MODULE - II

PART - A

① Given, $(D^2+4) \cdot y = \sin 2x$

form: $f(D) \cdot y = Q(x)$

$f(D) = D^2+4$; $Q(x) = \sin 2x$

y_c: Auxillary equation : $f(m)=0$

$$m^2+4=0$$

$$m = \pm 2i$$

$\alpha=0$; $\beta=2$ $\qquad \alpha \pm i\beta$

$$\therefore y_c = e^{\alpha x}(c_1 \cos \beta x + c_2 \sin \beta x).$$

$$y_c = e^{0x}(c_1 \cos 2x + c_2 \sin 2x)$$

$$y_c = c_1 \cos 2x + c_2 \sin 2x$$

y_p: $y_p = \frac{1}{f(D)} \cdot Q(x) = \frac{\sin 2x}{D^2+4}$

$$= \frac{1}{D^2+2^2} \cdot \sin 2x.$$

form: $\frac{1}{D^2+a^2} \sin ax = -x/2a \cos ax.$

$$\Rightarrow \frac{-x}{2(2)} \cos 2x = -\frac{x}{4} \cos 2x.$$

$\therefore$ General solution is $y = y_c + y_p$

$$y = c_1 \cos 2x + c_2 \sin 2x - \frac{x}{4} \cos 2x$$

(2) Given; $(D^2-2D)\cdot y = e^x \sin x$ — (1)

Method of Variation of Parameters

(1) Auxillary equation is $m^2-2m=0$ $\because f(m)=0$

$m(m-2)=0$

$m=0$ or $m=2$

$y_c = e^{0x}c_1 + c_2 e^{2x} = c_1 + c_2 e^{2x}$

(2) $y_c = c_1 \cdot 1 + c_2 e^{2x}$

form: $y_c = c_1 u(x) + c_2 v(x)$

$u(x)=1$; $v(x)=e^{2x}$

Eq (1) is of form $D^2 + P(x)\cdot D + Q(x)\cdot y = R(x)$

$P(x) = -2y$

$Q(x) = 0$

$R(x) = e^x \sin x$

(3) Find $uv' - u'v = 1 \cdot 2e^{2x} - e^{2x}(0)$

$= 2e^{2x}$

(4) $y_p = Au + Bv$

where,

$$A = -\int \frac{VR}{uv' - u'v}\,dx \qquad B = \int \frac{UR}{uv' - u'v}\,dx$$

$$A = -\int \frac{e^{2x} \cdot e^{x} \sin x}{2e^{2x}}\,dx \qquad B = \int \frac{e^{x} \sin x}{2e^{2x}}\,dx$$

$$A = -\frac{1}{2}\int e^{x} \sin x\,dx \qquad B = \frac{1}{2}\int e^{-x} \sin x\,dx$$

$$A = -\frac{1}{4} e^{x} (\sin x - \cos x) \qquad B = \frac{-e^{-x}}{4} (\sin x + \cos x)$$

$$\therefore y_p = -\frac{1}{4} e^{x} (\sin x - \cos x) - \frac{e^{-x}}{4} (\sin x + \cos x) e^{2x}$$

$$y_p = -\frac{1}{2} e^{x} \sin x$$

$$y = y_c + y_p$$

$$y = c_1 + c_2 e^{2x} - \frac{1}{2} e^{x} \sin x$$

$$* \int e^{ax} \sin bx\,dx = \frac{e^{ax}}{a^2 + b^2} (a \sin bx - b \cos bx) + c$$

③ Given; $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$

form: $\frac{d^2y}{dx^2} + P(x) \cdot \frac{dx}{dy} + Q(x) \cdot y = R(x)$

Method of variation of Parameters

① $P(x) = 0$; $Q(x) = 1$; $R(x) = \operatorname{cosec} x$

② Auxillary equation → $f(m) = 0$

$m^2 + 1 = 0$

$m = \pm i$

$\alpha = 0$; $\beta = 1$

$y_c = e^{\alpha x}(C_1 \cos \beta x + C_2 \sin \beta x)$

$y_c = C_1 \cos x + C_2 \sin x$

form: $y_c = C_1 u(x) + C_2 v(x)$

$u(x) = \cos x$; $v(x) = \sin x$.

③ Let, $y_p = Au + Bv$

$A = -\int \frac{vR}{uv' - u'v} dx$ $\qquad B = \int \frac{uR}{uv' - u'v} dx$

$uv' - u'v = \cos x \cdot \cos x - (-\sin x) \cdot \sin x = \cos^2 x + \sin^2 x$

$uv' - u'v = 1$

$$A = -\int \frac{\sin x \cdot \operatorname{cosec} x}{1}\, dx = -\int dx = -x$$

$$\cancel{B = \int \cos x}$$

$$B = \int \frac{\cos x \cdot \operatorname{cosec} x}{1}\, dx$$

$$B = \int \cot x\, dx = \log|\sin x|$$

$$y_p = Au + Bv = -x\cos x + \log|\sin x| \cdot \sin x$$

The General solution is $y = y_c + y_p$

$$y = c_1 \cos x + c_2 \sin x - x\cos x + \sin x \log|\sin x|$$

(4) $y'''' + 8y'' + 16y = 0 \Rightarrow (D^4 + 8D^2 + 16)y = 0$

form: $f(D) \cdot y = 0$

y_c: Auxillary equation $\rightarrow f(m) = 0$

$$m^4 + 8m^2 + 16 = 0$$

$$(m^2 + 4)^2 = 0$$

$$(m - 2i)^2 (m + 2i)^2 = 0$$

$$m = 2i, 2i, -2i, -2i$$

$$\therefore y_c = (c_1 + c_2 x)\cos 2x + (c_3 + c_4 x)\sin 2x$$

The general solution is.

$$y = (c_1 + c_2 x)\cos 2x + (c_3 + c_4 x)\sin 2x$$

(5) $y'''' + 18y'' + 81y = 64\cos x + 108\cos 3x$

$(D^4 + 18D^2 + 81)y = 64\cos x + 108\cos 3x$

$\underline{y_c}$: $f(m) = 0$

$m^4 + 18m^2 + 81 = 0$

$(m^2+9)(m^2+9) = 0$

$m = \pm 3i, \pm 3i$

$y_c = (c_1 + c_2 x)\cos 3x + (c_3 + c_4 x)\sin 3x.$

$\underline{y_p}$: $y_p = \dfrac{1}{f(D)} \cdot Q(x)$

$y_p = \dfrac{1}{D^4 + 18D^2 + 81}(64\cos x + 108\cos 3x)$

$y_p = 64 \dfrac{\cos x}{(D^2+3^2)^2} + 108 \cdot \dfrac{\cos 3x}{(D^2+3^2)^2}$

$D^2 = -a^2 = -1$ $\qquad$ $D^2 = -9$ $\qquad$ $-9+9=0$

$\dfrac{\cos ax}{(D^2+a^2)(D^2+a^2)} = \dfrac{+x\sin ax}{2a}$

$$y_p = 64 \frac{\cos x}{(-1+9)^2} + 108 \frac{x}{2(3)} \frac{\sin 3x}{(D^2+3^2)}$$

