MODULE - I

PROBABILITY THEORY

PART-A

① The possible no. of Exhaustive cases $= {}^9C_3$

(i). Let 'A' be the event of 3 students belong to different classes.

The favourable no. of outcomes for the event 'A' is ${}^2C_1 \times {}^3C_1 \times {}^4C_1$

Therefore, the required probability is.

$$P(A) = \frac{{}^2C_1 \times {}^3C_1 \times {}^4C_1}{{}^9C_3} = \frac{\cancel{2} \times \cancel{3} \times \cancel{4}2}{\cancel{84}} = \frac{2}{7}$$

$\cancel{28}$
$\cancel{14}7$

(ii) Let 'B' be the event that two belong to the same class and third to the different class.

The favourable no. of outcomes for the event 'B' is $({}^2C_2 \times {}^7C_1) + ({}^3C_1 \times {}^6C_1) + ({}^4C_2 \times {}^5C_1)$.

Therefore, the required probability is

$$P(B) = \frac{({}^2C_2 \times {}^7C_1) + ({}^3C_1 \times {}^6C_1) + ({}^4C_2 \times {}^5C_1)}{{}^9C_3} = \frac{55}{84},$$

(iii) Let 'c' be the event that the three belong to the same class.

The favourable no. of outcomes. ~~for~~ the event 'c' is $\left(0 + {}^{3}C_{3} \times {}^{4}C_{0} + {}^{4}C_{3} \times {}^{4}C_{0}\right)$

Therefore, the required probability is,

$$P(C) = \frac{\left(0 + {}^{3}C_{3} \times {}^{4}C_{0} + {}^{4}C_{3} \times {}^{4}C_{0}\right)}{{}^{9}C_{3}} = \frac{5}{84}$$

PART-A (UPDATED)

(2) Given; 'A' be the event of getting sum of points is odd.

'B' be the event of getting atleast one die shows an ace (1).

'A∩B' is the event that the sum of points is odd and atleast one die shows an ace.

$$S = \begin{Bmatrix} (1,1), (1,2) \text{ ——— } (1,6) \\ (2,1), (2,2), \text{ ——— } \\ \text{——— ——— } (6,6) \end{Bmatrix}$$

Total No. of outcomes = 36

(a) $P(A) = ?$

fav No. of outcomes = 18

(1,2), (1,4); (1,6)
(2,1), (2,3), (2,5)
(3,2), (3,4), (3,6) → fav. No. of cases
(4,1), (4,3), (4,5)
(5,2), (5,4), (5,6)
(6,1); (6,3), (6,5)

$$P(A) = \frac{18}{36} = \frac{1}{2}$$

(b) $P(B) = ?$

$\nearrow$ counted twice.

1st die $\rightarrow$ ((1,1), (1,2) — (1,6)) $\rightarrow$ 6

2nd die $\rightarrow$ ((2,1), (3,1), --- (6,1)) $\rightarrow$ 6

Total fav. outcms $= 6 + 6 - 1 = 11$

$$P(B) = \frac{11}{36}$$

(c) $P(A/B) = ?$

(d) $P(A \cap B) = ?$

1st die $\rightarrow$ (1,2), (1,4), (1,6) $\rightarrow$ 3

2nd die $\rightarrow$ (2,1), (2,4), (2,6) $\rightarrow$ 3

Total fav outcomes $= 6$

$$P(A \cap B) = \frac{6}{36} = \frac{1}{6}$$

(c) $P(A/B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{1}{6} \times \dfrac{36}{11} = \dfrac{6}{11}$

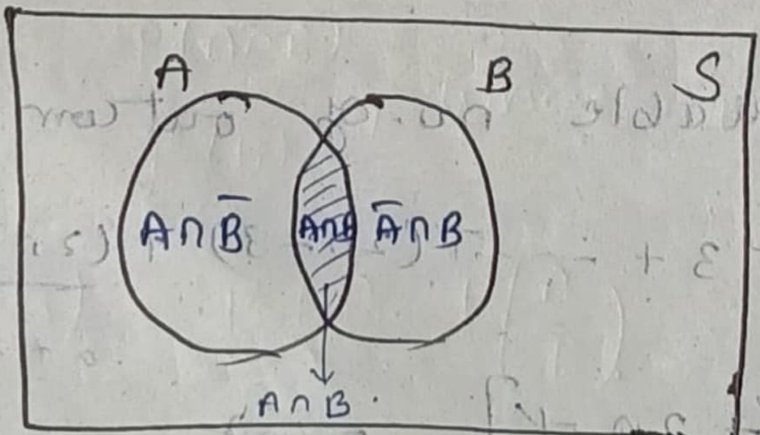
(e) $P(B/A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{1}{6} \times \dfrac{2}{1} = \dfrac{1}{3}$

(3) Law of addition of Probability for two events

Statement: Let 'A' and 'B' be any two events in the sample space 's' and are not the mutually exclusive, then.

