Electric Circuits

Definitions

- **Current** (I) : The rate of flow of electric charge or the flow of electrons.

$$\boxed{I = \frac{Q}{T}} \text{ (amperes)} \qquad i = \frac{dq}{dt}$$

where T = time and Q = charge

- **Electric Charge** (Q) : Property of matter that governs how it is affected by electric or magnetic fields. Can be positive or negative.

$1e^-$ charge $= 1.6 \times 10^{-19}$ Coulambs

- **Voltage** (V) : Potential difference between 2 charged particles is the force required to bring them as near as possible.

$$\boxed{V = W/Q} \text{ (volts)}$$

- Electrical Energy $E = QV$

- **Electrical Power** (P) : The rate at which electrical energy is converted to other form of energy

$$\boxed{P = \frac{W}{T}} \text{ (watt)} \qquad \boxed{P = V \times I} = \frac{dw}{dq} \times \frac{dq}{dt} = \frac{dw}{dt} \text{ J/s}$$

* Ohm's law

Statement : At constant temperature in an electric circuit, the current flowing through a conductor is directly proportional to the potential difference between its two sides.

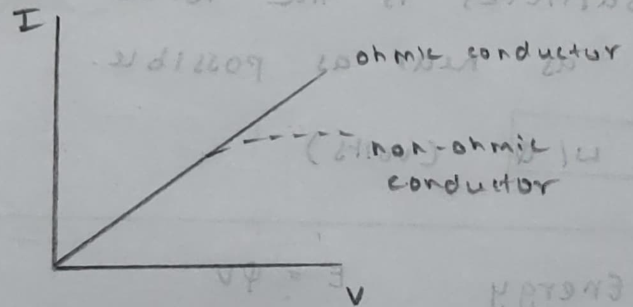
$$V \propto I \Rightarrow \boxed{V = IR}$$

where R is "resistance" = obstruction to the flow of current in a conductor or circuit.

- **Limitations** :

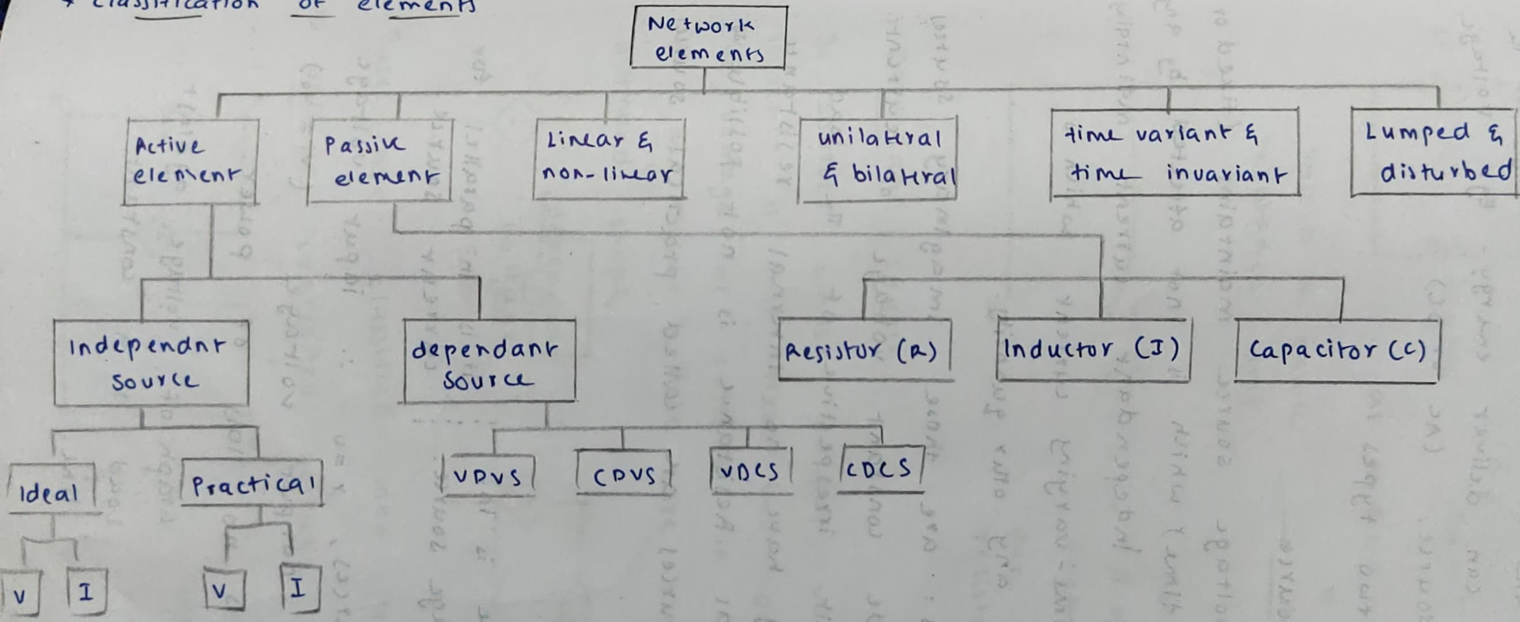
→ It is applicable only for metallic conductors.

→ It is not applicable for all electrical circuits such as semiconductor devices, transistor etc.