-3^2

$$y_p = \cancel{64} \cdot \frac{\cos x}{\cancel{64}} + \overset{\overset{3}{\cancel{18}}}{\cancel{108}} \cdot \frac{x}{\cancel{6}} \cdot \left(\frac{-x}{\cancel{2(3)}} \cos 3x\right)$$

$$y_p = \cos x + (-3x^2 \cos 3x)$$

$$y_p = \cos x - 3x^2 \cos 3x$$

$\therefore$ The general solution $y = y_c + y_p$

$$y = (C_1 + C_2 x)\cos 3x + (C_3 + C_4 x)\sin 3x + \cos x - 3x^2 \cos 3x$$

(6) Given; $(D^2+4)\cdot y = \sec 2x$

Method of Variation of Parameters

$$\frac{d^2y}{dx^2} + 4y = \sec 2x$$

form: $\frac{d^2y}{dx^2} + P(x)\cdot\frac{dy}{dx} + Q(x)\cdot y = R(x)$

(1) $P(x) = 0$; $Q(x) = 4$; $R(x) = \sec 2x$

Auxillary equation: $f(m) = 0$

$$m^2 + 4 = 0$$

$$m = \pm 2i$$

$$\alpha = 0; \beta = 2$$

(2) $y_c = e^{\alpha x}(c_1 \cos \beta x + c_2 \sin \beta x)$

$y_c = e^{0\cdot x}(c_1 \cos 2x + c_2 \sin 2x)$

$y_c = c_1 \cos 2x + c_2 \sin 2x$

$y_c = c_1 u(x) + c_2 v(x)$

$u(x) = \cos 2x$; $v(x) = \sin 2x$

$u'(x) = -2\sin 2x$; $v'(x) = 2\cos 2x$

$uv' - u'v = \cos 2x \cdot 2\cos 2x + 2\sin 2x \cdot \sin 2x$

$= 2(\underbrace{\cos^2 2x + \sin^2 2x}_{1}) = 2$

(3) Let; $y_p = Au + Bv$

$$A = -\int \frac{vR}{uv' - u'v}\,dx = -\int \frac{\sin 2x \cdot \sec 2x}{2}$$

$$= -\frac{1}{2}\int \tan 2x \, dx$$

$$= -\frac{1}{2}\left(-\frac{\log|\cos 2x|}{2}\right)$$

$$= \frac{\log|\cos 2x|}{4}$$

$$B = \int \frac{uR}{uv' - u'v}\,dx = \int \frac{\cos 2x \cdot \sec 2x}{2}\,dx$$

$$= \int \tfrac{1}{2}\,dx$$

$$= x/2$$

$$y_p = \frac{\log|\cos 2x|}{4} \cdot \cos 2x + \frac{x}{2}\sin 2x$$

$\therefore$ The general solution is given by.

$$y = y_c + y_p$$

$$y = c_1 \cos 2x + c_2 \sin 2x + \log\frac{|\cos 2x|}{4} \cos 2x + \frac{x}{2}$$

$\sin 2x$

(7) Given ; $(D^2+1)\cdot y = \tan x$

Method of variation of parameters.

Form: $\frac{d^2y}{dx^2} + P(x)\cdot\frac{dy}{dx} + Q(x)\cdot y = R(x)$

(1) $P(x) = 0$; $Q(x) = 1$; $R(x) = \tan x$

(2) The Auxillary function: $f(m) = 0$

$$m^2+1 = 0$$

$$m = \pm 1$$

$$\alpha = 0;\ \beta = 1$$

$y_c = e^{\alpha x}(c_1 \cos\beta x + c_2 \sin\beta x)$

$y_c = e^{ax}(c_1 \cos x + c_2 \sin x)$

$y_c = c_1 \cos x + c_2 \sin x$

$y_c = c_1 u(x) + c_2 v(x)$

$u(x) = \cos x$ & $v(x) = \sin x$

$u'(x) = -\sin x$ & $v'(x) = \cos x$

$uv' - u'v = \cos x \cdot \cos x + \sin x \cdot \sin x = 1$

(3) Let; $y_p = Au + Bv$

where; $A = -\int \frac{vR}{uv'-u'v}\,dx$

$$A = -\int \frac{\sin x \cdot \tan x}{1}\, dx = -\int \frac{\sin^2 x}{\cos x}\, dx$$

$$= -\int \left(\frac{1-\cos^2 x}{\cos x}\right) dx$$

$$= -\int (\sec x - \cos x)\, dx$$

$$= -\left\{\int \sec x\, dx - \int \cos x\, dx\right\}$$

$$= -\{\log|\sec x + \tan x| - \sin x\}$$

$$= \sin x - \log|\sec x + \tan x|$$

$$B = \int \frac{uR}{uv' - u'v}\, dx = \int \frac{\cos x \cdot \tan x}{1}\, dx = \int \sin x\, dx$$

$$= -\cos x$$

$$y_p = Au + Bv = (\sin x - \log|\sec x + \tan x|) \cdot \cos x - \cos x (\sin x)$$

$$y_p = \cancel{\sin x \cos x} - \log|\sec x + \tan x|(\cos x) - \cancel{\sin x \cos x}$$

$$y_p = -\log|\sec x + \tan x| \cdot \cos x$$

$\therefore$ The general solution is given by $y = y_c + y_p$

$$y = c_1 \cos x + c_2 \sin x - \cos x \cdot \log|\sec x + \tan x|$$

(8) $(D^2-2D+2)y = e^x \cdot \tan x$

Method of variation of parameters

(1) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = e^x \cdot \tan x$

form: $\frac{d^2y}{dx^2} + P(x)\cdot\frac{dy}{dx} + Q(x)\cdot y = R(x)$.

$P(x) = -2$; $Q(x) = 2$; $R(x) = e^x \cdot \tan x$;

Auxillary equation: $f(m) = 0$

(2) $m^2 - 2m + 2 = 0$

$m = 1 \pm i$

$\alpha \pm i\beta$

$\alpha = 1; \beta = 1$

$y_c = e^{\alpha x}(c_1 \cos\beta x + c_2 \sin\beta x)$

$y_c = e^{1\cdot x}(c_1 \cos x + c_2 \sin x)$.

$y_c = e^x(c_1 \cos x + c_2 \sin x)$.

$y_c = c_1 e^x \cdot \cos x + c_2 e^x \sin x$.

$y_c = c_1 u(x) + c_2 v(x)$.

