Hall Ticket No | | | | | | | | | | | Question Paper Code: AHSC12

INSTITUTE OF AERONAUTICAL ENGINEERING

(Autonomous)

Dundigal- 500 043, Hyderabad ,Telengana

MODEL QUESTION PAPER-I

B.Tech IVSemester End Examinations, July-2024

Regulations: IARE - UG20

COMPLEX ANALYSIS AND SPECIAL FUNCTIONS

ELECTRONICS AND COMMUNICATION ENGINEERING

Time: 3 hour Maximum Marks: 70

Answer ONE Question from each MODULE
All Questions Carry Equal Marks
All parts of the question must be answered in one place only

MODULE-I

1. (a) Define the term Analyticity of a complex variable function f (z). Show that the real part of an analytic function f (z) where

$$u = e^{-2xy}\sin\left(x^2 - y^2\right)$$

is a harmonic function. Hence find its harmonic conjugate. **[CO1:Understand 7M]**

(b) Define the term Continuity of a complex variable function f (z). Show that the function

$$f(z) = |z|$$

is continuous everywhere but nowhere differentiable **[CO1:Understand 7M]**

2. (a) Define the term Differentiability of a complex function f(z). Show that an analytic function with constant real part is always constant **[CO1:Understand7M]**

(b) Find an analytic function f (z) whose real part of an analytic function is

$$u = \frac{\sin 2x}{\cosh 2y - \cos 2x}$$

by Milne-Thompson method. **[CO1:Understand 7M]**

MODULE-II

3. (a) Solve the value of line integral to
$\int_c \frac{z}{(z-1)(z-2)^2}$dz where c is the circle
$|z-2|$=1/2 using Cauchy's integral formula.
[CO2:Understand 7M]

(b) Make use of vertices -1 ,1,1+i ,-1+i and verify Cauchy's theorem for the integral of

$$z^3$$

taken over the boundary of the rectangle formed . **[CO2:Understand 7M]**

4. (a) Solve the value of line integral to $\int\limits_C \left(y^2 + 2xy\right) dx + \left(y^2 - 2xy\right) dy$ where C is the boundary of the region$y = x^2 and x = y^2$. **[CO2:Understand 7M]**

   (b) Make use of Cauchy's integral formula and find the value of line integral

$$\int\limits_c \frac{z^4 - 3z^2 + 6}{(z+1)^3} dz$$

   where c is the circle

$$|z| = 2$$

[CO2:Understand 7M]

MODULE-III

5. (a) Extend the Laurent expansion of $f(z) = \frac{1}{z^2 - 4z + 3}$ for

$$1 < \ |z| < 3 \, (ii) \, |z| = 1 \, (iii) \, |z| > 3$$

[CO 3:Apply7M]

   (b) Define Cauchy's Residue theorem of an analytic function f(z) within and on the closed curve. . Find the value of $\oint\limits_c \frac{1}{\sinh z} dz$ using Residue theorem.

$$|z| = 4$$

[**CO4:Apply7M**]

6. (a) Define the following terms
   (i) The Isolated singularity of an analytic function f(z)
   (ii) Pole of order m of an analytic function f(z)
   (iii) Essential and Removable singularities of an analytic function f(z) **[CO 3:Apply7M]**

   (b) Solve the value of

$$|z| = 4 \int\limits_0^\pi \frac{d\theta}{(a + b\cos\theta)} using Residue theorem.$$

[CO 4:Apply 7M]

MODULE-IV

7. (a) Define Gamma and Beta functions. Solve the integral

$$\int\limits_0^\infty \sqrt{x} e^{-x/3} \, dx$$

   using Gamma function. **[CO5:Apply 7M]**

   (b) Solve the integral

$$\int\limits_0^2 (8 - x)^{1/3} dx$$

   using Beta-Gamma functions **[CO 5:Apply 7M]**

8. (a) State any three properties of Beta function. Show that

$$\beta(m,n) = \beta(m+1,n) + \beta(m,n+1)$$

using Gamma function. **[CO5:Apply 7M]**

(b) .Solve the integral

$$\int_0^1 x^m (\log x)^n dx = \frac{(-1)^m n!}{(m+1)^{n+1}}$$

where n is positive integer. **[CO 5:Apply 7M]**

MODULE-V

9. (a) What is Bessel differential equation and most general solution of Bessels differential equation? Show the Bessel's recurrence relation

$$xJ_n'(x) \; = \; nJ_n(x) \; - x\; J_{n+1}(x)\,.$$

[CO 6:Apply 7M]

