ELECTRICAL CIRCUITS

Course Code	: AEEC02
Category	: Foundation
Credits	: 3
Maximum Marks	: CIA (30) + SEE (70) = Total (100)

By
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COURSE OVERVIEW

The course introduces the basic concepts of circuit analysis which is the foundation for all subjects of the electrical and electronics engineering. It includes the basic fundamental laws of electricity and magnetism with an emphasis on resistors, inductors and capacitors (RLC) circuits applied to alternating current (AC) or direct current (DC) of electrical networks. Further This course provides network theorems with different excitations, two-port network and network topology to solve for real-time applications.

COURSE OBJECTIVES

The students will try to learn

I. The network reduction techniques such as source transformation, mesh analysis, nodal analysis and network theorems to solve different networks.

II. The basic concept of AC circuits for optimization of household and industrial circuitry.

III. The various configurations of electromagnetic induction used in magnetic circuits helps in the winding of electrical machines.

IV. The characteristics of two-port networks and network topologies suitable in power system.

COURSE SYLLABUS

MODULE-I: INTRODUCTION TO ELECTRICAL CIRCUITS

Circuit Concept: Basic definitions, Ohm‟s law at constant temperature, classifications of elements, independent and dependent sources, voltage and current relationships for passive elements,

Single Phase AC Circuits: Representation of alternating quantities, properties of different periodic wave forms, phase and phase difference, concept of impedance and admittance, power in AC circuits.

COURSE OUTCOMES

CO 1 : Define the various terminology used to study the characteristics of DC and AC electrical networks.

CO 2 : Discuss the different laws associated with electrical circuits to determine equivalent resistance and source currents.

CO 3 : Identify the alternating quantities with peak, average and root mean square values for different periodic wave forms.

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COURSE SYLLABUS

MODULE-II: ANALYSIS OF ELECTRICAL CIRCUITS

Circuit Analysis: Source transformation, Kirchhoff"s laws, total resistance, inductance and capacitance of circuits, Star - delta transformation technique, mesh analysis and nodal analysis, inspection method, super mesh, super node analysis.

COURSE OUTCOMES

CO 2 : Discuss the different laws associated with electrical circuits to determine equivalent resistance and source currents.

CO 4 : Discuss the indirect quantities associated with electrical circuit for branch currents and nodal voltages.

COURSE SYLLABUS

MODULE-III: NETWORK THEOREMS (DC AND AC)

Network Theorems (DC): Tellegen‟s, superposition, reciprocity, Thevenin‟s, Norton‟s, maximum power transfer, Milliman‟s and compensation theorems for DC excitations, numerical problems.

Network Theorems (AC): Tellegen‟s, superposition, reciprocity, Thevenin‟s, Norton‟s, maximum power transfer, Milliman‟s and compensation theorems for AC excitations, numerical problems.

COURSE OUTCOMES

CO 5 : Discuss the superposition principle, reciprocity and maximum power transfer condition for the electrical network with DC and AC excitation.

CO 6 : Summarize the procedure of thevenin’s, norton’s and milliman’s theorems to reduce complex network into simple equivalent network with DC and AC excitation.

COURSE SYLLABUS

MODULE-IV: MAGNETIC CIRCUITS

Magnetic circuits: Faraday"s laws of electromagnetic induction, concept of self and mutual inductance, dot convention, coefficient of coupling, composite magnetic circuit, analysis of series and parallel magnetic circuits.

COURSE OUTCOMES

CO 7 : Recall the faraday's laws of electromagnetic induction used in construction of magnetic circuit.

CO 8 : Describe the magnetic flux, reluctance, self and mutual inductance in the single coil and coupled coils magnetic circuits to know total magnetomotive force and total ampere turns values.

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COURSE SYLLABUS

MODULE-V: TWO PORT NETWORK AND GRAPH THEORY

Two Port Network: Two port parameters, interrelations, Two port Interconnections.

Network topology: Definitions, incidence matrix, basic tie set and basic cut set matrices for planar networks, duality and dual networks.

COURSE OUTCOMES

CO 9 : Discuss the two port parameters to be measure easily, without solving for all the internal voltages and currents in the different networks.

CO 10 : List the various network topology for graphical and digital representation of complex circuits to be utilized in power system

CO 11 : Define the importance of dual network for compare both mesh and nodal networks.

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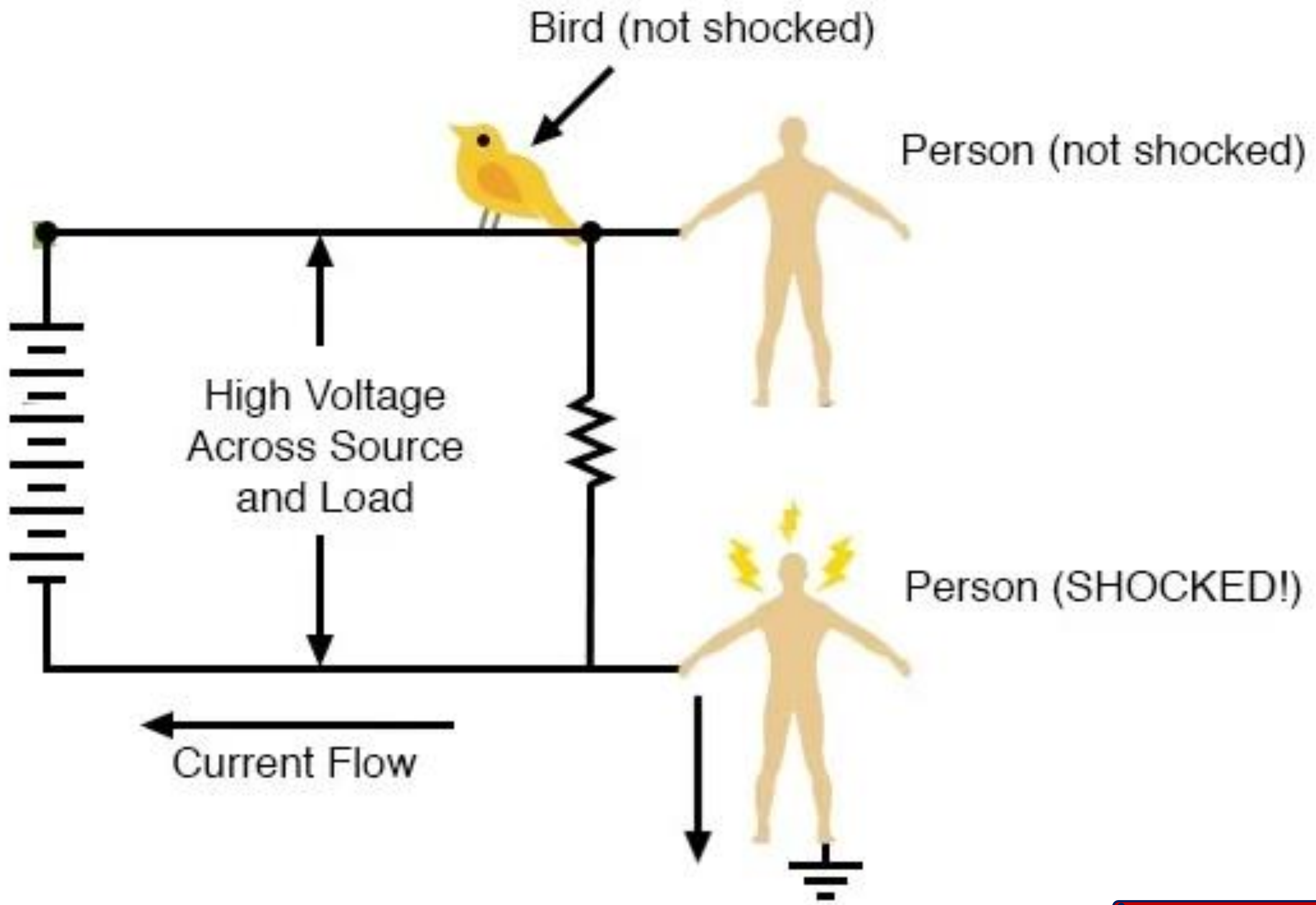
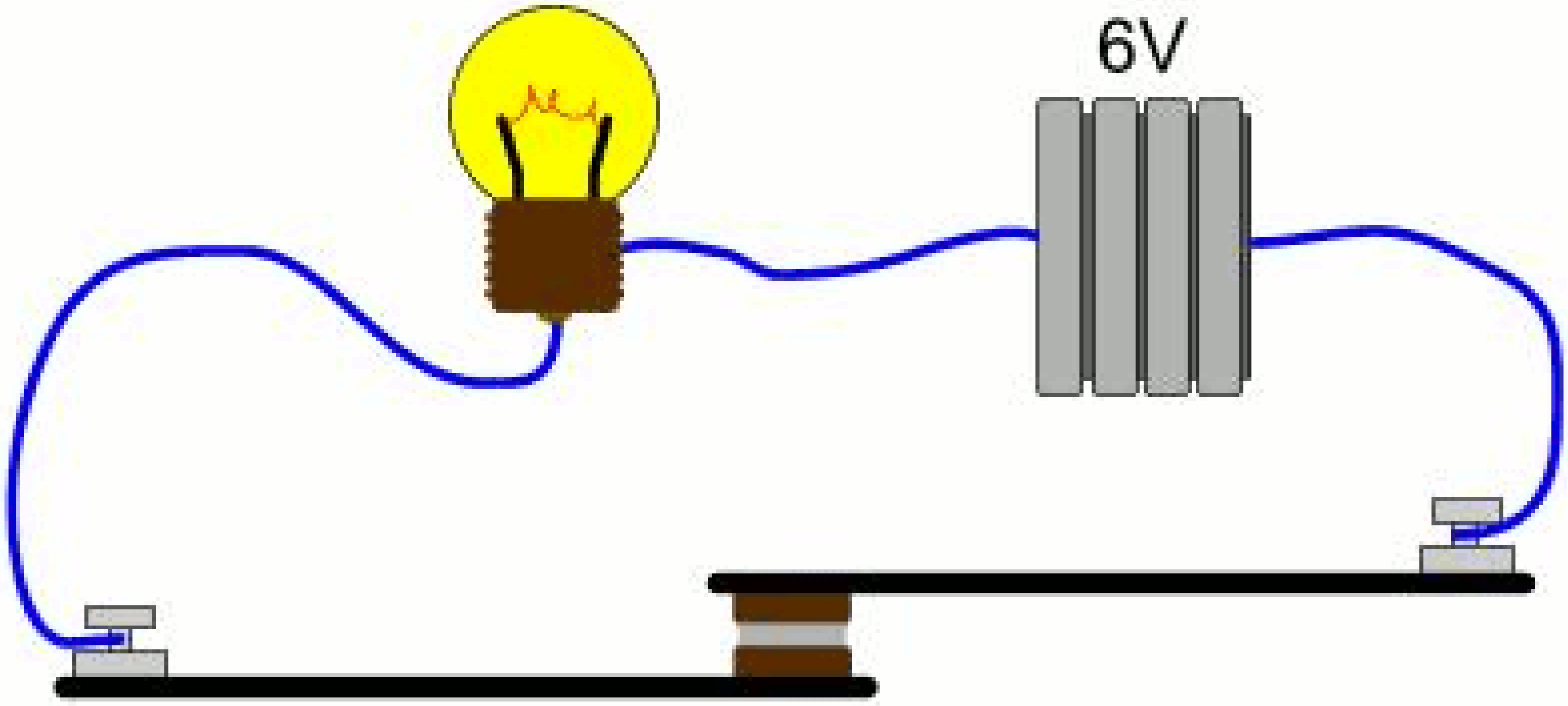
INTRODUCTION

INTRODUCTION



INTRODUCTION



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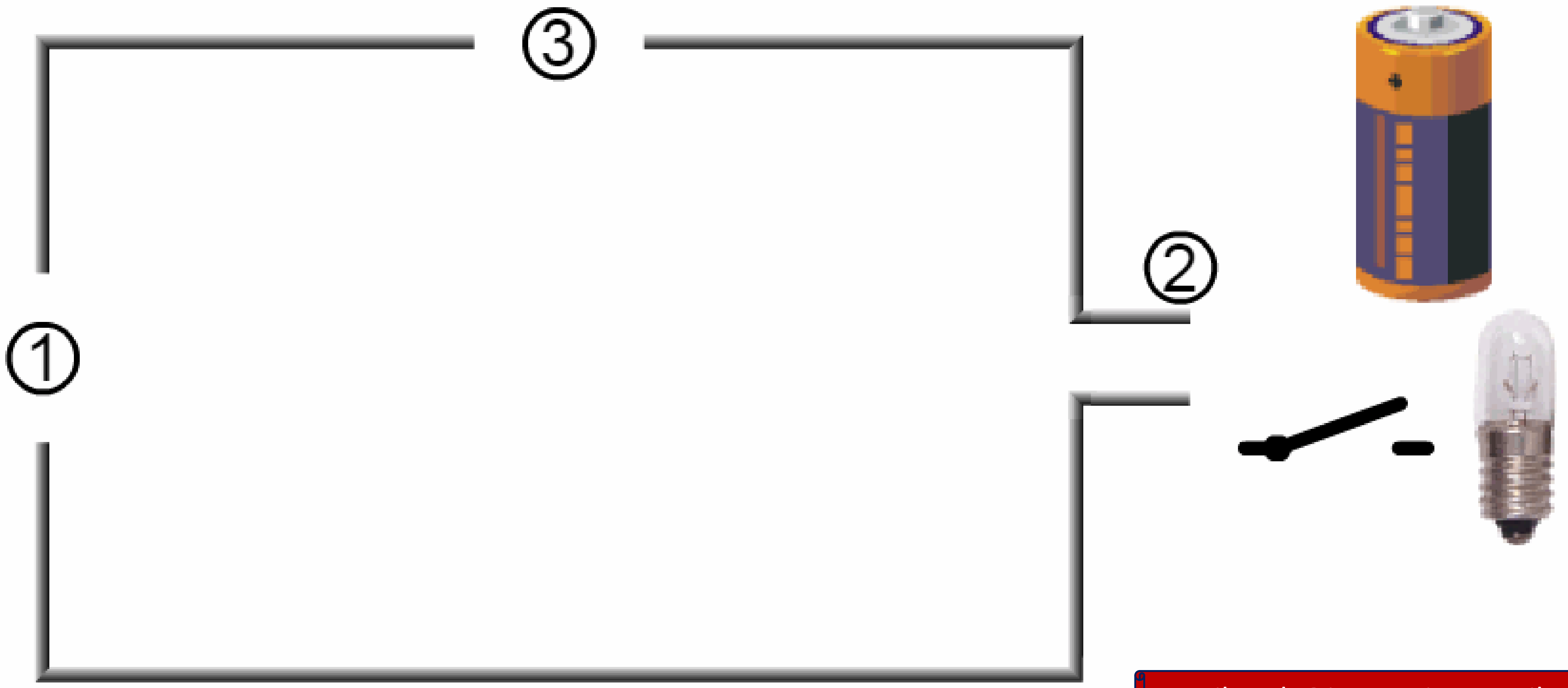
INTRODUCTION



OPEN CIRCUIT & CLOSED CIRCUIT

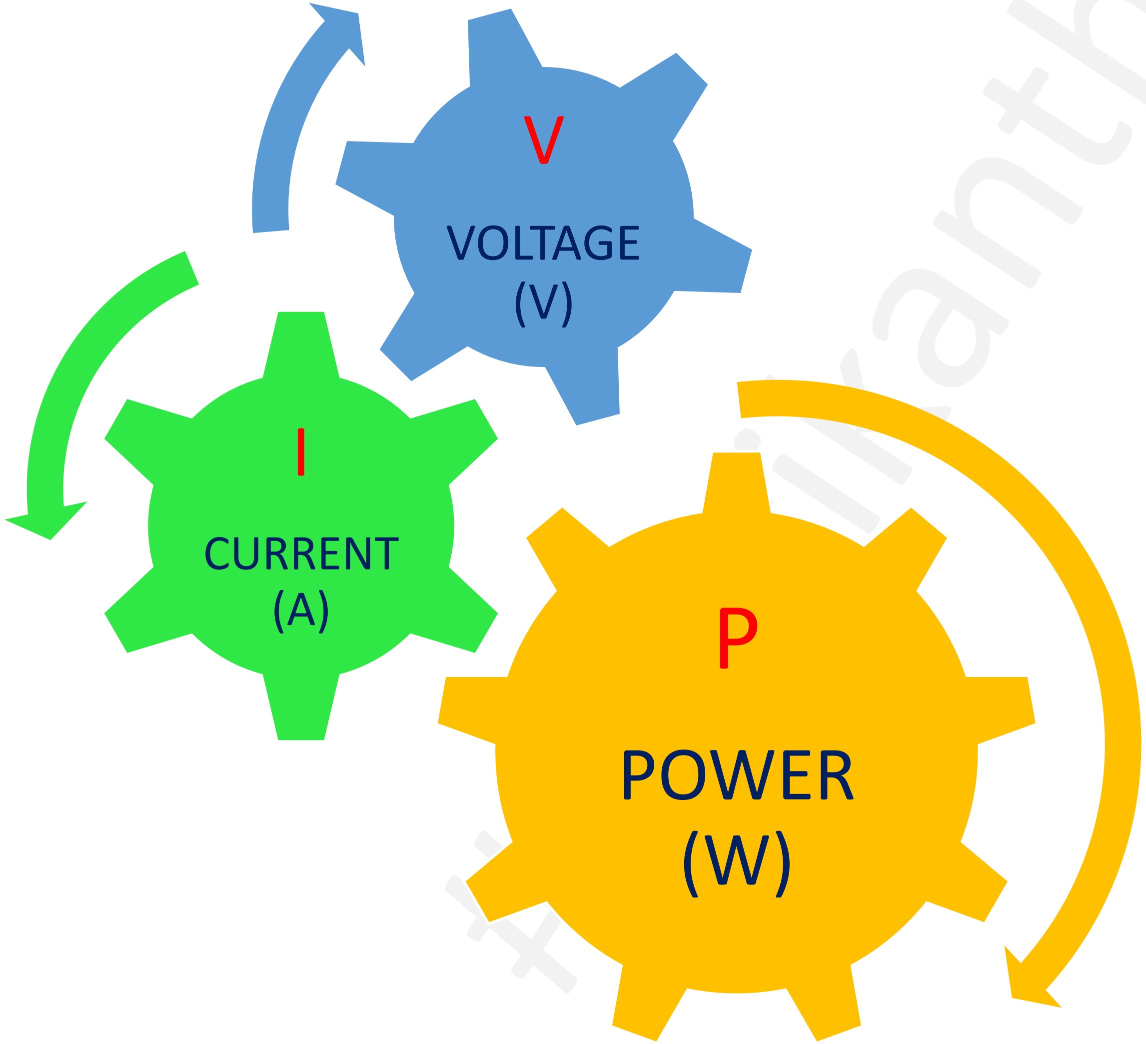
Open Circuit

- In Open Circuit, the electric current (charged particles) does not flow from an active energy source to the connected load or other components due to the incomplete path.
- An open circuit makes an incomplete path to flows the active energy from the source to load.
- In an electrical open circuit, the current does not flow.
- This circuit works as an OFF state position.

Closed Circuit

- In a Closed Circuit, the electric current (charged particles) flows from an active energy source to the connected load or other components due to the closed-loop path.
- A closed-circuit makes a complete path to flows the active energy from source to load.
- In an electric closed circuit, current flow from positive charge to the negative charge particles.
- This circuit works continuously ON state position.

V – I – P



Home 230V - AC

Controllers 5V to 12V - DC

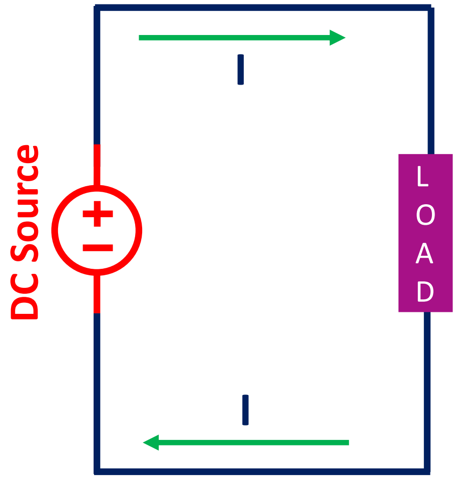
Home - ?A

Controllers - ?A

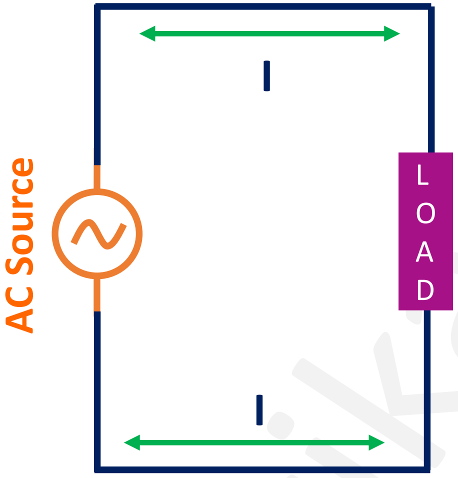
Home - V X I

Controllers – V X I

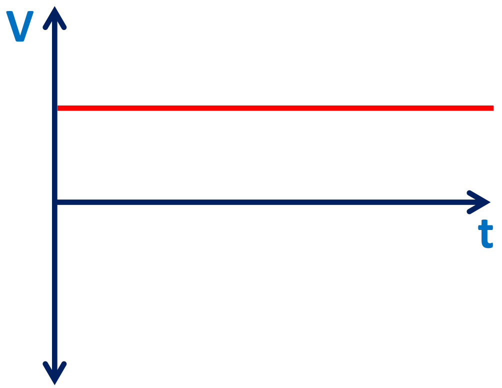
INTRODUCTION

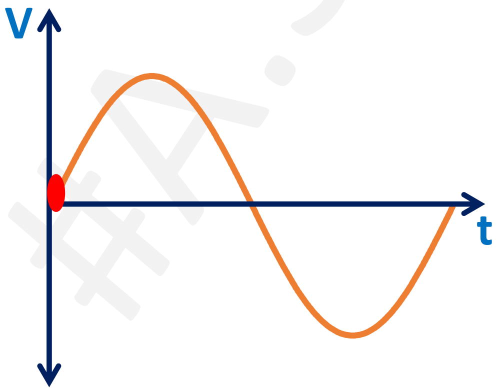


Direct Current (DC)



Alternating Current (AC)





DC Source

Voltage = 220v 1ϕ
Voltage = 0 – 12v IC
Frequency = 0 Hz

AC Source

Voltage = 230v 1ϕ
Voltage = 415v 3ϕ
Frequency = 50 – 60 Hz

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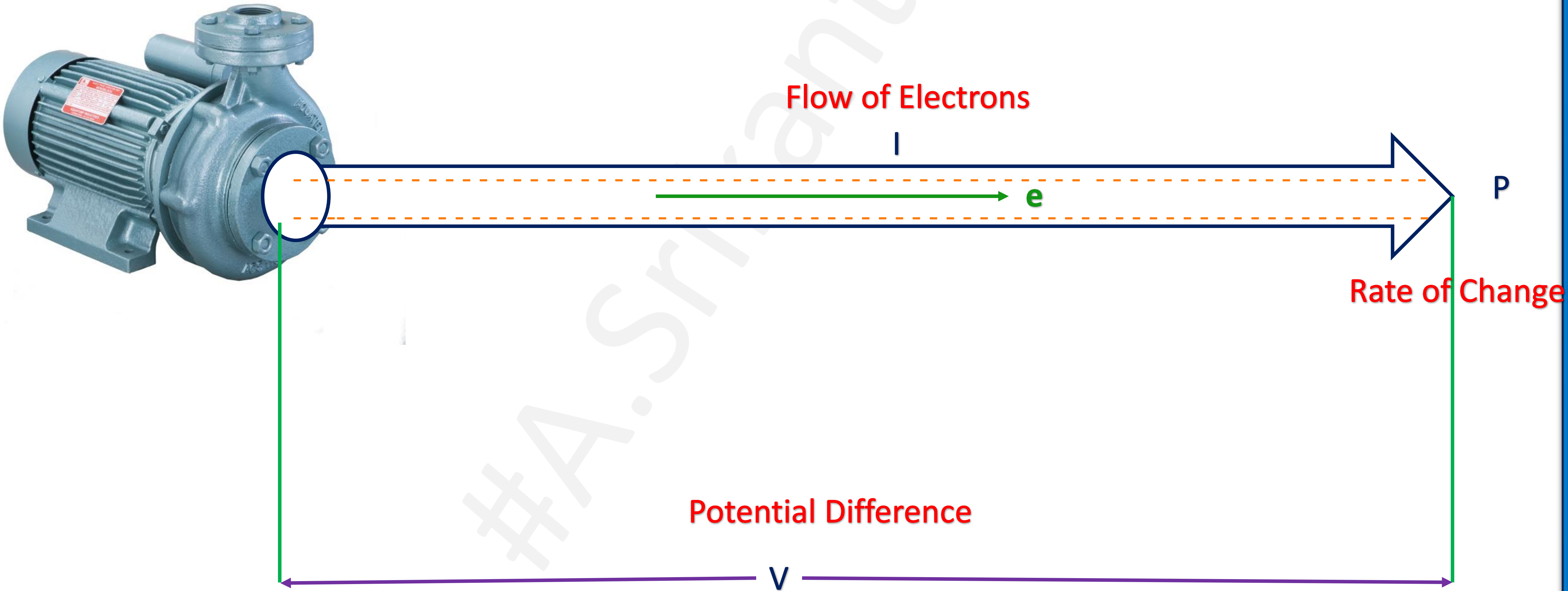
BASIC DEFINITIONS



BASIC DEFINITIONS

Current(I) : An electric current is the rate of flow of electric charge. or The flow of electrons develops the current.

A steady current can be expressed as $I = \frac{Q}{T}$ (Amperes)

A steady current can be expressed as $i = \frac{dq}{dt}$ (Amperes)

Where
T = time
Q = charge

BASIC DEFINITIONS

Electric charge, basic property of matter carried by some elementary particles that governs how the particles are affected by an electric or magnetic field. Electric charge, which can be positive or negative, occurs in discrete natural units and is neither created nor destroyed.

- ✓ An object with more electrons then protons is said to carry a negative charger.
- ✓ An object with more protons then electrons is said to carry a negative charger.
- ✓ The imbalance between P^{+} and e^{-} in a charged object is proportionally very small.
- ✓ 1 e =1.60 * 10^{-19} C

BASIC DEFINITIONS

Voltage(V) : The potential difference between force applied to two oppositely charged particles to bring them as near as possible is called as potential difference. (in electrical terminology it's voltage).

A steady voltage can be expressed as $V = \frac{W}{Q}\ (Volts)$

The time varying voltage can be expressed as $v = \frac{dw}{dq}\ (Volts)$

Where
W = work done
Q = charge

BASIC DEFINITIONS

Electrical Energy can be due to either kinetic energy or potential energy. Mostly it is due to potential energy, which is energy stored due to the relative positions of charged particles or electric fields.

$$E = QV \text{ (Electron-Volt(eV)}$$

Where,
Q is charge
V is the potential difference

BASIC DEFINITIONS

Power(P) : The rate at which electrical energy is converted to other form of energy, qual to the product of current and voltage drop.

Average power is given by $P = \frac{W}{T} = \mathbf{VI(watt)}$

Instantaneous power is given by $p = \frac{dw}{dt}\ \mathbf{(watt)}$

Where
T = time
W = work done

Ohm’s Law at Constant Temperature

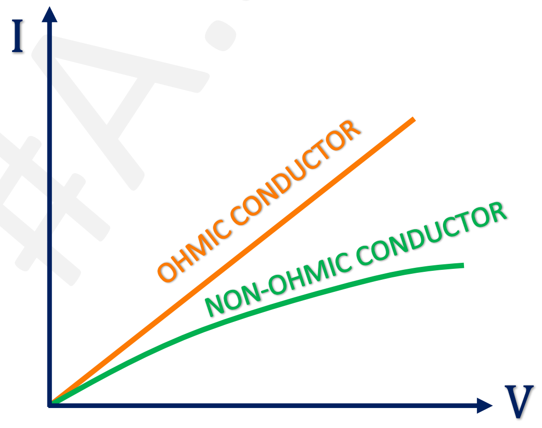
Ohm’s Law states that, at constant temperature in an electrical circuit the current (I) flowing through a conductor is directly proportional to potential difference (V) applied.

$$I \propto V \text{ or } V \propto I \Rightarrow V = IR$$

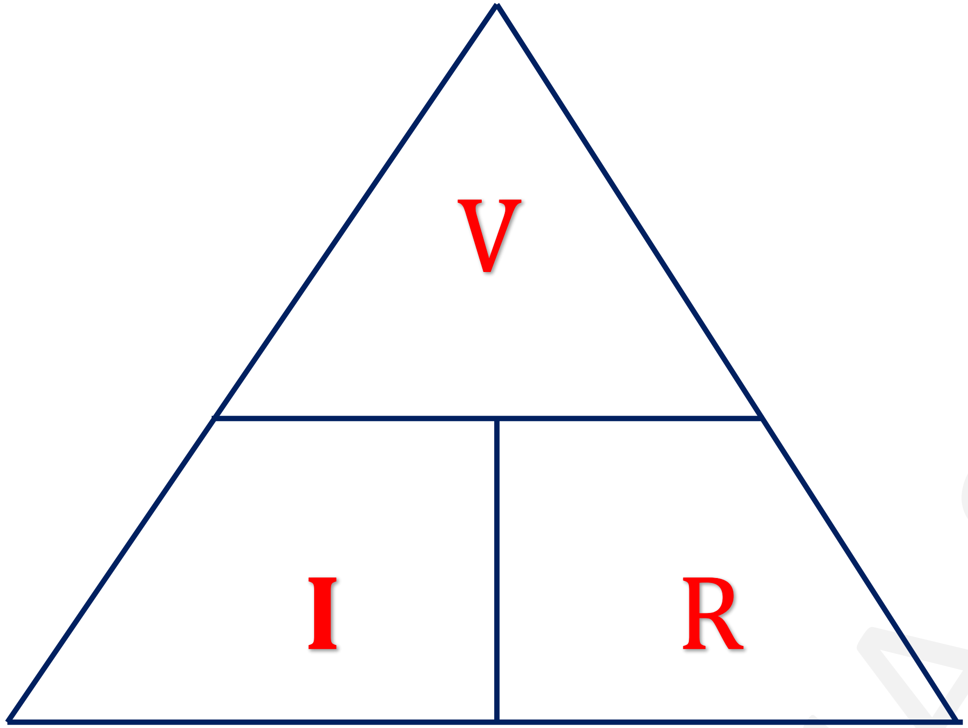
Where R = Resistance of the conductor

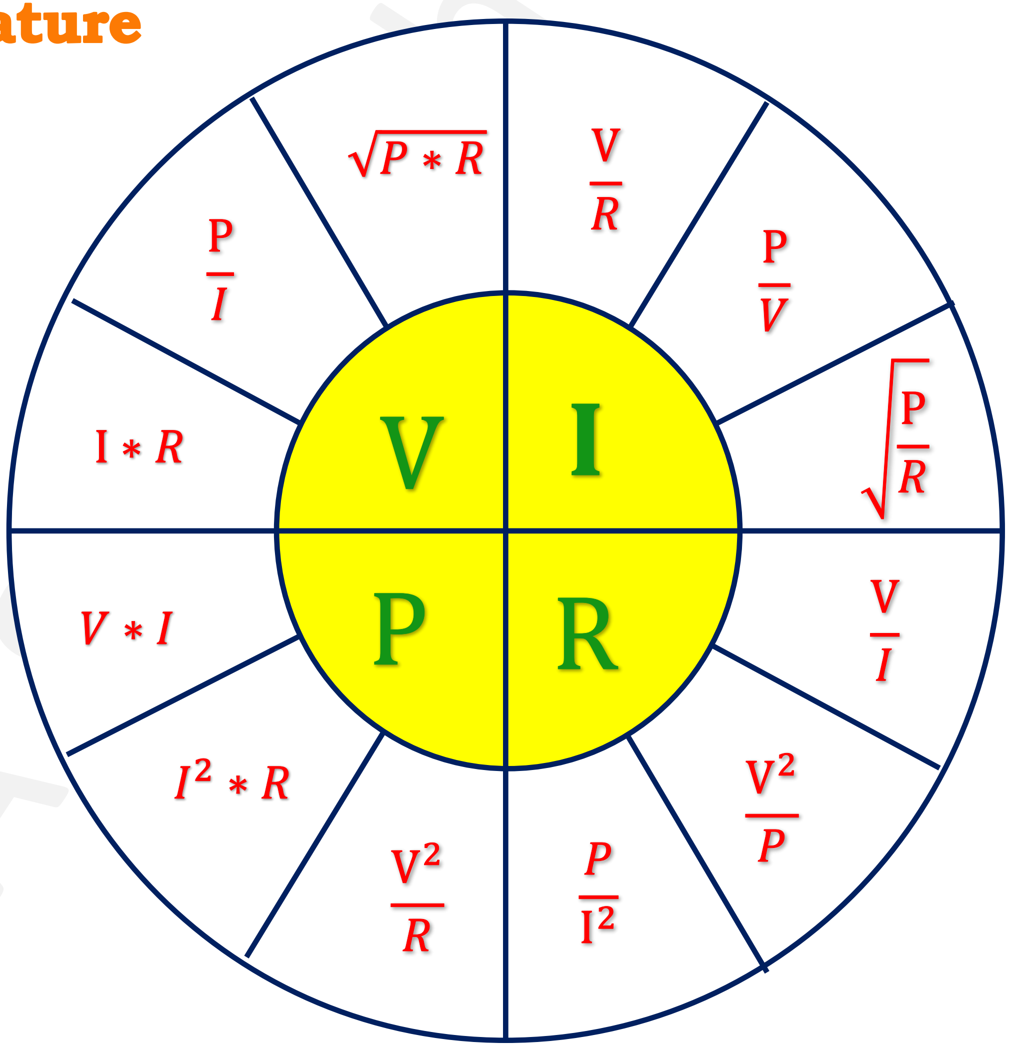
Limitations of Ohm’s Law

- It is applicable only for metallic conductor such as copper, silver etc.
- It is not applicable for all electrical circuit such semiconductor devices, transistors ect.



Ohm's Law at Constant Temperature





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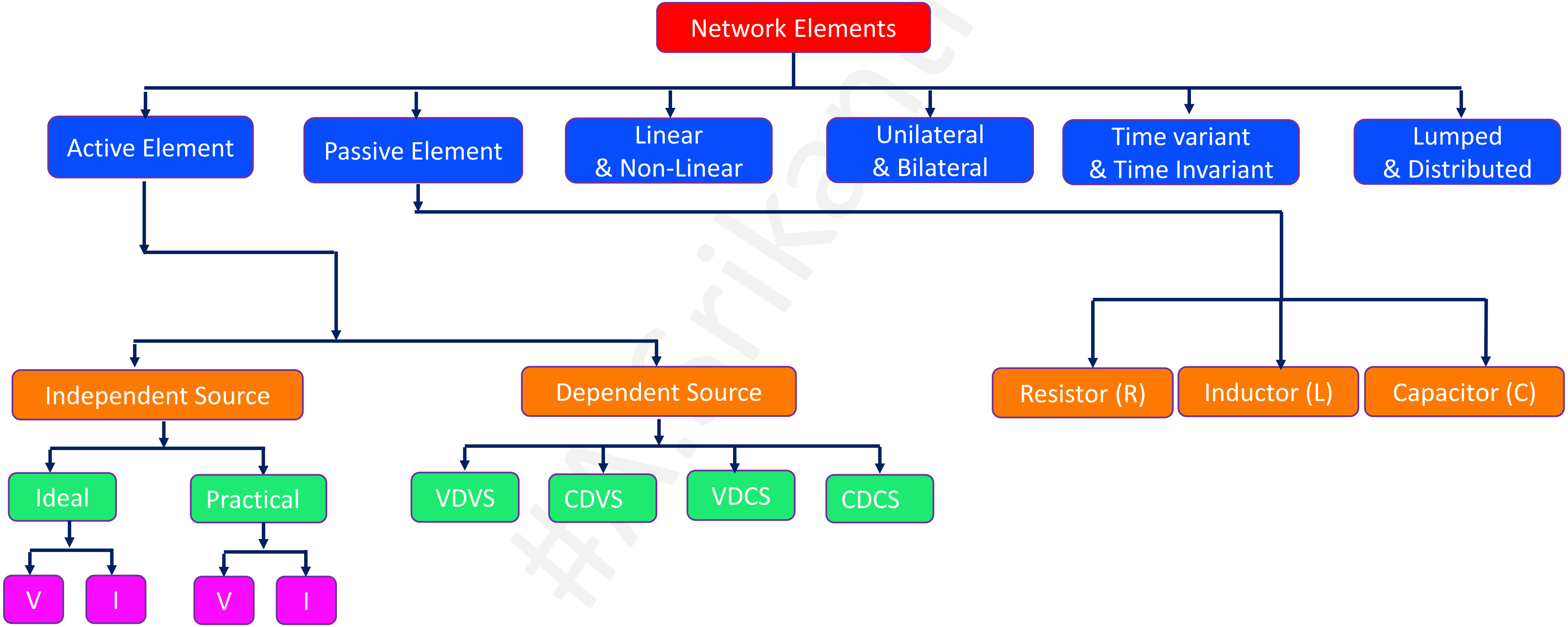
Classifications of Elements



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Classifications of Elements

Active Element : Active elements are the sources of energy or the element which can deliver energy are called active elements. **Example:** Voltage source and Current source.

Active Elements are

1. **Independent Source**
2. **Dependent Source**

Classifications of Elements

Independent Source : Active elements include independent voltage sources and independent current sources. An independent voltage source maintains (fixed or Varying with time), which is not affected by any other quantity. Similarly an independent current source maintains a current (fixed or time-varying) which is unaffected by any other quantity.

There are two independent energy sources

1. Ideal energy source

- A. Ideal voltage source
- B. Ideal current source

2. Practical energy source

- A. Practical voltage source
- B. Practical current source

Classifications of Elements

The electrical sources are those devices which provide active power to a circuit. There is two type of sources available in electrical networks, a voltage source or current source. The purpose of the voltage source is to provide voltage rather than the current and current source is to provide current rather voltage. Each source is then categorized as an ideal or practical source.

Ideal Sources are those imaginary electrical sources which provide constant voltage or current to the circuit irrespective of the load current. These ideal sources don’t have any internal resistance. Where it is impossible to build a source with zero internal resistance. So, all the real sources are called Practical Sources.

Classifications of Elements

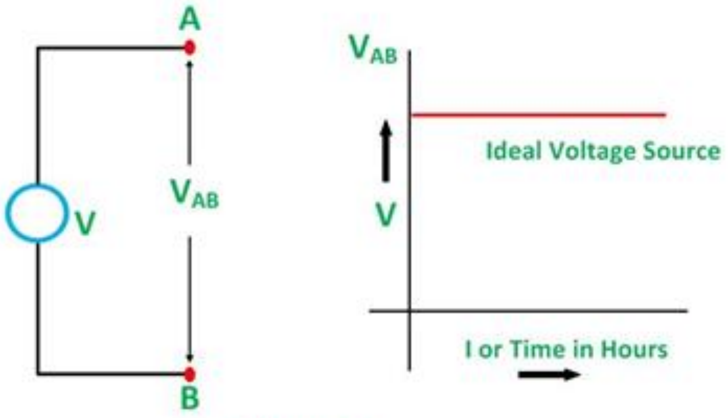


Figure A

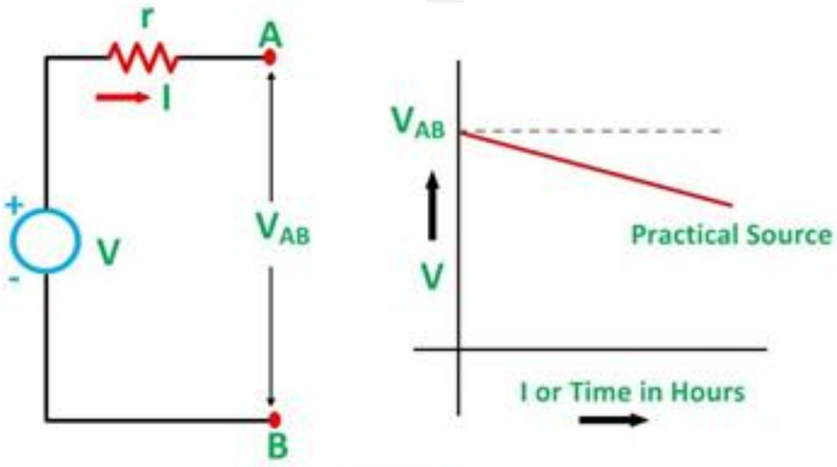


Figure B

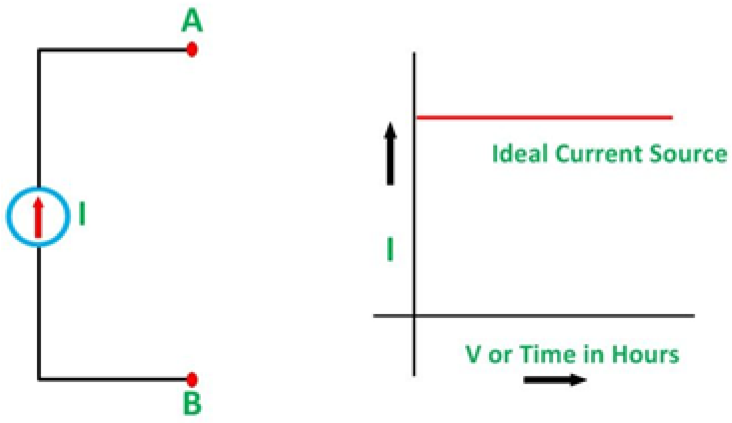


Figure C

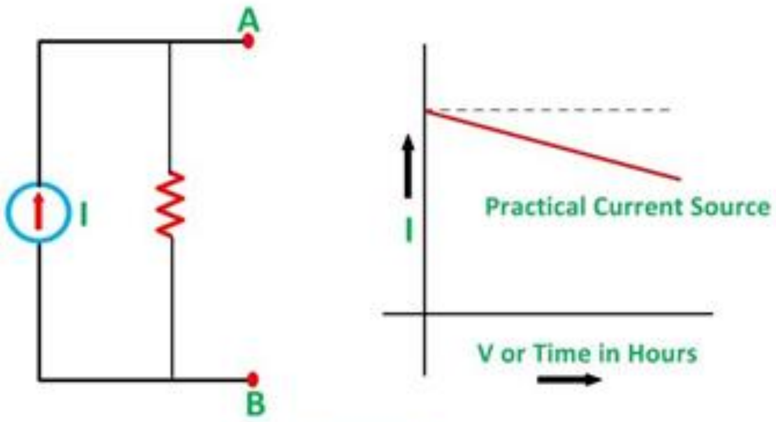


Figure D

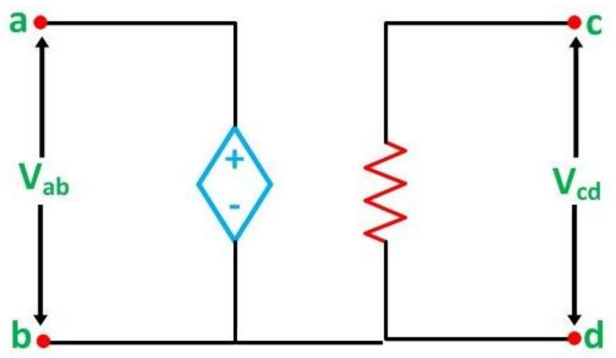
Classifications of Elements

Dependent Source : Active elements include dependent voltage sources and dependent current sources. The sources whose output voltage or current is not fixed but depends on the voltage or current in another part of the circuit is called Dependent or Controlled source. They are four terminal devices. When the strength of voltage or current changes in the source for any change in the connected network, they are called dependent sources. The dependent sources are represented by a diamond shape.

The dependent sources are further categorized as

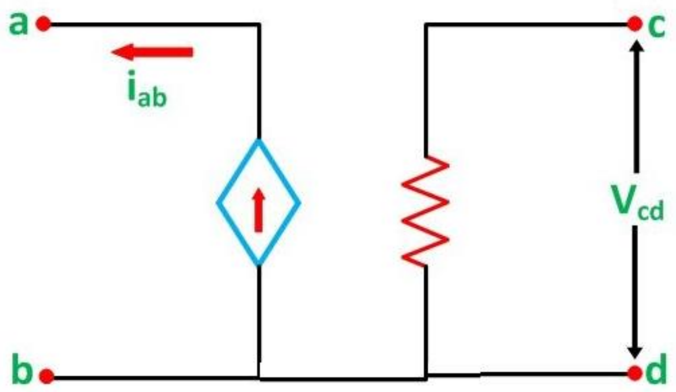
- Voltage Controlled Voltage Source (VCVS)
- Voltage Controlled Current Source (VCCS)
- Current Controlled Voltage Source (CCVS)
- Current Controlled Current Source (CCCS)

Classifications of Elements



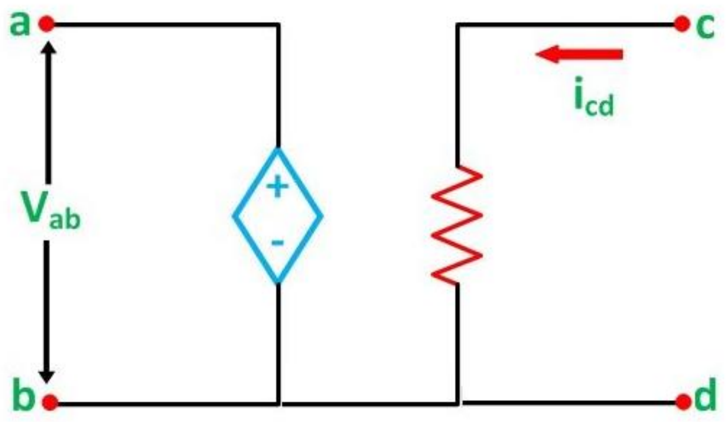
$$V_{ab} \propto V_{cd}$$
$$V_{ab} = kV_{cd}$$

- Voltage Controlled Voltage Source (VCVS)



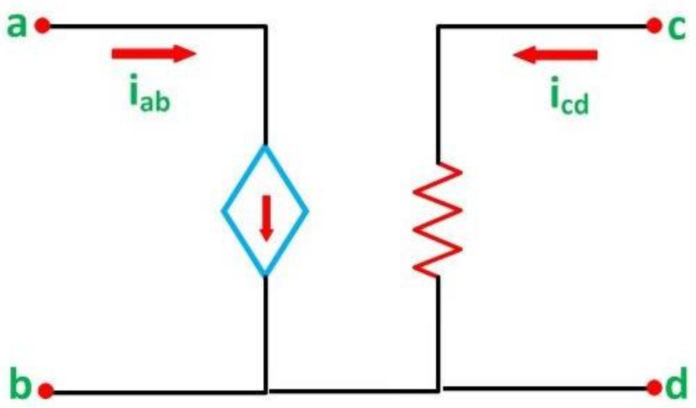
$$i_{ab} \propto V_{cd}$$
$$i_{ab} = \eta\, V_{cd}$$

- Voltage Controlled Current Source (VCCS)



$$V_{ab} \propto i_{cd}$$
$$V_{ab} = r\, i_{cd}$$

- Current Controlled Voltage Source (CCVS)



$$i_{ab} \propto i_{cd}$$
$$i_{ab} = \beta i_{cd}$$

- Current Controlled Current Source (CCCS)

Classifications of Elements

Passive Element : The elements which consume energy either by absorbing or storing are called passive elements. **Example:** Resistor, Inductor and Capacitor.

resistor

Inductor

capacitor

Classifications of Elements

Linear & Non-Linear : A circuit is said to be linear if it satisfies the relationship between voltage and current (i.e., OHM's Law). **Example :** Resistor, Inductor and Capacitor

If an element does not satisfy the OHM's Law relation, then it is called a Non Linear element.
Example : Diodes, transistors, thermistors etc.

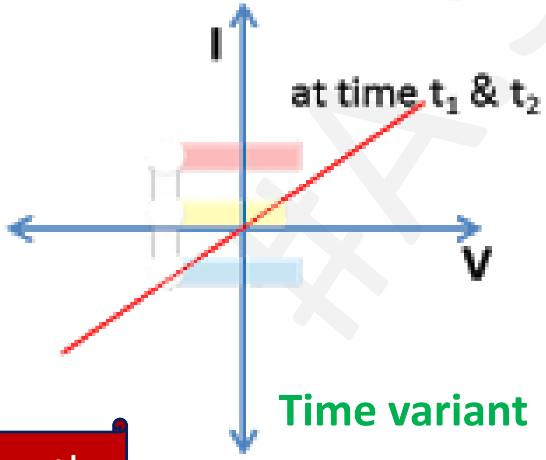
Unilateral Element : Conduction of current in one direction is termed as unilateral element.
Example: Diode, Transistor, etc.

Bilateral Element: Conduction of current in both directions in an element with same magnitude is termed as bilateral element. **Example:** Resistance, Inductance, Capacitance.

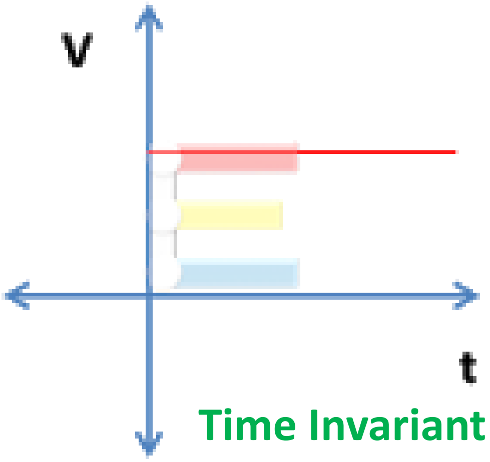
Classifications of Elements

Time Invariant : An element or a system is said to be time invariant if parameters of the element do not vary with time. For example a resistor is a time invariant element whose value of resistance R or any response by it remains same irrespective of the instant of time when the voltage or current is applied to it. Its value may change with change with change in voltage or current like in a non-linear resistor but still it is called as a time invariant element as long as its response won't with respect to time.

Time variant : The parameters of the time variant are not constant with respect to time and may change with change in the instant of time. The given below graph shows the example of V-I characteristics of a time variant element in which the slope of the characteristic of same element is higher at time t2 than at time t1.



Time variant



Time Invariant

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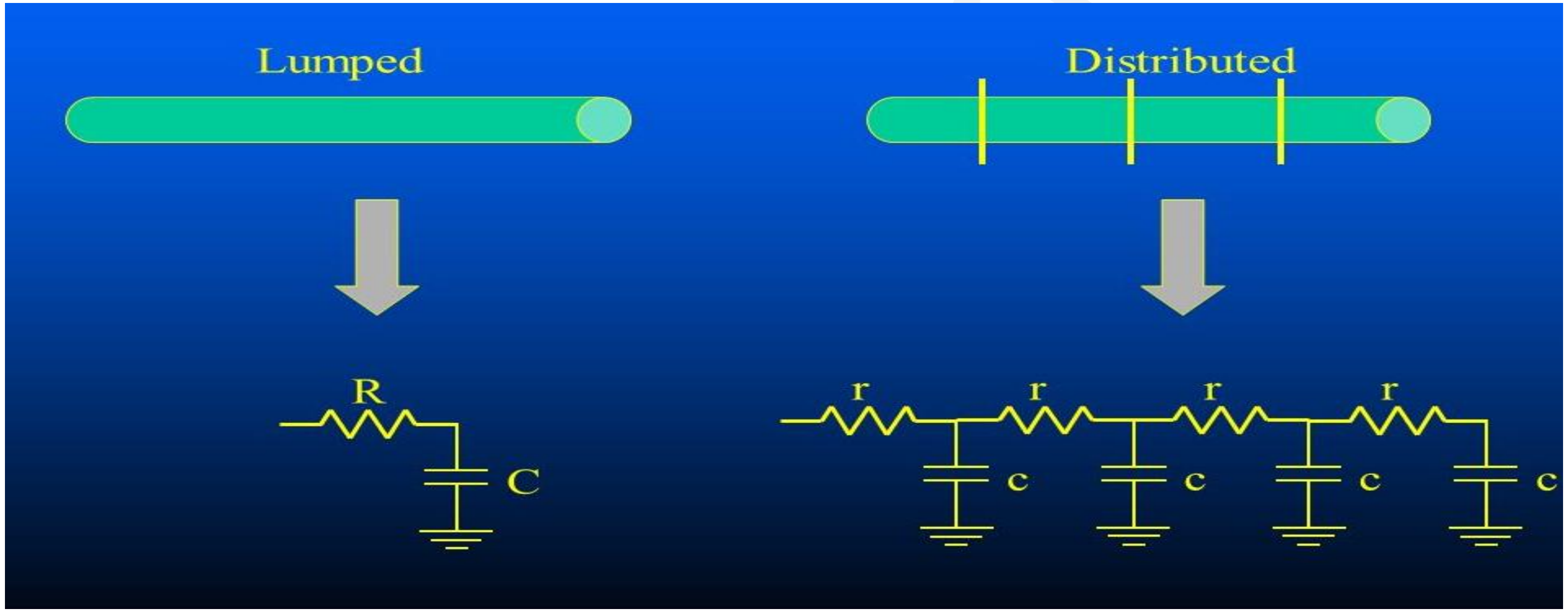
Classifications of Elements

Lumped Elements : The elements which are physically separable are called Lumped Elements.
Example: Resistor, Inductor, Capacitor.

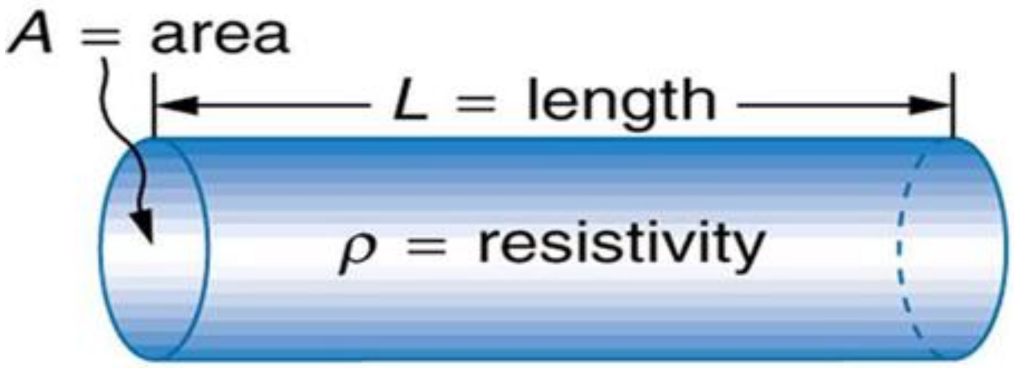
Distributed Elements : The elements which are physically un- separable are called Distributed Elements.
Example: Transmission Line, which has distributed resistance, inductance and Capacitance along its length.



R, L, C Parameters

Resistor (R) : A resistor is a passive two-terminal electrical component that implements electrical resistance as a circuit element. In electronic circuits, resistors are used to reduce current flow, adjust signal levels, to divide voltages, bias active elements, and terminate transmission lines, among other uses.

Resistance : Resistance is nothing but this property of resisting the flow of electrons or the current. The unit of resistance is **ohm (Ω)** . One ohm is equal to volt per ampere.



$$R = \rho \frac{L}{A}$$

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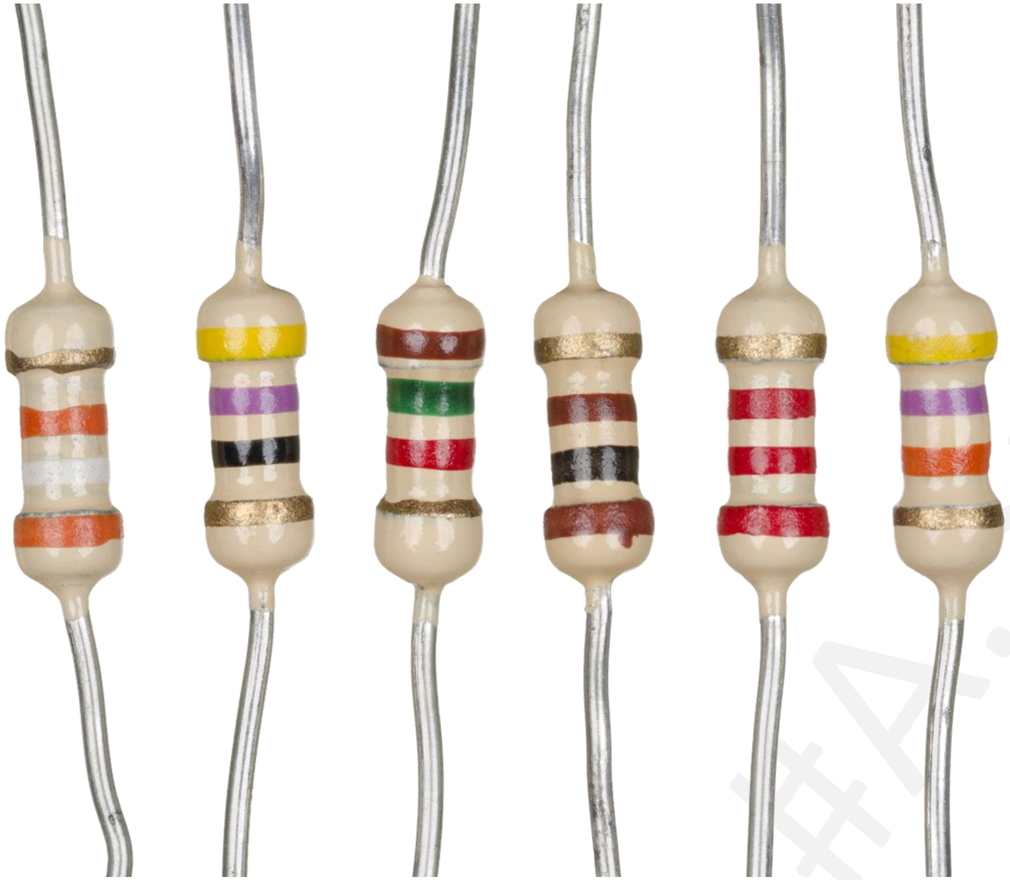
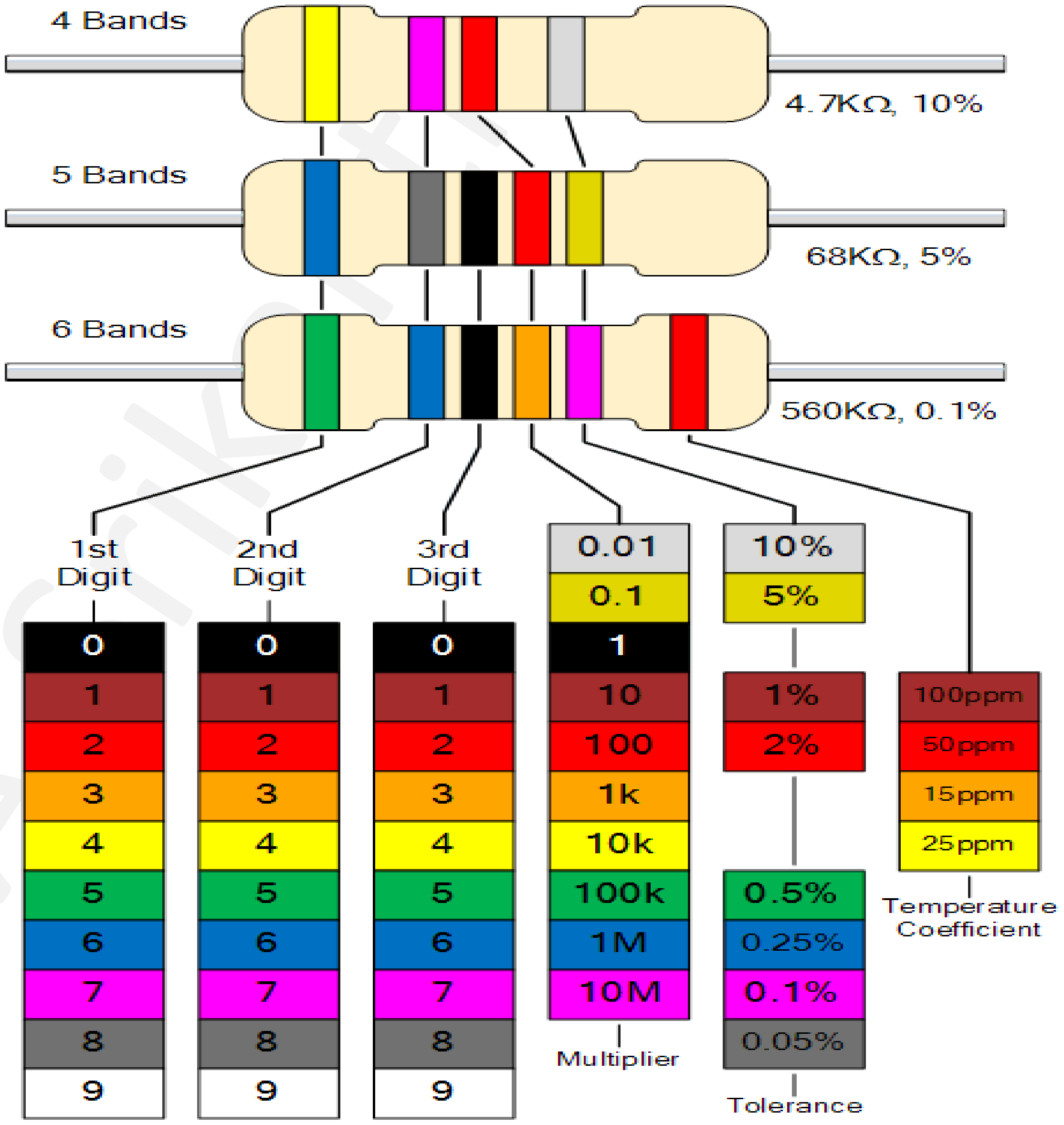
R, L, C Parameters



Color	Color	Value	Multiplier	Tolerance
	Black	0	X 1	N/A
	Brown	1	X 10	N/A
	Red	2	X 100	2%
	Orange	3	X 1000	N/A
	Yellow	4	X 10000	N/A
	Green	5	X 100000	N/A
	Blue	6	X 1000000	N/A
	Violet	7	X 10000000	N/A
	Gray	8	X 100000000	N/A
	White	9	X 1000000000	N/A
	Gold	N/A	X 0.1	5%
	Silver	N/A	X 0.01	10%

R, L, C Parameters



R, L, C Parameters

Resistance : Resistance is nothing but this property of resisting the flow of electrons or the current. The unit of resistance is ohm (Ω) . One ohm is equal to volt per ampere.

$$R = \rho \frac{L}{A}$$

From Ohm's law,

$$\mathrm{I} \propto \mathrm{V} \text{ or } \mathrm{V} \propto \mathrm{I} \Rightarrow \mathrm{V} = \mathrm{IR}$$

Where R = Resistance of the conductor

$$\mathrm{R} = \frac{V}{I}$$

$$V = IR$$

$$\mathrm{I} = \frac{V}{R}$$

Where V = Instantaneous Voltage in Volts

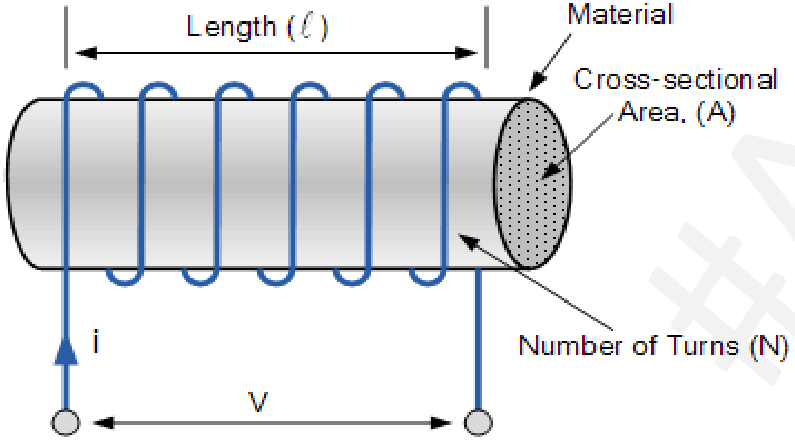
R = Resistance in Ohm's

I = Instantaneous current in Amps

R, L, C Parameters

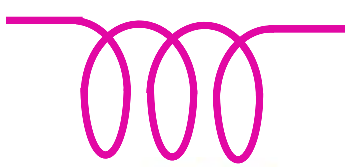
Inductor (L) : An inductor, also called a **coil, choke, or reactor**, is a passive two-terminal electrical component that stores energy in a magnetic field when electric current flows through it. An inductor typically consists of an insulated wire wound into a coil around a core.

The **inductance** is directly proportional to the number of turns in the coil. Inductance also depends on the radius of the coil and on the type of material around which the coil is wound. The units of Inductance is **Henry (H).**



$$L = \frac{\mu N^2 A}{l}$$

$$Energy = \frac{1}{2} L i^2$$



Inductor

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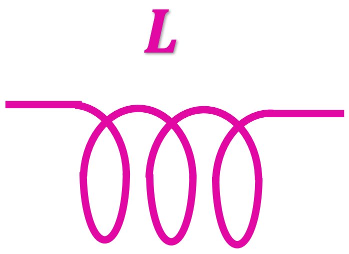
R, L, C Parameters

Inductors do not have a stable "resistance" as conductors do. However, there is a definite mathematical relationship between voltage and current for an inductor is

$$V = L\frac{di}{dt}$$

$$L = \frac{\mu N^2 A}{l}$$



$$I = \frac{1}{L}\int v\,dt$$

$$Energy = \frac{1}{2}Li^2$$

Where V = Instantaneous Voltage across the Inductor

L = Inductance in Henrys

$\frac{di}{dt}$ = Instantaneous rate of current change (amps per second)

Applications: Filters, Sensors, Transformers, Motors, Energy Storage etc.

R, L, C Parameters

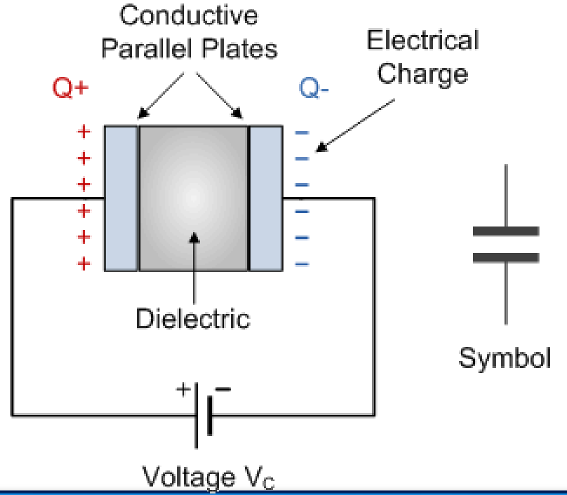
Capacitor (C) : The capacitor is a component which has the ability or "capacity" to store energy in the form of an electrical charge producing a potential difference (Static Voltage) across its plates, much like a small rechargeable battery. The **capacitor** is made of 2 close conductors (usually plates) that are separated by a dielectric material. The plates accumulate electric charge when connected to power source. One plate accumulates positive charge and the other plate accumulates negative charge.

The property of a capacitor to store charge on its plates in the form of an electrostatic field is called the **Capacitance** of the capacitor. . The units of capacitance is **Farads (F)**.

$$C = \frac{\varepsilon_0 A}{d}$$

$$\text{Energy} = \frac{1}{2} C v^2$$

C



R, L, C Parameters

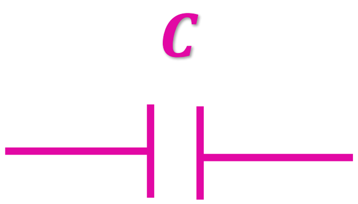
Capacitor (C) : Capacitors do not have a stable "**resistance**" as conductors do. However, there is a definite mathematical relationship between voltage and current for Capacitor is

$$I = C\frac{dv}{dt}$$

$$V = \frac{1}{C}\int i\,dt$$

$$C = \frac{\varepsilon_0 A}{d}$$

$$\text{Energy} = \frac{1}{2}Cv^2$$



Where I = Instantaneous Current through the capacitor

L = Capacitance in Farads

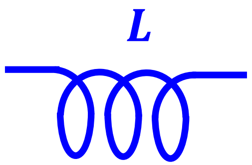
$\frac{dv}{dt}$ = Instantaneous rate of Voltage change (Volts per second)

Applications: Filters, Sensors, Energy Storage etc.

R, L, C Parameters

$$V = IR$$

$$I = \frac{V}{R}$$

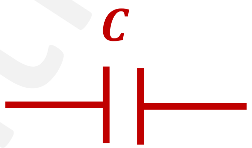


$$V = L\frac{di}{dt}$$

$$I = \frac{1}{L}\int v\,dt$$

$$L = \frac{\mu N^2 A}{l}$$

$$\text{Energy} = \frac{1}{2}Li^2$$



$$V = \frac{1}{C}\int i\,dt$$

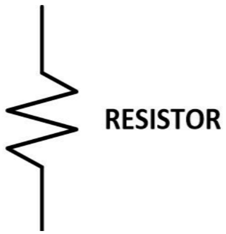
$$I = C\frac{dv}{dt}$$

$$C = \frac{\varepsilon_0 A}{d}$$

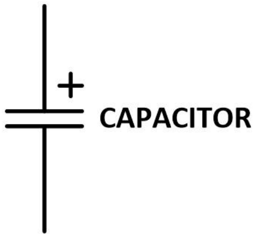
$$\text{Energy} = \frac{1}{2}Cv^2$$

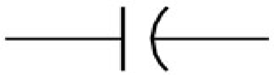
R, L, C Parameters

Standard Symbols for Electrical Components

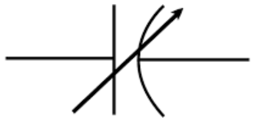


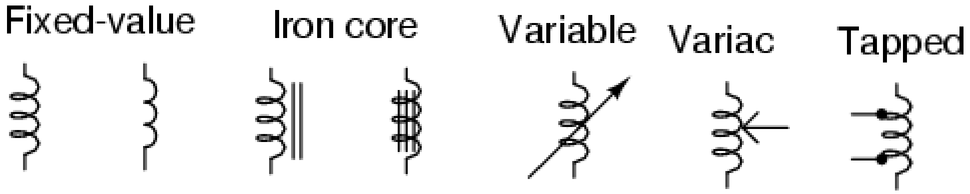
Variable Resistor or Potentiometer

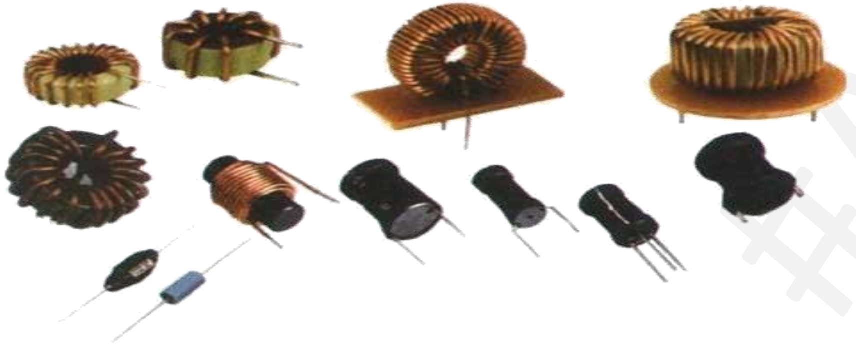




Capacitor







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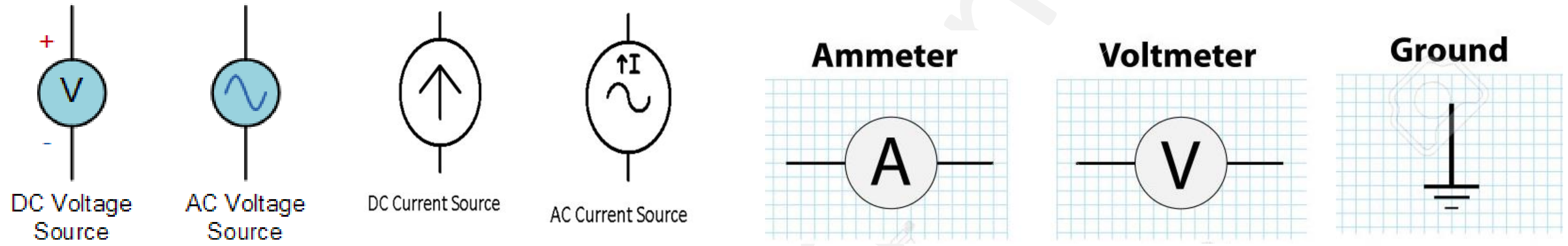
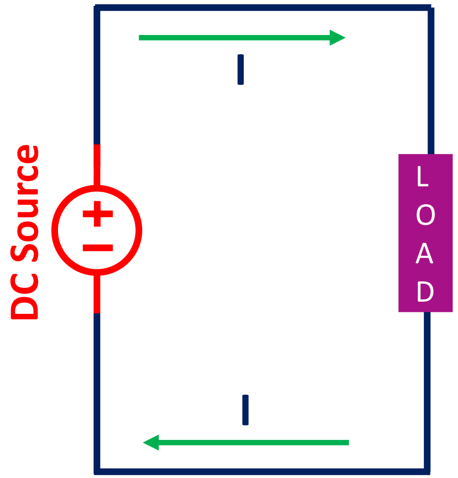
R, L, C Parameters

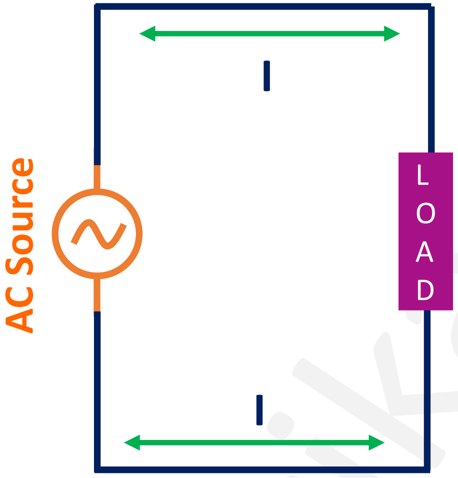
Standard Symbols for Electrical Components



INTRODUCTION TO ELECTRICAL CIRCUITS



Direct Current (DC)



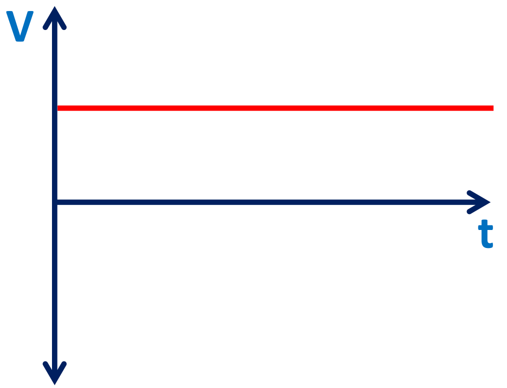
Alternating Current (AC)

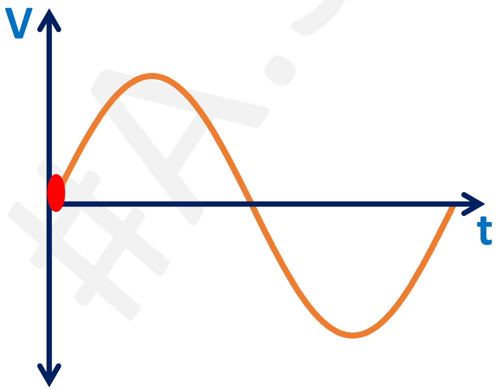
DC Source

Voltage = 220v 1ϕ
Voltage = 0 – 12v IC
Frequency = 0 Hz

AC Source

Voltage = 230v 1ϕ
Voltage = 415v 3ϕ
Frequency = 50 – 60 Hz





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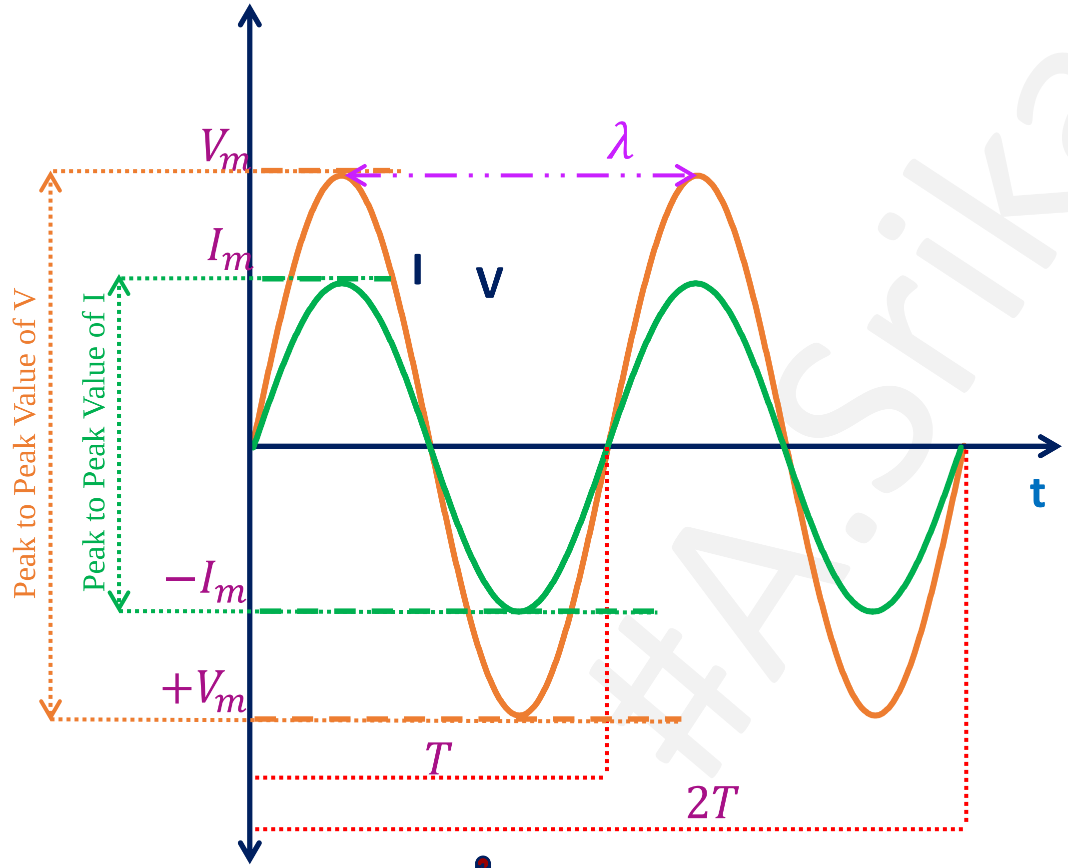
Representation of Alternating Quantities

The voltage that changes its polarity and magnitude at regular interval of time is called an alternating voltage. Similarly the direction of the current is changed and the magnitude of current changes with time it is called alternating current.



Cycle: *set of positive and negative*

Time Period : T

Wavelength : $\lambda = \frac{\vartheta}{f}$

Amplitude (or) Peak Value: $V_m\ (or)\ I_m$

Peak to Peak Value : $+V_m\ to\ -V_m\ (or) + I_m\ to\ -I_m$

Frequency: $f = \frac{1}{Period} = \frac{1}{Time\ Interval}$

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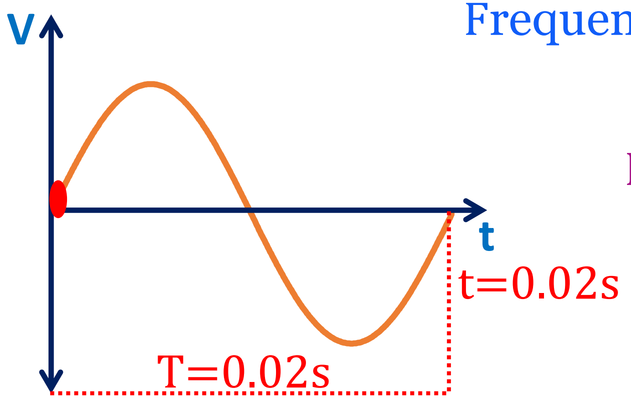
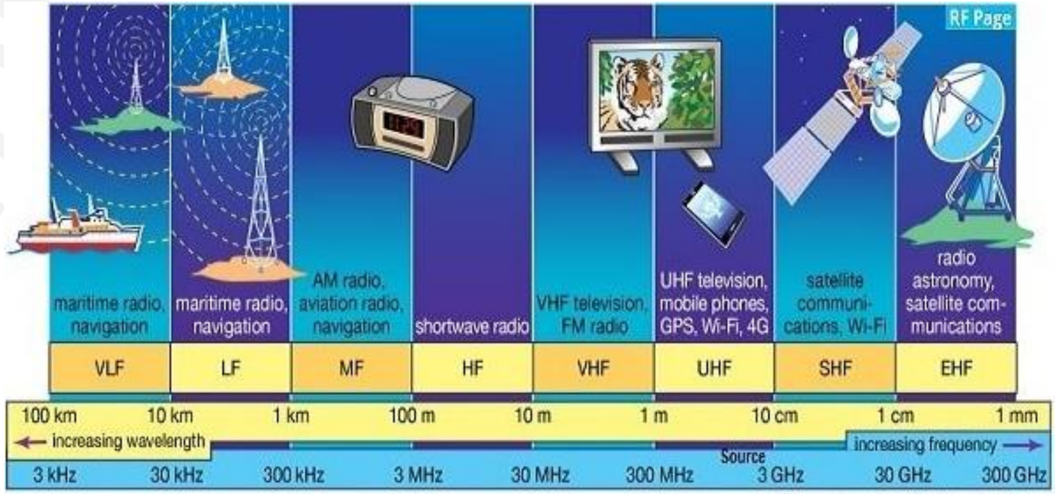
Frequency, in physics, the number of waves that pass a fixed point in unit time; also, the number of cycles or vibrations undergone during one unit of time by a body in periodic motion. A body in periodic motion is said to have undergone one cycle or one vibration after passing through a series of events or positions and returning to its original state.



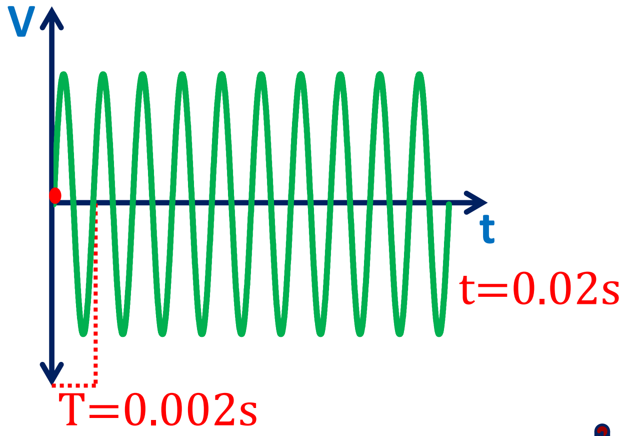
$$\text{Frequency(f)} = \frac{1}{\text{Period}} = \frac{1}{\text{Time Interval}}$$

$$\text{Frequency(f)} = \frac{1}{0.02} = 50Hz$$

Low Frequency waves



$$\lambda = \frac{\vartheta}{f}$$



$$\text{Frequency(f)} = \frac{1}{0.002} = 500Hz$$

High Frequency waves

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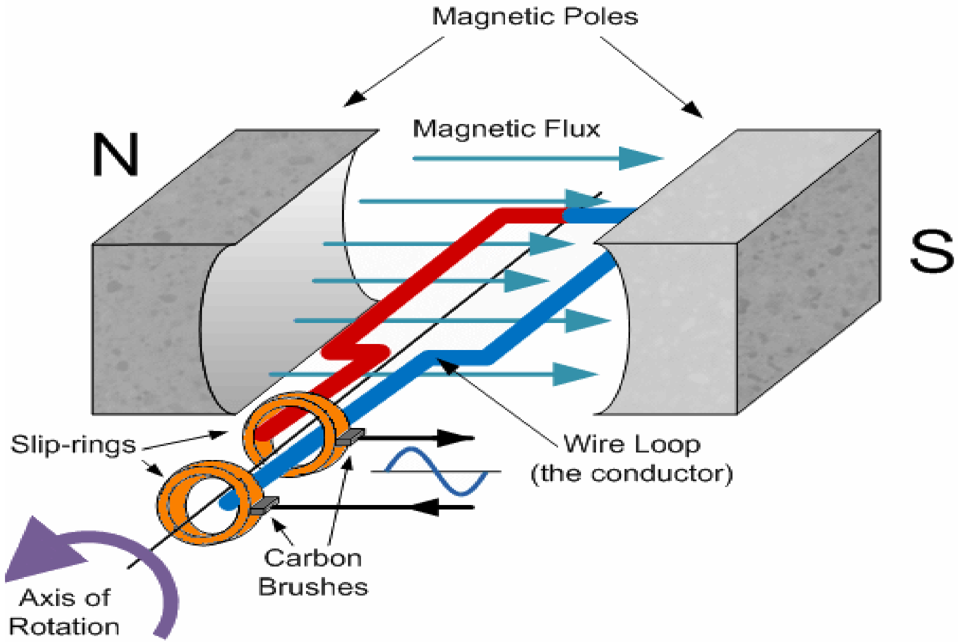
Representation of Alternating Quantities

If this single wire conductor is moved or rotated within a stationary magnetic field, an "EMF", (Electro-Motive Force) is induced within the conductor due to the movement of the conductor through the magnetic flux.

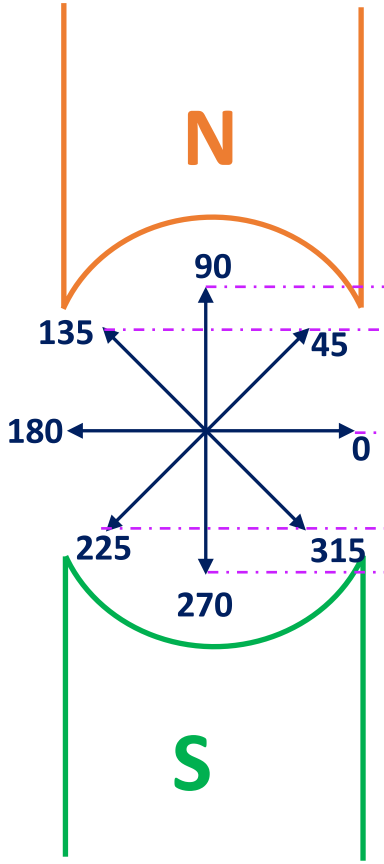
From this we can see that a relationship exists between Electricity and Magnetism giving us, as Michael Faraday discovered the effect of "Electromagnetic Induction".

