Differential Equations and Calculus SEM-II

Module-I

PART-A

1) $x^2y\,dx - (x^3+y^3)\,dy = 0$

sol: It is in the form of $M(x,y)dx + N(x,y)dy = 0$ is a homogeneous Equation so Integrating factor is $\frac{1}{Mx+Ny}$ of $Mdx + Ndy = 0$

$M = x^2y$ | $N = -(x^3+y^3)$

$\frac{\partial M}{\partial y} = x^2(1)$ | $\frac{\partial N}{\partial x} = -3x^2 + 0$

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ $\therefore$ Non exact and Homogeneous

Integrating factor I.F $= \frac{1}{x^2y(x) + -(x^3+y^3)y}$ $\left[\frac{1}{Mx+Ny}\right]$

$\Rightarrow \frac{1}{\cancel{x^3y} - \cancel{x^3y} - y^4}$

$\Rightarrow -\frac{1}{y^4}$

$$I.F \times \left[x^2 y\,dx - (x^3+y^3)dy\right] = 0$$

$$= \frac{-1}{y^4}\left[x^2 y\,dx - (x^3+y^3)dy\right] = 0$$

$$\Rightarrow \frac{-x^2 y}{y^4}dx + \frac{x^3+y^3}{y^4}dy = 0$$

$$= \frac{-x^2}{y^3}dx + \left(+\frac{x^3}{y^4} + \frac{1}{y}\right)dy = 0$$

$$\Rightarrow -\frac{x^2}{y^3}dx + \frac{x^3}{y^4}dy + \frac{1}{y}dy = 0$$

$$\Rightarrow \frac{-x^2}{y^3}dx + \left(\frac{x^3}{y^4} + \frac{1}{y}\right)dy$$

$$M_1\,dx + N_1\,dy = 0$$

$$M_1 = -\frac{x^2}{y^3} \qquad N_1 = \frac{x^3}{y^4} + \frac{1}{y}$$

$$\frac{\partial M_1}{\partial y} = \frac{+3x^2}{y^4} \qquad \frac{\partial N_1}{\partial x} = \frac{1}{y^4}\cdot 3x^2 + 0$$

$$\frac{\partial M_1}{\partial y} = \frac{\partial N_1}{\partial x}$$ exact and Homogeneous

$$\int M_1 dx + \int N_1 dy = c$$

$\downarrow$ y const. $\qquad$ $\downarrow$ free from "x" terms

$$\int -\frac{x^2}{y^3}dx + \int \frac{1}{y}\,dy = c$$

$$\Rightarrow -\frac{1}{y^3}\left[\frac{x^3}{3}\right] + \log y = c$$

$$= \boxed{\frac{-x^3}{3y^3} + \log y = c}$$

(20)

sol:- $2xy\,dy - (x^2+y^2+1)\,dx = 0$

$$\Rightarrow -(x^2+y^2+1)\,dx + 2xy\,dy = 0$$

$$M\,dx + N\,dy$$

$M = -(x^2+y^2+1)$ | $N = 2xy$

$\frac{\partial M}{\partial y} = -0 - 2y + 0$ | $\frac{\partial N}{\partial x} = 2y(1) = 2y$

$= -2y$

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ Non Exact

So

Non Homogeneous

$$\frac{1}{N}\left[\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right]$$

$$= \frac{1}{2xy}\left[-2y - 2y\right]$$

$$= \frac{-4y}{2xy} = \frac{-2}{x}$$

function of "x" alone

To check Homogeneous (or) not

if $f(kx, ky) = f(x,y)$ then It is Homogeneous

$$\frac{dy}{dx} = \frac{x^2+y^2+1}{2xy} = f(x,y)$$

$$f(kx,ky) = \frac{k^2x^2+k^2y^2+1}{2(kx)(ky)}$$

$$\neq f(x,y)$$

Scanned with OKEN Scanner

$$I.F = e^{\int F(x)\,dx}$$

$$= e^{\int -\frac{2}{x}dx}$$

$$\Rightarrow e^{-2\int \frac{1}{x}dx}$$

$$= e^{-2\log x}$$

$$= e^{\log_e x^{-2}}$$

$$= x^{-2}$$

$$\Rightarrow \frac{1}{x^2}\left[2xy\,dy - (x^2+y^2+1)\,dx\right] = 0$$

$$\Rightarrow \frac{2y}{x}dy - \left[\frac{x^2+y^2+1}{x^2}\right]dx] = 0$$

$$\Rightarrow \frac{2y}{x}dy - \left[\frac{x^2}{x^2} + \frac{y^2}{x^2} + \frac{1}{x^2}\right]dx = 0$$

$$\Rightarrow -\left[1 + \frac{y^2}{x^2} + \frac{1}{x^2}\right]dx + \left(\frac{2y}{x}\right)dy = 0$$

$$M_1\,dx + N_1\,dy = 0$$

$$M_1 = -\left[1 + \frac{y^2}{x^2} + \frac{1}{x^2}\right]$$

$$\frac{\partial M_1}{\partial y} = 0 - \frac{1}{x^2}\cdot 2y - 0$$

$$= -\frac{2y}{x^2}$$

$$N_1 = \frac{2y}{x}$$

$$\frac{\partial N_1}{\partial x} = 2y\cdot -1\cdot x^{-2}$$

$$= -\frac{2y}{x^2}$$

$$\frac{\partial M_1}{\partial y} = \frac{\partial N_1}{\partial x}$$

$$\int M_1 \, dx + \int N_1 \, dy = c$$

$\Downarrow$ y Const $\quad$ $\Downarrow$ free from "x" terms

$$\int -\left[\frac{1+y^2}{x^2} + \frac{1}{x^2}\right] dx + \int 0 \, dy = c$$

$$\Rightarrow -\left[x + y^2 \cdot \frac{x^{-2+1}}{-2+1} + \frac{x^{-2+1}}{-2+1}\right] + 0$$

$$\Rightarrow -\left[x + y^2 \cdot \frac{x^{-1}}{-1} + \frac{x^{-1}}{-1}\right] + 0$$

$$\Rightarrow -\left[x - \frac{y^2}{x} - \frac{1}{x}\right] + 0$$

$$\Rightarrow \left(-x + \frac{y^2}{x} + \frac{1}{x}\right) + 0 = c$$

$$\Rightarrow \frac{y^2}{x} + \frac{1}{x} - x = c$$

$$\Rightarrow \frac{y^2 + 1 - x^2}{x} = c$$

$$\boxed{y^2 + 1 - x^2 = xc}$$

Differential Equations and vector Calculus

module - I

Q ③

Sol:
$$(1+x^2)\frac{dy}{dx} + 2xy = 4x^2$$

$$\frac{dy}{dx} + \frac{2xy}{1+x^2} = \frac{4x^2}{1+x^2}$$

$$P = \frac{2x}{1+x^2}, \quad Q = \frac{4x^2}{1+x^2}$$

$$I.F = e^{\int P dx}$$

$$e^{\int \frac{2x}{1+x^2} dx}$$

$$\Rightarrow e^{\log_e(1+x^2)}$$

$$\Rightarrow 1+x^2$$

$$y.(1+x^2) = \int \frac{4x^2}{\cancel{1+x^2}} . \cancel{1+x^2}\, dx + c$$

$$\Rightarrow y(1+x^2) = 4.\frac{x^3}{3} + c$$

$$\Rightarrow \boxed{y(1+x^2) = \frac{4}{3}x^3 + c}$$

Q 4

Sol:
$$\frac{dy}{dx} + 2y = e^x + x$$

$$P = 2, \quad Q = e^x + x$$

$$I.F = e^{\int 2\, dx}$$

$$\Rightarrow e^{2x}$$

Or

$$y \cdot e^{2x} = \int e^{x} + x \cdot e^{2x} dx + C$$

$$y \cdot e^{2x} = \int (e^{3x} + x \cdot e^{2x}) dx$$

$$y \cdot e^{2x} = \frac{e^{4x}}{4} + \int x \cdot e^{2x} dx$$

$$\Rightarrow y \cdot e^{2x} = \frac{e^{4x}}{4} + x \cdot \frac{e^{2x}}{2} - 1 \cdot \frac{e^{2x}}{4}$$

$$\boxed{y \cdot e^{2x} = \frac{1}{4}(e^{4x} - e^{2x}) + x \cdot \frac{e^{2x}}{2} + C}$$