$u(x) = e^x \cos x$ $\quad$ $v(x) = e^x \sin x$

$u'(x) = -e^x \sin x + e^x \cos x$ $\quad$ $v'(x) = e^x \sin x + e^x \cos x$

$$uv' - u'v = e^x \cos x\,(e^x \sin x + e^x \cos x) - e^x \sin x\,(e^x \cos x - e^x \sin x)$$

$$= \cancel{e^x \cos x \cdot e^x \sin x} + (e^x \cos x)^2 - \cancel{e^x \sin x\, e^x \cos x} + (e^x \sin x)^2$$

$$\Rightarrow (e^x)^2 \cdot (\sin^2 x + \cos^2 x) = e^{2x}.$$

(3) $y_p = Au + Bv.$

$$A = -\int \frac{vR}{uv' - u'v}\,dx = -\int \frac{\cancel{e^x} \sin x \cdot \cancel{e^x} \tan x}{\cancel{e^{2x}}}\,dx.$$

$$= -\int \sin x \tan x\,dx$$

$$= -\int \frac{\sin^2 x}{\cos x}\,dx$$

$$= -\int \left(\frac{1 - \cos^2 x}{\cos x}\right) dx$$

$$= -\int (\sec x - \cos x)\,dx$$

$$= \sin x - \log|\sec x + \tan x|$$

$$B = \int \frac{uR}{uv' - u'v}\,dx = \int \frac{\cancel{e^x} \cos x \cdot \cancel{e^x} \tan x}{\cancel{e^{2x}}}\,dx$$

$$= \int \cos x \tan x\,dx$$

$$= \int \sin x\,dx$$

$$= -\cos x$$

$$y_p = (\sin x - \log|\sec x + \tan x|)\,e^x \cos x - \cos x \cdot e^x \sin x$$

$y_p = e^x \cancel{\sin x \cos x} - e^x \cos x \log|\sec x + \tan x| - e^x \sin x$

$y_p = -e^x \cos x \log|\sec x + \tan x|$

$\therefore$ The general solution is $y = y_c + y_p$

$y = c_1 e^x \cos x + c_2 e^x \sin x - e^x \cos x \log|\sec x + \tan x|$

$y = e^x (c_1 \cos x + c_2 \sin x - \cos x \cdot \log|\sec x + \tan x|)$

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(9) Given; $(D^2 - 6D + 9)y = e^{3x}/x^2$

Method of Variation of Parameters

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = e^{3x}/x^2.$$

form: $\frac{d^2y}{dx^2} + P(x).\frac{dy}{dx} + Q(x).y = R(x).$

(1) $P(x) = -6$; $Q(x) = 9$; $R(x) = e^{3x}/x^2$

(2) Auxillary equation : $f(m) = 0$.

$$m^2 - 6m + 9 = 0.$$

$$(m-3)^2 = 0$$

$$m = 3, 3.$$

$$y_c = (c_1 + c_2 x)e^{3x}.$$

$$y_c = c_1 e^{3x} + c_2 x.e^{3x}$$

form: $y_c = c_1 u(x) + c_2 v(x)$

ILATE $u'v + uv'$

$u(x) = e^{3x}$ $v(x) = x e^{3x}$

$u'(x) = 3e^{3x}$ $v'(x) = 1 \cdot e^{3x} + x \cdot e^{3x} \cdot 3$

$v'(x) = e^{3x} + 3x e^{3x}$

$uv' - u'v = e^{3x}(e^{3x} + 3x e^{3x}) - 3e^{3x}(x e^{3x})$

$uv' - u'v = (e^{3x})^2 + \cancel{3x(e^{3x})^2} - \cancel{3x(e^{3x})^2}$

$uv' - u'v = (e^{3x})^2 = e^{6x}$

③ Let; $y_p = Au + Bv$

$A = -\int \frac{vR}{uv' - u'v} dx = -\int \frac{\cancel{x e^{3x}} \cdot \frac{e^{3x}}{x^2}}{\cancel{(e^{3x})^2}} dx$

$= -\int \frac{1}{x} dx$

$= -\log|x|$

$B = \int \frac{uR}{uv' - u'v} dx = \int \frac{\cancel{e^{3x} \cdot e^{3x}}/x^2}{\cancel{(e^{3x})^2}} dx$

$= \int \frac{1}{x^2} dx$

$= -\frac{1}{x}$

$Au + Bv = -\log|x|(e^{3x}) - \frac{1}{\cancel{x}} \cdot \cancel{x} e^{3x}$

$$y_p = -e^{3x}(\log|x|+1)$$

$\therefore$ The general solution is given by.

$$y = y_c + y_p$$

$$y = (c_1 + c_2 x)e^{3x} - e^{3x}(\log|x|+1)$$

$$y = e^{3x}(c_1 + c_2 x - \log|x| - 1)$$

(10) Given; $(D^2 - 2D + 1)y = e^x \log x$

Method of variation of Parameters:

(1) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 1 \cdot y = e^x \log x$

form: $\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x) \cdot y = R(x).$

(1) $P(x) = -2$; $Q(x) = 1$; $R(x) = e^x \log x$

(2) Auxillary equation; $f(m) = 0$

$$m^2 - 2m + 1 = 0.$$

$$(m-1)^2 = 0$$

$$m = 1, 1$$

$$y_c = (c_1 + c_2 x)e^{1x}$$

$$y_c = c_1 e^x + c_2 x e^x.$$

form: $y_c = c_1 u(x) + c_2 v(x)$

$y_c = u(x) = e^x$ | $v(x) = x e^x$

$u'(x) = e^x$ | $v'(x) = 1 e^x + x e^x$

$v'(x) = e^x + x e^x$

$uv' - u'v = e^x(e^x + x e^x) - e^x(x e^x)$

$uv' - u'v = (e^x)^2 + x \cancel{e^x e^x} - x \cancel{e^x e^x}$

$uv' - u'v = (e^x)^2$

(3) Let $y_p = Au + Bv$

$A = -\int \frac{vR}{uv' - u'v} dx = -\int \frac{x \cancel{e^x} \cdot \cancel{e^x} \log x}{\cancel{(e^x)^2}} dx$

$= -\int \underset{v}{x} \, \underset{u}{\log x} \, dx$ ILATE

$\int uv\,dx = uv_1 - u'v_2 + u''v_3 - \cdots$

(or) $u \int v\,dx - \int (u' \int v\,dx)\,dx$

$\int x \log x\,dx = \log x \cdot \frac{x^2}{2} - \frac{1}{x} \cdot \frac{x^3}{4} + 0 - \cdots$

$A = -\left(\log x \cdot \left(\frac{x^2}{2}\right) - \frac{x^2}{4}\right) = \frac{x^2}{4} - \frac{x^2}{2}(\log x)$

