PART - A

1) Verify - Cayley-Hamilton

$A = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$ evalute $= 2A^4 - 5A^3 - 7A - 6I$.

Characteristic eauation.

$\lambda^2 - S_1\lambda + S_2 = 0$

~~$A^2 - 3A - 2 = 0$~~

$S_1 =$ Trace of matrix $=$

$S_1 = 1+2 = 3$

$S_2 =$ det of $A = 2 - 4 = -2 \quad \lambda^2 - 3\lambda + 2 = 0$

By cayley-Hamilton theorm =

$A^2 - 3A - 2I = 0$

$A^2 = \begin{bmatrix} 5 & 6 \\ 6 & 8 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 5 & 6 \\ 6 & 8 \end{bmatrix} - 3\begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 5 & 6 \\ 6 & 8 \end{bmatrix} - \begin{bmatrix} 3 & 6 \\ 6 & 6 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 5-3-2 & 6-6-0 \\ 6-6-0 & 8-6-2 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ = Verified.

$2A$

$A^2(2A^2 - 5A - 2) + (3A^2 - 7A - 6I)$

$A^2(A^2 + A^2 - 3A - 2A - 2) + (2A^2 - 7A - 6I)$

$A^2(\underbrace{A^2 - 3A - 2I}_{\underline{0}}) + 2A^2 - 7A - 6I + A^4 - 2A^3$

$2A^2 - 7A - 6I + A^4 - 2A^3$

$A^4 - 2A^3 + 2A^2 - 7A - 6I$

$A^2(A^2 - 2A + 2) - 7A - 6I$

$A^2(A^2 - 3A + A - 4 + 2) - 7A - 6I$

$A^2(\underbrace{A^2 - 3A - 2I}_{\underline{0}}) - 7A - 6I + A^3 + 4A^2$

$A^3 + 4A^2 - 7A - 6I$

~~$A(A^2 - 3A$~~

$A^3 + 7A^2 - 3A^2 - \underline{5A} - 2A - 6I$

$A(\underbrace{A^2 - 3A - 2}_{0}) - 6I + 7A^2 - 5A$

$7A^2 - 5A - 6I$

$$7\begin{bmatrix}5 & 6\\6 & 8\end{bmatrix} - 5\begin{bmatrix}1 & 2\\2 & 2\end{bmatrix} - 6\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

$$= \begin{bmatrix}35 & 42\\42 & 56\end{bmatrix} - \begin{bmatrix}5 & 10\\10 & 10\end{bmatrix} - \begin{bmatrix}6 & 0\\0 & 6\end{bmatrix}$$

$$\begin{bmatrix}24 & 32\\32 & 40\end{bmatrix}$$

(2) Cayley-Hamilton

A^3 and A^{-3}

$A = \begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix}$

The characteristic equation

$\lambda^2 - S_1\lambda + S_2 = 0$

$S_1 = \text{Trace} = 2+1 = 3$

$S_2 = \det \text{ of } A = |A| = 2-4 = -2$

$\lambda^2 - 3\lambda - 2 = 0$

$\Rightarrow$ To verify $= A^2 - 3A - 2I = 0$

$A^2 = \begin{bmatrix} 8 & 12 \\ 3 & 5 \end{bmatrix}$

$\begin{bmatrix} 8 & 12 \\ 3 & 5 \end{bmatrix} - 3\begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 8-6-2 & 12-12-0 \\ 3-3-0 & 5-3-2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

verified.

To find A^3.

$A^2 - 3A - 2I = 0$

$A^2 = 3A + 2I$

Multiply A on both sides

$A^2 A = A(3A + 2I)$

$A^3 = 3A^2 + 2A \rightarrow (1)$

$A^3 = 3\begin{bmatrix} 8 & 12 \\ 3 & 5 \end{bmatrix} + 2\begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix}$

$A^3 = \begin{bmatrix} 24+4 & 36+8 \\ 9+2 & 15+2 \end{bmatrix}$

$A^3 = \begin{bmatrix} 28 & 44 \\ 11 & 17 \end{bmatrix}$

$A^2 - 3A - 2I = 0$

$\Rightarrow$ multiply A^{-1} on B.S

$A^{-1}(A^2 - 3A - 2I) = A^{-1} 0$

$A^{-1}A^2 - 3A^{-1}A - 2A^{-1}I$

$= 0$

$A - 3I - 2A^{-1} = 0$

$A - 3I = 2A^{-1}$

$2A^{-1} = A - 3I$

$A^{-1} = \frac{1}{2}(A - 3I)$

$A^{-3} = (A^{-1})^3 = \frac{1}{8}(A - 3I)^3$

$\frac{1}{8}(A^3 - 9A^2 + 27A - 27I)$

from (1)

$\Rightarrow A^3 = 3A^2 + 2A$

$\cancel{26}$ 12

39

$A^3 = 3A^2 + 2A$

$A^2 - 3A - 2I = 0$

$A^2 = 3A + 2I$

$A^3 = 3(3A + 2I) + 2A$

$= 9A + 6I + 2A$

$A^3 = 11A + 6I$

$= \frac{1}{8}(11A + 6I - 9(3A + 2I) + 27A - 27I)$

$= \frac{1}{8}(11A + 6I - \cancel{27}A - 18I + 2\cancel{7}A - 27I)$

$\frac{1}{8}(11A - 39I)$

$A^{-3} = \frac{1}{8}(11A - 39I)$

$A^{-3} = \frac{1}{8}\left\{11\begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix} - 39\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right\}$

$A^{-3} = \frac{1}{8}\left\{\begin{bmatrix} 22 & 44 \\ 11 & 11 \end{bmatrix} - \begin{bmatrix} 39 & 0 \\ 0 & 39 \end{bmatrix}\right\}$

$A^{-3} = \frac{1}{8}\left\{\begin{bmatrix} 22-39 & 44-0 \\ 11-0 & 11-39 \end{bmatrix}\right\}$

$A^{-3} = \frac{1}{8}\left\{\begin{bmatrix} -17 & 44 \\ 11 & -28 \end{bmatrix}\right\}$

Scanned with OKEN Scanner

③ State Cayley Hamilton theorm.

Every square matrix Satisfies its own characteristic equa

eigen values and eigen vectors

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

The characteristic ean.

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$$

$S_1 = \text{Trace} = 0+0+0 = 0$

$S_2 =$ minors

$$\begin{bmatrix} 0 & 1 & 1 \\ & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$-1-1-1 = -3.$$

$$S_3 = \begin{bmatrix} \overset{+}{0} & \overset{-}{1} & \overset{+}{1} \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$+0 - 1(0-1) + 1(1-0)$$

$$0 - 1(-1) + 1(1)$$

$$\underline{\underline{1+1 = 2}}$$

$$1 \quad 0 \quad -3 \quad -2$$

$$\lambda^3 - 0\lambda^2 + (-3)\lambda - 2 = 0$$

$$\lambda^3 - 3\lambda - 2 = 0$$

$\cancel{\lambda = x} = \boxed{2, -1, -1} =$ Eigen values.

Eigen vector $(A - \lambda I)X = 0$

$$\begin{bmatrix} 0-\lambda & 1 & 1 \\ 1 & 0-\lambda & 1 \\ 1 & 1 & 0-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0-(-1) & 1 & 1 \\ 1 & 0-(-1) & 1 \\ 1 & 1 & 0-(-1) \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

if $\lambda = -1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 + x_2 + x_3 = 0 \longrightarrow (1)$

$x_1 + x_2 + x_3 = 0 \longrightarrow (2)$

$x_1 + x_2 + x_3 = 0 \longrightarrow (3)$

All equations are same

$x_1 + x_2 + x_3 = 0$

$\boxed{x_3 = K_1}$

$\boxed{x_2 = K_2}$

$x_1 = -(K_1 + K_2)$

eigenvector $= \begin{bmatrix} -(K_1+K_2) \\ K_2 \\ K_1 \end{bmatrix}$

$\lambda = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -K_1 - K_2 \\ K_2 \\ K_1 \end{bmatrix}$

$= K_1 \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} + K_2 \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$

if $\boxed{\lambda = 2}$

$$\begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow -2x_1 + x_2 + x_3 = 0 \longrightarrow (1)$

$x_1 - 2x_2 + x_3 = 0 \longrightarrow (2)$

$x_1 + x_2 - 2x_3 = 0 \longrightarrow (3)$

from eqn of (2) and (3)

$$\begin{array}{cccc} x_1 & x_2 & x_3 & x_1 \\ 1 & -2 & 1 & 1 \\ 1 & 1 & -2 & 1 \end{array}$$

$\dfrac{x_1}{4-1} = \dfrac{x_2}{1+2} = \dfrac{x_3}{1+2} = K$

$\dfrac{x_1}{\cancel{3}} = \dfrac{x_2}{\cancel{3}} = \dfrac{x_3}{\cancel{3}} = K$

1 1 1

eigenvector $= \begin{bmatrix} 1K \\ 1K \\ 1K \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

(4) $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 3 & 2 \\ 0 & 0 & 2 \end{bmatrix}$

The characteristic equation

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$$

$\Rightarrow S_1 = \text{Trace} = 1+3+2 = 6$

$S_2 =$ minors

$$\begin{bmatrix} 1 & 2 & -3 \\ 0 & 3 & 2 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 2 & -3 \\ 0 & 3 & 2 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 2 & -3 \\ 0 & 3 & 2 \\ 0 & 0 & 2 \end{bmatrix}$$

$6-0 + 2-0 + 3-0$

$6+2+3 = 11$

$$S_3 = |A| = \begin{array}{ccc} + & - & + \\ \end{array}\begin{vmatrix} 1 & 2 & -3 \\ 0 & 3 & 2 \\ 0 & 0 & 2 \end{vmatrix}$$

$+1(6-0) - 2(0-0) - 3(0-0)$

$S_3 = +6$

$\Rightarrow \lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$

$\boxed{\lambda = 1, 2, 3}$ = eigenvalues.

$A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 3 & 2 \\ 0 & 0 & 2 \end{bmatrix}$

$3A^3 + 5A^2 - 6A + 2I$

$A^3 = \begin{bmatrix} 1 & 26 & 3 \\ 0 & 27 & 38 \\ 0 & 0 & 8 \end{bmatrix}$

$$\Rightarrow A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 3 & 2 \\ 0 & 0 & 2 \end{bmatrix}$$

$|A - \lambda I| = 0$

$$\begin{vmatrix} 1-\lambda & 2 & -3 \\ 0 & 3-\lambda & 2 \\ 0 & 0 & 2-\lambda \end{vmatrix} = 0$$

$(1-\lambda)(3-\lambda)(2-\lambda) = 0$

$\boxed{\lambda = 1, 3, 2}$

$$A^2 = \begin{bmatrix} 1 & 8 & -5 \\ 0 & 9 & 10 \\ 0 & 0 & 4 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 1 & 26 & 3 \\ 0 & 27 & 38 \\ 0 & 0 & 8 \end{bmatrix}$$

Eigen values of $A^3 = 1, 27, 8$

eigen values of $A^2 = 1, 9, 4$

eigen values of $A = 1, 3, 2$

eigen values of $I = 1, 1, 1$

eigen values of matrix $B =$

$3A^3 + 5A^2 - 6A + 2I$

1) $3(1) + 5(1) - 6(1) + 2(1)$

$3 + 5 - 6 + 2 = 4$

2) $3(27) + 5(9) - 6(3) + 2(1)$

$81 + 45 - 18 + 2 = 110$

3) $3(8) + 5(4) - 6(2) + 2(1)$

$24 + 20 - 12 + 2$

$44 - 10$

$= 34$

$\boxed{\text{eigen values are} = 4, 110, 34}$

Part-A Module II

Diagonalization

(5) $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

diagonalizable or not.

characteristic eqn.

