MC

Module-2

PART-A

1. Determine the negations of the following statements, a)Jan will take a job in industry or go to graduate school b) James will bicycle or run tomorrow. c) If the processor is fast then the printer is slow

Sol:

a) P: Jan will take a job in industry

Q: Jan go to graduate school

$P \vee Q$

$\sim(P \vee Q) = \sim P \wedge \sim Q$

Jan will not take a job in industry and ~~not~~ won't go to graduate school.

b) P: James will bicycle tomorrow

Q: James run tomorrow

$P \vee Q$

$\sim(P \vee Q) = \sim P \wedge \sim Q$

James won't biclyee and run tomorrow.

c) P: The processor is fast

Q: the printer is slow

$P \rightarrow Q$

$\sim(P \rightarrow Q)$

If the processor is not fast then the printer is not slow.

2. Find the PDNF of $(p \wedge q) \vee (\sim p \vee r) \vee (q \vee r)$ using truth table.

Sol:

$(p \wedge q) \vee (\sim p \vee r) \vee (q \vee r) = x$

p	q	r	$p \wedge q$	$q \vee r$	$\sim p \vee r$	x	
T	T	T	T	T	T	T	– $(p \wedge q \wedge r)$
T	T	F	T	T	F	T	– $(p \wedge q \wedge \sim r)$
T	F	T	F	T	T	T	– $(p \wedge \sim q \wedge r)$
T	F	F	F	F	F	F	– —
F	T	T	F	T	T	T	– $(\sim p \wedge q \wedge r)$
F	T	F	F	T	T	T	– $(\sim p \wedge q \wedge \sim r)$
F	F	T	F	T	T	T	– $(\sim p \wedge \sim q \wedge r)$
F	F	F	F	F	T	T	– $(\sim p \wedge \sim q \wedge \sim r)$

PDNF: $(p \wedge q \wedge r) \vee (p \wedge q \wedge \sim r) \vee (p \wedge \sim q \wedge r) \vee (\sim p \wedge q \wedge r) \vee (\sim p \wedge q \wedge \sim r)$
$\vee (\sim p \wedge \sim q \wedge r) \vee (\sim p \wedge \sim q \wedge \sim r)$

3. Show that: a) $R \wedge (P \vee Q)$ is a valid conclusion from premises $P \vee Q$, $Q \to R$, $P \to M$, $\sim M$. b) R→S can be derived from the premises, P→(Q→S), $\sim R \vee P$ *and* Q

Sol:

Conclusion: $R \wedge (P \vee Q)$

Premises:- $P \vee Q$, $Q \to R$, $P \to M$, $\sim M$

1)	$P \to M$	Rule - P
2)	$\sim M$	Rule - P
3)	$\sim P$	Rule-T (Modus tollens 1 & 2)
4)	$P \vee Q$	Rule-P
5)	Q	Rule - T (disjunctive syllogism)
6)	$Q \to R$	Rule- P
7)	R	Rule - T (Modus ponens 6, 7)

8) $R \wedge (P \vee Q)$ Rule-T (7, 4) Conjunction rule.

Hence, the conclusion is valid.

b) Conclusion : $R \rightarrow S$

Premises: $P \rightarrow (Q \rightarrow S)$, $\sim R \vee P$, and Q.

Let us consider R as an additional premises -

1)	R	Rule-CP	Additional premise
2)	$\sim R \vee P$	Rule-P	
3)	P	Rule-T	1, 2 disjunctive syllogism.
4)	$P \rightarrow (Q \rightarrow S)$	Rule-P	
5)	$Q \rightarrow S$	Rule-T	3, 4 modus ponens.
6)	Q	Rule-P	5, 6 modus ponens
7)	S	Rule-T	

LHS of conclusion $\equiv$ RHS of conclusion.

$\therefore$ The conclusion is valid

4. Show that the following premises are inconsistent. (a) If jack misses many classes through illness, then he fails high school (b) If jack fails high school, then he is uneducated. (c) If jack reads lot of books, then he is not uneducated. Jack misses many classes through illness and lot of books

Sol:

P: Jack misses many classes

Q: Jack fails school

R: Jack is uneducated

S: Jack reads a lot of book.

1) $P \wedge S$	rule-P	
2) P	rule-T	simplification – 1
3) $P \rightarrow Q$	rule-P	
4) Q	rule-T	modus ponens 2 & 3
5) $Q \rightarrow R$	rule-P	
6) R	rule-T	modus ponens 4 & 5
7) $S \rightarrow \sim R$	rule-P	
8) S	rule-T	Simplification 1
9) $\sim R$	rule-T	7, 8 modus ponens
10) $R \wedge \sim R$	rule-T	6, 9 conjunction.

Hence the premises are inconsistent

5. Select p,q and r be the propositions p: you have the flee q: you miss the final examination r: you pass the course. Translate the following propositions into statement form. $(i) p \rightarrow q$ $(ii) \sim p \rightarrow r$ $(iii) q \rightarrow \sim r$

$(iv) p \vee q \vee r$ $(v) (p \rightarrow \sim r) \vee (q \rightarrow \sim r)$ $(vi) (p \wedge q) \vee (\sim q \wedge r)$.

