Assignment Part A

① $x+2y-3z=a$
$3x-y+2z=b$
$x-5y+8z=c$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & a \\ 3 & -1 & 2 & b \\ 1 & -5 & 8 & c \end{array}\right]$$

$\Rightarrow -a+x+2y-3z=0$
$-b+3x-y+2z=0$
$-c+x-5y+8z=0$

$$\left[\begin{array}{cccccc|c} -1 & 0 & 0 & 1 & 2 & -3 & 0 \\ 0 & -1 & 0 & 3 & -1 & 2 & 0 \\ 0 & 0 & -1 & 1 & -5 & 8 & 0 \end{array}\right]$$

Unique soln $\rho(A)=\rho(A|B)=n$
$n=3$

$\text{Det } A=0$

$$\text{Det } A_1 = \begin{bmatrix} a & 2 & -3 \\ b & -1 & 2 \\ c & -5 & 8 \end{bmatrix}$$

$a(-8+10)-2(8b-2c)$
$-3(-5b+c)$

$= -8a+10a-16b+4c$
$+15b-3c$

$= 2a-b+c$

$$\text{Det } A_2 = \begin{bmatrix} 1 & a & -3 \\ 3 & b & 2 \\ 1 & c & 8 \end{bmatrix}$$

$1(8b-2c)-a(24-2)-3(3c-b)$

$= 8b-2c-22a+3b-9c$

$= -22a+11b-11c$

$= 2a-b+c$

$$\text{Det } A_3 = \begin{bmatrix} 1 & 2 & a \\ 3 & -1 & b \\ 1 & -5 & c \end{bmatrix}$$

$\Rightarrow 1(-c+5b)-2(3c-b)$
$+a(-15+1)$

$-c+5b-6c+2b$
$-14a$

$= -14a+7b-7c$

$= 2a-b+c$

Condition should be $2a-b+c$.

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② $x+y+z=6$

$x+2y+3z=10$

$x+2y+az=b$

Sol ⇒ The given system in matrix form can be written as $AX=B$

$$\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & 3\\ 1 & 2 & a\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}6\\ 10\\ b\end{bmatrix}$$

The augumented matrix of the system

$$[A:B]=\begin{bmatrix}1 & 1 & 1 & 6\\ 1 & 2 & 3 & 10\\ 1 & 2 & a & b\end{bmatrix}$$

Reduce into echleon form

$R_2 \to R_2-R_1$
$R_3 \to R_3-R_1$
$$\begin{bmatrix}1 & 1 & 1 & 6\\ 0 & 1 & 2 & 4\\ 0 & 1 & a-1 & b-6\end{bmatrix}$$

$R_3 \to R_3-R_2$
$$\begin{bmatrix}1 & 1 & 1 & 6\\ 0 & 1 & 2 & 4\\ 0 & 0 & a-3 & b-10\end{bmatrix}$$

This matrix is in echleon form

(i) No Solution

The system will have no solution if $\rho(A) \neq \rho(A|B)$

$a-3=0 \Rightarrow a=3,\ b-10\neq 0 \Rightarrow b\neq 10$

(ii) unique solution: The system will have unique sol if $\rho(A)=\rho(A|B)=n$

$a-3\neq 0 \Rightarrow a\neq 3$, b may be have any value

(iii) Infinitely so) The system will have infinitely so if $\rho(A)=\rho(A|B)<n$ $a-3=0,\ b-10=0 \Rightarrow a=3, b=10$

(3) Calculate the rank

$$A = \begin{bmatrix} 2 & -4 & 3 & -1 & 0 \\ 1 & -2 & -1 & -4 & 2 \\ 0 & 1 & -1 & 3 & 1 \\ 4 & -7 & 4 & -4 & 5 \end{bmatrix}$$

$R_1 \leftrightarrow R_2$

$$\begin{bmatrix} 1 & -2 & -1 & -4 & 2 \\ 2 & -4 & 3 & -1 & 0 \\ 0 & 1 & -1 & 3 & 1 \\ 4 & -7 & 4 & -4 & 5 \end{bmatrix}$$

$R_2 \leftrightarrow R_3$

$$\begin{bmatrix} 1 & -2 & -1 & -4 & 2 \\ 0 & 1 & -1 & 3 & 1 \\ 2 & -4 & 3 & -1 & 0 \\ 4 & -7 & 4 & -4 & 5 \end{bmatrix}$$

$R_3 \to R_3 - 2R_1$, $R_4 \to R_4 - 4R_1$

$$\begin{bmatrix} 1 & -2 & -1 & -4 & 2 \\ 0 & 1 & -1 & 3 & 1 \\ 0 & 0 & 5 & 7 & -4 \\ 0 & 1 & 8 & 12 & -3 \end{bmatrix}$$

$R_4 \to R_4 - R_2$

$$\begin{bmatrix} 1 & -2 & -1 & -4 & 2 \\ 0 & 1 & -1 & 3 & 1 \\ 0 & 0 & 5 & 7 & -4 \\ 0 & 0 & 9 & 9 & -4 \end{bmatrix}$$

$R_4 \to 5R_4 - 9R_3$

$R_4 \to 5R_4 - 9R_3$

$$\begin{bmatrix} 1 & -2 & -1 & -4 & 2 \\ 0 & 1 & -1 & 3 & 1 \\ 0 & 0 & 5 & 7 & -4 \\ 0 & 0 & 9 & 9 & -4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & -1 & -4 & 2 \\ 0 & 1 & -1 & 3 & 1 \\ 0 & 0 & 5 & 7 & -4 \\ 0 & 0 & 0 & -18 & 16 \end{bmatrix}$$

Rank of $A = 4$

(4)

