MODULE-II

PART-A

1. Solve the differential equation $(D^2+4)y = \sin 2x$.

Sol) It is in the form of $f(D)y = Q(x)$

$y(x) = y_c + y_p$ — ①

Auxilary equation

$f(m) = 0$.

$m^2+4=0$

$m^2=-4$

$m=\pm 2i$

$m_1 = 2i$, $m_2 = -2i$

roots are different and complex.

$\alpha+\beta i = e^{\alpha x}(\cos\beta x + \sin\beta x)$

$y_c = e^{0x}(\cos 2x + \sin 2x)$

$y_c = \cos 2x + \sin 2x$ — ②

Particular Integral

$P.I = x\dfrac{1}{f'(D)}Q(x)$.

$f(D)=0$

$f'(D)\neq 0$.

$f'(D) = f'(-a^2)$

$P.I = x \times \dfrac{1}{2D}(\sin 2x)$

$P.I = x \times \dfrac{1}{2(-4)}\sin 2x$.

$PI = -\dfrac{x\sin 2x}{8}$

$y_p = -\dfrac{x\sin 2x}{8}$ — ③

from eq ①, eq ② and eq ③

$y(x) = y_c + y_p$

$y(x) = \cos 2x + \sin 2x - \dfrac{x\sin 2x}{8}$.

2. Apply the method of variation of parameters to solve $(D^2-2D)y = e^x\sin x$.

It is in the form $f(D)y = Q(x)$ $\boxed{R(x) = e^x\sin x}$.

$y(x) = y_c + y_p$ — ①

A.E : $f(D) = (D^2-2D)y = 0$

$f(m) = 0$

$f(m) = m^2-2m=0$

$m(m-2)=0$.

$m = 0, 2$

roots are different

$y_c = c_1e^{m_1x} + c_2e^{m_2x}$

$= c_1e^{0x} + c_2e^{2x}$

$= c_1 + c_2e^{2x}$.

$y_c = c_1u(x) + c_2v(x)$

$u(x) = 1$ $v(x) = e^{2x}$.

$u'(x) = 0$ $v'(x) = 2e^{2x}$.

for $W = \begin{vmatrix} u & v \\ u' & v' \end{vmatrix}$

$= uv' - vu'$

$= 1(2e^{2x}) - 0$

$W = 2e^{2x} \neq 0$.

$y_p = A u(x) + B v(x)$

$-A = -\int \frac{v(x) R(x)}{W} dx$

$-A = -\int \frac{e^{2x}(e^{x}\sin x)}{2e^{2x}} dx$

$-A = \frac{-1}{2}\int e^{x}\sin x \, dx$

$$\boxed{\int e^{ax}\sin bx \, dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)}$$

$A = \frac{-1}{2}\left[\frac{e^{x}}{1+1}(\sin x - \cos x)\right]$

$A = -\frac{e^{x}}{4}(\sin x - \cos x)$

$B = \int \frac{u(x) R(x)}{W} dx$

$= \int \frac{(1)\, e^{x} \sin x}{2e^{x}} dx$

$= \frac{1}{2}\int e^{-x}\sin x \, dx$

$\int e^{ax}\sin x = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)$

$B = \frac{1}{2}\left[\frac{e^{-x}}{1+1}(-\sin x - \cos x)\right]$

$B = \frac{+e^{-x}}{4}(-\sin x - \cos x)$

$= -\frac{e^{-x}}{4}(\sin x + \cos x)$

~~$y_p = y_c + y_p$~~

$y_p = A(u(x)) + B v(x)$

$y_p = -\frac{e^{x}}{4}(\sin x - \cos x)(1) - \frac{e^{-x}}{4}(\sin x + \cos x)$

$y_p = -\frac{e^{x}}{4}(\sin x - \cos x) - \frac{e^{x}}{4}(\sin x + \cos x)$

$y_p = -\frac{e^{x}}{4}(2\sin x)$

$y_p = -\frac{e^{x}}{2}(\sin x)$

$y(x) = y_c + y_p$

$= c_1 + c_2 e^{2x} - \frac{e^{x}}{2}\sin x$

3. Using the method of variation of parameters solve $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$.

It is in the form of

$f(D)y = Q(x)$

$y(x) = y_c + y_p$ —(1)

$A E = f(m) = 0.$

$(m^2+1) = 0$

$m = \pm i$

roots are complex

$\alpha + \beta i = e^{\alpha x}(c_1\cos\beta x + c_2\sin\beta x)$

$y_c = e^{0x}(c_1\cos x + c_2\sin x)$

$y_c = c_1\cos x + c_2\sin x$ —(2)

$y_c = c_1 u(x) + c_2 v(x)$ →

$u(x) = \cos$

$u'(x) = -\sin$

To find y

$= uv$

$W = (\cos$

$W = \cos^2$

$W = 1 + 0$

$y_p = A u($

$-A = -\int \frac{v(}{}$

$= -\int (\sin$

$= -\int (\frac{\sin}{\sin}$

$= -\int 1\, dx$

$= -x$

$B = \int \frac{u(x)}{}$

$= \int \frac{\cos}{}$

$= \int \frac{\cos}{\sin}$

$= \int \cot$

$B = \log[$

$u(x)$

$u'(x) = -\sin x \quad v'(x) = \cos x$

To find $W = \begin{vmatrix} u & v \\ u' & v' \end{vmatrix}$

$= uv' - vu'$

$W = (\cos x)(\cos x) - (\sin x)(-\sin x)$

$W = \cos^2 x + \sin^2 x$

$W = 1 \neq 0.$

$y_P = A\,u(x) + B\,v(x) \qquad \boxed{R(x) = \operatorname{cosec} x}$

$A = -\int \frac{v(x)R(x)}{W}\,\partial x$

$= -\int \frac{(\sin x)(\operatorname{cosec} x)}{1}\,\partial x$

$= -\int \frac{\sin x}{\sin x}\,\partial x$

$= -\int 1\,\partial x$

$= -x$

$B = \int \frac{u(x)R(x)}{W}\,\partial x$

$= \int \frac{(\cos x)(\operatorname{cosec} x)}{1}\,\partial x$

$= \int \frac{\cos x}{\sin x}\,\partial x$

$= \int \cot x\,\partial x$

$B = \log[\sin x]$

$y_P = -x\cos x + \log|\sin x| \sin x$ — (3)

$y(x) = y_c + y_p$

From eq (1), (2) & (3).

