Module - I → Probability

Probability, axiomatic approach, elementary theorems on Probability, conditional probability, multiplication theorem, Bayes Theorem (without proof).

Module - II → Random Variables

Random Variables, Discrete & Continuous random variables, probability distribution, probability mass function & probability density function.

Module - III → Probability Distributions :

Binomial distribution, Mean & Variance of Binomial distribution, Poisson distribution: Poisson distribution as a limiting case of Binomial distribution, Mean and ~~Various~~ Variance Of poisson distribution, Normal distribution: mean, Variance, mode, median of normal distribution.

Module - IV → Correlation and Regression

Correlation - Karl Pearson's coefficient of correlation, rank correlation, repeated ranks, Regression: Lines of reghression, regression coefficient, angle b/w two regression lines.

Module - V → Test Of Hypothesis

Population, sample, standard error; test of significance Null hypothesis, alternate hypothesis. Types of errors, level of significance.

Large sample Tests: Test of hypothesis for single mean, difference b/w mean, single proportion & difference b/w Proportions: Small sample tests: Student's t-distribution, F-distribution & chi-square distribution.

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Module - 1 Probability

What is Random Experiment?

Experiment:

An activity that produces a result or an outcome is called an experiment.

Random experiment:

- When we perform an activity or experiment, usually, we may get a different number of outcomes from an experiment.
- However, when an experiment satisfies the following two conditions, It is called a random experiment.

(i) It has more than one possible outcome

(ii) It is not possible to predict the outcome in advance.

Definition:

Probabilistic situation is referred to as a random experiment

Example:

Is picking a card from a well-shuffled deck of cards a random experiment?

Solution: We Know that a deck contains 52 cards and each of these cards has an equal chance to be selected

(i) The ~~expected~~ experi can be repeated since we can shuffle the deck of cards every time before picking a card and there are 52 possible outcomes.

(ii) It is possible to pick any of 52 cards & hence outcome is not Predictable before.

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Example: Of Random Experiment

Tossing a coin three times.

Number of possible outcomes = 8.

Sample space = S = {HHH, HHT, HTH, THH, TTH, THT, HTT, TTT}.

Rolling a pair of dice simultaneously.

Number of possible outcomes = 6.

Event: Each outcome of an experiment or trial that is conducted is termed an Event.

Sample space: The set of all the possible outcomes of an experiment is called the sample space of that experiment and is generally represented as S.

Sure Event: An event that will always occur is called a sure event. A sure event has a probability of 1.

Impossible Event: An event that will never occur is called an impossible event. An impossible event has a probability of 0.

Probability:

Favorable outcome: An event that produces the desired result in an experiment is called a favourable outcome for that experiment.

Independent Events: Two events are said to be independent if the occurrence of one event does not depend upon the other.

Overlapping Events: The events that can happen individually as well as jointly are said to be overlapping events.

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Types Of Events in Probability.

Equally likely Events:

These are those whose chances or probabilities of happening are equal. Both events are not related to one another.

Equally likely means that each outcome of an experiment occurs with equal probability.

Example: when we flip a coin there are equal possibilities of recieving either a head or a tail.

For example: If you toss a fair, a Head (H) and a Tail (T) are equally likely to occur.

six-sided die, each face (1, 2, 3, 4, 5, 6) is a likely to occur as any other face

Mutually Exclusive Events:

Mutually exclusive events cannot occur at the same time.

In other words, mutually exclusive events are called disjoint events.

If two events are considered disjoint events, then the probability of both events occuring at the same time will be zero.

For instance, the weather may be hot or chilly simultaneously.

We can't have the same weather at the same time.

While tossing a coin, the event of getting head & tail are mutually exclusive Because the prob of getting head & tail ~~is~~ simultaneously is 0.

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Exhaustive Events:

We call an event exhaustive when the set of all experiment results is the same as the sample space.

i.e Two events are exhaustive when their union is equal to the sample space.

Ex: The exp of throwing a die.

sample space $S = \{1, 2, 3, 4, 5, 6\}$

Assume that A, B & C are the events associated with this experiment. Also, let us define these events as:

A = event of getting a number greater than 3

Ans: $A = \{4, 5, 6\}$

B = event of getting a no. greater than 2 but less than 5

Ans: $B = \{3, 4\}$

C = event of getting a num. less than 3.

Ans: $C = \{1, 2\}$

we observe that

$A \cup B \cup C = \{4, 5, 6\} \cup \{3, 4\} \cup \{1, 2\}$

Probability

The probability of an event is written:

Probability can be defined as the possibility of occurance of an event. Probab is the likelihood or the chances that an uncertain event will occur.

Prob of an event always lies b/w 0 & 1.

$$P(\text{event}) = \frac{\text{no. of ways event can occur}}{\text{total no. of outcomes}}$$

$$P(A) = \frac{n(A)}{n(S)}$$

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1) Find the ~~pbo~~ prob of getting 3 on rolling a die. Calculate the ~~the~~ % give ans.