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B).}$$

Proof: Consider the following venn diagram



From the venn diagram,

we can write; $(A \cup B) = A \cup (\bar{A} \cap B)$.

Apply probability on both sides

$$P(A \cup B) = P(A \cup \underbrace{(\bar{A} \cap B)}_{B})$$

We know that; A & B are two mutually exclusive ~~evets~~ events; so; $P(A \cup B) = P(A) + P(B)$

$$P(A \cup B) = P(A) + P(\bar{A} \cap B) \quad \text{— (1)}$$

consider; $P(\bar{A} \cap B) = ?$

↳ From venn diagram

$$B = (\bar{A} \cap B) \cup (A \cap B)$$

Apply probability on both sides.

$$P(B) = P(\underbrace{(\bar{A} \cap B)}_{A} \cup \underbrace{(A \cap B)}_{B})$$

$$P(B) = P(\bar{A} \cap B) + P(A \cap B)$$

$$P(\bar{A} \cap B) = P(B) - P(A \cap B)$$

from (1)

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

Hence, Proved.

④ **Conditional Probability:** If the occurence of event 'A' effects the occurence of event 'B', then the two events are said to be dependent events.

→ In this case, we have to calculate the probability of 'A' on the assumption that event 'B' already occured and vice-versa, such a probability is called conditional probability.

Multiplication theorem on probability:

Statement: If $A_1, A_2, ---, A_n$ are 'n' events in the sample space 's', then.

$$P(A_1 \cap A_2 \cap ---- A_{n-1} \cap A_n) = P(A_1) \cdot P\left(\frac{A_2}{A_1}\right) ---- P\left(\frac{A_n}{A_1 \cap A_2 ---- A_{n-1}}\right)$$

Proof:

Strategy: By Mathematical Induction

We know that, By conditional probability,

$$\boxed{P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}; \text{ Where } P(B) \neq 0.}$$

$$P(A \cap B) = P(B) \cdot P\left(\frac{A}{B}\right) \quad \text{—— ①}$$

Now on taking $B = A_1$ and $A = A_2$

$$P(A_1 \cap A_2) = P(A_1) \cdot P\left(\frac{A_2}{A_1}\right) \quad \text{--- (2)}$$

This is true for $n = 2$ events.

Now on taking $A = A_3$ and $B = A_1 \cap A_2$

$$P(A_1 \cap A_2 \cap A_3) = P(A_1 \cap A_2) \cdot P\left(\frac{A_3}{A_1 \cap A_2}\right).$$

from (2)

$$P(A_1 \cap A_2 \cap A_3) = P(A_1) \cdot P\left(\frac{A_2}{A_1}\right) \cdot P\left(\frac{A_3}{A_1 \cap A_2}\right).$$

This is true for $n = 3$ events.

Therefore, By the principle of mathematical Induction, The theorem is true for all the integral values of 'n'

To culminate;

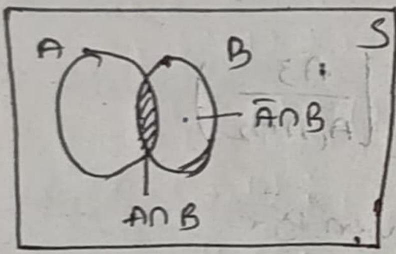
$$P(A_1 \cap A_2 \cap A_3 \cdots A_n) = P(A_1) \cdot P\left(\frac{A_2}{A_1}\right) \cdot P\left(\frac{A_3}{A_1 \cap A_2}\right) \cdots P\left(\frac{A_n}{A_1 \cap A_2 \cap A \cdots \cap A_{n-1}}\right)$$

Hence the proof.

(5) Given, that A and B are two independent events. Since, A and B are two independent events, $P(A \cap B) = P(A) \cdot P(B)$ — (1)

(i) To prove $\bar{A}$ and B are independent events.