$y(x) = c_1\cos x + c_2\sin x - x\cos x + \log|\sin x| \sin x.$

4. Find the general solution of

$y'''' + 8y'' + 16y = 0.$

Given equation is of the form $f(D)\cdot y = 0.$

$f(D) = (D^4 + 8D^2 + 16)y = 0.$

$f(m) = 0$

Auxiliary equation

$f(m) = 0$

$m^4 + 8m^2 + 16 = 0.$

$m = \pm 2i, \pm 2i$

roots are complex and repeated twice

$y = e^{0x}(c_1 + c_2x)\cos 2x + (c_3 + c_4x)\sin 2x)$

$y = (c_1 + c_2x)\cos 2x + (c_3 + c_4x)\sin 2x.$

General solution

$= (c_1 + c_2x)\cos 2x + (c_3 + c_4x)\sin 2x.$

$x.$

$)y$

$=0$

$)=0$

$x.$

$s3x.$

$n3x.$

6. Using the method of variation of parameters solve $(D^2+4)y = \sec 2x$.

$f(D)y = Q(x)$

$y(x) = y_c + y_p$

$A.E = f(m) = 0$

$m^2+4=0$

$m^2 = -4$

$m = \pm 2i$

$... x + \tan x| + \sin x.$ $\quad \int x e^{3x} / \frac{e^{3x}}{...}$

$y_c = c_1 \cos 2x + c_2 \sin 2x.$

$c_1 u(x) + c_2 v(x)$

$u(x) = \cos 2x$

$v(x) = \sin 2x.$

$u'(x) = -2\sin 2x$

$v'(x) = 2\cos 2x.$

$W = \begin{vmatrix} u & v \\ u' & v' \end{vmatrix} = uv' - vu'$

$= (\cos 2x)(2\cos 2x) + (2\sin 2x)(\sin 2x)$

$W = 2(1)$

$y_p = -A\, u(x) + B\, v(x)$

$-A = -\int \frac{v(x)(R(x))}{W} \partial x$

$= -\int \frac{(\sin 2x)(\sec(2x))}{2} \partial x$

$\Rightarrow \frac{-1}{2}\int \frac{\sin 2x}{\cos 2x} \partial x$

$= \frac{-1}{2}\int \tan 2x$

$= \frac{+1}{4} |\log|\cos 2x)$

$B = \int \frac{u(x) R(x)}{W} \partial x$

$= \int \frac{(\cos 2x)(\sec 2x)}{2} \partial x$

$= \frac{1}{2}\int \frac{\cos 2x}{\cos 2x} \partial x$

$= \frac{1}{2}\int 1 \partial x$

$= \frac{1}{2}(x)$

$y_p = -A\, u(x) + B\, v(x)$

$y_p = \frac{1}{4}\log|\cos 2x)\cos 2x + \frac{1}{2}x(\sin 2x)$

Solution

$y = y_c + y_p$

$y = c_1 \cos 2x + c_2 \sin 2x + \frac{1}{4}\log|\cos 2x)\cos 2x + \frac{1}{2}x \sin 2x.$

7. Using the method of variation of parameters solve $(D^2+1)y = \tan x.$

It is in the form $f(D)\,y = Q(x)$

$y(x) = y_c + y_p$

$f(D)y = (D^2+1)y = 0$

Auxiliary equation

$f(m) = m^2+1 = 0$

$m = \pm i$

roots are complex

$y_c = e^{\alpha x}(c_1 \cos \beta x + c_2 \sin \beta x)$

$= e^{0x}(c_1 \cos x + c_2 \sin ...$

$y_c = c_1 \cos x + c_2 \sin x$

$= c_1 u(x) + c_2 v(x)$

$u(x) = \cos x \quad u'(x) = -\sin x$

$v(x) = \sin x \quad v'(x) = \cos x$

$W = \begin{vmatrix} u & v \\ u' & v' \end{vmatrix} = uv' - vu'$

$= \cos^2 x + \sin^2 x$

$W = 1$

$y_p = A\,u(x) + B\,v(x)$

$A = -\int \frac{v(x)R(x)}{W}\,dx$

$= -\int \frac{(\sin x)(\tan x)}{1}\,dx$

$= -\int \frac{\sin x \times \sin x}{\cos x}\,dx$

$= -\int \frac{\sin^2 x}{\cos x}\,dx$

$= -\int \frac{1-\cos^2 x}{\cos x}\,dx$

$= -\int \sec x\,dx + \int \cos x\,dx$

$A = -\log|\sec x + \tan x| + \sin x$

$B = \int \frac{u(x)R(x)}{W}\,dx$

$= \int \frac{\cos(x)(\tan(x))}{1}\,dx$

$= \int \frac{\cos(x)\sin(x)}{\cos(x)}\,dx$

$= \int \sin(x)\,dx$

$B = -\cos x$

$y_p = A(u(x)) + B(v(x))$

$\Rightarrow (\log|\sec x + \tan x| + \sin x)\cos x - \cos x(\sin x)$

$y_p = -\log|\sec x + \tan x| \times \cos x$

General solution

$y(x) = y_c + y_p$

$= c_1\cos x + c_2\sin x - \log|\sec x + \tan x|\cos x$

8) Using the method of variation Parameters solve $(D^2 - 2D + 2)y = e^x \tan x$.

