TUE 10 SEPT

RANDOM VARIABLES

RANDOM VARIABLES:

In any rdm exp, the S.S associated w/ a real no. is called rdm var (r.v). It is denoted by X.

Ex: Tossing 2 coins (rdm exp)

X: Getting no. of heads

X: S ⟶ R

HH --> 2

HT --> 1

TH --> 1

TT --> 0

X : {0, 1, 2}

→ Types of Random Variables:

1. Discrete r.v
2. Continuous r.v

↳ Discrete Random Variable: A r.v is said to be discrete if the range of r.v is finite

Ex: Getting no. of heads if we toss 2 coins

X: {0, 1, 2}

→ Continuous Random Variable: A r.v is said to be cont. ~~r.v~~ if the range of the r.v is infinite or the interval of 2 real nos

Ex: The height of the students in a classroom may be 4 feet to 6 feet

$X : \{(4,6)\}$

PROBABILITY DISTRIBUTION (OR) PBT FUNCTION:

If X is a r.v then the ftn $f(x) = P(X = x_i)$ is called PD or PF.

→ Types of Probability Distribution:

1. Discrete PD (Pbt Mass Ftn)
2. Continuous PD (Pbt Density Ftn)

→ Discrete PD: If X is a discrete r.v then the pbt distribution $f(x) = P(x = x_i)$ is said to be pmf if it has the following props.

i) $f(x) \geq 0$

(ii) $\sum_{i=0}^{n} f(x) = 0$

→ Continuous PD : If X is cont rv then the p.d

$$f(x) = P(X = x_i)$$

is said to be pdf. If it has the following props:

(i) $f(x) \geq 0$

(ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

1. Obtain the prob distribution of getting no. of tails if we toss 2 coins.

Sol: Tossing 2 coins:

$S = \{HH, HT, TH, TT\}$

X = getting no. of tails $= \{0, 1, 2\}$

$P(0) = P(X = 0) = \frac{1}{4}$

$P(X = 1) = \frac{2}{4}$

$P(X = 2) = \frac{1}{4}$

The pd ftn is

X	0	1	2	total
$P(X = x_i)$	$1/2$	$2/4$	$1/2$	1

2. Obtained the ~~pbt~~ pd of getting no. of heads if we toss three coins.

Sol: $S = \{HHH, HHT, HTH, THH, TTH, THT, HTT, TTT\}$

$\quad\;\; 3 \quad 2 \quad 2 \quad 2 \quad 1 \quad 1 \quad 1 \quad 0$

X = getting no. of heads

$X = \{0, 1, 2, 3\}$

$P(0) = P(X=0) = \frac{1}{8}$

$P(1) = P(X=1) = \frac{3}{8}$

$P(2) = P(X=2) = \frac{3}{8}$

$P(3) = P(X=3) = \frac{1}{8}$

*

3. Out of 24 mangoes, 6 mangoes are rotten. If we
*
* draw 2 mangoes, obtain the pd of no. of rotten mangoes that can be drawn.

Sol: $n = {}^{24}C_2$

X = no. of rotten mangoes $= \{0, 1, 2\}$

$P(x=0) = \frac{{}^{6}C_0 \times {}^{18}C_2}{{}^{24}C_2}$

$P(X=1) = \frac{{}^{6}C_1 \times {}^{18}C_1}{{}^{24}C_2}$

$P(x=2) = \frac{{}^{6}C_2 \times {}^{18}C_0}{{}^{24}C_2}$

4. Find the probability distribution of sum of score on dice, if we throw 2 dice.

Sol:

$$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),$$
$$(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),$$
$$(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),$$
$$(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),$$
$$(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),$$
$$(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$$

