x^3}{3}\right)-\frac{7(2x)}{216}+\frac{(1+x^2)}{36}+\frac{(2)}{108}\right]$$

$$= \left[\frac{-x}{6}-\frac{x^3}{18}-\frac{14x}{216}+\frac{1}{36}+\frac{x^2}{36}+\frac{1}{54}\right]$$

$$y_p = \left[\frac{-x^3}{18}+\frac{x^2}{36}-\frac{25x}{108}+\frac{5}{108}\right]$$

$$y = c_1e^{3x}+c_2e^{-2x}+c_3e^{0x}+\left[\frac{-x^3}{18}+\frac{x^2}{36}-\frac{25x}{108}+\frac{5}{108}\right]$$

* Method - 4 & 5:-

Let $f(D)y = e^{ax}\cos bx$ (or) $e^{ax}\sin bx$ (or) $e^{ax}x^k$

P.I - $(y_p) = \frac{1}{f(D)} e^{ax}\cos bx$ (or) $e^{ax}\sin bx$ (or) $e^{ax}x^k$

In step-1, we can substitute "D" = D+a

$\Rightarrow e^{ax}\left[\frac{1}{f(D+a)}\right]\cos bx$ (or) $\sin bx$ (or) x^k

Remaining steps we can solve by earlier method

① Solve $(D^2-7D+6)y = e^{2x}(1+x)$

② solve $(D^2+2)y = e^{x}\cos x$

① Given $(D^2-7D+6)y = e^{2x}(1+x)$

Sol:-

$$A.E = m^2-7m+6$$
$$= m^2-6m-m+6$$
$$= m(m-6)-1(m-6)$$
$$= (m-1)(m-6)$$
$$= m = 1,6$$

$$y_c = c_1e^{x}+c_2e^{6x}$$

$$y_p = \frac{1}{D^2 - 7D + 6} e^{2x}(1+x) \qquad \left[\begin{array}{l} \because D = D + a \\ \text{Here } a = 2 \\ D = D + 2 \end{array}\right]$$

$$= e^{2x}\left[\frac{1}{(D+2)^2 - 7(D+2) + 6}\right](1+x)$$

$$= e^{2x}\left[\frac{1}{D^2 + 4D + 4 - 7D - 14 + 6}\right](1+x)$$

$$= e^{2x}\left[\frac{1}{D^2 - 3D \; \cancel{+} - 4}\right](1+x)$$

$$= e^{2x}\left[\frac{1}{-4\left[1 - \left[\frac{D^2 - 3D}{4}\right)\right]}\right](1+x)$$

$$= \frac{e^{2x}}{-4}\left[1 - \left(\frac{D^2 - 3D}{4}\right)\right]^{-1}(1+x)$$

$$= \frac{e^{2x}}{-4}\left[1 + \left(\frac{D^2 - 3D}{4}\right)\right](1+x)$$

$$= \frac{e^{2x}}{-4}\left[1 + \left(\frac{D^2}{4}\right) - \frac{3D}{4}\right](1+x)$$

$$= \frac{e^{2x}}{-4}\left[1 + x + \frac{D^2}{4}(1+x) - \frac{3D}{4}(1+x)\right]$$

$$= \frac{e^{2x}}{-4}\left[1 + x - \frac{3}{4}(1)\right] = \frac{e^{2x}}{-4}\left[x + \frac{1}{4}\right]$$

$$y = c_1 e^{x} + c_2 e^{6x} + \frac{e^{2x}}{-4}\left(x + \frac{1}{4}\right)$$

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4) $(D^2+2)y = e^{2x} + \cos 2x + x^2 + e^x(x) + e^{2x}\sin 2x$

2) Given $(D^2+2)y = e^x \cos x$

A.E $m^2+2=0$

$m = \pm i\sqrt{2}$

$y_c = c_1 \cos\sqrt{2}x + c_2 \sin\sqrt{2}x$

$$y_p = \frac{1}{(D^2+2)} e^x \cos x$$

$$= e^x \left[\frac{1}{(D+1)^2+2}\right] \cos x \quad (\because a=1)$$

$$= e^x \left(\frac{1}{D^2+1+2D+2}\right) \cos x \quad \left[\begin{array}{c} D^2=-b^2 \\ b=1 \\ D^2=-1 \end{array}\right]$$

$$= e^x \left(\frac{1}{D^2+2D+3}\right) \cos x$$

$$= e^x \left(\frac{1}{-1+2D+3}\right) \cos x$$

$$= \frac{e^x}{2} \left(\frac{1}{D+1}\right) \cos x$$

$$= \frac{e^x}{2} \left[\frac{D-1}{D^2-1}\right] \cos x$$

$$= \frac{e^x}{2} \left[\frac{-\sin x - \cos x}{-1-1}\right] = \frac{e^x}{2} \left[\frac{\sin x + \cos x}{2}\right]$$

$$= \frac{e^{x}(\sin x + \cos x)}{4}$$

$$y = y_c + y_p = c_1 \cos\sqrt{2}x + c_2 \sin\sqrt{2}x + \frac{e^{x}(\sin x + \cos x)}{4}$$

(5) $(D^2+2)y = e^{x}\cos 2x + x^2 e^{3x}$

$$y_c = c_1 \cos\sqrt{2}x + c_2 \sin\sqrt{2}x$$

$$y_{p_2} = \frac{1}{f(D)} e^{3x} x^2 \qquad y_{p_1} = \frac{1}{f(D)} e^{x}\cos 2x$$

$$y_{p_1} = \frac{1}{f(D)} e^{x}\cos 2x = e^{x}\left[\frac{1}{D^2+2}\right]\cos 2x$$

$$= e^{x}\left[\frac{1}{(D+1)^2+2}\right]\cos 2x$$

$$= e^{x}\left[\frac{1}{D^2+2D+1+2}\right]\cos 2x$$

$$= e^{x}\left[\frac{1}{D^2+2D+3}\right]\cos 2x \qquad \left(\begin{array}{l} D^2 = -b^2 \\ b = 2 \\ D^2 = -4 \end{array}\right)$$

$$= e^{x}\left[\frac{1}{-4+2D+3}\right]\cos 2x$$

$$= e^{x}\left[\frac{1}{2D-1}\right]\cos 2x$$

$$= e^{x}\left[\frac{2D+1}{4D^2-1}\right]\cos 2x$$

$$= e^{x}\left[\frac{2D\cos 2x + \cos 2x}{-17}\right]$$

$$= e^{x}\left[\frac{-4\sin 2x + \cos 2x}{-17}\right]$$

$$y_{p_2} = \frac{1}{(D^2+2)} e^{3x} x^2$$

$$= e^{3x}\left[\frac{1}{(D+3)^2+2}\right] x^2$$

$$= e^{3x}\left[\frac{1}{D^2+9+6D+2}\right] x^2$$

$$= e^{3x}\left[\frac{1}{D^2+6D+11}\right] x^2$$

$$= \frac{e^{3x}}{11}\left[\frac{1}{1+\frac{D^2+6D}{11}}\right] x^2$$

$$= \frac{e^{3x}}{11}\left[1+\frac{D^2+6D}{11}\right]^{-1} x^2 \quad \left[\because (1+x)^{-1} = 1-x+x^2 \dots\right]$$

$$= \frac{e^{3x}}{11}\left[1-\frac{(D^2+6D)}{11}+\frac{(D^2+6D)^2}{121}\right] x^2$$

$$= \frac{e^{3x}}{11}\left[x^2 - \frac{D^2x^2}{11} - \frac{6Dx^2}{11} + \left(\frac{D^4+36D^2+12D^3}{121}\right)x^2\right]$$

$$= \frac{e^{3x}}{11}\left[x^2 - \frac{2}{11} - \frac{12x}{11} + \frac{72}{121}\right]$$

$$y_p = y_{p_1} + y_{p_2} = e^{x}\left[\frac{-4\sin 2x + \cos 2x}{-17}\right] + \frac{e^{3x}}{11}\left[x^2 - \frac{2}{11} - \frac{12x}{11} + \frac{72}{121}\right]$$

$$y = y_c + y_p$$

$$= c_1\cos\sqrt{2}x + c_2\sin\sqrt{2}x + e^{x}\left[\frac{-4\sin 2x + \cos x}{-17}\right] + \frac{e^{3x}}{11}\left[x^2 - \frac{2}{11} - \frac{12x}{11} + \frac{72}{121}\right]$$