$f(D)y = g(x)$

$y(x) = y_c + y_p$

$f(D)y = (D^2 - 2D + 2)y = 0$

$f(m) = 0 \quad m^2 - 2m + 2 = 0$

$m = 1+i,\ 1-i$

roots are complex

$y = e^{\alpha x}(c_1\cos\beta x + c_2\sin\beta x)$

$y_c = e^x(c_1\cos x + c_2\sin x)$

$y_c = c_1 e^x\cos x + c_2 e^x \sin x$

$y_c = c_1 u(x) + c_2 v(x)$

$u(x) = e^x\cos x$

$v(x) = e^x\sin x$

$u'(x) = -e^x\sin x + e^x\cos x$

$v'(x) = e^x\cos x + e^x\sin x$

$w = \begin{vmatrix} u & v \\ u' & v' \end{vmatrix}$

$= uv' - vu'$

$= e^x\cos x(e^x\cos x + e^x\sin x) - e^x\sin x(-e^x\sin x + e^x\cos x)$

$W = e^{2x}\cos^2 x + e^{2x}\cos x\sin x + e^{2x}\sin^2 x - e^{2x}\cos x\sin x$

$w = e^{2x}(\cos^2 x + \sin^2 x)$

$w = e^{2x}(1)$

$w = e^{2x} \neq 0$

$y_p = Au(x) + Bv(x)$

$A = -\int \frac{v(x)R(x)}{w}\partial x$

$= -\int \frac{e^x\sin x(e^x\tan x)}{e^{2x}}\partial x$

$= -\int \sin x(\tan x)\partial x$

$= -\int \frac{\sin x \times \sin x}{\cos x}\partial x$

$= -\int \frac{\sin^2 x}{\cos x}\partial x$

$= -\int \frac{1-\cos^2 x}{\cos x}\partial x$

$= -\int \frac{1}{\cos x}\partial x + \int \cos x\,\partial x$

$= -\int \sec x\,\partial x + \int \cos x\,\partial x$

$A = -\log|\sec x + \tan x| + \sin x.$

$B = \int \frac{u(x)R(x)}{w}\partial x$

$B = \int \frac{e^x\cos x(e^x\tan x)}{e^{2x}}\partial x$

$= B = \int \cos x \tan x\,\partial x$

$= B = \int \cos x \times \frac{\sin x}{\cos x}\partial x$

$B = \int \sin x\,\partial x$

$B = -\cos x$

$y_p = Au(x) + Bv(x)$

$= (-\log|\sec x + \tan x| + \sin x)e^x\cos x + -\cos x(e^x\sin x)$

$y_p = -\log|\sec x + \tan x|$

$y(x) = y_c + y_p$

$= c_1e^x\cos x + c_2e^x\sin x - \log|\sec x + \tan x|$

Q. Using the method of of variation of parameters, solve ~~$(D^2-2D+1)y = e^x\log x$~~

$(D^2-6D+9)y = \frac{e^{3x}}{x^2}$

$f(D)y = Q(x)$

$y(x) = y_c + y_p$

$f(m) = 0$

$f(m) = m^2 - 6m + 9 = 0$

$m = 3, 3.$

$y_c = (c_1 + c_2 x)e^{3x}$

$= c_1 e^{3x} + c_2 x e^{3x}.$

$u(x) = e^{3x}$ $\quad v(x) = x e^{3x}$

$u'(x) = 3e^{3x}$ $\quad v'(x) = 3x e^{3x} + e^{3x}$

$W = \begin{vmatrix} u & v \\ u' & v' \end{vmatrix} = uv' - vu'$

$= e^{6x} \neq 0.$

$y_p = Au(x) + Bv(x)$

$A = -\int \frac{v(x)R(x)}{W} \partial x$

$= -\int \frac{x e^{3x}\left(\frac{e^{3x}}{x^2}\right)}{e^{6x}} \partial x$

$= -\int \frac{1}{x} \partial x$

$= -\log x$

$B = \int \frac{u(x)R(x)}{W} \partial x$

$= \int \frac{e^{3x} \times \frac{e^{3x}}{x^2}}{e^{6x}} \partial x$

$= \int \frac{1}{x^2} \partial x$

$= -\frac{1}{x}$

$y_p = Au(x) + Bv(x)$

$= -\log x(e^{3x})$
$\quad -\frac{1}{x} x e^{3x}$

$= -e^{3x}(\log x + 1)$

$y(x) = y_c + y_p$

$= (c_1 + c_2 x)e^{3x} -$
$e^{3x}\log x - e^{3x}$

10. Using the method of variation of parameters

Solve $(D^2 - 2D + 1)y = e^x \log x$

$f(D)y = Q(x)$

$y(x) = y_c + y_p$

A.E $f(m) = m^2 - 2m + 1 = 0$

$= (m-1)^2 = 0$

$= m - 1 = 0$

$m = 1, 1$

roots are real, repeated

$y_c = (c_1 + c_2 x)e^x.$

$y_c = c_1 e^x + c_2 x e^x$

$c_1 u(x) + c_2 v(x)$

$u(x) = e^x$ $\quad v(x) = xe^x$

$u'(x) = e^x$ $\quad v'(x) = xe^x + e^x$

$W = \begin{vmatrix} u & v \\ u' & v' \end{vmatrix} = uv' - vu'$

... $x + |\tan x| +$ ...

$= e^x(xe^x - e^x) -$ ...

$xe^x(e^x$ ...

$= xe^{2x} - e^{2x} -$ ...

$= -e^{2x} \neq 0.$

$y_p = A u(x) + B$ ...

$A = -\int \frac{v(x) R(x)}{W}$ ...

$= +\int \frac{xe^x(\ldots}{e \ldots}$

$= \int x \log x\, \partial$ ...

$=$ Integrat...

$= \log x \int x\, \partial x - \int \partial$ ...

$= \log x\left(\frac{x^2}{2}\right) - \int$ ...

$= \frac{x^2}{2}\log x$ ...

$A = \frac{x^2}{2}\log x -$ ...

$B = \int \frac{u(x)R(x)}{W}$ ...

$= \int \frac{e^x(e^x \log \ldots}{-e^{2x}}$

$= -\int \log x\, \partial x$

$= -(x \log x$ ...