Sol: $P(E) = \frac{1}{6}$ $\boxed{P(E) = \frac{n(E)}{n(S)}}$

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2) In a class there are 10 boys & 5 girls. A Committee of 4 students is to be selected from the class. Find prob for the comitte to contain atleast 3 girls.

Sol: no. of boys = 10
" " girls = 5

Total students in cls = 15

no. of ~~sel~~ students selected for comitte = 4.

$P(E) = \frac{n(A)}{n(S)}$

Sample space = $n(S) = {}^{15}C_4$.

$= \frac{15!}{11!\,4!}$ $\boxed{{}^nC_r = \frac{n!}{(n-r)!\,r!}}$

$= 1365$

$n(A) = (3G, 1B) + (4G, 0B)$

$= ({}^5C_3 \times {}^{10}C_1) + ({}^5C_4 \times {}^{10}C_0) = \left(\frac{5!}{2!\,3!} \times \frac{10!}{1!}\right) + \left(\frac{5!}{2!3!4!} \times\right.$

$= 100 + 5$

$= 105.$

$P(E) = \frac{105}{1365} = \frac{1}{13} = 0.0769.$

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3) In a class there are 10 boys and 5 girls. A comittee of 4 students to be selected from the class. Find the prob for comittee to contain at least 3 ~~girls~~ boys.

sol: no. of boys = 10

no. of girls = 5

Total no. of students in cls = 15

no. of students selected for comittee = 4

$\boxed{{}^{n}C_0 = 1}$

$n(A) = (3B, 1G) + (4B, 0G)$

$= ({}^{10}C_3 \times {}^{5}C_1) + ({}^{10}C_4 \times {}^{5}C_0)$

$\cancel{= \left(\frac{10!}{3\times2\times1} \times \frac{5!}{1}\right) + \left(\frac{10!}{4\times3\times2\times1} \times 1\right)}$

$\cancel{= 4\times5\times6\times7\times}$

$= \left(\frac{10!}{7!\,3!} \times \frac{5!}{4!\,1!}\right) + \left(\frac{10!}{6!\,4!} \times 1\right)$

$= \left(\frac{8\times9\times10}{3\times2\times1} \times 5\right) + \left(\frac{7\times8\times9\times10}{4\times3\times2\times1}\right)$

$\begin{array}{r} 120 \\ \times 5 \\ \hline 600 \end{array}$

$= 600 + 140 = 740.$

$n(S) = {}^{15}C_4$

$= \frac{15!}{11!\,4!} = 1365$

$P(E) = \frac{n(A)}{n(S)} = \frac{740}{1365}$

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4) A class consists of 6 girls and 10 boys. If a committe of 3 is chosen at random from a class, find the prob that

(i) 3 boys are selected. (ii) exactly 2 girls are selec

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sol: (i) $n(S) = {}^{16}C_3$

$= \frac{16!}{13!\,3!} = 560.$

$n(A) = (3B, 0G) \cancel{+ (2B, 1G)}$

$= ({}^{10}C_3 \times {}^6C_0) \cancel{+ ({}^{10}C_2 + {}^6C_1)}$

$= \left(\frac{10!}{7!\,3!} \times 1\right) \cancel{+ \left(\frac{10!}{8!\,2!} + \frac{6!}{5!\,1!}\right)}$

$= 120$ $\quad \therefore P(E) = \frac{120}{560} =$

(ii) $n(A) = (2G, 1B)$ $\qquad \boxed{{}^nC_1 = n}$

$= ({}^6C_2 \times {}^{10}C_1)$

$= \frac{6!}{4!\,2!} \times \frac{10!}{9!\,1!}$

$= \frac{5 \times \cancel{6}^3}{\cancel{2} \times 1} \times 10$

$= 150.$

$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{150}{560}$

Q) 2 Cards are selected at random from 10 each numbered 1 to 10 find prob that the sum is even if

(i) the 2 cards are drawn together

(ii) the 2 cards are drawn one after another with replacements.

Sol: $n(S) = {}^{10}C_2 = \frac{10!}{8!\,2!} = \frac{9 \times \cancel{10}^5}{\cancel{2} \times 1} = 45.$

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(i) $n(A)$

even $\begin{cases} \text{even} + \text{even} \\ \text{odd} + \text{odd} \end{cases}$

~~no. of ways drawing 2 cards at a~~

$n(A) = {}^5C_2 + {}^5C_2$

$= 10 + 10 = 20$

$P(E) = \frac{20}{45}$

(ii) $n(S) = {}^{10}C_1 \times {}^{10}C_1 = \cancel{10+10} \; \cancel{=20} \; 10 \times 10 = 100$

$n(A) = ({}^5C_1 \times {}^5C_1) + ({}^5C_1 \times {}^5C_1)$

$= (5 \times 5) + (5 \times 5)$

$= 50$

$P(E) = \frac{50}{100} = \frac{1}{2}$

Q) Five digit numbers are formed with 0,1,2,3,4 (not allowing a digit being repeated in any num). Find the prob of getting of getting 2 in ten's place & 0 in the units place always?