Sol:

5) p: you have the flee

q: You miss the final examination.

r: you pass the course

i) $p \rightarrow q$: If you have the flee, then you will miss the final examination

ii) $\sim p \rightarrow r$: If you don't have the flee, then. you will pass the course

iii) $q \rightarrow \sim r$: If you miss the ~~the~~ final examination, then you won't Pass the Course

iv) $p \vee q \vee r$: you have the flee. or you miss the final examination or you ~~will~~ pass the course

v) $(p \rightarrow \sim r) \vee (q \rightarrow \sim r)$: If you have the flee, then you won't miss the final examination or If you miss the final examination then you will pass the course.

vi) $(p \wedge q) \vee (\sim q \wedge r)$: 'You have the flee and you miss the final examination' or 'You won't miss the final examination and you pass the test.

6. Translate the following proposition in symbolic form, and find its negation: "If all triangles are right angled, then no triangle is equiangular"

Sol:

6) $P(x)$: x is right angled

$q(x)$: x is equiangular

Symbolic form:

$$[\forall x \in T, P(x)] \longrightarrow [\forall x \in T, \sim q(x)]$$

Negation:

$$[\forall x \in T, P(x)] \wedge [\exists x \in T, q(x)]$$

All triangles are right angled and some triangles are equiangular.

7. Rephrase an equivalent formula $\sim (p \rightarrow (q \rightarrow (r \vee p)))$ which does not contain any conditional (→) and biconditional (↔)

Sol:

$$\sim(p \leftrightarrow (q \rightarrow (r \vee p)))$$

$$\sim(p \leftrightarrow (\sim q \vee (r \vee p)))$$

$$\sim(p \rightarrow (\sim q \vee (r \vee p))) \wedge ((\sim q \vee (r \vee p)) \rightarrow p)$$

$$\sim((\sim p \vee (p \vee \sim q \vee r)) \wedge (\sim(p \vee \sim q \vee r) \vee p))$$

$$\sim((T \vee (\sim q \vee r)) \wedge (T \vee (\sim q \vee r)))$$

$$\sim(\sim q \vee r) \equiv q \wedge \sim r.$$

8. Demonstrate principle conjunctive normal form and principle disjunctive normal form with its procedural steps. Obtain the principal disjunctive normal form of $\boldsymbol{P} \rightarrow ((\boldsymbol{P} \rightarrow \boldsymbol{Q}) \wedge \sim (\sim \boldsymbol{Q} \vee \sim \boldsymbol{P}))$.

Sol: PDNF : $P \wedge Q$, $\sim P \wedge \sim Q$, $\sim P \wedge Q$, $P \wedge \sim Q$ are called maxterms. For a given formula, an equivalent formula consisting of disjunction of minterms only is known as principal disjunctive normal form. This is also called as sum of elementary products.

PCNF: The duality of minterms are called maxterms. For a given formula, an equivalent formula consisting of conjunction of maxterms only is known as principal conjunctive normal form This is also called as product of elementary sums.

<u>Steps for finding PDNF:</u>

- Replace the Connectives Conditional & Biconditional in the given Statement formula, by their equivalent formulas only ^, V, ~
- Apply negations to the variables by using. Demorgan's Law, followed by Distributive Law.
- Any elementary product which is a contradiction is dropped.
- Minterms are obtained in the disjunction by introducing missing factors through the Complement Law (PV~P = T) and then apply Distributive Law.

- Identical minterms appearing in the disjunctions are then deleted.

Steps for finding PCNF:

- Replace the Connectives Conditional & Biconditional in the given Statement formula, by their equivalent formulas only ^, V, ~
- Apply negations to the variables by using. Demorgan's Law, followed by Distributive Law.
- Any elementary sum which is a tautology is dropped.
- Maxterms are obtained in the conjunction by introducing missing factors through the Complement Law (PV~P = T) and then apply Distributive Law.
- Identical minterms appearing in the disjunctions are then deleted.

PDNF of $P \rightarrow ((P \rightarrow Q) \wedge \sim(\sim Q \vee P))$

$P \rightarrow ((\sim P \vee Q) \wedge (Q \wedge \sim P))$

$\sim P \vee ((\sim P \wedge (Q \wedge \sim P)) \vee (Q \wedge Q \wedge \sim P))$

$\sim P \vee ((\sim P \wedge Q) \vee (\sim P \wedge Q))$

$\sim P \vee (\sim P \wedge Q)$

$(\sim P \wedge Q) \vee (\sim P \wedge \sim Q) \vee (\sim P \wedge Q)$

$(\sim P \wedge Q) \vee (\sim P \wedge \sim Q)$

$\sim P = \sim P \wedge (Q \vee \sim Q)$

$= (\sim P \wedge Q) \vee (\sim P \wedge \sim Q)$

9. Show the following implication without constructing the truth table: $(P \to Q) \to Q \quad \Rightarrow \quad P \vee Q$

Sol:

9) $(P \to Q) \to Q \equiv P \vee Q$

$(\sim P \vee Q) \to Q$

$\sim(\sim P \vee Q) \vee Q$

$(P \wedge \sim Q) \vee Q$ $\quad$ ∵ Distributive law

$(P \vee Q) \wedge (Q \vee \sim Q)$

$(P \vee Q) \wedge T$ $\quad$ (∵ Negation law)

$P \vee Q$. $\quad$ (∵ Identity law)

10. Explain the validity of the argument:

$[(P \rightarrow Q) \wedge (\sim R \vee S) \wedge (P \vee R)] \rightarrow [\sim Q \rightarrow S]$ Using the rule of contradiction.