$B = x\log x + x$ ...

$= e^x(xe^x - e^x) - xe^x(e^x)$

$= xe^{2x} - e^{2x} - xe^{2x}$

$= -e^{2x} \neq 0.$

$y_p = A\,u(x) + B\,v(x)$

$A = -\int \frac{v(x)\,R(x)}{W}\,dx$

$= +\int \frac{xe^x(\log x)e^x}{e^{2x}}\,dx$

$= \int x\log x\,dx.$

= Integrating by parts

$= \log x \int x\,dx - \int \frac{d(\log x)}{dx}\int x\,dx\,dx$

$= \log x\left(\frac{x^2}{2}\right) - \int \frac{1}{x}\left(\frac{x^2}{2}\right)dx$

$= \frac{x^2}{2}\log x - \frac{1}{2}\int (x)\,dx$

$A = \frac{x^2}{2}\log x - \frac{x^2}{4} + c.$

$B = \int \frac{u(x)\,R(x)}{W}\,dx$

$= \int \frac{e^x(e^x\log x)}{-e^{2x}}\,dx$

$= -\int \log x\,dx$

$= -(x\log x - x)\,dx$

$B = x\log x + x.$

$y_p = A\,u(x) + B\,v(x)$

$= \left(\frac{x^2}{2}\log x - \frac{x^2}{4}\right)e^x + (x\log x + x)\,xe^x$

$= \frac{x^2}{2}\log x\,e^x - \frac{x^2}{4}e^x + x^2\log x\,e^x + x^2e^x$

$= \log x\,e^x\left(\frac{3x^2}{2}\right) + e^x\left(\frac{3x^2}{4}\right)$

$= \frac{3e^x}{2}\left[\log x(x^2) + \frac{x^2}{2}\right]$

$y(x) = y_c + y_p$

$= c_1e^x + c_2xe^x + \frac{3e^x}{2}\left(\log(x)(x^2) + \frac{x^2}{2}\right)$

PART-B

1. Solve the DE

$(D^2+3D+2)y = 2\cos(2x+3) + 2e^x + x^2.$

$f(D)y = Q(x).$

$y(x) = y_c + y_p.$

$f(D)y = (D^2+3D+2)y = 0.$

Auxiliary equation

$f(m) = m^2+3m+2 = 0$

$m^2+2m+m+2 = 0$

$m(m+2)+1(m+2) = 0$

$\boxed{m = -1, -2}$

$y_c = c_1e^{-x} + c_2e^{-2x}.$

P.I:

$$y_p = \frac{1}{f(D)} 2\cos(2x+3) + \frac{1}{f(D)} 2e^{x} + \frac{1}{f(D)} x^2$$

$$= \frac{1}{D^3+3D+2} 2\cos(2x+3) + 2\frac{1}{(D^3+3D+2)}(e^x) + \frac{1}{D^3+3D+2}(x^2)$$

$D^2 = -a^2$

$$= \frac{2 \times \cos(2x+3)}{-4D+3D+2} + \frac{2e^x}{1+3+2} + \frac{1}{2}\left[\frac{1+3D+D^3}{2}\right]^{-1} x^2$$

$$= \frac{2\times\cos(2x+3)}{2-D} + \frac{2e^x}{6} + \frac{1}{2}\left(1+\frac{3D+D^3}{2}\right)^{-1} x^2$$

$$= \frac{2(2+D)\cos(2x+3)}{2^2-(-4)} + \frac{e^x}{3}$$

$$\cancel{+\frac{1}{2}\left[1-\left[\frac{3D+D^3}{2}\right]^2\right]}$$

$$+\frac{1}{2}\left(1-\left(\frac{3D+D^3}{2}\right)+\left(\frac{3D+D^3}{2}\right)^2\right)x^2$$

$$= \frac{2}{4}\cos(2x+3) + \frac{2D}{8}\cos(2x+3) + \frac{e^x}{3} + \frac{x^2}{2} - \frac{3(2x)}{4} + \frac{9}{4}$$

$$= \frac{1}{4}(2\cos(2x+3) - 2\sin(2x+3)) + \frac{e^x}{3} + \frac{x^2}{2} - \frac{3x}{2} + 9/4$$

$$y_p = \frac{1}{2}(\cos(2x+3) - \sin(2x+3)) + \frac{e^x}{3} + \frac{x^2}{2} - \frac{3x}{2} + 9/4$$

$$y(x) = y_c + y_p$$

$$= c_1 e^{-x} + c_2 e^{-2x} + \frac{1}{2}(\cos(2x+3) - \sin(2x$$

2. Solve the differential equation

$$(D^2+4)y = 96x^2 + \sin 2x +$$

$$f(D)y = g(x)$$

$$y(x) = y_c + y_p$$

$$f(D)y = (D^2+4)y = 0$$

$$AE = m^2 = -4$$

$$m = \pm 2i$$

$$y_c = e^{\alpha x}(c_1\cos\beta x + c_2 \sin\beta x)$$

$$= e^{0x}(c_1 \cos 2x + c_2 \sin 2x)$$

$$y_c = c_1\cos 2x + c_2 \sin 2x$$

$y = y_c + y_p$

$y = e^{x/2}\left(c_1 \cos \frac{\sqrt{3}}{2}x + c_2 \sin \frac{\sqrt{3}}{2}x\right)$

$-\frac{1}{13}(2\cos 2x + 3\sin 2x)$

12) By using method of variation of parameters solve $(D^2+4)y = \sec 2x$.