V	P/I	$\sqrt{PR}$
	$I \cdot R$	
I	$\frac{V}{R}$	P/V
	$\sqrt{P/R}$	
P	VI	I^2R
	V^2/R	
R	V/I	V^2/P
	P/I^2	

* Classification of elements



→ Active Elements

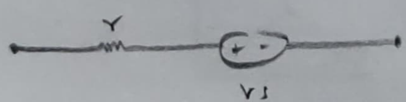
Active elements are the sources of energy or the element which can deliver energy. Eg: the voltage and current source. (AC or DC)

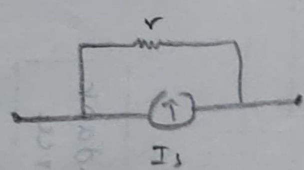
They are of two types:

- Independant source

independant voltage source maintains (fixed or varying with time) which is not affected by any other quantity. Independant current maintains a fixed or time-varying current which is unaffected by any other quantity.

⇒ Ideal sources: are those imaginary sources which provide constant voltage or current to the circuit irrespective of the load current. They have no internal resistance. Zero internal Resistance is not possible so all real sources are called practical sources.

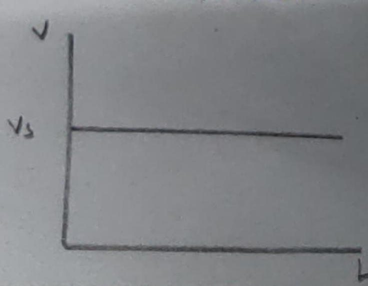




internal resistance is in series for voltage source.

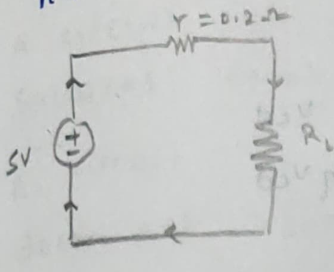
it is in parallel for current source.

In ideal sources, $r = 0$ ∴ input voltage is equal to output voltage ($V_i = V_o$) since there is no loss of power.



Graph of voltage against Load is a constant line.

⇒ practical sources : real - life sources which have some internal resistance.

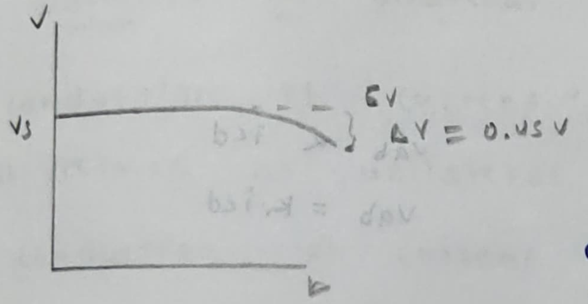


$$I = \frac{V}{(R_L + r)} = \frac{5}{2.2} = 2.27\,A$$

at $r = 0.2\,\Omega$, $V_r = I \cdot r$

$V_r = 2.27 \times 0.2$

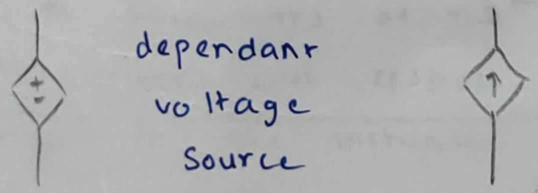
This is the ohmic resistance drop ⟶ $V_r = 0.45V$



There is a drop of 0.45V from vs to V_L due to internal resistance.

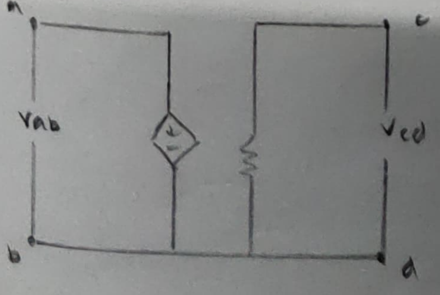
• Dependant source

The sources whose output voltage or current is not fixed but depends on the voltage or current in another part of the circuit. These are represented by diamond shape.



These are classified into 4 types:

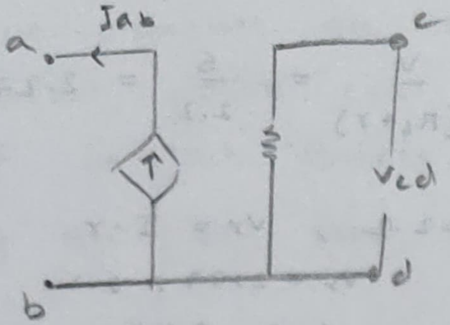
⇒ voltage controlled voltage source



$V_{ab} \propto V_{cd}$

$V_{ab} = K V_{cd}$

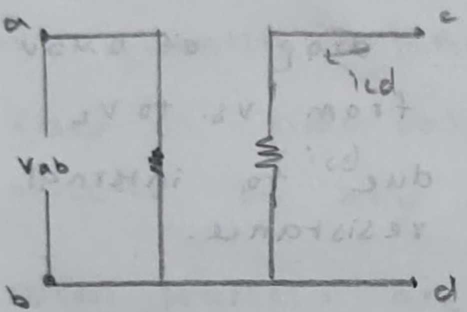
⇒ voltage controlled current source



$i_{ab} \propto V_{cd}$

$i_{ab} = \eta V_{cd}$

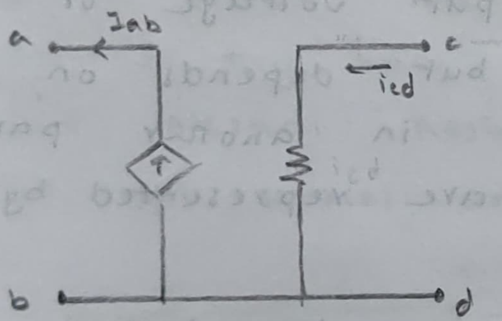
⇒ current controlled voltage source



$V_{ab} \propto i_{cd}$

$V_{ab} = k \cdot i_{cd}$

⇒ current controlled current source



$i_{ab} \propto i_{cd}$

$i_{ab} = k \cdot i_{cd}$

→ Passive Elements

Passive elements are recievers of energy. They consume energy in a circuit.