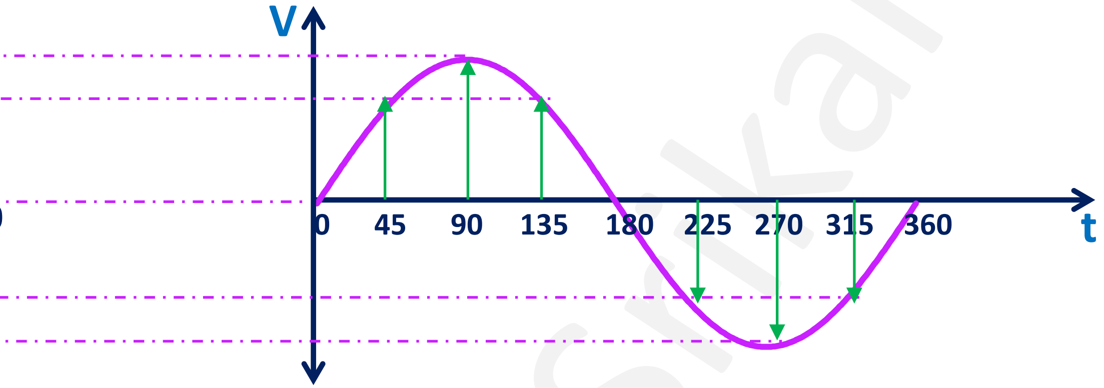
An AC generator uses the principal of Faraday's electromagnetic induction to convert a mechanical energy such as rotation, into electrical energy, a Sinusoidal Waveform.

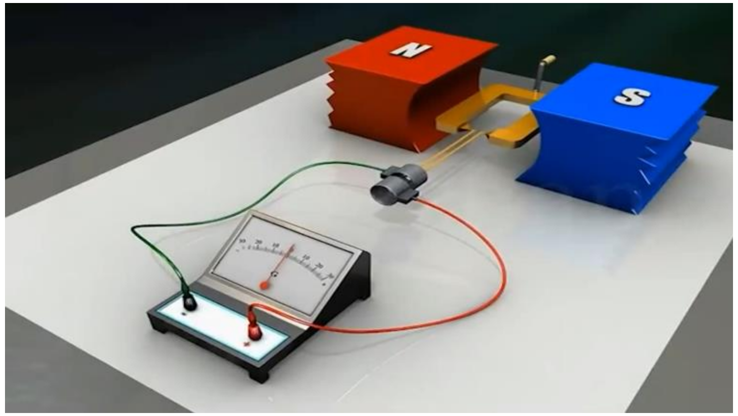


Representation of Alternating Quantities

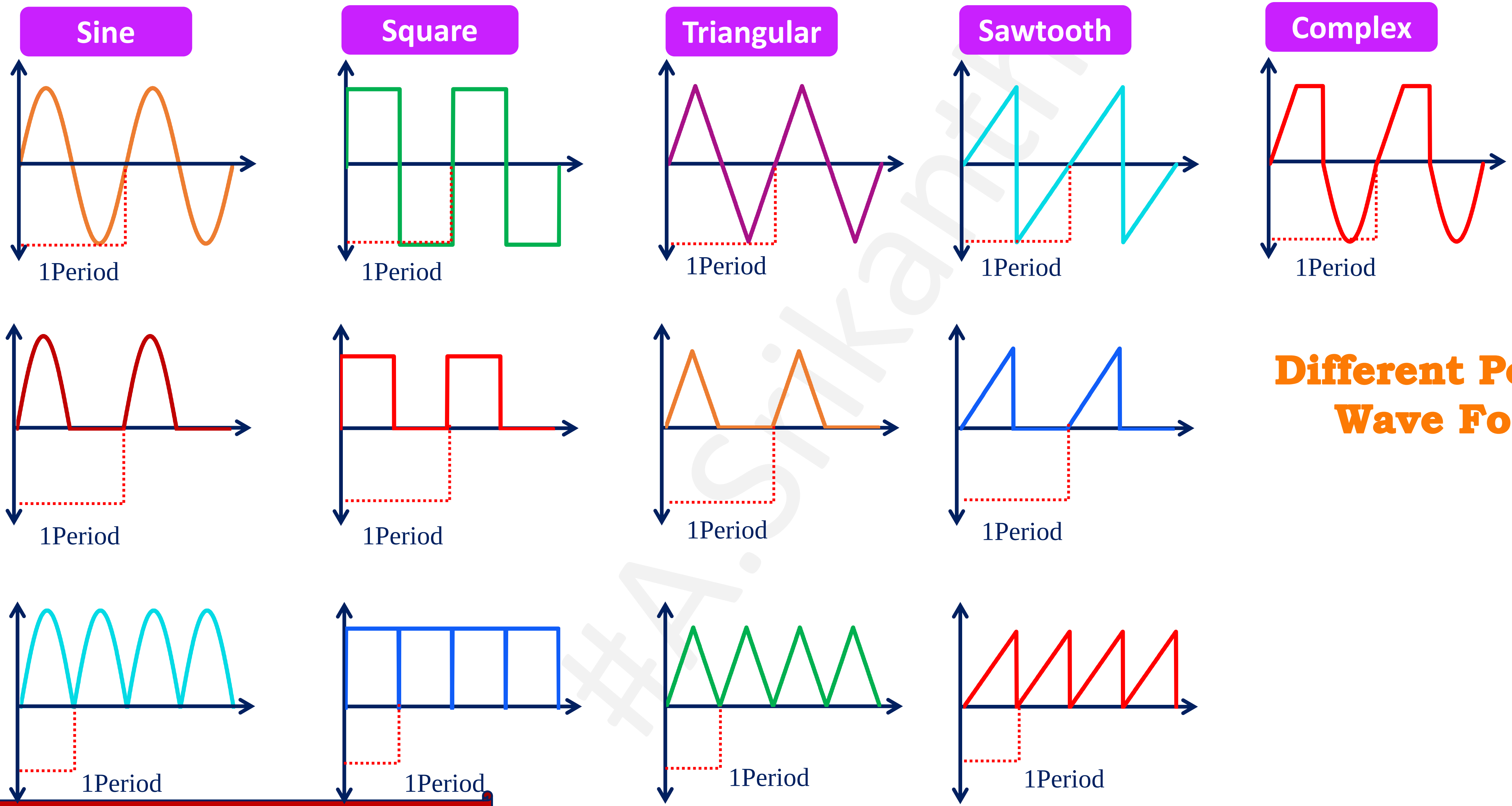
$$V(t) = V_m \sin(\omega t) = V_m \sin(\theta)$$







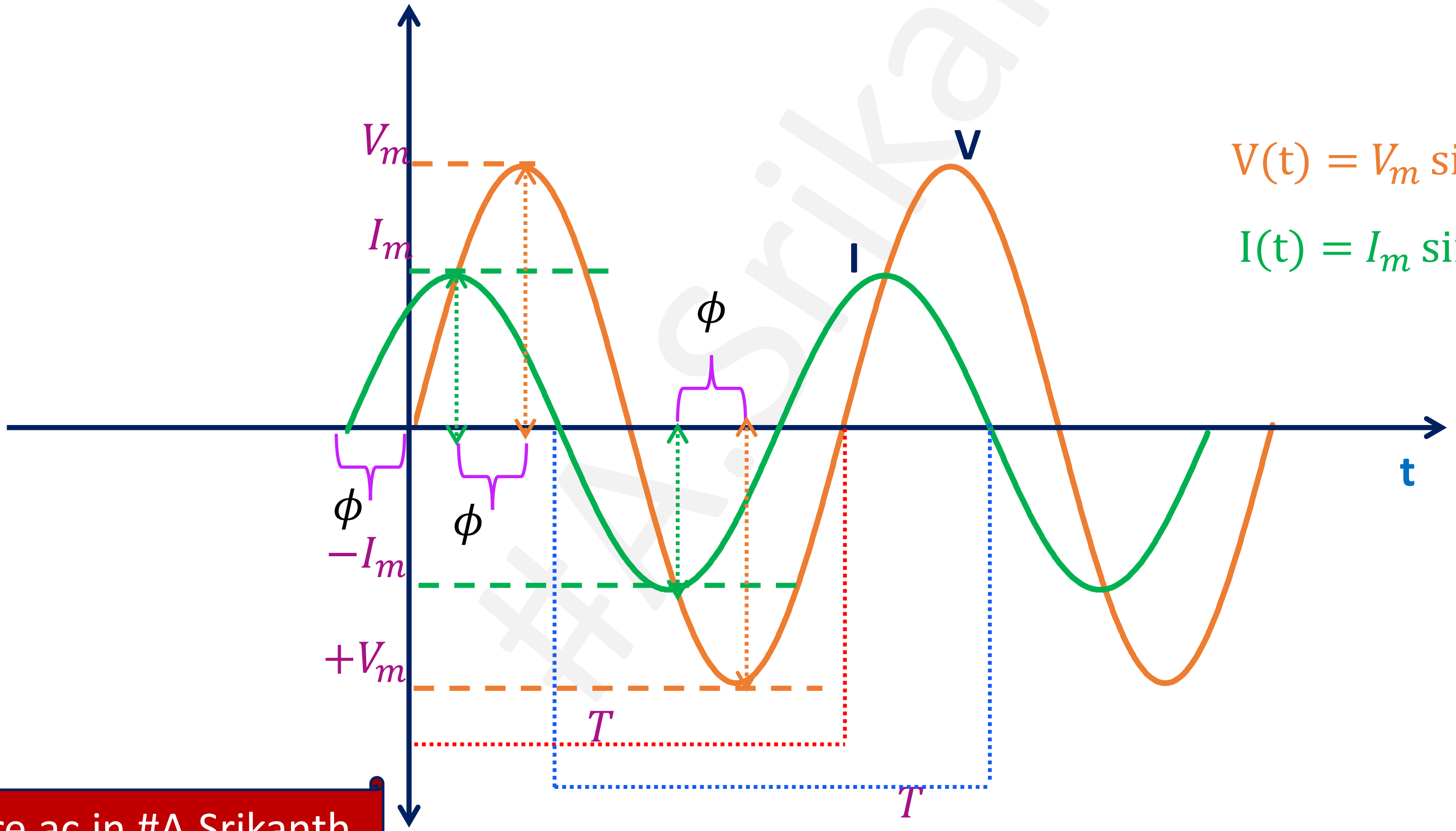
Speed : the speed at which the coil rotates inside the magnetic field.
Strength : the strength of the magnetic field.
Length : the length of the coil or conductor passing through the magnetic field.



Different Periodic Wave Forms

Phase and Phase Difference

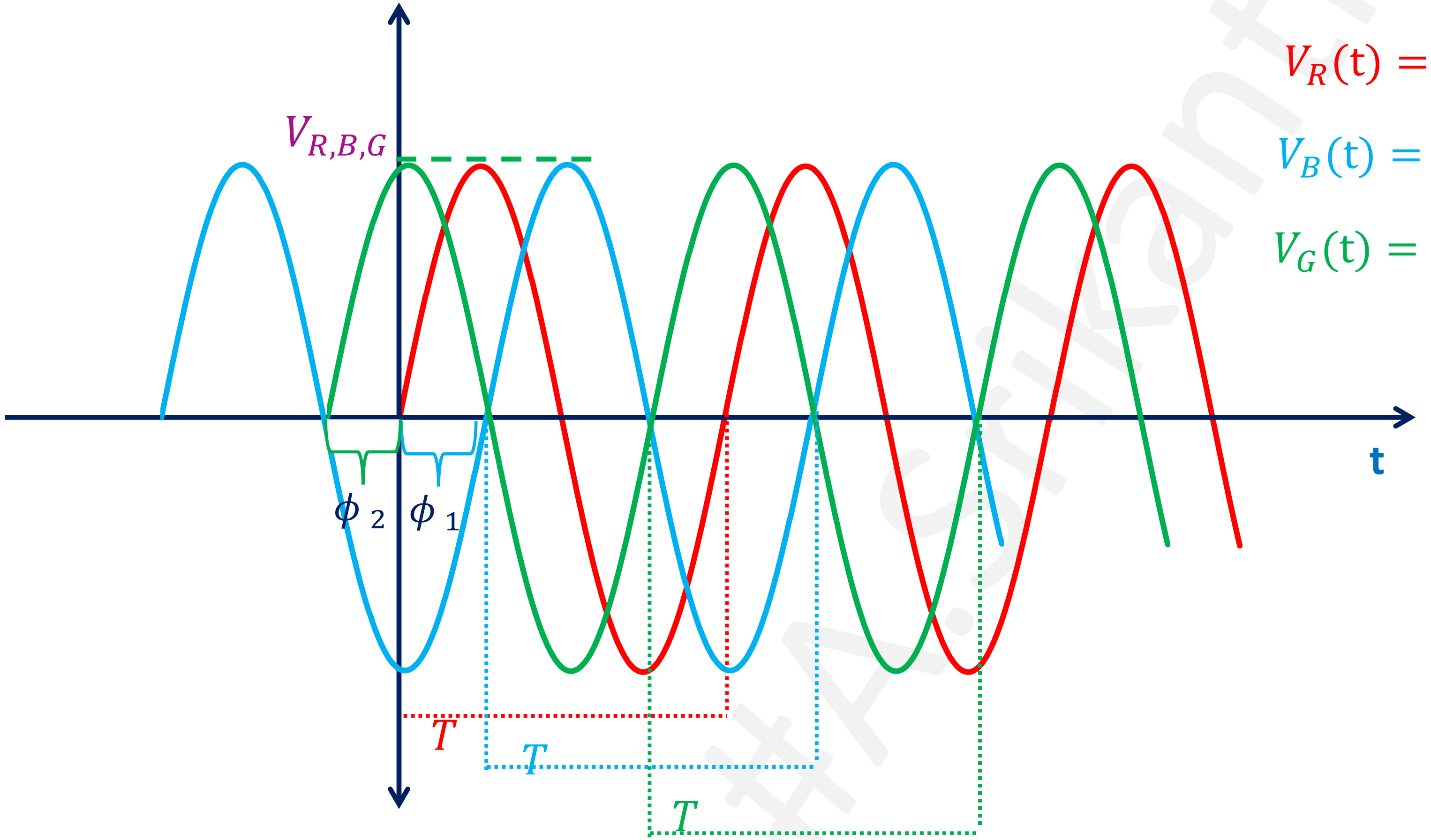
The phase difference between the two electrical quantities is defined as the angular phase difference between the maximum possible value of the two alternating quantities having the same frequency. In other words, the two alternating quantities have phase difference when they have the same frequency, but they attain their zero value at the different instant. The angle between zero points of two alternating quantities is called angle of phase differences.



$$V(t) = V_m \sin(\omega t) = V_m \sin(\theta)$$

$$I(t) = I_m \sin(\omega t + \phi) = I_m \sin(\theta + \phi)$$

Phase and Phase Difference

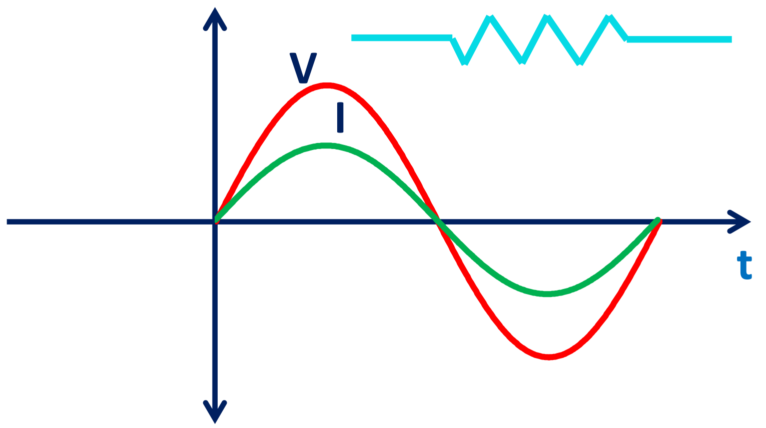


$$V_R(\mathrm{t}) = V_R \sin(\omega t) = V_R \sin(\theta)$$

$$V_B(\mathrm{t}) = V_B \sin(\omega t - \phi_1) = V_B \sin(\theta - \phi_1)$$

$$V_G(\mathrm{t}) = V_G \sin(\omega t + \phi_2) = V_G \sin(\theta + \phi_2)$$

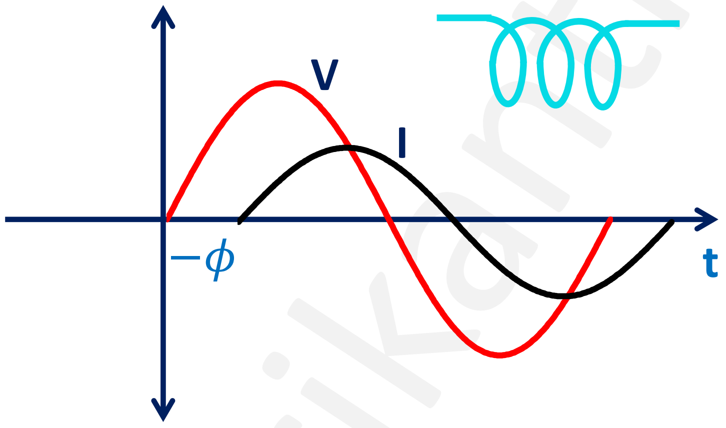
Phase and Phase Difference



Resistive circuit $\phi = 0$

Unity Power Factor

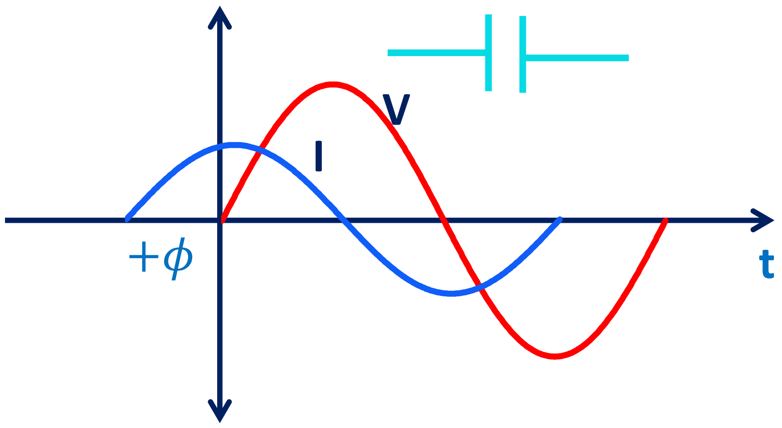
$V_R(\mathrm{t}) = V_R \sin(\omega t)$



Inductive circuit $\phi = -\phi$

Lagging Power Factor

$I_L(\mathrm{t}) = I_L \sin(\omega t - \phi)$



Capacitive circuit $\phi = +\phi$

Leading Power Factor

$I_C(\mathrm{t}) = I_C \sin(\omega t + \phi)$

Average Value for Periodic Wave

Average voltage, as the name indicates, is the average of instantaneous voltages that are chosen at appropriately timed intervals in the half cycle of the wave could be sinusoidal, Triangular, trapezoidal or any other shape. Average value represents the quotient of the area under AC wave form with respect to time. It is also known as DC Value.

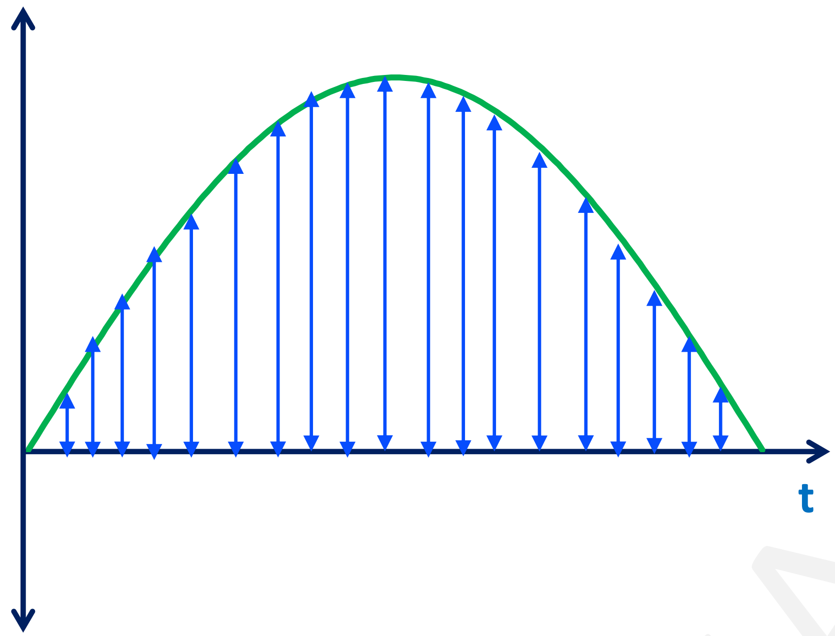
1. Average value is defines as that constant value, which produces the same amount of flux in case of voltage or same amount of charge in case of current as produced by alternating voltage or current when both are applied to the same circuit for the same period.
2. The average value of voltage is the average of all the instantaneous values during one complete cycle. They are actually dc values.
3. The average value is the amount of voltage that would be indicated by a DC voltmeter if it were connected across the load resistor.

Instantaneous value (either voltage or current) of an alternating waveform is the value at any particular instant of time. The voltage of a waveform at a given instant in time is called “Instantaneous voltage”.

Instantaneous voltage = Maximum voltage x sin θ

Average Value for Periodic Wave

The average value of an alternating wave (both sinusoidal and non sinusoidal) can be determined graphically by taking the arithmetic mean of the ordinates at equal internals over a half cycle of the wave.



$$V_{Avg} = \frac{V_1 + V_2 + V_3 + \cdots + V_N}{N}$$

The average value of periodic function f(t)

$$F_{Avg} = \frac{1}{T}\int_0^T \mathrm{f(t)\,d}t$$

RMS Value for Periodic Wave

While calculating Root Mean Square (RMS) value of alternating quantity heat energy produced by constant voltage. RMS value of an ac voltage is defined as that constant voltage, which produce the same amount of heat energy as produced by AC voltage, when both are applied to the same circuit for the same period.

The RMS value of an AC is considerable importance in practice because the ammeters and voltammeters record the RMS value of current and voltage, respectively.

The average power dissipated in the resistor in the interval is

$$P = I^2 R$$

$$P = \frac{i_1^2 R + i_2^2 R + i_3^2 R + \cdots + i_N^2 R}{N}$$

$$I^2 = \frac{i_1^2 + i_2^2 + i_3^2 + \cdots + i_N^2}{N}$$

Therefore, RMS value of AC is

$$I_{RMS} = \sqrt{\frac{i_1^2 + i_2^2 + i_3^2 + \cdots + i_N^2}{N}}$$

$$F_{Avg} = \frac{1}{T}\int_0^T \mathrm{f(t)\,dt}$$

The RMS value of periodic function f(t)

$$F_{RMS} = \sqrt{\frac{1}{T}\int_0^T \mathrm{f(t)^2\,dt}}$$

Form Factor and Pear Factor

Form Factor is defined as the ratio of RMS value to the average value of the wave.

$$Form\ Factor = \frac{RMS\ value}{Average\ value}$$

Peak or crest Factor is defined as the ratio of Peak value to the RMS value of the wave.

$$Peak\ Factor = \frac{Peak\ value}{RMS\ value}$$

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$$F_{Avg} = \frac{1}{T}\int_0^T \mathrm{f(t)\,dt}$$

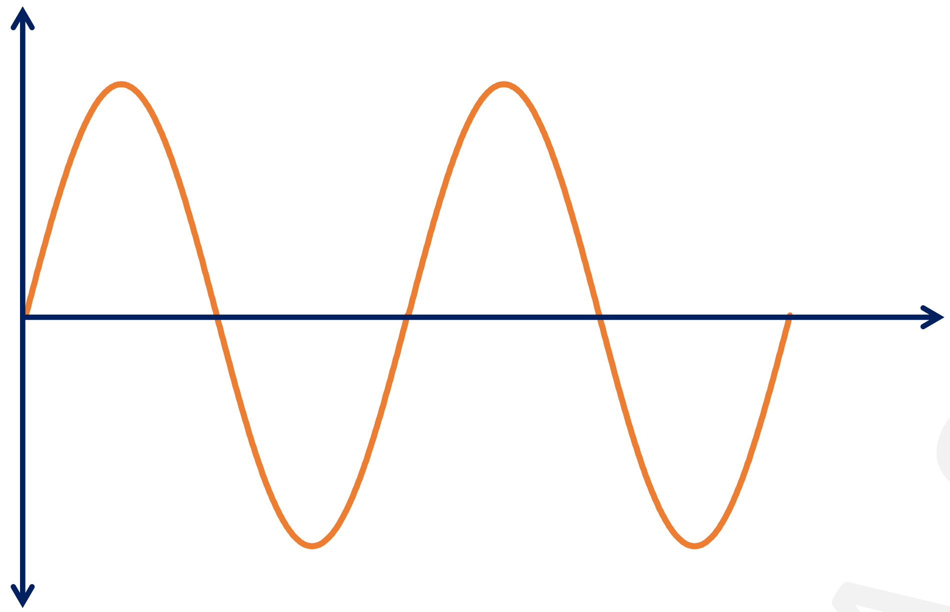
$$Form\ Factor = \frac{RMS\ value}{Average\ value}$$

$$F_{RMS} = \sqrt{\frac{1}{T}\int_0^T \mathrm{f(t)^2\,dt}}$$

$$Peak\ Factor = \frac{Peak\ value}{RMS\ value}$$

Average Value, RMS Value, form Factor and Peak Factor for Sinusoidal Wave

$$V(t) = V_m \sin(wt) = V_m \sin(\theta)$$



$$F_{Avg} = \frac{1}{T}\int_0^T f(t)\, dt$$

Time period T= π

$$F_{RMS} = \sqrt{\frac{1}{T}\int_0^T f(t)^2\, dt}$$

Time period T= 2π

$$V_{Avg} = \frac{1}{\pi}\int_0^{\pi} V_m \sin(\theta)\, d\theta$$

$$V_{Avg} = \frac{V_m}{\pi}[\cos(\theta)]_0^{\pi}$$

$$V_{Avg} = \frac{2V_m}{\pi}$$

$$V_{RMS} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} (V_m \sin(\theta))^2 d\theta}$$

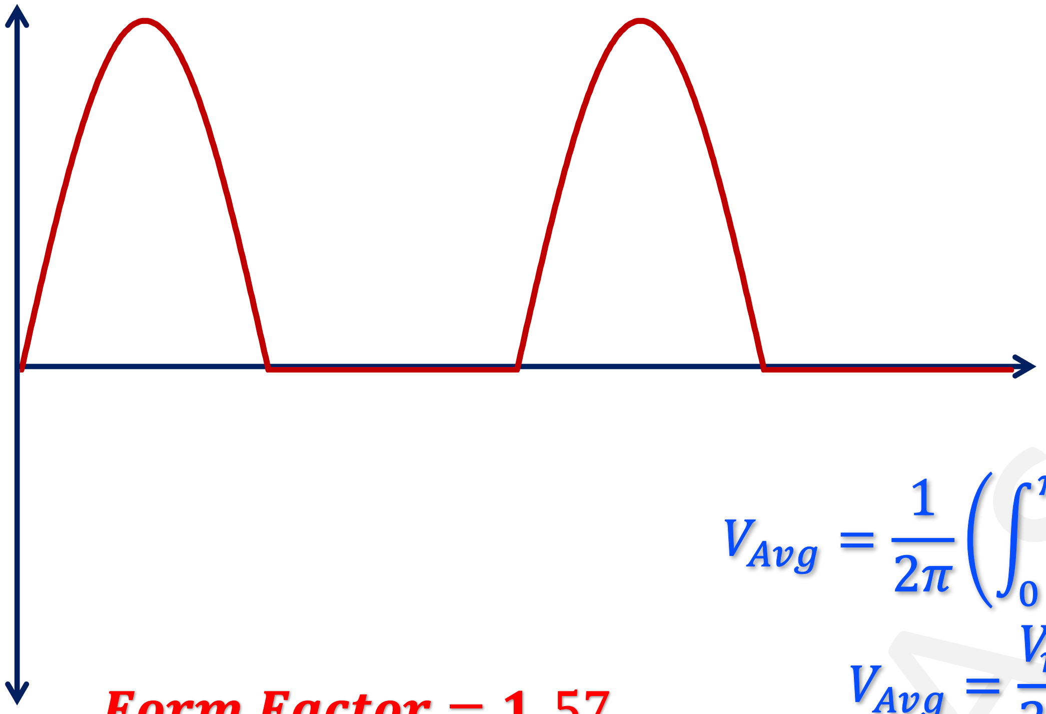
$$V_{RMS} = \sqrt{\frac{{V_m}^2}{2\pi}\int_0^{2\pi} \frac{1-\cos(2\theta)}{2} d\theta}$$

$$V_{RMS} = \frac{V_m}{\sqrt{2}}$$

$$Form\ Factor = 1.11$$

$$Peak\ Factor = 1.414$$

Average Value, RMS Value, form Factor and Peak Factor for given Wave



$$V(t) = V_m \sin(wt) = V_m \sin(\theta)$$

$$F_{Avg} = \frac{1}{T}\int_0^T f(t)\,dt \qquad F_{RMS} = \sqrt{\frac{1}{T}\int_0^T f(t)^2\,dt}$$

Time period T= 2π

Time period T= 2π

$$but\ (\pi\ \textbf{to}\ 2\pi)\ the\ value\ of\ V(t) = 0$$

$$V_{Avg} = \frac{1}{2\pi}\left(\int_0^\pi V_m \sin(\theta)\,d\theta + \int_\pi^{2\pi} 0\,d\theta\right)$$

$$V_{Avg} = \frac{V_m}{2\pi}[\cos(\theta)]_0^\pi$$

$$V_{Avg} = \frac{V_m}{\pi}$$

$$V_{RMS} = \sqrt{\frac{1}{2\pi}\left(\int_0^\pi (V_m \sin(\theta))^2 d\theta + 0\right)}$$

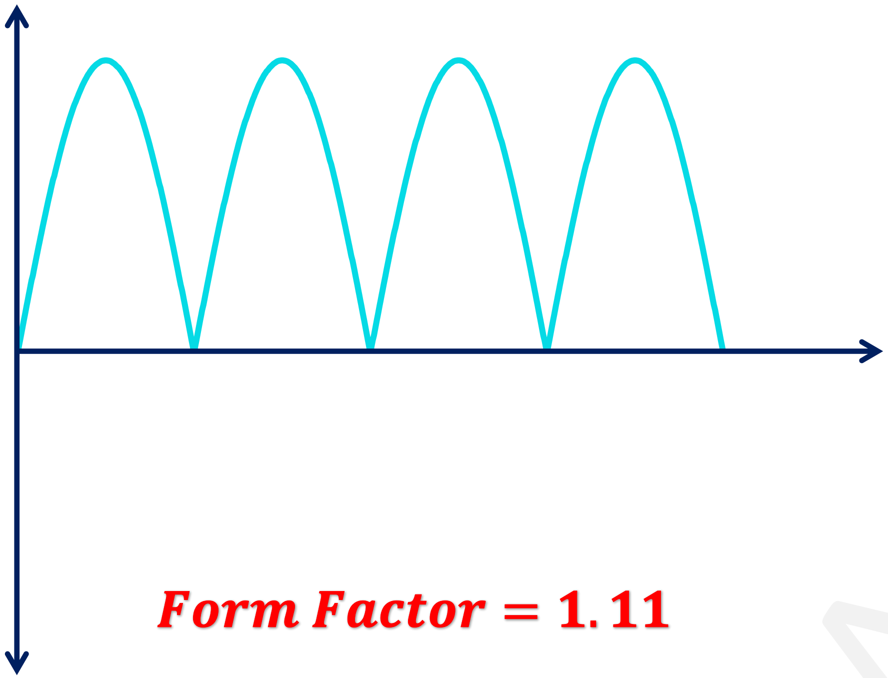
$$V_{RMS} = \sqrt{\frac{{V_m}^2}{2\pi}\int_0^\pi \frac{1-\cos(2\theta)}{2}d\theta}$$

$$V_{RMS} = \frac{V_m}{2}$$

$$\textbf{Form Factor} = \mathbf{1}.57$$

$$\textbf{Peak Factor} = 2$$

Average Value, RMS Value, form Factor and Peak Factor for given Wave



$$V(t) = V_m \sin(wt) = V_m \sin(\theta)$$

$$F_{Avg} = \frac{1}{T}\int_0^T f(t)\,dt$$

Time period T= π

$$F_{RMS} = \sqrt{\frac{1}{T}\int_0^T f(t)^2\,dt}$$

Time period T= π

$$V_{Avg} = \frac{1}{\pi}\int_0^{\pi} V_m \sin(\theta)\,d\theta$$

$$V_{Avg} = \frac{V_m}{\pi}[\cos(\theta)]_0^{\pi}$$

$$V_{Avg} = \frac{2V_m}{\pi}$$

$$V_{RMS} = \sqrt{\frac{1}{\pi}\int_0^{\pi} (V_m \sin(\theta))^2 d\theta}$$

$$V_{RMS} = \sqrt{\frac{V_m{}^2}{\pi}\int_0^{\pi} \frac{1-\cos(2\theta)}{2}\,d\theta}$$

$$V_{RMS} = \frac{V_m}{\sqrt{2}}$$

$$\textit{Form Factor} = 1.11$$

$$\textit{Peak Factor} = 1.414$$

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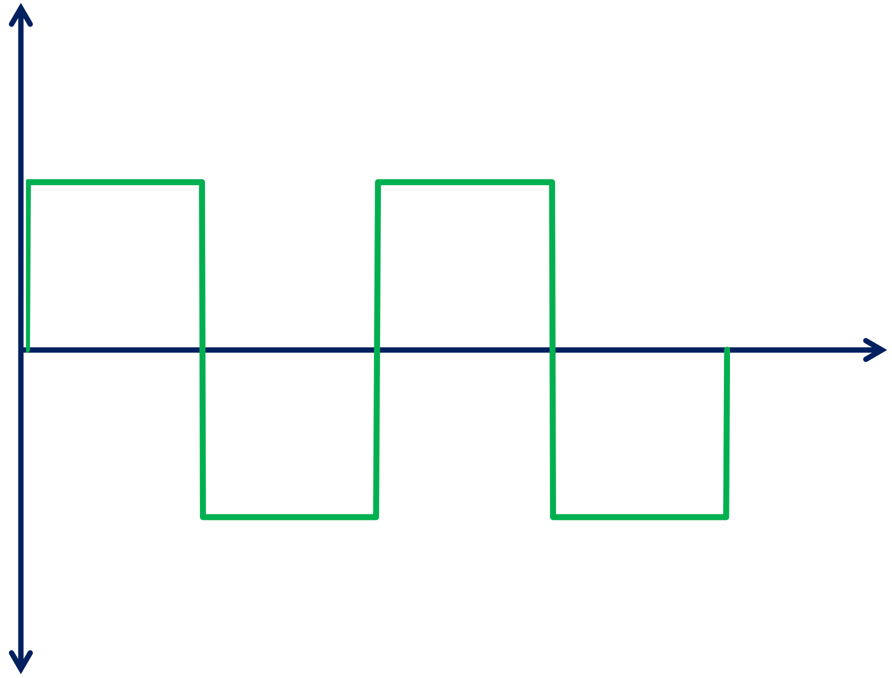
Average Value, RMS Value, form Factor and Peak Factor for given Wave



$$V(t) = V_m$$

$$F_{Avg} = \frac{1}{T}\int_0^T f(t)\,dt$$

Time period T= π

$$F_{RMS} = \sqrt{\frac{1}{T}\int_0^T f(t)^2\,dt}$$

Time period T= π

$$V_{Avg} = \frac{1}{\pi}\int_0^{\pi} V_m d\theta$$

$$V_{Avg} = \frac{V_m}{\pi} * \pi$$

$$V_{Avg} = V_m$$

$$V_{RMS} = \sqrt{\frac{1}{\pi}\int_0^{\pi} (V_m)^2 d\theta}$$

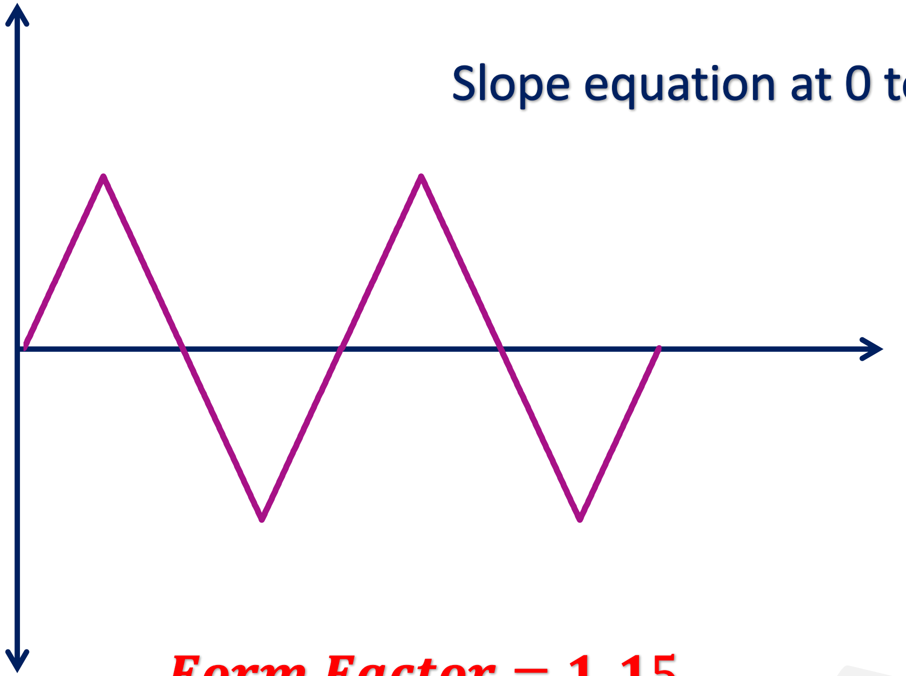
$$V_{RMS} = \sqrt{\frac{{V_m}^2}{\pi} * \pi}$$

$$V_{RMS} = V_m$$

$$\textit{Form Factor} = 1$$

$$\textit{Peak Factor} = 1$$

Average Value, RMS Value, form Factor and Peak Factor for given Wave



Slope equation at 0 to $\frac{\pi}{2}$ $V(t) = \frac{2V_m t}{\pi}$

$$F_{Avg} = \frac{1}{T}\int_0^T f(t)\,dt$$

Time period T=$\frac{\pi}{2}$

$$V_{Avg} = \frac{1}{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}} \frac{2V_m t}{\pi}\,dt$$

$$V_{Avg} = \frac{2}{\pi} * \frac{2V_m}{\pi}\int_0^{\frac{\pi}{2}} t\,dt$$

$$V_{Avg} = \frac{V_m}{2}$$

$$F_{RMS} = \sqrt{\frac{1}{T}\int_0^T f(t)^2\,dt}$$

Time period T=$\frac{\pi}{2}$

$$V_{RMS} = \sqrt{\frac{1}{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}} \left(\frac{2V_m t}{\pi}\right)^2 dt}$$

$$V_{RMS} = \sqrt{\frac{2}{\pi} * \left(\frac{2V_m}{\pi}\right)^2 \int_0^{\frac{\pi}{2}} (t)^2 dt}$$

$$V_{RMS} = \frac{V_m}{\sqrt{3}}$$

$$\textit{Form Factor} = 1.15$$

$$\textit{Peak Factor} = 1.73$$

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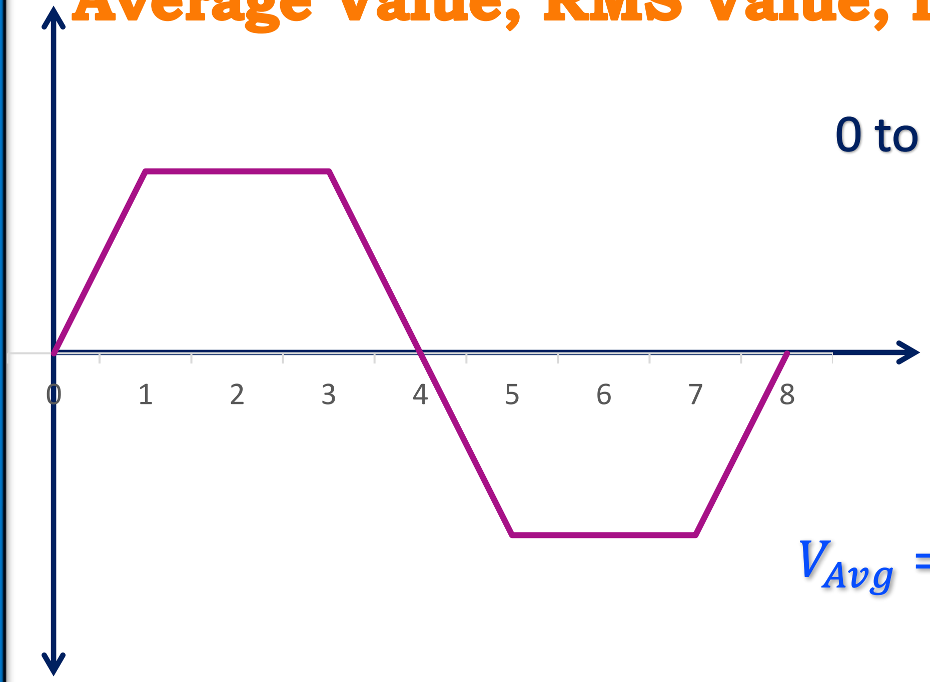
Average Value, RMS Value, form Factor and Peak Factor given Wave



0 to 1 $V(t) = 5t$ 1 to 3 $V(t) = 5$ 3 to 4 $V(t) = 5(4 - t)$

$$F_{Avg} = \frac{1}{T}\int_0^T f(t)\,dt$$

Time period T=4

$$F_{RMS} = \sqrt{\frac{1}{T}\int_0^T f(t)^2\,dt}$$

Time period T=4

$$V_{Avg} = \frac{1}{4}\left(\int_0^1 5t\,dt + \int_1^3 5\,dt + \int_3^4 5(4-t)\,dt\right)$$

$$V_{Avg} = 3.75V$$

$$V_{RMS} = \sqrt{\frac{1}{4}\left(\int_0^1 5t^2\,dt + \int_1^3 5^2\,dt + \int_3^4 (5(4-t))^2\,dt\right)}$$

$$V_{RMS} = 4.0825V$$

$$\boldsymbol{Form\ Factor} = \mathbf{1}.0887$$

$$\boldsymbol{Peak\ Factor} = \mathbf{1}.3333$$

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ELECTRICAL CIRCUITS

Course Code	: AEEC02
Category	: Foundation
Credits	: 3
Maximum Marks	: CIA (30) + SEE (70) = Total (100)

By
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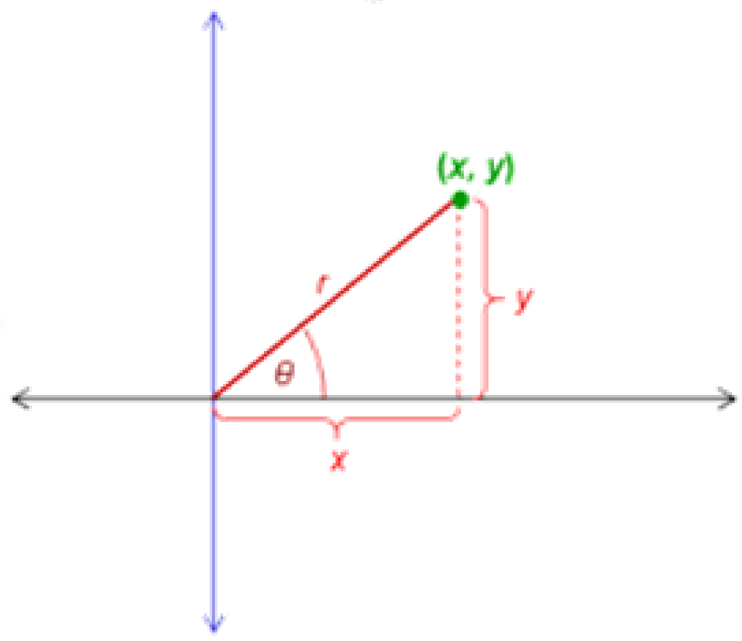
Assistant Professor
Electrical and Electronics Engineering

Concept of Impedance and Admittance



$$r = \sqrt{x^2 + y^2}$$

$$x = r\cos\theta \qquad y = r\sin\theta$$

$$\theta = \tan^{-1}\frac{x}{y}$$

Concept of Impedance and Admittance

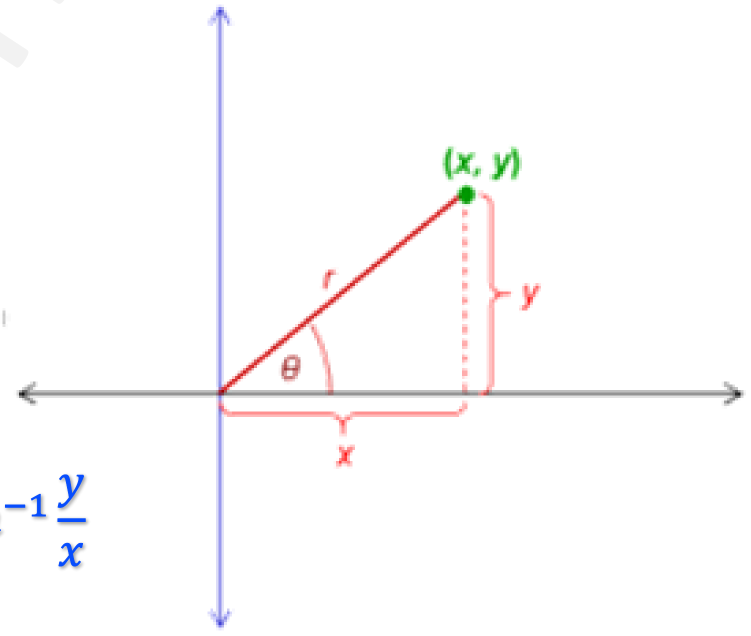
Representation of Rectangular and Polar Forms : Sinusoids are easily expressed in terms of phasors, which are more convenient to work with the sine and cosine functions. Phasors in the complex form can be represented polar and rectangular forms.

Rectangular or Cartesian or Complex Form : $z = x \pm jy$

Polar form : $z = \mathrm{r}\angle \pm \theta$

Trigonometrical Form : $z = \mathrm{r}(cos\theta \pm jsin\theta)$

Exponential : $z = re^{\pm j\theta}$

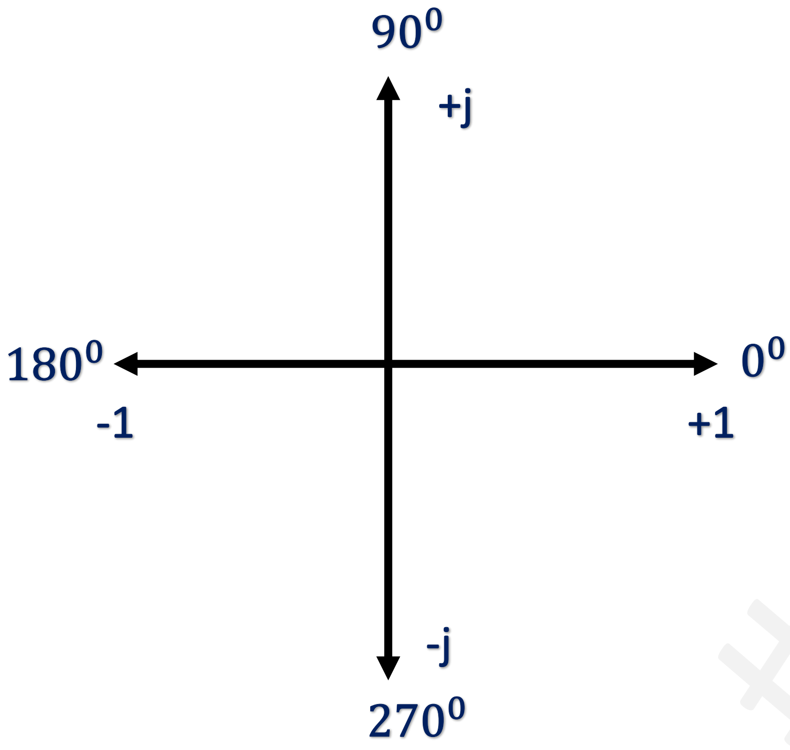


$\mathbf{r}=\sqrt{x^2+y^2}$ $\quad \boldsymbol{\theta}=tan^{-1}\frac{y}{x}$

$x=r\ cos\boldsymbol{\theta}$ $\quad y=r\ sin\boldsymbol{\theta}$

Concept of Impedance and Admittance

j Notation : The ‘j’ operator is a vector operator which when operates on a given phase, produces a Counter clockwise rotation of the phasor by 90^0, without changing its magnitude.



$$0^\circ = \pm 360^\circ = +1 = 1\angle 0^\circ = 1 + j0$$

$$+90^\circ = +\sqrt{-1} = +j = 1\angle +90^\circ = 0 + j1$$

$$-90^\circ = -\sqrt{-1} = -j = 1\angle -90^\circ = 0 - j1$$

$$\pm 180^\circ = \left(\sqrt{-1}\right)^2 = -1 = 1\angle \pm 180^\circ = -1 + j0$$

Concept of Impedance and Admittance

1. $z = 5 + j10$

2. $z = 10 - j3$

3. $z = 11.18\angle 63.435$

4. $z = 10.44\angle -16.7$

6. $Z_1 = 5 + j10\ ;\ \ Z_2 = 6 + j9$

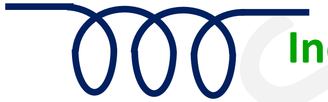
$$Z_1 + Z_2$$
$$Z_1 - Z_2$$
$$Z_1 * Z_2$$
$$\frac{Z_1}{Z_2}$$

Concept of Impedance and Admittance

In electric and electronic systems, **reactance** is the opposition of a circuit element to a change in current or voltage, due to that element's inductance or capacitance.

Reactance is measured in **Ohm's but is given the symbol "X"** to distinguish it from a purely resistive "R" value and as the component in question is an inductor, the reactance of an inductor is called **Inductive Reactance**, (XL) and is measured in Ohms.

L



$$X_L = \omega L = 2\pi f L$$

Inductive Reactance (X_L)

As the capacitor charges or discharges, a current flows through it which is restricted by the internal impedance of the capacitor. This internal impedance is commonly known as **Capacitive Reactance** and is given the symbol (XC) in Ohms.

C

Capacitive Reactance (X_C)

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

Concept of Impedance and Admittance

The impedance is defined as the ratio of sinusoidal voltage to the sinusoidal current. It is also defined as the total opposition offered to the flow of sinusoidal current. Hence the **impedance is measured in OHMS**.

The real part of the impedance is resistance and the imaginary part is reactance.

Impedance for series Resistive and Inductive :

$$Z = R + jX_L \text{ or } Z = \sqrt{R^2 + X_L^2}\,\angle \tan^{-1}\left(\frac{X_L}{R}\right)$$

Impedance for series Resistive and Capacitive :

$$Z = R - jX_C \text{ or } Z = \sqrt{R^2 + X_C^2}\,\angle \tan^{-1}\left(\frac{-X_C}{R}\right)$$

Impedance for series Resistive, Inductive and Capacitive :

$$Z = R + j(X_L - X_C) \text{ or } Z = \sqrt{R^2 + X^2}\,\angle \tan^{-1}\left(\frac{X}{R}\right)$$

Concept of Impedance and Admittance

In parallel circuit the inverse of the parameters will be useful for analysis. The inverse of impedance is Admittance. It is also defined as the ratio of sinusoidal current to voltage.

Admittance : $Y = \frac{1}{Z}$

Conductance : $G = \frac{1}{R}$

Susceptance : $B = \frac{1}{X}$

Admittance for series Resistive, Inductive and Capacitive :

$$Y = G + j(B_c - B_L) \text{ or } Y = \sqrt{G^2 + B^2} \angle \tan^{-1}\left(\frac{B}{G}\right)$$

Power in AC Circuits

Power is the rate at which is done, or power is the rate of energy transfer. In circuit excited by DC sources, the Voltage and current are constant, so the power is constant. This constant power is called Average Power (P). $P = VI.$

In circuit excited by AC source, the voltage and current are Sinusoidal quantities which varies with time. When voltage and current are time varying quantities the power is also a time varying quantity, is called Instantaneous power (p). p = vi

The average value of Sinusoidal function for full cycle is Zero, we can take the RMS value of voltage and current. The RMS value of voltage and current are complex and so the power is also complex. *The complex power is given by* $s = VI\angle\phi$

Where ϕ is the phase difference between V and I

$$s = VI\cos\phi + j\,VI\sin\phi$$

$$s = P + jQ = \sqrt{P^2 + Q^2}$$

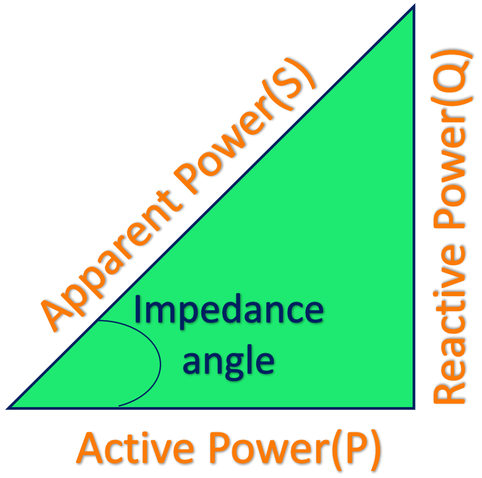
The magnitude of Complex power is Apparent Power (S). S=$|s|$ in VA or KVA

P = Active Power in KW = VI $\cos\phi$

Q= Reactive Power in KVAR = VI $\sin\phi$

Power in AC Circuits

Power Triangle is the representation of a right - angle triangle showing the relation between active power, reactive power and apparent power. When each component of the current that is the active component (Icosφ) or the reactive component (Isinφ) is multiplied by the voltage V



Power Factor is the ratio of watts that are converted to the volt amperes are fed into the circuit.

$$\text{Power factor} = \cos(\phi) = \frac{\text{Active power}}{\text{Apparent power}} = \frac{P}{S}$$

It is also defined as the cosine of phase angle (φ) between the voltage and current. depending on weather the current lags or leads the voltage, the power factor is also taken as **Unity or Lagging or Leading**.

COURSE SYLLABUS

MODULE-I: INTRODUCTION TO ELECTRICAL CIRCUITS

Circuit Concept: Basic definitions, Ohm‟s law at constant temperature, classifications of elements, independent and dependent sources, voltage and current relationships for passive elements,

Single Phase AC Circuits: Representation of alternating quantities, properties of different periodic wave forms, phase and phase difference, concept of impedance and admittance, power in AC circuits.

COURSE OUTCOMES

CO 1 : Define the various terminology used to study the characteristics of DC and AC electrical networks.

CO 2 : Discuss the different laws associated with electrical circuits to determine equivalent resistance and source currents.

CO 3 : Identify the alternating quantities with peak, average and root mean square values for different periodic wave forms.

ELECTRICAL CIRCUITS

Course Code	: AEEC02
Category	: Foundation
Credits	: 3
Maximum Marks	: CIA (30) + SEE (70) = Total (100)

By
A. Srikanth
Assistant Professor
Electrical and Electronics Engineering

COURSE SYLLABUS

MODULE-II: ANALYSIS OF ELECTRICAL CIRCUITS

Circuit Analysis: Source transformation, Kirchhoff"s laws, total resistance, inductance and capacitance of circuits, Star - delta transformation technique, mesh analysis and nodal analysis, inspection method, super mesh, super node analysis.

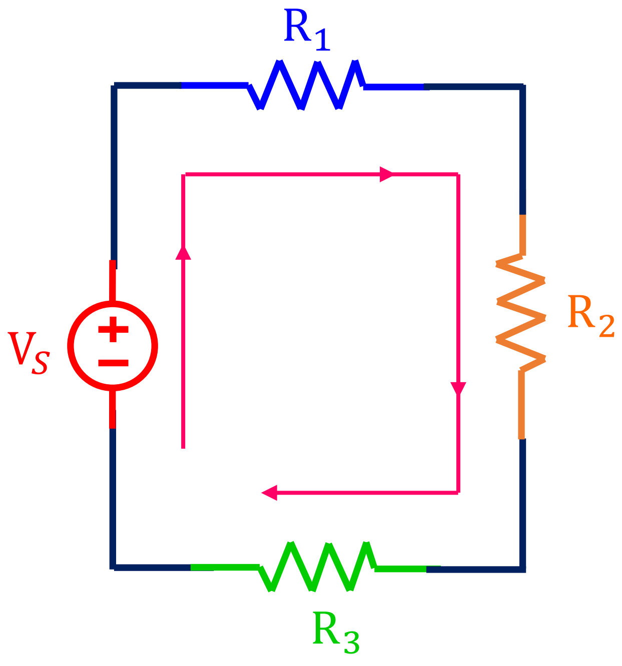
COURSE OUTCOMES

CO 2 : Discuss the different laws associated with electrical circuits to determine equivalent resistance and source currents.

CO 4 : Discuss the indirect quantities associated with electrical circuit for branch currents and nodal voltages.

Source Transformation

Voltage Divider Circuits are useful in providing different voltage levels from a common supply voltage.
The voltage divider equation is very useful for determining the relationships in a series circuit.



$$I = \frac{V_S}{R_1+R_2+R_3} = \frac{V_S}{R_T}$$

$$V_{R_1} = I * R_1 = \frac{V_S}{R_T} * R_1$$

$$V_{R_2} = I * R_2 = \frac{V_S}{R_T} * R_2$$

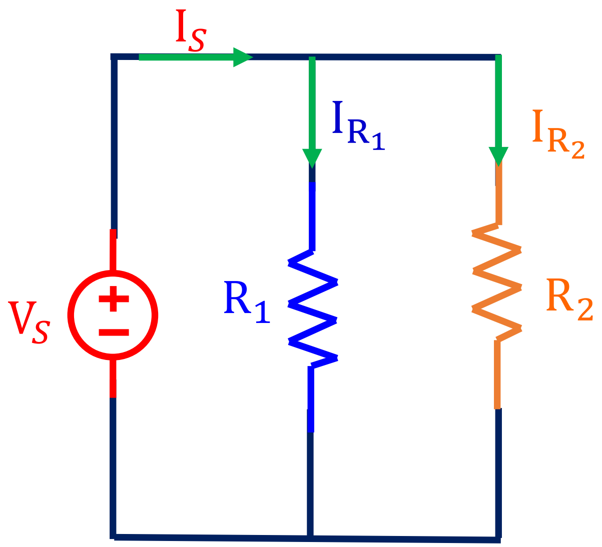
$$V_{R_3} = I * R_3 = \frac{V_S}{R_T} * R_3$$

$$\boxed{V_N = \frac{V_S}{R_T} * R_N}$$

Voltage Division Rule

Source Transformation

Current Divider Circuits are parallel circuits in which the source or supply current divides into a number of parallel paths. The definite portion of total current shared by any of the parallel paths can easily be calculated if the impedance of that path and the equivalent impedance of the parallel system are known to us.



$$I_{R_1} = \frac{V_S}{R_1} \qquad I_{R_2} = \frac{V_S}{R_2} \qquad I_S = \frac{V_S}{R_T}$$

Current Division Rule

$$I_S = I_{R_1} + I_{R_2}$$

$$R_T = \frac{R_1 * R_2}{R_1 + R_2}$$

$$V_S = I_S * R_T$$

$$I_{R_1} = \frac{I_S * R_T}{R_1} \qquad I_{R_1} = \frac{I_S * \frac{R_1 * R_2}{R_1 + R_2}}{R_1}$$

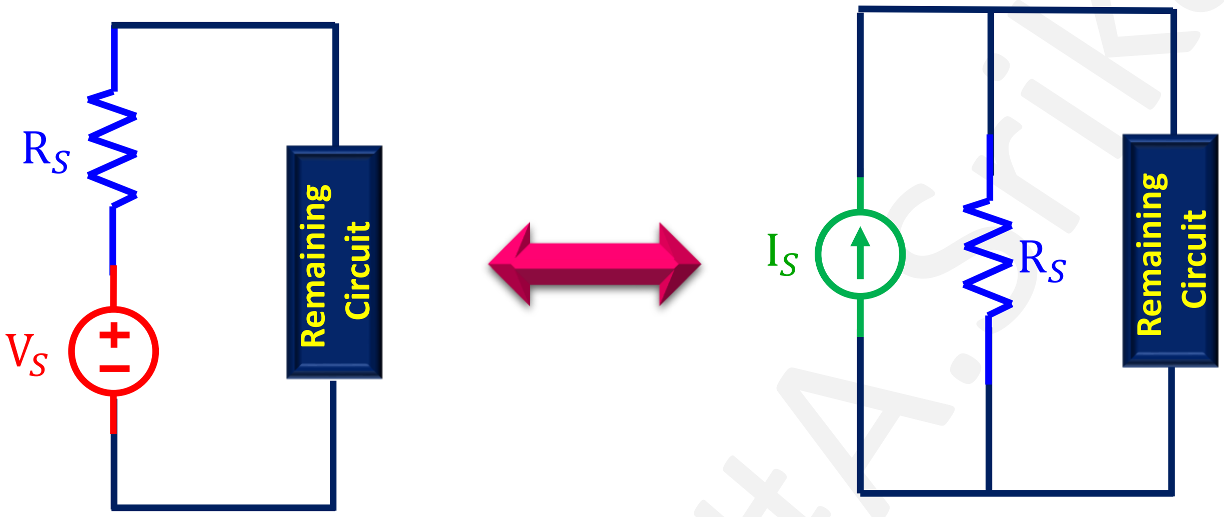
$$I_{R_1} = I_S * \frac{R_2}{R_1 + R_2} \qquad I_{R_2} = I_S * \frac{R_1}{R_1 + R_2}$$

$$I_{R_N} = I_S * \frac{R_{op}}{R_N + R_{op}}$$

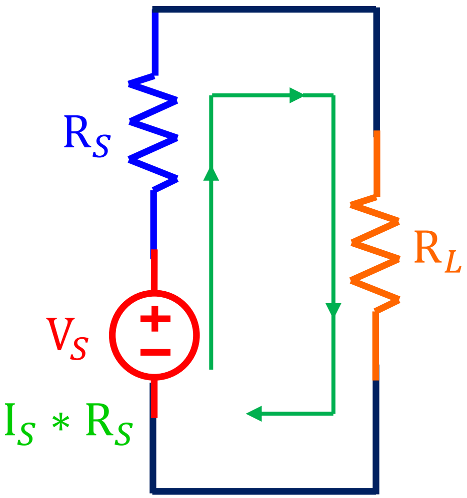
Source Transformation

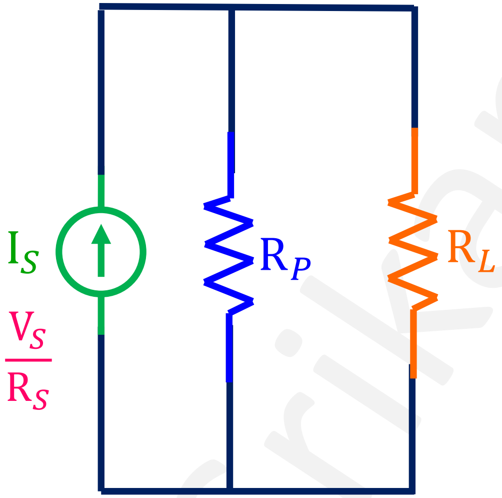
Source transformation methods are used for circuit simplification to modify the complex circuits by transforming independent voltage sources into independent current or independent current sources into independent voltage sources.

Source Transformation



Source Transformation





$$I_{R_L} = \frac{V_S}{R_S + R_L}$$

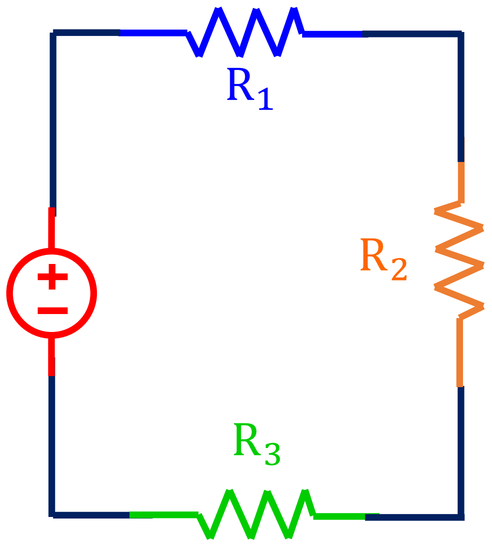
$$I_{R_L} = I_S * \frac{R_P}{R_P + R_L}$$

$$\frac{V_S}{R_S + R_L} = I_S * \frac{R_P}{R_P + R_L}$$

$$R_S = R_P$$

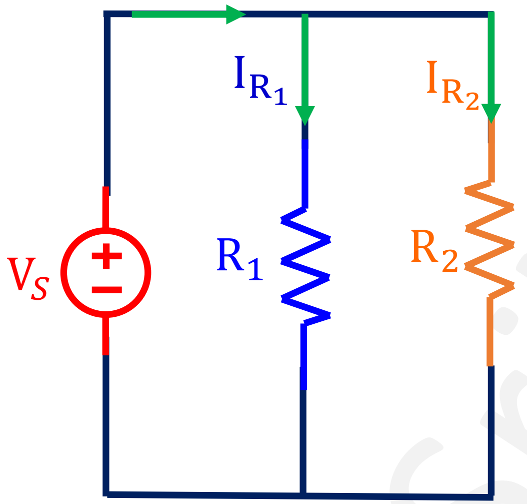
$$I_S = \frac{V_S}{R_S} \qquad V_S = I_S * R_S$$

Voltage Division Rule



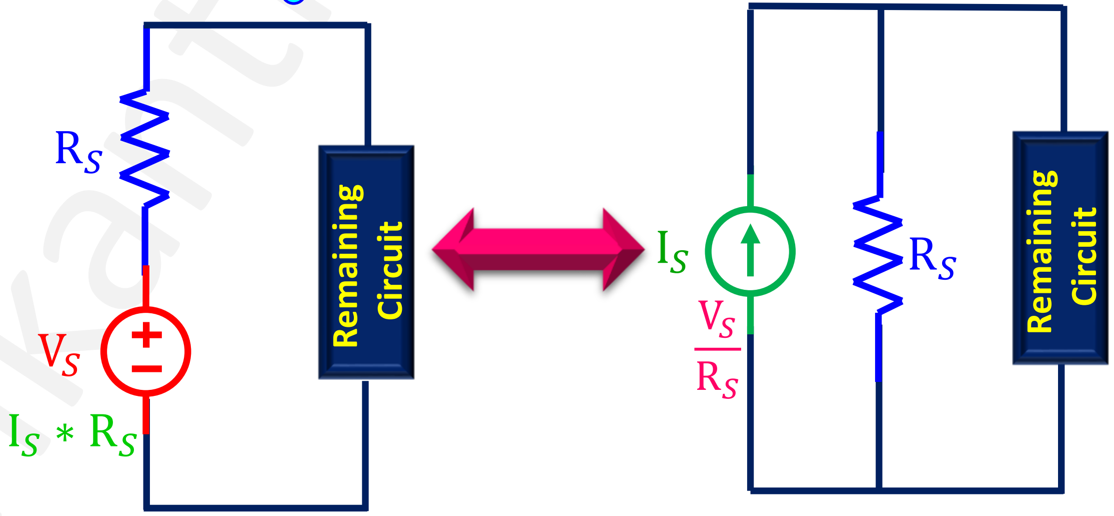
$$V_N = \frac{V_S}{R_T} * R_N$$

Current Division Rule



$$I_{R_N} = I_S * \frac{R_{op}}{R_N + R_{op}}$$

Source Transformation



$$V_S = I_S * R_S$$

$$I_S = \frac{V_S}{R_S}$$

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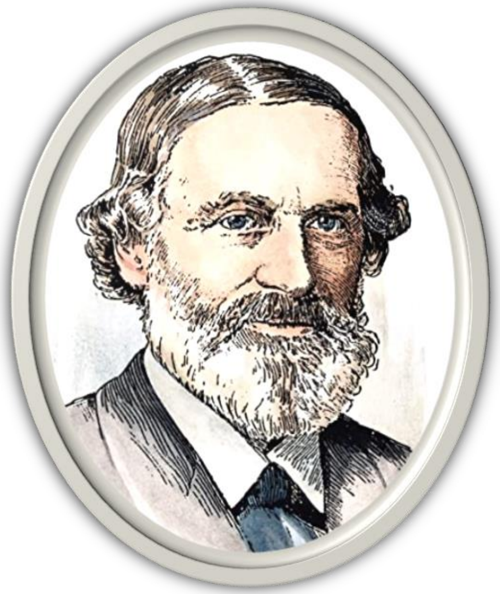
Kirchhoff's Laws

In 1845, German physicist Gustav Kirchhoff was described relationship of two quantities in Current and potential difference (Voltage) inside a circuit. This relationship or rule is called as Kirchhoff's circuit Law.

Kirchhoff's Circuit Law consist two laws, Kirchhoff's First law - which is related with current flowing, inside a closed circuit and called as Kirchhoff's current law (KCL) and the other one is Kirchhoff's Second law which is to deal with the voltage sources of the circuit, known as Kirchhoff's voltage law (KVL).



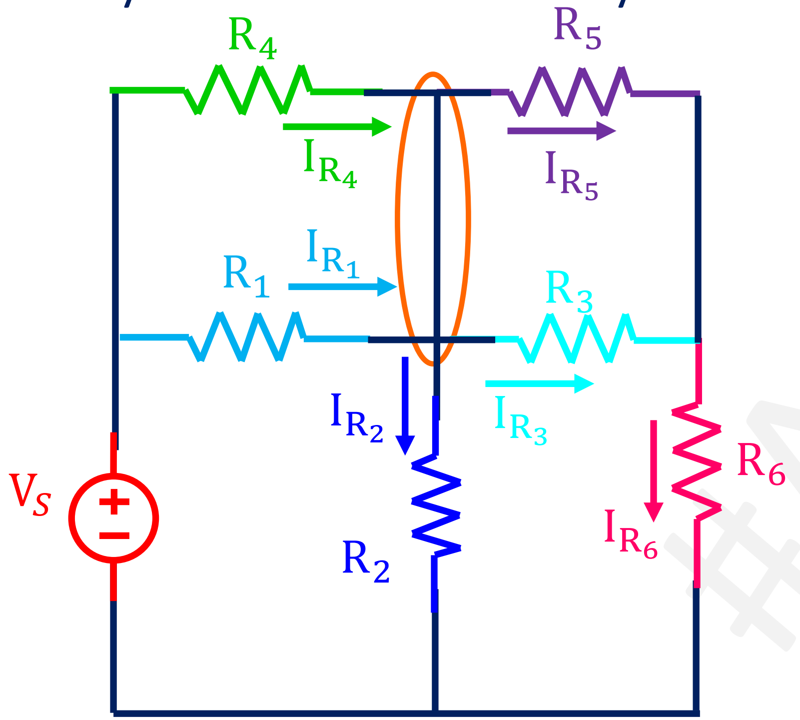
Kirchhoff's Laws

Kirchhoffs First Law – The Current Law, (KCL)

Kirchhoffs Current Law states that the "the algebraic sum of all the currents entering and leaving a node must be equal to zero. Or

Total current entering a junction or node is exactly equal to total current leaving the node.

This idea by Kirchhoff is commonly known as the Conservation of Charge.



$$I_{R_1} + I_{R_4} - I_{R_2} - I_{R_3} - I_{R_5} = 0$$

$$I_{R_1} + I_{R_4} = I_{R_2} + I_{R_3} + I_{R_5}$$

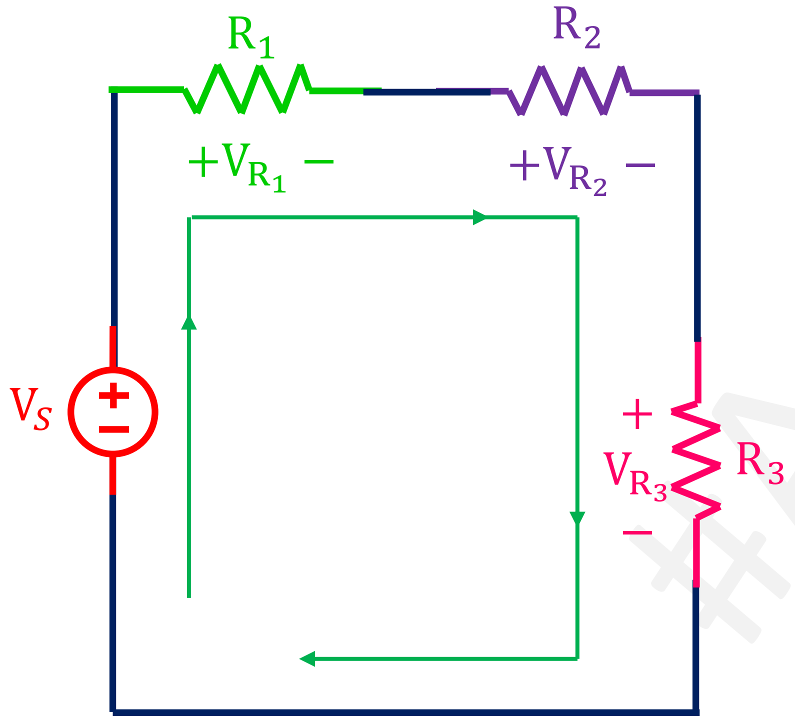
Kirchhoff's Laws

Kirchhoffs Second Law – The Voltage Law, (KVL)

Kirchhoffs Voltage Law, states that "in any closed loop network, the total voltage around the loop is equal to the sum of all the voltage drops within the same loop" which is also equal to zero. Or

The algebraic sum of all voltages within the loop must be equal to zero.

This idea by Kirchhoff is known as the Conservation of Energy.



$$V_S = V_{R_1} + V_{R_2} + V_{R_3}$$

$$-V_S + V_{R_1} + V_{R_2} + V_{R_3} = 0$$

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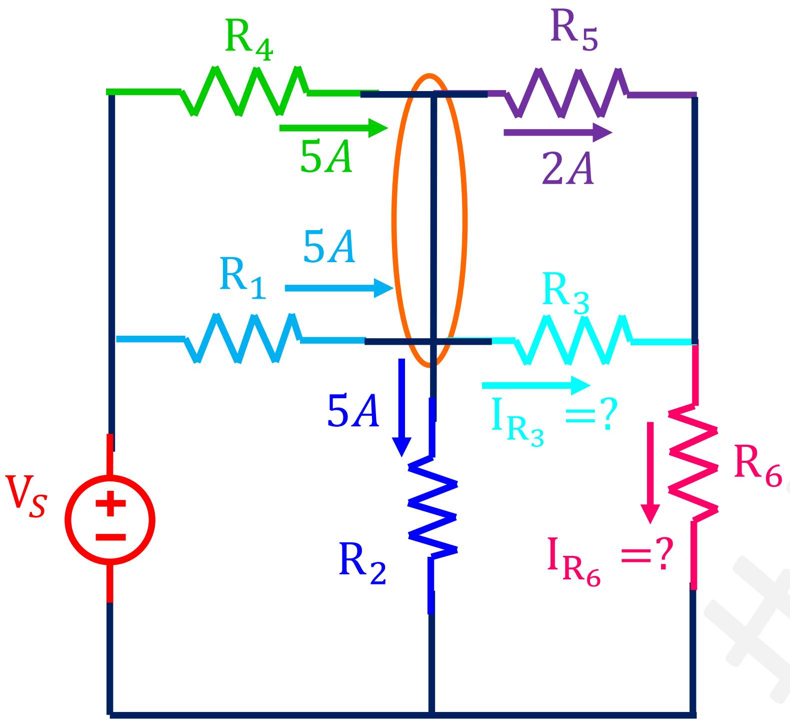
Kirchhoff's Laws

Kirchhoffs First Law – The Current Law, (KCL)

Kirchhoffs Current Law states that the "the algebraic sum of all the currents entering and leaving a node must be equal to zero.



$$I_{R_1} + I_{R_4} - I_{R_2} - I_{R_3} - I_{R_5} = 0$$

$$5 + 5 - 5 - I_{R_3} - 2 = 0$$

$$I_{R_3} = 3A$$

$$I_{R_3} + I_{R_5} - I_{R_6} = 0$$

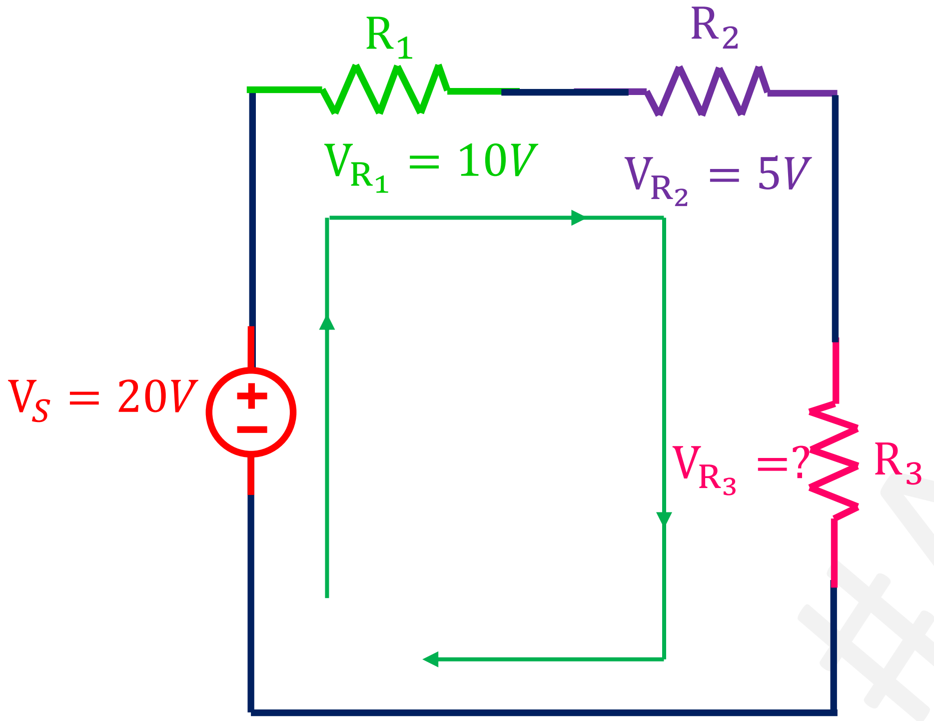
$$3 + 2 - I_{R_6} = 0$$

$$I_{R_6} = 5A$$

Kirchhoff's Laws

Kirchhoffs Second Law – The Voltage Law, (KVL)

Kirchhoffs Voltage Law, states that "in any closed loop network, the total voltage around the loop is equal to the sum of all the voltage drops within the same loop" which is also equal to zero.



$$-V_S + V_{R_1} + V_{R_2} + V_{R_3} = 0$$

$$-20 + 10 + 5 + V_{R_3} = 0$$

$$V_{R_3} = 5V$$

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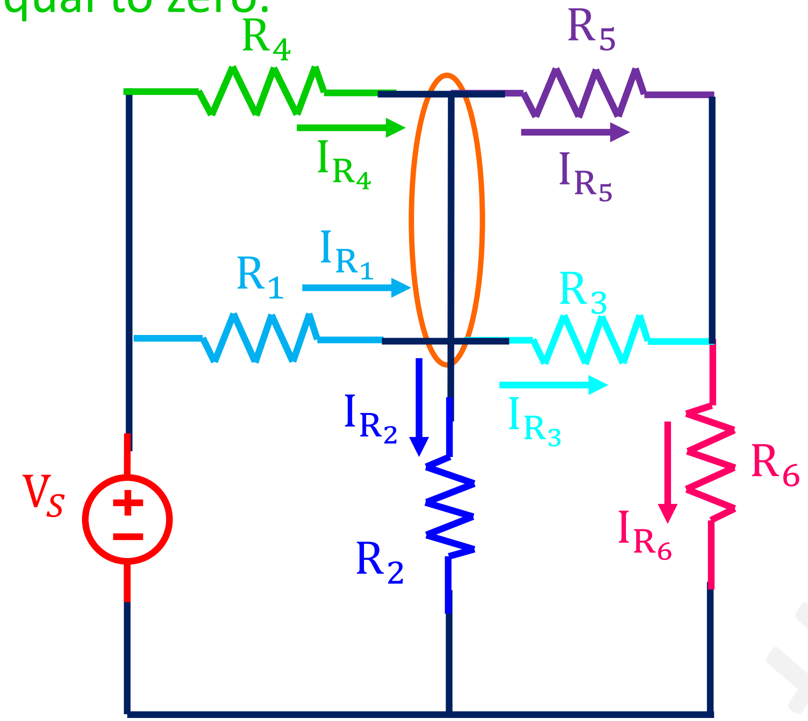
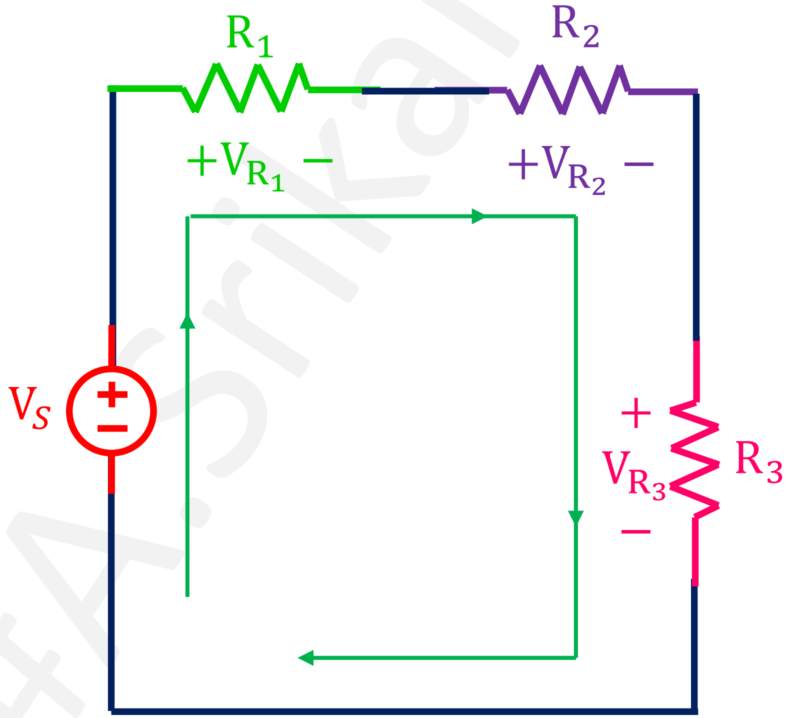
Kirchhoff's Laws

Kirchhoffs First Law – The Current Law, (KCL) : Total current entering a junction or node is exactly equal to total current leaving the node.

Kirchhoffs Second Law – The Voltage Law, (KVL) : The algebraic sum of all voltages within the loop must be equal to zero.



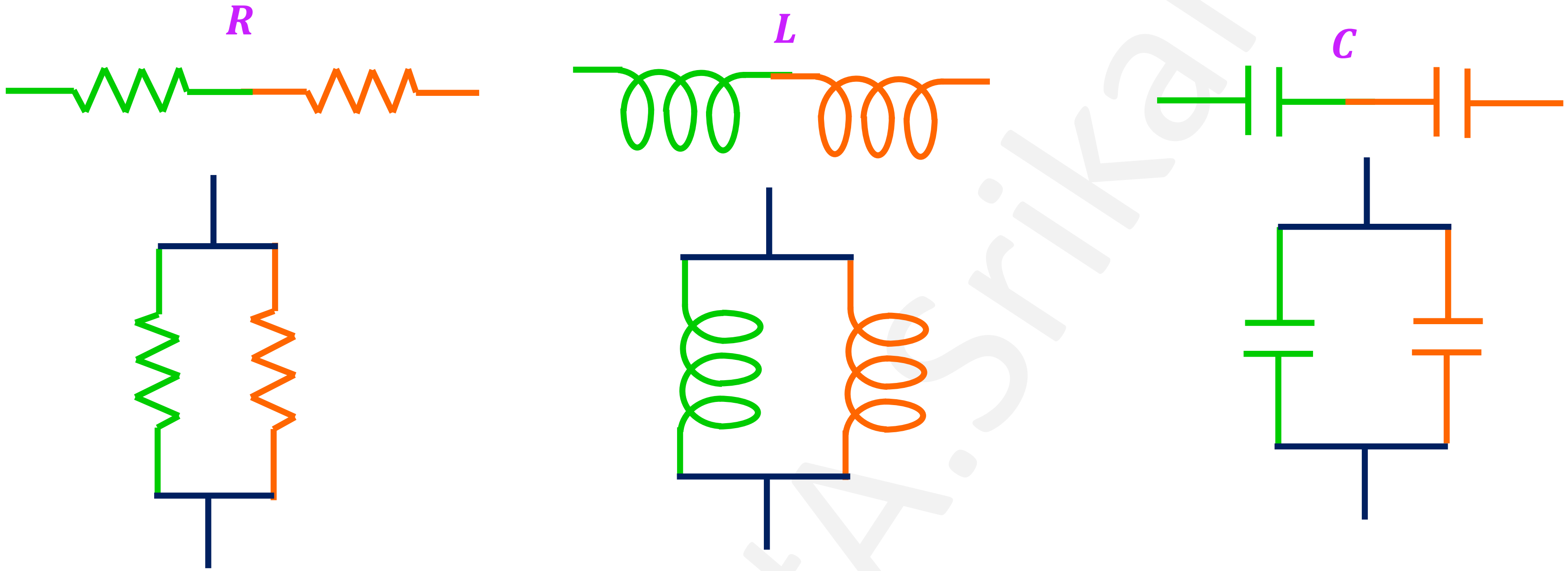


$$I_{R_1} + I_{R_4} - I_{R_2} - I_{R_3} - I_{R_5} = 0$$

$$-V_S + V_{R_1} + V_{R_2} + V_{R_3} = 0$$

Equivalent Values of Series, Parallel R, L & C Networks

The equivalent resistance is where the total resistance connected either in parallel or in series.
The equivalent resistance of a network is the single resistor which can replace the entire network in such a way that for a certain applied voltage as V we will get the same current as I.



Resistors in series carry the same current
Resistors in parallel have the same voltage across them

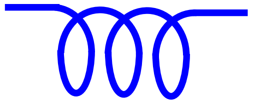
Equivalent Values of Series, Parallel R, L & C Networks

Ohm's law : It states that, at constant temperature in an electrical circuit the current (I) flowing through a conductor is directly proportional to potential difference (V) applied.

