

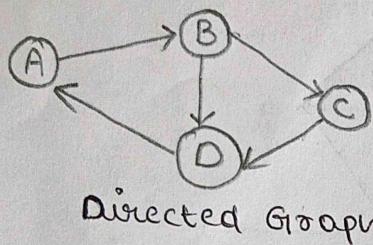
GRAPH THEORY

1. Define a graph?

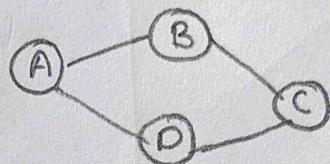
The closed loop is called as a graph.

There are different type of graphs:

1. Directed Graph: When the graph is represented by arrows then it is called directed graph.



Directed Graph



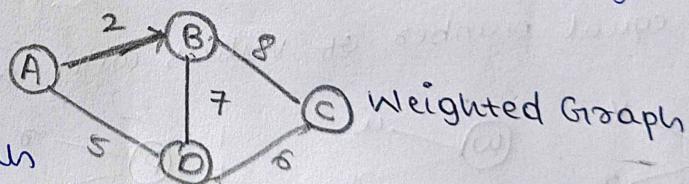
Undirected Graph

Graph:— A graph G_1 is a mathematical representation consisting of two sets vertices (V) and edges (E)

2. Trivial Graph: A graph consisting only one vertex and no edge.

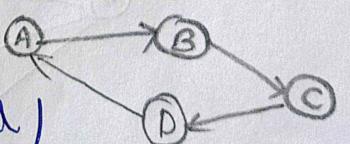
• Self loop graph: If the edge of graph starts at a vertex and ends at the same vertex then it is a loop.

3. Weighted graph: A graph in which the values are assigned to every edge of the graph



4. Cycle Graph: A graph which atleast contain one cycle which means the source and destination of graph are same.

• A cycle graph can be directional / undirectional ; weighted / unweighted



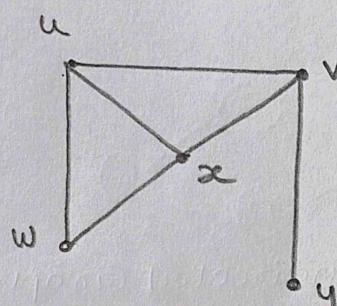
Cycle Graph

2. Define the conditions for two graphs G_1 and G_2 to be isomorphic with example? Define the following

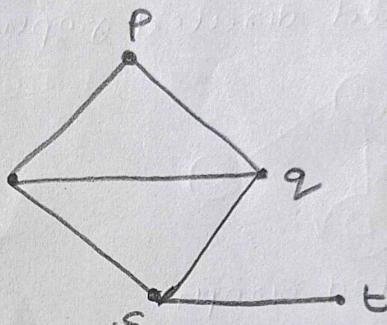
a. weighted Graph b. Complete Graph.

- Now if G_1 and G_2 two graphs are isomorphic if only
- No. of vertices are same
 - No. of edges are same
 - An equal number of vertices with given degree
 - Vertex correspondence and edge correspondence valid.

For example:



Graph G_1



Graph G_2

1. Number of vertices should be same

Graph G_1 vertices : 5

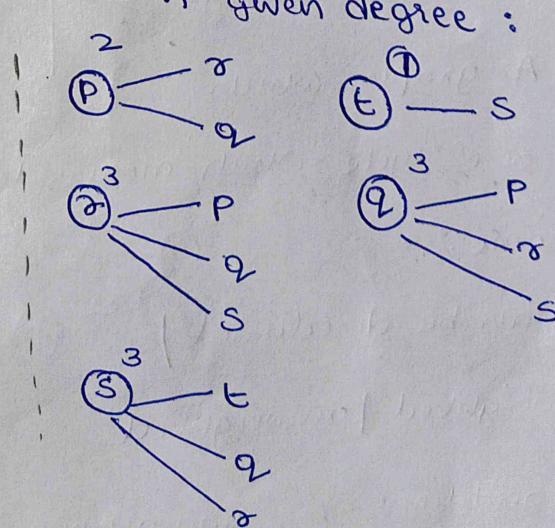
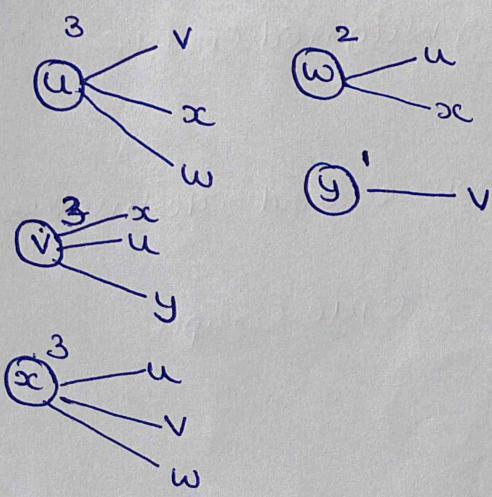
Graph G_2 vertices : 5

2. Number of edges are same

Graph G_1 : 6

Graph G_2 : 6

3. An equal number of vertices with given degree :



The equal no. of vertices with given degree are also same

4. equal vertex correspondence w edge correspondence:

$$(u \leftrightarrow s) \quad (w \leftrightarrow p)$$

$$(v \leftrightarrow s) \quad (y \leftrightarrow t)$$

$$(x \leftrightarrow q)$$

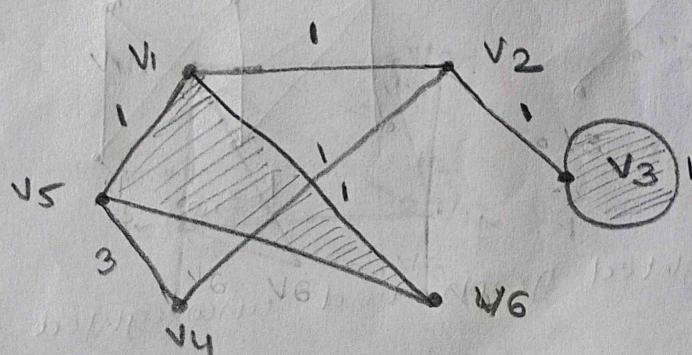
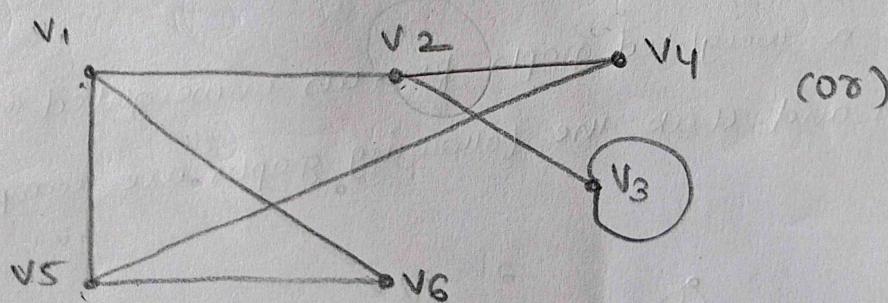
\therefore Both the graphs G_1 and G_2 are isomorphic.

→ Complete Graph: A complete graph is a simple undirected graph in which every pair of distinct vertices is connected by a unique edge.