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$$

$|A| = +1(-12-12) - 1(-4-6) + 3(-4+6)$

$-24+10+6$

$-24+16 = -8 \neq 0$

Inverse of a matrix by Gauss-Jordan

$AB = I = BA \Rightarrow B = A^{-1}$

consider $[A|I]$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 1 & 3 & -3 & 0 & 1 & 0 \\ -2 & -4 & -4 & 0 & 0 & 1 \end{array}\right]$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 + 2R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 0 & 2 & -6 & -1 & 1 & 0 \\ 0 & -2 & 2 & 2 & 0 & 1 \end{array}\right]$$

$R_3 \to R_3 + R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 0 & 2 & -6 & -1 & 1 & 0 \\ 0 & 0 & -4 & 1 & 1 & 1 \end{array}\right]$$

$R_3 \to R_3 / -4$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 0 & 2 & -6 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1/4 & -1/4 & -1/4 \end{array}\right]$$

$R_2 \to R_2 / 2$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -1/2 & 1/2 & 0 \\ 0 & 0 & 1 & -1/4 & -1/4 & -1/4 \end{array}\right]$$

$R_1 \to R_1 - R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 6 & \frac{3}{2} & -1/2 & 0 \\ 0 & 1 & -3 & -1/2 & 1/2 & 0 \\ 0 & 0 & 1 & -1/4 & -1/4 & -1/4 \end{array}\right]$$

$R_1 \to R_1 - 6R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 1 & 3/2 \\ 0 & 1 & -3 & -1/2 & -1/2 & 0 \\ 0 & 0 & 1 & -1/4 & -1/4 & -1/4 \end{array}\right]$$

$R_2 \to R_2 + 3R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 1 & 3/2 \\ 0 & 1 & 0 & -5/4 & -1/4 & -3/4 \\ 0 & 0 & 1 & -1/4 & -1/4 & -1/4 \end{array}\right]$$

Solution

$$\begin{bmatrix} 3 & 1 & 3/2 \\ -5/4 & -1/4 & -3/4 \\ -1/4 & -1/4 & -1/4 \end{bmatrix}$$

⑤ $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$

Rank by using echelon form

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 3R_1$

$R_4 - R_4 - 6R_1$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$R_2 \leftrightarrow R_3$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$R_4 \to R_4 - R_2$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & -3 & 3 \end{bmatrix}$$

$R_4 \to R_4 - R_3$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

⑥ Normal form

$$\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

$R_1 \leftrightarrow R_2$

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

$R_3 \to R_3 - 3R_1$

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

$R_4 \to R_4 - R_1$

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \end{bmatrix}$$

~~$R_3 \to$~~ $C_3 \to C_3 - C_4$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & -2 & -1 \\ 0 & 1 & -2 & -1 \\ 0 & 1 & -2 & -1 \end{bmatrix}$$

$C_4 \to C_4 - C_1$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & -1 \\ 0 & 1 & -2 & -1 \\ 0 & 1 & -2 & -1 \end{bmatrix}$$

(A)

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & -2 & -1 \end{bmatrix}$$

$R_4 \to R_4 - R_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_3 \to C_3 - 2C_4$

$= -2 - 2(-1)$

$= -2 + 2 = 0$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_4 \to C_4 + C_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix}$$

which is in normal form

$\boxed{\text{Rank of } A = 2}$

(7) $x-2y+z-w=0$

$x+y-2z+3w=0$

$4x+y-5z+8w=0$

$5x-7y+2z-w=0$

$\Rightarrow \begin{bmatrix} 1 & -2 & 1 & -1 & 0 \\ 1 & 1 & -2 & 3 & 0 \\ 4 & 1 & -5 & 8 & 0 \\ 5 & -7 & 2 & -1 & 0 \end{bmatrix}$

$\Rightarrow R_2 \to R_2 - R_1$

$\begin{bmatrix} 1 & -2 & 1 & -1 & 0 \\ 0 & 3 & -3 & 4 & 0 \\ 4 & 1 & -5 & 8 & 0 \\ 5 & -7 & 2 & -1 & 0 \end{bmatrix}$

$R_3 \to R_3 - 4R_1$

$R_4 \to R_4 - 5R_1$

$\begin{bmatrix} 1 & -2 & 1 & -1 & 0 \\ 0 & 3 & -3 & 4 & 0 \\ 0 & 9 & -9 & 12 & 0 \\ 0 & 3 & -3 & 4 & 0 \end{bmatrix}$

$R_3 \to R_3 - 3R_2$

$\begin{bmatrix} 1 & -2 & 1 & -1 & 0 \\ 0 & 3 & -3 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & -3 & 4 & 0 \end{bmatrix}$

$R_4 \to R_4 - R_2$

$\begin{bmatrix} 1 & -2 & 1 & -1 & 0 \\ 0 & 3 & -3 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$\Rightarrow$ There

$\rho(A) = 2 < 4$

Therefore infinitely many solution non-trivial.

let $w = k_1$

$z = k_2$

$3y - 3z + 4w = 0$

$3y - 3k_2 + 4k_1 = 0$

$3y = 3k_2 - 4k_1$

$\boxed{y = \frac{3k_2 - 4k_1}{3}}$

$x - 2y + z - w = 0$

$x - 2\left(\frac{3k_2 - 4k_1}{3}\right) + k_2 - k_1$

$x - \frac{6k_2 + 8k_1 + k_2 - k_1}{3} =$

$3x - 6k_2 + 8k_1 + 3k_2 - 3k$

$3x - 3k_2 + 5k_1 = 0$

$3x = 3k_2 - 5k_1$

$\boxed{x = \frac{3k_2 - 5k_1}{3}}$

8) $= \begin{bmatrix} 1 & -2 & -3 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$$\left[\begin{array}{ccc|ccc} 1 & -2 & -3 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 3 & 0 & 0 & 1 \end{array}\right]$$

$R_1 \to R_1 + R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -3 & 1 & 1 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 3 & 0 & 0 & 1 \end{array}\right]$$

$R_1 \to R_1 + R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 3 & 0 & 0 & 1 \end{array}\right]$$

