FOURIER SERIES

PART-A

1) Find the fourier series of the periodic function defined as $f(x) = \begin{cases} x, & 0<x<\pi/2 \\ \pi-x & \pi/2<x<\pi \end{cases}$ then

prove that $f(x) = \frac{\pi}{4}\left(\sin x - \frac{1}{3^2}\sin 3x + \frac{1}{5^2}\sin 5x - - - \right)$

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Given

$$f(x) = \begin{cases} x, & 0<x<\pi/2 \\ \pi-x & \pi/2<x<\pi \end{cases}$$

Given function is Half range sine series

$$\boxed{f(x) = \sum_{n=1}^{\infty} bn \sin(nx)\, dx}$$

$$\boxed{bn = \frac{2}{\pi}\int_0^{\pi} f(x) \sin nx\, dx}$$

$$bn = \frac{2}{\pi}\int_0^{\pi/2} x\sin nx\, dx + \frac{2}{\pi}\int_{\pi/2}^{\pi} (\pi-x)\sin nx\, dx$$

$$bn = \frac{2}{\pi}\left[(x)\left(\frac{-\cos nx}{n}\right) - (1)\left(\frac{-\sin nx}{n^2}\right)\right]_0^{\pi/2} + \frac{2}{\pi}\left[(\pi-x)\left(\frac{-\cos nx}{n}\right) - (-1)\left(\frac{-\sin nx}{n^2}\right)\right]_{\pi/2}^{\pi}$$

$$bn = \frac{2}{\pi}\left[\left(\frac{\pi}{2}\right)\left(\frac{-\cos(n\pi/2)}{n}\right) - (1)\left(\frac{-\sin(n\pi/2)}{n^2}\right) - \left\{\right.\right.$$

$(0)\left(-\frac{\cos n(0)}{n}\right) - (1)\left(-\frac{\sin n(0)}{n^2}\right)\Big\}\Big] + \frac{2}{\pi}$

$\Big[(\pi-\pi)\left(-\frac{\cos n\pi}{n}\right) - (-1)\left(-\frac{\sin n\pi}{n^2}\right) - \Big\{(\pi-\pi/2)$

$\left(-\frac{\cos n(\pi/2)}{n}\right) - (-1)\left(-\frac{\sin(n\pi/2)}{n^2}\right)\Big\}\Big]$

$\Rightarrow \frac{2}{\pi}\left[0 + \frac{1}{n^2} - (0)\right] + \frac{2}{\pi}\left[0 - 0 + 0 + \frac{1}{n^2}\right]$

$Bn = \frac{2}{\pi}\left[\frac{1}{n^2} + \frac{1}{n^2}\right]$

$bn = \frac{2}{\pi}\left(\frac{2}{n^2}\right)$

$bn = \frac{4}{\pi n^2}$

$f(x) = \sum_{n=1}^{\infty} bn \sin nx$

$f(x) = \sum_{n=1}^{\infty} \frac{4}{\pi n^2} \sin nx$

If $n = 1, 3, 5, 7$

$f(x) = \cancel{\sum_{n=1}^{\infty}}$

$f(x) = \frac{4}{\pi}\left(\frac{\sin(1)x}{(1)^2} + \frac{\sin 3x}{3^2} + \frac{\sin 5x}{5^2} + \dots\right) - ①$

putting $x = \pi/2$ in Eq ①

$f(x) = \frac{4}{\pi}\left(\sin\pi x + \frac{1}{3^2}\sin\left(\pi + \frac{\pi}{2}\right) + \frac{1}{5^2}\sin\left(2\pi + \frac{\pi}{2}\right) + \dots\right)$

$f(x) = \frac{4}{\pi}\Big(1 -$

$\sin(\pi + \frac{\pi}{2}) = (-1)$

$\sin(2\pi + \frac{\pi}{2}) = 1$

$f(x) = \frac{4}{\pi}\Big(\sin x - \frac{1}{3^2}\sin 3x + \frac{1}{5^2}\sin 5x) \cdots \Big)$

Hence proved.

2) If

$$f(x) = \begin{cases} -\pi & -\pi < x < 0 \\ x & 0 < x < 0 \end{cases}$$ Hence deduce that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\cdots = \frac{\pi^2}{8}$$

Given $f(x) = \begin{cases} -\pi & -\pi < x < 0 \\ x & 0 < x < 0 \end{cases}$

Given function in $(-\pi, \pi)$ it is niether function

$$\boxed{f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)}$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx.$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{0}(-\pi)\,dx + \frac{1}{\pi}\int_{0}^{\pi}(x)\,dx$$

$$= \frac{1}{\pi}(-\pi)\left[x\right]_{-\pi}^{0} + \frac{1}{\pi}\frac{1}{2}\left[x^2\right]_{0}^{\pi}$$

$$= -(\pi) + \frac{1}{2\pi}\left[\pi^2 - 0\right]$$

$$= -\pi + \frac{\pi}{2}$$

$$\boxed{a_0 = \frac{-\pi}{2}}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{0}(-\pi)\cos nx\,dx + \frac{1}{\pi}\int_{0}^{\pi}(x)\cos nx\,dx$$

$$a_n = \frac{1}{\pi}\left[(-\pi)\left(\frac{\sin nx}{n}\right)\right]_{-\pi}^{0} + \frac{1}{\pi}\left[(x)\left(\frac{\sin nx}{n}\right) - (1)\left(\frac{-\cos nx}{n^2}\right)\right]_{0}^{\pi}$$

a_n 17 $\quad \sin(0) = 0$

$\sin(\pi) = 0$

$$a_n = 0 + \frac{1}{\pi}\left[\frac{\cos nx}{n^2}\right]_{0}^{\pi}$$

$$a_n = \frac{1}{\pi}\left[\frac{(-1)^n}{n^2} - \frac{1}{n^2}\right]$$

$$\boxed{a_n = \frac{(-1)^n - 1}{\pi n^2}}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{0}(-\pi)\sin nx\,dx + \frac{1}{\pi}\int_{0}^{\pi}(x)\sin nx\,dx$$

$$= \frac{1}{\pi}\left[(-\pi)\left(\frac{-\cos nx}{n}\right)\right]_{-\pi}^{0} + \frac{1}{\pi}\left[(x)\left(\frac{-\cos nx}{n}\right) - (1)\left(\frac{-\sin nx}{n^2}\right)\right]_{0}^{\pi}$$

$$= -\left[\cancel{(-1) - \frac{(-1)^n}{n}}\right] + \cancel{\frac{1}{\pi}}\left[\cancel{-\pi\,\frac{(-1)^n}{n} + 0 - (0+0)}\right]$$

$$= -\left[-\frac{\cos nx}{n}\right]_{-\pi}^{0} + \frac{1}{\pi}\left[(x)\left[-\frac{\cos nx}{n}\right]\right]_{0}^{\pi}$$

$$= \frac{1-(-1)^n}{n} + \frac{1}{\pi}\left[\left\{-\pi\,\frac{(-1)^n}{n} + 0\right\} - \{0+0\}\right]$$

$$= \frac{1-(-1)^n}{n} + \frac{1}{\pi}\left[-\pi\,\frac{(-1)^n}{n}\right]$$

$$= \frac{1-(-1)^n}{n} - \frac{(-1)^n}{n}$$

$$\boxed{b_n = \frac{1-2(-1)^n}{n}}$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$$

$$f(x) = -\frac{\pi}{4} + \sum_{n=1}^{\infty}\left(\frac{(-1)^n - 1}{\pi n^2}\cos nx + \frac{1-2(-1)^n}{n}\sin nx\right)$$

__Deduce:-__

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$$

$$f(x) = -\frac{\pi}{4} + \sum_{n=1}^{\infty}\left(\frac{(-1)^n - 1}{\pi n^2}\cos nx + \frac{1-2(-1)^n}{n}\sin nx\right)$$

if $f(x) = 0$

$$\frac{-\pi + 0}{2} = -\frac{\pi}{4} + \sum_{n=1}^{\infty}\frac{(-1)^n - 1}{n^2\pi}$$

$$-\frac{\pi}{2} + \frac{\pi}{4} = \frac{1}{\pi}\left[\frac{-2}{1^2} + \frac{(-2)}{3^2} + \frac{(-2)}{5^2} + \cdots\right]$$

$$-\frac{\pi}{4} = \frac{-2}{\pi}\left(\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\right]$$

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$$

Hence proved.

3) If $f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi \le x \le 0 \\ 1 - \frac{2x}{\pi} & 0 \le x \le \pi \end{cases}$ Then find the values of a_0, a_n, b_n.

$f(x)$ is even function

$b_n = 0$

$$\boxed{f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nx)}$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$a_0 = \frac{2}{\pi}\int_0^{\pi}\left(1 - \frac{2x}{\pi}\right)dx$$

$$= \frac{2}{\pi}\left[x - \frac{x^2}{\pi}\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[\pi - \frac{\pi^2}{\pi}\right]$$

$$\boxed{a_0 = 0}$$

$$a_n = \frac{2}{\pi}\int_{-\pi}^{\pi} f(x)\,\cancel{dx}\cos nx\,dx$$

$$a_n = \frac{2}{\pi}\int_0^{\pi}\left(1-\frac{2x}{\pi}\right)\cos nx\,dx$$

$$= \frac{2}{\pi}\left[\left(1-\frac{2x}{\pi}\right)\left\{\frac{\sin nx}{n}\right) - \left(-\frac{2}{\pi}\right)\left(-\frac{\cos nx}{n^2}\right)\right]_0^{\pi}$$

$$= \left(-\frac{4}{\pi^2}\right)\left[\frac{(-1)^n}{n^2} - \frac{1}{n^2}\right]$$

$$a_n = \begin{cases} 0 \ ; & n \text{ is even} \\ -\frac{4}{n^2\pi^2} \ ; & n \text{ is odd} \end{cases}$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nx)$$

$$f(x) = 0 + \sum_{n=1}^{\infty}\left(\frac{-4}{\pi^2}\right)\left[\frac{(-1)^n-1}{n^2}\right]\cos nx$$

$$f(x) = \sum_{n=1}^{\infty}\left(\frac{-4}{\pi^2}\right)\left[\frac{(-1)^n-1}{n^2}\right]\cos nx$$

4) Find the fourier series ~~to~~ of the periodic function defined

as $f(x) = \begin{cases} -k & -\pi < x < 0 \\ k & 0 < x < \pi \end{cases}$ and hence show

that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \frac{\pi}{4}$.

