MODULE - V

TESTING OF HYPOTHESIS.

PART - A.

① Given; $n = 49$, $\bar{x} = 15200$, $\mu = 15150$, $\sigma = 1200$

(1) Null Hypothesis : H_0 $\mu = 15150$

(2) Alternate Hypothesis : H_1 $\mu \neq 15150$

(3) level of significance : $\alpha = 0.05$

(4) Critical region : Accept the null hypothesis if $-1.96 < z < 1.96$.

(5) Test static : $z = \dfrac{\bar{x} - \mu}{\left(\frac{\sigma}{\sqrt{n}}\right)} = \dfrac{15200 - 15150}{\left(\frac{1200}{\sqrt{49}}\right)}$

$= 0.2917$

Since, $|z| < 1.96$.

$\therefore$ we accept the null hypothesis

(2) Given; $n = 200$

No. of pieces confirming to specification
$= 200 - 18$
$= 182$

$\therefore p$ = proportion of pieces confirming to specifications $= \frac{182}{200} = 0.91$

P = population proportion $= \frac{95}{100} = 0.95$

(1) H_0 : $P = 95\%$

(2) H_1 : $P < 0.95$ (left tail test)

(3) Test static : $Z = \frac{p - P}{\sqrt{\frac{PQ}{n}}} = \frac{0.91 - 0.95}{\sqrt{\frac{0.95 \times 0.05}{200}}}$

$= -2.59$

Since, alternative hypothesis is left tailed, the tabulated value of z at 5% LOS is 1.645
level of significance

Since; Cal $|Z| = 2.6 > 1.645$

H_0 is rejected at 5% LOS and

Conclude, The manufacturer's claim is rejected.

(3) Let P1 and P2 be the proportions of defective items in the population of two sample items produced by factory.

Let the Null Hypothesis be $H_0 : P_1 = P_2$

Then the Alternate Hypothesis is $H_1 : P_1 \neq P_2$

Here; $n_1 = 500$; $n_2 = 400$, $x_1 = 15$, $x_2 = 20$.

$\therefore P_1 = \frac{x_1}{n_1} = 0.03$; $P_2 = \frac{x_2}{n_2} = 0.05$

we have;

$$P = \frac{n_1 P_1 + n_2 P_2}{n_1 + n_2} = \frac{x_1 + x_2}{n_1 + n_2} = 0.039$$

$$q = 1 - P = 0.961$$

Assuming that H_0 is true, the test static is

$$Z = \frac{P_1 - P_2}{\sqrt{Pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.03 - 0.05}{\sqrt{0.039 \times 0.961\left(\frac{1}{500} + \frac{1}{400}\right)}}$$

$$= -1.54$$

$\therefore |Z| = 1.54 < 1.96$

We accept the Null Hypothesis at 5% LOS. (level of significance)

Conclusion: No significant difference between the two proportions.

(4) Here; the sample size is small (10 < 30).

$\bar{x} = 0.742$ inches.

$\mu = 0.700$ inches

S.D = 0.040 inches.

$\therefore$ we use Student's t-Test

(1) Null Hypothesis H_0 : The product is confirming to specification.

(2) Alternate Hypothesis H_1 : $\mu \neq 0.700$

(3) LOS : $\alpha = 0.05$

(4) Test static : $z = \dfrac{\bar{x} - \mu}{s/\sqrt{n-1}}$

$$z = \frac{0.742 - 0.700}{\frac{0.040}{\sqrt{10-1}}} = 3.15$$

Cal $|z| = 3.15$ → Cal val of t

But; $t_{0.05} = 2.26$ → Tab val of t

Now; Cal val of t > Tab val of t

$\therefore$ H_0 is Rejected

Conclusion :

Product is not meeting the specification

→ 1 column is missed in question - mistake in Q.B.