Classified into :

- Resistor (R)
- Inductor (L)
- Capacitor (C)

→ Linear or non-Linear Elements

A circuit is said to be linear if it satisfies ohm's law. eg: Resistor, indultor.

A circuit is said to be non-linear if it does not satiffies ohm's law. eg: diodes transistors etc.

→ Unilaterral or Bilateral Elements.

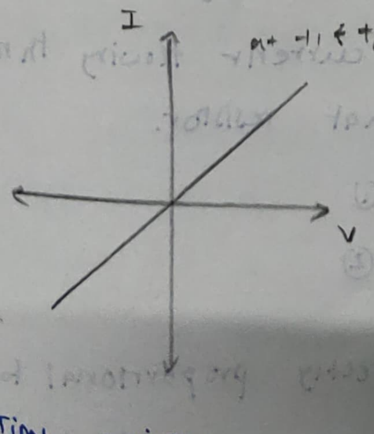
conduction of current in one direction is termed as unilateral. eg: diode, transistor.

Conduction of current in both directions in an element with same magnitude is termed as bilateral. eg: Resistor, Inductor.

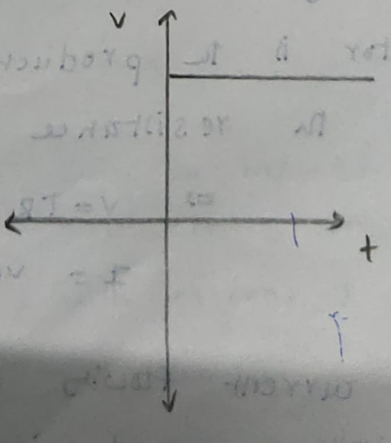
→ Time variant and time invariant

If the parameters of a element do not vary with time, then it is termed as time invariant. Eg: Resistor

If the parameters of the time variant are not constant with respect to time and may change in the instance of time.



Time variant



Time invariant

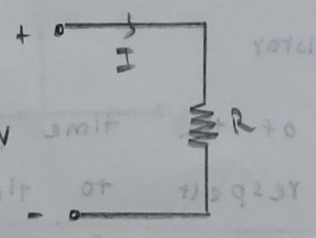
→ Lumped & Distributed Elements

The elements which are physically separable is called lumped elements. Eg: Inductor, Resistor.

The elements which are physically un-seperable are termed as distributed elements. Eg: transmission line.

R, L, C parameters:

Resistor: The main functionality of a resistor is to oppose or restrict the flow of electric current. Hence, the resistors are used in order to limit the amount of current flow and / or dividing voltage. Let the current flowing through the resistor is I amperes and the voltage across it is V volts. The symbol of resistor along with current I and voltage V are shown followingly.



According to Ohm's law, The voltage across the resistor is the product of current flowing through it and the resistance of that resistor.

$\Rightarrow$ $V = IR$ — (1)

$I = V/R$ — (2)

(2) $\Rightarrow$ current flowing is directly proportional to voltage and inversely proportional to resistance

Power in an electric circuit element can be represented as:

$$P = VI \quad \text{— (3)}$$

substitute eq (1) in eq (3)

$$P = (IR)I$$
$$P = I^2 R$$

substitute eq (2) in eq (3)

$$P = V(V/R)$$
$$P = V^2/R$$

So we can calculate the amount of power dissipated in resistor by using one of the formulae mentioned in eq (3) to eq (5)

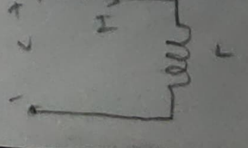
Inductor: In general inductors will have number of turns. Hence, they produce magnetic flux when current flow through it. So, the amount of total magnetic flux produced by an indicator depends on the. mathematically, an be written as

$$\psi \propto I$$
$$\psi = LI$$

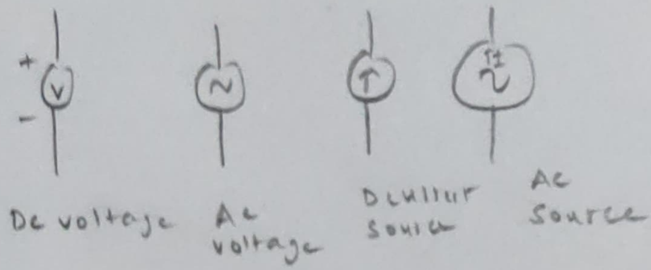
where,

- ψ is total magnetic flux
- L is inductance of an inductor

let the current flowing through the inductor is I amperes and the voltage across it is v volts. the symbol of inductor along with current I and voltage v are shown in following figure.

