Part A

1) $a_n - 4a_{n-1} + 4a_{n-2} = 0$, $n \geq 2$, $a_0 = 5/2$,
$a_1 = 8$

let $a_n = x^n$

$x^n - 4x^{n-1} + 4x^{n-2} = 0$

$x^2 - 4x + 4 = 0$

The characteristic equation for above homogenous equation is $(x-2)^2 = 0$

roots are $x = 2, 2$

$a_n = a(x)^n + b.n(x)^n$

$a_n = a(2)^n + b.n(2)^n$

let $n = 0$

$a_0 = 5/2 = a(2)^0 + b(0)(2)^0$

$a = 5/2$

let $n = 1$

$a_1 = 8 = 2a + 2b$

$8 = 2(5/2) + 2b$

$b = 3/2$

$a_n = 5/2(2)^n + 3/2\, n(2)^n$

2) $a_n + a_{n-3} = 0 \quad , n \geq 3$

let $a_n = x^n$

$x^n + x^{n-3} = 0$

$x^3 + 1 = 0$

The characteristic equation for above homogeneous relations is $x^3 + 1 = 0$

roots are $x_1 = -1 \; ; \; x_2 = \frac{1+\sqrt{3}i}{2} \; , \; x_3 = \frac{1-\sqrt{3}i}{2}$

$= (x+1)(x^2 - x + 1)$

$a_n = a(x_1)^n + [b\cos(n\theta) + c\sin(n\theta)]r^n$

$x + 1 = 0 \longrightarrow x_1 = -1$

$\frac{1 \pm \sqrt{3}i}{2}$ $\qquad \theta = 60^\circ = \pi/3$

$r = \sqrt{\frac{1+3}{4}} = 1$

$a_n = a(-1)^n + (1)^n \left[b\cos\left(\frac{n\pi}{3}\right) + c\sin\left(\frac{n\pi}{3}\right) \right]$

The general solution for above equation is

$a_n = a(-1)^n + (1)^n \left[b\cos\left(\frac{n\pi}{3}\right) + c\sin\left(\frac{n\pi}{3}\right) \right]$

3) $a_{n+2} + 3a_{n+1} + 2a_n = 3a_n \quad , n \geq 0, a_0 = 0, a_1 = 1$

$$a_{n+2} + 3a_{n+1} + 2a_n - 3a_n = 0$$

$$a_{n+2} + 3a_{n+1} - 1a_n = 0 \quad \text{——} \quad (1)$$

The characteristic equation for (1) is

$$x^2 + 3x - 1 = 0$$

roots are $\frac{-3}{2} - \frac{\sqrt{13}}{2}$, $\frac{-3+\sqrt{13}}{2}$

$$a_n = a(x_1)^n + b(x_2)^n$$

$$a_n = a\left(\frac{-3-\sqrt{13}}{2}\right)^n + b\left(\frac{-3+\sqrt{13}}{2}\right)^n$$

let $n = 0$

$$a_0 = 0 = a\left(\frac{-3-\sqrt{13}}{2}\right)^0 + b\left(\frac{-3+\sqrt{13}}{2}\right)^0$$

$$a + b = 0 \Rightarrow \boxed{a = -b}$$

let $n = 1$

$$a_1 = 1 = a\left(\frac{-3-\sqrt{13}}{2}\right) + b\left(\frac{-3+\sqrt{13}}{2}\right)$$

since $a = -b$

$$1 = -b\left(\frac{-3-\sqrt{13}}{2}\right) + b\left(\frac{-3+\sqrt{13}}{2}\right)$$

$$1 = \frac{\cancel{3b} + b\sqrt{13}}{2} + \left(\frac{-\cancel{3b} + b\sqrt{13}}{2}\right)$$

$$b\sqrt{13} = 1 \qquad \boxed{b = 1/\sqrt{13}}$$

4) Coefficient of x^{52} in

i) $(x^4+x^5+x^6+\cdots\cdots)^5$

$(x^4)^5(1+x+x^2+\cdots\cdots)^5$

$x^{20}\left((1-x)^{-1}\right)^5 \Rightarrow x^{20}\cdot\sum_{r=0}^{\infty}\binom{5+r-1}{r}x^r$