$R_2 \to R_2/2$

$R_3 \to R_3/3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1/3 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/3 \end{bmatrix}$$

Matrices

(1) $A = \begin{bmatrix} 4 & 3 & 2 & 1 \\ 5 & 1 & -1 & 2 \\ 0 & 1 & 2 & 3 \\ 1 & -1 & 3 & -2 \end{bmatrix}$

$R_4 \leftrightarrow R_1$

$$\begin{bmatrix} 1 & -1 & 3 & -2 \\ 5 & 1 & -1 & 2 \\ 0 & 1 & 2 & 3 \\ 4 & 3 & 2 & 1 \end{bmatrix}$$

$R_2 \to R_2 - 5R_1$

$R_3 \to R_3 - 4R_1$

$$\begin{bmatrix} 1 & -1 & 3 & -2 \\ 0 & 6 & -16 & 12 \\ 0 & 1 & 2 & 3 \\ 0 & 7 & -10 & 9 \end{bmatrix}$$

$C_2 + C_1,\ C_3 - 3C_1,\ C_4 + 2C_1$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 6 & -16 & 12 \\ 0 & 1 & 2 & 3 \\ 0 & 7 & -10 & 9 \end{bmatrix}$$

$R_3 \leftrightarrow R_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 \\ 0 & 3 & -8 & 6 \\ 0 & 7 & -10 & 9 \end{bmatrix}$$

$R_3 - 3R_2$

$R_4 - 7R_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & -4 & -3 \\ 0 & 0 & -24 & -30 \end{bmatrix}$$

$C_3 - 2C_2$

$C_4 - 3C_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -4 & -3 \\ 0 & 0 & -24 & 12 \end{bmatrix}$$

$C_4 - C_3,\ C_3 / -4$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 6 & 4 \end{bmatrix}$$

~~$C_4 - C_3$~~

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 6 & -2 \end{bmatrix}$$

$R_4 \mid -2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -3 & 1 \end{bmatrix}$$

$R_4 = R_4 + 3R_3$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rank of Given = 4

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\boxed{\text{Rank} = 3}$

(b) $x+y+z=6$

$x+2y+3z=14$

$x+4y+7z=30$

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 14 \\ 1 & 4 & 7 & 30 \end{bmatrix}$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 3 & 6 & 24 \end{bmatrix}$$

$R_3 \to R_3 - 3R_2$

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = 2 < 3$ (no of unknowns) which is consistent

which is Infinitely many solutions.

no of parameters $n - r = 3 - 2 = 1$

$\boxed{z = k}$

$y + 2z = 8$

$y + 2k = 8$

$\boxed{y = 8 - 2k}$

$x + y + z = 6$

$x + 8 - 2k + k = 6$

$x - k = 6 - 8$

$x - k = -2$

$x = -2 + k$

$\boxed{x = k - 2}$

$$\boxed{\begin{aligned} x &= k - 2 \\ y &= 8 - 2k \\ z &= k \end{aligned}}$$

Part-B

① $x+2y+z=8$
$2x+y+3z=13$
$3x+4y-az=b$

$$\begin{bmatrix}1&2&1\\2&1&3\\3&4&-a\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}8\\13\\b\end{bmatrix}$$

$$[A:B]=\begin{bmatrix}1&2&1&8\\2&1&3&13\\3&4&-a&b\end{bmatrix}$$

$R_2\to R_2-2R_1$
$R_3\to R_3-3R_1$

$$\begin{bmatrix}1&2&1&8\\0&-3&1&-3\\0&-2&-a-3&b-24\end{bmatrix}$$

$R_3\to 3R_3-2R_2$

$$\begin{bmatrix}1&2&1&8\\0&-3&1&-3\\0&0&-3a-11&3b-66\end{bmatrix}$$

(i) No solution
The system will have no solution
if $\rho(A)\neq\rho(A:B)$

$-3a-11=0$
$-(3a+11)=0$
$-3a=11$
$\boxed{a=\frac{-11}{3}}$

$3b-66\neq0$
$3b\neq66$
$\boxed{b\neq\frac{66}{3}}$

(ii) If the system will have unique solution
The system will have
if $\rho(A)=\rho(A:B)=n$
$-3a-11\neq0$, $3b$ may be have any value

(iii) Infinitely many
if $\rho(A)=\rho(A:B)<n$
$-3a-11=0$, $3b-66=0$
$-3a=11$ $3b=66$
$a=-\frac{11}{3}$ $b=\frac{66}{3}$ $\cancel{22}$

②
x
3
4
R_2
R_3
R

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② ~~$x+2y+z=8$~~
= ~~$2x+y+3$~~

$x-y+2z+t=2$
$3x+2y+t=1$
$4x+y+2z+2t=3$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 3 & 2 & 0 & 1 & 1 \\ 4 & 1 & 2 & 2 & 3 \end{array}\right]$$

$R_2 \to R_2 - 3R_1$
$R_4 \to R_4 - 4R_1$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 5 & -6 & -2 & -5 \\ 0 & 5 & -6 & -2 & -5 \end{array}\right]$$

$R_3 \to R_3 - R_2$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 5 & -6 & -2 & -5 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$\rho(A) = \rho(A|B) < n$

~~$\boxed{t = t}$~~, ~~$\boxed{t = k}$~~

$t = k_1 \quad z = k_2$

$5y - 6z - 2t = -5$
$5y - 6k_2 - 2k_1 = -5$
$5y = 6k_2 + 2k_1 - 5$

$$\boxed{y = \frac{6k_2 + 2k_1 - 5}{5}}$$

$x - y + 2z + t = 2$
$x - \left(\frac{6k_2 + 2k_1 - 5}{5}\right) + 2k_2 + k_1 = 2$

