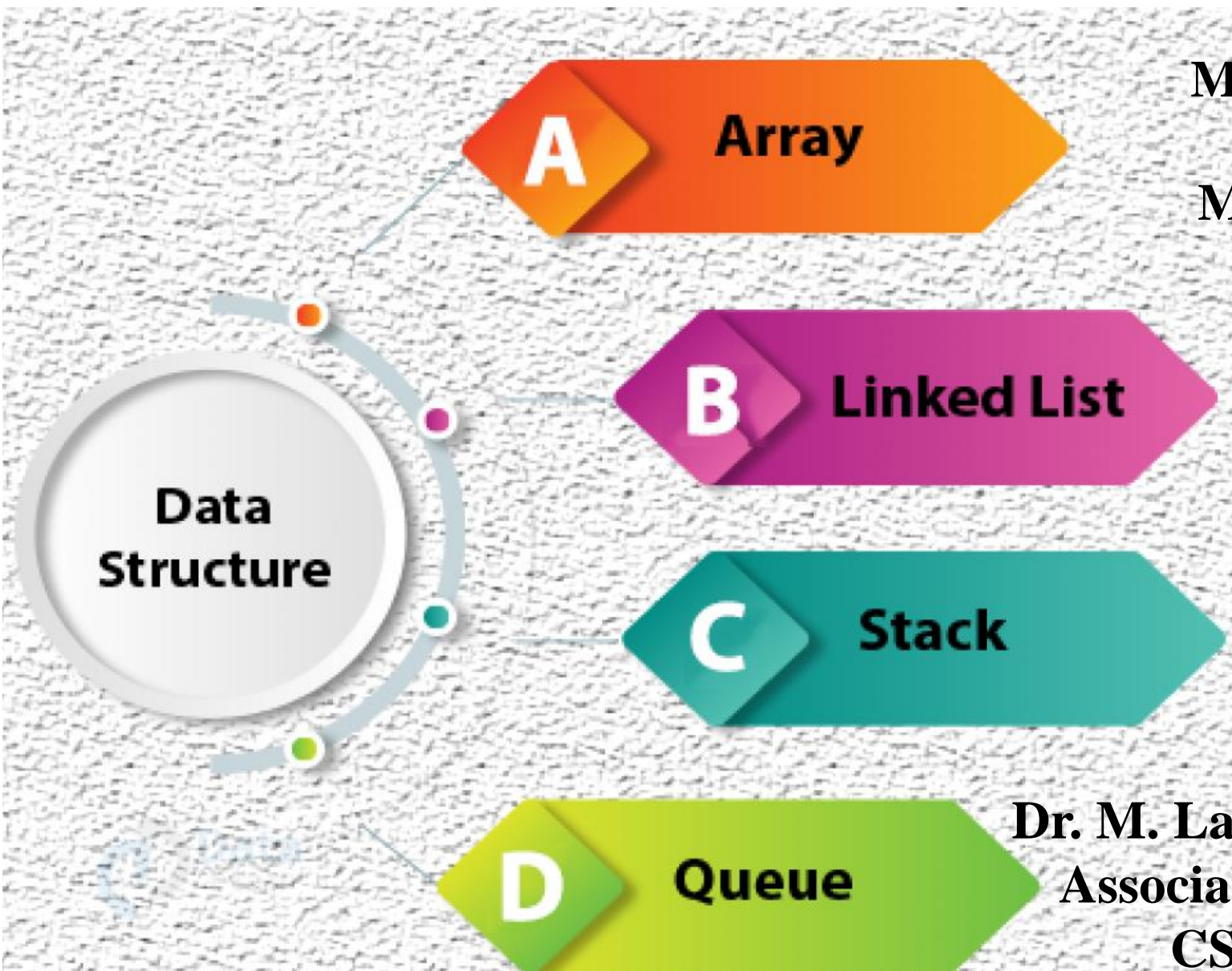


LINEAR DATA STRUCTURES

**MODULE – II
&
MODULE-III**



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MODULE – II

LINEAR DATA STRUCTURES

Abstract Data Type

- **Abstract Data Type (or ADT)**, there is a set of rules or description of the operations that are allowed on data.
- It is based on a user point of view i.e., how a user is interacting with the data.
- It does not specify how data will be organized in memory and what algorithms will be used for implementing the operations.
- The process of providing only the essentials and hiding the details is known as abstraction. However, we can choose to implement those set of rules differently.

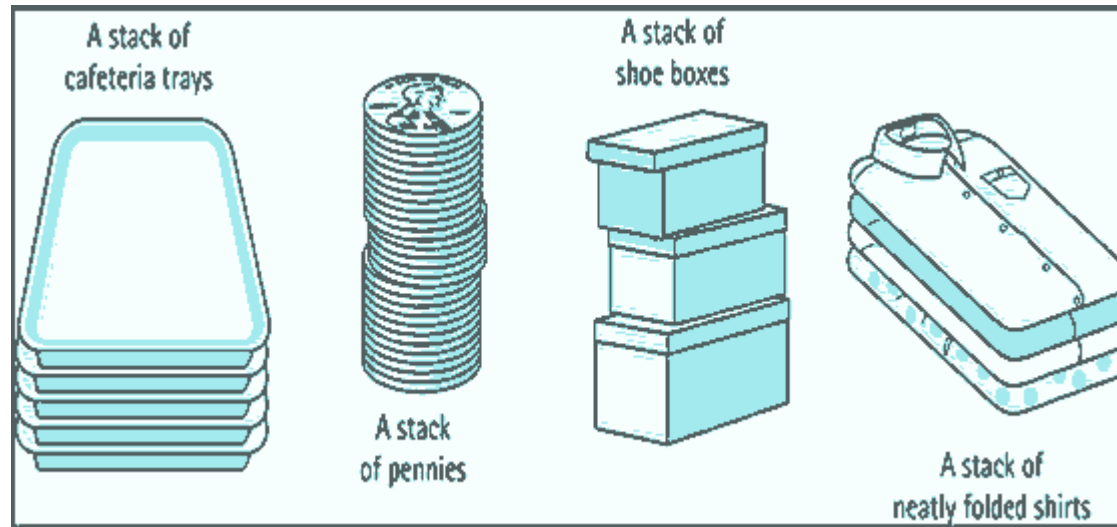
Interface Stack

```
{  
    public Boolean empty();  
        //Boolean, true when the stack is empty  
    public void push (String str);  
        //Add new element into stack  
    public String pop()  
        //Delete element from stack  
    public String peek ( )  
        //return top element of stack  
}
```

STACK ADT

- A stack is an ordered list in which all insertions and deletions are made at one end called the top.

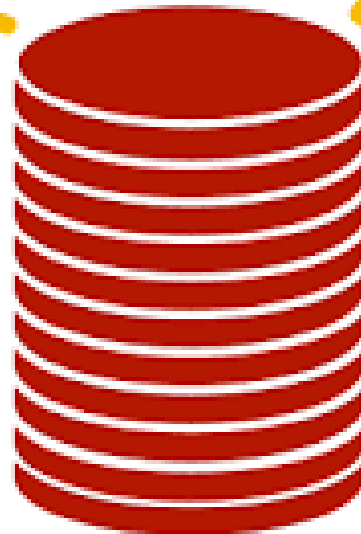
Example:



STACK ADT

- In stack always the last item to be put in to the stack is the first item to be removed. So stack is a **Last In First Out** or **LIFO** data structure.

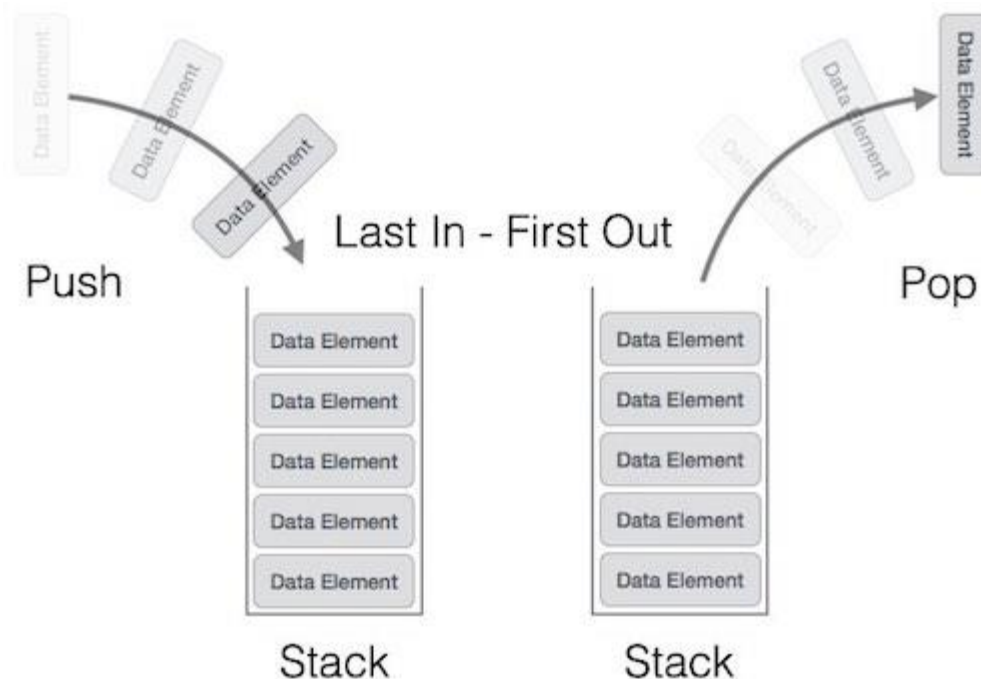
Any item will be removed from the top



New item will be added on the top

STACK ADT

➤ **Push** and **Pop** are the operations that are provide for insertion of an element in to the stack and the removal of an element from the stack.



STACK ADT

- The **add** operation of the stack is called **push** operation
- The **delete** operation is called as **pop** operation.
- **Push operation** on a **full stack** causes **stack overflow**.
- **Pop operation** on an **empty stack** causes **stack underflow**.
- **SP** is a pointer, which is used to access the top element of the stack.
- If you push elements that are added at the top of the stack;
- In the same way when we pop the elements, the element at the top of the stack is deleted.