R

$$V = IR$$
$$I = \frac{V}{R}$$

L



$$V = L\frac{di}{dt}$$
$$I = \frac{1}{L}\int v\,dt$$

C

$$V = \frac{1}{C}\int i\,dt$$
$$I = C\frac{dv}{dt}$$

Kirchhoffs First Law – The Current Law, (KCL)

Total current entering a junction or node is exactly equal to total current leaving the node.

$$I_{R_1} + I_{R_4} - I_{R_2} - I_{R_3} - I_{R_5} = 0$$

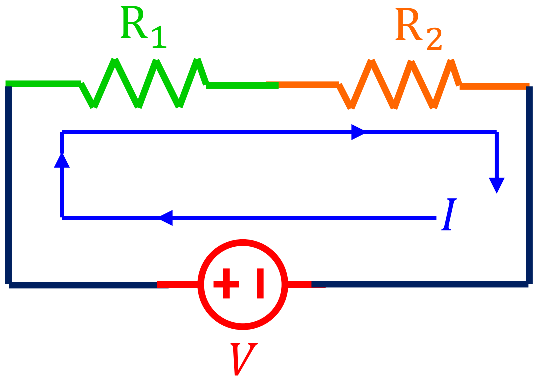
Kirchhoffs Second Law – The Voltage Law, (KVL)

The algebraic sum of all voltages within the loop must be equal to zero.

$$-V_S + V_{R_1} + V_{R_2} + V_{R_3} = 0$$

Equivalent Values of Series, Parallel R, L & C Networks

Resisters are in Series



$$V = I * R$$

$$V_{R_1} = I * R_1 \quad V_{R_2} = I * R_2$$

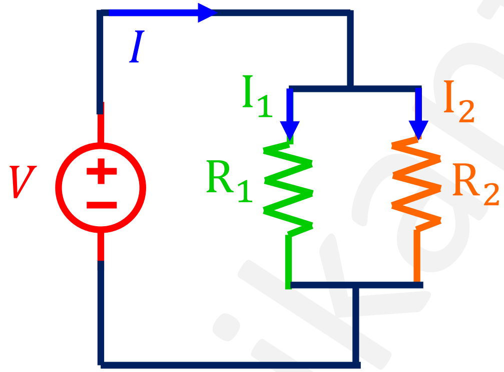
$$-V + V_{R_1} + V_{R_2} = 0$$

$$-I * R + I * R_1 + I * R_2 = 0$$

$$-R + R_1 + R_2 = 0$$

$$R_{eq} = R_1 + R_2$$

Resisters are in Parallel

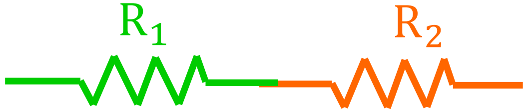


$$I = \frac{V}{R} \quad I_1 = \frac{V}{R_1} \quad I_2 = \frac{V}{R_2}$$

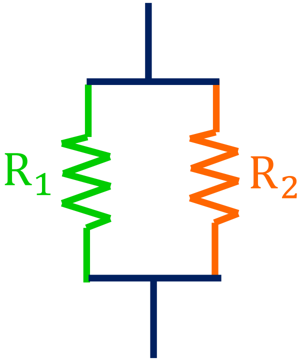
$$I = I_1 + I_2 \quad \frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2}$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$



$$R_{eq} = R_1 + R_2$$



$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

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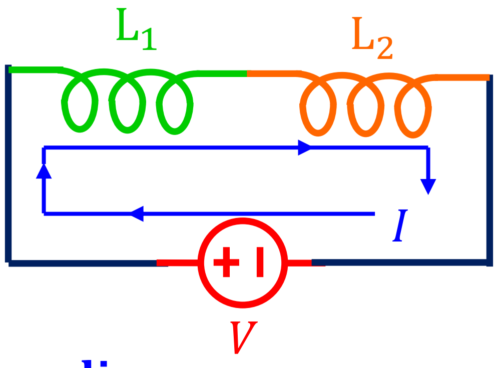
Equivalent Values of Series, Parallel R, L & C Networks

Inductors are in Series



$$V = \frac{di}{dt} * L$$

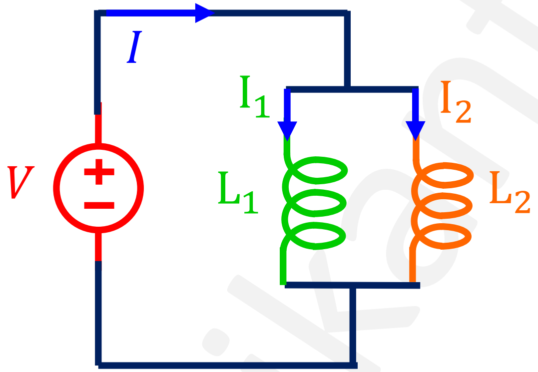
$$V_{L_1} = \frac{di}{dt} * L_1 \quad V_{L_2} = \frac{di}{dt} * L_2$$

$$-V + V_{L_1} + V_{L_2} = 0$$

$$-\frac{di}{dt} * L + \frac{di}{dt} * L_1 + \frac{di}{dt} * L_2 = 0$$

$$-L + L_1 + L_2 = 0 \qquad L_{eq} = L_1 + L_2$$

Inductors are in Parallel



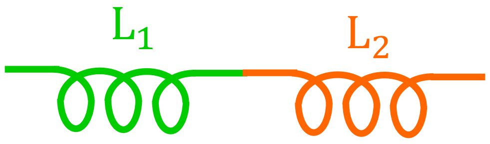
$$I = \frac{1}{L}\int v\,dt$$

$$I_1 = \frac{1}{L_1}\int v\,dt \qquad I_2 = \frac{1}{L_2}\int v\,dt$$

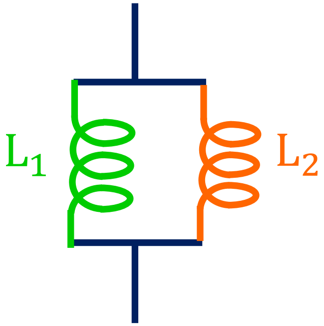
$$I = I_1 + I_2$$

$$\frac{1}{L}\int v\,dt = \frac{1}{L_1}\int v\,dt + \frac{1}{L_2}\int v\,dt$$

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$$



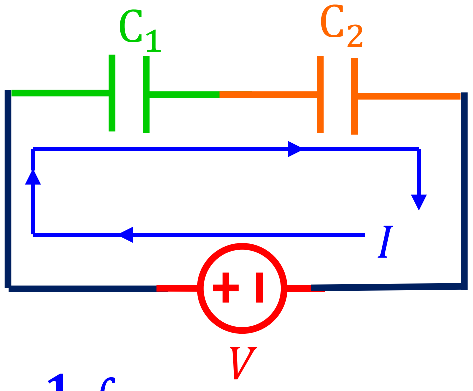
$$L_{eq} = L_1 + L_2$$



$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$$

Equivalent Values of Series, Parallel R, L & C Networks

Capacitors are in Series



$$V = \frac{1}{C}\int \mathbf{i\,dt}$$

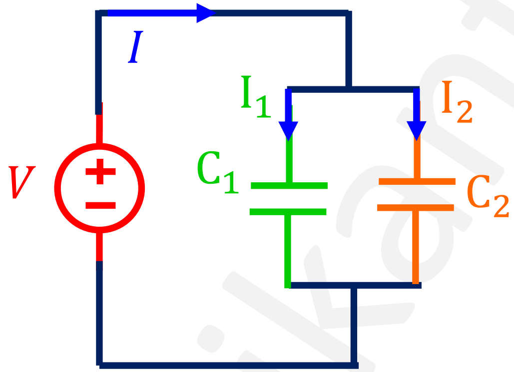
$$V_{C_1} = \frac{1}{C_1}\int \mathbf{i\,dt} \qquad V_{C_2} = \frac{1}{C_2}\int \mathbf{i\,dt}$$

$$-V + V_{R_1} + V_{R_2} = 0$$

$$-\frac{1}{C}\int \mathbf{i\,dt} + \frac{1}{C_1}\int \mathbf{i\,dt} + \frac{1}{C_2}\int \mathbf{i\,dt} = 0$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

Capacitors are in Parallel



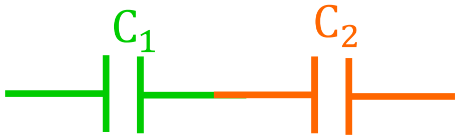
$$I = \mathbf{C}\frac{\mathbf{dv}}{\mathbf{dt}}$$

$$I_1 = C_1\frac{\mathbf{dv}}{\mathbf{dt}} \qquad I_2 = C_2\frac{\mathbf{dv}}{\mathbf{dt}}$$

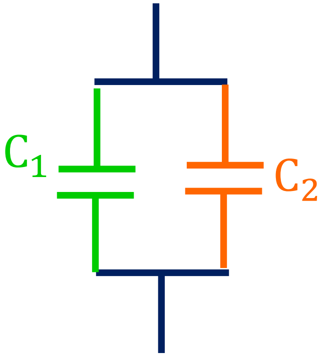
$$I = I_1 + I_2$$

$$\mathbf{C}\frac{\mathbf{dv}}{\mathbf{dt}} = C_1\frac{\mathbf{dv}}{\mathbf{dt}} + C_2\frac{\mathbf{dv}}{\mathbf{dt}}$$

$$C_{eq} = C_1 + C_2$$



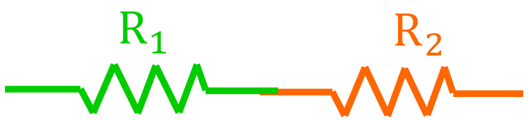
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$



$$C_{eq} = C_1 + C_2$$

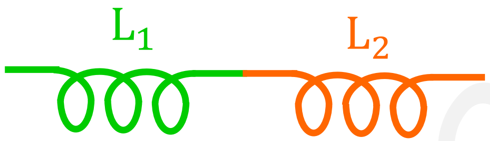
Equivalent Values of Series, Parallel R, L & C Networks

R



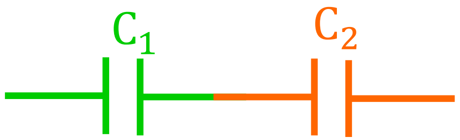
$$R_{eq} = R_1 + R_2$$

L

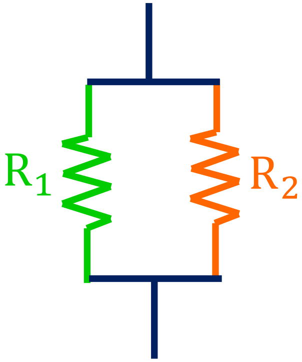


$$L_{eq} = L_1 + L_2$$

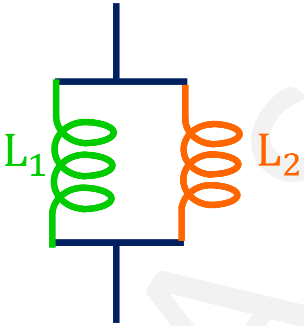
C



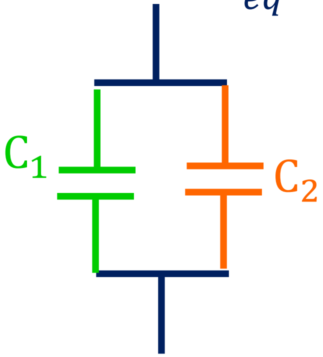
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$



$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$



$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$$



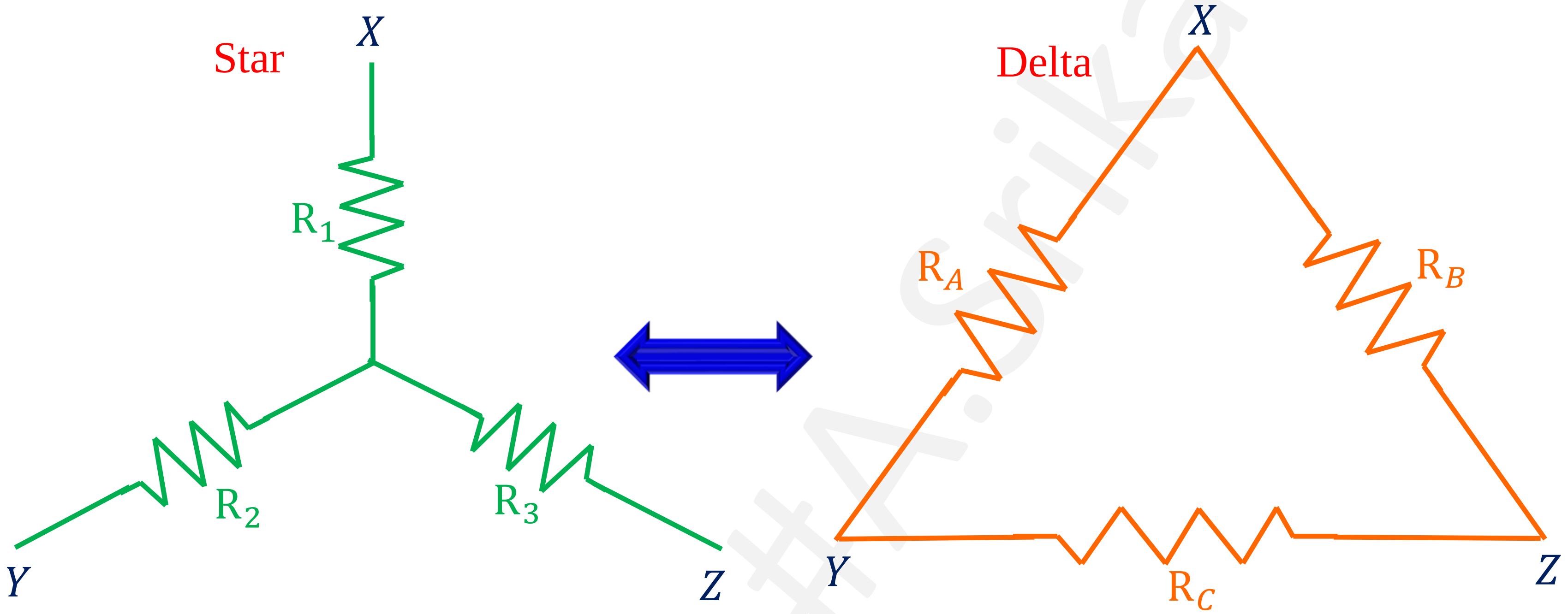
$$C_{eq} = C_1 + C_2$$

Resistors in series carry the same current

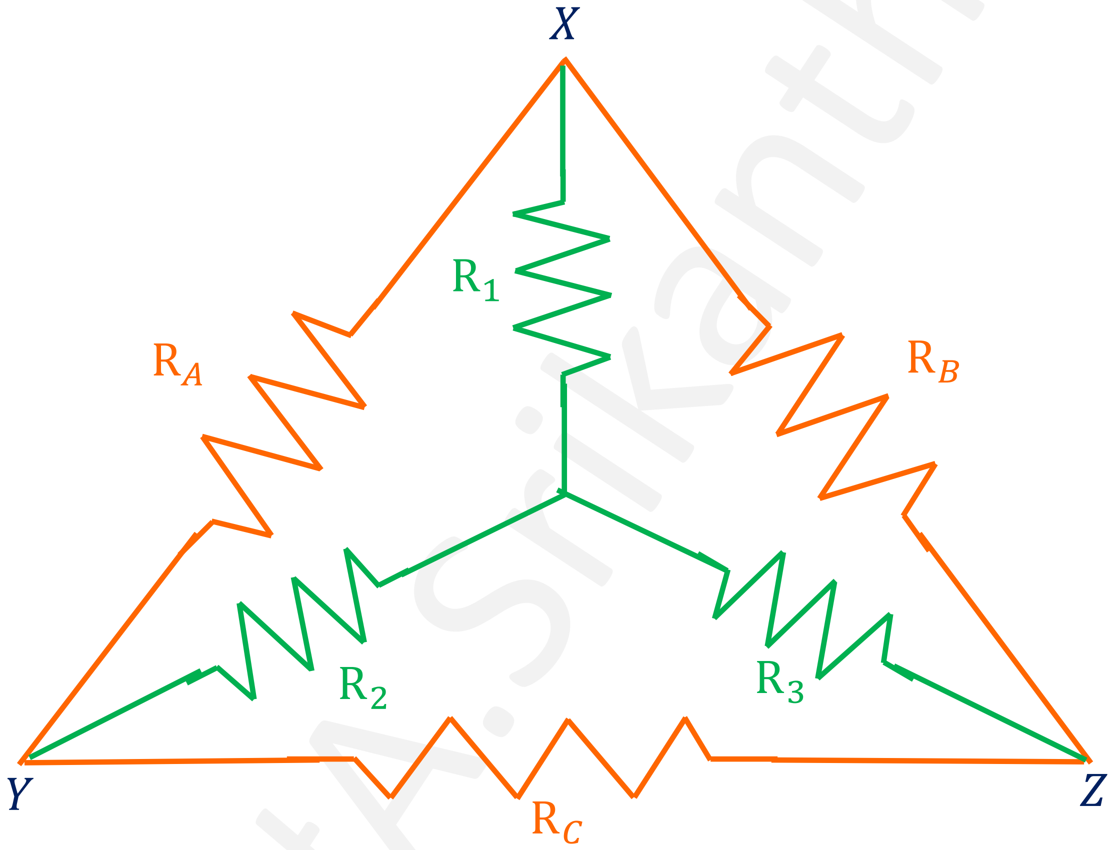
Resistors in parallel have the same voltage across them

Star - Delta Transformation Technique

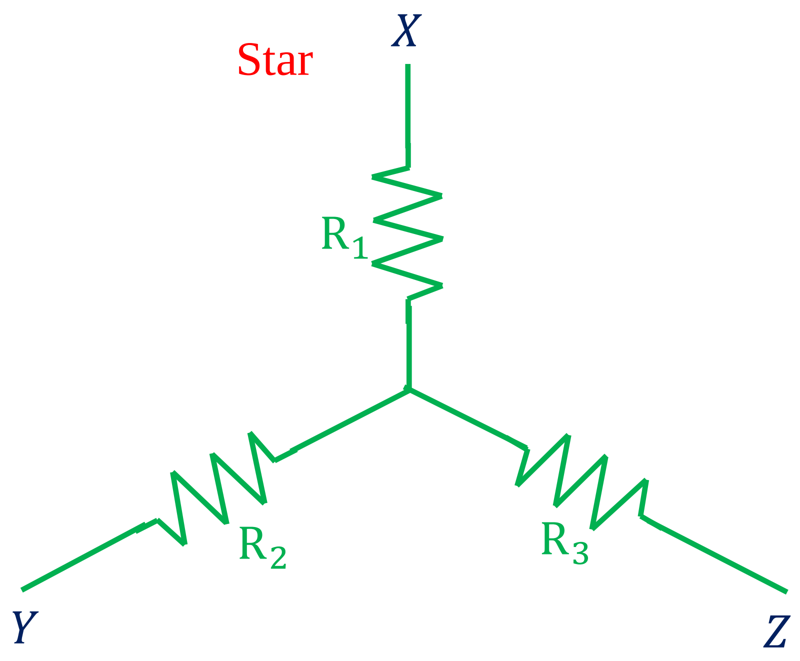
Delta-star and star-delta transformation is quite useful to simplify certain network problems. If three elements meet at a node then the three elements are said to be in star connection, whereas if three elements form closed path then they are said to be in delta connection.

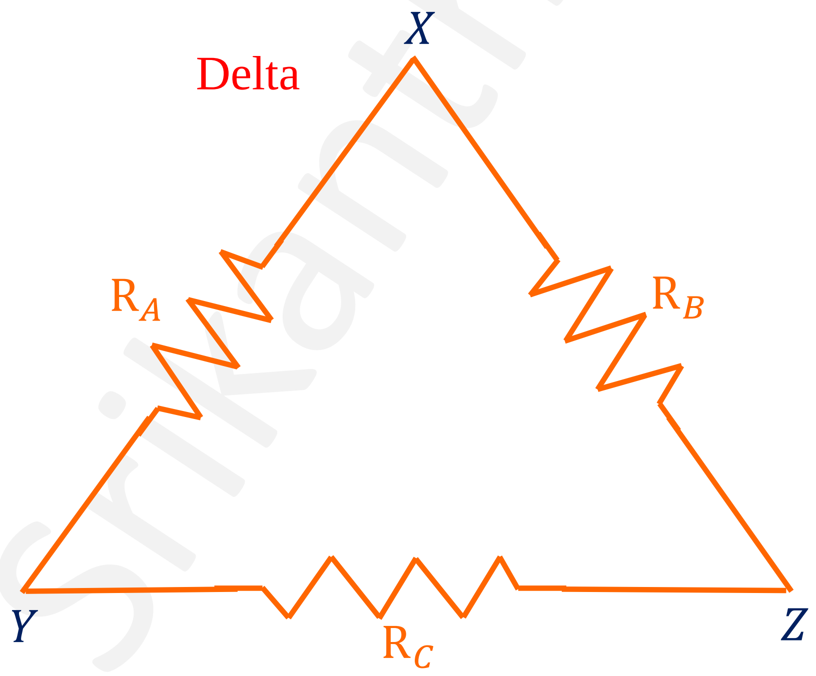


Star - Delta Transformation Technique



Star - Delta Transformation Technique



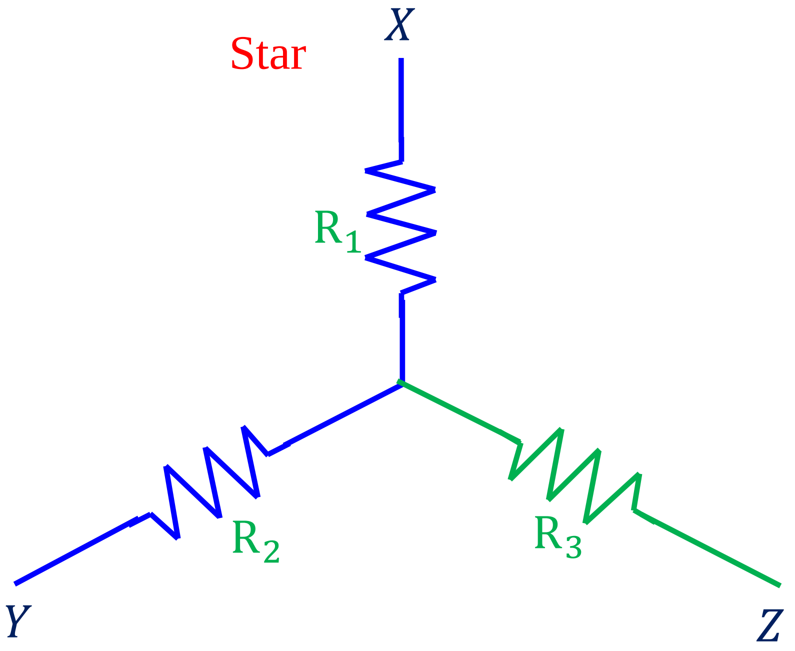


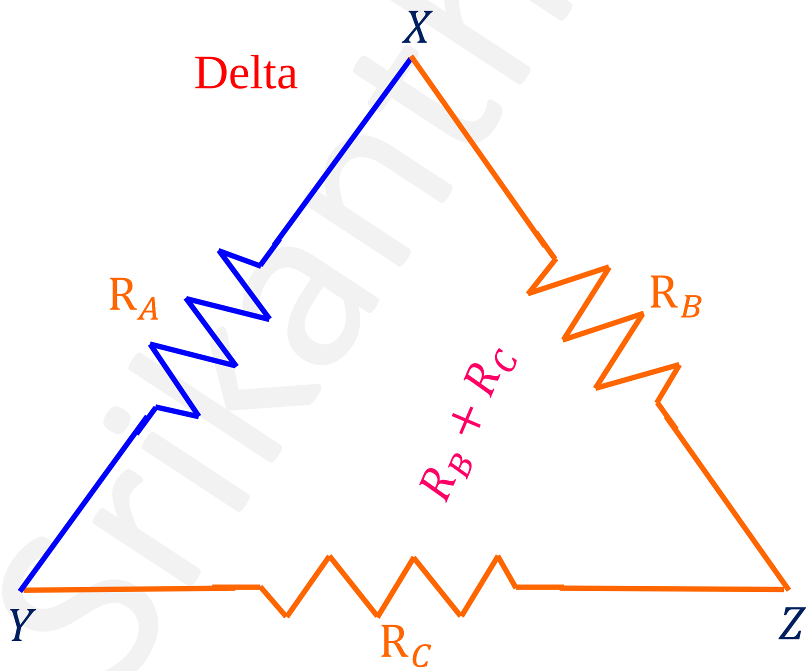
The equivalent resistance between any Two terminals

for Star
At X-Y

for Delta
At X-Y

Star - Delta Transformation Technique





$$S//P = \frac{R_S * R_P}{R_S + R_P}$$

The equivalent resistance between any Two terminals

for Star

At X-Y $\quad R_1 + R_2$

At Y-Z $\quad R_2 + R_3$

At X-Z $\quad R_1 + R_3$

for Delta

At X-Y $\quad \frac{R_A(R_B + R_C)}{R_A + R_B + R_C}$

At Y-Z $\quad \frac{R_C(R_A + R_B)}{R_A + R_B + R_C}$

At X-Z $\quad \frac{R_B(R_A + R_C)}{R_A + R_B + R_C}$

Star - Delta Transformation Technique

for Star

At X-Y $R_1 + R_2$

At Y-Z $R_2 + R_3$

At X-Z $R_1 + R_3$

for Delta

At X-Y $\frac{R_A(R_B+R_C)}{R_A+R_B+R_C}$

At Y-Z $\frac{R_C(R_A+R_B)}{R_A+R_B+R_C}$

At X-Z $\frac{R_B(R_A+R_C)}{R_A+R_B+R_C}$

$$R_1 + R_2 = \frac{R_A(R_B + R_C)}{R_A + R_B + R_C} \quad (1)$$

$$R_2 + R_3 = \frac{R_C(R_A + R_B)}{R_A + R_B + R_C} \quad (2)$$

$$R_1 + R_3 = \frac{R_B(R_A + R_C)}{R_A + R_B + R_C} \quad (3)$$

Equation (1) – (2)

$$R_1 + R_2 - (R_2 + R_3) = \frac{R_A(R_B + R_C)}{R_A + R_B + R_C} - \frac{R_C(R_A + R_B)}{R_A + R_B + R_C}$$

$$R_1 - R_3 = \frac{R_A R_B + R_A R_C - R_C R_A - R_C R_B}{R_A + R_B + R_C} = \frac{R_A R_B - R_C R_B}{R_A + R_B + R_C}$$

$$R_1 - R_3 = \frac{R_A R_B - R_C R_B}{R_A + R_B + R_C} \quad (4)$$

Star - Delta Transformation Technique

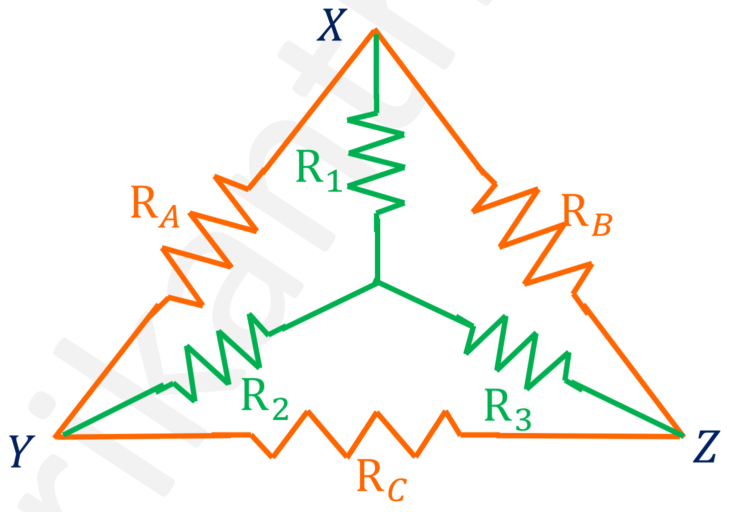
$$R_1 + R_3 = \frac{R_B(R_A + R_C)}{R_A + R_B + R_C} \dashrightarrow (3)$$

$$R_1 - R_3 = \frac{R_A R_B - R_C R_B}{R_A + R_B + R_C} \dashrightarrow (4)$$

Equation (3) + (4)

$$R_1 + R_3 + (R_1 - R_3) = \frac{R_B R_A + R_B R_C}{R_A + R_B + R_C} + \frac{R_A R_B - R_C R_B}{R_A + R_B + R_C}$$

$$2R_1 = \frac{2R_A R_B}{R_A + R_B + R_C} \qquad R_1 = \frac{R_A R_B}{R_A + R_B + R_C}$$



Delta to Star

$$R_1 = \frac{R_A R_B}{R_A + R_B + R_C}$$

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C}$$

$$R_3 = \frac{R_B R_C}{R_A + R_B + R_C}$$

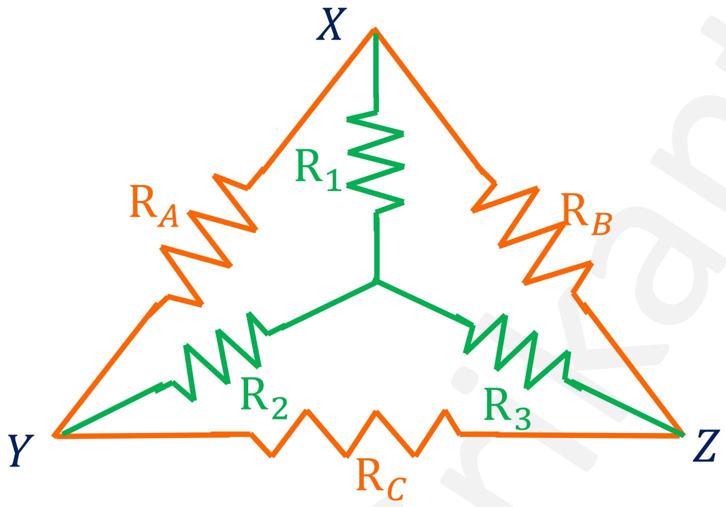
Star - Delta Transformation Technique

Delta to Star

$$R_1 = \frac{R_A R_B}{R_A + R_B + R_C}$$

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C}$$

$$R_3 = \frac{R_B R_C}{R_A + R_B + R_C}$$



Equation $(R_1) * (R_2)$ ------> (5)

Equation $(R_2) * (R_3)$ ------> (6)

Equation $(R_1) * (R_3)$ ------> (7)

$$R_1 * R_2 = \frac{(R_A R_B)(R_A R_C)}{(R_A + R_B + R_C)^2} \qquad R_2 * R_3 = \frac{(R_A R_C)(R_B R_C)}{(R_A + R_B + R_C)^2} \qquad R_1 * R_3 = \frac{(R_A R_B)(R_B R_C)}{(R_A + R_B + R_C)^2}$$

Equation (5) + (6) + (7)

$$R_1 * R_2 + R_2 * R_3 + R_1 * R_3 = \frac{(R_A R_B)(R_A R_C)}{(R_A + R_B + R_C)^2} + \frac{(R_A R_C)(R_B R_C)}{(R_A + R_B + R_C)^2} + \frac{(R_A R_B)(R_B R_C)}{(R_A + R_B + R_C)^2}$$

Star - Delta Transformation Technique

$$R_1 * R_2 + R_2 * R_3 + R_1 * R_3 = \frac{(R_A R_B)(R_A R_C)}{(R_A + R_B + R_C)^2} + \frac{(R_A R_C)(R_B R_C)}{(R_A + R_B + R_C)^2} + \frac{(R_A R_B)(R_B R_C)}{(R_A + R_B + R_C)^2}$$

$$R_1 * R_2 + R_2 * R_3 + R_1 * R_3 = \frac{(R_A R_B)(R_A R_C) + (R_A R_C)(R_B R_C) + (R_A R_B)(R_B R_C)}{(R_A + R_B + R_C)^2}$$

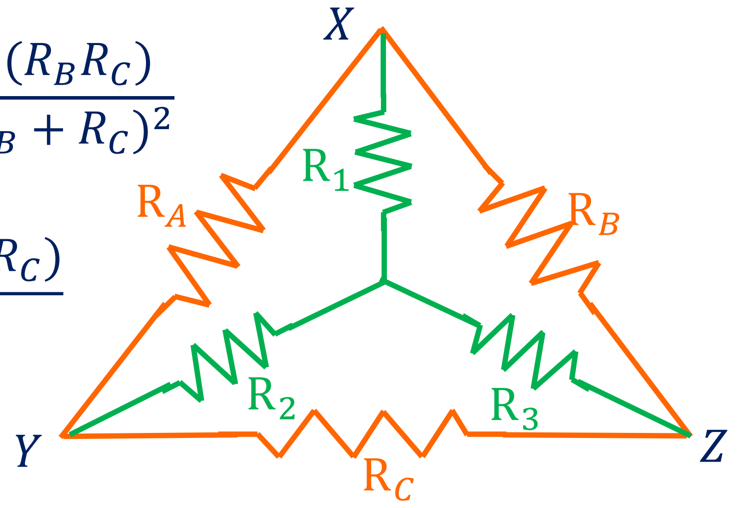
$$R_1 * R_2 + R_2 * R_3 + R_1 * R_3 = \frac{(R_A R_B R_C)(R_A + R_B + R_C)}{(R_A + R_B + R_C)^2}$$

$$R_1 * R_2 + R_2 * R_3 + R_1 * R_3 = \frac{R_A * R_B * R_C}{R_A + R_B + R_C}$$

$$R_1 = \frac{R_A R_B}{R_A + R_B + R_C}$$

$$R_1 * R_2 + R_2 * R_3 + R_1 * R_3 = R_1 * R_C$$

$$R_C = \frac{R_1 * R_2 + R_2 * R_3 + R_1 * R_3}{R_1}$$



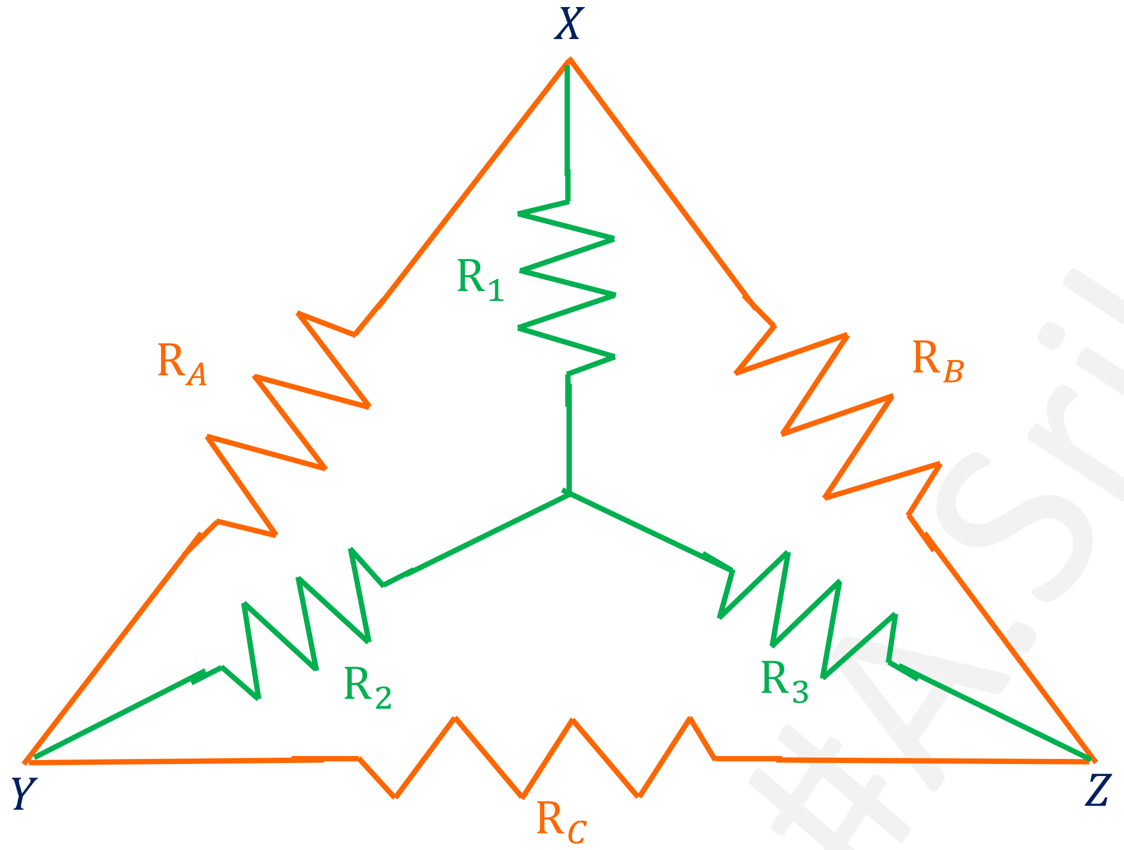
Star to Delta

$$R_C = \frac{R_1 * R_2 + R_2 * R_3 + R_1 * R_3}{R_1}$$

$$R_A = \frac{R_1 * R_2 + R_2 * R_3 + R_1 * R_3}{R_3}$$

$$R_B = \frac{R_1 * R_2 + R_2 * R_3 + R_1 * R_3}{R_2}$$

Star - Delta Transformation Technique



Delta to Star

$$R_1 = \frac{R_A R_B}{R_A + R_B + R_C}$$

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C}$$

$$R_3 = \frac{R_B R_C}{R_A + R_B + R_C}$$

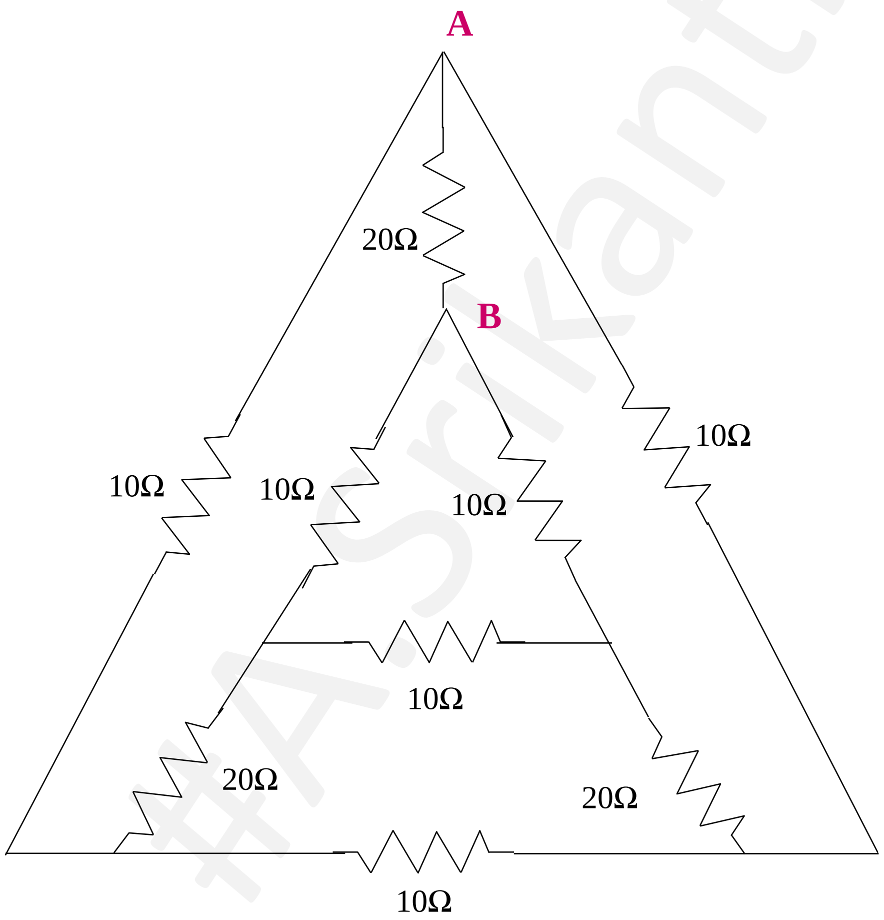
Star to Delta

$$R_C = \frac{R_1 * R_2 + R_2 * R_3 + R_1 * R_3}{R_1}$$

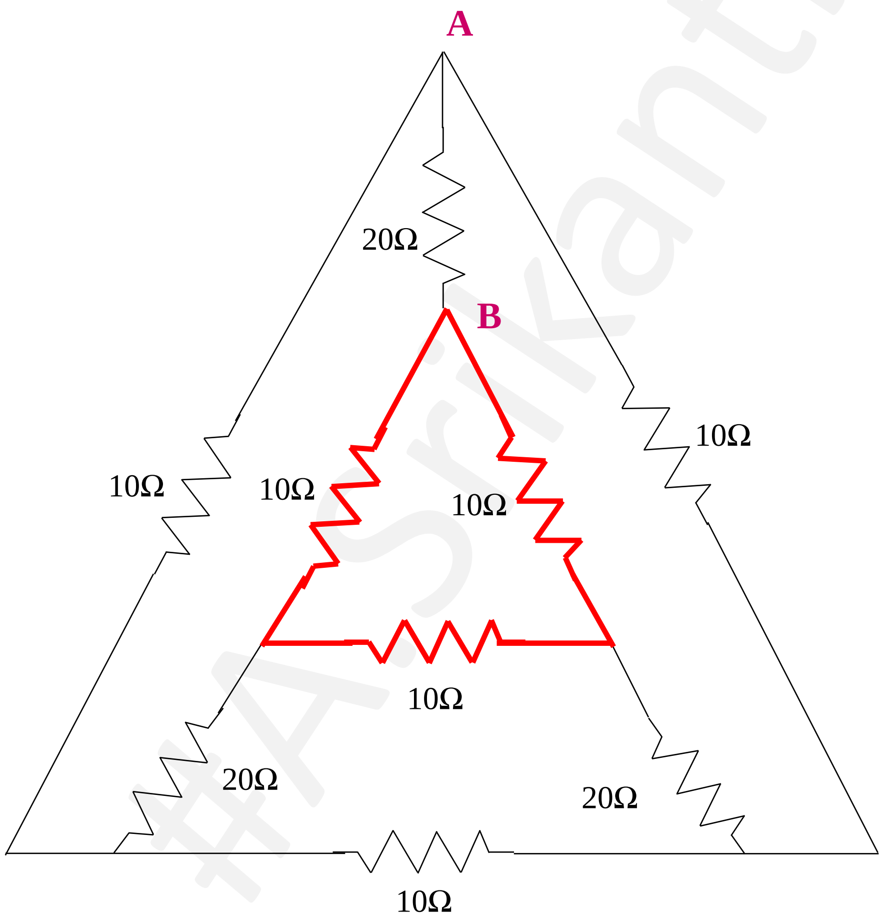
$$R_A = \frac{R_1 * R_2 + R_2 * R_3 + R_1 * R_3}{R_3}$$

$$R_B = \frac{R_1 * R_2 + R_2 * R_3 + R_1 * R_3}{R_2}$$

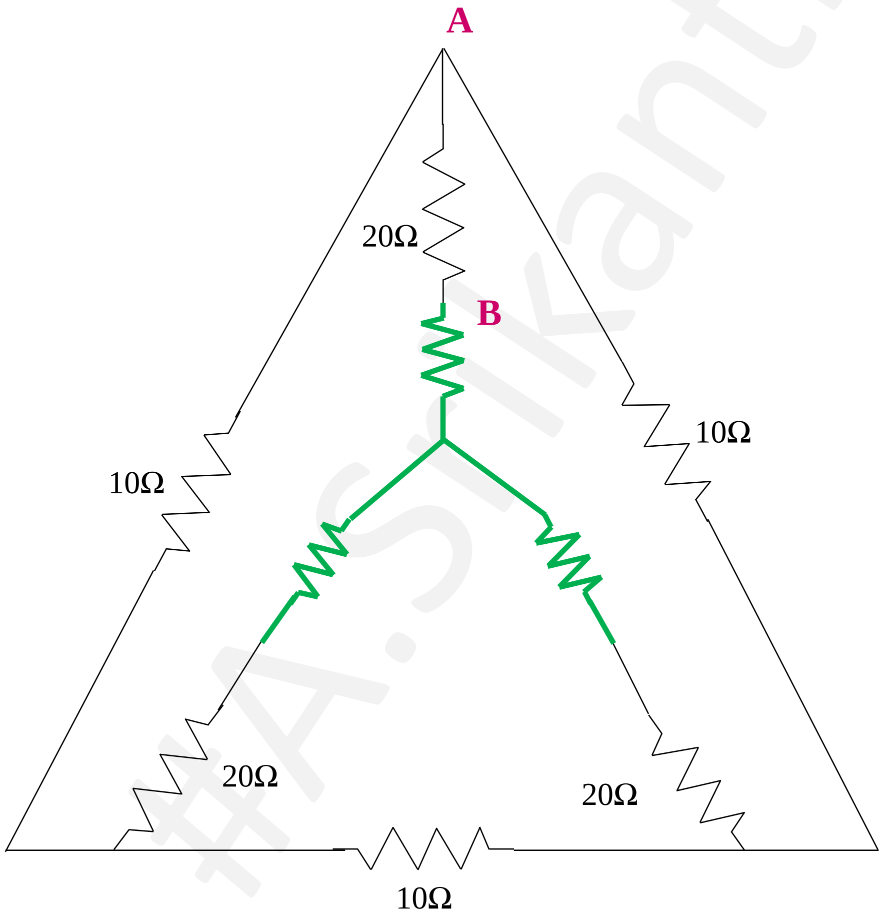
Find the equivalent resistance between points A and B.



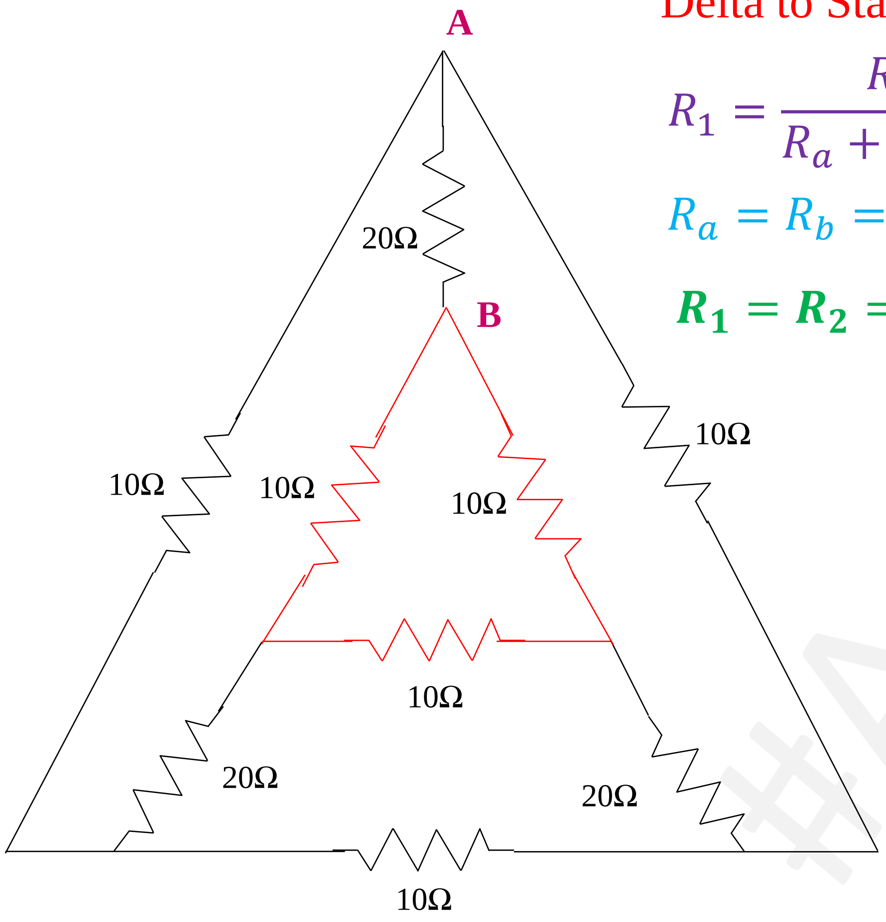
Find the equivalent resistance between points A and B.



Find the equivalent resistance between points A and B.



Find the equivalent resistance between points A and B.

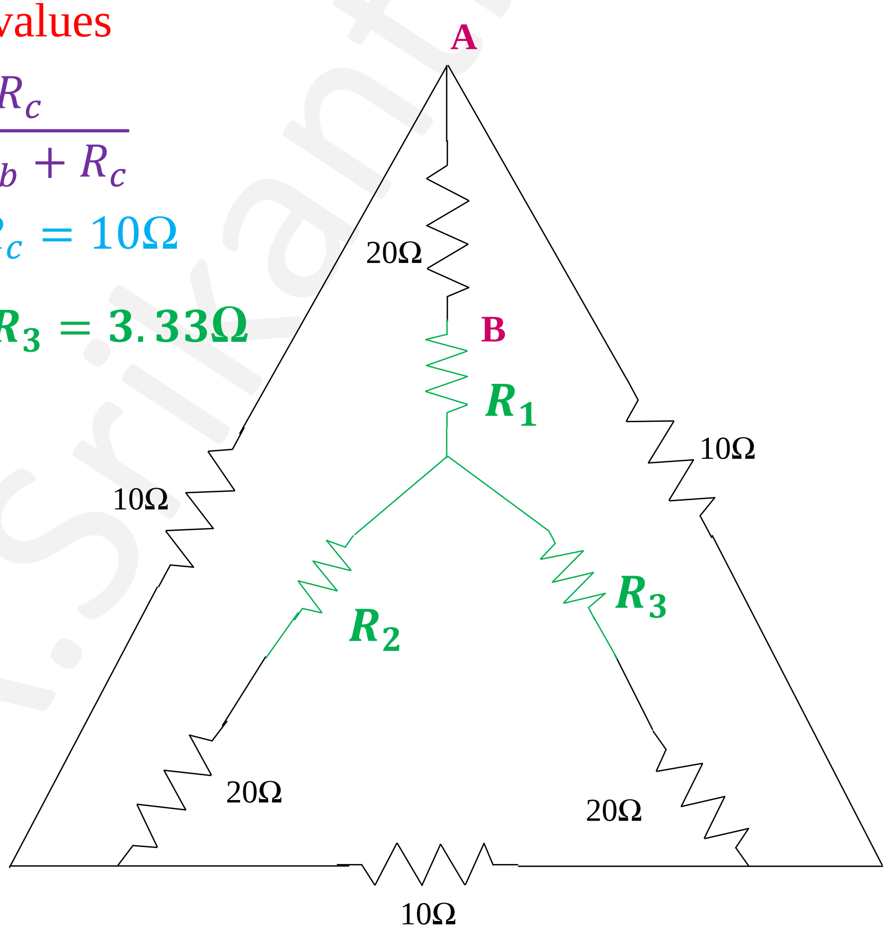


Delta to Star values

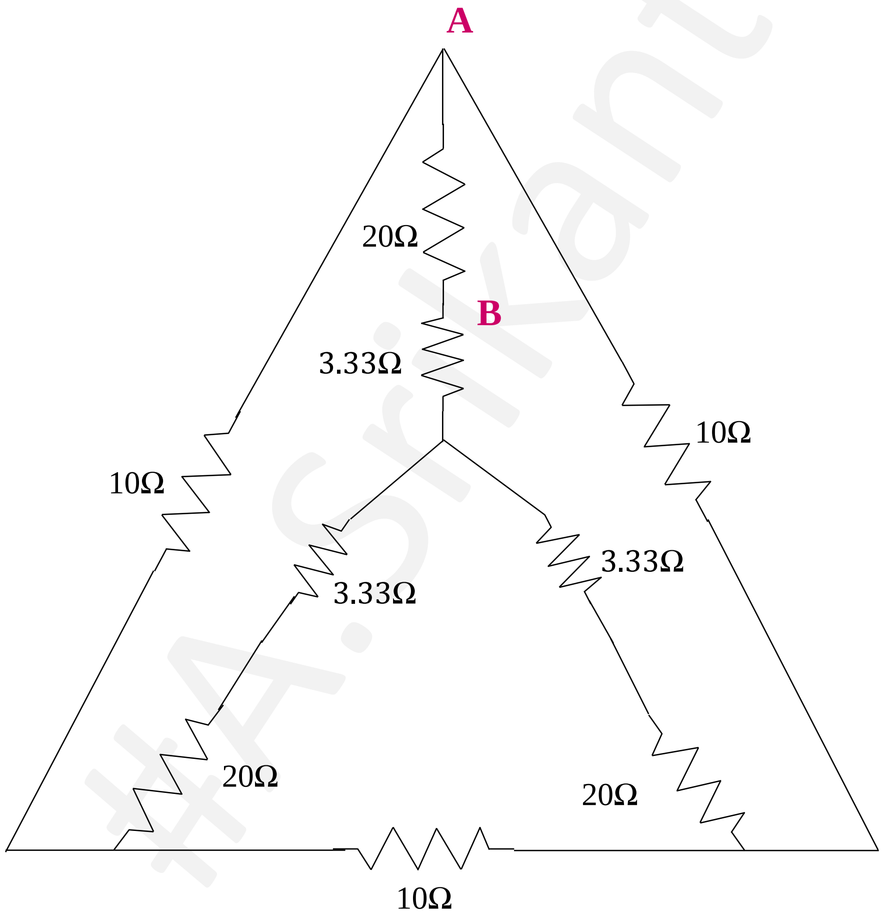
$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_a = R_b = R_c = 10\Omega$$

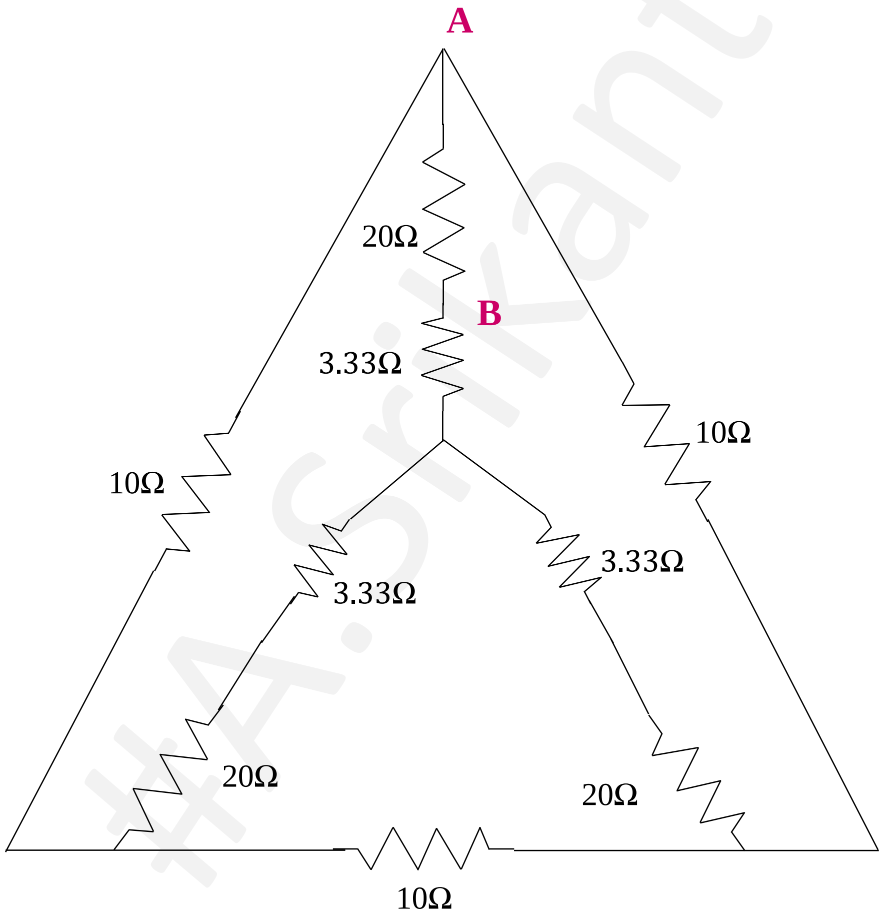
$$\boldsymbol{R_1 = R_2 = R_3 = 3.33\Omega}$$



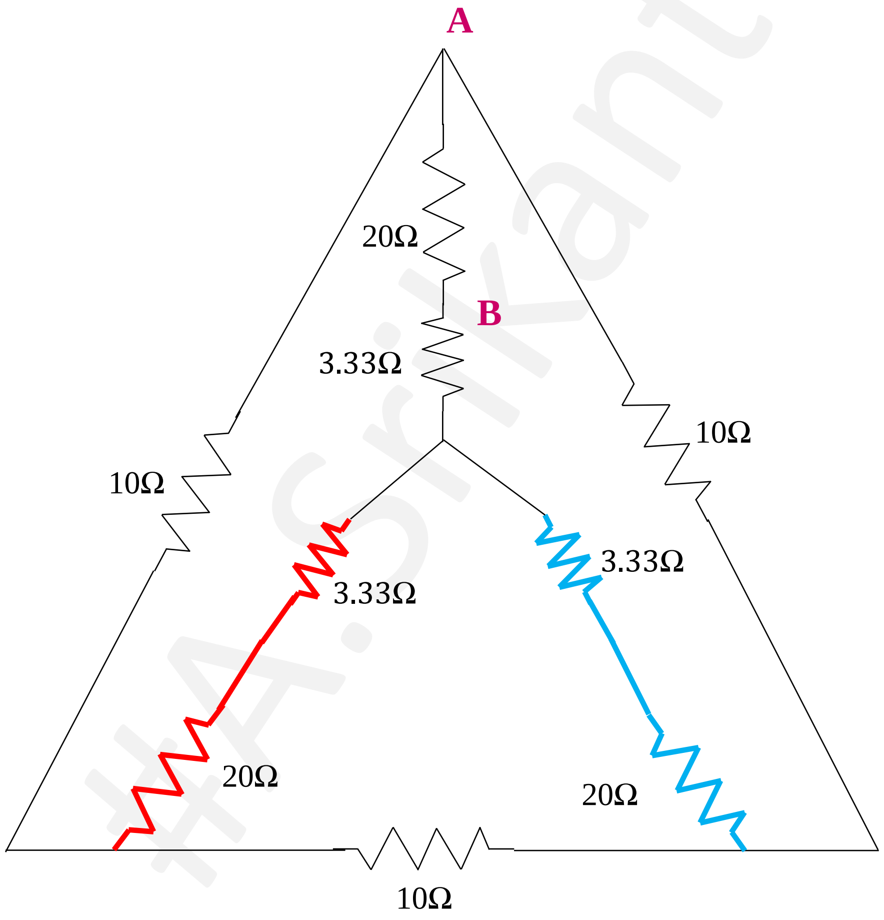
Find the equivalent resistance between points A and B.



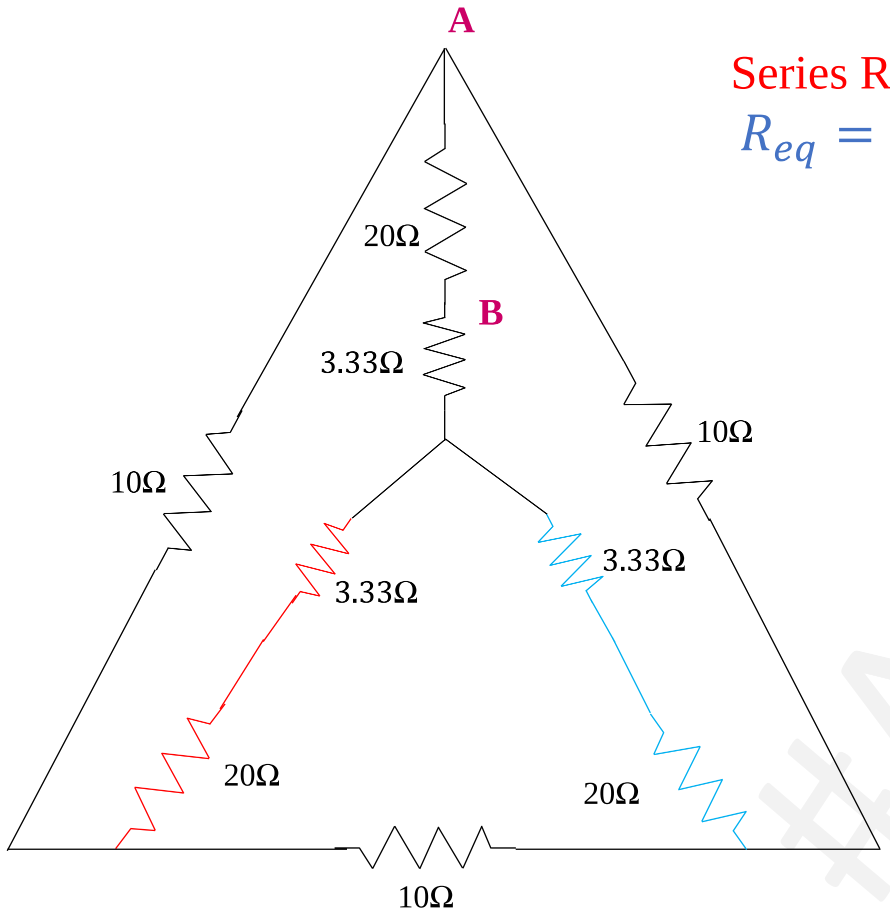
Find the equivalent resistance between points A and B.



Find the equivalent resistance between points A and B.

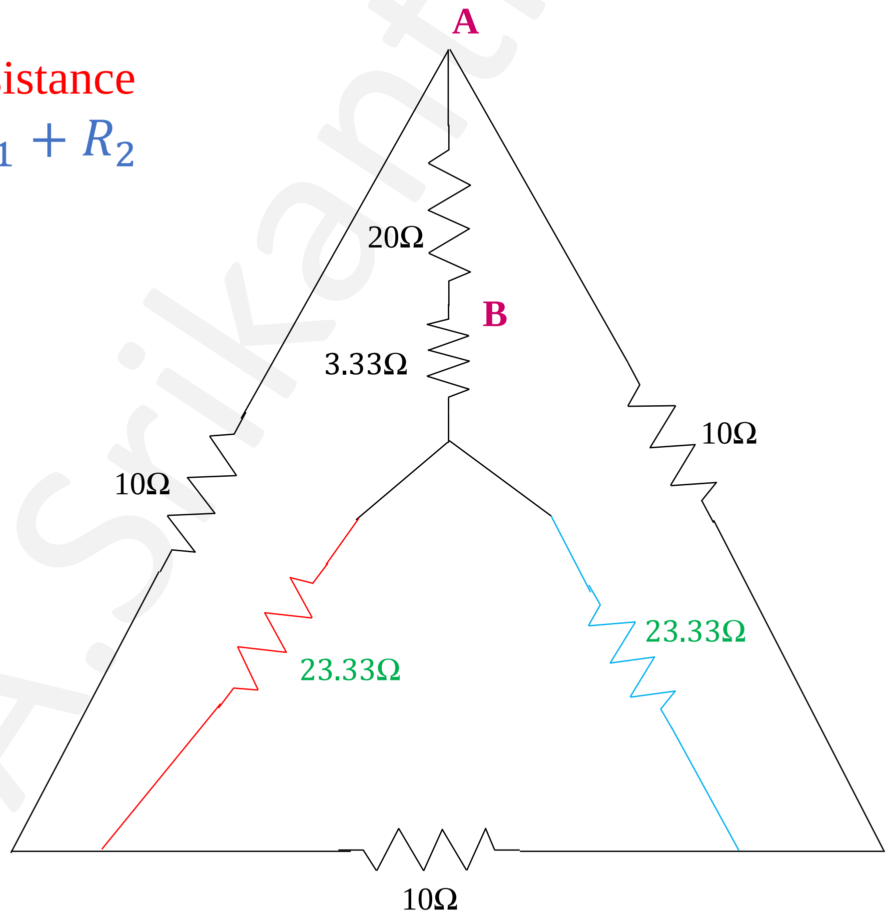


Find the equivalent resistance between points A and B.

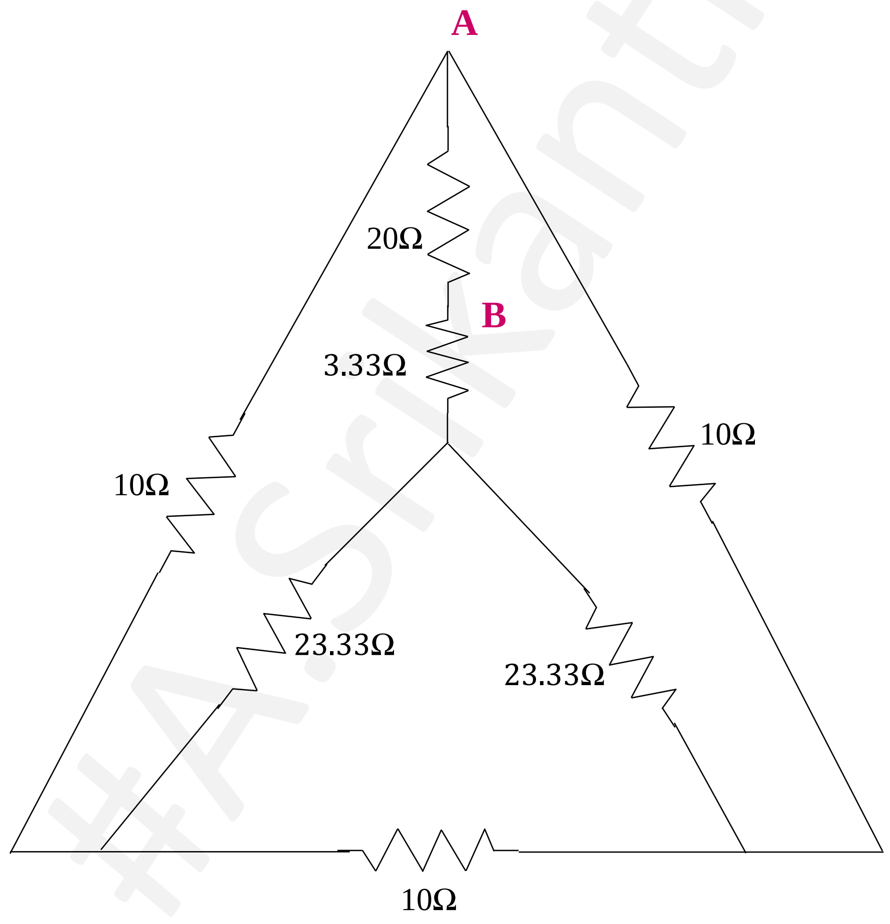


Series Resistance

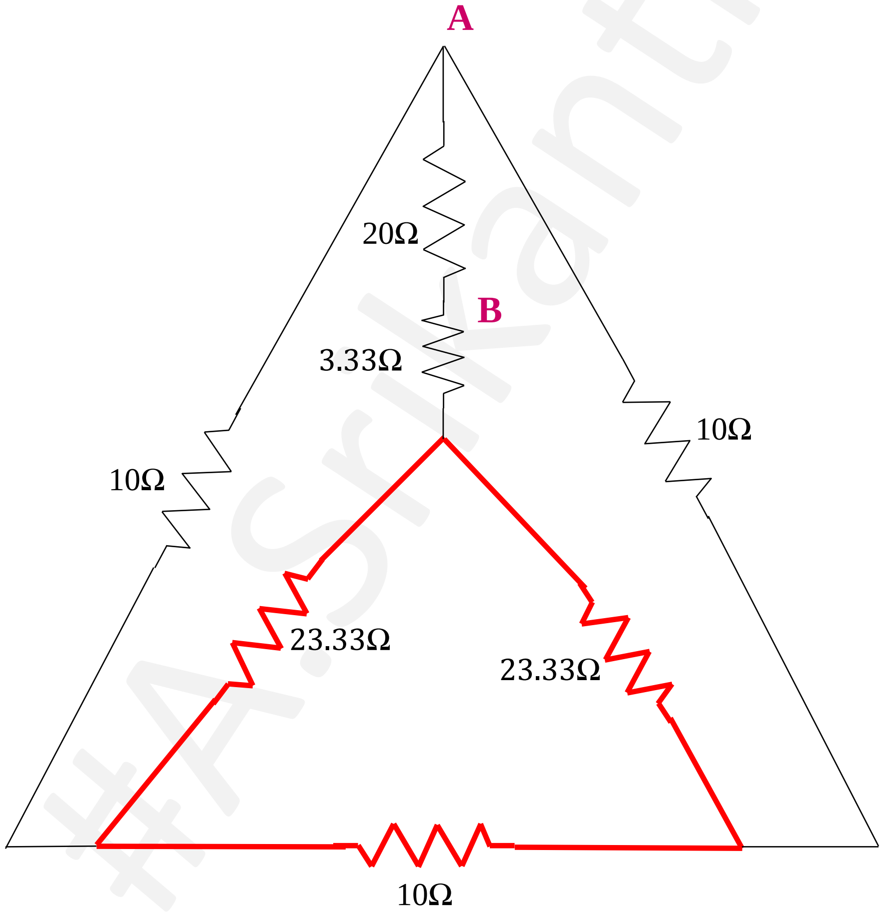
$$R_{eq} = R_1 + R_2$$



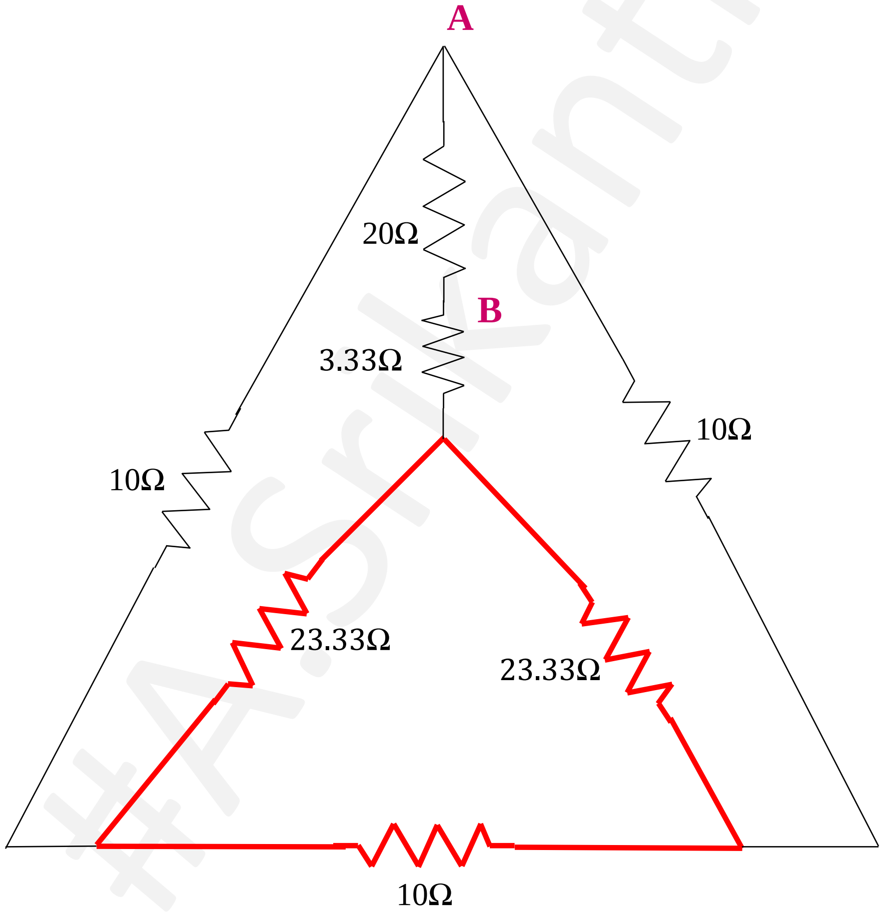
Find the equivalent resistance between points A and B.



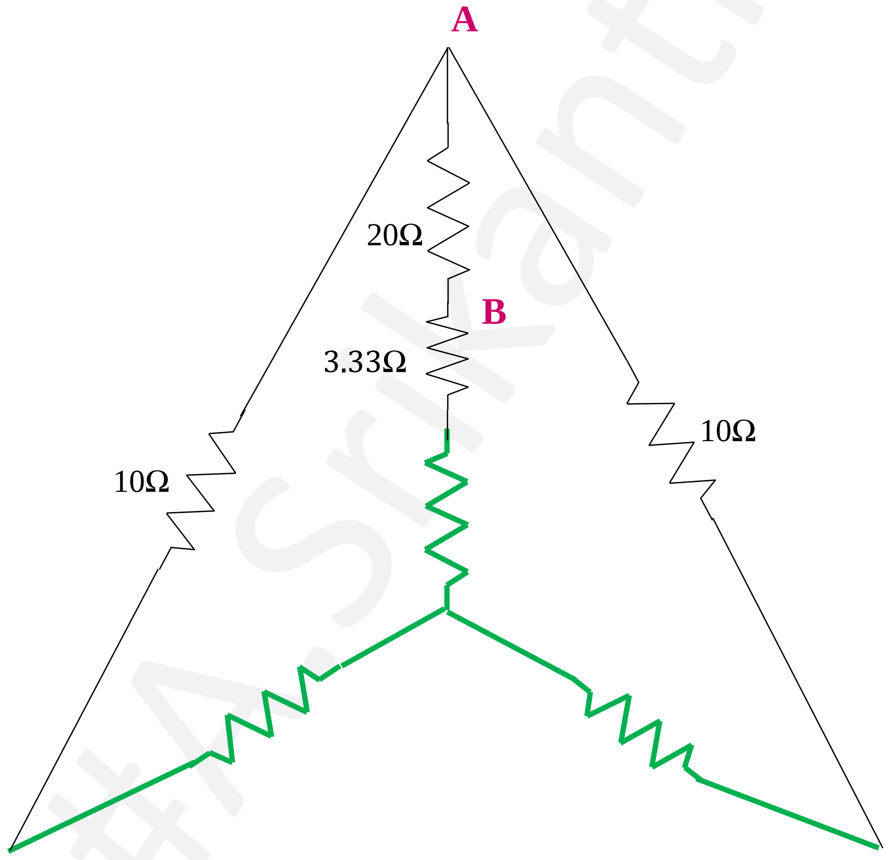
Find the equivalent resistance between points A and B.



Find the equivalent resistance between points A and B.



Find the equivalent resistance between points A and B.

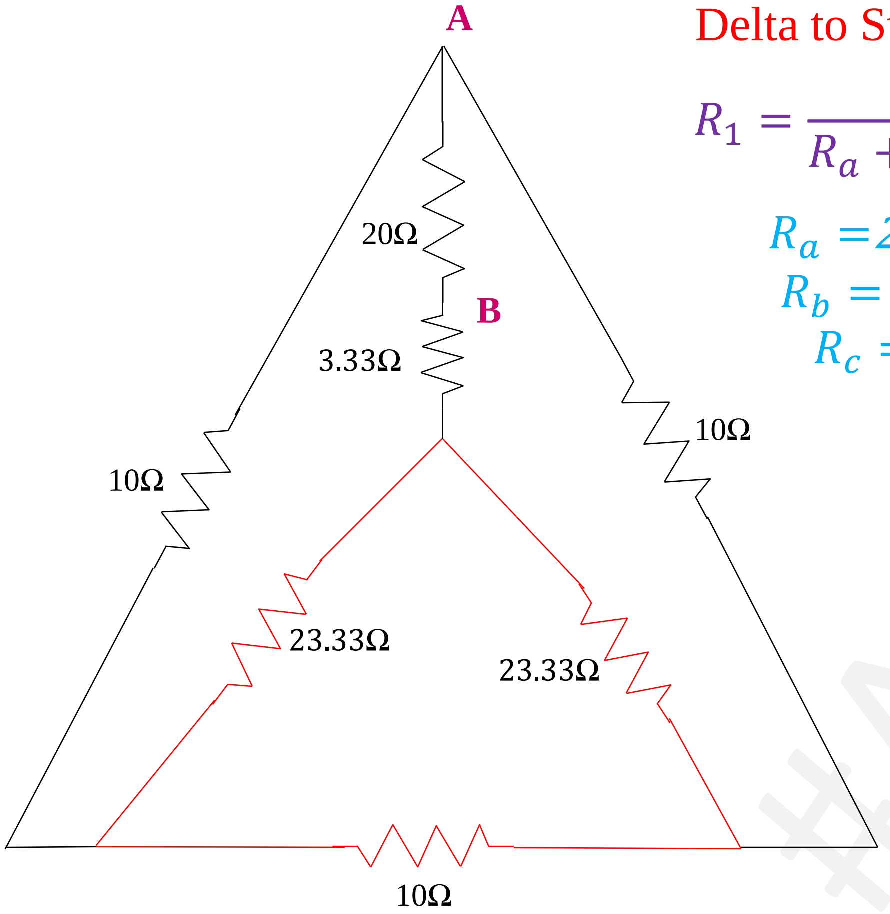


Find the equivalent resistance between points A and B.

$R_1 = 9.61\,\Omega$

$R_2 = 4.12\,\Omega$

$R_3 = 4.12\,\Omega$



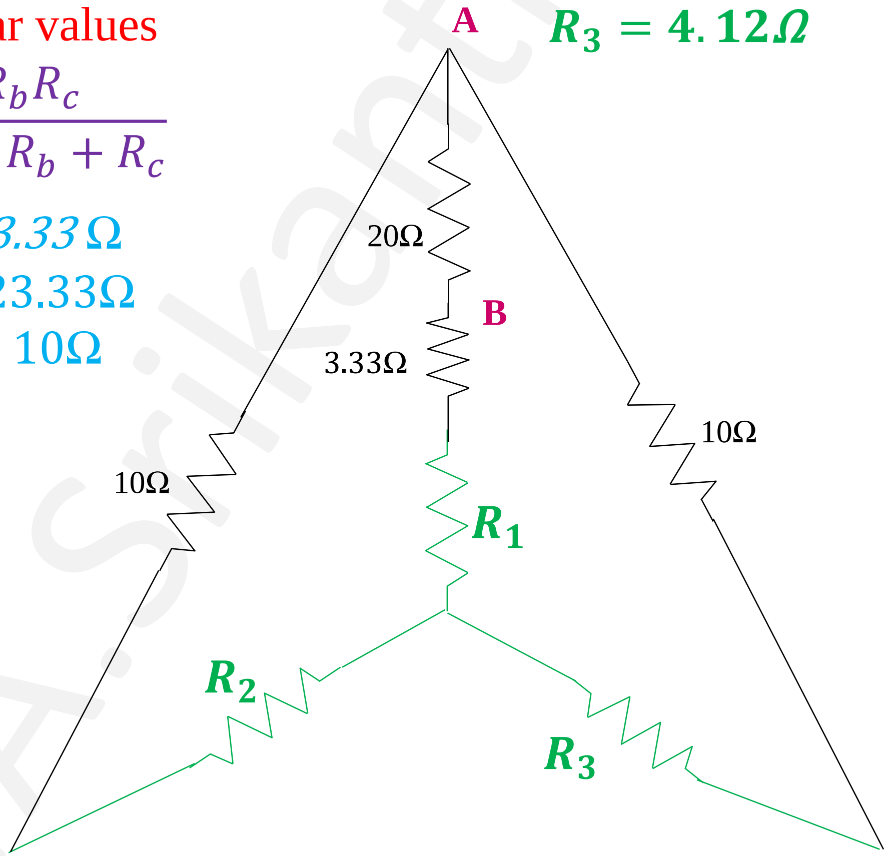
Delta to Star values

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

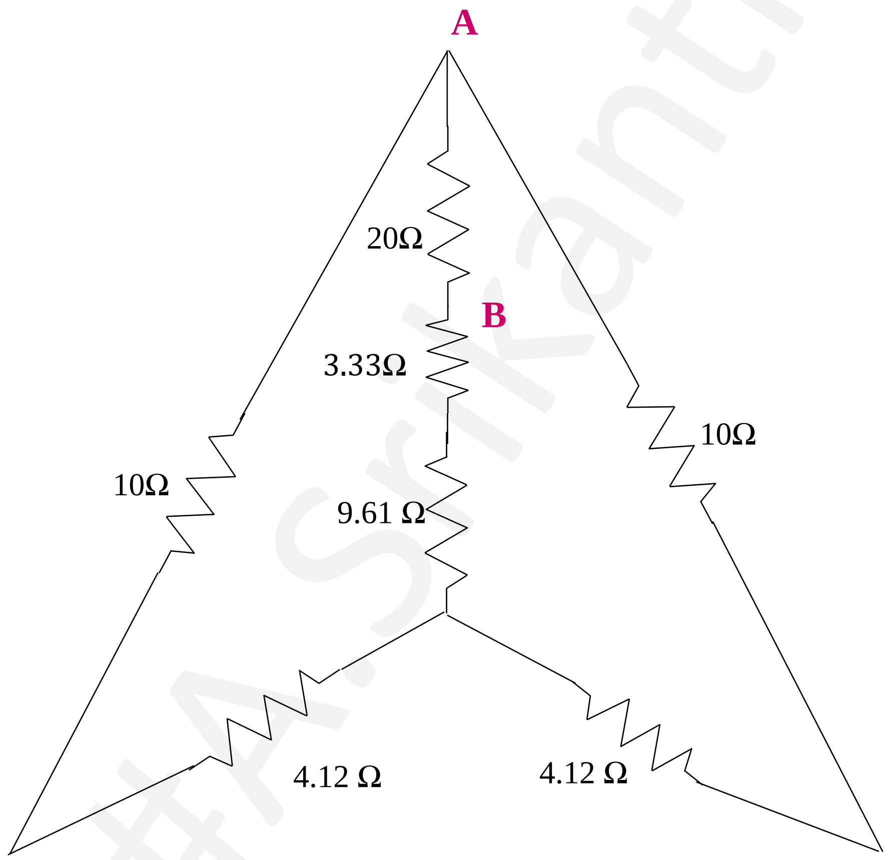
$R_a = 23.33\,\Omega$

$R_b = 23.33\Omega$

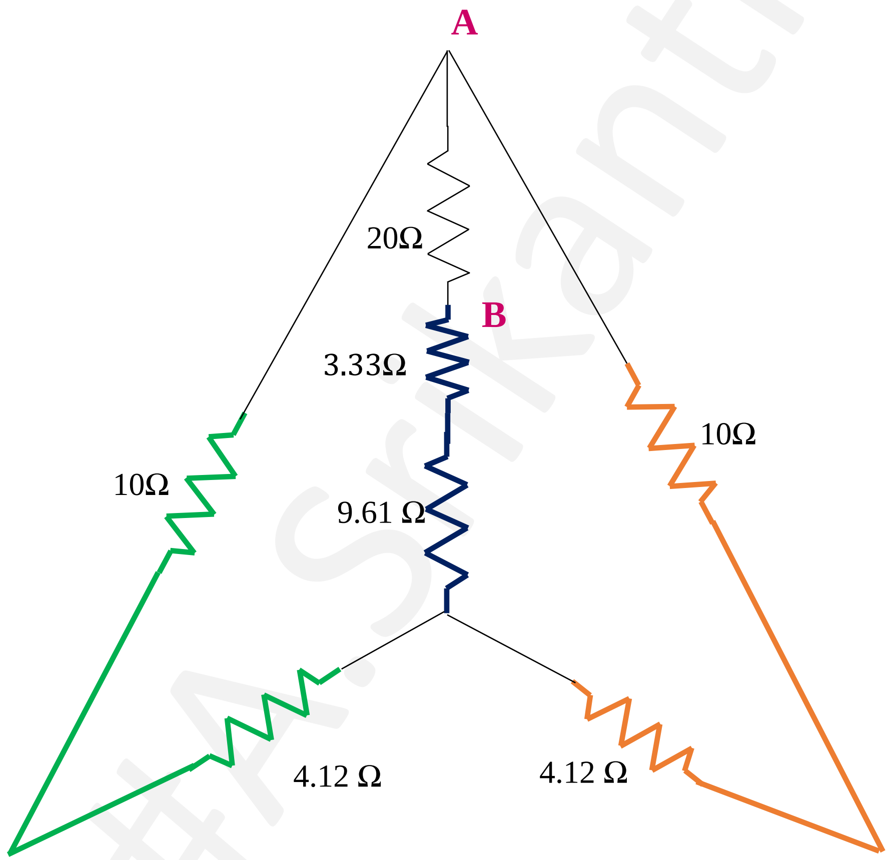
$R_c = 10\Omega$



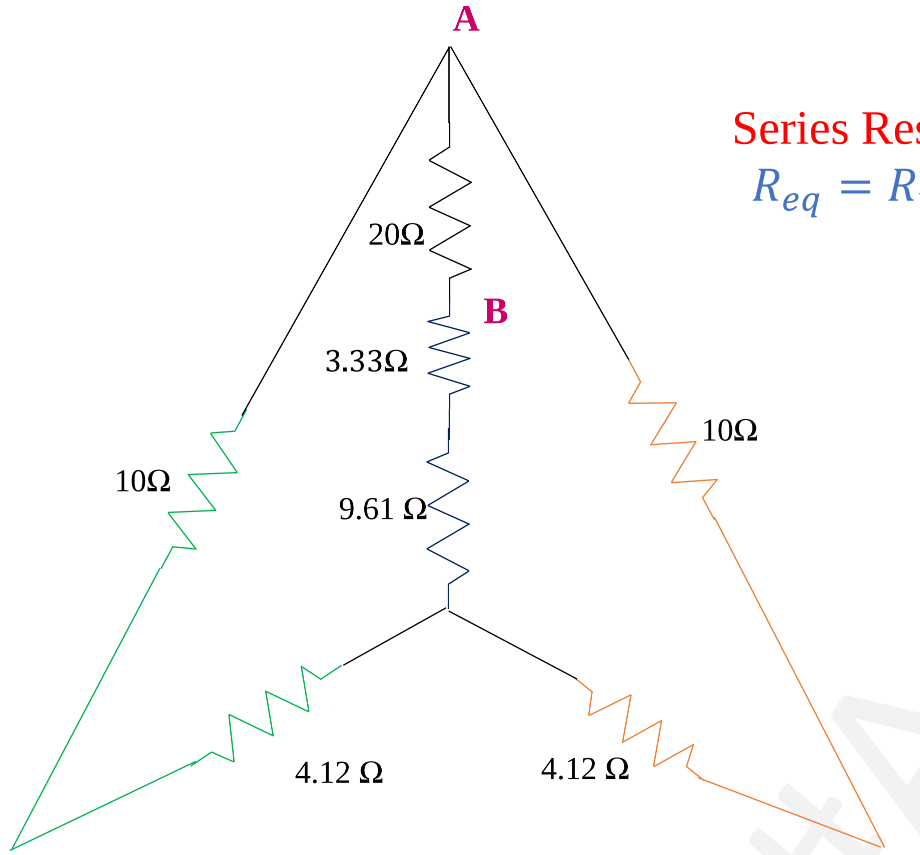
Find the equivalent resistance between points A and B.



Find the equivalent resistance between points A and B.