Sol: $n(S) = 5! - 4! = 96$

$n(A)$ = no. of favourable outcomes

$= 3! = 6$

$\therefore P(E) = \frac{n(A)}{n(S)} = \frac{6}{96} = \frac{1}{16}$

Complementary Events:

* Two events of a sample space whose intersection is ϕ whose union is the entire sample space are called complementary events.
* Thus if E is an event of sample space S, its complement is denoted by E' or $\bar{E}$.

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Axiomatic Approach

Def: Let S be a finite sample space. A real valued function P from the power set of S into R is called a prob function on S if the following three axioms are satisfied.

Axioms of Probability:

i) Axiom of positivity: $P(E) \geq 0$ for every subset E of S.

ii) Axiom of Certainty: $P(S) = 1$

iii) Axiom of union: If E_1 & E_2 are disjoint subsets of S, then

$$P(E_1 \cup E_2) = P(E_1) + P(E_2).$$

The image of $P(E)$ of E is called the prob of event E.

Note: If $E_1, E_2, E_3 \dots E_n$ are disjoint subsets of S, then

$$P(E_1 \cup E_2 \cup \dots \cup E_n) = P(E_1) + P(E_2) + \dots + P(E_n).$$

Example: In tossing 2 coins sample space

$$S = \{HH, HT, TH, TT\}$$

$$P(HH) = \frac{1}{4}$$

$$P(HT, TH) = \frac{1}{2}$$

$$P(TT) = \frac{1}{4}$$

$$P(HH) + P(HT, TH) + P(TT) = \frac{1}{4} + \frac{1}{2} + \frac{1}{4} = 1$$

These are mutually exclusive cases.

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1) What is the prob. that a card drawn at random from the pack of playing cards may be either a queen or a king.

Q) In a grp there are 3 men & 3 women 3 members are selected at random from this grp.

$P(A) = \frac{n(A)}{n(S)}$ $\quad n(S) = 5C_3$

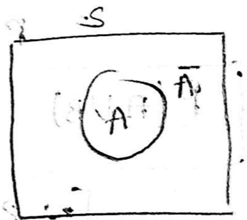
$= \frac{3C_1 2C_2}{5C_3}$ $\quad P(B) = \frac{n(B)}{n(S)} = \frac{3C_2 \times 2C_1}{5C_3}$

$P(A \cup B) = P(A) + P(B) = \frac{3}{10} + \frac{6}{10} = \frac{9}{10}$

Elementary theorems:

Theorem 1: Prob of complementary event $\bar{A} = 1 - P(A)$

Proof: $S = A \cup \bar{A}$



A and $\bar{A}$ are mutually exclusive events or disjoint sets

$\therefore A \cap \bar{A} = \phi$

$\therefore P(S) = P(A) + P(\bar{A})$

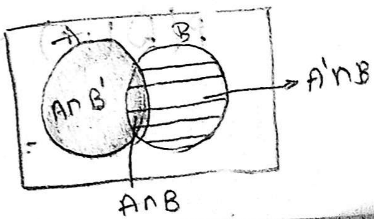
$P(\bar{A}) = 1 - P(A) \quad [P(S) = 1]$

Note: $\bar{A}$ can also be written as A' & A^c

Theorem 2: For any two events A and B, $P(\bar{A} \cap B) = P(B) - P(A \cap B)$

Proof: $A \cap B$ and $(A^c \cap B)$ are disjoint sets.

$\therefore$ These are mutually exclusive



$\therefore P([A\cap B]\cup[A^c\cap B]) = P(A\cap B) + P(A^c\cap B)$ (by Axioms)

But $B = [A\cap B]\cup[A^c\cap B]$

$P(B) = P[(A\cap B)\cup(A^c\cap B)]$

$P(B) = P(A\cap B) + P(A^c\cap B)$.

$$\boxed{P(A^c\cap B) = P(B) - P(A\cap B)}$$

Note: similarly $P(A\cap B^c) = P(A) - P(A\cap B)$.

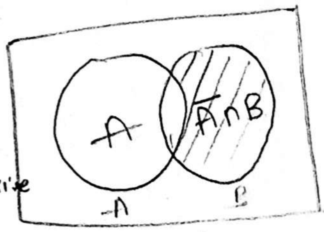
Theorem 3:

For any 2 events A and B;

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$.

Proof

A and $\bar{A}\cap B$ are disjoint sets



A and $\bar{A}\cap B$ are mutually exclusive

$\therefore P(A\cup[\bar{A}\cap B]) = P(A) + P(\bar{A}\cap B)$ $\quad A\cup(\bar{A}\cap B) = A\cup B$

$= P[A\cap(\bar{A}\cap B)]$

$= P(A) + P(\bar{A}\cap B)$

$= P(A) + P(B) - P(A\cap B)$

$$\boxed{P(A\cup B) = P(A) + P(B) - P(A\cap B)}$$

$[\because P(\bar{A}\cap B) = P(B) - P(A\cap B)]$

Theorem 4: For any three events A, B and C, $P(A\cup B\cup C)$
$= P(A) + P(B) + P(C) - P(A\cap B) - P(B\cap C) - P(C\cap A)$
$+ P(A\cap B\cap C)$.