$= \frac{x^2}{4}\left[1 - 2\log x\right]$

$$B = \int \frac{uR}{uv' - u'v}\, dx = \int \frac{\cancel{e^x} \cdot e^x \log x}{\cancel{(e^x)^2}}\, dx$$

$$B = \int \log x\, dx$$

$$B = x(\log x - 1)$$

$$y_p = Au + Bv = \frac{x^2}{4}\left[1 - 2\log x\right] \cdot e^x + x(\log x - 1) \cdot xe^x$$

$$= \frac{x^2 e^x}{4}\left[1 - 2\log x\right] + x^2 e^x (\log x - 1)$$

$$= x^2 e^x \left[\frac{1 - 2\log x}{4} + \log x - 1\right]$$

$$= x^2 e^x \left[\frac{2\log x - 3}{4}\right]$$

$$= x^2 e^x \left[\frac{\log x^2 - 3}{4}\right]$$

$\therefore$ The general solution is $y = y_c + y_p$

$$y = (C_1 + C_2 x)e^x + x^2 e^x \left[\frac{\log x^2 - 3}{4}\right]$$

$$y = \left(C_1 + C_2 x + \left(\frac{\log x^2 - 3}{4}\right) \cdot x^2\right) e^x$$

PART-B

① Given, $(D^2+3D+2).y = 2\cos(2x+3)+2e^x+x^2$

$f(D).y = \phi(x)$

Auxillary equation : $f(m)=0$

$$m^2+3m+2=0$$

$$(m+1)(m+2)=0$$

$$m=-1,-2$$

$\therefore y_c = c_1 e^{-x} + c_2 e^{-2x}$

$$y_p = \frac{1}{D^2+3D+2}\left[2\cos(2x+3)+2e^x+x^2\right]$$

$$y_p = \frac{1}{D^2+3D+2}\cdot 2\cos(2x+3) + \frac{1}{D^2+3D+2}2e^x + \frac{1}{D^2+3D+2}\cdot x^2$$

$D^2=-a^2$ $\quad f(D)=f(a)$ $\quad f(a)$ $\quad a=1$

$$y_p = \frac{1}{3D+(-4+2)}\,2\cos(2x+3) + \frac{1}{1^2+3(1)+2}2e^x + \frac{1}{2\left(1+\frac{D^2+3D}{2}\right)}\cdot x^2$$

$$y_p = 2\cdot\frac{1}{3D-2}\cos(2x+3) + \frac{1}{3}e^x + \frac{1}{2}\left(1+\frac{D^2+3D}{2}\right)^{-1}x^2$$

↓ Rationalise $\quad (1+D)^{-1}$

$$y_p = 2\cdot\frac{3D+2}{9D^2-4}\cos(2x+3) + \frac{1}{3}e^x + \frac{1}{2}\left[1-\frac{D^2+3D}{2}+\left(\frac{D^2+3D}{2}\right)^2\right]x^2$$

$D^2=-a^2$

$$y_p = 2\cdot\frac{3D+2}{9(-4)-4}\cos(2x+3) + \frac{1}{3}e^x + \frac{1}{2}\left[1-\frac{D^2+3D}{2}+\frac{9D^2}{4}\right]x^2$$

$y_p = \frac{-1}{20}\left[-6\sin(2x+3) + 2\cos(2x+3)\right) + \frac{1}{3}e^x +$

$\frac{1}{2}\left(x^2 - \frac{1}{2}(2+6x) + \frac{9}{4}(2)\right]$

$y_p = -\frac{1}{10}\left(\cos(2x+3) - 3\sin(2x+3)\right) + \frac{1}{3}e^x + \frac{1}{2}\left(x^2 - 3x + \frac{7}{2}\right)$

$\therefore$ The general solution is $y = y_c + y_p$

$y = c_1 e^{-x} + c_2 e^{-2x} - \frac{1}{10}\left(\cos(2x+3) - 3\sin(2x+3)\right) + \frac{e^x}{3} + \frac{x^2 - 3x + 7/2}{2}$

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Q) $D^2(D^2+4).y = 96x^2 + \sin 2x - K$

[~~Question Bird~~ Standard Question Textbook

Auxillary equation ; $f(m) = 0$

$m^2(m^2+4) = 0$

$m = 0, 0, \pm 2i$

$\alpha \pm i\beta \Rightarrow \alpha = 0, \beta = 2$

$\therefore y_c = e^{0x}(c_1 + c_2 x) + e^{0x}(c_3\cos 2x + c_4 \sin 2x)$

$y_c = (c_1 + c_2 x) + (c_3 \cos 2x + c_4 \sin 2x)$

$y_p = \frac{1}{D^2(D^2+4)}(96x^2 + \sin 2x - K)$

$y_p = \frac{1}{D^2(D^2+4)} \cdot 96x^2 + \frac{1}{D^2(D^2+4)} \cdot \sin 2x - \frac{1}{D^2(D^2+4)} K$

$$y_p = \frac{1}{4D^2\left(1+\frac{D^2}{4}\right)} 96x^2 + \frac{1}{(-4).(D^2+2^2)} \sin 2x - \frac{1}{D^2(D^2+4)} K.e^{0x}$$

$$y_p = \frac{1}{4D^2}\left(1+\frac{D^2}{4}\right)^{-1} 96x^2 - \frac{1}{4}\left(\frac{-x}{2(2)} \cos 2x\right) - \frac{K}{D^2(0+4)} e^0 . x$$

$$y_p = \frac{1}{4D^2}\left(1 - \frac{D^2}{4} + \frac{D^4}{16} - - -\right) 96x^2 + \frac{x}{16} \cos 2x - \frac{K}{4} . \frac{x^2}{2!} e^{0.x}$$

$$y_p = \frac{96}{4}\left(\frac{1}{D^2} - \frac{1}{4} + \frac{D^2}{16} - -\right) x^2 + \frac{x}{16} \cos 2x - \frac{K}{8} . x^2$$

$$y_p = 24\left(\frac{x^4}{12} - \frac{1}{4}.x^2 + \frac{1}{16}.2\right) + \frac{x}{16} \cos 2x - \frac{K}{8} x^2$$

$$y_p = (2x^4 - 6x^2 + 3) + \frac{x}{16} \cos 2x - \frac{K}{8} x^2$$

$\therefore$ The general solution is

$$y = y_c + y_p$$

$$y = c_1 + c_2 x + c_3 \cos 2x + c_4 \sin 2x + 2x^4 - 6x^2 + 3 + \frac{x}{16} \cos 2x - \frac{K}{8} x^2$$

③ $(D^2-2D+1)\cdot y = x^2 e^{3x}$

$f(D)\cdot y = \phi(x)$

Auxillary equation is : $f(m)=0$

$m^2-2m+1=0$

$(m-1)^2=0$

$m=1,1$

$\therefore y_c = (c_1+c_2 x)e^{x}$,

$y_p = \dfrac{1}{D^2-2D+1}\cdot x^2 e^{3x}$.