(5c)

To prove that the question is self orthogonal Here are the steps

$$y^2 = 4a(x+a) \quad (1)$$

diff wrt x on bs

$$2y \frac{dy}{dx} = 4a$$

let $\frac{dy}{dx} = y_1$

$$\cancel{2} y y_1 = \cancel{4}^{2} a$$

$$a = \frac{y y_1}{2}$$

Sub a value in Eq(1)

$$y^2 = \cancel{4}^{2} \cdot \frac{y y_1}{\cancel{2}} \left(x + \frac{y y_1}{2}\right)$$

$\Rightarrow \quad 2xyy_1 + (yy_1)^2 = y^2$

$\Rightarrow \quad 2xy_1 + yy_1^2 = y$

Suppose to check self orthogonal

let $y_1 = -\frac{1}{y_1}$

$$2x \cdot -\frac{1}{y_1} + y \cdot \left(\frac{-1}{y_1}\right)^2 = y$$

$$-\frac{2x}{y_1} + y \cdot \frac{1}{y_1^2} = y$$

$\Rightarrow \quad \frac{-2xy_1 + y}{y_1^2} = y$

$$-2xy_1 + y = yy_1^2$$

$\Rightarrow \quad \boxed{y = yy_1^2 + 2xy_1}$

Both the equations are equal so self orthogonal

(60)

to Convert Celcius to Faherea

$°F = C \times 9/5 + 32$

$18.4 \times 9 / 5 + 32$

$165.6 + 32$

197.6

$$\begin{array}{r} 18.4 \\ \times 9 \\ \hline 165.6 \end{array}$$

$T(t) = 92^\circ C$

$T(1) = 80^\circ C$

$$\frac{\times 9}{43.2}$$

$$\overset{48.4}{\cancel{92}} \times \frac{9}{\cancel{5}} + 32$$

$$\overset{16}{\cancel{80}} \times \frac{9}{\cancel{5}} + 32$$

$\cong 200^\circ F \cdot 197$

$144 + 32$

$\Rightarrow \cancel{177} \cdot 176$

$T(S) = 24^\circ C$

$$\overset{4.8}{\cancel{24}} \times \frac{9}{\cancel{5}} + 32$$

$43.2 + 32$

$75.2 = 75$

$$\boxed{T(t) = T_S + (T_0 - T_S) e^{-kt}}$$

$\cancel{197} = \cancel{75} + \cancel{1}$

$$147 = 75 + (197 - 75) e^{-k}$$

$$1\cancel{46} = 75 + (122) e^{-k}$$

$$\underset{72}{\cancel{146} - 75} = 122 e^{-k} \Rightarrow \log_e \frac{72}{122} = -k$$

$$\frac{\cancel{101}}{122} = e^{-k}$$

$$\log_e \frac{\cancel{101}}{122} = -k$$

$$\log \frac{122}{\cancel{101}} = k$$

$T(t) = 65^\circ C$

$$\overset{13}{\cancel{65}} \times \frac{9}{\cancel{5}} + 32$$

$197 + 32$

$$\Rightarrow 149 = 75 + (197 - 75) \cdot e^{-\left(\log \frac{122}{101}\right) t}$$

$$74 = 122 e^{-0.07t}$$

$0.6 = e^{-0.07t}$

$\log 0.6 = -0.07t$

$-\cancel{0}.2 = \cancel{+}0.07$

$\frac{2}{1\cancel{0}} = \frac{7}{10\cancel{0}} \times t$

$\cancel{1}.8 \quad 60$

$\frac{\cancel{20}}{\cancel{7}} = t$

$\boxed{t = 1.8 sec}$

7a)
Sol:
find orthogonal trajectory of $x^2+y^2=c^2$

$$2x + 2y \cdot \frac{dy}{dx} = 0$$

$$\therefore \frac{dy}{dx} = -\frac{x}{y}$$

For othogonal trajectory

$$\frac{dy}{dx} = \frac{-1}{\frac{dy}{dx}}$$

$$\Rightarrow \frac{-1}{-\frac{x}{y}} = y/x$$

$$\Rightarrow \frac{dy}{y} = \frac{dx}{x}$$

Integrate on bs

$$\int \frac{1}{y} dy = \int \frac{1}{x} dx$$

$$\Rightarrow \log y = \log x + \log a$$

$\boxed{\log y = \log xa}$ → This is orthogonal trajectory.

$y = Ax$

(8) Solve 1^{st} order d.E

Sol: $(x^4 e^x - 2mxy^2)dx + 2mx^2y\,dy = 0$

$M = x^4 e^x - 2mxy^2$, $N = 2mx^2y$

they given dE so to check it the condition is

$\frac{\partial M}{\partial y} = 0 - 2mx \times 2y = -4mxy$

$\frac{\partial N}{\partial x} = 2my \cdot 2x = 4mxy$

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

so DE is not exact

$\Rightarrow (x^4 e^x - 2mxy^2)dx + 2mx^2y\,dy = 0$

$\Rightarrow x^4 e^x dx - 2mxy^2 dx + 2mx^2 y\,dy = 0$

$= x^4 e^x dx - 2mxy(y\,dx - x\,dy) = 0$

divide with x^4

$\Rightarrow e^x dx - \frac{2my}{x}\frac{(y\,dx - x\,dy)}{x^2} = 0$

we can write this term as $d(e^x)$

$e^x dx + \frac{2my}{x}\left(\frac{x\,dy - y\,dx}{x^2}\right)$

this term as $d(y/x)$

$\Rightarrow d(e^x) + \frac{2my}{x} \cdot d(y/x) = d(c)$

Integrate on bs.

let $t = y/x$ then $\int t\,dt = \frac{t^2}{2}$

$e^x + \frac{2m}{2}\left(\frac{y}{x}\right)^2 = c$

$\Rightarrow \boxed{e^x + m(y/x)^2 = c}$

(9) DE of family of circles passing through origin and having centres on x-axis.

Sol:

we know that general equation of the circle passing through the origin and having centres on origin is

$$x^2+y^2+2gx=0$$

diff wrt x

$$2x+2y\frac{dy}{dx}+2g=0$$

$\div 2$

$$x+y\frac{dy}{dx}+g=0$$

$$\boxed{g=-x-y\frac{dy}{dx}}$$

$$x^2+y^2+2x\left(-x-y\frac{dy}{dx}\right)=0$$

$$\Rightarrow x^2+y^2-2x^2-2xy\frac{dy}{dx}=0$$

$$\Rightarrow -x^2+y^2-2xy\frac{dy}{dx}=0$$

$$\Rightarrow \boxed{y^2-x^2=2x\frac{dy}{dx}}$$

Scanned with OKEN Scanner

(10)

Sol:

$T_0 = 100°C$, $T(10) = 75°C$

$\Downarrow$ $\frac{20}{\cancel{100}} \times \frac{9}{\cancel{5}} + 32$ $\Rightarrow$ $\frac{32}{212\,F}$

$\Downarrow$ $\cancel{75}^{15} \times \frac{9}{\cancel{5}} + 32$

$135 + 32 = 167°F$

$T_S = 20°C$ $\Downarrow$ $\cancel{20}^{4} \times \frac{9}{\cancel{5}} + 32$

$36 + 32$

$68°F$

we know that

$$T(10) = T_S + (T_0 - T_S)\, e^{-kt}$$

$$\Rightarrow 68 + (212 - 68)\, e^{-k \times 10} = 167$$

$$68 + 144\, e^{-10k} = 167$$

$$144\, e^{-10k} = 99$$

$$e^{-10k} = \frac{99}{144}$$

$$\log\left(\frac{99}{144}\right) = -10k$$

$$\Rightarrow -\frac{\log\left(\frac{99}{144}\right)}{10} = k$$

$T_S = 25°C \Rightarrow \cancel{25}^{5} \times \frac{9}{\cancel{5}} + 32$

$45 + 32$

77

77
16.5
9 3.5

$$T(30) = 77 + (100 - 77)\, e^{-30k}$$

$$77 + 23\, e^{\cancel{30}^{3} \times \frac{\log_e (99/144)}{\cancel{10}}}$$

$$\Rightarrow 77 + \cancel{23} \times \frac{\cancel{99}}{\cancel{144}} \ldots$$

11.5
× 6
69.0

$93.5°F$

Part-B Long Answer Questions

① in class work.