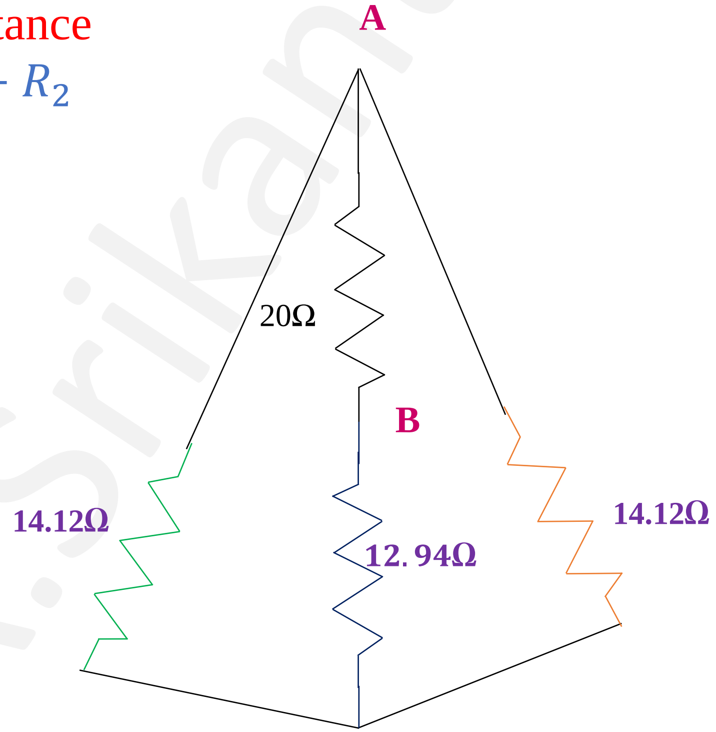


Find the equivalent resistance between points A and B.



Series Resistance

$$R_{eq} = R_1 + R_2$$



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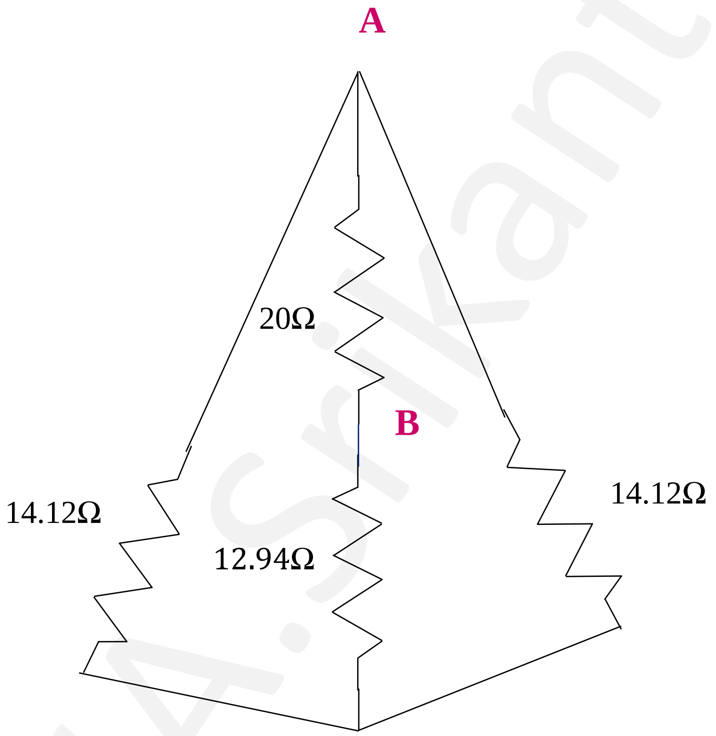
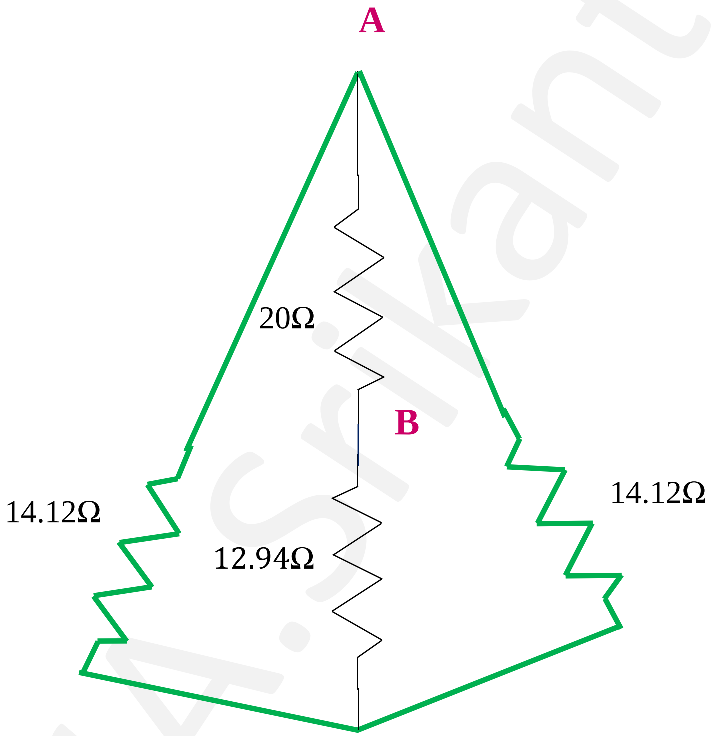
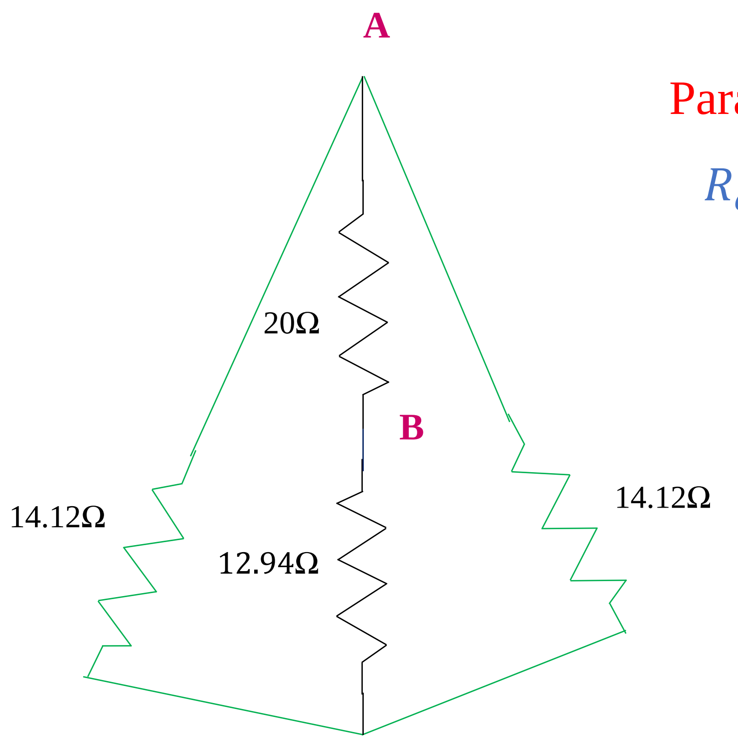
Find the equivalent resistance between points A and B.



Find the equivalent resistance between points A and B.

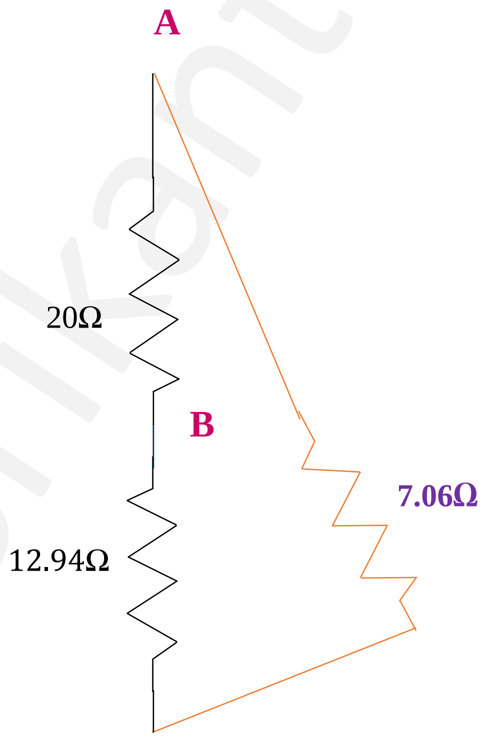


Find the equivalent resistance between points A and B.

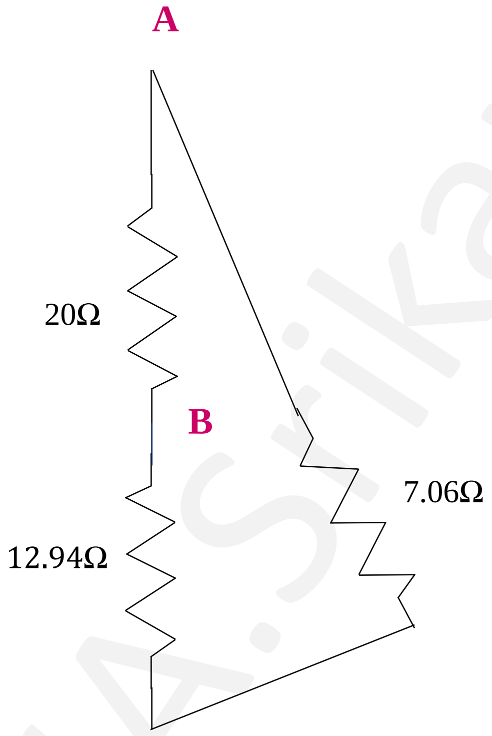


Parallel Resistance

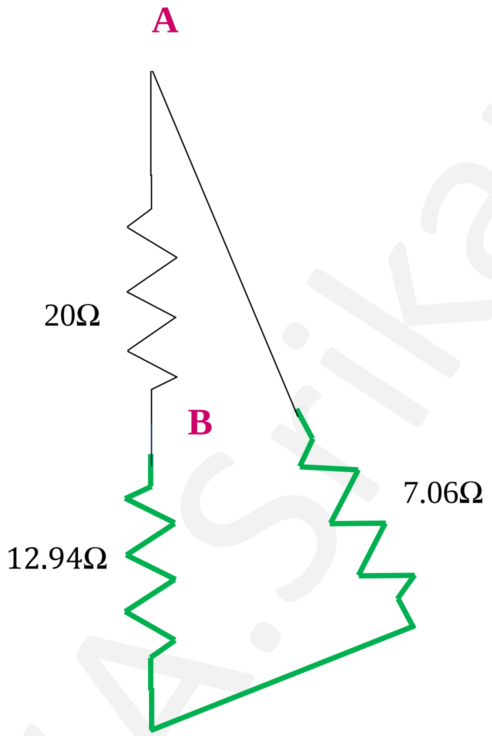
$$R_{eq} = \frac{R_1 * R_2}{R_1 + R_2}$$



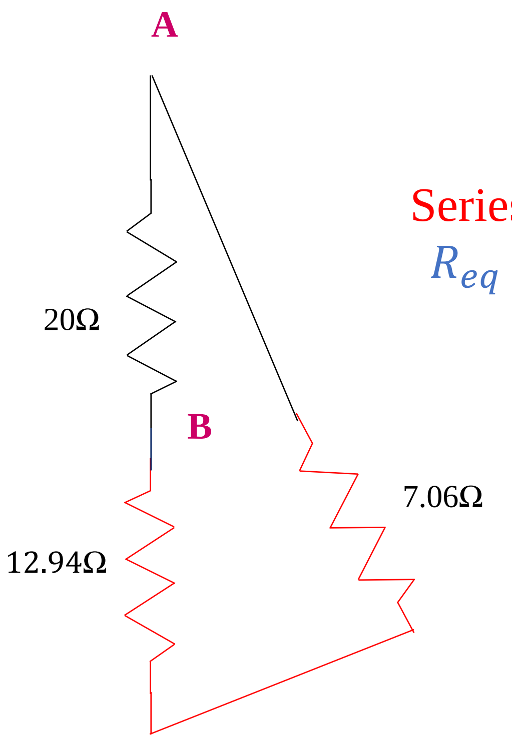
Find the equivalent resistance between points A and B.



Find the equivalent resistance between points A and B.

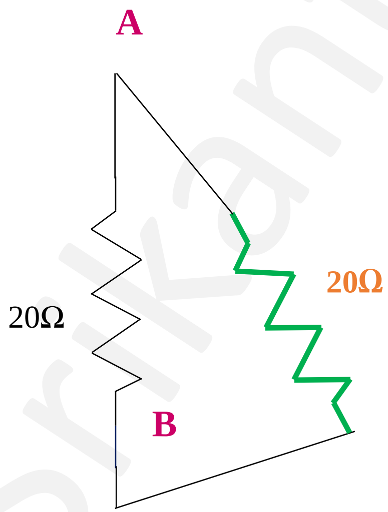


Find the equivalent resistance between points A and B.

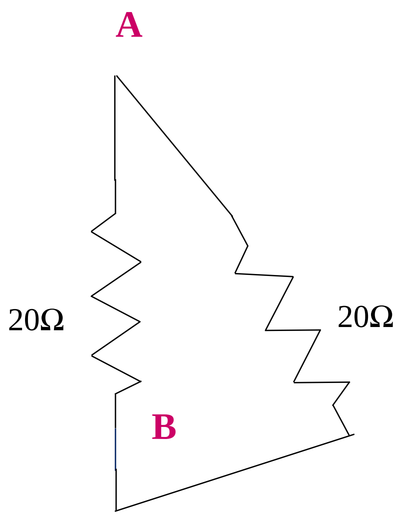


Series Resistance

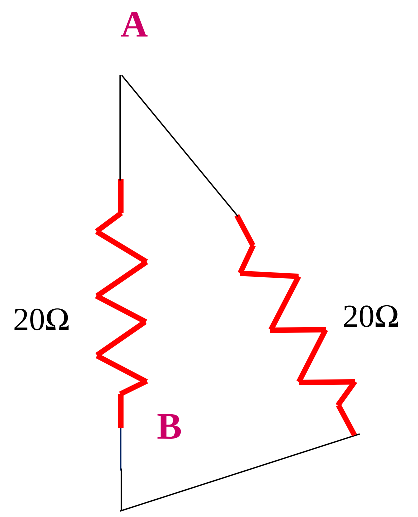
$$R_{eq} = R_1 + R_2$$



Find the equivalent resistance between points A and B.

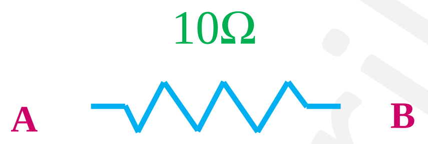


Find the equivalent resistance between points A and B.

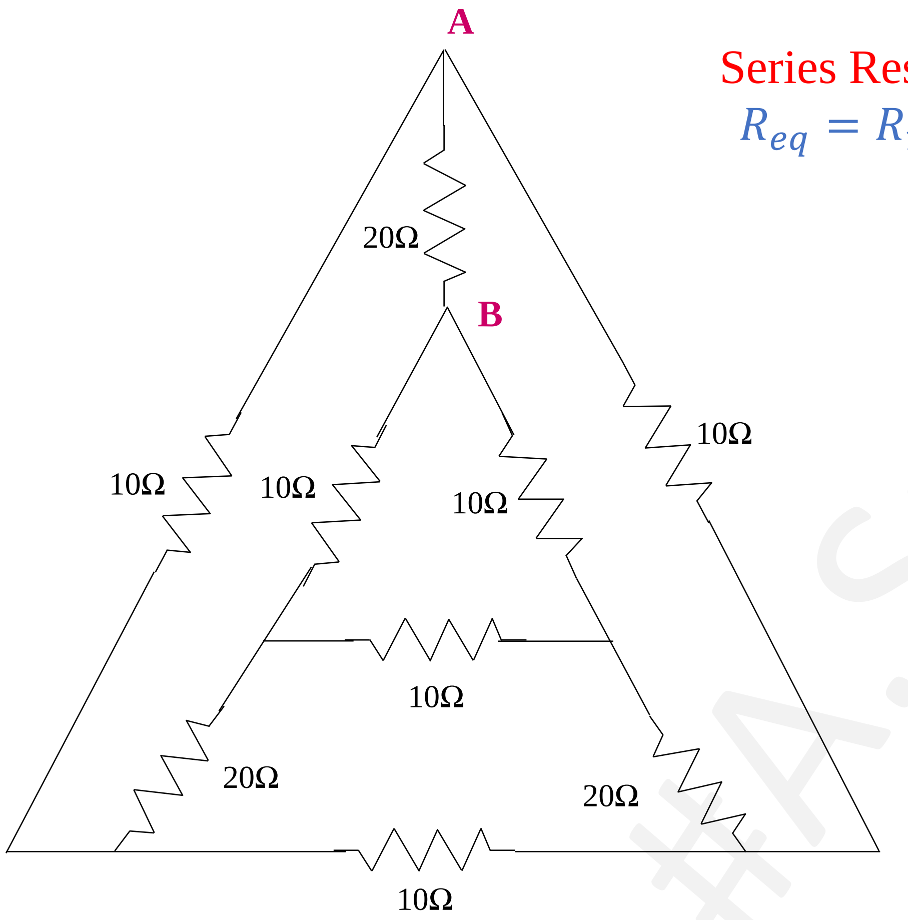


Parallel Resistance

$$R_{eq} = \frac{R_1 * R_2}{R_1 + R_2}$$



Find the equivalent resistance between points A and B.



Series Resistance

$$R_{eq} = R_1 + R_2$$

Parallel Resistance

$$R_{eq} = \frac{R_1 * R_2}{R_1 + R_2}$$

Delta to Star values

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

Mesh Analysis

Any closed electrical path is called loop. Mesh is defined as a loop which does not contain any other loops within it. If a network has a larger number of voltage sources, it is better to use mesh analysis, which mainly depends on KVL.

Steps

1. **Identify the number of loops**
2. **Give the mesh current directions**
3. **Apply the KVL in every loop**
4. **Find the unknow mesh currents**
5. **Find the unknow values**

Mesh Analysis

Steps

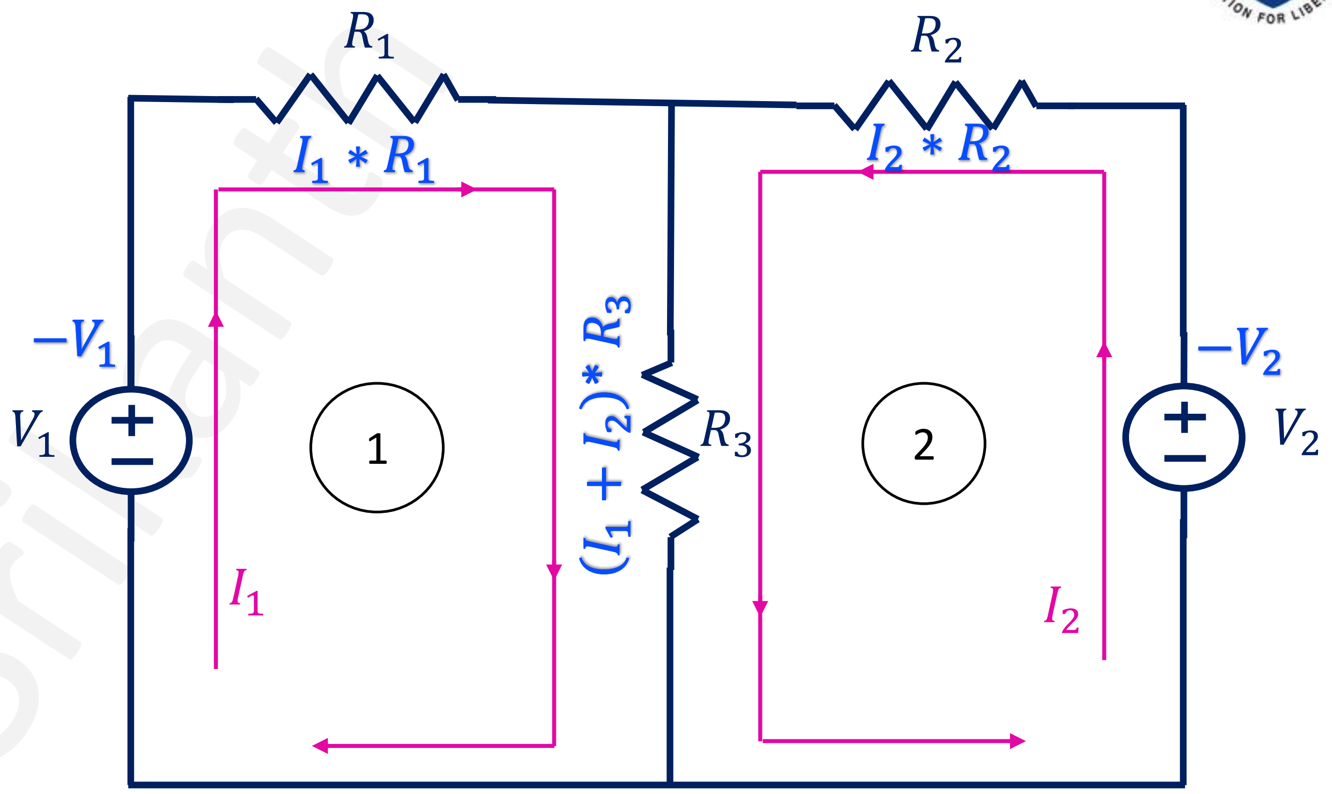
1. Identify the number of loops
2. Give the mesh current directions
3. Apply the KVL in every loop
4. Find the unknow mesh currents
5. Find the unknow values

$$-V_1 + I_1 * R_1 + (I_1 + I_2) * R_3 = 0$$

$$-V_2 + I_2 * R_2 + (I_1 + I_2) * R_3 = 0$$

$$I_1 * (R_1 + R_2) + I_2 * R_3 = V_1$$

$$I_1 * R_3 + I_2 * (R_2 + R_3) = V_2$$



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Mesh Analysis

Steps

1. Identify the number of loops
2. Give the mesh current directions
3. Apply the KVL in every loop
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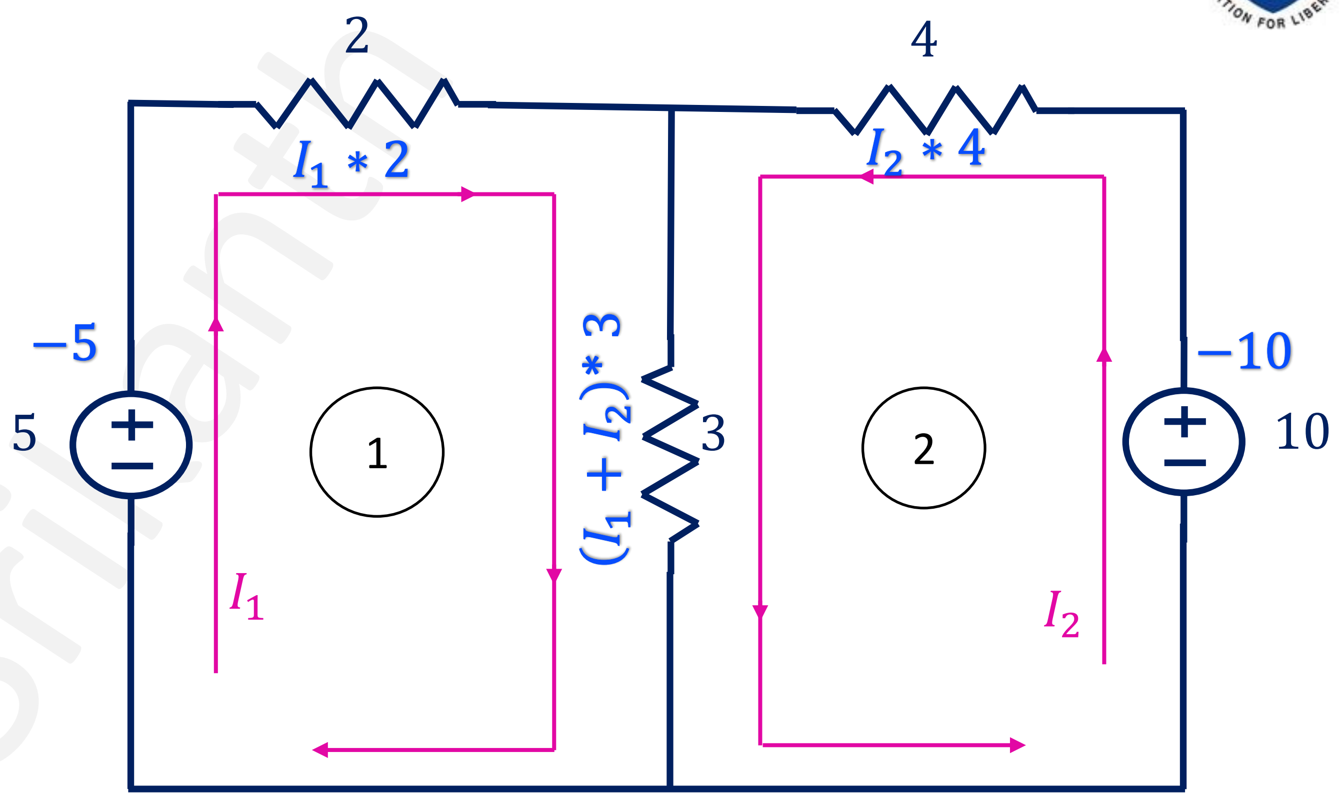

$$-5 + I_1 * 2 + (I_1 + I_2) * 3 = 0$$

$$-10 + I_2 * 4 + (I_1 + I_2) * 3 = 0$$

$$I_1 * 5 + I_2 * 3 = 5$$

$$I_1 * 3 + I_2 * 7 = 10$$

I_1=0.192A

I_2=1.346A

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Nodal Analysis

A node is a point in a network common to two or more circuit elements. If three or more elements meet at a node, that node is called a principle node. A node voltage is the voltage of given node with respective to one particulate node, called the reference node, which we assume at zero potential. If the network has more number of current sources, then the nodal analysis is useful method, mainly depends on KCL. An 'N' node circuit will be require (N-1) unknown voltages and (n-1) equations.

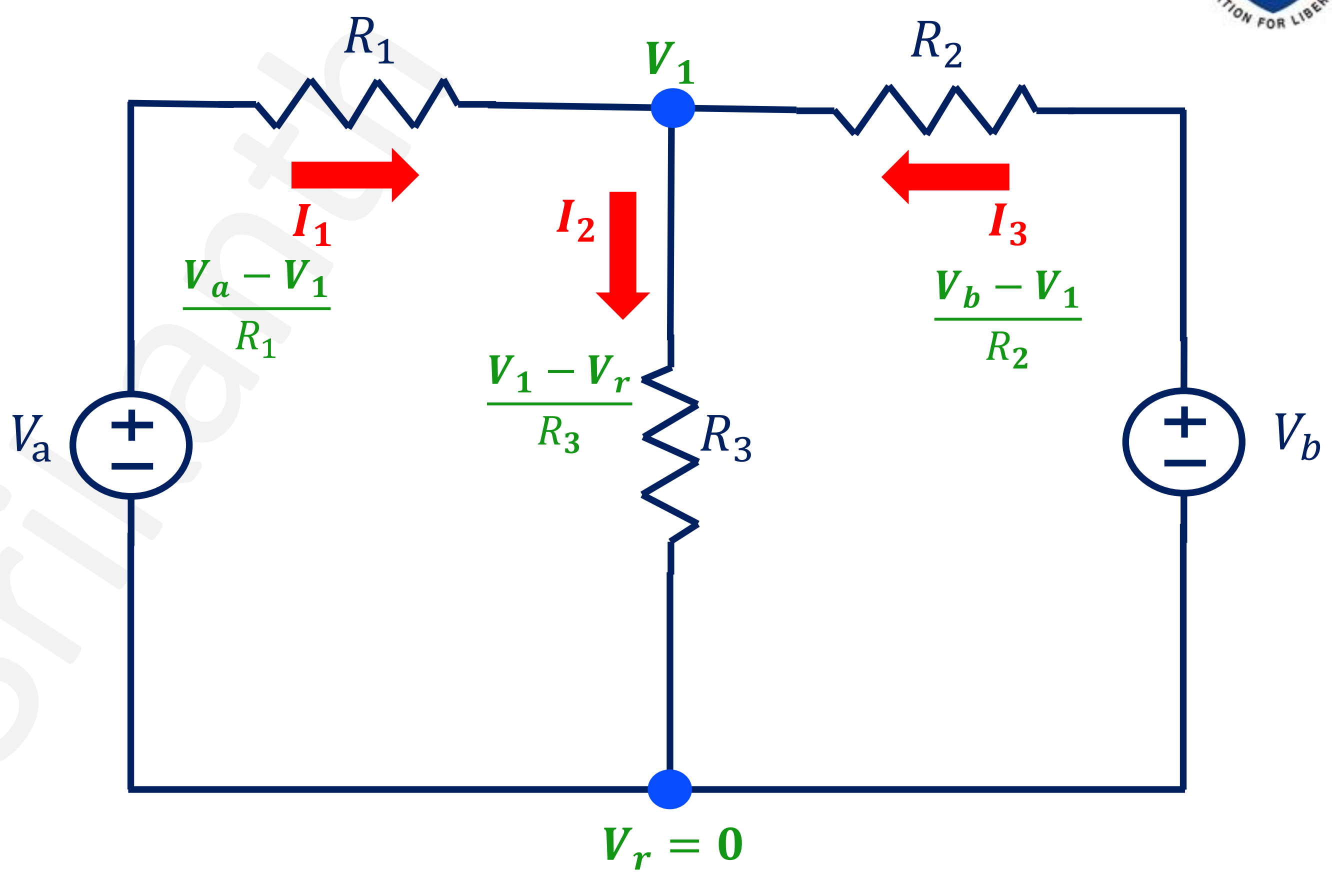
Steps

1. **Identify the number of Nodes**
2. **Mention the Node voltages**
3. **Apply the KCL at each Node**
4. **Find the unknow Node Voltage**
5. **Find the unknow values**

Nodal Analysis

Steps

1. Identify the number of Nodes
2. Mention the Node voltages
3. Apply the KCL at each Node
4. Find the unknow Node Voltage
5. Find the unknow values



At V_1

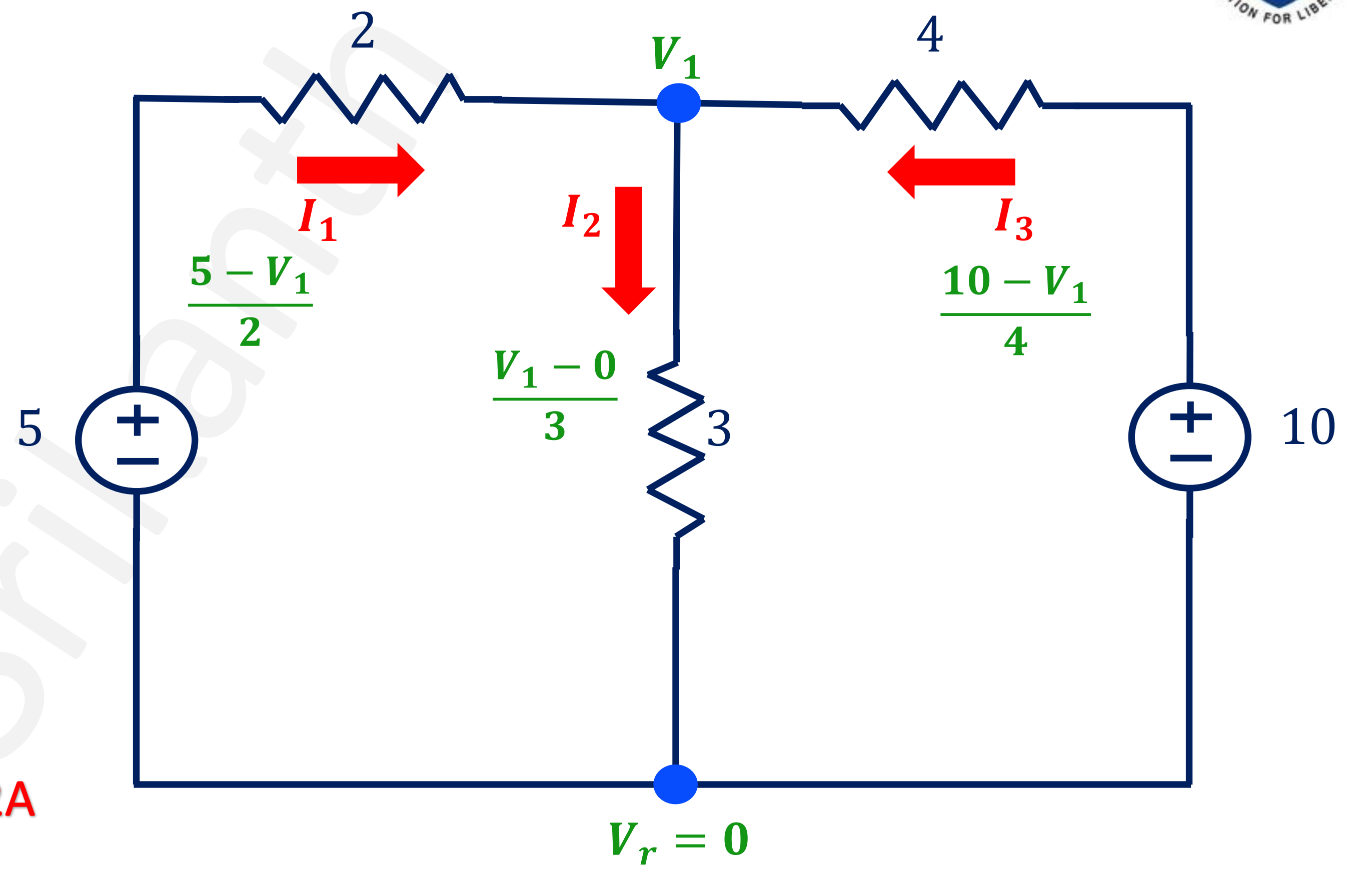
$$I_1 - I_2 + I_3 = 0$$

$$\frac{V_a - V_1}{R_1} - \frac{V_1 - V_r}{R_3} + \frac{V_b - V_1}{R_2} = 0$$

Nodal Analysis

Steps

1. Identify the number of Nodes
2. Mention the Node voltages
3. Apply the KCL at each Node
4. Find the unknow Node Voltage
5. Find the unknow values



At V_1

$$I_1 - I_2 + I_3 = 0$$

$$\frac{5-V_1}{2} - \frac{V_1-0}{3} + \frac{10-V_1}{4} = 0$$

$$V_1 = 4.615v$$

I_1=0.192A

I_3=1.346A

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Supermesh Analysis

Suppose any of branches in the network has a current source, then it is slightly difficult to apply mesh analysis. This difficulty can overcome by using Supper Mesh Technique. A Supper Mesh is constituted by two adjacent loop that have a common current source.

Steps

1. Identify the number of loops
2. Give the mesh current directions
3. Apply the KVL in every loop
4. Find the unknow mesh currents
5. Find the unknow values

Supermesh Analysis

Steps

1. Identify the number of loops
2. Give the mesh current directions
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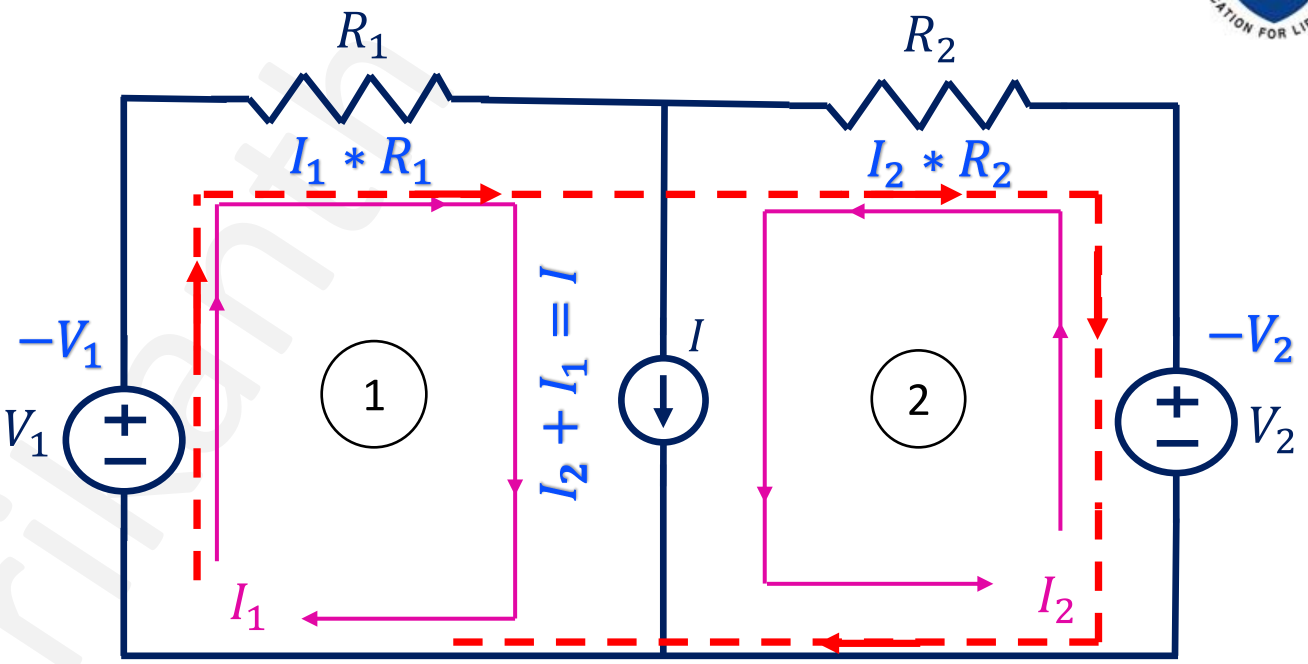


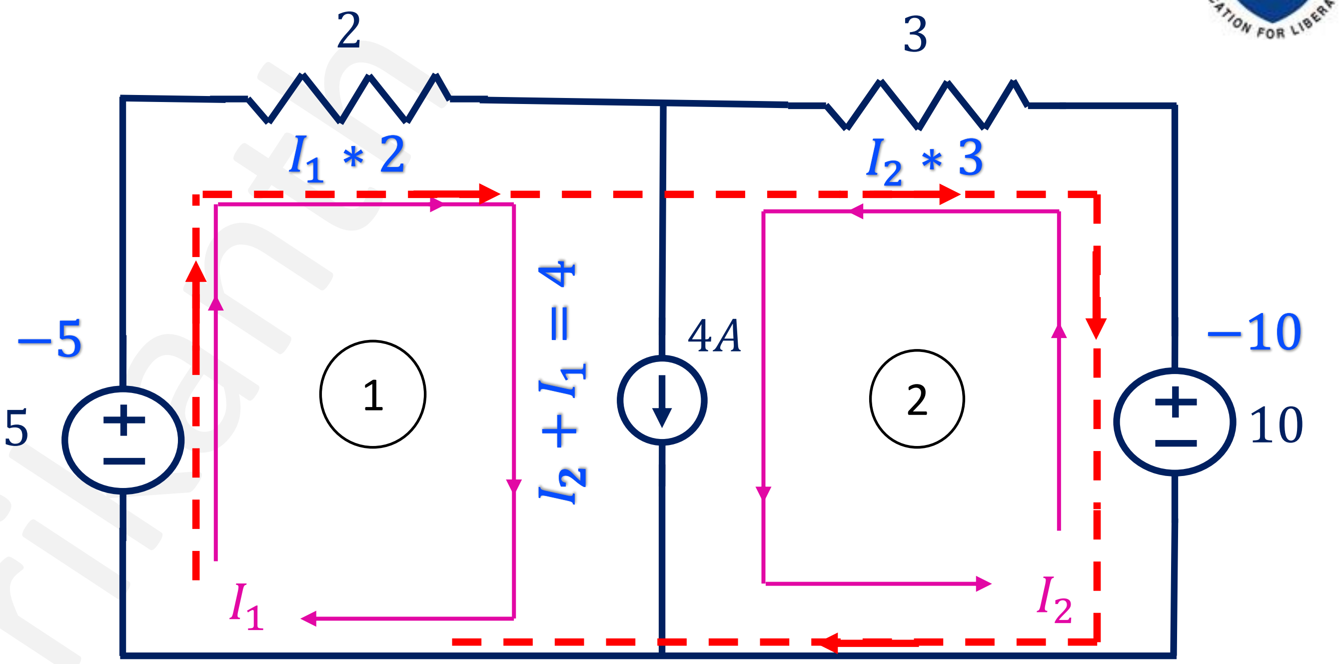
$$I_1 + I_2 = \mathrm{I}$$

$$-V_1 + I_1 * R_1 - I_2 * R_2 + V_2 = 0$$

Supermesh Analysis

Steps

1. Identify the number of loops
2. Give the mesh current directions
3. Apply the KVL in every loop
4. Find the unknow mesh currents
5. Find the unknow values



$$I_1 + I_2 = 4$$

$$-5 + I_1 * 2 - I_2 * 3 + 10 = 0$$

$$I_1 = 1.4A \quad I_1 = 2.6A$$

Supernode Analysis

Suppose any of branches in the network has single voltage source, then it is slightly difficult to apply node analysis. This difficulty can overcome by using Supper Node Technique. A Supper Node is constituted by two adjacent nodes that are connected by a voltage source are reduced to single node and the equations are formed as usual by applying KCL.

Steps

1. **Identify the number of Nodes**
2. **Mention the Node voltages**
3. **Apply the KCL at each Node**
4. **Find the unknow Node Voltage**
5. **Find the unknow values**

Supernode Analysis

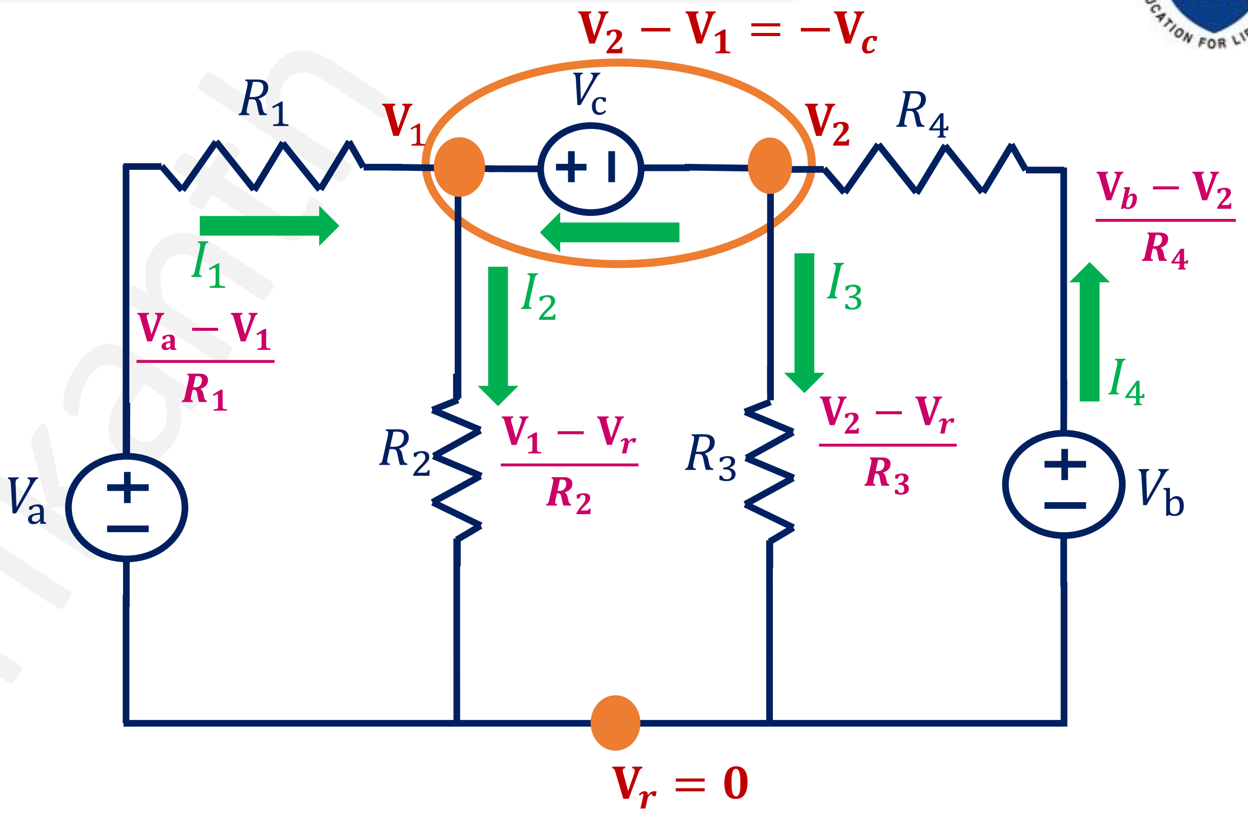
Steps

1. Identify the number of Nodes
2. Mention the Node voltages
3. Apply the KCL at each Node
4. Find the unknow Node Voltage
5. Find the unknow values

$$\mathbf{V}_2 - \mathbf{V}_1 = -\mathbf{V}_c$$

$$I_1 - I_2 - I_3 + I_4 = 0$$

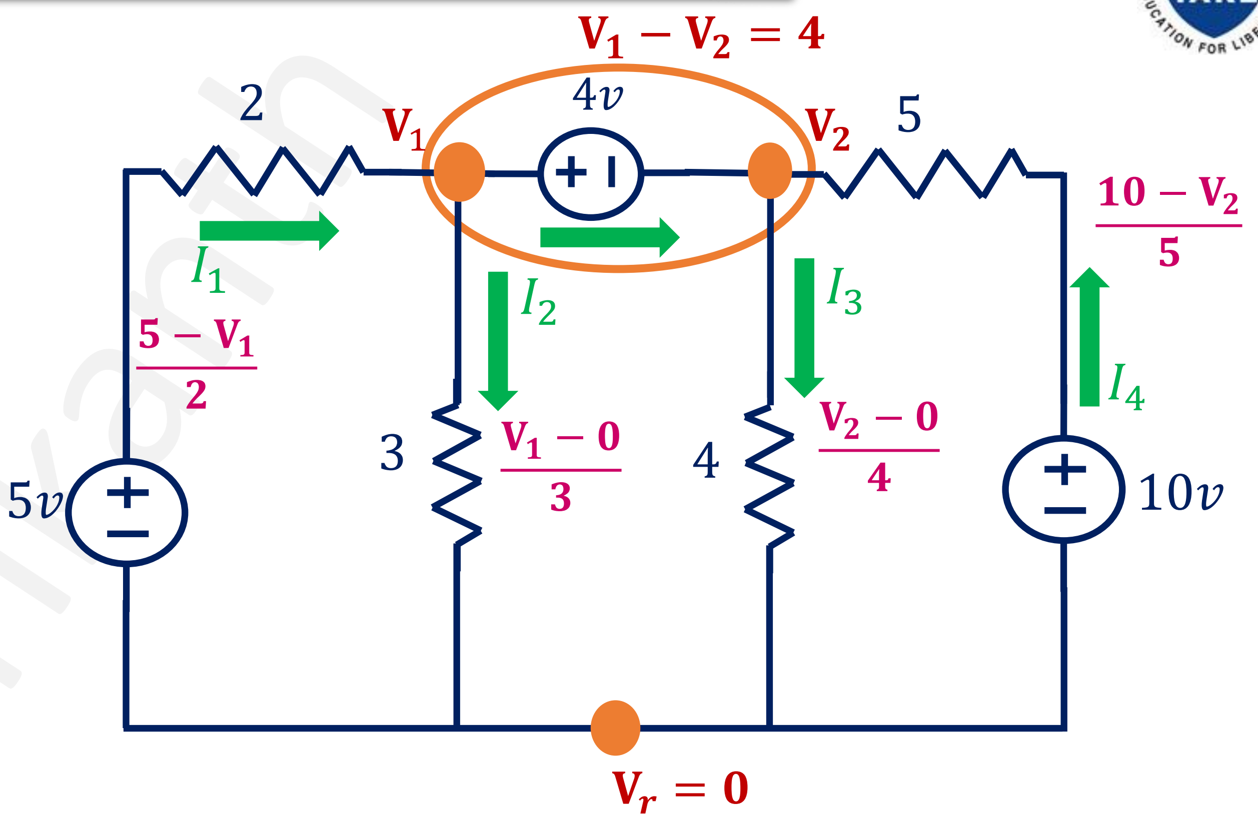
$$\frac{\mathbf{V_a} - \mathbf{V}_1}{R_1} - \frac{\mathbf{V}_1 - \mathbf{V}_r}{R_2} - \frac{\mathbf{V}_2 - \mathbf{V}_r}{R_3} + \frac{\mathbf{V}_b - \mathbf{V}_2}{R_4} = 0$$



Supernode Analysis

Steps

1. Identify the number of Nodes
2. Mention the Node voltages
3. Apply the KCL at each Node
4. Find the unknow Node Voltage
5. Find the unknow values



$$V_1 - V_2 = 4$$

$$I_1 - I_2 - I_3 + I_4 = 0$$

$$\frac{5 - V_1}{2} - \frac{V_1 - 0}{3} - \frac{V_2 - 0}{4} + \frac{10 - V_2}{5} = 0$$

$$V_1 = 4.909v$$

$$V_2 = 0.909v$$

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Inspection Method

The mesh or nodal equations are to be solved for finding loop currents or node voltages using Matrix form known as **Inspection Method**. These equations are algebraic equations of form [A][X]=[B], where [X] is unknown values. The Cramer's rule is a simple method used for solving these equations. It is also known as **Method of Determinations**. Consider the Equations

$$A_1X + B_1Y + C_1Z = D_1$$
$$A_2X + B_2Y + C_2Z = D_2$$
$$A_3X + B_3Y + C_3Z = D_3$$

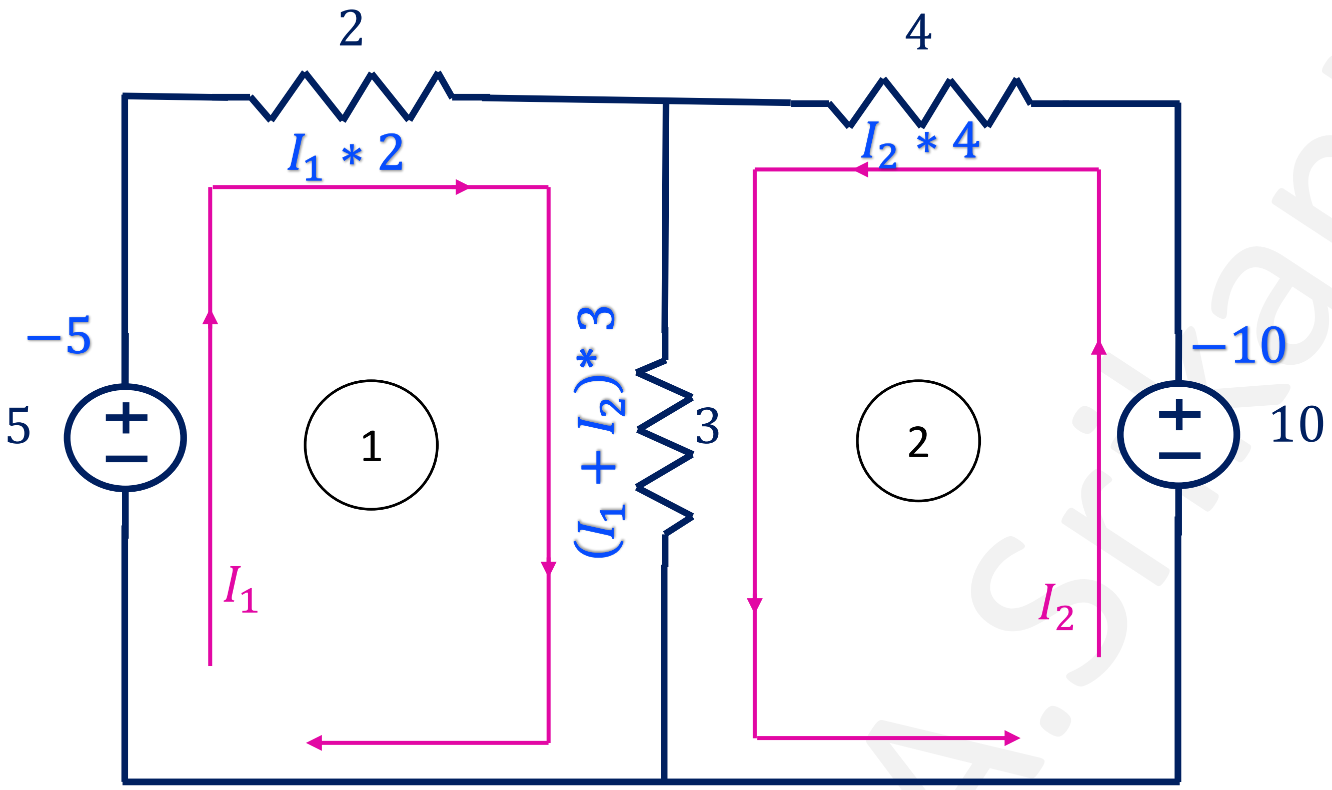
The above equations can be written matrix form as

$$\begin{bmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{bmatrix}\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix}$$

Using Cramer's rule $X = \frac{|\Delta_1|}{|\Delta|}$; $Y = \frac{|\Delta_2|}{|\Delta|}$; $Z = \frac{|\Delta_3|}{|\Delta|}$

$$\Delta = \begin{bmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{bmatrix};\ \Delta_1 = \begin{bmatrix} D_1 & B_1 & C_1 \\ D_2 & B_2 & C_2 \\ D_3 & B_3 & C_3 \end{bmatrix};\ \Delta_2 = \begin{bmatrix} A_1 & D_1 & C_1 \\ A_2 & D_2 & C_2 \\ A_3 & D_3 & C_3 \end{bmatrix}\ \Delta_3 = \begin{bmatrix} A_1 & B_1 & D_1 \\ A_2 & B_2 & D_2 \\ A_3 & B_3 & D_3 \end{bmatrix}$$

Inspection Method



$$\begin{bmatrix} 5 & 3 \\ 3 & 7 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$$

Using Cramer's rule $I_1 = \frac{|\Delta_1|}{|\Delta|}; I_2 = \frac{|\Delta_2|}{|\Delta|}$

$$\Delta = \begin{bmatrix} 5 & 3 \\ 3 & 7 \end{bmatrix} \quad \Delta_1 = \begin{bmatrix} 5 & 3 \\ 10 & 7 \end{bmatrix} \quad \Delta_2 = \begin{bmatrix} 5 & 5 \\ 3 & 10 \end{bmatrix}$$

$$I_1 = \frac{5}{26} = 0.192A \qquad I_2 = \frac{35}{26} = 1.346A$$

$$I_1 * 5 + I_2 * 3 = 5$$

$$I_1 * 3 + I_2 * 7 = 10$$

I_1=0.192A

I_2=1.346A

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ELECTRICAL CIRCUITS

Course Code	: AEEC02
Category	: Foundation
Credits	: 3
Maximum Marks	: CIA (30) + SEE (70) = Total (100)

By
A. Srikanth
Assistant Professor
Electrical and Electronics Engineering

COURSE SYLLABUS

MODULE-III: NETWORK THEOREMS (DC AND AC)

Network Theorems (DC): Tellegen"s, superposition, reciprocity, Thevenin"s, Norton"s, maximum power transfer, Milliman"s and compensation theorems for DC excitations, numerical problems.

Network Theorems (AC): Tellegen"s, superposition, reciprocity, Thevenin"s, Norton"s, maximum power transfer, Milliman"s and compensation theorems for AC excitations, numerical problems.

COURSE OUTCOMES

CO 5 : Discuss the superposition principle, reciprocity and maximum power transfer condition for the electrical network with DC and AC excitation.

CO 6 : Summarize the procedure of thevenin's, norton's and milliman's theorems to reduce complex network into simple equivalent network with DC and AC excitation.

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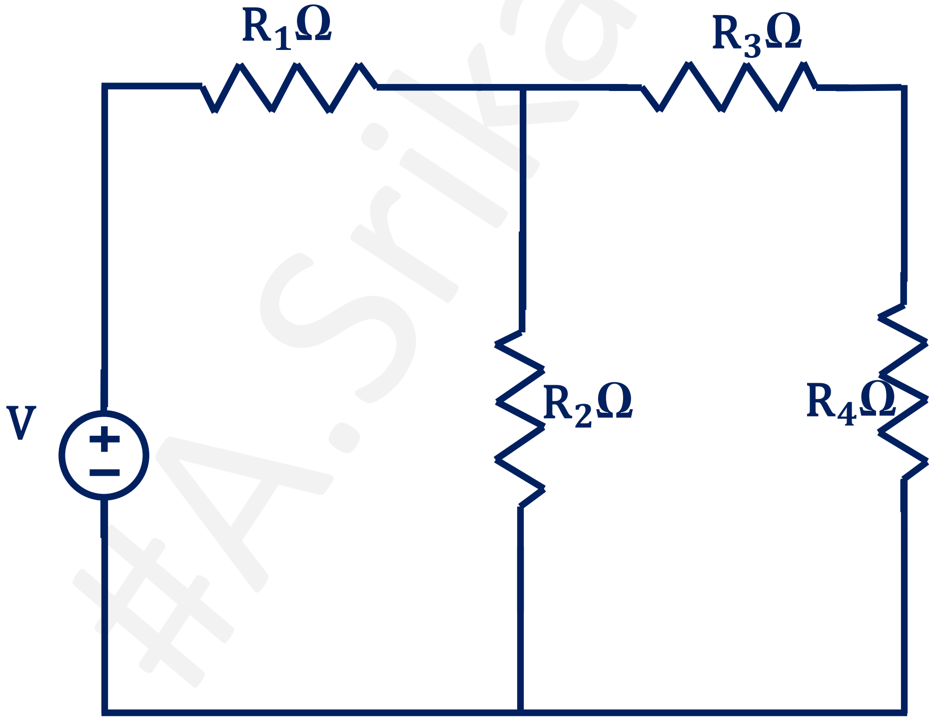
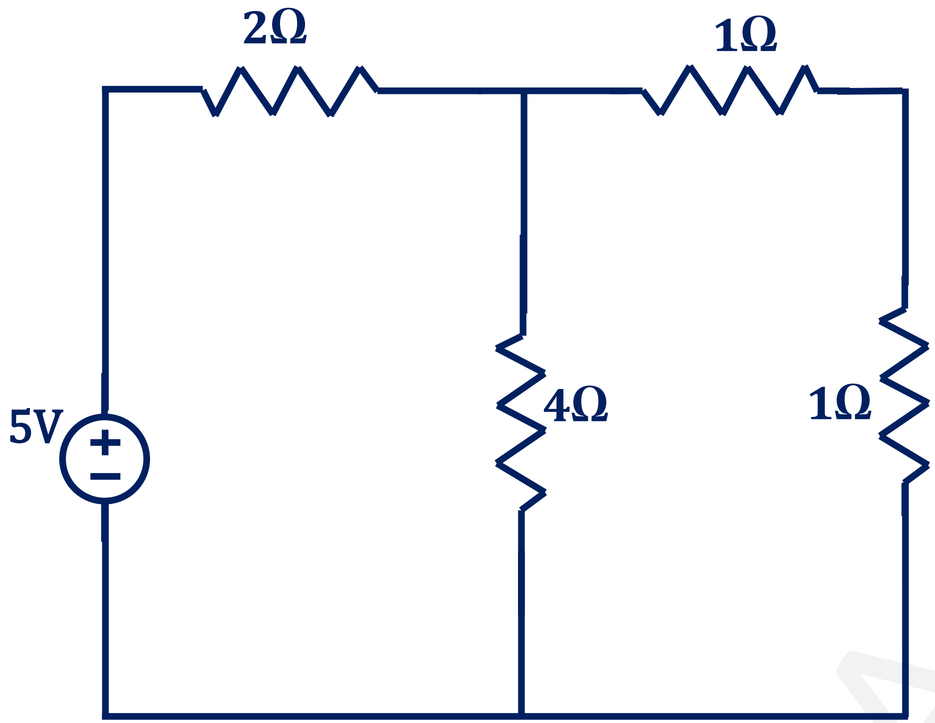
TELLEGEN's THEOREM

Tellegen's Theorem states that the summation of power delivered is zero for each branch of any electrical network at any instant of time. It is mainly applicable for designing the filters in signal processing. It is also used in complex operating systems for regulating stability. This theorem has been introduced in the year of 1952 by Dutch Electrical Engineer Bernard D.H. Tellegen. This is a very useful theorem in network analysis



$$\sum_{K=1}^{n} V_K * I_K = 0$$

TELLEGEN's THEOREM



$$\sum_{K=1}^{n} V_K * I_K = 0$$

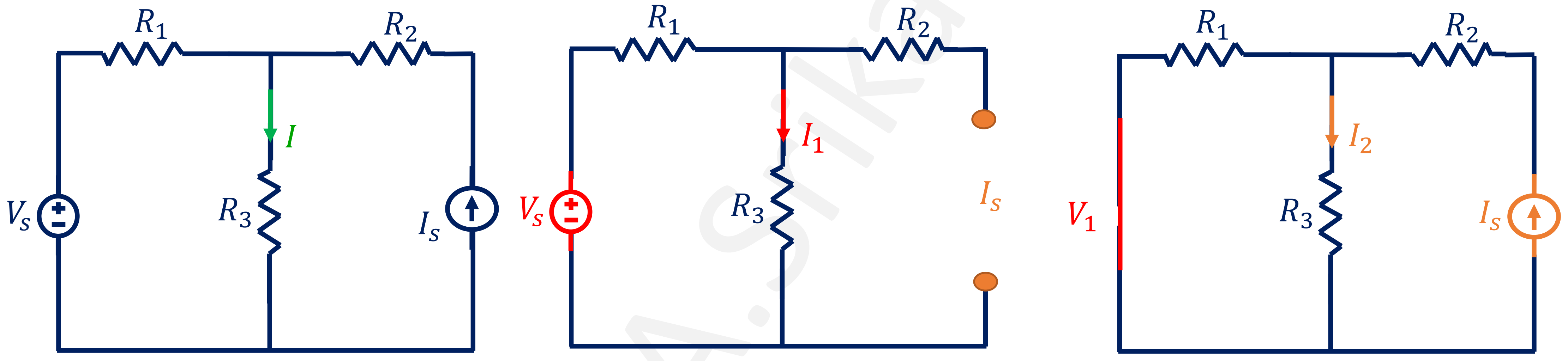
$$7.5 - 4.5 - 1 - 1 - 1 = 0$$

$I_1 = 1.5\ A$

$I_2 = 1A$

SUPERPOSITION THEOREM

Superposition theorem states that in any linear, active, bilateral network having more than one source, the response across any element is the sum of the responses obtained from each source considered separately and all other sources are replaced by their internal resistance. The superposition theorem is used to solve the network where two or more sources are present and connected



$$I = I_1 + I_2$$

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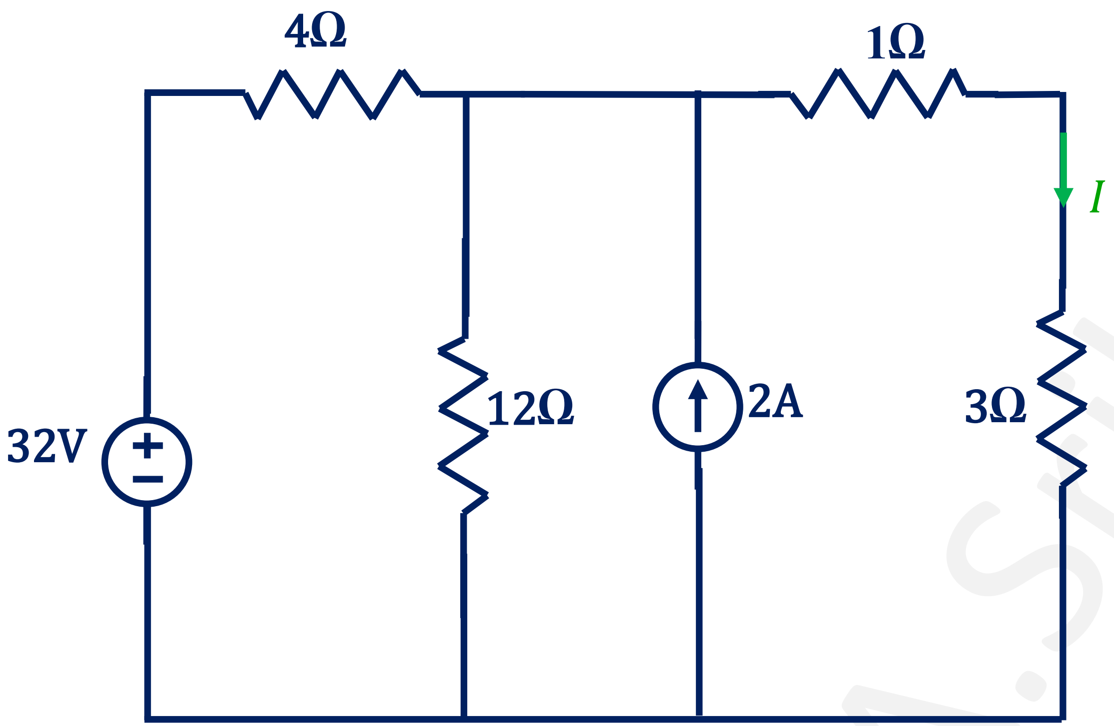
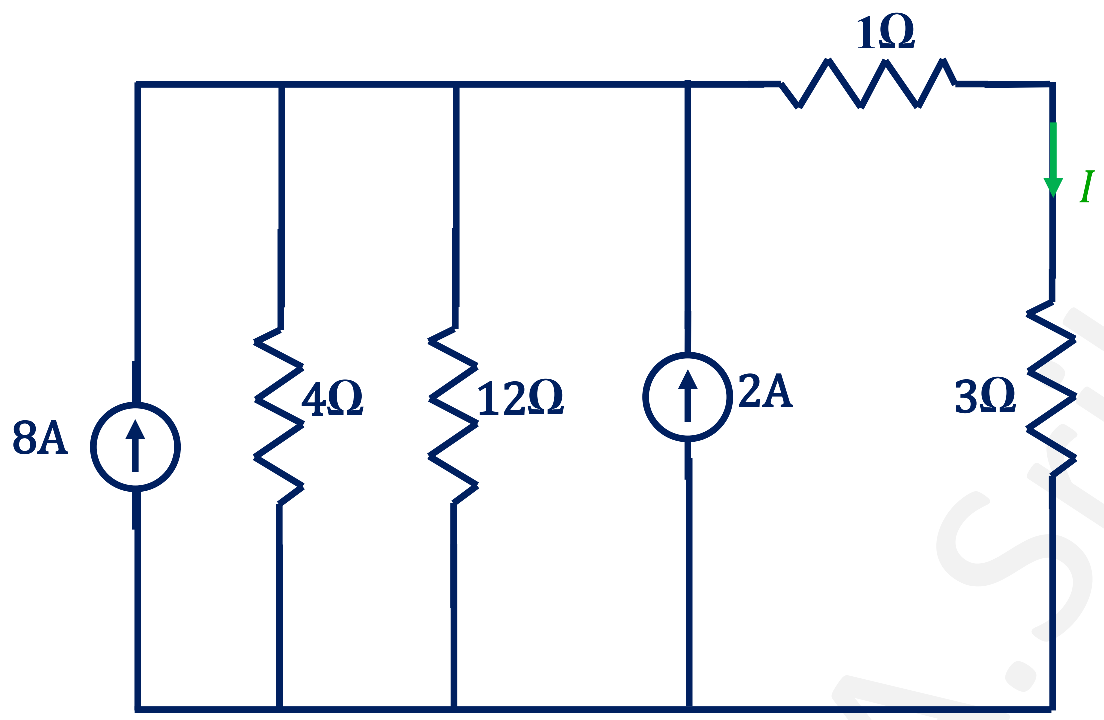
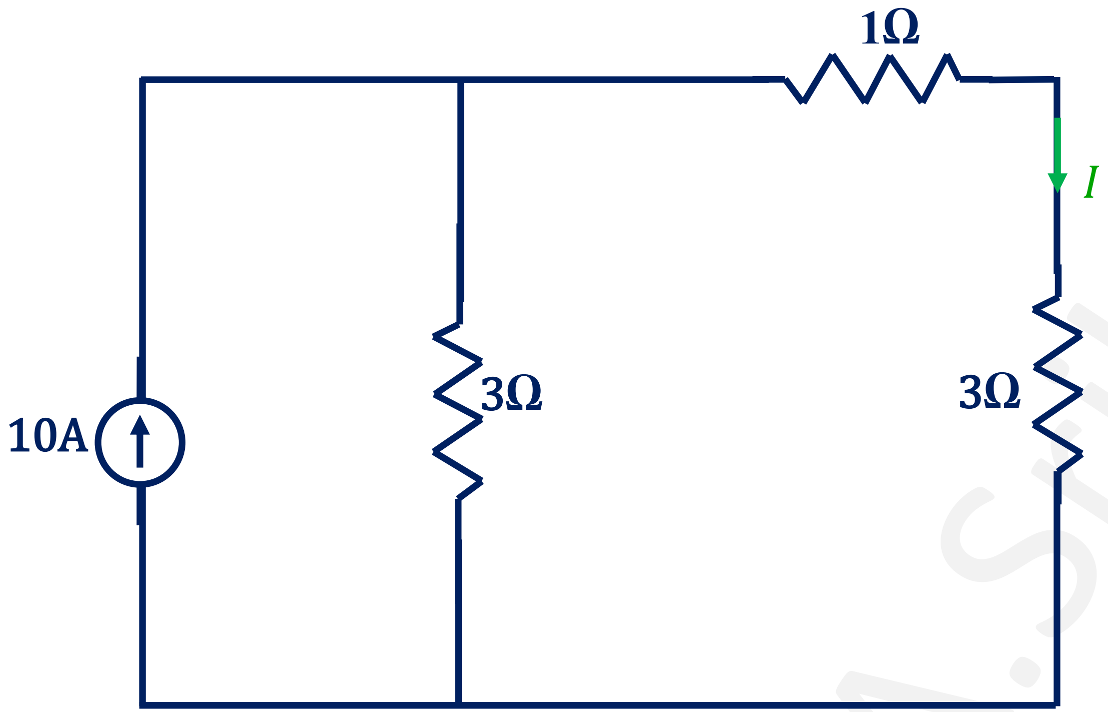
Prove the Superposition Theorem at 3 ohms resistor



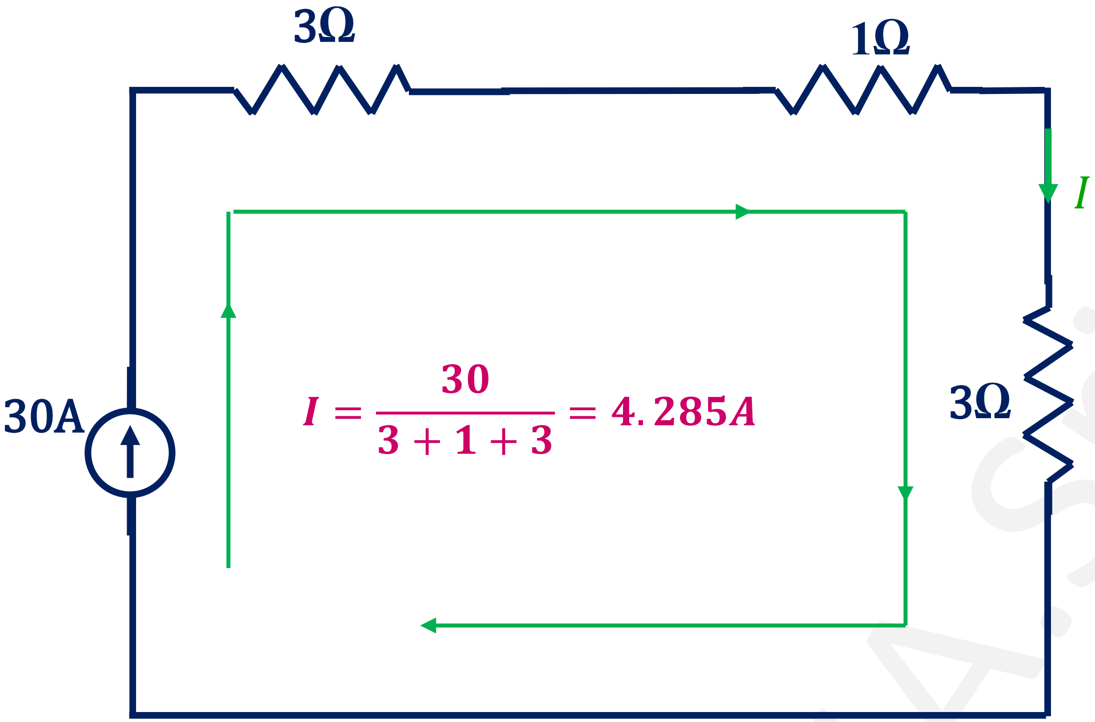
Prove the Superposition Theorem at 3 ohms resistor



Prove the Superposition Theorem at 3 ohms resistor



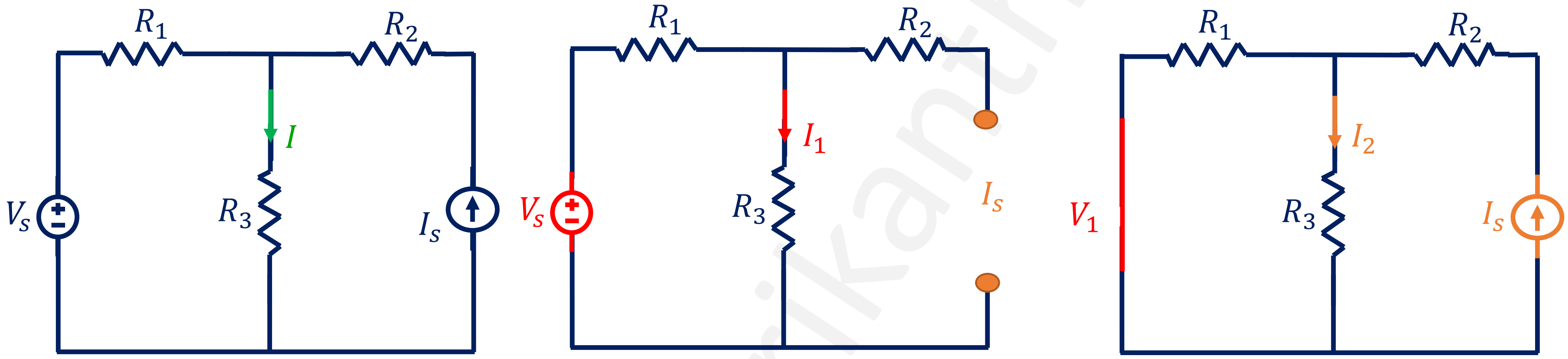
Prove the Superposition Theorem at 3 ohms resistor



$$I = \frac{30}{3+1+3} = 4.285A$$

$$I = 4.285A$$

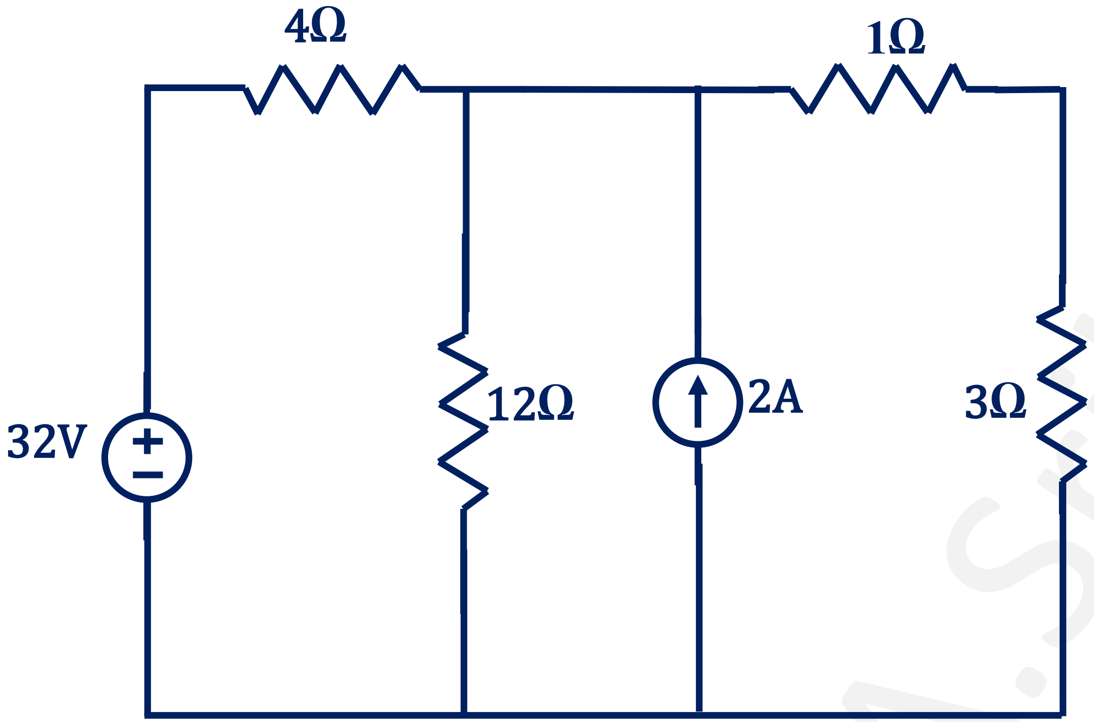
Prove the Superposition Theorem at 3 ohms resistor



$$I = I_1 + I_2$$

$$I = 4.285A$$

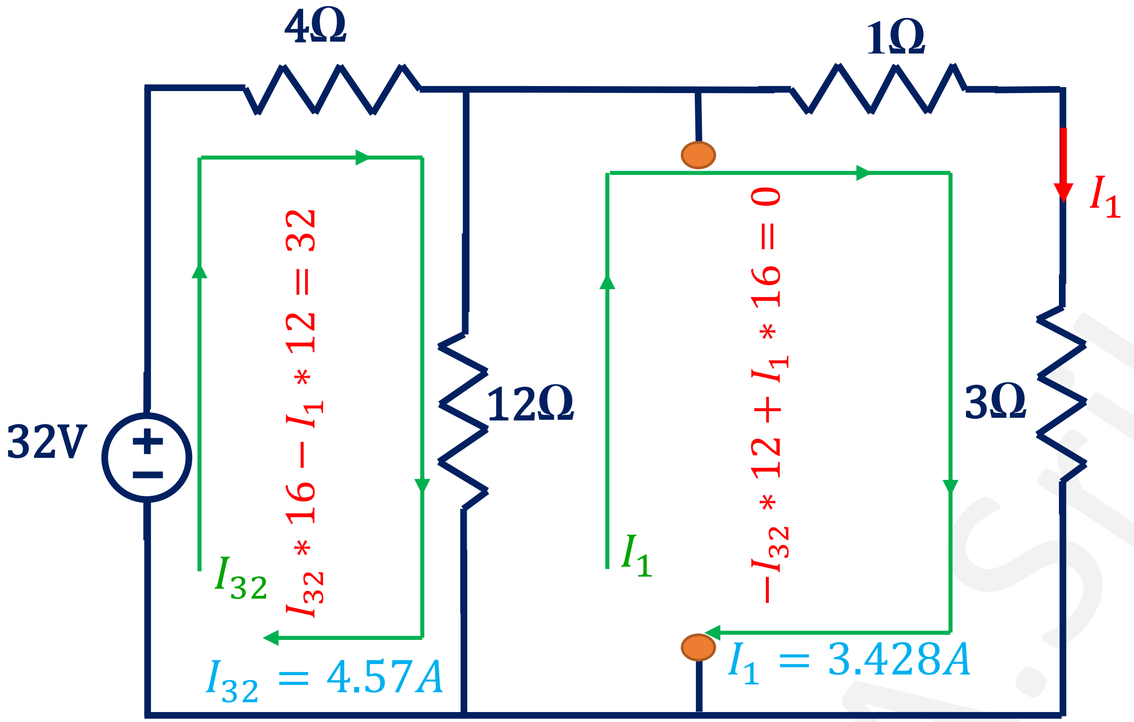
Prove the Superposition Theorem at 3 ohms resistor



$I = 4.285A$

V_{32} is acting source

Prove the Superposition Theorem at 3 ohms resistor



$I = 4.285A$

V_{32} is acting source

$I_1 = 3.428A$

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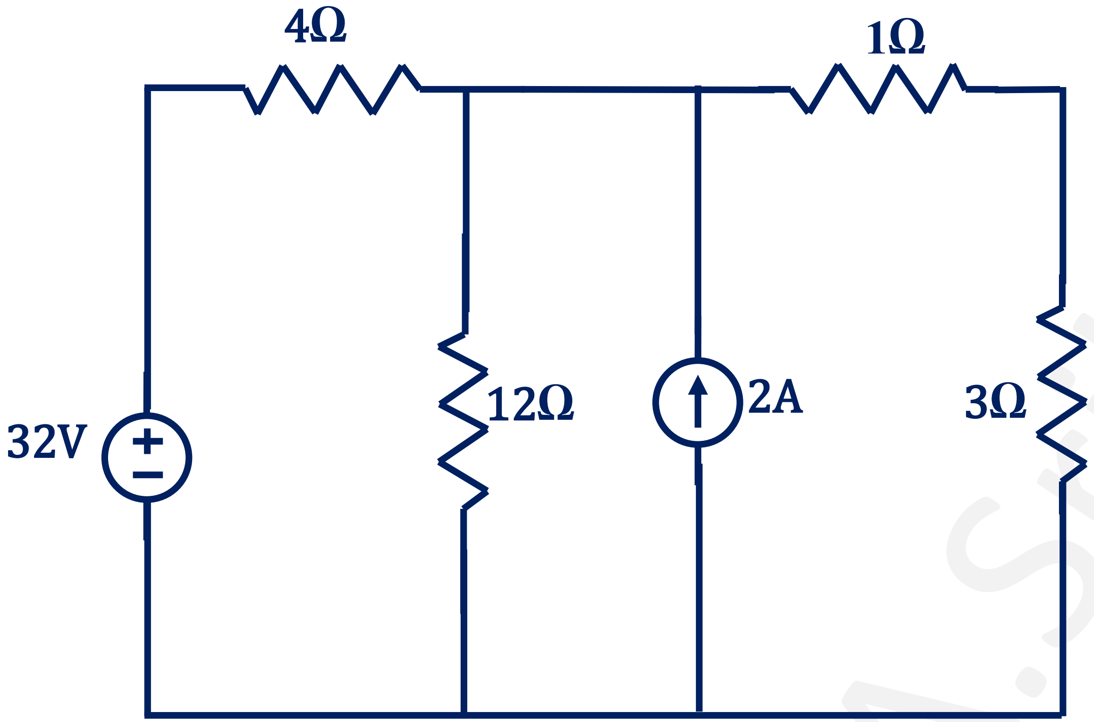
Prove the Superposition Theorem at 3 ohms resistor