STACK OPERATIONS

There are three operations applied on stack they are

1. PUSH
2. POP
3. PEEK

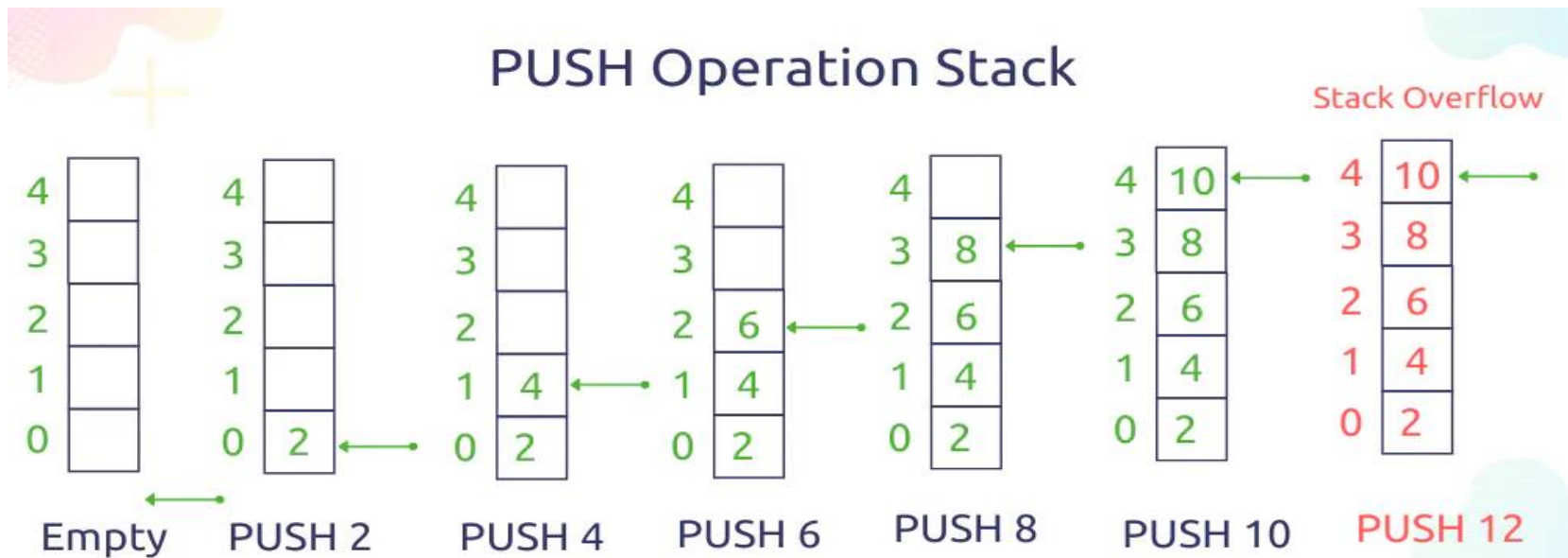
While performing push & pop operations the following test must be conducted on the stack.

1. Stack is empty or not
2. Stack is full or not

STACK OPERATIONS

Push:

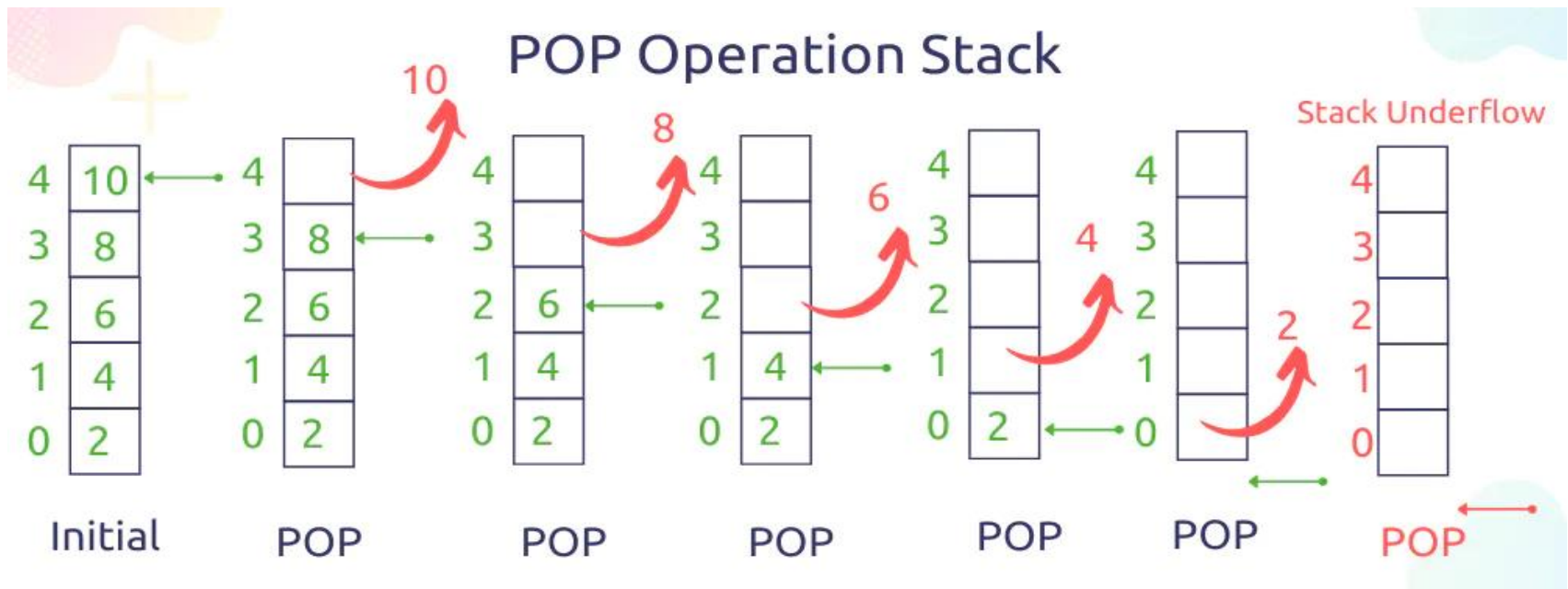
Push operation is used to add new elements in to the stack. At the time of adding first check the stack is full or not. If the stack is full it generates an error message "stack overflow".



STACK OPERATIONS

Pop:

Pop operation is used to delete elements from the stack. At the time of deletion first check the stack is empty or not. If the stack is empty it generates an error message "stack underflow".



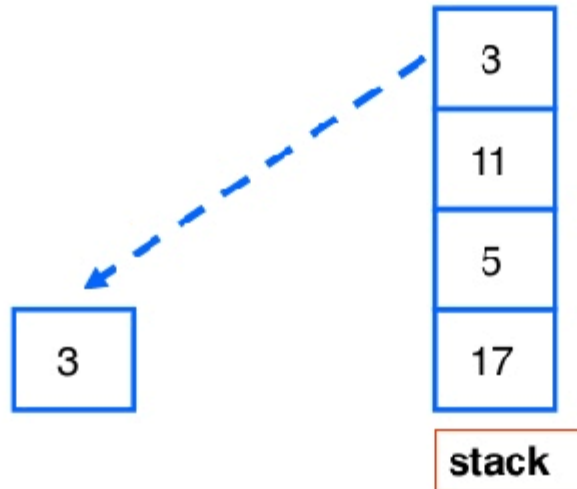
STACK OPERATIONS

Peep:

Find an element from the top of the stack.

Peek

- **Peek** means retrieve the top of the stack without removing it



REPRESENTATION OF A STACK USING ARRAYS



- Let us consider a stack with 6 elements capacity. This is called as the size of the stack.
- The number of elements to be added should not exceed the maximum size of the stack.
- If we attempt to add new element beyond the maximum size, we will encounter a stack overflow condition.
- Similarly, you cannot remove elements beyond the base of the stack. If such is the case, we will reach a stack underflow condition.

ALGORITHMS FOR PUSH AND POP

PUSH(STACK, TOP, MAXSIZE, ITEM): This procedure pushes an ITEM onto a stack

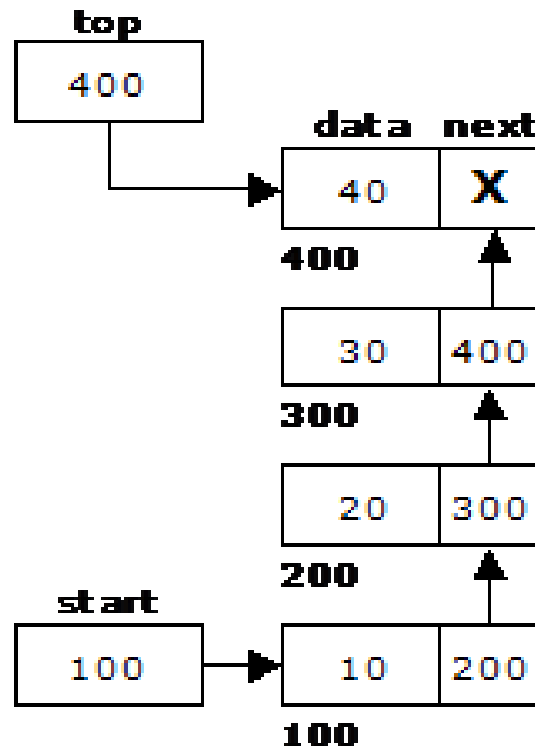
1. If $TOP == MAXSIZE$, then Print: Overflow, and return
2. Set $STACK[TOP] = ITEM$ [inserts ITEM in TOP Position]
3. Set $TOP = TOP + 1$ [increases TOP by 1]
4. Return

POP(STACK, TOP, ITEM): This procedure deletes the top element of STACK and assign it to the variable ITEM

1. If $TOP == 0$ then Print: UNDERFLOW, and return
2. Set $ITEM = STACK[TOP]$
3. Set $TOP = TOP - 1$ [Decreases TOP by 1]
4. Return

LINKED LIST IMPLEMENTATION OF STACK

We can represent a stack as a linked list. In a stack push and pop operations are performed at one end called top. We can perform similar operations at one end of list using top pointer.