$f(D)y = g(x)$

$y(x) = y_c + y_p$

$f(D)y = (D^2+4)y = 0$

A.E: $f(m) = 0$

$f(m) = m^2 + 4 = 0$

$m = \pm 2i$

$y_c = e^{0x}(c_1 \cos 2x + c_2 \sin 2x)$

$= c_1 \cos 2x + c_2 \sin 2x.$

$y_c = c_1 u(x) + c_2 v(x)$

$u(x) = \cos 2x \quad v(x) = \sin 2x$

$u'(x) = -2\sin 2x \quad v'(x) = 2\cos 2x$

$w = \begin{vmatrix} u & v \\ u' & v' \end{vmatrix} = uv' - vu'$

$= 2\cos^2(2x) + 2\sin^2(2x)$

$w = 2(1)$

$w = 2 \neq 0$

$y_p = A\, u(x) + B\, v(x)$

$A = -\int \frac{v(x) R(x)}{w} \partial x.$

$= -\int \frac{(\sin 2x)\sec 2x}{2} \partial x$

$= \frac{-1}{2} \int \frac{\sin 2x}{\cos 2x} \partial x$

$= \frac{-1}{2} \int \tan 2x \, \partial x$

$= \frac{-1}{2}\left[-\frac{\log|\cos 2x|}{2}\right]$

$= \frac{1}{4} \log|\cos 2x|.$

$B = \int \frac{u(x) R(x)}{w} \partial x.$

$= \int \frac{(\cos 2x)(\sec 2x)}{2} \partial x$

$= \frac{1}{2} \int \frac{\cos 2x}{\cos 2x} \partial x$

$= \frac{1}{2} \int 1 \, \partial x$

$B = \frac{x}{2}$

$y_p = \frac{1}{4}\log(\cos 2x]\cos 2x + \frac{x}{2}(\sin 2x)$

General solution

$y = y_c + y_p$

$= c_1 \cos 2x + c_2 \sin 2x + \frac{1}{4}\log(\cos 2x)\cos 2x + \frac{x}{2}\sin 2x.$

13) Solve the differential equation

$$(D^3-4D^2-D+4)y = e^{3x}\cos$$

$$f(D)y = Q(x)$$

$$y(x) = y_c + y_p$$

$$f(D)y = (D^3-4D^2-D+4)y$$

A.E $f(m)=0$

$$f(m) = m^3-4m^2-m+4=0.$$

$$(m-2)(m-2)$$

$$(m-2)=0.$$

$$m = 2, 2, 2$$

$$y_c = (c_1+c_2x+c_3x^2)e^{2x}$$

$$yp = \frac{1}{2} \times \frac{1}{5D-15} \cos 3x$$

$$- \frac{1}{2} \times \frac{1}{5D-31} \cos 5x$$

$$yp = \frac{1}{10} \times \frac{1}{D-3} \times \frac{D+3}{D+3} \cos 3x$$

$$- \frac{1}{2} \times \frac{5D+31}{25D^2-961} \cos 5x$$

$$yp = \frac{1}{10} \times \frac{D+3}{D^2-9} \cos 3x$$

$$- \frac{1}{2} \times \frac{5D+31}{25D^2-961} \cos 5x$$

$$yp = \frac{1}{10} \left(\frac{D+3}{-18} \right) \cos 3x$$

$$- \frac{1}{2} \left(\frac{5D+31}{-1586} \right) \cos 5x$$

$$yp = \frac{-1}{180} \left[-3\sin 3x + 3\cos 3x \right)$$

$$+ \frac{1}{3172} \left[5(5(-\sin 3x) + 31\cos 5x \right).$$

$$yp = \frac{1}{60} \left[\sin 3x - \cos 3x \right)$$

$$+ \frac{1}{3172} \left[31\cos 5x - 25\sin 3x \right)$$

$$y(x) = yc + yp$$

$$= c_1 e^{x} + c_2 e^{-6x}$$

$$+ \frac{1}{60} (\sin 3x - \cos 3x)$$

$$+ \frac{1}{3172} \left(31\cos 5x - 25\sin 3x \right)$$

11. Solve the differential equation,

$$(D^2+D+1)y = \sin 2x$$

$$f(D)y = g(x)$$

$$y(x) = y_c + yp.$$

$$f(D)y = (D^2+D+1)y = 0$$

A.E $f(m) = 0$

$$m^2 + m + 1 = 0.$$

$$m = \frac{-1 \pm \sqrt{3}i}{2}$$

$$yc = e^{-x/2} \left[c_1 \cos \frac{\sqrt{3}}{2} x + c_2 \sin \frac{\sqrt{3}}{2} x \right.$$