(5) we have ; $n_1 = 10$; $n_2 = 10$

$\bar{x} = \frac{1}{10}(117 + 105 + 97 + 105 + 123 + 109 + 86 + 78 + 103 + 107)$

$\bar{x} = 103$

$\bar{y} = \frac{1}{10}(106 + 98 + 87 + 104 + 116 + 95 + 90 + 69 + 108 + 85)$

$\bar{y} = 95.8$

Now we compute the standard deviations of both samples.

x	$x-\bar{x}$	y	$y-\bar{y}$	$(x-\bar{x})^2$	$(y-\bar{y})^2$
117	14	106	10.2	196	104.04
105	2	98	2.2	4	4.84
97	-6	87	-8.8	36	77.44
105	2	104	8.2	4	67.24
123	20	116	20.2	400	408.24
109	6	95	-0.8	36	0.64
86	-17	90	-5.8	289	33.64
78	-25	69	-26.8	625	718.24
103	0	108	12.2	0	148.84
107	4	85	-10.8	16	116.64
1030		958		1606	1679.6

Now; $s^2 = \frac{1}{n_1+n_2-2}\left[\Sigma(x_i-\bar{x})^2+\Sigma(y_i-\bar{y})^2\right]$

$s^2 = 182.53 \Rightarrow \boxed{s = 13.51}$

(1) Null Hypothesis : $H_0 \quad \mu_1 = \mu_2$

(2) Alternate Hypothesis : $H_1 \quad \mu_1 > \mu_2$

(Husbands are more intelligent than wives)

(3) L.O.S : $\alpha = 0.05$

(4) Test static : $t = \frac{\bar{x}-\bar{y}}{s\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = 1.19168$

$t_{cal} < t_{tab}$ (i.e $1.19168 < 1.734$)

We accept the Null Hypothesis H_0

i.e; There is no difference in I.Q's

Let the Null Hypothesis be $H_0 : \sigma_1^2 = \sigma_2^2$

The Alternate Hypothesis is $H_1 : \sigma_1^2 \neq \sigma_2^2$

Given; $n_1 = 11$; $n_2 = 9$; $\underbrace{s_1 = 0.8; \ s_2 = 0.5}_{\text{Sample varian}}$

$$S_1^2 = \frac{n_1 s_1^2}{n_1 - 1} = 0.704$$

Population variance

$$S_2^2 = \frac{n_2 s_2^2}{n_2 - 1} = 0.281$$

Test static $(F) = \dfrac{S_1^2}{S_2^2} = 2.5 \quad (\because S_1^2 > S_2^2)$

$F_{Cal} = 2.5$

$F_{Tab} = 3.35$ (at 5% LOS)

$F_{Cal} < F_{Tab}$

We accept Null Hypothesis at 5% Los

Conclusion: The variences of the two Populations is the same, and therefore, the two samples have the same Variance.

7 (1) Null Hypothesis H_0: There is a significant liking in the habit of taking soft drinks.

(2) Alternate Hypothesis H_1: There is no significant liking.

(3) L.O.S: $\alpha = 0.05$ (assumed)

(4) Computation:

Soft Drinks	Clerks	Teachers	Officers	Total
Pepsi	10	25	65	100
Thums Up	15	30	65	110
Fanta	50	60	30	140
Total	75	115	160	350

Table of expected frequencies:

$\frac{75 \times 100}{350} = 21.4$	$\frac{115 \times 100}{350} = 32.9$	$\frac{160 \times 100}{350} = 45.71$
$\frac{75 \times 110}{350} = 23.6$	$\frac{115 \times 110}{350} = 36.1$	$\frac{160 \times 110}{350} = 50.3$
$\frac{75 \times 140}{350} = 30$	$\frac{115 \times 140}{350} = 46$	$\frac{160 \times 140}{350} = 64.$

Calculations for χ^2

Observed	Expected frequency (O_i)	$(O_i - E_i)^2$ Frequency (E_i)	$\frac{(O_i - E_i)^2}{E_i}$
10	21.4	129.96	6.073
25	32.9	62.41	1.897
65	45.7	372.49	8.151
15	23.6	73.96	3.134
30	36.1	37.21	1.031
65	50.3	216.09	4.3
50	30	400	13.333
60	46	196	4.261
30	64	1156	18.062
			60.2425

Cal $\chi^2 = 60.2425$

Tab $\chi^2 = 9.488$ at 5% LOS.

$\therefore$ Cal $\chi^2 >$ Tab χ^2

we Reject Null hypothesis H_0.

Thus, we conclude that the habit of taking soft drinks depends on categories of employees.

(8) Let P_1 and P_2 be the proportions of defective units in the population of units inspected in machine 1 & machine 2 respectively.