Sol:

$(P \rightarrow Q) \wedge (\sim R \vee S) \wedge (P \vee R)$

1) $P \rightarrow Q \equiv \sim P \vee Q$ Rule-P
2) $P \vee R$ rule-P
3) $Q \vee R$ rule-T Resolution (1), (2)
4) $\sim R \vee S$ rule-P
5) $Q \vee S$ rule-T resolution (3), (4)

$Q \vee S \equiv \sim Q \rightarrow S.$

DMS

Module-1

PART-B

1.Define conditional proposition and logical equivalence with suitable examples.

Sol:

Sol: <u>Conditional</u> <u>proposition</u>: If p and q are statement variables, the compound proposition of p and q connected with '$\rightarrow$' is called a conditional proposition. The conditional Proposition is false, when p is true and q is false, otherwise it is true.

<u>Truth</u> <u>table</u> <u>for</u> <u>conditional</u>:

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

<u>Examples</u>:

1) If a polygon has exactly four sides, then it is a quadrilateral
2) If two angles are adjacent, then they have a common side
3) If you get good grades, then you will get into good college

Logical Equivalence:-

Two logical expressions are said to be logically equivalent, if they have the truth values for each possible substitution for their proposition variables.

⇒ The logical equivalence of statements p and q is denoted by $p \equiv q$.

Examples:-

The following statements are logically equivalent:

a) If lisa is in denmark, then she is in Europe $(D \Rightarrow E)$

b) If lisa is not in Europe, then she is not in Denmark $(\sim E \Rightarrow \sim D)$

2. a.Explain the term tautology? Show that [(p→q)→r]→[(p→q)→(p→r)] is tautology.

Sol:

Sol:- Let, $[(p \to q) \to r] \to [(p \to q) \to (p \to r)] = x$

p	q	r	$(p \to q)$	$(p \to r)$	$(p \to q) \to r$	$(p \to q) \to (p \to r)$	x
0	0	0	1	1	0	1	1
0	0	1	1	1	1	1	1
0	1	0	1	1	0	1	1
0	1	1	1	1	1	1	1
1	0	0	0	0	1	1	1
1	0	1	0	1	1	1	1
1	1	0	1	0	0	1	1
1	1	1	1	1	1	1	1

b. Define the converse, inverse and contrapositive of the following propositions: I. P → (Q →R) II. (PΛ (P → Q)) → Q.

Sol:

b) 1) $P \rightarrow (Q \rightarrow R)$

Converse: $(Q \rightarrow R) \rightarrow P$

Contrapositive: $[\sim(q \rightarrow r) \rightarrow (\sim p)]$

Inverse: $[(\sim P) \rightarrow \sim(q \rightarrow r)]$

2) $(P \wedge (P \rightarrow Q)) \rightarrow Q$

Converse: $Q \rightarrow (P \wedge (P \rightarrow Q))$

Contrapositive: $[(\sim Q) \rightarrow ((\sim P) \vee \sim(P \rightarrow Q))]$

Inverse: $[((\sim P) \vee \sim(P \rightarrow Q)) \rightarrow (\sim Q)]$

3. Show that $S \vee R$ is a tautologically implied by

$(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow s)$

Sol:

$(P \vee Q) \wedge [(P \rightarrow R) \wedge (Q \rightarrow S)]$

$\Rightarrow (\sim P \rightarrow Q) \wedge [(Q \rightarrow S) \wedge (P \rightarrow R)]$ ($\because p \rightarrow q = \sim p \vee q$)

($\because$ Commutative law) $P \wedge q \equiv q \wedge P$

$\Rightarrow [(\sim P \rightarrow Q) \wedge (Q \rightarrow S)] \wedge (P \rightarrow R)$ ($\because$ Associative law $\Rightarrow P \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$)

$\Rightarrow (\sim P \rightarrow S) \wedge (P \rightarrow R)$ ($\because$ Hypothetical Syllogism)

$$\begin{array}{c} P \rightarrow Q \\ Q \rightarrow S \\ \hline (P \rightarrow S) \end{array}$$

$\Rightarrow (P \vee S) \wedge (\sim P \vee R)$

$\Rightarrow (S \vee R)$ ($\because$ Resolution rule)

4. Show that $R \vee S$ is valid conclusion from the premises:

$$C \vee D, (C \vee D) \rightarrow \sim H), \sim H \rightarrow (A\ \mathrm{A}\!\sim B), (A\ \mathrm{A}\!\sim B) \rightarrow R \vee S$$