Proof:

$P(A\cup B\cup C) = P(D\cup C)$ where $D = A\cup B$

$P(D\cup C) = P(D) + P(C) - P(D\cap C)$ [$\because$ theorem 3]

$= P(A\cup B) + P(C) - P((A\cup B)\cap C)$

$= P(A) + P(B) - P(A\cap B) + P(C) - \cancel{P(P(A}$ $P[(A\cap C)\cup(B\cap C)]$

$= P(A) + P(B) - P(A\cap B) + P(C) - [P(A\cap C) + P(B\cap C) - P(A\cap B\cap C)]$

$= P(A) + P(B) + P(C) - P(A\cap B) - P(A\cap C) - P(B\cap C) + P(A\cap B\cap C)$

$$\boxed{P(A\cup B\cup C) = P(A) + P(B) + P(C) - P(A\cap B) - P(B\cap C) - P(A\cap C) + P(A\cap B\cap C)}$$

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Q) If $P(A) = \frac{1}{5}$, $P(B) = \frac{2}{3}$, $P(A\cap B) = \frac{1}{15}$, find

1) $P(A\cup B)$ 2) $P(\bar{A}\cap B)$ 3) $P(A\cap\bar{B})$ 4) $P(\bar{A}\cap\bar{B})$

5) $P(\bar{A}\cup\bar{B})$

Sol: $P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$= \frac{1}{5} + \frac{2}{3} - \frac{1}{15}$

$= \frac{4}{5}$

(ii) $P(\bar{A}\cap B) = P(B) - P(A\cap B)$

$= \frac{2}{3} - \frac{1}{15} = \frac{3}{5}$

(iii) $P(A\cap\bar{B}) = \frac{1}{5} - \frac{1}{15} = \frac{2}{15}$

(iv) $P(\bar{A}\cap\bar{B}) = P(\overline{A\cup B}) = 1 - P(A\cup B)$

$= 1 - \cancel{P}\frac{4}{5} = \frac{1}{5}$

(v) $P(\bar{A}\cup\bar{B}) = 1 - P(A\cap B)$

$= 1 - \frac{1}{15} = \frac{14}{15}$

Q) Two dice are thrown ~~let A~~

Let A be the even that the sum of the points on the faces is 9. Let B be the event that atleast one number is 6. Find Proba of following events

(i) $P(A\cap B)$ (ii) $A\cup B$ (iii) $A\cap\bar{B}$ (iv) $\bar{A}\cap B$ (v) $\bar{A}\cap\bar{B}$

(vi) $\bar{A}\cup\bar{B}$

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Sol: $A = (3,6)(4,5)(5,4)(6,3)$, $B = (1,6)(2,6)(3,6)(4,6)(5,6)(6,1)(6,2)(6,3)(6,4)(6,5)(6,6)$.

$P(A) = \frac{4}{36}$, $P(B) = \frac{11}{36}$

$n(S) = 36 = 6^2$

(i) $P(A\cap B) = \cancel{P(A)+P(B)}$ $(3,6)(6,3)$

$= \cancel{\frac{4}{36}+\frac{11}{36}}$ $\frac{2}{36}$

(ii) $P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$= \frac{4}{36} + \frac{11}{36} - \frac{2}{36}$

$= \frac{13}{36}$

(iii) $P(A\cap \bar{B}) = P(A) - P(A\cap B)$

$= \frac{4}{36} - \frac{2}{36} = \frac{2}{36} = \frac{1}{18}$

(iv) $P(\bar{A}\cap B) = P(B) - P(A\cap B)$

$= \frac{11}{36} - \frac{2}{36} = \frac{9}{36} = \frac{1}{4}$

(v) $\bar{A}\cap\bar{B} = \overline{A\cup B} = 1 - P(A\cup B)$

$= 1 - \frac{13}{36} = \frac{23}{36}$

(vi) ~~$\bar{A}\cap B$~~ $\bar{A}\cup\bar{B} = \overline{A\cap B} = 1 - \overline{A\cap B}$

$= 1 - \frac{2}{36} = \frac{17}{18}$

Q) Among 150 students, 80 are studying maths, 40 are studying physics and 30 are studying maths & physics if a student is chosen at random find the prob that the student i) studing maths or phy

ii) studying neither maths not phy.

Sol: $n(s) = 150$.

Let event A = student studying maths = 80

event B = " " phy = 40

$n(A) = 80$

$n(B) = 40$.

$P(A \cap B) = \frac{30}{150}$

$P(A) = \frac{80}{150}$, $P(B) = \frac{40}{150}$

i) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{80}{150} + \frac{40}{150} - \frac{30}{150}$

$= \frac{\cancel{90}}{\cancel{150}} = \frac{3}{5}$

ii) $P(\bar{A} \cap \bar{B}) = \overline{A \cup B}$

$= 1 - P(A \cup B)$

$= 1 - \frac{3}{5}$

$= \frac{2}{5}$.