$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

$S_1 = -2+1+0 = -1$

$S_2 = \begin{bmatrix} 1 & -6 \\ -2 & 0 \end{bmatrix} + \begin{bmatrix} -2 & -3 \\ -1 & 0 \end{bmatrix} + \begin{bmatrix} -2 & 2 \\ 2 & 1 \end{bmatrix}$,

$\Rightarrow 0-12-0-3-2-4$

$\Rightarrow 0-0-12-3-2-4$

$\Rightarrow -12-9 = -21$

$S_3 = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

$\Rightarrow -2(0-12)-2(0-6)-3(-4+1)$

$\Rightarrow -2(-12)-2(-6)-3(-3)$

$\Rightarrow 24+12+9 = 45$

$\Rightarrow \lambda^3+\lambda^2 - 21\lambda - 45 = 0$

$\boxed{\lambda = 5, -3, -3}$

So, the eigen values of $A = 5, -3, -3$

A.M of $-3 = 2$

G.M of $-3 = ?$

$\boxed{\lambda = -3}$

G.M of $\lambda = n - \rho(A - \lambda I)$

G.M of $(-1) = n - \rho(A+3I)$

$A+3I = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix} + 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 4 & -6 \\ -1 & -2 & 3 \end{bmatrix}$

$R_3 \to R_3 + R_1$

$R_2 \to R_2 - 2R_1$

$\begin{bmatrix} 1 & 2 & -3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\rho(A+3I) = 1$,

$n = 3$

G.M of $-3 = n - \rho(A+3I)$

$= 3-1$

$= 2$.

A.M = G.M.

given matrix A is diagonalizable.

⑥ $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$

Modal matrix (P)

ch equation =

$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

$S_1 = 1+5+1 = 7$

$S_2 = \begin{bmatrix} 5 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 5 \end{bmatrix}$

$= 5-1+1-9+5-1$

$4-8+4 = \underline{\underline{0}}$

$S_3 = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$

$+1(5-1)-1(1-3)$

$+3(1-15)$

$(4)-1(-2)+3(-14)$

$4+2-42$

$6-42$

$S_3 = \underline{\underline{-36}}$

$\lambda^3 - 7\lambda^2 + 0 + 36 = 0$

$\lambda^3 - 7\lambda^2 + 36 = 0$

$\boxed{\lambda = -2, 6, 3}$

$\boxed{\lambda = -2, 3, 6}$

eigenvector $(A - \lambda I)X = 0$

$\begin{bmatrix} 1-\lambda & 1 & 3 \\ 1 & 5-\lambda & 1 \\ 3 & 1 & 1-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\boxed{\lambda = -2}$

$\begin{bmatrix} 3 & 1 & 3 \\ 1 & 7 & 1 \\ 3 & 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = [$

$3x_1 + x_2 + 3x_3 = 0 \rightarrow ①$

$x_1 + 7x_2 + x_3 = 0; ②$

$3x_1 + x_2 + 3x_3 = 0 \rightarrow ③$

from ① & ②

x_1	x_2	x_3	x_1
3	1	3	3
1	7	1	1

$\frac{x_1}{1-10} = \frac{x_2}{3-3} = \frac{x_3}{10-1} = K$

$\frac{x_1}{-9} = \frac{x_2}{0} = \frac{x_3}{9} = K$

$\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

$\boxed{\lambda = 3}$ to the eigenvector

$$\begin{bmatrix} -2 & 1 & 3 \\ 1 & 2 & 1 \\ 3 & 1 & -2 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-2x_1 + x_2 + 3x_3 = 0 \rightarrow (1)$

$x_1 + 2x_2 + x_3 = 0 \rightarrow (2)$

$3x_1 + x_2 - 2x_3 = 0 \rightarrow (3)$.

from (2) & (3)

x_1	x_2	x_3	x_1
1	2	1	1
3	1	-2	3

$$\frac{x_1}{-4-1} = \frac{x_2}{3+2} = \frac{x_3}{1-6} = K$$

$$\frac{x_1}{-5} = \frac{x_2}{5} = \frac{x_3}{-5} = K.$$

$1 \quad -1 \quad 1$

$$\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$\boxed{\lambda = 6}$ to the eigen vector.

$$\begin{bmatrix} -5 & 1 & 3 \\ 1 & -1 & 1 \\ 3 & 1 & -5 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-5x_1 + x_2 + 3x_3 = 0 \rightarrow (1)$

$x_1 - x_2 + x_3 = 0 \rightarrow (2)$

$3x_1 + x_2 - 5x_3 = 0 \rightarrow (3)$.

from (2) & (3):

x_1	x_2	x_3	x_1
1	-1	1	1
3	1	-5	3

$$\frac{x_1}{5-1} = \frac{x_2}{3+5} = \frac{x_3}{1+3} = K$$

$$\frac{x_1}{\cancel{4}_{\,1}} = \frac{x_2}{\cancel{8}_{\,2}} = \frac{x_3}{\cancel{4}_{\,1}} = K$$

$$\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

P = modal matrix

$$P = \begin{bmatrix} -1 & 1 & 1 \\ 0 & -1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

(7) $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

$\Rightarrow$ The characteristic equation

$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

$S_1 = \text{Trace} = 8+7+3 = 18$

$S_2 = \text{minors}$.

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & 4 \\ 2 & -4 & 3 \end{bmatrix} + \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & 4 \\ 2 & -4 & 3 \end{bmatrix} + \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & 4 \\ 2 & -4 & 3 \end{bmatrix}$$

$21 - 16 + 24 - 4 + 56 - 36 =$

$S_2 = 5 + 20 + 20 = 45$

$S_3 = |A| = \begin{vmatrix} \overset{+}{8} & \overset{-}{-6} & \overset{+}{2} \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{vmatrix}$

$+8(21-16) + 6(-18+8) + 2(24-14)$

$8(5) + 6(-10) + 2(10)$

$40 - 60 + 20 = 0$

$\lambda^3 - 18\lambda^2 + 45\lambda = 0$

eigenvalues are

$\boxed{\lambda = 0, 3, 15}$

eigenvector

$(A - \lambda I)X = 0$

$$\begin{bmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\boxed{\lambda = 0}$

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$8x_1 - 6x_2 + 2x_3 = 0 \rightarrow (1)$

$-6x_1 + 7x_2 - 4x_3 = 0 \rightarrow (2)$

$2x_1 - 4x_2 + 3x_3 = 0 \rightarrow (3)$.

From (2) and (3)

x_1	x_2	x_3	x_1
-6	7	-4	-6
2	-4	3	2

$$\frac{x_1}{21-16} = \frac{x_2}{-8+18} = \frac{x_3}{24-14} = k.$$

$$\frac{x_1}{\cancel{5}_1} = \frac{x_2}{\cancel{10}_2} = \frac{x_3}{\cancel{10}_2} = k$$

$x_1 = k,\ x_2 = 2k,\ x_3 = 2k$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} k \\ 2k \\ 2k \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

eigenvector to $\boxed{\lambda = 3}$

$$\begin{bmatrix} 5 & -6 & 2 \\ -6 & 4 & -4 \\ 2 & -4 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$5x_1 - 6x_2 + 2x_3 = 0 \rightarrow (1)$

$-6x_1 + 4x_2 - 4x_3 = 0 \rightarrow (2)$

$2x_1 - 4x_2 = 0 \rightarrow (3)$

From (2) and (3)

x_1	x_2	x_3	x_1
-6	4	-4	-6
2	-4	0	2

$$\frac{x_1}{0-16} = \frac{x_2}{-8+0} = \frac{x_3}{24-8} = k$$

$$\frac{x_1}{\cancel{-16}_2} = \frac{x_2}{\cancel{-8}_1} = \frac{x_3}{\cancel{16}_{-2}} = k$$

$x_1 = 2k,\ x_2 = k,\ x_3 = -2k$

$$\begin{bmatrix} 2k \\ k \\ -2k \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$$

eigen vector to $\boxed{\lambda = 15}$

$$\begin{bmatrix} -7 & -6 & 2 \\ -6 & -8 & -4 \\ 2 & -4 & -12 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-7x_1 - 6x_2 + 2x_3 = 0 \rightarrow (1)$

$-6x_1 - 8x_2 - 4x_3 = 0 \rightarrow (2)$

$2x_1 - 4x_2 - 12x_3 = 0 \rightarrow (3)$

From (2) and (3)

x_1	x_2	x_3	x_1
-6	-8	-4	-6
2	-4	-12	2

$$\cancel{\frac{x_1}{24+16} = \frac{x_2}{96-16}} =$$

$$\frac{x_1}{96-16} = \frac{x_2}{-8-72} = \frac{x_3}{24+16} = k$$

$$\frac{x_1}{\cancel{80}_2} = \frac{x_2}{\cancel{-80}_{-2}} = \frac{x_3}{\cancel{40}_1} = k.$$

$$\begin{Bmatrix} x_1 = \\ x_2 = \\ x_3 = \end{Bmatrix} \begin{bmatrix} 2k \\ -2k \\ k \end{bmatrix} > \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$$

$$\boxed{Ax = \lambda x}$$

$\lambda^3 - 18\lambda^2 + 45\lambda = 0$

$A^3 - 18A^2 + 45A = 0$

$A^3 = 18A^2 - 45A$

Multiply A on both sides

$A^3 \cdot A = A(18A^2 - 45A)$

$A^4 = 18A^3 - 45A^2$

$$A^2 = \begin{bmatrix} 104 & -98 & 46 \\ -98 & 101 & -52 \\ 46 & -52 & 29 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 1512 & -1494 & 738 \\ -1494 & 1503 & -756 \\ 738 & -756 & 387 \end{bmatrix}$$

$$A^4 = 18 \begin{bmatrix} 1512 & -1494 & 738 \\ -1494 & 1503 & -756 \\ 738 & -756 & 387 \end{bmatrix} - 45 \begin{bmatrix} 104 & -98 & 46 \\ -98 & 101 & -52 \\ 46 & -52 & 29 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 27216 - 4680 & -26892 + 4410 & 13284 - 2070 \\ -26892 + 4410 & 27054 - 4545 & -13608 + 2340 \\ 13284 - 2070 & -13608 + 2340 & 6966 - 1305 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 22536 & -22482 & 11214 \\ -22482 & 22509 & -11268 \\ 11214 & -11268 & 5661 \end{bmatrix}$$

(Q) Verify theorem $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$

The characteristic eqn

$\lambda^2 - S_1\lambda + S_2 = 0$

$S_1 = \text{Trace} = 1+3 = 4$

$S_2 = |A| = 3-8 = -5$

$\lambda^2 - 4\lambda - 5 = 0 \rightarrow (1)$

By cayley theorem $A^2 - 4A - 5I = 0$

$A^2 = \begin{bmatrix} 9 & 16 \\ 8 & 17 \end{bmatrix}$

$\begin{bmatrix} 9 & 16 \\ 8 & 17 \end{bmatrix} - 4\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 9-4-5 & 16-16-0 \\ 8-8-0 & 17-12-5 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $\boxed{A^2 - 4A - 5I = 0}$

Verified.

