Module-1 : probability

What is random experiment?

Experiment :-

An activity that produces a result or an outcome is called an experiment.

Random experiment :-

when we perform an activity or experiment, usually, we may get a different number of outcome from an experiment.

* However, when an experiment satisfies the following conditions, it is called a random experiment.

i) it has more than one possible outcome.

ii) it has not possible to predict the outcome in advance.

Defination :- probablistic situation is referred to as an random experiment.

Ex 1:- Is picking a card from a well-shuffled deck of cards a random experiment?

Solution:- Total 52 cards, each of these cards has an equal chance to be selected.

ex:

1. Tossing a coin 3 times

Number of possible outcomes $= 2^3 = 8$.

sample space $= S = \{HHH, HHT, HTH, THH, TTH, THT, HTT, TTT\}$

2. Rolling a pair of dice simultaneously.

Number of possible outcomes $= 6^n = 6^2 = 36$

sample space $= S =$

$\{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6) \ldots\ldots (6,6)\}$

Event:- Each outcome of an experiment or trail that is conducted is event

sample space:- The set of all the possible outcomes of an experiment is called sample space.

Sure event:- An event that will always occur is called sure event. A sure event has probability of 1.

Impossible event:- An event that will never occur is called impossible event. the probability of impossible event is 0.

favourable outcome:- An event that produces the desired result in an experiment is called favourable outcome.

Independent outcome:- Two events are said to be independent if the occurence of one event does not depend upon the other.

overlapping event:- The event that can happen individually as well as jointly are said to be overlapping events.

Types of events in probability:-

Equally Likely events:- Equally likely events are those whose chances or probability of happening are equal. Both events are not related to one another.

Example:- when we flip a coin there are equal possibilities of recieving either a head or a tail.

Mutually exclusive events:- Mutually exclusive events cannot occur at the same time. In other words, mutually exclusive events are called disjoint events. If two events are considered disjoint events, then the probability of both events occuring at the same time is zero

* when tossing a coin, the event of getting head and tail are mutually exclusive

Exhaustive events:- we call an event exhaustive when the set of all experiment results is the same as the sample space.

i.e Two events are exhaustive when thier union is equal to the sample space.

Ex:- The experiment of throwing a dice.

Sample Space = $S = \{1, 2, 3, 4, 5, 6\}$

* Assume that A, B, and C are the events associated with this experiment. Also, let us define these events as:

A: = event of getting a number greater than 3 $A = \{4, 5, 6\}$

B = event of getting a number greater than 2 but less than 5 $B = \{3, 4\}$

C = event of getting a number less than 3 $C = \{1, 2\}$

$A \cup B \cup C = \{4, 5, 6\} \cup \{3, 4\} \cup \{1, 2\} = \{1, 2, 3, 4, 5, 6\}$

$\therefore$ ABC are called exhaustive events.

Probability:-

The probability of an event is written as:

Probability can be defined as the possibility of occurence of an event. probability is the likehood or chances that an uncertain event will occur.

The probability of an event lies between 0 and 1

$$p(\text{event}) = \frac{\text{number of ways event can occur}}{\text{total no. of outcomes}}$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{m}{n} = \frac{n(E)}{n(S)}$$

$$nP_r = \frac{n!}{(n-r)!}$$

$$nC_r = \frac{n!}{(n-r)!\,r!}$$

$n(S)$ = sample space

Q:- In a class there are 10 boys and 5 girls, a comittee of 4 students is to be selected from the class. Find the probability for the committee to contain atleast 3 girls.

Sol:- Given

no. of boys = 10 } total no. of students = 15
no. of girls = 5

Committee of 4 members to be selected

$4 \rightarrow 3G + 1B$

$4 \rightarrow 4G + 0B$

$$P(E) = \frac{n(E)}{n(S)}$$

Sample space $= n(S) = 15C_4$

$$= \frac{15!}{11!\,4!} = 1365$$

$n(E) = (3G + 1B) + (4G + 0B)$

$= (5C_3 \times 10C_1) + (5C_4 \times 10C_0)$

$= 100 + 5$

$= 105$

$P(E) = \frac{n(E)}{n(S)} = \frac{105}{1365} = \frac{1}{13} = 0.0769$

Q:- A class consists 6 girls and 10 boys, if a committee of 3 is choosen at random, from the class, find the probability that i) 3 boys are selected ii) exactacly 2 girls are selected.