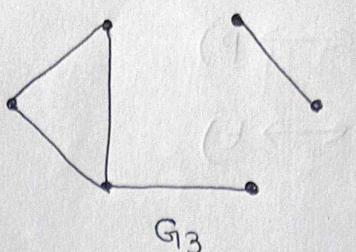
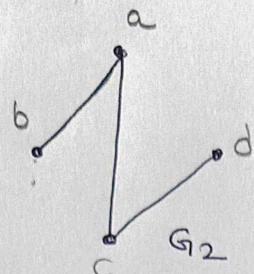
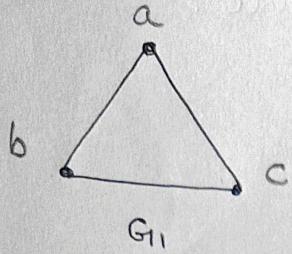
3. Draw the graph whose adjacency matrix is shown below?

	v_1	v_2	v_3	v_4	v_5	v_6
v_1	0	1	0	0	1	1
v_2	1	0	1	1	0	0
v_3	0	1	1	0	0	0
v_4	0	1	0	0	3	0
v_5	1	0	0	3	0	1
v_6	1	0	0	0	1	0

\therefore {this is a
undirected
graph}

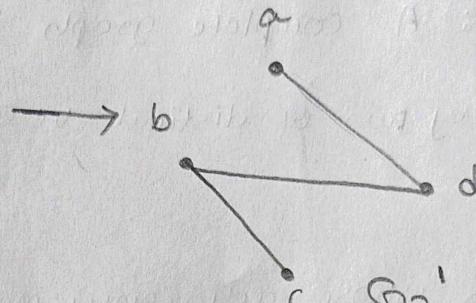
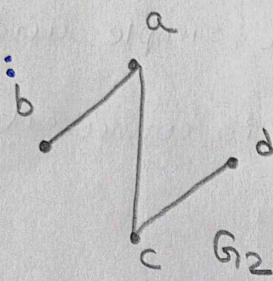


4. Find the complements of each graph show below



Graph G_1 does not have complements because it has equal no. of sides.

Graph G_2 :

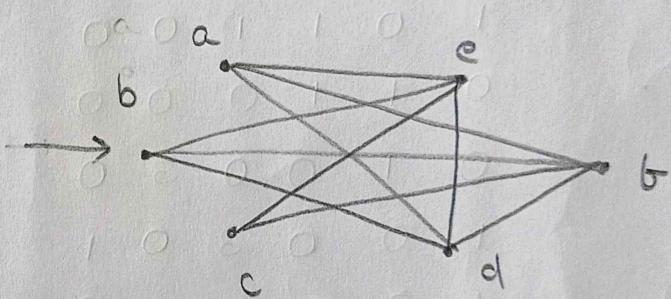
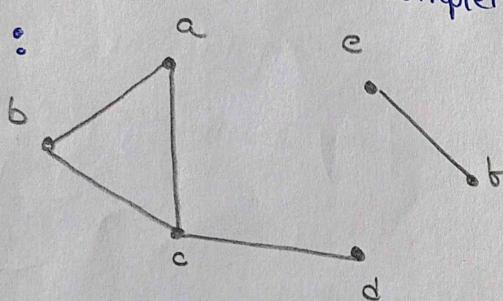


Now

graph G_2 has a

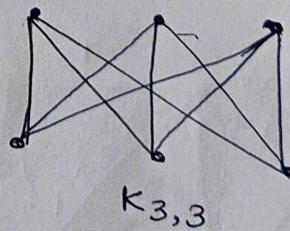
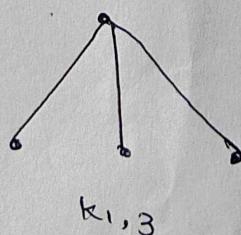
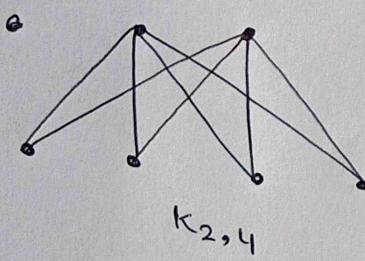
complementary graph G_2' !

Graph G_3 :



No, it does not have complementary graph.

5. What distinguishes a weighted graph from an unweighted one?
Define a bipartite graph and check the following graphs are complete
bipartite graphs or not



The difference between weighted graph and unweighted graph
is as follows:

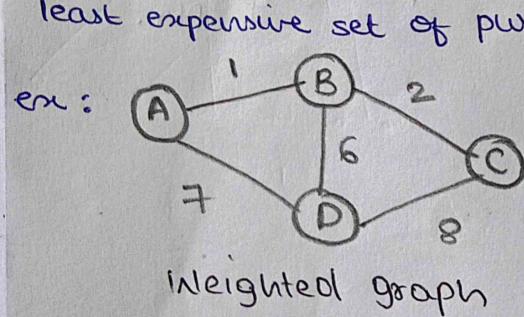
Weighted Graph

→ A weighted graph assigns a value or weight to each edge.

→ In weighted graph each edge connects a pair of vertices.

→ In weighted graph the assigning values are distance or time travel, shortest path etc

→ This are used in algorithms between nodes such as such as real-world problems like navigation system, least expensive set of purchase etc



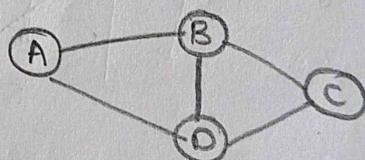
→ Whereas, in unweighted graph there is no value assigned to a graph.

→ In unweighted graph all edges are equal and there's no specific value associated with moving from one vertex to another.

→ There is no assigning value for a graph.

→ This are used in the presence or absence of a connection social network analysis, structure of the internet etc.

ex:



Unweighted graph.

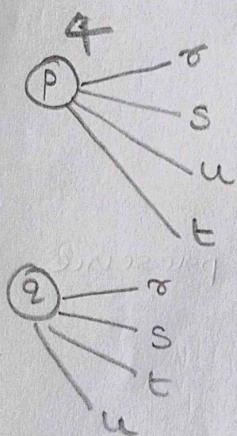
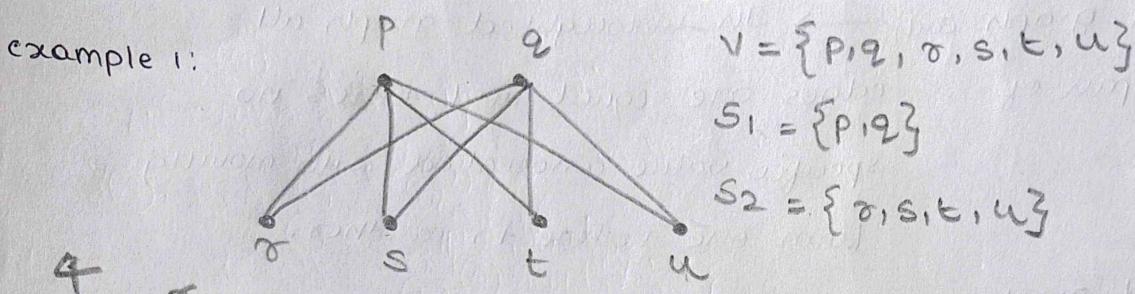
→ Definition of Bipartite graph: A graph is bipartite if the vertices can be partitioned into two sets X and Y so that every edge has one endpoint in X and the other in Y.

For a graph to be a bipartite it should satisfy some conditions:
1. All the vertices of the graph should be divided into two distinct

Sets of vertices X and Y

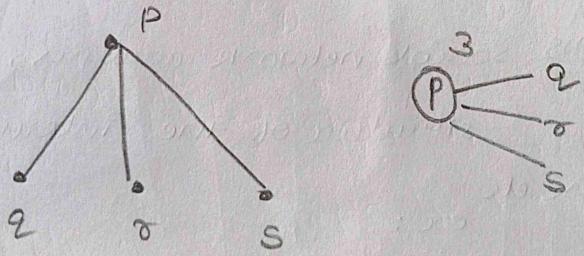
2. All the vertices present in the set X should only be connected to the vertices present in the set Y with some edges.
3. Both the sets that are created should be distinct that means both should not have same vertices in them.

example 1:



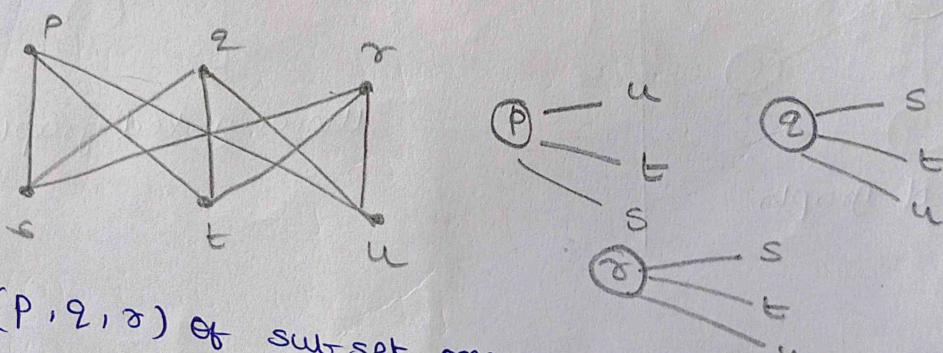
Now the vertices (P, Q) of sub set (S_1) are connected to the all the vertices of (s, t, u, v) of sub set (S_2) graph. Then this is called as complete bipartite graph.

example 2:



This is also a bipartite graph.

example 3:



The vertices (P, Q, R) of sub set one are connected to all the vertices (s, t, u, v) of sub set two. Then this is a complete bipartite graph.