$A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$

$A^3(A^2 - 4A - 5I) - 2A^3 + 11A^2 - A - 10I$

$A^3(0) - 2A^3 + 11A^2 - A - 10I$

$-2A^3 + 11A^2 - A - 10I$

$-2A(A^2 - 4A - 5I) + 3A^2 - 11A - 10I$

$-2A(0) + 3A^2 - 11A - 10I$ $\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

$3A^2 - 11A - 10I$

$3(A^2 - 4A - 5I) + A + 5I \Rightarrow$ $A + 5I = \begin{bmatrix} 6 & 4 \\ 2 & 8 \end{bmatrix}$

$3(0) + A + 5I =$

(9) Use Cayley-Hamilton

A^8

$$A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$$

Characteristic eqn

$\lambda^2 - S_1\lambda + S_2 = 0$

$S_1 = \text{Trace} = 1 - 1 = 0$

$S_2 = |A| = -1 - 4 = -5$

$\lambda^2 - 5 = 0$

By Cayley Hamilton

$A^2 - 5I = 0$

$\boxed{A^2 = 5I}$

multiply A^6

$A^8 = 5A^6$

$= 5A^2 \cdot A^2 \cdot A^2$

$5(5I)(5I)(5I)$

$= 625I.$

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(16) $\begin{bmatrix} 2 & 3 & 4 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix}$

Ch eqn $\Rightarrow \lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

$S_1 = 2+2+1 = 5$

$S_2 = \begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 4 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix}$

$S_2 = 2+0+2-0+4-0$

$2+2+4 = 8$

$S_3 = \begin{array}{ccc} + & - & + \end{array}$
$\begin{bmatrix} 2 & 3 & 4 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix}$

$+2(2+0) - 3(0+0) + 4(0-0)$

$2(2) = \underline{\underline{4}}$

$\lambda^3 - 5\lambda^2 + 8\lambda - 4 = 0.$

$\boxed{\lambda = 1, 2, 2}$

A.M of $2 = 2$

G.M of $2 = ?$

$\boxed{\lambda = 2}$

$n - \rho(A - \lambda I)$

$\boxed{n = 3}$

$n - \rho(A - \lambda I) \Rightarrow$

$\rho(A - 2I) \Rightarrow \begin{bmatrix} -4 & 2 & -3 \\ 2 & -1 & -6 \\ -1 & -2 & -2 \end{bmatrix}$

$\rho(A - 2I) = 3$

$n = 3$

$n - \rho(A - 2I)$

$3 - 3 = 0$

So, AM of $2 = 2$

G.M of $2 = 0$

So, A.M $\neq$ G.M

So, matrix A is not diagonalizable..

Part-B

① Diagonalize

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ -4 & 4 & 3 \end{pmatrix} \text{ find } A^4$$

chean = $\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3$

$S_1 = 1+2+3 = 6$

$$S_2 = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ -4 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$$

$S_2 = 6-4+3+4+2-0$

$2+7+2 = 11$

$$S_3 = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ -4 & 4 & 3 \end{bmatrix}$$

$+1(6-4) - 1(0+4) + 1(0+8)$

$1(2) - 1(4) + 1(8)$

$2-4+8 = 6$

$\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$

$\boxed{\lambda = 1, 2, 3}$

$\boxed{\lambda = 1}$ to the eigenvector

$$\begin{bmatrix} 1-\lambda & 1 & 1 \\ 0 & 2-\lambda & 1 \\ -4 & 4 & 3-\lambda \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ -4 & 4 & 2 \end{bmatrix}$$

$x_2 + x_3 = 0 \rightarrow$ ①

$x_2 + x_3 = 0 \rightarrow$ ②

$-4x_1 + 4x_2 + 2x_3 = 0 \rightarrow$ ③

from ② & ③

x_1	x_2	x_3	x_1
0	1	1	0
-4	4	2	-4

$$\frac{x_1}{2-4} = \frac{x_2}{-4-0} = \frac{x_3}{0+4} = k$$

$$\frac{x_1}{\cancel{-2}} = \frac{x_2}{\cancel{-4}} = \frac{x_3}{\cancel{4}} = k$$

$1 \quad 2 \quad -2$

$$= \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$$

$\boxed{\lambda = 2}$

$$\begin{bmatrix} -1 & 1 & 1 \\ 0 & 0 & 1 \\ -4 & 4 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-x_1 + x_2 + x_3 = 0 \rightarrow$ ①

$x_3 = 0 \rightarrow$ ②

$-4x_1 + 4x_2 + x_3 = 0 \rightarrow$ ③

x_1	x_2	x_3	x_1
0	0	1	0
-4	4	1	-4

$$\frac{x_1}{0-\cancel{4}} = \frac{x_2}{-4-0} = \frac{x_3}{0+0} = k$$

$$\frac{x_1}{-4} = \frac{x_2}{-4} = \frac{x_3}{0}$$

$$= \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ -2 & 0 & 1 \end{bmatrix}$$

$$P^{-1} = \begin{bmatrix} -1 & 1 & 0 \\ 4 & -3 & -1 \\ -2 & 2 & 1 \end{bmatrix}$$

$\boxed{\lambda = 4}$

$$\begin{bmatrix} -2 & 1 & 1 \\ 0 & -1 & 1 \\ -4 & 4 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-2x_1 + x_2 + x_3 = 0 \rightarrow (1)$

$-x_2 + x_3 = 0 \rightarrow (2)$

$-4x_1 + 4x_2 + 0x_3 = 0 \rightarrow (3)$

(1) (2) & (3)

$x_1 \quad x_2 \quad x_3 \quad x_1$

$0 \quad -1 \quad 1 \quad 0$

$-4 \quad 4 \quad 0 \quad -4$

$$\frac{x_1}{0-4} = \frac{x_2}{-4-0} = \frac{x_3}{0-4} = k$$

$$\frac{x_1}{-4} = \frac{x_2}{-4} = \frac{x_3}{-4} = k$$

$$\frac{x_1}{\cancel{-4}} = \frac{x_2}{\cancel{-4}} = \frac{x_3}{\cancel{-4}} = k$$

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$P^{-1}AP = \begin{bmatrix} -1 & 1 & 0 \\ 4 & -3 & -1 \\ -2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ -4 & 4 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ -2 & 0 & 1 \end{bmatrix}$$

$$P^{-1}AP = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} = D.$$

$$P^{-1}AP = D \Rightarrow A = PDP^{-1}$$

$$A^4 = PD^4P^{-1}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ -2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 81 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ 4 & -3 & -1 \\ -2 & 2 & 1 \end{bmatrix}$$

$$\Rightarrow A^4 = \begin{bmatrix} -99 & 115 & 65 \\ -100 & 116 & 65 \\ -160 & 160 & 81 \end{bmatrix}$$ check

② $$A = \begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix}$$

$-88 + 32 + 56$
$= 0$

$\Rightarrow$ modal matrix,

$\Rightarrow \lambda^3 S_1\lambda^2 + S_2\lambda - S_3 = 0$

$S_1 = 11 - 2 - 6 = 11 - 8 = 3$

$$S_2 = \begin{bmatrix} -2 & -5 \\ -4 & -6 \end{bmatrix} + \begin{bmatrix} 11 & -7 \\ 10 & -6 \end{bmatrix} + \begin{bmatrix} 11 & -4 \\ 7 & -2 \end{bmatrix}$$

$$S_2 = \left[\begin{array}{l} 12 - 20 + (-66) + 70 - 22 + 28 \\ -8 + 4 + 6 = 10 - 8 = 2 \end{array} \right.$$

$+ \quad - \quad +$

$$S_3 = \begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix} \rightarrow 11(12-20) + 4(-42+50) - 7(-28+20)$$

$11(-8) + 4(+8) - 7(-8)$

$\lambda^3 - 3\lambda^2 + 2\lambda - 0 = 0$

$\lambda = 2, 1, 0$

$\boxed{\lambda = 0, 1, 2}$

$(A - \lambda I)X = 0$

$$\begin{bmatrix} 11-\lambda & -4 & -7 \\ 7 & -2-\lambda & -5 \\ 10 & -4 & -6-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\boxed{\lambda = 0}$

$$\begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$11x_1 - 4x_2 - 7x_3 = 0 \rightarrow (1)$

$7x_1 - 2x_2 - 5x_3 = 0 \rightarrow (2)$

$10x_1 - 4x_2 - 6x_3 = 0 \rightarrow (3)$. (2) & (3)

$\Rightarrow$ $x_1 \; x_2 \; x_3 \; x_1$

$7 \; -2 \; -5 \; 7$

$10 \; -4 \; -6 \; 10$

$\dfrac{x_1}{12-20} = \dfrac{x_2}{-50+42} = \dfrac{x_3}{-28+20} = k$

$\dfrac{x_1}{-8} = \dfrac{x_2}{-8} = \dfrac{x_3}{-8} = k$

$= \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} k.$

$$\begin{bmatrix} 10 & -4 & -7 \\ 7 & -3 & -5 \\ 10 & -4 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$10x_1 - 4x_2 - 7x_3 = 0 \rightarrow (1)$

$7x_1 - 3x_2 - 5x_3 = 0 \rightarrow (2)$

$10x_1 - 4x_2 - 7x_3$

(2) & (3)

$x_1 \; x_2 \; x_3 \; x_1$

$7 \; -3 \; -5 \; 7$

$10 \; -4 \; -7 \; 10$

$\dfrac{x_1}{21-20} = \dfrac{x_2}{-50+49} = \dfrac{x_3}{-28+30}$

$\dfrac{x_1}{1} = \dfrac{x_2}{-1} = \dfrac{x_3}{+2}$

$\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$

$\boxed{\lambda = 2}$

$$\begin{bmatrix} 9 & -4 & -7 \\ 7 & -4 & -5 \\ 10 & -4 & -8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$9x_1 - 4x_2 - 7x_3 = 0 \rightarrow (1)$

$7x_1 - 4x_2 - 5x_3 = 0 \rightarrow (2)$

$10x_1 - 4x_2 - 8x_3 = 0 \rightarrow (3)$

$$\begin{array}{cccc} x_1 & x_2 & x_3 & x_1 \\ 7 & -4 & -5 & 7 \\ 10 & -4 & -8 & 10 \end{array}$$

$$\frac{x_1}{32-20} = \frac{x_2}{-50+56} = \frac{x_3}{-28+40}$$

$$\frac{x_1}{\cancel{12}\ 2} = \frac{x_2}{\cancel{6}\ 1} = \frac{x_3}{\cancel{12}\ 2}$$

$$\Rightarrow \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & 1 & 2 \\ 1 & -1 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

(6) Diagonalize

(8) Cayley-Hamilton Theorem

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$$

Characteristic ean $= \lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

$S_1 = \text{Trace} = 1+1+1 = 3$

$$S_2 = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$$

$1-4 + 1+2 + 1-4$

$\cancel{-3} + \cancel{3} - 3 = -3$

$$S_3 = |A| = \begin{vmatrix} 1 & 2 & -1 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{vmatrix}$$