② sol:- $(xe^{xy} + 2y)\frac{dy}{dx} + ye^{xy} = 0$

$\Rightarrow$ $\div$ with $xe^{xy}+2y$

$\frac{dy}{dx} + \frac{e^{xy}}{xe^{xy}+2y} \cdot y = 0$

$P = \frac{e^{xy}}{xe^{xy}+2y}$; $Q = 0$

$I.F = e^{\int \frac{e^{xy}}{xe^{xy}+2y} dx}$

$(xe^{xy} + 2y)dy + ye^{xy}dx = 0$

$(ye^{xy})dx + (xe^{xy}+2y)dy = 0$

$M dx + N dy = 0$

$M = ye^{xy}$, $N = xe^{xy}+2y$

$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(ye^{xy})$, $\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}(xe^{xy}+2y)$

$y \cdot e^{xy} \cdot x + e^{xy}$ $\qquad$ $x \cdot e^{xy} \cdot y + e^{xy} + 0$

$e^{xy}(xy-1)$ $\qquad \Rightarrow e^{xy}(xy+1) + 0$

So

then $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

Condition

$\therefore$ exact

$\int M dx + \int N dy = 0$

$\downarrow$ take "y" as Constant

$\downarrow$ without x terms

remove x terms

$\Rightarrow \int y e^{xy} dx + \int \cancel{x e^{xy}} + 2y \, dy$

$\Rightarrow \cancel{y}\left(\frac{e^{xy}}{\cancel{y}}\right) + \int 2y \, dy$

$\Rightarrow e^{xy} + 2 \cdot \frac{y^2}{2} = c$

$\boxed{e^{xy} + y^2 = c}$

③ ~~doubt~~

Sol:-

④

$x^3 \sec^2 y \, dy + 3x^2 \tan y \, dx = \cos x \, dx$

$x^3 \sec^2 y \, dy + (3x^2 \tan y - \cos x) dx = 0$

$M = 3x^2 \tan y - \cos x$, $N = x^3 \sec^2 y$

$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y} = 3x^2 \cdot \sec^2 y - 0$

$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}(x^3 \sec^2 y) = \sec^2 y \cdot 3x^2$

$3(\sec^2 y) x^2$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\int M\,dx + \int N\,dy = c$$

$$\int (3x^2 \tan y - \cos x)\,dx + \int \cancel{x^3 \sec^2 y}\,dy$$

0 free from x so take zero

$$\Rightarrow \cancel{3}\tan y \cdot \frac{x^3}{\cancel{3}} - \sin x + y = c$$

$$\Rightarrow x^3 \tan y - \sin x + y = c$$

(4)

Sol:
$$(x^2 - y^2)\,dx = 2xy\,dy$$

$$(x^2 - y^2)\,dx - 2xy\,dy = 0$$

$$M = x^2 - y^2, \quad N = -2xy$$

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(x^2 - y^2), \quad \frac{\partial N}{\partial x} = -2y$$

$$\Rightarrow 0 - 2y$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\Rightarrow \int M\,dx + \int N\,dy = c$$

$$\Rightarrow \int x^2 - y^2\,dx + \int 1\,dy = c$$

$$\Rightarrow \frac{x^3}{3} + y = c$$

Scanned with OKEN Scanner

(5)
sol:-

(6)
sol:- given curve

$xy = c^2$

diff wrt ~~y~~ x

$\Rightarrow x \cdot \frac{dy}{dx} + y = 0$

$\Rightarrow \frac{dy}{dx} = -y/x$

Replacing $\frac{dy}{dx}$ by $-\frac{dx}{dy}$

$-\frac{dx}{dy} = -y/x$

$\Rightarrow xdx = ydy$

~~$xdy - ydy = 0$~~

$\Rightarrow xdx - ydy = 0$

Integrate on bs

$\Rightarrow \int xdx - ydy = 0$

$\frac{x^2}{2} - \frac{y^2}{2} = 0$

$x^2 - y^2 = 2$

$x^2 - y^2 = (\sqrt{2})^2$

let $\sqrt{2} = a$

So there are required trajectory of the

(4)

7) $x(x-1)\frac{dy}{dx} - y = x^2(x-1)^2$

Sol: $\div$ with $x(x-1)$

$$\Rightarrow \frac{dy}{dx} - \frac{y}{x(x-1)} = x(x-1) \quad (1)$$

It is in the form of

$$\frac{dy}{dx} + Py = Q$$

$$P = \frac{-1}{x(x-1)}, \quad Q = x(x-1)$$

$$I.F = e^{+\int P\,dx}$$

$$\Rightarrow e^{-\int \frac{1}{x(x-1)}dx}$$

$$\Rightarrow e^{-\int\left(\frac{1}{x-1} - \frac{1}{x}\right)dx}$$

$$\Rightarrow e^{\int\left(\frac{1}{x} - \frac{1}{x-1}\right)dx} \Rightarrow e^{\log x - \log(x-1)}$$

$$e^{\log_e\left(\frac{x}{x-1}\right)} \Rightarrow \frac{x}{x-1} = I.F$$

multiply Eq (1) with I.F

$$\frac{x}{x-1}\frac{dy}{dx} - \frac{yx}{x(x-1)^2} = \frac{x}{x-1}\cdot x(x-1)$$

$$\frac{x}{x-1}dy - \frac{yx}{x(x-1)^2}dx = x^2\,dx$$

Integrate on b.s

$\Rightarrow y \cdot I.F = \int Q\,(I.F)\,dx + C$

$\Rightarrow y \cdot \frac{x}{x-1} = \int x\cancel{(x-1)} \times \frac{x}{\cancel{x-1}} + C$

$\Rightarrow \frac{yx}{x-1} = \int x^2$

$$\Rightarrow \boxed{\frac{yx}{x-1} = \frac{x^3}{3}}$$

(8)

soln $e^x \frac{dy}{dx} = 2xy^2 + y \cdot e^x$

$\div e^x$

$\Rightarrow \frac{dy}{dx} = \frac{2xy^2}{e^x} + y$

$\Rightarrow \frac{dy}{dx} - y = \frac{2xy^2}{e^x}$

$\Rightarrow \frac{1}{y^2}\frac{dy}{dx} - y^{-1} = \frac{2x}{e^x}$

$\Rightarrow$ let $-\frac{1}{y} = t$

diff w.r.t "x"

$\Rightarrow \frac{d}{dx}\left(-\frac{1}{y}\right) = \frac{dt}{dx}$

$\Rightarrow -y^{-1-1}(-1) \cdot \frac{dy}{dx} = \frac{dt}{dx}$

$\Rightarrow y^{-2}\frac{dy}{dx} = \frac{dt}{dx}$

$\frac{1}{y^2}\frac{dy}{dx} = \frac{dt}{dx}$

$\Rightarrow \frac{dt}{dx} + t = \frac{2x}{e^x} \Rightarrow \frac{dt}{dx} + 1 \cdot t = \frac{2x}{e^x}$