$$X = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$$

$P(X=2) = \frac{1}{36}$; $P(X=3) = \frac{2}{36} = \frac{1}{18}$

$P(X=4) = \frac{3}{36} = \frac{1}{13}$; $P(X=5) = \frac{4}{36} = \frac{1}{9}$

$P(X=6) = \frac{5}{36}$; $P(X=7) = \frac{6}{36} = \frac{1}{2}$

$P(X=8) = \frac{5}{36}$; $P(X=9) = \frac{4}{36} = \frac{1}{9}$

$P(X=10) = \frac{3}{36} = \frac{1}{12}$; $P(X=11) = \frac{2}{36} = \frac{1}{18}$

$P(X=12) = \frac{1}{36}$

5. Find the p.d of the max of each pair on dice if we throw 2 dice

Sol: $S = \{(1,1), \text{-----} (6,6)\}$

$X = \{1, 2, 3, 4, 5, 6\}$

$P(X=1) = \{1,1\} = \frac{1}{36}$

$P(X=2) = \{(1,2), (2,1), (2,2)\} = \frac{3}{36} = \frac{1}{12}$

$P(X=3) = \frac{5}{36}$; $P(X=4) = \frac{7}{36}$

$P(X=5) = \frac{9}{36} = \frac{1}{4}$; $P(X=6) = \frac{11}{36}$

6. Check whether the following ftns are pmf or not.

i) $f(x) = \frac{x-2}{2}$ for $x = 1, 2, 3, 4$

Sol: Cond^n for pmf:

i) $f(x) \geq 0$

$f(1) = -\frac{1}{2} \not\geq 0$

$\therefore$ The given fn is not a pmf.

(ii) $f(x) = \frac{x^2}{28}$ for $x = 0, 1, 2, 3, 4$

Sol: i) $f(x) \geq 0$

$f(0) = \frac{0}{28} = 0$

$f(1) = \frac{1}{28} \geq 0$

$f(2) = \frac{4}{28} \geq 0$

$f(3) = \frac{9}{28} \geq 0$

$f(4) = \frac{16}{28} \geq 0$

ii) $\sum_{i=0}^{4} f(x) = 1$

$\Rightarrow 0 + \frac{1}{28} + \frac{4}{28} + \frac{9}{28} + \frac{16}{28} = \frac{15}{48} \neq 1$

$\therefore$ The given fn is not pmf.

(iii) $f(x) = \frac{1}{6}$ for $x = 1, 2, 3, 4, 5, 6$

Sol: (i) $f(1) = \frac{1}{6} \geq 0$; $f(2) = \frac{1}{6} \geq 0$; $f(3) = \frac{1}{6} \geq 0$

$f(4) = \frac{1}{6} \geq 0$; $f(5) = \frac{1}{6} \geq 0$; $f(6) = \frac{1}{6} \geq 0$

(ii) $\sum_{i=0}^{6} f(x) = 1$

$\Rightarrow \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = 1$

$\therefore$ The given fn is a pmf.

7 A random var X has the following ~~pdf~~ pmf

$X = x_i$	0	1	2	3	4	5	6	7	8
$P(X = x_i)$	K	3K	5K	7K	9K	11K	13K	15K	17K

Find (i) K (ii) $P(X \geq 5)$ (iii) $P(X<4)$ (iv) $P(0<X<4)$

Sol: (i) $\sum_{i=0}^{8} f(x) = 1$

$\Rightarrow K + 3K + 5K + 7K + 9K + 11K + 13K + 15K + 17K = 1$

$\Rightarrow 81K = 1 \Rightarrow K = \frac{1}{81}$

(ii) $P(X \geq 5) = P(X=5) + P(6) + P(7) + P(8)$

$\Rightarrow 11K + 13K + 15K + 17K \Rightarrow 56K \Rightarrow \frac{56}{81}$

(iii) $P(X<4) \Rightarrow P(0) + P(1) + P(2) + P(3)$

$\Rightarrow K + 3K + 5K + 7K = \frac{16}{81}$

(iv) $P(0<X<4) \Rightarrow P(1) + P(2) + P(3)$

$\Rightarrow 3K + 5K + 7K \Rightarrow \frac{15}{81} \Rightarrow \frac{5}{27}$

8. A r.v X has the following pbt ~~distribution~~ fn

X	0	1	2	3	4	5	6
p(x)	0	K	2k	2K	3K	k^2	$7K^2+k$

Find i) K ii) $P(x<5)$ iii) $P(x \geq 5)$

Sol:

i) $\Sigma f(x) = 1$

$\Rightarrow 0 + K + 2K + 2K + 3K + K^2 + 7K^2 + K = 1$

$\Rightarrow 9K + 8K^2 - 1 = 0 \Rightarrow 8K^2 + 9K - 1 = 0$

$\Rightarrow K = 0.1$

ii) $P(x<5) \Rightarrow P(0) + P(1) + P(2) + P(3) + P(4)$

$\Rightarrow 0 + K + 2K + 2K + 3K \Rightarrow 8K \Rightarrow 0.8$

iii) $p(x \geq 5) \Rightarrow P(5) + P(6)$

$\Rightarrow K^2 + 7K^2 + K \Rightarrow 8K^2 + K \Rightarrow 0.18$.