Let the Null Hypothesis H_0 be : $P_1 = P_2$

Then Alternate Hypothesis H_1 be : $P_1 > P_2$

$p_1 = \frac{17}{375} = 0.045$; $p_2 = \frac{22}{450} = 0.049$; $p_1 > p_2$

Here ; $n_1 = 375$; $n_2 = 450$; $p_1 = 0.045$; $p_2 = 0.049$

$$P = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} = 0.047$$

$$S.E \text{ of } (p_1 - p_2) = \sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} = 0.015$$

Assuming H_0 is true, the test static is

$$Z = \frac{p_1 - p_2}{SE \text{ of } (p_1 - p_2)} = -0.267$$

$\therefore |Z| = 0.267 < 1.96$

we accept Null Hypothesis (H_0) at 5% LOS

Conclusion : There is no significant performance of two machines.

(1) Given; $\bar{x}_1 = 55$; $\bar{x}_2 = 57$; $n_1 = 10$; $n_2 = 20$

$S_1 = 10$; $S_2 = 15$

(1) Null Hypothesis H_0 : $\bar{x}_1 = \bar{x}_2$ (No difference)

(2) Alternate Hypothesis H_1 : $\bar{x}_1 \neq \bar{x}_2$

(3) L.O.S ; $\alpha = 0.05$ (assumed)

(4) Critical Region : Accept H_0, if

$-1.96 < z < 1.96$

(5) The test static : $z = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{S_1^2}{n_1} + \dfrac{S_2^2}{n_2}}} = -0.433$

$|z| = 0.43 < 1.96$

$\therefore$ we accept the Null Hypothesis (H_0)

at 5% L.O.S

Conclusion :

There is no significant difference between the means.

(10) Let the Null Hypothesis be $H_0 : \sigma_1^2 = \sigma_2^2$

The Alternative Hypothesis is $H_1 : \sigma_1^2 \neq \sigma_2^2$

Given ; $n_1 = 5$ and $n_2 = 5$

$$\bar{x} = \frac{1}{5}(14.1 + 10.1 + 14.7 + 13.7 + 14.0) = 13.32$$

$$\bar{y} = \frac{1}{5}(14.0 + 14.5 + 13.7 + 12.7 + 14.1) = 13.8$$

Computing Standard deviations of the Samples.

x	$x - \bar{x}$	$(x-\bar{x})^2$	y	$y - \bar{y}$	$(y-\bar{y})^2$
14.1	0.78	0.6084	14.0	0.2	0.04
10.1	-3.22	10.3684	14.5	0.7	0.49
14.7	1.38	1.9044	13.7	-0.1	0.01
13.7	0.38	0.1444	12.7	-1.1	1.21
14.0	0.68	0.4624	14.1	0.3	0.09
66.6		13.488	69		1.84

If s_1^2 and s_2^2 be the estimates of σ_1^2 and σ_2^2 then

$$s_1^2 = \frac{\Sigma(x_i - \bar{x})^2}{n_1 - 1} = 3.372$$

$$s_2^2 = \frac{\Sigma(y_i - \bar{y})^2}{n_2 - 1} = 0.46$$

✓

Let H_0 be true. Since $s_1^2 > s_2^2$, the test static is $F = \frac{s_1^2}{s_2^2}$

$F = 7.33.$

Degrees of freedom are $(n_1 - 1, n_2 - 1)$
$(4, 4)$

Tab. value of F for $(4, 4)$ at 10% LOS is 6.39

$\therefore F_{cal} > F_{tab}$

We Reject the Null Hypothesis H_0

Conclusion:

There is a significant difference between the variances.

PART - B

(1) Given; $n = 400$; $\bar{x} = 40$; $\mu = 38$; $\hat{\sigma} = 10$

(1) Null Hypothesis $H_0 : \mu = 38$.

(2) Alternative Hypothesis $H_1 : \mu \neq 38$

(3) L.O.S : $\alpha = 0.05$

(4) Test Static : $Z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}} = 4.$

i.e $Z = 4 > 1.96$

$\therefore$ we reject the Null Hypothesis H_0

i.e, the sample is not from the population whose mean is 38.

95% confidence interval is $\left(\bar{x} - 1.96 \frac{\sigma}{\sqrt{n}}, \bar{x} + 1.96 \cdot \sigma/\sqrt{n}\right)$

i.e $\left(40 - \dfrac{1.96(10)}{\sqrt{400}}, 40 + 1.96 \dfrac{(10)}{\sqrt{400}}\right)$

i.e, $(39.02, 40.98)$

(5) Let μ_1 & μ_2 be the means of two populations.