Sol: No. of boys = 10 } total no. of students = 16

No. of girls = 6

committe of 3 to be selected.

i) 3 boys are selected.

~~3 boys selec~~ $3 \longrightarrow 3B, 0G$

$P(E) = \frac{n(E)}{n(S)}$

$n(S) = 16C_3$

$n(E) = (3B, 0G)$

$= ({}^{10}C_3 \times {}^{6}C_0)$

$= {}^{10}C_3$

$P(E) = \frac{{}^{10}C_3}{{}^{16}C_3} = \frac{120}{560} = \frac{3}{14}$

ii) 2 girls are selected

$(1B, 2G)$

$n(S) = {}^{16}C_3 = 560$

$n(E) = (1B, 2G)$

$= ({}^{10}C_1 \times {}^{6}C_2) = 150$

$P(E) = \frac{n(E)}{n(S)} = \frac{150}{560} = \frac{15}{56}$

Q7 Two cards are selected ~~for~~ at random from 10 cards numbered 1 - 10. find the probability that sum is even

i) the two cards are drawn together

ii) the two cards are drawn one after other with replacement.

Sol,

2 cards are drawn at random from 10 cards numbered 1-10

~~sum is even = (1,3) (1,5), (1,7), (1,9), (2,4), (2,6) (2,8), (2,10), (3,1), (3,3), (3,5), (3,7), (3,9), (4,2), 6~~

i) two cards at a time

$n(S) = {}^{10}C_2$

Event = getting sum even

{1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

{1, 3, 5, 7, 9}

{2, 4, 6, 8, 10}

suppose two cards are drawn at a time number of cards.

sum 2 even numbers = even number

sum of 2 odd number = even number.

$n(E) = {}^5C_2 + {}^5C_2$

$= 10 + 10 = 20$

$P(E) = \frac{n(E)}{n(S)} = \frac{20}{45} \quad \frac{4}{9} = 0.444$

ii) suppose 2 cards are drawn one after the other with replacement for the sum to be even = both even or both odd

number of ways of selecting 2 even cards = $5C_1 \times 5C_1 = 25$

number of ways of selecting 2 odd cards = $5C_1 \times 5C_1 = 25$

total no. of favorable outcomes = $25 + 25 = 50$

$n(S) = 10C_1 \times 10C_1 = 100$

$P(E) = \frac{n(E)}{n(S)} = \frac{50}{100} = \frac{1}{2} = 0.5$

Q:- five digit numbers are formed with 0, 1, 2, 3, 4 (not allowing a digit bieng repeated in any number). find the probability of getting 2 in the ten's place and in the units place always?

sol: $n(S) = 5! - 4! = 96$

E = event of getting 2 in tenth place and 0 in the units place

No. of favourable outcomes = $3! = 6$

$P(E) = \frac{n(E)}{n(S)} = \frac{6}{96} = \frac{1}{16}$

Complementary events :-

* Two events of a sample space whose intersection is ϕ whose union is the entire sample space are called complementary events.

* Thus if E is an event of sample space S, its complement is denoted by E' or $\bar{E}$

Probability - Axiomatic Approach

Def :- Let S be a finite sample space. A real valued function P from the power set of S into R is called a probability function on S if the following three axioms are satisfied.

Axioms of Probability :-

i) Axiom of positivity : $P(E) \geq 0$ for every subset E of S

ii) Axiom of certainity : $P(S) = 1$

iii) Axiom of union : If E_1 and E_2 are disjoint subsets of S, then $P(E_1 \cup E_2) = P(E_1) + P(E_2)$

The image of $P(E)$ of E is called the probability of the Event (E).