$\frac{dy}{dx} + Py = Q$

$P = 1, \quad Q = \frac{2x}{e^x}$

Integration factor (I.F) = $e^{\int P dx}$

$= e^{\int 1 \cdot dx}$

$= e^{x}$

$t\ y \cdot (I.F) = \int Q \cdot I.F\, dx + c$

$t\ y \cdot e^{x} = \int \frac{2x}{\cancel{e^{x}}} \cdot \cancel{e^{x}}\, dx + c$

$t\ y \cdot e^{x} = \frac{2x^2}{\cancel{2}} + c$

$t\ y \cdot e^{x} = x^2 + c$

sub t value in above

$\Rightarrow \frac{-1}{y} e^{x} = x^2 + c$

$-e^{x} = yx^2 + cy$

$\Rightarrow \boxed{x^2 y + e^{x} + cy = 0}$

(9)

Sol:- To find the temperature at 35°C,

According to Newton's law of cooling the rate at which an object cools is directly proportional to the difference in the temperature of the object

$mc \frac{dT}{dt} = k(t - t_0)$

here dT means change temperature dt is change in

Here t is the average temperature, t_0 is temperature of surrounding

$$\Rightarrow mc \frac{t_1 - t_2}{t} = k\left(\frac{t_1+t_2}{2} - t_0\right)$$

$t \rightarrow$ temperature of body in air

$$\Rightarrow \frac{mc}{k} = \frac{\frac{t_1+t_2}{2} - t_0}{\frac{t_1-t_2}{t}}$$

Rough

$$\frac{60}{25} \times \frac{mc}{k} = 120-35$$

$$\frac{60}{25} \times \frac{mc}{k} = 85$$

$$\frac{mc}{k} \times \frac{85 \times 25}{60}$$

$$\Rightarrow \frac{\frac{140+80}{2} - 35}{\frac{140-80}{25}}$$

$$\frac{85}{5} \times \frac{25}{60}$$

$$\Rightarrow \frac{125}{12}$$

$$\Rightarrow \frac{240 - 70}{\frac{2}{\frac{60}{25}}} \Rightarrow \frac{170}{2} \times \frac{25}{60}$$

$$\Rightarrow \boxed{35.4}$$

$$\frac{mc}{k} \quad \frac{120-35}{\frac{60}{25}}$$

$$\frac{85 \times 25}{60}$$

35.4

now for 2^{nd} condition

$$mc \times \frac{t_1 - t_2}{t} = k\left(\frac{t_1+t_2}{2} - t_0\right)$$

$$\Rightarrow \frac{mc}{k} \times \frac{140-80}{t} = \frac{140+80}{2} - 25$$

$$\Rightarrow 35.4 \times \frac{60}{t} = \frac{240-50}{2} = \frac{190}{2}$$

$$\Rightarrow 35.4 \times \frac{60}{t} = \frac{190}{2}$$

$$\Rightarrow 19t = 35.4 \times 12$$

Scanned with OKEN Scanner

(9)

Sol:-

temp of air $\theta_0 = 25°C$

At $t = 0$ min, $\theta = 140°C$ (temperature of body) cools down to $80°$)

At $t = 20$ min, $T = 80°C$

$$\boxed{T = T_0 + e^{-kt+c}}$$ According to Newton's law of Cooling

Case : I

$$140 = 25 + e^{0+c}$$

$$e^c = \cancel{1}15$$

$$e^c = 115°C$$

Case :- II

$$80 = 25 + e^{-kt} \cdot e^c$$

$$\Rightarrow 80 = 25 + e^{-k \times 20} \cdot 115$$

$$\Rightarrow \frac{55}{115} = e^{-20k}$$

$$\Rightarrow -20k = \log \frac{55}{115}$$

$$\therefore k = \frac{-1}{20} \log_e \frac{55}{115}$$

Scanned with OKEN Scanner

Case :- (iv)

At what time the temperature of Body cools down to 35^0C

$T = 35$, $T_0 = 25^0$

$\Rightarrow T = T_0 + e^{-kt + c}$

$\Rightarrow 35 = 25 + e^{-kt} \cdot e^{c}$

$\Rightarrow 10 = e^{-\left(-\frac{1}{20}\log\frac{55}{115}\right)\times t} \cdot 115$

$\Rightarrow 10 = e^{\frac{t}{20}\log\frac{55}{115}} \cdot 115$

$\Rightarrow \frac{10}{115} = e^{t/20 \log\frac{55}{115}}$

$\Rightarrow \frac{t}{20}\log_e\frac{55}{125} = \log_e\frac{10}{115}$

$\Rightarrow \frac{t}{20} = \frac{\log\frac{10}{115}}{\log\frac{55}{125}} \Rightarrow \frac{\log 10 - \log 115}{\log 55 - \log 125}$

$\Rightarrow \frac{t}{20} = \frac{1 - 2.06}{\log 55 - 3\log 5}$

$\frac{t}{20} = 3.31$ $\quad \frac{-1.06}{1.740 - 3\times 0.6} \Rightarrow \frac{\cancel{-}1.06}{\cancel{-}0.3}$

$\boxed{t = 66.2 \text{ min}}$ //

$\Rightarrow \cong 3.31$

(10)

Sol:- $x(1-x^2)\frac{dy}{dx} + (2x^2-1)y = x^3$

$\div$ with $x(1-x^2)$

$$\Rightarrow \frac{dy}{dx} + \frac{2x^2-1}{x(1-x^2)}y = \frac{x^3}{x(1-x^2)}$$

Compare with

$$\frac{dy}{dx} + Py = Q$$

$$P = \frac{2x^2-1}{x(1-x^2)}, \quad Q = \frac{x^3}{x(1-x^2)}$$

$$I.F = e^{\int P\,dx}$$

$$e^{\underbrace{\int \frac{2x^2-1}{x(1-x^2)}dx}_{y}}$$

let

$$y = \int \frac{2x^2-1}{x(1-x^2)}dx$$

$$\Rightarrow \int \frac{2x^{\cancel{2}}}{\cancel{x}(1-x^2)}dx - \int \frac{1}{x(1-x^2)}dx$$

$$\Rightarrow \int \frac{2x}{\underbrace{(1-x^2)}_{t}}dx - \int \frac{1}{x(1-x^2)}dx$$

$$\Rightarrow 1-x^2 = t$$

$$\Rightarrow -2x\,dx = dt$$

$$\Rightarrow \int \frac{-dt}{t} = \int \frac{dx}{x(1^2-x^2)}$$

$$\Rightarrow \int -\frac{dt}{t} = \int \frac{1}{x(1-x)(1+x)}$$

$$\Rightarrow \int \frac{-dt}{t} = \int \frac{(1-x)+x}{x(1-x)(1+x)}\,dx$$

$$\Rightarrow -\int \frac{1}{t}\,dt = \int \frac{\cancel{1-x}}{x(\cancel{1-x})(1+x)}\,dx + \int \frac{x}{x(1-x)(1+x)}\,dx$$

$$\Rightarrow -\int \frac{1}{t}\,dt = \int \frac{1}{x(1+x)}\,dx + \int \frac{1}{(1-x)(1+x)}\,dx$$

$$\Rightarrow -\log|t| = \int \frac{(x+1)-x}{x(1+x)} + \int \frac{1}{1-x^2}\,dx$$

$$\Rightarrow -\log|t| = \int \left(\frac{1}{x} - \frac{1}{1+x}\right)dx + \frac{1}{2(1)} \log\left|\frac{1+x}{1-x}\right|$$

$$\Rightarrow -\log(1-x^2) = \log x - \log(1+x) \quad \left[\int \frac{1}{a^2-x^2}\,dx = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right|\right.$$

$$+ \frac{1}{2}\log\left(\frac{1+x}{1-x}\right)$$

$$\Rightarrow \log(1+x) - [\log(1-x^2) + \log x] = \frac{1}{2}\log\left|\frac{1+x}{1-x}\right|$$

$$\Rightarrow \cancel{\frac{\log(1+x)}{(\log(1-x^2)+\log x)} = \frac{1}{2}\log\left|\frac{1+x}{1-x}\right|}$$