Consider the following venn diagram,



From the venn diagram,

$$B = (A \cap B) \cup (\bar{A} \cap B)$$

$$P(B) = P(A \cap B) + P(\bar{A} \cap B).$$

$$P(\bar{A} \cap B) = P(B) - P(A \cap B).$$

$$P(\bar{A} \cap B) = P(B) - P(A) \cdot P(B)$$

$$P(\bar{A} \cap B) = P(B)\,(1 - P(A))$$

$$P(\bar{A} \cap B) = P(\bar{A}) \cdot P(B)$$

Therefore; $\bar{A}$ and B are also the independent events.

(ii) To prove A and $\bar{B}$ are independent events.

From the (i) venn diagram

$$A = (A \cap B) \cup (A \cap \bar{B})$$

$$P(A) = P(A \cap B) + P(A \cap \bar{B})$$

$$P(A \cap \bar{B}) = P(A) - P(A \cap B)$$

$$P(A \cap \bar{B}) = P(A) - P(A)P(B).$$

$$P(A \cap \bar{B}) = P(A)(1 - P(B))$$

$$P(A \cap \bar{B}) = P(A)\, P(\bar{B})$$

Therefore, A and $\bar{B}$ are independent events.

(iii) To prove $\bar{A}$ and $\bar{B}$ are independent events.

consider; $P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B})$

$$P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B)$$

$$P(\bar{A} \cap \bar{B}) = 1 - (P(A) + P(B) - P(A \cap B))$$

$$P(\bar{A} \cap \bar{B}) = 1 - P(A) - P(B) + P(A).P(B)$$

$$P(\bar{A} \cap \bar{B}) = 1 - P(A) - P(B)(1 - P(A))$$

$$P(\bar{A} \cap \bar{B}) = (1 - P(A))(1 - P(B))$$

$$P(\bar{A} \cap \bar{B}) = P(\bar{A}).P(\bar{B})$$

Therefore, $\bar{A}$ and $\bar{B}$ are also independent events.

(6) Bag '1' $\rightarrow$ 5 Red + 3 Black $\rightarrow$ 8

Bag '2' $\rightarrow$ 4 Red + 5 Black $\rightarrow$ 9

one Bag is selected and 2 balls are drawn from it

$P(\underbrace{1 \text{ Red} + 1 \text{ Black}}_{E}) = ?$

$P(\text{Bag } 1) = P(\text{Bag } 2) = \frac{1}{2}$

Balls from Bag "1"

$5C_1 \times 3C_1 \rightarrow$ fav cases

$8C_2 \rightarrow$ Total cases

$$P(E/\text{Bag } 1) = \frac{5C_1 \times 3C_1}{8C_2} = \frac{15}{28}$$

Balls from Bag '2'

fav cases $\rightarrow 4C_1 \times 5C_1$

Total cases $\rightarrow 9C_2$

$$P(E/\text{Bag } 2) = \frac{4C \times 5C_1}{9C_2} = \frac{20}{36} = \frac{5}{9}$$

Total Probability

$$P(E) = P\left(\frac{E}{Bag_1}\right) \cdot P(Bag\ 1) + P\left(\frac{E}{Bag_2}\right) \cdot P(Bag_2)$$

$$= \frac{15}{28} \cdot \frac{1}{2} + \frac{5}{9} \cdot \frac{1}{2}$$

$$= \frac{275}{504}$$

$\therefore$ The Probability that one ball is red and other is black is : $P(E) = \frac{275}{504}$

(16)

(7) Let 'A' and 'B' be the events that the sum is greater than '8' and neither '7' nor '11' respectively:-

If two dice are thrown, the Exhaustive no. of outcomes = 36

$$S = \begin{Bmatrix} (1,1); (1,2); (1,3); (1,4); (1,5); (1,6) \\ (2,1); (2,2); (2,3); (2,4); (2,5); (2,6) \\ (3,1); (3,2); (3,3); (3,4); (3,5); (3,6) \\ (4,1); (4,2); (4,3); (4,4); (4,5); (4,6) \\ (5,1); (5,2); (5,3); (5,4); (5,5); (5,6) \\ (6,1); (6,2); (6,3); (6,4); (6,5); (6,6) \end{Bmatrix}$$