$y_p = e^{3x}\cdot\dfrac{1}{D^2-2D+1}\cdot x^2$

Put $D = D+3$

$D^2-2D+1 = (D+3)^2-2(D+3)+1$

$\Rightarrow D^2+9+6D-2D-6+1 = D^2+4D+4$

$y_p = e^{3x}\cdot\dfrac{x^2}{D^2+4D+4} = \dfrac{e^{3x}}{4}\cdot\dfrac{x^2}{\left(1+\frac{D^2+4D}{4}\right)}$

$y_p = \dfrac{e^{3x}}{4}\cdot\left[1+\dfrac{D^2+4D}{4}\right]^{-1}\cdot x^2$

$(1+D)^{-1}$

$$y_p = \frac{e^{3x}}{4}\left[1 - \frac{D^2+4D}{4} + \left(\frac{D^2+4D}{4}\right)^2 - \cdots\right] \cdot x^2$$

$D(x^2) = 2x$

$D^2(x^2) = 2$

$$y_p = \frac{e^{3x}}{4}\left[1 + \frac{3}{4}D^2 \mp D\right] \cdot x^2$$

$$y_p = \frac{e^{3x}}{4}\left[x^2 + \frac{3}{4} \cdot 2 \mp 2x\right] = \frac{e^{3x}}{4}\left[x^2 \mp 2x + \frac{3}{2}\right].$$

$\therefore$ The general solution is $y = y_c + y_p$

$$y = (C_1 + C_2 x)e^x + \frac{e^{3x}}{4}(x^2 + 2x - 3/2).$$

(4) $(D^3 - 6D^2 + 11D - 6)y = e^{-2x} + e^{-3x}$

Auxillary equation is : $f(m) = 0$.

$$m^3 - 6m^2 + 11m - 6 = 0$$

$$(m-1)(m^2 - 5m + 6) = 0$$

$$(m-1)(m-2)(m-3) = 0$$

$$m = 1, 2, 3.$$

$$y_c = c_1 e^x + c_2 e^{2x} + c_3 e^{3x}.$$

$$y_p = \frac{1}{D^3 - 6D^2 + 11D - 6}(e^{-2x} + e^{-3x})$$

$$y_p = \frac{e^{-2x}}{D^3 - 6D^2 + 11D - 6} + \frac{e^{-3x}}{D^3 - 6D^2 + 11D - 6}$$

$$y_p = \frac{e^{-2x}}{(-2)^3 - 6(-2)^2 + 11(-2) - 6} + \frac{e^{-3x}}{(-3)^3 - 6(-3)^2 + 11(-3) - 6}$$

$$y_p = \frac{e^{-2x}}{-60} + \frac{e^{-3x}}{-120}$$

$\therefore$ The general solution is

$$y = y_c + y_p$$

$$y = c_1 e^x + c_2 e^{2x} + c_3 e^{3x} - \frac{e^{-2x}}{60} - \frac{e^{-3x}}{120}$$

$$y = c_1 e^x + c_2 e^{2x} + c_3 e^{3x} - \frac{1}{120}\left(2e^{-2x} + e^{-3x}\right).$$

(5) $(D^2+1) \cdot y = \sin x \sin 2x + e^x x^2$

Auxillary equation ; $m^2 + 1 = 0$

$$m = \pm i$$

$$y_c = c_1 \cos x + c_2 \sin x$$

$$y_p = \frac{1}{D^2+1}\left(\sin x \sin 2x + e^x x^2\right).$$

$$y_p = \frac{\sin x \sin 2x}{D^2+1} + \frac{e^x \cdot x^2}{D^2+1}$$

$$= \frac{1}{2} \cdot \frac{\cos x - \cos 3x}{D^2+1} + e^x \cdot \frac{1}{(D+1)^2+1} \cdot x^2$$

$$D^2+2D+2$$

$$= \frac{1}{2}\left(\frac{\cos x}{D^2+1} - \frac{\cos 3x}{D^2+1}\right) + \frac{e^x}{2}\left(1 + \frac{D^2+2D}{2}\right)^{-1} x^2$$

$$= \frac{1}{2}\left(\frac{x}{2}\sin x - \frac{\cos 3x}{-9+1}\right) + \frac{e^x}{2}\left[1 - \frac{D^2+2D}{2} + \left(\frac{D^2+2D}{2}\right)^2\right] \cdot x^2$$

$$= \frac{1}{2}\left(\frac{x}{2}\sin x + \frac{1}{8}\cos 3x\right) + \frac{e^x}{2}\left(1 - \frac{D^2+2D}{2} + D^2\right) \cdot x^2$$

$$= \frac{x}{4}\sin x + \frac{1}{16}\cos 3x + \frac{e^x}{2}\left(1 - D + \frac{D^2}{2}\right) \cdot x^2$$

$$= \frac{x}{4}\sin x + \frac{1}{16}\cos 3x + \frac{e^x}{2}\left(x^2 - 2x + 1\right).$$

$\therefore$ The Solution is $y = y_c + y_p$

$$y = c_1 \cos x + c_2 \sin x + \frac{x}{4}\sin x + \frac{1}{16}\cos 3x + \frac{e^x}{2}(x^2 - 2x + 1).$$

(7) $(D^2-4)y = 2\cos^2 x$

Auxillary equation: $f(m)=0$

$$m^2-4=0$$

$$m=2,-2$$

$$y_c = c_1 e^{2x} + c_2 e^{-2x}$$

$$y_p = \frac{1}{D^2-4}(2\cos^2 x) = \frac{1}{D^2-4}(1+\cos 2x)$$

$$y_p = \frac{e^{0x}}{D^2-4} + \frac{\cos 2x}{D^2-4}$$

$D^2 = -a^2$

$$y_p = \frac{e^{0x}}{-4} + \frac{\cos 2x}{-8}$$

$(D=0)$ $(D^2=-4)$

$\therefore$ General Solution is $y = y_c + y_p$

$$y = c_1 e^{2x} + c_2 e^{-2x} - \frac{1}{4} - \frac{\cos 2x}{8}$$

(8) $(D^2+1)y = \sin x \sin 2x$

Auxillary equation: $f(m) = 0$

$$m^2+1=0$$

$$m = \pm i$$

$$\alpha = 0;\ \beta = 1$$

$$y_c = c_1 \cos x + c_2 \sin x$$

$$y_p = \frac{1}{(D^2+1)} \sin x \sin 2x$$

$$y_p = \frac{1}{2} \frac{\cos x - \cos 3x}{D^2+1}$$

$$y_p = \frac{1}{2}\left\{ \frac{\cos x}{D^2+1} - \frac{\cos 3x}{D^2+1} \right\}$$

put $D^2 = -1$ $\qquad$ $D^2 = -9.$

$D^2+1=0$

$$y_p = \frac{1}{2}\left\{ \frac{x \sin x}{\cancel{1}} - \frac{\cos 3x}{-9+1} \right\}$$

$$y_p = \cancel{\tfrac{1}{2}}\left\{ \frac{x \sin x}{2} + \frac{\cos 3x}{16} \right\}$$