Q) An integer is choosen at random from the first ~~two~~ 200 positive integers. What is the prob that the integer choosen is divisible by 6 or 8.

Sol: sample space $\longrightarrow n(s) = 200$.

Suppose A and B are the events that the number chosen is 6 or 8.

A = divisible by 6.

= 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84,

$n(A) = 33$.

B = divisible by 8.

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$n(B) = 25$

$P(A) = \frac{33}{200}$, $P(B) = \frac{25}{200}$, $P(A \cap B) = \frac{8}{200}$.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{33}{200} + \frac{25}{200} - \frac{8}{200}$$

$$= \frac{\cancel{50}}{\cancel{200}_4} = \frac{1}{4}.$$

Addition Theorem On Probability:

If S is a sample spase and A, B are any 2 events in S. then ~~P(A)~~ $P(A \cup B)$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

Note: If A, B are 2 mutually exclusive events then according to the addition theorem

$$P(A \cup B) = P(A) + P(B).$$

- If there are more than 2 events then

~~P(AUB~~

$$P(E_1 \cup E_2 \cup \dots \cup E_n) = P(E_1) + P(E_2) + \dots + P(E_n)$$

This rule is known as the theorem of ~~RoE~~ addition for mutually exclusive events.

Example:

If A & B are events of occurance of 2 & 3 on dies respectively then calculate the prob of occurance of 2 or 3 on dies

Sol: Let A = getting 2

B = getting 3.

~~Prob of~~ $P(A) = \frac{1}{6}$, $P(B) = \frac{1}{6}$

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A and B are independent and mutually exclusive events then according to addition theorem.

$$P(A \cup B) = P(A) + P(B) \cancel{- P(A \cap B)}$$
$$= \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$$

Theorem:

If A, B, C are mutually independent events then $A \cup B$ and C are also independent.

$$P[(A \cup B) \cap C] = P[(A \cap C) \cup (B \cap C)] \quad [\because \text{by distributive law}]$$
$$= P(A \cap C) + P(B \cap C) - P(A \cap B \cap C)$$
$$= P(A).P(C) + P(B).P(C) - P(A).P(B).P(C)$$

$\because$ since A, B, C mutually independent.

$$= P(C)\left[P(A) + P(B) - P(A)P(B)\right]$$
$$= P(C)\left[P(A) + P(B) - P(A \cap B)\right]$$
$$= P(C).P(A \cup B)$$

Hence $(A \cup B)$ and C are mutually independent.

Theorem:

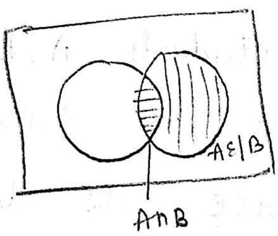
If A and B are independent then show that

(i) A^c and B

(ii) A and B^c

(iii) A^c and B^c

are also independent.



Proof

(i) ~~A^c and B~~.

$$B = (A \cap B) \cup (\bar{A} \cap B)$$

$$P(B) = P\left[(A \cap B) \cup [\bar{A} \cap B]\right]$$

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$= P(A \cap B) + P(\bar{A} \cap B)$

$P(\bar{A} \cap B) = P(B) - P(A \cap B)$.

i) A^c and B

$P(\bar{A} \cap B) = P(B) - P(A \cap B)$

$= P(B) - P(A) . P(B)$

$= P(B)[1 - P(A)]$

$= P(B) . P(\bar{A})$.

$\therefore$ A^c and B are mutually independent

ii) A and B^c

$P(A \cap \bar{B}) = P(A) - P(A \cap B)$

$= P(A) - P(A) . P(B)$

$= P(A)[1 - P(B)]$

$= P(A) . P(\bar{B})$.

$\therefore$ A & B^c are mutually independent.

iii) A^c & B^c

$P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B})$

$= 1 - P(A \cup B)$

$= 1 - [P(A) + P(B) - P(A \cap B)]$

$= 1 - [P(A) + P(B) - P(A) . P(B)]$

$= [1 - P(A)] - P(B)[1 - P(A)]$

$= [1 - P(A)][1 - P(B)]$

$= P(\bar{A}) . P(\bar{B})$.

3

Q) 3 students A, B, C are in running race A & B have the same prob of wining and each is twice as likely to win as C. Find the prob that B or C wins.

Sol: $P(A) = P(B)$

$P(A) = P(B) = 2P(C)$.

$$P(B \cup C) = P(B) + P(C) - P(B \cap C)$$

$$= \frac{2}{5} + \frac{1}{5} - 0$$

$$= \frac{3}{5}$$

$\because P(A)+P(B)+P(C) = 1$

$2P(C)+2P(C)+P(C) = 1$

$5P(C) = 1$

$P(C) = \frac{1}{5}$

$\therefore P(A) = \frac{2}{5}, P(B) = \frac{2}{5}$

Conditional ~~Probability~~: Event:

If E_1, E_2 are events of a sample space 's' and if E_2 occurs after the occurance of E_1, then the event of occurance of E_2 after the Event E_1 is called conditional event of E_2 given E_1, denoted by E_2/E_1. similarly we define E_1/E_2, occurance of E_1 after the event E_2.