~~Type 6:~~ Method 6 :-

let $f(D)y = xV$

Type (i) :- Here $V = \sin bx / \cos bx$

P.I $\Rightarrow y_p = \frac{1}{f(D)} xV$

$$y_p = \left[x - \frac{f'(D)}{f(D)}\right]\frac{1}{f(D)} V$$

Type (ii) :- let $f(D)y = x^k V$

Here $V = \sin bx / \cos bx$ & $k > 1$

Here $y_p = \frac{1}{f(D)} x^k V$

By using imaginary Part & Real Part of the procedure of V, we can write it as.

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i) $\Rightarrow$ $y_p = \frac{1}{f(D)} x^k \sin bx$

I.P (Imaginary part)

$= \text{I.P of } \frac{1}{f(D)} x^k (\cos bx + i \sin bx)$

$= \text{I.P of } \frac{1}{f(D)} x^k e^{ibx}$

ii) $\Rightarrow$ $y_p = \frac{1}{f(D)} x^k \cos bx$

R.P (Real part)

$= \text{real part of } \frac{1}{f(D)} x^k e^{ibx}$

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① $(D^2+4)y = x \sin x$

② $(D^2-5D+6)y = x \cos 2x$

Ex ① Sol:- Given $(D^2+4)y = x \sin x$

A.E $m^2+4=0$

$m = \pm i\sqrt{4} = \pm i(2)$

$y_c = C_1 \cos\sqrt{4}x + C_2 \sin\sqrt{4}x$

$y_c = C_1 \cos 2x + C_2 \sin 2x$

$y_p = \frac{1}{f(D)} Q(x)$

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$$= \frac{1}{(D^2+4)}(x\sin x)$$

$$y_p = \left[x - \frac{f'(D)}{f(D)}\right]\frac{1}{f(D)}(V)$$

$$= \left[x - \frac{2D}{D^2+4}\right]\frac{1}{D^2+4}(\sin x) \qquad \left[\because D^2 = -b \quad D^2 = -1\right]$$

$$= \left[x - \frac{2D}{D^2+4}\right]\frac{1}{-1+4}\sin x$$

$$= \left[x - \frac{2D}{D^2+4}\right]\left[\frac{1}{3}\sin x\right]$$

$$= \frac{1}{3}\left[x\sin x - \left[\frac{2D}{D^2+4}\right]\sin x\right] \qquad \left[\because D^2 = -b^2 \quad D = -1\right]$$

$$= \frac{1}{3}\left[x\sin x - \left[\frac{2D}{-1+4}\right]\sin x\right]$$

$$= \frac{1}{3}\left[x\sin x - \frac{2\cos x}{3}\right]$$

$$y_p = \frac{1}{9}\left[3x\sin x - 2\cos x\right]$$

(2)

Sol:- Given

$$(D^2 - 5D + 6)y = x\cos 2x$$

$$A.E = m^2 - 5m + 6 = 0$$

$$\Rightarrow m^2 - 3m - 2m + 6 = 0$$

$$\Rightarrow m(m-3) - 2(m-3) = 0$$

$$\Rightarrow m = 2, 3$$

$$y_c = c_1 e^{2x} + c_2 e^{3x}$$

$$y_p = \frac{1}{f(D)} Q(x)$$

$$= \frac{1}{(D^2-5D+6)} (x\cos 2x)$$

$$y_p = \left[x - \frac{f'(D)}{f(D)}\right] \frac{1}{f(D)} (V)$$

$$= \left[x - \frac{(2D-5)}{(D^2-5D+6)}\right] \frac{1}{D^2-5D+6} (\cos 2x) \quad \left[\because D^2 = -b^2 \quad D^2 = -4\right]$$

$$= \left[x - \frac{(2D-5)}{D^2-5D+6}\right] \frac{1}{-4-5D+6} (\cos 2x)$$

$$= \left[x - \frac{(2D-5)}{D^2-5D+6}\right] \frac{1}{-5D+2} \cos 2x$$

$$= \left[x - \frac{(2D-5)}{D^2-5D+6}\right] \frac{(2+5D)\cos 2x}{(2-5D)(2+5D)}$$

$$= \left[x - \frac{(2D-5)}{D^2-5D+6}\right] \left[\frac{2\cos 2x + 5(-2\sin 2x)}{104}\right]$$

$$= \left[x - \frac{(2D-5)}{D^2-5D+6}\right] \left[\frac{2\cos 2x - 10\sin 2x}{104}\right]$$

$$= x\left(\frac{2\cos 2x - 10\sin 2x}{104}\right) - \frac{(2D-5)}{D^2-5D+6} \left(\frac{2\cos 2x - 10\sin 2x}{104}\right)$$

108 (or else)

$$\left[\frac{2x\cos 2x - 10x\sin 2x}{104}\right] - \frac{2}{104}\left[\frac{(2D-5)\cos 2x}{D^2-5D+6}\right] + \frac{10}{104}\left[\frac{(2D-5)\sin 2x}{D^2-5D+6}\right]$$

$$= \left[\frac{2x\cos 2x - 10x\sin 2x}{104}\right] - \frac{1}{52}\left[\frac{(2D-5)\cos 2x}{-4-5D+6}\right] + \frac{5}{52}\left(\frac{(2D-5)(\sin 2x)}{-4-5D+6}\right)$$

$$=$$

$$\Rightarrow \left(\frac{2x\cos 2x - 10x\sin 2x}{104}\right) - \frac{2}{52}\left[\frac{D\cos 2x}{-5D+2}\right] + \frac{5}{52}\left[\frac{\cos 2x}{-5D+2}\right] + \frac{10}{52}\frac{(D\sin 2x)}{-5D+2} - \frac{25}{52}\left(\frac{\sin 2x}{-5D+2}\right)$$

$$\Rightarrow \left[\frac{2x\cos 2x - 10x\sin 2x}{104}\right] - \frac{1}{26}\left[\frac{D\cos 2x\,(2+5D)}{-25D^2+4}\right] + \frac{5}{52}\frac{\cos 2x\,(2+5D)}{52(-25D^2+4)} + \frac{5}{26}\left[\frac{D\sin 2x\,(2+5D)}{-25D^2+4}\right] - \frac{25}{52}\left[\frac{\sin 2x\,(2+5D)}{-25D^2+4}\right]$$

