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Module - II

RANDOM VARIABLES

In the study of random variables we are usually interested in their probability distribution namely, in the probabilities with which they take on the various values in their range.

Random variable

A random variable 'X' on a sample space 'S' is function from S into the set R of real numbers such that the preimage of every element of R is an event of S.

- A random variable is a function that assigns a real number to each sample point ~~into~~ in the sample space of a random experiment (or)
- A real valued function 'X' defined on a sample space S of a random experiment ~~that~~ is i.e, $X : S \to R$ is called a random variable.

Ex:- In the experiment of tossing two coins

$S = \{HH, HT, TH, TT\}$

consider X = No. of heads

$X(HH) = 2$

$X(HT) = X(TH) = 1$

$X(TT) = 0$

$X : S \to R \Rightarrow X : \{HH, HT, TH, TT\} \to \{0, 1, 2\}$

Types of random variables

There are two types of random variables

1. Discrete random variables
2. continuous random variables

① Discrete random variable

A random variable which can take on only a finite number is called discrete random variable
- Countable

(or)

A discrete random variable is a random variable with a finite or countable infinite range ~~with a finite~~(or) quantities which are capable of taking integral values

Ex:- (i) the no. of children in the family
(ii) No. of printing mistakes in each page of book

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② Continuous random variable

A random variable 'x' is said to be continuous if it can take all possible values in an interval.

- A continuous random variable is a random variable that can be measured to any desired degree of accuracy.

Ex:- Age, weight, height etc.
- The weight of the student of a class
- The height of a child as it grows.

Example

② Suppose two coins are tossed simultaneously, then sample space is

$$S = \{HH, HT, TH, TT\}$$

Let 'x' ~~the~~ denotes the heads then,

if $x = 0 \quad = \{TT\}$

$P(x=0) = \frac{1}{4}$

if $x = 1 \quad = \{HT, TH\}$

$P(x=1) = \frac{2}{4} = \frac{1}{2}$

if $x = 2 \quad = \{HH\}$

$P(x=2) = \frac{1}{4}$

NOTE:

Usually discrete random variable gives or generate by counting numbers $\{1, 2, 3 - - -\}$

- Continuous random variables generated by measuring problem $\{$value in $[1, 2]\}$

Probability distribution function of a

~~If of~~ discrete random variable

For a discrete random variable 'X', the real valued function such that

$$\boxed{P(X = x) = P(x)}$$

is called probability function of a discrete random variable.

- Probability function $p(x)$ gives the measure of probability for different values of x.

- Properties of a probability function

1. $P(x) \geq 0$ for all x
2. $\Sigma P(x) = 1$
3. $P(x)$ can not be negative for any value of x.

Probability distribution function

Let x be a random variable the probability distribution function associated with x is defined as the probability that the outcome of an experiment will be one of the outcome for which $X(s) \leq x$ for all $x \in R$

i.e, the function $f(x)$ of a random variable X is defined by $\boxed{F_X(x) = P(X \leq x) = P\{s ; X(s) \leq x\}}$

is called distribution function of X.

Discrete probability ~~funct~~ distribution

Probability distribution of a random variable is the set of its possible values together with their respective probabilities.

Suppose 'X' is a discrete random variable with possible outcomes $x_1, x_2, x_3, ---$.

the probability of each outcome $P(X = x_i) = P(x_i)$ for $i = 1, 2, 3 ---$

If $P(x_i)$ satisfy the two conditions

(i) $P(x_i) > 0$ for all values of i

(ii) $\Sigma P(x_i) = 1$

then the function $P(X)$ is called the probability mass function of a random variable X and the set of values $\{P(x_i)\}$ is called the discrete probability distribution of discrete random variable.

- the probability distribution of random variable x is given by

probability distribution table

x	1	2	3	4	5	---	n
$p(x)$	$p(1)$	$p(2)$	$p(3)$	$p(4)$	$p(5)$	---	$p(n)$

Ex:- Two dies are thrown. Let X assigned to each point A,B in S. The maximum of its numbers i.e, $X(A,B) = \max(a,b)$

Find the probability distribution.

$X = \{1, 2, 3, 4, 5, 6\}$

$P(x=1) = (1,1) = \frac{1}{36}$

$P(x=2) = (1,2), (2,1), (2,2) = \frac{3}{36}$

$P(x=3) = (1,3), (3,1), (2,3), (3,2), (3,3) = \frac{5}{36}$

$p(X=4) = (1,4),(2,4),(3,4),(4,4),(4,3),(4,2),(4,1)$

$= \frac{7}{36}$

$P(X=5) = (1,5),(2,5),(3,5),(4,5),(5,5),(5,4),(5,3)$
$(5,2),(5,1)$

$= \frac{9}{36}$

$P(X=6) = (1,6),(2,6),(3,6),(4,6),(5,6),(6,6),(6,5)$
$(6,4),(6,3),(6,2),(6,1)$

$= \frac{11}{36}$

distribution table

X	1	2	3	4	5	6
p (x)	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$

Mathematical expectation

Let X be a discrete random variable taking value $x = 0, 1, 2, ---$ then the mathematical expectation of X is denoted by $E(X)$ and is defined as $E(X) = \sum x \, P(X=x)$

$$= \sum x_i p(x_i)$$

$$E(X) = \sum x_i p_i$$

similarly $\boxed{E(X^r) = \sum p_i x_i^r}$

In general the expected value of function $g(x)$ of a random variable X is given by

$$E[g(x)] = \sum p_i g(x)$$

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→ The behaviour of a random variable is completely characterized by distribution function $E(x)$

→ Instead of function a more compact description can be made by a. ________

→ Expectation place a very important role in decision making, because most of the time we take decision making based on what is expected to happen,

Expectation of a discrete variable (Repeated)

Suppose a random variable 'x' assumes the value $x_1, x_2, \ldots x_n$ with respect to probabilities $p_1, p_2, \ldots p_n$ then the mathematical expectation or mean or expected value of X is denoted by $E(x)$. Therefore

$$\boxed{E(x) = x_1 p_1 + x_2 p_2 + \cdots + x_n p_n}$$

$$= \sum_{i=1}^{n} x_i p_i$$

Similarly $E(x^r) = \sum_{i=1}^{n} p_i x_i^r$

Mean

The mean value 'μ' of discrete distribution function is given by

$$\mu = \frac{\sum p_i x_i}{\sum p_i}$$

$$\Rightarrow \boxed{\mu = \Sigma p_i x_i = E(X)} \quad (\because \Sigma p_i = 1)$$

E(x) is also called mean or arithmetic mean of 'x' denoted by μ.