Note:- If $E_1, E_2, E_3 \ldots E_n$ are disjoint subsets of S, then

$$P(E_1 \cup E_2 \cup E_3 \ldots \cup E_n) = P(E_1) + P(E_2) - - P(E_n)$$

* Example: In tossing two coins sample space

$S = \{HH, HT, TH, TT\}$

$P(HH) = \frac{1}{4}$

$P(HT, TH) = \frac{1}{2}$

$P(TT) = \frac{1}{4}$

$P(HH) + P(HT, TH) + P(TT) = \frac{1}{4} + \frac{1}{2} + \frac{1}{4} = 1$

These are mutually exclusive cases.

Q:- what is probability that a card drawn at random from pack of playing cards may be either a queen or a king.

sol:- $n(S) = 52$

E = Event of getting a king or queen

there are 4 kings & 4 queens in the deck of card

so, $n(E) = 8$

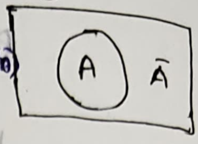
$P(E) = \frac{n(E)}{n(S)} = \frac{8}{52} = \frac{4}{26} = \frac{2}{13}$

Elementary theorems:-

*** Theorem 1:** probability of complementary event

$\bar{A} = 1 - P(A)$

proof:- $S = A \cup \bar{A}$ $\quad P(S) = P(A \cup \bar{A}) = P(A) + P(\bar{A})$



A and $\bar{A}$ are mutually exclusive events or disjoint sets.

$\therefore A \cap \bar{A} = \phi$

$P(S) = P(A \cup \bar{A})$

$= P(A) + P(\bar{A})$ [Axiom 3]

$1 = P(A) + P(\bar{A})$ [Axiom 2]

$$\boxed{P(\bar{A}) = 1 - P(A)}$$

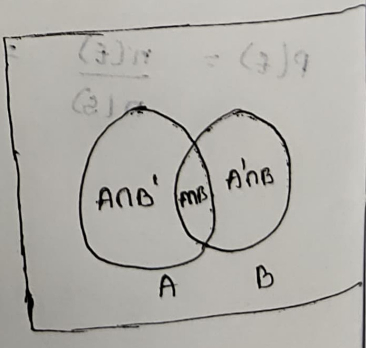
Note:- $\bar{A}$ also written as A' and A^c

*** Theorem 2:-** For any two events A and B:

$$P(A^c \cap B) = P(B) - P(A \cap B)$$

proof:- $A \cap B$ and $(A^c \cap B)$ are disjoint sets

$\therefore$ These are mutually exclusive



$\therefore P([A \cap B] \cup [A^c \cap B]) = P(A \cap B) + P(A^c \cap B)$

[by Axiom 3]

But $[A \cap B] \cup [A^c \cap B] = B$

$\therefore P(B) = P(A \cap B) + P(A^c \cap B)$

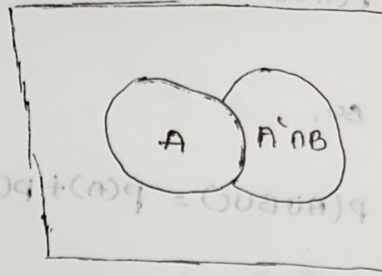
$\therefore P(A^c \cap B) = P(B) - P(A \cap B)$

Note:- similarly $P(A \cap B^c) = P(A) - P(A \cap B)$

* **Theorem 3** :- for any two events A and B ;

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

proof :- A and $A^c \cap B$ are disjoint sets



A and $A^c\cap B$ are mutually exclusive

$\therefore P(A\cup[A^c\cap B]) = P(A) + P(A^c\cap B)$

$$A\cup[A^c\cap B] = A\cup B$$

$$P[A\cup(A^c\cap B)] = P(A\cup B)$$

$$= P(A) + P(\bar{A}\cap B)$$

$$= P(A) + P(B) - P(A\cap B)$$

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

* **Theorem 4** :- For any three events A, B and C,

$$P(A\cup B\cup C) = P(A) + P(B) + P(C) - P(A\cap B) - P(B\cap C) - P(C\cap A) + P(A\cap B\cap C).$$

proof :- $P(A\cup B\cup C) = P(D\cup C)$ where $D = A\cup B$

$P(D\cup C) = P(D) + P(C) - P(D\cap C)$ [∵ theorem 3]

$$P(A\cup B\cup C) = P(A\cup B) + P(C) - P((A\cup B)\cap C)$$

$$P(A\cup B\cup C) = P(A) + P(B) - P(A\cap B) + P(C) - P[(A\cup B)\cap C)]$$

W.k.T

$\cancel{(A}\ (A\cup B)\cup C = (A\cap C)\cup(B\cap C)$ [distributive law]

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P[(A \cap B) \cup (B \cap C)]$

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - [P(A \cap C) + P(B \cap C) - P(A \cap B \cap C)]$

so,

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C)$
$+ P(A \cap B \cap C)$.