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$$= \left[\frac{2x\cos 2x - 10x\sin 2x}{104}\right] - \frac{1}{26}\left[\frac{2(-2\sin 2x) + 5(-4\cos 2x)}{104}\right] +$$

$$\frac{5}{52}\left[\frac{2\cos 2x + (-10\sin 2x)}{104}\right] + \frac{5}{26}\left[\frac{4\cos 2x - 20\sin 2x}{104}\right]$$

$$-\frac{25}{52}\left[\frac{2\sin 2x + 10\cos 2x}{104}\right]$$

③ $(D^2 + 2D + 1)y = x\cos x$

Sol:- A.E $m^2 + 2m + 1 = 0$

$m^2 + m + m + 1 = 0$

$m(m+1) + 1(m+1) = 0$

$(m+1)(m+1)$

$m = -1, -1$

$y_c = (C_1 + C_2 x)e^{-x}$

$$y_p = \frac{1}{D^2 + 2D + 1} x\cos x$$

$$= \left[x - \frac{2D+2}{D^2+2D+1}\right]\frac{1}{D^2+2D+1}\cos x \quad \left[\because D^2 = -b^2, \; D^2 = -1\right]$$

$$= \left[x - \frac{2\cancel{(D+1)}}{(D+1)^{\cancel{2}}}\right]\frac{1}{\cancel{-1} + 2D + \cancel{1}}\cos x$$

$$= \left[x - \frac{2}{D+1}\right]\frac{1}{2D}\cos x$$

$$=\frac{1}{2}\left[x\sin x-\frac{2(D-1)\cdot\sin x}{(D+1)(D-1)}\right]$$

$$=\frac{1}{2}\left[x\sin x-\frac{(2\cos x-2\sin x)}{-1-1}\right]$$

$$y_p=\frac{1}{4}\left[2x\sin x+2\cos x-2\sin x\right]$$

$$y=y_c+y_p=(c_1+c_2x)e^{-x}+\frac{1}{4}\left[2x\sin x+2\cos x-2\sin x\right]$$

⑨ Solve $(D^2-4D+4)y=x^2\sin x$

Sol:- A.E $=m^2-4m+4=0$

$=m^2-2m-2m+4=0$

$=m(m-2)-2(m-2)=0$

$\Rightarrow m=2,2$

$$y_c=(c_1+c_2x)e^{2x}$$

$$y_p=\frac{1}{D^2-4D+4}x^2\sin x$$

$$=\text{I.P of }\left(\frac{1}{D^2-4D+4}\right)x^2e^{ix}$$

$$=\text{I.P of }e^{ix}\left[\frac{1}{(D+i)^2-4(D+i)+4}\right]x^2$$

$\left[\begin{array}{l}\because D=D+a\\ \text{Here } a=i\\ D=D+i\\ i^2=-1\end{array}\right]$

$$=\text{I.P of }e^{ix}\left[\frac{1}{D^2+i^2+2iD-4D-4i+4}\right]x^2$$

$$y_p = \frac{1}{(D^2-4D+4)}\, x^2 \sin x$$

$$= I.P \text{ of } \left[\frac{1}{(D-2)^2}\right] e^{ix} x^2$$

$$= I.P \text{ of } e^{ix} \left[\frac{1}{(D+i-2)^2}\right] x^2$$

$\left[\because D = D+a, \text{ Here } a = i, D = D+i, i^2 = -1\right]$

$$= I.P \text{ of } e^{ix} \left[\frac{1}{\left[-2\left(1-\left(\frac{D+i}{2}\right)\right)\right]^2}\right] x^2$$

$$= I.P \text{ of } \frac{e^{ix}}{4} \left[\frac{1}{\left(1-\left(\frac{D+i}{2}\right)\right)^2}\right] x^2$$

$$= I.P \text{ of } \frac{e^{ix}}{4} \left[1-\left(\frac{D+i}{2}\right)\right]^{-2} x^2$$

$$= I.P \text{ of } \frac{e^{ix}}{4} \left[1+2\left[\frac{D+i}{2}\right]+3\left[\frac{D+i}{2}\right]^2\right] x^2$$

$$= I.P \text{ of } \frac{e^{ix}}{4} \left[x^2+(2x+ix^2)+\frac{3}{4}(2+4xi-x^2)\right]$$

$$= I.P \text{ of } \frac{e^{ix}}{4} \left[\frac{x^2}{4}+2x+ix^2+\frac{3}{2}+3xi\right]$$

$$= I.P \text{ of } \frac{(\cos x+i\sin x)}{4} \left[\frac{x^2}{4}+2x+ix^2+\frac{3}{2}+3xi\right]$$

$$I.P \text{ of } \left(\frac{-1}{4}\right)\left[\frac{x^2\cos x}{4} + 2x\cos x + ix^2\cos x + \frac{3}{2}(\cos x)\right.$$

$$+ 3xi\cos x + \frac{ix^2\sin x}{4} + i2x\sin x - x^2\sin x + i\frac{3}{2}\sin x$$

$$\left. - 3x\sin x\right]$$

$$= I.P \text{ of } \left(\frac{-1}{4}\right)\left[\frac{x^2\cos x}{4} + 2x\cos x + \frac{3}{2}\cos x - x^2\sin x -\right.$$

$$3x\sin x + i\left(x^2\cos x + 3x\cos x + \frac{x^2\sin x}{4} + 2x\sin x + \right.$$

$$\left.\frac{3}{2}\sin x\right)$$

$$= \frac{-1}{4}\left[x^2\cos x + 3x\cos x + \frac{x^2\sin x}{4} + 2x\sin x + \frac{3}{2}\sin x\right]$$

$$y = y_c + y_p$$

$$= (c_1 + c_2x)e^{2x} + \frac{1}{4}\left(x^2\cos x + 3x\cos x + \frac{x^2\sin x}{4} + \right.$$

$$\left. 2x\sin x + \frac{3}{2}\sin x\right)$$

01/02/20

* **Method - 7 :-**

① $(D^2+3D+2)y = e^x x^2 \sin x$

Sol:-

$$AE = m^2 + 3m + 2$$

$$= m^2 + 2m + m + 2$$

$$= m(m+2) + 1(m+2)$$

$m = -1, -2$

$y_c = c_1 e^{-x} + c_2 e^{-2x}$

$y_p = \dfrac{1}{D^2+3D+2} e^{x} x \sin x$ $\quad \left[\begin{array}{l} \because D = D+a \\ \text{Here } a = 1 \\ D = D+1 \end{array}\right]$

$= e^{x}\left[\dfrac{1}{(D+1)^2+3(D+1)+2}\right] x \sin x$

$= e^{x}\left[\dfrac{1}{D^2+2D+1+3D+3+2}\right] x \sin x$

$= e^{x}\left[\dfrac{1}{D^2+5D+6}\right] x \sin x$

$= e^{x}\left[x - \dfrac{2D+5}{D^2+5D+6}\right] \dfrac{1}{D^2+5D+6} (\sin x)$ $\quad \left[\begin{array}{l} \because D^2 = -b^2 \\ D^2 = -1 \end{array}\right]$

$= e^{x}\left[x - \dfrac{2D+5}{D^2+5D+6}\right] \dfrac{1}{5D+5} (\sin x)$

$= \dfrac{e^{x}}{5}\left[x - \dfrac{2D+5}{D^2+5D+6}\right] \left(\dfrac{D-1}{D^2-1}\right) \sin x$

$= \dfrac{e^{x}}{5}\left[x - \dfrac{2D+5}{D^2+5D+6}\right] \dfrac{(\cos x - \sin x)}{-2}$

$= \dfrac{e^{x}}{-10}\left(x(\cos x - \sin x) - \dfrac{(2D+5)(\cos x - \sin x)}{D^2+5D+6}\right)$

$$= \frac{e^x}{-10}\left[x(\cos x - \sin x)\right] + \frac{e^x}{10}\left[\frac{-2\sin x - 2\cos x + 5\cos x - 5\sin x}{D^2+5D+6}\right]$$

$$= \frac{e^x}{-10}\left[x(\cos x - \sin x)\right] + \frac{e^x}{10}\left[\frac{3\cos x - 7\sin x}{D^2+5D+6}\right]$$

$$= \frac{e^x}{-10}\left[x(\cos x - \sin x)\right] + \frac{3e^x}{10}\left[\frac{\cos x}{D^2+5D+6}\right] - \frac{7e^x}{10}\left[\frac{\sin x}{D^2+5D+6}\right]$$