$\Rightarrow \cancel{\log\left(\frac{1+x}{1}\right)}$

$\Rightarrow \log(1+x) - [\log x(1-x^2)] - \frac{1}{2}\log\frac{1+x}{1-x}$

$\Rightarrow \log\left|\left(\frac{1+x}{x(1-x^2)}\right)\right| - \frac{1}{2}\log\left|\frac{1+x}{1-x}\right|$

$\Rightarrow \log\left|\frac{1+x}{x(1-x^2)}\right| - \log\left|\left(\frac{1+x}{1-x}\right)\right|^{1/2}$

$\Rightarrow \log\left|\frac{\frac{1+x}{x(1-x^2)}}{\left(\frac{1+x}{1-x}\right)^{1/2}}\right| \Rightarrow \log\left|\frac{1+x}{x(1-x^2)} \times \left(\frac{1-x}{1+x}\right)^{1/2}\right|$

$\Rightarrow \log\left|\frac{\cancel{1+x}^{\sqrt{1+x}}}{x(\cancel{1+x})(\cancel{1-x})_{\sqrt{1+x}\sqrt{1-x}}} \times \frac{\cancel{\sqrt{1-x}}}{\cancel{\sqrt{1+x}}}\right|$

$\Rightarrow \log\left|\frac{1}{x\sqrt{1+x}\sqrt{1-x}}\right|$

$\Rightarrow \log\left|\frac{1}{x(1-x^2)^{1/2}}\right|$

$I.F = e^{\log\left|\frac{1}{x(1-x^2)^{1/2}}\right|}$

$= \frac{1}{x(1-x^2)^{1/2}}$

$$y.(I.F) = \int (I.F) \times (Q)\,dx + c$$

$$\Rightarrow y \cdot \frac{1}{x\sqrt{(1-x^2)}} = \int \frac{1}{x\sqrt{1-x^2}} \times \frac{ax^2}{1-x^2}\,dx + c$$

$$\Rightarrow y \cdot \frac{1}{x(1-x^2)^{1/2}} = a\int \frac{x}{(1-x^2)^{3/2}}\,dx + c$$

let

$$1-x^2 = t$$

$$-2x\,dx = dt$$

$$x\,dx = \frac{-dt}{2}$$

$$\Rightarrow \frac{y \cdot 1}{x(1-x^2)^{1/2}} = a\int \frac{-dt}{2(t)^{3/2}} + c$$

$$\Rightarrow \frac{y}{x(1-x^2)^{1/2}} = \frac{-a}{2}\left[\frac{t^{-3/2+1}}{\frac{-3}{2}+1}\right]$$

$$\Rightarrow \frac{y}{x(1-x^2)^{1/2}} = \frac{-a}{2} \cdot \frac{t^{-1/2}}{-1/2} + c$$

$$\Rightarrow \frac{y}{x(1-x^2)^{1/2}} = +a \cdot t^{1/2} + c$$

$$\Rightarrow \frac{y}{x(1-x^2)^{1/2}} = a \cdot \sqrt{1-x^2} + c$$

multiply $x\sqrt{1-x^2}$ on b.s

$$\frac{y\,(\cancel{x\sqrt{1-x^2}})}{\cancel{x\sqrt{1-x^2}}} = a\,x\sqrt{1-x^2}\cdot\sqrt{1-x^2} + c\,(x\sqrt{1-x^2})$$

$$\boxed{y = ax\cdot(1-x^2) + c\,x\sqrt{1-x^2}}$$

(11)

Sol:- $\frac{dy}{dx}(x^2y^3 + xy) = 1$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{x^2y^3+xy}$$

$$\Rightarrow \frac{dx}{dy} = x^2y^3 + xy$$

$$\Rightarrow \frac{dx}{dy} - xy = x^2y^3$$

$$\Rightarrow \frac{1}{x^2}\frac{dx}{dy} - \frac{y\,x}{x^2} = y^3$$

$$\Rightarrow \frac{1}{x^2}\frac{dx}{dy} - \frac{y}{x} = y^3$$

let $1/x = u$ $\Rightarrow$ diff on bs w.r.t y $\quad -\frac{1}{x^2}\frac{dx}{dy} = \frac{du}{dy}$

$$\cancel{u^2\frac{dx}{dy}} - \cancel{uy} = \cancel{y^3}$$

$$\Rightarrow -\frac{du}{dy} - uy = y^3$$

$$\Rightarrow \frac{du}{dy} + uy = -y^3$$

$$\Rightarrow \frac{du}{dy} + (y)u = -y^3$$

$P = y,\ Q = -y^3$

$I.F = e^{\int y\,dy}$

$= e^{y^2/2}$

$\Rightarrow u\cdot(I.F) = \int Q\cdot I.F\,dx + C$

$\Rightarrow u\cdot(e^{y^2/2}) = \int -y^3 \cdot e^{y^2/2}\,dx + C$

But Here y's u

So replace y with term u

$\Rightarrow u\cdot e^{y^2/2} = \int -y^3 \cdot \int e^{y^2/2} - (-3y^2)\iint e^{y^2/2} + (-6y)\iiint e^{y^2/2}$

$- (-6)\iiiint e^{y^2/2}$

$+0$

$\Rightarrow u\cdot e^{y^2/2} = -y^3 \cdot e^{y^2/2} \cdot \frac{1}{2}\cdot\frac{y}{3} + 3y^2\cdot\frac{1}{6}\cdot\int e^{\frac{y^2}{2}}\cdot y^3 - 6y\iint e^{y^2/2}\cdot y^3$

$+6\iiiint e^{y^2/2}$

$\Rightarrow u\cdot e^{y^2/2} = -\frac{1}{6}\cdot e^{y^2/2}\cdot y^6 + \frac{y^2}{2} + \dots$

$\Rightarrow u\cdot e^{y^2/2} = -2\left(\frac{y^2}{2} - 1\right) e^{y^2/2} + C$

$\Rightarrow$ $\div$ whole expression with $e^{y^2/2}$

$$\frac{u \cdot \cancel{e^{y^2/2}}}{\cancel{e^{y^2/2}}} = -2\frac{\left(\frac{y^2}{2}-1\right)e^{y^2/2}}{e^{y^2/2}} + \frac{c}{e^{y^2/2}}$$

$$\Rightarrow u = -2\left(\frac{y^2}{2}-1\right) + \frac{c}{e^{y^2/2}}$$

$$\Rightarrow \frac{1}{x} = -2\left(\frac{y^2}{2}-1\right) + \frac{c}{e^{y^2/2}}$$

$\Rightarrow$ multiply x on b.s

$$\Rightarrow \frac{\cancel{x}}{\cancel{x}} = -2\left(\frac{y^2}{2}-1\right)x + \frac{c \cdot x}{e^{y^2/2}}$$

$$\Rightarrow 1 = (-y^2+2)x + c \cdot x \cdot e^{-y^2/2}$$

$$\Rightarrow \boxed{1 = (2-y^2)x + cxe^{-y^2/2}}$$

(12.)