$5x - 6k_2 + 2k_1 - 5 + 10k_2 + 5k_1 = 10$

$5x + 4k_2 + 7k_1 = 15$
$5x = 15 - 4k_2 - 7k_1$

$$\boxed{x = \frac{15 - 4k_2 - 7k_1}{5}}$$

✗

$5x - 6k_2 - 2k_1 + 5 + 10k_2 + 5k_1 = 10$

$5x + 4k_2 + 3k_1 = 5$
$5x = -4k_2 - 3k_1 + 5$
$x = \frac{-4k_2 - 3k_1 + 5}{5}$ ✓

(3)

$$= \begin{bmatrix} 1 & 0 & -4 & 5 \\ 2 & -1 & 3 & 0 \\ 8 & 1 & 0 & 7 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 8R_1$

$$\begin{bmatrix} 1 & 0 & -4 & 5 \\ 0 & -1 & 11 & -10 \\ 0 & 1 & 32 & -33 \end{bmatrix}$$

$R_2 \leftrightarrow R_3$

$$\begin{bmatrix} 1 & 0 & -4 & 5 \\ 0 & -1 & 11 & -10 \\ 0 & 1 & 32 & -33 \end{bmatrix}$$

$\Rightarrow C_3 \to C_3 + 4C_1$

$C_4 \to C_4 + 5C_1$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 11 & -10 \\ 0 & 1 & 32 & -33 \end{bmatrix}$$

$R_2 \leftrightarrow R_3$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 32 & -33 \\ 0 & -1 & 11 & -10 \end{bmatrix}$$

$R_3 \to R_3 + R_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 32 & -33 \\ 0 & 0 & 43 & -43 \end{bmatrix}$$

$C_3 \to C_3 - 32C_2$

$C_4 \to C_4 + 33C_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 43 & -43 \end{bmatrix}$$

$R_3 / 43$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$\boxed{\text{Rank} = 3}$

(4)
$$\begin{bmatrix} 1 & -2 & 3 & 1 \\ 2 & 1 & -1 & 2 \\ 6 & -2 & a & b \end{bmatrix}$$

Rank = 3 -

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 6R_1$

$$\begin{bmatrix} 1 & -2 & 3 & 1 \\ 0 & 5 & -7 & 0 \\ 0 & 10 & a-18 & b-6 \end{bmatrix}$$

$R_3 \to R_3 - 2R_2$

$$\begin{bmatrix} 1 & -2 & 3 & 1 \\ 0 & 5 & -7 & 0 \\ 0 & 0 & a-4 & b-6 \end{bmatrix}$$

nosolution

$a-4 \neq 0 \quad b \in R$

$\boxed{a \neq 4}$

(5) $x+y+z=3$

$x+2y+2z=6$

$x+9y+az=b$

$$\begin{bmatrix} 1 & 1 & 1 & 3 \\ 1 & 2 & 2 & 6 \\ 1 & 9 & a & b \end{bmatrix}$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & 1 & 1 & 3 \\ 0 & 8 & a-1 & b-3 \end{bmatrix}$$

$R_3 \to R_3 - 8R_2$

$$\begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & a-9 & b-27 \end{bmatrix}$$

(i) Nosolution

$\rho(A) \neq \rho(A|B)$

$a-9=0, b-27 \neq 0$

$a=9, b \neq 27$

(ii) unique solution

$\rho(A) = \rho(A|B) = n$

$a-9 \neq 0 \quad b \in R$

$a \neq 9$ b may be have any value

(iii) Infinitely many

$\rho(A) = \rho(A|B) < n$

$a-9=0, b-27=0$

$\boxed{a=9, b=27}$

⑥ $\Rightarrow \begin{bmatrix} 4 & 4 & -3 & 1 \\ 1 & 1 & -1 & 0 \\ K & 2 & 2 & 2 \\ 9 & 9 & K & 3 \end{bmatrix}$

Rank = 3
if $k+6=0$
$\boxed{k=-6}$

$R_1 \leftrightarrow R_2$

$\begin{bmatrix} 1 & 1 & -1 & 0 \\ 4 & 4 & -3 & 1 \\ K & 2 & 2 & 2 \\ 9 & 9 & K & 3 \end{bmatrix}$

Det = 0

$R_2 \to R_2 - 4R_1$
$R_4 \to R_4 - 9R_1$
$R_3 \to R_3 - KR_1$

$\begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & (2-k)(2+k) & 2 \\ 0 & 0 & k+9 & 3 \end{bmatrix}$

$R_2 \leftrightarrow R_3$

$\begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & 0 & (2-k)(2+k) & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & k+9 & 3 \end{bmatrix}$

$R_4 \to 3R_3$

$\begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & (2-k)(2+k) & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & k+6 & 0 \end{bmatrix}$

Part B

7) $\begin{bmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix}$

find rank and reduce into Normal form.

$R_1 \leftrightarrow R_3$

$$\begin{bmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 & 3 \\ 4 & 2 & 0 & 2 \\ 2 & -2 & 0 & 6 \\ 1 & -2 & 1 & 2 \end{bmatrix}$$

$C_2 \to C_2 + C_1$

$C_4 \to C_4 - 3C_1$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 4 & 6 & 0 & -10 \\ 2 & 0 & 0 & 0 \\ 1 & -1 & 1 & -1 \end{bmatrix}$$

$R_2 \to R_2 - 2R_3$

$R_4 \to R_4 - R_1$

$R_4 \leftrightarrow R_3$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 6 & 0 & -10 \\ 0 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2 \leftrightarrow R_3$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & -1 \\ 0 & 6 & 0 & -10 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_4 \to C_4 + C_3$

$C_3 \to C_3 + C_2$

$R_3 \to R_3 - 6R_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -10 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2 \to R_2 / -1$

$R_3 \to R_3 / -10$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_3 \leftrightarrow C_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} I_3 & 0 \\ 0 & 0 \end{bmatrix}$$

which is in normal form

$\boxed{\text{Rank of } A = 3}$

(8) $x+2y+3z=\lambda x$

$3x+y+2z=\lambda y$

$2x+3y+z=\lambda z$

have a nontrivial solution.