$+1(1-4) - 2(2+4) - 1(-4-2)$

$1(-3) - 2(6) - 1(-6)$

$-3 - 12 + 6$

$-6 - 3 = -9$

$\lambda^3 - 3\lambda^2 - 3\lambda + 9 = 0$

$\Rightarrow$ By cayley $\boxed{A^3 - 3A^2 - 3A + 9I = 0}$

$$A^3 = \begin{bmatrix} 3 & 24 & -21 \\ 6 & 21 & -24 \\ 6 & -6 & 3 \end{bmatrix}, \quad A^2 = \begin{bmatrix} 3 & 6 & -6 \\ 0 & 9 & -6 \\ 0 & 0 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 24 & -21 \\ 6 & 21 & -24 \\ 6 & -6 & 3 \end{bmatrix} - 3\begin{bmatrix} 3 & 6 & -6 \\ 0 & 9 & -6 \\ 0 & 0 & 3 \end{bmatrix} - 3\begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix} + 9\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 24 & -21 \\ 6 & 21 & -24 \\ 6 & -6 & 3 \end{bmatrix} - \begin{bmatrix} 9 & 18 & -18 \\ 0 & 27 & -18 \\ 0 & 0 & 9 \end{bmatrix} - \begin{bmatrix} 3 & 6 & -3 \\ 6 & 3 & -6 \\ 6 & -6 & 3 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$\begin{bmatrix} 3-9-3+9 & 24-18-6+0 & -21+18+3+0 \\ 6-0-6+0 & 21-27-3+9 & -24+18+6+0 \\ 6-0-6+0 & -6-0+6+0 & 3-9-3+9 \end{bmatrix} = \begin{bmatrix} 0 & 0 & \\ 0 & 0 & \\ 0 & 0 & \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Verified.

$A^3-3A^2-3A+9I=0$

multiply A^{-1} on BS

$A^{-1}(A^3-3A^2-3A+9I)=A^{-1}0$

$A^2-3A-3I+9A^{-1}=0$

$9A^{-1}=-A^2+3A+3I$

$A^{-1}=\frac{1}{9}(-A^2+3A+3I)$

$$A^{-1}=\frac{1}{9}\left(-\begin{bmatrix} 3 & 6 & -6 \\ 0 & 9 & -6 \\ 0 & 0 & 3 \end{bmatrix} + 3\begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix} + 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right)$$

$$A^{-1}=\frac{1}{9}\left[\begin{bmatrix} -\cancel{3}+\cancel{3}+3 & -6+6+0 & 6-3+0 \\ -0+6+0 & -9+3+3 & 6-6+0 \\ -0+6+0 & -0-6+0 & -3+3+3 \end{bmatrix}\right]$$

$$\boxed{A^{-1}=\frac{1}{9}\left[-\begin{pmatrix} 3 & 0 & 3 \\ +6 & -3 & 0 \\ 6 & -6 & 3 \end{pmatrix}\right]}$$

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A^4 ?

$A^3 - 3A^2 - 3A + 9I = 0$

$A^3 = 3A^2 + 3A - 9I$

multiply A on both sides

$A(A^3) = A(3A^2 + 3A - 9I)$

$A^4 = 3A^3 + 3A^2 - 9A$

$$A^4 = 3\begin{bmatrix} 3 & 24 & -21 \\ 6 & 21 & -24 \\ 6 & -6 & 3 \end{bmatrix} + 3\begin{bmatrix} 3 & 6 & -6 \\ 0 & 9 & -6 \\ 0 & 0 & 3 \end{bmatrix} - 9\begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 9 & 72 & -63 \\ 18 & 63 & -72 \\ 18 & -18 & 9 \end{bmatrix} + \begin{bmatrix} 9 & 18 & -18 \\ 0 & 27 & -18 \\ 0 & 0 & 9 \end{bmatrix} - \begin{bmatrix} 9 & 18 & -9 \\ 18 & 9 & -18 \\ 18 & -18 & +9 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 9+9-9 & 72+18-18 & -63 - 18 + 9 \\ 18+0-18 & 63+27-9 & -72 - 18 + 18 \\ 18+0-18 & -18+0+18 & 9+9-9 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 9 & 72 & -72 \\ 0 & 81 & -72 \\ 0 & 0 & 9 \end{bmatrix}$$

(4) Find eigen values and eigen vectors

$\Rightarrow A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$

Characteristic eqn.

$\lambda^2 - S_1\lambda + S_2 = 0$

$S_1 = \text{Trace} = 5 + 2 = 7$

$S_2 = |A| = 10 - 4 = 6$

$\lambda^2 - 7\lambda + 6 = 0$

$\lambda^2 - 6\lambda - \lambda + 6 = 0$

$\lambda(\lambda - 6) - 1(\lambda - 6) = 0$

$(\lambda - 6)(\lambda - 1) = 0$

$\boxed{\lambda = 6, 1}$ = eigenvalues

$|A - \lambda I|X = 0$

$\boxed{\lambda = 1}$ to eigenvector

$$A = \begin{bmatrix} 5-\lambda & 4 \\ 1 & 2-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\boxed{\lambda = 1}$ To the eigen vector.

$|A - \lambda I| \lambda = 0$

$$\begin{bmatrix} 4 & 4 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$4x_1 + 4x_2 = 0 \rightarrow (1)$

$x_1 + x_2 = 0 \rightarrow (2)$

Corresponding eigen vector

$(4, 1)$

$\boxed{\lambda = 6}$ To the eigen vector

$$\begin{bmatrix} -1 & 4 \\ 1 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$-x_1 + 4x_2 = 0$

$x_1 - 4x_2 = 0$

Corresponding to eigen vector

$(-1, 1)$

(5) $A = \begin{bmatrix} 2 & 3+4i \\ 3-4i & 2 \end{bmatrix}$

Characteristic eqn $\lambda^2 - S_1\lambda + S_2 = 0$

$S_1 = 2+2 = 4$

$S_2 = |A| = \left[4 - [(3)^2 - (4i)^2]\right]$

$[4 - (9+16)] = [4 - 25] = -21$

$\lambda^2 - 4\lambda - 21 = 0$

$\lambda^2 - 7\lambda + 3\lambda - 21 = 0$

$\lambda(\lambda - \cancel{3}) + 3(\lambda - 7) = 0$

$(\lambda - 7)(\lambda + 3) = 0$

$\boxed{\lambda = 7, -3}$

If there exists a nonzero vector λ such that

$\boxed{AX = \lambda X}$

$\Rightarrow \begin{bmatrix} 2-\lambda & 3+4i \\ 3-4i & 2-\lambda \end{bmatrix}$

$\boxed{\lambda = 7}$ to eigen vector

$\begin{bmatrix} -5 & 3+4i \\ 3-4i & -5 \end{bmatrix}$

$\boxed{\lambda = -3}$ to eigenvector

$\begin{bmatrix} 5 & 3+4i \\ 3-4i & 5 \end{bmatrix}$

eigenvector corresponding.

$V_1 \Rightarrow (+5, 3+4i)$

$V_2 = (-3-4i, 5)$.

(7) $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

characteristic equation

$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

$S_1 = \text{Trace} = 6+3+3 = 12$

$S_2 = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

$S_2 = 9-1+18-4+18-4$

$S_2 = 8+14+14$

$S_2 = 36$

$S_3 = |A| = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

$+6(9-1)+2(-6+2)+2(2-6)$

$6(8)+2(-4)+2(-4)$

$48-8-8 = 48-16$

$= 32$

$\lambda^3 - 12\lambda^2 + 36\lambda - 32 = 0$

$x_1 = 8$

$x_2 = 2$

$x_3 = 2$

$\boxed{\lambda = 8, 2, 2}$ eigenvalues

$(A-\lambda I)X = 0$

$\begin{bmatrix} 6-\lambda & -2 & 2 \\ -2 & 3-\lambda & -1 \\ 2 & -1 & 3-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\boxed{\lambda = 2}$ sus to eigenvector

$\begin{bmatrix} 4 & -2 & 2 \\ -2 & 1 & -1 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$4x_1 - 2x_2 + 2x_3 = 0 \longrightarrow (1)$

$-2x_1 + x_2 - x_3 = 0 \longrightarrow (2)$

$2x_1 - x_2 + x_3 = 0 \longrightarrow (3)$

from (2) and (3)

x_1	x_2	x_3	x_1
-2	1	-3	-2
2	-1	1	+2

$\frac{x_1}{1-3} = \frac{x_2}{+6+2} = \frac{x_3}{2-2} = k$

$\frac{x_1}{-2} = \frac{x_2}{1} = \frac{x_3}{0} = k$

$x_1 = k$

$x_2 = 2k$

$x_3 = 0k$

from (1) and (2)

x_1	x_2	x_3	x_1
4	-2	2	4
-2	1	-1	-2

$\frac{x_1}{2-2} = \frac{x_2}{-4+4} = \frac{x_3}{}$

Method 2

$$\begin{bmatrix} 4 & -2 & 2 \\ -2 & 1 & -1 \\ 2 & -1 & 1 \end{bmatrix}$$

Reduce matrix.

$R_1/4$

$$\begin{bmatrix} 1 & -1/2 & 1/2 \\ -2 & 1 & -1 \\ 2 & -1 & 1 \end{bmatrix}$$

$R_2 \to R_2 + 2R_1$

$R_3 \to R_3 - 2R_1$

$$\begin{bmatrix} 1 & -1/2 & 1/2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\Rightarrow x_1 - \frac{1}{2}x_2 + \frac{1}{2}x_3 = 0$

$x_1 = \frac{1}{2}x_2 - \frac{1}{2}x_3$

eigenvector

$$\begin{bmatrix} 0.5x_2 - 0.5x_3 \\ x_2 \\ x_3 \end{bmatrix}$$

let $x_2 = 1, x_3 = 0$

$$v_1 = \begin{bmatrix} 0.5 \\ 1 \\ 0 \end{bmatrix}$$

let $x_2 = 0, x_3 = 1$

$$v_2 = \begin{bmatrix} -0.5 \\ 0 \\ 1 \end{bmatrix}$$

$\boxed{\lambda = 8}$ to eigenvector

$$\begin{bmatrix} -2 & -2 & 2 \\ -2 & -5 & -1 \\ 2 & -1 & -5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-2x_1 - 2x_2 + 2x_3 = 0 \to (1)$

$-2x_1 - 5x_2 - x_3 = 0 \to (2)$

$2x_1 - x_2 - 5x_3 = 0 \to (3)$ from (2) and (3)

$$\Rightarrow \begin{array}{cccc} x_1 & x_2 & x_3 & x_1 \\ -2 & -5 & -1 & -2 \\ 2 & -1 & -5 & 2 \end{array}$$

$\cancel{\dfrac{x_1}{25-2} = \dfrac{x_2}{-2-10} =}$

$\Rightarrow \dfrac{x_1}{25-1} = \dfrac{x_2}{-2-10} = \dfrac{x_3}{2+10} = k$

$\dfrac{x_1}{\cancel{24}\ 2} = \dfrac{x_2}{\cancel{-12}\ -1} = \dfrac{x_3}{\cancel{12}\ 1} = k$

$x_1 = 2k$

$x_2 = -k$

$x_3 = k$

$$= \begin{bmatrix} 2k \\ -k \\ k \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

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(8) Characteristic polynomial

$$A = \begin{bmatrix} 3 & 1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix} \qquad A^2 = \begin{bmatrix} 9 & 7 & 5 \\ -9 & 25 & -9 \\ 7 & -7 & 11 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 25 & 39 & 17 \\ -61 & 125 & -61 \\ 39 & -39 & 47 \end{bmatrix}$$

A be square matrix

$|A - \lambda I|$ is characteristic polynomial.