Given function is ~~even function~~ odd function

constant even function

$\cancel{b_n = 0}$

$\boxed{f(-x) = -f(x)}$ odd

$$\cancel{f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx}$$

i) $\cancel{a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx}$

$\cancel{a_0 = \frac{2}{\pi}\int_{0}^{\pi} k\,dx}$

$a_0 = 0 \quad a_n = 0$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx\,dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} f(x)\sin nx\,dx + \int_0^{\pi} f(x)\sin nx\,dx\right]$$

$$= \frac{1}{\pi}\left[-k\int_{-\pi}^{0}\sin nx\,d + k\int_0^{\pi}\sin nx\,dx\right]$$

$$= \frac{k}{\pi}\left[-\left(-\frac{\cos nx}{n}\right)_{-\pi}^{0} + 1\left[-\frac{\cos nx}{n}\right]_0^{\pi}\right]$$

$$= \frac{k}{\pi}\left[-\left(-\frac{\cos 0}{n} + \frac{\cos n\pi}{n}\right) + \left(-\frac{\cos n\pi}{n} + \frac{\cos 0}{n}\right)\right]$$

$$= \frac{k}{\pi}\left[\frac{1}{n} - \frac{\cos n\pi}{n} - \frac{\cos n\pi}{n} + \frac{1}{n}\right]$$

$$= \frac{k}{\pi}\left[\frac{2 - 2\cos n\pi}{n}\right]$$

$$= \frac{2k}{\pi}\left[\frac{1-(-1)^n}{n}\right]$$

$$b_n = \begin{cases} \text{if } n \text{ odd} & \frac{4k}{n\pi} \\ \text{if } n \text{ Even} & 0 \end{cases}$$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx$$

$$\boxed{f(x) = \sum_{n=1}^{\infty} \frac{4k}{n\pi} \sin nx}$$

Deduce :-

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - - -$$

$$f(x) = \frac{4k}{n\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin nx$$

$$= \frac{4k}{\pi}\left[\frac{1}{1}\sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x + \cdots\right]$$

put $x = \pi/2$

$$k = \frac{4k}{\pi}\left[\frac{1}{1}\sin\frac{\pi}{2} + \frac{1}{3}\sin\frac{3\pi}{2} + \frac{1}{5}\sin\frac{5\pi}{2} + \cdots\right]$$

$$\cancel{k} = \frac{4\cancel{k}}{\pi}\left[\frac{1}{1}(1) + \frac{1}{3}(-1) + \frac{1}{5}(1) + \frac{1}{7}(-1) - - -\right]$$

$$\frac{\pi}{4} = \left[1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - - -\right]$$

Hence proved.

7) In the Expansion of $f(x) = \left(\frac{\pi - x}{2}\right)^2$ $0 < x < 2\pi$, find the value of an, bn

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx$$

$$= \frac{1}{\pi}\int_0^{2\pi}\left(\frac{\pi - x}{2}\right)^2$$

$$= \frac{1}{4\pi}\int_0^{2\pi}(\pi - x)^2$$

$$= \frac{1}{4\pi}\left[\frac{(\pi - x)^3}{-3}\right]_0^{2\pi}$$

$$= \frac{-1}{4\pi}\left[\frac{(\pi - x)^3}{3}\right]_0^{2\pi}$$

$$= -\frac{1}{4\pi}\left[\frac{(\pi - 2\pi)^3}{3} - \frac{(\pi - 0)^3}{3}\right]$$

$$= -\frac{1}{4\pi}\left[\frac{(-\pi)^3}{3} - \frac{\pi^3}{3}\right]$$

$$= -\frac{1}{4\pi}\left(\frac{-2\pi^3}{3}\right]$$

$$= \frac{2\pi^{\cancel{3}}}{12\cancel{\pi}} = \frac{\pi^2}{6}$$

$$\boxed{a_0 = \frac{\pi^2}{6}}$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\int_0^{2\pi}\frac{(\pi - x)^2}{4}\cos nx\,dx$$

$$= \frac{1}{4\pi}\left[(\pi-x)^2\left(\frac{\sin nx}{n}\right) - (-2(\pi-x))\left(-\frac{\cos nx}{n^2}\right) + (-2(0-1)\left(\frac{-\sin x}{n^3}\right)\right]_0^{2\pi}$$

$$= \frac{1}{4\pi}\left[(-2(\pi-x))\left(-\frac{\cos nx}{n^2}\right)\right]_0^{2\pi}$$

$$= -\frac{2}{4\pi}\left[(\pi-x)\left(\frac{\cos nx}{n^2}\right)\right]_0^{2\pi}$$

$$= -\frac{1}{2\pi}\left[\pi-2\pi\ \frac{1}{n^2} - \left(\frac{\pi}{n^2}\right)\right]$$

$$= -\frac{1}{2\pi}\left[\frac{\pi-2\pi}{n^2} - \frac{\pi}{n^2}\right]$$

$$= -\frac{1}{2\pi}\left(\frac{-2\pi}{n^2}\right)$$

$$\boxed{a_n = \frac{1}{n^2}}$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx\,dx$$

$$= \frac{1}{\cancel{4}\pi}\int_0^{2\pi}\left(\frac{\pi-x}{2}\right)^2 \sin nx\,dx$$

$$= \frac{1}{4\pi}\left[(\pi-x)^2\left(-\frac{\cos nx}{n}\right) - (-2(\pi-x))\left(-\frac{\sin nx}{n^2}\right) + -2(0-1)\left(\frac{\cos nx}{n^3}\right)\right]_0^{2\pi}$$

$$= \frac{1}{4\pi}\left[(\pi-x)^2\left(-\frac{\cos nx}{n}\right) + \left(\frac{2\cos nx}{n^3}\right)\right]_0^{2\pi}$$

$$= \frac{1}{4\pi}\left[-(\pi-2\pi)^2\frac{1}{n} + \frac{2}{n^3} - \left\{-(\pi-0)^2\frac{1}{n} + \frac{2}{n^3}\right)\right.$$

$$\frac{1}{4\pi}\left[-\frac{\pi^2}{n}+\frac{2}{n^3}+\frac{\pi^2}{n}-\frac{2}{n^3}\right]$$

$$\frac{1}{4\pi}[0]$$

$$\boxed{bn=0}$$

$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x) \sin nx \, dx$

$= \frac{1}{4\pi}\int_0^{2\pi} (\pi - x)^2 \sin nx \, dx.$

$= \frac{1}{4\pi}\left[(\pi-x)^2\left(-\frac{\cos nx}{n}\right) - (-2)(\pi - x)\left(-\frac{\sin nx}{n^2}\right) + -2(-1)\left(\frac{\cos nx}{n^3}\right)\right.$

$+ -2(-1)\left(\frac{\cos 2n\pi}{n^3}\right)\Big]_0^{2\pi}$

$= \frac{1}{4\pi}\left[(-\pi)^2 - \pi^2\left(-\cos\frac{n2\pi}{n}\right.\right.$

$= \frac{1}{4\pi}\left[-\frac{\pi^2}{n} + \frac{2}{n^3} + \frac{\pi^2}{n} - \frac{2}{n^3}\right] = 0$

$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx.$

$= \frac{\pi^2}{12} + \sum_{n=1}^{\infty}\left[\frac{1}{n^2}\cos(nx)\right]$

8) Determine the Fourier series representation

$x(t) = \begin{cases} t & 0 \le t < \pi \\ 2\pi - t & \pi \le t < 2\pi \end{cases}$

$a_0 = \frac{1}{\pi}\left[\int_0^{\pi} t \, dt + \int_{\pi}^{2\pi} (2\pi - t)\, dt\right.$

$= \frac{1}{\pi}\left[\left[\frac{t^2}{2}\right]_0^{\pi} + \left[\left(2\pi t - \frac{t^2}{2}\right]_{\pi}^{2\pi}\right] dt\right.$

$= \frac{1}{\pi}\left[\frac{\pi^2}{2} + \left[4\pi^2 - \frac{4\pi^2}{2} - \left[2\pi^2 - \frac{\pi^2}{2}\right]\right]\right)$

$= \frac{1}{\pi}\left[\frac{\pi^2}{2} + \left[2\pi^2 - \left[\frac{3\pi^2}{2}\right]\right]\right]$