Conditional Probability:

If E_1 and E_2 are two events of a sample space 's' and $P(E_1) \neq 0$ then the prob of E_2 after the event E_1 has occured is called conditional Prob. of event of E_2 given E_1 and is denoted by $P\left(\frac{E_2}{E_1}\right)$ or $P(E_2/E_1)$.

$$P(E_2/E_1) = \frac{P(E_1 \cap E_2)}{P(E_1)}, \quad P(E_1/E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)}$$

Example:

Q) 2 dies are thrown simultaneously, and the sum of the numbers obtained is found to be 7. What is the Prob that the number 3 has appeared at least once?

Sol: Let A = sum of the numbers obtained found to be 7

B = getting 3 event atleast once

$A = (1,6)(2,5)(3,4)(4,3)(5,2)(6,1)$

$B = (1,3)(2, \quad (3,4)(4,3)($

$P(B/A) = \dfrac{P(A \cap B)}{P(A)}$ $\quad n(A) = 6$

$n(A \cap B) = 2$

(or) $\dfrac{n(A \cap B)}{n(A)} = \dfrac{2}{6} = \dfrac{1}{3}$

Q) 15 numbered cards are there from 1 to 15, and 2 cards are chosen at random such that the sum of the numbers on both the cards is even. Find the prob that the chosen cards are odd numbered.

Sol: Let A = event of selecting 2 odd numbered cards.

B = event of selecting cards whose sum is even.

$P(A/B) = \dfrac{P(A \cap B)}{P(B)}$

$= \dfrac{n(A \cap B)}{n(B)}$

1 2 3 4 5 6 7
8 9 10 11 12 13
14 15

$n(B) = {}^8C_2 + {}^7C_2$

$= \dfrac{8!}{6!\,2!} + \dfrac{7!}{5!\,2!}$

$= 49$

$n(A \cap B) = {}^8C_2$

$= 28$

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$\frac{n(A \cap B)}{n(B)} = \frac{28}{49} = \frac{4}{7}$

Q) The prob of a student passing in science is $4/5$ and the of the student passing in both science and maths is $1/2$. What is the prob of that student passing in maths ~~that~~ knowing that he passed in science?

Sol: Given: Let A = event of passing in science

$$P(A) = \frac{4}{5}$$

B = event of passing in maths

$$P(B) = ?$$

$A \cap B$ = passing in both science and maths

$$P(A \cap B) = \frac{1}{2}$$

$$P(B/A) = ?$$

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{\frac{1}{2}}{\frac{4}{5}}$$

$$= \frac{5}{8}$$

Multiplication Theorem: Of Probability:

In a random experiment if E_1, E_2 are two events such that $P(E_1) \neq 0$ and $P(E_2) \neq 0$ then

$$P(E_1 \cap E_2) = P(E_1) \cdot P(E_2/E_1)$$

$$P(E_2 \cap E_1) = P(E_2) \cdot P(E_1/E_2)$$

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Proof: Let S be the sample space associated with the random experiment.

Let E_1, E_2 be 2 events of S such that $P(E_1) \neq 0$, $P(E_2) \neq 0$. by the definition of conditional prob of E_2 given E_1

$$P\left(\frac{E_2}{E_1}\right) = \frac{P(E_1 \cap E_2)}{P(E_1)}.$$

$$\therefore P(E_1 \cap E_2) = P(E_1) \cdot P\left(\frac{E_2}{E_1}\right)$$

since $P(E_2) \neq 0$ $\quad P\left(\frac{E_1}{E_2}\right) = \frac{P(E_1 \cap E_2)}{P(E_2)}$.

$$\therefore P(E_1 \cap E_2) = P(E_2) \cdot P\left(\frac{E_1}{E_2}\right).$$

Note: If A, B are 2 events then ~~$P(\bar{A})$~~

$$P(\bar{A} \cup \bar{B}) = 1 - P(A \cap B).$$

$$\cancel{= 1 - P(B) \cdot P(A/B)}$$

$$= 1 - P(A) \cdot P(B/A)$$

- It can be extended to 3 events E_1, E_2, E_3 as

$$P(E_1 \cap E_2 \cap E_3) = P(E_1) \cdot P\left(\frac{E_1}{E_2}\right) \cdot P\left(\frac{E_3}{E_1 \cap E_2}\right).$$

this result can be extended to four or more events.

Q) Find the prob of drawing 2 red balls in succession from a bag containing 4 red and 5 black balls when the ball that is drawn first is
(i) not replaced (ii) replaced.

Sol: Let A = event of drawing a red ball in the first draw.

B = event of drawing a red ball in second draw.

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the first ball can be drawn in 9 ways.

second ball " " 8 ways.