Sol:

Sol: $(C \vee D)$, $(C \vee D) \rightarrow \sim H$, $\sim H \rightarrow (A \wedge \sim B)$, $(A \wedge \sim B) \rightarrow (R \vee S)$

1) $C \vee D$ Rule – P

2) $(C \vee D) \rightarrow \sim H$ Rule – P

3) $\sim H$ Rule – T ($\because$ using Modus ponens)

4) $\sim H \rightarrow (A \wedge \sim B)$ Rule – P

5) $A \wedge \sim B$ Rule – T ($\because$ using modus ponens)

6) $(A \wedge \sim B) \rightarrow (R \vee S)$ Rule – P

7) $R \vee S$ Rule – T ($\because$ using modus ponens)

5. a) Prove that i) $\sim(\boldsymbol{P}\ \mathrm{t}\ \boldsymbol{Q}) \Rightarrow (\sim \boldsymbol{P} \downarrow \sim \boldsymbol{Q})$ $\boldsymbol{ii})$ $\sim(\boldsymbol{P} \downarrow \boldsymbol{Q}) \Rightarrow$ $(\sim \boldsymbol{P}\ \mathrm{t} \sim \boldsymbol{Q})$ **Without using truth table.**

Sol:

a) i) Prove that $\sim(P \uparrow Q) \equiv \sim P \downarrow \sim Q$

Sol:

$$\sim(P \uparrow q) \Leftrightarrow \sim(\sim(P \wedge q))$$

$$\sim(\sim P \vee \sim q)$$

$$(\sim P) \downarrow (\sim q) \qquad (\because \text{NOR} = \sim(P \vee q))$$

ii) Prove that $\sim(P \downarrow Q) \equiv \sim P \uparrow \sim Q$

$$\sim(P \downarrow q) \Leftrightarrow \sim(\sim(P \vee q))$$

$$\sim(\sim P \wedge \sim q)$$

$$(\sim P) \uparrow (\sim q) \qquad (\because \text{NAND} = \sim(P \wedge q))$$

b)Express p→(~p→q) i)in terms of '↑' only. ii)in terms of '↓' only.

Sol:

b) Express $p \to (\sim p \to q)$

i) In terms of $\uparrow$

$p \to (p \vee q)$

$p \to \sim(\sim(p \vee q))$

$p \to \sim(\sim(p \vee q))$

$p \to \sim(\sim p \wedge \sim q)$

$p \to (\sim p \uparrow \sim q)$

$\sim p \vee (\sim p \uparrow \sim q)$

$\sim p \wedge \sim(\sim p \uparrow \sim q))$

$p \uparrow \sim(\sim p \uparrow \sim q)$

ii) In terms of $\downarrow$

$p \to (p \vee q)$

$\sim p \vee (p \vee q)$

$\sim(\sim(\sim p \vee (p \vee q)))$

$\sim(\sim p \downarrow (p \vee q))$

$\sim(\sim p \downarrow \sim(p \downarrow q))$

6. (a) Evaluate the proposition $(p \wedge q) \sim (p \vee q)$ is a contradiction?

Sol: $(p \wedge q) \wedge (\sim(p \vee q))$

$(p \wedge q) \wedge (\sim p \wedge \sim q)$

$(q \wedge p) \wedge (\sim p \wedge \sim q)$ ∵ Commutative law

$(q \wedge p \wedge \sim p) \wedge \sim q$ ∵ Associative law

$(q \wedge F) \wedge \sim q$ ∵ Negation or Inverse law

$F \wedge \sim q$ ∵ "

$\Rightarrow F$

Hence, the given proposition is a contradiction.

(b) Evaluate the following statements in symbolic form? i. all men are good ii. no men are good iii. some men are good iv. some men are not good

Sol:

Sol:- i) all men are good

$P(x)$: x is a man

$q(x)$: x is good

(x) [[redacted] $(x) \rightarrow q(x)$]

ii) No men are good

This can be written as

For all x, if x is a man, then x is not good

$$(x)\,[P(x) \rightarrow \sim(q(x))]$$

iii) Some men are good

$$\exists(x)\,[P(x) \wedge q(x)]$$

iv) Some men are not good

$$\exists(x)\,[P(x) \wedge \sim q(x)]$$

7. Demonstrate the disjunctive normal form of the formula:

$$P \to ((P \to Q) \wedge \sim(\sim Q \vee \sim P))?$$

Sol:

$P \to ((P \to Q) \wedge \sim(\sim Q \vee \sim P))$

$P \to ((P \to Q) \wedge (Q \wedge P))$

$P \to ((\sim P \vee Q) \wedge (Q \wedge P))$ $\quad$ ($\because P \to Q \equiv \sim P \vee Q$)

$P \to ((Q \wedge P) \wedge (\sim P \vee Q))$ $\quad$ ($\because$ Commutative law)

$P \to (((Q \wedge P) \wedge \sim P) \vee ((Q \wedge P) \wedge Q))$ $\quad$ ($\because$ Associative law)