Q:- If $P(A) = \frac{1}{5}$, $P(B) = \frac{2}{3}$, $P(A \cap B) = \frac{1}{15}$

find 1) $P(A \cup B)$ 2) $P(\bar{A} \cap B)$ 3) $P(A \cap \bar{B})$

4) $P(\bar{A} \cap \bar{B})$ 5) $P(\bar{A} \cup \bar{B})$.

Sol:- Given $P(A) = \frac{1}{5}$, $P(B) = \frac{2}{3}$, $P(A \cap B) = \frac{1}{15}$

so,

1) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{5} + \frac{2}{3} - \frac{1}{15} = \frac{4}{5} = 0.8$

2) $P(\bar{A} \cap B) = P(B) - P(A \cap B)$

$= \frac{2}{3} - \frac{1}{15} = \frac{3}{5} = 0.6$

3) $P(A \cap \bar{B}) = P(A) - P(A \cap B)$

$= \frac{1}{5} - \frac{1}{15} = \frac{2}{15} = 0.133$

4) $P(\bar{A}\cap\bar{B}) = P(\overline{A\cup B}) = 1 - P(A\cup B)$

$= 1 - \frac{4}{5} = \frac{1}{5}$

5) $P(\bar{A}\cup\bar{B}) = P(\overline{A\cap B}) = 1 - P(A\cap B)$

$= 1 - \frac{1}{15} = \frac{14}{15}$

Q:- Two dice are thrown let A be the event that sum of the ~~face~~ point on the faces is 9. Let B be the event that atleast 1 number is 6. find probability of following events

1) $P(A\cap B)$ 2) $P(A\cup B)$ 3) $P(A\cap\bar{B})$ 4) $P(\bar{A}\cap B)$

5) $P(\overline{A\cap B})$ 6) $P(\overline{A\cup B})$.

Sol:- A = event of getting sum as 9.

B = event of getting atleast 1 number as 9.

Sample space = 36

$P(A) = \frac{4}{36} = \frac{1}{9}$

$P(B) = \frac{11}{36}$

1) $P(A\cap B) = \frac{2}{36} = \frac{1}{18}$

2) $P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$= \frac{1}{9} + \frac{11}{36} - \frac{1}{18} = \frac{13}{36}$

3) $P(A \cap \bar{B}) = P(A) - P(A \cap B)$

$= \frac{1}{9} - \frac{1}{18} = \frac{1}{18}$

4) $P(\bar{A} \cap B) = P(B) - P(A \cap B)$

$= \frac{11}{36} - \frac{1}{18} = \frac{1}{4}$

5) $P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$

$= 1 - \frac{13}{36} = \frac{23}{36}$

6) $P(\overline{A \cup B}) = 1 - P(A \cap B)$

$= 1 - \frac{1}{18} = \frac{17}{18}$

Q:- Among 150 students 80 are studing maths, 40 are studing physics & 30 are studing maths & physics. If a student is choosen at random find the probability that the student studying maths or physics. studing niether maths nor physics.

Sol: Total students = 150

studing physics = 40

studing Maths = 80

studing both = 30

A = event of getting a student reading maths or physics.

B = event of getting a student studing neither maths nor physics

$n(S) = 150$

$n(A) = 80 \cancel{+40}$, $\cancel{n(B) = 0}$ $n(B) = 40$

$\cancel{p(A) = \frac{80}{150} + \frac{40}{150} = \frac{120}{150} = \frac{4}{5}}$

$\cancel{p(B) = \frac{0}{150} = 0}$.