$[D^2 = -1]$ $[\because D^2 = -1]$

$$= \frac{e^x}{-10}\left[x(\cos x - \sin x)\right] + \frac{3e^x}{10}\left[\frac{\cos x}{5D+5}\right] - \frac{7e^x}{10}\left[\frac{\sin x}{5D+5}\right]$$

$[D^2 = -1]$

$$= \frac{e^x}{-10}\left[x(\cos x - \sin x)\right] + \frac{3e^x}{50}\left[\frac{(D-1)\cos x}{-2}\right] - \frac{7e^x}{50}\left[\frac{(D-1)\sin x}{-2}\right]$$

$$= \frac{e^x}{-10}(x\cos x - x\sin x) + e^x\left[\frac{3\sin x + 3\cos x + 7\cos x - 7\sin x}{100}\right]$$

$$y_p = \frac{e^x}{10}(x\sin x - x\cos x) + \frac{e^x}{100}(10\cos x - 4\sin x)]$$

$$y = y_c + y_p = c_1 e^{-x} + c_2 e^{-2x} + \frac{e^x}{10}(x\sin x - x\cos x) + \frac{e^x}{100}(10\cos x - 4\sin x)$$

*** Q) Solve $(D^2 - 4D + 4)y = 8x^2 e^{2x}\sin 2x$

11^{0} (a) $(D^2+4)y = e^{10x} + \sin 3x + x^2 + x\sin x + e^x x + e^x \cos x + x^2\cos x$
$+ xe^x \sin x + x^2 e^x \cos x$

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Sol:- Given

$AE \Rightarrow m^2 - 4m + 4 = 0$

$(m-2)^2 = 0$

$(m-2)(m-2) = 0$

$m = 2, 2$

Roots are real & equal

$\therefore y_c = \cancel{c_1 e^{2x} + c_2 e^{2x}} \quad (c_1 + c_2 x)e^{2x}$

$y_p = \dfrac{1}{f(D)} Q(x)$

$\left[\begin{array}{l} D = D+a \\ \text{Here } a = 2 \\ D = D+2 \end{array}\right]$

$= \dfrac{1}{D^2 - 4D + 4} 8e^{2x} x^2 \sin 2x$

$= \dfrac{1}{(D-2)^2} 8e^{2x} x^2 \sin 2x$

$= 8e^{2x} \left[\dfrac{1}{(D+2-2)^2}\right] x^2 \sin 2x$

$= 8e^{2x} \left[\dfrac{1}{D^2}\right] x^2 \sin 2x$

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$$= 8e^{2x}\, I.P \text{ of } e^{i2x}\left[\frac{1}{D^2}\right]x^2$$

$$= 8e^{2x}\, I.P \text{ of } \left[\frac{1}{D^2}\right]e^{i2x}x^2 \qquad \left[\begin{array}{l}\because D = D+a \\ \text{Here } a = 2i \\ D = D+2i\end{array}\right]$$

$$= 8e^{2x}\, I.P \text{ of } e^{i2x}\left[\frac{1}{(D+2i)^2}\right]x^2 \qquad [\because i^2 = -1]$$

$$= 8e^{2x}\, I.P \text{ of } e^{i2x}\left[\frac{1}{4i^2\left[1+\frac{D}{2i}\right]^2}\right]x^2$$

$$= 8e^{2x}\cdot I.P \text{ of } \frac{e^{i2x}}{-4}\left[\left(1+\frac{D}{2i}\right)\right]^{-2}x^2$$

$$= -2e^{2x}\, I.P \text{ of } e^{i2x}\left[1-2\left(\frac{D}{2i}\right)+3\left(\frac{D}{2i}\right)^2\right]x^2$$

$$= -2e^{2x}\, I.P \text{ of } e^{i2x}\left[x^2-\frac{2x}{i}-\frac{3(\cancel{2})}{\cancel{4}_{2}}\right]$$

$$= -2e^{2x}(\cos 2x + i\sin 2x)\left[x^2-\frac{2x}{i}-\frac{3}{2}\right]$$

$$= -2e^{2x}\Big[x^2\cos 2x - \frac{2x\cos 2x}{i} - \frac{3}{2}\cos 2x + ix^2\sin 2x - 2x\sin 2x - i\frac{3}{2}\sin 2x\Big]$$

$$= -2e^{2x}\Big[x^2\cos 2x - \frac{3}{2}\cos 2x - 2x\sin 2x + i\Big(2x\cos 2x + x^2\sin 2x - \frac{3}{2}\sin 2x\Big)\Big]$$

$$= -2e^{2x}\left[2x\cos 2x + x^2\sin 2x - \frac{3}{2}\sin 2x\right] \;//$$

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$y = y_c + y_p$

$= (c_1 + c_2 x)e^{2x} - 2e^{2x}\left[\cancel{2x}\cos 2x + x^2 \sin 2x - \frac{3}{2}\sin 2x\right]$

Method-8:-

* General method:-
* Inverse operator:-
* Formulae:-

$$y_p = \frac{1}{D-\alpha} Q(x) = e^{\alpha x}\int Q e^{-\alpha x}\,dx$$

(or)

$$\frac{1}{D+\alpha} Q(x) = e^{-\alpha x}\int Q e^{\alpha x}\,dx$$

① Solve $\frac{1}{D}(x^2) = \int x^2 = \frac{x^3}{3}$

② Here $\frac{1}{D}$ is a inverse operator of D and denominator is Numerator is Derivative and denominator is integration

② Solve $\frac{1}{D+2}(\cos 3x)$

Sol:-

$$y_p = \frac{1}{D+2}\cos 3x \qquad [\because \alpha = 2]$$

$$= e^{-2x}\int \cos 3x\, e^{2x}\,dx$$

$$= e^{-2x}\left[\frac{e^{2x}}{4+9}(2\cos 3x + 3\sin 3x)\right]$$

$$= \frac{1}{13}\left[2\cos 3x + 3\sin 3x\right]$$

(or)

$$\frac{D-2}{D^2-4}(\cos 3x)$$

$$= \frac{-3\sin 3x - 2\cos 3x}{-9-4}$$

$$= \left[\frac{3\sin 3x + 2\cos 3x}{13}\right] \,//$$

③ Solve $(D^2+4D+3)y = e^{e^x}$

Sol:-

$$A.E = m^2+4m+3$$

$$= m^2+3m+m+3$$

$$= m(m+3)+1(m+3)$$

$$= (m+1)(m+3)$$

$$m = -1, -3$$

$$y_c = c_1 e^{-x} + c_2 e^{-3x}$$

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$$y_p = \frac{1}{(D+1)(D+3)} e^{e^x}$$

$$= \frac{1}{2}\left[\frac{1}{D+1} - \frac{1}{D+3}\right] e^{e^x}$$ [$\because$ By using partial fraction]

$$\left[\frac{1}{(D+1)(D+3)} = \frac{A}{(D+1)} + \frac{B}{(D+3)}\right.$$

$$\left. 1 = A(D+3) + B(D+1)\right]$$

$$= \frac{1}{2}\left[\frac{1}{D+1}\right] e^{e^x} + \frac{1}{2}\left[\frac{1}{D+3}\right] e^{e^x}$$

$$= \frac{1}{2} e^{-x} \int e^{e^x} e^x \, dx - \frac{1}{2} e^{-3x} \int e^{e^x} e^{3x} \, dx$$

put $e^x = t$

$e^x dx = dt$

$$e^{3x} = e^x e^x e^x dx$$
$$= t^2 dt$$

$$\left[\int uv = u\int v - u'\iint v + u''\iiint v\right]$$

$$= \frac{1}{2} e^{-x} \int e^t dt - \frac{1}{2} e^{-3x} \int e^t t^2 dt$$

$$= \frac{1}{2} e^{-x} e^t - \frac{1}{2} e^{-3x} \left[t^2 e^t - 2te^t + 2e^t\right]$$

$$= \frac{1}{2} e^{-x} e^{e^x} - \frac{1}{2} e^{-3x} \left[e^{2x} e^{e^x} - 2e^x e^{e^x} + 2e^{e^x}\right]$$

$$= \frac{1}{2} e^{e^x} \left[\cancel{e^{-x}} - \cancel{e^{-x}} + 2e^{-2x} - 2e^{-3x}\right]$$

$$= e^{e^x} \left[e^{-2x} - e^{-3x}\right]$$

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* Variation of Parameters :-