Given; $n_1 = 1000, n_2 = 2000$; $\bar{x}_1 = 67.5$ inches

$\bar{x}_2 = 68$ inches

S.D = 2.5 inches

(1) Null Hypothesis H_0 : $\mu_1 = \mu_2$ & $\sigma = 2.5$ inches

(2) Alternative Hypothesis H_1 : $\mu_1 \neq \mu_2$

(3) The test static is :

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma^2}{n_1^2} + \frac{\sigma^2}{n_2^2}}}$$

$$z = -5.16$$

$|z| = 5.16 > 1.96$

$z_{cal} > z_{tab}$

The Null Hypothesis is rejected at 5% LOS

Conclusion : The samples are not drawn from the same population of S.D 2.5 inches.

(3) Given; $n = 50$; $\mu = 8.9$; $\bar{x} = 9.2$; $\alpha = 0.05$

$\sigma = \sqrt{1.6} = 1.26$ (Not Specified in Question properly)

(1) Null Hypothesis H_0 : $\mu = 8.9$

(2) Alternative Hypothesis H_1 : $\mu < 8.9$

(3) L.O.S : $\alpha = 0.05$

(4) Test Static : $Z = \dfrac{\bar{x} - \mu}{\left(\frac{\sigma}{\sqrt{n}}\right)} = \dfrac{9.2 - 8.9}{\left(\frac{1.26}{\sqrt{50}}\right)}$

$= 1.76$

$Z_{cal} = 1.76$

$Z_{tab} = 1.64$

$\therefore Z_{cal} > Z_{tab}$.

$1.76 > 1.64$

$\therefore$ we Reject the null hypothesis (H_0)

If $\sigma = 1.6$ then

$Z_{cal} < Z_{tab}$

we accept H_0

(4) Given; $n = 40$; $\mu = 73.2$; $\bar{x} = 76.9$

$\sigma = 8.6$

(1) Null Hypothesis H_0 : $\mu = 73.2$

(2) Alternative Hypothesis H_1 : $\mu > 73.2$

(3) Level of significance : $\alpha = 99\%$

(4) The test statistic : $z = \dfrac{\bar{x} - \mu}{\left(\frac{\sigma}{\sqrt{n}}\right)} = 2.59$

z_{tab} at 99% LOS = 2.33

z_{cal} at 99% LOS = 2.59

$z_{cal} > z_{tab}$

∴ The Null Hypothesis H_0 is Rejected.

Conclusion : $\bar{x}$ and μ differ significantly.

⑤ Given;

$n_1 = 100$; $n_2 = 75$

$\bar{x}_1 = 1190$; $\bar{x}_2 = 1230$

$\sigma_1 = 90$; $\sigma_2 = 120$

$\alpha = 0.05$

① Null Hypothesis $H_0 : \mu_1 = \mu_2$

② Alternative Hypothesis $H_1 : \mu_1 \neq \mu_2$

③ L.O.S : $\alpha = 0.05$

④ Test static : $$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$z = -2.42$$

$z_{cal} = 2.42$; $z_{tab} = 1.96$

$z_{cal} > z_{tab}$

∴ The Null Hypothesis H_0 is Rejected.

Conclusion: Yes, There is a significant difference between the means of two batches.

(6) $n_1 = 200$; $n_2 = 100$

$p_1 = \frac{42}{200} = 0.21$; $p_2 = \frac{18}{100} = 0.18$

$P_1 - P_2 = 8\% = 0.08$

(1) Null Hypothesis $H_0 : P_1 - P_2 = 0.08$

(2) Alternative Hypothesis $H_1 : P_1 - P_2 \neq 0.08$

(2 Tail Test)

(3) L.O.S ; $\alpha = 0.05$

(4) Test - static : $z = \frac{(p_1 - p_2) - (P_1 - P_2)}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$

$p = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} = 0.2$

$q = 1 - p = 0.8$

$z = \frac{-0.05}{0.0489} = -1.02$

$|z|_{cal} = 1.02$; $z_{tab} = 1.96$

$z_{cal} < z_{tab}$

$\therefore$ we Accept the null hypothesis (H_0)

Conclusion : The 8% difference in the sale of two brands of cigarettes is a valid claim.