NOTE: $\mu = E(x)$

> If $E(x) = \mu$ then $E(x) - \mu = 0$

Variance

Variance 'σ^2' of discrete distribution function is given by ~~P(x)~~ =

$$var(x) = V(x) = \sigma^2 = E(x^2) - (E(x))^2$$

$$\boxed{\sigma^2 = E(x^2) - \mu^2}$$

Standard deviation

The positive square root of the variance is defined as standard deviation. And it is denoted by σ.

$$S.D = \sigma = \sqrt{E(x^2) - \mu^2}$$

$$\boxed{\sigma = \sqrt{\Sigma p_i x_i^2 - \mu^2}}$$

Q:- Let x denoted the number of heads in a single toss of 4 fair coins. Determine (i) $P(x < 2)$
(ii) $P(1 < x \leq 3)$

Sol:-
x = no. of heads

X	0	1	2	3	4
p(x)	$\frac{1}{16}$	$\frac{4}{16}$	$\frac{6}{16}$	$\frac{4}{16}$	$\frac{1}{16}$

Total no. of possibilities $= 2^n = 2^4 = 16$

$P(X=0) = \{TTTT\}$ ~~TT, TT~~ }

$P(X=0) = \frac{1}{16} = \frac{1}{16}$

$P(X=1) = \{HTTT, HHTT, TTHT, TTTH\}$

$P(X=1) = \frac{4}{16} = \frac{1}{4}$

$P(X=2) = \{HHTT, THHT, TTHH, HTHT, THTH, HTTH\}$

$= \frac{6}{16} = \frac{3}{8}$

$P(X=3) = \{HHHT, THHH, HTHH\}\ HHTH\}$

$P(X=3) = \frac{4}{16}$

$P(X=4) = \{HHHH\}$

$= \frac{1}{16}$

(i) $P(X<2) = P(0) + P(1)$

$= \frac{1}{16} + \frac{4}{16} = \frac{5}{16}$

(ii) $P(1<X\leq 3) = P(X=2) + P(X=3)$

$= \frac{6}{16} + \frac{4}{16} = \frac{10}{16} = \frac{5}{8}$

1) Two dice are thrown. Let X assigned to each point A, B in S. The maximum of its numbers i.e. $X(A, B) = \max(a, b)$. $X = \{1, 2, 3, 4, 5, 6\}$. Find the probability distribution. Also find mean and variance and standard deviation of distribution.

X	1	2	3	4	5	6
p(x)	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$

(i) Mean $= \sum p_i x_i$

$$= 1\left(\frac{1}{36}\right) + 2\left(\frac{3}{36}\right) + 3\left(\frac{5}{36}\right) + 4\left(\frac{7}{36}\right) + 5\left(\frac{9}{36}\right) + 6\left(\frac{11}{36}\right)$$

$$= \frac{1}{36} + \frac{6}{36} + \frac{15}{36} + \frac{28}{36} + \frac{45}{36} + \frac{66}{36}$$

Mean $= \frac{161}{36}$

(ii) Variance $(\sigma^2) = E(x^2) - \mu^2$

$4 \quad 25 \quad 9 \quad 225 \qquad 4 \quad 16 \quad 7 \quad 112 \qquad 36 \quad 11 \quad 36 \quad 36 \quad 6$

$E(x^2) = \sum p_i x_i^2$

$$= \frac{1}{36}(1) + \frac{3}{36}(2^2) + \frac{5}{36}(3^2) + \frac{7}{36}(4^2) + \frac{9}{36}(5^2) + \frac{11}{36}(6^2)$$

$$= \frac{1}{36} + \frac{12}{36} + \frac{45}{36} + \frac{112}{36} + \frac{225}{36} + \frac{396}{36}$$

$$= \frac{791}{36}$$

variance $= E(x^2) - \mu^2$

$$= \frac{791}{36} - \left(\frac{161}{36}\right)^2$$

$\sigma^2 = 21.97 - 20.000$

$\sigma^2 = 1.97$

(iii) standard deviation $(\sigma) = \sqrt{1.97}$
$= 1.40$

② A random variable 'x' has the following probability function

x	0	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

Find (i) K (ii) $P(X < 6)$ (iii) $P(X \geq 6)$
(iv) $P(0 < X < 5)$ and $P(0 \leq X \leq 4)$
(v) If $P(X \leq k) > \frac{1}{2}$ find minimum value of k

sol:-
(i) Sum of the probability $(\Sigma p_i) = 1$

$0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$

$9k + 10k^2 = 1$

$10k^2 + 9k - 1 = 0$

$k = \frac{1}{10}, -1$

$\therefore \boxed{k = \frac{1}{10}}$

(vi) Determine the distribution function of X

(vii) calculate mean and variance

(vi)

x	0	1	2	3	4	5	6	7
$p(x)$	0	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{1}{100}$	$\frac{2}{100}$	$\frac{7}{100}+\frac{1}{10}$