* Linear eqns of the 2nd order with variable coefficients :-

The general form of the variation of parameters is

$$\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = R(x) \longrightarrow (1)$$

Here P, Q are funcns of "x" and 'R' is only the funcns of "x" (or constants)

Consider "y_c",

Here $y_c = c_1 u(x) + c_2 v(x) \longrightarrow (2)$

and $y_p = A(x)\, u(x) + B(x)\, v(x) \longrightarrow (3)$

∴ The general soln $y = y_c + y_p$

Where $A(x) = -\int \frac{v(x)\, R(x)}{uv' - vu'}\, dx$

and $B(x) = \int \frac{u(x)\, R(x)}{uv' - vu'}\, dx$

(1) By applying the method of variation of parameters to solve $\frac{d^2(y)}{dx^2} + y = \operatorname{cosec} x$

Sol: Given: $m^2+1=0$

$$m=\pm i$$

$$y_c=(c_1\cos 1x+c_2\sin 1x)e^{0x}$$

$$y_c=c_1\cos x+c_2\sin x \longrightarrow (1)$$

Here $u(x)=\cos x$, $v(x)=\sin x$

$$uv'-vu'=\cos x(\cos x)-\sin x(-\sin x)$$

$$=\cos^2 x+\sin^2 x=1$$

$$\therefore y_p=A(x)u(x)+B(x)v(x) \longrightarrow (2)$$

where $A(x)=\displaystyle\int\frac{v(x)R(x)}{uv'-vu'}\,dx$

$$=-\int\frac{(\sin x)\operatorname{cosec} x}{1}\,dx$$

$$A(x)=-\int 1\,dx=-x$$

$$B(x)=\int\frac{u(x)R(x)\,dx}{uv'-vu'}$$

$$=\int\frac{\cos x\operatorname{cosec} x}{1}\,dx$$

$$B(x)=\int\cot x\,dx$$

$$B(x)=\log|\sin x|$$

(2) $\Rightarrow \therefore y_p=-x\cos x+\log|\sin x|\sin x$

$$\therefore y=y_c+y_p=c_1\cos x+c_2\sin x-x\cos x+\log|\sin x|\sin x$$

② $\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}$

Sol: A.E $= m^2 - 1 = 0$

$m^2 = +1$

$m = \pm 1$

$y_c = c_1 e^x + c_2 e^{-x}$

$u(x) = e^x, \quad v(x) = e^{-x}$

$uv' - vu' = e^x(-e^{-x}) - e^{-x}(e^x)$

$= -1 - 1 = -2$

$A(x) = -\int \frac{e^{-x}\left(\frac{2}{1+e^x}\right)}{-2} dx$

$= \int \frac{e^{-x}}{1+e^x} dx$

$= \int \frac{e^{-x} e^x \, dx}{(1+e^x) e^x}$

$= \int \frac{1}{e^x(1+e^x)} dx$

$1 = A(1+e^x) + B(e^x)$

$A = 1, \; B = -1$

$$= \int \left(\frac{1}{e^x} - \frac{1}{1+e^x} \right) dx$$

$$A(x) = -e^{-x} - \log(1+e^x).$$

$$B(x) = \int \frac{e^x \left(\frac{2}{1+e^x} \right)}{-2}$$

$$= -\int \frac{e^x}{1+e^x} \, dx$$

put $1+e^x = t$

$$e^x dx = dt$$

$$= -\int \frac{dt}{t} = -\log|t|$$

$$= -\log|1+e^x|$$

$$y_p = -e^{-x} - \log(1+e^x) e^x - \log|1+e^x| e^{-x}$$

$$y = y_c + y_p = c_1 e^x + c_2 e^{-x} - e^{-x} - \log(1+e^x) e^x - \log|1+e^x| e^{-x}$$

05/01/20

* Homogeneous linear eq^ns (Euler cauchy eq^ns) :-

An eq^n of the form

$$x^n \frac{d^n y}{dx^n} + P_1 x^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + \dots + P_n y = \phi(x)$$

where $P_1 \, P_2, \dots, P_n$ are real constants and

$\phi(x)$ is a func^n of x.

Here $\frac{d}{dx} = D$ then the independent variable of eq^n - ① is $x = e^z$ (or) $z = \log x$, here $x > 0$

Continuing this process upto several steps we get, $\frac{d}{dz} = \theta$

$\Rightarrow xD = \theta$, $x^2D^2 = \theta(\theta-1)$

$x^3D^3 = \theta(\theta-1)(\theta-2)$ — — —

— — — — — — — —

putting these substitutions in the given eq^n finally we get general sol^n $y = y_c + y_p$

Here solve required sol^n by the previous methods

① solve $(x^2D^2 - 4xD + 6)y = x^2$.

Sol:- Given $(x^2D^2 - 4xD + 6)y = x^2 \longrightarrow$ ①

$xD = \theta$

$x^2D^2 = \theta(\theta-1)$

$x = e^z$ (or) $z = \log x$

① can be converted

$(\theta(\theta-1) - 4(\theta) + 6)y = e^{2z}$

$(\theta^2 - \theta - 4\theta + 6)y = e^{2z}$

$(\theta^2 - 5\theta + 6)y = e^{2z} \longrightarrow ②$

A.E $\Rightarrow$ $m^2 - 5m + 6 = 0$

$m^2 - 3m - 2m + 6 = 0$

$m(m-3) - 2(m-3) = 0$

$(m-3)(m-2) = 0$

$m = 2, 3$

$y_c = c_1 e^{2z} + c_2 e^{3z}$

$$y_p = \frac{1}{f(\theta)} Q(z)$$

$$= \frac{1}{\theta^2 - 5\theta + 6} e^{2z} \quad [\because \theta = 2]$$

$$= \frac{1}{(\theta-3)(\theta-2)} e^{2z}$$

$$= \frac{z e^{2z}}{(-1)1!}$$

$\therefore y_p = -z e^{2z}$

$y = y_c + y_p = c_1 e^{2z} + c_2 e^{3z} - z e^{2z}$

$y = c_1 x^2 + c_2 x^3 - \log x (x^2)$

② Solve $(x^2 D^2 - xD + 1)y = \log x \longrightarrow ①$

Sol:

$xD = \theta$

$x^2 D^2 = \theta(\theta - 1)$

$x = e^z$ (or) $z = \log x$

① Can be converted

$(\theta(\theta-1)-(\theta)+1)y = z$

$(\theta^2-\theta-\theta+1)y = z$

$(\theta^2-2\theta+1)y = z$

$A.E \Rightarrow m^2-2m+1 = 0$

$m = 1, 1$

$y_c = (c_1 + c_2 z)e^z$

$y_p = \frac{1}{f(\theta)} \cdot Q(z)$

$= \frac{1}{(\theta-1)^2} z$

$= \left[\frac{1}{(-(1-\theta))^2}\right] z$

$= (1-\theta)^{-2} z$ $\quad \left[(1-x)^{-2} = 1+2x+3x^2 \cdots\right]$

$= (1+2\theta) z$

$= (z+2(1)) = z+2$

$y = (c_1+c_2 z)e^z + z + 2$

$= (c_1+c_2 \log x)x + \log x + 2$

③ $(x^3D^3+2x^2D^2+2)y = 10\left(x+\frac{1}{x}\right)$

④ $(x^2D^2+3xD+1)y = \frac{\log x \sin(\log x)+1}{x}$

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Q/10 $(D^2+4)y = e^{10x} + \sin 3x + x^2 + x\sin x + e^x x + e^x\cos x + x^2\cos x + xe^x\sin x + x^2e^x\cos x$

Sol:-

$A.E = m^2+4=0$

$m^2 = -4$

$m = \pm 2i$

Roots are complex.