$\sim P \vee ((Q \wedge F) \vee (P \wedge Q \wedge Q))$ $\quad$ ($\because$ Domination and Idempotent law)

$\sim P \vee ((Q \wedge F) \vee (P \wedge Q))$

$\sim P \vee (F \vee (P \wedge Q))$ $\quad$ ($\because$ Idempotent law)

$\sim P \vee (P \wedge Q)$ $\quad$ ($\because$ Identity law)

$(\sim P \wedge Q) \vee (\sim P \wedge \sim Q) \vee (P \wedge Q)$

$\sim P = \sim P \wedge (Q \vee \sim Q)$

$= (\sim P \wedge Q) \vee (\sim P \wedge \sim Q)$

8. Define free and bound variables with an example?

Sol: Free variables: A **free variable** is a variable that has no limitations. It can represent whatever number it needs to represent.

For example, the *x* in this function is a free variable:

- $f(x) = 3x - 1$

Bound Variables: A **bound variable**, on the other hand, is a variable with limitations. A bound variable can't represent whatever number you need it to. Instead, its possible values have already been specified.

An example of a bound variable is this one:

$$\sum_{x=1}^{4} x + 4$$

9. (a) Show that if " m is an even integer then m+7 is an odd integer" by using direct proof?

(b) Show each of the following in symbolic form

i.all monkeys have tails

ii. no monkey has tail   iii. Some monkey has tails

iv. some monkey has no tails

Sol: a)

if m is an even integer, then $m+7$ is an odd integer

So, $m = 2k$, where k is an integer.

$m+7 = 2k+7$

$\Rightarrow 2(k+3)+1$

This is in the form of $2n+1$

So, we can say that $m+7$ is odd

b)

i) all monkeys have tails.

$P(x)$: x is a monkey

$Q(x)$: x has a tail

$(x)\,[P(x) \rightarrow Q(x)]$

ii) No monkey has tail

$(x)\,[P(x) \rightarrow \sim(Q(x))]$

iii) Some monkey has tails

$\exists(x)\,[P(x) \wedge Q(x)]$

iv) Some men are not good

$\exists(x)\,[P(x) \wedge \sim Q(x)]$

10. Demonstrate proof by contradiction with example.

Sol:

1. Proof by contradiction: This method is based on the fact that a statement x can only be true or false (and not both). The idea is to prove that the Proposition x is true by showing that it cannot be false. This is done by assuming that x is false, and proving that this leads to a contradiction. when this happens, we can conclude that the assumption x is false is incorrect. so x is true.

Ex: A classic proof by contradiction from mathematics is the proof that $\sqrt{2}$ is irrational.

Let us consider that $\sqrt{2}$ is a rational number

So we can write $\sqrt{2} = \frac{a}{b}$; where a/b is an irreducible fraction.

squaring on B.S

$$2 = \frac{a^2}{b^2}$$

$$a^2 = 2b^2 \rightarrow ①$$

we can see a is even, since it is twice of some number.

so, consider $a = 2k$; k is a whole number $\rightarrow ②$

Substitute eq-② in ①

$$(2k)^2 = 2b^2.$$

11. i. Explain the direct proof of the statement "The square of an odd integer is an odd integer".

ii. Explain the indirect proof of the statement. "If n2 is odd, then n is odd"

Sol: i)

Sol The square of an odd integer is an odd integer

Let n be any odd integer, so

$n = 2k + 1$; where k is an integer

Squaring on B.S

$n^2 = (2k+1)^2$

$n^2 = 4k^2 + 4k + 1$

$n^2 = 2(2k^2 + 2k) + 1$

Here n^2 is not divisible by 2

Hence, the square of an odd integer is an odd integer.

ii)

If n^2 is odd, then n is odd.

⇒ Indirectly we can prove that n is not odd if n^2 is not odd.

So, we need to prove, if n is even, then n^2 is even.

Let n be an even integer

$n = 2k$

$n^2 = 4k^2$ (∵ sq on B.S)

$n^2 = 2(2k^2)$

∴ n^2 is an even integer.

Hence, indirectly we can say if n^2 is odd, then n is odd.

12. Define the converse for the statement "If a quadrilateral is a parallelogram, then its diagonals bisect each other".

Sol:

Given statement,

⇒ if a quadrilateral is a parallogram, then it's diagonals bisect each other

Let, P : A quadrilateral is a parallogram.

q : Diagonals of a quadrilateral. bisect each other.

the compound proposition for the statement is

$P \rightarrow q$ and converse is $q \rightarrow P$.

$q \rightarrow p$: If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallogram.

13. Define the inverse for the statement "If a triangle is not isosceles, then it is not equilateral".

Sol:

Given statement,

If a triangle is not isosceles, then it is not equilateral.

Let, p: A triangle is ~~not~~ isosceles

q: A triangle is equilateral

$\sim p \rightarrow \sim q \Rightarrow$ Compound statement

Inverse: $p \rightarrow q$

$p \rightarrow q$: If a triangle is isosceles, then it is equilateral.