L distribution function of X

(ii) $P(X<6) = P(0)+P(1)+P(2)+P(3)+P(4)+P(5)$

$= 0+\frac{1}{10}+\frac{2}{10}+\frac{2}{10}+\frac{3}{10}+\frac{1}{100}$

$= \frac{8}{10}+\frac{1}{100}$

$P(X<6) = \frac{81}{100}$

(iii) $P(X\geq 6) = P(X=6)+P(X=7)$

$= \frac{2}{100}+\frac{7}{100}+\frac{1}{10}$

$= \frac{9+10}{100} = \frac{19}{100}$

(iv) $P(0<X<5) = P(1)+P(2)+P(3)+P(4)$

$= \frac{1}{10}+\frac{2}{10}+\frac{2}{10}+\frac{3}{10} = \frac{8}{10}$

and $P(0\leq X\leq 4) = P(0)+P(1)+P(2)+P(3)+P(4)$

$= 0+\frac{1}{10}+\frac{2}{10}+\frac{2}{10}+\frac{3}{10}$

$= \frac{8}{10}$

(v) $P(X\leq K) > \frac{1}{2}$

$K=0$

$P(X\leq 0) = P(0) < \frac{1}{2}$

$= 0 < \frac{1}{2}$

$P(X\leq 1) = P(0)+P(1) = 0+0.1 = 1$

$P(X\leq 2) = P(0)+P(1)+P(2) = 0+0.1+0.2 = 0.3$

$P(\cancel{X}\leq 3) = P(0)+P(1)+P(2)+P(3) = 0.5$

$P(X \le 4) = P(0) + P(1) + P(2) + P(3) + P(4)$

$= 0 + 0.1 + 0.2 + 0.3 + 0.3$

$= 0.8$

Minimum value of $k = \cancel{d}\ p$

Mean $(\mu) = \Sigma p_i x_i$

$= 0 + 1\left(\frac{1}{10}\right) + 2\left(\frac{2}{10}\right) + 3\left(\frac{3}{10}\right) + 4\left(\frac{3}{10}\right) +$

$5\left(\frac{1}{100}\right) + 6\left(\frac{2}{100}\right) + 7\left(\frac{7}{100} + \frac{1}{10}\right)$

$= \frac{1}{10} + \frac{4}{10} + \frac{6}{10} + \frac{12}{10} + \frac{5}{100} + \frac{12}{100} + \frac{49}{100} + \dots$

$= \frac{30}{10} + \frac{66}{100} = \frac{300+66}{100} = \frac{366}{100}$

variance $(\sigma^2) = E(x^2) - \mu^2$

$E(x^2) = \Sigma p_i x_i^2$

$= 0 + \frac{1}{10} + \frac{2}{10}(2^2) + \frac{2}{10}(3^2) + \frac{3}{10}(4^2) +$

$\frac{1}{100}(5^2) + \frac{2}{100}(6^2) + \left(\frac{7}{100} + \frac{1}{10}\right)7^2$

$= \frac{1}{10} + \frac{8}{10} + \frac{18}{10} + \frac{48}{10} + \frac{25}{100} + \frac{72}{100}$

$+ \frac{343}{100} + \frac{49}{10}$

$= \frac{124}{10} + \frac{440}{100} = \frac{1240+440}{100} = \frac{1680}{100}$

$$\sigma^2 = \frac{168}{10} - \left(\frac{366}{100}\right)^2$$

$$= 16.8 - 13.39 = \cancel{3.79}\ 3.41$$

$$\underline{\sigma^2 = 3.41}$$

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(3) find mean and variance of uniform probabili is given by $f(x) = 1/n$, for $x = 1, 2, 3, \ldots$

sol:-

x	1	2	3	...	n
$f(x)$	$1/n$	$1/n$	$1/n$	...	$1/n$

$$\text{Mean} = \Sigma p_i x_i$$

$$= 1\left(\frac{1}{n}\right) + 2\left(\frac{1}{n}\right) + 3\left(\frac{1}{n}\right) + \cdots + n\left(\frac{1}{n}\right)$$

$$= \frac{1}{n}(1 + 2 + 3 + \cdots + n)$$

$$= \frac{1}{n}\left(\frac{n(n+1)}{2}\right)$$

$$\text{Mean}(\mu) = \frac{n+1}{2}$$

$$\text{variance } (\sigma^2) = \Sigma p_i x_i^2 - \mu^2$$

$$= \left[\frac{1}{n} + 4\left(\frac{1}{n}\right) + 9\left(\frac{1}{n}\right) + 16\left(\frac{1}{n}\right) + \cdots + n^2\left(\frac{1}{n}\right)\right] - \left(\frac{n+1}{2}\right)^2$$

$$= \frac{1}{n}(1 + 4 + 9 + 16 + \cdots + n^2) - \frac{(n+1)^2}{4}$$

$$= \frac{1}{\cancel{n}}\left(\frac{\cancel{n}(n+1)(2n+1)}{6}\right) - \frac{(n+1)^2}{4}$$

$$= \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4}$$

$$= \frac{4(n+1)(2n+1) - 6(n+1)^2}{24}$$

$$= \frac{4(2n^2+n+2n+1) - 6(n^2+1+2n)}{24}$$

$$= \frac{8n^2 - 6n^2 + 12n - 12n + 4 - 6}{24}$$

$$= \frac{2n^2 - 2}{24} = \boxed{\frac{n^2-1}{12}}$$

(4) A random variable 'x' has following probability function

x	1	2	3	4	5	6
p(x)	k	3k	5k	7k	9k	11k

(i) find the value of k

(ii) Mean and variance

(iii) $P(x \geq 3)$ (iv) $P(1 < x \leq 5)$

(i) Sum of probability = 1

$$k + 3k + 5k + 7k + 9k + 11k = 1$$

$$36k = 1$$

$$k = \frac{1}{36}$$

(ii) Mean $(\mu) = \sum p_i x_i$

$$= 1\left(\frac{1}{36}\right) + 2\left(\frac{3}{36}\right) + 3\left(\frac{5}{36}\right) + 4\left(\frac{7}{36}\right) + 5\left(\frac{9}{36}\right) + 6\left(\frac{11}{36}\right)$$

$$= \frac{1}{36}(1+6+15+28+45+66)$$

$$\mu = \frac{161}{36}$$

variance $(\sigma^2) = \Sigma p_i x_i^2 - \mu^2$

$$= 1\left(\frac{1}{36}\right) + 4\left(\frac{3}{36}\right) + 9\left(\frac{5}{36}\right) + 16\left(\frac{7}{36}\right) + 25\left(\frac{9}{36}\right) + 36\left(\frac{11}{36}\right)$$

$$= \frac{1}{36}(1+12+45+112+225+396) - \left(\frac{161}{36}\right)^2$$

$$= \frac{791}{36} - \left(\frac{161}{36}\right)^2$$

$$\sigma^2 = 21.97 - 20.00 = 1.97$$

(iii) $P(X \geq 3) = P(X=3) + P(X=4) + P(X=5) + P(X=6)$

$$= \frac{5}{36} + \frac{7}{36} + \frac{9}{36} + \frac{11}{36}$$

$$= \frac{\cancel{32}}{\cancel{36}} = \boxed{\frac{8}{9}}$$

(iv) $P(1 < X \leq 5) = P(X=2) + P(X=3) + P(X=4) + P(X=5)$

$$= \frac{3}{36} + \frac{5}{36} + \frac{7}{36} + \frac{9}{36}$$

$$= \frac{\cancel{24}}{\cancel{36}} = \boxed{\frac{2}{3}}$$

(5) A sample of 3 items is selected at random from a box containing 10 items in which four are defective. find the expected no. of defective items?