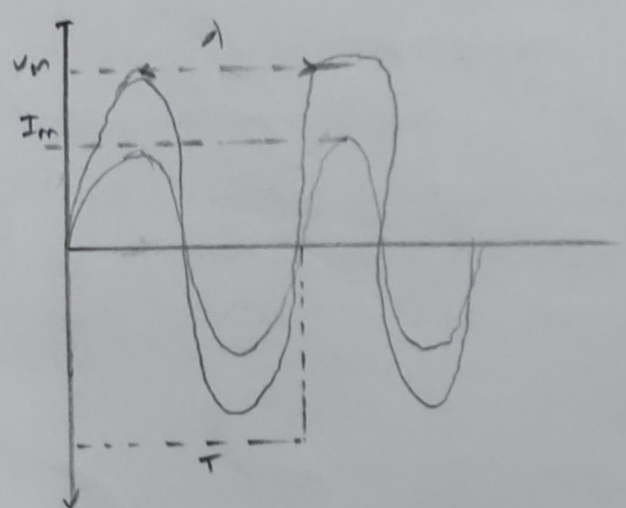


Standard symbols for electrical components



* Representation of Alternating quantities

The voltage that changes its polarity and magnitude at regular interval of time. similarly change in direction and magnitude of current. These are alternating quantities.



cycle : set of +ve & -ve

Time period : T

Wavelength : $\lambda = v/f$

Amplitude : Vm or Im

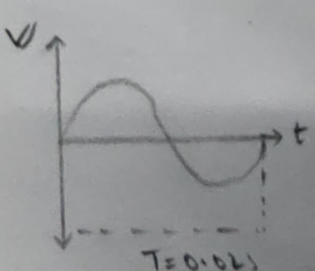
Peak to Peak value :

+Vm to - Vm (2Vm)

+Im to - Im (2Im)

frequency : $f = \frac{1}{\text{period}}$

Frequency is the number of waves that pass a fixed point in unit time ; no. of cycles or vibrations undergone during unit time



frequency $(f) = \frac{1}{\text{Time period}} = \frac{1}{0.02} = 50\ Hz$

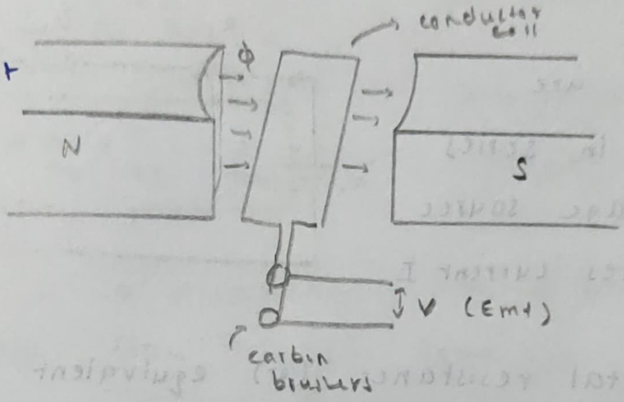
50 Hz ⇒ 50 cycles per second.

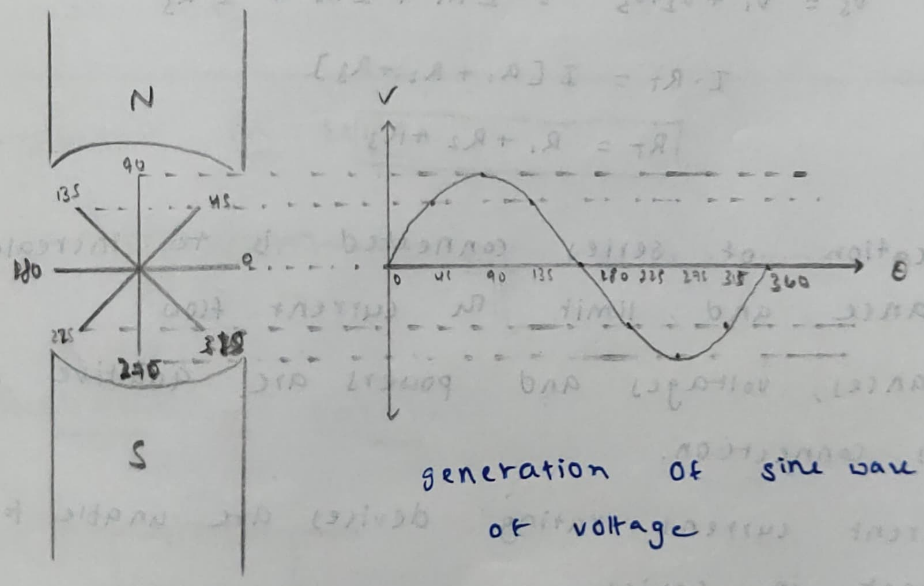
An AC generator uses the principle of Faraday's EMI to convert mechanical rotation into electrical energy.

EMF is induced in the conductor when it cuts a magnetic field.

$v(t) = V_m \sin \omega t$

$= V_m \sin(\theta)$



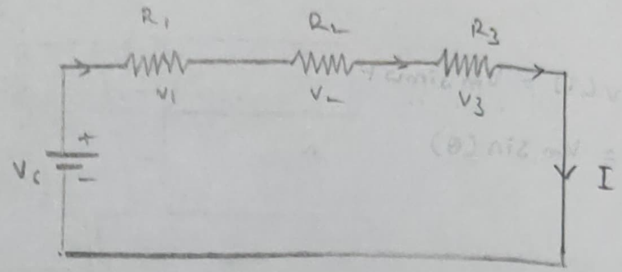


generation of sine wave of voltage

Resistors Series Connection

In a series circuit, all components are connected such that there is only one closed path & same current flows through all the components.

R_1, R_2, R_3 are connected in series with a voltage source V_s & causes current I



Total resistance (or) equivalent resistance

$$V_s = V_1 + V_2 + V_3 = IR_1 + IR_2 + I \cdot R_3$$

$$\cancel{I} \cdot R_T = I[R_1 + R_2 + R_3]$$

$$\boxed{R_T = R_1 + R_2 + R_3}$$

- Application of series connected is to increase the resistance and limit the current flow.
- Resistances, voltages and powers are additive in series connection.
- Different current rating devices are unable to connect in series

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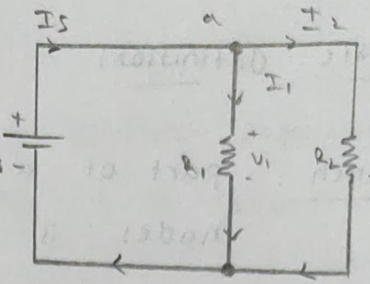
Resistors Parallel Connection

In parallel circuit all components in the circuit are connected in a manner that there are more than one path (or) branches in which current can flow.