$x^{20+r}\cdot\sum_{r=0}^{\infty}\binom{4+r}{r}$

$r=32$

$\frac{36!}{(32!)(4!)}x^{52} = \frac{36\times35\times34\times33}{4!}x^{52}$

$\Rightarrow 58905.$

ii) $(x^4+2x^5+3x^6+\cdots\cdots)^5$

$(x^4)^5(1+2x+3x^2+\cdots\cdots)^5$

$x^{20}\left((1-x)^{-2}\right)^5$

$x^{20}\cdot\sum_{r=0}^{\infty}\binom{9+r}{r}x^r$

$x^{20+r}\left(\sum_{r=0}^{\infty}\binom{9+r}{r}\right)$

$r=32.$

$\left(\sum_{r=0}^{\infty}\binom{41}{32}\right)\cdot x^{52}$

$\frac{41!}{32!\,9!}x^{52} = \frac{41\times40\times39\times\cdots\times32}{9!}x^{52}$

6) $a_n - 3a_{n-1} - 2a_{n-2} = 0$, $n \geq 2$, $a_0 = 5$, $a_1 = 3$

The characteristic equation for above homogenous an equation is

$$x^2 - 3x - 2 = 0$$

roots are $\frac{3-\sqrt{17}}{2}$, $\frac{3+\sqrt{17}}{2}$

$$a_n = a(x_1)^n + b(x_2)^n$$

$$a_n = a\left(\frac{3-\sqrt{17}}{2}\right)^n + b\left(\frac{3+\sqrt{17}}{2}\right)^n$$

let $n=0$

$$a_0 = 5 = a\left(\frac{3-\sqrt{17}}{2}\right)^0 + b\left(\frac{3+\sqrt{17}}{2}\right)^0$$

$$a+b=5 \quad \text{——— (1)}$$

let $n=1$

$$a_1 = 3 = a\left(\frac{3-\sqrt{17}}{2}\right) + b\left(\frac{3+\sqrt{17}}{2}\right)$$

$$3a+3b+b\sqrt{17}-a\sqrt{17} = 6 \quad \text{——— (2)}$$

from (1) and (2)

$$a = \frac{5\sqrt{17}+11}{2\sqrt{17}} \qquad b = \frac{5\sqrt{17}-11}{2\sqrt{17}}$$

$$a_n = \frac{5\sqrt{17}+11}{2\sqrt{17}}\left(\frac{3-\sqrt{17}}{2}\right)^n + \frac{5\sqrt{17}-11}{2\sqrt{17}}\left(\frac{3+\sqrt{17}}{2}\right)^n$$

6) $a_{n+3} - 3a_{n+2} + 3a_{n+1} - a_n = 3+5n, \quad n \geq 0$

$a_{n+3} - 3a_{n+2} + 3a_{n+1} - a_n = 3+5n$ is a nonhomogenous equation

The homogenous part of above equation is

$$a_{n+3} - 3a_{n+2} + 3a_{n+1} - a_n = 0$$

The characteristic equation is

$$x^3 - 3x^2 + 3x - 1 = 0$$
$$= (x-1)^3 = 0$$

The roots are 1, 1, 1 (repeated roots)

$$a_n = a_h + a_t$$

$$a_h = c_1 (x_1)^n + c_2 \cdot n \cdot (x_1)^n + c_3 \cdot n^2 \cdot (x_1)^n$$
$$= c_1 (1)^n + c_2 \cdot n \cdot (1)^n + c_3 \cdot n^2 \cdot (1)^n$$

$a_t =$

for $f(n) = 3+5n$

$$a_n^p = k_1 + 5n^3(k_2 n + k_3) \text{ --- (2)} \quad [k_1, k_2, k_3 = \text{const}]$$

substitute $a_n^{(p)}$ in a_n of given relation

$k_1 + 5(n+3)^3(k_2(n+3) + k_3)] - 3[k_1 + 5(n+2)^3(k_2(n+2) + k_3)] + 3[k_1 + 5(n+3)^3(k_2(n+1) + k_3)] - [k_1 + 5n^3(k_2 n + k_3)] = 3+5n$