APPLICATIONS OF STACK

- Stack is used by compilers to check for balancing of **parentheses, brackets and braces**.
- Stack is used to evaluate a **postfix expression**.
- Stack is used to convert an **infix expression into postfix/prefix** form.
- In **recursion**, all intermediate arguments and return values are stored on the processor's stack.
- During a function call the return address and arguments are pushed onto a stack and on return they are popped off.
- **Depth first search** uses a **stack data structure** to find an element from a **graph**.

APPLICATIONS OF STACK

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ALGEBRAIC EXPRESSIONS

- An algebraic expression is a legal combination of **operators** and **operands**.
- Operand is the **quantity** on which a **mathematical operation** is performed.
- Operand may be a **variable** like x, y, z or a **constant** like $5, 4, 6$ etc.
- Operator is a symbol which signifies a **mathematical or logical operation** between the **operands**.
- Examples of familiar operators include $+, -, *, /, ^$ etc.
- An algebraic expression can be represented using three different notations. They are
- **infix, postfix and prefix notations:**

INFIX NOTATION

Infix: It is the form of an arithmetic expression in which we fix (place) the arithmetic operator in between the two operands.

Example:

$$(A + B) * (C - D)$$

$$a+b$$

$$((6-(3+2)*5)^2+3)$$

$$(a+b+c)/2$$

$$a*b/2$$

PREFIX NOTATION

Prefix: It is the form of an arithmetic notation in which we fix (place) the arithmetic operator before (pre) its two operands. The prefix notation is called as polish notation (due to the **polish mathematician Jan Lukasiewicz** in the year 1920).

Example:

+ab

+^ -abc2

/*ab2

+ A B – C D

POSTFIX NOTATION

Postfix: It is the form of an arithmetic expression in which we fix (place) the arithmetic operator after (post) its two operands. The postfix notation is called as *suffix notation* and is also referred to *reverse polish notation*.

Example:

$ab+$

$632+5*-2^3+$

$ab+c+2/$

$ab*2/$

$A B + C D - *$

OPERATOR PRECEDENCE

We consider five binary operations: $+$, $-$, $*$, $/$ and $\$$ or \uparrow (exponentiation). For these binary operations, the following in the order of precedence (highest to lowest):

| OPERATOR | PRECEDENCE | VALUE |
|---|--------------|-------|
| Exponentiation ($\$$ or \uparrow or \wedge) | Highest | 3 |
| $*$, $/$ | Next highest | 2 |
| $+$, $-$ | Lowest | 1 |

CONVERTING EXPRESSIONS

Let us convert the expressions from one type to another. These can be done as follows:

1. Infix to postfix
2. Infix to prefix
3. Postfix to infix
4. Postfix to prefix
5. Prefix to infix
6. Prefix to postfix

ALGORITHM - INFIX TO POSTFIX

- 1) If the character is **LEFT PARENTHESIS**, push to the **STACK**.
- 2) If the character is an **OPERAND**, **ADD** to the **POSTFIX EXPRESSION**.
- 3) If the character is an **OPERATOR**, check whether **STACK** is Empty
 - a) If the **STACK** is empty, push operator into **STACK**.
 - b) If the **STACK** is not empty, check the **priority of the operator**.
 - i) If the **priority of scanned operator > operator present at TOP of the STACK**, then **push operator into STACK**.
 - ii) If the **priority of scanned operator <= operator present at the TOP of the STACK**, then pop operator from **STACK** and **ADD** to **POSTFIX EXPRESSION** and go to **step i**.
- 4) If the character is **RIGHT PARENTHESIS**, then **POP** all operator and operands from **STACK** until it **reaches LEFT PARENTHESIS** and **ADD** to **POSTFIX EXPRESSION**.
- 5) After reading all characters, if **STACK** is **not empty** then **POP** and **ADD** to **POSTFIX EXPRESSION**.

EXAMPLE - INFIX TO POSTFIX

Convert the following infix expression $A + B * C - D / E * H$ into its equivalent postfix expression.

| SYMBOL | POSTFIX STRING | STACK | REMARKS |
|---------------|-----------------------|--|---------|
| A | A | | |
| + | A | + | |
| B | A B | + | |
| * | A B | + * | |
| C | A B C | + * | |
| - | A B C * + | - | |
| D | A B C * + D | - | |
| / | A B C * + D | - / | |
| E | A B C * + D E | - / | |
| * | A B C * + D E / | - * | |
| H | A B C * + D E / H | - * | |
| End of string | A B C * + D E / H * - | The input is now empty. Pop the output symbols from the stack until it is empty. | |

EXAMPLE - INFIX TO POSTFIX

Convert $((A - (B + C)) * D) \uparrow (E + F)$ infix expression to postfix form:

| SYMBOL | POSTFIX STRING | STACK | REMARKS |
|---------------|-----------------------|--|---------|
| (| | (| |
| (| | ((| |
| A | A | ((| |
| - | A | ((- | |
| (| A | ((- (| |
| B | A B | ((- (| |
| + | A B | ((- (+ | |
| C | A B C | ((- (+ | |
|) | A B C + | ((- | |
|) | A B C + - | (| |
| * | A B C + - | (* | |
| D | A B C + - D | (* | |
|) | A B C + - D * | | |
| ^ | A B C + - D * | ^ | |
| (| A B C + - D * | ^ (| |
| E | A B C + - D * E | ^ (| |
| + | A B C + - D * E | ^ (+ | |
| F | A B C + - D * E F | ^ (+ | |
|) | A B C + - D * E F + | ^ | |
| End of String | A B C + - D * E F + ^ | The input is now empty. Pop the output Symbols from the stack until it is empty. | |

EXAMPLE - INFIX TO POSTFIX

Convert $a + b * c + (d * e + f) * g$ the infix expression into postfix form:

| SYMBOL | POSTFIX STRING | STACK | REMARKS |
|---------------|---------------------------|--|---------|
| a | a | | |
| + | a | + | |
| b | a b | + | |
| * | a b | + * | |
| c | a b c | + * | |
| + | a b c * + | + | |
| (| a b c * + | + (| |
| d | a b c * + d | + (| |
| * | a b c * + d | + (* | |
| e | a b c * + d e | + (* | |
| + | a b c * + d e * | + (+ | |
| f | a b c * + d e * f | + (+ | |
|) | a b c * + d e * f + | + | |
| * | a b c * + d e * f + | + * | |
| g | a b c * + d e * f + g | + * | |
| End of string | a b c * + d e * f + g * + | The input is now empty. Pop the output symbols from the stack until it is empty. | |

INFIX TO PREFIX

- The precedence rules for converting an expression from infix to prefix are identical.
- The only change from postfix conversion is that **traverse** the expression **from right to left** and the **operator** is placed **before the operands** rather than after them.

EXAMPLE - INFIX TO PREFIX

Convert the infix expression $A + B - C$ into prefix expression

| SYMBOL | POSTFIX STRING | STACK | REMARKS |
|---------------|----------------|--|---------|
| C | C | | |
| - | C | - | |
| B | B C | - | |
| + | - B C | + | |
| A | A - B C | + | |
| End of String | + A - B C | The input is now empty. Pop the output symbols from the stack until it is empty. | |

EXAMPLE - INFIX TO PREFIX

Convert the infix expression $(A + B) * (C - D)$ into prefix expression.

| SYMBOL | POSTFIX STRING | STACK | REMARKS |
|---------------|----------------|--|---------|
|) | |) | |
| D | D |) | |
| - | D |) - | |
| C | C D |) - | |
| (| - C D | | |
| * | - C D | * | |
|) | - C D | *) | |
| B | B - C D | *) | |
| + | B - C D | *) + | |
| A | A B - C D | *) + | |
| (| + A B - C D | * | |
| End of String | * + A B - C D | The input is now empty. Pop the output symbols from the stack until it is empty. | |

INFIX TO PREFIX

The following algorithm must be followed for infix to prefix conversion.

1. Reverse the input string.
2. Convert the reversed string into POSTFIX expression.
3. Now reverse the resulting postfix expression obtained from the previous step. The resulting expression is prefix expression

EXAMPLE - INFIX TO PREFIX

Convert the infix expression $A \uparrow B * C - D + E / F / (G + H)$ into prefix expression.

Step by step output for " $A \uparrow B * C - D + E / F / (G + H)$ " expression

1. Reversed string: $(H + G) / F / E + D - C * B \uparrow A$

2. Postfix of $(H + G) / F / E + D - C * B \uparrow A$:

$H G + F / E / D + C B A ^ * -$

3. Reversed string of $H G + F / E / D + C B A ^ * -$:

$- * ^ A B C + D / E / F / + G H$

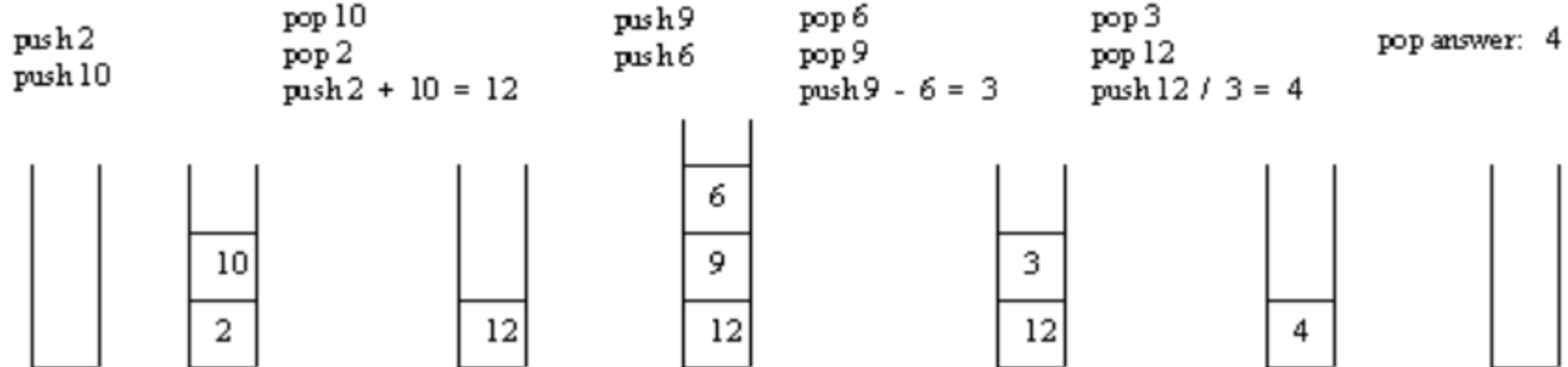
The postfix expression is evaluated easily by the use of a stack.