$y_c = (c_1\cos 2x + c_2\sin 2x)e^{0x}$.

$y_c = c_1\cos 2x + c_2\sin 2x$.

$y_{P_1} = \frac{1}{f(D)} Q(x)$

$\begin{bmatrix} D=a \\ a=10 \\ D=10 \end{bmatrix}$

$= \frac{1}{D^2+4} e^{10x}$

$= \frac{1}{100+4} e^{10x}$

$$\boxed{y_{P_1} = \frac{e^{10x}}{104}}$$

$y_{P_2} = \frac{1}{D^2+4} \sin 3x$

$\begin{bmatrix} D^2 = -b^2 \\ b=3 \\ D^2 = -9 \end{bmatrix}$

$= \frac{1}{-9+4} \sin 3x$

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$$= \frac{1}{-5}\sin 3x$$

$$\boxed{y_{p_2} = \frac{\cancel{\sin 3x}}{-5}}$$

$$y_{p_3} = \frac{1}{D^2+4}x^2$$

$$= \frac{1}{\left(4\left(1+\frac{D^2}{4}\right)\right)}x^2 = \frac{1}{4}\left[\left(1+\frac{D^2}{4}\right)\right]^{-1}x^2$$

$$= \frac{1}{4}\left[1-\left(\frac{D^2}{4}\right)+\left(\frac{D^2}{4}\right)^2\right]x^2$$

$$= \frac{1}{4}\left[1-\frac{D^2}{4}+\frac{D^4}{16}\right]x^2$$

$$= \frac{1}{4}\left[\cancel{x^2}\right. = \frac{1}{4}\left[1-\frac{D^2}{4}\right]x^2$$

$$= \frac{1}{4}\left[x^2-\frac{\cancel{2}\,1}{\cancel{16}\,8}\right]$$

$$= \frac{1}{4}\left[x^2-\frac{1}{18}\right]$$

$$y_{P_4} = \frac{1}{f(D)} Q(x)$$

$$= \frac{1}{D^2+4} x \sin x$$

$$= \left[x - \frac{f'(D)}{f(D)}\right] \frac{1}{f(D)} (V)$$

$$= \left[x - \frac{2D}{D^2+4}\right] \frac{1}{D^2+4} (\sin x) \quad \left[\because D^2 = -b^2 \\ D^2 = -1\right]$$

$$= \left[x - \frac{2D}{D^2+4}\right] \left(\frac{1}{3} \sin x\right)$$

$$= \frac{1}{3}\left[x \sin x - \left[\frac{2D}{D^2+4}\right] \sin x\right]$$

$$= \frac{1}{3}\left[x \sin x - \frac{2D}{3} \sin x\right]$$

$$= \frac{1}{3}\left[x \sin x - \frac{2\cos x}{3}\right]$$

$$y_{P_4} = \frac{1}{9}\left[3x \sin x - 2\cos x\right]$$

$$y_{P_5} = \frac{1}{f(D)} Q(x)$$

$$= \frac{1}{D^2+4} e^x . x \quad \left[\because D = a \\ a = 1 \\ D = 1\right]$$

$$= e^x \left(\frac{1}{(D+1)^2+4}\right) x \quad \left[\begin{aligned} = D &= D+a \\ &= D+1 \end{aligned}\right]$$

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$$= e^x \left[\frac{1}{D^2+1+2D+4} \right] x$$

$$= e^x \left[\frac{1}{D^2+2D+5} \right] x$$

$$= e^x \left[\frac{1}{5\left(1+\frac{2D}{5}+\frac{D^2}{5}\right)} \right] x$$

$$= e^x \left[\frac{1}{5\left(1+\frac{2D+D^2}{5}\right)} \right] x$$

$$= \frac{e^x}{5} \left(\left(1+\frac{2D+D^2}{5}\right)^{-1} \right) x$$

$$= \frac{e^x}{5} \left(1 - \frac{2D+D^2}{5} \right) x$$

$$= \frac{e^x}{5} \left(1 - \frac{2D}{5} \right) x$$

$$= \frac{e^x}{5} \left(x - \frac{2}{5} \right)$$

$$y_{P_6} = \frac{1}{(D^2+4)} e^x \cos x \qquad (\because D = D+a)$$

$$\left(\begin{matrix} D^2 = -b^2 \\ b = 1 \\ D^2 = -1 \end{matrix} \right)$$

$$= e^x \left[\frac{1}{(D^2+2D+5)} \right] \cos x$$

$$e^{x}\left(\frac{1}{-1+2D+5}\right)\cos x$$

$$= e^{x}\left(\frac{1}{2D+4}\right)\cos x$$

$$= \frac{e^{x}}{2}\left[\frac{1}{D+2}\right]\cos x$$

$$= \frac{e^{x}}{2}\left[\frac{D-2}{D^2-2}\right]\cos x \qquad [\because D^2=-1]$$

$$= \frac{e^{x}}{2}\left[\frac{D-2}{-1-2}\right]\cos x$$

$$yP_6 = \frac{e^{x}}{2}\left(\frac{-\sin x - 2\cos x}{-3}\right)$$

$$yP_7 = \frac{1}{D^2+4}\,x^2\cos x$$

$$R.P \text{ of } \left(\frac{1}{D^2+4}\right) x^2 e^{ix}$$

$$R.P \text{ of } e^{ix}\left(\frac{1}{(D+i)^2+4}\right)x^2$$

$$R.P \text{ of } e^{ix}\left(\frac{1}{D^2-1+2Di+4}\right)x^2$$

$$R.P \text{ of } e^{ix}\left(\frac{1}{D^2+2Di+3}\right)x^2$$

$$\text{R.P } 9\,\frac{e^{ix}}{3}\left(\frac{1}{1+\frac{D^2+2Di}{3}}\right)x^2$$

$$\text{R.P } 9\,\frac{e^{ix}}{3}\left(\left(1+\frac{D^2+2Di}{3}\right)^{-1}\right)x^2$$

$$\text{R.P } 9\,\frac{e^{ix}}{3}\left(1-\left(\frac{D^2+2Di}{3}\right)+\left(\frac{D^2+2Di}{3}\right)^2\right)x^2$$

$$\text{R.P } 9\,\frac{e^{ix}}{3}\left(x^2-\frac{(D^2+2Di)x^2}{3}+\frac{(D^4+4D^2(-1)+4D^3i)x^2}{9}\right)$$

$$\text{R.P } 9\,\frac{e^{ix}}{3}\left[x^2-\frac{(2+4ix)}{3}+\frac{-4(2)}{9}\right]$$

$$\text{R.P } 9\,\frac{e^{ix}}{3}\left[\frac{\cos x+i\sin x}{3}\right]\left[x^2-\frac{2}{3}-\frac{4ix}{3}-\frac{8}{9}\right]$$

$$=\frac{x^2\cos x}{3}-\frac{2\cos x}{9}-\frac{8\cos x}{27}+\frac{4x\sin x}{9}$$

$$=\frac{1}{3}\left(x^2\cos x-\frac{2\cos x}{3}-\frac{8\cos x}{9}+\frac{4x\sin x}{3}\right)$$