$$k_1 = 0, \quad k_2 = \frac{1}{24}, \quad k_3 = -3/20$$

$$a_n^{(p)} = \frac{5}{24} n^4 - \frac{3}{4} n^3$$

$$a_n = a_n^{(h)} + a_n^{(p)}$$

$$a_n = c_1 + c_2(n) + c_3(n)^3 + 5/24\, n^4 - 3/4\, n^3$$

7) $a_n + 4a_{n-1} + 4a_{n-2} = 8$, $n \geq 2$, $a_0 = 1, a_1 = 2$

The homogenous part of above eq^n is

$$a_n + 4a_{n-1} + 4a_{n-2} = 0$$

The characteristic equation is

$$x^2 + 4x + 4 = 0$$

$$(x+2)^2 = 0$$

The roots are $-2, -2$

$$a_n = a_n^{h} + a_n^{p}$$

a_n^h - solution for homogenous part

a_n^p - solution for particular part

$$a_n^h = a(x_1)^n + b.n(x_1)^n$$

$$= a(-2)^n + b.n(-2)^n$$

The particular solution

$f(n) = 8$, polynomial of degree 0 and 1 is not the root of characteristic equation

Then solution is

$$a_n^p = A_0$$

$$\Rightarrow A_0 + 4A_0 + 4A_0 = 8$$

$$A_0 = 8/9$$

$$a_n = a_n^h + a_n^p$$

$$a_n = a(-2)^n + b.n(-2)^n + 8/9$$

let $n = 0$

$$a_0 = 1 = a + b + 8/9 \quad \text{---} \; (1)$$

put $n=1$

$$a_1 = 2 = -2a - 2b + 8/9 \quad \text{—— (2)}$$

using (1) and (2)

$$a = 1/9 \quad b = -2/3$$

$$a_n = 1/9\,(-2)^n + \left(-2/3\right)(-2)^n + 8/9$$

8) $s_K - 3s_{K-1} - 4s_{K-2} = 4^K, \; K \geq 2$

The homogenous part of above equation is

$$s_K - 3s_{K-1} - 4s_{K-2} = 0$$

The characteristic equation is

$$x^2 - 3x - 4 = 0$$

roots are $4, -1$

$$a_n^h = a(4)^n + b(-1)^n$$

$f(n) = 4^n \Rightarrow f(n) = \alpha b^n$ where α is constant

and

since $4 = 4$ and $4 \neq -1$ then $a_n^p = Anx^n$

$$a_n = a_n^h + a_n^p$$

$$= a(4)^n + b(-1)^n + a_n^p$$

$$s_K - 3s_{K-1} - 4s_{K-2} = 4^K$$

$$An4^n = 3A(n-1)4^{n-1} + 4A(n-2)4^{n-2} + 4^n$$

$$n = \frac{3(n-1)}{4} + \frac{(n-2)}{4} + 1/A$$

$$4(n) = 3(n-1) + (n-2) + \frac{4}{A}$$

$$\cancel{4n} = \cancel{3n} - 3 + \cancel{n} - 2 + \frac{4}{n}$$

$$\frac{4}{n} = 5 \qquad \boxed{n = \frac{4}{5}}$$

$$a_n = a_n^h + a_n^p$$

$$= a(4)^n + b(-1)^n + \left(\frac{4}{5}\right) n (4)^n$$

9) $a_{n+1} - a_n = n^2, n \geq 0$

The given relation is in the form of

$a_{n+1} = ca_n + \phi(n)$ where $c = 1$, $\phi(n) = n^2$

The generating function for relation is given by

$$f(x) = \frac{a_0 + xg(x)}{1 - cx} = \frac{a_0 + xg(x)}{1 - x}$$

$$g(x) = \sum_{n=0}^{\infty} \phi(n) x^n = \sum_{n=0}^{\infty} n^2 x^n$$

$g(x)$ is generating function for sequence n^2

$$g(x) = \frac{x(1+x)}{(1-x)^3}$$

given $a_0 = 1$

$$f(x) = \left\{1 + \frac{x^2(1+x)}{1-x^3}\right\} \frac{1}{1-x}$$

$$= \frac{(1-x)^3 + x^2(1+x)}{(1-x)^4}$$

$$= \frac{1 - 3x + 4x^2}{(1-x)^4}$$

10) $a_n - a_{n-1} - 6a_{n-2} = 0, n \geq 2, a_0 = 2, a_1 = 1$

The given relation can be written as

$$a_{n+2} - a_{n+1} - 6a_n = 0, n \geq 0.$$

The coefficients of a_{n+1} and a_n are $A = -1$

$B = -6$

and the relation is homogenous.