Both balls " " 9×8 ways.

There are 4 ways in which A can occur and 3 ways in which B can occur. so A & B can occur in 4×3 ways.

i) Drawing the ball is not replaced.

$$\frac{4}{9} \times \frac{3}{8} = \frac{1}{6}.$$

$$\Rightarrow \frac{1}{6}.$$

ii) replaced:

$$= \frac{4}{9} \times \frac{4}{9}$$

$$= \frac{16}{81}$$

Q) A problem in statistics is given to 3 students A, B, C whose chances of solving it are $1/2, 3/4, 1/4$. What is the prob that the problem is solved.

Sol: Let $P(A)$ = the prob that A to solve the problem

$$P(A) = \frac{1}{2}.$$

$P(B)$ = " B "

$$P(B) = \frac{3}{4}$$

$P(C)$ = the prob that C to solve the problem

$$P(C) = \frac{1}{4}.$$

The req prob is

$$P(A \cup B \cup C) = 1 - (P(A \cup B \cup C))^c = 1 - [P(A^c \cap B^c \cap C^c)]$$

$$= 1 - [P(A)^c . P(B)^c . P(C)^c]$$

$$= 1 - [(1 - P(A))(1 - P(B))(1 - P(C))]$$

$$= 1 - [(1 - \tfrac{1}{2})(1 - \tfrac{3}{4})(1 - \tfrac{1}{4})]$$

$$= 1 - \left(\frac{1}{2} \cdot \frac{1}{4} \cdot \frac{3}{4}\right) = 29/32.$$

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Baye's Theorem

Suppose $E_1, E_2 \ldots E_n$ are mutually exclusive events of sample space S. such that $P(E_i) > 0$, $i = 1, 2, 3 \ldots n$ and A is any orbitary event of S such that $P(A) > 0$ and $A \subseteq \bigcup_{i=1}^{n} E_i$ then conditional Prob of E_i given $P(A)$ for $i = 1, 2 \ldots n$ is

$$P(E_i/A) = \frac{P(E_i).P(A/E_i)}{\sum_{i=1}^{n} P(E_i).P(A/E_i)}$$

Problem:

Q) Bag A contains 2 white and 3 red balls. Bag B contains 4 white and 5 red balls. One ball is ~~stan~~ drawn at random from one the bag and it is found to be red. Find the prob that the red ball is drawn from bag B.

Sol: Let A = event of selecting bag A.
B = event of selecting bag B.
Let R = event of drawing a red ball.

$$P(B/R) = \frac{P(B).P(R/B)}{P(B).P(R/B) + P(A).P(R/A)}$$

$P(A) = \frac{1}{2}$, $P(B) = \frac{1}{2}$.

$P(R/B) = \frac{5}{9}$, $P(R/A) = \frac{3}{5}$

$$P(B/R) = \frac{\frac{1}{2}\left(\frac{5}{9}\right)}{\frac{1}{2}\left(\frac{5}{9}\right) + \frac{1}{2}\left(\frac{3}{5}\right)} = \frac{\frac{5}{18}}{\frac{5}{18} + \frac{3}{10}}$$

$$= \frac{\frac{5}{18}}{\frac{26}{45}} = \frac{25}{52}$$

Q) In a Bolt factory machines A, B, C manufacture 20%, 30%, 50% of the total of their output and 6%, 3% and 2% are defective. A bolt is drawn at random and found to be defective. What is the prob that is manufactured by machines A, B, C.

Sol: Let A = the bolt was manufactured by machine A

$P(A) = 20\% = 20/100$.

B = the bolt was manufactured by machine B.

C = " " C.

$P(B) = \cancel{50}\ 30\%$, $P(C) = 50\%$

$= \frac{30}{100}$, $P(C) = \frac{50}{100}$.

Let D = the bolt is defective.

So defective of machine A.

$\cancel{P(A)}$ $P(D/A) = 6\% = 6/100$

similly, $P(D/B) = 3\% = 3/100$

, $P(D/C) = 2\% = 2/100$.

(i) if bolt is defective then the prob that it is from machine A.

$$P(A/D) = \frac{P(A).P(D/A)}{P(A).P(D/A) + P(B).P(D/B) + P(C).P(D/C)}$$

$$= \frac{\frac{20}{100}\left(\frac{6}{100}\right)}{\frac{20}{100}\left(\frac{6}{100}\right) + \frac{30}{100}\left(\frac{3}{100}\right) + \frac{50}{100}\left(\frac{2}{100}\right)}$$

$$= \frac{\frac{3}{250}}{\frac{31}{1000}} = \frac{0.012}{0.031}$$

$$= \frac{12}{31}$$

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(ii) $P(B/D) = \dfrac{P(B) \cdot P(D/B)}{P(A) \cdot P(D/A) + P(B) \cdot P(D/B) + P(C) \cdot P(D/C)}$