(b) Show that $J_{5/2}(x) = \sqrt{\frac{2}{\pi x}}\left(\frac{3-x^2}{x^2}\sin x - \frac{3}{x}\cos x\right)$ **[CO 6:Apply 7M]**

10. (a) Show that $\int_0^1 xJ_n(\alpha\, x)J_n(\beta\, x)\;dx = \begin{cases} 0, if\ \alpha = \beta \\ \frac{1}{2}[J_{n+1}(\alpha)]^2, if\ \alpha \neq \beta \end{cases}$ **[CO 6:Apply 7M]**

(b) Show that

$$J_n(x) = \frac{1}{\pi}\int_0^\pi \cos(n\theta - x\sin\theta)d\theta$$

where

$$J_n(x)$$

Bessel's function, 'n' being a integer. **[CO 6:Apply 7M]**

****END OF EXAMINATION****

COURSE OBJECTIVES:

The students will try to learn:

I	The applications of complex variable and conformal mapping in two dimensional complex potential theories..
II	The fundamental calculus theorems and criteria for the independent path on contour integral used in problems of engineering.
III	The concepts of special functions and its application for solving the partial differential equations in physics and engineering.
IV	The mathematics of combinatorial enumeration by using generating functions and complex analysis for understanding the numerical growth rates.

COURSE OUTCOMES:

After successful completion of the course, students should be able to:

CO 1	**Identify** the fundamental concepts of analyticity and differentiability for finding complex conjugates , conformal mapping of complex transformations.	Understand
CO 2	**Apply** integral theorems of complex analysis and its consequences for the analytic function with derivatives of all orders in simple connected region.	Apply
CO 3	**Extend** the Taylor and Laurent series for expressing the function in terms of complex power series.	Apply
CO 4	**Apply** Residue theorem for computing definite integrals by using the singularities and poles of real and complex analytic functions over closed curves.	Apply
CO 5	**Determine** the characteristics of special functions for obtaining the proper and improper integrals for obtaining the proper and improper integrals.	Apply
CO 6	**Apply** the role of Bessel functions in the process of obtaining the series solutions for second order differential equation	Apply

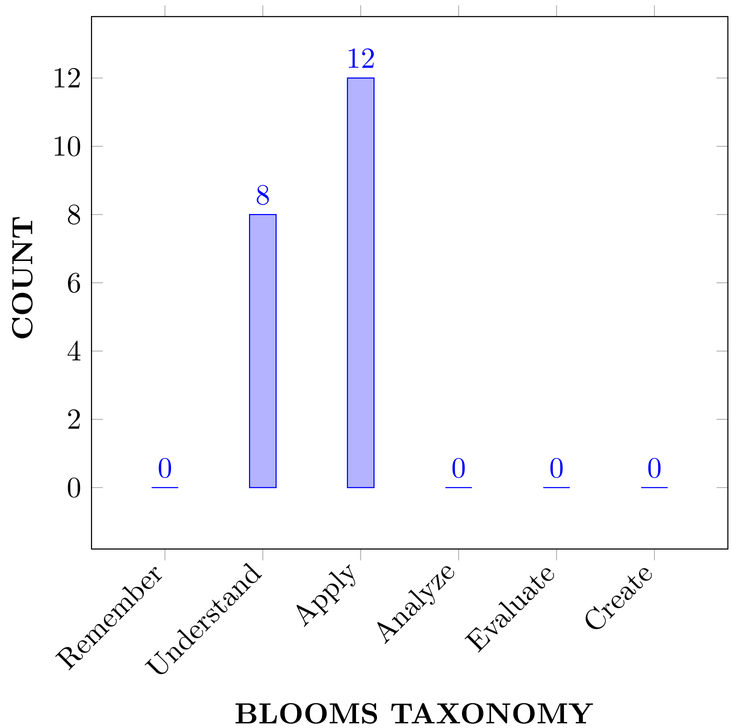
QUESTION PAPER 1: MAPPING OF SEMESTER END EXAMINATION QUESTIONS TO COURSE OUTCOMES