The generating function is

$$f(x) = \frac{a_0 + (a_1 + a_0 A)x}{1 + Ax + Bx^2}$$

$$= \frac{a_0 + (a_1 - a_0)x}{1 - x - 6x^2} \quad \text{--- (1)}$$

substitute $a_0 = 2$ and $a_1 = 1$ in eq (1)

$$= \frac{2 + (1-2)x}{1 - x - 6x^2}$$

$$= \frac{2 - x}{1 - x - 6x^2}$$

$$= \frac{x - 2}{6x^2 + x - 1}$$

Part – B

1) generating functions

i) 1, 2, 3, 4

$a_0 = 1$, $a_1 = 2$, $a_2 = 3$, $a_3 = 4$

$f(x) = a_0 + a_1 x + a_2 x^2 + \dots a_{n-1} x^{n-1} + \sum_{r=0}^{\infty} x^r a_r$

$= 1 + 2x + 3x^2 + 4x^3 + \dots$

$= (1-x)^{-2}$ is the generating function for 1, 2, 3, 4

ii) 1, −2, 3, −4

$a_0 = 1$, $a_1 = -2$, $a_2 = 3$, $a_3 = -4$

$f(x) = 1x^0 - 2x^1 + 3x^2 + (-4)x^3 + \dots$

$= 1 - 2x + 3x^2 - 4x^3 + \dots$

$= (1+x)^{-2}$ is the generating function for 1, −2, 3, −4

iii) 0, 1, 2, 3

$a_0 = 0$, $a_1 = 1$, $a_2 = 2$, $a_3 = 3$

$f(x) = 0x^0 + 1x^1 + 2x^2 + 3x^3 + \dots$

$= x + 2x^2 + 3x^3 + \dots$

$= x(1 + 2x + 3x^2 + \dots)$

$= x(1-x)^{-2}$ is the generating function for 0, 1, 2, 3

2) generating function for following

i) $1^2, 2^2, 3^2$

$a_0 = 1^2, a_1 = 2^2, a_2 = 3^2$

$f(x) = 1^2 x^0 + 2^2 x^1 + 3^2 x^2 + \dots$

$= 1 + 4x + 9x^2 + \dots$

$(1-x)^{-1} = 1 + x + x^2 + x^3 + \dots$

$(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \dots$

$x(1-x)^{-2} = x + 2x^2 + 3x^3 + 4x^4 + \dots$

taking derivative of both sides

LHS & RHS

differentiate LHS & RHS wrt x.