$= \frac{1}{\pi}\left[\frac{\pi^2}{2} + \left[\frac{\pi^2}{2}\right]\right]$

$= \frac{2\pi^2}{2}\left(\frac{1}{\pi}\right]$

$\boxed{= \pi}$

$a_n =$

$a_n = \frac{1}{\pi}$

$a_n = \frac{1}{\pi}$

9)

$$a_n = \frac{1}{\pi}\int_0^{\pi} t\cos nt\,dt + \frac{1}{\pi}\int_{\pi}^{2\pi}(2\pi - t)\cos nt\,dt$$

$$a_n = \frac{1}{\pi}\left[t\left(\frac{\sin nt}{n}\right) + \frac{\cos nt}{n^2}\right]_0^{\pi} + \frac{1}{\pi}\left[(2\pi - t)\left(\frac{\sin nt}{n}\right) - (-1)\left(\frac{-\cos nt}{n^2}\right)\right]_{\pi}^{2\pi}$$

$$a_n = \frac{1}{\pi}\left[0 + \frac{(-1)^n}{n^2} + \frac{1}{n^2}\right] + \frac{1}{\pi}\left[-\frac{(-1)^n}{n^2} - \left[0 - \frac{(-1)^n}{n^2}\right]\right]$$

$$= \frac{1}{\pi}\left(\frac{(-1)^n + 1}{n^2}\right] + \cancel{\frac{1}{\pi}(-1)^{n+1}} + \frac{1}{\pi}\left[\frac{-(-1)^n + (-1)^n}{n^2}\right]$$

$$a_n = \frac{1}{\pi}\left[\frac{(-1)^n + 1}{n^2}\right]$$

$$b_n = \frac{1}{\pi}\int_0^{\pi} t\sin nt\,dt + \frac{1}{\pi}\int_{\pi}^{2\pi}(2\pi - t)\sin nt\,dt$$

$$= \frac{1}{\pi}\left[\frac{-t\cos nt}{n} + \frac{\sin nt}{n^2}\right]_0^{\pi} + \frac{1}{\pi}\left[(2\pi - t)\frac{-\cos nt}{n} - (-1)\left(\frac{-\sin nt}{n^2}\right)\right]_{\pi}^{2\pi}$$

$$= \frac{1}{\pi}\left[-\frac{\pi(-1)^n}{n}\right] + \frac{1}{\pi}\left[0 - \left[-\pi\frac{(-1)^n}{n}\right]\right]$$

$$= \frac{1}{\cancel{\pi}}\left[-\frac{\cancel{\pi}(-1)^n}{n}\right] + \frac{1}{\cancel{\pi}}\left(\frac{\cancel{\pi}(-1)^n}{n}\right)$$

$$= \frac{-(-1)^n + (-1)^n}{n} = 0$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n\cos nx + b_n\sin nx\right]$$

$$= \frac{\pi}{2} + \sum_{n=1}^{\infty}\frac{1}{\pi}\left(\frac{(-1)^n + 1}{n^2}\right]\cos nx$$

9) Find the Fourier series representation

$$f(x) = \begin{cases} 0, & -\pi < x < 0 \\ x^2, & 0 < x < \pi. \end{cases}$$

9) Find the Fourier series of the periodic function defined as

$$f(x) = \begin{cases} 0, & -\pi < x < 0 \\ x^2 & 0 < x < \pi \end{cases}$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

i) $$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\,dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} 0\,dx + \int_{0}^{\pi} x^2\,dx\right]$$

$$= \frac{1}{\pi}\left[\frac{x^3}{3}\right]_0^{\pi}$$

$$= \frac{1}{3\pi}\left[\pi^3 - 0\right]$$

$$\boxed{a_0 = \frac{\pi^2}{3}}$$

ii) $$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx\,dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} (0) \cos nx\,dx + \int_{0}^{\pi} x^2 \cos nx\,dx\right]$$

$$= \frac{1}{\pi}\left[x^2\left(\frac{\sin nx}{n}\right) - 2x\left(-\frac{\cos nx}{n^2}\right) + 2\left(\frac{-\sin nx}{n^3}\right)\right]_0^{\pi}$$

$$= \frac{1}{\pi}\left[+2x\frac{\cos nx}{n^2}\right]_0^{\pi}$$

$$= \frac{1}{\pi}\left[\frac{2\pi(-1)^n}{n^2} - 0\right]$$

$$\boxed{a_n = \frac{2(-1)^n}{n^2}}$$

iii) $b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$

$$b_n = \frac{1}{\pi}\left[\int_{-\pi}^{0}(0)\sin nx\,dx + \int_{0}^{\pi} x^2 \sin nx\,dx\right]$$

$$= \frac{1}{\pi}\left[x^2\frac{-\cos nx}{n} - (2x)\left(-\frac{\sin nx}{n^2}\right) + (2)\left(\frac{\cos nx}{n^3}\right)\right]_0^{\pi}$$

$$= \frac{1}{\pi}\left[(x^2)\left(-\frac{\cos nx}{n}\right) + 2\left(\frac{\cos nx}{n^3}\right)\right]_0^{\pi}$$

$$= \frac{1}{\pi}\left[\frac{-\pi^2(-1)^n}{n} + \frac{2(-1)^n}{n^3} - \left(-0 + \frac{2}{n^3}\right)\right]$$

$$= \frac{1}{\pi}\left(\frac{-\pi^2(-1)^n}{n} + \frac{2(-1)^n}{n^3} - \frac{2}{n^3}\right)$$

$$b_n = \frac{1}{\pi}\left(\frac{-\pi^2(-1)^n}{n} + \frac{2(-1)^n - 2}{n^3}\right)$$

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i) 10) Find the Fourier Series of the periodic function defined as

$$f(x) = \begin{cases} -\frac{1}{2}(\pi+x), & -\pi < x < 0 \\ \frac{1}{2}(\pi - x) & 0 < x < \pi \end{cases}$$

Given function is Niether function.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n\cos nx + b_n\sin nx)$$

$$a_0 = \frac{1}{\pi}\int_{0}^{\pi} f(x)\,dx.$$

$$a_0 = \frac{1}{\pi}\left[\int_{-\pi}^{0} -\frac{1}{2}(\pi+x)\,dx + \int_{0}^{\pi}\frac{1}{2}(\pi-x)\,dx\right]$$

$$a_0 = \frac{1}{2\pi}\left[\int_{-\pi}^{0} -\pi-x\,dx + \int_{0}^{\pi}\pi-x\,dx\right]$$

$$= \frac{1}{2\pi}\left[\left[-\pi(x)-\frac{x^2}{2}\right]_{-\pi}^{0} + \left[\pi x - \frac{x^2}{2}\right]_{0}^{\pi}\right]$$

$$= \frac{1}{2\pi}\left[-\pi(0)-\frac{0}{2}-\left\{-\pi(-\pi)-\frac{(-\pi)^2}{2}\right\} + \pi(\pi)-\frac{\pi^2}{2}-0\right]$$

$$= \frac{1}{2\pi}\left[\left(-\pi^2+\frac{\pi^2}{2}\right)+\pi^2-\frac{\pi^2}{2}\right]$$

$$= \frac{1}{2\pi}\left[-\pi^2+\cancel{\frac{\pi^2}{2}}+\pi^2-\cancel{\frac{\pi^2}{2}}\right]$$

$$= \frac{2\pi^2}{2\cancel{\pi}}$$

$$\boxed{a_0 = 0}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} -\frac{1}{2}(\pi+x)\cos nx\,dx + \int_{0}^{\pi}\frac{1}{2}(\pi-x)\cos nx\,dx\right]$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{0} -(\pi+x)\cos nx\,dx + \int_{0}^{\pi}(\pi+x)\cos nx\,dx\right]$$

$$= \frac{1}{2\pi}\left[\left[-(\pi+x)\left(\frac{\sin nx}{n}\right) - (-1)\left(-\frac{\cos nx}{n^2}\right)\right]_{-\pi}^{0} + \left[(\pi+x)\left(\frac{\sin nx}{n}\right) - (1)\left(-\frac{\cos nx}{n^2}\right)\right]_{0}^{\pi}\right]$$

$$= \frac{1}{2\pi}\left[\cancel{-(\pi+)} \frac{-1}{n^2} + \frac{(-1)^n}{n^2} + \frac{(-1)^n}{n^2} - \frac{1}{n^2}\right]$$

$$= \frac{1}{2\pi}\left[\frac{-2+2(-1)^n}{n^2}\right]$$

$$= \frac{2}{2\pi n^2}\left(-1+(-1)^n\right)$$

$$\boxed{a_n = \frac{1}{\pi n^2}\left(-1+(-1)^n\right)}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{0} -(\pi+x)\sin nx\,dx + \int_{0}^{\pi}(\pi+x)\sin nx\,dx\right]$$

$$= \frac{1}{2\pi}\left[\left[-(\pi+x)\left(-\frac{\cos nx}{n}\right) - (-1)\left(-\frac{\sin nx}{n^2}\right)\right]_{-\pi}^{0} + \left[(\pi+x)\left(-\frac{\cos nx}{n}\right) - (1)\left(-\frac{\sin nx}{n^2}\right)\right]_{0}^{\pi}\right]$$

$$= \frac{1}{2\pi}\left[+(\pi+0)\frac{1}{n} - 0 - \{0\} + \left[\cancel{+\pi^2} - \frac{\pi^2(-1)^n}{n}\right.\right.$$

$\frac{1}{2\pi}\left(\frac{\pi}{n} - \frac{\pi^2(-1)^n}{n}\right)$

$= \frac{\pi}{2n\pi}\left(1 - \pi(-1)^n\right).$

$b_n = \frac{1}{2n}\left(1 - \pi(-1)^n\right)$

$b_n =$

PART-B

1) Obtain the fourier series expansion of $f(x)$ given that $f(x) = (\pi - x)^2$ in $0 < x < 2\pi$ and deduce the value of

$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots = \frac{\pi^2}{6}$

Given

$f(x) = (\pi - x)^2$

$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx$

$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\,dx$

$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx\,dx$

$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left((a_n\cos nx) + (b_n\sin nx)\right)$