Particular Integral

$$yp = \frac{1}{f(D)} \sin 2x$$

$$= \frac{1}{(D^2+D+1)} \sin 2x$$

$$yp = \frac{1}{-4+D+1} \sin 2x$$

$$yp = \frac{1}{D-3} \sin 2x.$$

$$yp = \frac{D+3}{D^2-9} \sin 2x$$

$$yp = \frac{D+3}{-4-9} \sin 2x$$

$$yp = \frac{D+3}{-13} \sin 2x.$$

$$yp = -\frac{1}{13} (2\cos 2x + 3\sin 2x)$$

$$yp = \frac{-1}{13} (2\cos 2x + 3\sin 2x)$$

7. Solve the differential equation $(D^2-4)y = 2\cos^2 x$.

$f(D)y = Q(x)$

$y(x) = y_c + y_p$

A.E $f(m) = 0$

$m^2 - 4 = 0$

$m^2 = 4$

$m = \pm 2$

$m = 2, -2$

$y_c = c_1 e^{2x} + c_2 e^{-2x}$

Particular Integral

$y_p = \frac{1}{f(D)} Q(x)$

$y_p = \frac{1}{D^2-4}(2\cos^2 x)$

$\boxed{\cos^2 x = \frac{1+\cos 2x}{2}}$

$y_p = \frac{1}{D^2-4}[1+\cos 2x)$

$= \frac{1}{D^2-4}(1 e^{0x})$

$+ \frac{1}{D^2-4}(\cos 2x)$

$= \frac{1}{-4} e^{0x} + \frac{1}{-4-4} \cos 2x$

$= \frac{-1}{4} - \frac{1}{8}\cos 2x$

$= \frac{-1}{4}\left[1 + \frac{\cos 2x}{2}\right]$

8. Solve the differential equation

$(D^2+1)y = \sin x \sin 2x$

$f(D)y = Q(x)$

$y(x) = y_c + y_p$

A.E $f(D)y = (D^2+1)y = 0$

$f(m) = 0$

$f(m) = m^2 + 1 = 0$

$m = \pm i$

$y_c = e^{0x}(c_1\cos x + c_2 \sin x)$

$y_c = c_1 \cos x + c_2 \sin x$

Particular Integral

$y_p = \frac{1}{f(D)} \sin x \sin 2x$

$\sin A \sin B$

$= \frac{1}{2}(\cos(A-B) - \cos(A+B))$

$y_p = \frac{1}{D^2+1}\left[\frac{\cos x - \cos 3x}{2}\right.$

$y_p = \frac{1}{2} \times \frac{1}{D^2+1}\cos x -$

$\frac{1}{2} \times \frac{1}{D^2+1}\cos 3x$

$y_p = \frac{x}{2} \times \frac{1}{2D}\cos x -$

$\frac{1}{2} \times \frac{1}{-9+1}\cos 3x$

$y_p = \frac{x}{4}[\sin x]$

$- \frac{1}{2} \times \frac{1}{-8}\cos 3x$

$y_p = \frac{x\sin x}{4} + \frac{\cos 3x}{16}$

General solution

$y(x) = y_c + y_p$

$y = c_1\cos x + c_2\sin x$

$+ \frac{x\sin x}{4} + \frac{\cos 3x}{16}$

9. Solve the differential equation

$(D^2+9)y = \cos 3x + \sin 2x.$

$f(D)y = \phi(x)$

$y = y_c + y_p.$

$f(D)y = (D^2+9)y = 0$

$f(m) = 0$

$f(m) = m^2+9 = 0$

$m = \pm 3i$

$y_c = e^{\alpha x}(c_1\cos\beta x + c_2\sin\beta x)$

$y_c = e^{0x}(c_1\cos 3x + c_2\sin 3x)$

$y_c = c_1\cos 3x + c_2\sin 3x.$

Particular Integral.

$y_p = \frac{1}{f(D)}\cos 3x + \frac{1}{f(D)}\sin 2x.$

$= \frac{1}{(D^2+9)}\cos 3x + \frac{1}{(D^2+9)}\sin 2x$

$f(a) = 0 \quad f'(-a^2) \neq 0$

$= x \times \frac{1}{2D}\cos 3x + \frac{1}{-4+9}\sin 2x.$

$= x \times \frac{1}{2}\frac{\sin 3x}{3} + \frac{1}{5}\sin 2x.$

$= \frac{x\sin 3x}{6} + \frac{1}{5}\sin 2x.$

$y_p = \frac{x\sin 3x}{6} + \frac{\sin 2x}{5}$

$y = y_c + y_p$

$= c_1\cos 3x + c_2\sin 3x$

$+ \frac{x\sin 3x}{6} + \frac{\sin 2x}{5}$

10. Solve the differential equation

$(D^2+5D-6)y = \sin 4x\sin x$

$f(D)y = \phi(x)$

$y = y_c + y_p$

~~A.E~~ $f(D)y = (D^2+5D-6)y = 0$

$f(D)$ ~~$y=$~~

A.E $f(m) = 0$

$m^2+5m-6 = 0$

$(m-1)(m+6) = 0$

$m = 1, -6$

roots are different.

$y_c = c_1e^{x} + c_2e^{-6x}$

Particular Integral

$y_p = \frac{1}{f(D)}\sin 4x\sin x$

$y_p = \frac{1}{(D^2+5D-6)}\left(\frac{\cos 3x - \cos 5x}{2}\right)$

$y_p = \frac{1}{2}\times\frac{1}{D^2+5D-6}\cos 3x$

$-\frac{1}{2}\times\frac{1}{D^2+5D-6}(\cos 5x)$

$y_p = \frac{1}{2}\times\frac{1}{-9+5D-6}\cos 3x$

$-\frac{1}{2}\times\frac{1}{-25+5D-6}\cos 5x$

Particular Integral

$y_p = \frac{1}{f(D)} \sin x \sin 2x$

$+ \frac{1}{f(D)} e^x x^2$

$y_p = \frac{1}{D^2+1}\left[\frac{\cos x - \cos 3x}{2}\right]$

$+ e^x \frac{1}{((D+1)^2+1)} x^2$

$= \frac{1}{2(D^2+1)} \cos x - \frac{1}{2} \times \frac{1}{D^2+1} \cos 3x$

$+ e^x \left(\frac{1}{D^2+2D+2}\right] x^2$

$= \frac{x \sin x}{4} + \frac{\cos 3x}{16}$

$+ e^x \left[\frac{1}{2\left[1+\left[\frac{D^2+2D}{2}\right]\right]}\right] x^2$

$= \frac{x \sin x}{4} + \frac{\cos 3x}{16}$

$+ \frac{e^x}{2}\left[1+\left(\frac{D^2+2D}{2}\right)\right]^{-1} x^2$

$= \frac{x \sin x}{4} + \frac{\cos 3x}{16}$

$+ \frac{e^x}{2}\left[1-\left[\frac{D^2+2D}{2}\right]+\left[\frac{D^2+2D}{2}\right]^2\right] x^2$

$= \frac{x \sin x}{4} + \frac{\cos 3x}{16}$

$+ \frac{e^x}{2}\left[x^2 - \frac{2}{2} - \frac{2(2x)}{2}\right.$

$\left. + \frac{4(2)}{2}\right]$.