$1 \cdot (1-x)^{-2} + (+2)(1-x)^{-3} x$

$$\frac{(1-x)^2 + 2x(1-x)}{(1-x)^4} = 1 + 4x + 9x^2 + 16x^3 + \dots$$

$$\frac{(1-x)(1+x)}{(1-x)4} = 1 + 4x + 9x^2 + 16x^3 + \dots$$

$(1+x)(1-x)^{-3} = 1 + 4x + 9x^2 + 16x^3 + \dots$

The generating function for 1, 4, 9, 16 is

$(1+x)(1-x)^{-3}$

ii) $0^2, 1^2, 2^2, 3^2 \ldots$

$a_0 = 0, \; a_1 = 1, \; a_2 = 4, \; a_3 = 9$

$f(x) = 0x^0 + 1x^1 + 4x^2 + 9x^3 + \cdots$

$= x + 4x^2 + 9x^3 + \cdots$

$= x(1 + 4x + 9x^2 + \cdots)$

$= x((1+x)(1-x)^{-3})$

$= x(1+x)(1-x)^{-3}$

3) i) $1^3, 2^3, 3^3 \ldots$

$a_0 = 1^3, \quad a_1 = 2^3, \quad a_2 = 3^3$

$f(x) = 1^3 x^0 + 2^3 x^1 + 3^3 x^2 + \ldots$

$= 1 + 8x + 27x^2 + \ldots$

let $\dfrac{x(1+x)}{(1-x)^3} = x + 4x^2 + 9x^3 + \ldots$ — ①

differentiate eq ① on both sides wrt x

$\dfrac{x^2+4x+1}{(1-x)^4} = 1 + 8x + 27x^3 + \ldots$

The generating function for $1, 8, 27 \ldots$ is $\dfrac{x^2+4x}{(1-x)}$

ii) $0, 1^3, 2^3, 3^3 \ldots$

$a_0 = 0, \quad a_1 = 1, \quad a_2 = 8, \quad a_3 = 27$

$f(x) = 0x^0 + 1x^1 + 8x^2 + 27x^3$

$= x + 8x^2 + 27x^3 + \ldots$

$= x(1 + 8x + 27x^2 + \ldots)$ — ①

let $1 + 8x + 27x^2 + \ldots = \dfrac{x^2+4x+1}{(1-x)^4}$ — ②

substitute ② in ①

$= x\left(\dfrac{x^2+4x+1}{(1-x)^4}\right)$ is the generating function

4) generating function for 1, 1, 0, 1, 1, 1

$a_0 = 1,\ a_1 = 1,\ a_2 = 0,\ a_3 = 1,\ a_4 = 1,\ a_5 = 1$

$f(x) = 1x^0 + 1x^1 + 0x^2 + 1x^3 + 1x^4 + 1x^5$

$= 1 + x + x^3 + x^4 + x^5 + \text{---}$

$= 1 + x + x^3(1 + x + x^2 + \text{---})$

let $1 + x + x^2 + \text{---} = (1-x)^{-1}$

$= 1 + x + x^3((1-x)^{-1})$

$= 1 + x + \dfrac{x^3}{1-x}$

$= \dfrac{(1+x)(1-x) + x^3}{1-x}$

$= \dfrac{x^3 - x^2 + 1}{1-x}$

The generating function for 1, 1, 0, 1, 1, 1 is

$\dfrac{x^3 - x^2 + 1}{1-x}.$

5) coeffecient of x^{12} in $x^3(1-2x)^{10}$

$$(1-x)^n = \sum_{r=0}^{n} {}^nC_r (1)^{n-r} (-x)^r$$

$$(a+b)^n = \sum_{r=0}^{n} {}^nC_r (a)^{n-r} (b)^r$$

$$(a-b)^n = \sum_{r=0}^{n} {}^nC_r (a)^{n-r} (-b)^r$$

$$= x^3 (1-2x)^{10}$$

$$= x^3 \sum_{r=0}^{n} {}^{10}C_r (1)^{10-r} (-2x)^r$$

we need coeffecient of x^{12}

$$\Rightarrow x^3 \left[\sum_{r=0}^{n} {}^{10}C_r (1)^{10-r} (-2x)^r \right]$$

$$\Rightarrow {}^{10}C_r (-2)^r \sum_{r=0}^{n} (1)^{10-r} (x)^{3+r}$$

$$3 + r = 12$$

$$\boxed{r = 9}$$

$\Rightarrow$ ${}^{10}C_9 (-2)^9$ (is the coeff of x^{12})

$\Rightarrow$ $10 \times (-521)$

$\Rightarrow$ -5210 is coeffecient of x^{12}

6) coeffecient of x^5 in $(1-2x)^{-7}$

$$(a-b)^n = \sum_{r=0}^{n} {}^nC_r\, a^{n-r}\, b^r$$

r is the power of required variable coeffecient

for $(1-2x)^{-7}$

$r = 5$

$$(a-b)^{-n} = \sum_{r=0}^{\infty} {}^{n+r-1}C_r\, a^{n-1}\, b^r$$

$n=7$, $r=5$ in given question

$= {}^{7+5-1}C_5\, (1)^6 (2x)^5$

$= {}^{11}C_5\, (1)^6 (2x)^5$

The coeffecient of x^5 is $= {}^{11}C_5 (2)^5$

$= 14,784$

7) i) coefficient of x^{27} in $(x^4+x^5+x^6 \dots)^5$

$(x^4+x^5+x^6+\dots)^5 = x^{20}(1+x+x^2 \dots)^5$

$= x^{20}((1-x)^{-1})^5$

$= x^{20}(1-x)^{-5}$

$\because 1+x+x^2+\_\_\_ = (1-x)^{-1}$

coefficient of x^r in $(1-x)^{-n}$ is ${}^{n+r-1}C_r x^r$

we need coefficient of x^{27} in $(1-x)^{-5} x^{20}$

$= x^{20} \times \sum_{r=0}^{\infty} {}^{4+r}C_r x^r$

$r=7$

$= {}^{11}C_7 = 330$

The coefficient of x^{27} is 330.