$$= \frac{\frac{30}{100}\left(\frac{3}{100}\right)}{\frac{20}{100}\left(\frac{6}{100}\right) + \frac{30}{100}\left(\frac{3}{100}\right) + \frac{50}{100}\left(\frac{2}{100}\right)} = \frac{\frac{9}{1000}}{\frac{31}{1000}}$$

$$= \frac{9}{31}.$$

(iii) $P(C/D) = \dfrac{P(C) \cdot P(D/C)}{P(A) P(D/A) + P(B) \cdot P(D/B) + P(C) \cdot P(D/C)}$

$$= \frac{\frac{50}{100}\left(\frac{2}{100}\right)}{\frac{20}{100}\left(\frac{6}{100}\right) + \frac{30}{100}\left(\frac{3}{100}\right) + \frac{50}{100}\left(\frac{2}{100}\right)} = \frac{\frac{1}{100}}{\frac{31}{1000}}$$

$$= \frac{10}{31}$$

Q. An Insurance company has insured 4000 doctors, 8000 teachers, and 12000 businessmen. The probabilities of doctors, teacher and businessman dying before the age of 58 are 0.01, 0.03 and 0.05 respectively. If one of the insured individuals dies before 58, find the prob that he is a doctor.

Sol:

$$P(E_1) = \frac{4000}{4000 + 8000 + 12000} = \frac{4000}{24000}$$

$$= \frac{1}{6}$$

$$P(E_2) = \frac{8000}{4000 + 8000 + 12000} = \frac{8000}{24000}$$

$$= \frac{1}{3}$$

$$P(E_3) = \frac{12000}{24000} = \frac{1}{2}$$

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$P(A/E_1) = 0.01$, ~~$P(B \cdot P(A/E_2), P(A$~~

$P(A/E_2) = 0.03$, $P(A/E_3) = 0.05$

By Baye's theorem,

$$P(E_1/A) = \frac{P(E_1) \cdot P(A/E_1)}{P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2) + P(E_3) \cdot P(A/E_3)}$$

$$= \frac{\frac{1}{6}(0.01)}{\frac{1}{6}(0.01) + \frac{1}{3}(0.03) + \frac{1}{2}(0.05)}$$

$$= \frac{\frac{1}{600}}{\frac{11}{300}} = \frac{1}{22}.$$

Q) A businessman goes to hotels x, y, z 20%, 50%, 30% of the time respectively. It is known that 5%, 4%, 8% of the rooms in x, y, z hotels have ~~quality~~ fault plumbing. What is the prob that businessman room having ~~q~~ fault plumbing is assinged to hotel z.

Sol: Let the prob of businessman going to hotels x, y, z be respectively $P(x), P(y), P(z)$

then, $P(x) = \frac{20}{100}$; $P(y) = \frac{50}{100}$, $P(z) = \frac{30}{100}$.

Let E be the event that the hotel room has fault plumbing.

$P(E/x) = \frac{5}{100}$, $P(E/y) = \frac{4}{100}$, $P(E/z) = \frac{8}{100}$.

$$P(z/E) = \frac{P(z) \cdot P(E/z)}{P(x) \cdot P(E/x) + P(y)(E/y) + P(z) \cdot P(E/z)}$$

$$= \frac{\frac{30}{100}\left(\frac{8}{100}\right)}{\frac{20}{100}\left(\frac{5}{100}\right) + \frac{50}{100}\left(\frac{4}{100}\right) + \frac{30}{100}\left(\frac{8}{100}\right)} = \frac{\frac{3}{125}}{\frac{27}{500}} = \frac{4}{9}$$

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8) There are 3 bags, first bag contains 1 white, 2 red, 3 green balls, second bag contains, 2 white, 3 red, 1 green balls and third bag contains 3 white, 1 red, 2 green balls. 2 balls are drawn from a bag chosen at random these are found to be 1 white and 1 red. Find the prob that the balls so drawn came from the second bag.

Sol: $P(E_1) = \frac{1}{3}$, $P(E_2) = \frac{1}{3}$, $P(E_3) = \frac{1}{3}$.

Let A = event that 2 balls drawn are 1 white and 1 red respectively.

$$P(A/E_1) = \frac{{}^1C_1 \times {}^2C_1}{{}^6C_2} = \frac{2}{15}, \quad P(A/E_2) = \frac{{}^2C_1 \times {}^3C_1}{{}^6C_2} = \frac{6}{15}$$

$$P(A/E_3) = \frac{{}^3C_1 \times {}^1C_1}{{}^6C_2} = \frac{3}{15}$$

$$P(E_2/A) = \frac{P(E_2) \cdot P(A/E_2)}{P(E_1) \cdot P(A/E_1) + P(E_2) P(A/E_2) + P(E_3) \cdot P(A/E_3)}$$

$$= \frac{\frac{1}{3}\left(\frac{6}{15}\right)}{\frac{1}{3}\left(\frac{2}{15}\right) + \frac{1}{3}\left(\frac{6}{15}\right) + \frac{1}{3}\left(\frac{3}{15}\right)} = \frac{\frac{2}{15}}{\frac{11}{45}}$$

$$= \frac{6}{11}$$