$\lambda x+2y-3z=0$

$3x+y-\lambda y+2z=0$

$2x+3y+z-\lambda z=0$

$x(1-\lambda)+2y+3z=0$

$3x+y(1-\lambda)+2z=0$

$2x+3y+z(1-\lambda)=0$

nontrivial solution

$$\begin{vmatrix} 1-\lambda & 2 & 3 \\ 3 & 1-\lambda & 2 \\ 2 & 3 & 1-\lambda \end{vmatrix}=0$$

$R_1 \to R_1+R_2+R_3$

$$\begin{vmatrix} 6-\lambda & 6-\lambda & 6-\lambda \\ 3 & 1-\lambda & 2 \\ 2 & 3 & 1-\lambda \end{vmatrix}=0$$

$$(6-\lambda)\begin{vmatrix} 1 & 1 & 1 \\ 3 & 1-\lambda & 2 \\ 2 & 3 & 1-\lambda \end{vmatrix}$$

$C_2 \to C_2-C_1$

$C_3 \to C_3-C_1$

$$(6-\lambda)\begin{vmatrix} 1 & 0 & 0 \\ 3 & -\lambda-2 & -1 \\ 2 & 1 & -\lambda-1 \end{vmatrix}=0$$

$(6-\lambda)\left[(\lambda+1)(\lambda+2)+1\right]=0$

$(6-\lambda)=0$

$6=\lambda$

$\boxed{\lambda=6}$

$\lambda^2+2\lambda+\lambda+2+1=0$

$\boxed{\lambda=6}$ $\lambda^2+3\lambda+3=0$

9 inverse matrix A

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$$

by Guass Jordan Method.

$AB = I = BA = B = A^{-1}$.

$$A = \begin{array}{c} \begin{matrix} + & - & + \end{matrix} \\ \begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix} \end{array}$$

$+2(12-2) - 3(16-1) + 4(8-3)$

$2(10) - 3(15) + 4(5)$

$20 - 45 + 20$

$40 - 45 = -5 \neq 0$ A^{-1} exists

Consider $[A|I]$

$$\left[\begin{array}{ccc|ccc} 2 & 3 & 4 & 1 & 0 & 0 \\ 4 & 3 & 1 & 0 & 1 & 0 \\ 1 & 2 & 4 & 0 & 0 & 1 \end{array}\right]$$

$R_1 \to R_1/2$

$$\left[\begin{array}{ccc|ccc} 1 & 3/2 & 2 & 1/2 & 0 & 0 \\ 4 & 3 & 1 & 0 & 1 & 0 \\ 1 & 2 & 4 & 0 & 0 & 1 \end{array}\right]$$

$R_2 \to R_2 - 4R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 3/2 & 2 & 1/2 & 0 & 0 \\ 0 & -3 & -7 & -2 & 1 & 0 \\ 1 & 2 & 4 & 0 & 0 & 1 \end{array}\right]$$

$R_3 \to R_3 - R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 3/2 & 2 & 1/2 & 0 & 0 \\ 0 & -3 & -7 & -2 & 1 & 0 \\ 0 & 1/2 & 2 & -1/2 & 0 & 1 \end{array}\right]$$

$R_2 \to R_2/-3$

$$\left[\begin{array}{ccc|ccc} 1 & 3/2 & 2 & 1/2 & 0 & 0 \\ 0 & 1 & +7/3 & 2/3 & -1/3 & 0 \\ 0 & 1/2 & 2 & -1/2 & 0 & 1 \end{array}\right]$$

$R_1 \to R_1 - \frac{3}{2}R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{3}{2} & -\frac{1}{2} & 1/2 & 0 \\ 0 & 1 & 7/3 & 2/3 & -1/3 & 0 \\ 0 & 1/2 & 2 & -1/2 & 0 & 1 \end{array}\right]$$

$R_3 \to R_3 - 1/2 R_2$

$R_3 \to R_3 / 5/6$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{3}{2} & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 1 & 7/3 & \frac{2}{3} & -\frac{1}{3} & 0 \\ 0 & 0 & 5/6 & -\frac{5}{6} & \frac{1}{6} & 1 \end{array}\right]$$

$R_3 \to R_3 / \frac{5}{6}$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{3}{2} & -1/2 & 1/2 & 0 \\ 0 & 1 & 7/3 & 2/3 & -1/3 & 0 \\ 0 & 0 & 1 & -1 & \frac{1}{5} & \frac{6}{5} \end{array}\right]$$

$R_1 \to R_1 + \frac{3}{2} R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & \frac{4}{5} & \frac{9}{5} \\ 0 & 1 & 7/3 & 2/3 & -1/3 & 0 \\ 0 & 0 & 1 & -1 & \frac{1}{5} & 6/5 \end{array}\right]$$

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$R_2 \to R_2 - \frac{7}{3} R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 4/5 & 9/5 \\ 0 & 1 & 0 & 3 & -4/5 & -14/5 \\ 0 & 0 & 1 & -1 & 1/5 & 6/5 \end{array}\right]$$

$$B = A^{-1} \begin{bmatrix} -2 & 4/5 & 9/5 \\ 3 & -4/5 & -14/5 \\ -1 & 1/5 & 6/5 \end{bmatrix}$$

(6) $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

inverse

$A = IA$

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$

$R_1 \leftrightarrow R_2$

$R_3 \to R_3 - 3R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -5 & -8 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -3 & 1 \end{bmatrix}$$

$R_1 \to R_1 - 2R_2$

$R_3 \to R_3 + 5R_2$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 5 & -3 & 1 \end{bmatrix}$$

$R_2 \to R_2 - R_3$

$R_3 \to R_3/2$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ -4 & 3 & -1 \\ 5/2 & -3/2 & 1/2 \end{bmatrix}$$