MEAN, VARIANCE AND STANDARD DEVIATION

→ PMF:

↳ Mean (μ): $E(x) = \mu = \sum_{i=0}^{n} x_i f(x_i)$

↳ Variance: $\sigma^2 = V(x) = \sum_{i=0}^{n} x^2 f(x) - \mu^2$

↳ Standard Deviation: $\sigma = \sqrt{\sigma^2}$

9. Let X be a discrete r.v taking the values $1, 2, \ldots, 6$ & $f(x_i) = \frac{1}{6}$, then find mean & variance.

Sol: $f(x_i) = \frac{1}{6}$ for $x = 1, 2, \ldots, 6$

$X = x_i$	1	2	3	4	5	6	Total
$f(x_i) = P(X = x_i)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	1

(i) Mean $(\mu) = \sum_{i=1}^{6} x_i f(x_i)$

$\Rightarrow 1.\frac{1}{6} + 2.\frac{1}{6} + 3.\frac{1}{6} + 4.\frac{1}{6} + 5.\frac{1}{6} + 6.\frac{1}{6}$

$\Rightarrow \frac{1}{6} + \frac{2}{6} + \frac{3}{6} + \frac{4}{6} + \frac{5}{6} + \frac{6}{6} \Rightarrow \frac{7}{2} = 3.5$

(ii) Variance, $\sigma^2 = \sum_{i=1}^{6} x^2 f(x) - \mu^2$

$\Rightarrow \left[1.\frac{1}{6} + 4.\frac{1}{6} + 9.\frac{1}{6} + 16.\frac{1}{6} + 25.\frac{1}{6} + 36.\frac{1}{6}\right] - (3.5)^2$

$\Rightarrow \frac{1}{6} + \frac{4}{6} + \frac{9}{6} + \frac{16}{6} + \frac{25}{6} + \frac{36}{6} - (3.5)^2$

$\Rightarrow \frac{91}{6} - (3.5)^2 \Rightarrow 2.916$

10. Two coins are tossed simultaneously. Let X denotes the no. of heads then find expectation of X & variance of X.

Sol:

X	0	1	2
$P(x=x_i)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

expection $E(X) = 0\cdot\frac{1}{4} + 1\cdot\frac{1}{2} + 2\cdot\frac{1}{4} \Rightarrow 1$

Variance $V(x) = 0\cdot\frac{1}{4} + 1\cdot\frac{1}{2} + 4\cdot\frac{1}{4} - 1$

$\Rightarrow \frac{1}{2} + 1 - 1 \Rightarrow \frac{1}{2}$

11. Let X denotes the min of the 2 nos that appear when a pair of fair dice is thrown once. Determine

(i) Discrete p.d (ii) Expectation (iii) Variance

Sol: $S = \{(1,1) \ldots (6,6)\}$

X = getting min of 2 nos

(i)

X	1	2	3	4	5	6
$P(X=x_i)$	$\frac{11}{36}$	$\frac{1}{4}$	$\frac{7}{36}$	$\frac{5}{36}$	$\frac{1}{12}$	$\frac{1}{36}$

(ii) $E(x) = 1\cdot\frac{11}{6} + 2\cdot\frac{1}{4} + 3\cdot\frac{7}{36} + 4\cdot\frac{5}{36} + 5\cdot\frac{1}{12} + 6\cdot\frac{1}{36}$

$\Rightarrow 2.527$

(iii) $V(x) = 1 \cdot \frac{11}{36} + 4 \cdot \frac{1}{4} + 9 \cdot \frac{7}{36} + 16 \cdot \frac{5}{36} + 25 \cdot \frac{1}{12} + 36 \cdot \frac{1}{36} - (2.$