Consider a circuit with resistors R_1 and R_2 connected in parallel.

current gets divided at node (point) a : $I = I_1 + I_2$



$$I_s = I_1 + I_2 + I_3 \cdots I_n$$

$$\frac{V_s}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} \cdots \frac{V}{R_n}$$

$$\frac{V_s}{R_{eq}} = V\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \cdots\right]$$

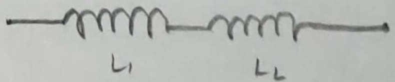
$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \cdots \frac{1}{R_n}}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2}$$

Voltage is same
current is divided

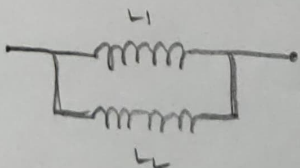
- If there is a discontinuity in a branch, current will still flow in other branches.
- used in Industries and home household connections.

Inductors in Series



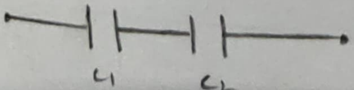
$$L_{eq} = L_1 + L_2$$

Inducton in parallel



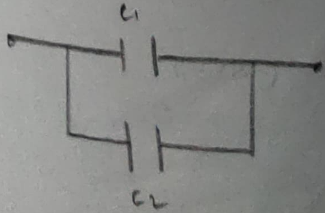
$$L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$$

Capacitors in Series



$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

Capacitors in parallel



$$C_{eq} = C_1 + C_2$$

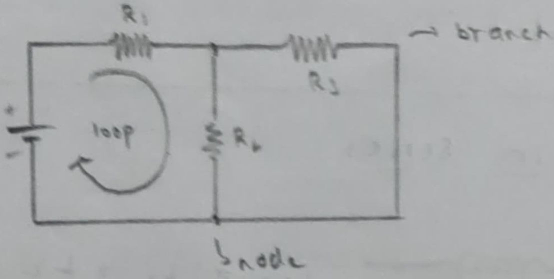
* Basic definitions

- Branch : part of a network connected between two nodes is a branch.

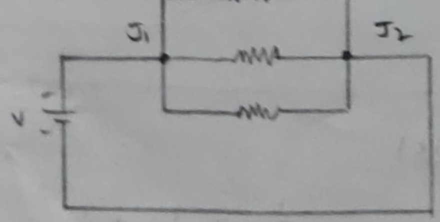
- node : 2 branches meeting at a point.

- principle node : 2 or more branches meeting at a point

- Mesh (or) loop : A closed path which starts and ends at same node which is not passing any node (or) branch twice.



- junction point : A point of a network at which three (or) more no. of circuit elements are joined together.



* kirchoff's Laws

1. kirchoff's current law (KCL)

2. kirchoff's voltage law (KVL)

→ Kirchoff's current law (KCL)

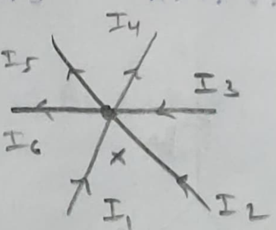
statement: KCL states that algebraic sum of currents at any node is equal to zero.

(or)

sum of incoming currents at a node or junction is equal to sum of total current leaving.

i.e $\boxed{\sum_{node} I = 0}$ (or) $\boxed{\sum I_{ent} = \sum I_{leav}}$

explanation:



At node x, 6 branches carrying currents I_1, I_2, I_3 I_4, I_5, I_6 are connected.

By KCL, algebraic sum = 0

$$\Rightarrow I_1 + I_2 + I_3 - I_4 - I_5 - I_6 = 0$$

$$I_1 + I_2 + I_3 = I_4 + I_5 + I_6$$

current entering = current leaving

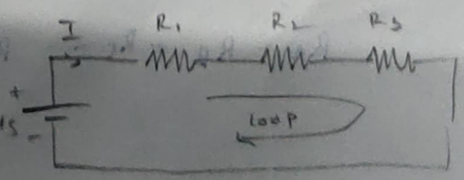
→ kirchoff's voltage Law (KVL)

statement: Algebraic sum of th voltages around a closed loop is equal to zero

i.e $\boxed{\sum_{In\ loop} V = 0}$ (or) $\boxed{\sum_{loop} V_s = \sum_{loop} (IR)\ drop}$

Algebraic sum of voltage sources is equal to voltage drops in a clossed loop or circuit.