$R_1 \to R_1 + R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{-1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & \frac{-3}{2} & 1/2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{-1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix}$$

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(11) $3x - y + 4z = 3$

$x + 2y - 3z = -2$

$6x + 5y + az = -3$

having solutions

$$D = \begin{vmatrix} 3 & -1 & 4 \\ 1 & 2 & -3 \\ 6 & 5 & a \end{vmatrix} = 0$$

$\Rightarrow 3(2a+15) + 1(a+18) + 4(5-12)$

$7a + 35 = 0$

$7a = -35$

$\boxed{a = -5}$

$3x - y + 4z = 3$

$x + 2y - 3z = -2$

$\boxed{x = k}$

$3k - y + 4z = 3$

$1k + 2y - 3z = -2$

$y = 3k - 3 + 4z$

$k + 2(3k - 3 + 4z) - 3z = -2$

$k + 6k - 6 + 8z - 3z = -2$

$5z = 4 - 7k$

$z = \frac{4 - 7k}{5}$

$y = 3k - 3 + 4z$

$y = 3k - 3 + 4\left(\frac{4 - 7k}{5}\right)$

$y = 3k - 3 + \frac{16 - 28k}{5}$

$y = \frac{-13k + 1}{5}$

The sol $\left(k, \frac{-13k+1}{5}, \frac{4-7k}{5}\right)$

(12) rank of matrix into echleon form.

$$\begin{bmatrix} 1 & 4 & 3 & -2 & 1 \\ -2 & -3 & -1 & 4 & 3 \\ -1 & 6 & 7 & 2 & 9 \\ -3 & 3 & 6 & 6 & 12 \end{bmatrix}$$

$R_2 \to R_2 + 2R_1$

$R_3 \to R_3 + R_1$

$R_4 \to R_4 + 3R_1$

$$\begin{bmatrix} 1 & 4 & 3 & -2 & 1 \\ 0 & 5 & 5 & 0 & 5 \\ 0 & 10 & 10 & 0 & 10 \\ 0 & 15 & 15 & 0 & 15 \end{bmatrix}$$

$R_3 \to R_3 - 2R_2$

$R_4 \to R_4 - 3R_2$

$R_2 \to R_2 / 5$

$$\begin{bmatrix} 1 & 4 & 3 & -2 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$\boxed{\text{Rank of } A = 2}$

Converted into echleon form

13 $(\lambda-1)x+(3\lambda+1)y+2\lambda z=0$

$(\lambda-1)x+(4\lambda-2)y+(\lambda+3)z=0$

$2x+(3\lambda+1)y+3(\lambda-1)z=0$

have a non trivial sol.

$$\Rightarrow \begin{vmatrix} \lambda-1 & 3\lambda+1 & 2\lambda \\ \lambda-1 & 4\lambda-2 & \lambda+3 \\ 2 & 3\lambda+1 & 3\lambda-3 \end{vmatrix} = 0$$

$$(\lambda-1)[(4\lambda-2)(3\lambda-3)-(\lambda+3)(3\lambda+1)]$$
$$-(3\lambda+1)[(\lambda-1)(3\lambda-3)-2(\lambda+3)]$$
$$+2\lambda((\lambda-1)(3\lambda+1)-2(4\lambda-2)]$$

$$\cancel{9\lambda^3}-37\lambda^2+31\lambda\cancel{+3}-\cancel{9\lambda^3}+21\lambda^2+17\lambda+\cancel{3}$$
$$-16\lambda^2+48\lambda+6+6\lambda^3-20\lambda^2+6\lambda$$
$$6\lambda^3+54\lambda-36\lambda^2=0$$
$$6\lambda^3-36\lambda^2+54\lambda=0$$
$$\boxed{\lambda=3,3,0}$$

(14) $A = \begin{bmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{bmatrix}$

$\Rightarrow R_2 \to R_2 - 2R_1$

$R_4 \to R_4 - R_3$

$$\begin{bmatrix} 2 & 1 & 3 & 5 \\ 0 & 0 & -5 & -7 \\ 8 & 4 & 7 & 13 \\ 0 & 0 & -10 & -14 \end{bmatrix}$$

$R_3 \to R_3 - 4R_1$

$$\begin{bmatrix} 2 & 1 & 3 & 5 \\ 0 & 0 & -5 & -7 \\ 0 & 0 & -5 & -7 \\ 0 & 0 & -10 & -14 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$R_4 \to R_4 - 2R_2$

$$\begin{bmatrix} 2 & 1 & 3 & 5 \\ 0 & 0 & -5 & -7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\boxed{\text{Rank of } A = 2.}$

Reduced into echleon form

(15) $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$

Normal form.

$R_1 \leftrightarrow R_2$

$$\begin{bmatrix} 1 & -1 & -2 & -4 \\ 2 & 3 & -1 & -1 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 3R_1$

$R_4 \to R_4 - 6R_1$

$$\begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & +3 & 7 \\ 0 & 4 & 9 & 10 \\ 0 & 9 & 12 & 17 \end{bmatrix}$$

$R_3 \to R_3 + R_2$

$$\begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 9 & 12 & 17 \\ 0 & 9 & 12 & 17 \end{bmatrix}$$

$R_4 \to R_4 - R_3$

$$\begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 9 & 12 & 17 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_2 \to C_2 + C_1$

$C_3 \to C_3 + 2C_1$

$C_4 \to C_4 + 4C_1$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 5 & 3 & 7 \\ 0 & 9 & 12 & 17 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\cancel{C_3 \to 2C_3 - C_2}$ $C_3 \to 2C_3 - C_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 5 & 1 & 7 \\ 0 & 9 & 15 & 17 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_2 \leftrightarrow C_3$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 5 & 7 \\ 0 & 15 & 9 & 17 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_3 \to C_3 - 5C_2$

$C_4 \to C_4 - 7C_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 15 & -66 & -88 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_3 \to R_3 - 15R_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -66 & -88 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_3 / -11$

$C_4 \to 6C_4 - 8C_3$

$R_3 \to R_3 / 6$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} I_3 & 0 \\ 0 & 0 \end{bmatrix}$$

$\boxed{\text{Rank of } A = 3}$

Reduced to normal form.