$$=\frac{1}{3}\left(x^2\cos x-\frac{6\cos x}{9}-\frac{8\cos x}{9}+\frac{4x\sin x}{3}\right)$$

$$y_{p_1}=\frac{1}{3}\left(x^2\cos x-\frac{14\cos x}{9}+\frac{4x\sin x}{3}\right)$$

$$y_{p} = \left(\frac{1}{D^2+4}\right) x e^x \sin x$$

$$= e^x \left(\frac{1}{(D+1)^2+4}\right) x \sin x$$

$$= e^x \left(\frac{1}{D^2+2D+5}\right) x \sin x$$

$$= e^x \left(x - \frac{2D+2}{D^2+2D+5}\right) \frac{1}{D^2+2D+5} \sin x$$

$$= e^x \left[x - \frac{2D+2}{D^2+2D+5}\right] \cdot \frac{1}{2D+4} \sin x$$

$$= \frac{e^x}{2} \left[x - \frac{2D+2}{D^2+2D+5}\right] \frac{D-2}{D^2-4} \sin x$$

$$= \frac{e^x}{2} \left[x - \frac{2D+2}{D^2+2D+5}\right] \frac{\cos x - 2\sin x}{-5}$$

$$= \frac{e^x}{-10} \left[x\cos x - 2x\sin x - \frac{2D+2}{D^2+2D+5} \cos x + \frac{2D+2}{D^2+2D+5} 2\sin x\right]$$

$$= \frac{e^x}{-10} \left[x\cos x - 2x\sin x - \frac{2(D+1)}{2D+4} \cos x + \frac{2(D+1)}{2D+4} 2\sin x\right]$$

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$$= \frac{e^x}{-10}\left[x\cos x - 2x\sin x - \frac{(D+1)(D-2)}{D^2-4}\cos x + \frac{(D+1)(D-2)}{D^2-4}2x\sin x\right]$$

$$= \frac{e^x}{-10}\left[\cancel{x\cos x - 2x\sin x - \frac{(D+1)(D-2)}{D^2-4}\cos x +}\right.$$

$$\cancel{\frac{(D+1)(D-1)}{}}$$

$$= \frac{e^x}{-10}\left[x\cos x - 2x\sin x - \frac{D^2-2D+D-2}{-5}\cos x + \frac{D^2-2D+D-2}{-5}2\sin x\right]$$

$$= \frac{e^x}{-10}\left[x\cos x - 2x\sin x - \frac{(-\cos x + \sin x - 2\cos x)}{-5} + \frac{2}{-5}(-\sin x - \cos x - 2\sin x)\right]$$

$$= \frac{e^x}{-10}\left(x\cos x - 2x\sin x + \frac{\cos x - \sin x + \cancel{2\cos x}}{-5} + \frac{-2\sin x - 2\cos x - 4\sin x}{-5}\right).$$

$$y_{p_8} = \frac{e^x}{-10}\left[x\cos x - 2x\sin x + \frac{\cos x - 7\sin x}{5}\right]$$

$$y_{p_9} = \left(\frac{1}{D^2+4}\right)x^2 e^x \cos x$$

$$= e^x\left(\frac{1}{(D+1)^2+4}\right)x^2\cos x$$

$$= e^x\left[\frac{1}{D^2+2D+1+4}\right]x^2\cos x$$

$$= e^x\left[\frac{1}{D^2+2D+5}\right]x^2\cos x$$

$$= e^x\left[\frac{1}{D^2+2D+5}\right]x^2 e^{ix}$$

$$= e^x \text{ R.P of } e^{ix}\left[\frac{1}{(D+i)^2+2(D+i)+5}\right]x^2$$

$$= e^x \text{ R.P of } e^{ix}\left(\frac{1}{D^2+2Di-1+2D+2i+5}\right)x^2$$

$$= e^x \text{ R.P of } e^{ix}\left(\frac{1}{D^2+2D+2i(D+1)+4}\right)x^2$$

$$= e^x \text{ R.P of } \frac{e^{ix}}{4}\left(\frac{1}{1+\frac{D^2+2D+2i(D+1)}{4}}\right)x^2$$

$$= e^x \, R.P \text{ of } \frac{e^{ix}}{4}\left[1+\frac{D^2+2D+2i(D+1)}{4}\right]^{-1} x^2$$

$$= e^x \, R.P \text{ of } \frac{e^{ix}}{4}\left[1-\frac{D^2+2D+2iD+2i}{4}+\frac{(D^2+2D+2iD+2i)^2}{16}\right]x^2$$

$$= e^x\left(\frac{\cos x+i\sin x}{4}\right)\left[1-\frac{D^2+2D+2iD+2i}{4}+\frac{D^4+12D^2i+4D^3-8D-4}{16}\right]x^2$$

$$= e^x\left(\frac{\cos x+i\sin x}{4}\right)\left[x^2-\frac{(2+4x+4ix+2ix^2)}{4}+\frac{24i-16x-4x^2}{16}\right]$$

$$= e^x\left(\frac{\cos x+i\sin x}{4}\right)\left[x^2-\frac{2}{4}-\frac{4x}{4}-\frac{16x}{16}-\frac{4x^2}{16}-\frac{4ix}{4}-\frac{\cancel{6ix^2}}{4}+\frac{24i}{16}\right]$$

$$= e^{x}\left(\frac{\cos x + i\sin x}{4}\right)\left[\cancel{x^2} - \frac{1}{2} - 2x - \cancel{x^2} - i\left(x^3 + x^2 - \frac{3}{2}\right)\right]$$

$$= e^{x}\left(\frac{\cos x + i\sin x}{4}\right)\left[\left(-\frac{1}{2} - 2x\right) - i\left(x + x^2 - \frac{3}{2}\right)\right]$$

$$= e^{x}\left[\frac{\cos x}{4}\left(-\frac{1}{2} - 2x\right) + \frac{\sin x}{4}\left(x + x^2 - \frac{3}{2}\right)\right]$$

$$\boxed{y_{p_2} = -\frac{e^{x}\cos x}{8} - \frac{2xe^{x}\cos x}{4} + \frac{e^{x}x\sin x}{4} + \frac{e^{x}x^2\sin x}{4} - \frac{e^{x}3\sin x}{8}}$$

07/02/20

Given

45 Q:- $(x^2D^2 - 3xD + 1)y = \frac{\log x \sin(\log x) + 1}{x}$

$$(\theta^2 - 4\theta + 1)y = \frac{z\sin z + 1}{e^{z}}$$

$\frac{4 \pm \sqrt{16-4}}{2}$, $\frac{2 \pm (2\sqrt{3})}{2}$, $= 2 \pm \sqrt{3}$

$y_c = (c_1\cos\sqrt{3}\ln x + c_2\sin\sqrt{3}\ln x)\, e^{2x}$

$$y_p = \frac{1}{(\theta^2 - 4\theta + 1)}\left(z\sin z\, e^{-z} + e^{-z}\right)$$

$$= \frac{1}{(\theta^2 - 4\theta + 1)} z\sin z\, e^{-z} + \frac{1}{(\theta^2 - 4\theta + 1)} e^{-z}$$

$$= (e^{-z})\frac{1}{(\theta-1)^2 - 4(\theta-1) + 1}(z\sin z) + \frac{1}{6}e^{-z}$$