6. Let G be a graph with vertex set $V(G) = \{a, b, c, d, e, f\}$ and edge set $E(G) = \{ab, ae, bc, cc, de, ed\}$

a. Draw G

b. Is G simple?

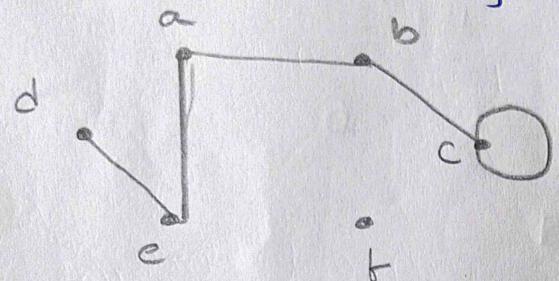
c. List the degree of every vertex

d. find all edges incident of b

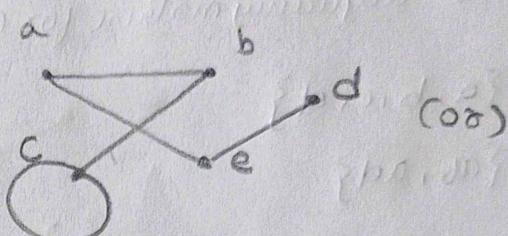
e. List all neighbours of a

f. Give the adjacency matrix for G_1 .

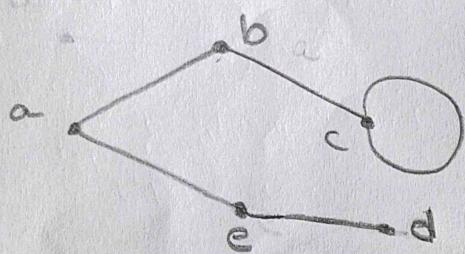
a. $V(G_1) = \{a, b, c, d, e, f\}$



(Q3)

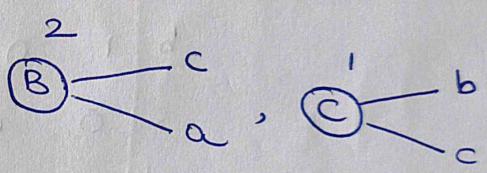
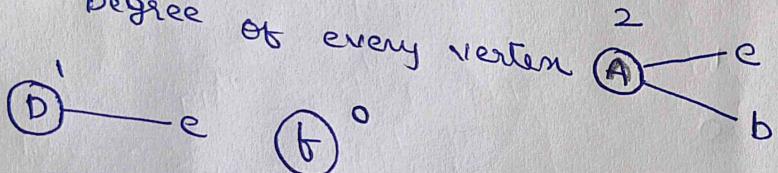


(Q3)



b. No, G_1 is not simple
(because it has a loop)

c. Degree of every vertex



d. Edges incident from b are $\{ba, bc\}$

e.

f.

	a	b	c	d	e	f
a	0	1	0	0	0	0
b	1	0	1	0	0	0
c	0	1	1	0	0	0
d	0	0	0	0	0	0
e	1	0	0	1	0	0
f	0	0	0	0	0	0

Adjacency matrix of given G_1 .

7. Let G_1 be a graph with vertex set $V(G_1) = \{a, b, c, d, f\}$ and

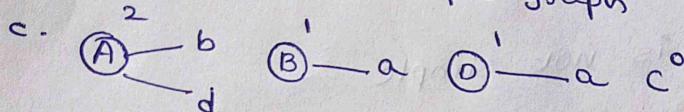
edge set $E(G_1) = \{ab, ad\}$

- Draw G_1
- Is G_1 simple?
- List the degree of every vertex
- Give the adjacency matrix for G_1 .

a. $V(G_1) = \{a, b, c, d\}$

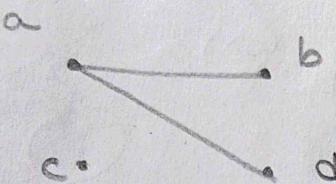
$E(G_1) = \{ab, ad\}$

b. Yes, it is a simple graph



d. Adjacency matrix

	a	b	c	d
a	0	1	0	1
b	1	0	0	1
c	0	0	0	0
d	1	0	0	0



$v = 4$

$E = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$

$|E| = 3$

${}^6C_2 = \frac{6 \times 5}{2} = 30$

8. Let G_1 be a graph with the vertex set $V(G_1) = \{a, b, c, d, e, f\}$

edge set $E(G_1) = \{ad, ae, bd, cd, ce, cf\}$

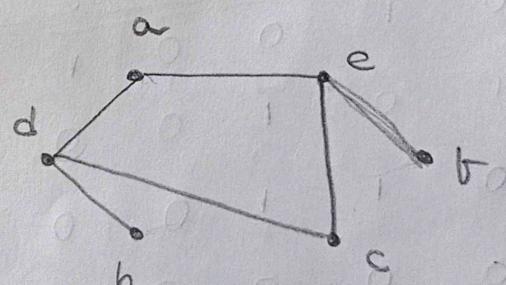
- Draw G_1
- Is G_1 simple?
- Is G_1 bipartite?
- List the degree of every vertex
- Give the adjacency matrix for G_1 .

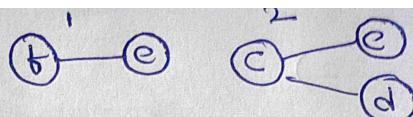
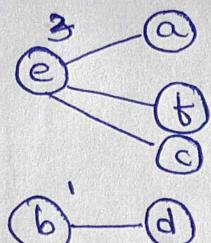
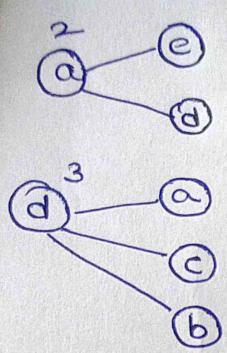
a. $V(G_1) = \{a, b, c, d, e, f\}$

$E(G_1) = \{ad, ae, bd, cd, ce, cf\}$

b. It is a simple graph

c. It is not a bipartite graph.