Sol:

$$\frac{2dy}{dx} - y\sec x = y^3 \tan x$$

$\div$ with 2

$$\frac{dy}{dx} - \frac{y\sec x}{2} = \frac{y^3 \tan x}{2}$$

$$P = -\frac{\sec x}{2}, \quad Q = \frac{y^3 \tan x}{2}$$

Scanned with OKEN Scanner

$\cos x \quad \cos x \qquad \overline{\cos^2 x}$

$$y \cdot (\sec x + \tan x) = \frac{1}{2}\int \frac{\sin x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x}\, dx + c$$

$$= \frac{1}{2}\int \frac{1}{\cos^2 x}(\sin x + \sin^2 x)\, dx + c$$

(or)

(12) sol:-

$$\Rightarrow \frac{2dy}{dx} - y\sec x = y^3 \tan x$$

$$\Rightarrow \frac{2}{y^3}\frac{dy}{dx} - \frac{\sec x}{y^2} = \tan x$$

$\div$ with 2

$$\Rightarrow \frac{dy}{dx}\cdot\frac{1}{y^3} - \frac{\sec x}{2y^2} = \frac{\tan x}{2}$$

let $1/y^2 = v$

$$-2/y^3 \cdot \frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{1}{y^3}\frac{dy}{dx} = -\frac{1}{2}\frac{dv}{dx}$$

$$\Rightarrow -\frac{1}{2}\frac{dv}{dx} - \frac{\sec x}{2}\cdot v = \tan x$$

$$\Rightarrow \frac{1}{2}\frac{dv}{dx} + \frac{\sec x \cdot v}{2} = -\tan x$$

$P = \sec x$, $Q = -\tan x$

$I.F = e^{\int \sec x\, dx} = e^{\ln(\sec x + \tan x)} = \sec x + \tan x$

$V \cdot I.F = \int Q \cdot I.F\, dx + c$

$V \cdot (\sec x + \tan x) = \int -\tan x \cdot (\sec x + \tan x)\, dx + c$

$= -\int (\sec x \tan x + \tan^2 x)\, dx + c$

$= -\int (\sec x \tan x + \sec^2 x - 1)\, dx$

$V(\sec x + \tan x) = -[\sec x + \tan x - x] + c$

$\Rightarrow \frac{1}{y^2}(\sec x + \tan x) = -(\sec x + \tan x) + x + c$

$\Rightarrow (\sec x + \tan x)\frac{1}{y^2} + \sec x + \tan x = x + c$

$\Rightarrow \boxed{\sec x + \tan x\left(\frac{1}{y^2} + 1\right) = x + c}$

(3)

sol: $(1-x^2)\frac{dy}{dx} + xy = y^3 \sin^{-1}x$

$\div$ with $1-x^2$

$$\frac{dy}{dx} + \frac{x}{1-x^2}y = \frac{y^3 \sin^{-1}x}{1-x^2}$$

$\div$ with y^3

$$\Rightarrow \frac{1}{y^3}\frac{dy}{dx} + \frac{x}{1-x^2}\cdot\frac{y}{y^3} = \frac{\sin^{-1}x}{\cancel{y^3}(1-x^2)}$$

$$\Rightarrow \frac{1}{y^3}\frac{dy}{dx} + \frac{x}{1-x^2}\cdot\frac{1}{y^2} = \frac{\sin^{-1}x}{(1-x^2)}$$

let $\frac{1}{y^2} = t$

$$\Rightarrow -2y^{-3}\frac{dy}{dx} = \frac{dt}{dx}$$

$$\Rightarrow \frac{-2}{y^3}\frac{dy}{dx} = \frac{dt}{dx}$$

$$\Rightarrow \frac{dy}{dx}\cdot\frac{1}{y^3} = \frac{dt}{dx}\cdot\frac{-1}{2}$$

$$\Rightarrow -\frac{1}{2}\frac{dt}{dx} + \frac{x}{1-x^2}\cdot t = \frac{\sin^{-1}x}{\cancel{y^3}(1-x^2)}$$

multiply with -2

$$\Rightarrow -\cancel{2}\cdot-\frac{1}{\cancel{2}}\frac{dt}{dx} + \frac{-2x}{1-x^2}\cdot t = \frac{-2\sin^{-1}x}{(1-x^2)}$$

$$\Rightarrow \frac{dt}{dx} - \left(\frac{2x}{1-x^2}\right)t = \frac{-2\times\sin^{-1}x}{(1-x^2)}$$

$P = \frac{-2x}{1-x^2}$, $Q = \frac{-2\sin^{-1}x}{1-x^2}$

$I.F = e^{\int P\,dx}$ $\quad \because \int \frac{f'(x)}{f(x)} = \log(f(x))+c$

$e^{\int \frac{-2x}{1-x^2}dx}$

$e^{\log_e(1-x^2)}$

$\Rightarrow 1-x^2$

$\Rightarrow t\cdot(I.F) = \int Q\cdot I.F\,dx + C$

$\Rightarrow t\cdot(1-x^2) = \int \frac{-2\sin^{-1}x}{\cancel{1-x^2}}\cdot \cancel{1-x^2}\,dx + C$

$\Rightarrow t(1-x^2) = -2\cdot\int \sin^{-1}x\,dx + C$ $\quad \because \int \sin^{-1}x = x\sin^{-1}x + \sqrt{1-x^2}$

$\Rightarrow t(1-x^2) = -2[x\sin^{-1}x + \sqrt{1-x^2}] + C$

$\Rightarrow \boxed{\frac{1}{y^2}(1-x^2) = -2[+x\sin^{-1}x + \sqrt{1-x^2}] + C}$

(14) *

Sol: $(xy^2 - x^2)dx + (3x^2y^2 + x^2y - 2x^3)dy = 0$

let the Equation is in the form of $Mdx + Ndy = 0$.

x^2y^2

(14)

Sol:- $(xy^2 - x^2)\,dx + (3x^2y^2 + x^2y - 2x^3 + y^2)\,dy = 0$ — (1)

$M = xy^2 - x^2$, $N = 3x^2y^2 + x^2y - 2x^3 + y^2$

$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(xy^2 - x^2)$

$\Rightarrow x\cdot(2y) - 0$

$= 2xy$

$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}(3x^2y^2 + x^2y - 2x^3 + y^2)$

$= 3y^2\cdot(2x) + y\cdot(2x) - 2\cdot(3x^2) + 0$

$= 6xy^2 + 2xy - 6x^2$

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

given Eqn is Non-exact

&

Non-Homogeneous

Scanned with OKEN Scanner

The Equation ① is not in the form of

$$y\, f(x,y)\,dx + x\, g(xy)\, dy = 0$$

So,

$$\Rightarrow \frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) = \frac{1}{xy^2 - x^2}\left[6xy^2 + 2xy - 6x^2 - 2xy\right]$$

$$\Rightarrow \frac{1}{xy^2 - x^2}\left[6xy^2 + \cancel{2xy} - 6x^2 - \cancel{2xy}\right]$$

$$\Rightarrow \frac{1}{\cancel{xy^2 - x^2}}\, 6\left[\cancel{xy^2 - x^2}\right]$$

$$= 6$$

Integrating factor $= e^{\int 6\,dy} = e^{6\int dy}$

$\therefore I.F = e^{\int k\,dy}$

$\Rightarrow \cancel{e^{6y}}$

e^{6y}

multiply eq① with I.F

$$\Rightarrow e^{6y}\left[(6y^2 - x^2)dx + (3x^2y^2 + x^2y - 2x^3 + y^2)dy\right] = 0$$

$$\Rightarrow (xy^2e^{6y} - x^2e^{6y})\,dx + \left[3x^2y^2e^{6y} + x^2ye^{6y} - 2x^3e^{6y} + y^2e^{6y}\right]dy = 0$$