sol:- X = No. of defective items

x	0	1	2	3
$P(x)$	1/6	1/2	3/10	1/30

Total no. of cases $= {}^{10}C_3 = \frac{10\times 9\times 8\times 7!}{7!(3\times 2\times 1)}$

$= 120$

$P(x=0) = \frac{{}^6C_3}{{}^{10}C_3} = \frac{\frac{6\times 5\times 4\times 3!}{3!(3\times 2\times 1)}}{120} = \frac{20}{120} = \frac{1}{6}$

$P(x=1) = \frac{{}^6C_2\, {}^4C_1}{{}^{10}C_3} = \frac{\frac{6\times 5\times 4!}{4!(2\times 1)} \cdot \frac{4\times 3!}{3!}}{120} = \frac{60}{120} = \frac{1}{2}$

$P(x=2) = \frac{{}^6C_1\, {}^4C_2}{{}^{10}C_3} = \frac{\frac{6\times 5!}{5!} \cdot \frac{4\times 3\times 2!}{2!(2\times 1)}}{120} = \frac{36}{120} = \frac{3}{10}$

$P(x=3) = \frac{{}^6C_0\, {}^4C_3}{{}^{10}C_3} = \frac{\frac{4\times 3\times 2\times 1!}{1!(3\times 2\times 1)}}{120} = \frac{4}{120} = \frac{1}{30}$

$\frac{15}{4}$
$\frac{}{60}$

$E(x) = \sum p_i x_i$

$= 0\left(\frac{1}{6}\right) + 1\left(\frac{1}{2}\right) + 2\left(\frac{3}{10}\right) + 3\left(\frac{1}{30}\right)$

$= 0 + \frac{1}{2} + \frac{6}{10} + \frac{3}{30}$

$= \frac{1}{2} + \frac{3}{5} + \frac{1}{10} = \frac{5+6+1}{10} = \frac{12}{10} = \frac{6}{5}$

$E(x) = 1.2$

Continuous probability distribution

Probability density function

Let 'x' be a continuous random variable X $a \le x \le b$ then

$\boxed{f(x) = f_X(x) = P(X = x)}$ is defined as

probability density function of x and statisfy the following condition

① $f(x) \ge 0$

② $\int_a^b f(x)\,dx = 1$

Properties of probability density function

① $f(x) \ge 0$

② $\int_a^b f(x)\,dx = 1$

③ The probability $P(E)$ is given by

$$P(E) = \int_E f(x)\,dx$$ is well defined for any event E

④ In case of continuous random variable we associate the probabilities with intervals.

In this case the probability of variable at a particular point is always 0.

Continuous distribution function (or) cumulative distribution function

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(x)\,dx$$

$$P(X \geq x) = \int_{x}^{\infty} f(x)\,dx$$

Total probability

~~Mean of~~

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

If we take a, b values then

$$\int_{a}^{b} f(x)\,dx = 1$$

Mean

Mean of distribution is given by

$$\boxed{\mu = E(x) = \int_{-\infty}^{\infty} x f(x)\, dx}$$

If x is defined from a to b then

$$\mu = E(x) = \int_{a}^{b} x f(x)\, dx$$

In general, mean (or) expectation of any function $g(x)$

$$\boxed{E\left(g(x)\right) = \int_{-\infty}^{\infty} g(x) f(x)\, dx}$$

Variance

$$\text{Variance } (\sigma^2) = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\, dx \quad \text{(or)}$$

$$\boxed{\sigma^2 = \int_{-\infty}^{\infty} x^2 f(x)\, dx - \mu^2}$$

Suppose x is defined from a to b

$$\sigma^2 = \int_{a}^{b} (x-\mu)^2 f(x)\, dx \quad \text{or}$$

$$\boxed{\sigma^2 = \int_{a}^{b} x^2 f(x)\, dx - \mu^2}$$

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Mediance

Median (M) is given by

$$\int_a^M f(x)\,dx = \int_M^b f(x)\,dx = \frac{1}{2}$$

since x defined from a to b

NOTE: Median is the point which divides the entire distribution into two equal parts.

Mode

Mode is the value of x for which $f(x)$ is maximum, mode is given by

$$f'(x) = 0 \text{ and } f''(x) < 0 \text{ for } a < x < b$$

Mean deviation

Mean deviation about the mean(μ) is given by

$$\int_{-\infty}^{\infty} |x-\mu|\, f(x)\,dx$$

Q:- A continuous random variable X has the probability density function is given by

$$f(x) = \begin{cases} kx, & 0 \le x \le 1 \\ k, & 1 \le x \le 2 \\ -kx+3k, & 2 \le x \le 3 \\ 0, & \text{otherwise} \end{cases}$$

find the value of k and also calculate $P(X \leq 1.5)$

Sol:- Given probability density function

$$f(x) = \begin{cases} kx, & 0 \leq x \leq 1 \\ k, & 1 \leq x \leq 2 \\ -kx+3k, & 2 \leq x \leq 3 \\ 0, & \text{otherwise} \end{cases}$$

since total probability = 1

i.e, $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_{-\infty}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx + \int_{1}^{2} f(x)\,dx + \int_{2}^{3} f(x)\,dx + \int_{3}^{\infty} f(x)\,dx = 1$$

$$0 + \int_{0}^{1} kx\,dx + \int_{1}^{2} k\,dx + \int_{2}^{3} (-kx+3k)\,dx + 0 = 1$$

$$k\int_{0}^{1} x\,dx + k\int_{1}^{2} dx + (-k)\int_{2}^{3} x\,dx + 3k\int_{2}^{3} dx = 1$$