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} (\pi-x)^2 dx$$

$$= \frac{1}{\pi}\int_0^{2\pi} (\pi^2+x^2-2\pi x)\, dx$$

$$= \frac{1}{\pi}\left[\pi^2(x) + \frac{x^3}{3} - 2\pi\left(\frac{x^2}{2}\right)\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\pi^2(2\pi) + \frac{8\pi^3}{3} - \pi(4\pi^2) - [\pi^2(0)+0-0]\right]$$

$$= \frac{1}{\pi}\left[2\pi^3 + \frac{8\pi^3}{3} - 4\pi^3\right]$$

$$= \frac{1}{\pi}\left[\frac{8\pi^3}{3} - 2\pi^3\right]$$

$$= \frac{1}{\pi}\left[\frac{8\pi^3 - 6\pi^3}{3}\right]$$

$$= \frac{1}{\pi}\left(\frac{2\pi^3}{3}\right)$$

$$\boxed{a_0 = \frac{2\pi^2}{3}}$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} (\pi-x)^2 \cos nx\, dx$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} (\pi^2+x^2-2\pi x) \cos nx\, dx$$

$$= \frac{1}{\pi}\left[(\pi^2+x^2-2\pi x)\frac{\sin nx}{n} - (2x-2\pi)\left(\frac{-\cos nx}{n^2}\right) + (2)\left(\frac{-\sin nx}{n^3}\right)\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[(\pi-x)^2\left(\frac{\sin nx}{n}\right) - 2(x-\pi)\left(\frac{-\cos nx}{n^2}\right) + 2\left(\frac{-\sin nx}{n^3}\right)\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[(\pi-x)^2\left(\frac{\sin nx}{n}\right) - 2(x-\pi)\left(\frac{-\cos nx}{n^2}\right) + (2)\left(-\frac{\sin nx}{n^3}\right)\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[(\pi-2\pi)^2\left(\frac{\sin 2n\pi}{n}\right) - 2(2\pi-\pi)\left(-\frac{\cos 2n\pi}{n^2}\right) + 2\left(-\frac{\sin 2n\pi}{n^3}\right) - \left\{((\pi-0))^2\left(\frac{\sin n(0)}{n}\right) - 2(0-\pi)\left(\frac{-\cos n(0)}{n^2}\right) + 2\left(-\frac{\sin n(0)}{n^3}\right)\right\}\right]$$

$$= \frac{1}{\pi}\left[\pi^2(0) - (2\pi)\left(\frac{-1}{n^2}\right) + 2(0) - \left\{(\pi^2)(0) \cancel{+2\pi} - (2\pi)\left(\frac{1}{n^2}\right) + 2(0)\right\}\right]$$

$$= \frac{1}{\pi}\left[\frac{2\pi}{n^2} - \left\{\cancel{0} - \frac{2\pi}{n^2}\right.\right]$$

$$= \frac{1}{\pi}\left[\frac{4\pi}{n^2}\right]$$

$$= \frac{4}{n^2}$$

$$bn = \frac{1}{\pi}\int_0^{2\pi}(\pi-x)^2\left(\frac{\sin nx}{n}\right)$$

$$= \frac{1}{\pi}\left[(\pi-x)^2\left(\frac{-\cos nx}{n}\right) - 2(x-\pi)\left(-\frac{\sin nx}{n^2}\right) + (2)\left(\frac{\cos nx}{n^3}\right)\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\pi^2\left(\frac{-1}{n}\right) + 2\left(\frac{1}{n^3}\right) - \left\{(\pi^2)\left(\frac{-1}{n}\right) + 2\left(\frac{1}{n^3}\right)\right\}\right]$$

$$= \frac{1}{\pi}\left[-\frac{\pi^2}{n} + \frac{2}{n^3} + \frac{\pi^2}{n} - \frac{2}{n^3} \right]$$

$$\boxed{b_n = 0}$$

$$f(x) = \frac{\pi^3}{3} + \sum_{n=1}^{\infty} \frac{4}{n^2} \cos nx$$

deduce

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots = \frac{\pi^2}{6}$$

$$(\pi - x)^2 = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{4}{n^2} \cos nx$$

If $\boxed{x = 0}$.

$$\pi^2 = \frac{\pi^2}{3} + 4 \sum_{n=1}^{\infty} \frac{1}{n^2}$$

$$\pi^2 - \frac{\pi^2}{3} = 4 \sum_{n=1}^{\infty} \frac{1}{n^2}$$

$$\frac{3\pi^2 - \pi^2}{3} = 4 \sum_{n=1}^{\infty} \frac{1}{n^2}$$

$$\frac{2\pi^2}{12} = \sum_{n=1}^{\infty} \frac{1}{n^2}$$

$$\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2}$$

Hence proved.

2) Find the fourier series to represent the function $f(x) = x^3$ in $-\pi < x < \pi$

Given function is odd function in $-\pi$ to π

$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx$$

$$b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin nx\,dx.$$

$$b_n = \frac{2}{\pi}\int_0^{\pi} x^3 \sin nx\,dx$$

$$= \frac{2}{\pi}\left[x^3\left(-\frac{\cos nx}{n}\right) - (3x^2)\left(-\frac{\sin nx}{n^2}\right) + (6x)\left(\frac{\cos nx}{n^3}\right) - (6)\left(\frac{\sin nx}{n^4}\right)\right]_0^{\pi}.$$

$$= \frac{2}{\pi}\left[x^3\left(-\frac{\cos nx}{n}\right) + 6x\left(\frac{\cos nx}{n^3}\right)\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[-\left(\pi^3\frac{(-1)^n}{n}\right) + \frac{6\pi(-1)^n}{n^3} - \{0\}\right].$$

$$= \frac{2\cancel{\pi}}{\cancel{\pi}}\left(-\frac{\pi^2(-1)^n}{n} + \frac{6(-1)^n}{n^3}\right)$$

$$= \frac{2(-1)^n}{n}\left(-\pi^2 + \frac{6}{n^2}\right)$$

$$f(x) = \sum_{n=1}^{\infty} \frac{2(-1)^n}{n}\left(-\pi^2 + \frac{6}{n^2}\right)\sin nx$$

If $f(x) = x$, in $(-\pi, \pi)$, Find the Fourier series expansion for the function.

Given function is odd function in $(-\pi, \pi)$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx$$

$$b_n = \frac{2}{\pi} \int_0^{\pi} f(x) \sin nx \, dx$$

$$b_n = \frac{2}{\pi} \int_0^{\pi} x \sin nx \, dx$$

$$= \frac{2}{\pi} \left[x\left(-\frac{\cos nx}{n}\right) - (1)\left(-\frac{\sin nx}{n^2}\right) \right]_0^{\pi}$$

$$= \frac{2}{\pi} \left[\frac{-\pi(-1)^n}{n} + 0 - \{-0 + 0\} \right]$$

$$= \frac{2}{\pi} \left(-\frac{\pi(-1)^n}{n} \right)$$

$$= \frac{-2\cancel{\pi}}{n\cancel{\pi}} (-1)^n$$

$$b_n = \frac{-2}{n} (-1)^n$$

$$f(x) = \sum_{n=1}^{\infty} \frac{-2}{n} (-1)^n \sin nx$$

(1) Find the fourier series to represent the function

$f(x) = e^{ax}$ in $0 < x < 2\pi$.

Given function $f(x) = e^{ax}$ in $(0, 2\pi)$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$$a_0 = \frac{1}{\pi} \int_0^{2\pi} f(x)\, dx$$

$$a_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \cos nx\, dx$$

$$b_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \sin nx\, dx$$

$$a_0 = \frac{1}{\pi} \int_0^{2\pi} e^{ax}\, dx$$

$$= \frac{1}{\pi} \left[\frac{e^{ax}}{a}\right]_0^{2\pi}$$

$$= \frac{1}{\pi} \left[\frac{e^{a2\pi}}{a} - \frac{e^{a(0)}}{a}\right]$$

$$= \frac{1}{\pi} \left[\frac{e^{2\pi a}}{a} - \frac{1}{a}\right]$$

$$= \frac{1}{a\pi} \left(e^{2\pi(a)} - 1\right)$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} e^{ax}\cos nx\,dx$$

$$\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)$$

$$a_n = \frac{1}{\pi}\left[\frac{e^{ax}}{a^2+n^2}(a\cos nx + n\sin nx)\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\frac{e^{2\pi(a)}}{a^2+n^2}(a\cos n(2\pi) + n\sin n(2\pi)) - \left\{\frac{e^0}{a^2+n^2}(a\cos n(0) + n\sin n(0)\right\}\right]$$

$$= \frac{1}{\pi}\left[\frac{e^{2\pi(a)}}{a^2+n^2}(a(1)+0) - \left(\frac{1}{a^2+n^2}(a(1)+0)\right)\right]$$

$$= \frac{1}{\pi}\left[\frac{e^{2\pi(a)}}{a^2+n^2}(a) - \frac{1}{a^2n^2}(a)\right]$$

$$= \frac{a}{(a^2+n^2)\pi}\left(e^{2\pi(a)} - 1\right]$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} e^{ax}\sin nx\,dx.$$

$$\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\cancel{\cos}\,bx - b\sin bx)$$

$$= \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} e^{ax}\sin nx\,dx$$

$$= \frac{1}{\pi}\left[\frac{e^{ax}}{a^2+n^2}(a\sin nx - n\cos nx)\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\frac{e^{2\pi(a)}}{a^2+n^2}(a(0)-n(1)) - \left\{\frac{e^{a(0)}}{a^2+n^2}(a(0)-n(1))\right\}\right]$$

$$= \frac{1}{\pi}\left[\frac{e^{2\pi(a)}}{a^2+n^2}(-n) + \frac{1}{a^2+n^2}(n)\right]$$

$$= \frac{n}{(a^2+n^2)\pi}\left(e^{2\pi(a)} + 1\right)$$

5) Determine the Half range ~~series~~ sine Fourier series for the function $f(x) = \cos x$ for $0<x<\pi$