(3)

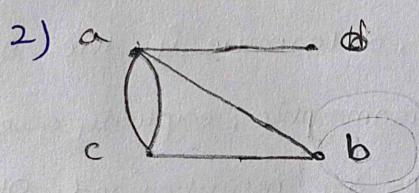
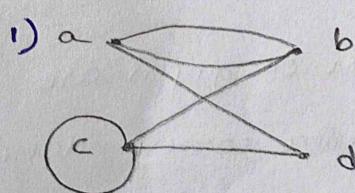
e. Adjacency matrix

	a	b	c	d	e	f
a	0	0	0	1	1	0
b	0	0	0	1	0	0
c	0	0	0	1	0	0
d	1	1	1	0	0	0
e	1	0	1	0	0	1
f	0	0	0	0	1	0

9. Draw the graph for each of the following of adjacency matrices given below:

$$\begin{matrix}
 & a & b & c & d \\
 a & 0 & 2 & 0 & 1 \\
 b & 2 & 0 & 1 & 0 \\
 c & 0 & 1 & 1 & 1 \\
 d & 1 & 0 & 1 & 0
 \end{matrix}$$

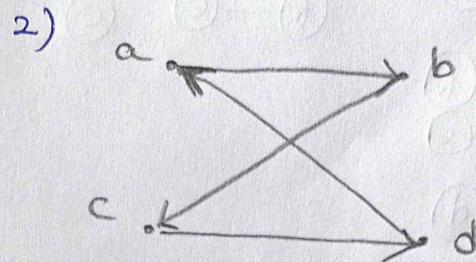
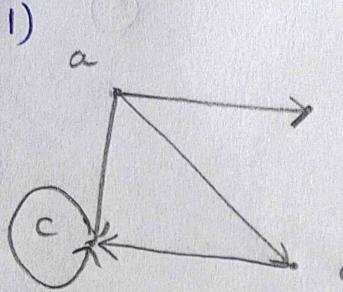
$$\begin{matrix}
 & a & b & c & d \\
 a & 0 & 1 & 2 & 1 \\
 b & 1 & 2 & 1 & 0 \\
 c & 2 & 1 & 0 & 0 \\
 d & 1 & 0 & 0 & 0
 \end{matrix}$$



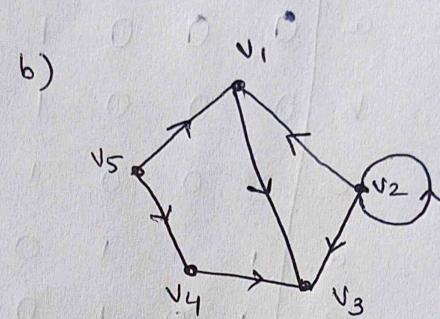
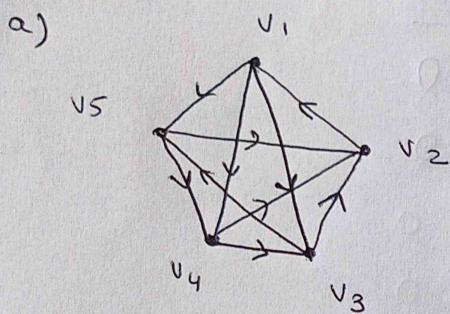
10. Draw the diagraph for each of the adjacency matrices given below:

$$\begin{matrix}
 & a & b & c & d \\
 a & 0 & 1 & 1 & 1 \\
 b & 0 & 0 & 0 & 0 \\
 c & 0 & 0 & 1 & 0 \\
 d & 0 & 0 & 1 & 0
 \end{matrix}$$

$$\begin{matrix}
 & a & b & c & d \\
 a & 0 & 1 & 0 & 0 \\
 b & 0 & 0 & 1 & 0 \\
 c & 0 & 0 & 0 & 1 \\
 d & 1 & 0 & 0 & 0
 \end{matrix}$$



11. Find the adjacency matrix for each digraphs or tournaments given below.



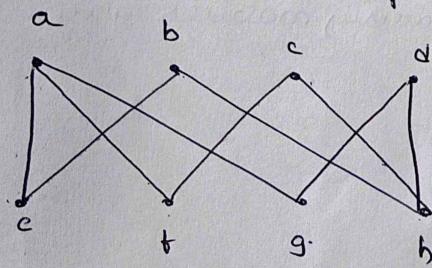
1)

	v_1	v_2	v_3	v_4	v_5
v_1	0	0	1	1	1
v_2	1	0	0	0	0
v_3	0	1	0	0	1
v_4	0	1	1	0	0
v_5	0	1	0	1	0

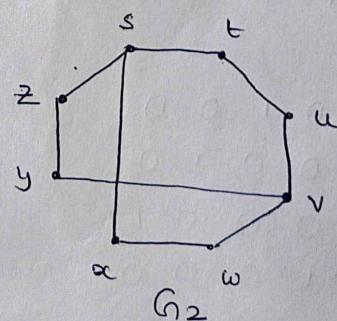
2)

	v_1	v_2	v_3	v_4
v_1	0	0	1	0
v_2	1	1	1	0
v_3	0	0	0	0
v_4	0	0	1	0
v_5	1	0	0	1

12. For each problems below, determine if the given pair of graphs are isomorphic. For those that are isomorphic, explicitly give the vertex correspondence and check that edge relationships are maintained. Otherwise, provide reasoning for why the pair of graphs are not isomorphic.



G_1



G_2

To say the Graphs G_1 and G_2 are isomorphic

The two graphs have same vertex and edges.

i.e. the given graph G_1 and another graph G_2 has same vertices (8) and same edges (10).

To say both the graphs are isomorphic

1. Same vertices = 8

2. Same edges = 10

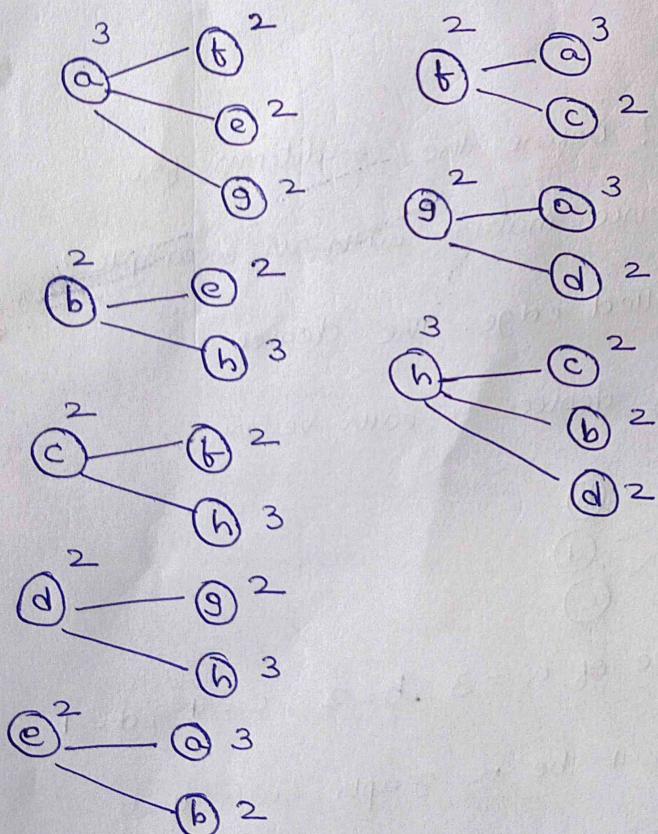
3. Degree sequence:

$$(a, b, c, d, e, f, g, h) = (3, 2, 2, 2, 2, 2, 2, 3)$$

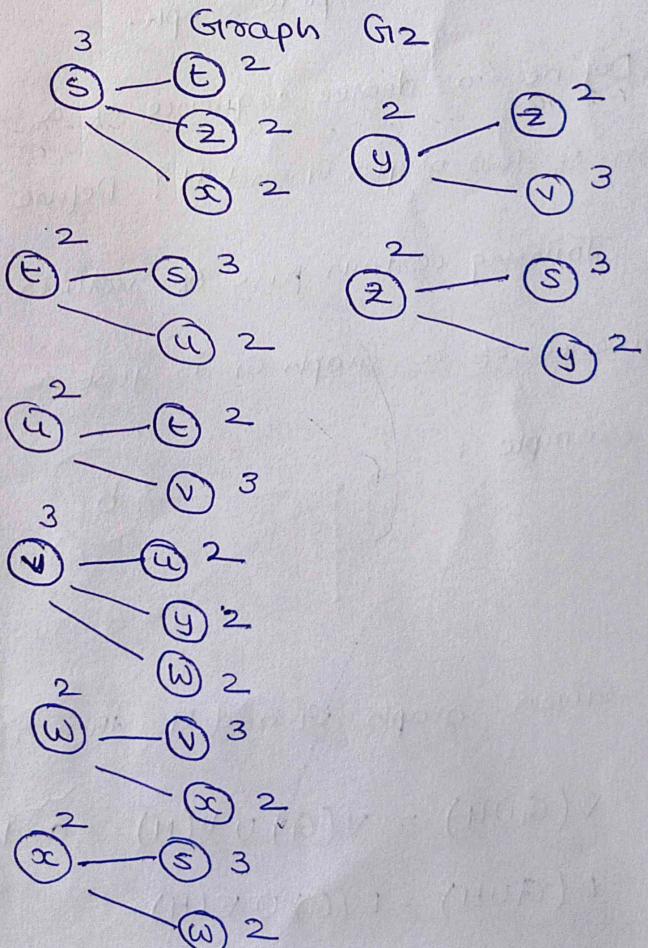
$$(s, t, u, v, w, x, y, z) = (3, 2, 2, 3, 2, 2, 2, 2)$$