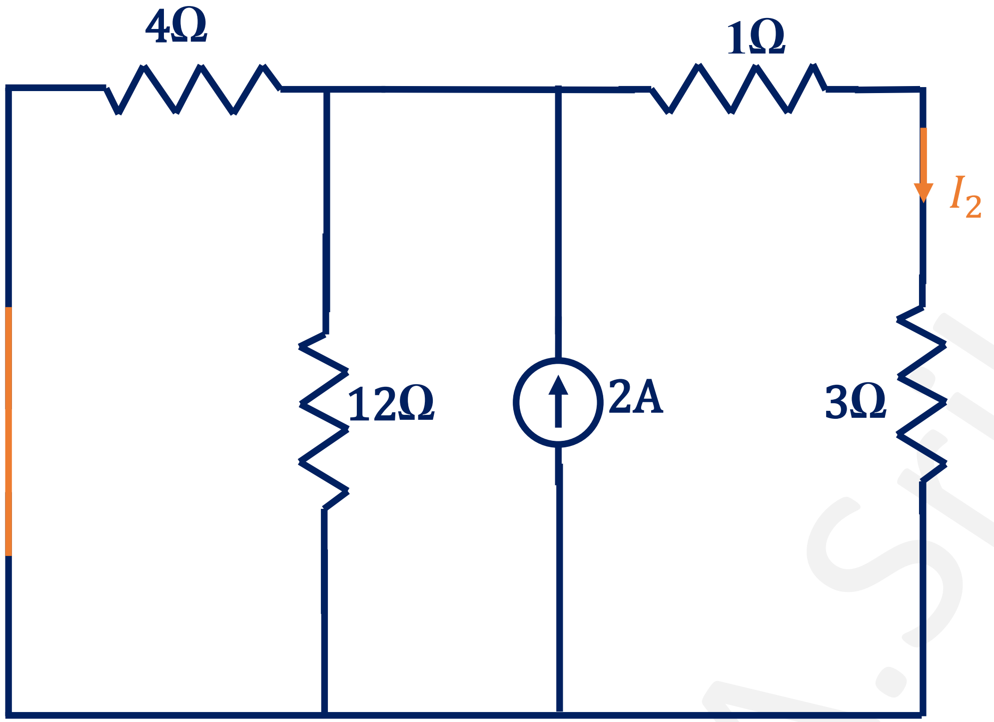
$I = 4.285A$

V_{32} is acting source

$I_1 = 3.428A$

$2A$ is acting source

Prove the Superposition Theorem at 3 ohms resistor



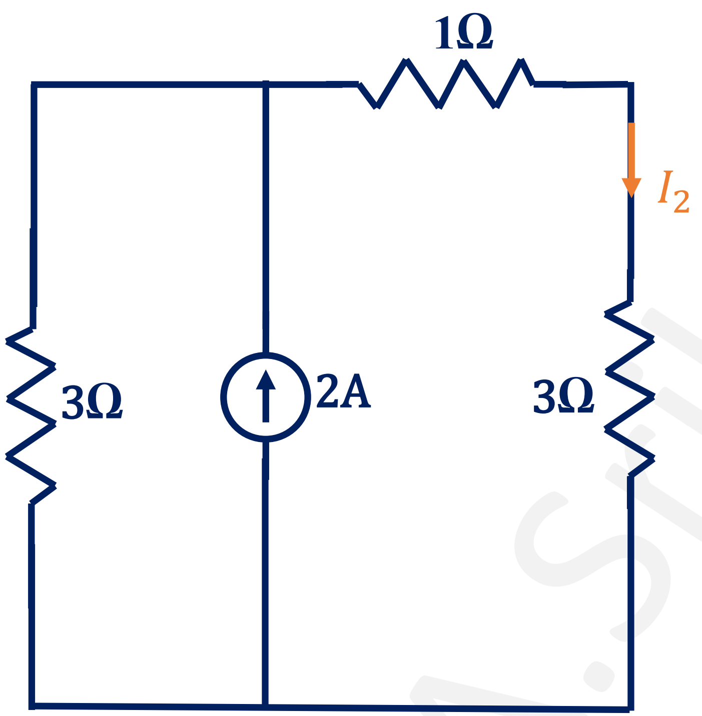
$I = 4.285A$

V_{32} is acting source

$I_1 = 3.428A$

$2A$ is acting source

Prove the Superposition Theorem at 3 ohms resistor



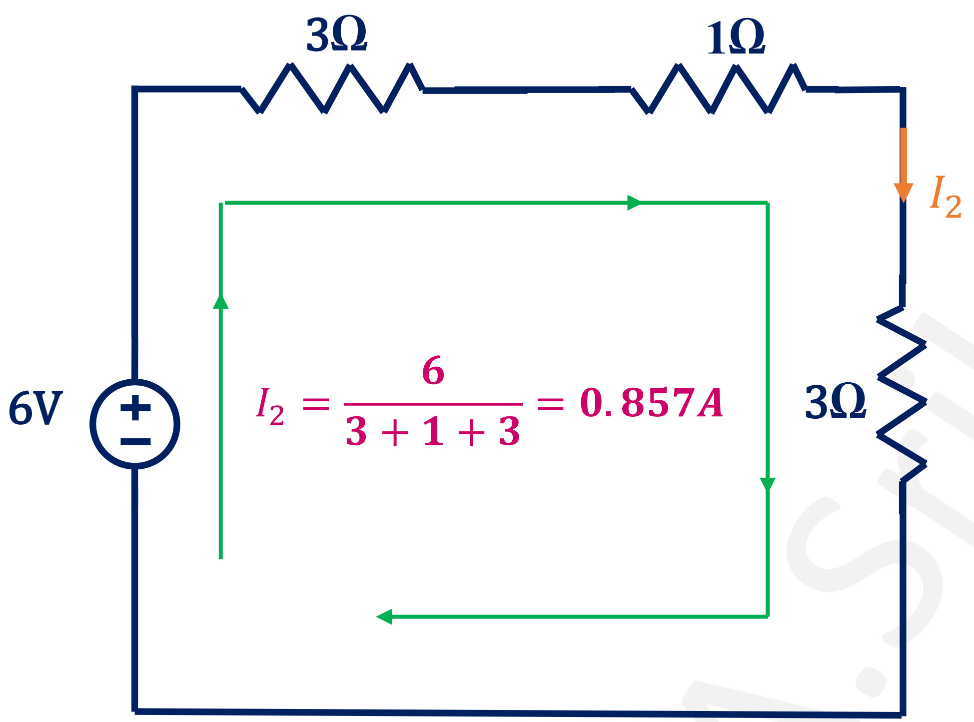
$I = 4.285A$

V_{32} is acting source

$I_1 = 3.428A$

$2A$ is acting source

Prove the Superposition Theorem at 3 ohms resistor



$I = 4.285A$

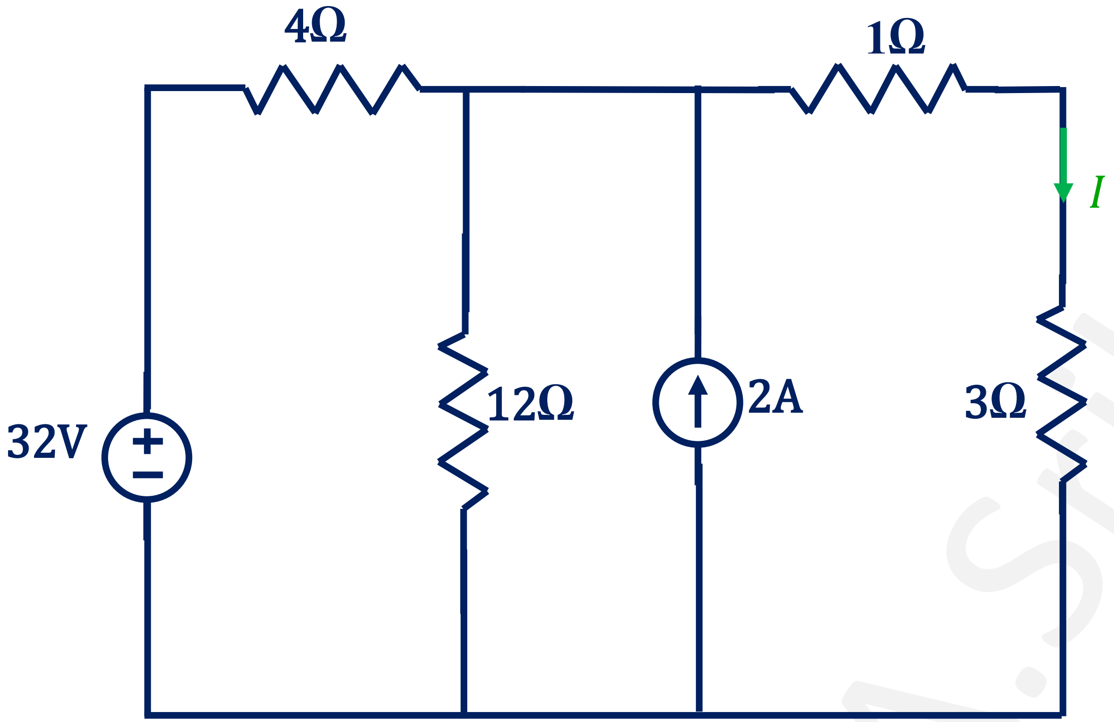
V_{32} is acting source

$I_1 = 3.428A$

$2A$ is acting source

$I_2 = 0.857A$

Prove the Superposition Theorem at 3 ohms resistor



$I = 4.285A$

V_{32} is acting source

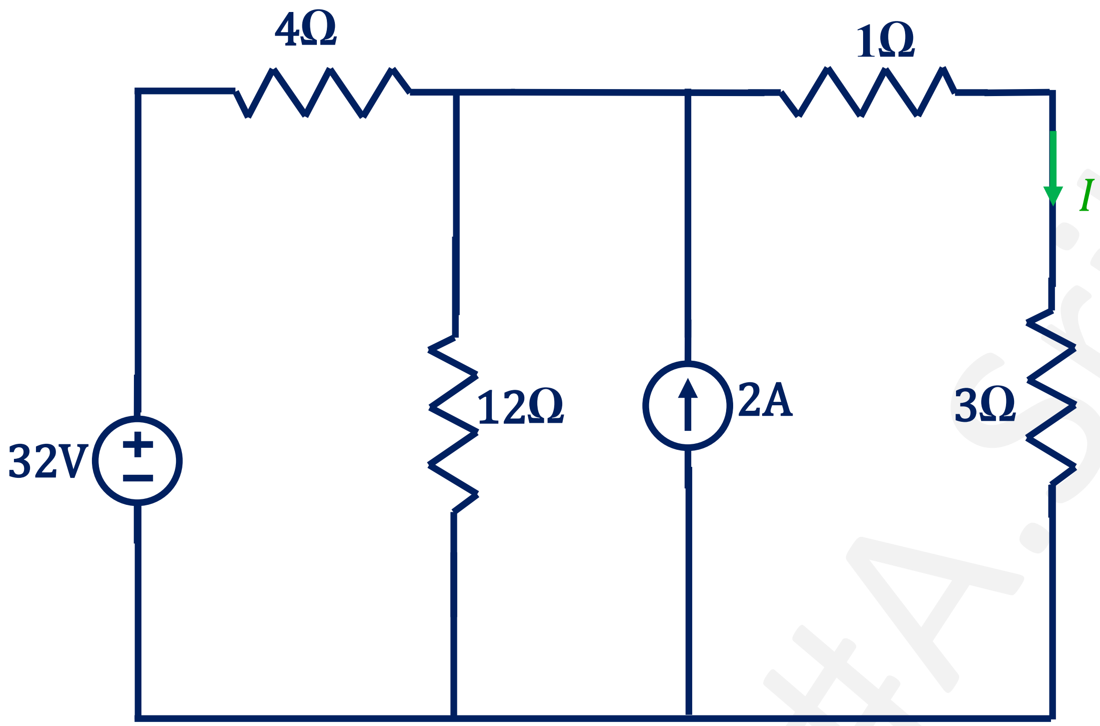
$I_1 = 3.428A$

$2A$ is acting source

$I_2 = 0.857A$

$$I = I_1 + I_2 = 3.428 + 0.857 = 4.285A$$

Superposition theorem states that in any linear, active, bilateral network having more than one source, the response across any element is the sum of the responses obtained from each source considered separately and all other sources are replaced by their internal resistance. The superposition theorem is used to solve the network where two or more sources are present and connected



$I = 4.285A$

V_{32} is acting source

$I_1 = 3.428A$

$2A$ is acting source

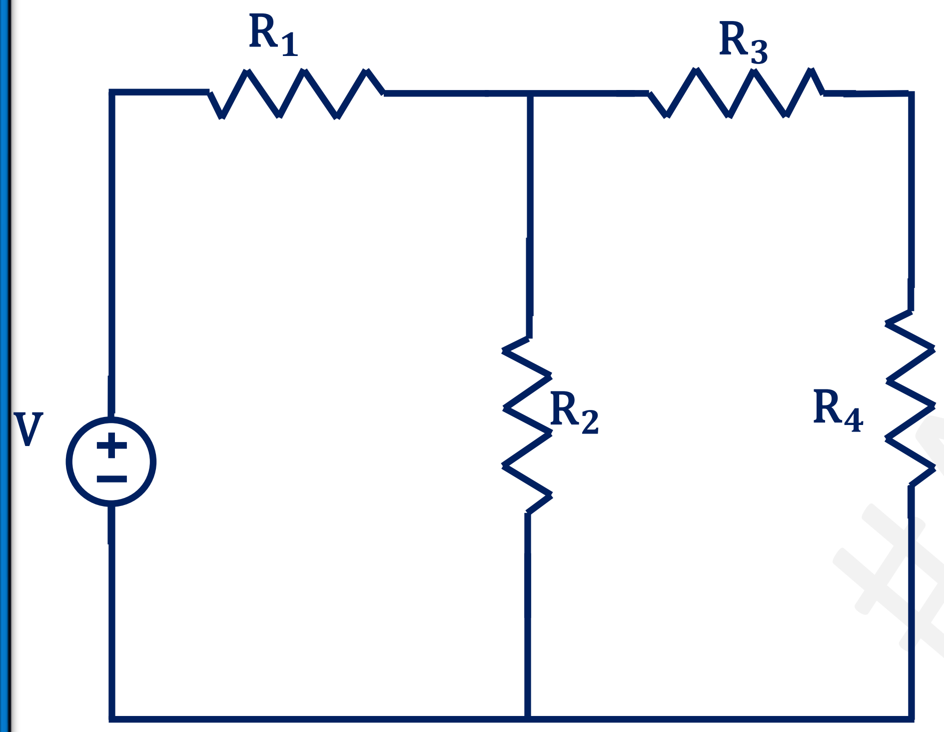
$I_2 = 0.857A$

$$I = I_1 + I_2$$

Prove the Superposition Theorem at 3 ohms resistor

TELLEGEN's THEOREM

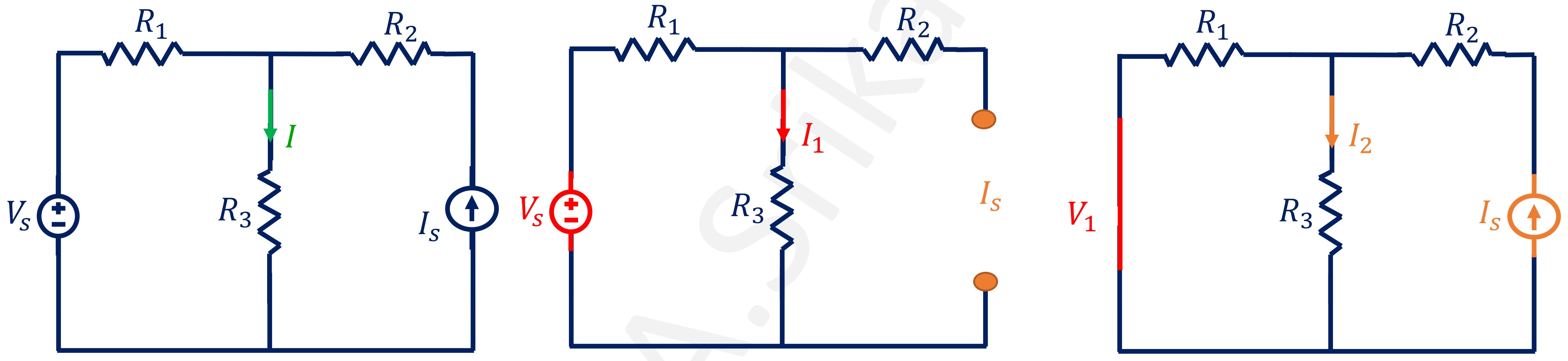
Tellegen's Theorem states that the summation of power delivered is zero for each branch of any electrical network at any instant of time. It is mainly applicable for designing the filters in signal processing. It is also used in complex operating systems for regulating stability. This theorem has been introduced in the year of 1952 by Dutch Electrical Engineer Bernard D.H. Tellegen. This is a very useful theorem in network analysis



$$\sum_{K=1}^{n} V_K * I_K = 0$$

SUPERPOSITION THEOREM

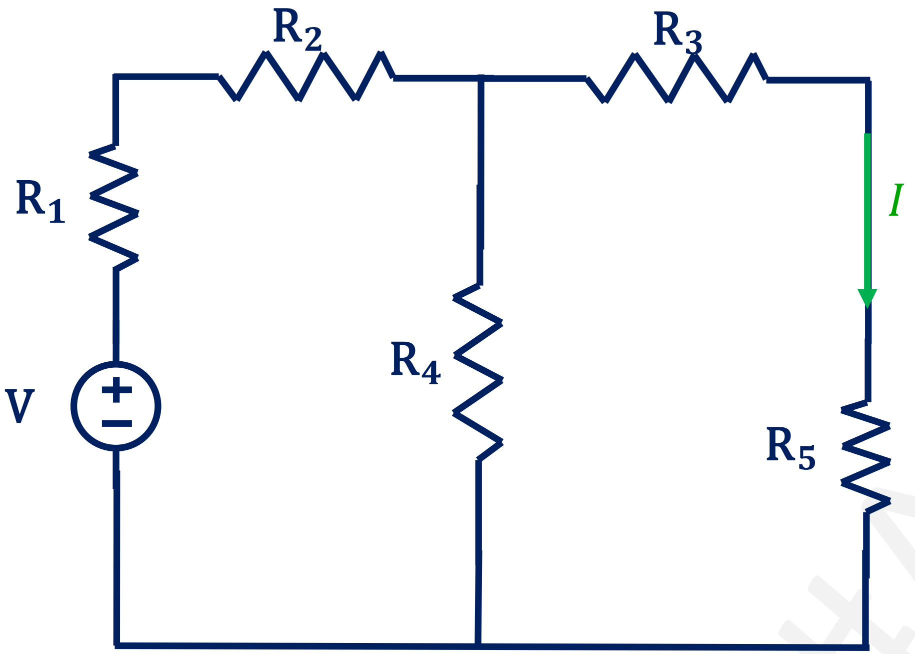
Superposition theorem states that in any linear, active, bilateral network having more than one source, the response across any element is the sum of the responses obtained from each source considered separately and all other sources are replaced by their internal resistance. The superposition theorem is used to solve the network where two or more sources are present and connected

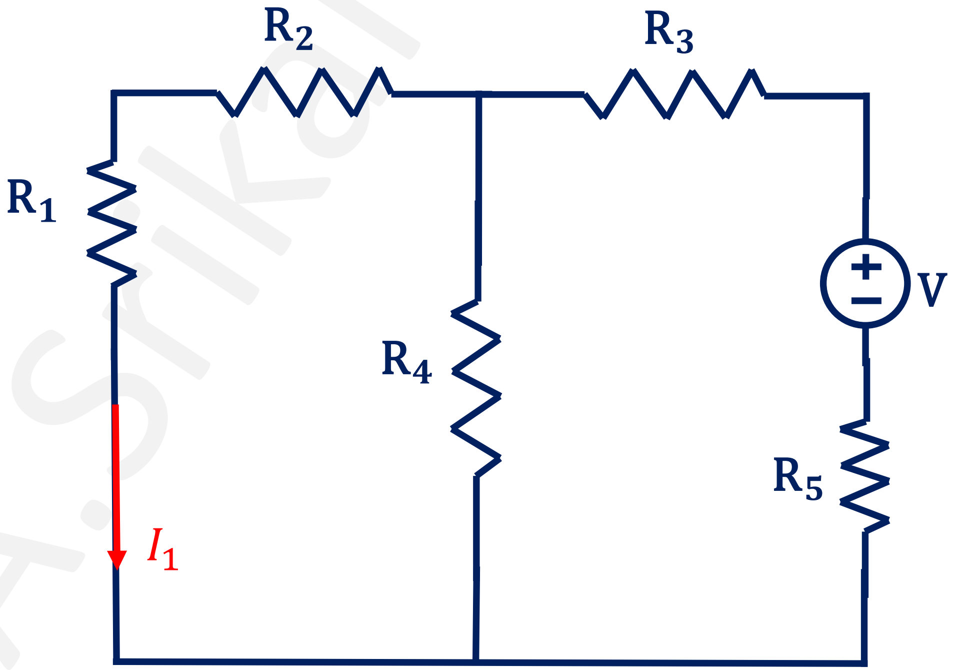


$$I = I_1 + I_2$$

RECIPROCITY THEOREM

Reciprocity Theorem states that, the value of current due to a single source in any particular branch of circuit is equal to the value of current in the original branch where the source was placed when the source is shifted to that particular branch of circuit.

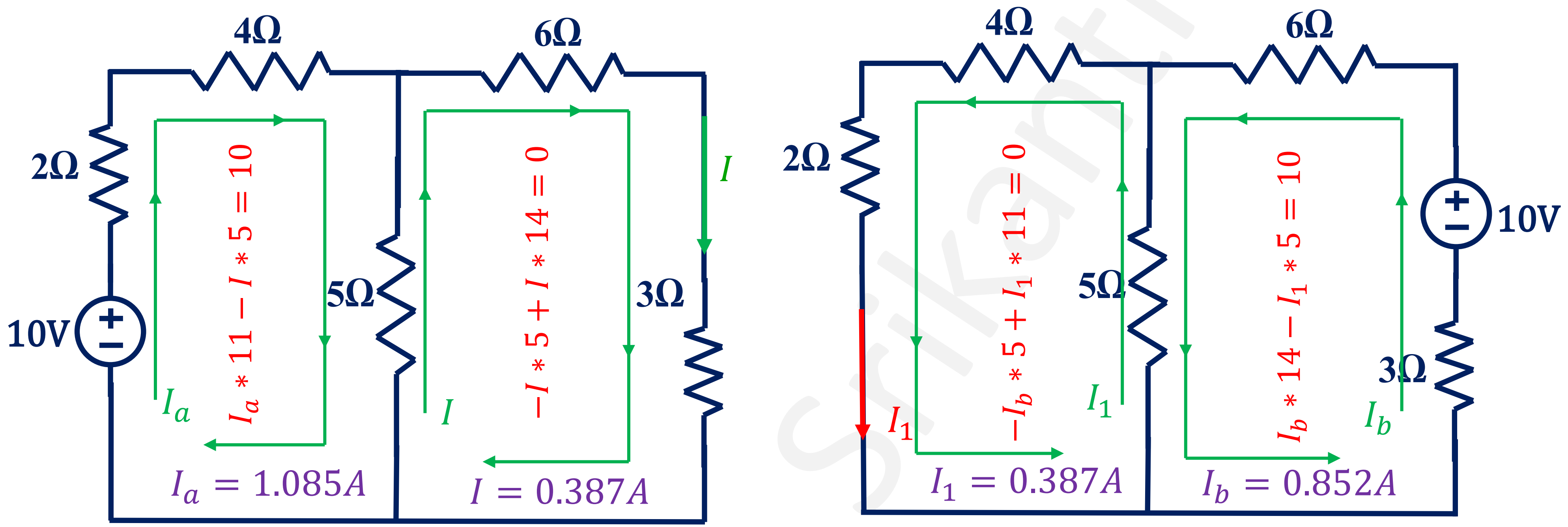




$$I = I_1$$

The ratio $\frac{V}{I}$ is known as the transfer resistance

RECIPROCITY THEOREM



$$I = I_1 = 0.387$$

The ratio $\frac{V}{I}$ is known as the transfer resistance $= \frac{10}{0.387} = \mathbf{25.83\Omega}$

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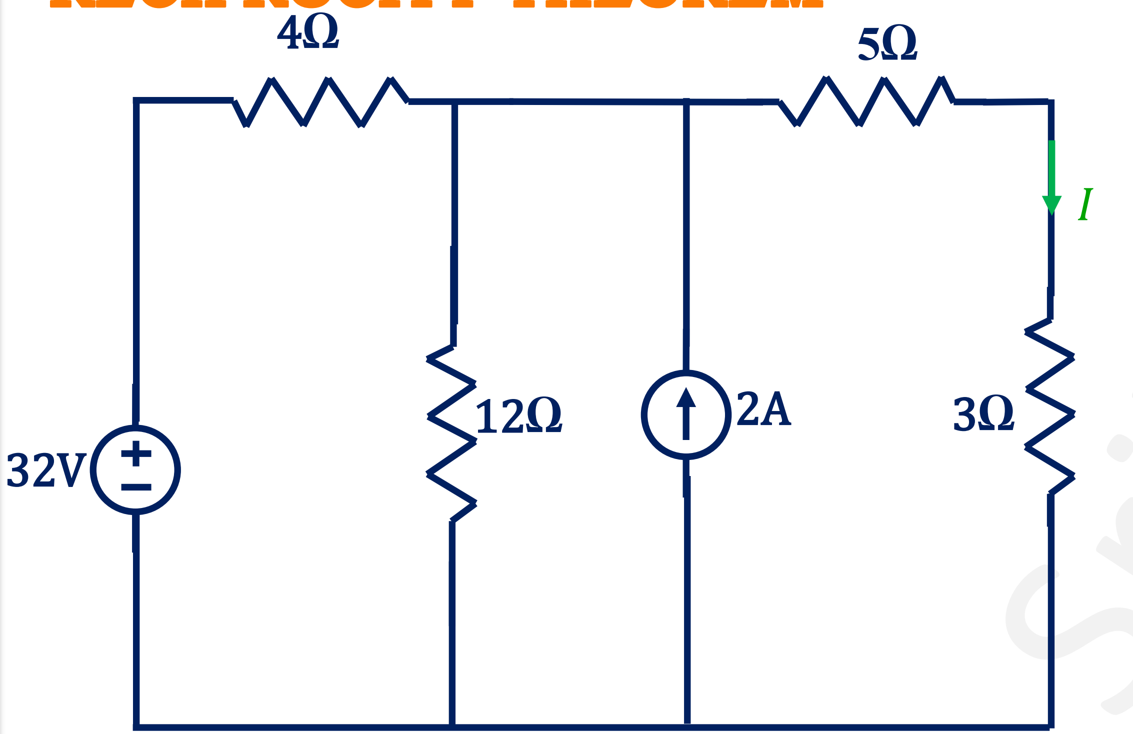
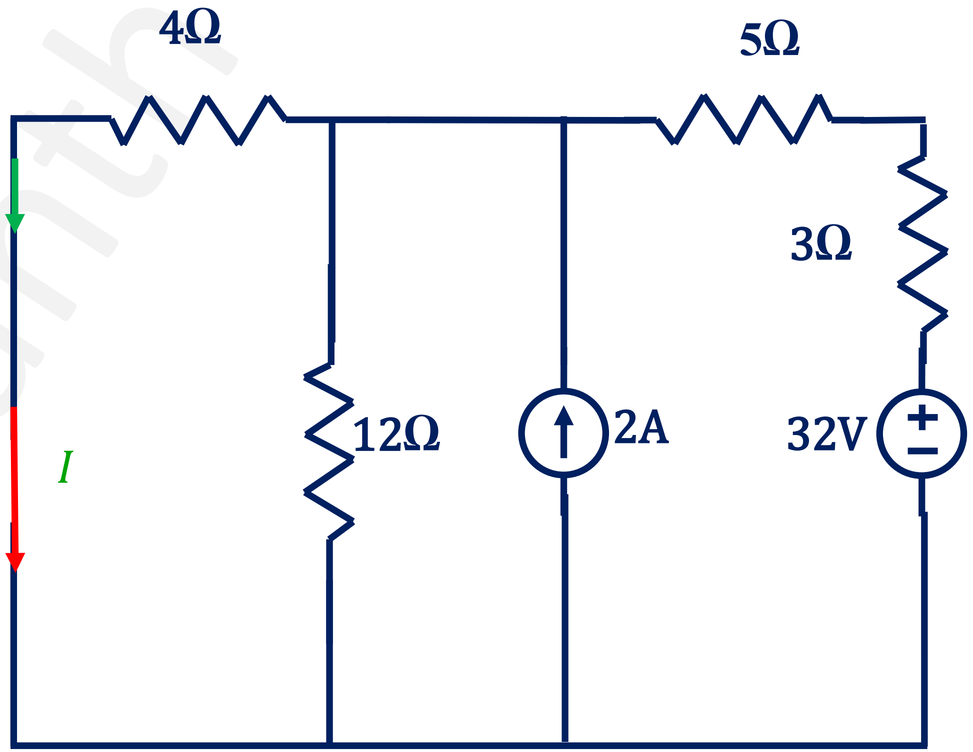
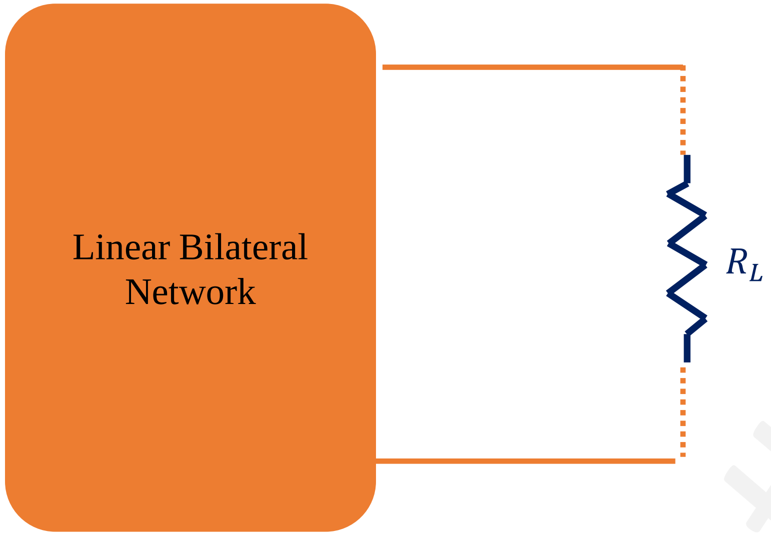
RECIPROCITY THEOREM

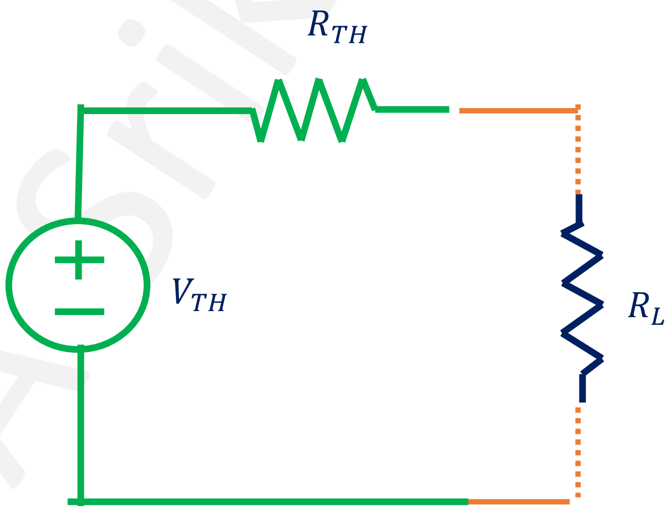




THEVENIN'S THEOREMS

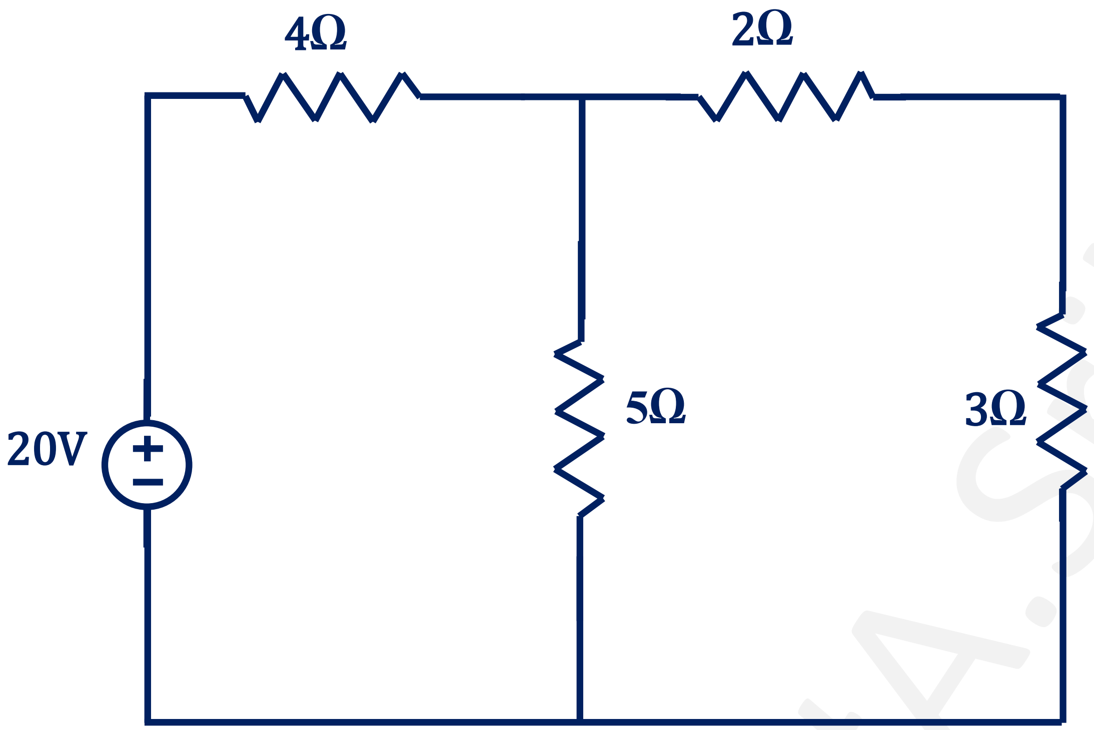
Thevenin's Theorem states that "Any linear bilateral network (AC or DC) containing several voltages, currents and resistances can be replaced by just one single voltage source (V_{th}) in series with a single resistance (R_{th}) connected across the load". Where V_{th} or V_{oc} is the open circuited voltage measured between the load terminals & R_{th} is the Thevenin's equivalent resistance measured across the load when all the voltage sources are replaced by short circuit and current sources are replaced by open circuit.

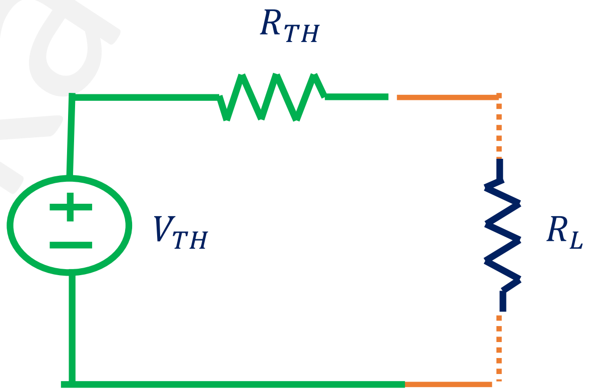




THEVENIN'S THEOREMS

Calculate the current flowing through 3 ohms resistor using Thevenin's Theorems





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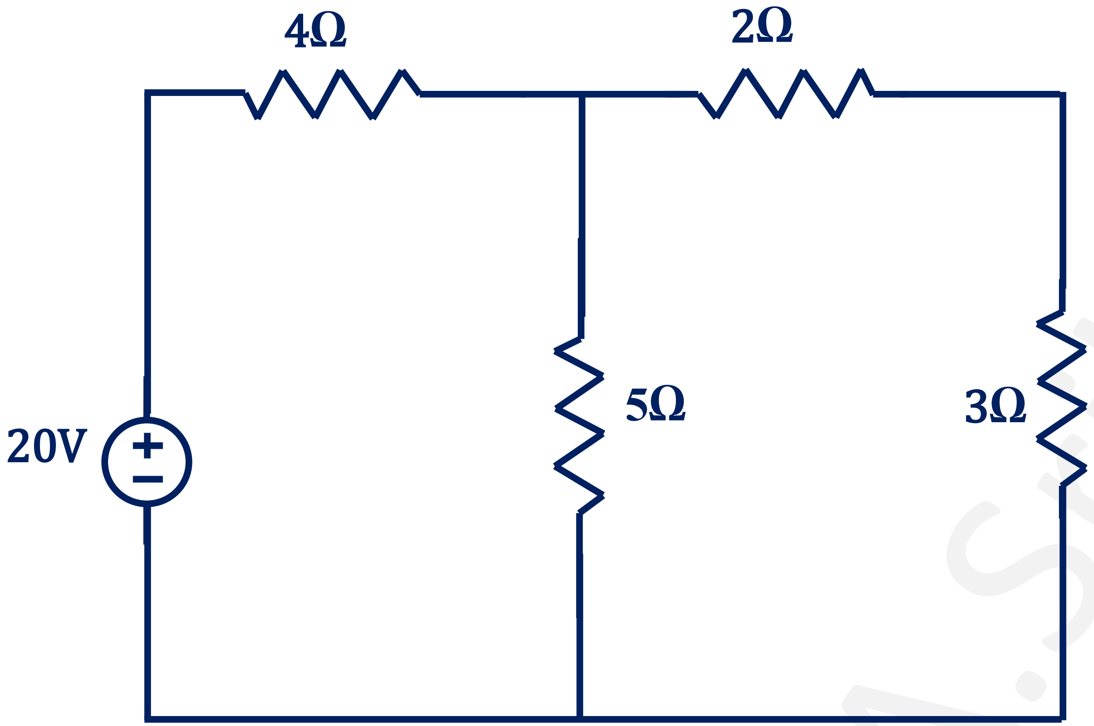
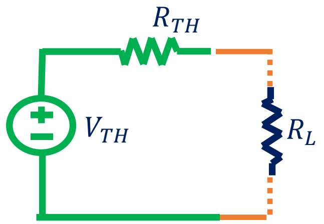
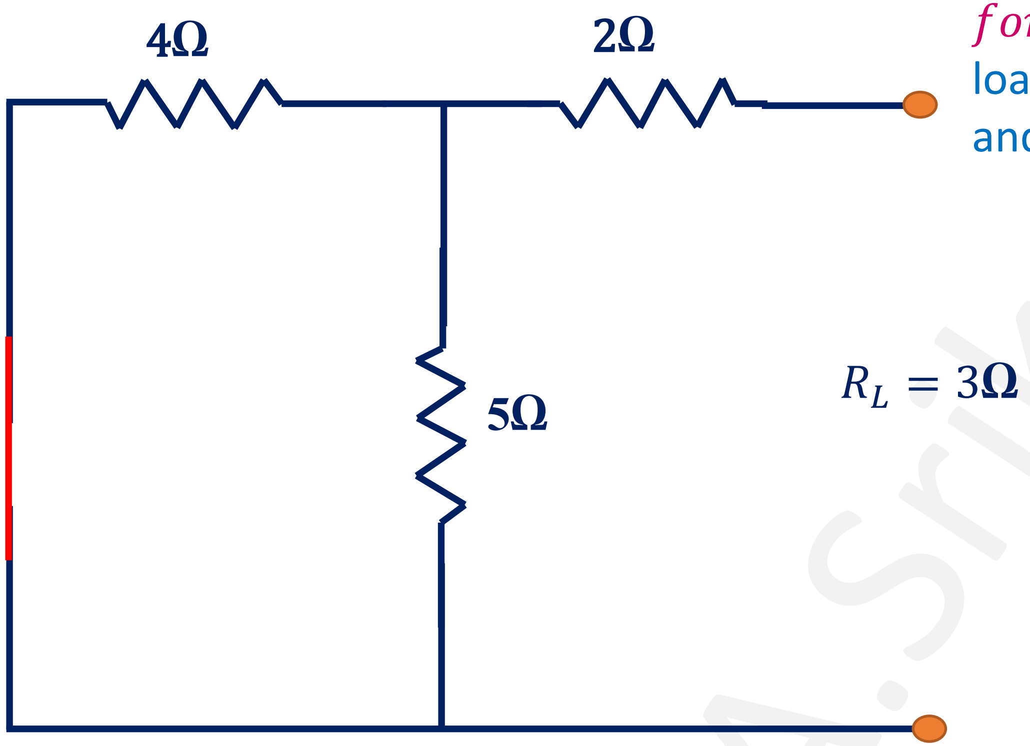
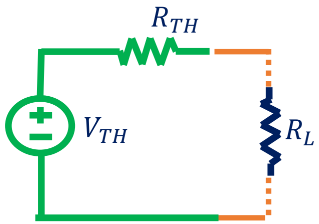




$for\ R_{TH}$ Thevenin's equivalent resistance measured across the load when all the voltage sources are replaced by short circuit and current sources are replaced by open circuit.

Calculate the current flowing through 3 ohms resistor using Thevenin's Theorems





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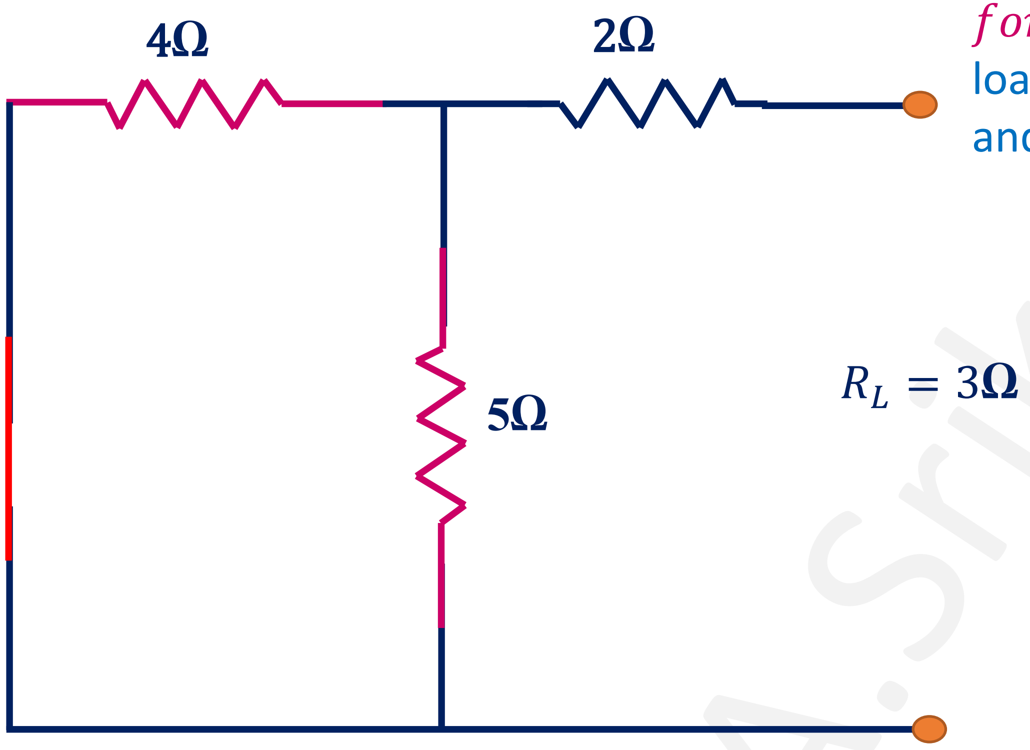

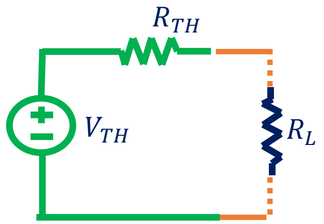

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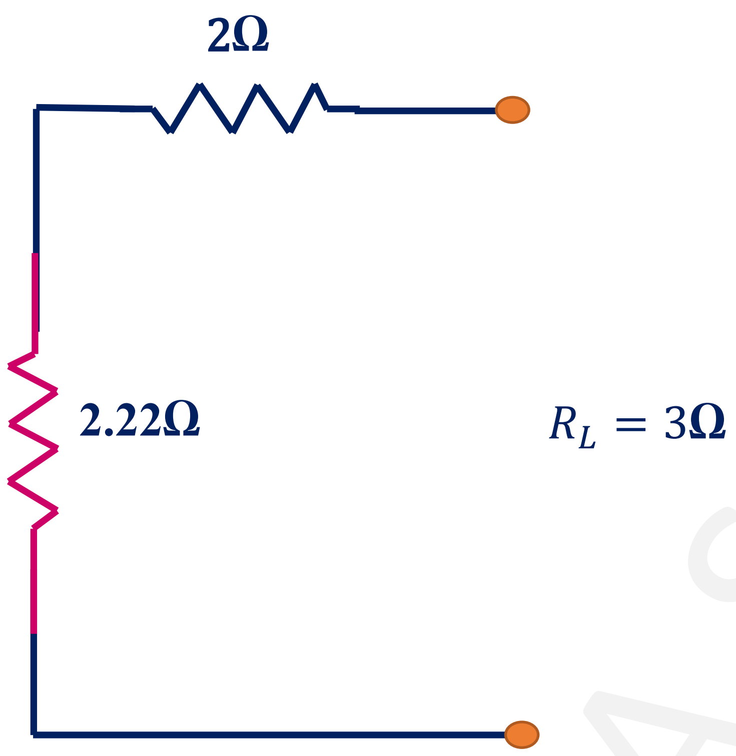
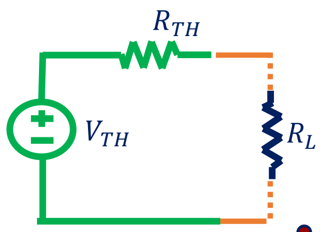
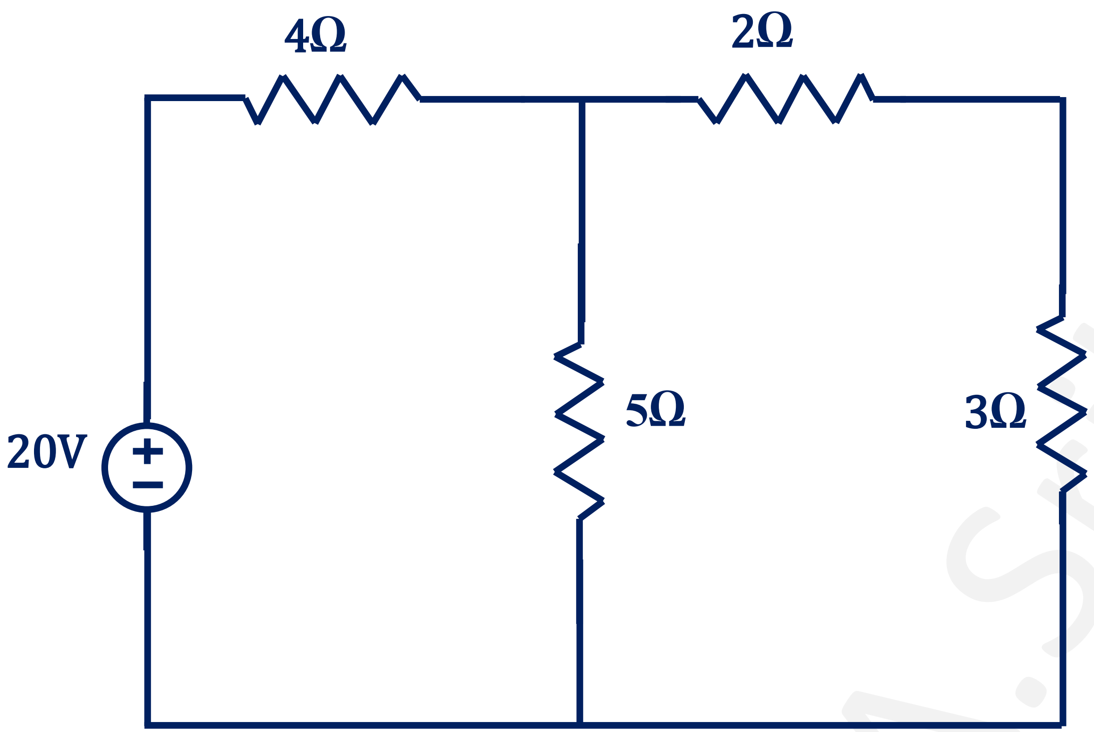




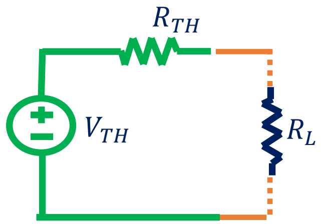
$for\ R_{TH}$ Thevenin's equivalent resistance measured across the load when all the voltage sources are replaced by short circuit and current sources are replaced by open circuit.

$$R_{TH} = 4.22\Omega$$

Calculate the current flowing through 3 ohms resistor using Thevenin's Theorems



V_{th} or V_{oc} is the open circuited voltage measured between the load terminals



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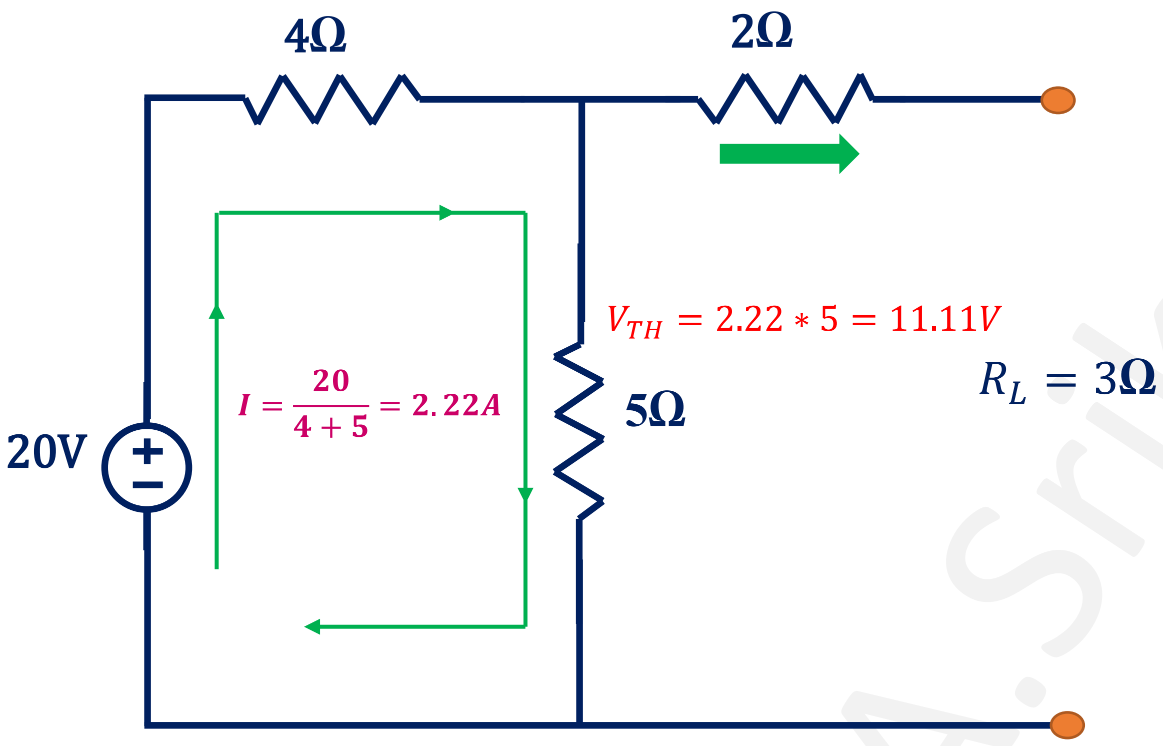
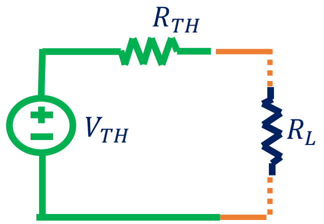
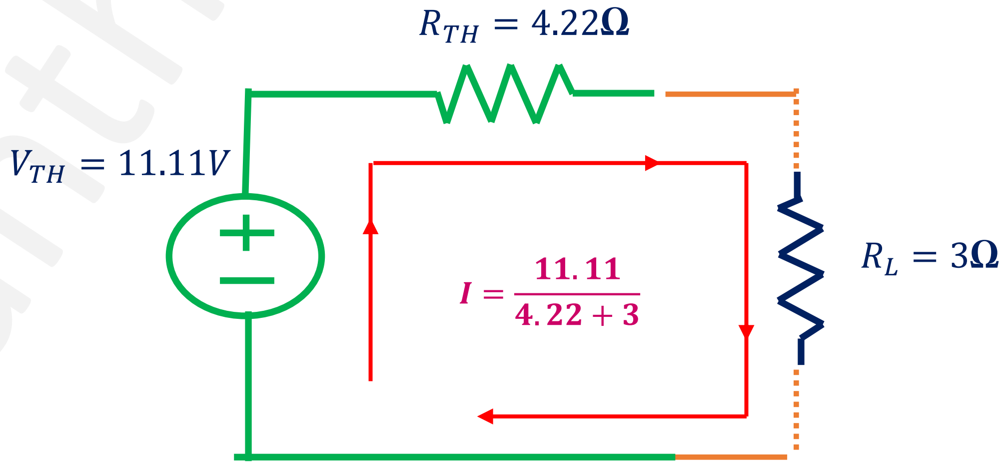
Calculate the current flowing through 3 ohms resistor using Thevenin's Theorems





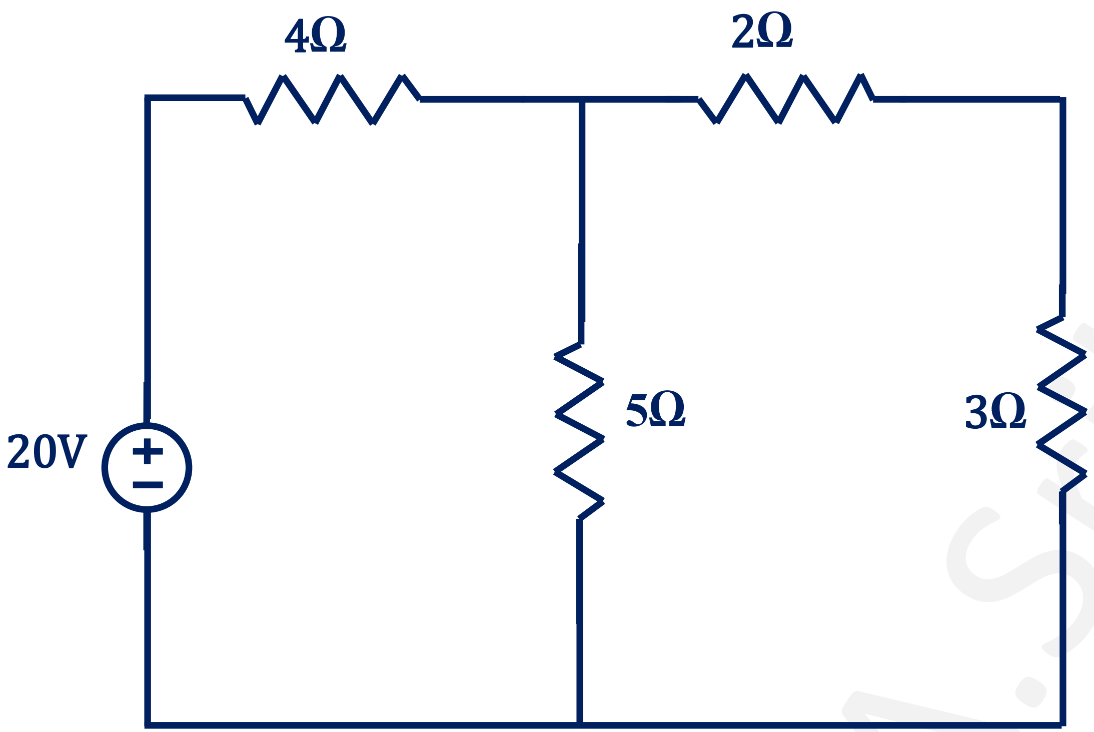
$R_{TH} = 4.22\Omega$

$V_{TH} = 11.11V$



$I_{3\Omega} = 1.538A$

Calculate the current flowing through 3 ohms resistor using Thevenin's Theorems



$R_{TH} = 4.22\Omega$

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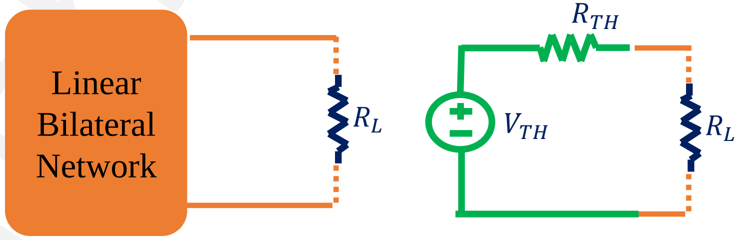

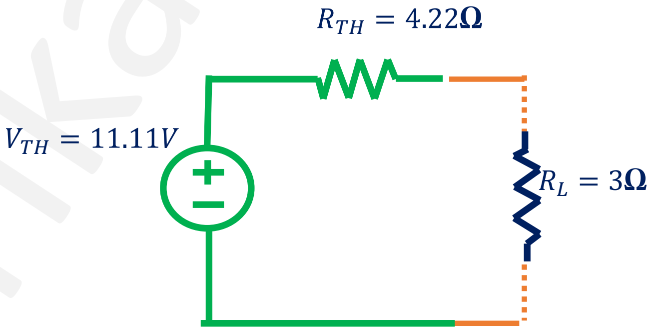

$I_{3\Omega} = 1.538A$

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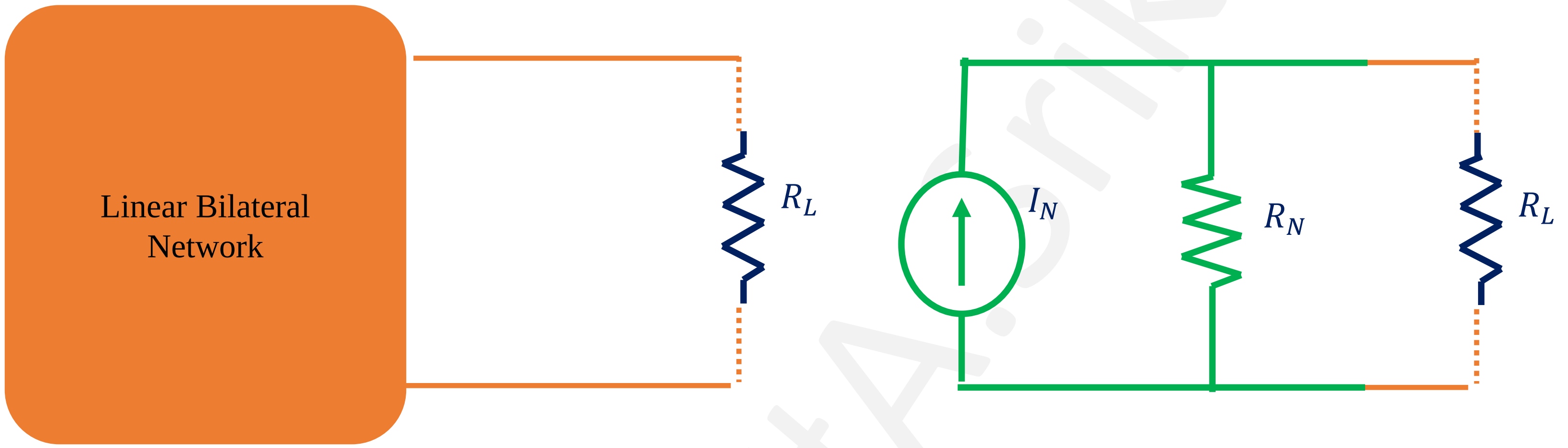
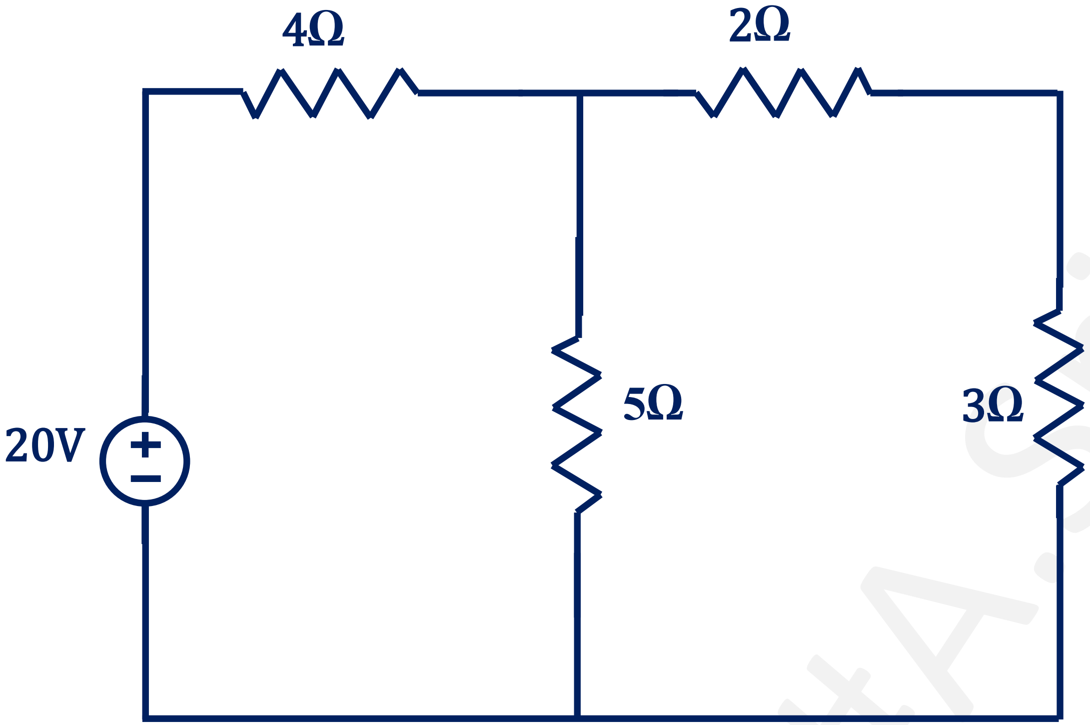
NORTON'S THEOREMS

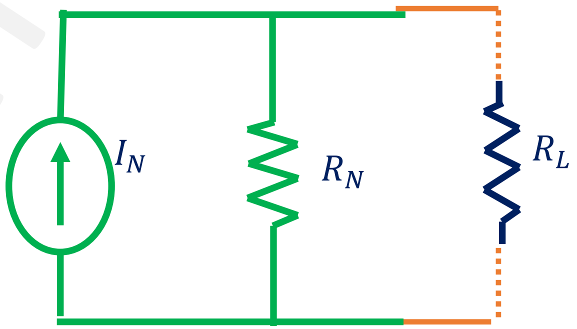
Norton's Theorems states that "Any linear bilateral network (AC or DC) containing several voltages, currents and resistances can be replaced by just one single current source (I_N) in parallel with a single resistance (R_N) connected across the load". Where I_N or I_{sc} is the short-circuited current measured between the load terminals & R_N is the Norton's equivalent resistance measured across the load when all the voltage sources are replaced by short circuit and current sources are replaced by open circuit.



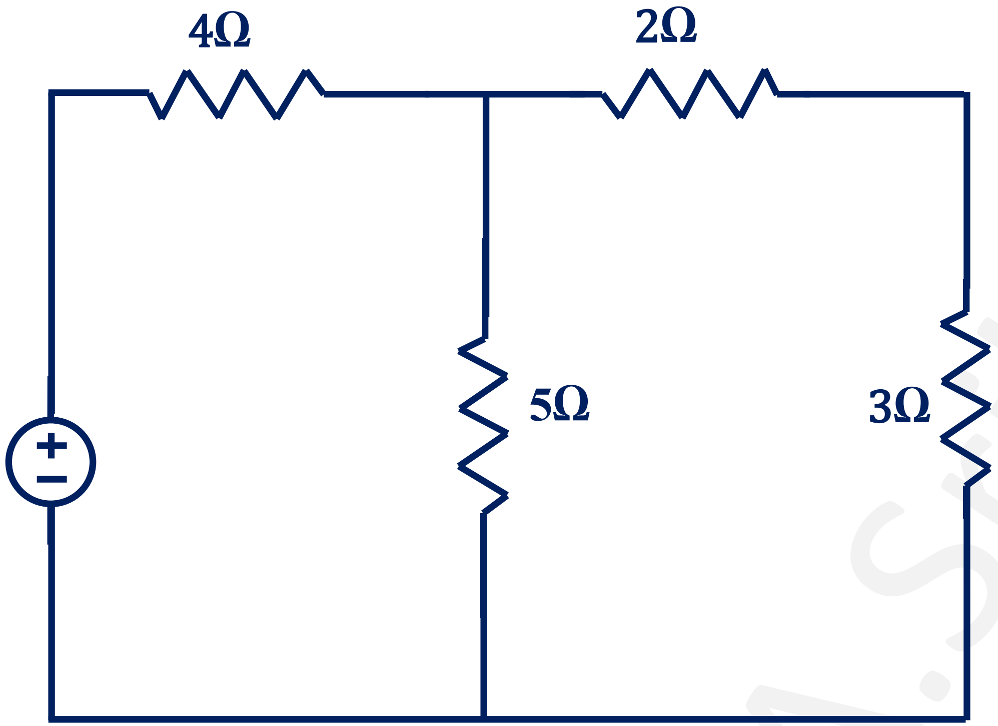
NORTON'S THEOREMS

Calculate the current flowing through 3 ohms resistor using Norton's Theorems

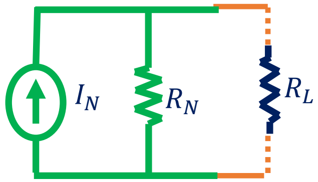




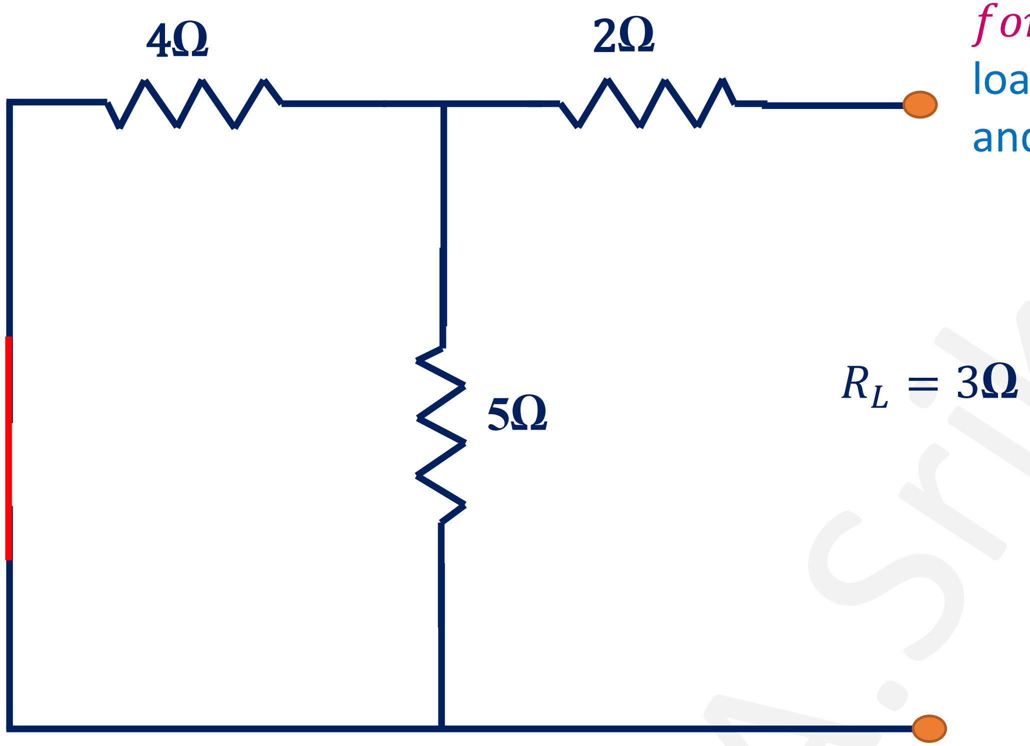
Calculate the current flowing through 3 ohms resistor using Norton's Theorems

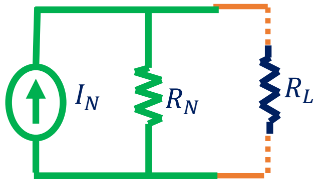


$for\ R_N$ Norton's equivalent resistance measured across the load when all the voltage sources are replaced by short circuit and current sources are replaced by open circuit.



Calculate the current flowing through 3 ohms resistor using Norton's Theorems





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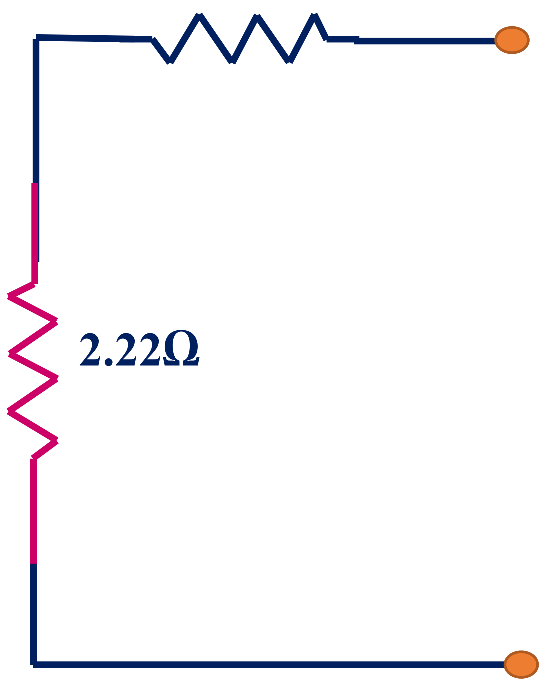
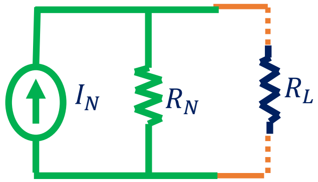
Calculate the current flowing through 3 ohms resistor using Norton's Theorems



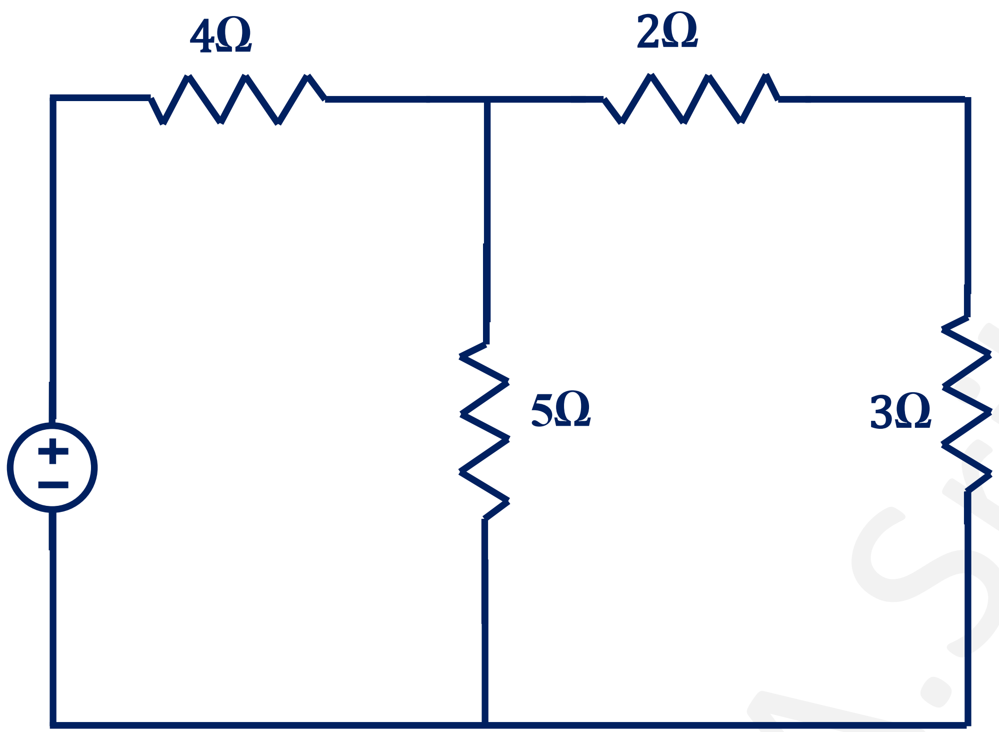
$R_L = 3\Omega$



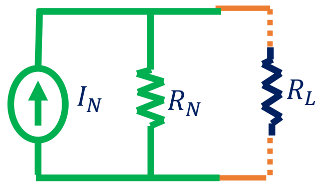
$for\ R_N$ Norton's equivalent resistance measured across the load when all the voltage sources are replaced by short circuit and current sources are replaced by open circuit.

$$R_N = 4.22\Omega$$

Calculate the current flowing through 3 ohms resistor using Norton's Theorems

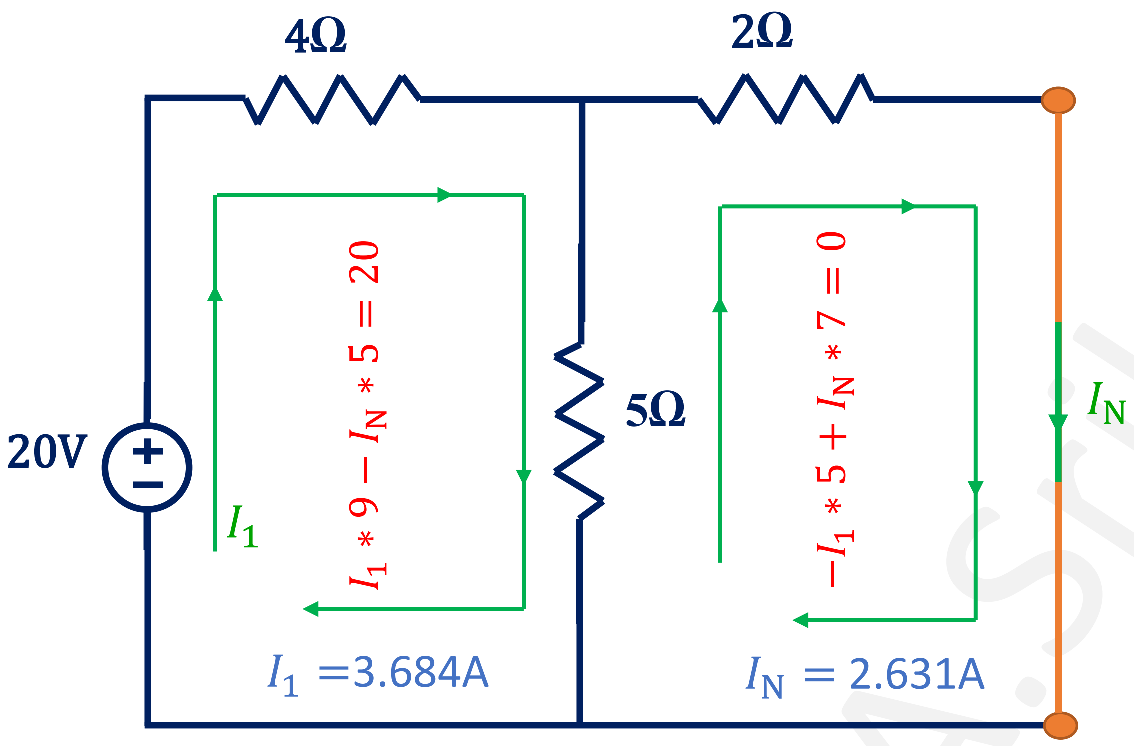


I_N or I_{sc} is the short-circuited current measured between the load terminals



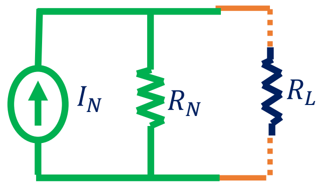
$R_N = 4.22\Omega$

Calculate the current flowing through 3 ohms resistor using Norton's Theorems



I_1 =3.684A

I_N = 2.631A



$R_N = 4.22\Omega$

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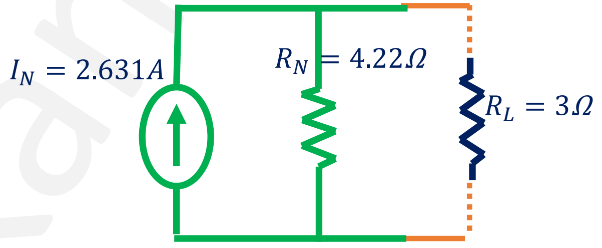
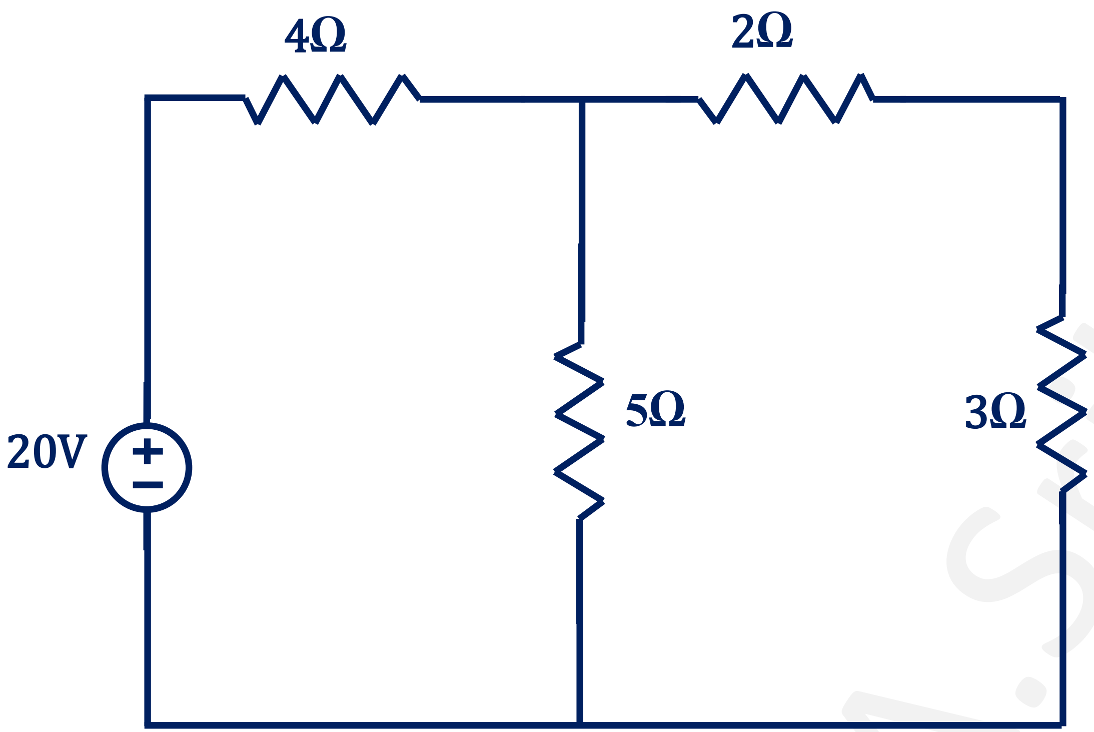
I_N or I_{sc} is the short-circuited current measured between the load terminals



$$I_{3\Omega} = I_N * \frac{R_N}{R_N + R_L} = 2.631 * \frac{4.22}{4.22 + 3}$$

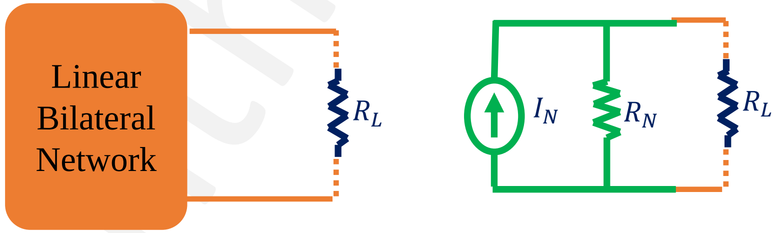
$$I_{3\Omega} = 1.538A$$

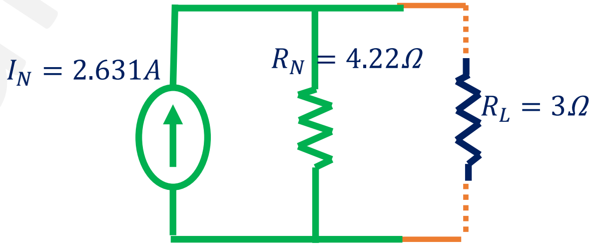
Calculate the current flowing through 3 ohms resistor using Norton's Theorems



$R_N = 4.22\Omega$

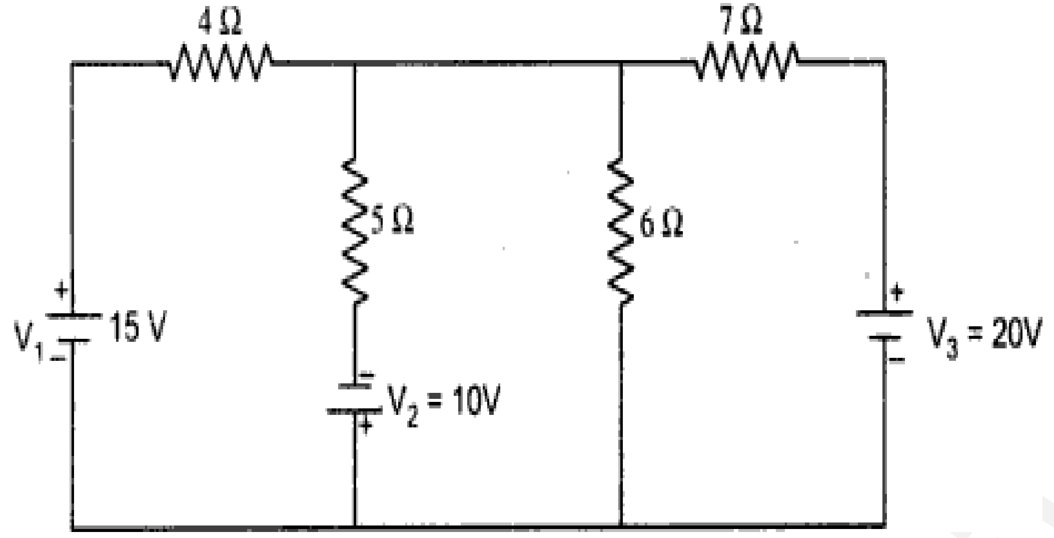
$I_N = 2.631A$





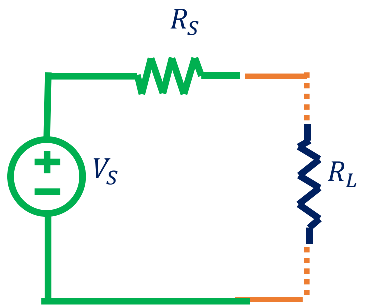
$I_{3\Omega} = 1.538A$

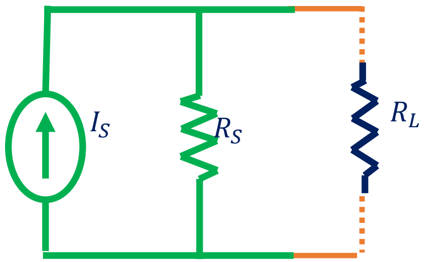
Determine the current through the 6Ω resistor using Thevenin's theorem.



Maximum Power Transfer Theorem

The maximum power transfer theorem states that in a linear , bilateral network , maximum power is delivered to the load when the load resistance is equal to the internal resistance of a source





$$R_S = R_L$$

$$\frac{d}{dx}\frac{u}{v} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

If it is an independent voltage source, then its series resistance (internal resistance Rs) or if it is independent current source, then its parallel resistance (internal resistance Rs) must equal to the load resistance RL to deliver maximum power to the load.

$$I = \frac{V_S}{R_S + R_L}$$

$$P = {V_S}^2 * \frac{R_L}{(R_S + R_L)^2}$$

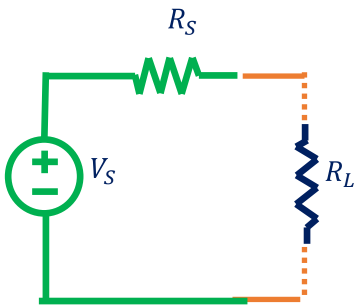
$$\frac{dP}{dR_L} = 0$$

$$\frac{d}{dR_L}{V_S}^2 * \frac{R_L}{(R_S + R_L)^2} = 0$$

$$P = I^2 * R = \left(\frac{V_S}{R_S + R_L}\right)^2 * R_L$$

$${V_S}^2 * \frac{(R_S + R_L)^2 - 2R_L(R_S + R_L)}{(R_S + R_L)^4} = 0$$

Maximum Power Transfer Theorem



$R_S = R_L$

$V_S = 10v, R_S = 5\Omega$

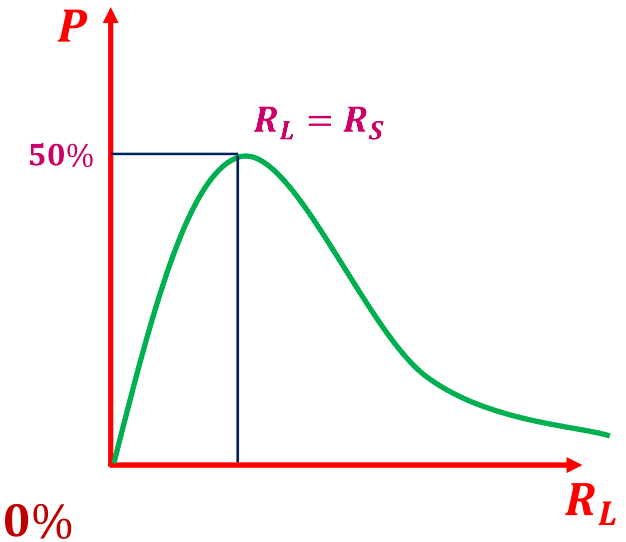
$R_L = 1; I = 1.667A; P = 2.778W$

$R_L = 3; I = 1.25A; P = 4.68W$

$R_L = 5; I = 1A; P = 5W$

$R_L = 7; I = 0.833A; P = 4.86W$

$R_L = 9; I = 0.714A; P = 4.59W$



$$I = \frac{V_S}{R_S + R_L}$$

$$P = \left(\frac{V_S}{R_S + R_L}\right)^2 * R_L$$

$P = 10 * 1 = 10W$ $\quad P = \left(\frac{10}{5+5}\right)^2 * 5 = 5W$ $\quad$ Efficiency $(\eta) = 50\%$

$R_S = R_L = R$

$$\text{Efficiency } (\eta) = \frac{P_o}{P_i} * 100 = \frac{I^2 * R_L}{I^2(R_S + R_L)} * 100$$

$$P = \frac{{V_S}^2}{4R}$$

$$\text{Efficiency } (\eta) = \frac{1}{2} * 100$$

Efficiency $(\eta) = 50\%$

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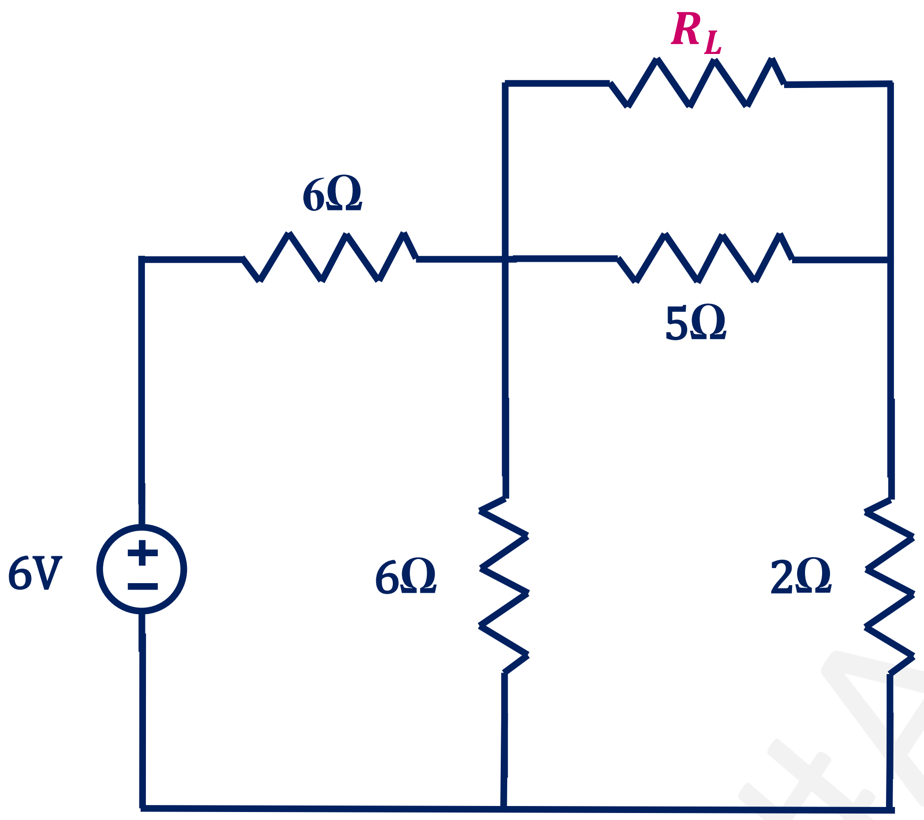
Find the value of R_L for the given network below that the power is maximum? And also find the Max Power through load-resistance R_L by using maximum power transfer theorem?