Given function half range cosine series

$f(x) = \cos x$

a_0 $a_n = 0$ ~~$b_n = 0$~~ b_n

$$\cancel{a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx}$$

$$\cancel{a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx.}$$

$a_0 = \frac{2}{\pi} \int_0^\pi \cos x \, dx$

$a_0 = \frac{2}{\pi} \left[\sin x \right]_0^\pi$

$a_0 = \frac{2}{\pi} [0]$

$\boxed{a_0 = 0}$

$a_n = \frac{2}{\pi} \int_0^\pi \cos x \cos(nx) \, dx$

$= \frac{2}{\pi}$

$b_n = \frac{2}{\pi} \int_0^\pi \cos x \sin nx \, dx$

$b_n = \frac{2}{\pi} \Big[\cos x \left(-\frac{\cos nx}{n} \right) - (\sin x) \left(-\frac{\sin nx}{n^2} \right)$

$+ (-\cos x) \left(\frac{\cos nx}{n^3} \right) \Big]_0^\pi$

$b_n = \frac{2}{\pi} \Big[\cos(\pi) \left(-\frac{\cos n(\pi)}{n} \right) + (-\cos \pi) \left(\frac{\cos n\pi}{n^3} \right)$

$- \left\{ \cos(0) \left(-\frac{\cos n(0)}{n} \right) + (-\cos(0)) \left(\frac{\cos n(0)}{n^3} \right) \right\} \Big]$

$b_n = \frac{2}{\pi} \left[\frac{-1 \, (-1)^n}{n} - (1) \left(\frac{(-1)^n}{n^3} \right) - \frac{(+1)}{n} + \frac{1}{n^3} \right]$

$\cancel{b_n} =$

5) Find the Fourier series expansion for the function $f(x) = x^2 \quad 0 < x < 2\pi$.

Given function $f(x) = x^2$

a_0, a_n, b_n

$$a_0 = \frac{1}{\pi} \int_0^{2\pi} f(x)\, dx$$

$$a_0 = \frac{1}{\pi} \int_0^{2\pi} x^2\, dx$$

$$= \frac{1}{\pi} \left[\frac{x^3}{3}\right]_0^{2\pi}$$

$$\frac{1}{\pi} \left[\frac{8\pi^3}{3} - 0\right]$$

$$a_0 = \frac{8\pi^2}{3}$$

$$a_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \cos nx\, dx$$

$$= \frac{1}{\pi} \int_0^{2\pi} x^2 \cos nx\, dx$$

$$= \frac{1}{\pi} \left[x^2\left(\frac{\sin nx}{n}\right) - (2x)\left(-\frac{\cos nx}{n^2}\right) + 2\left(-\frac{\sin nx}{n^3}\right)\right]_0^{2\pi}$$

$$= \frac{1}{\pi} \left[2x \frac{\cos nx}{n^2}\right]_0^{2\pi}$$

$$= \frac{1}{\pi} \left[4\pi \frac{1}{n^2} - 0\right]$$

$$a_n = \frac{4}{n^2}$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx\,dx$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} x^2 \sin nx\,dx$$

$$b_n = \frac{1}{\pi}\left[x^2\left(\frac{-\cos nx}{n}\right) - (2x)\left(\frac{-\sin nx}{n^2}\right) + (2)\left(\frac{\cos nx}{n^3}\right)\right]_0^{2\pi}$$

$$b_n = \frac{1}{\pi}\left[-x^2\left(\frac{\cos nx}{n}\right) + 2\left(\frac{\cos nx}{n^3}\right)\right]_0^{2\pi}$$

$$b_n = \frac{1}{\pi}\left[-4\pi^2\frac{1}{n} + \cancel{2\frac{1}{n^3}} - 0 - \cancel{\frac{2}{n^3}}\right]$$

$$= \frac{1}{\pi}\left(\frac{-4\pi^2}{n}\right)$$

$$= \frac{-4\pi}{n}$$

7) If $f(x) = \cos x$, Expand $f(x)$ as a Fourier series in the interval $(-\pi, \pi)$

Given function is even function

a_0 a_n $b_n = 0$

$$a_0 = \frac{1}{\pi}\int_0^{\pi} f(x)\,dx$$

$$a_0 = \frac{1}{\pi}\int_0^{\pi} \cos x\,dx$$

$$= \frac{1}{\pi}\left[\sin x\right]_0^{\pi}$$

$$\boxed{a_0 = 0}$$

$$a_n = \frac{1}{\pi}\int_0^{\pi} f(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\int_0^{\pi} \cos x \cos nx\,dx$$

$$\left(\cos a \cos b = \frac{\cos(a-b)}{2} + \frac{\cos(a+b)}{2}\right)$$

$$= \cancel{\frac{1}{2\pi}\int_0^{\pi} \cos(a-b)}$$

$$= \frac{1}{2\pi}\int_0^{\pi}\left[\frac{\cos(x-nx)}{2} + \frac{\cos(x+nx)}{2}\right]$$

$$= \frac{1}{2\pi}\left[\frac{\sin(1-n)x}{1-n} + \frac{\sin(1+n)x}{1+n}\right]_0^{\pi}$$

$$= \frac{1}{2\pi}\left[\frac{\sin\pi - \sin n\pi}{1-n} + \frac{\sin\pi + \sin n\pi}{1+n} - \left\{\frac{\sin(0) - \sin n(0)}{1-n} + \frac{\sin 0 + \sin n(0)}{1-n}\right\}\right]$$

$$= 0$$

8) Express the function $f(x) = x - \pi$ as Fourier series in the interval $(-\pi, \pi)$

Given function is Niether function in $(-\pi, \pi)$

$a_0 \quad a_n \quad b_n$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi}(x-\pi)\,dx$$

$$= \frac{1}{\pi}\left[\frac{x^2}{2} - \pi[x]\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[\frac{\pi^2}{2} - \pi^2 - \left\{\frac{\pi^2}{2} + \pi^2\right\}\right]$$

$$= \frac{1}{\pi}\left[\cancel{\frac{\pi^2}{2}} - \pi^2 - \cancel{\frac{\pi^2}{2}} - \pi^2\right]$$

$$= \frac{-2\cancel{\pi}^2}{\cancel{\pi}}$$

$$\boxed{a_0 = -2\pi}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi}(x-\pi)\cos nx\,dx$$

$$= \frac{1}{\pi}\left[(x-\pi)\left(\frac{\sin nx}{n}\right) - (1)\left(-\frac{\cos nx}{n^2}\right)\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[(\pi-\pi)(0) + \frac{(-1)^n}{n^2} - \left\{\frac{(-1)^n}{n^2}\right\}\right]$$

$$= \frac{1}{\pi}\left(\frac{(-1)^n}{n^2} - \frac{(-1)^n}{n^2}\right]$$

$$\boxed{a_n = 0}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi}(x-\pi)\sin nx\,dx.$$

$$= \frac{1}{\pi}\left[(x-\pi)\left(\frac{-\cos nx}{n}\right) - (1)\left(\frac{-\sin nx}{n^2}\right)\right]$$

$$= \frac{1}{\pi}\left(0 - 0 - \left\{-2\pi \frac{(-1)^n}{n} + 0\right\}\right)$$

$$= \frac{2\pi(-1)^{n+1}}{\pi n}$$

$$\boxed{b_n = \frac{2(-1)^{n+1}}{n}}$$

9) Find the Fourier series to represent the function $f(x) = e^{-ax}$ from $x = (-\pi, \pi)$ and hence deduce that

$$\frac{\pi}{\sinh \pi x} = 2\left[\frac{1}{2^2+1} - \frac{1}{3^2+1} + \frac{1}{4^2+1} - \cdots\right]$$

Given function is Niether function

a_0, a_n, b_n

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} e^{-ax}\,dx$$

$$= \frac{1}{\pi}\left[\frac{e^{-ax}}{-a}\right]_{-\pi}^{\pi}$$

$$= \frac{1}{a\pi} - \left[e^{-a(\pi)} - e^{+a\pi}\right]$$

$$\boxed{\sinh\theta = \frac{e^{\theta} - e^{-\theta}}{2}}$$

$$a_0 = \frac{2\sinh a\pi}{a\pi}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cdot \cos nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} e^{-ax}\cos nx\,dx$$

$$= \frac{1}{\pi}\left[\frac{e^{-ax}}{a^2+n^2}\left(-a\cos nx + n\,\cancel{\sin nx}\right)\right]_{-\pi}^{\pi}$$

$$= \frac{-a}{\pi(a^2+n^2)}\left[e^{-ax}\cos nx\right]_{-\pi}^{\pi}$$

$$= \frac{-a}{\pi(a^2+n^2)}\left[e^{-a\pi}\cos n\pi - e^{-a(-\pi)}\cos n(-\pi)\right]$$

$$= \frac{-a}{\pi(a^2+n^2)}\left[e^{-a\pi}(-1)^n - e^{a\pi}(-1)^n\right]$$

$$= \frac{a(-1)^n}{\pi(a^2+n^2)}\left[\frac{e^{a\pi}-e^{-a\pi}}{2}\right]\times 2$$

$$= \frac{2a(-1)^n}{\pi(a^2+n^2)}\sinh(a\pi)$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} e^{-ax}\sin nx\,dx$$

$$= \frac{1}{\pi}\left[\frac{e^{-ax}}{a^2+n^2}\left(-a\sin nx - n\cos nx\right)\right]_{-\pi}^{\pi}$$