$\Rightarrow 8.361 - (2.527)^2$

$\Rightarrow 1.975$

12. A fair die is ~~tossed~~. Let the r.v X denote the twice the no. appearing on the die:

(i) Write pd of X (ii) Mean & (iii) Variance

Sol: When a fair die is thrown, tot. no. of outcomes is $n(S) = 6$

$S = \{1, 2, 3, 4, 5, 6\}$

X denotes the ~~no. appears~~ twice the no. So, 2S

$2S = \{2, 4, 6, 8, 10, 12\}$

(i) Pd

X	2	4	6	8	10	12
$P(X=x_i)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	0	0	0

(ii) Mean, $\mu = 2 \cdot \frac{1}{6} + 4 \cdot \frac{1}{6} + 6\frac{1}{6}$

$\Rightarrow 2$

(iii) Variance, $\sigma^2 = 4 \cdot \frac{1}{6} + 16 \cdot \frac{1}{6} + 36 \cdot \frac{1}{6} - 2^2$

$\Rightarrow 9.33 - 4 \Rightarrow 5.333$

MATHEMATICAL EXPECTATION:

Let X be a discrete r.v. Then,

$$E(x) = \Sigma x f(x)$$
$$= \sum_{i=0}^{n} x_i (f(x_i))$$

is called ME of X

$$E(x^2) = \Sigma x^2 f(x)$$

$$E(x^3) = \Sigma x^3 f(x)$$

Note:
$$V(x) = E(x^2) - [E(x)]^2$$

→ Properties (Mean):

1. $E(a) = a$ (const.)
2. $E(ax) = aE(x)$
3. $E(x+y) = E(x) + E(y)$
4. $E(x-y) = E(x) - E(y)$
5. $E(ax+b) = aE(x) + b$
6. $E(ax+by) = aE(x) + bE(y)$

→ Properties (Variance)

1. $V(a) = 0$
2. $V(ax) = a^2 F(x)$
3. $V(x+y) = V(x) + V(y)$
4. $V(x-y) = V(x) - V(y)$
5. $V(ax+b) = a^2 V(x) + b$
6. $V(ax+by) = a^2 V(x) + b^2 (V(y))$

Here a & b are consts.

13. Let X be a r.v of the following distribution table

$X = x_i$	3	6	9
$P(X = x_i)$	$1/6$	$1/2$	$1/3$

Find i) $E(x)$ ii) $E(x^2)$ iii) $E(1+2x^2)^2$

Sol)

(i) $E(x) = \sum x f(x)$

$= 3(1/6) + 6(1/2) + 9(1/3)$

$= \frac{1}{2} + \frac{1}{3} + 3 = 6.5$

(ii) $E(x^2) = \sum x^2 f(x)$

$= 9(1/2) + 36(1/2) + 81(1/3)$

$= \frac{9}{2} + 18 + 27 = 46.5$

(iii) $E(1+2x)^2 = E(1 + 4x^2 + 4x)$

$\Rightarrow E(1) + E(4x^2) + E(4x)$

$\Rightarrow 1 + 4E(x^2) + 4E(x) \Rightarrow 1 + 4(46.5) + 6.5$

$\Rightarrow 1 + 186.0 + 6.5 = 193.5$

14. Let X be a r.v for the following values of $X = \{1, 2, 3\}$ if $f(x) = \frac{x}{6}$. Find (i) $E(x)$ (ii) $E(x^2)$ (iii) $E(x^3)$ (iv) $E(x^3+2x+7)$

Sol:

X	1	2	3
$P(x)$	$1/6$	$2/6$	$3/6$

(i) $E(x) = \sum x f(x) = 1(1/6) + 2(2/6) + 3(3/6) = \frac{7}{3} = 2.33$

(ii) $E(x^2) = \sum x^2 f(x) = 1^2(1/6) + 2^2(2/6) + 3^2(3/6) = 6$

(iii) $E(x^3) = \sum x^3 f(x) = 1(1/6) + 8(2/6) + 27(3/6) = \frac{49}{3} = 16.33$

(iv) $E(x^3+2x+7) = E(x^3) + E(2x) + E(7)$

$\Rightarrow 16.33 + 2(E(x)) + 7 \Rightarrow \frac{49}{3} + 2\left(\frac{7}{3}\right) + 7 = 28$