$$k\left.\frac{x^2}{2}\right|_0^1 + k\,x\Big|_1^2 - k\left.\frac{x^2}{2}\right|_2^3 + 3k\,x\Big|_2^3 = 1$$

$$\frac{k}{2}(1-0) + k(2-1) - \frac{k}{2}(3^2-2^2) + 3k(3-2) = 1$$

$$\frac{k}{2} + k - \frac{k}{2}(9-4) + 3k(1) = 1$$

$$\frac{k}{2} + k - \frac{5k}{2} + 3k = 1$$

$$4k + \left(\frac{k}{2} - \frac{5k}{2}\right) = 1$$

$$4k + \left(\frac{-4k}{2}\right) = 1$$

$$8k - 4k = 2$$

$$4k = 2$$

$k = \frac{2}{4}$

$$\boxed{k = \frac{1}{2}}$$

$$P(X \le 1.5) = \int_{-\infty}^{x} f(x)\,dx$$

$$= \int_{-\infty}^{1.5} f(x)\,dx$$

$$= \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx + \int_{1}^{1.5} f(x)\,dx$$

$$= 0 + \int_{0}^{1} kx\,dx + \int_{1}^{1.5} k\,dx$$

$$= k\int_{0}^{1} x\,dx + k\int_{1}^{1.5} dx$$

$$= k\left.\frac{x^2}{2}\right|_0^1 + k\,x\Big|_1^{1.5}$$

$$= \frac{k}{2}(1-0) + k(1.5-1)$$

$$= k\left(\frac{1}{2}\right) + k\,(0.5)$$

$$\frac{\cancel{5}^1}{\cancel{10}_2}$$

$$= \frac{k}{2} + \frac{k}{2}$$

$$= \frac{\cancel{2}k}{\cancel{2}} = k = \boxed{\frac{1}{2}}$$

(2) If the probability density function of a random variable is given by

$$f(x) = \begin{cases} k(1-x^2), & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

find the value of k and the probabilities that a random variable having this probability density will take on a value

(i) between 0.1 and 0.2

(ii) > 0.5 (greater than 0.5) and

mean and variance

sol:- Given

$$f(x) = \begin{cases} k(1-x^2), & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

since total probability = 1

i.e., $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_{-\infty}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx + \int_{1}^{\infty} f(x)\,dx = 1$$

$$0 + \int_{0}^{1} k(1-x^2)\,dx + 0 = 1$$

$$k \int_{0}^{1} 1 - x^2\,dx = 1$$

$$k \left(x\Big|_0^1 - \frac{x^3}{3}\Big|_0^1 \right) = 1$$

$$k \left((1-0) - \frac{1}{3} \right) = 1$$

$$k \left(1 - \frac{1}{3} \right) = 1$$

$$k \left(\frac{2}{3} \right) = 1$$

$$\boxed{k = \frac{3}{2}}$$

(ii) between 0.1 and 0.2

$$P(0.1 < x < 0.2) = \int_{a}^{b} f(x)\,dx$$

$$= \int_{0.1}^{0.2} f(x)\,dx$$

$$= \int_{0.1}^{0.2} k(1-x^2)\,dx$$

$$= k \left(x - \frac{x^3}{3} \right)_{0.1}^{0.2}$$

$$= \frac{3}{2}\left[(0.2-0.1) - \frac{1}{3}\left((0.2)^3 - (0.1)^3\right)\right]$$

$$= \frac{3}{2}\left[0.1 - \frac{1}{3}(0.008 - 0.001)\right]$$

$$= \frac{3}{2}\left[0.1 - \frac{1}{3}(0.007)\right]$$

$$= \frac{3}{2}(0.1 - 0.002)$$

$$= \frac{3}{2}(0.098)$$

$$= \underline{0.147}$$

$$\begin{array}{r} 0.2 \\ \times 0.2 \\ \hline 0.04 \end{array} \qquad \begin{array}{r} 0.008 \\ 0.001 \\ \hline 0.007 \end{array}$$

(iii) $P(X > 0.5) = \int_x^\infty f(x)\,dx$

$$= \int_{0.5}^{\infty} f(x)\,dx$$

$$\begin{array}{r} 0.25 \\ \times 0.5 \\ \hline .125 \end{array}$$

$$= \int_{0.5}^{1} f(x)\,dx + \int_{1}^{\infty} f(x)\,dx$$

$$= \int_{0.5}^{1} k(1-x^2)\,dx + 0$$

$$= k\left((1-0.5) - \frac{1}{3}\left[1^3 - (0.5)^3\right]\right)$$

$$= \frac{3}{2}\left[0.5 - \frac{1}{3}(1 - 0.125)\right]$$

$$= \frac{3}{2}(0.5 - 0.291)$$

$$= \underline{0.3135}$$

$$\text{Mean}(\mu) = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$= \int_{-\infty}^{0} x f(x)\,dx + \int_{0}^{1} x f(x)\,dx + \int_{1}^{\infty} x f(x)\,dx$$

$$= \int_0 x k (1 - x^2)\, dx$$

$$\cancel{= \frac{3}{2}\left(x - \frac{x^3}{3}\right)_0^1} = k \int_0^1 (x - x^3)\, dx$$

$$\cancel{= \frac{3}{2}\left(1 - \frac{1}{3}\right)} \qquad = k \left(\frac{x^2}{2} - \frac{x^4}{4}\right)_0^1$$

$$\cancel{= \frac{3}{2} \cdot \frac{2}{3} = 1} \qquad = \frac{3}{2}\left(\frac{1}{2} - \frac{1}{4}\right)$$

$$\cancel{\mu = 1} \qquad = \frac{3}{2}\left(\frac{1}{4}\right) = \boxed{\frac{3}{8}}$$

$$\text{variance}(\sigma^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx - \mu^2$$

$$= \int_{-\infty}^{0} x^2 f(x)\, dx + \int_0^1 x^2 f(x)\, dx + \int_1^{\infty} x^2 f(x)\, dx - \mu^2$$