$y_p = \frac{x \sin x}{4} + \frac{\cos 3x}{16}$

$+ \frac{e^x}{2}(x^2 - 2x + 3)$

$y(x) = y_c + y_p$

$= c_1 \cos x + c_2 \sin x$

$+ \frac{x \sin x}{4} + \frac{\cos 3x}{16}$

$+ \frac{e^x}{2}(x^2 - 2x + 3)$

6) Solve the differential equation $(D^3+1)y = 3+5e^x$

$f(D)y = Q(x)$

$y(x) = y_c + y_p$

Auxiliary equation

$f(m) = 0$

$m^3 + 1 = 0.$

$m = -1, \frac{1 \pm \sqrt{3}i}{2}$

$y_c = c_1 e^{-x} + e^{\frac{1}{2}x}\left(c_2 \cos \frac{\sqrt{3}}{2} x + c_3 \sin \frac{\sqrt{3}}{2} x\right)$

Particular Integral

$y_p = \frac{1}{f(D)} 3 e^{0x} + \frac{1}{f(D)} 5 e^x$

$y_p = \frac{1}{(D^3+1)} 5 e^{0x} + \frac{1}{(D^3+1)} 5 e^x$

$y_p = 3 \times \frac{1}{1} e^{0x}$

$+ \frac{1}{1+1} 5 e^x$.

$y_p = 3 + \frac{5e^x}{2}$

$y(x) = y_c + y_p$

$= c_1 e^{-x} + e^{x/2}\left(c_2 \cos \frac{\sqrt{3}}{2} x + c_3 \sin \frac{\sqrt{3}}{2} x\right)$

$+ 3 + 5e^{x}/2$

$yp = \frac{e^{3x}}{6}(2x^2-4x+3)$

$-\frac{4}{25}\cos 2x + \frac{3}{25}\sin 2x - 8.$

General solution

$y(x) = yc + yp.$

$y(x) = c_1e^x + c_2e^x x +$

$\frac{e^{3x}}{6}(2x^2-4x+3)$

$-\frac{4}{25}\cos 2x + \frac{3}{25}\sin 2x - 8$

4) Solve the differential equation

$(D^3-6D^2+11D-6)y = e^{-2x}+e^{-3x}.$

It is in the form of

$f(D)y = Q(x)$

$y(x) = yc + yp.$

$f(D)y = (D^3-6D^2+11D-6)y = 0.$

$f(m) = m^3-6m^2+11m-6 = 0.$ A.E

$(m-1)(m-2)(m-3) = 0.$

$m = 1, 2, 3.$

roots are different.

$yc = c_1e^x + c_2e^{2x} + c_3e^{3x}$

particular Integral ~~part~~

$yp = \frac{1}{f(D)} e^{ax}$

$= \frac{1}{(D^3-6D^2+11D-6)} e^{-2x} +$

$\frac{1}{(D^3-6D^2+11D-6)} e^{-3x}.$

$= \frac{1}{-8-24-22-6} e^{-2x} +$

$\frac{1}{-27-6(9)-33-6} e^{-3x}$

$= \frac{1}{-60} e^{-2x} + \frac{1}{-120} e^{-3x}$

$= -\frac{1}{60}\left(e^{-2x} + \frac{e^{-3x}}{2}\right)$

General solution

$y = yc + yp$

$= c_1e^x + c_2e^{2x} + c_3e^{3x}$

$-\frac{1}{60}\left(e^{-2x} + \frac{e^{-3x}}{2}\right)$

5. Solve the DE

$(D^2+1)y = \sin x \sin 2x + e^x$

$f(D)y = Q(x)$

$y(x) = yc + yp.$

$f(m) = 0$

$f(m) = m^2+1 = 0$

$m^2 = -1$

$m = \pm i$

$yc = e^{\alpha x}(c_1\cos \beta x + c_2\sin \beta x)$

$yc = e^{0x}(c_1\cos x + c_2\sin x)$

$yc = c_1\cos x + c_2\sin x.$

$\sin A \sin B = \frac{\cos(A-B) - \cos(A+}{2}$

P.I:

$$y_p = \frac{1}{f(D)} 2\cos(2x+3) + \frac{1}{f(D)} 2e^x + \frac{1}{f(D)} x^2$$

$$= \frac{1}{D^3+3D+2} 2\cos(2x+3) + 2\frac{1}{(D^3+3D+2)}(e^x) + \frac{1}{D^3+3D+2}(x^2)$$

$D^2 = -a^2$

$$= \frac{2\times\cos(2x+3)}{-4D+3D+2} + \frac{2e^x}{1+3+2} + \frac{1}{2}\left[\frac{1+3D+D^3}{2}\right]^{-1} x^2$$

$$= \frac{2\times\cos(2x+3)}{2-D} + \frac{2e^x}{6} + \frac{1}{2}\left(1+\frac{3D+D^3}{2}\right)^{-1} x^2$$

$$= \frac{2(2+D)\cos(2x+3)}{2^2-(-4)} + \frac{e^x}{3} + \frac{1}{2}\left[1-\left[\frac{3D+D^3}{2}\right]^2\right]$$

$$+\frac{1}{2}\left(1-\left(\frac{3D+D^3}{2}\right)+\left(\frac{3D+D^3}{2}\right)^2\right)x^2$$

$$= \frac{2}{4}\cos(2x+3) + \frac{2D}{8}\cos(2x+3) + \frac{e^x}{3} + \frac{x^2}{2} - \frac{3(2x)}{4} + \frac{9}{4}$$

$$= \frac{1}{4}(2\cos(2x+3) - 2\sin(2x+3)) + \frac{e^x}{3} + \frac{x^2}{2} - \frac{3x}{2} + 9/4$$

$$y_p = \frac{1}{2}(\cos(2x+3) - \sin(2x+3)) + \frac{e^x}{3} + \frac{x^2}{2} - \frac{3x}{2} + 9/4$$

$$y(x) = y_c + y_p$$

$$= c_1e^{-x} + c_2e^{-2x} + \frac{1}{2}(\cos(2x+3) - \sin(2x ...$$

2. Solve the differential equation

$$(D^2+4)y = 96x^2 + \sin 2x + ...$$

$$f(D)y = g(x)$$

• $y(x) = y_c + y_p$

$f(D)y = (D^2+4)y = 0$

A.E: $m^2 = -4$

$m = \pm 2i$

$$y_c = e^{\alpha x}(c_1\cos\beta x + c_2\sin\beta x)$$

$$= e^{0x}(c_1\cos 2x + c_2\sin 2x)$$

$$y_c = c_1\cos 2x + c_2\sin 2x$$

Particular Integral

$$y_p = \frac{1}{f(D)}96(x^2) + \frac{1}{f(D)}\sin 2x - \frac{1}{f(D)} ...$$

$$y_p = \frac{1}{D^2+4}96(x^2 ...$$

$$+\frac{1}{D^2+4}\sin 2x - \frac{1}{D^2+4}e^{...}$$

$y_p = \frac{1}{4\left(1+\frac{D^2}{4}\right)} 96x^2 + x \times \frac{1}{2D} \times \sin(2x) - \frac{1}{0+4} e^{0x}(K)$