15. If $E(x) = 1$ & $E(x^2) = 4$, find mean & variance of $2x-3$

Sol: $E(2x-3) = 2[E(x)] - 3 \Rightarrow 2(1) - 3 = -1$

$V(2x-3) = 2^2[V(x)] - 3 \Rightarrow 4[E(x^2) - (E(x))^2] - 3$

$\Rightarrow 4[4-1] - 3 \Rightarrow 12 - 3 = 9$

16. If $E(x) = 10$ & $V(x) = 1$, find $E\{2x(x+20)\}$

Sol: $E\{2x^2 + 40x\} \Rightarrow E(2x^2) + E(40x) \Rightarrow 2E(x^2) + 40E(x)$

$\Rightarrow 2[V(x) + [E(x)]^2] + 40E(x) \Rightarrow 2(1 + (10)^2) + 40 \times 10$

$\Rightarrow 602$

17. Is a function defined by $f(x) = \begin{cases} 0, & x < 2 \\ \frac{3+2x}{18}, & 2 \le x \le 4 \\ 0, & x > 4 \end{cases}$

pdf or not.

Sol: Here $f(x) \geq 0$

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{2} f(x)\,dx + \int_{2}^{4} f(x)\,dx + \int_{4}^{\infty} f(x)\,dx$$

$$\Rightarrow 0 + \int_{2}^{4} \frac{3+2x}{18}\,dx + 0$$

$$\Rightarrow \frac{1}{18}\int_{2}^{4} (3+2x)\,dx$$

$$\Rightarrow \frac{1}{18}\left[3x + \frac{2x^2}{2}\right]_2^4$$

$$\Rightarrow \frac{1}{18}\left[(12+16) - (6+4)\right]$$

$$\Rightarrow \frac{1}{18}[28-10] = \frac{18}{18} = 1$$

∴ The given fn is pdf

Note:

- $\int uv\,dx = uv_1 - u'v_2 + u''v_3 - u'''v_4$
- $e^0 = 1$
- $e^{-0} = 1$
- $e^{\infty} = \infty$
- $e^{-\infty} = 0$

18. The p.d.f is

$$f(x) = \begin{cases} k(3x^2 - 2), & -1 < x < 2 \\ 0, & \text{otherwise} \end{cases}$$

then find k.

Sol: $$\int_{-\infty}^{-1} 0\,dx + \int_{-1}^{2} k(3x^2 - 2)\,dx + \int_{2}^{\infty} 0\,dx = 1$$

$$\Rightarrow 0 + k\int_{-1}^{2} (3x^2 - 2)\,dx + 0 = 1$$

$$\Rightarrow k\left[(x^3)_{-1}^{2} - \left(\frac{2x^2}{2}\right)_{-1}^{2}\right] = 1$$

$$\Rightarrow k\left[[8+1] - [4+2]\right] = 1$$

$$\Rightarrow k(9-6) = 1 \Rightarrow 3k = 1 \Rightarrow k = \frac{1}{3}\text{ y.}$$

19. Find the const. k so that the ftn $f(x)$ is defined as

$$f(x) = \begin{cases} \frac{1}{k}, & a < x < b \\ 0, & \text{otherwise} \end{cases}$$

Sol: $$\int_{-\infty}^{a} 0\,dx + \int_{a}^{b} \frac{1}{k}\,dx + \int_{b}^{\infty} 0\,dx = 1$$

$$\Rightarrow 0 + \frac{1}{k}(x)_{a}^{b} + 0 = 1$$

$\Rightarrow \frac{1}{k}(b-a) = 1 \Rightarrow k = b-a$

<u>Note.</u>

If $f(x)$ is a density ftn, then $P(a<x<b) = \int_a^b f(x)\,dx$

Q0. A random var. X has the ~~pdf~~ pdf

$$f(x) = \begin{cases} 2e^{-2x}, & \text{if } x>0 \\ 0, & \text{if } x<0 \end{cases}$$

Find i) $P(1<x<3)$ ii) $P(x>0.5)$

Sol. (i) $\int_1^3 2e^{-2x}\,dx \Rightarrow \left(2\frac{e^{-2x}}{-2}\right)_1^3 \Rightarrow \left(-e^{-2x}\right)_1^3$