- When a number is seen, it is pushed onto the stack;
- When an operator is seen, the operator is applied to the two numbers that are popped from the stack and the result is pushed onto the stack.
- When an expression is given in postfix notation, there is no need to know any precedence rules; this is our obvious advantage.

EXAMPLE - EVALUATION OF POSTFIX

Evaluate the postfix expression: 2 10 + 9 6 - /

2 10 + 9 6 - /



EXAMPLE - EVALUATION OF POSTFIX

Evaluate the postfix expression: 6 5 2 3 + 8 * + 3 + *

| SYMBOL | OPERAND 1 | OPERAND 2 | VALUE | STACK | REMARKS |
|--------|--------------|-----------|-------|------------|---|
| 6 | | | | 6 | |
| 5 | | | | 6, 5 | |
| 2 | | | | 6, 5, 2 | |
| 3 | | | | 6, 5, 2, 3 | The first four symbols are placed on the stack. |
| + | 2 | 3 | 5 | 6, 5, 5 | Next a '+' is read, so 3 and 2 are popped from the stack and their sum 5, is pushed |
| 8 | 2 | 3 | 5 | 6, 5, 5, 8 | Next 8 is pushed |
| * | 5 | 8 | 40 | 6, 5, 40 | Now a '*' is seen, so 8 and 5 are popped as $8 * 5 = 40$ is pushed |
| + | 5 | 40 | 45 | 6, 45 | Next, a '+' is seen, so 40 and 5 are popped and $40 + 5 = 45$ is pushed |
| 3 | 5 | 40 | 45 | 6, 45, 3 | Now, 3 is pushed |
| + | 45 | 3 | 48 | 6, 48 | Next, '+' pops 3 and 45 and pushes $45 + 3 = 48$ is pushed |
| * | 6 | 48 | 288 | 288 | Finally, a '*' is seen and 48 and 6 are popped, the result $6 * 48 = 288$ is pushed |

EXAMPLE - EVALUATION OF POSTFIX

Evaluate the following postfix expression: 6 2 3 + - 3 8 2 / + * 2 ↑ 3 +

| SYMBOL | OPERAND 1 | OPERAND 2 | VALUE | STACK |
|--------|-----------|-----------|-------|------------|
| 6 | | | | 6 |
| 2 | | | | 6, 2 |
| 3 | | | | 6, 2, 3 |
| + | 2 | 3 | 5 | 6, 5 |
| - | 6 | 5 | 1 | 1 |
| 3 | 6 | 5 | 1 | 1, 3 |
| 8 | 6 | 5 | 1 | 1, 3, 8 |
| 2 | 6 | 5 | 1 | 1, 3, 8, 2 |
| / | 8 | 2 | 4 | 1, 3, 4 |
| + | 3 | 4 | 7 | 1, 7 |
| * | 1 | 7 | 7 | 7 |
| 2 | 1 | 7 | 7 | 7, 2 |
| ↑ | 7 | 2 | 49 | 49 |
| 3 | 7 | 2 | 49 | 49, 3 |
| + | 49 | 3 | 52 | 52 |

QUEUE ADT

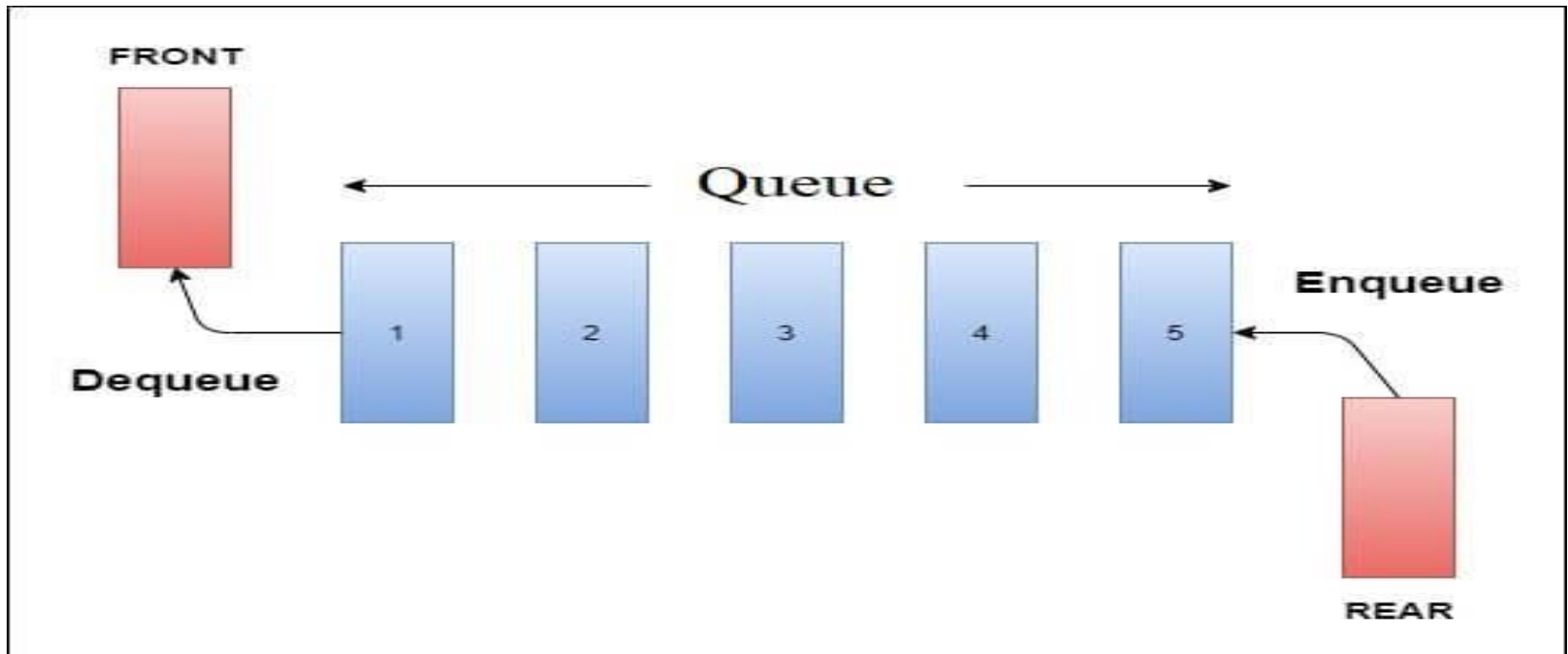
- A **queue** can be defined as an ordered list which enables **insert operations** to be performed at one end called **REAR** and **delete operations** to be performed at another end called **FRONT**.
- Queue is referred to be as **First In First Out (FIFO)** list.

Example: people waiting in line for a rail ticket form a queue.



QUEUE ADT

- **Enqueue:** The enqueue operation is used to **insert** the element at the rear end of the queue.
- **Dequeue:** The dequeue operation performs the **deletion** from the front-end of the queue. It also returns the element which has been removed from the front-end.



Queue ADT

```
public interface QueueADT
{
    //Adds one element to the rear of the queue
    public void enqueue(int element);
    //removes and returns the element at the front of the queue
    public int dequeue();
    //returns without removing the element at the front of the
queue
    public int first();
    //returns true if the queue contains no elements
    public boolean isEmpty();
    //returns the number of elements in the queue
    public int size();
}
```

QUEUE OPERATIONS

A queue is an object (an abstract data structure - ADT) that allows the following operations:

- **Enqueue:** Add an element to the end of the queue
- **Dequeue:** Remove an element from the front of the queue
- **IsEmpty:** Check if the queue is empty
- **Size:** Returns numbers of elements in the queue
- **First:** Get the value of the front of the queue without removing it

REPRESENTATION OF QUEUES

There are two ways to represent a queue in a memory:

Using an array

It uses a **one-dimensional array** and it is a better choice where a queue of **fixed size** is require.

Using an linked list

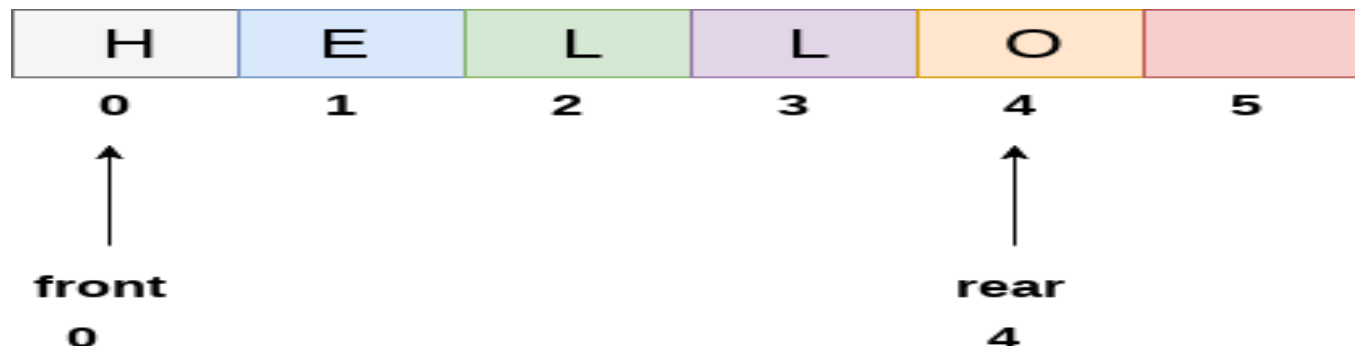
It uses a **double linked list** and provides a queue whose **size can vary during processing**.