ii) coefficient of x^{27} in $(x^4+2x^5+3x^6 \dots)^5$

$(x^4+2x^5+3x^6+\dots)^5 = x^{20}(1+2x+3x^2+\dots)^5$

$[1+2x+3x^2+\dots = (1-x)^{-2}]$

$= x^{20}[(1-x)^{-2}]^5$

$$= x^{20} \times (1-x)^{-10}.$$

$$= x^{20} \times \sum_{r=0}^{\infty} {}^{9+r}C_r x^r$$

$$\left[(1-x)^{-n} = \sum_{r=0}^{\infty} {}^{n+r-1}C_r x^r\right]$$

$$r = 7$$

$$= {}^{16}C_7 = 11,440$$

The coeffecient of x^{27} is 11,440.

8) $a_n = a_{n-1} + n^3,\ n \geq 1,\ a_0 = 5.$

using substitution method :

$n = 01$

$a_{01} = a_0 + (1)^3 \Rightarrow a_1 = a_0 + 1$

$= 5 + 1$

$\boxed{a_1 = 6}$

$n = 2$

$a_2 = a_1 + 2^3$ $[a_1 = a_0 + 1^3]$

$= a_0 + 1^3 + 2^3$

$= 14$

$a_3 = a_2 + (3)^3$ $\left[\begin{array}{l} a_2 = a_1 + 2^3 \\ a_1 = a_0 + 1^3 \end{array}\right]$

$= a_1 + 2^3 + 3^3$

$= a_0 + 1^3 + 2^3 + 3^3$ $[a_0 = 5]$

$= 41$

$a_n = a_0 + 1^3 + 2^3 + \ldots n^3$

$= a_0 + \Sigma n^3$

$= a_0 + \left(\frac{n(n+1)}{2}\right)^2$

$= 5 + \frac{n^2(n+1)^2}{4} = \frac{20 + n^2(n+1)^2}{4}$

10) $a_{n+1} = 8a_n, n \geq 0, a_0 = 4$

$$a_n = \frac{a_{n+1}}{8}$$

The characteristic equation is $x - 8 = 0$

$$x = 8$$

$$a_n = a(x)^n$$

$$a_n = a(8)^n$$

$$n = 0$$

$$a_0 = 4 = a(8)^0$$

$$\boxed{a = 4}$$

$$a_n = 4(8)^n$$

11) $a_n + a_{n-1} + 10a_{n-2} = 7$, $n \geq 2$, $a_0 = 10$, $a_1 = 4$

The characteristic equation for homogenous part is $x^2 + x + 10 = 0$

$$x_1 = \frac{-1+\sqrt{39}\,i}{2}, \quad x_2 = \frac{-1-\sqrt{39}\,i}{2}$$

$$a_n = r^n(a\cos(n\theta) + b\sin(n\theta))$$

$$\theta = \pi - \tan^{-1}(\sqrt{39})$$

$$r = \sqrt{10}$$

$$a_n = (\sqrt{10})^n\left(a\cos(n(\pi - \tan^{-1}(\sqrt{39}))) + b\sin(n(\pi - \tan^{-1}(\sqrt{39})))\right)$$

$$a_n^p = A_0$$

$$\Rightarrow A_0 + A_0 + 10A_0 = 7 \Rightarrow A_0 = 7/12$$

$$a_n = a_n^{(p)} + a_n^{(h)}$$

$$= (\sqrt{10})^n\left(\cos(n(\pi - \tan^{-1}(\sqrt{39}))) + \sin(n(\pi - \tan^{-1}(\sqrt{39})))\right) + 7/12$$

13) $a_n = 3a_{n-1} + 2n$, $a_1 = 3$ — I

characteristic equation for homogeneous eqn

$a_n = 3a_{n-1}$ ie $x = 3$

The solution is $a_n = a_n^{(h)} + a_n^{p}$

$a_n^{(h)} = A(x_1)^n$

$a_n^{(h)} = A(3)^n$

$a_n^{p} = A_0 + A_1 n$ — ①

put eq ① in — I

$A_0 + A_1 n = 3[A_0 + A_1(n-1)] + 2n$

$A_0 + A_1 n = 3A_0 + 3A_1 n - 3A_1 + 2n$

$-2A_0 - 2A_1 n + 3A_1 = 2n$

$2A_1 n + 2A_0 - 3A_1 = -2n$

$2A_1 n - 3A_1 + 2A_0 + 2n = 0$

$2A_1 n - 3A_1 + 2A_0 = -2n$

equate the corresponding terms on both side

$2A_1 = -2$, $-3A_1 + 2A_0 = 0$

$\boxed{A_1 = -1}$ $A_1 = \frac{2}{3} A_0$

$\boxed{A_0 = \frac{3}{2}(-1)}$

$A_1 = -1$, $A_0 = \frac{-3}{2}$ — put them in equation — ①.