$\Rightarrow -\left[e^{-6} - e^{-1}\right] \Rightarrow e^{-6} + e^{-1}$

(ii) $\int_{0.5}^{\infty} 2e^{-2x}\,dx \Rightarrow \left(-e^{-2x}\right)_{0.5}^{\infty} \Rightarrow -\left[e^{-2(\infty)} - e^{-2(0.5)}\right]$

$\Rightarrow -\left[0 - e^{-1}\right] \Rightarrow \frac{1}{e}$

21. A cont. r.v has the following distribution

$$f(x) = \begin{cases} 0, & \text{if } x<1 \\ k(1-x)^4, & \text{if } 1<x<3 \\ 0, & \text{if } x>3 \end{cases}$$

then find k

Sol:

$$\int_{-\infty}^{1} 0\,dx + \int_{1}^{3} k(1-x)^4\,dx + \int_{3}^{\infty} 0\,dx$$

$$\Rightarrow k\int_{1}^{3}(1-x)^4\,dx \Rightarrow k\int_{0}^{-2} -(t)^4\,dt \Rightarrow k\left(-\frac{t^5}{5}\right)_0^{-2}$$

$$\Rightarrow \cancel{k[-2-0]} \Rightarrow \cancel{2k=1} \Rightarrow \cancel{k=\frac{1}{2}}.$$

$$\Rightarrow -k\left(\frac{(-2)^5}{5} - 0\right) \Rightarrow -k\left(\frac{-32}{5}\right) \Rightarrow \frac{32k}{5} = 1$$

$$\Rightarrow k = \frac{5}{32}.$$

22. A cont. r.v X is defined as

$$f(x) = \begin{cases} \frac{(3+x)^2}{16}, & \text{if } -3<x<-1 \\ \frac{6-2x^2}{16}, & \text{if } -1<x<1 \\ \frac{(3-x)^2}{16}, & \text{if } 1<x<3 \\ 0, & \text{otherwise} \end{cases}$$

Sol: $\int_{-\infty}^{-3} 0\,dx + \int_{-3}^{-1} \frac{(3+x)^2}{16} + \int_{-1}^{1} \frac{6-2x^2}{16}\,dx + \int_{1}^{3} \frac{(3-x)^2}{16}dx + \int_{3}^{\infty} 0\,dx$

$= 0 + \int_{-3}^{-1} \frac{9+x^2+6x}{16}\,dx + \int_{-1}^{1} \frac{6-2x^2}{16}\,dx + \int_{1}^{3} \frac{9+x^2-6x}{16}\,dx + \int_{3}^{\infty} 0\,dx$

$\Rightarrow \left[\frac{9x}{16} + \frac{x^3}{3.16} + \frac{6x^2}{2.16}\right]_{-3}^{-1} + \left[\frac{6x}{16} - \frac{2x^3}{3.16}\right]_{-1}^{1} + \left[\frac{9x}{16} + \frac{x^3}{3.16} - \frac{6x^2}{2.16}\right]_{1}^{3}$

$\Rightarrow \frac{9[-1+3]}{16} + \frac{[-1+27]}{3.16} + \frac{6[1-9]}{2.16} + \frac{6[1+1]}{16} - \frac{2[1+1]}{3.16}$

$+ \frac{9[3-1]}{16} + \frac{[27-1]}{3.16} - \frac{6[9-1]}{2.16}$

$\Rightarrow \frac{9(2)}{16} + \frac{(26)}{3.16} + \frac{6(-8)}{2.16} + \frac{6(2)}{16} - \frac{4}{3.16} + \frac{9(2)}{16} + \frac{26}{3.16} - \frac{6(8)}{2.16}$

$\Rightarrow \frac{9}{8} + \frac{13}{24} - \frac{3}{2} + \frac{3}{4} - \frac{1}{12} + \frac{9}{8} + \frac{13}{24} - \frac{3}{2}$

$\Rightarrow 1$

$\therefore$ The given ftn is a p.d.f.