Explanation:

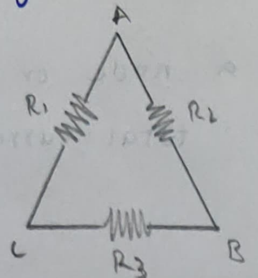


According to KVL,

$$V_s = IR_1 + IR_2 + IR_3$$

* Delta to Star transformation

conversion of resistors in Delta (Δ) formation to star ($*$) formation in order to simplify a given circuit

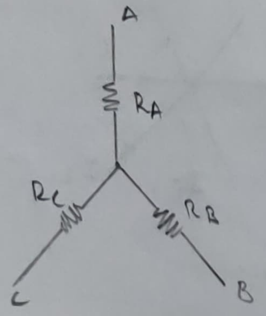


$$R_{AB} = \frac{(R_1+R_3)R_2}{R_1+R_2+R_3} \quad [R_2 \parallel (R_1+R_3)]$$

$$R_{BC} = \frac{(R_1+R_2)R_3}{R_1+R_2+R_3} \quad [R_3 \parallel (R_1+R_2)]$$

$$R_{CA} = \frac{(R_2+R_3)R_1}{R_1+R_2+R_3} \quad [R_1 \parallel (R_2+R_3)]$$

equivalent star network is as follows:



Here, $R_{AB} = R_A + R_B$

$R_{BC} = R_B + R_C$

$R_{CA} = R_C + R_A$

equating the 2 sets of eqns, we get:

$$R_A + R_B = \frac{(R_1+R_3)R_2}{R_1+R_2+R_3} \quad -① $$

$$R_B + R_C = \frac{(R_1+R_2)R_3}{R_1+R_2+R_3} \quad -② $$

$$R_C + R_A = \frac{(R_2+R_3)R_1}{R_1+R_2+R_3} \quad -③ $$

By adding ①, ② and ③ equations,

$$R_A + R_B + R_C = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1+R_2+R_3} \quad -④ $$

subtracting eqns ①, ② and ③ from ④ we get:

$$\boxed{R_A = \frac{R_1 R_2}{R_1+R_2+R_3}} \quad \boxed{R_B = \frac{R_2 R_3}{R_1+R_2+R_3}} \quad \boxed{R_C = \frac{R_3 R_1}{R_1+R_2+R_3}}$$

These are the corresponding star network eqns from delta network resistances.

* Star to Delta transformation

conversion of star network into equivalent delta network

from above equations,

$$R_A = \frac{R_1 R_2}{R_1+R_2+R_3} \quad R_B = \frac{R_2 R_3}{R_1+R_2+R_3} \quad R_C = \frac{R_3 R_1}{R_1+R_2+R_3}$$

multiply each set of 2 eqns and add,

$$R_A R_B + R_B R_C + R_C R_A = \frac{R_1 R_2 R_3}{R_1+R_2+R_3} \quad \text{— ④}$$

dividing ④ with ①, ② and ③ we get:

$$\boxed{R_1 = R_A + \frac{R_C R_A}{R_B} + R_C}$$

$$\boxed{R_2 = R_A + R_B + \frac{R_A R_B}{R_C}}$$

$$\boxed{R_3 = \frac{R_B R_C}{R_A} + R_B + R_C}$$

These equations can be used to find corresponding delta network resistances from a star network

* Mesh Analysis

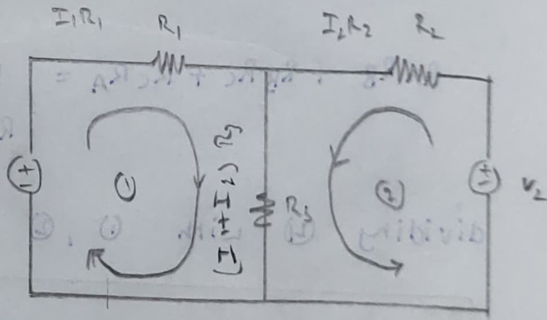
Any closed electrical path is called loop, Mesh is defined as a loop which does not contain any other loops within it, if a network has a larger number of voltage sources, it is better to use mesh analysis (depends on KVL)

$$m = b - (j - 1) \quad \{b\text{-branches} \quad j\text{-junctions}\}$$

Steps :

1. Identify the number of loops
2. Give the mesh current directions
3. Apply the KVL in every loop and get eqn.
4. Find the unknown mesh currents.
5. Find the unknown values

Eg:



In loop ①,

$V_1 - I_1R_1 - (I_1 + I_2)R_3 = 0$

$V_1 = I_1R_1 + (I_1 + I_2)R_3$

$V_1 = I_1(R_1 + R_3) + I_2R_3$

In loop ②,

$V_2 = I_2R_2 - (I_1 + I_2)R_3$

$V_2 = I_1R_3 + I_2(R_2 + R_3)$

Given, $R_1 = 2\Omega$, $R_2 = 4\Omega$, $R_3 = 3\Omega$, $V_1 = 5\Omega$, $V_2 = 10\Omega$

$5 = I_1(2+3) + I_2 \cdot 3 \Rightarrow 5 = 5I_1 + 3I_2$

$10 = I_1 \cdot 3 + I_2(4+3) \Rightarrow 10 = 3I_1 + 7I_2$

$50 = 15I_1 + 35I_2$

$-15 = 15I_1 + 9I_2$

$35 = 26I_2 \Rightarrow I_2 = \frac{35}{26}$

$\boxed{I_1 = 0.172\ A}$ $\boxed{I_2 = 1.346\ A}$

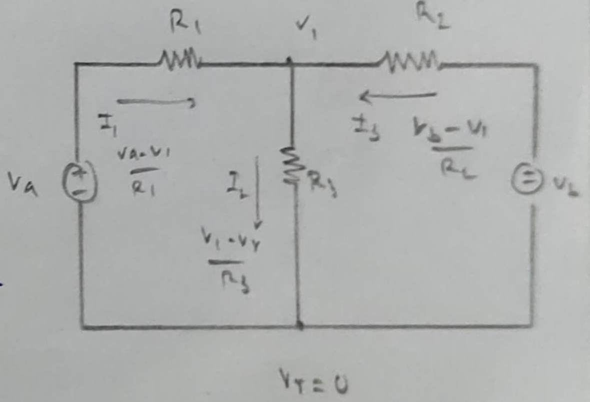
* Nodal Analysis

A node is a point in a network of common to two or more circuit elements. If 3 or more elements meet at a node, its called a principle node. A node voltage is the voltage of given node with respect to a particular node, called the reference node, which we assume at zero potential. If the network has more no. of current sources, then nodal analysis is useful.