(16) $A = \begin{bmatrix} -2 & 1 & 3 \\ 0 & -1 & 1 \\ 1 & 2 & 0 \end{bmatrix}$

Inverse

$A = IA$

$$\begin{bmatrix} -2 & 1 & 3 \\ 0 & -1 & 1 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$

$R_1 \leftrightarrow R_3$

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 1 \\ -2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \cdot A$$

$R_3 \to R_3 + 2R_1$, $R_3 \to R_3 + 5R_2$

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 1 \\ 0 & 5 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix} \cdot A$$

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 8 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 5 & 2 \end{bmatrix}$$

$R_3 \to R_3/8$

$R_2 \to R_2 - R_3$

$R_3 \to R_3/8$

$R_2 \to R_2 - R_3$

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ -1/8 & 3/8 & -2/8 \\ 1/8 & 5/8 & 2/8 \end{bmatrix}$$

$R_1 \to R_1 + 2R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2/8 & 6/8 & 4/8 \\ -1/8 & 3/8 & -2/8 \\ 1/8 & 5/8 & 2/8 \end{bmatrix}$$

$R_2 / -1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{-2}{8} & \frac{6}{8} & \frac{4}{8} \\ \frac{1}{8} & \frac{-3}{8} & \frac{2}{8} \\ \frac{1}{8} & \frac{5}{8} & \frac{2}{8} \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{-2}{8} & \frac{6}{8} & \frac{4}{8} \\ \frac{1}{8} & \frac{-3}{8} & \frac{2}{8} \\ \frac{1}{8} & \frac{5}{8} & \frac{2}{8} \end{bmatrix}$$

(17) values of eqn.

$x+y+z=1$

$x+2y+4z=a$

$x+4y+10z=a^2$

maybe consistent

$x+y=1-z$

~~x+2y~~

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 10 \end{vmatrix}$$

$R_1 \to R_1 - R_2$

$R_2 \to R_2 - R_3$

$$\begin{vmatrix} 0 & -1 & -3 \\ 0 & -2 & -6 \\ 1 & 4 & 10 \end{vmatrix}$$

$\Delta = 0$

not a unique sol.

Infinite sol

$\Delta_1 = 0, \Delta_2 = 0, \Delta_3$

$$\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ a & 2 & 4 \\ a^2 & 4 & 10 \end{vmatrix}$$

$C_3 \to C_3 - C_2$

$\cancel{+1(20-16)-1(10a-4a^2)}$

$1(12-8)-1(6a-2a^2)=0$

$a^2-3a+2=0 \quad a=1,2$

(18)

$$A = \begin{bmatrix} 1 & 1 & -3 & -1 \\ 1 & 2 & 1 & 1 \\ 3 & 1 & 3 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

reducing into echelon form.

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - 3R_1$

$R_4 \to R_4 - R_1$

$$\begin{bmatrix} 1 & 1 & -3 & -1 \\ 0 & 1 & 4 & 2 \\ 0 & -2 & 12 & 5 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$R_3 \leftrightarrow R_4$

$$\begin{bmatrix} 1 & 1 & -3 & -1 \\ 0 & 1 & 4 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & -2 & 12 & 5 \end{bmatrix}$$

$R_4 \to R_4 + 2R_2$

$$\begin{bmatrix} 1 & 1 & -3 & -1 \\ 0 & 1 & 4 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 20 & 9 \end{bmatrix}$$

$R_4 \to R_4 - 20R_3$

$$\begin{bmatrix} 1 & 1 & -3 & -1 \\ 0 & 1 & 4 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & -11 \end{bmatrix}$$

$\boxed{\text{Rank of } A = 4}$

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(19) $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 5 \\ 1 & 5 & 5 & 7 \\ 8 & 1 & 14 & 17 \end{bmatrix}$

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - R_1$
$R_4 \to R_4 - 4R_2$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -3 & -2 & -3 \\ 0 & 3 & 2 & 3 \\ 0 & -3 & -2 & -3 \end{bmatrix}$$

$R_3 \to R_3 + R_2$
$R_4 \to R_4 + R_3$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -3 & -2 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank of (A) = 2 by elementary row operations.

20) Solve the system of eqns.

$x + 3y - 2z = 0$
$2x - y + 4z = 0$
$x - 11y + 14z = 0$

This is a Homogeneous eqn from non trivial solution

So, $\begin{bmatrix} 1 & 3 & -2 & 0 \\ 2 & -1 & 4 & 0 \\ 1 & -11 & 14 & 0 \end{bmatrix} = [A : 0]$

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & 3 & -2 & 0 \\ 0 & -7 & 8 & 0 \\ 0 & -14 & 16 & 0 \end{bmatrix}$$

$R_3 \to R_3 - 2R_2$

$$\begin{bmatrix} 1 & 3 & -2 & 0 \\ 0 & -7 & 8 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The given sys. non-trivial sol.

$\boxed{z = K}$

$-7y + 8z = 0, \; x + 3y - 2z = 0$

$-7y + 8K = 0$

$+7y = +8K$

$\boxed{y = \frac{8K}{7}}, \; \boxed{x = \frac{-10K}{7}}$

$\boxed{z = K}$

$x + 3y - 2z = 0$

$x + 3\left(\frac{8K}{7}\right) - 2K = 0$

$7x + 10K = 0$

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Part-C

Write about Consistency

In linear equation there are two system which are divided into Homogeneous and non-Homogeneous System.

Non Homogeneous system

Consistence | Inconsistence

Consistence have the condition $\rho(A) = \rho([A|B])$

Consistence have two solutions unique and Infinite Solutions

If it has no solution is called Inconsistency.