Representation of a Queue using Array

A one dimensional array, say $Q[1..N]$, can be used to represent a queue.

Two pointers, namely FRONT and REAR, are used to indicate the two ends of the queue.

Insertion of new element, the pointer REAR will be the consultant and for deletion the pointer FRONT will be the consultant



Queue

Algorithm for Enqueue

1. If $\text{REAR} == \text{SIZE}$ then
2. print “Queue is full”
3. Exit
4. Else
5. If ($\text{REAR} == 0$) and ($\text{FRONT} == 0$) then // Queue
is empty
6. $\text{FRONT} = 1$
7. End If
8. $\text{REAR} = \text{REAR} + 1$
9. $\text{Q}[\text{REAR}] = \text{ITEM}$
10. End If
11. Stop

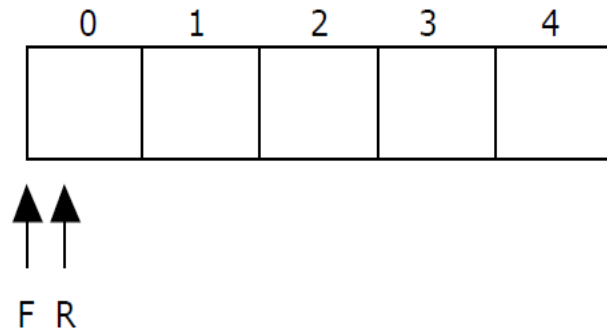
Algorithm for Dequeue

1. If (FRONT == 0) then
2. print “Queue is Empty”
3. Exit
4. Else
5. ITEM = QUEUE[FRONT]
6. If (FRONT == REAR) then
7. FRONT = 0
8. REAR = 0
9. Else
10. FRONT = FRONT + 1
11. End If
12. End If
13. Stop

Example

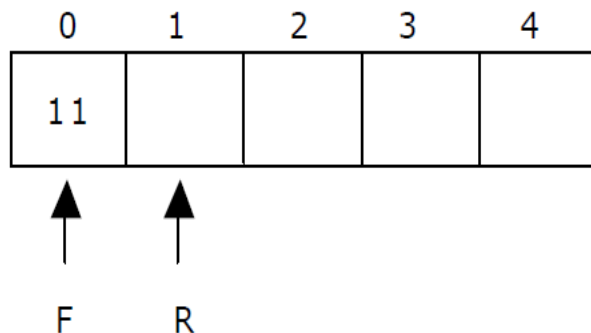
Representation of a Queue using Array

Let us consider a queue, which can hold maximum of five elements. Initially the queue is empty.



Queue Empty
FRONT = REAR = 0

Now, insert 11 to the queue. Then queue status will be:

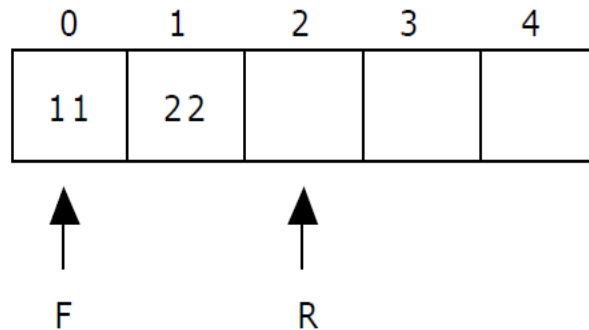


REAR = REAR + 1 = 1
FRONT = 0

Example

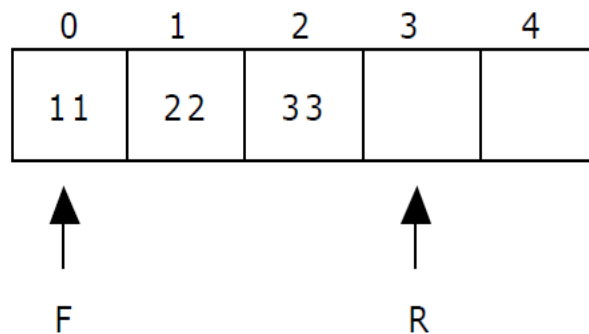
Representation of a Queue using Array

Next, insert 22 to the queue. Then the queue status is:



$REAR = REAR + 1 = 2$
 $FRONT = 0$

Again insert another element 33 to the queue. The status of the queue is:

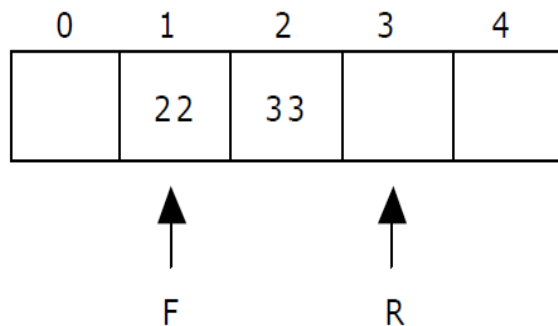


$REAR = REAR + 1 = 3$
 $FRONT = 0$

Example

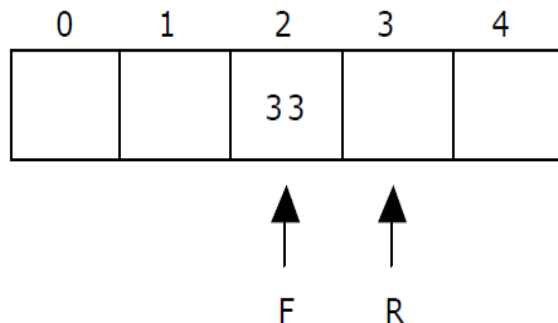
Representation of a Queue using Array

Now, delete an element. The element deleted is the element at the front of the queue. So the status of the queue is:



$REAR = 3$
 $FRONT = FRONT + 1 = 1$

Again, delete an element. The element to be deleted is always pointed to by the FRONT pointer. So, 22 is deleted. The queue status is as follows:

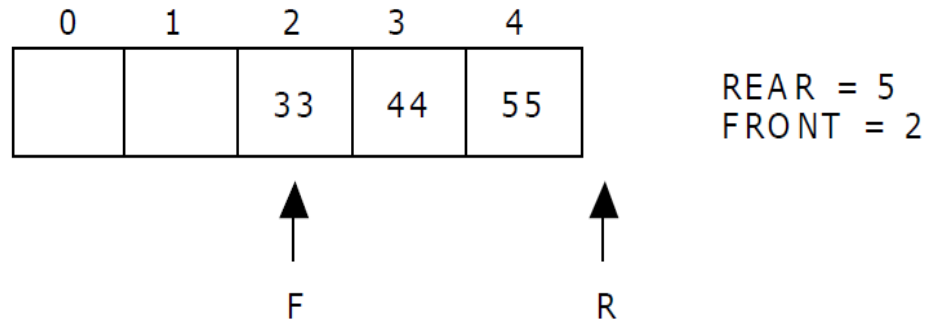


$REAR = 3$
 $FRONT = FRONT + 1 = 2$

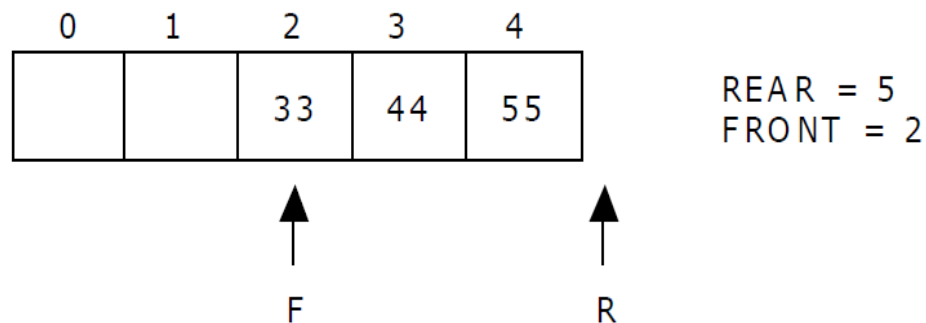
Example

Representation of a Queue using Array

Now, insert new elements 44 and 55 into the queue. The queue status is:



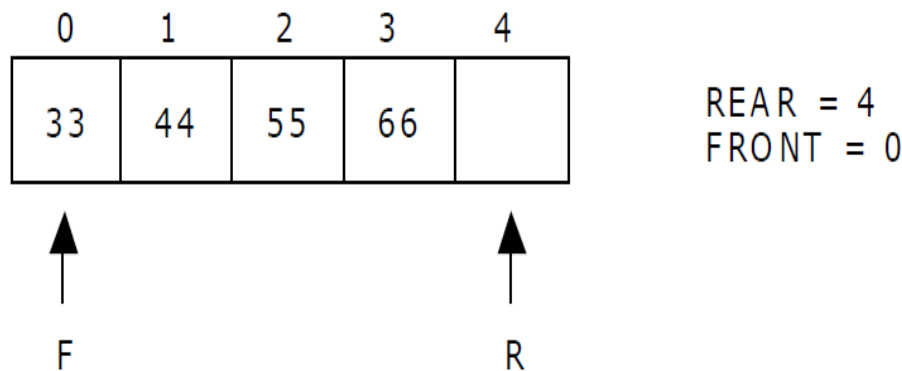
Next insert another element, say 66 to the queue. We cannot insert 66 to the queue as the rear crossed the maximum size of the queue (i.e., 5). There will be queue full signal. The queue status is as follows:



Example

Representation of a Queue using Array

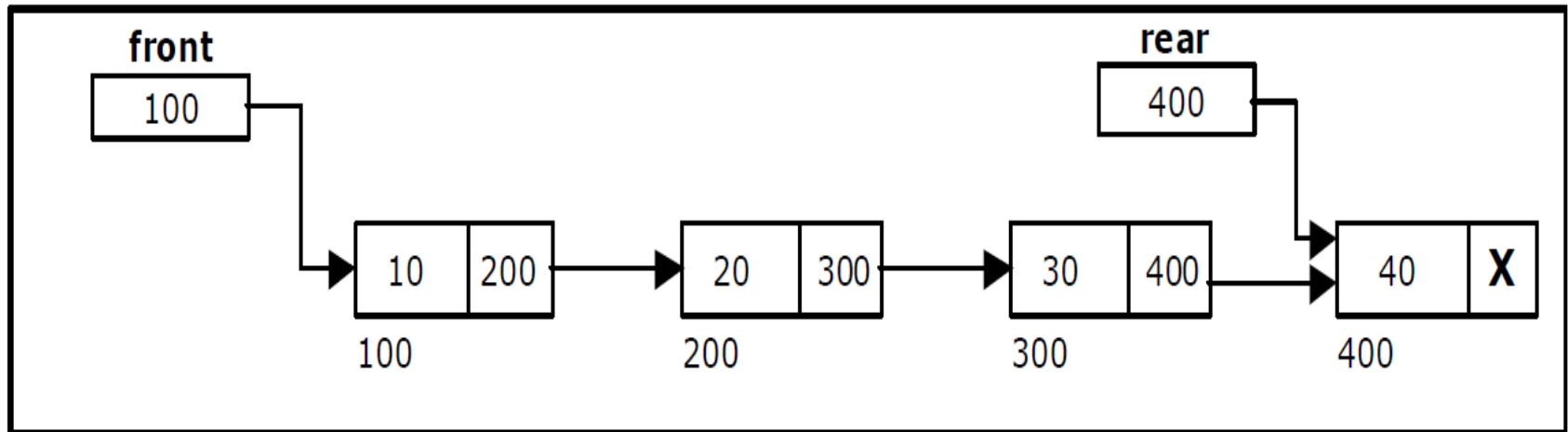
Now it is not possible to insert an element 66 even though there are two vacant positions in the linear queue. To overcome this problem the elements of the queue are to be shifted towards the beginning of the queue so that it creates vacant position at the rear end. Then the FRONT and REAR are to be adjusted properly. The element 66 can be inserted at the rear end. After this operation, the queue status is as follows:



This difficulty can overcome if we treat queue position with index 0 as a position that comes after position with index 4 i.e., we treat the queue as a **circular queue**.

Representation of a Queue using Linked List

Queue can be represented using **linked list**. In a queue data is deleted from the front end and inserted at the rear end. We can perform similar operations on the two ends of a list. We use two pointers **front** and **rear** for our linked queue implementation.



Applications of Queue

- It is used to schedule the jobs to be processed by the CPU.
- When multiple users send print jobs to a printer, each printing job is kept in the printing queue. Then the printer prints those jobs according to first in first out (FIFO) basis.
- Breadth first search uses a queue data structure to find an element from a graph.

Types of Queue

Queue in data structure is of the following types

Simple Queue (or) Linear Queue

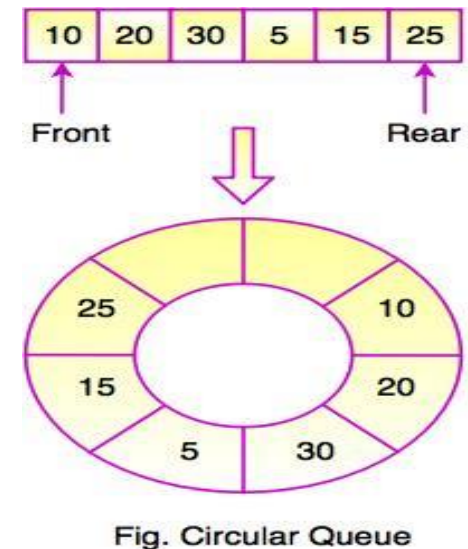
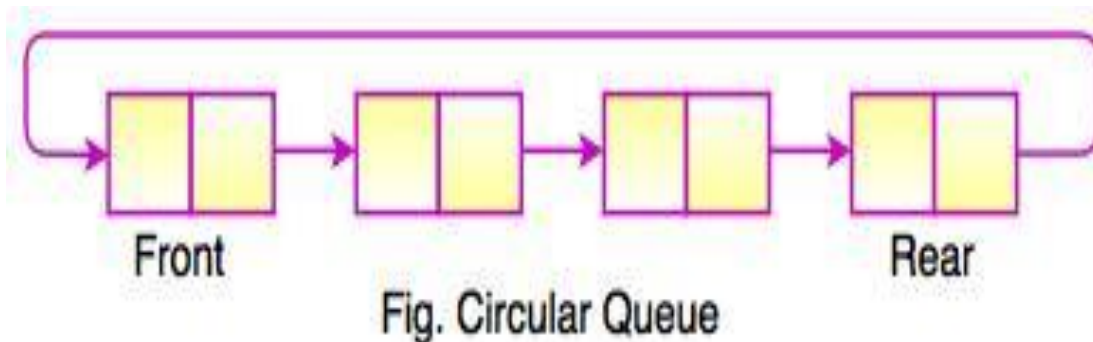
- A Queue is a linear structure which follows a particular order in which the operations are performed. The order is First In First Out (FIFO).
- Insertion occurs at the rear (end) of the queue and deletions are performed at the front (beginning) of the queue list.



Types of Queue

Circular Queue

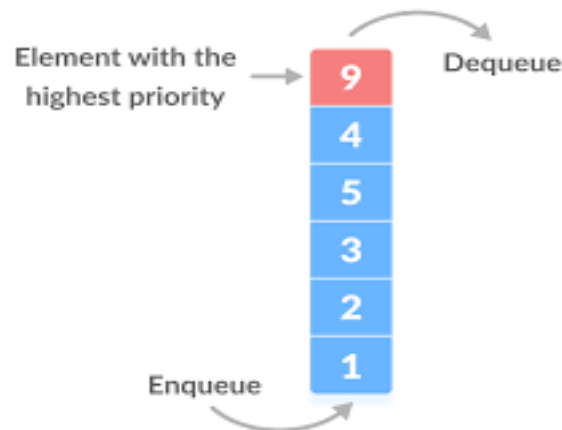
- In a circular queue, the last element points to the first element making a circular link.
- The main advantage of a circular queue over a simple queue is better memory utilization.
- If the last position is full and the first position is empty, we can insert an element in the first position. This action is not possible in a simple queue.



Types of Queue

Priority Queue

- A priority queue is a special type of queue in which each element is associated with a priority and is served according to its priority.
- If elements with the same priority occur, they are served according to their order in the queue.
- Insertion occurs based on the arrival of the values and removal occurs based on priority.



Types of Queue

Deque (Double Ended Queue)

- Double Ended Queue is also a Queue data structure in which the insertion and deletion operations are performed at both the ends (front and rear).
- That means, we can insert at both front and rear positions and can delete from both front and rear positions.



Circular Queue

- A circular queue is one in which the insertion of new element is done at the very first location of the queue if the last location of the queue is full.
- Suppose if we have a Queue of n elements then after adding the element at the last index i.e. $(n-1)$ th , as queue is starting with 0 index, the next element will be inserted at the very first location of the queue which was not possible in the simple linear queue.

Circular Queue Operations

- The Basic Operations of a circular queue are
 - **Insertion:** Inserting an element into a circular queue results in $\text{Rear} = (\text{Rear} + 1) \% \text{MAX}$, where MAX is the maximum size of the array.
 - **Deletion :** Deleting an element from a circular queue results in $\text{Front} = (\text{Front} + 1) \% \text{MAX}$, where MAX is the maximum size of the array.
 - **Travers:** Displaying the elements of a circular Queue.
- Circular Queue Empty: $\text{Front} = \text{Rear} = 0$.

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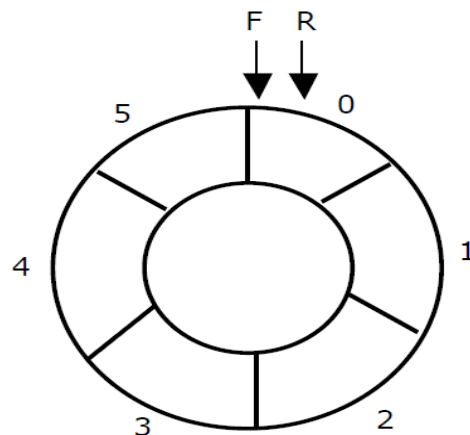
Representation of Circular Queue

Circular Queue can be created in three ways they are

1. Using single linked list
2. Using double linked list
3. Using arrays

Representation of Circular Queue using arrays:

Let us consider a circular queue, which can hold maximum (MAX) of six elements. Initially the queue is empty.