$\textcircled{1}$ $M_1 dx + N_1 dy = 0$

$\Rightarrow M_1 = xy^2e^{6y} - x^2e^{6y}$, $N_1 = 3x^2y^2e^{6y} + x^2ye^{6y} - 2x^3e^{6y} + y^2e^{6y}$

$$\cancel{\frac{\partial M_1}{\partial y} = \frac{\partial N_1}{\partial}}$$

$$\frac{\partial M_1}{\partial y} = \frac{\partial}{\partial y}\left(xy^2e^{6y} - x^2e^{6y}\right)$$

$\Rightarrow x\frac{\partial}{\partial y}(y^2e^{6y}) - x^2 \cdot e^{6y} \cdot 6$

$\Rightarrow$

$$\frac{\partial M_1}{\partial y} = x\left[y^2 \cdot e^{6y} \cdot 6 + e^{6y} \cdot 2y\right] - 6x^2 \cdot e^{6y}$$

$\Rightarrow 6xy^2e^{6y} + 2xye^{6x} - 6x^2e^{6y}$

$$\frac{\partial N_1}{\partial x} = \frac{\partial}{\partial x}\left(3x^2y^2e^{6y} + x^2ye^{6y} - 2x^3e^{6y} + y^2e^{6y}\right)$$

$\Rightarrow 3y^2e^{6y} \cdot 2x + ye^{6y} \cdot 2x - 2e^{6y} \cdot 3x^2 + 0$

$$\frac{\partial N_1}{\partial x} = 6xy^2e^{6y} + 2xye^{6y} - 6x^2e^{y}$$

$$\frac{\partial M_1}{\partial y} = \frac{\partial N_1}{\partial x}$$

$\Rightarrow \underbrace{\int M_1 \, dx}_{y \text{ is Const}} + \underbrace{\int N_1 \, dy}_{\text{free from "x" term}}$

$\Rightarrow \int\left(xy^2e^{6y} - x^2e^{6y}\right)dx + \int\left(\cancel{3x^2y^2e^{6y}} + \cancel{x^2ye^{6y}} - \cancel{2x^3e^{6y}} - y^2e^{6y}\right)dx$

$$\Rightarrow y^2 e^{6y}\int x\,dx - e^{6y}\int x^2\,dx + \int -y^2 e^{6y}\,dy$$

$$\Rightarrow y^2 e^{6y}\cdot\frac{x^2}{2} - e^{6y}\cdot\frac{x^3}{3} - \int \underbrace{y^2}_{u}\underbrace{e^{6y}dy}_{dv} = c$$

$$\Rightarrow \frac{x^2y^2}{2}e^{6y} - \frac{x^3}{3}e^{6y} \cancel{\int y^3\, 6dy} = c$$

assume y^2 as "u"

$\Rightarrow$ $e^{6y}dy$ as "dv"

$\cancel{\int e^{6y}dy = v}$

$\Rightarrow \cancel{\frac{e^{6y}}{6} = dv}$

$\cancel{e^{6y} = 6\,dv}$

$u = y^2$	$dv = e^{6y}dy$
$u' = 2y$	$v = \int e^{6y}dy$
$u'' = 2$	$= e^{6y}/6$
$u''' = 0$	$v_1 = \frac{1}{6}e^{6y}\cdot\frac{1}{6} = \frac{1}{36}e^{6y}$
	$v_2 = \frac{1}{36}\cdot\frac{e^{6y}}{6}$

$[\int u\,dv = uv - u'v_1 + u''v_2 - u'''v_3 - - -]$

$$\int u\,dv = \int \underbrace{y^2}_{u}\cdot\underbrace{e^{6y}dy}_{dv} = y^2\left(\frac{e^{6y}}{6}\right) - 2y\left(\frac{1}{36}\cdot e^{6y}\right) + 2\left(\frac{1}{36}\frac{e^{6y}}{6}\right) + 0 - -$$

$$\Rightarrow \frac{y^2e^{6y}}{6} - \frac{ye^{6y}}{18} + \frac{e^{6y}}{108}$$

$\therefore$ The general solution of (1) is

$$\Rightarrow \frac{x^2y^2}{2}e^{6y} - \frac{x^3}{3}e^{6y} - \left[\frac{y^2e^{6y}}{6} - \frac{ye^{6y}}{18} + \frac{e^{6y}}{108}\right] = c$$

(15)

Sol: $x^2\frac{dy}{dx} = e^y - x$

$\div$ Both sides by x^2

$$\frac{dy}{dx} = \frac{e^y}{x^2} - \frac{1}{x}$$

let $e^y = t$

$$\frac{dy}{dx} = \frac{t}{x^2} - \frac{1}{x}$$

$$x^2\frac{dy}{dx} + x = e^y$$

$$x^2dy + xdx = e^ydx$$

$\div x^2e^y$

$$\frac{dy}{e^y} + \frac{dx}{xe^y} = \frac{dx}{x^2}$$

(16)

Sol: $$\frac{dy}{dx} + \frac{y\cos x + \sin y + y}{\sin x + x\cos y + x} = 0$$

$\Rightarrow (y\cos x + \sin y + y)dx + (\sin x + x\cos y + x)dy = 0$

$M = y\cos x + \sin y + y$, $N = \sin x + x\cos y + x$

$$\frac{\partial M}{\partial y} = \cos x + \cos y + 1, \quad \frac{\partial N}{\partial x} = \cos x + \cos y + 1$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\underbrace{\int M\,dx}_{y\text{ const}} + \underbrace{\int N\,dy}_{\text{free from } x \text{ terms}}$$

$$\int y\cos x + \sin y + y + \int \sin x + \cancelto{0}{x\cos y + x} = c$$

$$\boxed{y \cdot \sin x + x \sin y + xy = C}$$

17

Sol: $x^2 + y^2 + 2gx + c = 0$ —— (1)

to get the Eq^n

diff w.r.t x

$$2x + 2y \cdot \frac{dy}{dx} + 2g = 0$$

$$\Rightarrow \quad 2g = -\left(2x + 2y\left(\frac{dy}{dx}\right)\right)$$

Sub $2g$ in Eq (1)

$$x^2 + y^2 - \left(2x + 2y\frac{dy}{dx}\right)x + c = 0$$

$$\Rightarrow x^2 + y^2 - 2x^2 - 2xy\frac{dy}{dx} + c = 0$$

replace $\frac{dy}{dx}$ by $-\frac{dx}{dy}$

$$x^2 + y^2 - 2x^2 - 2xy\left(-\frac{dx}{dy}\right) + c = 0$$

$$\Rightarrow y^2 - x^2 + 2xy\frac{dx}{dy} + c = 0$$

$$\Rightarrow y^2 dy - x^2 dy + 2xy\,dx + c\,dy = 0$$

$$\Rightarrow 2xy\,dx + (y^2 - x^2 + c)dy = 0$$ —— (2)

$M = 2xy$, $N = y^2 - x^2 + c$

$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(2xy)$, $\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}(y^2 - x^2 + c)$

$= 2x$, $0 - 2x$

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

$\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)\frac{1}{M} = (-2x - 2x)\frac{1}{2xy}$

$-4x \times \frac{1}{2xy} = -\frac{2}{y}$

$I.F = e^{\int -\frac{2}{y} dy}$

$= e^{-2\log y} = e^{\log y^{-2}}$

$\Rightarrow y^{-2}$

multiplying I.F with Equation in (2)

$I.F\,[2xy\,dx + (y^2 - x^2 + c)dy]$

$\Rightarrow \frac{1}{y^2}[2xy\,dx + (y^2 - x^2 + c)dy] = 0$

$\Rightarrow \frac{2x}{y}dx + \left(1 - \frac{x^2}{y^2} + \frac{c}{y^2}\right)dy = 0$ — (3)