Q.No		All Questions carry equal marks	Taxonomy	CO's	PO's
1	a	Define the term Analyticity of a complex variable function f (z). Show that the real part of an analytic function f (z) where $$u = e^{-2xy} \sin\left(x^2 - y^2\right)$$ is a harmonic function. Hence find its harmonic conjugate.	Understand	CO 1	PO 1
	b	Define the term Continuity of a complex variable function f (z). Show that the function $$f\left(z\right) = \left\|z\right\|$$ is continuous everywhere but nowhere differentiable	Understand	CO 1	PO 1,4
2	a	Define the term Differentiability of a complex function f(z). Show that an analytic function with constant real part is always constant	Understand	CO 1	PO 1,4
	b	Find an analytic function f (z) whose real part of an analytic function is $$= \frac{\sin 2x}{\cosh 2y - \cos 2x}$$ by Milne-Thompson method.	Understand	CO 1	PO 1,4
3	a	Solve the value of line integral to $\int\limits_c \frac{z}{(z-1)(z-2)^2}$dz where c is the circle $\|z-2\|$=1/2 using Cauchy's integral formula.	Understand	CO 2	PO 1,2
	b	Make use of Cauchy's integral formula and find the value of line integral $$\int\limits_c \frac{z^4 - 3z^2 + 6}{(z+1)^3} dz$$ where c is the circle $$\|z\| = 2$$.	Understand	CO 2	PO 1

4	a	Solve the value of line integral to $\int\limits_{C} \left(y^2 + 2xy\right) dx + \left(y^2 - 2xy\right) dy$ where C is the boundary of the region$y = x^2 and x = y^2$. **[7M]**	Understand	CO 2	PO 1,2
	b	Make use of Cauchy's integral formula and find the value of line integral $$\int\limits_{c} \frac{z^4 - 3z^2 + 6}{(z+1)^3} dz$$ where c is the circle $$\lvert z \rvert = 2$$.	Apply	CO 2	PO 1,2
5	a	Extend the Laurent expansion of $f(z) = \frac{1}{z^2 - 4z + 3}$ for $$1 < \ \lvert z \rvert < 3 \, (ii) \, \lvert z \rvert = 1 \, (iii) \, \lvert z \rvert > 3$$	Apply	CO3	PO 1,4
	b	Define Cauchy's Residue theorem of an analytic function f(z) within and on the closed curve. . Find the value of $\oint\limits_{c} \frac{1}{\sinh z} dz$ using Residue theorem. $$\lvert z \rvert = 4$$	Apply	CO4	PO 1,4
6	a	Define the following terms (i) The Isolated singularity of an analytic function f(z) (ii) Pole of order m of an analytic function f(z) (iii) Essential and Removable singularities of an analytic function f(z)	Understand	CO 3	PO 1,4
	b	Solve the value of $\int\limits_{0}^{\pi} \frac{d\theta}{(a + b\cos\theta)}$ using Residue theorem.	Apply	CO 4	PO 1,4
7	a	Define Gamma and Beta functions. Solve the integral $$\int\limits_{0}^{\infty} \sqrt{x} e^{-x/3} \, dx$$ using Gamma function.	Apply	CO 5	PO 1,2

	b	Solve the integral $$\int_0^2 (8-x)^{1/3} dx$$ using Beta-Gamma functions.	Apply	CO 5	PO 1
8	a	State any three properties of Beta function. Show that $$\beta(m,n) = \beta(m+1,n) + \beta(m,n+1)$$ using Gamma function.	Apply	CO 5	PO 1,2
	b	Solve the integral $$\int_0^1 x^m (\log x)^n dx = \frac{(-1)^m n!}{(m+1)^{n+1}}$$	Apply	CO 5	PO 1, 2
9	a	What is Bessel differential equation and most general solution of Bessels differential equation? Show the Bessel's recurrence relation $$xJ'_n(x) = nJ_n(x) - x\, J_{n+1}(x).$$	Apply	CO 6	PO 1
	b	Show that $J_{5/2}(x) = \sqrt{\frac{2}{\pi x}} \left(\frac{3-x^2}{x^2} \sin x - \frac{3}{x} \cos x \right)$	Apply	CO 6	PO 1, 4
10	a	Show that $\int_0^1 xJ_n(\alpha\, x) J_n(\beta\, x)\ dx = \begin{cases} 0, if\ \alpha = \beta \\ \frac{1}{2}[J_{n+1}(\alpha)]^2, if\ \alpha \neq \beta \end{cases}$	Apply	CO 6	PO 1,2
	b	Show that $$J_n(x) = \frac{1}{\pi} \int_0^\pi \cos(n\theta - x \sin\theta) d\theta$$ where $$J_n(x)$$ Bessel's function, 'n' being a integer.	Apply	CO 6	PO 1

KNOWLEDGE COMPETENCY LEVELS OF MODEL QUESTION PAPER



Signature of Course Coordinator **HOD,ECE**