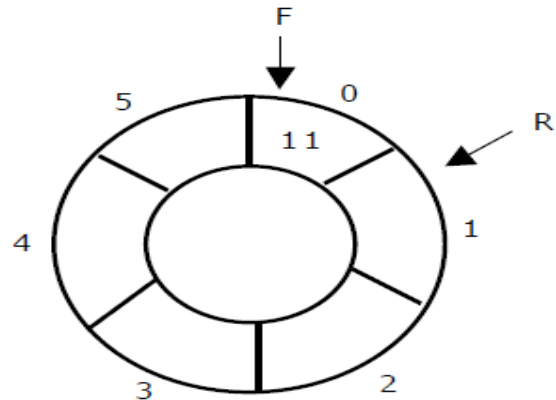


Circular Queue

```
Queue Empty
MAX = 6
FRONT = REAR = 0
COUNT = 0
```

Representation of Circular Queue

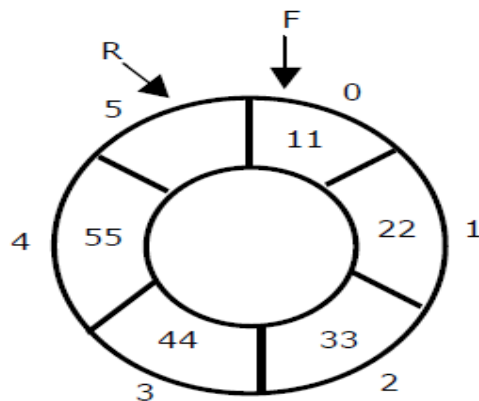
Now, insert 11 to the circular queue. Then circular queue status will be:



Circular Queue

FRONT = 0
 $REAR = (REAR + 1) \% 6 = 1$
 COUNT = 1

Insert new elements 22, 33, 44 and 55 into the circular queue. The circular queue status is:

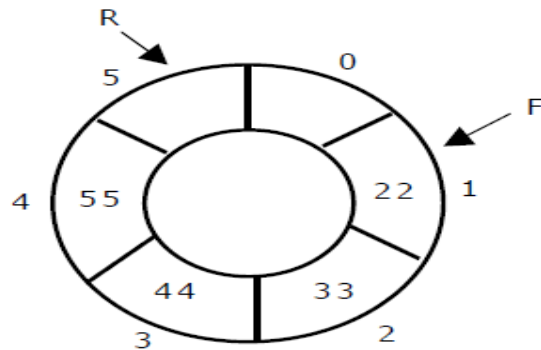


Circular Queue

FRONT = 0
 $REAR = (REAR + 1) \% 6 = 5$
 COUNT = 5

Representation of Circular Queue

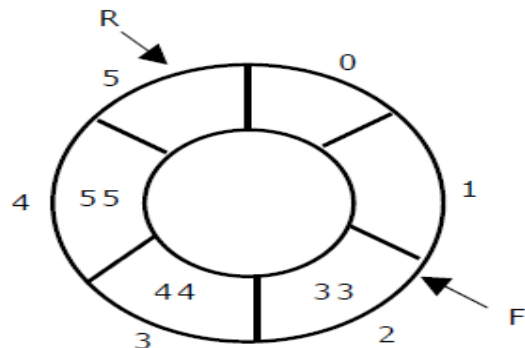
Now, delete an element. The element deleted is the element at the front of the circular queue. So, 11 is deleted. The circular queue status is as follows:



Circular Queue

$$\begin{aligned}\text{FRONT} &= (\text{FRONT} + 1) \% 6 = 1 \\ \text{REAR} &= 5 \\ \text{COUNT} &= \text{COUNT} - 1 = 4\end{aligned}$$

Again, delete an element. The element to be deleted is always pointed to by the FRONT pointer. So, 22 is deleted. The circular queue status is as follows:

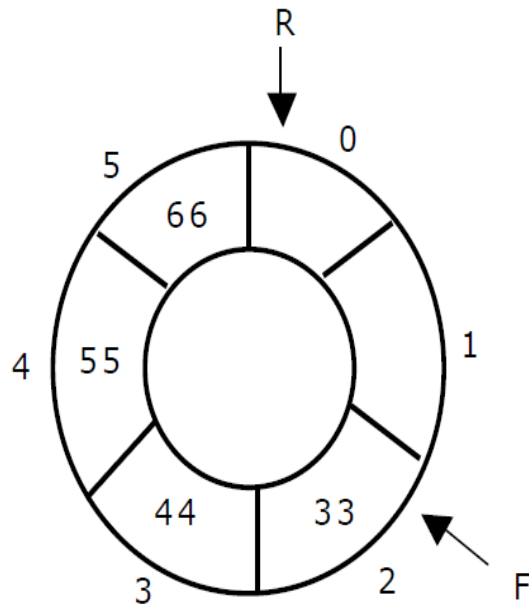


Circular Queue

$$\begin{aligned}\text{FRONT} &= (\text{FRONT} + 1) \% 6 = 2 \\ \text{REAR} &= 5 \\ \text{COUNT} &= \text{COUNT} - 1 = 3\end{aligned}$$

Representation of Circular Queue

Again, insert another element 66 to the circular queue. The status of the circular queue is:

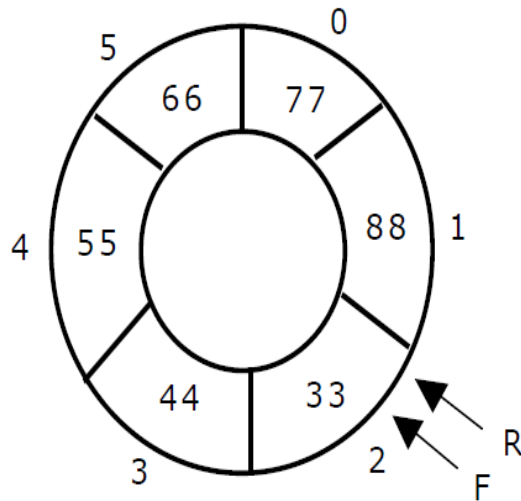


Circular Queue

```
FRONT = 2  
REAR = (REAR + 1) % 6 = 0  
COUNT = COUNT + 1 = 4
```

Representation of Circular Queue

Now, insert new elements 77 and 88 into the circular queue. The circular queue status is:



Circular Queue

FRONT = 2, REAR = 2
 $REAR = REAR \% 6 = 2$
 COUNT = 6

Now, if we insert an element to the circular queue, as $COUNT = MAX$ we cannot add the element to circular queue. So, the circular queue is *full*.

Algorithm for Enqueue into Circular Queue

Algorithm:

```
if (front == (rear+1)/max_size )           // queue is full
    write queue over flow
else:
    take the value
    if (front == -1) // queue is empty
        set front=rear=0
        queue[rear] = item
    else // queue is not empty
        rear=(rear+1)/max_size
        queue[rear]=item
end if
```


Algorithm for Dequeue into Circular Queue

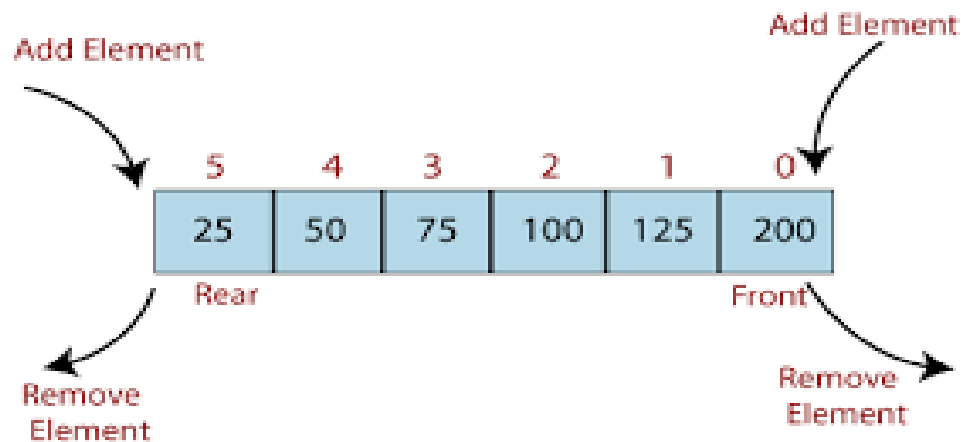
Algorithm:

```
if (front == -1) //queue is empty
    write queue is under flow
else:
    item=queue[front]
    if (front == rear) // queue contains a single element
        set front=rear=-1
    else
        front = (front+1)/max_size
end if
```

Deque

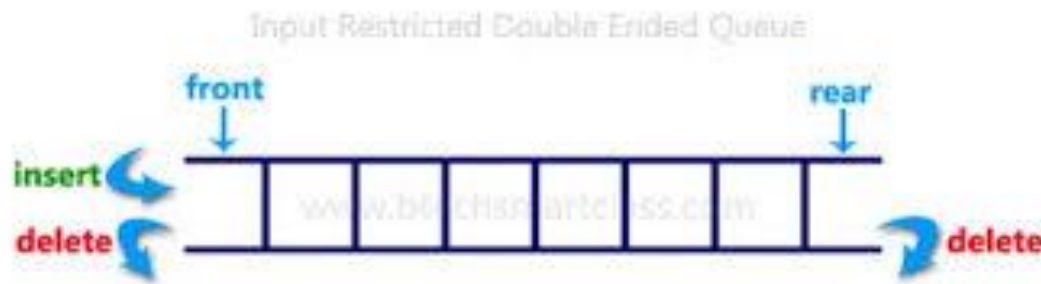
A **double-ended queue** (**dequeue**, often abbreviated to **deque**, pronounced *deck*) generalizes a queue, for which elements can be added to or removed from either the front (head) or back (tail).

It is also often called a **head-tail linked list**.



Types of Deque

- There are two variations of deque. They are:
 - Input restricted deque (IRD)
 - Output restricted deque (ORD)
- An Input restricted deque is a deque, which allows insertions at one end but allows deletions at both ends of the list.



Types of Deque

- An output restricted deque is a deque, which allows deletions at one end but allows insertions at both ends of the list.