$\therefore$ The general solution: $y = y_c + y_p.$

$$y = c_1 \cos x + c_2 \sin x + \frac{x \sin x}{2} + \frac{\cos 3x}{16}.$$

(9) $(D^2+9).y = \cos 3x + \sin 2x$

Auxillary equation: $f(m) = 0$

$$m^2+9=0$$

$$m = \pm 3i$$

$$\alpha \pm i\beta$$

$$\alpha = 0;\ \beta = 3$$

$$y_c = e^{\alpha x}(c_1 \cos \beta x + c_2 \sin \beta x)$$

$$y_c = e^{0.x}(c_1 \cos 3x + c_2 \sin 3x),$$

$$y_c = c_1 \cos 3x + c_2 \sin 3x$$

$$y_p = \frac{1}{D^2+9}(\cos 3x + \sin 2x)$$

$$y_p = \frac{1}{D^2+9}\cdot \cos 3x + \frac{1}{D^2+9}\sin 2x$$

$$-3^2 \qquad D^2 = -a^2 = -4$$

$$y_p = \frac{x}{6}\sin 3x + \frac{\sin 2x}{-4+9}$$

$$y_p = \frac{x}{6}\sin 3x + \frac{1}{5}\sin 2x$$

$\therefore$ General solution is $y = y_c + y_p$

$$y = c_1 \cos 3x + c_2 \sin 3x + \frac{x}{6}\sin 3x + \frac{1}{5}\sin 2x$$

(10) $(D^2+5D-6)\cdot y = \sin 4x \sin x$

Auxillary equation: $f(m)=0$

$$m^2+5m-6=0$$

$$(m+6)(m-1)=0$$

$$m=-6,1$$

$$y_c = c_1 e^{-6x} + c_2 e^{x}$$

$$y_p = \frac{1}{D^2+5D-6} \sin 4x \sin x$$

$$y_p = \frac{1}{2}\left[\frac{\cos 3x - \cos 5x}{D^2+5D-6}\right] = \frac{1}{2}\left[\frac{\cos 3x}{D^2+5D-6} - \frac{\cos 5x}{D^2+5D-6}\right]$$

$$D^2=-9 \qquad D^2=-25$$

$$= \frac{1}{2}\left[\frac{\cos 3x}{5D-15} - \frac{\cos 5x}{5D-31}\right]$$

$$= \frac{1}{2}\left[\frac{1}{5}\cdot\frac{\cos 3x}{D-3} - \frac{\cos 5x}{5D-31}\right]$$

$$= \frac{1}{2}\left[\frac{1}{5}\cdot\frac{(D+3)\cos 3x}{D^2-9} - \frac{\cos 5x\,(5D+31)}{25D^2-961}\right]$$

$$D^2=-9 \qquad D^2=-25$$

$$\begin{array}{r} -625 \\ -961 \\ \hline -1586 \end{array}$$

$$= \frac{1}{2}\left[\frac{1}{5}\cdot\frac{-3\sin 3x + 3\cos 3x}{-18} - \left(\frac{-25\sin 5x + 31\cos 5x}{-1586}\right)\right]$$

$$= \frac{-\sin 3x + \cos 3x}{-60} + \frac{25\sin 5x - 31\cos 5x}{3172}$$

$$= \frac{\sin 3x - \cos 3x}{60} + \frac{25\sin 5x - 31\cos 5x}{3172}$$

$\therefore$ General Solution is $y = y_c + y_p$

$$y = c_1 e^{-6x} + c_2 e^{x} + \frac{\sin 3x - \cos 3x}{60} + \frac{25\sin 5x - 31\cos 5x}{3172}$$

(11) $(D^2 + D + 1) \cdot y = \sin 2x$

Auxillary equation; $f(m) = 0$

$$m^2 + m + 1 = 0$$

$$m = \frac{-1 \pm i\sqrt{3}}{2}$$

$$y_c = e^{\alpha x}(c_1 \cos \beta x + c_2 \sin \beta x)$$

$$y_c = e^{-\frac{x}{2}}\left(c_1 \cos \frac{\sqrt{3}}{2}x + c_2 \sin \frac{\sqrt{3}}{2}x\right)$$

$$y_p = \frac{1}{(D^2+D+1)} \sin 2x.$$

put $D^2 = -2^2 = -4$.

$$y_p = \frac{1}{(-4+D+1)} \sin 2x$$

$$y_p = \frac{1}{(D-3)} \sin 2x \times \frac{D+3}{D+3}$$

$$y_p = \frac{1}{D^2-9} (D+3) \sin 2x.$$

$-4 \qquad D^2 = -4$

$$y_p = \frac{1}{-4-9} (2\cos 2x + 3\sin 2x),$$

$$y_p = -\frac{1}{13} (2\cos 2x + 3\sin 2x),$$

$\therefore$ The general solution is $y = y_c + y_p$

$$y = e^{-\frac{x}{2}} \left(c_1 \cos \frac{\sqrt{3}}{2}x + c_2 \sin \frac{\sqrt{3}}{2}x\right) - \frac{1}{13}(2\cos 2x + 3\sin 2x).$$

(12) $(D^2+4).y = \sec 2x$

$\downarrow$

Repeated 'Part - A 6^{th} Q (Refer).

(13) $(D^3-4D^2-D+4)y = e^{3x}\cos 2x$.

Auxillary equation: $m^3-4m^2-m+1=0$.

$$(m-1)(m^2-3m-4)=0$$

$$(m-1)(m+1)(m-4)=0$$

$$m=1,-1,4$$

$$y_c = c_1e^x + c_2e^{-x} + c_3e^{4x},$$

$$y_p = \frac{1}{D^3-4D^2-D+4}\, e^{3x}\cos 2x$$

$$y_p = e^{3x}\frac{1}{(D+3)^3-4(D+3)^2-(D+3)+4}\cos 2x$$

$$= e^{3x}\frac{1}{D^3+5D^2+2D-8}\cos 2x$$

$$D^2=-2^2$$

$$= e^{3x}\frac{1}{-4D+5(-4)+2D-8}\cos 2x$$

$$= e^{3x}\frac{1}{-2D-28}\cos 2x = -\frac{1}{2}e^{3x}\frac{1}{D+14}\cos 2x$$

$$= -\frac{e^{3x}}{2} \frac{D-14}{(D-14)(D+14)} \cos 2x$$

$$= -\frac{e^{3x}}{2} \frac{D-14}{D^2-196} \cos 2x$$

$D^2 = -4$

$$\Rightarrow \frac{e^{3x}}{400}(D-14)\cos 2x = \frac{e^{3x}}{400}(-2\sin 2x - 14\cos 2x)$$

$$= -\frac{2e^{3x}}{400}(\sin 2x + 7\cos 2x)$$

$$y_p = -\frac{e^{3x}}{200}(\sin 2x + 7\cos 2x)$$

$\therefore$ The general solution is $y = y_c + y_p$

$$y = c_1 e^{x} + c_2 e^{-x} + c_3 e^{4x} - \frac{e^{3x}}{200}(\sin 2x + 7\cos 2x),$$