$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3$ $\Rightarrow$

$\Rightarrow S_1 = \text{Trace} = 3+5+3$
$= 8+3 = 11$

$S_2 =$ minors

$$\begin{bmatrix} \cancel{3} & \cancel{1} & \cancel{1} \\ \cancel{-1} & 5 & -1 \\ \cancel{1} & -1 & 3 \end{bmatrix} + \begin{bmatrix} 3 & \cancel{1} & 1 \\ \cancel{-1} & \cancel{5} & \cancel{-1} \\ 1 & \cancel{-1} & 3 \end{bmatrix} + \begin{bmatrix} 3 & 1 & \cancel{1} \\ -1 & 5 & \cancel{-1} \\ \cancel{1} & \cancel{-1} & \cancel{3} \end{bmatrix}$$

$\Rightarrow 15-1 + 9-1 + 15+1$

$14 + 8 + 16 = 38$

$$\begin{array}{c} \quad + \quad - \quad + \\ S_3 = |A| = \begin{bmatrix} 3 & 1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix} \end{array}$$

$\Rightarrow +3(15-1) - 1(-3+1) + 1(1-5)$

$3(14) - 1(-2) + 1(-4)$

$42 + 2 - 4$

$= 40$

$\Rightarrow \lambda^3 - 11\lambda^2 + 38\lambda - 40I = 0$

By Cayley Hamilton theorm $= \boxed{A^3 - 11A^2 + 38A - 40I = 0}$

$\boxed{\lambda = 2, 5, 4}$ = eigen values.

$\rightarrow$ (1)

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$A^2 =$

$$\begin{bmatrix} 25 & 39 & 17 \\ -61 & 125 & -61 \\ 89 & -39 & 47 \end{bmatrix} - 11\begin{bmatrix} 9 & 7 & 5 \\ -9 & 25 & -9 \\ 7 & -7 & 11 \end{bmatrix} + 38\begin{bmatrix} 3 & 1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix} - 40\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 25 & 39 & 17 \\ -61 & 125 & -61 \\ 89 & -39 & 47 \end{bmatrix} - \begin{bmatrix} 99 & 77 & 55 \\ -99 & 275 & -99 \\ 77 & -77 & 121 \end{bmatrix} + \begin{bmatrix} 114 & 38 & 38 \\ -38 & 190 & -38 \\ 38 & -38 & 114 \end{bmatrix} - \begin{bmatrix} 40 & 0 & 0 \\ 0 & 40 & 0 \\ 0 & 0 & 40 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0. \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Cayley Hamilton theorm verifed.

⇒ By ① ean.

$A^3 - 11A^2 + 38A - 40I = 0,$

Multiply A^{-1} on both sides.

$A^{-1}(A^3 - 11A^2 + 38A - 40I) = A^{-1}.0$

$A^2 - 11A + 38I - 40A^{-1} = 0$

$A^2 - 11A + 38I = 40A^{-1}$

$A^{-1} = \frac{1}{40}\left[A^2 - 11A + 38I\right]$

$$A^{-1} = \frac{1}{40}\left[\begin{bmatrix} 9 & 7 & 5 \\ -9 & 25 & -9 \\ 7 & -7 & 11 \end{bmatrix} - 11\begin{bmatrix} 3 & 1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix} + 38\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right]$$

$$A^{-1} = \frac{1}{40}\begin{bmatrix} 14 & -4 & -6 \\ 2 & 8 & 2 \\ -4 & 4 & 16 \end{bmatrix}$$

14.

$$A^3 - 11A^2 + 38A - 40I = 0$$

$$A^3 = 11A^2 - 38A + 40I$$

Multiply A on both sides

$$A \cdot A^3 = A(11A^2 - 38A + 40I)$$

$$A^4 = 11A^3 - 38A^2 + 40A$$

$$A^4 = 11\begin{bmatrix} 45 & 39 & 17 \\ -61 & 125 & -61 \\ 39 & -39 & 47 \end{bmatrix} - 38\begin{bmatrix} 9 & 7 & 5 \\ -9 & 25 & -9 \\ 7 & -7 & 11 \end{bmatrix} + 40\begin{bmatrix} 3 & 1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 495 & 429 & 187 \\ -671 & 1375 & -671 \\ 429 & -429 & 517 \end{bmatrix} - \begin{bmatrix} 342 & 266 & 190 \\ -342 & 950 & -342 \\ 266 & -266 & 418 \end{bmatrix} + \begin{bmatrix} 120 & 40 & 40 \\ -40 & 200 & -40 \\ 40 & -40 & 120 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 58 & 203 & 37 \\ -369 & 625 & -369 \\ 203 & -203 & 219 \end{bmatrix}$$

(9) Diagonalize

$A = \begin{bmatrix} 1 & -8 \\ -5 & 4 \end{bmatrix}$

Chear $= \lambda^2 - S_1\lambda + S_2 = 0$

$S_1 = 1+4 = 5$

$S_2 = 4 - 40 = -36.$

$\lambda^2 - 5\lambda - 36 = 0$

$\lambda^2 - 9\lambda + 4\lambda - 36 = 0$

$\lambda(\lambda - 9) + 4(\lambda - 9) = 0$

$(\lambda - 9)(\lambda + 4)$

$\boxed{\lambda = 9, -4}$

eigenvector $= \boxed{\lambda = 9}$

$\begin{bmatrix} 1-\lambda & -8 \\ -5 & 4-\lambda \end{bmatrix}$

$$= \begin{bmatrix} -8 & -8 \\ -5 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$-8x_1 - 8x_2 = 0$

$-5x_1 - 5x_2 = 0$

$-8(x_1 + x_2) = 0$

$-5(x_1 + x_2) = 0$

$x_2 = K_1$

$x_1 + x_2 = 0$

$x_1 + K_1 = 0$

$x_1 = -K_1$

$(x_1, x_2) = (-1, 1)$

$\boxed{\lambda = -4}$

$$\begin{bmatrix} 5 & -8 \\ -5 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$5x_1 - 8x_2 = 0$

$-5x_1 + 8x_2 = 0$

$+$

$10x_1 - 16x_2 = 0$

$2(5x_1 - 8x_2) = 0$

$5x_1 - 8x_2 = 0$

$x_2 = K_1$

$5x_1 - 8K_1 = 0$

$5x_1 = 8K_1$

$x_1 = \frac{8}{5}$

$(x_1, x_2) = (\frac{8}{5}, 1)$

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$$= \begin{bmatrix} -1 & \frac{8}{5} \\ 1 & 1 \end{bmatrix}$$

$\boxed{\lambda = 6} \Rightarrow \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$

$$5)8(1.6$$
$$\frac{5}{30}$$
$$\frac{30}{0}$$

modal matrix

$$P = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 1.6 \\ 1 & 1 \end{bmatrix}$$

$$P^{-1} = \frac{1}{6} \begin{bmatrix} 3 & 0 & -3 \\ 2 & 2 & 2 \\ 1 & -2 & 1 \end{bmatrix}.$$

$$^{-1} = \begin{bmatrix} -1 & -8/5 \\ -1 & -1 \end{bmatrix}$$

$$\cancel{AP^{-1} \Rightarrow \begin{bmatrix} -1 & 8/5 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -8 \\ -5 & 4 \end{bmatrix} \begin{bmatrix} 1 & -8/5 \\ -1 & -1 \end{bmatrix}}$$

$$P^{-1}AP = \begin{pmatrix} -4 & 0 \\ 0 & 9 \end{pmatrix} = D_1$$

$$\cancel{D \Rightarrow \begin{bmatrix} -\frac{117}{5} & -\frac{T}{5} \\ 0 & -\frac{12}{5} \end{bmatrix}}$$

(14) $\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$

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(6) $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$

characteristic equation

$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

$S_1 = \text{Trace} = 2+2+2 = 6$

$S_2 = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$

$S_2 = 4-0+4-1+4-0$

$4+3+4 = 11$

$S_3 = \begin{array}{ccc} + & - & + \end{array}$
$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} + 2(4-0) - 0 + 1(0-2)$

$2(4) - 0 + 1(-2)$

$8-0-2 = 8-2 = 6$ //

$$\boxed{\lambda^3-6\lambda^2+11\lambda-6=0}$$

⇓

characteristic equation

$$\boxed{\lambda = 1, 2, 3}$$

For finding eigenvector the system is

$[A-\lambda I]X=0$

$$\begin{bmatrix} 2-\lambda & 0 & 1 \\ 0 & 2-\lambda & 0 \\ 1 & 0 & 2-\lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\boxed{\lambda=1}$ = eigenvector

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1+x_3=0 \longrightarrow (1)$

$x_2=0 \longrightarrow (2)$

$x_1+x_3=0 \longrightarrow (3)$

from (2) and (3)

$$\begin{matrix} x_1 & x_2 & x_3 & \cancel{x_1} \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \end{matrix}$$

$$\frac{x_1}{1-0} = \frac{x_2}{0-0} = \frac{x_3}{0-1} = k$$

$$\frac{x_1}{1} = \frac{x_2}{0} = \frac{x_3}{-1} = k$$

eigenvector is $\begin{bmatrix} k \\ 0k \\ -k \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

$\boxed{\lambda=2}$ to the eigenvector

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_3=0 \longrightarrow (1)$

$x_1=0 \longrightarrow (2)$

From (1) and (2)

$$\begin{matrix} x_1 & x_2 & x_3 & x_1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{matrix}$$

$$\frac{x_1}{0-0} = \frac{x_2}{1-0} = \frac{x_3}{0-0} = k$$

$$\frac{x_1}{0} = \frac{x_2}{1} = \frac{x_3}{0} = k$$

$x_1=0,\ x_2=1k,\ x_3=0$

eigenvector = $\begin{bmatrix} 0\cdot k \\ 1\cdot k \\ 0\cdot k \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

⇒ $\boxed{\lambda=3}$ eigen vectors

$$\begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-x_1+x_3=0 \longrightarrow (1)$

$-x_2=0 \longrightarrow (2)$

$x_1-x_3=0 \longrightarrow (3)$

from (2) and (3)

$$\begin{array}{cccc} x_1 & x_2 & x_3 & x_1 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & -1 & 1 \end{array}$$

$$\frac{x_1}{1-0} = \frac{x_2}{0+0} = \frac{x_3}{0+1} = k$$

$x_1 = k$
$x_2 = 0$
$x_3 = k$

$$\Rightarrow \begin{bmatrix} k \\ 0 \\ k \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

(1) $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

characteristic roots.