(7)

Urban	Rural
$n_1 = 500$	$n_2 = 400$
$x_1 = 120$	$x_2 = 48$
$p_1 = \frac{120}{500} = 0.24$	$p_2 = \frac{48}{400} = 0.12$
$q_1 = 0.76$	$q_2 = 0.88$

(1) Null Hypothesis (H_0): $P_1 = P_2$

(2) Alternative Hypothesis (H_1): $P_1 \neq P_2$

(2 tail Test)

(3) L.O.S : $\alpha = 0.05$

(4) Test statistic :

$$Z = \frac{P_1 - P_2}{\sqrt{\frac{P_1 q_1}{n_1} + \frac{P_2 q_2}{n_2}}}$$

$$Z = 4.78$$

$Z_{cal} = 4.78$; $Z_{tab} = 1.96$

$Z_{cal} > Z_{tab}$

The Null hypothesis is Rejected.

∴ The Proportion of 'cell' phones in rural and urban areas is not same.

(1) Given; $\bar{x}_1 = 55$, $n_1 = 400$, $s_1 = 10$
$\bar{x}_2 = 57$, $n_2 = 100$, $s_2 = 15$

(1) Null Hypothesis (H_0) : $\bar{x}_1 = \bar{x}_2$

(2) Alternative Hypothesis (H_1) : $\bar{x}_1 \neq \bar{x}_2$

(3) L.O.S : $\alpha = 0.05$ (assumed)

(4) Critical region : Accept H_0 if
$-1.96 < z < 1.96$

(5) Test statistic :
$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$z = -1.26$$

$\therefore |z|_{cal} = 1.26$; $z_{tab} = 1.96$

$z_{cal} < z_{tab}$

We Accept Null Hypothesis (H_0).

Conclusion : There is no significant difference between the means.

(9) $n = 600$; $p = \frac{325}{600} = 0.54$

$P = 1/2 = 0.5$

$Q = 1 - P = 0.5$

(1). Null Hypothesis H_0 : $P = Q$

(2) Alternative Hypothesis : $P > 0.5$ (Right tailed)

(3). Test Statistic : $Z = \frac{p - P}{\sqrt{\frac{PQ}{n}}}$

$Z = 2.04.$

$Z_{cal} = 2.04$; $Z_{tab} = 1.64$

$Z_{cal} > Z_{tab}$

We Reject the Null Hypothesis (H_0)

Conclusion :

The majority of men in the city are smokers.

small sample

(c) $n = 26$; $\bar{x} = 990$; $\sigma = 20$; $\mu = 1000$

Degree of freedom $= n-1 = 25$

Student's t test :

(1) Null Hypothesis $H_0 : \mu = 1000$

(2) Alternative Hypothesis $H_1 : \mu < 1000$

(3) L.O.S : $\alpha = 0.05$

(4) Test Statistic : $t = \frac{\bar{x} - \mu}{s/\sqrt{n-1}} = -2.5$

$|t| = 2.5$

$t_{cal} = 2.5$; $t_{Tab} = 1.70$

$t_{cal} > t_{tab}$

We Reject the Null hypothesis H_0.

Conclusion :

The sample is not upto the standard.

(11) We have to determine S.D's and mean as follows.

$$\bar{x} = \frac{70+120+110+101+88+83+95+98+107+100}{10}$$

$\bar{x} = 97.2$

x	$x-\bar{x}$	$(x-\bar{x})^2$
70	-27.2	739.84
120	22.8	519.84
110	12.8	163.84
101	3.8	14.44
88	-9.2	84.64
83	-14.2	201.64
95	-2.2	4.84
98	0.8	0.64
107	9.8	96.04
100	2.8	7.84
972		1833.60

$$s^2 = \frac{1}{n-1} \sum_i (x_i - \bar{x})^2 = \frac{1833.60}{9}$$

$$s = \sqrt{203.73} = 14.27$$

(1) Null Hypothesis (H_0) : $\mu = 100$

(2) Alternative Hypothesis (H_1) : $\mu \neq 100$

(3) L.O.S ; $\alpha = 0.05$

(v) Test Statistic : $t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = -0.62$

$|t| = 0.62$

$t_{cal} = 0.62$

$10-1 = 9$

$t_{tab} = 2.26$ (Two Tailed Test)

$t_{cal} < t_{tab}$

We Accept the Null Hypothesis (H_0).