M, V & SD:

→ PDF:

↳ Mean: $\mu = E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$

↳ Variance: $\sigma^2 = V(x) = \int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu^2$

↳ SD: $\sigma = \sqrt{\sigma^2}$

Note:

$\int UV\,dx = UV_1 - U'V_{11} + U''V_{111} - \dots$

22. A cont. r.v has the following pdf

$$f(x) = \begin{cases} kxe^{-\lambda x}, & \text{for } x \geq 0, \lambda > 0 \\ 0, & \text{otherwise} \end{cases}$$

Find (i) K (ii) Mean & (iii) Variance

Sol: (i) $\int_{-\infty}^{0} 0\,dx + \int_{0}^{\infty} kxe^{-\lambda x}\,dx = 1$

$\Rightarrow k\left[x\frac{e^{-\lambda x}}{-\lambda} - 1\frac{e^{-2x}}{+\lambda^2}\right]_0^{\infty} = 1$

$\Rightarrow -k\left[\frac{xe^{-\lambda x}}{\lambda} + \frac{e^{-\lambda x}}{\lambda^2}\right]_0^{\infty} = 1$

$$\Rightarrow -K\left[\left[\frac{\infty . e^{-\lambda\infty}}{\lambda} + \frac{e^{-\lambda\infty}}{\lambda^2}\right] - \left[0 + \frac{e^{-\lambda 0}}{\lambda^2}\right]\right]$$

$$\Rightarrow -K\left[(0-0) - \left(0 + \frac{1}{\lambda^2}\right)\right] = 1 \quad \Rightarrow -K\left(\frac{-1}{\lambda^2}\right) = 1$$

$$\Rightarrow K = +\lambda^2 .$$

(ii) $$\mu = \int_{-\infty}^{\infty} x\, f(x)\, dx$$

$$K\int_0^{\infty} x\,(x e^{-\lambda x})\, dx \quad \Rightarrow K\int_0^{\infty} x^2 e^{-\lambda x}\, dx$$

$$\Rightarrow K\left[x^2\left(\frac{e^{-\lambda x}}{-\lambda}\right) - 2x\left(\frac{e^{-\lambda x}}{\lambda^2}\right) + 2\left(\frac{e^{-\lambda x}}{-\lambda^3}\right)\right]_0^{\infty}$$

$$\Rightarrow K\left[\left[-\frac{\infty^2\, e^{-\lambda.\infty}}{-\lambda} - 2.\infty\left(\frac{e^{-\lambda.\infty}}{\lambda^2}\right) + 2\left(\frac{e^{-\lambda.\infty}}{-\lambda^3}\right)\right]\right.$$

$$\Rightarrow \cancel{K\left[0\right.} - \left[0 - 0 + 2\,\frac{e^{-\lambda . 0}}{-\lambda^3}\right]\Big]$$

$$\Rightarrow K\left[(0-0+0) - \left(0 - 0 - \frac{2}{\lambda^3}\right)\right] \Rightarrow \lambda^2\left(\frac{2}{\lambda^3}\right) \Rightarrow \frac{2}{\lambda} .$$

(iii) $$\sigma^2 = \int_{-\infty}^{\infty} x^2 f(x)\, dx - \mu^2 \Rightarrow K\int_0^{\infty} x^3 e^{-\lambda x}\, dx - \frac{4}{\lambda^2}$$

$$\Rightarrow K\left[x^3\left(\frac{e^{-\lambda x}}{-\lambda}\right) - 3x^2\left(\frac{e^{-\lambda x}}{\lambda^2}\right) + 6x\left(\frac{e^{-\lambda x}}{-\lambda^3}\right) - 6\left(\frac{e^{-\lambda x}}{\lambda^4}\right)\right]_0^{\infty} - \frac{4}{\lambda^2}$$

$$\Rightarrow -k\left[\frac{x^3e^{-\lambda x}}{\lambda}+\frac{3x^2e^{-\lambda x}}{\lambda^2}+\frac{6xe^{-\lambda x}}{\lambda^3}+\frac{6e^{-\lambda x}}{\lambda^4}\right]_0^\infty-\frac{4}{\lambda^2}$$

$$\Rightarrow -k\left[(0+0+0+0)-\left(0+0+0+\frac{6}{\lambda^4}\right)\right]-\frac{4}{\lambda^2}$$

$$\Rightarrow -\lambda^2\left(-\frac{6}{\lambda^4}\right)-\frac{4}{\lambda^2}\Rightarrow \frac{6}{\lambda^2}-\frac{4}{\lambda^2}\Rightarrow \frac{2}{\lambda^2}.$$