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$$

$S_1 = \text{trace} = 1+1+1 = 3$

$$S_2 = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}$$

$S_2 = 0$

$$S_3 = |A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$+1(1-1) - 1(1-1) + 1(1-1)$

$1(0) - 1(0) + 1(0)$

$S_3 = 0$

$\lambda^3 - 3\lambda^2 + 0 + 0 = 0$

$\lambda^3 - 3\lambda^2 = 0$

eigenvalue

$$\boxed{\lambda = 3, 0, 0}$$

Corresponding eigenvectors

$$\begin{bmatrix} 1-\lambda & 1 & 1 \\ 1 & 1-\lambda & 1 \\ 1 & 1 & 1-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\boxed{\lambda = 3}$ to the eigenvector

$$\begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-2x_1 + x_2 + x_3 = 0 \longrightarrow (1)$

$x_1 - 2x_2 + x_3 = 0 \longrightarrow (2)$

$x_1 + x_2 - 2x_3 = 0 \longrightarrow (3)$

from (2) and (3) eqn's.

$$\begin{array}{cccc} x_1 & x_2 & x_3 & x_1 \\ 1 & -2 & 1 & 1 \\ 1 & 1 & -2 & 1 \end{array}$$

$$\frac{x_1}{4-1} = \frac{x_2}{1+2} = \frac{x_3}{1+2} = k$$

$$\frac{x_1}{\cancel{3}} = \frac{x_2}{\cancel{3}} = \frac{x_3}{\cancel{3}} = k$$

$$= \begin{bmatrix} k \\ k \\ k \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$\boxed{\lambda = 0}$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1+x_2+x_3=0 \longrightarrow (1)$

$x_1+x_2+x_3=0 \longrightarrow (2)$

$x_1+x_2+x_3=0 \longrightarrow (3)$

All eqn are same

$x_1+x_2+x_3=0$

$x_3=K_1$

$x_2=K_2$

$x_1=-(K_1+K_2)$

eigenvector $=\begin{bmatrix}-(K_1+K_2)\\ K_2\\ K_1\end{bmatrix}$

$X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}-K_1-K_2\\ K_2\\ K_1\end{bmatrix}$

$\Rightarrow \begin{bmatrix}-1\\ 0\\ 1\end{bmatrix}K_1+\begin{bmatrix}-1\\ 1\\ 0\end{bmatrix}K_2$

(12) $A=\begin{bmatrix}1 & -2 & 1\\ 1 & -2 & 3\\ 0 & -1 & 2\end{bmatrix}$

ch equation =

$\lambda^3-S_1\lambda^2+S_2\lambda-S_3=0$

$\Rightarrow S_1=\text{Trace}=1-2+2$

$= 1-0=1$

$S_2=\begin{bmatrix}\cancel{1} & \cancel{-2} & \cancel{1}\\ \cancel{1} & -2 & 3\\ \cancel{0} & -1 & 2\end{bmatrix}+\begin{bmatrix}1 & \cancel{-2} & 1\\ \cancel{1} & \cancel{-2} & \cancel{3}\\ 0 & \cancel{-1} & 2\end{bmatrix}+\begin{bmatrix}1 & -2 & \cancel{1}\\ 1 & -2 & \cancel{3}\\ \cancel{0} & \cancel{-1} & \cancel{2}\end{bmatrix}$

$S_2=-4+3+2-0+-2+2$

$-1+2+0=1$

$\boxed{S_2=1}$

$S_3=\begin{array}{c}\begin{matrix}+ & - & +\end{matrix}\\ \begin{bmatrix}1 & -2 & 1\\ 1 & -2 & 3\\ 0 & -1 & 2\end{bmatrix}\end{array}$

$+1(-4+3)+2(2-0)+1(-1+0)$

$1(-1)+2(2)+1(-1)$

$-1+4-1=4-2=2$

$S_3=2$

$\lambda^3-\lambda^2+\lambda-2=0$

By cayley $A^3-A^2+A-2I=0$

$A^3=\begin{bmatrix}0 & 3 & -4\\ -2 & 3 & -2\\ -1 & 1 & 1\end{bmatrix}$

$A^2=\begin{bmatrix}-1 & 1 & -3\\ -1 & -1 & 1\\ -1 & 0 & 1\end{bmatrix}$

$$\begin{bmatrix} 0 & 3 & -4 \\ -2 & 3 & -2 \\ -1 & 1 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 & -3 \\ -1 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -2 & 1 \\ 1 & -2 & 3 \\ 0 & -1 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

verified (satisfies)

$\Rightarrow A^3 - A^2 + A - 2I = 0$

Multiply A^{-1} on both sides

$A^{-1}(A^3 - A^2 + A - 2I) = A^{-1} \cdot 0$

$A^2 - A + I - 2A^{-1} = 0$

$2A^{-1} = A^2 - A + I$

$A^{-1} = \frac{1}{2}\left[A^2 - A + I\right]$

$$A^{-1} = \frac{1}{2}\left\{ \begin{bmatrix} -1 & 1 & -3 \\ -1 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & -2 & 1 \\ 1 & -2 & 3 \\ 0 & -1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right\}$$

$$A^{-1} = \frac{1}{2}\begin{bmatrix} -1-1+1 & 1+2+0 & -3-1+0 \\ -1-1+0 & -1+2+1 & 1-3+0 \\ -1-0+0 & 0+1+0 & 1-2+1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{2}\begin{bmatrix} -1 & 3 & -4 \\ -2 & 2 & -2 \\ -1 & 1 & 0 \end{bmatrix}$$

(13) Use Cayley-Hamilton theorem

$2A^5 - 3A^4 + A^2 - 4I$

$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

The characteristic equation of A

$|A - \lambda I| = 0$

$\begin{vmatrix} 3-\lambda & 1 \\ -1 & 2-\lambda \end{vmatrix} = 0$

$(3-\lambda)(2-\lambda) + 1 = 0$

$6 - 3\lambda - 2\lambda + \lambda^2 + 1 = 0$

$\lambda^2 - 5\lambda + 7 = 0$

Applying Cayley Hamilton theorem

$A^2 - 5A + 7I = 0$

$2A^5 - 3A^4 + A^2 - 4I$

$2A^3(A^2 - 5A + 7I) + 7A^4 - 14A^3 - 4I + A^2$

$\cancel{A^2}$.

$\underbrace{2A^3(A^2 - 5A + 7I)}_{0} + 10A^4 - 14A^3 - 3A^4 + A^2 - 4I$

$\cancel{10A^4 - 14A^3 - 3A^4 + A^2 - 4I}$

$7A^4 - 14A^3 + A^2 - 4I$

$7A^4 - 14A^3 + A^2 - 4I$

$\underbrace{7A^2(A^2 - 5A + 7I)}_{0} + 21A^3 - 48A^2 - 4I$

$21A^3 - 48A^2 - 4I$

$21A(A^2 - 5A + 7I) + 57A^2 - 147A - 4I$

$\underbrace{21A(0)}_{0} + 57A^2 - 147A - 4I$

$57A^2 - 147A - 4I$

$57(\underbrace{A^2 - 5A + 7I}_{0})$
$+ 138A - 403I$

$= \boxed{138A - 403I}$

$$\begin{bmatrix} 0 & \frac{-10}{5} \end{bmatrix}$$

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$$= \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$

chea =

$$\lambda^3 - 11\lambda^2 + 36\lambda - 36 = 0$$

$$\boxed{\lambda = 2, 3, 6}$$

$$\boxed{\lambda = 2} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$$\boxed{\lambda = 3}$$

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

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$$P^{-1}AP = \frac{1}{6}\begin{bmatrix} 3 & 0 & -3 \\ 2 & 2 & 2 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

(15)

$$A = \begin{bmatrix}1&3&4\\0&2&5\\0&0&3\end{bmatrix}$$

characteristic eqn

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$$

$$S_1 = \text{trace} = 1+2+3 = 6$$

$$S_2 = \begin{bmatrix}\cancel{1}&\cancel{3}&\cancel{4}\\\cancel{0}&2&5\\\cancel{0}&0&3\end{bmatrix} + \begin{bmatrix}1&\cancel{3}&4\\\cancel{0}&\cancel{2}&\cancel{5}\\0&\cancel{0}&3\end{bmatrix} + \begin{bmatrix}1&3&\cancel{4}\\0&2&\cancel{5}\\\cancel{0}&\cancel{0}&\cancel{3}\end{bmatrix}$$

$$6-0+3-0+2-0$$

$$6+3+2 = 11$$

$$S_3 = \begin{bmatrix}1&3&4\\0&2&5\\0&0&3\end{bmatrix}$$

$$+1(6-0)-3(0-0)+4(0-0)$$

$$1(6) = 6$$

$$\Rightarrow \lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$$

$\boxed{\lambda = 1, 2, 3}$ = eigenvalues

$\Rightarrow \boxed{\lambda = 1}$ to the eigenvector

$$\begin{bmatrix}1-\lambda&3&4\\0&2-\lambda&5\\0&0&3-\lambda\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}0&3&4\\0&1&5\\0&0&2\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$3x_2 + 4x_3 = 0 \to (1)$

$x_2 + 5x_3 = 0 \to (2)$

$2x_3 = 0 \to (3)$

$$\begin{array}{cccc} x_1 & x_2 & x_3 & x_1 \\ 0 & 3 & 4 & 0 \\ 0 & 1 & 5 & 0 \end{array}$$

$$\frac{x_1}{15-4} = \frac{x_2}{0-0} = \frac{x_3}{0-0}$$

$$\frac{x_1}{11} = \frac{x_2}{0} = \frac{x_3}{0} = k$$

$x_1 = 11k,\ x_2 = 0,\ x_3$

$$\begin{bmatrix}11k\\0\\0\end{bmatrix} = \begin{bmatrix}1\\0\\0\end{bmatrix}$$

$\boxed{\lambda = 2}$ to the eigen vector

$$\begin{bmatrix}1-2&3&4\\0&2-2&5\\0&0&3-2\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}$$

$$\begin{bmatrix}-1&3&4\\0&0&5\\0&0&1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$-x_1 + 3x_2 + 4x_3 = 0 \to (1)$

$5x_3 = 0 \to (2)$

$x_3 = 0 \to (3)$

from (1) and (2)

$$\begin{array}{cccc} x_1 & x_2 & x_3 & x_1 \\ -1 & 3 & 4 & -1 \\ 0 & 0 & 5 & 0 \end{array}$$

$$\frac{x_1}{15-0} = \frac{x_2}{0+5} = \frac{x_3}{0-0} = k$$

$\frac{x_1}{\cancel{15}\,3} = \frac{x_2}{\cancel{5}\,1} = \frac{x_3}{0} = k$

$x_1 = 3k$

$x_2 = k$

$x_3 = 0$

$$\begin{bmatrix} 3k \\ k \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$$

$\boxed{\lambda=3}$ to the eigenvector

$$\begin{bmatrix} -2 & 3 & 4 \\ 0 & -1 & 5 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-2x_1 + 3x_2 + 4x_3 = 0 \rightarrow (1)$

$-x_2 + 5x_3 = 0 \rightarrow (2)$

$\frac{x_1}{15+4} = \frac{x_2}{0+10} = \frac{x_3}{2+0}$

$\frac{x_1}{19} = \frac{x_2}{10} = \frac{x_3}{\cancel{2}\,1} = k$

$$\begin{bmatrix} \frac{19}{2}k \\ 5k \\ k \end{bmatrix} = \begin{bmatrix} \frac{19}{2} \\ 5 \\ 1 \end{bmatrix}$$