$p(A) = \frac{80}{150}$, $P(B) = \frac{40}{150}$, $p(A \cap B) = \frac{30}{150}$

1) $P(A \cup B) = \frac{80}{150} + \frac{40}{150} - \frac{30}{150} = \frac{80+40-30}{150}$

$= \frac{90}{150} = \frac{3}{5}$

2) $p(\bar{A} \cap \bar{B}) = p(\overline{A \cap B}) = 1 - p(A \cup B)$

$= 1 - \frac{3}{5} = \frac{2}{5}$

Q:- An integer is choosen from the first 200 positive number. what is the probability that the integer choosen is divisible by 6 or 8?

Sol:- Sample space $n(S) = 200$

$P(A)$ = probability that the number is divisible by 6

$P(B)$ = probability that the number is divisible by 8

$n(A) = 33$, $n(B) = 25$

$P(A) = \frac{n(A)}{n(S)}$, $P(B) = \frac{n(B)}{n(S)}$

$= \frac{33}{200}$, $P(B) = \frac{25}{200}$

$P(A \cap B) = \frac{8}{200}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{33}{200} + \frac{25}{200} - \frac{8}{200} = \frac{50}{200}$

$= \frac{1}{4} = 0.25$

Addition theorem on probability:-

If S is a sample space A, B are any two events in S then $P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Note:- If A, B are mutually exclusive events then according to the addition theorem

$$P(A \cup B) = P(A) + P(B)$$

* If there are more than two events then

$$P(A \cup B \cup \dots Z) = P(A) + P(B) + \dots \dots P(Z)$$

This rule is known as the Theorem of Addition for Mutually exclusive events

Q:- If A and B are occurence of 2 and occurence of 3 on a dice respectively, then calculate the probability of occurence of A or B i.e getting 2 or 3 on dice

Sol:- probability of getting $2 = \frac{1}{6}$

probabability of getting $3 = \frac{1}{6}$

As A and B are independent and mutually exclusive events, using the theorem of addition.

$$P(A \cup B) = P(A) + P(B)$$

$$P(A \cup B) = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$$

Theorem:- If A, B, C are mutually independent events then $A \cup B$ and C are also independent

$$p[(A \cup B) \cap C] = p[(A \cap C) \cup (B \cap C)] \quad [\because \text{distributive laws}]$$

$$= p[(A \cap C) + (B \cap C) - (A \cap B \cap C)]$$

$$= p(A \cap C) + p(B \cap C) - p(A \cap B \cap C) \quad [\because \text{Addition theorem}]$$

$$= p(A)\,p(C) + p(B)\,p(C) - p(A)\,p(B)\,(C)$$

$$= p(C)[p(A) + p(B) - p(A)p(B)]$$

$$= p(C)[p(A) + p(B) - p(A \cap B)]$$

$$= p(C)[p(A \cup B)]$$

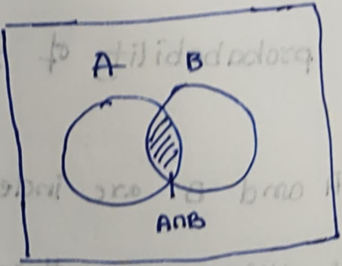
$\therefore$ $A \cup B$ and C are mutually independent.

Theorem: If A and B are independent then show that

i) A^c and B ii) A and B^c iii) A^c and B^c

are also independent.