(i) The favourable no. of outcomes for event 'A' is 10

$$\begin{Bmatrix} (3,6); (4,5); (4,6); (5,4); (5,5); (5,6); \\ (6,3); (6,4); (6,5); (6,6). \end{Bmatrix}$$

$$P(A) = \frac{\text{Favourable No. of outcomes}}{\text{Total No. of outcomes}}$$

$$P(A) = \frac{10}{36} \checkmark$$

(ii) The possible cases for the sum '7'

is $\{(1,6)\ (2,5)\ (3,4)\ (4,3)\ (5,2)\ (6,1)\}$

↳ 6 cases

The possible cases for the sum '11' is

2 cases only i.e $\{(5,6);(6,5)\}$

$P(\underbrace{\text{getting } 7}_{(\text{let } C)}) = P(C) = 6/36.$

$P(\underbrace{\text{getting } 11}_{(\text{let } D)}) = P(D) = 2/36$

P(neither 7 nor 11) $= P(\bar{C} \cap \bar{D}) = P(\overline{C \cup D})$

$P(B) = 1 - P(C \cup D)$

mutually exclusive

$P(B) = 1 - (P(C) + P(D))$

$P(B) = 1 - \left(\frac{6}{36} + \frac{2}{36}\right)$

$P(B) = 1 - \frac{8}{36}$

$P(B) = \frac{28}{36}$ ✓

(8) Group → 4 men & 2 women

Total → 6

Three persons are selected at random

Total No. of outcomes $= {}^6C_3 = 20$

(i) 1 man & 2 women

Fav outcomes → ${}^4C_1 \times {}^2C_2 = 4 \cdot 1 = 4$

$P(\text{1 man \& 2 women}) = \frac{4}{20} = \frac{1}{5}$

(ii) 2 men & 1 women

Fav outcomes → ${}^4C_2 \times {}^2C_1 = 6 \times 2 = 12$

$P(\text{2 men \& 1 women}) = \frac{12}{20} = \frac{3}{5}$

(9) Given; $P(A) = \frac{1}{2}$; $P(B) = \frac{3}{4}$; $P(C) = \frac{1}{4}$

The problem will be solved if anyone solves the problem that means we need to calculate $P(A \cup B \cup C)$.

Since A & B are independent events;

$$P(A \cap B) = P(A) \cdot P(B).$$

We know that;

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A) \cdot P(B) - P(B) \cdot P(C) - P(C) \cdot P(A) + P(A) \cdot P(B) \cdot P(C)$$

$$P(A \cup B \cup C) = \frac{1}{2} + \underbrace{\frac{3}{4} + \frac{1}{4}}_{1} - \frac{1}{2} \cdot \frac{3}{4} - \frac{3}{4} \cdot \frac{1}{4} - \frac{1}{4} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{1}{4}$$

$$P(A \cup B \cup C) = \frac{1}{2} + 1 - \frac{3}{8} - \frac{3}{16} - \frac{1}{8} + \frac{3}{32}$$

$$P(A \cup B \cup C) = 1 - \underbrace{\frac{6}{32} + \frac{3}{32}}_{} = 1 - \frac{3}{32}$$

$$\boxed{P(A \cup B \cup C) = \frac{29}{32}}$$

(10) Let E_1, E_2, E_3 denotes the events that boxes are selected at random (1, 2, 3)

then $P(E_1) = 1/3$

$P(E_2) = 1/3$

$P(E_3) = 1/3$

Let 'A' denotes the event of getting a red ball then

$P(A/E_1) = \frac{3C_1}{8C_1} = \frac{3}{8}$

$P(A/E_2) = \frac{1C_1}{6C_1} = 1/6$

$P(A/E_3) = \frac{4C_1}{9C_1} = \frac{4}{9}$

The required probability that it comes from box 2 is given by,

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2) \cdot P\left(\frac{A}{E_2}\right)}{\sum_{i=1}^{3} P(E_i) \cdot P\left(\frac{A}{E_i}\right)}$$

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2) \cdot P(A/E_2)}{P(E_1) \cdot P\left(\frac{A}{E_1}\right) + P(E_2) \cdot P\left(\frac{A}{E_2}\right) + P(E_3) \cdot P\left(\frac{A}{E_3}\right)}$$

$$P\left(\frac{E_2}{A}\right) = \frac{\left(\frac{1}{3} \times \frac{1}{6}\right)}{\left(\frac{1}{3} \times \frac{3}{8}\right) + \left(\frac{1}{3} \times \frac{1}{6}\right) + \left(\frac{1}{3} \times \frac{4}{9}\right)} = \frac{12}{71}$$

(Take $1/3$ common for simple Calculation

8

PART-B (UPDATED)

20
15
24

~~12~~

60 13
1/5, 20

$\boxed{15}$ ~~12~~

~~64~~
15

① 60 Boys & 20 girls

half of Boys → 30
half of girls → 10

Total students who play cricket → 40

Total Students in class → 80.