$$= \int_0^1 x^2 (k(1 - x^2))\, dx - \mu^2$$

$$= k \int_0^1 (x^2 - x^4)\, dx - \mu^2$$

$$= k \left(\frac{x^3}{3} - \frac{x^5}{5}\right)_0^1 - \mu^2$$

$$= \frac{3}{2}\left(\frac{1}{3} - \frac{1}{5}\right) - \frac{9}{64}$$

$$= \frac{3}{2}\left(\frac{2}{15}\right) - \frac{9}{64}$$

$$= \frac{3}{15} - \frac{9}{64} = \underline{0.0593}$$

③ A continuous random variable has the probability density function

$$f(x) = \begin{cases} kxe^{-\lambda x}, & \text{for } x \geq 0, \lambda > 0 \\ 0, & \text{otherwise} \end{cases}$$

find (i) k (ii) mean (iii) variance

sol:- Given

$$f(x) = \begin{cases} kxe^{-\lambda x}, & \text{for } x \geq 0, \lambda > 0 \\ 0, & \text{otherwise} \end{cases}$$

since total probability = 1

$$\text{i.e., } \int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{-\infty}^{0} f(x)\,dx + \int_{0}^{\infty} f(x)\,dx = 1$$

$$0 + \int_{0}^{\infty} f(x)\,dx = 1$$

$$\int_{0}^{\infty} kxe^{-\lambda x}\,dx = 1$$

$$k\int_{0}^{\infty} xe^{-\lambda x}\,dx = 1$$

$$k\left(\left(x\frac{e^{-\lambda x}}{x}\right)_0^{\infty} - \int_0^{\infty}\frac{e^{-\lambda x}}{\lambda}\right) = 1$$

$$k\left[\frac{xe^{-\lambda x}}{\lambda} - \frac{e^{-\lambda x}}{\lambda^2}\right]_0^{\infty} = 1$$

$$k\left[\left(\frac{\infty e^{-\lambda\infty}}{\lambda} - 0\right)\left(\frac{e^{-\lambda\infty}}{\lambda^2} - \frac{e^{-0}}{\lambda^2}\right)\right] = 1$$

$$k\left(0-0+\frac{1}{\lambda^2}\right)=1 \Rightarrow \frac{k}{\lambda^2}=1$$

$$\boxed{k=\lambda^2}$$

(ii) Mean $(\mu) = \int_{-\infty}^{\infty} x f(x)\,dx$

$$= \int_{-\infty}^{0} x f(x)\,dx + \int_{0}^{\infty} x f(x)\,dx$$

$$0 + \int_{0}^{\infty} x\, k x e^{-\lambda x}\,dx$$

$$= k \int_{0}^{\infty} x^2 e^{-\lambda x}\,dx$$

$$= k\left[\left[x^2 \frac{e^{-\lambda x}}{\lambda} - \frac{2x e^{-\lambda x}}{\lambda^2}\right]_0^{\infty} + 2\frac{e^{-\lambda x}}{-\lambda^3}\Big|_0^{\infty}\right]$$

$$= k\left[\left(\frac{\infty^2 e^{-\lambda\infty}}{\lambda} - 0\right) - \left(\frac{2\infty e^{-\lambda\infty}}{\lambda^2} - 0\right)\right] + 2k \frac{e^{-\lambda x}}{\lambda^3}\Big|_0^{\infty}$$

$$= 0 + 2k\left(\frac{e^{-\lambda\infty}}{\lambda^3} - \frac{e^{-0}}{-\lambda^3}\right)$$

$$= 2\lambda^2\left(+\frac{1}{\lambda^3}\right)$$

$$= \frac{+2}{\lambda} \qquad \boxed{\mu = \frac{2}{\lambda}}$$

(iii) Variance $(\sigma^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu^2$

$$= \int_{-\infty}^{0} x^2 f(x)\,dx + \int_{0}^{\infty} x^2 f(x)\,dx - \mu^2$$

$$= 0 + \int_{0}^{\infty} x^2\, k x e^{-\lambda x}\,dx - \mu^2$$

$$= k \int_{0}^{\infty} x^3 e^{-\lambda x}\,dx - \mu^2$$

$$= \lambda^2 \int_0^\infty x^3 e^{-\lambda x} dx - \mu^2$$

$$= \lambda^2 \left[\left(x^3 \frac{e^{-\lambda x}}{-\lambda} \right) - \left(3x^2 \frac{e^{-\lambda x}}{\lambda^2} \right) + \left(6x \frac{e^{-\lambda x}}{-\lambda^3} \right) - 6 \left(\frac{e^{-\lambda x}}{\lambda^4} \right) \right]_0^\infty - \left(\frac{2}{\lambda}\right)^2$$

$x^3 \quad e^{-\lambda x}$

$3x^2 \quad \frac{e^{-\lambda x}}{-\lambda}$

$6x \quad \frac{e^{-\lambda x}}{\lambda^2}$

$6 \quad \frac{e^{-\lambda x}}{-\lambda^3}$

$0 \quad \frac{e^{-\lambda x}}{\lambda^4}$

$$= \lambda^2 \left[\left(\infty^3 \frac{e^{-\lambda \infty}}{-\lambda} - 0 \right) - \left(3\infty^2 \cdot \frac{e^{-\lambda \infty}}{\lambda^2} - 0 \right) + \left(6\infty \frac{e^{-\lambda \infty}}{-\lambda^3} - 0 \right) - 6 \left(\frac{e^{-\lambda \infty}}{\lambda^4} - \frac{e^0}{\lambda^4} \right) \right] - \left(\frac{2}{\lambda}\right)^2$$

$$= \lambda^2 \left[0 - 0 + 0 - 6 \left(0 - \frac{1}{\lambda^4} \right) \right] - \frac{4}{\lambda^2}$$

$$= \cancel{\lambda^2} \frac{6}{\lambda^{\cancel{4}2}} - \frac{4}{\lambda^2} = \frac{6-4}{\lambda^2} = \boxed{\frac{2}{\lambda^2}}$$

$$\boxed{\sigma^2 = \frac{2}{\lambda^2}}$$

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(4) Probability density function of a random variable X $f(x) = \begin{cases} \frac{1}{2} \sin x, & \text{for } 0 \le x \le \pi \\ 0, & \text{otherwise} \end{cases}$