Conclusion :

The data support the assumption of mean I.Q of 100 in the population.

---

(12) (i) Null Hypothesis (H_0) : $\mu_1 = \mu_2$ and $\sigma_1^2 = \sigma_2^2$

Here we have to use two tests

(i) To test equality of variances by F-test.
(ii) To test equality of means by t-test.

(i) F - test :

$n_1 = 10$; $n_2 = 12$
$\bar{x}_1 = 15$; $\bar{x}_2 = 14$.

$\Sigma (x_i - \bar{x})^2 = 90$; $\Sigma (y_i - \bar{y})^2 = 108$

$$s_1^2 = \frac{\Sigma(x_i-\bar{x})^2}{n_1-1} = 10$$

$$s_2^2 = \frac{\Sigma(y_i-\bar{y})^2}{n_2-1} = 9.82$$

$$F = \frac{s_1^2}{s_2^2} = 1.018$$

$F_{cal} = 1.018$; $F_{tab} = 2.90$ (5% LOS for (9, 11))

$F_{cal} < F_{tab}$

we accept Null Hypothesis (H_0).

$\therefore \sigma_1^2 = \sigma_2^2$,

i.e, The samples came from the same normal populations with same variance.

(ii) t-test:

① Null Hypothesis (H_0) : $\mu_1 = \mu_2$

Given ; $\bar{x}_1 = 15$; $\bar{x}_2 = 14$.

$n_1 = 10$; $n_2 = 12$

$\Sigma(x_i-\bar{x})^2 = 90$; $\Sigma(y_i-\bar{y})^2 = 108$

$$S^2 = \frac{1}{n_1+n_2-2}\left(\Sigma(x_i-\bar{x})^2 + \Sigma(y_i-\bar{y})^2\right) = 9.9$$

$$S = 3.15$$

The test statistic

$$Z = \frac{\bar{x} - \bar{y}}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$Z = 0.74$$

$Z_{cal} = 0.74$

$Z_{tab} = 2.08$ (t for 20 at 5% LOS)

$Z_{cal} < Z_{tab}$

$\therefore$ we Accept the Null hypothesis (H_0).

i.e, $\mu_1 = \mu_2$

from (i) & (ii), The given Samples have been drawn from the same normal population. Hence, we accept the null hypothesis that $\mu_1 = \mu_2$ and $\sigma_1^2 = \sigma_2^2$.

(13) Given; $n_1 = 8$; $n_2 = 7$

$$\bar{x} = \frac{1}{8}(11+11+13+11+15+9+12+14) = 12$$

$$\bar{y} = \frac{1}{7}(9+11+10+13+9+8+10) = 10$$

x	$x-\bar{x}$	$(x-\bar{x})^2$	y	$y-\bar{y}$	$(y-\bar{y})^2$
11	-1	1	9	-1	1
11	-1	1	11	1	1
13	1	1	10	0	0
11	-1	1	13	3	9
15	3	9	9	-1	1
9	-3	9	8	-2	4
12	0	0	10	0	0
14	2	4	-	-	-
96		26	70		16

$$s^2 = \frac{1}{n_1+n_2-2}\left(\Sigma(x_i-\bar{x})^2 + \Sigma(y_i-\bar{y})^2\right) = 3.23$$

$$S = 1.8$$

(1) Null Hypothesis (H0) : $\mu_1 = \mu_2$

(2) Alternative Hypothesis (H_1) : $\mu_1 \neq \mu_2$

(3) L.O.S : $\alpha = 0.05$

(4) Test Statistic ;

$$t = \frac{\bar{x}-\bar{y}}{S\cdot\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = 2.15$$

$t = 8+7-2 = 13$

$t_{tab} = 2.16$ (13 d.f at 5% LOS)