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$.

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{30}{40} + \frac{10}{40} - \frac{40}{80}$

$= 1/2$

∴ The Person to be a boy or a girl who plays cricket → 1/2.

(2) let 'A' be the event that the later box contains 2 red and 6 black balls

Total No. of balls = 15

No. of ways of choosing 8 balls from 15 is $^{15}C_8$ → Total no. of cases.

No. of ways of choosing 2 red balls out of 5 is $^5C_2$.

No. of ways of choosing 6 black balls out of 10 is $^{10}C_6$.

Favourable no. of cases = $^5C_2 \times {}^{10}C_6$.

$$P(A) = \frac{\text{Favourable no. of cases}}{\text{Total No. of Cases}}$$

$$P(A) = \frac{^5C_2 \times {}^{10}C_6}{^{15}C_8}$$

$$P(A) = \frac{140}{429}$$ ✓

(3) BOX I → 2 Black + 3 Red + 1 White → 6

BOX II → 1 Black + 1 Red + 2 White → 4

BOX III → 5 Black + 3 Red + 4 White → 12

$P(\text{Box I}) = P(\text{Box II}) = P(\text{Box III}) = \frac{1}{3}.$

$P(\text{Red}) = ?$

$P(R) = P\left(\frac{R}{B_1}\right).P(B_1) + P\left(\frac{R}{B_2}\right).P(B_2) + P\left(\frac{R}{B_3}\right).P(B_3)$

$P(R) = \frac{3}{6} \cdot \frac{1}{3} + \frac{1}{4} \cdot \frac{1}{3} + \frac{3}{12} \cdot \frac{1}{3}$

$P(R) = \frac{4}{12} = \frac{1}{3}$

∴ The Probability that the ball drawn is Red is $1/3$.

(4) Let 'A' be the event of drawing 4 gold coins in 1^{st} draw. Let 'B' be the event of drawing 4 silver coins in 2^{nd} draw.

In a bag there contains 10 gold and 8 silver coins. Two successive drawings of 4 coins are made such that

(i) Coins are replaced before second trail,

$P(A \cap B) = P(A) \cdot P(B)$ {∵ A & B - independent events}

$$P(A \cap B) = \frac{^{10}C_4}{^{18}C_4} \times \frac{^{8}C_4}{^{18}C_4}$$

$$P(A \cap B) = \frac{49}{31212}$$

(ii) Coins are not replaced before second trail;

$$P(A \cap B) = P(A) \cdot P(B/A)$$

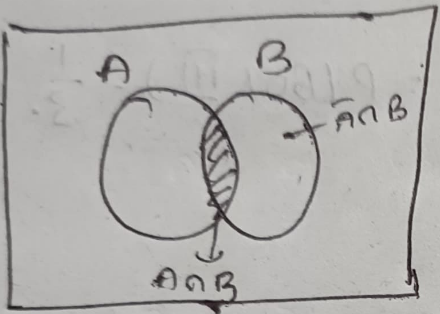
$$P(A \cap B) = \frac{^{10}C_4}{^{18}C_4} \times \frac{^{8}C_4}{^{14}C_4}$$

$$P(A \cap B) = \frac{35}{7293}$$

2

(5) let A & B be independent events

$$P(A\cap B) = P(A)\cdot P(B) \quad \text{--- (1)}$$



From venn diagram

$$B = (A\cap B) \cup (\bar{A}\cap B)$$

$$P(B) = P(A\cap B) + P(\bar{A}\cap B)$$

$$P(\bar{A}\cap B) = P(B) - P(A\cap B)$$

Hence proved

⑥ BOX → 6 red, 4 white & 5 black balls

Total - 15 balls.

4 balls are drawn at random and the ball drawn. There is at least 1 ball of each colour.