(14) $(D^2+9) \cdot y = \cos 3x$

Auxillary equation $\rightarrow m^2 + 9 = 0 \rightarrow m = \pm 3i$

$\alpha = 0;\ \beta = 3$

$$y_c = e^{0x}(c_1 \cos 3x + c_2 \sin 3x)$$

$e^{0x} = 1$

$$y_p = \frac{1}{D^2+9} \cos 3x = \frac{x}{2(3)} \sin 3x = \frac{x}{6} \sin 3x,$$

$D = 3$

$\therefore$ The general solution is $y = y_c + y_p$

$$y = c_1 \cos 3x + c_2 \sin 3x + x/6 \sin 3x,$$

(15) $(D^3 - 7D^2 + 14D - 8)y = e^x \cos 2x$

Auxillary equation: $f(m) = 0$

$$m^3 - 7m^2 + 14m - 8 = 0$$

$$(m-1)(m^2 - 6m + 8) = 0$$

$$(m-1)(m-2)(m-4) = 0$$

$$m = 1, 2, 4.$$

$$y_c = c_1 e^{x} + c_2 e^{2x} + c_3 e^{4x}$$

$$y_p = \frac{1}{D^3 - 7D^2 + 14D - 8} \cdot e^x \cos 2x$$

$$y_p = e^x \cdot \frac{1}{(D+1)^3 - 7(D+1)^2 + 14(D+1) - 8} \cos 2x.$$

$$y_p = e^x \frac{1}{D^3 - 4D^2 + 3D} \cos 2x.$$

$D^2 = -2^2$

$$y_p = e^x \frac{1}{-4D + 16 + 3D} \cos 2x.$$

$$y_p = e^x \frac{1}{16 - D} \cos 2x$$

$$y_p = e^x \frac{16 + D}{256 - \underset{-4}{D^2}} \cos 2x = e^x \cdot \frac{16 + D}{260} \cos 2x$$

$$y_p = \frac{e^x}{260} (16 \cos 2x - 2 \sin 2x).$$

$\therefore$ The general Solution is $y = y_c + y_p$

$$y = c_1 e^{x} + c_2 e^{2x} + c_3 e^{4x} + \frac{e^{x}}{260}(16\cos 2x - 2\sin 2x)$$

$$y = c_1 e^{x} + c_2 e^{2x} + c_3 e^{4x} + \frac{e^{x}}{130}(8\cos 2x - \sin 2x)$$

(18) $(D^3 - 4D^2 - D + 4)y = e^{3x}\cos 2x$ — Standard Text book question.

No change

Repeat - 13^{th} Question (part B).

(17) $(D^3 + 4D).y = \sin 2x$

Auxillary equation: $m^3 + 4m = 0$

$$m(m^2 + 4) = 0$$

$$m = 0;\ m = \pm 2i$$

$$y_c = c_1 + c_2\cos 2x + c_3 \sin 2x$$

$$y_p = \frac{1}{D^3 + 4D}\sin 2x = \frac{1}{D(D^2+4)}\sin 2x$$

$$\Rightarrow \frac{-\cos 2x}{2(D^2+4)} = \frac{-1}{2}\left(\frac{x\sin 2x}{4}\right)$$

$$= -\frac{x\sin 2x}{8}$$

$\therefore$ The general solution is $y = y_c + y_p$

$$y = c_1 + c_2\cos 2x + c_3\sin 2x - \frac{x\sin 2x}{8}$$

(18) $(D^2+4D+4).y = 3\sin x + 4\cos x.$

$\underline{y_c}$: $f(m) = 0$

$m^2+4m+4=0$

$m^2+2m+2m+4=0$

$m=-2,-2$

$y_c = (c_1+c_2x)e^{-2x}$

$\underline{y_p}$: $y_p = \frac{1}{f(D)} . Q(x)$

$y_p = \frac{1}{(D+2)(D+2)} 3\sin x + 4\cos x$

$y_p = 3. \frac{\sin x}{D^2+4D+4} + 4 \frac{\cos x}{D^2+4D+4}$

$D^2=-1^2$ $(-1-7)$

$y_p = 3. \frac{\sin x}{-1+4D+4} + 4 \frac{\cos x}{-1+4D+4}$

$y_p = 3 \frac{\sin x}{4D+3} + 4 \frac{\cos x}{4D+3}$

$y_p = 3 \frac{\sin x(4D-3)}{16D^2-9} + 4 \frac{\cos x(4D-3)}{16D^2-9}$

$D^2=-1^2$

$$y_p = \frac{3(40\sin x - 3\sin x)}{-16-9} + 4\frac{(40\cos x - 3\sin x)}{-25}$$

$$y_p = \frac{3(4\cos x - 3\sin x)}{-25} + \frac{4(+4\sin x - 3\cos x)}{-25}.$$

$$y_p = \frac{-1}{25}\left[12\cos x - 9\sin x + 16\sin x + 12\cos x\right]$$

$$y_p = \frac{1}{25}\left[24\cos x + 7\sin x\right]$$

$\therefore$ The general solution is $y = y_c + y_p$

$$y = (c_1 + c_2 x)e^{-2x} + \frac{1}{25}(24\cos x + 7\sin x),$$

20) $(D^3 - 1)y = e^x + \sin 3x + 2$

$\underline{y_c}$: $m^3 - 1 = 0$

$(m-1)(m^2 + m + 1) = 0$

$m = 1, \ \frac{-1 \pm \sqrt{3}i}{2}$

$$y_c = c_1 e^x + e^{-x/2}\left[c_2 \cos\frac{\sqrt{3}x}{2} + c_3 \sin\frac{\sqrt{3}x}{2}\right]$$