$t_{cal} < t_{tab}$

$\therefore$ we Accept the Null Hypothesis (H_0)

i.e, the difference between the means of Sample is not significant

(14) Let the Null hypothesis be $H_0: \sigma_1^2 = \sigma_2^2$

The Alternative hypothesis is $H_1: \sigma_1^2 \neq \sigma_2^2$

Calculation of Sample variances

x	$x-\bar{x}$	$(x-\bar{x})^2$	y	$y-\bar{y}$	$(y-\bar{y})^2$
20	-2.33	5.4289	27	-7.4	54.76
16	-6.33	40.0689	33	-1.4	1.96
27	4.67	21.8089	42	7.6	57.76
23	0.67	0.4489	35	0.6	0.36
22	-0.33	0.1089	32	-2.4	5.76
26	3.67	13.4689	34	-0.4	0.16
—	—	—	38	3.6	12.96
134		81.3334	241		133.72

$\bar{x} = 22.33$; $\Sigma(x_i - \bar{x})^2 = 81.3334$

$\bar{y} = 34.4$; $\Sigma(y_i - \bar{y})^2 = 133.72$

$n_1 = 6$; $n_2 = 7$

$$s_1^2 = \frac{\Sigma (x_i - \bar{x})^2}{n_1 - 1} = 13.55$$

$$s_2^2 = \frac{\Sigma (y_i - \bar{y})^2}{n_2 - 1} = 22.29$$

let H_0 be true.

$s_2^2 > s_1^2$, the statistic is

$$F = \frac{s_2^2}{s_1^2} = 1.64$$

$(n_1 - 1, n_2 - 1)$ i.e $(5, 6)$ at 5% LOS

$F_{tab} = 4.39$

$F_{cal} < F_{tab}$

We Accept the null hypothesis H_0

i.e there is no significant difference between the variances of the time distribution by the workers.

Null Hypothesis (H_0) : The die is unbiased.

The expected frequency of each number 1, 2, 3, 4, 5, 6 is $\frac{264}{6} = 44$.

Calculations for χ^2

Observed	Expected frequency (O_i)	$(O_i - E_i)^2$ frequency (E_i).	$\frac{(O_i - E_i)^2}{E_i}$
40	44	16	0.3636
32	44.	144	3.2727
28	44	256	5.8181
58	44	196	4.4545
54	44	100	2.2727
52	44.	64.	1.4545
			17.6362
264	264.		

$$\therefore \chi^2 = \Sigma \frac{(O_i - E_i)^2}{E_i} = 17.6362$$

No. of degree of freedom = $n - 1 = 5$

χ^2_{tab} for 5 d.f at 5% LOS = 11.07

$\chi^2_{cal} > \chi^2_{tab}$

The Null Hypothesis (H_0) is Rejected

i.e, The Die is Biased

(16) Null Hypothesis (H_0) : The digits were distributed in equal number in Tables.

The expected frequency of each digit $= \frac{200}{10} = 20$

Calculations for χ^2

Observed	Expected frequency(O_i)	$(O_i - E_i)^2$ frequency(E_i)	$\frac{(O_i - E_i)^2}{E_i}$
18	20	4	0.2
19	20	1	0.05
23	20	9	0.45
21	20	1	0.05
16	20	16	0.8
25	20	25	1.25
22	20	4	0.2
20	20	0	0
21	20	1	0.05
15	20	25	1.25
200	200		4.3

$\therefore \chi^2_{cal} = 4.3$

$D.f = 10 - 1 = 9$

χ^2 at 9 d.f at 5% LOS is 16.919

$\chi^2_{cal} < \chi^2_{tab}$

We Accept the Null Hypothesis H_0.

i.e, The digits are distributed in equal number in the tables.

(ii) $n = 14$; $\bar{x} = 17.85$, $s = 1.955$; $\mu = 18.5$

D.o.f $= n-1 = 13$

(1) Null Hypothesis (H_0) : Not significant result

(2) Alternative Hypothesis (H_1) : $\mu \neq 18.5$

(3) L.O.S : $\alpha = 0.05$

(4) The test statistic : $t = \dfrac{\bar{x}-\mu}{s/\sqrt{n-1}} = -1.199$

$|t| = 1.199$

$t_{cal} = 1.199$

d.f 13, t at 5% L.O.S $t_{tab} = 2.16$

$t_{cal} < t_{tab}$

$\therefore$ We Accept the Null hypothesis H_0

i.e; The Result of the experiment is not significant.