Case - 1 : R R W B   Case - 2 : W W R B   Case - 3 : B B R W

Each possibility, we have the following ways of arranging the balls $= \frac{4!}{2!} = 12$ ways

Case - 1 : $P_1 = \frac{6}{15} \times \frac{5}{14} \times \frac{4}{13} \times \frac{5}{12} \times 12 = \frac{600}{2730}$

Case - 2 : $P_2 = \frac{4}{15} \times \frac{3}{14} \times \frac{6}{13} \times \frac{5}{12} \times 12 = \frac{360}{2730}$

Case - 3 : $P_3 = \frac{5}{15} \times \frac{4}{15} \times \frac{6}{13} \times \frac{4}{12} \times 12 = \frac{480}{2730}$

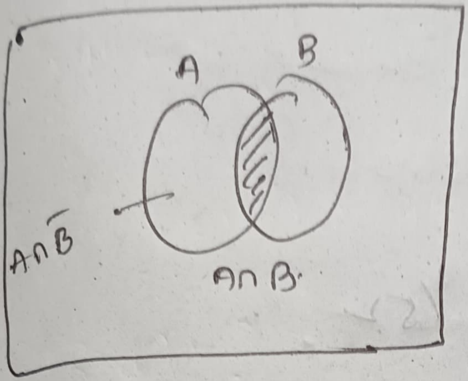
The required probability $= P_1 + P_2 + P_3$

$= \frac{48}{91}$

$=$

(7) let A & B be independent events.

$P(A \cap B) = P(A) \cdot P(B)$ — (1)



From venn diagram.

$A = (A \cap \bar{B}) \cup (A \cap B)$

$P(A) = P(A \cap \bar{B}) + P(A \cap B)$

$P(A \cap \bar{B}) = P(A) - P(A \cap B)$

Hence proved

(8) Total 52 cards

$P(\text{Spade}) = 13/52$

$P(\text{Ace}) = 4/52$

$P(\text{Ace of Spades}) = 1/52$

$P(\text{Ace or Spades}) = ?$

$P(\text{Ace or Spades}) = P(\text{Ace}) + P(\text{Spade}) - P(\text{Ace of Spades})$

$\Rightarrow \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52}$

$= \frac{4}{13}$

(a) Let $P(A)$; $P(B)$, $P(C)$ be the probabilities of the events that the bolts are manufactured by the machines A, B, C respectively. Then

$$P(A) = \frac{20}{100} = \frac{1}{5}; \quad P(B) = \frac{30}{100} = \frac{3}{10}; \quad P(C) = \frac{50}{100} = \frac{1}{2}.$$

Let 'D' denote the bolt is defective, then

$$P(D/A) = \frac{6}{100}; \quad P(D/B) = \frac{3}{100}; \quad P(D/C) = \frac{2}{100}$$

If bolt is defective, then the probability that it is from

(i) machine - A:

$$P(A/D) = \frac{P(D/A) \cdot P(A)}{P(D/A) \cdot P(A) + P(D/B) \cdot P(B) + P(D/C) \cdot P(C)}$$

$$P(A/D) = \frac{12}{31}.$$

(ii) machine - B:

$$P(B/D) = \frac{P(D/B) \cdot P(B)}{P(D/A) \cdot P(A) + P(D/B) \cdot P(B) + P(D/C) \cdot P(C)}$$

$$P(B/D) = \frac{9}{31}.$$

(iii) machine - C:

$$P(C/D) = \frac{P(D/C) \cdot P(C)}{P(D/A) \cdot P(A) + P(D/B) \cdot P(B) + P(D/C) \cdot P(C)}$$

$$P(C/D) = \frac{10}{31}.$$

(10) Let the probabilities of business man going to hotels X, Y, Z be respectively $P(X), P(Y), P(Z)$. Then,

$$P(X) = \frac{20}{100} = \frac{2}{10} ; \quad P(Y) = \frac{50}{100} = \frac{5}{10} ;$$

$$P(Z) = \frac{30}{100} = \frac{3}{10} .$$

Let 'E' be the event that the hotel room has faulty plumbing. Then the probabilities that hotels x, y, z have faulty plumbing are: $P(E/x) = \frac{5}{100} = \frac{1}{20}$; $P(E/y) = 4/100 = 1/25$

$$P(E/z) = 8/100 = 2/25 .$$

The probability that the business man's room having faulty plumbing is assigned to hotel z

$$= P(z/E) = \frac{P(z) . P(E/z)}{P(x) . P\left(\frac{E}{x}\right) + P(y) . P\left(\frac{E}{y}\right) + P(z) . P\left(\frac{E}{z}\right)}$$

$$= \frac{\frac{3}{10} \times \frac{2}{25}}{\frac{2}{10} \times \frac{1}{20} + \frac{5}{10} \times \frac{1}{25} + \frac{3}{10} \times \frac{2}{25}}$$

$$= \frac{4}{9} .$$

(11) Given; $P(A) = \frac{1}{2}$; $P(B) = \frac{1}{3}$; $P(A \cap B) = \frac{1}{5}$