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$$= \frac{e^{-z}}{6} + e^{-z}\left[\frac{1}{\theta^2 - 2\theta + 1 - 4\theta + 4 + 1}\right] z \sin z$$

$$= \frac{e^{-z}}{6} + e^{-z}\left[\frac{1}{\theta^2 - 6\theta + 6}\right] z \sin z$$

$$= \frac{e^{-z}}{6} + e^{-z}\left[z - \frac{2\theta - 6}{\theta^2 - 6\theta + 6}\right] \frac{1}{-1 - 6\theta + 6} \sin z$$

$$= \frac{e^{-z}}{6} + e^{-z}\left[z - \frac{2\theta - 6}{\theta^2 - 6\theta + 6}\right] \frac{\sin z}{-6\theta + 5}$$

$$= \frac{e^{-z}}{6} + e^{-z}\left[z - \frac{2\theta - 6}{\theta^2 - 6\theta + 6}\right] \frac{(5 + 6\theta)\sin z}{61}$$

$$= \frac{e^{-z}}{6} + e^{-z}\left[z - \frac{2\theta - 6}{\theta^2 - 6\theta + 6}\right] \frac{5\sin z + 6\cos z}{61}$$

$$= \frac{e^{-z}}{6} + \frac{e^{-z}}{61}\left(z(5\sin z + 6\cos z) - \frac{(2\theta - 6)(5\sin z + 6\cos z)}{\theta^2 - 6\theta + 6}\right)$$

$$= \frac{e^{-z}}{6} + \frac{e^{-z}}{61}\left(z(5\sin z + 6\cos z) - \left[\frac{10\cos z - 12\sin z - 30\sin z - 36\cos z}{\theta^2 - 6\theta + 6}\right]\right)$$

$$= \frac{e^{-z}}{6} + \frac{e^{-z}}{61}\left[5z\sin z + 6z\cos z + \left[\frac{26\cos z + 42\sin z}{\theta^2 - 6\theta + 6}\right]\right]$$

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$$= \frac{e^{-z}}{6} + \frac{e^{-z}}{61}\left[5z\sin z + 6z\cos z + \left(\frac{26\cos z + 42\sin z}{\theta^2 - 6\theta + 6}\right)\right]$$

$$= \frac{e^{-z}}{6} + \frac{e^{-z}}{61}\left[5z\sin z + 6z\cos z + \frac{26\cos z + 42\sin z}{-6\theta + 5}\right]$$

$$= \frac{e^{-z}}{6} + \frac{e^{-z}}{61}\left[5z\sin z + 6z\cos z + \frac{(6\theta+5)(26\cos z + 42\sin z)}{61}\right]$$

$$= \frac{e^{-z}}{6} + \frac{e^{-z}}{61}\left[5z\sin z + 6z\cos z + \left(\frac{-156\sin z + 252\cos z + 130\cos z + 210\sin z}{61}\right)\right]$$

$$= \frac{e^{-z}}{6} + \frac{e^{-z}}{61}\left[5z\sin z + 6z\cos z + \left(\frac{54\sin z + 382\cos z}{61}\right)\right]$$

$$= \frac{1}{6x} + \frac{1}{61x}\left[5\log x\sin(\log x) + 6\log x\cos(\log x) + \left(\frac{54\sin(\log x) + 382\cos(\log x)}{61}\right)\right]$$

* LEGENDER'S LINEAR EQUATION:-

An eq^n of the form

$$(a+bx)^n\frac{d^n y}{dx^n} + P_1(a+bx)^{n-1}\frac{d^{n-1}y}{dx^{n-1}} + \cdots + P_n y = Q(x)$$

where $P_1, P_2, P_3 \ldots P_n$ are real constants & $Q(x)$ is a func^n of 'x'

This can be solved by the substitution of

$a+bx = e^z$ (or) $z = \log(a+bx)$ and $\theta = \frac{d}{dz}$

~~(1) solve $(x+1)^2 \frac{d^2y}{dx^2} - 3(x+1)$~~

then $(a+bx) Dy = b\theta y$

$(a+bx)^2 D^2 y = b^2\theta(\theta-1)y$

$(a+bx)^3 D^3 y = b^3\theta(\theta-1)(\theta-2)y$

(1) Solve $(x+1)^2 \frac{d^2y}{dx^2} - 3(x+1)\frac{dy}{dx} + 4y = x^2+x+1$

Sol:- $((x+1)^2 D^2 - 3(x+1) D + 4) y = x^2+x+1$

here $a=1$, $b=1$

$(\theta(\theta-1) - 3\theta + 4)y = (e^z-1)^2 + e^z$

$(\theta^2 - \theta - 3\theta + 4)y = e^{2z} + 1 - 2e^z + e^z$

$(\theta^2 - 4\theta + 4)y = e^{2z} - e^z + 1$

A.E $\Rightarrow m^2 - 4m + 4 = 0$

$m = 2, 2$

$y_c = (c_1 + c_2 z) e^{2z}$

$$y_p = \frac{1}{D^2-4D+4}\left(e^{2z}-e^{z}+1\right)$$

$$= \left(\frac{1}{D^2-4D+4}e^{2z}\right) - \left(\frac{1e^{z}}{(D^2-4D+4)}\right) + \left(\frac{e^{0z}}{D^2-4D+4}\right)$$

$$= \frac{1}{(D-2)^2}e^{2z} - \frac{1e^{z}}{1} + \frac{e^{0z}}{4}$$

$$= \frac{e^{2z}z^2}{(1)2!} - e^{z} + \frac{1}{4}$$

$$= \frac{z^2e^{2z}}{2} - e^{z} + \frac{1}{4}$$

$$\therefore y = (c_1 + c_2\log(1+x))(1+x)^2 + \frac{[\log(1+x)]^2(1+x)^2}{2} - (1+x) + \frac{1}{4}$$

② Solve $(2x-1)^3\frac{d^3y}{dx^3} + (2x-1)\frac{dy}{dx} - 2y = x$

~~Sol: $(2x-1)^3\frac{d^3y}{dx^3} + (2x-1)\frac{dy}{dx} - 2y = x$~~

~~$a=-1,\ b=2$~~

$z = \log(a+bx)$

$= \log(2x-1)$

$a+bx = e^{z}$

$(-1+2x) = e^{z}$

$2x = e^{z}+1$

$x = e^{z}+1/2$

$(2x-1)^3 D^3 y = 8\theta(\theta-1)(\theta-2)$

$(2x-1) D y = 2\theta$

$= [8\theta(\theta-1)(\theta-2) + 2\theta - 2] y = \frac{e^z}{2} + \frac{1}{2}$

$= [8(\theta^2-\theta)(\theta-2) + 2\theta - 2] y = \frac{e^z}{2} + \frac{1}{2}$

$= [8(\theta^3 - 2\theta^2 - \theta^2 + 2\theta) + 2\theta - 2] y = \frac{e^z}{2} + \frac{1}{2}$

$= [8\theta^3 - 24\theta^2 + 18\theta - 2] y = \frac{e^z}{2} + \frac{1}{2}$

$M = \frac{2-\sqrt{3}}{2}, \frac{2+\sqrt{3}}{2}, 1$

$y_p = \frac{1}{2}\left[\frac{e^z}{8\theta^3 - 24\theta^2 + 8\theta - 2}\right] + \frac{1}{2}\left[\frac{e^{0z}}{8\theta^3 - 24\theta^2 + 8\theta - 2}\right]$

$= \frac{1}{2}\left[\frac{e^z}{(\theta-1)(8\theta^2 - 16\theta + 2)}\right] + \frac{1}{2}\left[\frac{e^{0z}}{-2}\right]$

$M=1$ | 8 -24 18 -2

0 8 -16 2

8 -16 2 0

$= \frac{1}{2}\left[\frac{e^z}{(-6)} \frac{z}{1!(1)}\right] - \frac{1}{4}$

$\Rightarrow \frac{ze^z}{-12} - \frac{1}{4}$

$$y = c_1(2x-1) + \left(c_2 \cosh \frac{\sqrt{3}}{2} \log(2x-1) + c_3 \sinh \frac{\sqrt{3}}{2} \log(2x-1)\right)(2x-1) + \frac{\log(2x-1)(2x-1)}{-12} - \frac{1}{4}$$