4. One-to-one mapping

Graph G_1



Graph G_2



$$a \leftrightarrow s \quad f \leftrightarrow y$$

$$b \leftrightarrow t \quad g \leftrightarrow z$$

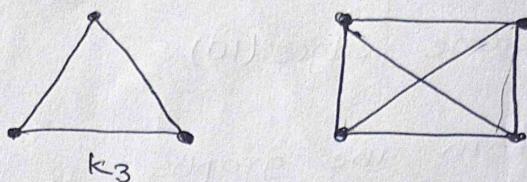
$$c \leftrightarrow u \quad h \leftrightarrow v$$

$$d \leftrightarrow w$$

$$e \leftrightarrow x$$

Hence, graph G_1 and graph G_2 are isomorphic.

13. List the properties of complete graph and identify the complete graphs from the following:



A complete graph is a graph in which every vertex is connected to every other vertex.

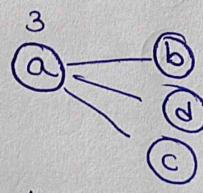
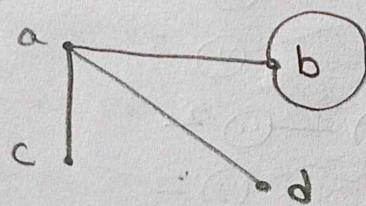
A complete graph is an undirected graph where every pair of distinct vertices is connected by a unique edge.

1. Not a complete graph
2. Yes, it is a complete graph
3. Yes, it is a complete graph
4. Yes, it is a complete graph.

14. Define a degree sequence of a graph? Define the conditions for union of two graphs G and H? Define incidence matrix with an example?

→ Joining certain pair of vertices is called edge. The degree sequence of a graph G is just a list of degrees of each vertex.

For example :



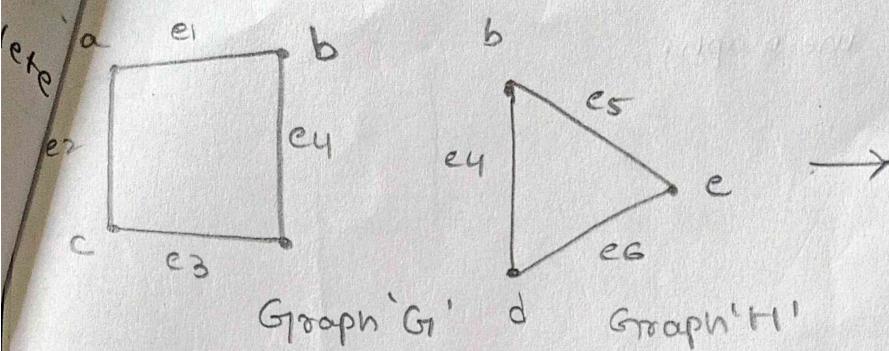
degree of a = 3, b = 2, c = 1, d = 1

→ Given graph G and H their union will be a graph such that

$$V(G \cup H) = V(G) \cup V(H) \text{ and}$$

$$E(G \cup H) = E(G) \cup E(H)$$

The union of G and H is denoted as G ∪ H



$$V(G_1) = \{a, b, c, d\}$$

$$V(H) = \{b, d, e\}$$

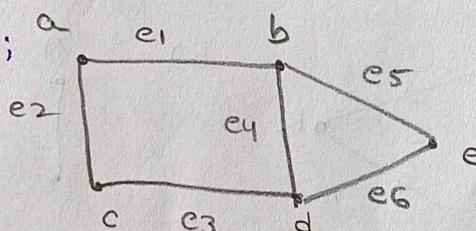
$$V(G_{1H}) = \{a, b, c, d, e\}$$

$$E(G_1) = \{e_1, e_2, e_3, e_4\}$$

$$E(H) = \{e_4, e_5, e_6\}$$

$$E(G_{1H}) = \{e_1, e_2, e_3, e_4, e_5, e_6\}$$

Now the union graph G_{1H} ;



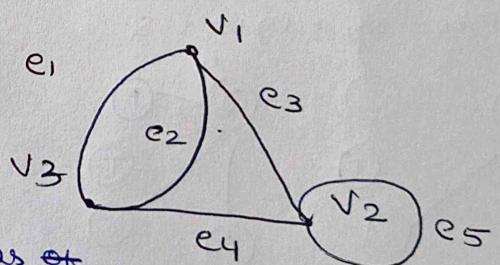
15. Why matrix representations of graphs are useful for computer programs. Also write the adjacency matrix for following graph
Mainly computers are often linked together

into a network such that as a local area network. A network can also be represented mathematically as a graph. By adding properties of

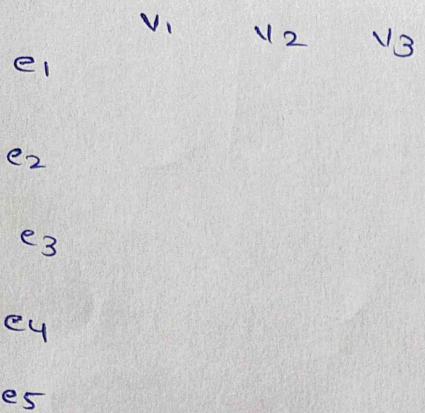
to the nodes and edges to represent the capacities of the network.

We As far how graph theory works, at the present time is not fully aligned with computer. We have methods of drawing graph networks so that the human eye can judge the properties for themselves.

We have ways to describe the structures inside graphs. We have algorithms about how to compare the data in the graph and see the

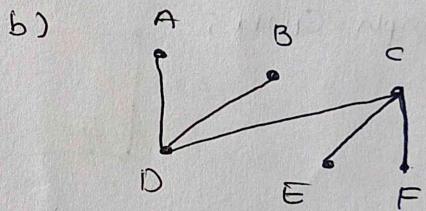
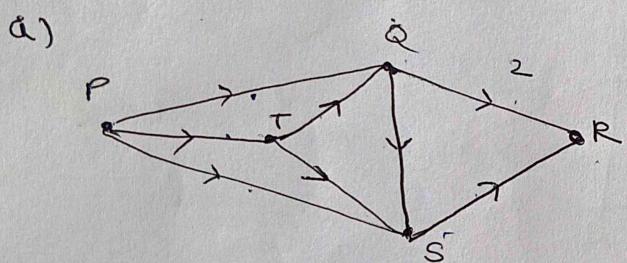


relationships b/w diff parts of the graph.