$$B = (A \cap B) \cup (\bar{A} \cap B)$$



$$P(B) = p[(A \cap B) \cup (\bar{A} \cap B)]$$

$$= p[A \cap B) + p(\bar{A} \cap B)$$

$$p(\bar{A} \cap B) = p(B) - p(A \cap B)$$

(i) A^c and B

$P(\bar{A} \cap B) = P(B) - P(A \cap B)$

$= P(B) - P(A)(P(B))$

$= P(B)[\cancel{P} \; [1 - P(A)]$

$= P(B) \, P(\bar{A})$

$\therefore$ B and A^c are independent

(ii) A and B^c

$P(A \cap \bar{B}) = P(A) - P(A \cap B)$

$= P(A) - P(A)P(B)$

$= P(A) \; [1 - P(B)]$

$= P(A) \cdot P(\bar{B})$

$\therefore$ A and $\bar{B}$ are independent

(iii) A^c and B^c

$P(\bar{A} \cap \bar{B}) = P(\overline{A \cap B}) = 1 - P(A \cup B)$

$= \cancel{\text{deep}(A)p(B)})$

$= 1 - P(A) + P(B) + P(A \cap B)$

$= 1 - [P(A) + P(B) + P(A)(B)]$

$= [1 - P(A)] - P(B) \, [1 - P(A)]$

$= \cancel{(A} \; [1 - P(A)] \; [1 - P(B)]$

$= P(\bar{A}) \, P(\bar{B})$

$\therefore$ A^c and B^c are independent.

Q:- 3 students A, B, C are in running race

A and B have the same probability

and each is twice as likely to win as C.

Find the probability that B or C wins.

Sol:- $P(A) = P(B)$

$P(A) = P(B) = 2P(C)$

$P(B \cup C) = P(B) + P(C) - P(B \cap C)$

$P(A) + P(B) + P(C) = 1$

$2P(C) + 2P(C) + P(C) = 1$

$5P(C) = 1$

$P(C) = 1/5$, $P(A) = \frac{2}{5}$, $P(B) = \frac{2}{5}$

Now,

$P(B \cup C) = P(B) + P(C) - P(B \cap C)$

$= \frac{2}{5} + \frac{1}{5} - 0 = \boxed{3/5}$

Conditional event:-

If e_1, e_2 are events of a sample space S and if e_2 occurs after the occurence of e_1 then the event of occurence of e_2 after the event e_1 is called ~~contion~~ conditional event of e_2 given e_1.

* it is denoted by $\frac{e_2}{e_1}$, similarly we define

$\frac{e_1}{e_2}$ occurence of e_1 after the event e_2

conditional probability:-

If E_1 and E_2 are two events in a sample space S and $P(E_1) \neq 0$ then the probability of E_2 after the event E_1 has occured is called conditional probability of event of E_2 given E_1 and it is denoted by $P\left(\frac{E_2}{E_1}\right)$ or

$\cancel{P\left(\frac{E_1}{E_2}\right)}$ $P(E_2/E_1)$

$$P(E_2/E_1) = P(E_1 \cap E_2)/P(E_1)$$

$$P(E_1/E_2) = P(E_1 \cap E_2)/P(E_2)$$

$$= \frac{\frac{n(E_1 \cap E_2)}{\cancel{n(S)}}}{\frac{n(E_2)}{\cancel{n(S)}}} = \frac{n(E_1 \cap E_2)}{n(E_2)}$$

$$P(E_2/E_1) = \frac{\frac{n(E_1 \cap E_2)}{\cancel{n(S)}}}{\frac{n(E_1)}{\cancel{n(S)}}} = \frac{n(E_1 \cap E_2)}{n(E_1)}$$

Q:- Two dice are thrown simultaneously and the sum of the numbers obtained is found to be 7 what is the probability that the number 3 has appeared at least once.

Sol:- Two dice are thrown

sum of numbers should be 7 by getting 3 atleast once.

Let A = sum of the numbers obtained to be 7

B = getting atleast 3 at once.

$P(B) = 11/36$

$P(A) = 6/36$

$A \cap B = 2$

$P(A \cap B) = 2/36$

$P(A/B) = P(A \cap B)/P(B) = \frac{\cancel{2}}{\cancel{36}} \times \frac{\cancel{36}}{\cancel{6}\,3} = \frac{1}{3}$

Fifteen

Q:- ~~Ten~~ numbered cards are there from 1 to 15, and two cards are chosen at random such that numbers on both the cards are even. Find the probability that the choosen cards are odd

Sol: Let A = event of selecting two odd numbered cards

B = event of selecting cards whose sum is even

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

B = sum of even

$n(B)$ = no. of ways of choosing two numbers whose sum is even

$= {}^8C_2 + {}^7C_2$

$= 49$

$n(A \cap B) = {}^8C_2 = 28$

$$P(A/B) = \frac{n(A \cap B)}{n(B)} = \frac{28}{49} = \frac{4}{7}$$

Q:- The probability of a student passing in Science is 4/5 and of the student passing in both science and maths is 1/2. What is the probability of that student passing in maths knowing that he passed in science.