$A^3 - 6A^2 + 11A - 6 = 0$

Multiply A^{-1} on both sides

$A^{-1}(A^3 - 6A^2 + 11A - 6) = A^{-1} 0$

$A^2 - 6A + 11I - 6A^{-1} = 0$

$A^2 - 6A + 11I = 6A^{-1}$

$A^{-1} = \frac{1}{6}\left[A^2 - 6A + 11I\right]$

$$A^2 = \begin{bmatrix} 1 & 9 & 31 \\ 0 & 4 & 25 \\ 0 & 0 & 9 \end{bmatrix}$$

$$A^{-1} = \frac{1}{6}\left[\begin{bmatrix} 1 & 9 & 31 \\ 0 & 4 & 25 \\ 0 & 0 & 9 \end{bmatrix} - 6\begin{bmatrix} 1 & 3 & 4 \\ 0 & 2 & 5 \\ 0 & 0 & 3 \end{bmatrix} + 11\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right]$$

$$A^{-1} = \frac{1}{6}\begin{bmatrix} 6 & -9 & 7 \\ 0 & 3 & -5 \\ 0 & 0 & 2 \end{bmatrix}$$

(16)

$$\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

characteristic equation

$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

$\Rightarrow S_1 = \text{Trace} = 2+2+2 = 6$

$$S_2 = \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix}$$

$4-1+4-1+4-1$

$\Rightarrow 3+3+3 = 9$

$$S_3 = \begin{vmatrix} \overset{+}{2} & \overset{-}{-1} & \overset{+}{1} \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{vmatrix}$$

$+2(4-1) + 1(-2+1) + 1(1-2)$

$2(3) + 1(-1) + 1(-1)$

$6-1-1 = 6-2 = 4.$

$\lambda^3 - 6\lambda^2 + 9\lambda - 4 = 0$

$\boxed{\lambda = 4, 1, 1}$

$A^3 - 6A^2 + 9A - 4I = 0$

Multiply A^{-1} on both sides

$A^{-1}(A^3 - 6A^2 + 9A - 4I) = A^{-1} 0$

$A^2 - 6A + 9I - 4A^{-1} = 0$

$4A^{-1} = A^2 - 6A + 9I$

$A^{-1} = \frac{1}{4}\left[A^2 - 6A + 9I\right]$

$A^{-1} = \frac{1}{4}\left(\begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - 6\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} + 9\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right)$

$A^{-1} = \frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$

(17) $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$

$S_3 = \begin{array}{ccc} + & - & + \end{array}$
$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$

$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

$+1(-12-12) - 1(-4-6) + 3(-4+6)$

$-24 - 1(-10) + 3(2)$

$-24 + 10 + 6$

$-24 + 16 = -8$

2) $S_1 = 1 + 3 - 4 = 0$

$S_2 =$

$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 3 \\ 1 & 2 & -3 \\ -2 & -4 & -4 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ 2 & -4 & -4 \end{bmatrix}$

2) $-12 - 12$

$-12 - 12 + -4 + 6 + 3 - 1$

$-24 + 12 + 2 \quad -\cancel{34+2} = -20$

$\Rightarrow \lambda^3 - 0\lambda^2 - 20\lambda + 8 = 0$

$\lambda^3 - 20\lambda + 8 = 0$.

By cayley $A^3 - 20A + 8I = 0 \longrightarrow (1)$

$$A^3 = \begin{bmatrix} 12 & 20 & 60 \\ 20 & 52 & -60 \\ -40 & -80 & -88 \end{bmatrix}$$

$$\begin{bmatrix} 12 & 20 & 60 \\ 20 & 52 & -60 \\ -40 & -80 & -88 \end{bmatrix} - 20\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix} + 8\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 12 & 20 & 60 \\ 20 & 52 & -60 \\ -40 & -80 & -88 \end{bmatrix} - \begin{bmatrix} 20 & 20 & 60 \\ 20 & 60 & -60 \\ -40 & -80 & -80 \end{bmatrix} + \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$ cayley-Hamilton theorm verified.

$\Rightarrow$ By (1) eqn.

$A^3 - 20A + 8I = 0$ multiply A^{-1} on both sides

$A^{-1}(A^3 - 20A + 8I) = A^{-1} \cdot 0$

$A^2 - 20I + 8A^{-1} = 0$

$8A^{-1} = -A^2 + 20I$

$A^{-1} = \frac{1}{8}\left[-A^2 + 20I\right]$

$$A^{-1} = \frac{1}{8}\left(-\begin{bmatrix} -4 & -8 & -12 \\ 10 & 22 & 6 \\ 2 & 2 & 22 \end{bmatrix} + \begin{bmatrix} 20 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 20 \end{bmatrix}\right)$$

$$A^{-1} = \frac{1}{8}\begin{bmatrix} 24 & 8 & 12 \\ -10 & -2 & -6 \\ -2 & -2 & -2 \end{bmatrix}$$

(18) continuation

$$P^{-1}AP = D, \quad A = PDP^{-1}$$

$$A^4 = PD^4P^{-1}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -2 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 16 & 0 & 0 \\ 0 & 81 & 0 \\ 0 & 0 & 1296 \end{bmatrix} \frac{1}{6} \begin{bmatrix} 3 & 0 & -3 \\ 2 & 2 & 2 \\ 1 & -2 & 1 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 251 & -405 & 235 \\ -405 & 891 & -405 \\ 235 & -405 & 251 \end{bmatrix}$$

(19) $\begin{bmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{bmatrix}$

ch equation $= \lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

S_1 = Trace $= 7-1-1 = 7-2 = 5$

S_2 = minor

$\begin{bmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{bmatrix} + \begin{bmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{bmatrix} + \begin{bmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{bmatrix}$

$1-4-7+12-7+12$

$= -3+5+5 = 10-3 = 7$

$S_3 = \begin{bmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{bmatrix}$

$+7(1-4)-2(6-12)$

$-2(-12+6)$

$7(-3)-2(-6)-2(-6)$

$-21+12+12$

$24-21 = 3$

$\lambda^3 - 5\lambda^2 + 7\lambda - 3 = 0$

By Cayley $A^3 - 5A^2 + 7A - 3I = 0$

$A^3 = \begin{bmatrix} 79 & 26 & -26 \\ -78 & -25 & 26 \\ 78 & 26 & -25 \end{bmatrix}$

$A^2 = \begin{bmatrix} 25 & 8 & -8 \\ -24 & -7 & 8 \\ 24 & 8 & -7 \end{bmatrix}$

$\begin{bmatrix} 79 & 26 & -26 \\ -78 & -25 & 26 \\ 78 & 26 & -25 \end{bmatrix} - 5\begin{bmatrix} 25 & 8 & -8 \\ -24 & -7 & 8 \\ 24 & 8 & -7 \end{bmatrix} + 7\begin{bmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 79 & 26 & -26 \\ -78 & -25 & 26 \\ 78 & 26 & -25 \end{bmatrix} - \begin{bmatrix} 125 & 40 & -40 \\ -120 & -35 & 40 \\ 120 & 40 & -35 \end{bmatrix} + \begin{bmatrix} 49 & 14 & -14 \\ -42 & -7 & 14 \\ 42 & 14 & -7 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(Satisfies)

$\Rightarrow A^3 - 5A^2 + 7A - 3I = 0$

multiply A^{-1} on both sides

$A^{-1}(A^3 - 5A^2 + 7A - 3I) = A^{-1}0$

$A^2 - 5A + 7I - 3A^{-1} = 0$

$3A^{-1} = A^2 - 5A + 7I$

$A^{-1} = \frac{1}{3}(A^2 - 5A + 7I)$

$$A^{-1} = \frac{1}{3}\left[\begin{bmatrix} 25 & 8 & -8 \\ -24 & -7 & 8 \\ 24 & 8 & -7 \end{bmatrix} - 5\begin{bmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right]$$

$$A^{-1} = \frac{1}{3}\left[\begin{bmatrix} 25 & 8 & -8 \\ -24 & -7 & 8 \\ 24 & 8 & -7 \end{bmatrix} - \begin{bmatrix} 35 & 10 & -10 \\ -30 & -5 & 10 \\ 30 & 10 & -5 \end{bmatrix} + \begin{bmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix}\right]$$

$$A^{-1} = \frac{1}{3}\begin{bmatrix} -3 & -2 & 2 \\ 6 & 5 & -2 \\ -6 & -2 & 5 \end{bmatrix}$$

20 Cayley-Hamilton theorm and

$$A = \begin{bmatrix} 8 & -8 & 2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$$

$$S_2 = \begin{bmatrix} \cancel{8} & \cancel{-8} & \cancel{2} \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix} + \begin{bmatrix} 8 & -8 & 2 \\ \cancel{4} & \cancel{-3} & \cancel{-2} \\ 3 & -4 & 1 \end{bmatrix} + \begin{bmatrix} 8 & -8 & 2 \\ 4 & -3 & -2 \\ \cancel{3} & \cancel{-4} & \cancel{1} \end{bmatrix}$$

$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$ = chan

$S_1 = 8 - 3 + 1 = 6$

$-3 + \cancel{8} + \cancel{8} - 6 - 24 + 32$

$\cancel{-3 + 8 + 8 - 6 - 24 + 32}$

$-3 - 6 - 24 + 32$

$-9 - 24 + 32 = 32 - 33$

$= -1$

24/25

$$S_3 = \begin{vmatrix} \overset{+}{8} & \overset{-}{-8} & \overset{+}{2} \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{vmatrix}$$

$$+8(-3-8)+8(4+6)$$
$$+2(-16+9)$$

$$8(-11)+8(10)+2(-7)$$

$$-88+80-14=$$
$$-8-14=-22$$

$$\lambda^3-6\lambda^2-\lambda+22=0$$

By cayley.

$$A^3-6A^2-A+22I=0$$

$$A^3=\begin{bmatrix} 214 & -296 & 206 \\ 88 & -115 & 70 \\ 69 & -100 & 69 \end{bmatrix}$$

$$A^2=\begin{bmatrix} 38 & -48 & 34 \\ 14 & -15 & 12 \\ 11 & -16 & 15 \end{bmatrix}$$

$$\begin{bmatrix} 214 & -296 & 206 \\ 88 & -115 & 70 \\ 69 & -100 & 69 \end{bmatrix} - 6\begin{bmatrix} 38 & -48 & 34 \\ 14 & -15 & 12 \\ 11 & -16 & 15 \end{bmatrix} - \begin{bmatrix} 8 & -8 & 2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix} + \begin{bmatrix} 22 & 0 & 0 \\ 0 & 22 & 0 \\ 0 & 0 & 22 \end{bmatrix}$$

$$\begin{bmatrix} 214-228-8+22 & -296+288+8+0 & 206-204-2+0 \\ 88-84-4 & -115+90+3+22 & 70-72+2+0 \\ 69-66-3+0 & -100+96+4+0 & 69-60-1+22 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$ verified.

$$A^3 - 6A^2 - A + 22I = 0$$

multiply A^{-1} on B.S.

$$A^{-1}(A^3 - 6A^2 - A + 22I) = A^{-1} 0$$

$$A^2 - 6A - I + 22A^{-1} = 0$$

$$22A^{-1} = -A^2 + 6A + I.$$

$$A^{-1} = \frac{1}{22}\left[-A^2 + 6A + I\right].$$

$$A^{-1} = \frac{1}{22}\left[-\begin{bmatrix} 38 & -48 & 34 \\ 14 & -15 & 12 \\ 11 & -16 & 15 \end{bmatrix} + 6\begin{bmatrix} 8 & -8 & 2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right]$$

$$A^{-1} = \frac{1}{22}\begin{bmatrix} 11 & 0 & -22 \\ 10 & -2 & -24 \\ 7 & -8 & -8 \end{bmatrix}.$$

Part C

Part c

① eigenvalues of

$$\begin{bmatrix} 4 & 1-3i \\ 1+3i & 7 \end{bmatrix}$$

characteristic eqn. $\lambda^2 - S_1\lambda + S_2 = 0$

$S_1 = 4 + 7 = 11$

$S_2 = |A| = [28 - [(1)^2 - (3i)^2]]$

$S_2 = |A| = [28 - [1 + 9]]$

$S_2 = |A| = [28 - 10]$

$S_2 = 18$

$\lambda^2 - 11\lambda + 18 = 0$

$\boxed{\lambda = 9, 2}$ eigenvalues.