0

$$b_n = \frac{1(-n)}{(a^2+n^2)\pi}\left[e^{-ax}\cos nx\right]_{-\pi}^{\pi}$$

$$= \frac{-n}{\pi(a^2+n^2)}\left[e^{-a\pi}\cos(n\pi) - e^{-a(-\pi)}\cos n(-\pi)\right]$$

$$= \frac{-n}{\pi(a^2+n^2)}\left[e^{-a\pi}(-1)^n - e^{a\pi}(-1)^n\right]$$

$$= \frac{n(-1)^n}{\pi(a^2+n^2}\left[\frac{e^{a\pi} - e^{-a\pi}}{2}\right] \times 2$$

a) $$b_n = \frac{2n(-1)^n}{\pi(a^2+n^2)}\sinh a\pi.$$

$$f(x) = e^{-ax} = \left(\frac{\frac{\cancel{2}}{\pi a}\sinh a\pi}{\cancel{2}} + \sum_{n=1}^{\infty}\frac{2a(-1)^n}{\pi(a^2+n^2)}\sinh a\pi\cos nx + \sum_{n=1}^{\infty}\frac{2n(-1)^n}{\pi(a^2+n^2)}\sinh a\pi\sin nx\right)$$

$$e^{-ax} = \frac{\sinh(a\pi)}{\pi}\left[\frac{1}{a} + \sum_{n=1}^{\infty}\frac{2a\cos nx(-1)^n}{(a^2+n^2)} + \frac{2n\sin nx(-1)^n}{a^2+n^2}\right]$$

Now put

$a=1$ & $x=0$

$$e^{-1(0)} = \frac{\sinh \pi}{\pi}\left(\frac{1}{1} + \sum_{n=1}^{\infty} \frac{-2(1).(1)(-1)^n}{(1^2+n^2)} + \frac{2n \cdot 0(-1)^n}{1^2+n^2}\right)$$

$$\frac{\pi}{\sinh \pi} = \frac{1}{1} + 2\sum_{n=1}^{\infty} \frac{(-1)^n}{1+n^2}$$

$$\frac{\pi}{\sinh \pi} = 1 + 2\left(\frac{-1}{1+1} + \frac{1}{1+2^2} + \frac{1}{1+3^2} + \frac{1}{1+4^2} \cdots\right)$$

$$\frac{\pi}{\sinh \pi} = 1 + 2\left(\frac{-1}{1^2+1} + \frac{1}{1+2^2} - \frac{1}{1+3^2} + \frac{1}{1+4^2} \cdots\right)$$

Hence proved.

10) Expand the function $f(x) = \left(\frac{\pi - x}{2}\right)^2$ as a Fourier series in the interval $0<x<2\pi$ hence deduce that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \cdots \frac{\pi^2}{12}$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\, dx$$

$$= \frac{1}{\pi}\int_0^{2\pi} \left(\frac{\pi - x}{2}\right)^2$$

$$= \frac{1}{4\pi}\left[\frac{(\pi - x)^3}{-3}\right]_0^{2\pi}$$

$$= \frac{-1}{4\pi}\left[\frac{(\pi - x)^3}{3}\right]_0^{2\pi}$$

$$= \frac{1}{4\pi}\left[\frac{(-\pi)^3}{3} - \frac{\pi^3}{3}\right]$$

$$= \frac{1}{4\pi}\left(-\frac{2\pi^3}{3}\right]$$

$$\frac{2\pi^3}{12\pi} = \frac{\pi^2}{6}$$

$$\boxed{a_0 = \frac{\pi^2}{6}}$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\int_0^{2\pi}\left(\frac{\pi-x}{2}\right)^2 \cos nx\,dx$$

$$= \frac{1}{4\pi}\ (\pi-x)^2\left(\frac{\sin nx}{n}\right) - \left(-2(\pi-x)\right)\left(-\frac{\cos nx}{n^2}\right)$$

$$+ \left(-2(0-1)\right)\left(-\frac{\sin nx}{n^3}\right)\Bigg]_0^{2\pi}$$

$$= \frac{1}{4\pi}\left[-\left(-2(\pi-x)\right)\left(-\frac{\cos nx}{n^2}\right)\right]_0^{2\pi}$$

$$= -\frac{1}{2\pi}\left[\left((\pi-x)\right)\frac{\cos nx}{n^2}\right]_0^{2\pi}$$

$$= \frac{1}{2\pi}\left[\frac{\pi}{n^2} + \frac{\pi}{n^2}\right]$$

$$\boxed{a_n = \frac{1}{n^2}}$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx\,dx$$

$$= \frac{1}{\pi}\int_0^{2\pi} \left(\frac{\pi-x}{2}\right)^2 \sin nx\,dx$$

$$= \frac{1}{4\pi}\int_0^{2\pi} (\pi-x)^2 \sin nx\,dx$$

$$= \frac{1}{4\pi}\left[(\pi-x)^2\left(\frac{\sin nx}{n}\right) - (-2(\pi-x))\left(-\frac{\cos nx}{n^2}\right) + (-2(0-1))\left(-\frac{\sin nx}{n^3}\right)\right]_0^{2\pi}$$

$$= \frac{1}{4\pi}\left[-(-2(\pi-x))\left(-\frac{\cos nx}{n^2}\right)\right]_0^{2\pi}$$

$$= \frac{-1}{2\pi}\left[((\pi-x)\right.$$

$$= \frac{1}{4\pi}\left[(\pi-x)^2\left(-\frac{\cos nx}{n}\right) - (-2(\pi-x))\left(-\frac{\sin nx}{n^2}\right) + 2(0-1)\left(\frac{\cos nx}{n^3}\right)\right]_0^{2\pi}$$

$$= \frac{1}{4\pi}\left[(\pi-x)^2\left(-\frac{\cos nx}{n}\right) + (-2(0-1))\left(\frac{\cos nx}{n^3}\right)\right]_0^{2\pi}$$

$$= \frac{1}{4\pi}\left[-\frac{\pi^2}{n} + \cancel{\frac{2}{n^3}} + \frac{\pi^2}{n} - \cancel{\frac{2}{n^3}}\right]$$

$$b_n = 0$$

Deduces

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \cdots = \frac{\pi^2}{12}$$

$$f(x) = \frac{\pi^2}{12} + \sum_{n=1}^{\infty}\left[\frac{1}{n^2}\cos nx\right]$$

$$\left(\frac{\pi - x}{2}\right)^2 = \frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{1}{n^2}\cos nx$$

If $x = \pi$

$$0 = \frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$$

$$-\frac{\pi^2}{12} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$$

$$-\frac{\pi^2}{12} = \frac{-1}{1^2} + \frac{2}{2^2} - \frac{1}{3^2} \cdots$$

$$\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{\cancel{2}}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} \cdots$$

Hence proved.

11) Find the Fourier series to represent the function $f(x) = x - x^2$ in $(-\pi, \pi)$

It is a niether function

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi}(x-x^2)\,dx$$

$$= \frac{1}{\pi}\left[\frac{x^2}{2}-\frac{x^3}{3}\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[\frac{\pi^2}{2}-\frac{\pi^3}{3}-\left\{\frac{\pi^2}{2}+\frac{\pi^3}{3}\right\}\right]$$

$$= \frac{1}{\pi}\left[\cancel{\frac{\pi^2}{2}}-\frac{\pi^3}{3}-\cancel{\frac{\pi^2}{2}}-\frac{\pi^3}{3}\right]$$

$$= \frac{-2\pi^3}{3\pi}$$

$$\boxed{a_0 = -\frac{2\pi^2}{3}}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi}(x-x^2)\cos nx\,dx$$

$$= \frac{1}{\pi}\left[(x-x^2)\overbrace{\left(\frac{\sin nx}{n}\right)}^{0} - (1-2x)\left(-\frac{\cos nx}{n^2}\right) + (-2)\overbrace{\left(-\frac{\sin nx}{n^3}\right)}^{0}\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[(1-2x)\frac{\cos nx}{n^2}\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[(1-2\pi)\frac{(-1)^n}{n^2} - (1+2\pi)\left(\frac{(-1)^n}{n^2}\right)\right]$$

$$= \frac{(-1)^n}{\pi n^2}\left(\cancel{1}-2\pi-\cancel{1}-2\pi\right)$$

$$= \frac{(-1)^n}{\pi n^2}(-4\pi)$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} (x-x^2)\sin nx\,dx$$

$$= \frac{1}{\pi}\left[(x-x^2)\left(\frac{-\cos nx}{n}\right) - (1-2x)\left(\frac{-\sin nx}{n^2}\right)^{0} + (-2)\left(\frac{\cos nx}{n^3}\right)\right]$$

$$= \frac{1}{\pi}\left[(x-x^2)\left(\frac{-\cos nx}{n}\right) - 2\left(\frac{\cos nx}{n^3}\right)\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left((\pi-\pi^2)\frac{-(-1)^n}{n} - 2\frac{(-1)^n}{n^3} - \left\{(-\pi-\pi^2)\cdot\frac{-(-1)^n}{n} - 2\frac{(-1)^n}{n^3}\right)\right.$$

$$= \frac{1}{\pi}\left(-(\pi-\pi^2)\frac{(-1)^n}{n} - \cancel{2\frac{(-1)^n}{n^3}} - (\pi+\pi^2)\frac{(-1)^n}{n} + \cancel{2\frac{(-1)^n}{n^3}}\right)$$

$$= \frac{1}{\pi}\left((-\pi+\cancel{\pi^2}-\pi-\cancel{\pi^2})\frac{(-1)^n}{n}\right)$$

$$= \frac{-2\cancel{\pi}}{\cancel{\pi}}\frac{(-1)^n}{n}$$

$$b_n = \frac{2(-1)^{n+1}}{n}$$

(8) Determine the Half range sine series for $f(x) = x(\pi - x)$, $0 < x < \pi$. Deduce that

$$\frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} \cdots = \frac{\pi^3}{32}$$