$$a_n^{(p)} = \left(\frac{-3}{2}\right) + (-1)^n$$

$$a_n = a_n^h + a_n^p$$

$$a_n = A(3)^n + (-1)^n + \left(\frac{-3}{2}\right)$$

let $n=1$

$$a_1 = A(3)^1 + (-1)(1) + \left(\frac{-3}{2}\right)$$

given $a_1 = 3$

$$3 = 3A - 1 - 3/2$$

$$3 = 3A - 5/2$$

$$3A = \frac{11}{2} \qquad \boxed{A = 11/6}$$

$$a_n = 11/6\,(3)^n + (-1)^n + \left(\frac{-3}{2}\right)$$

14) $a_n - 3a_{n-1} = n$, $a_0 = 1$, $n \geq 1$ — using generating function

let $n = 1$

$a_1 = (1) + 3(a_0)$

$= 1 + 3(1)$

$= 4$

let $n = 2$

$a_2 = 3a_1 + 2$

$= 3[3(a_0) + 1] + 2$

$= 9(a_0) + 2 + 3$

let $n = 3$

$a_3 = 3a_2 + 3$

$= 3[3a_1 + 2] + 3$

let $n = 1$

$a_1 = 8 = -3a + 2b$

$-3a + 2b = 8$ ——— (2)

from (1) and (2) we have

$a = -2 \quad , b = 1$

$a_n = -2(-3)^n + 1(2)^n$

9) $a_n = a_{n-1} + 3a_{n-2} + 3n \quad , n \geq 1$

15) $a_{n+1} - a_n = 3^n \quad , n \geq 0 , \ a_0 = 1$

let $n = 0$

$a_1 = 3^0 + a_0$

$a_1 = 3^0 + a_0$

let $n = 1$

$a_2 = 3^1 + a_1$

$= 3^1 + 3^0 + a_0$

let $n = 2$

$a_3 = 3^2 + a_2$

$= 3^2 + 3^1 + 3^0 + a_0$

let $n = r$

$a_r = a_0 + 3^0 + 3^1 + \ldots 3^r$

$= a_0 + \sum_{r=0}^{n} 3^r$

$= a_0 + \frac{3^{n+1} - 1}{2}$

$a_0 = 1$

$= 1 + \frac{3^{n+1} - 1}{2}$

$\Rightarrow \frac{2 + 3^{n+1} - 1}{2} = \frac{3^{n+1} + 1}{2}$

16) coeff of x^{15} in $(1+x)^4 (1-x)^4$

$\Rightarrow (1+x)^4 (1-x)^4$

$\Rightarrow (1-x^2)^4$

The above expansion is

$^4C_0 (1)^4 (-x^2)^0 + {}^4C_1 (1)^3 (-x^2)^1 + {}^4C_2 (1)^2 (-x^2)^2$
$+ {}^4C_3 (1)^1 (-x^2)^3 + {}^4C_4 (1)^0 (-x^2)^4$

$\Rightarrow {}^4C_0 + {}^4C_1 (-x^2) + {}^4C_2 (x^4) + {}^4C_3 (-x^6)$
$+ {}^4C_4 (-x^8)$

since the variable x has highest degree of 8, we cannot find the coefficient of x^{15} in $(1+x)^4 (1-x)^4$

(2) ~~coeff of x^{10} in $(x^3)($~~

(2) coeff of x^{10} in $(x^3-5x)/(1-x)^3$

$\Rightarrow \dfrac{(x^3-5x)}{(1-x)^3}$

$\Rightarrow (x^3-5x)(1-x)^{-3}$

$\Rightarrow x^3(1-x)^{-3} - 5x(1-x)^{-3}$ —— (I)

coefficient of x^{10} in eq (I)