(i) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

$P(A \cup B) = \frac{1}{2} + \frac{1}{3} - \frac{1}{5} = \frac{5}{6} - \frac{1}{5}$

$\Rightarrow P(A \cup B) = \frac{19}{30}$ ✓

(ii) $P(\bar{A} \cap B) = P(B) - P(A \cap B)$

$P(\bar{A} \cap B) = \frac{1}{3} - \frac{1}{5} = \frac{5-3}{15}$

$P(\bar{A} \cap B) = \frac{2}{15}$ ✓

(iii) $P(A \cap \bar{B}) = P(A) - P(A \cap B)$

$P(A \cap \bar{B}) = \frac{1}{2} - \frac{1}{5} = \frac{5-2}{10}$

$P(A \cap \bar{B}) = \frac{3}{10}$ ✓

(iv) $P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B})$

$P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B) = 1 - \frac{19}{30}$

$P(\bar{A} \cap \bar{B}) = \frac{11}{30}$ ✓

(12) 6 Men, 4 ladies → Total → 10

Committee members → 5

A Committee of 5 members out of 10 Can be formed in $^{10}C_5$ ways = 252.

The favourable cases for one lady,

	Men	ladies	
1 lady + 4 men →	$^6C_4$ ×	$^4C_1$	= 60
2 ladies + 3 men →	$^6C_3$ ×	$^4C_2$	= 120
3 ladies + 2 men →	$^6C_2$ ×	$^4C_3$	= 60
4 ladies + 1 men →	$^6C_1$ ×	$^4C_4$	= 6

Total — 246 ways

P(atleast one lady) = $\frac{\text{fav. no. of cases}}{\text{Total no. of cases}}$

$= \frac{246}{252}$

$= \frac{41}{42}$

(13) let 'C' → Disease x correctly diagnosed by Doctor 'A'.

'W' → Disease x wrongly diagnosed by Doctor 'A'

'D' → Patient died

$P(C/D) = ? = \frac{P(D/C) \cdot P(C)}{P(D)}$

$P(C) = 0.60$

$P(W) = 1 - P(C) = 0.40$

$P(D/C) = 0.40$

$P(D/W) = 0.70$

$P(D) = P(D/C) \cdot P(C) + P(D/W) \cdot P(D)$

$P(D) = 0.24 + 0.28 = 0.52$

$P(C/D) = \frac{(0.40)(0.60)}{0.52} = 6/13$

∴ The probability that the disease was correctly diagnosed given that the patient died is $6/13$.

(14) Bag → 5 white, 7 Black and 4 Red

Total → 16 balls.

If 3 balls are drawn at random $= {}^{16}C_3 = 560$

a) P(All 3 are different) = ?

Fav outcomes $= {}^5C_1 \times {}^7C_1 \times {}^4C_1 = 140.$

$$P(\text{All different}) = \frac{140}{560} = \frac{1}{4}$$

b) P(All 3 same colour) = ?

Fav outcomes $= {}^5C_3 + {}^7C_3 + {}^4C_3 = 49$

$$P(\text{All Same}) = \frac{49}{560} = \frac{7}{80}.$$

c) P(2 ball same, 1 different) = ?

i) 2 white → ${}^5C_2 = 10.$

1 from remaining → ${}^{11}C_1 = 11$

Total $= 11 \cdot 10 = 110.$

(ii) 2 Black → $^7C_2 = 21$

1 from remaining → $^9C_1 = 9$

Total = $21 \times 9 = 189$

(iii) 2 Red → $^4C_2 = 6$

1 from remaining → $^{12}C_1 = 12$.

Total = $6 \cdot 12 = 72$

Fav. outcomes = $110 + 189 + 72 = 371$

$P(2 \text{ same}, 1 \text{ different}) = \frac{371}{560}$

(15) F → Flu ; M → measles ; R → Rashes

$$P(F/R) = ? = \frac{P(R/F) \cdot P(F)}{P(R)}$$

Given: $P(F) = 0.90$

$P(M) = 0.10$

$P(R/F) = 0.08$

$P(R/M) = 0.95$

$$P(R) = P(R/F) \cdot P(F) + P(R/M) \cdot P(M)$$

$$= 0.072 + 0.095$$

$$= 0.167$$

$$P(F/R) = \frac{0.08 \times 0.90}{0.167} = 0.431$$

∴ The probability that the child has flu given that rashes are observed approximately $\approx 43.1\%$

(16) A Committee of 4 students out of 15 can be formed in ${}^{15}C_4$ ways.
(10 boys + 5 girls.)