14. What is principal disjunctive normal form for

$$(\boldsymbol{p} \wedge \boldsymbol{q}) \vee (\sim \boldsymbol{p} \wedge \boldsymbol{r})\ \boldsymbol{V}\ (\boldsymbol{q} \wedge \boldsymbol{r})?$$

Sol:

$$(P\wedge q) \vee (\sim P\wedge r) \vee (q\wedge r)$$

$$(P\wedge q) = (P\wedge q) \wedge (R\vee \sim R)$$

$$= (P\wedge q\wedge r) \vee (P\wedge q\wedge \sim r)$$

$$(\sim P\wedge r) = (\sim P\wedge r) \wedge (Q\vee \sim Q)$$

$$= (\sim P\wedge Q\wedge r) \vee (\sim P\wedge \sim Q\wedge r)$$

$$(q\wedge r) = (q\wedge r) \wedge (P\vee \sim P)$$

$$= (P\wedge q\wedge r) \vee (\sim P\wedge q\wedge r)$$

$$(P\wedge q)\vee(\sim P\wedge r)\vee(q\wedge r) \equiv (P\wedge q\wedge r)\vee(P\wedge q\wedge \sim r)\vee(\sim P\wedge q\wedge r)\vee (P\wedge q\wedge r)\vee(\sim P\wedge \sim q\wedge r)\vee(\sim P\wedge q\wedge r)$$

$$(P\wedge q)\vee(\sim P\wedge r)\vee(q\wedge r) \equiv (P\wedge q\wedge r)\vee(P\wedge q\wedge \sim r)\vee(\sim P\wedge q\wedge r)\vee(\sim P\wedge \sim q\wedge r)$$

15. What is PCNF of $(P \vee R) \wedge (P \vee\sim Q)$ Also find its PDNF, with using truth table.

Sol:

$$(P\vee R) \wedge (P\vee \sim Q)$$

$$(P\vee R) = (P\vee R) \vee (Q\wedge \sim Q)$$

$$= (P\vee Q\vee R) \wedge (P\vee \sim Q\vee R)$$

$$(P\vee \sim Q) = (P\vee \sim Q) \vee (R\wedge \sim R)$$

$$= (P\vee \sim Q\vee R) \wedge (P\vee \sim Q\vee \sim R)$$

$$(P\vee R)\wedge(P\vee \sim Q) = (P\vee Q\vee R)\wedge(P\vee \sim Q\vee R)\wedge(P\vee \sim Q\vee R)\wedge(P\vee \sim Q\vee \sim R)$$

$$= (P\vee Q\vee R)\wedge(P\vee \sim Q\vee R)\wedge(P\vee \sim Q\vee \sim R)$$

PDNF

p	q	r	$\sim q$	$(p\vee r)$	$(p\vee \sim q)$	$(p\vee r)\wedge(p\vee \sim q)$	
0	0	0	1	0	1	0	
0	0	1	1	1	1	1	$-(\sim p\wedge \sim q\wedge r)$
0	1	0	0	0	0	0	
0	1	1	0	1	0	0	
1	0	0			1		$-(p\wedge \sim q\wedge \sim r)$
1	0	1	1	1	1	1	$-(p\wedge \sim q\wedge r)$
1	1	0	0	1	1	1	$-(p\wedge q\wedge \sim r)$
1	1	1	0	1	1	1	$-(p\wedge q\wedge r)$

PDNF: $(p\wedge q\wedge r)\vee(p\wedge q\wedge \sim r)\vee(p\wedge \sim q\wedge \sim r)\vee(\sim p\wedge \sim q\wedge r)\vee(p\wedge \sim q\wedge r)$

16. Show that the following argument is valid: p→ q, r→ q, r ⇒p

Sol:

$p\to q,\ r\to q,\ r$

$\therefore p$

1) r — rule - P

2) $r\to q$ — rule - P

3) q — rule - T (Modus ponens)

4) $p\to q$ — rule - P

5) p — rule - T (∵ Modus ponens)

∴ The given argument is valid

17. Show that the hypothesis, "It is not sunny this afternoon and it is colder than yesterday", "we will go swimming only if it is sunny", "If we do not go swimming, then we will take a canoe trip" and "If we take a canoe trip, then we will be home by sunset" lead to the conclusion "We will be home by sunset"

Sol:

p: It is sunny this afternoon

q: We will go swimming

r: It is colder than yesterday

s: We will take a canoe trip.

Conclusion: We will be home by sunset : t

Hypothesis: $(\sim p \wedge q)$, $r \rightarrow p$, $\sim r \rightarrow s$, $s \rightarrow t$.

1) $\sim p \wedge q$ rule-P

2) $\sim p$ rule-T (∵ using simplification rule)

3) $r \to p$ rule-P

4) $\sim r$ rule-T (∵ Modus tollens)

5) $\sim r \to s$ rule-P

6) s rule-T (∵ Modus ponens)

7) $s \to t$ rule-P

8) t rule-T (∵ Modus ponens)

18. Determine the validity of the following argument: If 7 is less than 4, then 7 is not a prime number, 7 is not less than 4. Therefore 7 is a prime number.