find (i) Mean (ii) Mode and (iii) Median of the distribution and also find the probability between 0 and $\frac{\pi}{2}$

sol:- Given

$$f(x) = \begin{cases} \frac{1}{2}\sin x, & \text{for } 0 \le x \le \pi \\ 0, & \text{otherwise} \end{cases}$$

$$\text{Mean } (\mu) = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$= \int_{-\infty}^{0} x f(x)\,dx + \int_{0}^{\pi} x f(x)\,dx + \int_{\pi}^{\infty} x f(x)\,dx$$

$$= 0 + \int_{0}^{\pi} x f(x)\,dx + 0$$

$$= \int_{0}^{\pi} x \frac{1}{2}\sin x\,dx$$

$$= \frac{1}{2}\int_{0}^{\pi} x \sin x\,dx$$

x	$\sin x$
1	$-\cos x$
	$-\sin x$

$$= \frac{1}{2}\left[-x\cos x + \sin x\right]_0^{\pi}$$

$$= \frac{1}{2}\left[(-\pi\cos\pi + \sin\pi) - (0 + \sin 0)\right]$$

$$= \frac{1}{2}\left((\pi + 0) - 0\right)$$

$$\text{Mean} = \frac{\pi}{2}$$

(ii) Mode $= f'(x) = 0$ and $f''(x) < 0$ for $a < x < b$

$$\cancel{x\cos x + \sin x = 0}$$

$$\frac{1}{2}\cos x = 0$$

$$\cos x = 0$$

$$\cos x = \cos\frac{\pi}{2}$$

$$\boxed{x = \frac{\pi}{2}} \text{ mode}$$

$$f''(x) = \frac{1}{2} - \sin x = -\frac{1}{2}\sin\frac{\pi}{2}$$

$$= -\frac{1}{2} < 0$$

so this is the mode.

(iii) Median $(M) = \int_a^M f(x)dx = \int_M^b f(x)dx = \frac{1}{2}$

$$\int_0^M f(x)dx = \int_M^{\pi} f(x)dx = \frac{1}{2}$$

$$\int_0^M f(x)dx = \frac{1}{2}$$

$$\int_0^M \frac{1}{2}\sin x\,dx = \frac{1}{2}$$

$$\frac{1}{2}\int_0^M \sin dx = \frac{1}{2}$$

$$\cancel{\frac{1}{2}}\left[-\cos x\right]_0^M = \cancel{\frac{1}{2}}$$

$$\left[-\cos x\right]_0^M = 1$$

$$-\cos M + \cos 0 = 1$$

$$-\cos M = 0$$

$$\cos M = 0$$

Median $\boxed{M = \frac{\pi}{2}}$

(iv) $P\left(0 < x < \frac{\pi}{2}\right) = \int_a^b f(x)dx$

$$= \int_0^{\pi} \frac{1}{2}\sin x\,dx$$

$$= \frac{1}{2}\int_0^{\pi} \sin x\,dx$$

$$= \frac{1}{2}\left(-\cos x\Big|_0^{\pi}\right)$$

$$= \frac{1}{2}\left(-\cos\pi + \cos 0\right)$$

$$= \frac{1}{2}(1+1)$$

$$= 1$$

Mean of the poison Distribution

$$\mu = E(x) = \sum_{x=0}^{\infty} x\,P(x) = \sum_{x=0}^{\infty} \frac{x\, e^{-\lambda} \lambda^{x}}{x!}$$

$$= e^{-\lambda} \sum_{x=0}^{\infty} \frac{x \cdot \lambda^{x}}{x!}$$

$$= e^{-\lambda} \sum_{x=0}^{\infty} \frac{x\, \lambda^{x}}{x(x-1)!}$$

$$= e^{-\lambda} \sum_{x=1}^{\infty} \frac{\lambda^{x}}{(x-1)!}$$

$$= e^{-\lambda} \sum_{y=0}^{\infty} \frac{\lambda^{y+1}}{y!} \quad [\because y = x-1]$$

$$= e^{-\lambda} \sum_{y=0}^{\infty} \frac{\lambda^{y} \cdot \lambda}{y!}$$

$$= \lambda e^{-\lambda} \sum_{y=0}^{\infty} \frac{\lambda^{y}}{y!}$$

$$= \lambda e^{-\lambda} \cdot e^{\lambda} \quad \left[\because \sum_{y=0}^{\infty} \frac{\lambda^{y}}{y!} = e^{\lambda}\right]$$

$$= \lambda$$

$$\boxed{\mu = \lambda}$$

Variance of the poisson Distribution

$$V(X) = E(X^2) - [E(X)]^2$$

$$= \sum_{x=0}^{\infty} x^2 P(x) - \mu^2$$

$$= \sum_{x=0}^{\infty} x^2 P(x) - \lambda^2 \quad [\because \mu = \lambda]$$

$$= \sum_{x=0}^{\infty} x^2 \cdot \frac{e^{-\lambda} \cdot \lambda^{x}}{x!} - \lambda^2$$

$$= \sum_{x=0}^{\infty} \cancel{x^2}\, \frac{e^{-\lambda} \lambda^{x}}{x(x-1)!} - \lambda^2 \quad [\because x! = x(x-1)!]$$

$$= e^{-\lambda} \sum_{x=1}^{\infty} x\, \frac{\lambda^{x}}{(x-1)!} - \lambda^2$$

$$= e^{-\lambda} \sum_{x=1} \frac{[(x-1)+1]\lambda^x}{(x-1)!} - \lambda^2$$

$$= e^{-\lambda}\left[\sum_{x=1}^{\infty} \frac{(x-1)\lambda^x}{(x-1)!} + \frac{\lambda^x}{(x-1)!}\right] - \lambda^2$$

Put $y = x-1$.

$$= e^{-\lambda}\left[\sum_{x=1}^{\infty} \frac{\cancel{(x-1)}\lambda^x}{\cancel{(x-1)}(x-2)!} + \sum_{y=0}^{\infty} \frac{\lambda^{y+1}}{y!}\right] - \lambda^2$$

$$= e^{-\lambda}\left[\sum_{x=2}^{\infty} \frac{\lambda^x}{(x-2)!} + \sum_{y=0}^{\infty} \frac{\lambda^{y+1}}{y!}\right] - \lambda^2$$