Let 'E' be the event of forming a committe with atleast 3 girls
1 boy 3 girls + 0 boy 4 girls.
No. of ways of forming the committee is = The no. of favourable ways to E.

$= \left({}^{10}C_1 \times {}^{5}C_3\right) + \left({}^{10}C_0 \times {}^{5}C_4\right)$

$= 100 + 5$

$= 105$

$$P(E) = \frac{\text{No. of Favourable ways to 'E'}}{\text{Exhaustive ways to 'E'}}$$

$$P(E) = \frac{105}{{}^{15}C_4} = 0.0769$$

(17) Given that 5 men out of 100 and 25 women out of 10000 are color blind.

Now, A color blind person is choosen at random.

The probability that the chosen person is male $= P(m) = \frac{1}{2}$.

The probability that the chosen person is female $= P(W) = \frac{1}{2}$.

Let B represent a blind person. Then,

$$P(B/M) = 5/100 = 0.05$$

$$P(B/W) = 25/10000 = 1/400 = 0.0025$$

The probability that the chosen person is male is given by,

$$P(M/B) = \frac{P(B/m) \cdot P(M)}{P(m) \cdot P(B/m) + P(W) \cdot P(B/W)}$$

$$= \frac{0.05 \times 0.5}{(0.05 \times 0.5) + (0.5 \times 0.0025)}$$

$$= 0.95$$

(18) Given: $P(H/M) = \frac{4}{100}$

$P(H/W) = 1/100$

$P(W) = \frac{60}{100} = \frac{3}{5}$ $\qquad P(W/H) = ?$

$P(M) = 1 - P(W)$ $\quad (\because P(M) + P(W) = 1)$

$P(M) = 1 - \frac{3}{5}$

$P(M) = \frac{2}{5}$

From Baye's theorem,

$$P(W/H) = \frac{P(W) \cdot P(H/W)}{P(M) \cdot P\left(\frac{H}{M}\right) + P(W) \cdot P\left(\frac{H}{W}\right)}$$

$$P(W/H) = \frac{\frac{3}{5} \cdot \frac{4}{100}}{\frac{2}{5} \cdot \frac{4}{100} + \frac{3}{5} \cdot \frac{1}{100}} = \frac{3}{8+3}$$

$$P(W/H) = \frac{3}{11}$$

(19) Let $P(A)$ be the probability of 'A' hitting target.

$P(B)$ be the probability of 'B' hitting target.

$P(C)$ be the probability of 'C' hitting target.

Given; $P(A) = \frac{3}{5}$; $P(B) = 2/5$; $P(C) = 3/4$

$P(\bar{A}) = \frac{2}{5}$; $P(\bar{B}) = \frac{3}{5}$; $P(\bar{C}) = 1/4$

$P(A \cup B \cup C) = ?$

The probability of that none of A, B, C hits the target $= P(\bar{A} \cap \bar{B} \cap \bar{C})$

$= P(\bar{A}) . P(\bar{B}) . P(\bar{C})$

$\Rightarrow \frac{2}{5} . \frac{3}{5} . \frac{1}{4} = \frac{3}{50}$

$\therefore$ Required probability $= P(A \cup B \cup C)$

= The probability of atleast one of A, B, C hitting the target.

$\Rightarrow 1 - P(\bar{A} \cap \bar{B} \cap \bar{C}) = 1 - \frac{3}{50}$

$= \frac{47}{50}$.

(10) S → Spam ; N → Not spam ; T → Tagged as spam

$$P(N/T) = ? = \frac{P(T/N) \cdot P(N)}{P(T)}$$

Given ;

$P(S) = 0.50$; $P(N) = 0.50$

$P(T/S) = 0.99$; $P(T/N) = 0.05$

$$P(T) = P(T/S) \cdot P(S) + P(T/N) \cdot P(N)$$

$$= 0.495 + 0.025$$

$$= 0.52$$

$$P(N/T) = \frac{0.05 \times 0.50}{0.52} = 0.0481$$

∴ The probability that the mail is not spam given that it is tagged as spam is approximately 4.81%.

Verified ! & updated !

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