2) If it has no solution is called Inconsistency which from non-Homogeneous system

3) Define Trivial Solutions ?

A) The Homogeneous system is always that Constant

$\rho(A) = \rho[A:B] = \rho[A:0]$

If $\rho(A) = n$, no of unknowns, the system $Ax = 0$ has unique solution

In this case $x = 0$, ie $x_1 = x_2 = \cdots x_n = 0$

which is zero solution (or) trivial solution $\{|A| \neq 0\}$

4) Define Non Trivial solns?

A) If $\rho(A) < n$, no of unknowns The system has non trivial soln's (or) infinite sol $\{|A| = 0\}$. [Homogeneous system from 3Q)]

5) Define about non-Homogeneous equations.

The given system which is the form of $Ax = B$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix} \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad B = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Variable matrix   Const matrix

6) write about the rank of a matrix

A) It is the maximum number of linearly independent rows | columns of a matrix.

7) Explain elementary operations:

There are two types

Elementary row operations

Elementary ~~row~~ column operations

⇒ Interchange of any two rows and columns

⇒ Multiplication of any row and column by a non zero scalar k

⇒ Add a multiple of one row to another and of one column to another

8) Explain about the rank of a matrix briefly.

A) A non-negitive integer r is said to be the rank of a matrix A if it posses the following properties

(i) There is atleast one minor of order "r" which is non-zero

(ii) which is greater than "r" is zero

Denoted by $\rho(A)$

10Q) Write about conditions for normal form of a matrix.

A) Every non-zero matrix can be reduced to the form of $\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$ called the normal form or canonical form with the help of elementary Transformations

11 Solve the system eqns

$x+2y-5z=0$

$3x+4y+6z=0$

$x+y+z=0$

A) The eqn of form of $Ax=0$

$$\begin{bmatrix} 1 & 2 & -5 \\ 3 & 4 & 6 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$[A:B] = [A:0] = \begin{bmatrix} 1 & 2 & -5 & 0 \\ 3 & 4 & 6 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

$R_2 \to R_2 - 3R_1$

$R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & 2 & -5 & 0 \\ 0 & -2 & 21 & 0 \\ 0 & -1 & 6 & 0 \end{bmatrix}$$

$R_3 \to 2R_3 - R_2$

$$\begin{bmatrix} 1 & 2 & -5 & 0 \\ 0 & -2 & 21 & 0 \\ 0 & -1 & 6 & 0 \end{bmatrix}$$

$\rho(A) = \rho(A:B) = 3$

$|A| \neq 0$

$\boxed{\text{So, } x=y=z=0}$

Unique solution

Trivial

12) $x+y+w=0$

$y+z=0$

$x+y+z+w=0$

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$\rho(A) = 3$

So, $x=0, y=0, z=0, w=0$

which is Trivial solution.

(unique soln)

13. Explain Gauss Jordan method working rule.

Inverse of matrix $AB = I = BA \Rightarrow B = A^{-1}$

let A be any non-singular matrix

Steps :

1) write A as $A = IA$, $AB = I = BA \Rightarrow B = A^{-1}$

let A be any non-singular matrix.

Apply elementary row transformations on A on L.H.S and reduce it to I (Identity matrix)

5) Apply same row transformation on
I on R.H.S (reduces to B)

7) Find the inverse of the given matrix by Gauss-Jordan Method

14) Explain about the system of linear eqns.

→ A system of linear eqns is a collection of one (or) more linear equation involving the same of set variables

$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$

$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$

$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m$

linear equations have Homogeneous and non-Homogeneous eqn's.

15) Rank of matrix:

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 8 & 7 & 0 & 5 \end{bmatrix}$$

6-5
6-5(2)
6-10
7-5(3)
7-15
8-5(4)
8-20

$R_2 \to R_2 - 5R_1$

$R_3 \to R_3 - 8R_1$

7-8(2)
7-16
-9
0-8(3)
5-8(4)
5-32
-27

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -4 & -8 & -12 \\ 0 & -9 & -24 & -27 \end{bmatrix}$$

$R_3 \to 4R_3 - 9R_2$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -4 & -8 & -12 \\ 0 & 0 & -24 & 0 \end{bmatrix}$$

$\boxed{\text{Rank} = 3}$

16) rank of is 3 value of k

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & k & 7 \\ 3 & 6 & 10 \end{bmatrix}$$

$+1(10k-42)-2(20-21)$

$+3(12-3k)=0$

$10k-42-2(-1)+36-9k=0$

$k-42+2+36-9k=0$

$k-42+38=0$

$k-4=0$

$\boxed{k=4}$

$|D| = 4(2k-18)-4(k-18)-3($

$8k-72-4k+72-$

$+54$

$4k+27=0$

$k=-27/4$

17)

$$\begin{bmatrix} 1 & 1 & -1 \\ 1 & -1 & k \\ 3 & 1 & 0 \end{bmatrix}$$

$|A| = +1(0-k)-1(0-3k)$

$-1(1+3)$

$\Rightarrow -k+3k-4$

$2k-4=0$

$2k=4$

$\boxed{k=2}$

18)

$$\begin{bmatrix} 4 & 4 & -3 \\ 1 & 2 & 2 \\ 9 & 9 & k \end{bmatrix}$$

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19)

$$\begin{bmatrix} 4 & 4 & 4 & 4 \\ 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 9 & 9 & 9 & 9 \end{bmatrix}$$

$R_1 \to R_1/4$

$R_2 \to R_3/2$

$R_{3} \to R_4/9$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$\rho(A) = 1$

A-20

$$A = \begin{bmatrix} -3 & 0 & 0 \\ 0 & 5 & 6 \\ 0 & 0 & 7 \end{bmatrix}$$

$A = -3(35-0) = |A|$

$|A| = -105 \neq 0$

$\rho(A) = 3$