(18) $\bar{x} = \frac{230}{5} = 46$; $\bar{y} = \frac{399}{7} = 57$

x	$x-\bar{x}$	$(x-\bar{x})^2$	y	$y-\bar{y}$	$(y-\bar{y})^2$
42	-4	16	38	-19	361
39	-7	49	42	-15	225
48	2	4	56	-1	1
60	14	196	64	7	49
41	-5	25	68	11	21
			69	12	44
			62	5	25
230	0	290	399	0	926

$\Sigma(x_i-\bar{x})^2 = 290$; $\Sigma(y_i-\bar{y})^2 = 926$

$$S^2 = \frac{1}{n_1+n_2-2}\left(\Sigma(x_i-\bar{x})^2 + \Sigma(y_i-\bar{y})^2\right) = 121.6$$

$S = 11.03$

(1) Null Hypothesis (H_0) : $\mu_1 = \mu_2$

(2) Alternative Hypothesis (H_1) : $\mu_1 > \mu_2$

(3) L.O.S ; $\alpha = 0.05$

(4) Test statistic : $t = \frac{\bar{x}-\bar{y}}{S\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = -1.7$

$t_{cal} = |t| = 1.7$

$D.f = 5 + 7 - 2$

$= 10$

$\therefore t_{tab}$ with d.f 10 at 5% L.O.S is

$t_{tab} = 1.81$

$t_{cal} < t_{tab}$

$\therefore$ we Accept the Null Hypothesis (H_0)

i.e; The Medicines A and B do not differ significantly as regards their effect on increase in weight.

---

(19) Let the Null hypothesis (H_0) be: $\sigma_1^2 = \sigma_2^2$

The Alternative hypothesis is: $\sigma_1^2 \neq \sigma_2^2$

Given; $n_1 = 10$; $\Sigma(x_i - \bar{x})^2 = 120$

$n_2 = 12$; $\Sigma(y_i - \bar{y})^2 = 314$

$s_1^2 = \frac{120}{10-1} = 13.33$

$s_2^2 = \frac{314}{12-1} = 28.54$

Let H_0 be true. Since $s_2^2 > s_1^2$, the test statistic is

$$F = \frac{S_2^2}{S_1^2} = 2.14$$

$D.f = (n_1-1, n_2-1) = (9, 11)$

F_{tab} at D.f at 5% LOS is

$F_{tab} = 2.90$

$F_{cal} = 2.14$

$F_{cal} < F_{tab}$

$\therefore$ we Accept the null hypothesis H_0

i.e, The Difference is not significant at 5% level.

---

(20) Here; $n_1 = 5$; $n_2 = 6$

$$\bar{x} = \frac{\Sigma x_i}{n_1} = 8230$$

$$\bar{y} = \frac{\Sigma y}{n_2} = 7940$$

$\Sigma (x_i - \bar{x})^2 = 63000$

$\Sigma (y_i - \bar{y})^2 = 54600$ } from below Table.

Calculation for mean's & S.D's of samples

Mine 1 Mine 2

x	$x-\bar{x}$	$(x-\bar{x})^2$	y	$y-\bar{y}$	$(y-\bar{y})^2$
8260	30	900	7950	10	100
8130	-100	10000	7890	-50	2500
8350	120	14400	7900	-40	1600
8070	-160	25600	8140	200	40000
8340	110	12100	7920	-20	400
-	-	~~63000~~	7840	-100	10000
41150	-	63000	47640	-	54600

$$S_1^2 = \frac{\Sigma(x_i-\bar{x})^2}{n_1-1} = 15750$$

$$S_2^2 = \frac{\Sigma(y_i-\bar{y})^2}{n_2-1} = 10920$$

(1) Null Hypothesis H_0 : $S_1^2 = S_2^2$

(2) Alternative Hypothesis H_1 : $S_1^2 \neq S_2^2$

(3) L.O.S : $\alpha = 0.02$

Verified

(4) Test statistic : $F = S_1^2 / S_2^2$ $(S_1^2 > S_2^2)$

$$F = \frac{15750}{10920} = 1.44$$

$F_{cal} = 1.44$ $F_{tab} = 5.19$. → $\overset{5-1}{(4,}\overset{6-1}{5)}$ D.o.f at 2% LOS.

$F_{cal} < F_{tab}$

→ we Accept Null Hypothesis (H_0)

i.e; variances of 2 populations are equal.

Prepared by:
M. Sai Charan
AIML-C