It is exact first-order diff eqⁿ, so solution is

$\Rightarrow \int\limits_{y\,\text{Const.}} M\,dx + \int (\text{terms not containing } x \text{ terms})\, dy = c$

$$\Rightarrow \int \frac{2x}{y}\,dx + \int \left(1+\frac{c}{y^2}\right)dy = c$$

$$\Rightarrow \frac{\cancel{2}x^2}{\cancel{2}y} + y + c\,\frac{y^{-2+1}}{-2+1}$$

$$\Rightarrow \frac{x^2}{y} + y + c\cdot\frac{y^{-1}}{-1} = c$$

$$\frac{x^2}{y} + y + \frac{c}{y} = c$$

$$x^2 + y^2 - c = cy \Rightarrow x^2 + y^2 - cy - c = 0 \quad (4)$$

$\Rightarrow$ The general Equation of circle is

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

Compare eq (4) and general eq^n

$$x^2 + y^2 - (-2f)y - c = 0$$

$$\boxed{x^2 + y^2 + 2fy - c = 0}$$

(a)
18) Sol:

$$1+x\,\frac{dy}{dx} - y = e^{3x}(1+x)^2$$

$\div (1+x)$ on both sides

Scanned with OKEN Scanner

$$\frac{dy}{dx} - \frac{y}{(1+x)} = e^{3x}(1+x)$$

Compare with $\frac{dy}{dx} + Py = Q$

$$P = -\frac{1}{(1+x)}, \quad Q = e^{3x}(1+x)$$

$$I.F = e^{\int P dx} = e^{\int \frac{-1}{(1+x)} dx}$$

$$= e^{-\log(1+x)} = e^{\log_e (1+x)^{-1}}$$

$$= (1+x)^{-1} = 1/1+x$$

$$\Rightarrow y.(I.F) = \int Q . IF \, dx + c$$

$$y . \frac{+1}{(1+x)} = \int e^{3x}\cancel{(1+x)} . \frac{1}{\cancel{1+x}} + c$$

$$y\left(\frac{1}{1+x}\right) = \frac{e^{3x}}{3} + c$$

$$\boxed{y = \frac{e^{3x}}{3}(1+x) + C(x+1)}$$

19)

Sol:- $(x^2-ay)\,dx = (ax-y^2)\,dy$

$\Rightarrow (x^2-ay)\,dx - (ax-y^2)\,dy = 0 \quad \text{---}(1)$

$M = x^2-ay \quad \Big| \quad N = -(ax-y^2)$

$\frac{\partial M}{\partial y} = 0-a \quad , \quad \frac{\partial N}{\partial x} = -a+0$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

$\underset{\substack{\downarrow \\ y \text{ const}}}{\int M\,dx} + \underset{\substack{\downarrow \\ \text{removing } x \text{ terms}}}{\int N\,dy} = C$

$\int x^2-ay\,dx + \int y^2\,dy = c$

$\Rightarrow \frac{x^3}{3} - ay \cdot x + \frac{y^3}{3} = c$

$\cancel{x^2} - \cancel{xdy}$ $\boxed{x^3 - 3ayx + y^3 = 3c}$

20)
Sol: $y(x^2y^2+2)dx + x(2-2x^2y^2)dy = 0$ — ①

$\Rightarrow (x^2y^3+2y)dx + (2x - 2x^3y^2)dy = 0$

$M = x^2y^3+2y$, $N = 2x - 2x^3y^2$

$\frac{\partial M}{\partial y} = x^2 \cdot 3y^2 + 2$ $= 2 + 3x^2y^2$, | $\frac{\partial N}{\partial x} = 2 - 2y^2 \cdot 3x^2$ $= 2 - 6x^2y^2$

So we know when

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ ∴ Non exact & Non Homogeneous

~~the integrating factor is $\frac{1}{Mx+Ny}$~~

~~$\Rightarrow \frac{1}{(x^2-ay)x + (-ax+y^2)y}$~~

~~$\Rightarrow \frac{1}{x^3 - axy - axy + y^3}$~~

the Equation ① is in the form of

$y \cdot f_1(x,y)dx + x \cdot f_2(x,y)dy = 0$

So Integrating factor for Equation ① is

$I.F = \frac{1}{Mx - Ny}$

$$\frac{1}{(x^2y^3+2y)x-(2x-2x^3y^2)y}$$

$$\Rightarrow \frac{1}{x^3y^3+\cancel{2xy}-\cancel{2xy}+2x^3y^3}$$

$$\Rightarrow \frac{1}{3x^3y^3} = I.F$$

multiply I.F with Eq ①

$$\frac{1}{3x^3y^3}\left[y(x^2y^2+2)\,dx + x(2-2x^2y^2)\,dy\right]=0 \quad ②$$

multiply ② with 3 on both sides

$$\Rightarrow \frac{\cancel{3}}{\cancel{3}x^3y^3}\left[y(x^2y^2+2)\,dx + x(2-2x^2y^2)\,dy\right]=0$$

$$\Rightarrow \frac{1}{x^3y^3}\left[(x^2y^3+2y)\,dx+(2x-2x^3y^2)\,dy\right]=0$$

$$\Rightarrow \left(\frac{x^2y^3+2y}{x^3y^3}\right)dx + \left(\frac{2x-2x^3y^2}{x^3y^3}\right)dy = 0$$

$$\Rightarrow \left(\frac{\cancel{x^2y^3}}{\cancel{x^3y^3}}+\frac{2y}{x^3y^3}\right)dx + \left(\frac{2x}{x^3y^3}-\frac{2\cancel{x^3}y^2}{\cancel{x^3}y^3}\right)dy = 0$$

$$\Rightarrow \left(\frac{1}{x}+\frac{2}{x^3y^2}\right)dx + \left(\frac{2}{x^2y^3}-\frac{2}{y}\right)dy = 0$$

$$M_1\,dx + N_1\,dy = 0$$

$$M_1 = \frac{1}{x} + \frac{2}{x^3y^2}, \quad N_1 = \left(\frac{2}{x^2y^3} - \frac{2}{y}\right)$$

$$\Rightarrow \frac{\partial M_1}{\partial y} = \frac{\partial}{\partial y}\left(\frac{1}{x} + \frac{2}{x^3y^2}\right)$$

$$\Rightarrow 0 + \frac{2}{x^3}\frac{\partial(y^{-2})}{\partial y}$$

$$\Rightarrow \frac{2}{x^3} \cdot -2y^{-3}$$

$$\Rightarrow \frac{-4}{x^3y^3}$$

$$\frac{\partial N_1}{\partial x} = \frac{2}{y^3} \cdot -2x^{-3} - 0$$

$$= \frac{-4}{x^3y^3}$$

$$\frac{\partial M_1}{\partial y} = \frac{\partial N_1}{\partial x}$$

then we know that

$$\underbrace{\int M_1\,dx}_{y\ \text{Const}} + \underbrace{\int N_1\,dy}_{\text{free from } x \text{ terms}} = c$$

$$\int\left(\frac{1}{x} + \frac{2}{y^2}\cdot\frac{1}{x^3}\right)dx + \int -\frac{2}{y}\,dy = c$$

$$\Rightarrow \int \frac{1}{x}\,dx + \int \frac{2}{y^2}\cdot\frac{1}{x^3}\,dx + -2\int \frac{1}{y}\,dy = c$$

$$\log x + \frac{2}{y^2} \cdot \frac{x^{-3+1}}{-3+1} - 2\log y = c$$

$$\log x + \frac{\cancel{2}}{y^2}\left(-\frac{x^{-2}}{\cancel{2}}\right) - 2\log y = c$$

$$\Rightarrow \log x - \frac{1}{x^2 y^2} - 2\log y = c$$

$$\Rightarrow \log x - \log y^2 - \frac{1}{x^2 y^2} = c$$

$$\boxed{\log\left(\frac{x}{y^2}\right) - \frac{1}{x^2 y^2} = C}$$