The maximum power transfer theorem states that in a linear , bilateral network , maximum power is delivered to the load when the load resistance is equal to the internal resistance of a source

$$R_S = R_L$$

Maximum Power Transfer Theorem

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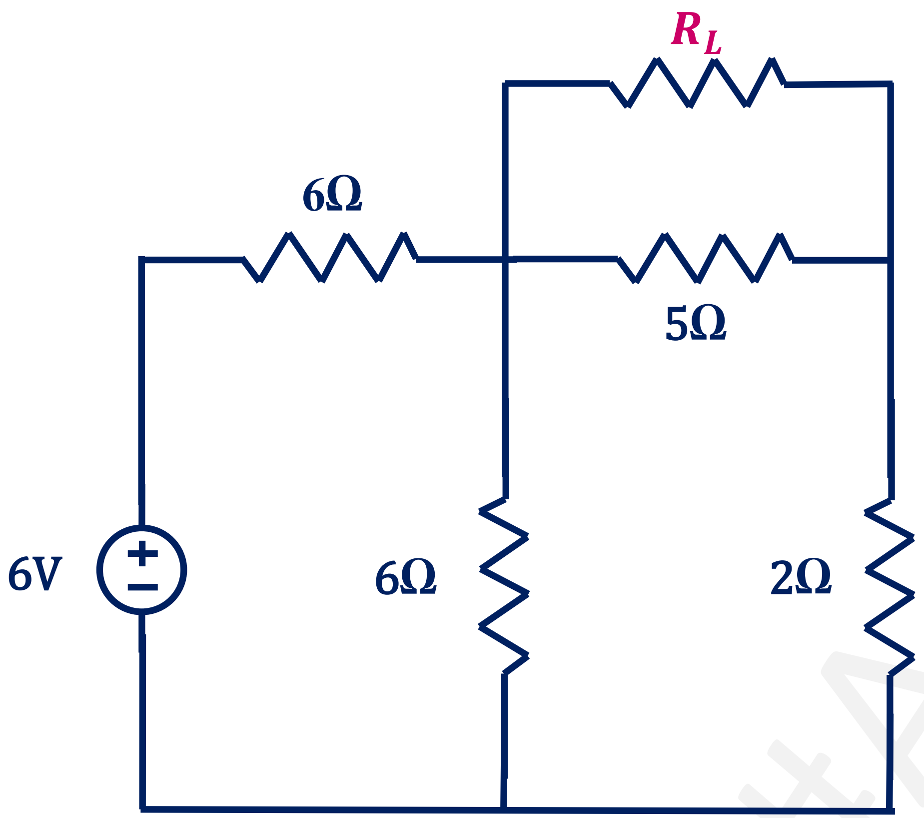

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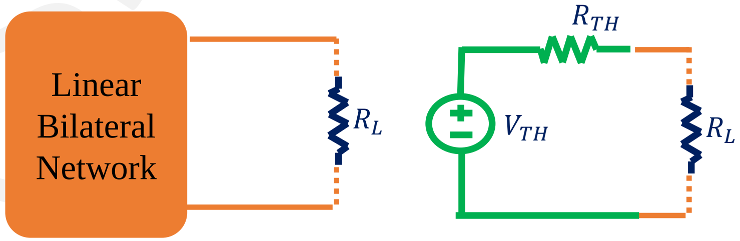

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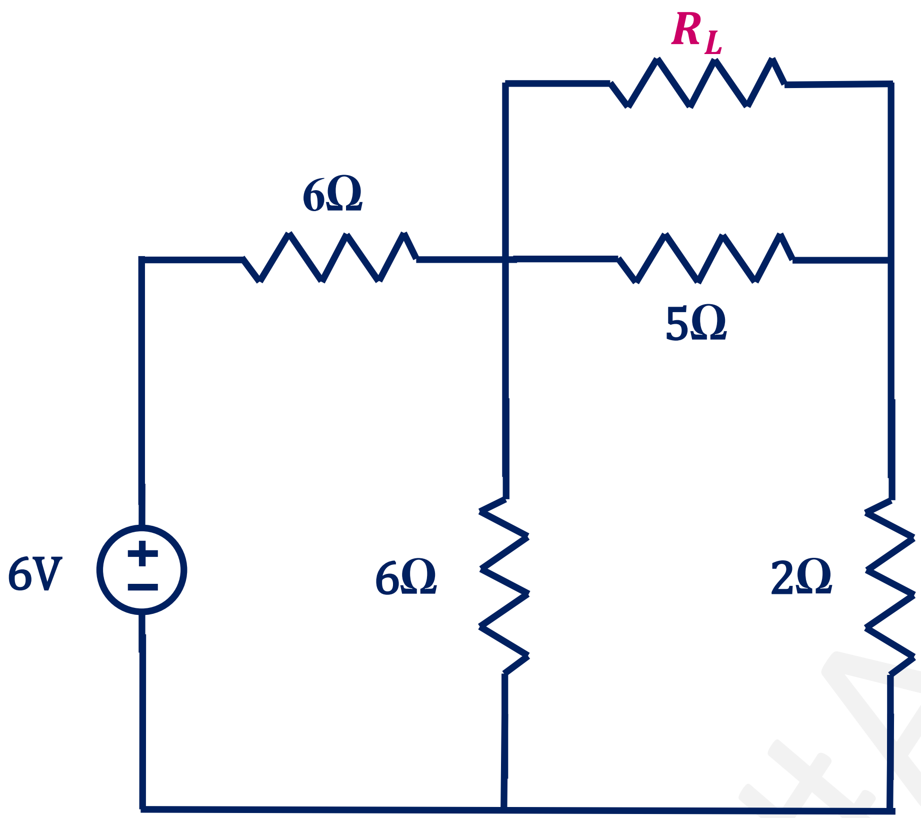
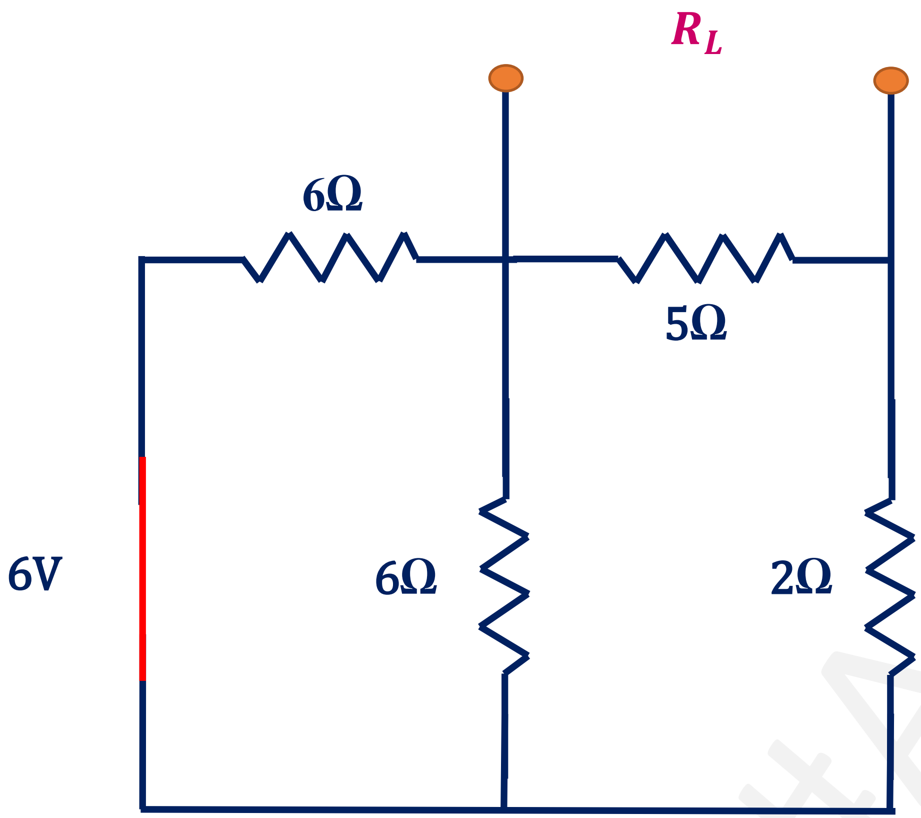


$for\ R_{TH}$ Thevenin's equivalent resistance measured across the load when all the voltage sources are replaced by short circuit and current sources are replaced by open circuit.

Find the value of R_L for the given network below that the power is maximum? And also find the Max Power through load-resistance R_L by using maximum power transfer theorem?



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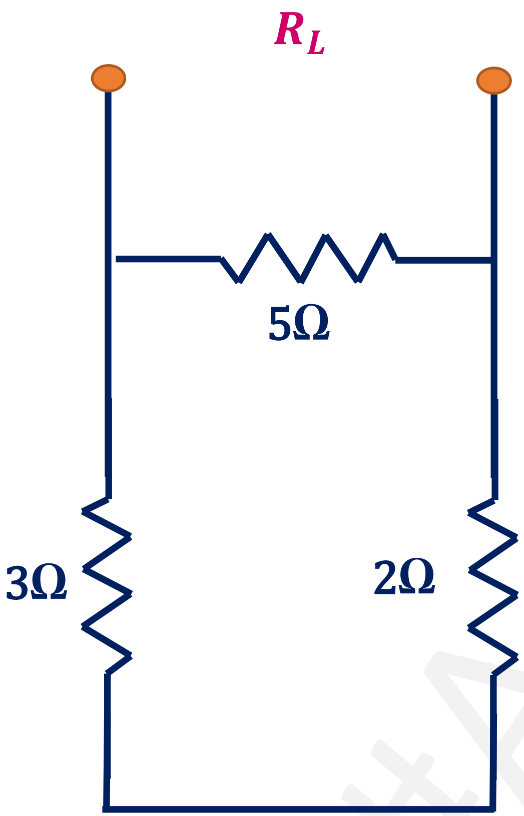

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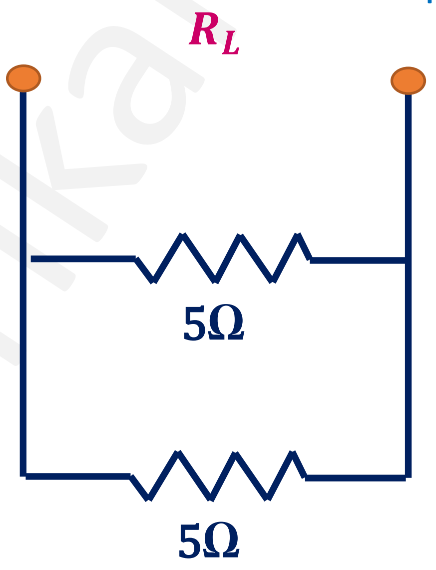
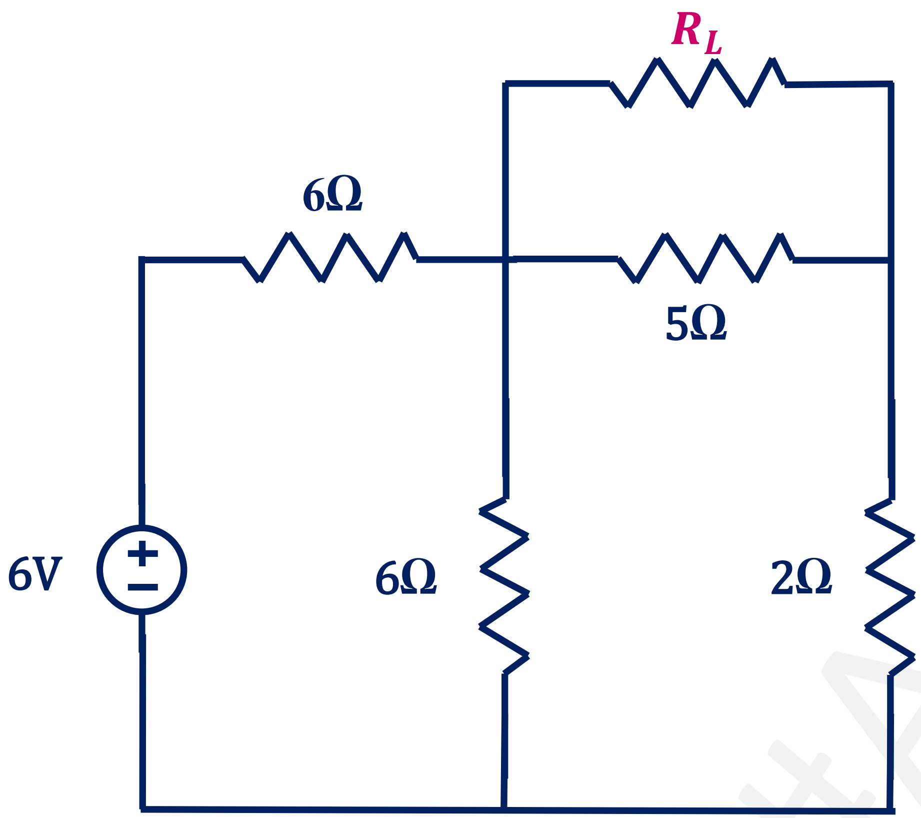


$$R_L = 2.5\Omega$$

Maximum Power Transfer Theorem

Find the value of R_L for the given network below that the power is maximum? And also find the Max Power through load-resistance R_L by using maximum power transfer theorem?



$$R_{th} = 2.5\Omega$$

V_{th} or V_{oc} is the open circuited voltage measured between the load terminals

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Maximum Power Transfer Theorem

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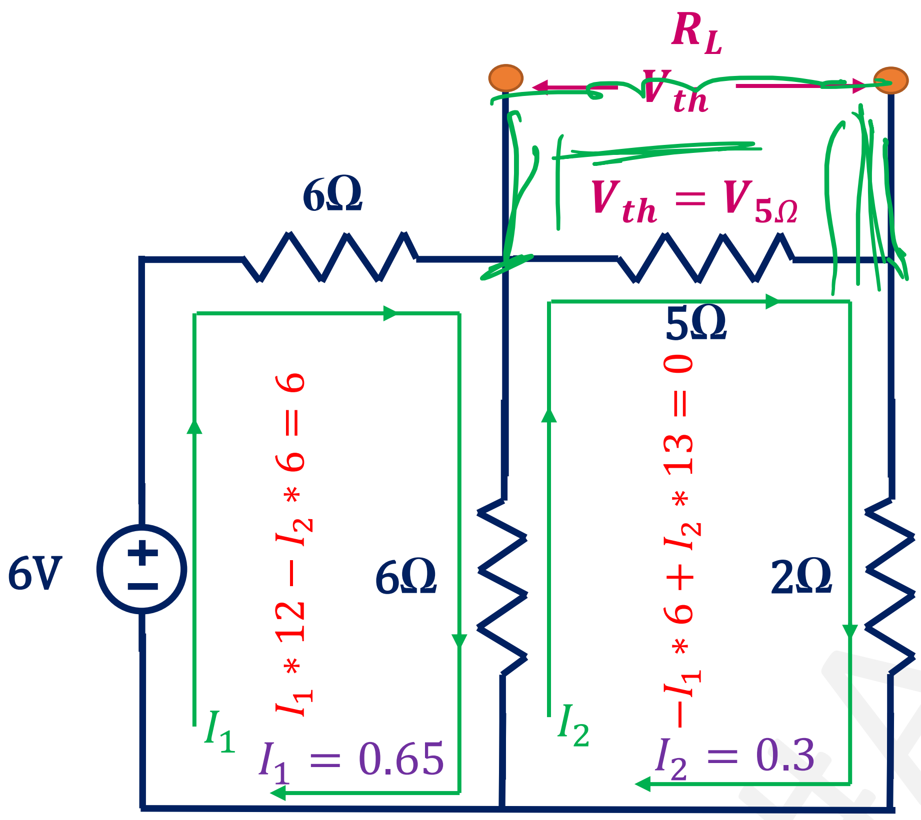
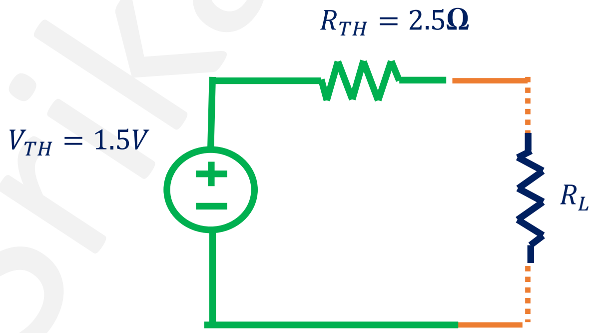


$$R_{th} = 2.5\Omega$$

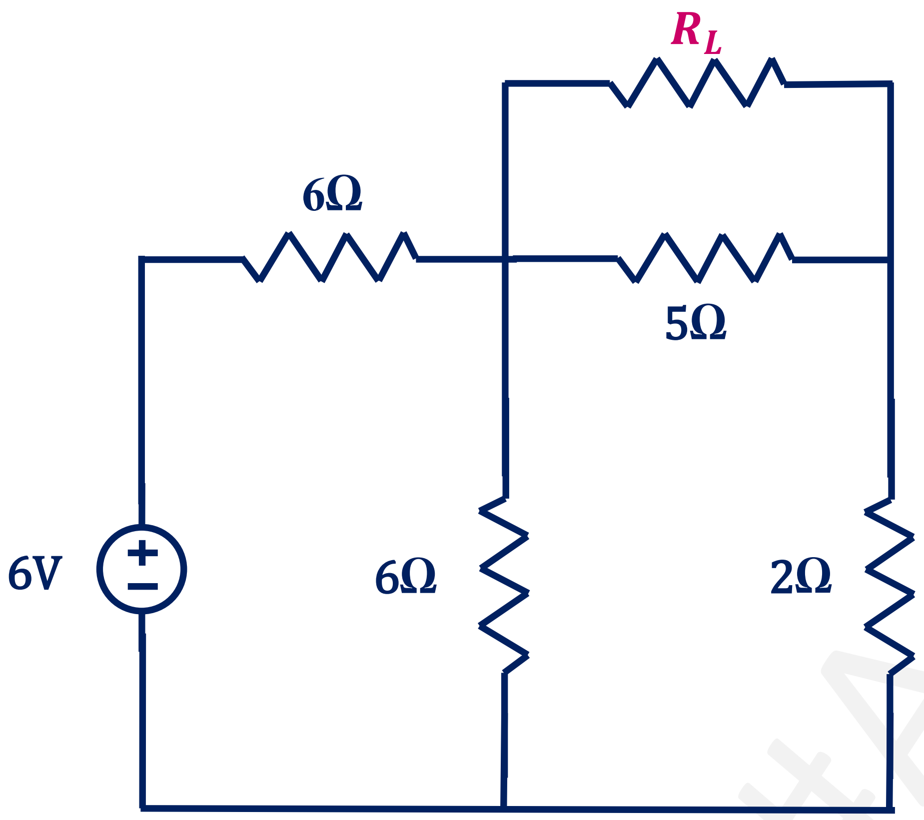
V_{th} or V_{oc} is the open circuited voltage measured between the load terminals

$$V_{th} = V_{5\Omega} = 0.3 * 5 = 1.5V$$



Maximum Power Transfer Theorem

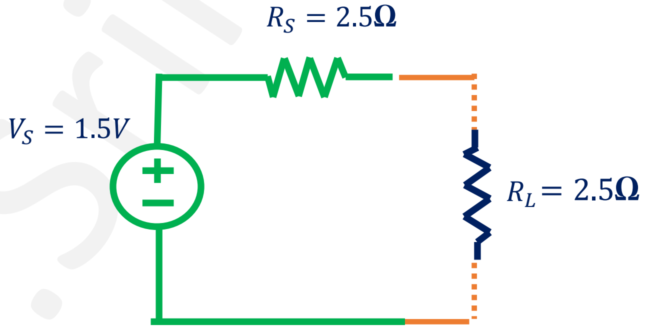
Find the value of R_L for the given network below that the power is maximum? And also find the Max Power through load-resistance R_L by using maximum power transfer theorem?



The maximum power transfer theorem states that in a linear , bilateral network , maximum power is delivered to the load when the load resistance is equal to the internal resistance of a source

$$R_S = R_L$$

$$R_S = R_L = 2.5$$



$$P = \frac{{V_S}^2}{4R} = \frac{1.5^2}{4 * 2.5} = 0.225W$$

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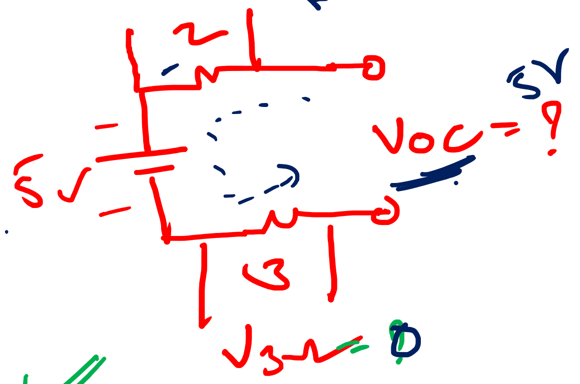
1



$V_{2\Omega} = 0$

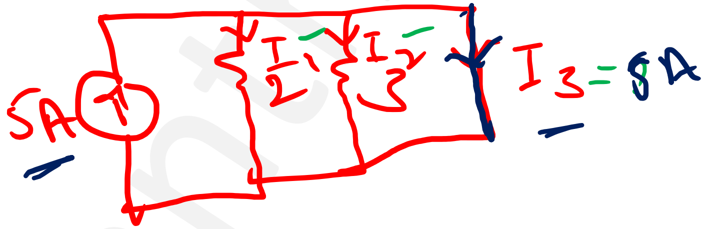
$V_{3\Omega} = 0$

$V_{oc} = 5V$

O/P = $V = V_{max}$, $I = 0$

S/C: $V = 0$, $I = I_{max}$

2

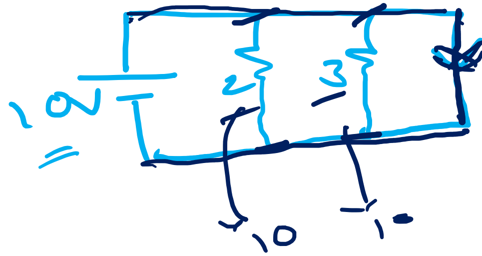


$I_1 = 0$

$I_2 = 0$

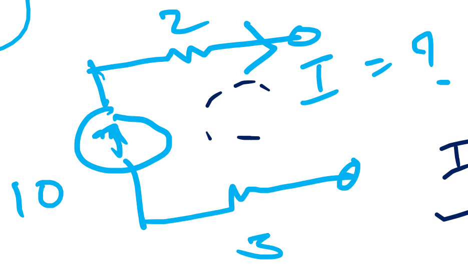
$I_3 = 5A$

3

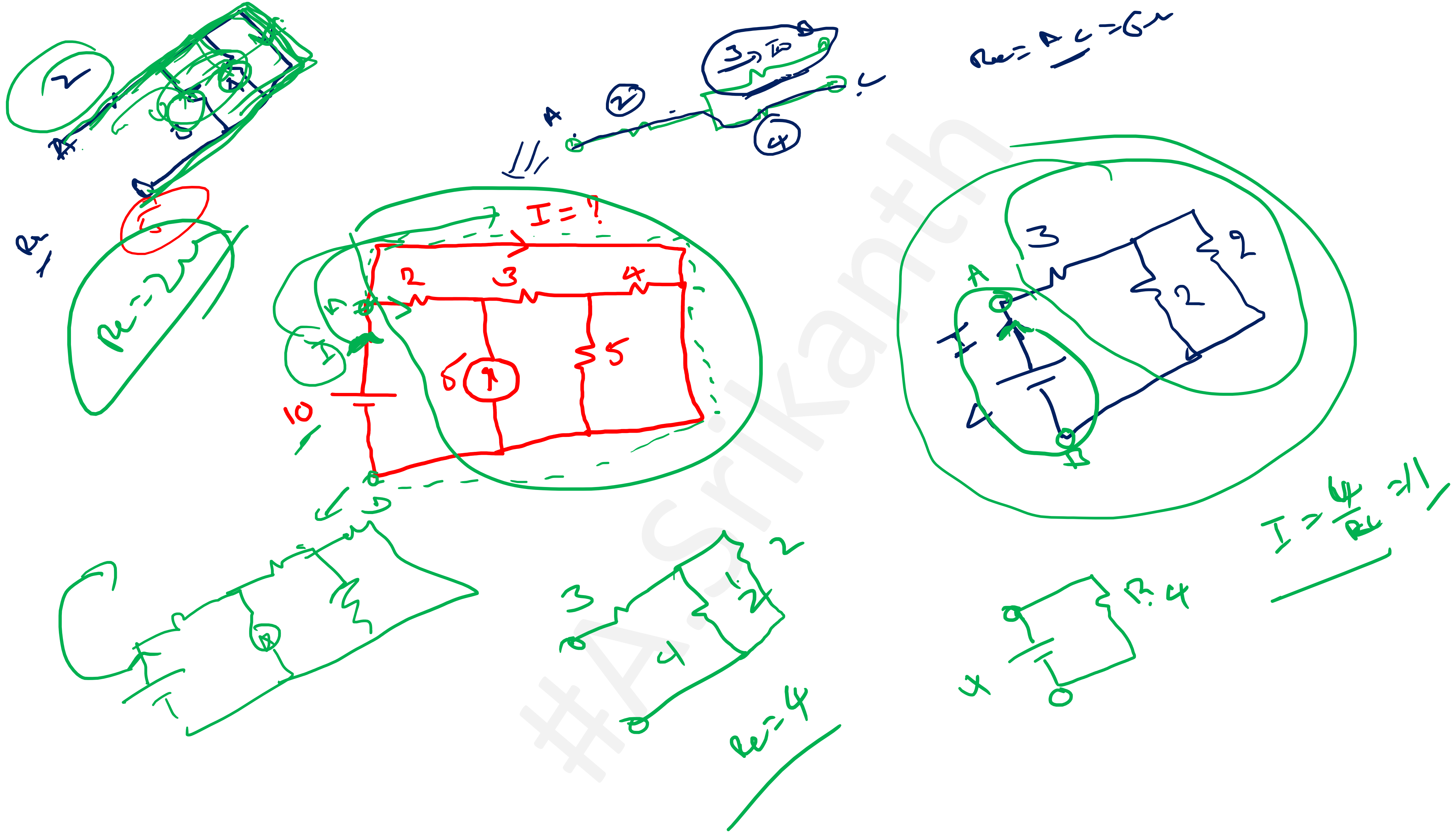


$V = 0$

4

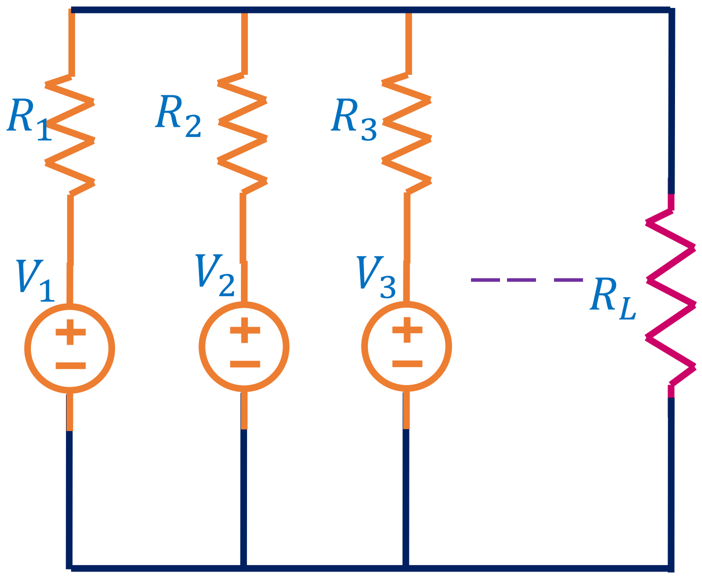


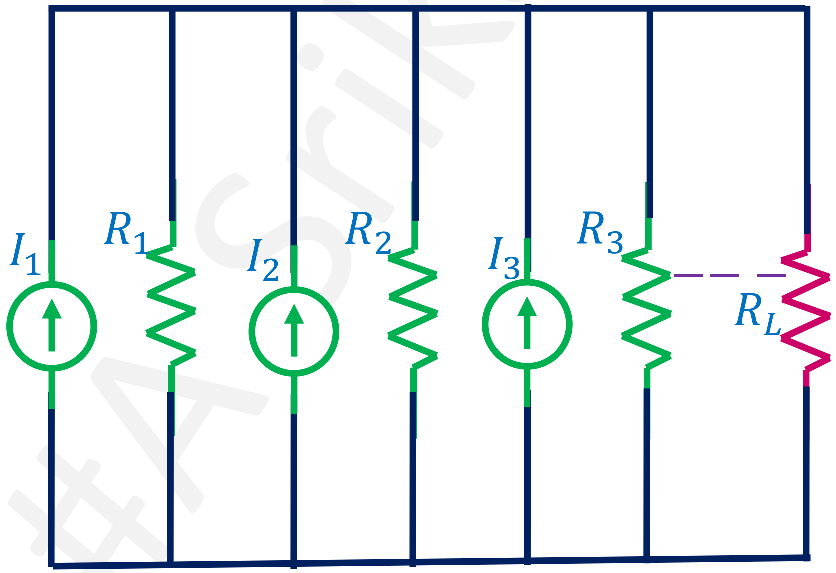
$I = 0$

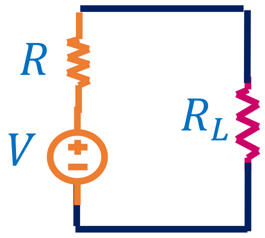

I = ?
2
3
4
5
10
Re = 2
Re = 4
Re = 4 - 6
I = 4/Re = 1

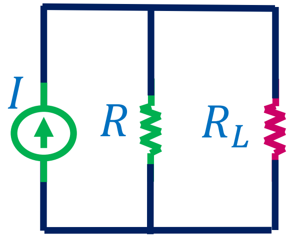
Millman's Theorem | Parallel Generator Theorem

The Millman's Theorem states that When a number of voltage sources are in parallel having with their internal resistance respectively or number of current sources are in parallel having with their internal resistance respectively, the arrangement can replace by a single equivalent voltage source series with an equivalent resistance or current parallel with an equivalent resistance. This theorem is nothing but a combination of Thevenin's Theorem and Norton's Theorem. It is very useful theorem to find out voltage across the load and current through the load.









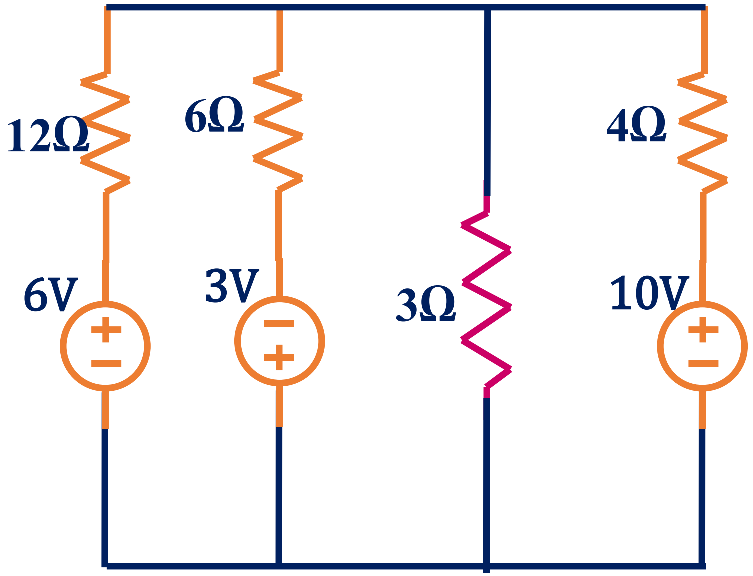
$$I = I_1 + I_2 + I_3 ---$$

$$\frac{1}{R} = \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} ---$$

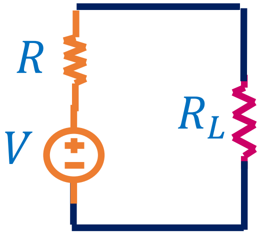
$$\boldsymbol{V} = \boldsymbol{I} * \boldsymbol{R} = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} --- * R_{eq} = \frac{\sum \frac{V}{R}}{\sum \frac{1}{R_{eq}}}$$

Millman's Theorem | Parallel Generator Theorem

Calculate the current flowing through 3 ohms resistor using Millman's Theorems



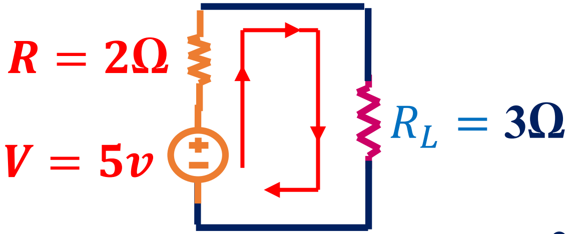
$$\frac{1}{R} = \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} --- -$$



$$\frac{1}{R} = \frac{1}{12} + \frac{1}{6} + \frac{1}{4} = \frac{1}{2} \qquad \boldsymbol{R = 2\Omega}$$

$$V = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} ---- * R_{eq} = \frac{\sum \frac{V}{R}}{\sum \frac{1}{R_{eq}}}$$

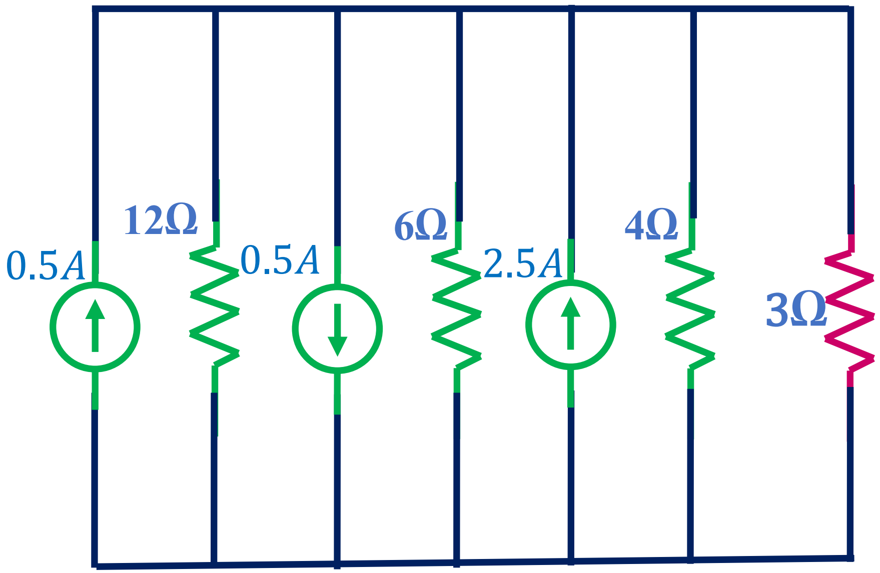
$$V = \frac{6}{12} - \frac{3}{6} + \frac{10}{4} * 2 \qquad \boldsymbol{V = 5v}$$



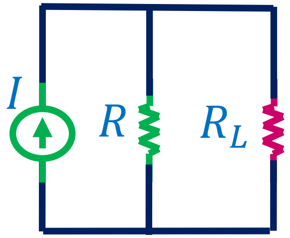
$$I_{3\Omega} = \frac{5}{2+3} = 1A \qquad \boldsymbol{I_{3\Omega} = 1A}$$

Millman's Theorem | Parallel Generator Theorem

Calculate the current flowing through 3 ohms resistor using Millman's Theorems



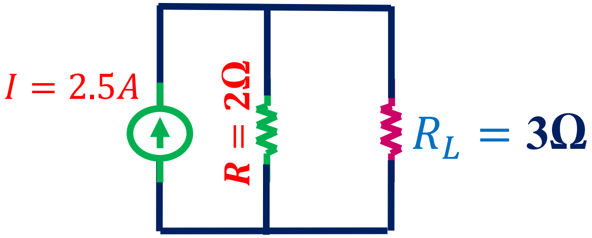
$$\frac{1}{R} = \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} --- -$$



$$\frac{1}{R} = \frac{1}{12} + \frac{1}{6} + \frac{1}{4} = \frac{1}{2} \qquad \boldsymbol{R = 2\Omega}$$

$$I = I_1 + I_2 + I_3$$

$$I = 0.5 - 0.5 + 2.5 \qquad I = 2.5A$$

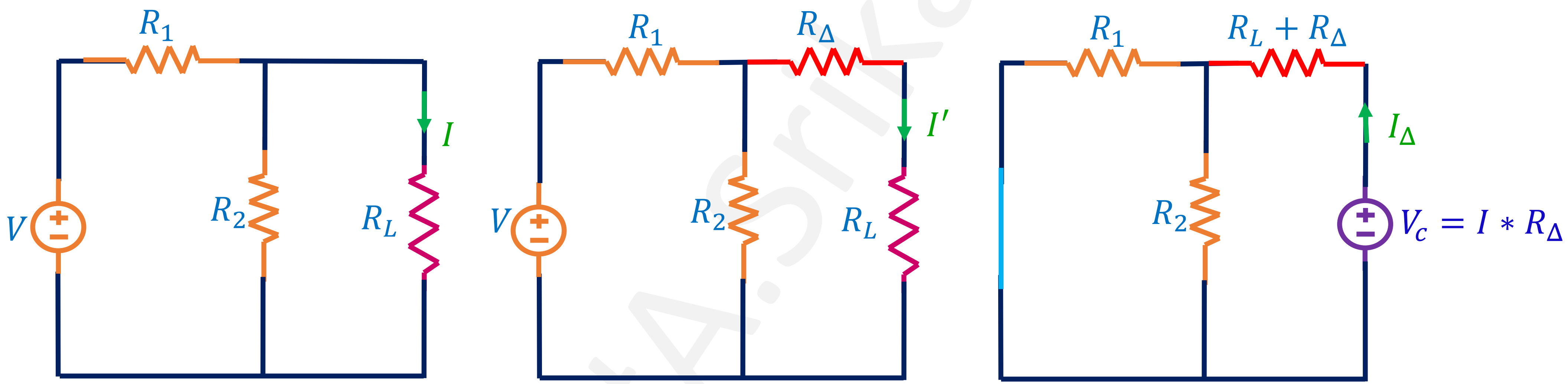


$$I_{3\Omega} = I_N * \frac{R_N}{R_N + R_L} = 2.5 * \frac{2}{2+3}$$

$$I_{3\Omega} = 1A$$

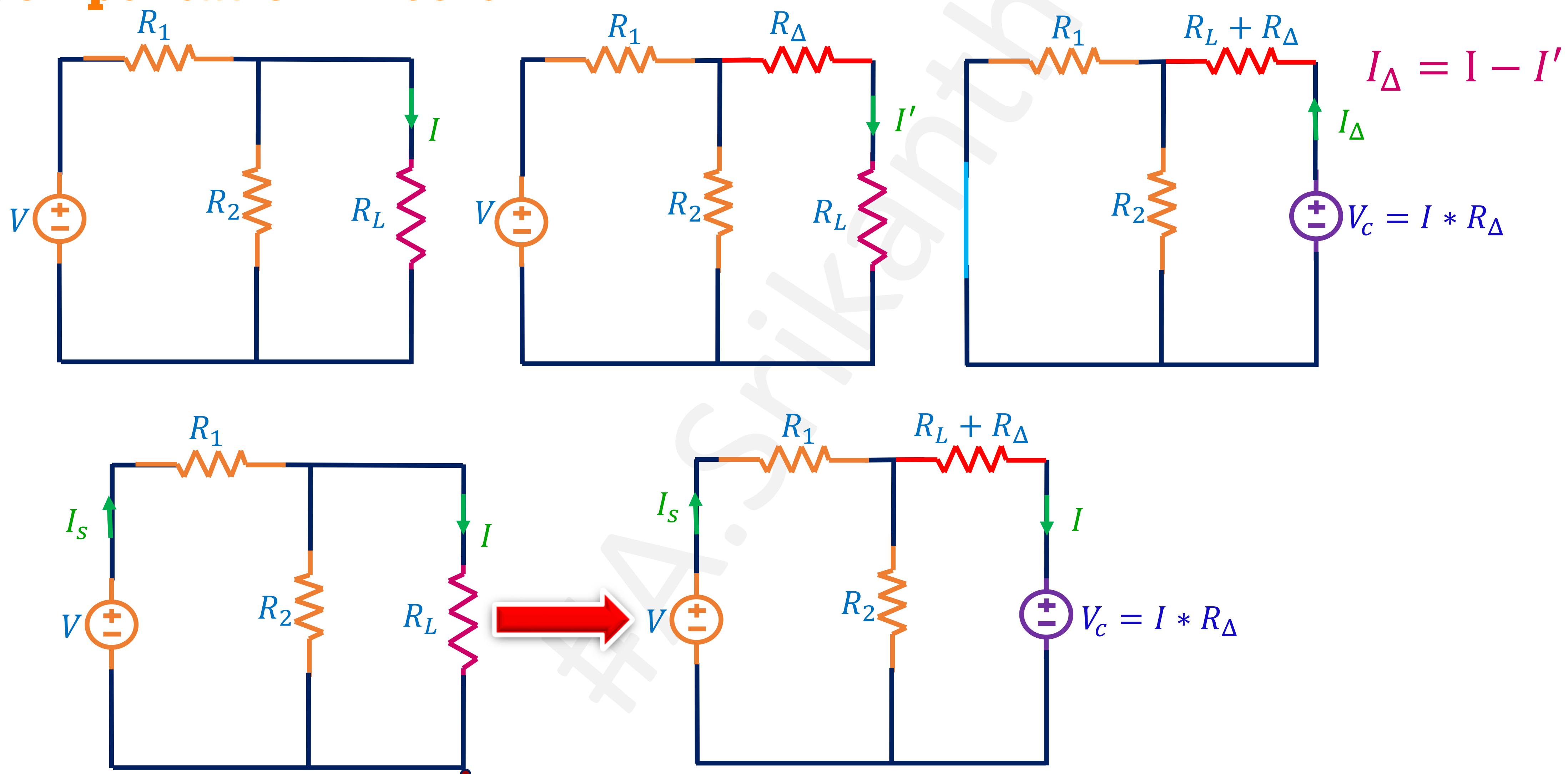
Compensation Theorem

Compensation Theorem states that in a linear time-invariant network when the resistance (R_L) of an uncoupled branch, carrying a current (I), is changed by (R_Δ), then the currents in all the branches would change and can be obtained by assuming that an ideal voltage source of (V_c) has been connected such that V_c = I (R_Δ) in series with (R_L + R_Δ) when all other sources in the network are replaced by their internal resistances.

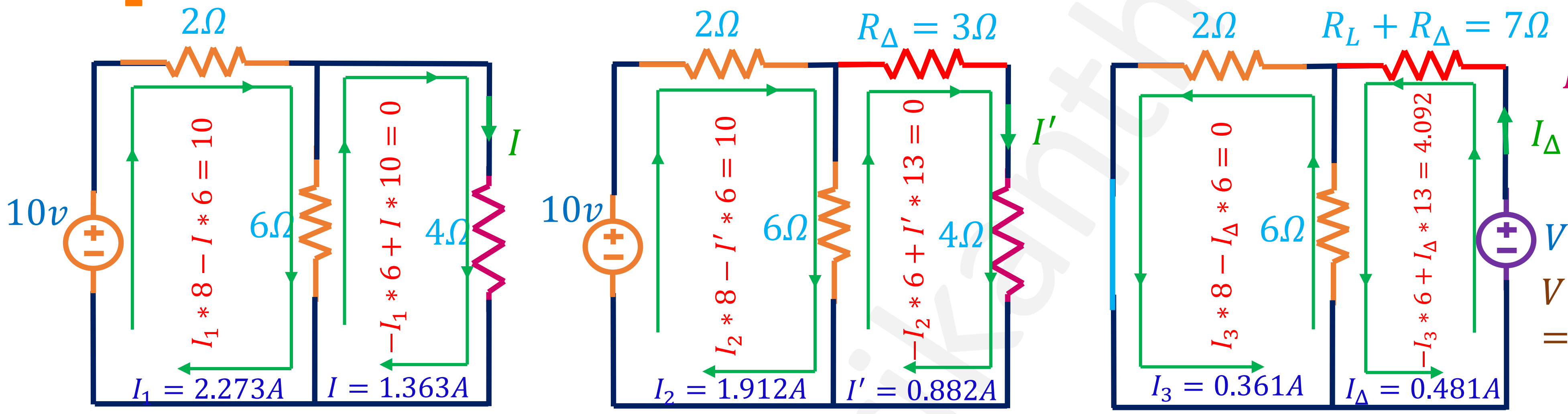


$$I_\Delta = \mathrm{I} - I'$$

Compensation Theorem



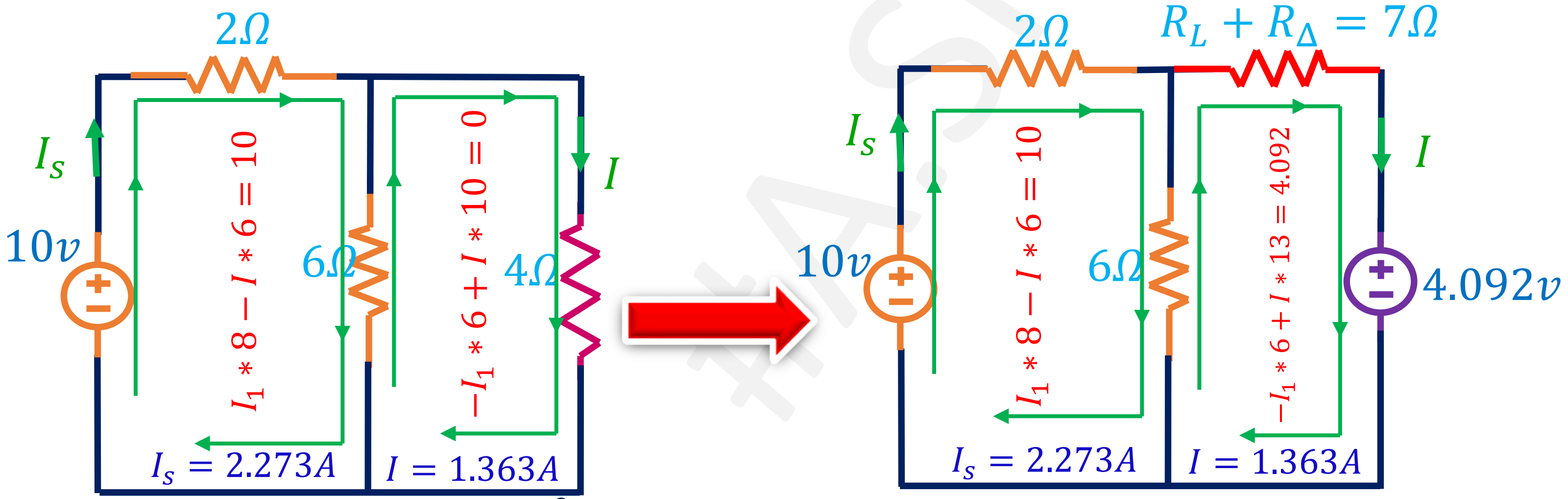
Compensation Theorem



$$I_\Delta = \mathrm{I} - I'$$

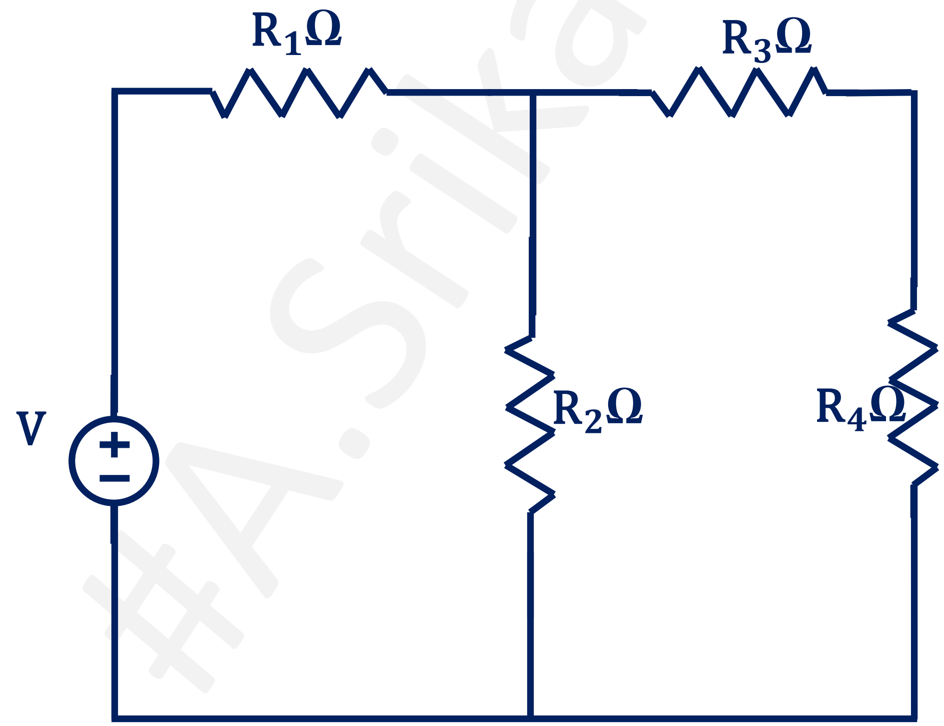
$$V = I * R_\Delta$$

$$V = 1.364 * 3$$
$$= 4.092v$$



TELLEGEN's THEOREM

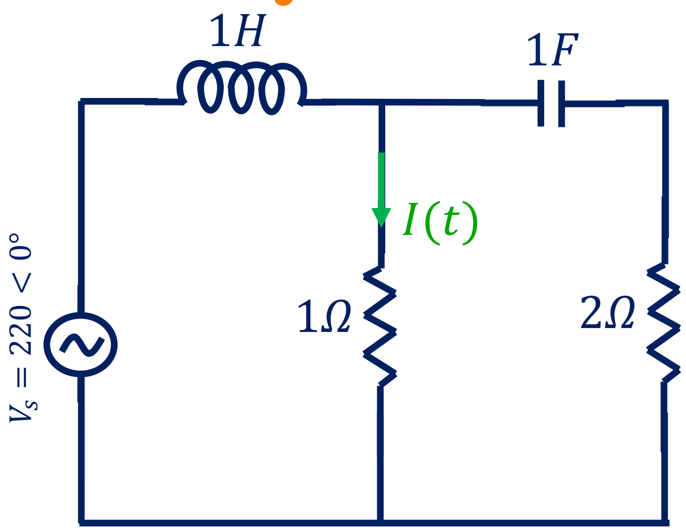
Tellegen's Theorem states that the summation of power delivered is zero for each branch of any electrical network at any instant of time. It is mainly applicable for designing the filters in signal processing. It is also used in complex operating systems for regulating stability. This theorem has been introduced in the year of 1952 by Dutch Electrical Engineer Bernard D.H. Tellegen. This is a very useful theorem in network analysis



$$\sum_{K=1}^{n} V_K * I_K = 0$$

AC CIRCUIT PROBLEM

Mesh Analysis



$V_s = 220 < 0°$ $L = 1H$ $C = 1F$

$X_L = \omega L$

$= 2\pi f L = 2\pi * 50 * 1 = 314.15\Omega$

$X_C = \frac{1}{\omega C}$

$= \frac{1}{2\pi f C} = \frac{1}{2\pi * 50 * 1} = 0.00318\Omega$

$$(1 + j314.15)I_1 - 1I_2 = 220 < 0°$$

$$-1I_1 + (3 - j0.00318)I_2 = 0$$

$$\begin{bmatrix} 1 + j314.15 & -1 \\ -1 & 3 - j0.00318 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 220 < 0° \\ 0 \end{bmatrix}$$

Using Cramer's rule $I_1 = \frac{|\Delta_1|}{|\Delta|}$; $I_2 = \frac{|\Delta_2|}{|\Delta|}$

$$\Delta = \begin{bmatrix} 314.15 < 89.81° & 1 < 180° \\ 1 < 180° & 3 < -0.0607° \end{bmatrix}$$

$$|\Delta| = 942.45 < 89.74° - 1 < 0°$$

$$|\Delta| = 4.276 + j942.44 - 1 + j0$$

$$|\Delta| = 942.44 < 89.80°$$

$$\Delta_1 = \begin{bmatrix} 220 < 0° & 1 < 180° \\ 0 & 3 < -0.0607° \end{bmatrix}$$

$$\Delta_2 = \begin{bmatrix} 314.15 < 89.81° & 220 < 0° \\ 1 < 180° & 0 \end{bmatrix}$$

$$|\Delta| = 942.44 < 89.80°$$

$$|\Delta_1| = 660 < -0.0607°$$

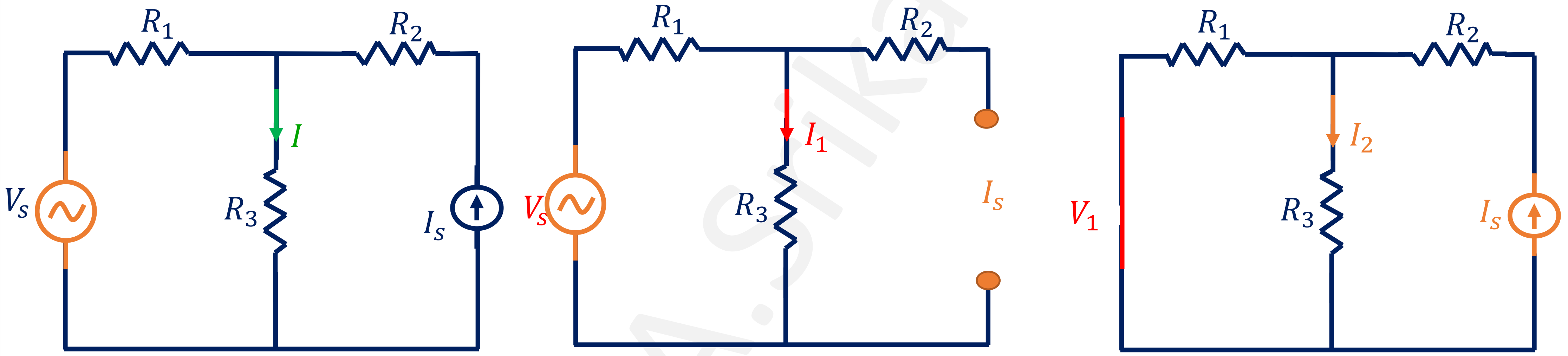
$$|\Delta_2| = -220 < 180°$$

$$I_1 = \mathbf{0.7} < -89.86° \boldsymbol{A}$$

$$I_2 = \mathbf{-0.233} < 90.2° \boldsymbol{A}$$

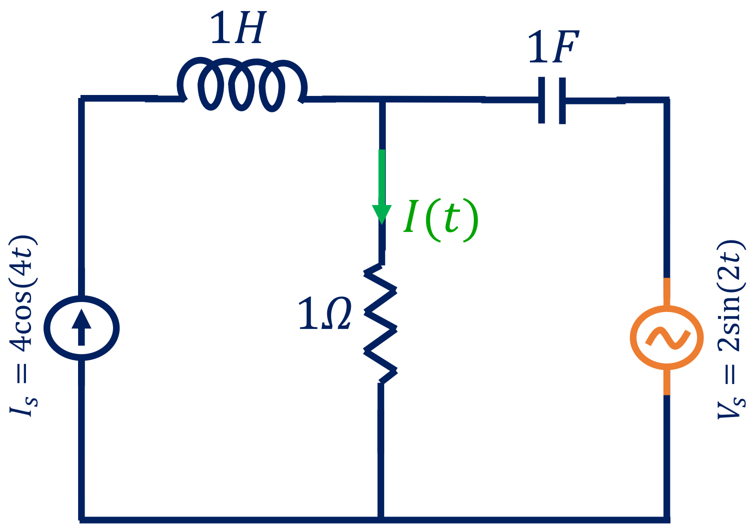
SUPERPOSITION THEOREM

Superposition theorem states that in any linear, active, bilateral network having more than one source, the response across any element is the sum of the responses obtained from each source considered separately and all other sources are replaced by their internal resistance. The superposition theorem is used to solve the network where two or more sources are present and connected



$$I = I_1 + I_2$$

SUPERPOSITION THEOREM



$L = 1H$ $\quad C = 1F$

$I_s = 4\cos(4t)$

$V_s = 2\sin(2t)$

$X_L = \omega L$ $\quad X_C = \dfrac{1}{\omega C}$

$\mathrm{V(t)} = V_m \sin(\omega t) = V_m \sin(\theta)$

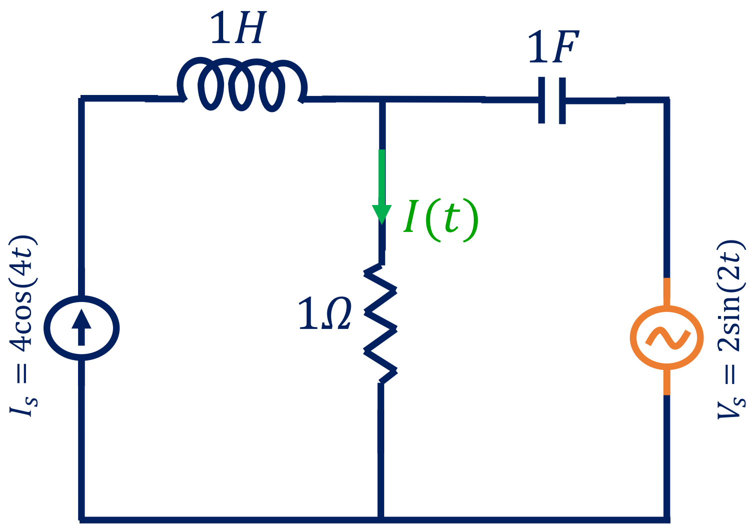
$X_L = 4$

$\mathrm{I(t)} = I_m \sin(\omega t) = I_m \sin(\theta)$

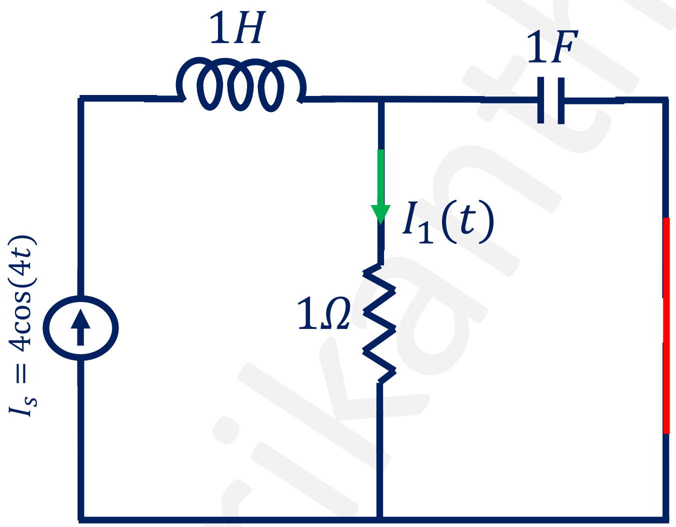
$V_m = 2$

$I_m = 4$

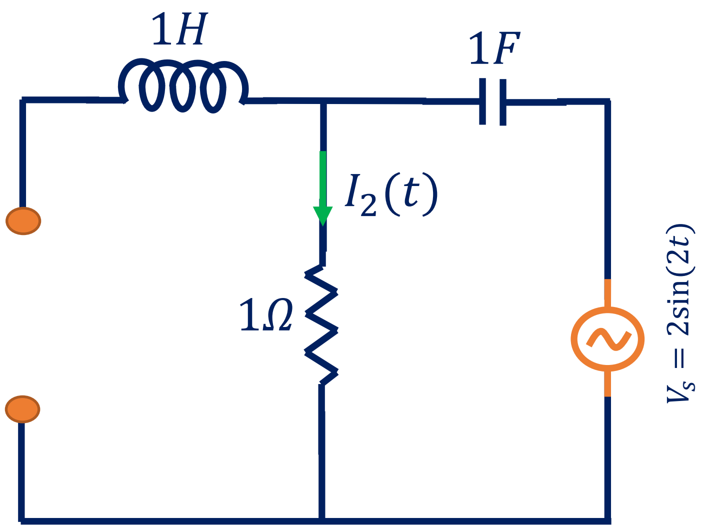
SUPERPOSITION THEOREM



I_s is acting source



V_s is acting source



$I_s = 4\cos(4t)$

$V_s = 2\sin(2t)$

$\mathrm{V(t)} = V_m \sin(\omega t) = V_m \sin(\theta)$

$\mathrm{I(t)} = I_m \sin(\omega t) = I_m \sin(\theta)$

$1H$

$V_m = 2$

$I_m = 4$

$X_L = wL$

$I_{1\Omega} = I_N * \frac{R_N}{R_N + R_L}$

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$k\Omega$ → $10^3\ \Omega$

$L = 10\,mH$
$= 10\times10^{-3}\ H$

$R = 5\Omega$ $L = 10H$

$f = 50\,Hz$

$I = \frac{V}{Z} = \frac{220\angle0}{3141.59\angle1.56}$

$\Rightarrow Z = R + jX_L$

$X_L = \omega L = 2\pi f L$

$X_L = 2\times\pi\times50\times10$

$X_L = 3141.59\,\Omega$

$Z = R + jX_L$

$\boxed{Z = 5 + j\,3141.59}$

$\boxed{Z = 3141.59\angle1.56}$

$r = \sqrt{5^2 + 3141.59^2}$

$\theta = \tan^{-1}\left(\frac{3141.59}{5}\right)$

$C = 0.1\,MF$
$= 0.1\times10^{-6}\ F$

$R = 10\Omega$ $C = 0.1F$

$V = I\times Z$
$= 22\angle0 \times 10\angle-3.179$

$Z = R - jX_C$

$X_C = \frac{1}{\omega C}$
$= \frac{1}{2\pi f C}$

$X_C = \frac{1}{2\times\pi\times50\times0.1}$

$X_C = 0.0318\,\Omega$

$\boxed{Z = 10 - j\,0.0318}$

$\boxed{Z = 10\angle-3.179}$

$f = 50\,Hz$

$R = 2k\Omega$ $L = 1\,mH$ $C = 0.01\,MF$

$318315\angle-1.56$

$\boxed{Z = R + jX_L - jX_C}$

$Z = R + j(X_L - X_C)$

$X_L = 2\pi f L$
$= 2\times\pi\times50\times1\times10^{-3}$
$X_L = 0.3141\,\Omega$

$X_C = \frac{1}{2\times\pi\times50\times0.01\times10^{-6}}$

$X_C = 318309.88\,\Omega = 318.31\,k\Omega$

$\boxed{Z = 2000 + j(0.3141 - 318309.58)}$

$\boxed{= 2000 - j\,318309}$

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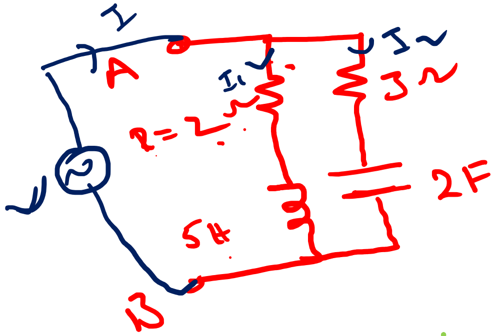
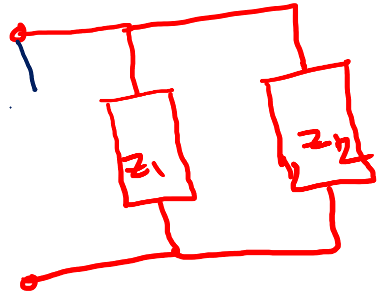
$\frac{R_1 R_2}{R_1+R_2}$





$$Z_3 = \frac{Z_1 \times Z_2}{Z_1 + Z_2}$$

$X_L = 2\pi f L$

$= 2\pi \times 50 \times 5$

$X_L = 1570.79\,\Omega$

$X_C = \frac{1}{2\pi f C}$

$X_C = \frac{1}{2\pi \times 50 \times 2}$

$X_C = 0.00159$

$I = \frac{V}{Z} = \frac{220\angle 0}{3\angle -5.74}$

$Z_1 = R + jX_L$

$\boxed{Z_1 = 2 + j1570.79}$

$Z_1 = 1570\angle 1.56$

$Z_2 = R - jX_L$

$\boxed{Z_2 = 3 - j0.00159}$

$Z_2 = 3\angle -5.29$

$$Z_3 = \frac{1570\angle 1.56 \times 3\angle -5.29}{2 + j1570.79 + 3 - j0.00159}$$

$$Z_3 = \frac{4710\angle -3.73}{5 + j1570}$$

$$Z_3 = \frac{4710\angle -3.73}{1570\angle 1.56}$$

$\boxed{Z_3 = 3\angle -5.74}$

$X_L = j1 = 1\angle 90$

$= -j1$

$X_C = 1$

$\boxed{X_C = 1\angle -90}$

$\boxed{Z = 0 - j30}$

$Z = 30\angle -90$

$X_C = 30$

$= -j30$

$Z_1 = 10 + j20 = 22.36\angle 1.10$

$\frac{Z_1 \times X_C}{Z_1 + X_C} = \frac{22.36\angle 1.10 \times 30\angle -90}{10 + j20 - j30}$

Z_2

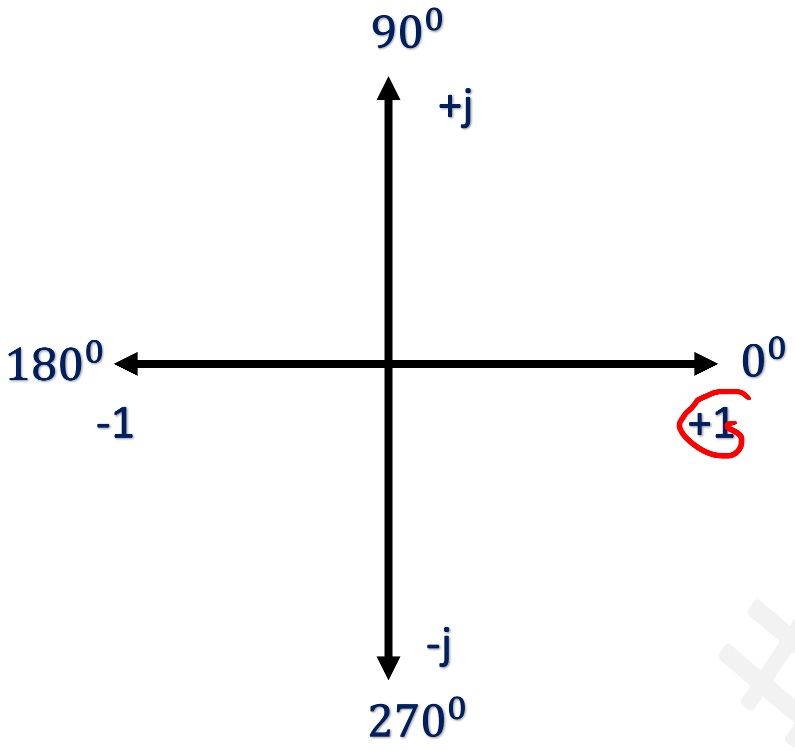
(1.57)

$\boxed{X_L = \angle 90}$

$\boxed{X_C = \angle -90}$

Concept of Impedance and Admittance

j Notation : The ‘j’ operator is a vector operator which when operates on a given phase, produces a Counter clockwise rotation of the phasor by 90^0, without changing its magnitude.



$$0^\circ = \pm 360^\circ = +1 = 1\angle 0^\circ = 1 + j0$$

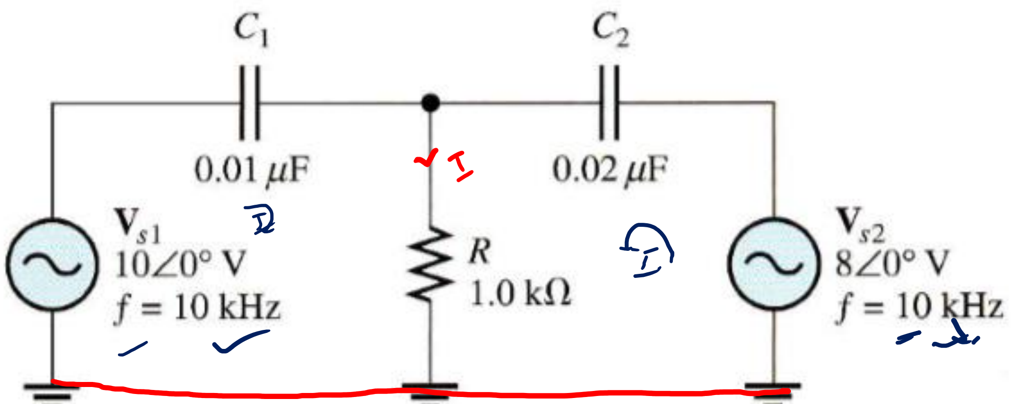
$$+90^\circ = +\sqrt{-1} = +j = 1\angle +90^\circ = 0 + j1$$

$$-90^\circ = -\sqrt{-1} = -j = 1\angle -90^\circ = 0 - j1$$

0 - j30

$30\angle -90^\circ$

$$\pm 180^\circ = \left(\sqrt{-1}\right)^2 = -1 = 1\angle \pm 180^\circ = -1 + j0$$



$V_{s1} \Rightarrow I_1$ at R $\quad V_{s2} \Rightarrow I_2$ at R

$\boxed{I_R = I_1 + I_2}$

$I_s \times \frac{R_{op}}{R_T}$

$= 6.28\times10^{-3}\angle 90 \times \frac{530\angle 90}{1000 - j530.51}$

$I_1 =$

$R = 1\,k\Omega = 1000\,\Omega$

$X_{C1} = \frac{1}{\omega C_1} = \frac{1}{2\pi f C_1} = \frac{1}{2\pi \times 10\times10^{3} \times 0.01 \times 10^{-6}} = 6.28\times10^{-4}$

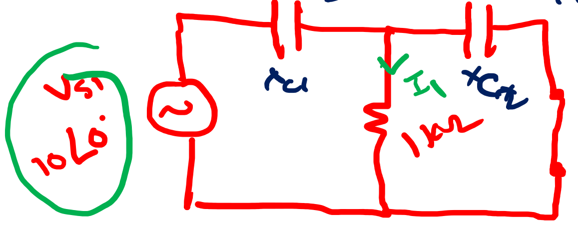
$X_{C1} = 1591.54\,\Omega$

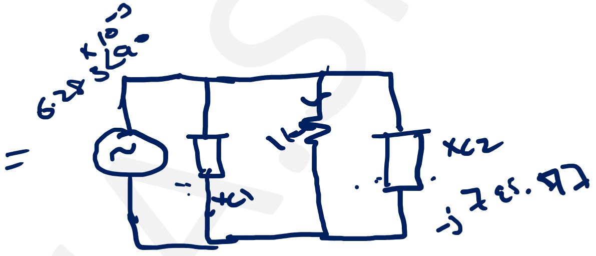
$X_{C2} = \frac{1}{2\pi f C_2} = \frac{1}{2\times\pi\times10\times10^{3}\times0.02\,\mu F} = 795.77\,\Omega$

I_1 at V_{s1}

0.01μF $X_{C1} = 1591.54\,\Omega$ 0.02μF $X_{C2} = 795.77\,\Omega$

$V = I \times R$
$I = V/R$

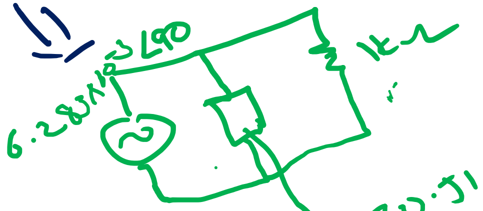




$\frac{Z_{C1} \times Z_{C2}}{Z_{C1} + Z_{C2}} = \frac{(-j1591.54)(-j795.77)}{-j1591.54 - j795.77}$

$= \frac{(1591.54\angle -90)(795.77\angle -90)}{2387.31\angle -90}$

$= 530.51\angle -90$



$Z_{C1} = 0 + (-j1591.54)$

$Z_{C1} = -j1591.54$

$= \frac{10\angle 0}{X_{C1}} = \frac{10\angle 0}{-j1591.54}$

$= \frac{10\angle 0}{1591.54\angle -90}$

$= 6.283\times10^{-3}\angle 90$

COURSE SYLLABUS

MODULE-III: NETWORK THEOREMS (DC AND AC)

Network Theorems (DC): Tellegen"s, superposition, reciprocity, Thevenin"s, Norton"s, maximum power transfer, Milliman"s and compensation theorems for DC excitations, numerical problems.

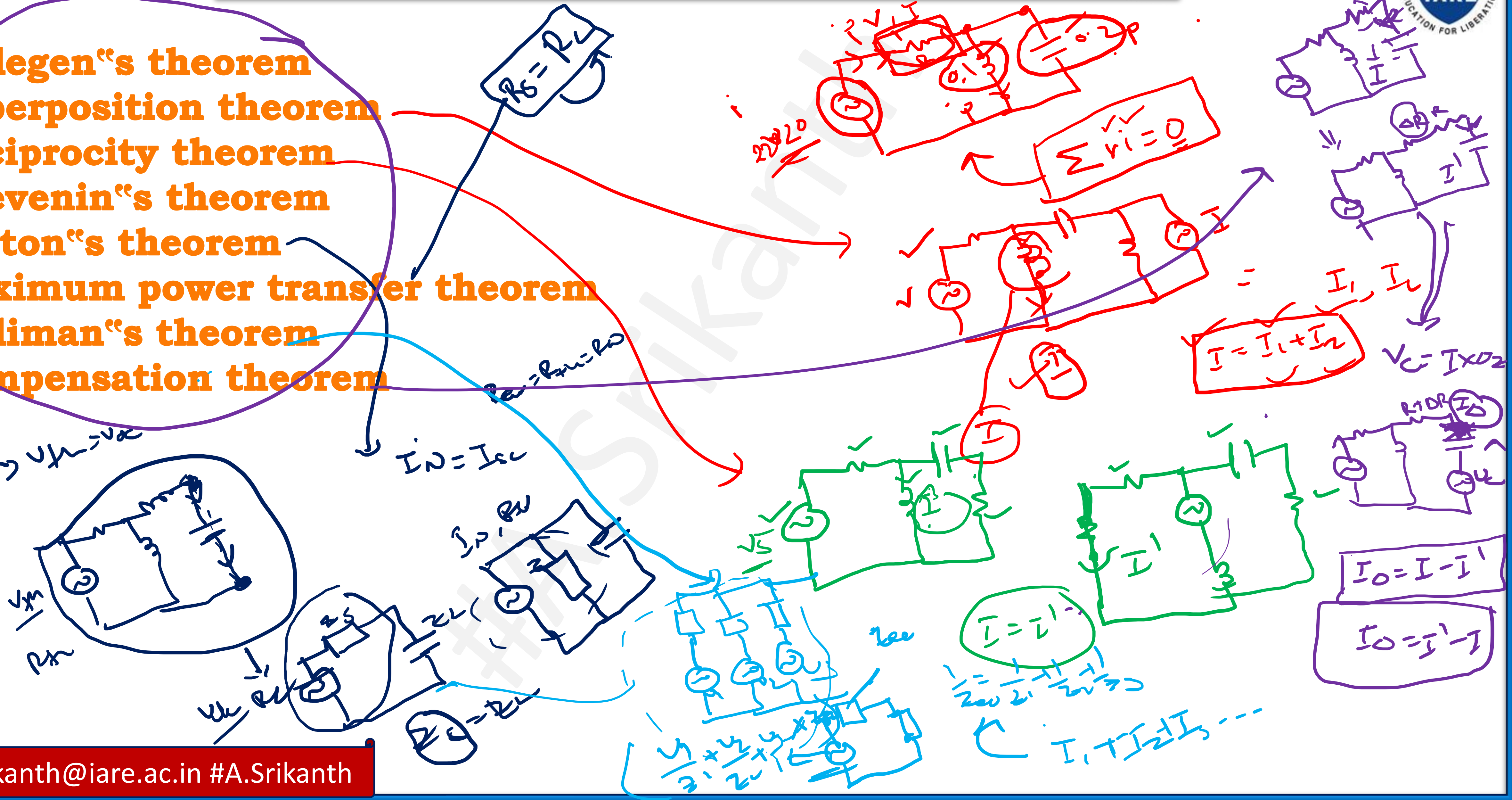
Network Theorems (AC): Tellegen"s, superposition, reciprocity, Thevenin"s, Norton"s, maximum power transfer, Milliman"s and compensation theorems for AC excitations, numerical problems.

COURSE OUTCOMES

CO 5 : Discuss the superposition principle, reciprocity and maximum power transfer condition for the electrical network with DC and AC excitation.

CO 6 : Summarize the procedure of thevenin's, norton's and milliman's theorems to reduce complex network into simple equivalent network with DC and AC excitation.

Tellegen"s theorem
Superposition theorem
Reciprocity theorem
Thevenin"s theorem
Norton"s theorem
Maximum power transfer theorem
Milliman"s theorem
Compensation theorem



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$\dfrac{Z_1 \times Z_2}{Z_1 + Z_2}$

$V = 200\angle 0$

$I = 2\angle 45^\circ$

$P = 400\angle 45^\circ$

$Z = 10\angle 90^\circ$

$4+3j$

$2+j5$

$4+j5$

$2+j3$

$\times$, $\%$, $+$, $-$

$(x\angle\theta_1)(y\angle\theta_2) = xy\angle\theta_1+\theta_2$

$(2+j3)(2+j3)$

$200\angle 0 + 400\angle 25$

$(x_1+jy_1)+(x_2+jy_2)$

$(x_1+x_2)+j(y_1+y_2)$

$r\angle\theta$

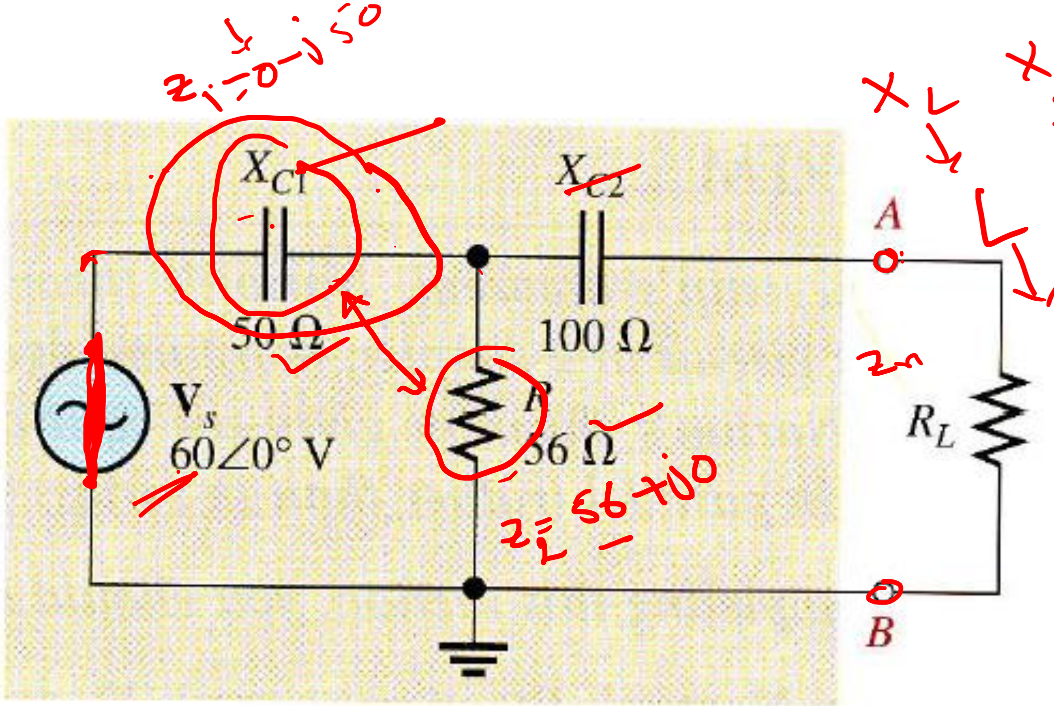
$Z = R+jX$

Y

$\angle\theta_1 \times \angle\theta_2 = \angle\theta_1+\theta_2$

$\dfrac{2+j3}{4+j5} = ?$

$\dfrac{\angle\theta_1}{\angle\theta_2} = \angle\theta_1-\theta_2$



$X_L \to L \to 2\pi f L$

$X_C \to \frac{1}{\omega C} = \frac{1}{2\pi f C}$

$Z_n = ?$

50 Hz

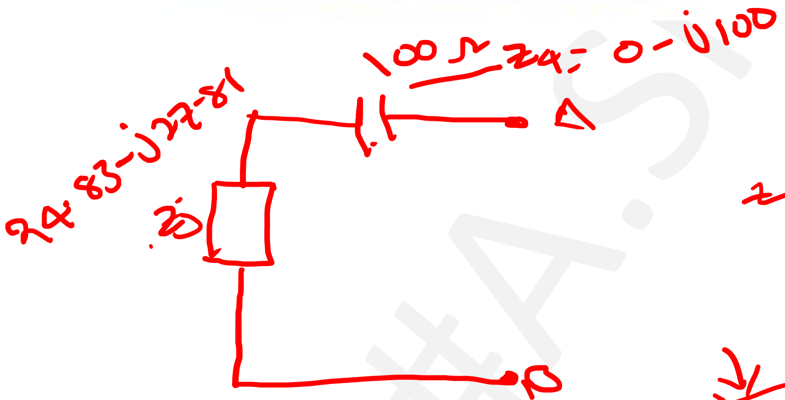
$$Z_3 = \frac{Z_1 \times Z_2}{Z_1 + Z_2} = \frac{(0-j50)(56+j0)}{0-j50+56+j0}$$

$$= \frac{50\angle -90 \times 56\angle 0}{56 - j50}$$

$$= \frac{2800\angle -90}{75.07\angle -41.76} \quad 37.29$$

$$Z_3 = 37.29\angle -48.24$$

$$\boxed{Z_3 = 24.83 - j\,27.81}$$



$$Z_n = Z_3 + Z_4$$

$$= 24.83 - j27.81 + 0 - j100$$

$$\boxed{Z_n = 24.83 - j127.81}$$

$$Z_n = 130.19\angle -79.005$$

ELECTRICAL CIRCUITS

Course Code	: AEEC02
Category	: Foundation
Credits	: 3
Maximum Marks	: CIA (30) + SEE (70) = Total (100)

By
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Electrical and Electronics Engineering

COURSE SYLLABUS

MODULE-IV: MAGNETIC CIRCUITS

Magnetic circuits: Faraday"s laws of electromagnetic induction, concept of self and mutual inductance, dot convention, coefficient of coupling, composite magnetic circuit, analysis of series and parallel magnetic circuits.

COURSE OUTCOMES

CO 7 : Recall the faraday's laws of electromagnetic induction used in construction of magnetic circuit.

CO 8 : Describe the magnetic flux, reluctance, self and mutual inductance in the single coil and coupled coils magnetic circuits to know total magnetomotive force and total ampere turns values.

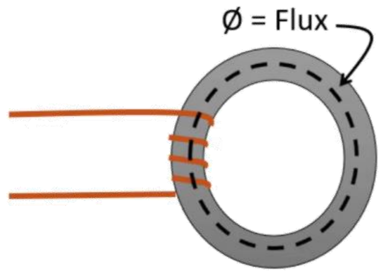
MAGNETIC CIRCUITS AND ELECTRIC CIRCUIT

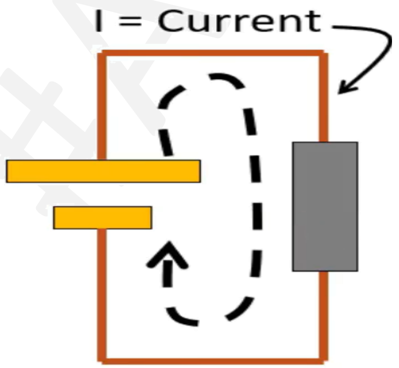
Magnetic Circuit

A closed path of magnetic flux
Flux = mmf/reluctance
Flux is measured in "Wb"
The flux does not flow but sets up in magnetic circuit
MMF is force because of which flux setup in magnetic circuit
Reluctance opposes the flow of flux
Flux density is B=Ø/A wb/m2
Magnetic lines of force moves from N to S pole

Electric Circuit

A closed path of current
Current = emf/resistance
Current is measured in "Ampere"
The current flow through the circuit
EMF is force because of which current flows throw circuit
Resistance opposes the flow of current
Current density is J=I/A A/m2
Electric lines of force moves outward for positive and inward for negative charge

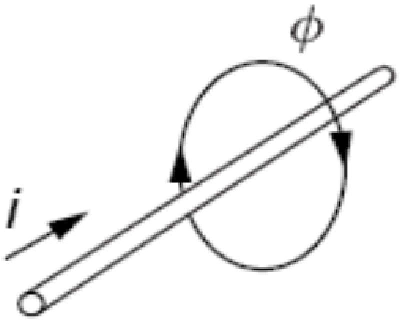


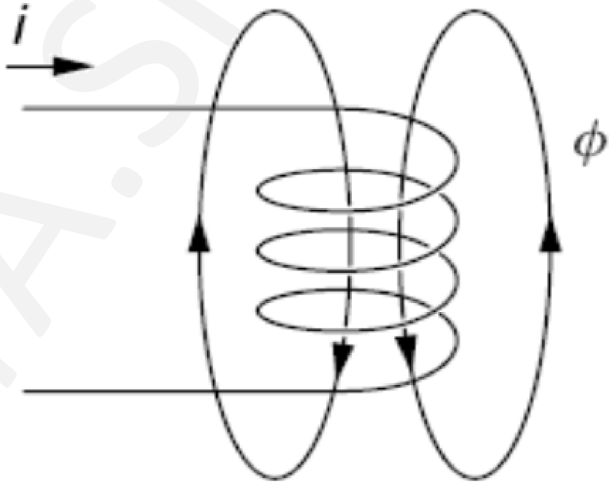


MAGNETIC CIRCUITS

A closed path followed by magnetic flux is known as a magnetic circuit. In a magnetic circuit, flux starts from one point and finishes at the same point. A magnetic circuit usually consists of magnetic materials having high permeability such as iron, soft-steel, etc. since they offer small opposition to magnetic flux.

Electrical current flowing along a wire creates a magnetic field around the wire, That magnetic field can be visualized by showing lines of magnetic flux, which are represented with the symbol ϕ. The direction of that field that can be determined using the “right hand rule”





Faraday"s Laws of Electromagnetic Induction

Electromagnetic induction was discovered independently by Michael Faraday in 1831 and Joseph Henry in 1832. Faraday was the first to publish the results of his experiments in 1831 August 29. Michael Faraday, was an English physicist who gave, one of the most basic laws of electromagnetism called Faraday's law of electromagnetic induction. At first, the law was rejected due to lack of mathematical and theoretical calculations. This law says about the electric circuit and magnetic field. This principle is used in most of the electrical utility. As we know some of the applications names are electrical motors, generators, electrical transformers and magnetic control circuits such as contactor, relays etc.

Faraday"s Laws of Electromagnetic Induction

Faraday's law of electromagnetic induction (referred to as Faraday's law) is a basic law of electromagnetism predicting how a magnetic field will interact with an electric circuit to produce an electromotive force (EMF). This phenomenon is known as electromagnetic induction.

Faraday's law states that a current will be induced in a conductor which is exposed to a changing magnetic field. Lenz's law of electromagnetic induction states that the direction of this induced current will be such that the magnetic field created by the induced current opposes the initial changing magnetic field which produced it. The direction of this current flow can be determined using Fleming's right-hand rule.

Faraday's law of induction explains the working principle of transformers, motors, generators, and inductors. The law is named after Michael Faraday, who performed an experiment with a magnet and a coil. During Faraday's experiment, he discovered how EMF is induced in a coil when the flux passing through the coil changes.

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Faraday"s Laws of Electromagnetic Induction

Faraday's law of electromagnetic induction, also known as Faraday's law, is the basic law of electromagnetism which helps us to predict how a magnetic field would interact with an electric circuit to produce an electromotive force (EMF). This phenomenon is known as electromagnetic induction.

Faraday's Laws of Electromagnetic Induction consists of two laws. The first law describes the induction of emf in a conductor and the second law quantifies the emf produced in the conductor.