N nodes will give (N - 1) equations.

Steps :

1. Identify no. of nodes
2. Mention the node voltages
3. Apply KCL at each node
4. Find unknown node voltage
5. Find unknown values.



at V_1 node, KCL ⇒

$$I_1 - I_2 + I_3 = 0$$

$$\frac{V_a - V_1}{R_1} - \frac{V_1 - V_2}{R_3} + \frac{V_b - V_1}{R_2} = 0$$

Given,
$V_a = 5\,\Omega$
$V_b = 10\,\Omega$
$R_1 = 2\,\Omega$
$R_2 = 4\,\Omega$
$R_3 = 3\,\Omega$

$$V_1 = 4.615\,V$$

$$\frac{5 - V_1}{2} - \frac{V_1 - 0}{3} + \frac{10 - V_1}{4} = 0$$

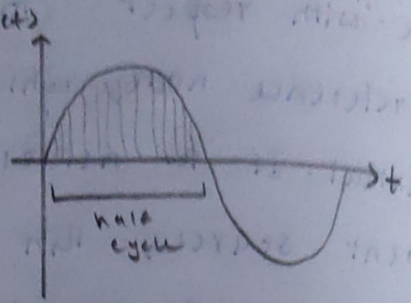
$\boxed{I_1 = 0.192\ A}$

$\boxed{I_2 = 1.346\ A}$

* Average value

The average value of an alternating quantity is defined as that value which is obtained by averaging all the instantaneous values over a period of a half cycle.

$$\boxed{f_{avg} = \frac{1}{t}\int_0^t f(t)\,dt}$$



at $t = \frac{2\pi}{2} = \pi$

* RMS value (Root Mean Square)

The RMS value of alternating current is equivalent to steady current (DC current) which produces the same amount of heat as that produced by an alternating current when it passing through a same circuit for a specified time.

$$\boxed{f_{rms} = \sqrt{\frac{1}{t}\int_0^t f^2(t)\,dt}}$$

square root of average of the squares of its instantaneous values over a one-complete cycle.

If $f(t) = (a_0 + a_1\cos\omega t + a_2\cos 2\omega t + \dots) + (b_1\sin\omega t + \dots)$

$$= \sqrt{a_0^2 + \frac{a_1^2 + a_2^2}{2} \dots \frac{b_1^2 + b^2}{2}}$$

$$f_{rms} = \sqrt{\frac{f_1^2 + f_2^2 + f_3^2 + \dots + f_n^2}{n}}$$

* **Form factor**

ratio of rms and average value is called form factor.

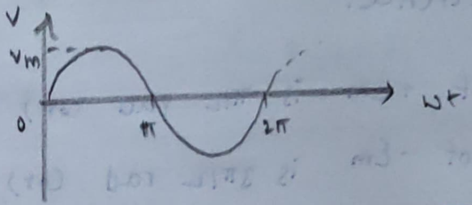
$$\boxed{F.F = K_f = \frac{\text{RMS value}}{\text{Average value}}}$$

* **Peak factor (or) crest factor :**

ratio of peak (or) max value to RMS value.

$$\boxed{P.F = K_p = \frac{\text{Max value}}{\text{RMS value}}}$$

Q. Derive Avg, RMS value and form and peak factor for given signal.



Sol

$V = V_m \sin \omega t$

$f_{avg} = \frac{1}{T}\int_0^T f(t)\,dt$

$f_{avg} = \frac{1}{\pi}\int_0^{\pi} V_m \sin(\omega t)\,d(\omega t)$

$f_{avg} = \frac{V_m}{\pi}\left[-\cos\omega t\right]_0^{\pi}$

$f_{avg} = \frac{2V_m}{\pi}$

$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi}(V_m \sin\omega t)^2\,d\omega t}$

$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V_m^2 \sin^2(\omega t)\,d\omega t}$

$= \sqrt{\frac{V_m^2}{2\pi}\int_0^{2\pi}\left[\frac{1-\cos 2\omega t}{2}\right]d(\omega t)}$

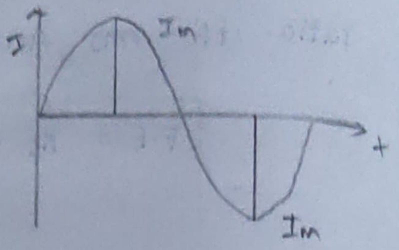
$= \frac{V_m}{\sqrt{2}}$

form factor $= \frac{RMS}{AVG} = \frac{V_m/\sqrt{2}}{2V_m/\pi} = \frac{V_m}{\sqrt{2}} \cdot \frac{\pi}{2V_m} = \frac{\pi}{2\sqrt{2}} \approx 1.11$

Peak factor $= \frac{max}{RMS} = \frac{V_m}{V_m/\sqrt{2}} = V_m \cdot \frac{\sqrt{2}}{V_m} = \sqrt{2} = 1.414$

* Time period of AC (or) peak value

Highest value reached by a quantity in the cycle is called maximum (or) peak (or) crest value.