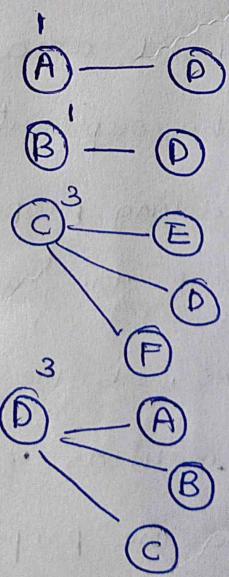
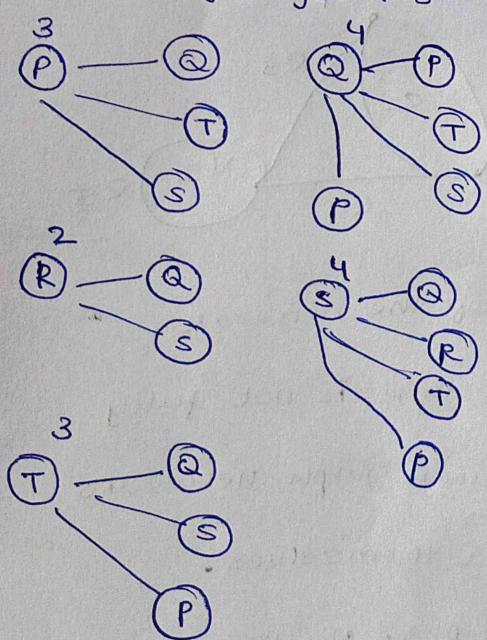
PART-B.

1. Write down the number of vertices, number of edges, and the degree of each vertex



- a) number of vertices : 5
 $\{P, Q, R, S, T\}$

number of edges : 8



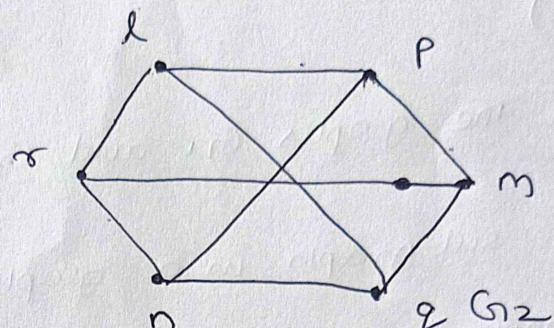
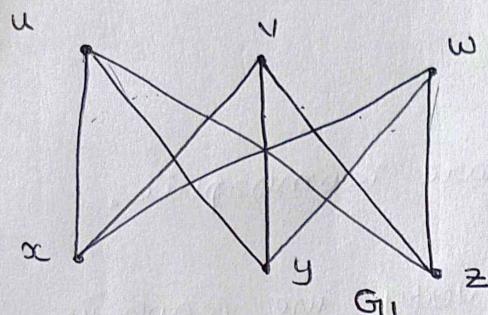
degree of each vertex :

$$\{P, Q, R, S, T\} = \{3, 4, 2, 4, 3\}$$

8

$$\{A, B, C, D\} = \{1, 1, 3, 3\}$$

Draw Define isomorphism of graphs? State that the two labelled graphs are isomorphic or not with reasons.

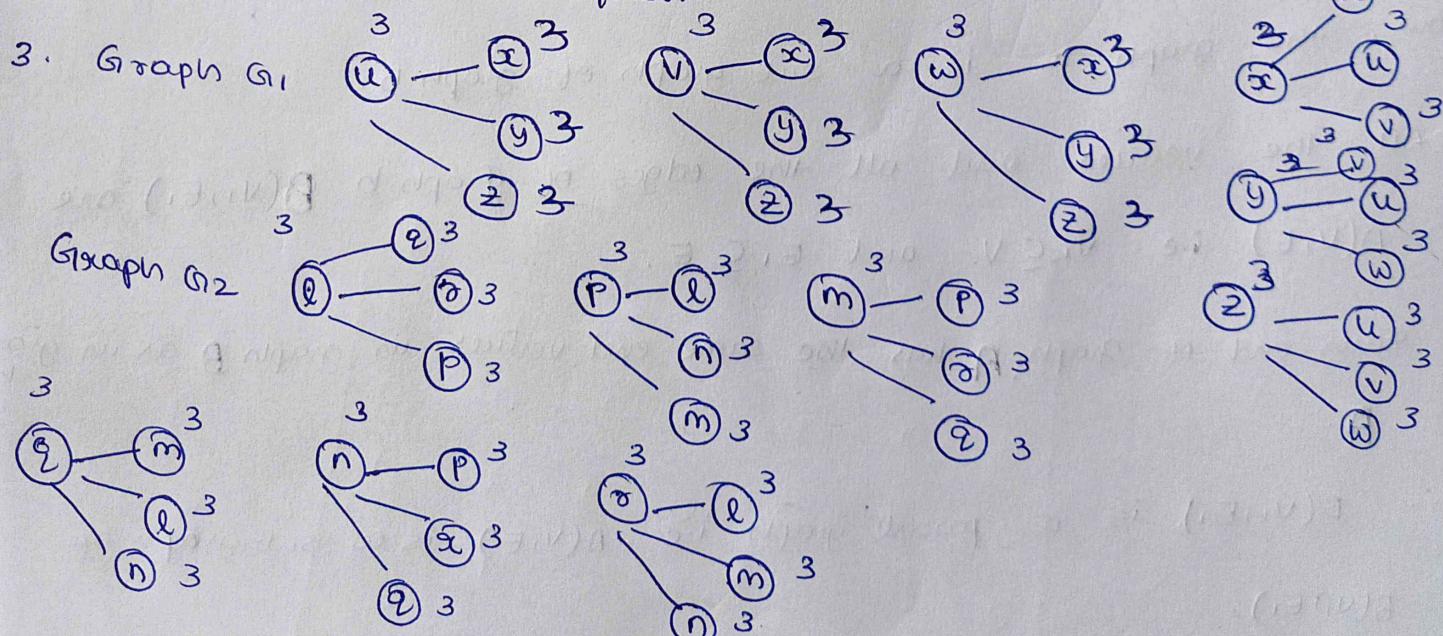


Isomorphisms graphs are those which have same^{no. of} vertices, same no. of edges, an equal number vertices with even degree and vertex correspondance w/ edge correspondance valid.

The above G_1 and G_2 we can say that:

1. The Graph G_1 has 6 vertices $\{u, v, w, x, y, z\}$ and Graph G_2 has also 6 vertices $\{l, p, m, q, n, r\}$. The both graphs have same no. of vertices $\{6\}$.
2. The Graph G_1 has 9 edges and graph G_2 has 9 edges.

The no. of edges are also equal.



Both the graphs G_1 and G_2 the vertices have same no. of degree

4. $(u \leftrightarrow l)$ $(y \leftrightarrow n)$

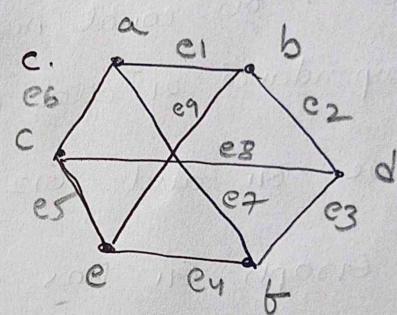
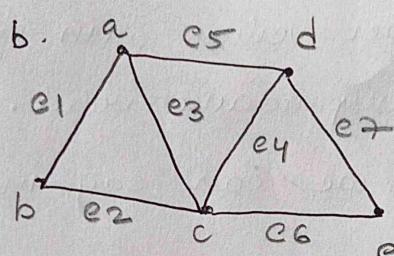
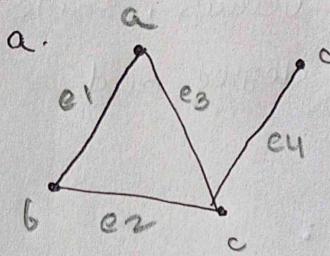
$(v \leftrightarrow p)$ $(z \leftrightarrow q)$

$(w \leftrightarrow m)$

$(x \leftrightarrow o)$

\therefore The Both the graphs G_1 and G_2 are isomorphic.

3. Define a subgraph in a graph? Verify the graph in
a. is a subgraph of the graph in (b), but it is not a subgraph
of the graph c.



Def'n:

A subgraph H of a graph G is said to be a proper subgraph of G if either $V(H)$ is a proper subset of $V(G)$ or $E(H)$ is a proper subset of $E(G)$. (or) a graph all of whose points and lines are connected in a larger graph.

3. Now, the graph a is a subgraph of graph b.

- All the vertices and all the edges of graph b $B(V_1, E_1)$ are in $A(V, E)$ i.e. $V_1 \subseteq V$ and $E_1 \subseteq E$.
- Each end of graph A has the same end vertices in graph B as in graph A .