Sol:

p: 7 is less than 4

q: 7 is a prime number

Hypothesis: $p \to \sim q$, $\sim p$

Conclusion: q

p	q	$\sim p$	$\sim q$	$p \to \sim q$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

19. Apply rules of inferences to obtain the conclusion of the following arguments: "Babu is a student in this class, knows how to write programs in JAVA". "Everyone who knows to write programs in JAVA can get a high-paying job".Therefore, "someone in this class can get a high-paying job".

Sol:

P(x) : x knows java

q(x) : x gets high paying jobs

a: Babu.

$(x)(P(x) \rightarrow q(x))$, $(\exists x)(P(x))$

Conclusion : $\exists(x)\, q(x)$

1) $(\exists x)\ P(x)$ · rule - P

2) $P(a)$ removing Existential quantifier

3) $(x)(P(x) \rightarrow q(x))$ rule - P

4) $P(a) \rightarrow q(a)$ removing universal quantifier

5) $q(a)$ rule - T (∵ modus ponens 2 & 4)

6) $\exists(x)\ q(x)$ combining with existential quantifier

20. Use the indirect method to prove that the conclusion $\exists z \boldsymbol{Q}(z)$ follows form the premises $\forall x\, (\boldsymbol{P}(x) \rightarrow \boldsymbol{Q}(x))$and $\exists y \boldsymbol{P}(y)$

Sol:

Sol:- The given Conclusion is $\exists z\, Q(z)$

Negation of the conclusion can be taken as additional Premise

$$\sim(\exists z\, Q(z))$$

this additional premise with given premises derives a contra-diction $(P \wedge \sim P)$

Given premises,

$$\forall x\, (P(x) \rightarrow Q(x)),\ \exists y\, (P(y)),\ \sim(\exists z\, Q(z))$$

1) $\sim(\exists x\, Q(x))$ Rule-P (Additional premise)

2) $\forall z\, (\sim Q(z))$ Rule-T (Negation law)

3) $\forall x\, (P(x) \rightarrow Q(x))$ Rule-P

4) $P(a) \rightarrow Q(a)$ Rule-US -3

5) $\sim Q(a)$ Rule-US - (2)

6) $\sim P(a)$ Rule-T (Modus tollens of (4) and (5)

7) $\exists y \, P(y)$ Rule-P

8) $P(a)$ Rule-US - (7)

9) $P(a) \wedge (\sim(P(a)))$ Rule-T (~~Identity rule~~) Conjunction rule

10) F contradiction

∴ The given premises are inconsistent

∴ The conclusion is valid.

DMS

Module-1

PART-C

1. Define statement and atomic statement.

Sol: Statement: statement is any declarative sentence which is either true or false

Atomic statement: In logic, a statement which cannot be broken down into smaller statements, also simply called an atomic statement

2. Explain logical equivalence with an example.

Sol: Logical Equivalence: Two statement forms are called **logically equivalent if**, and only if, they have identical truth values for each possible substitution for their proposition.

Ex: The following statements are logically equivalent:

1. If Lisa is in Denmark, then she is in Europe
2. If Lisa is not in Europe, then she is not in Denmark

3. Define tautology.

Sol: A tautology is a logical statement in which the conclusion is equivalent to the premise. More colloquially, it is formula in propositional calculus which is always true

4. Write the converse, inverse and contra positive for the following Proposition : P →(Q →R).

Sol:

b) 1) $P \rightarrow (Q \rightarrow R)$

Converse: $(Q \rightarrow R) \rightarrow P$

Contrapositive: $[\sim(q \rightarrow r) \rightarrow (\sim p)]$

Inverse: $[(\sim p) \rightarrow \sim(q \rightarrow r)$

5. Illustrate NAND and NOR with examples.

Sol: NAND: **Logical NAND** (“Not And”) is an operation on two logical values, typically the values of two propositions, that produces a value of *false* if and only if both of its operands are true. In other words, it produces a value of *true* if and only if at least one of its operands is false.

NOR:

A predicate in logic equivalent to the composition NOT OR that yields false if any condition is true, and true if all conditions are false.

Write your own examples

6. **Demonstrate conditional and biconditional statements**

Sol: Conditional Statement:

Let p and q are two statements then "if p then q" is a compound statement, denoted by p→ q and referred as a conditional statement, or implication. The implication p→ q is false only when p is true, and q is false; otherwise, it is always true.

Biconditional Statement:

If p and q are two statements then "p if and only if q" is a compound statement, denoted as p ↔ q and referred as a biconditional statement or an equivalence. The equivalence p ↔ q is true only when both p and q are true or when both p and q are false.

7.Define contradiction.

Sol: Contradiction:

A statement that is always false is known as a contradiction.

Example: Show that the statement p A~p is a contradiction.

Solution:

p	~p	p A~p
T	F	F
F	T	F

8. Recall the definition for contradiction and provide a proof by contradiction of the following statement: For every integer n, " if n^2 is odd then n is odd."

Sol:

If n^2 is odd, then n is odd.

$\Rightarrow$ Indirectly we can prove that n is not odd if n^2 is not odd.

So, we need to prove, if n is even, then n^2 is even.