$\Rightarrow$ coeff of x^{10} in $(x^3(1-x)^{-3})$ + coeff of x^{10} in $(-5x(1-x)^{-3})$

$\Rightarrow$ coefficient of x^{10} in $x^3(1-x)^{-3}$

$\Rightarrow$ coefficient of x^7 in $(1-x)^{-3}$

$$(1-x)^{-n} = \sum_{r=0}^{\infty} {}^{n+r-1}C_r x^r$$

$\Rightarrow {}^{3+r-1}C_r x^r$

$\Rightarrow {}^{2+r}C_r x^r$

since we need coefficient of x^7 $r=7$

$\Rightarrow {}^9C_7$

The coefficient of x^{10} in $x^3(1-x)^{-3}$ is ${}^9C_2$

$\Rightarrow 36$

coefficient of x^{10} in $-5x(1-x)^{-3}$

$\Rightarrow$ coefficient of x^9 in $(1-x)^{-3}$

$$(1-x)^{-n} = \sum_{r=0}^{\infty} {}^{n+r-1}C_r x^r$$

$$= {}^{3+r-1}C_r x^r$$

$$= {}^{2+r}C_r x^r$$

since we need coefficient of x^9 $r=9$

$$= {}^{11}C_9$$

The coefficient of x^{10} in $x(1-x)^{-3}$ is $(-5)({}^{11}C_9)$

$$= -275$$

The coefficient of x^{10} in $(x^3-5x)(1-x)^{-3}$ is

$$36-(275)$$

$$= -239$$

18) $a_n + a_{n-1} - 6a_{n-2} = 0$, $a_0 = -1$, $a_1 = 8$

The characteristic equation for above is

$$x^2 + x - 6 = 0$$

roots are $x_1 = -3$, $x_2 = 2$

$$a_n = a(x_1)^n + b(x_2)^n$$
$$= a(-3)^n + b(2)^n$$

let $n = 0$

$$a_0 = -1 = a + b$$

$$a + b = -1 \quad \text{—— (1)}$$

20) $a_n = a_{n-1} + n$, $a_0 = 2$, using substitution method

let $n = 1$

$a_1 = a_0 + 1$

$= 2 + 1$

$= 3$

$n = 2$

$a_2 = a_1 + 2$

$= a_0 + 1 + 2$

$n = 3$

$a_3 = a_2 + 3$

$= a_0 + 1 + 2 + 3$

$n = n$

$a_n = a_0 + \Sigma n$

$= a_0 + \frac{n(n+1)}{2}$

$2a_n = 2a_0 + n(n+1)$

Part - C

1) part-B - 1(i)
2) part B - 1(ii)
3) part B - 1(iii)
4) part B - 1(iv)
5) part B - 5
6) part B - 6
7) part B - 7(i)
8) part B - 2(i)
9) part B - 2(ii)
10) part B - 7(ii)
11) part B - 3(i)
12) part B - 8
13) ———
14) part B - 3(ii)
15) part B - 10
16) ———
17) part B - 19(i)
18) ———
19) ———
20) part B - 7(ii)
21) part B - 8

Part C

18) generating function for 1, 1, 1, 1 ...

$a_0 = 1,\ a_1 = 1,\ a_2 = 1 \ldots$

$f(x) = 1x^0 + 1x^1 + 1x^2 + 1x^3 + \ldots$

$(1-x)^{-1} = 1 + x + x^2 + x^3 + \ldots$

The generating function is $(1-x)^{-1}$

19) generating function for 1, -1, 1, -1 ...

$a_0 = 1,\ a_1 = -1,\ a_2 = 1,\ a_3 = -1$

$f(x) = 1x^0 - x + x^2 - x^3 + \ldots$

$= 1 - x + x^2 - x^3 + \ldots$

$(1+x)^{-1} = 1 - x + x^2 - x^3 + x^4 \ldots$

The generating function is $(1+x)^{-1}$

16) $a_{n+1} = 8a_n,\ n \geq 0,\ a_0 = 6$

The characteristic equation is $x - 8 = 0$

$x = 8$

$a_n = a(x)^n$

$a_n = a(8)^n$

let $n = 0$

$a_0 = 6 = a(8)^0$

$\boxed{a = 6}$

$a_n = 6(8)^n$