- $\therefore B(V_1, E_1)$ is a parent graph i.e. $A(V, E)$ is a subgraph of $B(V_1, E_1)$.

The graph a is not a subgraph c because the vertices and

(9)

edges of graph c are different from graph. Hence graph a is a sub graph of graph b but not graph c.

4. Explain the following : a. Adjacency matrix b. Incidence matrix
Write the adjacency & incidence matrix for the following graph:

a. The adjacency matrix, also called as

connection matrix, is a matrix containing

rows and columns which is used to represent a simple labelled graph with 0 or 1 in the position of (v_i, v_j)

b. The order of an incidence matrix of n number of nodes & b number of branches $n \times b$. For each and every graph, incidence matrix can be obtained

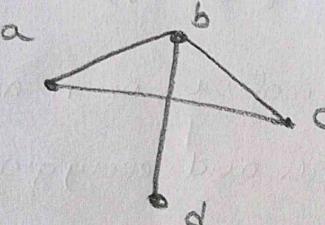
Adjacency matrix :

$$\begin{matrix} & 1 & 2 & 3 & 4 \\ 1 & 0 & 1 & 0 & 1 \\ 2 & 1 & 0 & 1 & 2 \\ 3 & 0 & 1 & 0 & 1 \\ 4 & 1 & 2 & 1 & 0 \end{matrix} \quad (\text{doubt})$$

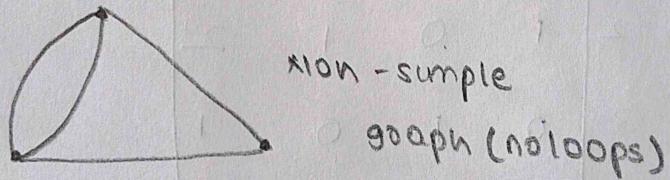
Incidence Matrix :

$$\begin{matrix} & 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 & 1 & 1 \\ 3 & 0 & 1 & 1 & 0 & 0 & 0 \\ 4 & 0 & 0 & 1 & 1 & 1 & 1 \end{matrix}$$

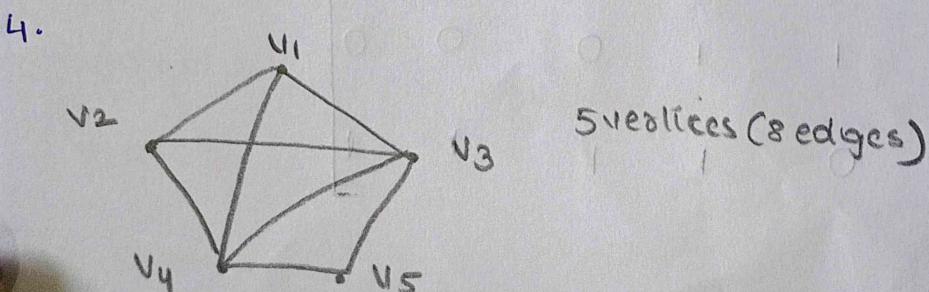
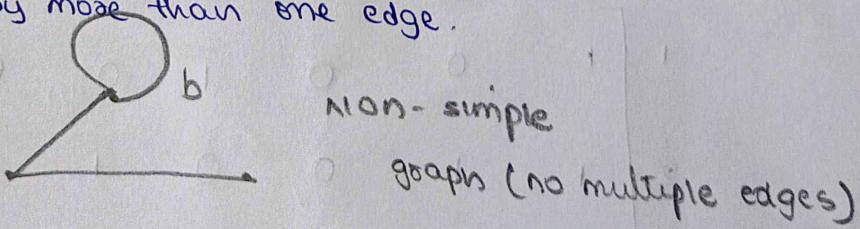
5. Explain and draw the following graphs:

- (i) a simple graph
 - (ii) a non-simple graph with no loops.
 - (iii) a non-simple graph with no multiple edges.
 - (iv) a non-simple graph with no multiple edges, each with five vertices and eight edges.
1. A simple graph is a graph that does not have more than one edge between two vertices and no edge starts and ends at the same vertex. In simple words, a simple graph is a graph without loops and multiple edges.
- 
- simple graph

2. A non-simple graph with no loops means a type of graph that have multiple edges which connect same two vertices but there are still no loops.



3. A non-simple graph has loops so, where the edge can be connected to a vertex to itself creating a loop where there are no pair of vertices connected by more than one edge.



(10)

Show that the two graphs in fig a are

isomorphic by suitably labelling the vertices, and also explain why the two graphs in fig (b) are not isomorphic.

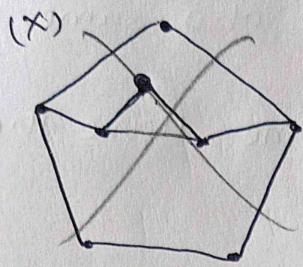


Fig a : a

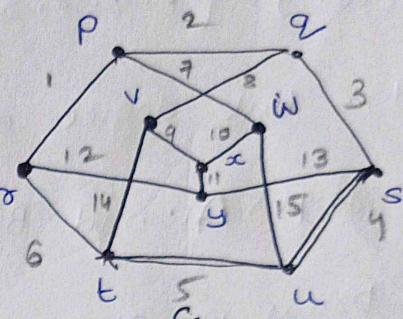
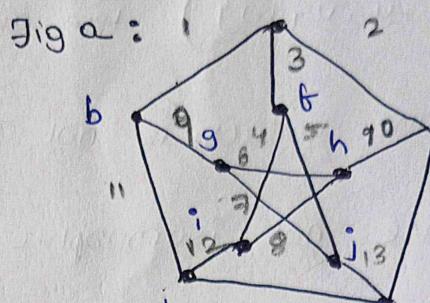
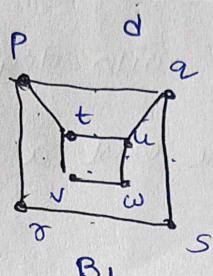
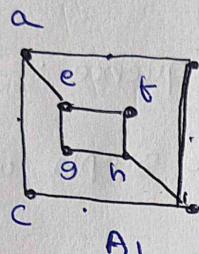
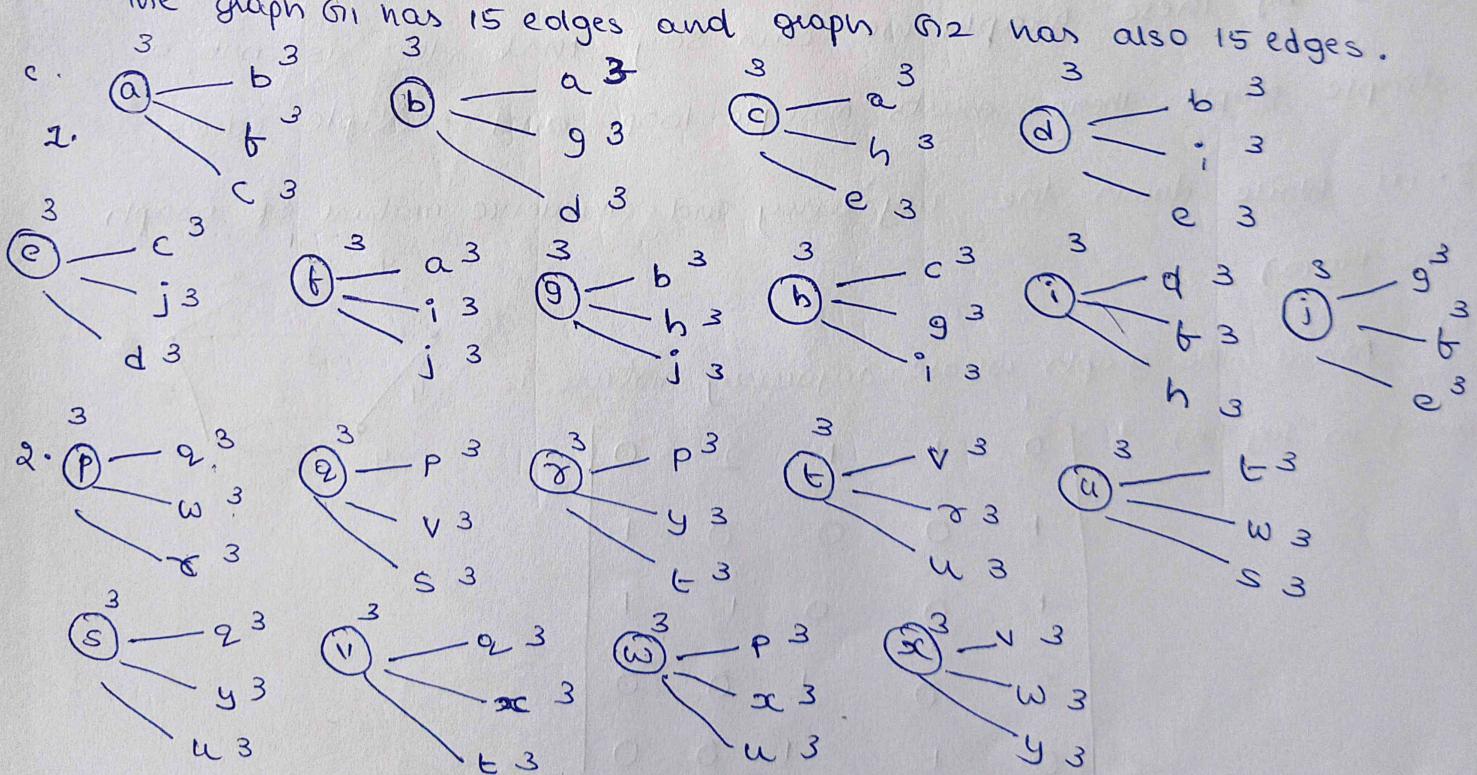