Sol: Let

A = event of passing in science

$P(A) = 4/5$

B = event of passing in maths

$A \cap B$ = passing in both science and maths.

$P(A \cap B) = 1/2$

$$P(B/A) = \frac{P(A \cap B)}{P(A)} = 1/2 \times \frac{5}{4}$$

$$= \frac{5}{8}$$

∴ the probability of passing in maths is 5/8

Multiplication theorem of probability :-

In a random experiment If E_1, E_2 are two events such that $P(E_1) \neq 0$ & $P(E_2) \neq 0$

then $P(E_1 \cap E_2) = P(E_1) \cdot P(E_2/E_1)$

also, $P(E_2 \cap E_1) = P(E_2) \cdot P(E_1/E_2)$

proof: Let S be the sample space associated with the random experiment

Let E_1, E_2 be two events of S such that $P(E_1) \neq 0$, $P(E_2) \neq 0$ by the defination the conditional probability E_2 given E_1

$$P\left(\frac{E_2}{E_1}\right) = \frac{P(E_1 \cap E_2)}{P(E_1)}$$

$$\therefore P(E_1 \cap E_2) = P(E_1) \cdot P(E_2 / E_1)$$

since $P(E_2) \neq 0$

$$P\left(\frac{E_1}{E_2}\right) = \frac{P(E_1 \cap E_2)}{P(E_2)}$$

$$P(E_1 \cap E_2) = P(E_2) \cdot P(E_1 / E_2)$$

Note :- * If A, B are two events then ~~$P(\bar{A} \cup \bar{B})$~~

$$P(\bar{A} \cup \bar{B}) = 1 - P(A \cap B)$$

$$= 1 - P(A) \cdot P(B/A)$$

* It can be extended to three events E_1, E_2 and E_3 as $P(E_1 \cap E_2 \cap E_3) = P(E_1) \cdot P(E_1/E_2) \cdot P\left(\frac{E_3}{E_1 \cap E_2}\right)$ or

this result can be extended to four more events.

Q :- Find the probability of drawing 2 red balls in succession from bag containing 4 red and 5 black balls when the ball that is drawn first is i) not replaced ii) replaced.

Sol:

Let A = event of drawing a red ball in the ~~the~~ first draw

B = event of drawing a red ball in the second draw

After first draw the ball is not replaced

the first ball can be drawn in 9 ways. $^9C_1$

Second ball ~~be~~ can be drawn in 8 ways. $^8C_1$

both balls can be drawn in $^9C_1 \times {}^8C_1$ ways

$^9C_1 \times {}^8C_1 = 72$

there are 4 ways in which A can occur

B can occur in 3 ways.

So A and B can occur in 4×3 ways.

$^4C_1 \times {}^3C_1 = 12$

$$P(A \cap B) = \frac{12}{72} = \frac{1}{6}$$

ii) suppose that ball is replaced after the first draw then

$$P(A \cap B) = \frac{4}{9} \times \frac{4}{9} = \frac{16}{81}$$

Q: A problem in statistics is given to 3 students A, B, C whose chances of solving it are $\frac{1}{2}$, $\frac{3}{4}$ and $\frac{1}{4}$ respectively. What is the probability that the problem is solve.

Sol) Let $P(A)$ = the probability A to solve the problem

$P(A) = \frac{1}{2}$

$P(B)$ = the probability ~~of~~ B to solve the problem

$P(B) = \frac{3}{4}$

$P(C)$ = the probability of C to solve the problem

$P(C) = \frac{1}{4}$

the required probability is

$P(A \cup B \cup C) = 1 - [P(A \cup B \cup C)^c]$

$= 1 - [P(A^c \cap B^c \cap C^c)]$

$= 1 - [P(A^c) \cdot P(B^c) \cdot P(C^c)]$

$= 1 - [[1 - P(A)] \cdot [1 - P(B)] \cdot [1 - P(C)]]$

$= 1 - [(1 - \frac{1}{2}) \cdot (1 - \frac{3}{4}) \cdot (1 - \frac{1}{4})]$

$= 1 - [\frac{1}{2} \cdot \frac{1}{4} \cdot \frac{3}{4}] = 1 - (\frac{3}{32}) = \frac{29}{32}$