$y_p = \frac{96}{4}\left[1+\frac{D^2}{4}\right]^{-1} x^2 + x \frac{1}{2(-4)} \sin 2x - \frac{K}{4}$

$y_p = 24\left[1 - \frac{D^2}{4} + \cancel{\frac{D^4}{256}}\right] x^2 - \frac{x}{8} \sin 2x - \frac{K}{4}$

$y_p = 24x^2 - 12 - \frac{x}{8}\sin 2x - \frac{K}{4}$

General solution:

$y(x) = y_c + y_p$

$y(x) = c_1 \cos 2x + c_2 \sin 2x + 24x^2 - 12 - \frac{x}{8}\sin 2x - \frac{K}{4}$

3. Solve the differential equation $(D^2-2D+1)y = x^2 e^{3x} - \sin 2x + 3.$

It is in the form

$f(D)y = Q(x)$

$y(x) = y_c + y_p.$

$f(D)y = (D^2-2D+1)y = 0$

A.E: $f(m)$ $(m-1)^2 = 0$

$m-1=0$

$m = 1, 1$

m_1 and m_2 are equal

$y_c = (c_1 + c_2 x)e^{x}.$

Particular Integral

$y_p = \frac{1}{(D^2-2D+1)} x^2 e^{3x} - \frac{1}{(D^2-2D+1)} \sin 2x + 3 \times \frac{1}{D^2-2D+1} e^{0x}$

$y_p = e^{3x} \frac{1}{((D+3)^2 - 2(D+3)+1)} x^2 - \left\{\frac{1}{-4-2D+1}\right\} \sin 2x + 3 \times \frac{1}{0-2+1} e^{0x}$

$y_p = e^{3x} \times \frac{1}{(D+2)^2} x^2 - \frac{1}{(-2D-3)} \sin 2x - 3$

$y_p = \frac{e^{3x}}{4} \times \left(1+\frac{D}{2}\right)^{-2} x^2 + \frac{1}{2D+3} \times \frac{2D-3}{2D-3}(\sin 2x) - 3$

$y_p = \frac{e^{3x}}{4}\left[1 - \frac{2D}{2} + 3\left(\frac{D}{2}\right)^2\right] x^2 + \frac{1(2D-3)}{4D^2-9} \sin 2x - 3$

$y_p = \frac{e^{3x}}{4}\left(x^2 - 2x + \frac{3}{4}(2)\right] - \frac{1(2D)\sin 2x}{25} + \frac{3}{25}\sin 2x - 3$

$y_p = \frac{e^{3x}}{4}\left(x^2 - 2x + \frac{3}{2}\right] - \frac{2(2)\cos 2x}{25} + \frac{3}{25}\sin 2x - 3$

$= e^{x}(xe^{x}-e^{x}) - xe^{x}(e^{x})$

$= xe^{2x} - e^{2x} - xe^{2x}$

$= -e^{2x} \neq 0.$

$y_p = A\,u(x) + B\,v(x)$

$A = -\int \frac{v(x)\,R(x)}{W}\,dx$

$= +\int \frac{xe^{x}(\log x)e^{x}}{e^{2x}}\,dx$

$= \int x\log x\,dx$

= Integrating by parts

$= \log x \int x\,dx - \int \frac{d(\log x)}{dx}\int x\,dx\,dx$

$= \log x\left(\frac{x^2}{2}\right) - \int \frac{1}{x}\left(\frac{x^2}{2}\right)dx$

$= \frac{x^2}{2}\log x - \frac{1}{2}\int (x)\,dx$

$A = \frac{x^2}{2}\log x - \frac{x^2}{4} + c$

$B = \int \frac{u(x)\,R(x)}{W}\,dx$

$= \int \frac{e^{x}(e^{x}\log x)}{-e^{2x}}\,dx$

$= -\int \log x\,dx$

$= -(x\log x - x)\,dx$

$B = -x\log x + x.$

$y_p = Au(x) + Bv(x)$

$= \left(\frac{x^2}{2}\log x - \frac{x^2}{4}\right)e^{x} + (x\log x + x)xe^{x}$

$= \frac{x^2}{2}\log x\,e^{x} - \frac{x^2}{4}e^{x} + x^2\log x\,e^{x} + x^2e^{x}$

$= \log x\,e^{x}\left(\frac{3x^2}{2}\right) + e^{x}\left(\frac{3x^2}{4}\right)$

$= \frac{3e^{x}}{2}\left[\log x\,(x^2) + \frac{x^2}{2}\right]$

$y(x) = y_c + y_p$

$= c_1e^{x} + c_2xe^{x} + \frac{3e^{x}}{2}\left(\log(x)(x^2) + \frac{x^2}{2}\right)$

PART-B

1. Solve the DE

$(D^2+3D+2)y = 2\cos(2x+3) + 2e^{x} + x^2$

$f(D)y = g(x)$

$y(x) = y_c + y_p.$

$f(D)y = (D^2+3D+2)y = 0$

Auxiliary equation

$f(m) = m^2+3m+2 = 0$

$m^2+2m+m+2 = 0$

$m(m+2)+1(m+2) = 0$

$\boxed{m = -1, -2}$

$y_c = c_1e^{-x} + c_2e^{-2x}$