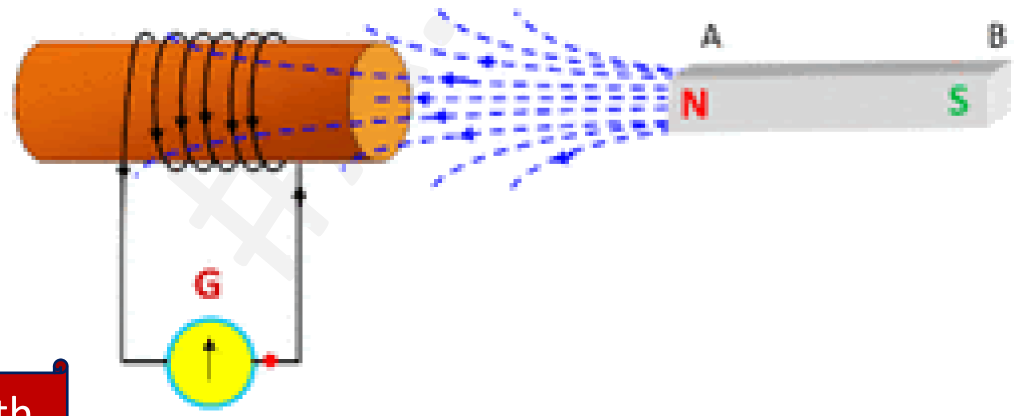
- Faraday's First Law of Electromagnetic Induction
- Faraday's Second Law of Electromagnetic Induction

Faraday"s Laws of Electromagnetic Induction

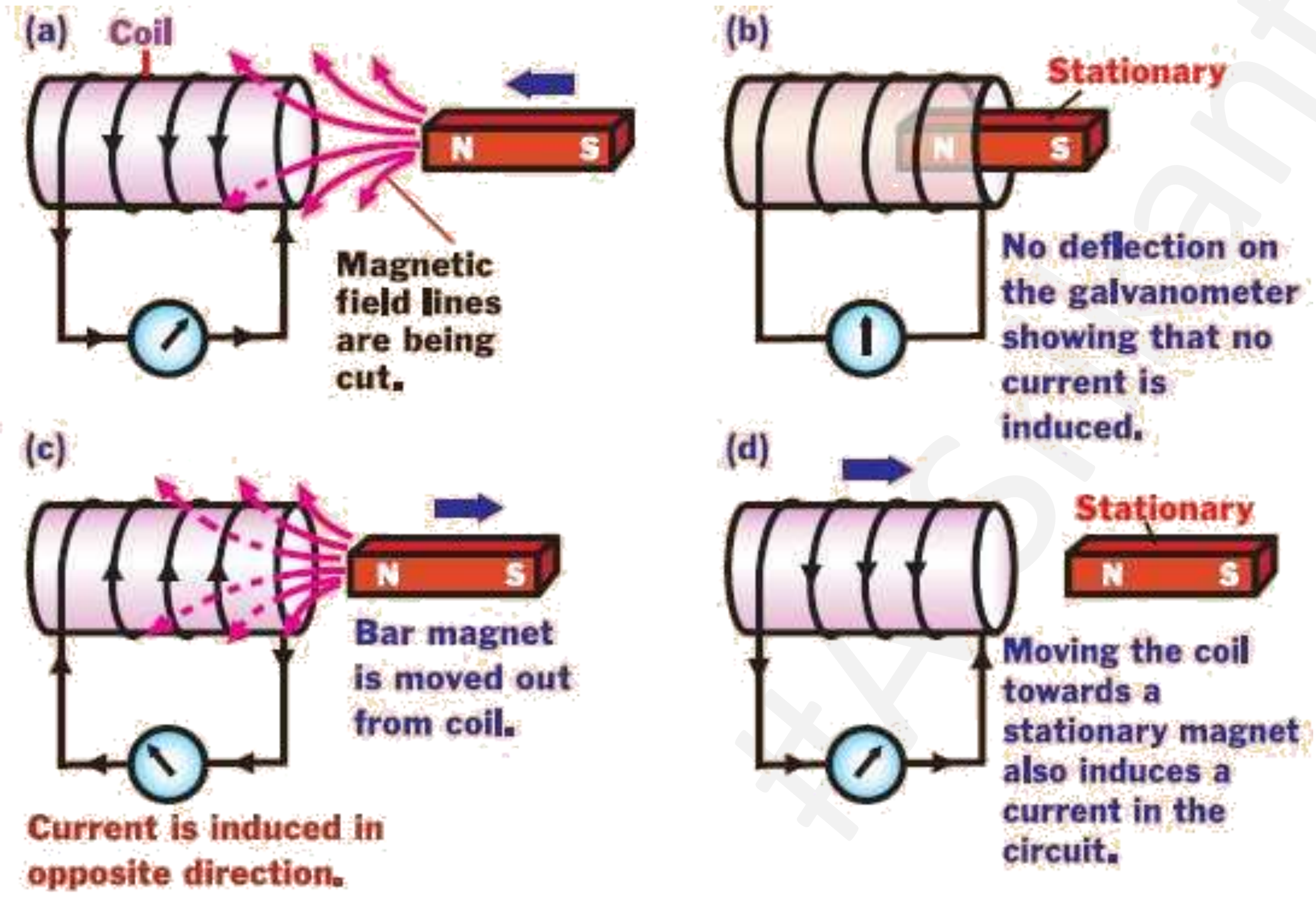
Faraday's First Law of Electromagnetic Induction

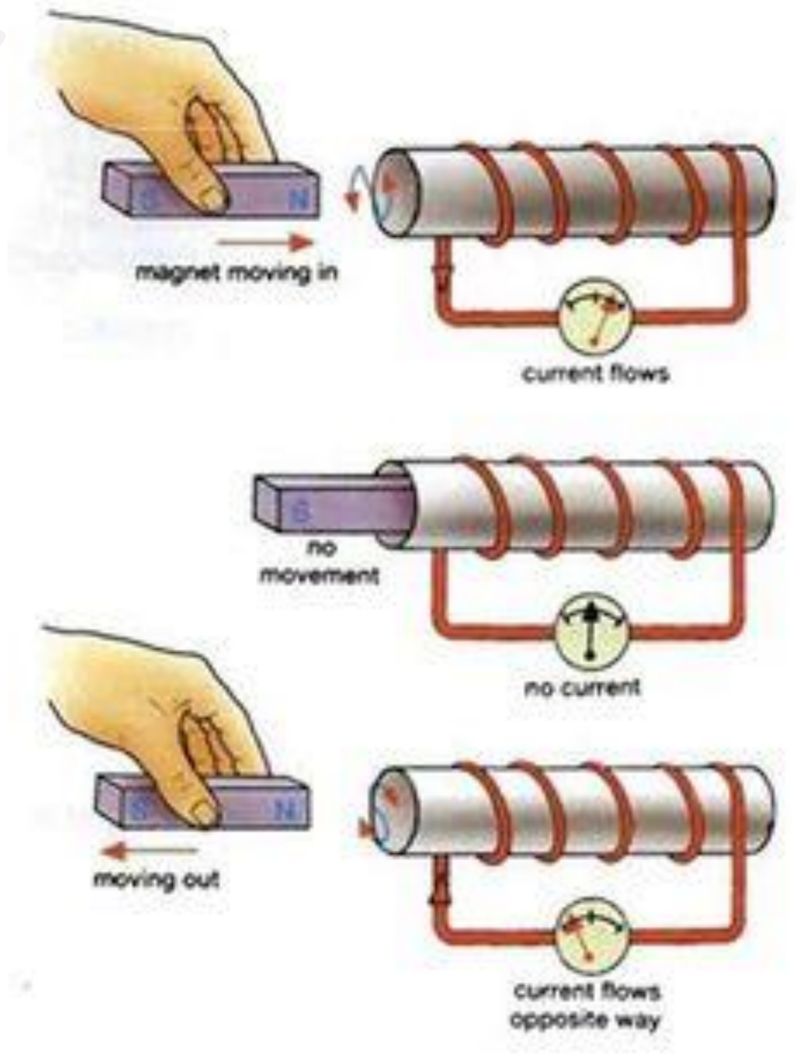
The discovery and understanding of electromagnetic induction are based on a long series of experiments carried out by Faraday and Henry. From the experimental observations, Faraday concluded that an emf is induced in the coil when the magnetic flux across the coil changes with time.

Whenever a conductor is placed in a varying magnetic field, an electromotive force is induced. If the conductor circuit is closed, a current is induced, which is called induced current.



Faraday"s Laws of Electromagnetic Induction





Faraday's First Law of Electromagnetic Induction

Faraday"s Laws of Electromagnetic Induction

Faraday's First Law of Electromagnetic Induction

Few ways to change the magnetic field intensity in a closed loop:

- By rotating the coil relative to the magnet.
- By moving the coil into or out of the magnetic field.
- By changing the area of a coil placed in the magnetic field.
- By moving a magnet towards or away from the coil.

Faraday‟s Laws of Electromagnetic Induction

Faraday’s Second Law of Electromagnetic Induction

Faraday’s second law of electromagnetic induction states that the induced emf in a coil is equal to the rate of change of flux linkage. The flux is the product of the number of turns in the coil and the flux associated with the coil.

$$\varepsilon = -N\frac{\Delta\phi}{\Delta t}$$

Where ε is the electromotive force, Φ is the magnetic flux, and N is the number of turns.

Faraday"s Laws of Electromagnetic Induction

Lenz's Law

- ✓ The German physicist Heinrich Friedrich Lenz deduced a rule known as Lenz's law that describes the polarity of the induced emf.
- ✓ Lenz's law states that "The polarity of induced emf is such that it tends to produce a current which opposes the change in magnetic flux that produced it."
- ✓ The negative sign in the formula represents this effect. Thus, the negative sign indicates that the direction of the induced emf and change in the direction of magnetic fields have opposite signs.

Faraday"s Laws of Electromagnetic Induction

Faraday's Law Derivation

Consider, a magnet is approaching towards a coil. Here we consider two instants at time T1 and time T2

- Flux linkage with the coil at the time T1 is given by $T_1 = N\phi_1$
- Flux linkage with the coil at the time T2 is given by $T_2 = N\phi_2$
- Change in the flux linkage is given by $N(\phi_2 - \phi_1)$
- Let us consider this change in flux linkage as $\phi = \phi_2 - \phi_1$
- Hence, the change in flux linkage is given by $N\phi$
- The rate of change of flux linkage is given by $\frac{N\phi}{t}$
- Taking the derivative of the above equation, we get $N\frac{d\phi}{dt}$

Faraday"s Laws of Electromagnetic Induction

Faraday's Law Derivation

- ❖ According to Faraday's second law of electromagnetic induction, we know that the induced emf in a coil is equal to the rate of change of flux linkage. Therefore, $\varepsilon = N\frac{\Delta\phi}{\Delta t}$
- ❖ Considering Lenz's law, $\varepsilon = -N\frac{\Delta\phi}{\Delta t}$

  From the above equation, we can conclude the following

- ➢ Increase in the number of turns in the coil increases the induced emf
- ➢ Increasing the magnetic field strength increases the induced emf
- ➢ Increasing the speed of the relative motion between the coil and the magnet, results in the increased emf

Concept of Self and Mutual Inductance

Inductance

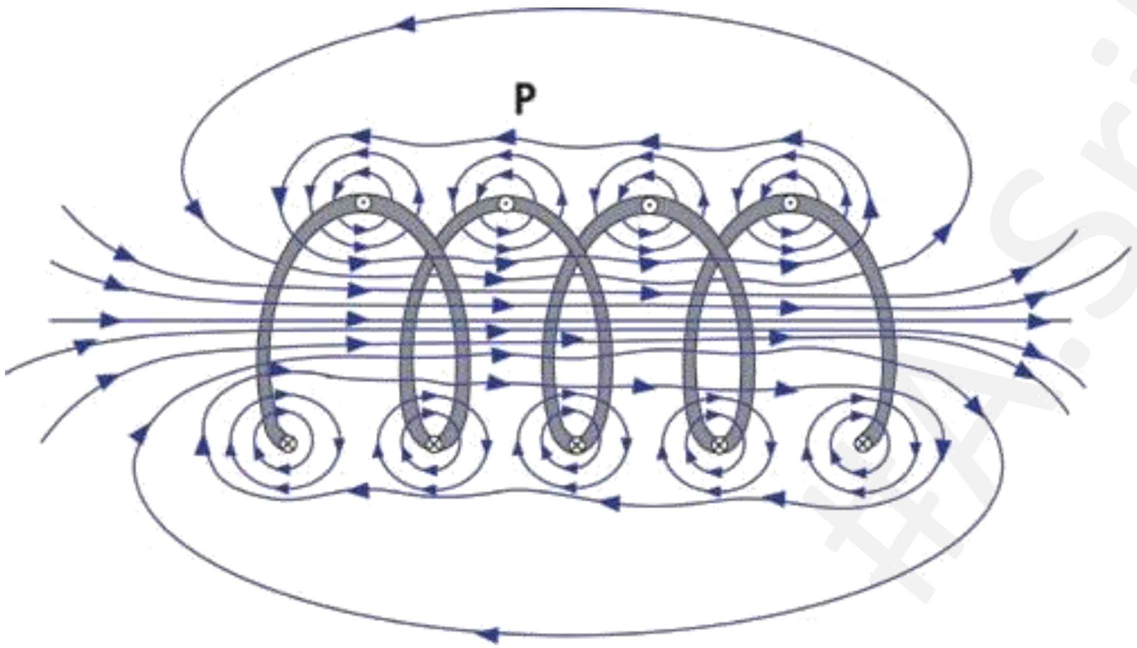
- ✓ Induction is the magnetic field which is proportional to the rate of change of the magnetic field. Induction is also known as inductance. L is used to represent the inductance and Henry is the SI unit of inductance.
- ✓ Factors Affecting Inductance
  - ➢The number of turns of the wire used in the inductor.
  - ➢The material used in the core.
  - ➢The shape of the core.

$$\textit{Electromotive force} = -L\frac{\Delta I}{\Delta t}$$

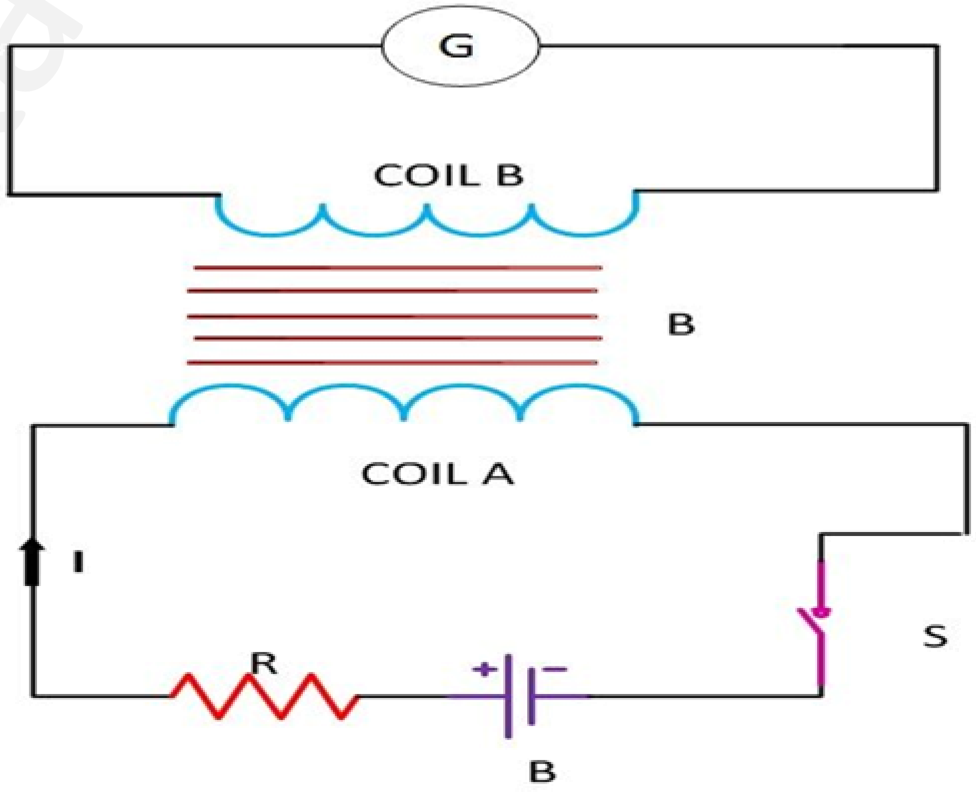
Concept of Self and Mutual Inductance

Types of Inductance

1. Self Induction



2. Mutual Induction



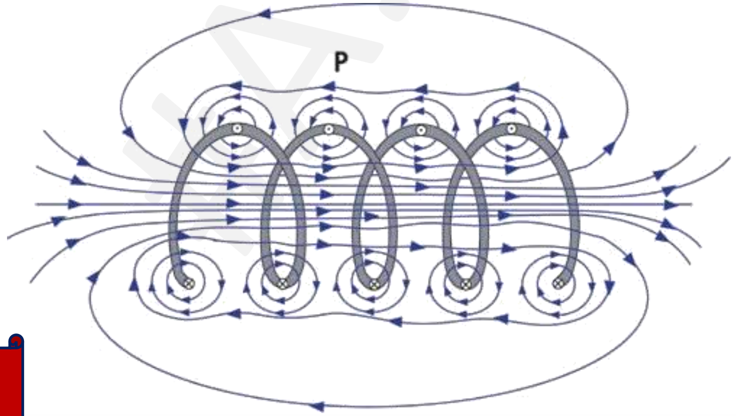
Concept of Self and Mutual Inductance

Self Induction

Self-induction means the coils induce the emf themselves. There is a change in the magnetic flux through that coil and because of this, the current will be induced in the coil by itself. So once the current get induced, the current tries to oppose the flux.

When there is a change in the current or magnetic flux of the coil, an opposed induced electromotive force is produced. This phenomenon is termed as Self Induction. When the current starts flowing through the coil at any instant, it is found that, that the magnetic flux becomes directly proportional to the current passing through the circuit.

$$\phi \propto I$$



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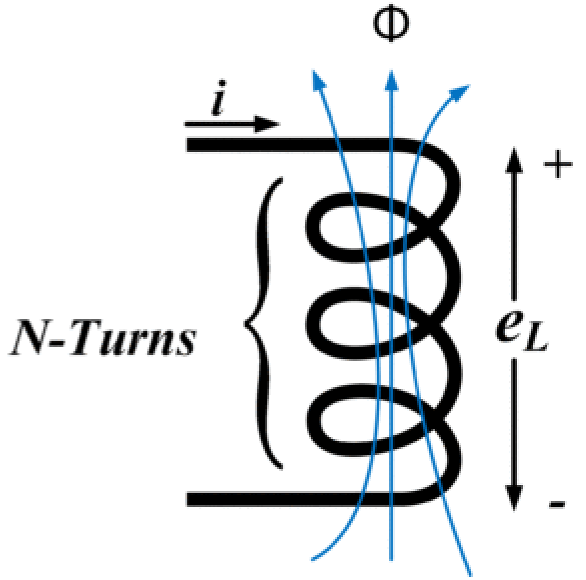
Concept of Self and Mutual Inductance

Self Induction

The rate of change of magnetic flux in the coil is given as $= -N\dfrac{d\phi}{dt}$

Electromotive force in a inductance $= -L\dfrac{dI}{dt}$

$$-N\frac{d\phi}{dt} = -L\frac{dI}{dt}$$



$$\boxed{e_L = N\frac{\Delta\phi}{\Delta t}}$$

$$\boxed{L = N\frac{\Delta\phi}{\Delta i}}$$

$$L = N\frac{\phi}{I}$$

$$B = \frac{\phi}{A}$$

$$B = \mu_r\mu_o\frac{NI}{l}$$

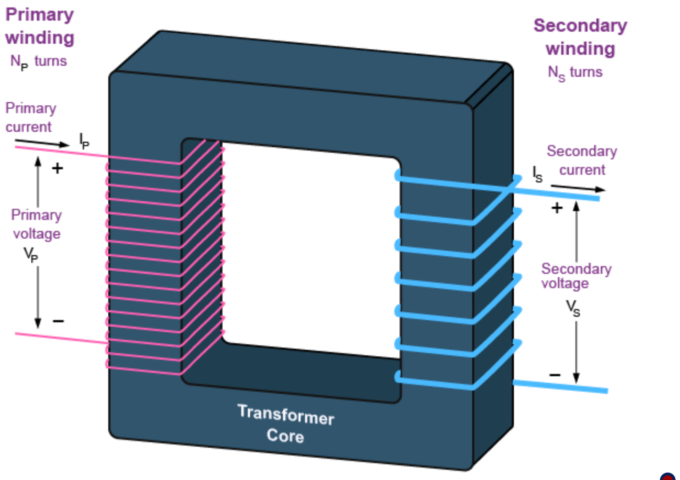
Where,

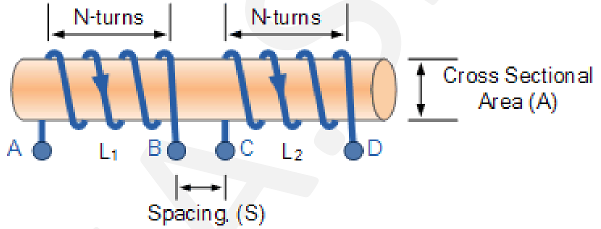
- L is the self inductance in Henries
- N is the number of turns
- ϕ is the magnetic flux
- I is the current in amperes

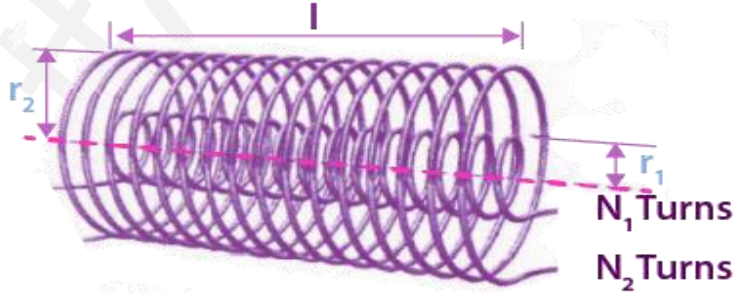
Concept of Self and Mutual Inductance

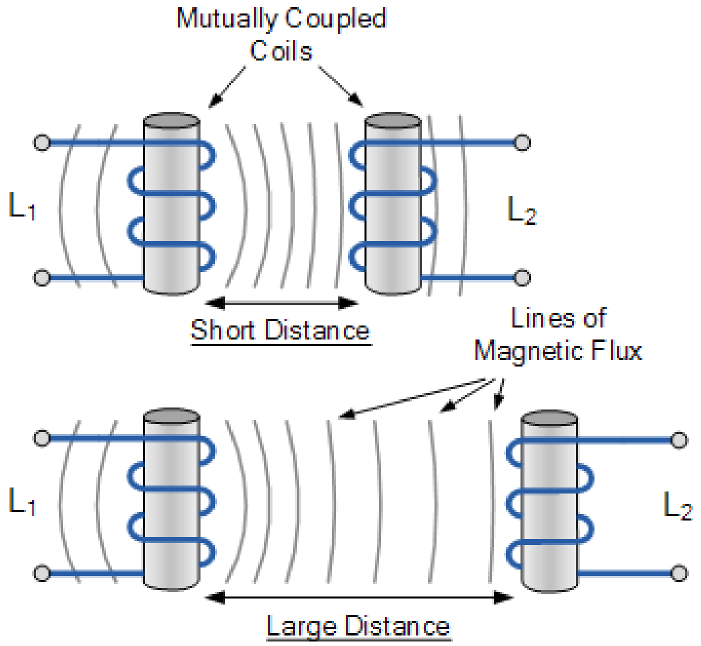
Mutual Induction

Mutual Inductance is the interaction of one coils magnetic field on another coil as it induces a voltage in the adjacent coil. Mutual Inductance between the two coils is defined as the property of the coil due to which it opposes the change of current in the other coil. When the current in the neighboring coil changes, the flux sets up in the coil and because of this, changing flux emf is induced in the coil called Mutually Induced emf and the phenomenon is known as Mutual Inductance.









Concept of Self and Mutual Inductance

Mutual Induction

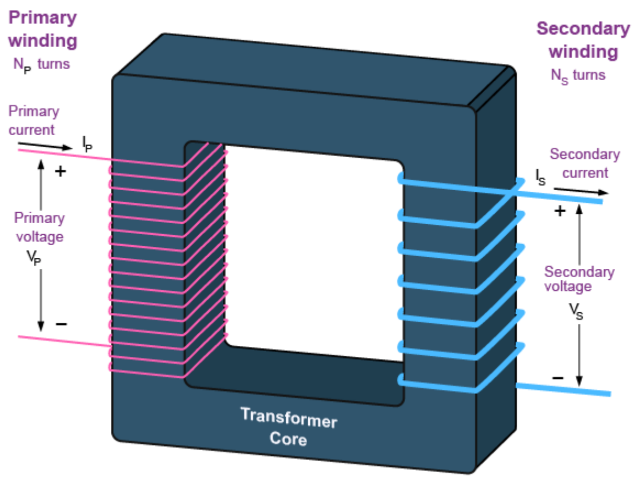
We take two coils, and they are placed close to each other. The two coils are P- coil (Primary coil) and S- coil (Secondary coil). To the P-coil, a battery, and a key is connected wherein the S-coil a galvanometer is connected across it. When there is a change in the current or magnetic flux linked with two coils an opposing electromotive force is produced across each coil, and this phenomenon is termed as Mutual Induction.

$\phi = MI$ $\quad$ $L = N\frac{\phi}{I}$

Electromotive force in a inductance $= -M\frac{dI}{dt}$

$M12 = (N2\phi12)/I1$ $\quad$ $M21 = (N1\phi21)/I2$

$M12 = M21 = M$



Concept of Self and Mutual Inductance

Mutual Induction

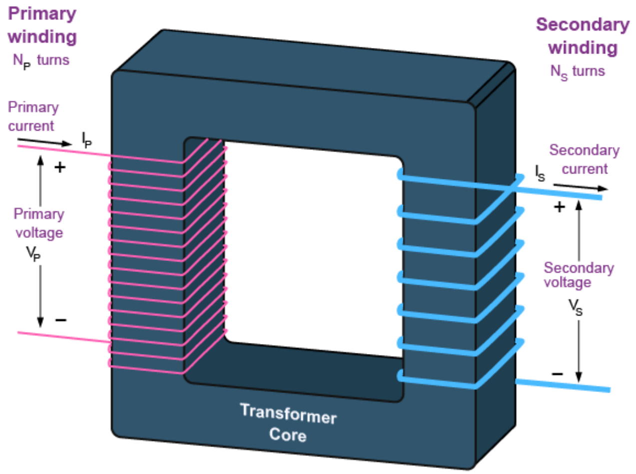
$$\phi = MI \qquad L = N\frac{\phi}{I}$$

Electromotive force in a inductance $= -M\dfrac{dI}{dt}$

$M12 = (N2\phi12) / I1 \qquad M21 = (N1\phi21) / I2$

$$v_1 = M_{12}\frac{di_2}{dt}$$

$$v_2 = M_{21}\frac{di_1}{dt}$$



$$B = \frac{\phi}{A}$$

$$M = \frac{\mu_0 \mu_r N_1 N_2 A}{\ell}$$

$$B = \mu_r \mu_o \frac{NI}{l}$$

$M12 = M21 = M$

Where:
μ_o is the permeability of free space ($4.\pi.10^{-7}$)
μ_r is the relative permeability of the soft iron core
N is in the number of coil turns
A is in the cross-sectional area in m^2
ℓ is the coils length in meters

$$M = \sqrt{L_1 L_2} \ \ H$$

Coupling Factor Between Coils $\quad k = \dfrac{M}{\sqrt{L_1 L_2}}$

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$$\varepsilon = -N\frac{\Delta\phi}{\Delta t}$$

$$B = \frac{\phi}{A}$$

$$B = \mu_r \mu_o \frac{NI}{l}$$

$$L = N\frac{\phi}{I}$$

$$M = N\frac{\phi}{I}$$

$$\varepsilon = -M\frac{dI}{dt}$$

$$M = \frac{\mu_o \mu_r N_1 N_2 A}{\ell}$$

$$M12 = M21 = M$$

$$k = \frac{M}{\sqrt{L_1 L_2}}$$

$$M = \sqrt{L_1 L_2} \text{ H}$$

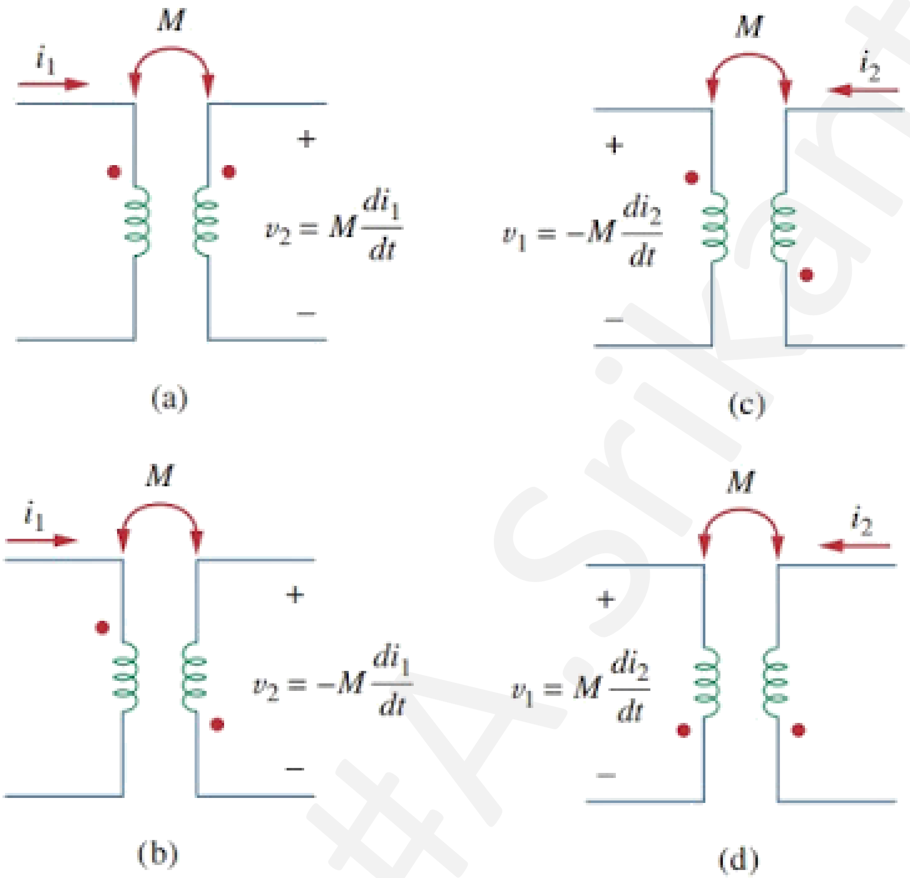
dot Convention

Dot convention is a technique, which gives the details about voltage polarity at the dotted terminal. This information is useful, while writing KVL equations.

•If the current enters at the dotted terminal of one coil (or inductor), then it induces a voltage at another coil (or inductor), which is having positive polarity at the dotted terminal.

•If the current leaves from the dotted terminal of one coil (or inductor), then it induces a voltage at another coil (or inductor), which is having negative polarity at the dotted terminal.

dot Convention



Analysis of Series and Parallel Magnetic Circuits

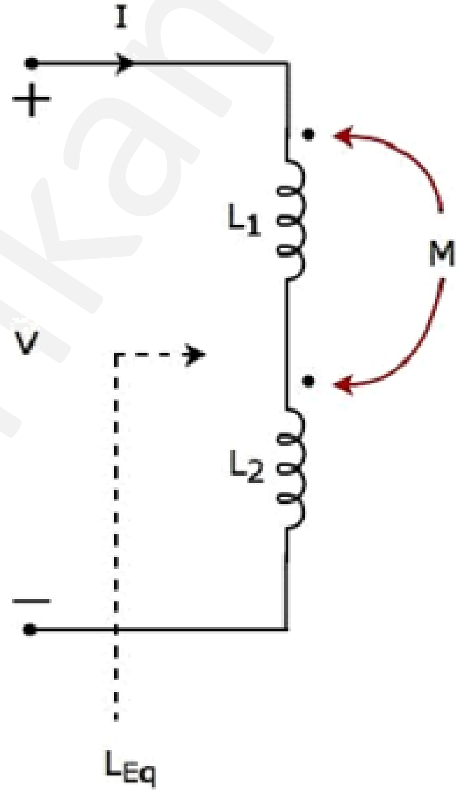
Two inductors that are connected in series

$$V - L_1\frac{dI}{dt} - M\frac{dI}{dt} - L_2\frac{dI}{dt} - M\frac{dI}{dt} = 0$$

$$V = L_1\frac{dI}{dt} + L_2\frac{dI}{dt} + 2M\frac{dI}{dt}$$

$$V = (L_1 + L_2 + 2M)\frac{dI}{dt}$$

$$L_{Eq} = L_1 + L_2 + 2M$$



Analysis of Series and Parallel Magnetic Circuits

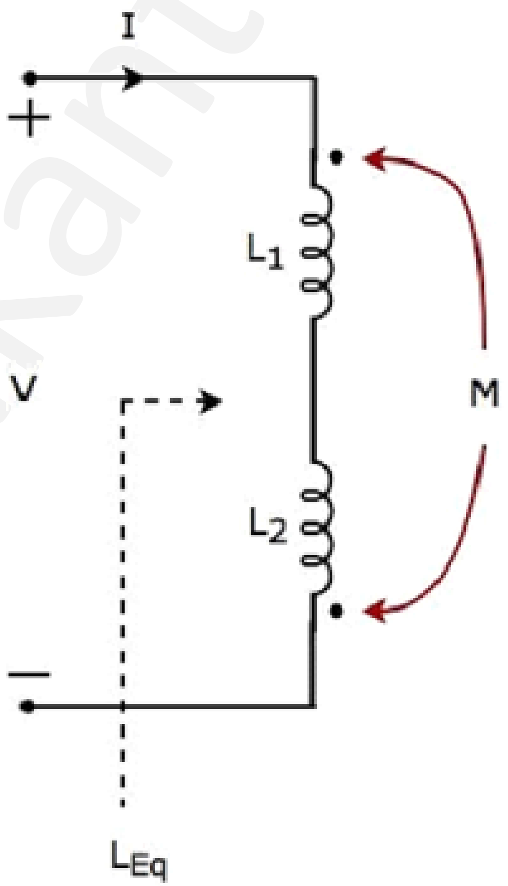
Two inductors that are connected in series

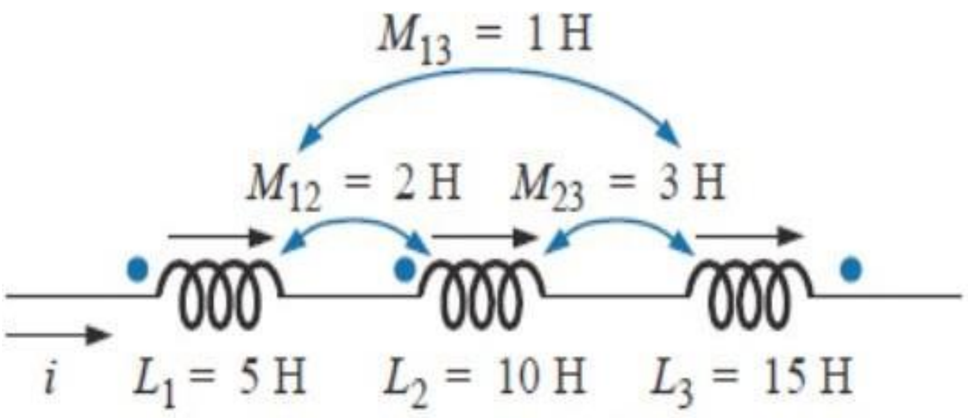
$$V - L_1\frac{dI}{dt} + M\frac{dI}{dt} - L_2\frac{dI}{dt} + M\frac{dI}{dt} = 0$$

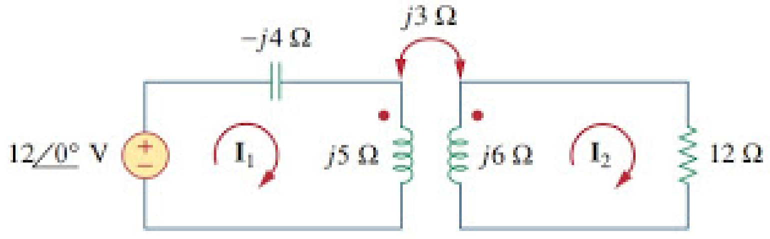
$$\Rightarrow V = L_1\frac{dI}{dt} + L_2\frac{dI}{dt} - 2M\frac{dI}{dt}$$

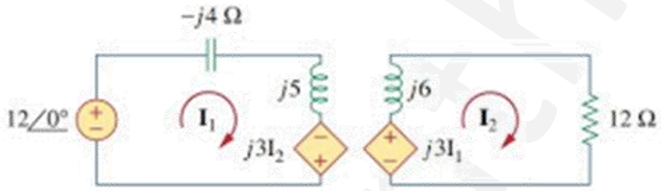
$$\Rightarrow V = (L_1 + L_2 - 2M)\frac{dI}{dt}$$

$$L_{Eq} = L_1 + L_2 - 2M$$




M_{13} = 1 H
M_{12} = 2 H
M_{23} = 3 H
i
L_1 = 5 H
L_2 = 10 H
L_3 = 15 H





$$-12 + (-j4 + j5)\mathbf{I}_1 - j3\mathbf{I}_2 = 0$$

$$-j3\mathbf{I}_1 + (12 + j6)\mathbf{I}_2 = 0$$

Coefficient of Coupling

For different current assignments, the instantaneous energy stored is given as

$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 \pm M i_1 i_2$$

But $w > 0$ for any case,

$$\frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 - M i_1 i_2 \geq 0$$

$$\frac{1}{2}\left(i_1\sqrt{L_1} - i_2\sqrt{L_2}\right)^2 + i_1 i_2\left(\sqrt{L_1 L_2} - M\right) \geq 0$$

$$\Rightarrow \sqrt{L_1 L_2} - M \geq 0$$

$$\Rightarrow \boxed{M \leq \sqrt{L_1 L_2}}$$

The *coupling coefficient k* is defined as

$$\boxed{k = \frac{M}{\sqrt{L_1 L_2}}}$$

or $\quad M = k\sqrt{L_1 L_2} \quad (0 \leq k \leq 1)$

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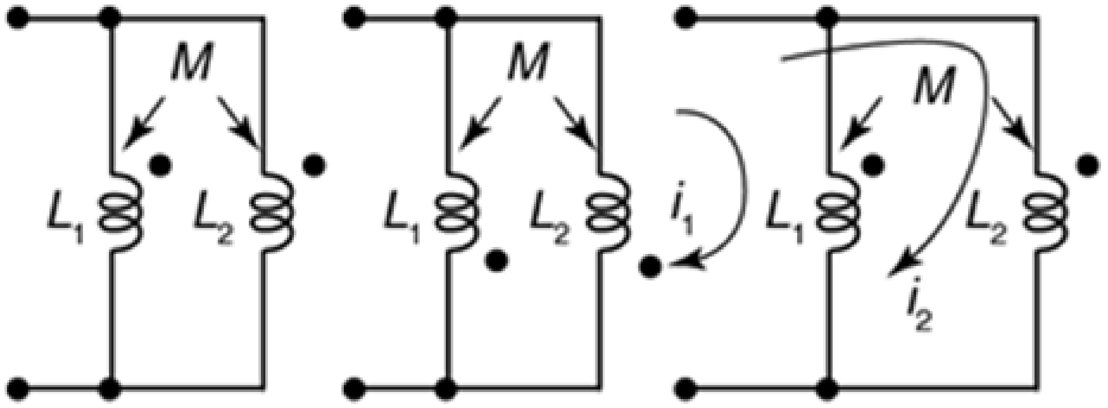
Analysis of Series and Parallel Magnetic Circuits

Two inductors that are connected in Parallel

When two coils of self-inductances L_1 and L_2 are connected in parallel, two types of connection are possible:

In this connection, the two coils are connected in parallel such a way that their induced e.m.f.'s are of same polarities.



$$L_T = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$

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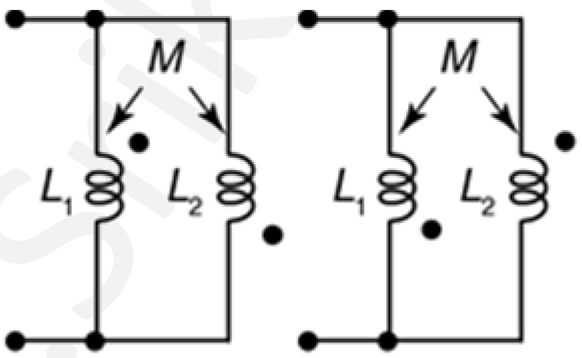
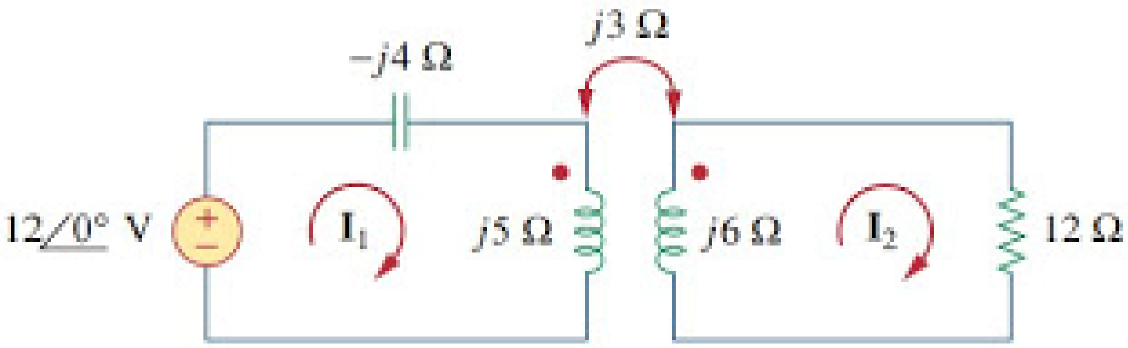
Analysis of Series and Parallel Magnetic Circuits

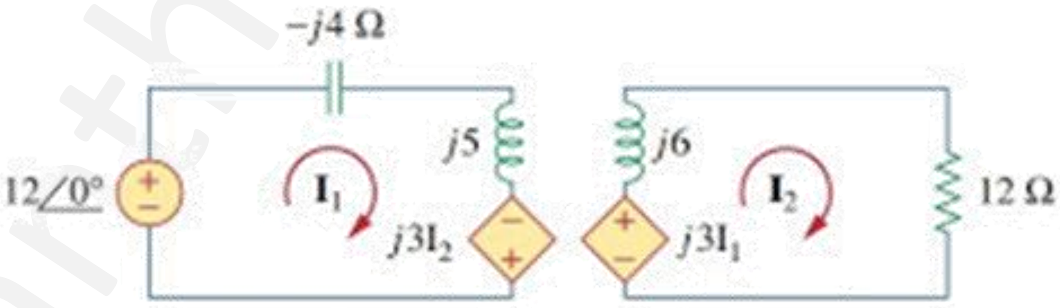
Two inductors that are connected in Parallel

In this connection, the two coils are connected in parallel such a way that their induced e.m.f.'s are of opposite polarities.

$$L_T = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$$

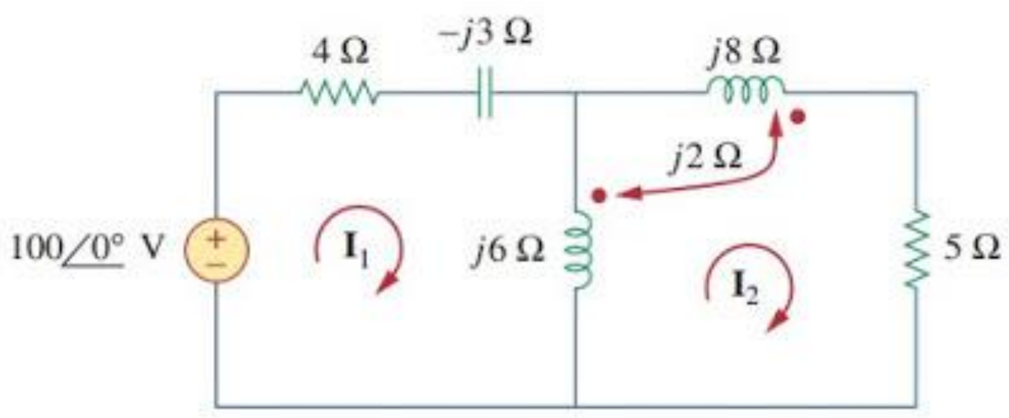


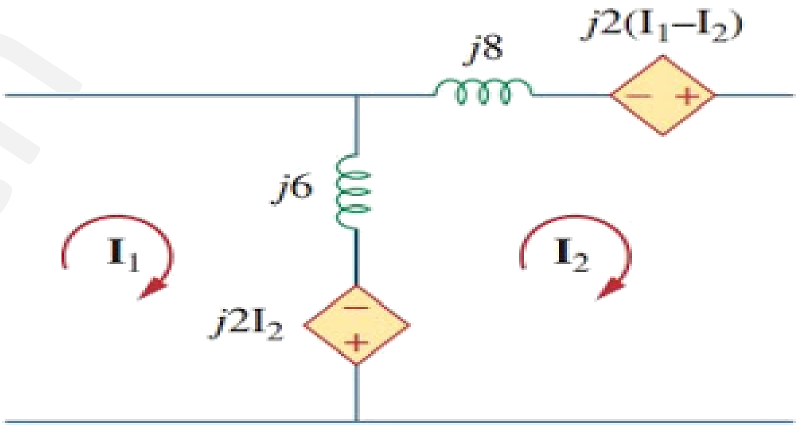




$$-12 + (-j4 + j5)\mathbf{I}_1 - j3\mathbf{I}_2 = 0$$

$$-j3\mathbf{I}_1 + (12 + j6)\mathbf{I}_2 = 0$$





$$-100 + \mathbf{I}_1(4 - j3 + j6) - j6\mathbf{I}_2 - j2\mathbf{I}_2 = 0$$

$$0 = -2j\mathbf{I}_1 - j6\mathbf{I}_1 + (j6 + j8 + j2 \times 2 + 5)\mathbf{I}_2$$

Composite Magnetic Circuit

Consider a circular ring made from different materials of lengths l1, l2 and l3 cross-sectional areas a1, a2 and a3 and relative permeability μr1, μr2 and μr3 with a cut of length lg known as air gap. The total is the arithmetic sum of individual reluctances as they are joined in series

$$S = \frac{l_1}{\mu_0\mu_r a_1} + \frac{l_2}{\mu_0\mu_r a_2} + \frac{l_3}{\mu_0\mu_r a_3} + \frac{l_g}{\mu_0 a_g}$$

$$Total\, mmf = \phi \times S$$

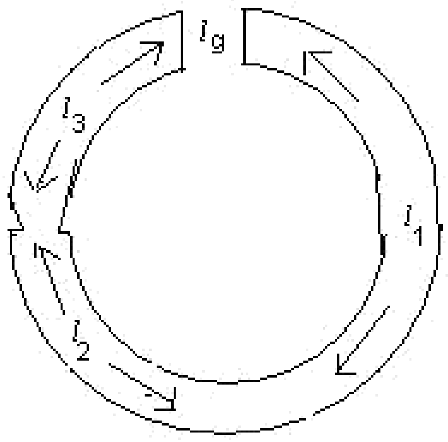
$$B = \mu_r \mu_o \frac{NI}{l}$$

$$= \phi\left[\frac{l_1}{\mu_0\mu_r a_1} + \frac{l_2}{\mu_0\mu_r a_2} + \frac{l_3}{\mu_0\mu_r a_3} + \frac{l_g}{\mu_0 a_g}\right]$$

Or Total ampere-turns required

$$= H_1 l_1 + H_2 l_2 + H_3 l_3 + H_g + l_g$$

= Sum of ampere-turns required for individual parts of the magnetic circuit.



Composite Magnetic Ciruit

MAGNETIC CIRCUITS

$$\varepsilon = -N\frac{\Delta\phi}{\Delta t}$$

$$B = \frac{\phi}{A}$$

$$M12 = M21 = M$$

$$L = N\frac{\phi}{I}$$

$$B = \mu_r\mu_o\frac{NI}{l}$$

$$M = \sqrt{L_1 L_2}\ \text{H}$$

$$k = \frac{M}{\sqrt{L_1 L_2}}$$

$$M = N\frac{\phi}{I} \qquad \varepsilon = -M\frac{dI}{dt}$$

$$M = \frac{\mu_0\mu_r N_1 N_2 A}{\ell}$$

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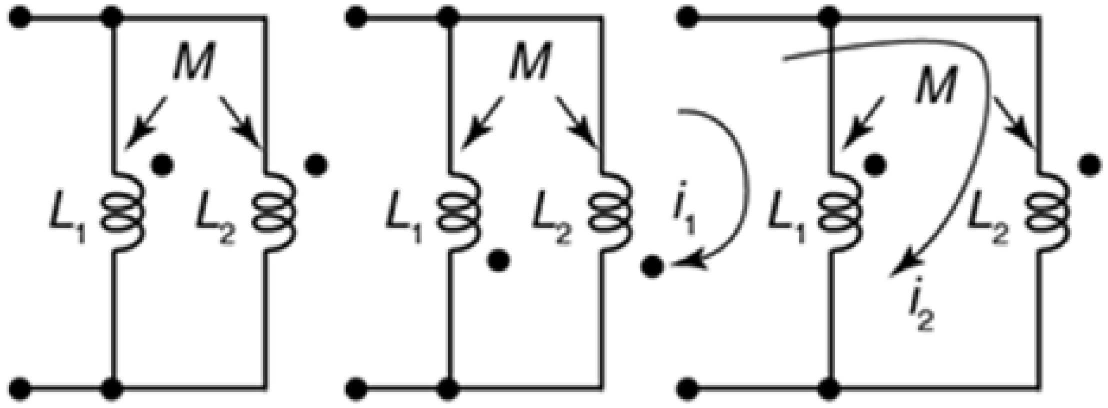
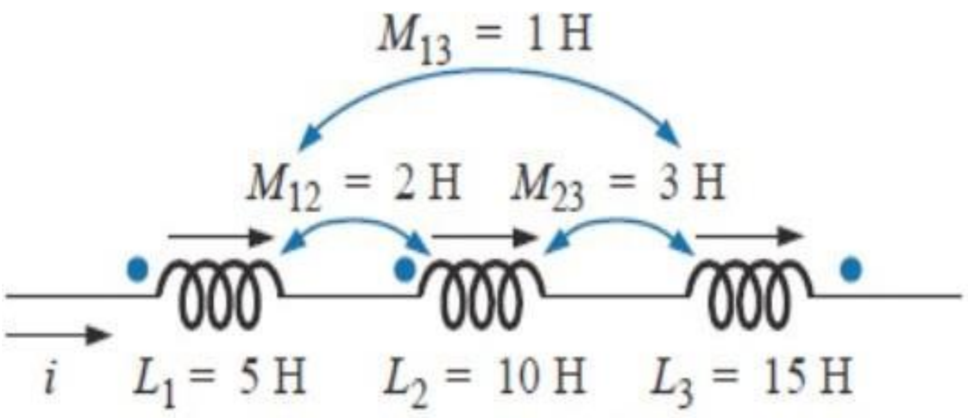
Analysis of Series and Parallel Magnetic Circuits

Two inductors that are connected in Parallel



$$L_T = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$



An iron ring of mean length 100cm and cross sectional area of 10cm 2 has an air gap of 1mm cut in it. it is wound with a coli of 100 turns. Assume relative permeability of iron is 500.calculate inductance of coil.

Two coupled coils with L1 =0.02H,L2=0.01H,k=0.5 are connected in four different ways: series aiding, series opposing and parallel with both arrangements of the winding sense. what area the four equivalent inductances?

ELECTRICAL CIRCUITS

Course Code	: AEEC02
Category	: Foundation
Credits	: 3
Maximum Marks	: CIA (30) + SEE (70) = Total (100)

By
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Electrical and Electronics Engineering

COURSE SYLLABUS

MODULE-V: TWO PORT NETWORK AND GRAPH THEORY

Two Port Network: Two port parameters, interrelations, Two port Interconnections.

Network topology: Definitions, incidence matrix, basic tie set and basic cut set matrices for planar networks, duality and dual networks.

COURSE OUTCOMES

CO 9 : Discuss the two port parameters to be measure easily, without solving for all the internal voltages and currents in the different networks.

CO 10 : List the various network topology for graphical and digital representation of complex circuits to be utilized in power system

CO 11 : Define the importance of dual network for compare both mesh and nodal networks.

Two Port Network

Many complex, such as amplification circuits and filters, can be modeled by a two-port network model. A two-port network is represented by four external variables voltage V_1 and current I_1 at the input port, and voltage V_2 and current I_2 at the output port, so that the two-port network can be treated as a black box modeled by the relationships between the four variables V_1, V_2, I_1 and I_2. There exist six different ways to describe the relationships between these variables, depending on which two of the four variables are given, while the other two can always be derived.

Two Port Network

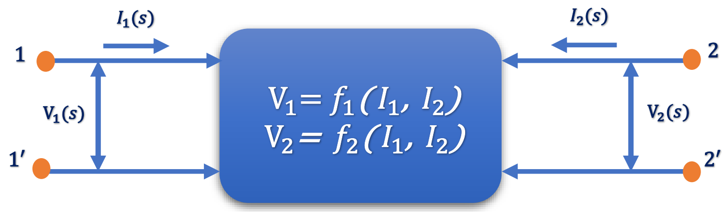
There are different parameters, needed to analyze a two port network.

If the network is linear, i.e., each variable can be expressed as a linear function of some two other variables, then we have the following models

- **Z (or) Impedance Model**
- **Y (or) Admittance Model**
- **ABCD (or) Transmission Model**
- **H (or) Hybrid Model**

Two Port Network

Z parameters are also known as impedance parameters. When we use Z parameter for analyzing two part network, the voltages are represented as the function of currents.



$$V_1 = f_1(I_1, I_2) \text{ and } V_2 = f_2(I_1, I_2)$$

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = Z \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$Z_{11} = Input\ impedance\ keeping\ output\ open = \frac{V_1}{I_1}\ ;\ I_2 = 0.$$

$$Z_{12} = Reverse\ transfer\ impedance\ keeping\ input\ open = \frac{V_1}{I_2}\ ;\ I_1 = 0.$$

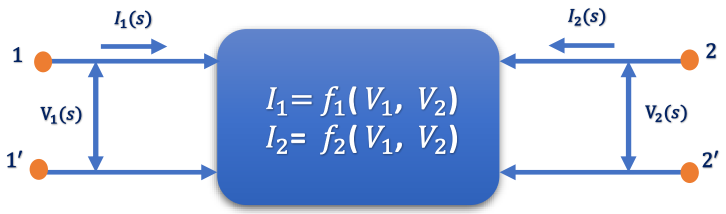
$$Z_{22} = output\ impedance\ keeping\ input\ open = \frac{V_2}{I_2}\ ;\ I_1 = 0.$$

$$Z_{21} = Forward\ transfer\ impedance\ keeping\ output\ open = \frac{V_2}{I_1}\ ;\ I_2 = 0.$$

Two Port Network

Y parameters are also known as admittance parameters. When we use Y parameter for analyzing two part network, the current are represented as the function of voltage. Y parameter is dual of Z parameters.

$I_1 = f_1(V_1, V_2)$ and $I_2 = f_2(V_1, V_2)$



$$I_1 = Y_{11}V_1 + Y_{12}V_2$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = Y \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$Y_{11} = Input\ admittance\ keeping\ output\ short\ circuited = \frac{I_1}{V_1}\ ;\ V_2 = 0.$$

$$Y_{12} = Reverse\ transfer\ admittance\ keeping\ input\ short\ circuited = \frac{I_1}{V_2}\ ;\ V_1 = 0.$$

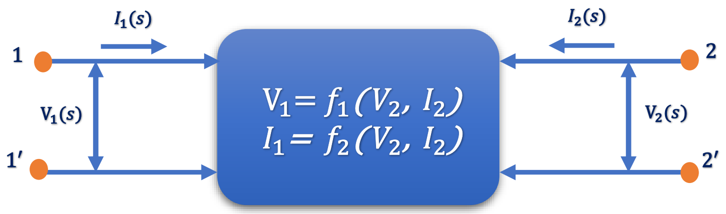
$$Y_{22} = output\ admittance\ keeping\ input\ short\ circuited = \frac{I_2}{V_2}\ ;\ V_1 = 0.$$

$$Y_{21} = Forward\ transfer\ admittance\ keeping\ output\ short\ circuited = \frac{I_2}{V_1}\ ;\ V_2 = 0.$$

Two Port Network

ABCD parameters are also called Transmission parameters. Here, voltage and current and of input part are expressed in term of output part.

$V_1 = f_1(V_2, I_2)$ and $I_1 = f_2(V_2, I_2)$



$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$A = Reverse\ voltage\ gain\ keeping\ output\ open\ circuited = \frac{V_1}{V_2}\ ;\ I_2 = 0.$$

$$B = Reverse\ transfer\ impedance\ keeping\ output\ short\ circuited = \frac{V_1}{I_2}\ ;\ V_2 = 0.$$

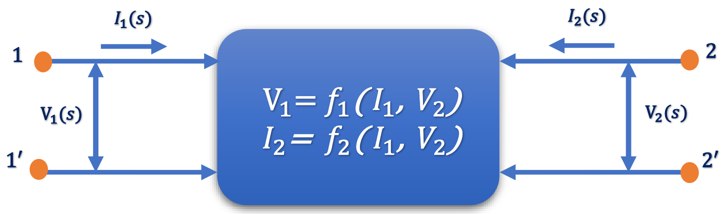
$$C = Reverse\ transfer\ admittance\ keeping\ output\ opencircuited = \frac{I_1}{V_2}\ ;\ I_2 = 0.$$

$$D = Reverse\ current\ gain\ keeping\ output\ short\ circuited = \frac{I_1}{I_2}\ ;\ V_2 = 0.$$

Two Port Network

H parameters also known as hybrid parameters. In hybrid parameter circuit, voltage gain, current gain, impedance and admittance are used to determines relation between current and voltage of two port network.

$$V_1 = f_1(I_1, V_2) \text{ and } I_2 = f_2(I_1, V_2)$$



$$V_1 = H_{11} I_1 + H_{12} V_2$$
$$I_2 = H_{21} I_1 + H_{22} V_2$$

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = H \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$H_{11} = Input\ impedance\ keeping\ output\ short\ circuited = \frac{V_1}{I_1}\ ;\ V_2 = 0.$$

$$H_{12} = Reverse\ voltage\ gain\ keeping\ input\ open = \frac{V_1}{V_2}\ ;\ I_1 = 0.$$

$$H_{22} = output\ admittance\ keeping\ input\ open = \frac{I_2}{V_2}\ ;\ I_1 = 0.$$

$$H_{21} = Forward\ current\ gain\ keeping\ output\ short\ circuited = \frac{I_2}{I_1}\ ;\ V_2 = 0.$$

Two Port Network Interrelations

Z parameters

$V_1 = f_1(I_1, I_2)$ and $V_2 = f_2(I_1, I_2)$

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$Z_{11} = \frac{V_1}{I_1}$; $I_2 = 0.$ $\quad Z_{12} = \frac{V_1}{I_2}$; $I_1 = 0.$

$Z_{22} = \frac{V_2}{I_2}$; $I_1 = 0.$ $\quad Z_{21} = \frac{V_2}{I_1}$; $I_2 = 0.$

Y parameters

$I_1 = f_1(V_1, V_2)$ and $I_2 = f_2(V_1, V_2)$

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$
$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

$Y_{11} = \frac{I_1}{V_1}$; $V_2 = 0.$ $\quad Y_{12} = \frac{I_1}{V_2}$; $V_1 = 0.$

$Y_{22} = \frac{I_2}{V_2}$; $V_1 = 0.$ $\quad Y_{21} = \frac{I_2}{V_1}$; $V_2 = 0.$

ABCD parameters

$V_1 = f_1(V_2, I_2)$ and $I_1 = f_2(V_2, I_2)$

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

$A = \frac{V_1}{V_2}$; $I_2 = 0.$ $\quad B = \frac{V_1}{I_2}$; $V_2 = 0.$

$C = \frac{I_1}{V_2}$; $I_2 = 0.$ $\quad D = \frac{I_1}{I_2}$; $V_2 = 0.$

H parameters

$V_1 = f_1(I_1, V_2)$ and $I_2 = f_2(I_1, V_2)$

$$V_1 = H_{11} I_1 + H_{12} V_2$$
$$I_2 = H_{21} I_1 + H_{22} V_2$$

$H_{11} = \frac{V_1}{I_1}$; $V_2 = 0.$ $\quad H_{12} = \frac{V_1}{V_2}$; $I_1 = 0.$

$H_{22} = \frac{I_2}{V_2}$; $I_1 = 0.$ $\quad H_{21} = \frac{I_2}{I_1}$; $V_2 = 0.$

Two Port Network Interrelations

Z parameters

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

Y parameters

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$
$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = Y \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Using Cramer's rule $V_1 = \dfrac{\begin{vmatrix} I_1 & Y_{12} \\ I_2 & Y_{22} \end{vmatrix}}{\begin{vmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{vmatrix}};\ V_2 = \dfrac{\begin{vmatrix} Y_{11} & I_1 \\ Y_{21} & I_2 \end{vmatrix}}{\begin{vmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{vmatrix}};$

$$V_1 = \frac{(I_1 * Y_{22}) - (I_2 * Y_{12})}{(Y_{11} * Y_{22}) - (Y_{12} * Y_{21})} = \frac{Y_{22} I_1}{|\Delta_Y|} - \frac{Y_{12} I_2}{|\Delta_Y|}$$

$$|\Delta_Y| = (Y_{11} * Y_{22}) - (Y_{12} * Y_{21})$$

$$V_1 = \frac{Y_{22} I_1}{|\Delta_Y|} - \frac{Y_{12} I_2}{|\Delta_Y|}$$

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$Z_{11} = \frac{Y_{22}}{|\Delta_Y|} \qquad Z_{12} = \frac{-Y_{12}}{|\Delta_Y|}$$

$$Z_{21} = \frac{-Y_{21}}{|\Delta_Y|} \qquad Z_{22} = \frac{Y_{11}}{|\Delta_Y|}$$

Two Port Network Interrelations

Y parameters

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$
$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

$$\mathbf{I_1} = \frac{(\mathbf{V_1} * \mathbf{Z_{22}}) - (\mathbf{V_2} * \mathbf{Z_{12}})}{(\mathbf{Z_{11}} * \mathbf{Z_{22}}) - (\mathbf{Z_{12}} * \mathbf{Z_{21}})} = \frac{\mathbf{Z_{22} V_1}}{|\Delta_Z|} - \frac{\mathbf{Z_{12} V_2}}{|\Delta_Z|}$$

$$\mathbf{Y_{11}} = \frac{\mathbf{Z_{22}}}{|\Delta_Z|} \qquad \mathbf{Y_{12}} = \frac{-\mathbf{Z_{12}}}{|\Delta_Z|}$$

$$\mathbf{Y_{21}} = \frac{-\mathbf{Z_{21}}}{|\Delta_Z|} \qquad \mathbf{Y_{22}} = \frac{\mathbf{Z_{11}}}{|\Delta_Z|}$$

$$|\Delta_Z| = (\mathbf{Z_{11}} * \mathbf{Z_{22}}) - (\mathbf{Z_{12}} * \mathbf{Z_{21}})$$

Z parameters

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$\mathbf{V_1} = \frac{\mathbf{Z_{22}} V_1}{|\Delta_Z|} - \frac{\mathbf{Z_{12} V_2}}{|\Delta_Z|}$$

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = Z \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Using Cramer's rule $I_1 = \frac{\begin{vmatrix} V_1 & Z_{12} \\ V_2 & Z_{22} \end{vmatrix}}{\begin{vmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{vmatrix}}$; $I_2 = \frac{\begin{vmatrix} Z_{11} & V_1 \\ Z_{21} & V_2 \end{vmatrix}}{\begin{vmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{vmatrix}}$;

Two Port Network Interrelations

Z parameters

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

ABCD parameters

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

$$I_1 = \frac{V_2 - Z_{22} I_2}{Z_{21}}$$

$$I_1 = \frac{V_2}{Z_{21}} - \frac{Z_{22} I_2}{Z_{21}}$$

$$V_1 = Z_{11}\left(\frac{V_2 - Z_{22} I_2}{Z_{21}}\right) + Z_{12} I_2$$

$$V_1 = \frac{Z_{11}}{Z_{21}} V_2 - \frac{|\Delta_Z|}{Z_{21}} I_2$$

$$A = \frac{Z_{11}}{Z_{21}} \qquad B = \frac{|\Delta_Z|}{Z_{21}}$$

$$C = \frac{1}{Z_{21}} \qquad D = \frac{Z_{22}}{Z_{21}}$$

$$V_2 = \frac{I_1 + DI_2}{C}$$

$$V_2 = \frac{I_1}{C} + \frac{DI_2}{C}$$

$$V_1 = \frac{A}{C} I_1 + \frac{(AD - BC)}{C} I_2$$

$$Z_{11} = \frac{A}{C} \qquad Z_{12} = \frac{AD - BC}{C}$$

$$Z_{21} = \frac{1}{C} \qquad Z_{22} = \frac{D}{C}$$

Two Port Network Interrelations

Z parameters

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

H parameters

$$V_1 = H_{11} I_1 + H_{12} V_2$$
$$I_2 = H_{21} I_1 + H_{22} V_2$$

$$I_2 = -\frac{Z_{21}}{Z_{22}} I_2 + \frac{1}{Z_{22}} V_2$$

$$V_1 = Z_{11}\left(-\frac{Z_{21} I_2}{Z_{22}} + \frac{V_2}{Z_{22}}\right) + Z_{12} I_2$$

$$V_1 = \frac{|\Delta_Z|}{z_{22}} I_1 + \frac{Z_{12}}{Z_{22}} I_2$$

$$H_{11} = \frac{|\Delta_Z|}{Z_{22}} \qquad H_{12} = \frac{Z_{12}}{Z_{22}}$$

$$H_{21} = -\frac{Z_{21}}{Z_{22}} \qquad H_{22} = \frac{1}{Z_{22}}$$

$$V_2 = \frac{-\mathbf{H}_{21} I_1}{\mathbf{H}_{22}} + \frac{\mathbf{I}_2}{\mathbf{H}_{22}}$$

$$V_1 = \frac{H_{11}\, H_{22} - H_{21}\, H_{12}}{H_{22}} I_1 + \frac{H_{12}}{H_{22}} I_2$$

$$Z_{11} = \frac{|\Delta_H|}{H_{22}} \qquad Z_{12} = \frac{H_{12}}{H_{22}}$$

$$Z_{21} = \frac{-\mathbf{H}_{21}}{\mathbf{H}_{22}} \qquad Z_{22} = \frac{1}{\mathbf{H}_{22}}$$

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Two Port Network Interrelations

Y parameters

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$
$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

$$V_1 = -\frac{Y_{22}}{Y_{21}} V_2 + \frac{1}{Y_{21}} I_2$$

$$I_1 = \frac{-|\Delta_Y|}{Y_{21}} V_2 + \frac{Y_{11}}{Y_{21}} I_2$$

$$A = -\frac{Y_{22}}{Y_{21}} \qquad B = -\frac{1}{Y_{21}}$$

$$C = -\frac{|\Delta_Y|}{Y_{21}} \qquad D = -\frac{Y_{11}}{Y_{21}}$$

ABCD parameters

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

$$I_2 = \frac{-1}{B} V_1 + \frac{A}{B} V_2$$

$$I_1 = \frac{D}{B} V_1 - \frac{(AD - BC)}{B} V_2$$

$$Y_{11} = \frac{D}{B} \qquad Y_{12} = -\frac{(AD - BC)}{B}$$

$$Y_{21} = \frac{-1}{B} \qquad Y_{22} = \frac{A}{B}$$

Two Port Network Interrelations

Y parameters

$I_1 = Y_{11} V_1 + Y_{12} V_2$
$I_2 = Y_{21} V_1 + Y_{22} V_2$

$$V_1 = \frac{1}{Y_{11}} I_1 - \frac{Y_{12}}{Y_{11}} V_2$$

$$I_2 = \frac{Y_{21}}{Y_{11}} I_1 + \frac{|\Delta_Y|}{Y_{11}} V_2$$

$$H_{11} = \frac{1}{Y_{11}} \qquad H_{12} = -\frac{Y_{12}}{Y_{11}}$$

$$H_{21} = \frac{Y_{21}}{Y_{11}} \qquad H_{22} = \frac{|\Delta_Y|}{Y_{11}}$$

H parameters

$V_1 = H_{11} I_1 + H_{12} V_2$
$I_2 = H_{21} I_1 + H_{22} V_2$

$$I_1 = \frac{1}{H_{11}} V_1 - \frac{H_{12}}{H_{11}} V_2$$

$$I_2 = \frac{H_{21}}{H_{11}} V_1 + \frac{(H_{11} H_{22} - H_{12} H_{21})}{H_{11}} V_2$$

$$Y_{11} = \frac{1}{H_{11}} \qquad Y_{12} = -\frac{H_{12}}{H_{11}}$$

$$Y_{21} = \frac{H_{21}}{H_{11}} \qquad Y_{22} = \frac{|\Delta_H|}{H_{11}}$$

Two Port Network Interrelations

ABCD parameters

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

H parameters

$$V_1 = H_{11} I_1 + H_{12} V_2$$
$$I_2 = H_{21} I_1 + H_{22} V_2$$

$$I_2 = -\frac{1}{D} I_1 + \frac{C}{D} V_2$$

$$V_1 = \frac{B}{D} I_1 + \frac{(AD - BC)}{D} V_2$$

$$H_{11} = \frac{B}{D} \qquad H_{12} = \frac{(AD - BC)}{D}$$

$$H_{21} = -\frac{1}{D} \qquad H_{22} = \frac{C}{D}$$

$$I_1 = -\frac{H_{22}}{H_{21}} V_2 + \frac{1}{H_{21}} I_2$$

$$V_1 = -\frac{(H_{11} H_{22} - H_{12} H_{21})}{H_{21}} V_2 + \frac{H_{11}}{H_{21}} I_2$$

$$A = -\frac{|\Delta_H|}{H_{21}} \qquad B = -\frac{H_{11}}{H_{21}}$$

$$C = -\frac{H_{22}}{H_{21}} \qquad D = -\frac{1}{H_{21}}$$

Two Port Network Interrelations

Z parameters

$$Z_{11}=\frac{Y_{22}}{|\Delta_Y|}\qquad Z_{12}=\frac{-Y_{12}}{|\Delta_Y|}$$

$$Z_{21}=\frac{-Y_{21}}{|\Delta_Y|}\qquad Z_{22}=\frac{Y_{11}}{|\Delta_Y|}$$

$$Z_{11}=\frac{A}{C}\qquad Z_{12}=\frac{AD-BC}{C}$$

$$Z_{21}=\frac{1}{C}\qquad Z_{22}=\frac{D}{C}$$

$$Z_{11}=\frac{|\Delta_H|}{H_{22}}\qquad Z_{12}=\frac{H_{12}}{H_{22}}$$

$$Z_{21}=\frac{-H_{21}}{H_{22}}\qquad Z_{22}=\frac{1}{H_{22}}$$

Y parameters

$$Y_{11}=\frac{Z_{22}}{|\Delta_Z|}\qquad Y_{12}=\frac{-Z_{12}}{|\Delta_Z|}$$

$$Y_{21}=\frac{-Z_{21}}{|\Delta_Z|}\qquad Y_{22}=\frac{Z_{11}}{|\Delta_Z|}$$

$$Y_{11}=\frac{D}{B}\qquad Y_{12}=-\frac{(AD-BC)}{B}$$

$$Y_{21}=\frac{-1}{B}\qquad Y_{22}=\frac{A}{B}$$

$$Y_{11}=\frac{1}{H_{11}}\qquad Y_{12}=-\frac{H_{12}}{H_{11}}$$

$$Y_{21}=\frac{H_{21}}{H_{11}}\qquad Y_{22}=\frac{|\Delta_H|}{H_{11}}$$

ABCD parameters

$$A=\frac{Z_{11}}{Z_{21}}\qquad B=\frac{|\Delta_Z|}{Z_{21}}$$

$$C=\frac{1}{Z_{21}}\qquad D=\frac{Z_{22}}{Z_{21}}$$

$$A=-\frac{Y_{22}}{Y_{21}}\qquad B=-\frac{1}{Y_{21}}$$

$$C=-\frac{|\Delta_Y|}{Y_{21}}\qquad D=-\frac{Y_{11}}{Y_{21}}$$

$$A=-\frac{|\Delta_H|}{H_{21}}\qquad B=-\frac{H_{11}}{H_{21}}$$

$$C=-\frac{H_{22}}{H_{21}}\qquad D=-\frac{1}{H_{21}}$$

H parameters

$$H_{11}=\frac{|\Delta_Z|}{Z_{22}}\qquad H_{12}=\frac{Z_{12}}{Z_{22}}$$

$$H_{21}=-\frac{Z_{21}}{Z_{22}}\qquad H_{22}=\frac{1}{Z_{22}}$$

$$H_{11}=\frac{1}{Y_{11}}\qquad H_{12}=-\frac{Y_{12}}{Y_{11}}$$

$$H_{21}=\frac{Y_{21}}{Y_{11}}\qquad H_{22}=\frac{|\Delta_Y|}{Y_{11}}$$

$$H_{11}=\frac{B}{D}\qquad H_{12}=\frac{(AD-BC)}{D}$$

$$H_{21}=-\frac{1}{D}\qquad H_{22}=\frac{C}{D}$$

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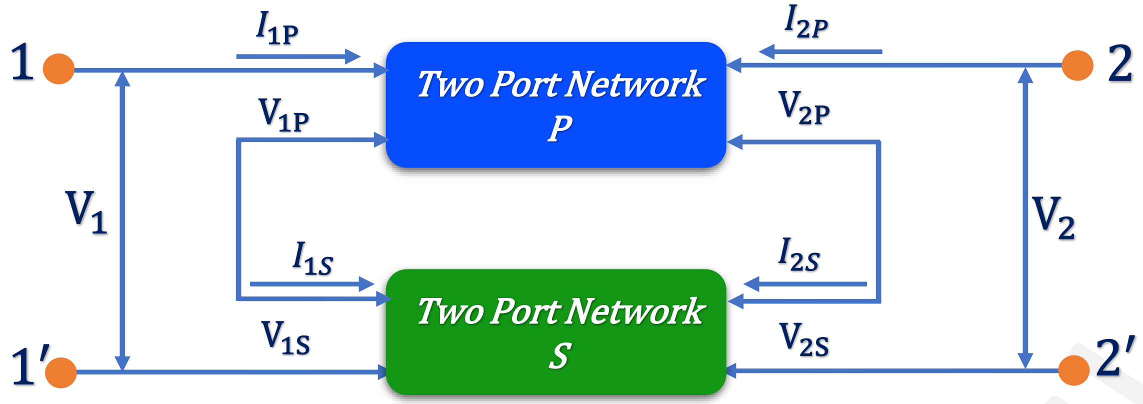
Two Port Interconnections

Interconnections permit the description of complex systems in terms of simpler components or subsystems.

Various types of Interconnection of Two Port Network such as series connection of two ports, parallel connection of two ports, cascade connection of two ports etc.

- Series Connection of Two Ports
- Parallel Connection of Two Ports
- Cascade Connection of Two Ports
- Series and Parallel Permissibility of Connection

Two Port Interconnections



Consider two networks P and S are connected in series. When two ports are connected in series, we can add their z parameters to get overall z-parameter of the overall series connection.

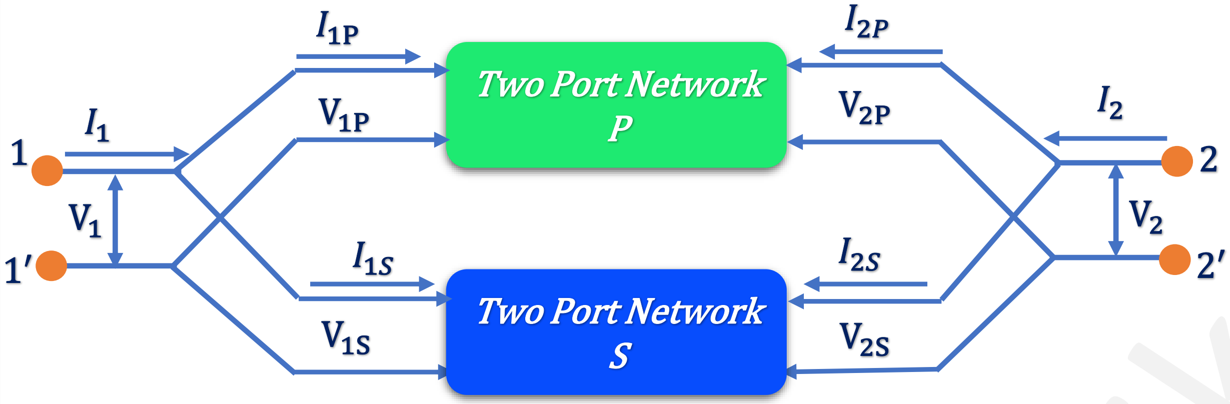
Current are same, Voltage of interconnection is the sum of voltages.

$$V_{1P} = Z_{11P} I_{1P} + Z_{12P} I_{2P}$$
$$V_{2P} = Z_{21P} I_{1P} + Z_{22P} I_{2P}$$

$$V_{1S} = Z_{11S} I_{1S} + Z_{12S} I_{2S}$$
$$V_{2S} = Z_{21S} I_{1S} + Z_{22S} I_{2S}$$

$$V_1 = (Z_{11P} + Z_{11S}) I_1 + (Z_{12P} + Z_{12S}) I_2$$
$$V_2 = (Z_{21P} + Z_{21S}) I_1 + (Z_{22P} + Z_{22S}) I_2$$

Two Port Interconnections



Consider two networks P and S are connected in parallel. When two ports are connected in parallel, we can add their y-parameters to get overall y-parameters of the parallel connection.

Voltages are same, Current of Interconnection is the sum of current.

$$I_{1P} = Y_{11P} V_{1P} + Y_{12P} V_{2P}$$
$$I_{2P} = Y_{21P} V_{1P} + Y_{22P} V_{2P}$$

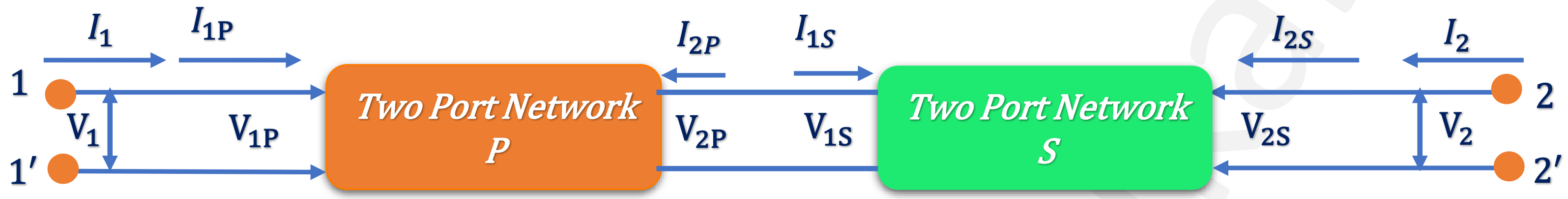
$$I_{1S} = Y_{11S} V_{1S} + Y_{12S} V_{2S}$$
$$I_{2S} = Y_{21S} V_{1S} + Y_{22S} V_{2S}$$

$$I_1 = (Y_{11P} + Y_{11S})V_1 + (Y_{12P} + Y_{12S})V_2$$
$$I_2 = (Y_{21P} + Y_{21S})V_1 + (Y_{22P} + Y_{22S})V_2$$

Two Port Interconnections

The cascade connection is also called Tandem connection. Consider two networks P and S are connected in cascade. When two ports are connected in cascade, we can multiply their individual transmission parameters to get overall transmission parameters of the cascade connection.

Output of first subsystem acts as input for the second.



$$V_{1P} = A_P V_{2P} - B_P I_{2P} \qquad V_{1S} = A_S V_{2S} - B_S I_{2S}$$
$$I_{1P} = C_P V_{2P} - D_P I_{2P} \qquad I_{1S} = C_S V_{2S} - D_S I_{2S}$$

$$V_1 = (A_P A_S) V_2 - (B_P B_S) I_{2S}$$
$$I_1 = (C_P C_S) V_2 - (D_P D_S) I_{2S}$$

Two Port Network

Z parameters

$V_1 = f_1(I_1, I_2)$ and $V_2 = f_2(I_1, I_2)$

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$Z_{11} = \frac{V_1}{I_1}$; $I_2 = 0.$ $\quad Z_{12} = \frac{V_1}{I_2}$; $I_1 = 0.$

$Z_{22} = \frac{V_2}{I_2}$; $I_1 = 0.$ $\quad Z_{21} = \frac{V_2}{I_1}$; $I_2 = 0.$

Y parameters

$I_1 = f_1(V_1, V_2)$ and $I_2 = f_2(V_1, V_2)$

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$
$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

$Y_{11} = \frac{I_1}{V_1}$; $V_2 = 0.$ $\quad Y_{12} = \frac{I_1}{V_2}$; $V_1 = 0.$

$Y_{22} = \frac{I_2}{V_2}$; $V_1 = 0.$ $\quad Y_{21} = \frac{I_2}{V_1}$; $V_2 = 0.$

ABCD parameters

$V_1 = f_1(V_2, I_2)$ and $I_1 = f_2(V_2, I_2)$

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

$A = \frac{V_1}{V_2}$; $I_2 = 0.$ $\quad B = \frac{V_1}{I_2}$; $V_2 = 0.$

$C = \frac{I_1}{V_2}$; $I_2 = 0.$ $\quad D = \frac{I_1}{I_2}$; $V_2 = 0.$

H parameters

$V_1 = f_1(I_1, V_2)$ and $I_2 = f_2(I_1, V_2)$

$$V_1 = H_{11} I_1 + H_{12} V_2$$
$$I_2 = H_{21} I_1 + H_{22} V_2$$

$H_{11} = \frac{V_1}{I_1}$; $V_2 = 0.$ $\quad H_{12} = \frac{V_1}{V_2}$; $I_1 = 0.$

$H_{22} = \frac{I_2}{V_2}$; $I_1 = 0.$ $\quad H_{21} = \frac{I_2}{I_1}$; $V_2 = 0.$

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Two Port Network Interrelations

Z parameters

$$Z_{11}=\frac{Y_{22}}{|\Delta_Y|} \qquad Z_{12}=\frac{-Y_{12}}{|\Delta_Y|}$$

$$Z_{21}=\frac{-Y_{21}}{|\Delta_Y|} \qquad Z_{22}=\frac{Y_{11}}{|\Delta_Y|}$$

$$Z_{11}=\frac{A}{C} \qquad Z_{12}=\frac{AD-BC}{C}$$

$$Z_{21}=\frac{1}{C} \qquad Z_{22}=\frac{D}{C}$$

$$Z_{11}=\frac{|\Delta_H|}{H_{22}} \qquad Z_{12}=\frac{H_{12}}{H_{22}}$$

$$Z_{21}=\frac{-H_{21}}{H_{22}} \qquad Z_{22}=\frac{1}{H_{22}}$$

Y parameters

$$Y_{11}=\frac{Z_{22}}{|\Delta_Z|} \qquad Y_{12}=\frac{-Z_{12}}{|\Delta_Z|}$$

$$Y_{21}=\frac{-Z_{21}}{|\Delta_Z|} \qquad Y_{22}=\frac{Z_{11}}{|\Delta_Z|}$$

$$Y_{11}=\frac{D}{B} \qquad Y_{12}=-\frac{(AD-BC)}{B}$$

$$Y_{21}=\frac{-1}{B} \qquad Y_{22}=\frac{A}{B}$$

$$Y_{11}=\frac{1}{H_{11}} \qquad Y_{12}=-\frac{H_{12}}{H_{11}}$$

$$Y_{21}=\frac{H_{21}}{H_{11}} \qquad Y_{22}=\frac{|\Delta_H|}{H_{11}}$$

ABCD parameters

$$A=\frac{Z_{11}}{Z_{21}} \qquad B=\frac{|\Delta_Z|}{Z_{21}}$$

$$C=\frac{1}{Z_{21}} \qquad D=\frac{Z_{22}}{Z_{21}}$$

$$A=-\frac{Y_{22}}{Y_{21}} \qquad B=-\frac{1}{Y_{21}}$$

$$C=-\frac{|\Delta_Y|}{Y_{21}} \qquad D=-\frac{Y_{11}}{Y_{21}}$$

$$A=-\frac{|\Delta_H|}{H_{21}} \qquad B=-\frac{H_{11}}{H_{21}}$$

$$C=-\frac{H_{22}}{H_{21}} \qquad D=-\frac{1}{H_{21}}$$

H parameters

$$H_{11}=\frac{|\Delta_Z|}{Z_{22}} \qquad H_{12}=\frac{Z_{12}}{Z_{22}}$$

$$H_{21}=-\frac{Z_{21}}{Z_{22}} \qquad H_{22}=\frac{1}{Z_{22}}$$

$$H_{11}=\frac{1}{Y_{11}} \qquad H_{12}=-\frac{Y_{12}}{Y_{11}}$$

$$H_{21}=\frac{Y_{21}}{Y_{11}} \qquad H_{22}=\frac{|\Delta_Y|}{Y_{11}}$$

$$H_{11}=\frac{B}{D} \qquad H_{12}=\frac{(AD-BC)}{D}$$

$$H_{21}=-\frac{1}{D} \qquad H_{22}=\frac{C}{D}$$

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Problems on Two Port Network

The parameters of two port network are Z11=20ohms,Z22 =30 ohms,Z12=Z21=10ohm find Y and ABCD parameters of the net work

$Z_{11}=20\Omega$ $Z_{22}=30\Omega$ $Z_{12}=10\Omega$ $Z_{21}=10\Omega$

$|\Delta_Z| = (Z_{11}Z_{22} - Z_{12}Z_{21})$

$= (20\times 30) - (10\times 10)$

$|\Delta_Z| = 500$

$$Y_{11}=\frac{Z_{22}}{|\Delta_Z|} \quad Y_{12}=\frac{-Z_{12}}{|\Delta_Z|}$$

$$Y_{21}=\frac{-Z_{21}}{|\Delta_Z|} \quad Y_{22}=\frac{Z_{11}}{|\Delta_Z|}$$

$$A=\frac{Z_{11}}{Z_{21}} \quad B=\frac{|\Delta_Z|}{Z_{21}}$$

$$C=\frac{1}{Z_{21}} \quad D=\frac{Z_{22}}{Z_{21}}$$

$Y_{11} = \frac{30}{500} = 0.06\,\Omega^{-1}$

$Y_{12} = \frac{-10}{500} = -0.02$

$Y_{21} = -0.02$

$Y_{22} = \frac{20}{500} = 0.04$

$\frac{1}{\Omega}$

$A = \frac{20}{10} = 2$

$B = \frac{500}{10} = 50$

$C = \frac{1}{10} = 0.1$

$D = \frac{30}{10} = 3$

$Y = I_1 \; I_2$

$V_1 \; V_2$

$V_2, -I_2$

ABCD

V_1, I_1 — Z

$V_1 = Z_{11}I_1 + Z_{12}I_2$

$V_2 = Z_{21}I_1 + Z_{22}I_2$

Y: V_1, V_2

$I_1 \rightarrow V_1, V_2$

I_2

$V_1 = 20I_1 + 10I_2$

$V_2 = 10I_1 + 30I_2$

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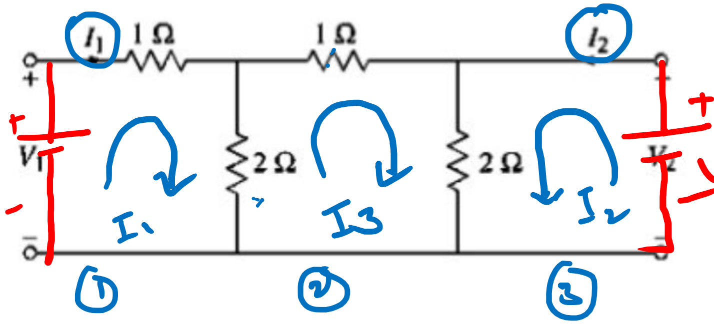
Problems on Two Port Network

Find the transmission parameters for the network shown in figure.