Baye's theorem :-

Suppose $E_1, E_2, \dots E_n$ are mutually exclusive events of a sample S, such that $P(E_i) > 0$, $i = 1, 2, 3 \dots n$ and A is any arbitary event at S such that $P(A) > 0$ and $A \subseteq \bigcup_{i=1}^{n} E_i$, then conditional probability of E_i, given $P(A)$ for $i = 1, 2, \dots n$ is equal to

$$P(E_i/A) = \frac{P(E_i) \cdot P(A/E_i)}{\sum_{i=1}^{n} P(E_i) \cdot P(A/E_i)}$$

Q:- Bag A contains ~~outside~~ 2 white and 3 red balls and Bag B is contains 4 white and 5 red balls are ball is drawn at random from one of the bag and it is found to be red find the probability that the red ball drawn is from bag B.

Sol: Let A = event of selecting bag A

B = event of selecting bag B

Let R = event of drawing a Red ball

$P(B/R) = ?$

$$P(B/R) = \frac{P(B) \cdot P(R/B)}{P(A) \cdot P(R/A) + P(B) \cdot P(R/B)}$$

$P(A) = \frac{1}{2}$, $P(B) = \frac{1}{2}$

$P(R/B) = 5/9$

$P(R/A) = 3/5$

$$P(B/R) = \frac{1/2 \cdot 5/9}{\frac{1}{2} \cdot \frac{3}{5} + \frac{1}{2} \cdot 5/9} = \frac{\frac{5}{18}}{\frac{3}{10} + \frac{5}{18}}$$

$$= \frac{\frac{5}{18}}{\frac{164}{180}} = \frac{5}{18} \times \frac{180}{164} = \frac{900}{1872}$$

$$\begin{array}{r} 180 \\ \times 5 \\ \hline 900 \end{array} \qquad \begin{array}{r} 104 \\ \times 18 \\ \hline 832 \\ 104\times \\ \hline 1872 \end{array}$$

Q:- An insurance company has insured 4000 doctors, 8000 teachers, and 12000 businessmen. The probabilities of a doctor, teacher, and business man dying before the age of 58 are 0.01, 0.03, and 0.05 respectively. If one of the insured individuals dies before 58, find the probability that he is a doctor.

Sol:- Let E_1 = person is a doctor.

E_2 = person is a teacher

E_3 = person is a businessman

A = The death of an insured person

$P(E_1) = \frac{4}{24} = \frac{1}{6}$, $P(E_2) = \frac{8}{24} = \frac{1}{3}$, $P(E_3) = \frac{1}{2}$

$P(A/E_1) = 0.01$, $P(A/E_2) = 0.03$, $P(A/E_3) = 0.05$

$$P(E_1/A) = \frac{P(E_1) \cdot P(A/E_1)}{P(E_1) \cdot P(A/E_1) + P(E_2) P(A/E_2) + P(E_3) P(A/E_3)}$$

$$= \frac{\frac{1}{6} \times \frac{1}{100}}{\left(\frac{1}{6} \cdot \frac{1}{100}\right) + \left(\frac{1}{3} \cdot \frac{3}{100}\right) + \left(\frac{1}{2} \cdot \frac{5}{100}\right)} = \frac{\frac{1}{600}}{\frac{1}{600} + \frac{3}{300} + \frac{5}{200}}$$

$$= \frac{\frac{1}{600}}{\frac{1 + 6 + 15}{600}} = \frac{1}{22}$$

$$P(E_2/A) = \frac{\frac{1}{3} \cdot \frac{3}{100}}{\frac{22}{600}} = \frac{\frac{3}{300}}{\frac{22}{600}} = \frac{6}{22}$$

$$P(E_3/A) = \frac{\frac{15}{600}}{\frac{22}{600}} = \frac{15}{22}$$

$\frac{5}{2} \cdot \frac{5}{100}$ $\frac{5}{200}$ $\frac{22}{600}$ $\frac{\frac{5}{200}}{\frac{22}{600}} = \frac{15}{22}$