$\boxed{Ax = \lambda x}$

$$\Rightarrow \begin{bmatrix} 4-\lambda & 1-3i \\ 1+3i & 7-\lambda \end{bmatrix}$$

$\boxed{\lambda = 9}$ to the eigenvector

$$\Rightarrow \begin{bmatrix} -5 & 1-3i \\ 1+3i & -2 \end{bmatrix}$$

② Cayley-Hamilton theorem

Every square matrix satisfies its own characteristic equation

③ eigenvalues of $A^3 - 2A^2 - A - 5$

$A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$

char. eqn $\Rightarrow \lambda^2 - S_1\lambda + S_2 = 0$

$S_1 = \text{Trace} = 2 + 1 = 3$

$S_2 = |A| = 2 - 0 = 2.$

$\lambda^2 - 3\lambda + 2 = 0$

$\lambda^2 - 2\lambda - \lambda + 2 = 0$

$\lambda(\lambda - 2) - 1(\lambda - 2) = 0$

$(\lambda - 2)(\lambda - 1) = 0$

$\boxed{\lambda = 1, 2}$

$A^2 = \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix}$

$A^3 = \begin{bmatrix} 8 & 0 \\ 0 & 1 \end{bmatrix}$

eigenvalues of $A^3 = 8, 1$

eigenvalues of $A^2 = 4, 1$

eigenvalues of $A = 2, 1$

eigenvalues of $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 1, 1$

$A^3 - 2A^2 - A - 5I$

1) $(8)^3 - 2(4) - 2 - 5$

$512 - 8 - 7$

$512 - 15 = \underline{\underline{497}}$

2) $(1)^3 - 2(1) - 1 - 5 = 1 - 2 - 1 - 5$

$= \underline{\underline{-7}}$

⑤ 2, 3, 4 ⇒ eigenvalues

$= 2 \times 3 \times 4$

$= 24 = |A|$

$\lambda_1, \lambda_2, \lambda_3$ are eigen values of A of adj A

$\frac{|A|}{\lambda_1}, \frac{|A|}{\lambda_2}$ & $\frac{|A|}{\lambda_3}$ adj A

$\cancel{\frac{24}{2}}, \cancel{\frac{24}{3}}, \cancel{\frac{24}{4}}$ 12, 8, 6

$\Rightarrow$ $\boxed{12, 8, 6}$ eigenvalues of A.

⑥ Sum of eigenvalues

$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

Ch eqn $\Rightarrow \lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

$S_1 = \text{Trace} = 2+3+2 = 7$

$S_2 = \begin{bmatrix} 3 & 1 \\ 2 & 2 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$

$6 - 2 + 4 - 1 + 6 - 2$

$4 + 3 + 4 = 11$

$S_3 = |A| = \begin{vmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{vmatrix}$

$+2(6-2) - 2(2-1) + 1(2-3)$

$2(4) - 2(1) + 1(-1)$

$8 - 2 - 1 = 8 - 3 = 5.$

$\lambda^3 - 7\lambda^2 + 11\lambda - 5 = 0$

$\boxed{\lambda = 5, 1, 1}$ = eigenvalues.

Sum of eigen = $5 + 1 + 1 = 7$.

⑦ verify Cayley-Hamilton

$A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

Ch eqn $= \lambda^2 - S_1\lambda + S_2 = 0$

$S_1 = \text{Trace} = 1 - 1 = 0$

$S_2 = -1 - 4 = -5$

$\lambda^2 - 0\lambda + (-5) = 0$

$\lambda^2 - 5 = 0$

By cayley $= A^2 - 5I = 0$

$A^2 = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

$\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Verified Cayley-Hamilton

(9) $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

chearn $\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

$S_1 = $ Trace $= 6+3+3 = 12$

$S_2 =$ minors

$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

$9-1+18-4+18-4$

$8+14+14$

$= 36$

$S_3 = |A| = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

$+6(9-1)+2(-6+2)+2(2-6)$

$6(8)+2(-4)+2(-4)$

$48-8-8 = 48-16$

$= 32$

$\boxed{\lambda^3 - 12\lambda^2 + 36\lambda - 32 = 0}$ chean

(11) eigen values.

$A = \begin{bmatrix} 3i & 2+i \\ -2+i & -i \end{bmatrix}$

$\Rightarrow$ chear $= \lambda^2 - S_1\lambda + S_2 = 0$

$S_1 = 3i - i = 2i$

$S_2 = |A| = [-3(i^2) - (4+i^2)]$

$S_2 = |A| = [-3(-1) - [-4-1]]$

$S_2 = |A| = (3 - [-5]) = (3+5)$

$= 8$

$\lambda^2 - 2i + 8 = 0$

$\lambda^2 - 4i + 2i + 8 = 0$

$\lambda(\lambda - 4i) + 2i(\lambda - 4i)$

$(\lambda + 2i)(\lambda - 4i) = 0$

$\boxed{\lambda = -2i \quad \lambda = 4i}$ $\Rightarrow$ eigen value

8

4 2

(13) Let A be square matrix of order ...
= is called characteristic equation of matrix

(14) The roots of the characteristic equation $|A-\lambda I|=0$
= Called eigen values of matrix.
⇒ If λ is an eigenvalue of the square matrix A. If there exists nonzero vector x such that $\boxed{Ax=\lambda x}$ is said to be eigen vector

(15) $A=\begin{bmatrix}1&2\\2&1\end{bmatrix}$

-3, λ, $-3\ 1$

ch eqn $=\lambda^2-s_1\lambda+s_2=0$

$s_1=2$

$s_2=1-4=-3$

$\lambda^2-2\lambda-3=0$

$\lambda^2-3\lambda+\lambda-3=0$

$\lambda(\lambda-3)+1(\lambda-3)=0$

$(\lambda-3)(\lambda+1)=0$

$\lambda-3=0$ } $\lambda=3$

$\lambda+1=0$ } $\lambda=-1$

$\boxed{\lambda=3,-1}$ = characteristic values.

(17) $A=\begin{bmatrix}1&4\\3&2\end{bmatrix}$

ch eqn $=\lambda^2-s_1\lambda+s_2=0$

$s_1=3$

$s_2=2-12=-10$

$\lambda^2-3\lambda-10=0$

-10, λ, $-5\ 2$

$\lambda^2-5\lambda+2\lambda-10=0$

$\lambda(\lambda-5)+2(\lambda-5)=0$

$(\lambda-5)(\lambda+2)=0$

$\boxed{\lambda=5,-2}$ = eigenvalues.

(19) $\begin{bmatrix}2&5&7\\1&4&6\\2&-2&3\end{bmatrix}$

ch eqn $=\lambda^3-s_1\lambda^2+s_2\lambda-s_3=0$

$s_1=2+4+3=9$

$s_2=\begin{vmatrix}2&5&7\\1&4&6\\2&-2&3\end{vmatrix}+\begin{vmatrix}2&5&7\\1&4&6\\2&-2&3\end{vmatrix}+\begin{vmatrix}2&5&7\\1&4&6\\2&-2&3\end{vmatrix}$

$12+12+6-14+8-5$

$24-8+3$

$27-8=19$

$s_3=\begin{vmatrix}2&5&7\\1&4&6\\2&-2&3\end{vmatrix}$

$+2(12+12)-5(3-12)+7(-2-8)$

$2(24)-5(-9)+7(-10)$

~~48+45-112~~

~~193-112=~~

~~48+45-112=~~

~~93-112=~~

$24\times2=48$

$48+45-70$

$=23$

$\lambda^3-9\lambda^2+19\lambda-23=0$

$\lambda_1 = 6.667811696$

$$\boxed{\begin{aligned}\lambda_1 &= 6.668\\ \lambda_2 &= 1.166 + i\cdot(1.446)\\ \lambda_3 &= 1.166 - i\,(1.446)\end{aligned}}$$

Summand product

$6.668 + 1.166 + i\,\cancel{(1.446)} + 1.166 - i\,\cancel{(1.446)}$

$6.668 + 1.166 + 1.166 = 9$

product =

$(6.668) \times \underbrace{(1.166 + i\,(1.446)}_{(a+b)}\underbrace{(1.166 - i\,(1.446))}_{(a-b)}$

$(6.668) \times ((1.166)^2 + (1.446)^2)$

$(6.668) \times (1.359556 + 2.090916)$

$(6.668) \times (3.450472)$

$= 23.$

(20)

$$\begin{bmatrix} -2 & 3 & 6 \\ 0 & 3 & 6 \\ 0 & 0 & 1 \end{bmatrix}$$

chean $\Rightarrow \lambda^3 - 2\lambda^2 - 5\lambda + 6 = 0$

$\boxed{\lambda = -2, 3, 1}$

$\Downarrow$

characteristic roots

$S_1 = -2 + 3 + 1 = 4 - 2 = 2$

$S_2 = \begin{bmatrix} 3 & 6 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -2 & 6 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -2 & 3 \\ 0 & 3 \end{bmatrix}$

$S_2 = 3 - 0 + (-2) - 0 - 6 - 0$

$S_2 = 3 - 2 - 6$

$S_2 = 3 - 8 = -5$

$S_3 = \begin{bmatrix} -2 & 3 & 6 \\ 0 & 3 & 6 \\ 0 & 0 & 1 \end{bmatrix}$

$-2(3-0) - 3(0-0) + 6(0-0)$

$-2(3) = -6$

57

Part C

(4) $\boxed{D = P^{-1} A P}$

P is Known as modal matrix

D is Known as diagonal matrix as Spectral matrix.

(A) Use Cayley Hamilton

A^8

Part - A - 6

$A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

Characteristic eqn

$\lambda^2 - S_1\lambda + S_2 = 0$

$S_1 = \text{Trace} = 1 - 1 = 0$

$S_2 = |A| = -1 - 4 = -5$

$\lambda^2 - 5 = 0$

By cayley Hamilton

$A^2 - 5I = 0$

$\boxed{A^2 = 5I}$

multiply A^6

$A^8 = 5A^6$

$= 5A^2 \cdot A^2 \cdot A^2$

$5(5I)(5I)(5I)$

$= 625I.$