Deque representation using Arrays

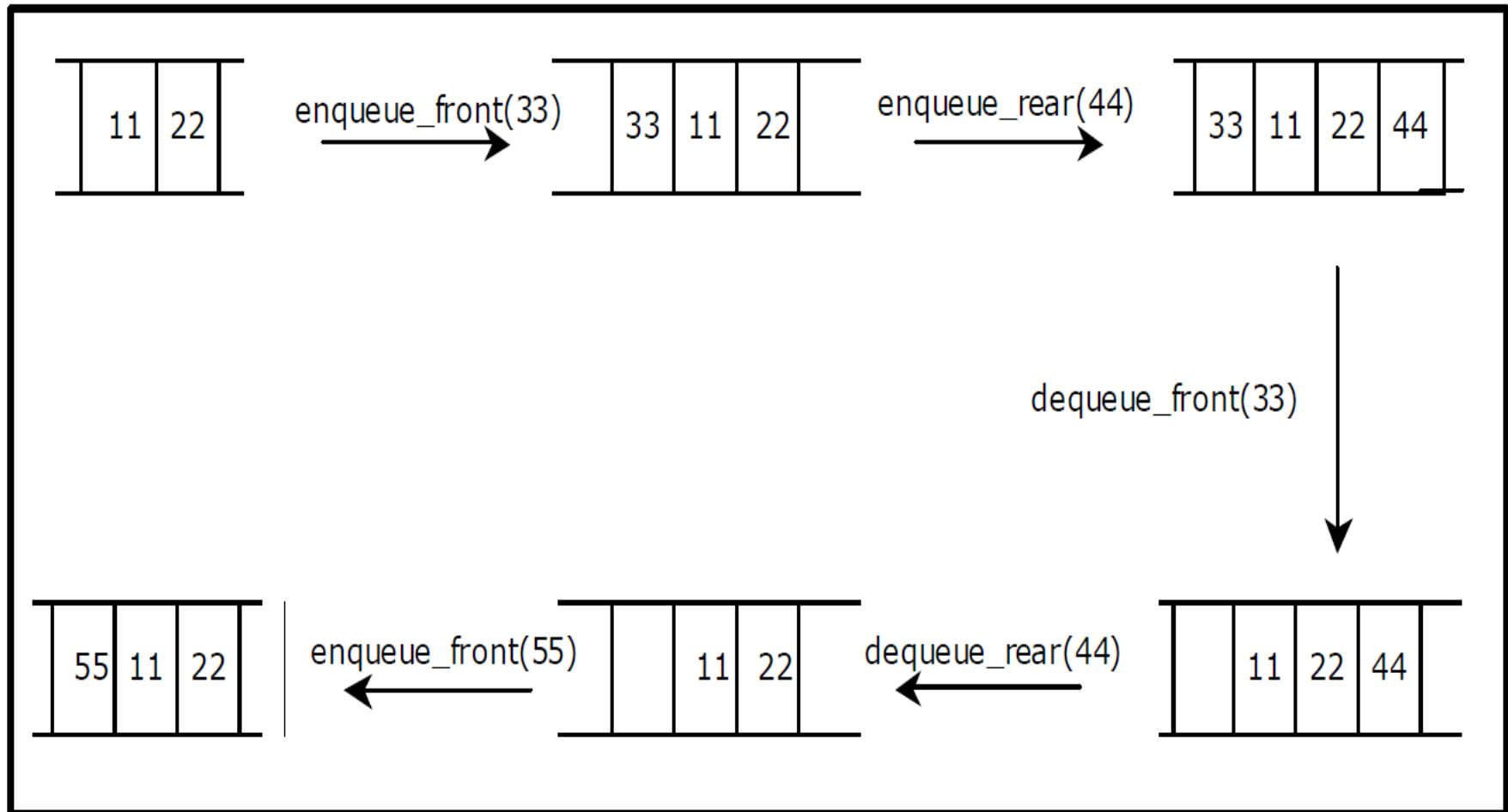
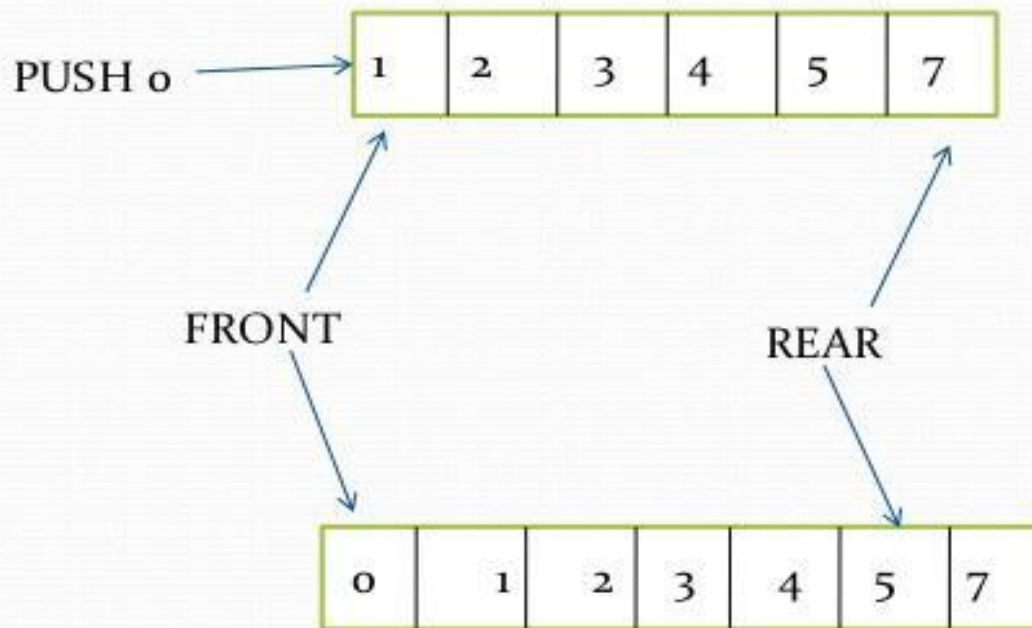


Figure 4.6. Basic operations on deque

Deque Operations

Insert_front

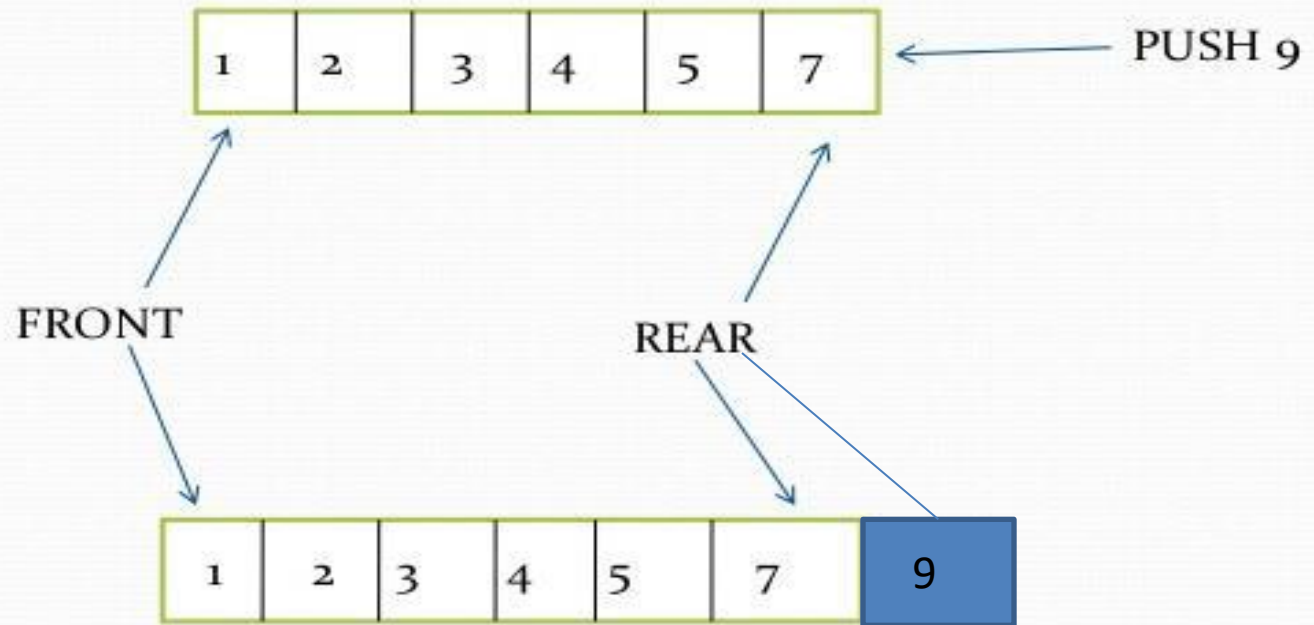
- `insert_front()` is a operation used to push an element into the front of the *Deque*.



Deque Operations

Insert_back

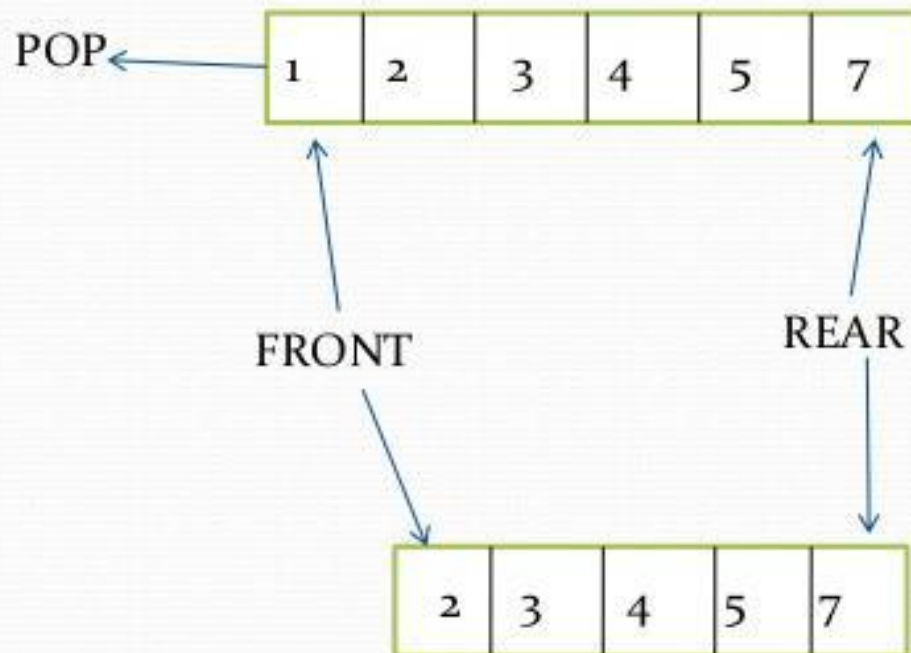
- insert_back() is a operation used to push an element at the back of a *Deque*.



Deque Operations

Remove_front

- remove_front() is a operation used to pop an element on front of the *Deque*.



Deque Operations

Remove_back

- `remove_front()` is a operation used to pop an element on front of the *Deque*.