$V_1 = f(V_2, I_2)$

$I_1 = f(V_2, I_2)$

(1) → $1I_1 + 2(I_1 - I_3) = V_1$

$V_1 = 3I_1 - 2I_3$ — (1)

(2) → $2(I_3 - I_1) + 1I_3 + 2(I_3 + I_2) = 0$

$-2I_1 + 2I_2 + 5I_3 = 0$ — (2) ⇒

(3) → $V_2 = 2I_2 + 2I_3$ — (3) ⇒ $I_3 = \frac{V_2}{2} - I_2$

$-2I_1 + 2I_2 + 5I_3 = 0$

$2I_1 = 2I_2 + 5I_3$

$I_1 = \frac{2I_2}{2} + \frac{5}{2}I_3$

$V_1 = 3\left[I_2 + \frac{5}{2}I_3\right] - 2I_3$

$V_1 = 3\left[I_2 + \frac{5}{2}\left(\frac{V_2}{2} - I_2\right)\right] - 2\times\left[\frac{V_2}{2} - I_2\right]$

$I_1 = I_2 + \frac{5}{2}\left[\frac{V_2}{2} - I_2\right]$

$V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$

$A = 11/4$

$B = 5/2$

$C = 5/4$

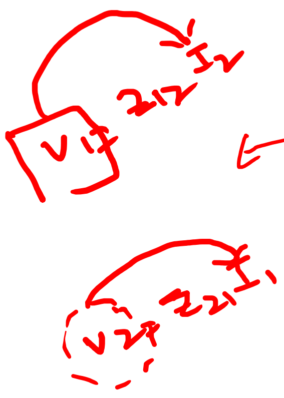
$D = 3/2$

Problems on Two Port Network

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

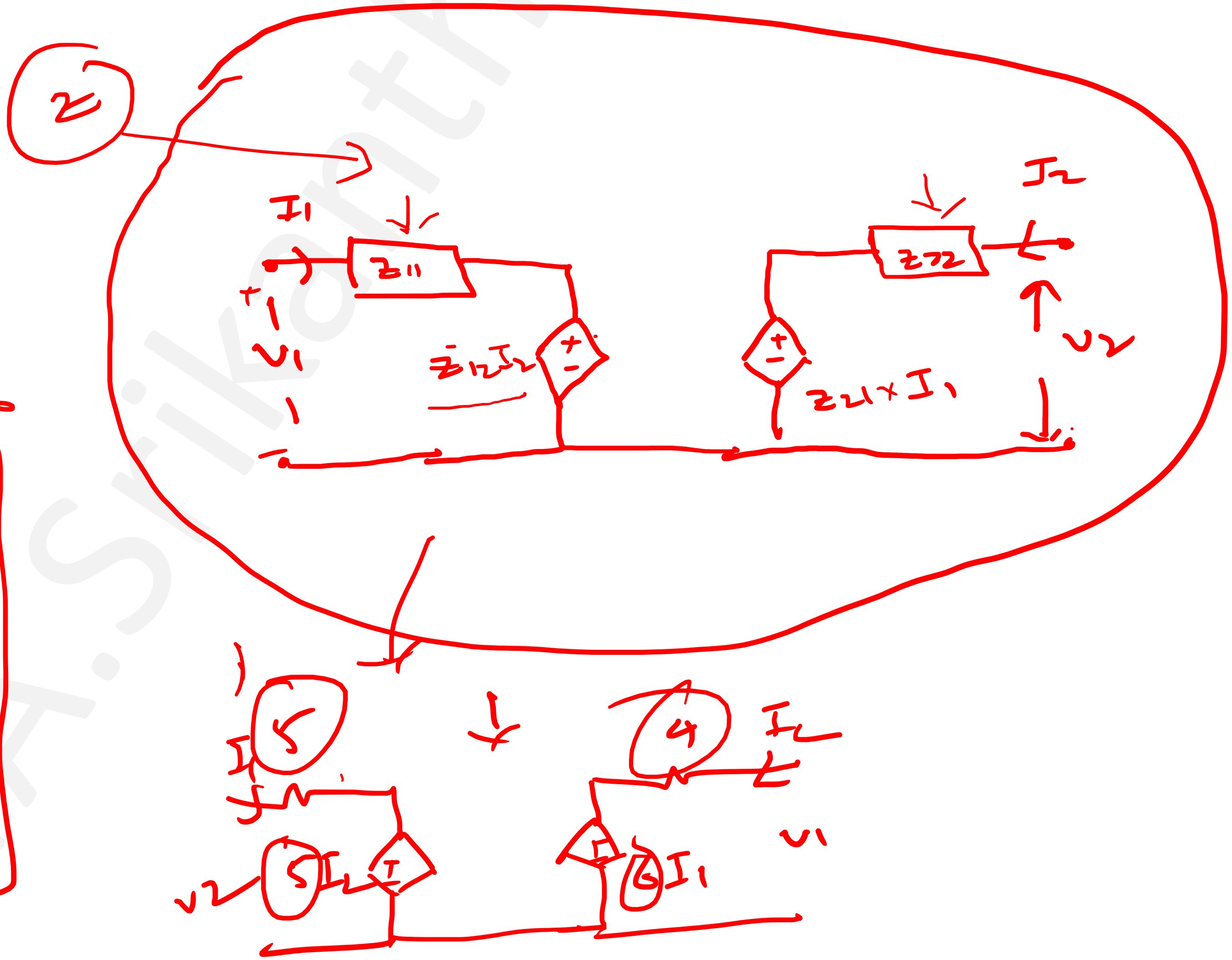
$$Z_{11} = \frac{V_1}{I_1} ; \quad I_2 = 0$$



$$Z_{12} = \frac{V_1}{I_2} ; \quad I_1 = 0$$

$$Z_{21} = \frac{V_2}{I_1} ; \quad I_2 = 0$$

$$Z_{22} = \frac{V_2}{I_2} ; \quad I_1 = 0$$



Problems on Two Port Network

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

Y

$$Y_{11} = \frac{I_1}{V_1} \quad V_2 = 0$$

$$I_1 = Y_{12}V_2 \Leftarrow Y_{12} = \frac{I_1}{V_2} \quad V_1 = 0$$

$$Y_{21} = \frac{I_2}{V_1}; \quad V_2 = 0$$

$$Y_{22} = \frac{I_2}{V_2}; \quad V_1 = 0$$

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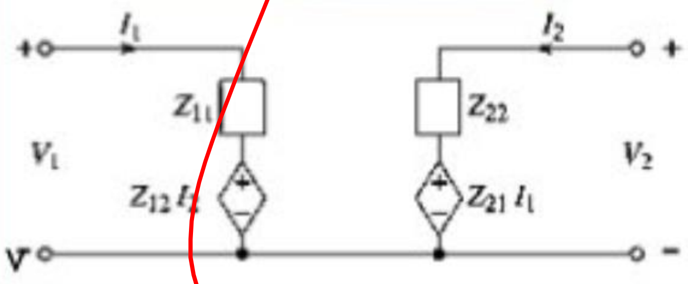
Problems on Two Port Network

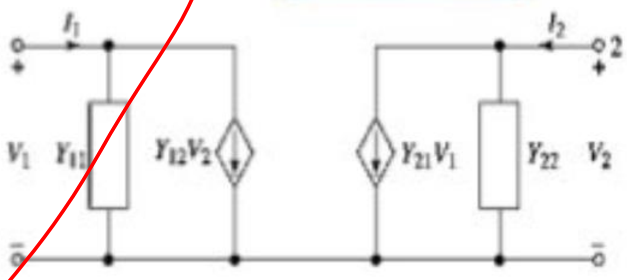
Impedance Z parameters

$$V_1 = z_{11}I_1 + z_{12}I_2$$
$$V_2 = z_{21}I_1 + z_{22}I_2$$



Admittance Y parameters

$$I_1 = y_{11}V_1 + y_{12}V_2$$
$$I_2 = y_{21}V_1 + y_{22}V_2$$



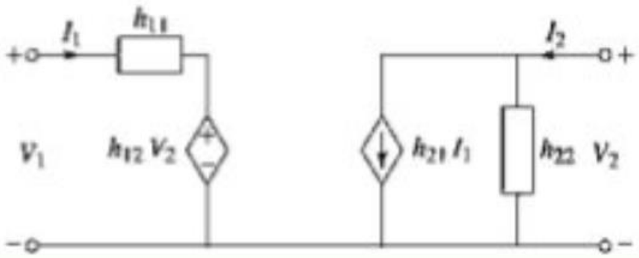
Transmission line parameters

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

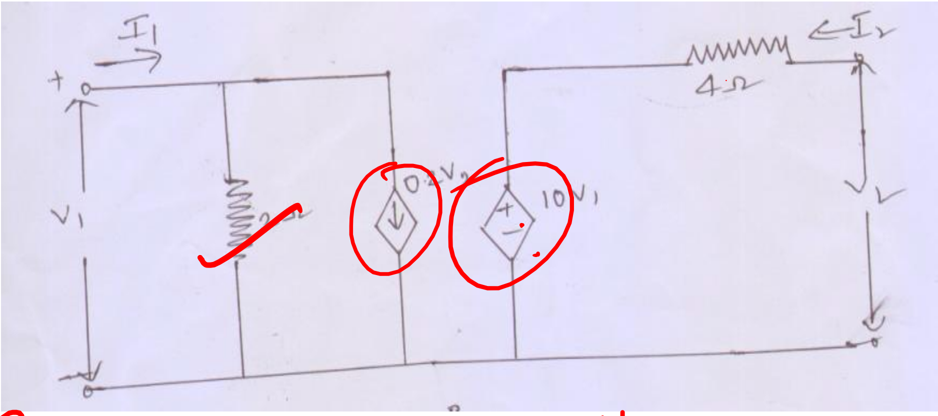
NOT POSSIBLE

Hybrid H parameters

$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2$$



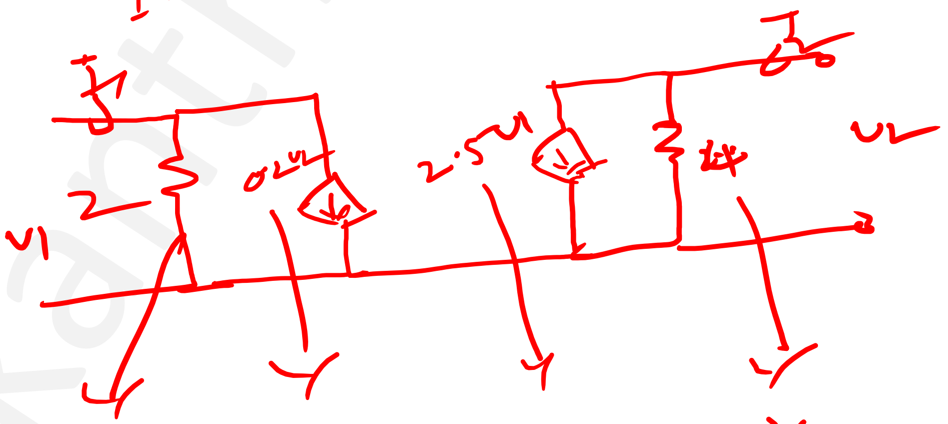
Problems on Two Port Network



Y

$v = I \times R$

$I = v/R = \frac{10V_1}{4}$



Y_{11}	Y_{12}	Y_{21}	Y_{22}
2	0.2	2.5	4

$Y_{11} = 2\Omega$

$Y_{12} = 0.2\Omega$

Y_{21}

Y_{22}

Z → V_1 V_2 → f(I_1, I_2)

Y → I_1, I_2 → f(V_1 V_2)

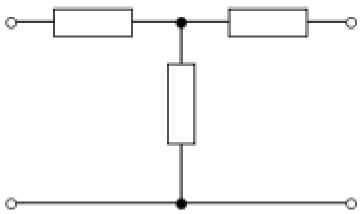
T → V_1 I_1 → f(V_2 I_2)

h → V_1, I_2 → f(V_2 I_2)

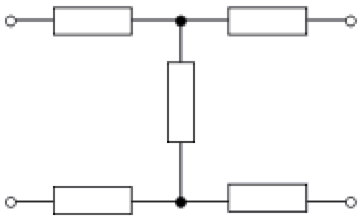
I

Network Topology or Graph Theory

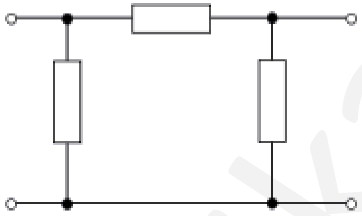
Network topology is a graphical representation of electric circuits. It is useful for analyzing complex electric circuits by converting them into network graphs. Network topology is also called as Graph theory.



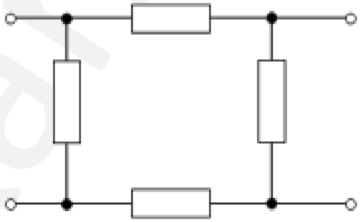
T topology



H topology

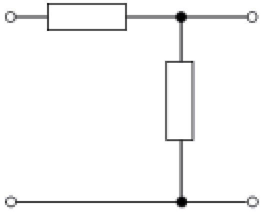


Π topology



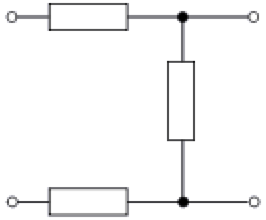
Box topology

Unbalanced form

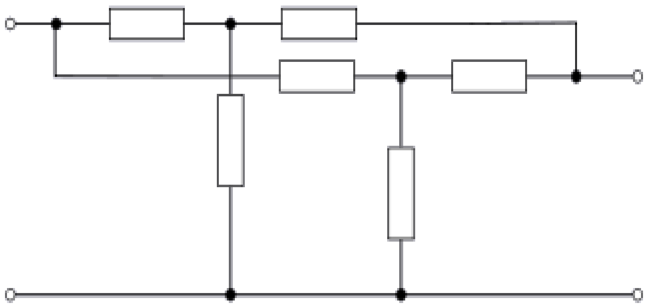


L topology

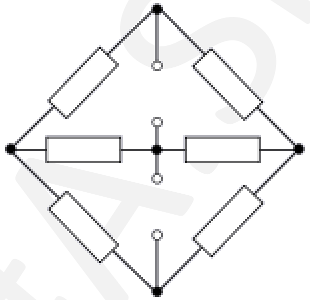
Balanced form



C topology



Twin T topology

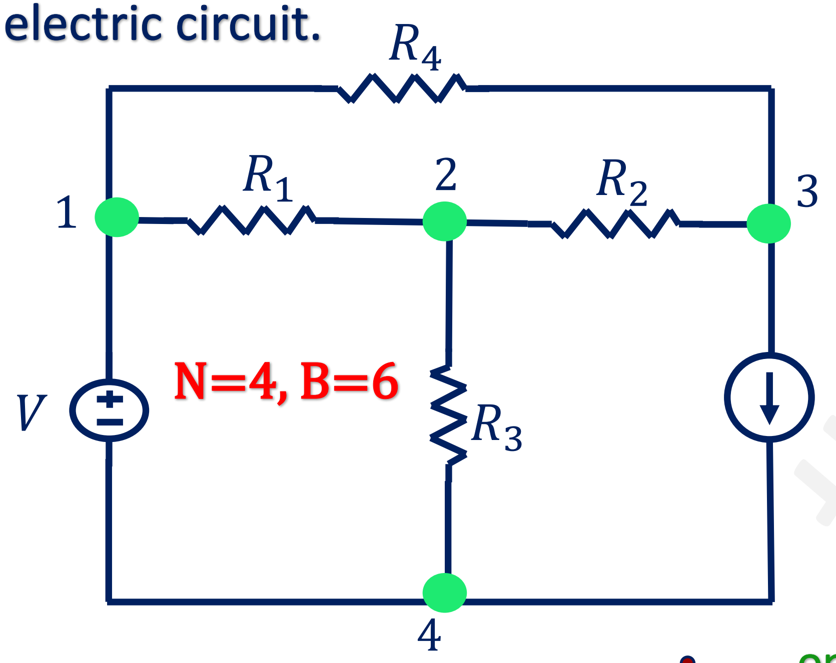


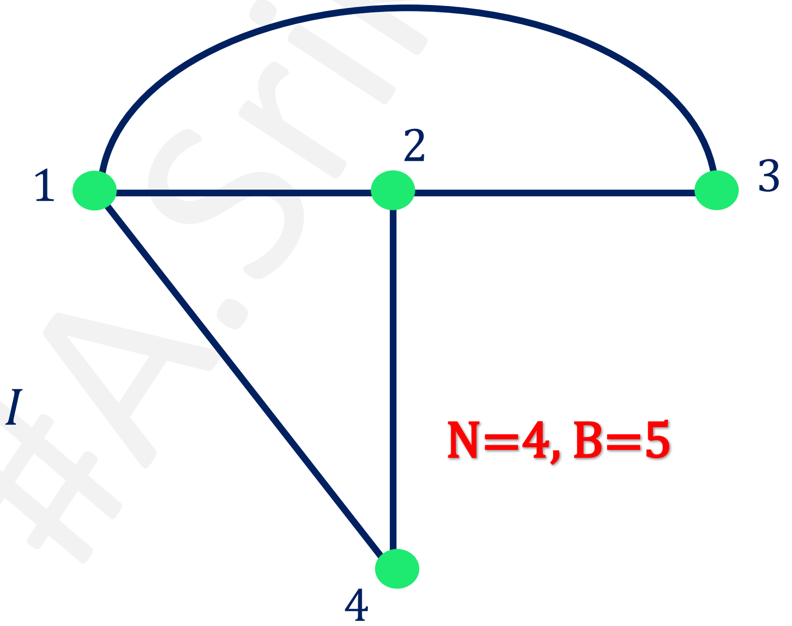
Twin T topology drawn as a bridge

Network Topology or Graph Theory

Graph consists of a set of nodes connected by branches. In graphs, a node is a common point of two or more branches. Sometimes, only a single branch may connect to the node. A branch is a line segment that connects two nodes.

Any electric circuit or network can be converted into its equivalent graph by *replacing the passive elements and voltage sources with short circuits and the current sources with open circuits.* That means, the line segments in the graph represent the branches corresponding to either passive elements or voltage sources of electric circuit.





one branch less in the graph because the current source is made as open circuit

Network Topology or Graph Theory

Graph

The number of nodes present in a graph will be equal to the number of principal nodes present in an electric circuit.

The number of branches present in a graph will be less than or equal to the number of branches present in an electric circuit.

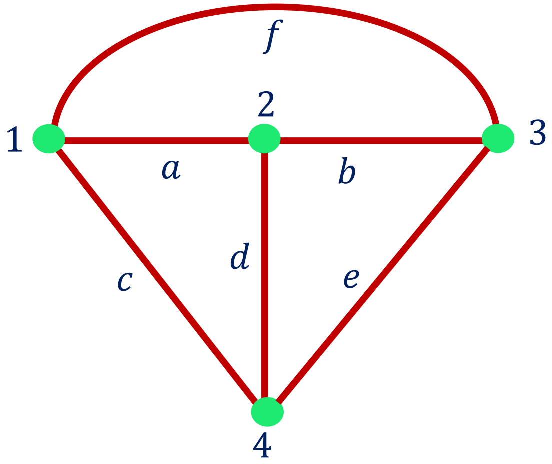
Types of Graphs

- Connected Graph
- Unconnected Graph
- Directed Graph
- Undirected Graph

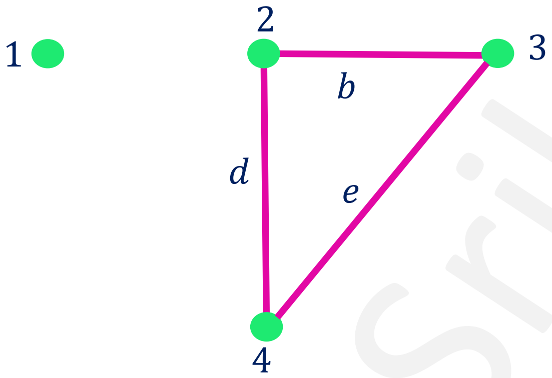
Network Topology or Graph Theory

Types of Graphs

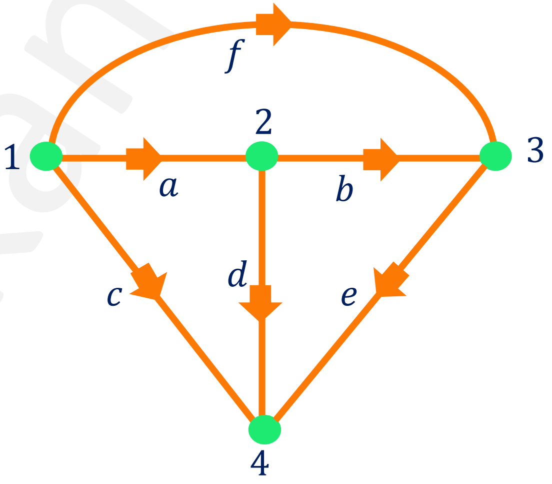
- Connected Graph



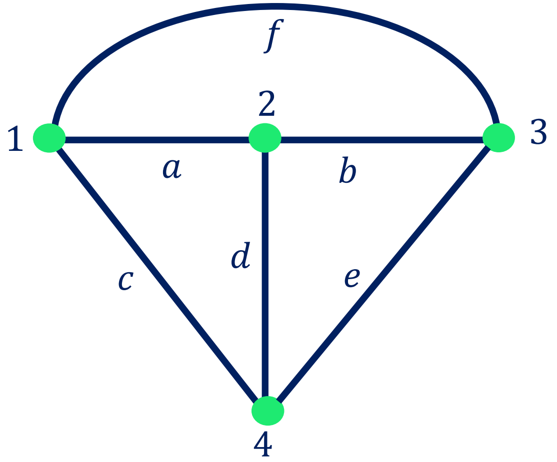
- Unconnected Graph



- Directed Graph



- Undirected Graph



Network Topology or Graph Theory

Subgraph

A part of the graph is called as a **subgraph**. We get subgraphs by removing some nodes and/or branches of a given graph. So, the number of branches and/or nodes of a subgraph will be less than that of the original graph. Hence, we can conclude that a subgraph is a subset of a graph.

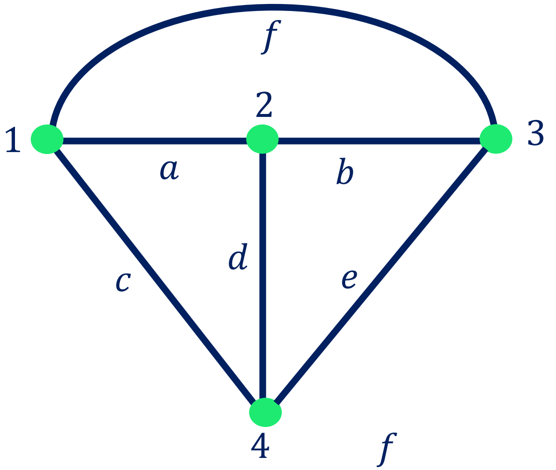
Types of Subgraph

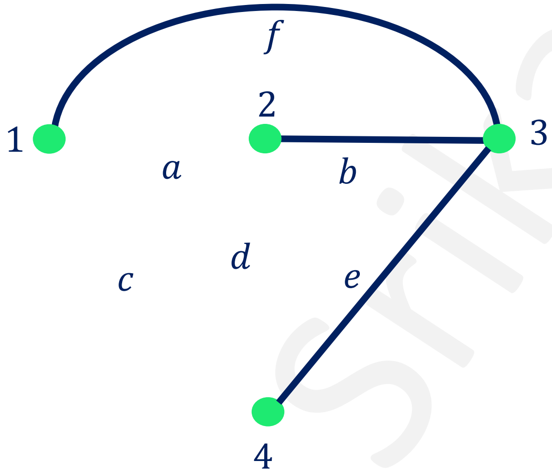
- Tree
- Co-Tree

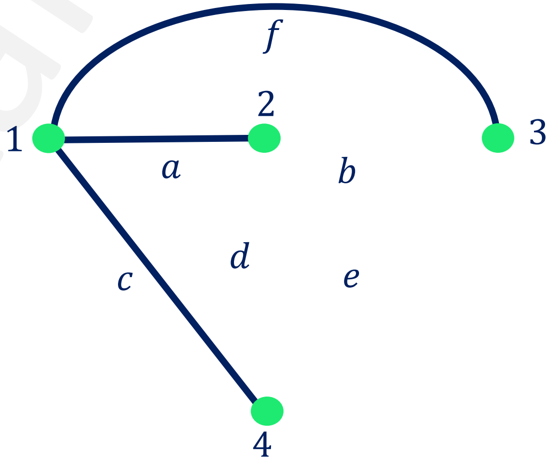
Network Topology or Graph Theory

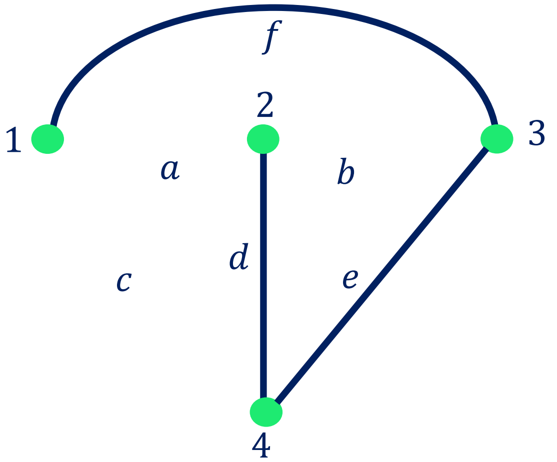
Subgraph

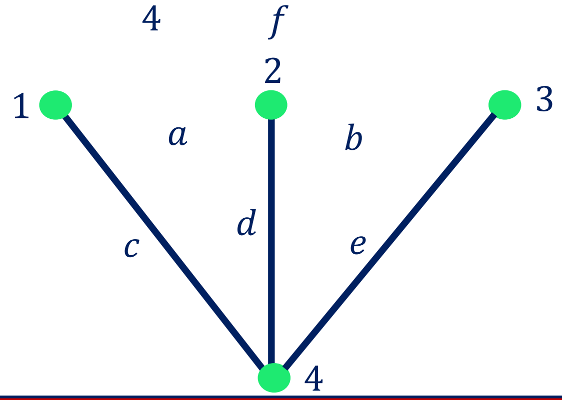
Tree is a connected subgraph of a given graph, which **contains all the nodes of a graph**. But, there **should not be any loop** in that subgraph. The branches of a tree are called as twigs

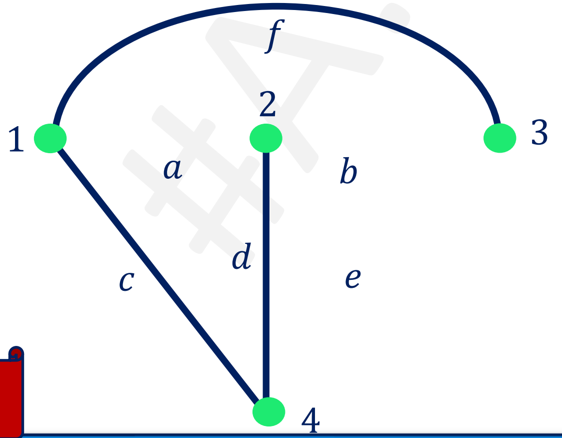










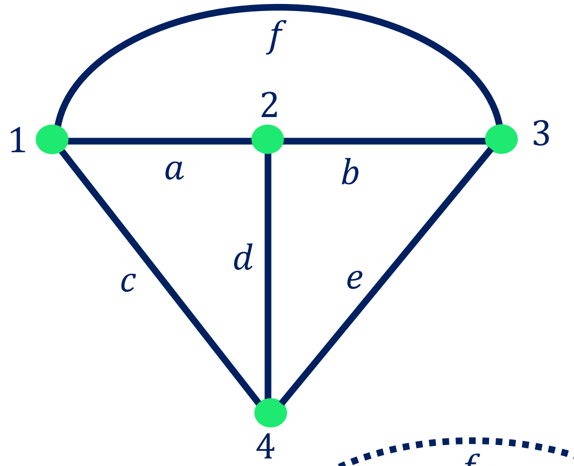


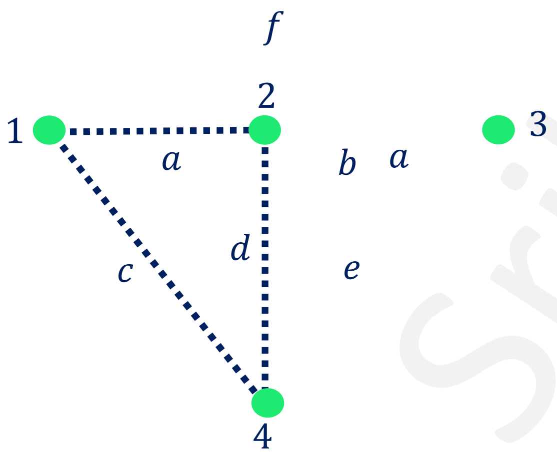
Tree should be equal to N-1 where 'N' is the number of nodes of the given graph.

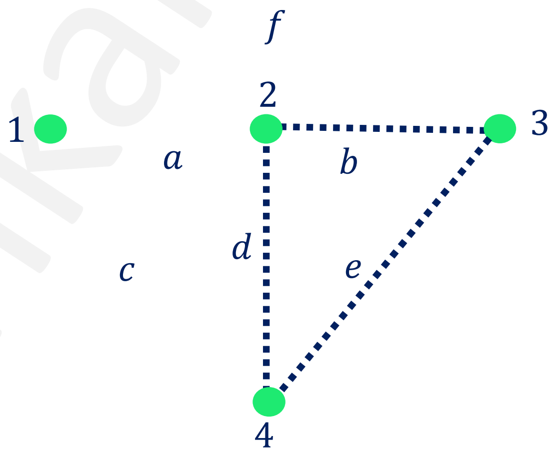
Network Topology or Graph Theory

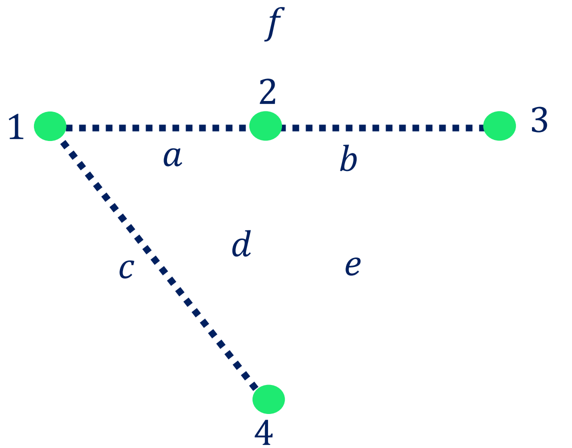
Subgraph

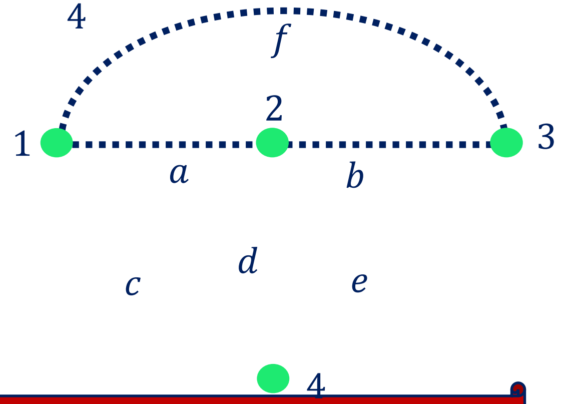
Co-Tree is a subgraph, which is formed with the branches that are removed while forming a Tree. Hence, it is called as Complement of a Tree. For every Tree, there will be a corresponding Co-Tree and its branches are called as links(l) or chords. In general, the links are represented with dotted lines.

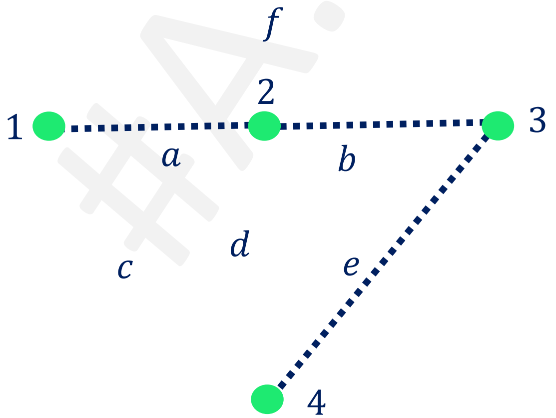












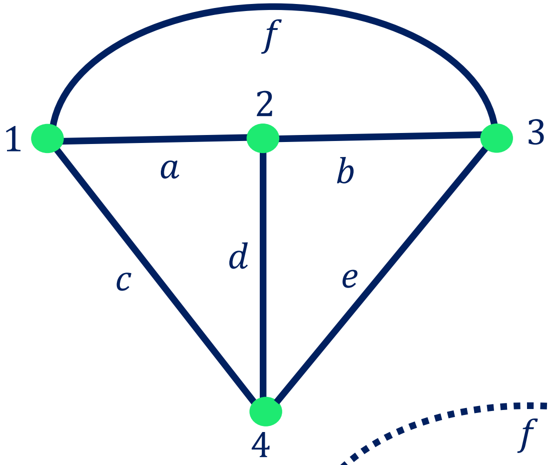
$$l = B - N + 1$$

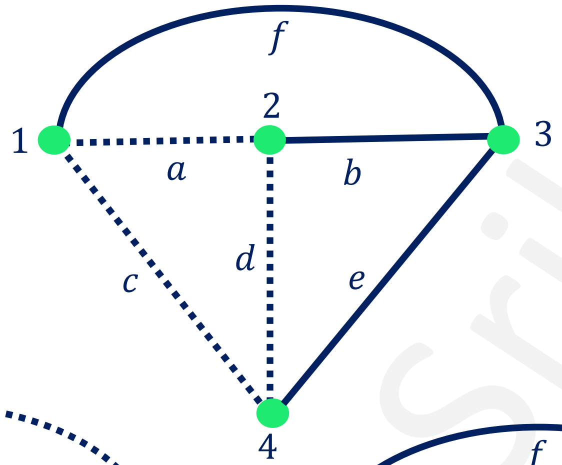
l = Number of links.
B =Number of branches present in a given graph.
N=Number of nodes present in a given graph.

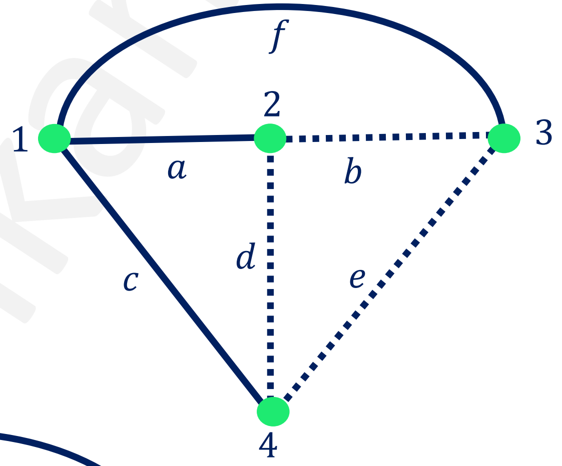
Network Topology or Graph Theory

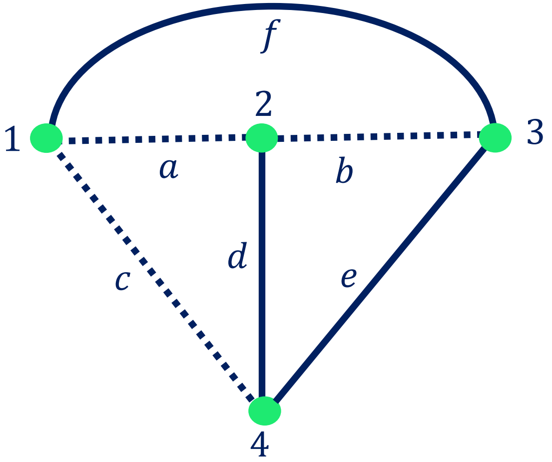
Original Graph

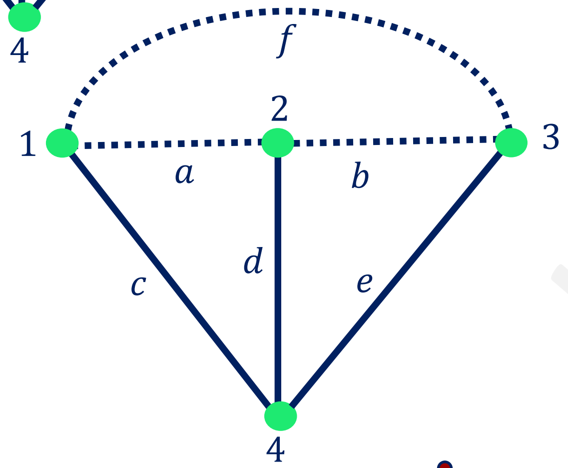
If we combine a Tree and its corresponding Co-Tree, then we will get the original graph

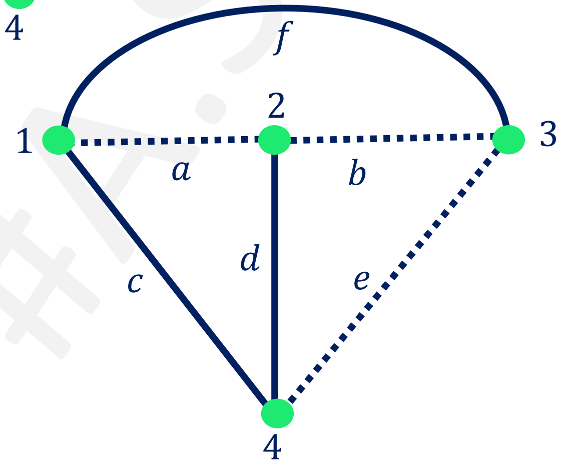


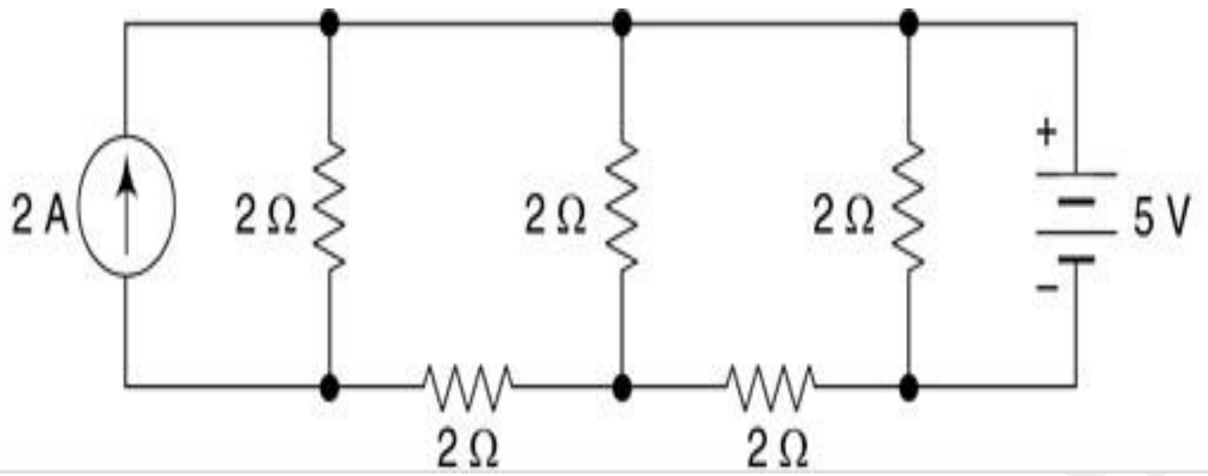











2 A
2 Ω
2 Ω
2 Ω
+
5 V
-
2 Ω
2 Ω

Network Topology or Graph Theory

Incidence Matrix

Incidence Matrix is represented with the letter A.
The order of incidence matrix will be N × B.

Basic Tie set Matrices

Basic Tie set Matrices is represented with the letter B.
The order of Basic Tie set matrix will be l × B.

Basic cut set Matrices

Basic Cut set Matrices is represented with the letter C.
The order of Basic Cut set matrix will be T × B.

The elements of matrix will be having one of these three values, +1, -1 and 0.
If the branch current is entering towards a selected node, then the value of the element will be +1.
If the branch current is leaving from a selected node, then the value of the element will be -1.
If the branch current neither enters at a selected node nor leaves from a selected node, then the value of element will be 0.

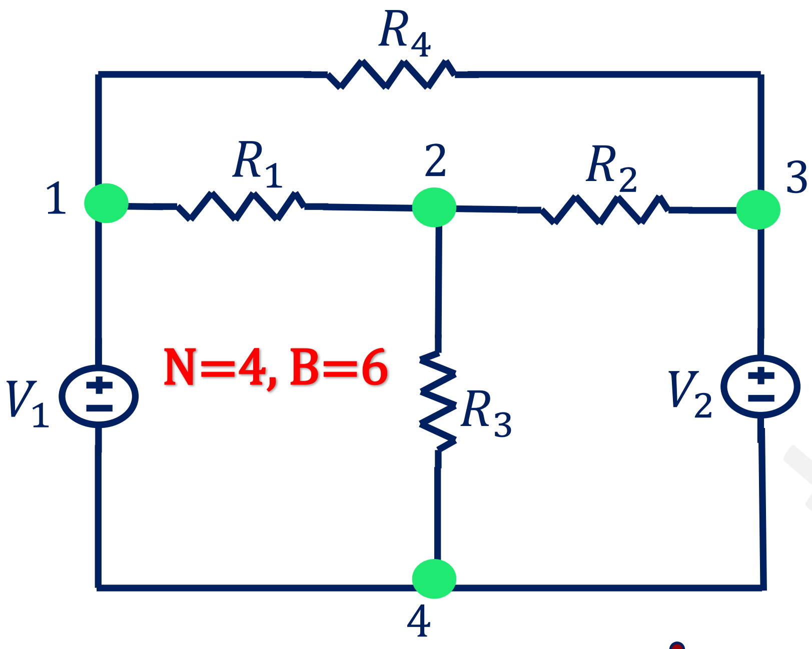
Network Topology or Graph Theory

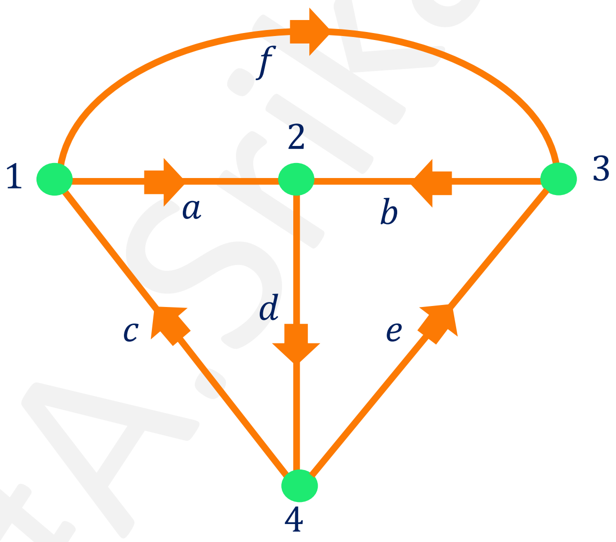
Incidence Matrix

Incidence Matrix is represented with the letter A.
The order of incidence matrix will be N × B.

If there are 'N' nodes and 'B' branches are present in a directed graph, then the incidence matrix will have 'N' rows and 'B' columns. Here, rows and columns are corresponding to the nodes and branches of a directed graph. Hence, the order of incidence matrix will be N × B .

$$A = \begin{array}{c} \\ N_1 \\ N_2 \\ N_3 \\ N_4 \end{array} \begin{array}{c} \begin{matrix} a & b & c & d & e & f \end{matrix} \\ \begin{bmatrix} -1 & 0 & 1 & 0 & 0 & -1 \\ 1 & 1 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 & -1 & 0 \end{bmatrix} \end{array}$$





N=4, B=6

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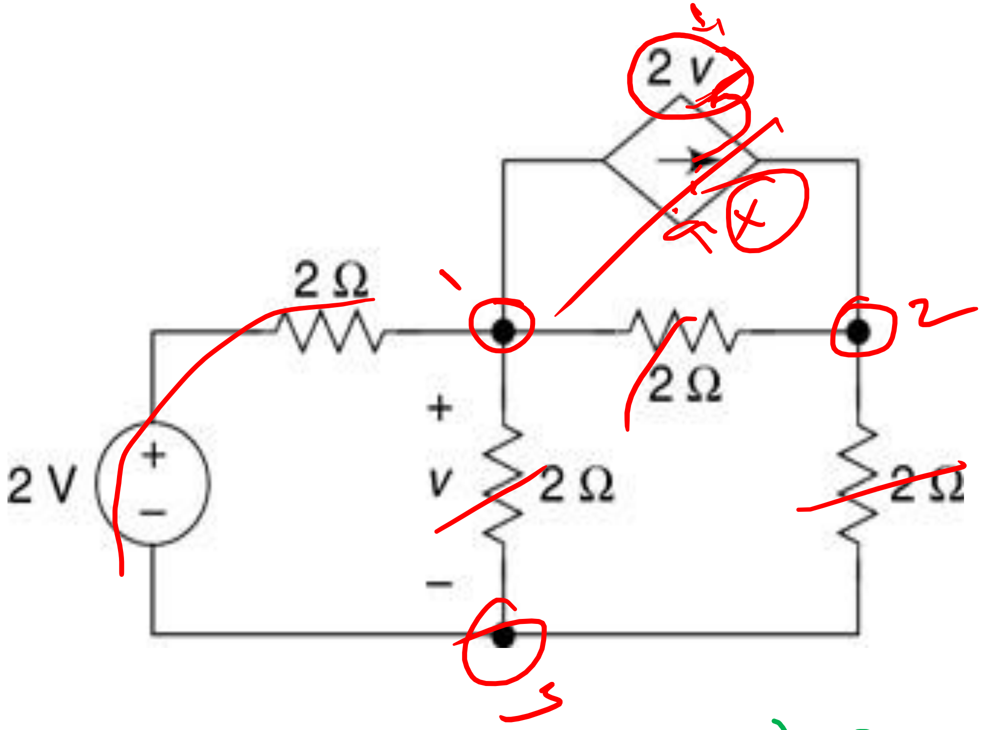
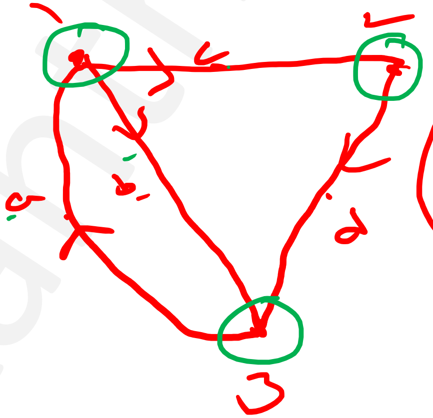


$$A = \begin{array}{c} \\ n_1 \\ n_2 \\ n_3 \\ n_4 \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ -1 & 1 & 0 & -1 & 1 \\ 0 & -1 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

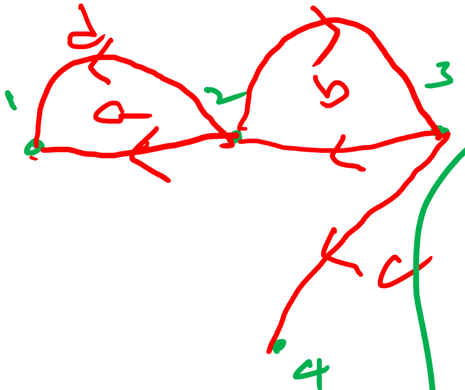
p = 4
b = 5



p = 3
b = 4

$$A = \begin{array}{c} \\ n_1 \\ n_2 \\ n_3 \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} +1 & -1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ -1 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

p × b
3 × 4

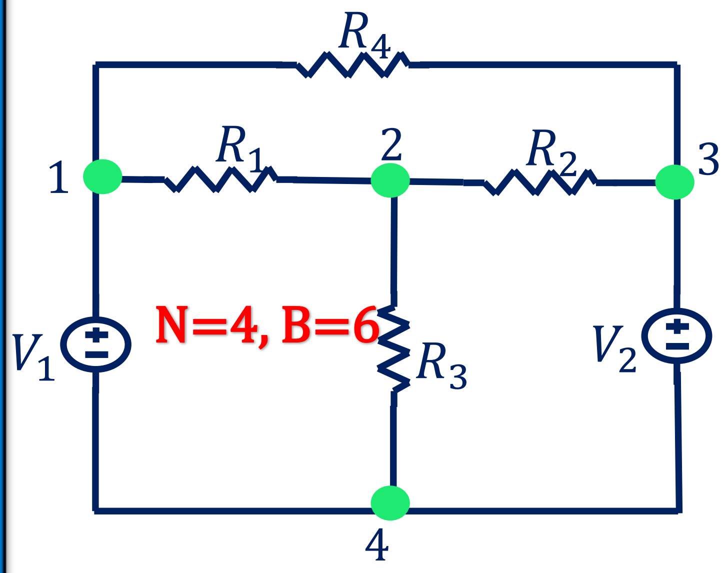


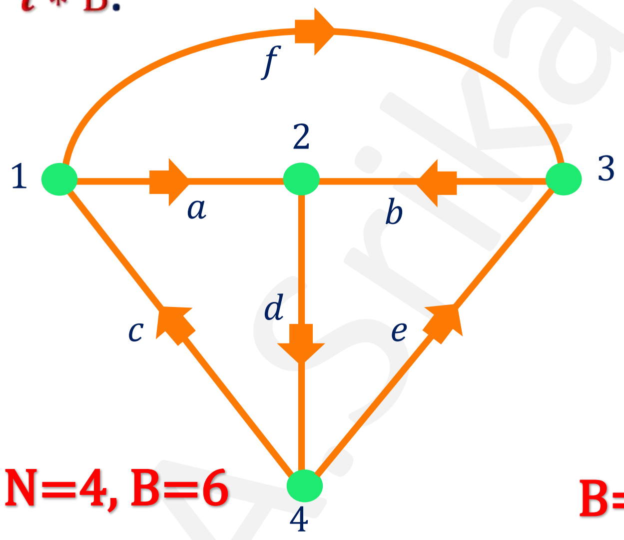
Network Topology or Graph Theory

Basic Tie set Matrices

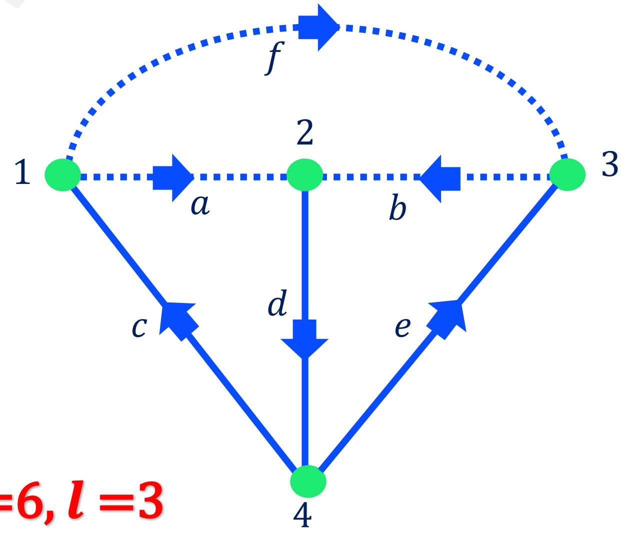
Basic Tie set Matrices is represented with the letter B.
The order of incidence matrix will be $l \times$ B.

If there are 'N' nodes, 'B' branches and l co-trees are present in a directed graph, then the Basic Tie set matrix will have 'l' rows and 'B' columns. Here, rows and columns are corresponding to the co-trees and branches of a directed graph. Hence, the order of incidence matrix will be $l * \mathrm{B}$.





N=4, B=6



B=6, l =3

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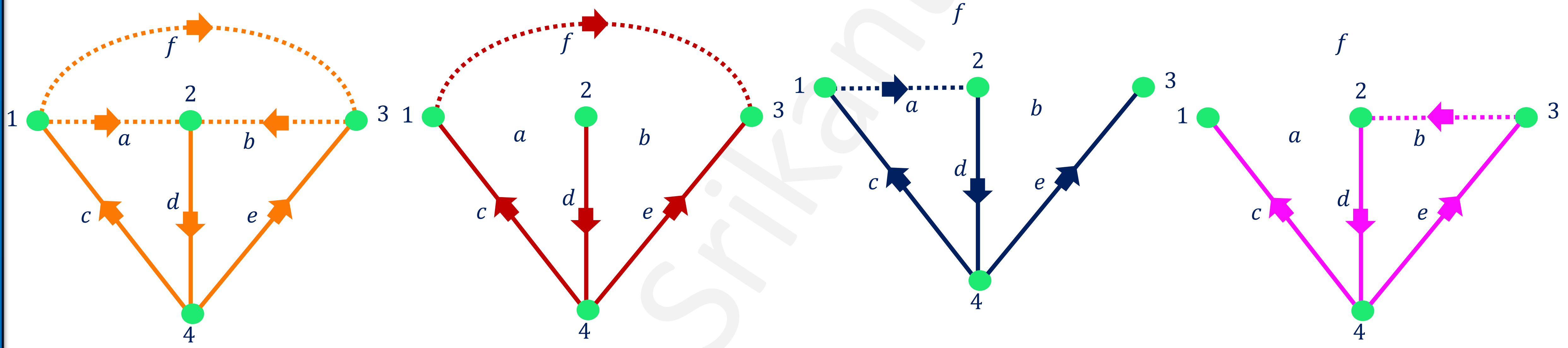
Network Topology or Graph Theory

Basic Tie set Matrices

Basic Tie set Matrices is represented with the letter B.
The order of incidence matrix will be $l \times$ B.



N =4, B =6, l =3

$$B = \begin{array}{c} \\ l_1 \\ l_2 \\ l_3 \end{array} \begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 & -1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

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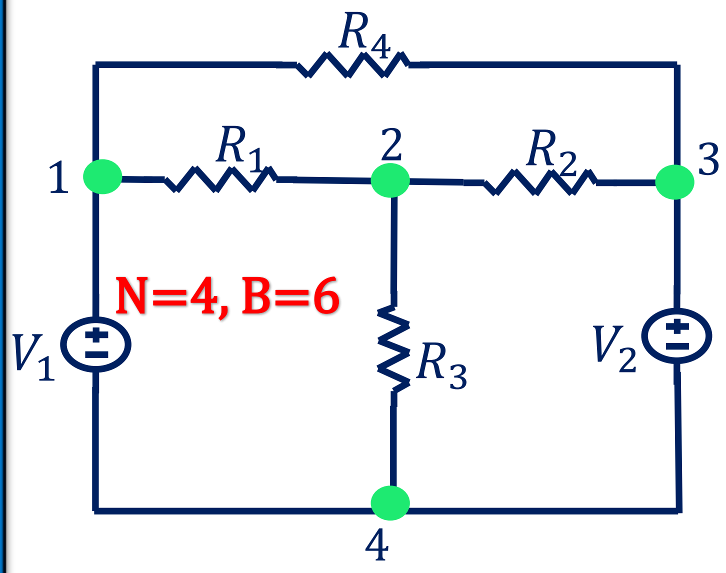
Network Topology or Graph Theory

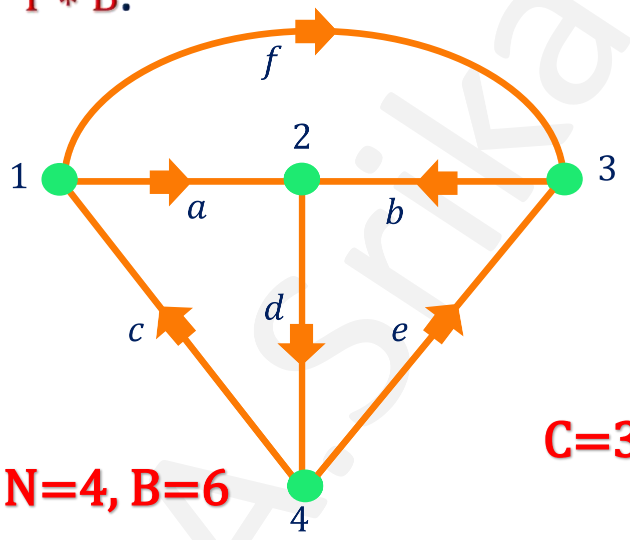
Basic cut set Matrices

Basic Cut set Matrices is represented with the letter C.
The order of incidence matrix will be T × B.

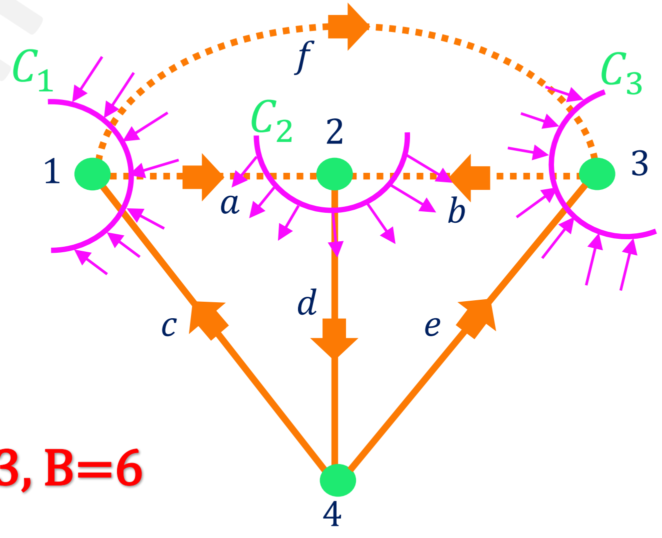
If there are ‘N’ nodes, ‘B’ branches and T trees are present in a directed graph, then the Basic Cut set matrix will have ‘T’ rows and ‘B’ columns. Here, rows and columns are corresponding to the trees and branches of a directed graph. Hence, the order of incidence matrix will be $T * B$.



N=4, B=6



N=4, B=6



C=3, B=6

Cut-set will be that node which will contain only one twig and any number of links.

$$C = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \end{array} \begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\ \begin{bmatrix} -1 & 0 & 1 & 0 & 0 & -1 \\ -1 & -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

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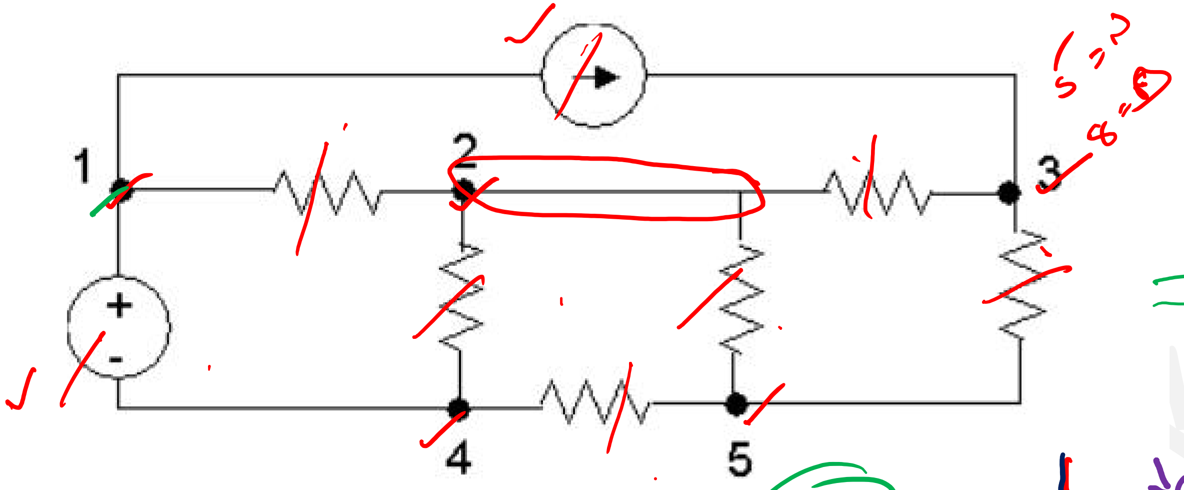
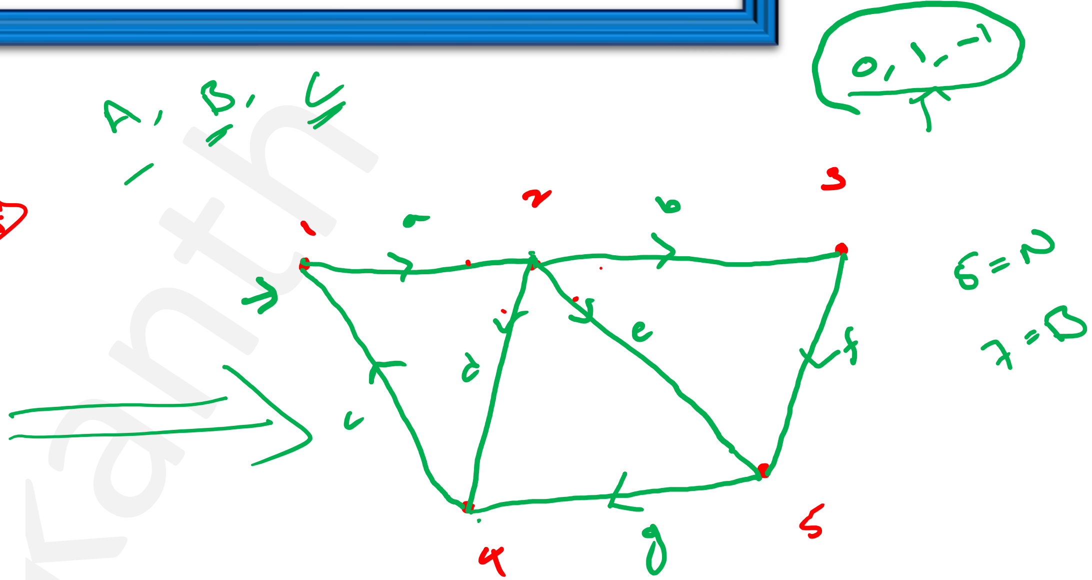
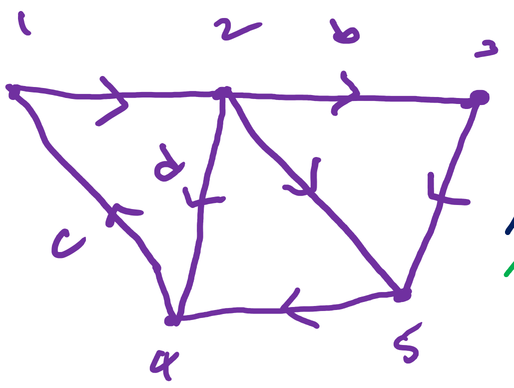
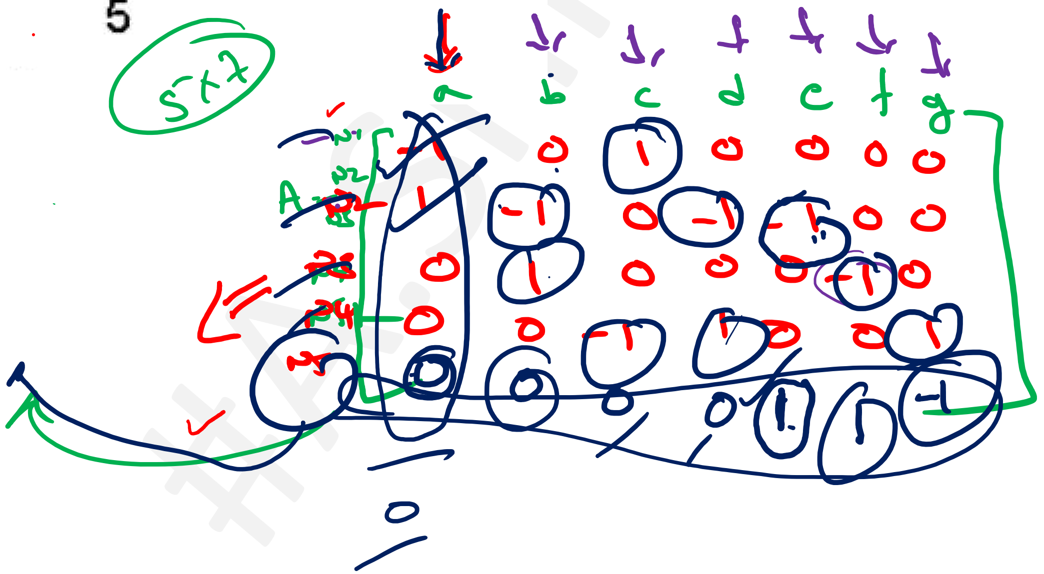
Network Topology or Graph Theory









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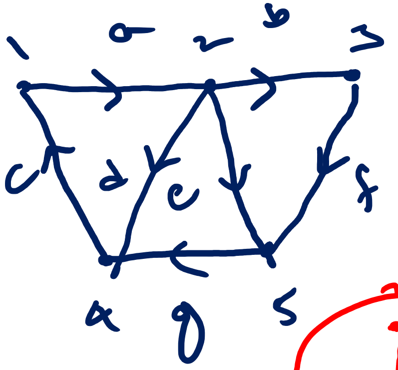
Network Topology or Graph Theory



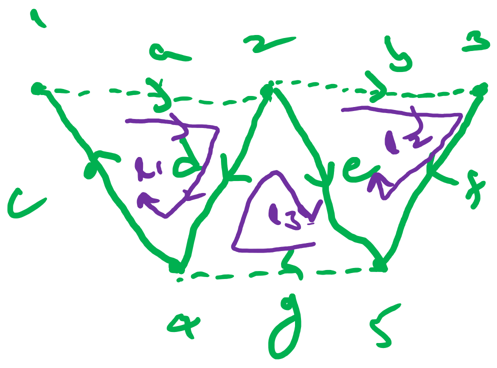
$5 = N$

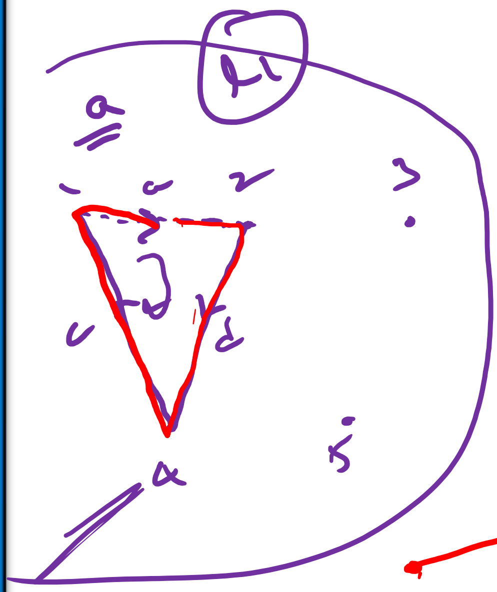
$7 = B$

$T = N - 1 = 5 - 1 = 4$

$l = B - N + 1 = 3$

$B = l \times B = 3 \times 7$





$$
B = \begin{array}{c|ccccccc}
 & a & b & c & d & e & f & g \\
\hline
l_1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\
l_2 & 0 & 1 & 0 & 0 & -1 & 1 & 0 \\
l_3 & 0 & 0 & 0 & -1 & 1 & 0 & 1
\end{array}
$$

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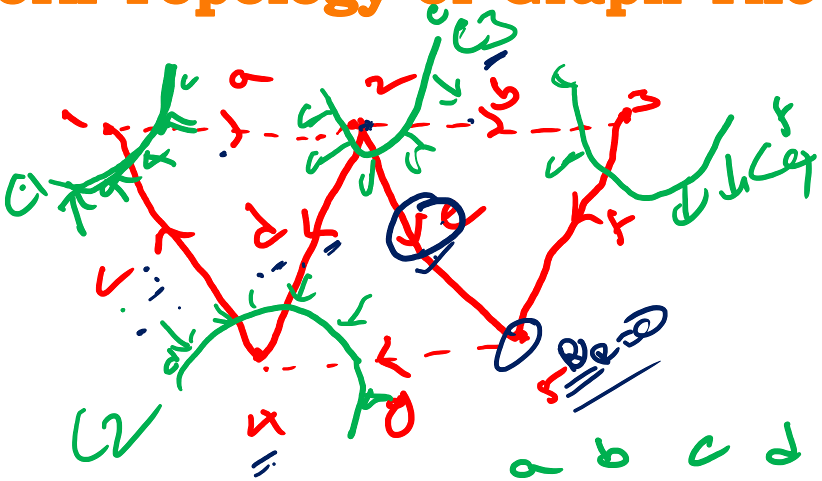
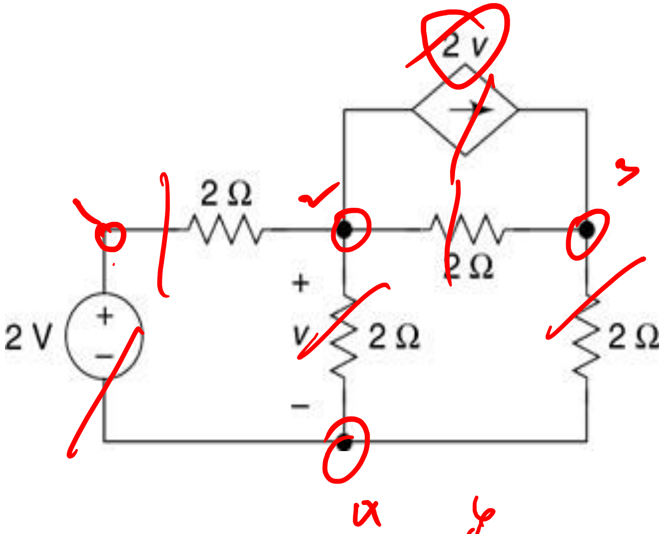
Network Topology or Graph Theory



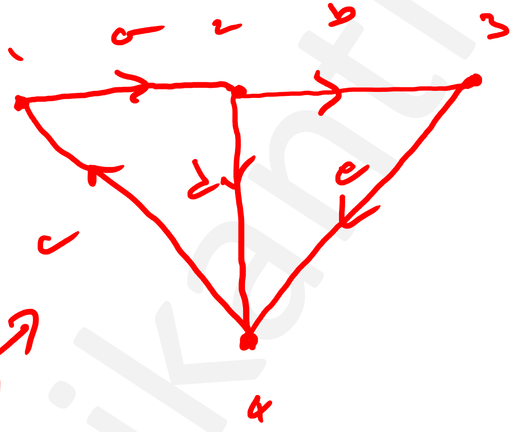
$$C = T \times B = 4 \times 7$$

$$C = \begin{array}{c|c} & \begin{matrix} a & b & c & d & e & f & g \end{matrix} \\ \begin{matrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{matrix} & \begin{bmatrix} -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 1 \\ -1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

Network Topology or Graph Theory

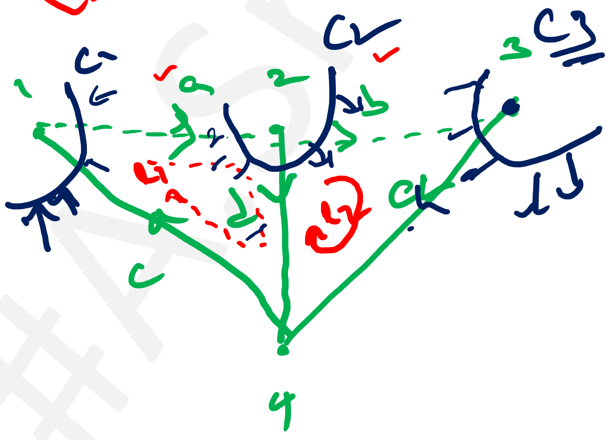


N = 4 ; B = 6



N = 4
B = 6

T = N − 1 = 3
l = B − N + 1 = 2



A = N × B = 4 × 5
B = l × B = 2 × 5
C = T × B = 3 × 5

Network Topology or Graph Theory

Network Topology or Graph Theory

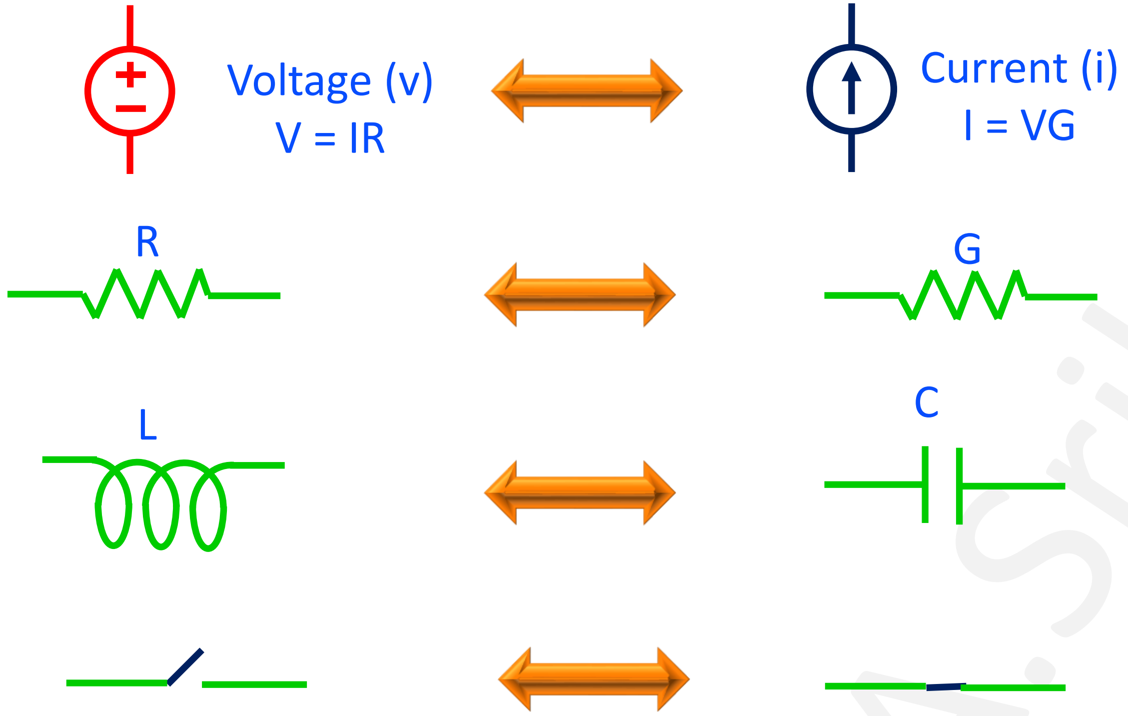
Duality and Dual Networks

It is interesting to know how systems relate to one another. How a mechanical system can be modelled as an electrical system and observed. The concept of duality in electrical circuits is of great importance. Two phenomena are said to be dual if they can be expressed by same form of mathematical equations.

Principle of Duality:

Principle of duality in context of electrical networks states that A dual of a relationship is one in which current and voltage are interchangeable. Two networks are dual to each other if one has mesh equation numerically identical to others node equation

Duality and Dual Networks



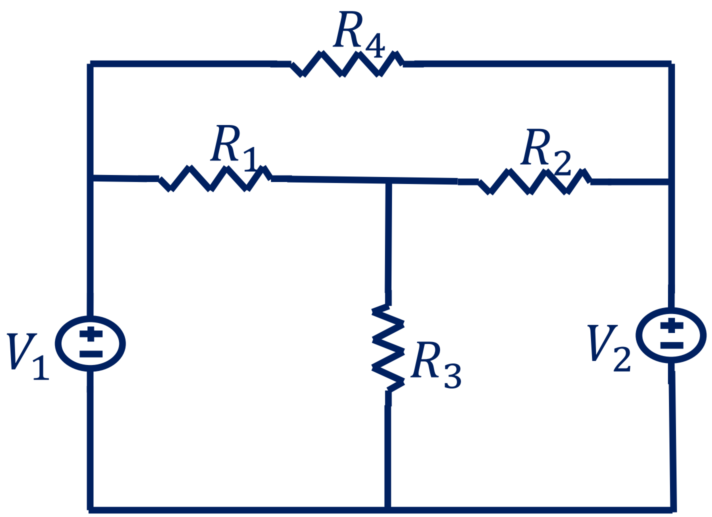
Series	Parallel
Impedance	Admittance
KVL	KCL

Duality and Dual Networks

List of Dual Pairs:

For evaluating a dual network, you should follow these points

- The number of meshes in a network is equal to number of nodes in its dual network
- The impedance of a branch common to two meshes must be equal to admittance between two nodes in the dual network
- Voltage source common to both loops must be replaced by a current source between two nodes
- Open switch in a network is replaced by a closed switch in its dual network or vice versa



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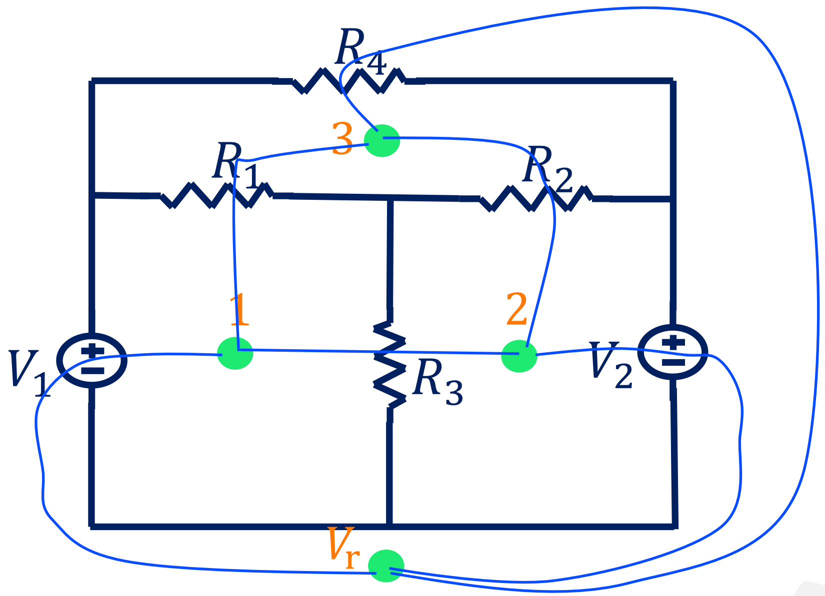
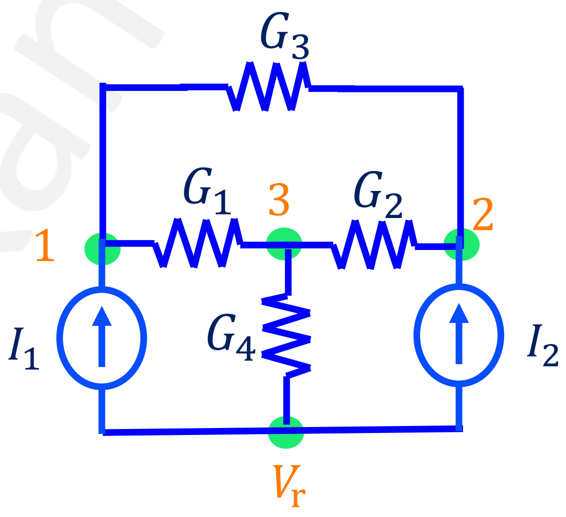
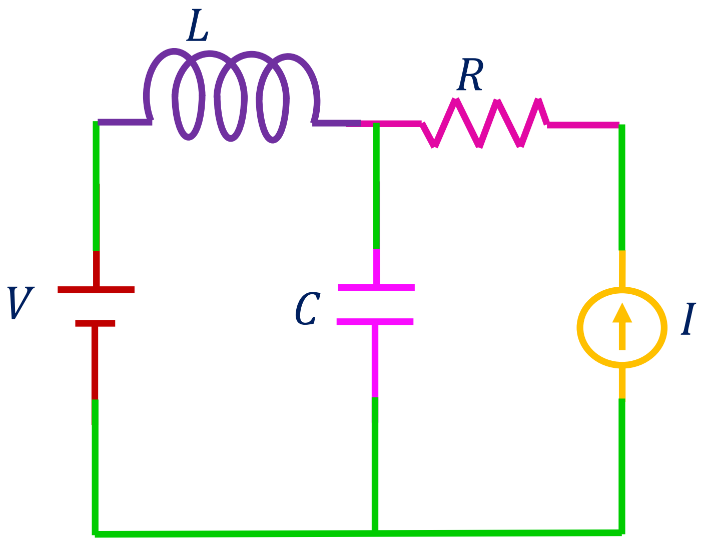
Duality and Dual Networks

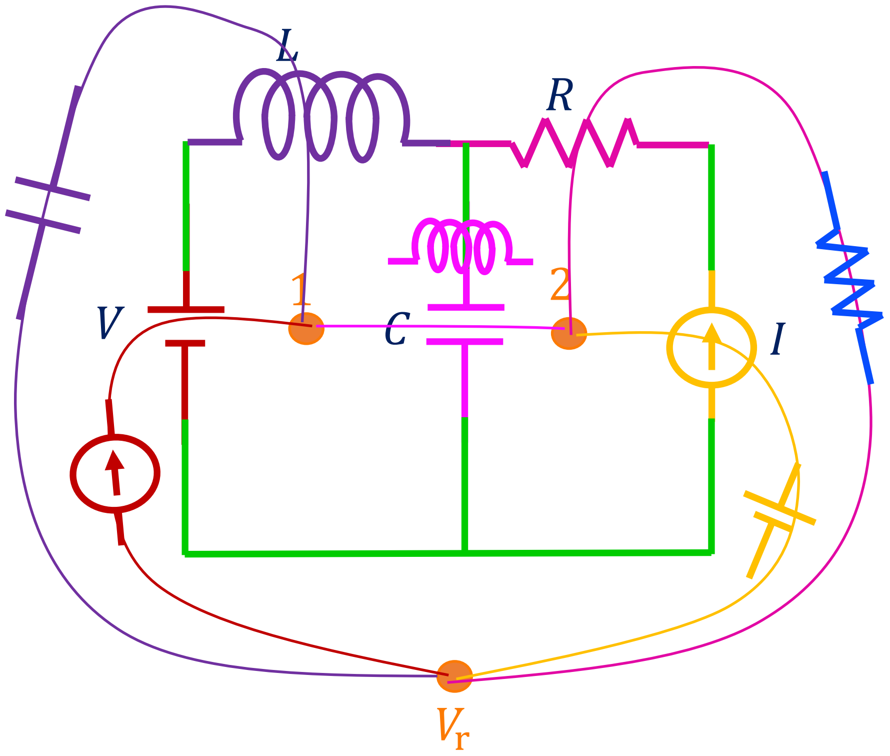


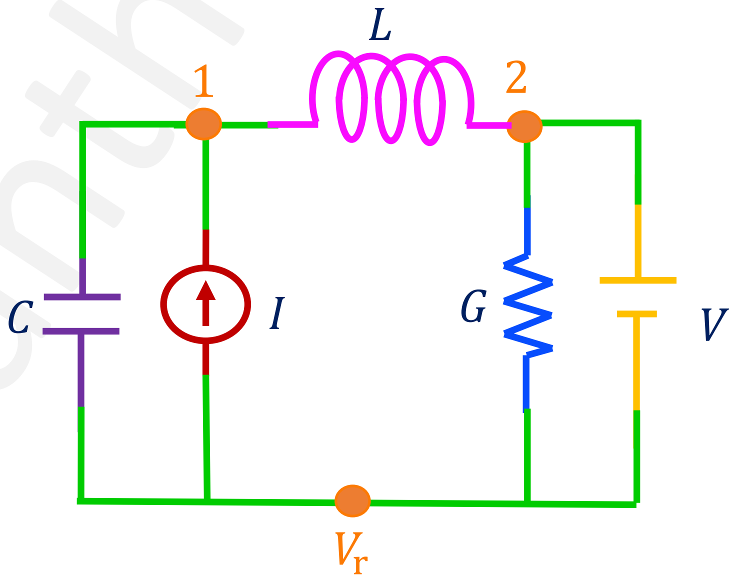


Duality and Dual Networks

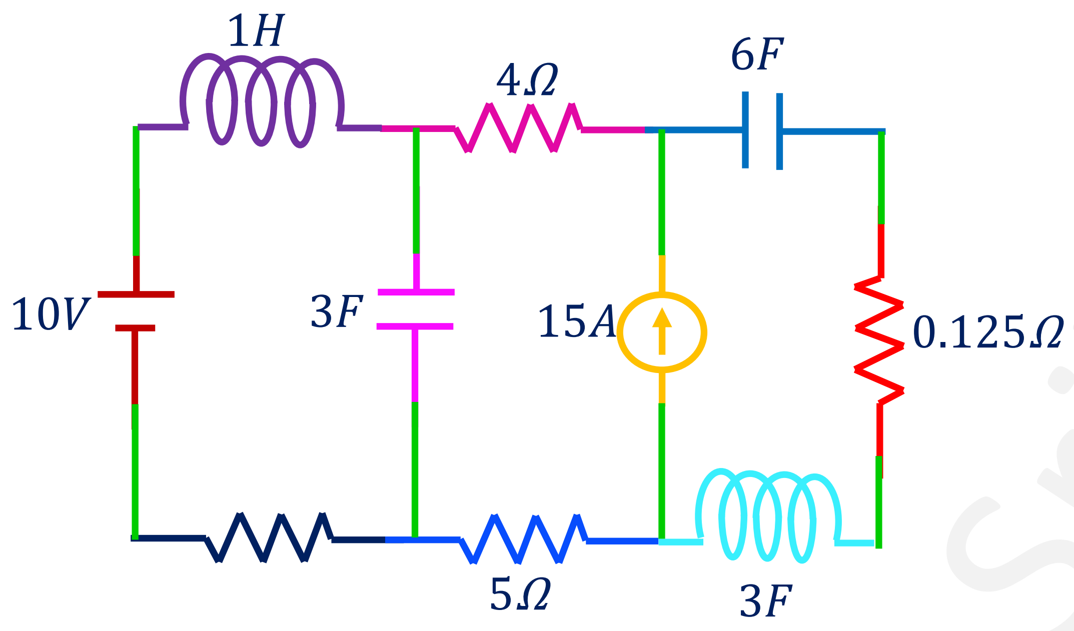


Duality and Dual Networks





Duality and Dual Networks



Duality and Dual Networks