Fig b :



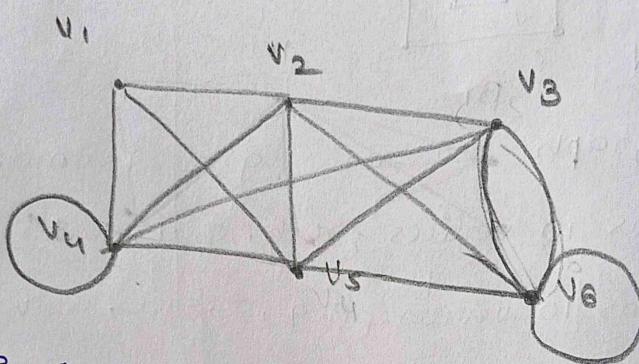
- Graph G_{11} and graph G_{12} in fig a isomorphic because :
 - The graph G_{11} has 10 vertices $\{a, b, c, d, e, f, g, h, i, j\}$ and even graph G_{12} has 10 vertices $\{p, q, r, s, t, u, v, w, x, y\}$
 - The graph G_{11} has 15 edges and graph G_{12} has also 15 edges.



Every vertex has even degree

∴ Both Graphs G_{11} and graph G_{12} are isomorphic

2. Graph A_1 and B_1 are not isomorphic because they have same vertices ($A_1 \in V_1 = \{a, b, c, d, e, f, g, h\}$), ($B_1 \in V_2 = \{p, q, r, s, t, u, v, w\}$) and also same edges but degree of the vertices is not same. Hence the two graphs A_1 and B_1 are not isomorphic.
7. Draw a graph on 6 vertices with degree sequence $(3, 3, 5, 5, 5, 5)$ to verify does there exist a simple graph with these degrees?

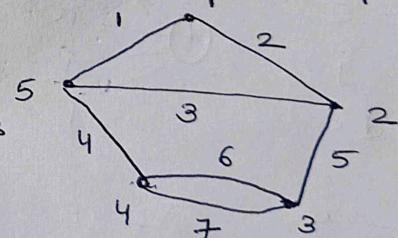


Hence by these graph we can say that this is not a simple graph there exists multiple loops and multiple edges.

8. (i) Write down the adjacency and incidence matrix of graph in Fig(a)

(ii) Draw the graph whose adjacency matrix is given in fig (b)

$$\begin{matrix} & 1 & 2 & 3 & 4 & 5 \\ 1 & 0 & 1 & 1 & 2 & 0 \\ 2 & 1 & 0 & 0 & 0 & 1 \\ 3 & 1 & 0 & 0 & 1 & 1 \\ 4 & 2 & 0 & 1 & 0 & 0 \\ 5 & 0 & 1 & 1 & 0 & 0 \end{matrix}$$



(iii) Draw the graph whose incidence matrix is given in Fig(c)

	a	b	c	d	e	f	g	h	
1	0	0	1	1	1	1	1	0	
2	0	1	0	1	0	0	0	1	
3	0	0	0	0	0	0	0	1	
4	1	0	1	0	1	0	1	0	
5	1	1	0	0	0	1	0	0	

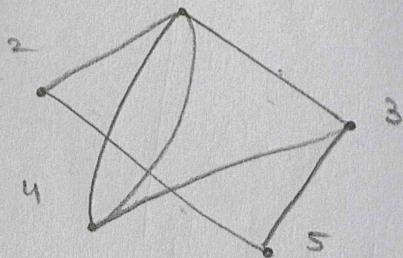
(i) adjacency matrix

	1	2	3	4	5	v ₁	v ₂	v ₃	v ₄	v ₅
1	0	1	0	0	1	0	1	0	0	1
2	1	0	1	0	1	1	0	0	0	1
3	0	1	0	2	0	0	1	0	2	0
4	0	0	2	0	1	0	0	2	0	1
5	1	1	0	1	0	1	1	0	1	0

(ii) incidence matrix :

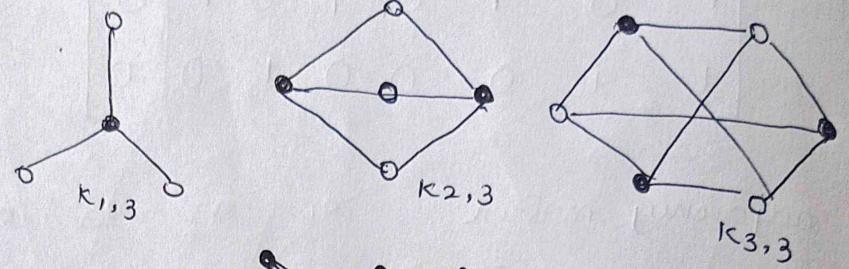
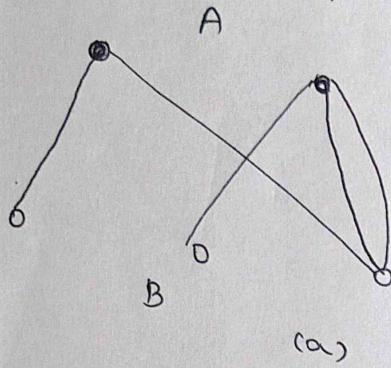
	1	2	3	4	5	6	7
1	1	1	0	0	0	0	0
2	0	1	1	0	1	0	0
3	0	0	0	0	1	1	1
4	0	0	0	0	1	1	1
5	1	0	1	1	0	1	1

(iii)



(iv)

9. Define bipartite graphs and complete bipartite graphs. Justify the graph in fig(a) is a bipartite graph or not w/ also the graphs in fig (b) are complete bipartite graphs or not.



10. Draw a digraph for the following :
- (a) Snakes eat frogs & birds eat spiders, birds and spiders both eat insects, frogs eat snails, spiders and insects. Draw a digraph representing this predatory behaviour.
- (b) John likes Joan, Jean and Jane; Joe likes Jane and Joan; Jean and Joan like each other. Draw a digraph illustrating these relationships b/w John, Joan, Jean, Jane, and Joe.

(iii)