Let n be an even integer

$$n = 2k$$

$$n^2 = 4k^2 \quad (\because \text{sq on BS})$$

$$n^2 = 2(2k^2)$$

$\therefore$ n^2 is an even integer.

Hence, indirectly we can say if n^2 is odd, then n is odd

9. Define converse, contra-positive and inverse of implication

Sol:

- The converse of the conditional statement is "If Q then P", $(Q \rightarrow P)$
- The contrapositive of the conditional statement is "If not Q then not P", $(\sim Q \rightarrow \sim P)$

- The inverse of the conditional statement is "If not P then not Q", $(\sim P \rightarrow \sim Q)$

10. Translate the following statements in to symbolic form: a) all men are good, b) no men are good

Sol:

i) all men are good

$P(x)$: x is a man

$q(x)$: x is good

$(x)\,[P(x) \rightarrow Q(x)]$

ii) No men are good

This can be written as

For all x, if x is a man, then x is not good

$(x)\,[P(x) \rightarrow \sim(q(x))]$

11. Write the disjunctive normal form of the formula: P↔Q.

Sol:

$p \leftrightarrow q$

$(p \to q) \wedge (q \to p)$

$(\sim p \vee q) \wedge (\sim q \vee p)$

$((\sim p \vee q) \wedge (\sim q)) \vee ((\sim p \vee q) \wedge p)$ (∵ Distributive law)

$((\sim p \wedge \sim q) \vee (q \wedge \sim q)) \vee ((\sim p \wedge p) \vee (q \wedge p))$
(∵ Distributive law)

$((\sim p \wedge \sim q) \vee F) \vee (F \vee (q \wedge p))$ (∵ Negation law)

$(\sim p \wedge \sim q) \vee (q \wedge p)$. (∵ Identity law)

12. Show the value of: P↔Q in terms of {~, ∨} only.

Sol:

$p \leftrightarrow q$

$(p \to q) \wedge (q \to p)$

$(\sim p \vee q) \wedge (\sim q \vee p)$

$((\sim p \vee q) \wedge (\sim q)) \vee ((\sim p \vee q) \wedge p)$ (∵ Distributive law)

$((\sim p \wedge \sim q) \vee (q \wedge \sim q)) \vee ((\sim p \wedge p) \vee (q \wedge p))$
(∵ Distributive law)

$((\sim p \wedge \sim q) \vee F) \vee (F \vee (q \wedge p))$ (∵ Negation law)

$(\sim p \wedge \sim q) \vee (q \wedge p)$. (∵ Identity law)

$[\sim (p \vee q) \vee \sim(\sim p \vee \sim q)]$

13. Define free and bound variables.

Sol: Free variables: A **free variable** is a variable that has no limitations. It can represent whatever number it needs to represent.

For example, the *x* in this function is a free variable:

- $f(x) = 3x - 1$

Bound Variables: A **bound variable**, on the other hand, is a variable with limitations. A bound variable can't represent whatever number you need it to. Instead, its possible values have already been specified.

An example of a bound variable is this one:

$$\sum_{x=1}^{4} x + 4$$

14. Explain about the statement "if m is an even integer then m+7 is an odd integer" by indirect proof.

Sol:

if m is an even integer, then $m+7$ is an odd integer

So, $m = 2k$, where k is an integer.

$m+7 = 2k+7$

$\Rightarrow 2(k+3)+1$

This is in the form of $2n+1$

So, we can say that $m+7$ is odd

15. Find the truth table for conjunction and conditional statements?

Sol:

Conjunction:

p	q	p∧q
T	T	T
T	F	F
F	T	F
F	F	F

Conditional:

p	q	p→q
T	T	T
T	F	F
F	T	T
F	F	T

16. Find the truth table for p→(q→r)?

Sol:

$p \rightarrow (q \rightarrow r)$

p	q	r	$(q \rightarrow r)$	$p \rightarrow (q \rightarrow r)$
T	T	T	T	T
T	T	F	F	F
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	F	T
F	F	T	T	T
F	F	F	T	T

17. show whether $p \vee [\sim (p \wedge q)]$ is tautology or not.

Sol:

p	q	$(p \wedge q)$	$\sim(p \wedge q)$	$p \vee [\sim(p \wedge q)]$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

$\therefore$ $p \vee [\sim(p \wedge q)]$ is tautology.

18. R: Mark is rich. H:Mark is happy Translate the statements into symbolic form a) mark is poor but happy b) mark is poor but not happy

Sol:

a) ~R^H **b)** ~R^~H

19. Translate the following statement into symbolic form: "the crop will be destroyed if there is a flood".

Sol: P → Q ≡ if there is a flood then the crops will be destroyed

20. show whether $((p \vee q) \vee \sim p)$ a tautology or not.

Sol:

$(P \vee q) \vee (\sim P)$

P	q	$\sim P$	$P \vee q$	$(P \vee q) \vee (\sim P)$
T	T	F	T	T
T	F	F	T	T
F	T	T	T	T
F	F	T	F	T

$\therefore (P \vee q) \vee (\sim P)$ is tautology