Given

$f(x) = x(\pi - x)$

$$b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin nx\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi} x(\pi - x)\sin nx\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi} (\pi x - x^2)\sin nx\,dx.$$

$$= \frac{2}{\pi}\left[(\pi x - x^2)\left(\frac{-\cos nx}{n}\right) - (\pi - 2x)\left(\frac{-\sin nx}{n^2}\right) + (-2)\frac{\cos nx}{n^3}\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[\frac{2}{n^3}(1 - \cos n\pi)\right]$$

$$= \frac{4}{n\pi^3}\left(1 - (-1)^n\right]$$

$$b_n = \begin{cases} 0 & \text{when } n \text{ is even} \\ \frac{8}{\pi n^3} & \text{when } n \text{ is odd} \end{cases}$$

Hence

$$x(\pi - x) = \sum_{n=1,3,5}^{\infty} \frac{8}{\pi n^3}\sin nx \ (\text{odd})$$

$$x(\pi-x) = \frac{8}{\pi}\left(\sin x + \frac{\sin 3x}{3^3} + \frac{\sin 5x}{5^3} + \cdots\right) \quad -(1)$$

Deduction

putting $x=\frac{\pi}{2}$ in (1), we get

$$\cancel{\frac{\pi}{2}\left(x-\frac{\pi}{2}\right)}$$

$$\frac{\pi}{2}\left(\pi-\frac{\pi}{2}\right) = \frac{8}{\pi}\left(\sin\frac{\pi}{2} + \frac{\sin 3\pi/2}{3^3} + \frac{\sin 5\pi/2}{5^3} \cdots\right)$$

$$\frac{\pi}{2}\left(\frac{\pi}{2}\right) = \cancel{\frac{8}{\pi}\left(1-\frac{1}{3^3}\right.}$$

$$\frac{\pi^2}{4} = \frac{8}{\pi}\left(\sin\left(\frac{\pi}{2}\right) + \frac{\sin\left(\pi+\frac{\pi}{2}\right)}{3^3} + \frac{\sin\left(2\pi+\frac{\pi}{2}\right)}{5^3} \cdots\right)$$

$$\frac{\pi^3}{32} = 8\left(\frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} \cdots\right)$$

Hence proved.

13) Express $f(x) = e^{-x}$ as a Fourier series in the interval $(-l, l)$.

$(c, c+2L) = (-l, l)$.

$c = -l$

$c+2L = l$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n\cos\left(\frac{n\pi x}{L}\right) + b_n\sin\left(\frac{n\pi x}{L}\right)\right]$$

$$a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)\,dx$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos nx\,dx$$

$bn = \frac{1}{l} \int_{-l}^{l} f(x) \sin n x$

$c+c$ $\quad c, c+2L = (-l, l)$

$c = -l$

$c+2l = -l + 2(L)$

$2L = +l$

$\boxed{L = +l/2}$

$l = 1$

$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) + b_n \sin\left(\frac{n\pi x}{l}\right)$

$a_0 = \frac{1}{l} \int_{-l}^{l} f(x)\,dx = \frac{1}{l} \int_{-1}^{1} e^{-x} dx = \left[\frac{e^{-x}}{-1}\right]_{-1}^{1}$

$a_0 = e - e^{-1} = 2\sinh$

$a_n = \frac{1}{l} \int_{-l}^{l} f(x) \cos\left(\frac{n\pi x}{l}\right) dx$

$= \int_{-1}^{1} e^{-x} \cos n\pi x \, dx$

$= \left[\frac{e^{-x}}{1+n^2\pi^2}\left(-\cos n\pi x + n\pi \sin \pi x\right)\right]_{-1}^{1}$

$= \frac{1}{1+n^2\pi^2}\left[-e^{-1}(-1)^n + e(-1)^n\right]$

$= \frac{(-1)^n}{1+n^2\pi^2}\left[e - e^{-1}\right]$

$a_n = \frac{(-1)^n 2\sinh 1}{1+n^2\pi^2}$

$$b_n = \frac{1}{l}\int_{-l}^{l} f(x)\sin\left(\frac{n\pi x}{l}\right)dx$$

$$= \int_{-1}^{1} e^{-x}\sin n\pi x\, dx$$

$$= \left[\frac{e^{-x}}{1+n^2\pi^2}\left(-\sin n\pi x - n\pi\cos n\pi x\right)\right]_{-1}^{1}$$

$$= \frac{1}{1+n^2\pi^2}\left[e^{-1}\left(-n\pi\cos n\pi - e\left(-n\pi\cos n\pi\right)\right)\right]$$

$$= \frac{(-1)^n n\pi}{1+n^2\pi^2}\left[e - e^{-1}\right]$$

$$b_n = (-1)^n \frac{2n\pi\sinh 1}{1+n^2\pi^2}$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos\frac{n\pi x}{l} + b_n\sin\frac{n\pi x}{l}$$

$$f(x) = \sinh 1\left[1 + 2\sum_{n=1}^{\infty}\left(\frac{(-1)^n\cos n\pi x}{1+n^2\pi^2} + \frac{(-1)^n n\pi\sin n\pi x}{1+n^2\pi^2}\right)\right]$$

Find the Fourier series of periodicity 3 for the function $f(x) = (2x - x^2)$ in $(0,3)$

$c, c+2l = (0,3)$

$\boxed{c = 0}$

$c + 2l = 3$

$0 + 2l = 3$

$\boxed{l = 3/2}$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx$$

$$a_0 = \frac{1}{l} \int_c^{c+2l} f(x)\,dx$$

$$= \frac{1}{3/2} \int_0^3 (2x - x^2)\,dx$$

$$= \frac{2}{3} \left[\frac{\cancel{2}x^2}{\cancel{2}} - \frac{x^3}{3} \right]_0^3$$

$$= \frac{2}{3} \left(9 - \frac{27}{3} - (0) \right]$$

$\boxed{a_0 = 0}$

$$a_n = \frac{1}{l} \int_c^{c+2l} f(x) \cos\left(\frac{n\pi x}{l}\right) dx$$

$$= \frac{2}{3} \int_0^3 (2x - x^2) \cos\left(\frac{n\pi x}{l}\right) dx$$

$$= \frac{2}{3} \left[(2x - x^2) \frac{\sin\left(\frac{2n\pi x}{3}\right)}{\left(\frac{2n\pi}{3}\right)} - (2 - 2x) \frac{-\cos\left(\frac{2n\pi x}{3}\right)}{\left(\frac{2n\pi}{3}\right)^2} + (-2) \frac{-\sin\left(\frac{2n\pi x}{3}\right)}{\left(\frac{2n\pi}{3}\right)^3} \right]_0^3$$

$$\frac{2}{3}\left[(2-2x)\frac{\cos\left(\frac{2n\pi x}{3}\right)}{\left(\frac{2n\pi}{3}\right)^2}\right]_0^3$$

$$\frac{\cancel{2}}{\cancel{3}}\times\frac{\cancel{3}^2}{(2n\pi)^2}\left[(2-2x)\cos\left(\frac{2n\pi x}{3}\right)\right]_0^3$$

$$\frac{3}{2n^2\pi^2}\left[2-2(3)\cos\frac{2n\pi(\cancel{3})}{\cancel{3}}-\left\{2-(2(0))\cos\left(\frac{2n\pi(0)}{3}\right)\right]\right.$$

if $n=2$

$$=\frac{3}{2n^2\pi^2}\left[(-4)(+1)^n-2(1)\right]$$

if $n=1$

$-4(-1)$

$4-2$

-6

$$=\frac{3}{\cancel{2}n^2\pi^2}\left(-\cancel{6}^3\right)$$

$$=\frac{-9}{n^2\pi^2}\text{ if } n \text{ is } \cancel{\text{even}}.$$

$$=\frac{\cancel{6}}{n^2\pi^2}\text{ if } n \text{ is odd.}$$

$$\boxed{a_n=\frac{-9}{n^2\pi^2}}$$

$$b_n=\frac{1}{L}\int_C^{C+2L} f(x)\sin\left(\frac{n\pi x}{l}\right)dx$$

$$=\frac{2}{3}\int_0^3(2x-x^2)\sin\left(\frac{2n\pi x}{3}\right)dx$$

$$=\frac{2}{3}\left[(2x-x^2)\left(\frac{-\cos\left(\frac{2n\pi x}{3}\right)}{\left(\frac{2n\pi}{3}\right)}\right)-(2-2x)\left(\frac{-\sin 2n\pi x/3}{(2n\pi/3)^2}\right)\right.$$

$$+(-2)\left(\frac{\cos\left(\frac{2n\pi x}{3}\right)}{(2n\pi/3)^3}\right)\Bigg]_0^3$$

$$\frac{2}{3}\left[\left[(6-9)\left(-\frac{\cos\left(\frac{2n\pi \cancel{3}}{\cancel{3}}\right)}{(2n\pi/3)}\right)+(-2)\frac{\cos\left(\frac{2n\pi 3}{3}\right)}{(2n\pi/3)^3}\right]\right.$$

$$-\left[0+\frac{(-2)\cos(0)}{(2n\pi/3)^3}\right]$$

$$=\frac{2}{3}\left[(-3)\frac{(-(1))}{2n\frac{\pi}{3}}-\cancel{2}\frac{1}{(2n\pi/3)^3}+\frac{\cancel{2}}{(2n\pi/3)^3}\right]$$

$$=\frac{\cancel{2}}{\cancel{3}}\left[\frac{(+3)(\cancel{3})}{2n\pi}-\frac{2\cancel{7}}{4\cancel{n}^2\pi^2}+\frac{2\cancel{7}}{4n^3\pi^3}\right]$$

$$\boxed{b_n=\frac{3}{n\pi}}$$

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}\left(a_n\cos\frac{n\pi x}{2}+b_n\sin\frac{n x\pi}{2}\right)$$

$$\boxed{2x-x^2=\frac{0}{2}+\sum_{n=1}^{\infty}\frac{-9}{n^2\pi^2}\cos\left(\frac{2n\pi x}{3}\right)+\frac{3}{n\pi}\sin\left(\frac{2n\pi x}{3}\right)}$$