Put $z = x-2$

$$= e^{-\lambda}\left[\sum_{z=0}^{\infty} \frac{\lambda^{z+2}}{z!} + \sum_{y=0}^{\infty} \frac{\lambda^{y+1}}{y!}\right] - \lambda^2$$

$$= e^{-\lambda}\left[\lambda^2 \sum_{z=0}^{\infty} \frac{\lambda^z}{z!} + \lambda \cdot \sum_{y=0}^{\infty} \frac{\lambda^y}{y!}\right] - \lambda^2$$

$$= e^{-\lambda}[\lambda^2 \cdot e^{\lambda} + \lambda e^{\lambda}] - \lambda^2$$

$$= e^{-\lambda}[\lambda^2 \cdot e^{\lambda} + \lambda e^{\lambda}] - \lambda^2$$

$$= e^{-\lambda} e^{\lambda}(\lambda^2 + \lambda) - \lambda^2$$

$$= \cancel{\lambda^2} + \lambda - \cancel{\lambda^2} = \lambda$$

$$\boxed{V(X) = \lambda}$$

Standard Deviation $(\sigma) = \sqrt{\lambda}$.

Note: Mean = Variance = $np = \lambda$ is finite.

Reccurence relation for poisson Distribution:

$$P(x+1) = \frac{\lambda}{x+1} p(x)$$

Proof: $p(x) = \dfrac{e^{-\lambda}\lambda^x}{x!} \longrightarrow (1)$

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$$P(x+1) = \frac{e^{-\lambda}\lambda^{x+1}}{(x+1)!} \longrightarrow (2)$$

$$\frac{(2)}{(1)} = \frac{P(x+1)}{P(x)} = \frac{\frac{e^{-\lambda}\lambda^{x+1}}{(x+1)!}}{\frac{e^{-\lambda}\lambda^{x}}{x!}}$$

$$= \frac{\frac{e^{-\lambda}\lambda^{x}\lambda}{(x+1)x!}}{\frac{e^{-\lambda}\lambda^{x}}{x!}} = \frac{\lambda}{x+1}$$

$$P(x+1) = \frac{\lambda}{x+1} P(x)$$

3) <u>Mode</u>: Mode is the value of x for which the prob $P(x)$ is max:

$$\therefore P(x) \geq P(x+1) \text{ and } P(x) \geq P(x-1)$$

$$P(x) \geq P(x+1) \implies x \geq \lambda - 1$$

$$P(x) \geq P(x-1) \implies x \leq \lambda$$

$$\therefore \lambda - 1 \leq x \leq \lambda$$

Hence mode of the poisson distribution lies b/w $\lambda-1$ and λ.

- If λ is an integer then modes are $\lambda-1$ and λ.
- If λ is not an integer then the mode is integral part of λ.

Q) Find the λ value of $P(3) = P(2)$

Sol:
$$\frac{P(x+1)}{P(x)} = \frac{\lambda}{x+1}$$

$$\frac{P(3)}{P(2)} = \frac{\lambda}{2+1}$$

$$\frac{P(3)}{P(2)} = \frac{\lambda}{3} \quad [\because P(3) = P(2)]$$

$$\frac{P(3)}{P(3)} = \frac{\lambda}{3}$$

$$\frac{\lambda}{3} = 1$$

$$\therefore \lambda = 3.$$

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① Fit a poisson distribution to the following:

x	0	1	2	3	4	5
$f(x)$	142	156	69	27	5	1

Sol: $n = 5$

$N = \sum f_i = 142 + 156 + 69 + 27 + 5 + 1$

$= 400$

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i} = \frac{0(142) + 1(156) + 2(69) + 3(27) + 4(5) + 5(1)}{400}$$

$$= \frac{400}{400} = 1$$

$$\mu = \lambda = 1$$

$$P(x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

$$P(x) = \frac{e^{-1} (1)^x}{x!}$$

$$F = N \cdot P(x)$$

x	f	Expect (or) Th $f = N \cdot P(x)$		
0	142	$F_1 = 400 \cdot P(0)$	$= 400 \cdot \frac{e^{-1}(1)^0}{0!}$	$= 0.37$
1	156	$F_2 = 400 \cdot P(1)$	$= 400 \cdot \frac{e^{-1}(1)^1}{1!}$	$= 0.37$
2	69	$F_3 = 400 \cdot P(2)$	$= 400 \cdot \frac{e^{-1}(1)^2}{2!}$	$= 0.18$
3	27	$F_4 = 400 \cdot P(3)$	$= 400 \cdot \frac{e^{-1}(1)^3}{3!}$	$= 0.061$
4	5	$F_5 = 400 \cdot P(4)$	$= 400 \cdot \frac{e^{-1}(1)^4}{4!}$	$= 0.015$
5	1	0.003		

$F = N \cdot P(x)$

$F = 400 \times 0.37 = 148$

$$\begin{array}{r} 148 \\ 72 \\ 24 \\ 6 \\ 1.2 \\ \hline 399.2 \\ \hline \end{array}$$

Q) 4 coins are tossed 160 times, the no. of times x heads occur is given below.

x	0	1	2	3	4
f(x)	8	34	69	43	6

Fit a poisson distribution to the following data on the hypothesis is that coins are unbiased

Sol: $n = 4 \quad N = \sum f_i = 160$

$$\text{mean} = \frac{\sum f_i x_i}{\sum f_i}$$

$$\mu = \frac{0 + 34 + 2(69) + 3(43) + 4(6)}{160} = \frac{325}{160}$$

$\mu = 2.03$

x	f(x)	P(x)	F = N.P(x)
0	8	$\frac{e^{-2.03}(2.03)^0}{0!} = 0.131$	21.01
1	34	$\frac{e^{-2.03}(2.03)^1}{1!} = 0.266$	42.56
2	69	$\frac{e^{-2.03}(2.03)^2}{2!} = 0.270$	43.2
3	43	$\frac{e^{-2.03}(2.03)^3}{3!} = 0.183$	29.28
4	6	$\frac{e^{-2.03}(2.03)^3}{3!} = 0.092$	14.72

$\sum F = 150.75$

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