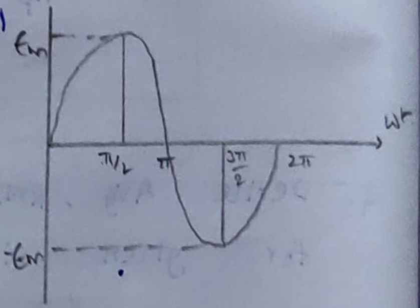


$$I = I_0 \sin(\omega t + \phi)$$

I_0 - Amplitude or peak value

* Phase:

Phase is defined as the fractional part of time period or cycle through which the quantity has advanced from the selected zero position of reference.



Phase of $+E_m$ is $\pi/2$ rad (or) $T/4$ sec

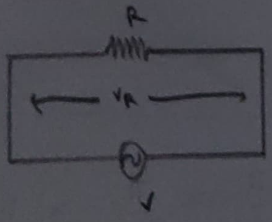
Phase of $-E_m$ is $3\pi/2$ rad (or) $3T/4$ sec

$$V = V_m \sin\omega t$$

$$I = I_m \sin\omega t$$

Two waveforms are said to be in phase when the phase difference b/w them is zero.

* AC circuit with pure resistance



$$V = V_m \sin\omega t$$

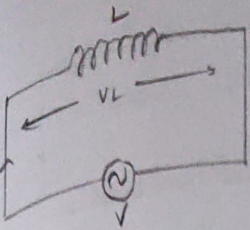
$$i = \frac{V}{R} = \frac{V_m \sin\omega t}{R}$$

$$i = I_m \sin\omega t \Rightarrow \boxed{I_m = \frac{V_m}{R}}$$

$V_m = V\sqrt{2}$

$I_m = \sqrt{2}\, I$

* AC circuit with pure inductance :



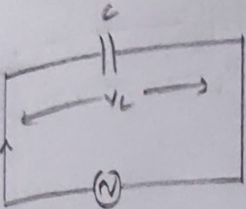
$v = V_m \sin \omega t$

$v = L\dfrac{di}{dt}$

$V_m \sin \omega t = L\dfrac{di}{dt}$

$i = I_m \sin(\omega t - \pi/2) \Rightarrow \boxed{I_m = \dfrac{V_m}{\omega L}}$

* AC circuit with pure capacitance :



$q = C V_m \sin \omega t$

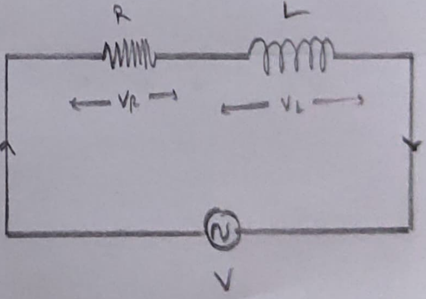
$i = C V_m \, \omega \cos \omega t$

$i = I_m \sin(\omega t + \pi/2) \Rightarrow \boxed{I_m = \omega C V_m}$

In Capacitor, $X_C = \dfrac{1}{2\pi f C}$

In Inductor, $X_L = 2\pi f L$

* R-L series Circuit



$V = \sqrt{V_R^2 + V_L^2}$

$V_R = IR \,;\, V_L = I X_L$

$V = \sqrt{(IR)^2 + (IX_L)^2}$

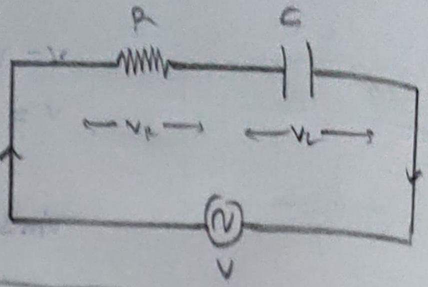
$V = I\sqrt{R^2 + X_L^2}$

$\boxed{V = IZ}$ where impedence Z, $\boxed{Z = \sqrt{R^2 + X_L^2}}$

formulae :

1. phase angle, $\theta = \tan^{-1}\left(\dfrac{V_L}{V_R}\right) = \tan^{-1}\left(\dfrac{X_L - X_C}{R}\right) = \tan^{-1}\left(\dfrac{X_L}{R}\right)$
2. Average power, $P = VI\cos\phi = I^2 R$
3. Impedence, $Z = \sqrt{R^2 + X_L^2}$
4. Power factor, $\cos\phi = R/Z$
6. $Z = \sqrt{R^2 + (X_L - X_C)^2}$

* R-C series circuit



$V = \sqrt{V_R^2 + V_C^2}$

$V = I\sqrt{R^2 + X_C^2}$

$V = IZ$

where impedence, $\boxed{Z = \sqrt{R^2 + X_C^2}}$

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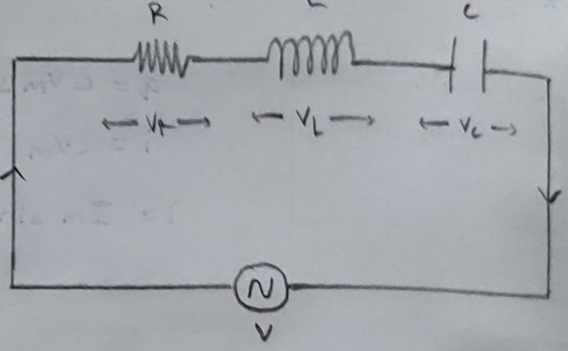
* RLC series circuit



$V = \sqrt{V_R^2 + (V_L - V_C)^2}$

$V = \sqrt{(IR)^2 + I^2(X_L - X_C)^2}$

$V = I\sqrt{R^2 + (X_L - X_C)^2}$

$V = IZ$

where impedence, $\boxed{Z = \sqrt{R^2 + (X_L - X_C)^2}}$

phase angle, $\phi = \tan^{-1}\left(\frac{V_L - V_C}{V_R}\right) = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$

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