Q 23. If $f(x) = ke^{-|x|}$ is a p.d.f in the interval $-\infty < x < \infty$ then find (i) K (ii) mean (iii) variance (iv) $P(0 < x < 4)$

Sol: Given $f(x) = ke^{-|x|}$

$$|x| = \begin{cases} x, & x > 0 \\ -x, & x < 0 \end{cases}$$

(i) $\int_{-\infty}^{\infty} f(x)\,dx = 1 \Rightarrow \int_{-\infty}^{\infty} k\cdot e^{-|x|}\,dx = 1$

$$\Rightarrow \int_{-\infty}^{0} f(x)dx + \int_{0}^{\infty} f(x)dx = 1$$

$$\Rightarrow k\left[\int_{-\infty}^{0} e^{-(-x)}\,dx + \int_{0}^{\infty} e^{-(x)}\,dx\right] = 1$$

$$\Rightarrow k\left[\int_{-\infty}^{0} e^{x}\,dx + \int_{0}^{\infty} e^{-x}\,dx\right] = 1 \Rightarrow k\left[(e^x)_{-\infty}^{0} + \left(\frac{e^{-x}}{-1}\right)_0^\infty\right] = 1$$

$$\Rightarrow k\left[(1-0)+(0-(-1))\right] = 1 \Rightarrow k(2) = 1 \Rightarrow k = \frac{1}{2}$$

(ii) Mean:

$$\mu = \int_{-\infty}^{\infty} x f(x)\, dx$$

$$\Rightarrow \int_{-\infty}^{0} x f(x)\, dx + \int_{0}^{\infty} x f(x)\, dx \Rightarrow K\left[\int_{-\infty}^{0} x\, e^{-(-x)}\, dx + \int_{0}^{\infty} x . e^{-x}\, dx\right]$$

$$\Rightarrow K\left[\left[x . e^{x} - 1 . e^{x}\right]_{-\infty}^{0} + \left[x . \frac{e^{-x}}{-1} - 1 . \frac{e^{-x}}{1}\right]_{0}^{\infty}\right]$$

$$\Rightarrow K\left[\left[(0-1) - (0-0)\right] + \left[(0-0) - (0-1)\right]\right] \Rightarrow K\left[-1+1\right] = 0$$

(iii) Variance:

$$\sigma^2 = \int_{-\infty}^{\infty} x^2 f(x)\, dx - \mu^2$$

$$\Rightarrow K\left[\int_{-\infty}^{0} x^2 . e^{-(-x)}\, dx + \int_{0}^{\infty} x^2 . e^{-x}\, dx\right] - \mu^2$$

$$\Rightarrow K\left[\left[x^2 . e^{x} - 2x . e^{x} + 2e^{x}\right]_{-\infty}^{0} + \left[x^2 . \frac{e^{-x}}{-1} - \frac{2x\, e^{-x}}{1} + \frac{2 . e^{-x}}{-1}\right]_{0}^{\infty}\right] - \mu^2$$

$$\Rightarrow K\left[\left[(0-0+2) - 0 - 0 + 0\right]_{-\infty}^{0} + \left[(0-0+0) - (0 - 0 + \frac{2}{-1})\right]\right] - \mu^2$$

$$\Rightarrow K\left[2+2\right] - (0)^2 \Rightarrow \frac{1}{2}(4) = 2$$

$$\sigma^2 = 2$$

(iv) $P(0 < x < 4) \Rightarrow \int_0^4 f(x)\,dx$

$$\Rightarrow \int_0^4 K e^{-|x|}\,dx \Rightarrow k \int_0^4 e^{-x}\,dx$$

$$\Rightarrow k \left[\frac{e^{-x}}{-1}\right]_0^4 \Rightarrow \left[\frac{e^{-4} - e^{-0}}{-2}\right]$$

$$\Rightarrow \frac{1 - \left[e^{-4}\right]}{\cancel{2}\,2}$$