$\underline{y_p}$: $\frac{1}{f(D)} \cdot Q(x)$

$\frac{1}{D^3 - 1}\ e^x + \sin 3x + 2$

$$y_p = \frac{e^x}{D^3-1} + \frac{\sin 3x}{D^3-1} + \frac{2}{D^3-1}$$

$\downarrow (D-1)(D^2+D+1)$ $\quad \downarrow$ $\quad \downarrow$

$D \to a \to 0.$

$f(D) \to f(D) = \phi(D)$ $\quad D^2 = -9$

$\phi(D) = \phi(a) = 1$

$$y_p = \frac{x}{1!} \cdot \frac{e^x}{(1^2+1+1)} + \frac{\sin 3x}{9D+1} - 2$$

$$y_p = \frac{x e^x}{3} - \frac{\sin 3x}{9D+1} \times \frac{9D-1}{9D-1} - 2$$

$$y_p = \frac{x e^x}{3} - \frac{(9D-1)\sin 3x}{81D^2-1} - 2$$

$\downarrow$

$D^2 = -9$

$$y_p = \frac{x e^x}{3} - \frac{(9D-1)\sin 3x}{-729-1} - 2$$

$$y_p = \frac{x e^x}{3} + \frac{(9D \sin 3x - \sin 3x)}{730} - 2$$

$$y_p = \frac{x e^x}{3} + \frac{27\cos 3x - \sin 3x}{730} - 2$$

$\therefore$ The general solution is $y = y_c + y_p$

$$y = c_1 e^{-x} + e^{-\frac{x}{2}}\left[c_2 \cos \frac{\sqrt{3}x}{2} + c_3 \sin \frac{\sqrt{3}x}{2}\right] + \frac{x e^x}{3} + \frac{1}{730}(27\cos 3x - \sin 3x) - 2.$$

(19) $(D^2+1)y = \sin x$

<u>Method</u> <u>of</u> <u>variation</u> <u>of</u> Para<u>meters</u>:

$$\frac{d^2y}{dx^2} + 1(y) = \sin x$$

<u>form</u>: $\frac{d^2y}{dx^2} + P(x) \cdot \frac{dy}{dx} + Q(x) \cdot y = R(x)$

(1) $P(x) = 0$; $Q(x) = 1$; $R(x) = \sin x$

(2) Auxillary equation : $f(m) = 0$

$$m^2 + 1 = 0$$

$$m = \pm i.$$

$$y_c = c_1 \cos x + c_2 \sin x$$

$$y_c = c_1 u(x) + c_2 v(x).$$

(3) Let; $y_p = Au + Bv$

$u(x) = \cos x$ $\qquad$ $v(x) = \sin x$

$u'(x) = -\sin x$ $\qquad$ $v'(x) = \cos x$

$uv' - u'v = \cos x \cos x + \sin x \cdot \sin x$

$uv' - u'v = 1.$

$$A = -\int \frac{vR}{uv' - u'v}\, dx = -\int \frac{\sin x \cdot \sin x}{1}\, dx$$

$$A = -\int \sin^2 x \, dx = -\frac{1}{2}\int (1-\cos 2x)\, dx$$

$$= -\frac{1}{2}\left(x - \frac{\sin 2x}{2}\right)$$

$$= -\frac{1}{2}\left(\frac{2x-\sin 2x}{2}\right)$$

$$= -\frac{1}{4}(2x - \sin 2x).$$

$$B = \int \frac{u \cdot R}{uv' - u'v}\, dx = \int \frac{\cos x \cdot \sin x}{1}\, dx$$

$$= \frac{1}{2}\int \sin 2x \, dx$$

$$\Rightarrow \frac{1}{2}\left(\frac{-\cos 2x}{2}\right) = \frac{-1}{4}\cos 2x.$$

$$y_p = \frac{-1}{4}(2x - \sin 2x)\cos x + \left(\frac{-1}{4}\cos 2x\right)\cdot \sin x$$

$\therefore$ The general solution is. $y = y_c + y_p$

$$y = c_1 \cos x + c_2 \sin x - \frac{\cos x}{4}(2x - \sin 2x) - \frac{\sin x}{4}\cos 2x.$$

(2) $(D^2+4)\cdot y = 96x^2 + \sin 2x - k$

$\underline{y_c}$: $f(m) = 0$

$$m^2 + 4 = 0$$

$$m = \pm 2i$$

$$y_c = e^0 (c_1 \cos 2x + c_2 \sin 2x)$$

$$y_c = c_1 \cos 2x + c_2 \sin 2x)$$

$\underline{y_p}$: ~~Q(x)~~ $y_p = \frac{1}{f(D)} \cdot Q(x)$

$$y_p = \frac{1}{D^2+4} 96x^2 + \sin 2x - k$$

$$y_p = 96 \cdot \frac{x^2}{D^2+4} + \frac{\sin 2x}{D^2 + 4} - \frac{k}{D^2+4}$$

$f(8)$ $\quad$ $f(\tau)$ $\quad$ $f(1)$

$$y_p = \frac{96}{4}\left[1 + \frac{D^2}{4}\right]^{-1} x^2 + \frac{-x}{2(2)} \sin 2a - \frac{k}{0^2+4}$$

$$y_p = 24\left[1 - \frac{D^2}{4} + \cancel{\frac{D^4}{4}}\right] x^2 - \frac{x}{4} \sin 2a - k/4$$

$$y_p = 24\left[x^2 - \frac{\cancel{2}}{\cancel{4}_2}\right] - \frac{x}{4} \sin 2a - k/4$$

$$y_p = 24x^2 - 12 - \frac{x}{4} \sin 2a - \frac{k}{4}$$

$\therefore$ The general Solution is $y = y_c + y_p$.

$$y = c_1 \cos 2x + c_2 \sin 2x + 24x^2 - 12 - \frac{x}{4} \sin 2x.$$

(6) $(D^3 + 1) \cdot y = 3 + 5e^x$

$\underline{y_c}$: $f(m) = 0$

$$m^3 + 1 = 0$$

$$m = -1, \frac{1 \pm \sqrt{3} i}{2}$$

$$y_c = c_1 e^{-x} + e^{\frac{x}{2}} \left[c_2 \cos \frac{\sqrt{3}x}{2} + c_3 \sin \frac{\sqrt{3}x}{2} \right]$$

$\underline{y_p}$: $y_p = \frac{1}{f(D)} \cdot Q(x)$

$$= \frac{1}{D^3 + 1} \, 3 + 5e^x$$

$$= \frac{3}{D^3 + 1} + 5 \frac{e^x}{D^3 + 1}$$

$D \to 0$ $\qquad D \to f(a) = 1$

$$= \frac{3}{0 + 1} + 5 \frac{e^x}{1 + 1}$$

$$= 3 + \frac{5e^x}{2}$$

$\therefore$ The general solution is

$$y = y_c + y_p$$

$$y = c_1 e^{-x} + e^{x/2}\left(c_2 \cos \frac{\sqrt{3}x}{2} + c_3 \sin \frac{\sqrt{3}x}{2}\right) + 3 + 5\frac{e^x}{2}$$

Verified !

Standard Textbook questions are included for practice purpose.

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