17) Find the Fourier series Expansion for the function $f(x) = |x|$, in $(-\pi, \pi)$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$$

$$a_0 = \frac{2}{\pi} \int_0^{\pi} f(x)\, dx$$

$$= \frac{2}{\pi} \int_0^{\pi} x\, dx$$

$$= \frac{2}{\pi} \left[\frac{x^2}{2}\right]_0^{\pi}$$

$$= \frac{\cancel{2}}{\pi} \left[\frac{\pi^2}{\cancel{2}}\right]$$

$$\boxed{a_0 = \pi}$$

$$a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx\, dx$$

$$= \frac{2}{\pi} \int_0^{\pi} x \cos nx\, dx$$

$$= \frac{2}{\pi} \left[(x)\left\{\frac{\sin nx}{n}\right\} - (1)\left\{-\frac{\cos nx}{n^2}\right\} \right]_0^{\pi}$$

$$= \frac{2}{\pi} \left[\left\{ \frac{\cos nx}{n^2} \right\} \right]_0^{\pi}$$

$$= \frac{2}{\pi} \left[\left\{ \frac{(-1)^n - 1}{n^2} \right\} \right]$$

$$a_n = \begin{cases} 0 & : n \text{ is Even} \\ \frac{-4}{\pi n^2} & : n \text{ is odd} \end{cases}$$

From the following function, fourier expansion for the function

$f(x) = x + x^2$ in $(-\pi, \pi)$

It is a niether function

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\, dx$$

$$= \frac{1}{\pi} \int_{-\pi}^{\pi} (x + x^2)\, dx$$

$$= \frac{1}{\pi} \left[\frac{x^2}{2} + \frac{x^3}{3} \right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi} \left[\cancel{\frac{\pi^2}{2}} + \frac{\pi^3}{3} - \cancel{\frac{\pi^2}{2}} + \frac{\pi^3}{3} \right]$$

$$= \frac{2\pi^3}{3\cancel{\pi}}$$

$$\boxed{a_0 = \frac{2\pi^2}{3}}$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx\, dx$$

$$= \frac{1}{\pi} \int_{-\pi}^{\pi} (x + x^2) \cos nx\, dx$$

$$= \frac{1}{\pi} \left[(x + x^2) \left(\frac{\sin nx}{n} \right) - (1 + 2x) \left(-\frac{\cos nx}{n^2} \right) + (2) \left(\frac{-\sin nx}{n^3} \right) \right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[(1+2x)\left(\frac{\cos nx}{n^2}\right)\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[1+2\pi\,\frac{(-1)^n}{n^2} - \left\{1-2\pi\,\frac{(-1)^n}{n^2}\right\}\right]$$

$$= \frac{1}{\pi}\left[\cancel{1}+2\pi\,\frac{(-1)^n}{n^2} - \cancel{1} + 2\pi\,\frac{(-1)^{n+1}}{n^2}\right]$$

$$= \frac{4\pi}{\cancel{\pi}}\left(\frac{(-1)^n}{n^2} + \frac{(-1)^{n+1}}{n^2}\right)$$

$$= \frac{4}{n^2}\left((-1)^n + (-1)^{n+1}\right)$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi}(x+x^2)\sin nx\,dx$$

$$= \frac{1}{\pi}\left[(x+x^2)\left(\frac{-\cos nx}{n}\right) - (1+2x)\overbrace{\left(\frac{-\sin nx}{n^2}\right)}^{0} + (2)\left(\frac{\cos nx}{n^3}\right)\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[-(\pi+\pi^2)\frac{(-1)^n}{n} + 2\frac{(-1)^n}{n^3} - \left\{-(-\pi+\pi^2)\frac{(-1)^n}{n} + 2\frac{(-1)^n}{n^3}\right\}\right]$$

$$= \frac{1}{\pi}\left[-\pi-\pi^2\frac{(-1)^n}{n} + \frac{2(-1)^n}{n^3} - \pi + \pi^2\frac{(-1)^n}{n} + 2(-1)^n/n^3\right]$$

Find the fourier Series of periodicity 5 for the function $f(x) = 2x - x^2$ in $(0,5)$.

$c, c+2L = (0,5)$

$c = 0 \quad c+2L = 5.$

$0 + 2L = 5$

$$\boxed{L = 5/2}$$

$$a_0 = \frac{1}{L} \int_c^{c+2L} f(x)\,dx.$$

$$= \frac{2}{5} \int_0^5 (2x - x^2)\,dx$$

$$= \frac{2}{5}\left[\frac{\cancel{2}x^2}{\cancel{2}} - \frac{x^3}{3}\right]_0^5$$

$$= \frac{2}{5}\left[5^2 - \frac{5^3}{3}\right] - 0$$

$$= \frac{2(\cancel{5})}{\cancel{5}}\left[5 - \frac{5^2}{3}\right]$$

$$= 2\left[\frac{15 - 25}{3}\right]$$

$$= 2\left[\frac{-10}{3}\right]$$

$$= \frac{-20}{3}$$

$$a_n = \frac{1}{L} \int_c^{c+2L} f(x) \cos nx\,dx$$

$$= \frac{2}{5} \int_0^5 (2x - x^2) \cos\left(\frac{n\pi x}{L}\right) dx$$

$$= \frac{2}{5} \int_0^5 (2x - x^2) \cos\left(\frac{2n\pi x}{5}\right) dx$$

$$= \frac{2}{5}\int_0^5 (2x-x^2)\cos\left(\frac{2n\pi x}{5}\right)dx$$

$$= \frac{2}{5}\left[(2x-x^2)\frac{\sin\left(\frac{2n\pi x}{5}\right)}{\left(\frac{2n\pi}{5}\right)} \xrightarrow{} 0 - (2-2x)\frac{-\cos\left(\frac{2n\pi x}{5}\right)}{\left(\frac{2n\pi}{5}\right)^2} + (-2)\frac{-\sin\left(\frac{2n\pi x}{5}\right)}{\left(\frac{2n\pi}{5}\right)^3} \xrightarrow{} 0\right]_0^5$$

$$= \frac{2}{5}\left[(2-2x)\cos\frac{2n\pi x}{5}\right]_0^5 \times \frac{5^2}{(2n\pi)^2}$$

$$= \frac{2}{5}\left[(2-10)(1)\right]\frac{5^{\cancel{2}}}{\cancel{2}n^2\pi^2}$$

$$= \frac{5}{\cancel{2}n^2\pi^2}\left(-\cancel{8}\right)4$$

$$\boxed{a_n = -\frac{20}{n^2\pi^2}}$$

$$b_n = \frac{1}{L}\int_c^{c+2L} f(x)\sin\left(\frac{n\pi x}{L}\right)dx$$

$$= \frac{2}{5}\int_0^5 (2x-x^2)\sin\left(\frac{2n\pi x}{5}\right)dx$$

$$= \frac{2}{5}\left[(2x-x^2)\left(\frac{-\cos\left(\frac{2n\pi x}{5}\right)}{\left(\frac{2n\pi}{5}\right)}\right) - (2-2x)\frac{-\sin\left(\frac{2n\pi x}{5}\right)}{\left(\frac{2n\pi}{5}\right)^2} \xrightarrow{} 0 + 2(-2)\frac{\cos\left(\frac{2n\pi x}{5}\right)}{\left(\frac{2n\pi}{5}\right)^3}\right]_0^5$$

$$= \frac{2}{5}\left[\left\{(2(5)-25)\frac{1}{\left(\frac{2n\pi}{5}\right)} - (2)\frac{1}{\left(\frac{2n\pi}{5}\right)^3}\right\} - \left\{2(0) - \cancel{0}\left(\frac{1}{\frac{2n\pi}{5}}\right) - 2\frac{1}{\left(\frac{2n\pi}{5}\right)^3}\right\}\right]$$

$$= \frac{2}{5}\left[15\frac{1}{\left(\frac{2n\pi}{5}\right)} - \cancel{\frac{2}{\left(\frac{2n\pi}{5}\right)^3}} + \cancel{\frac{2}{\left(\frac{2n\pi}{5}\right)^3}}\right]$$

$$= \frac{2}{\cancel{5}}\left[\frac{\overset{15}{\cancel{75}}}{2n\pi}\right]$$

$$\boxed{b_n = \frac{15}{n\pi}}$$

$$f(x) = \frac{-10}{3} + \sum_{n=1}^{\infty} \frac{-20}{n^2\pi^2}\cos\left(\frac{2n\pi x}{5}\right) + \frac{15}{n\pi}\sin\left(\frac{2n\pi x}{5}\right)$$

20) Find the Half Range Fourier cosine series for the function $f(x) = \sin x$, $0 < x < \pi$.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$$

$$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi} \sin x\,dx$$

$$= \frac{2}{\pi}\left(\frac{-\cos nx}{n}\right)_0^{\pi} \qquad \frac{2}{\pi}\left[-\cos x\right]_0^{\pi}$$

$$a_0 = \frac{-2}{n\pi}\left[(-1)^n - 1\right] \qquad -\frac{2}{\pi}\left[-1-1\right]$$

$$+\frac{4}{\pi}$$

$$a_0 = \frac{-2}{n\pi}\left[(-1)^n - 1\right] \qquad \boxed{a_0 = \frac{4}{\pi}}$$

$$a_n = \frac{2}{\pi}\int_0^\pi \sin x \cos nx\, dx \qquad \because 2SC = S + S$$

$$= \frac{1}{\pi}\int_0^\pi \left(\sin(1+n)x + \sin(1-n)x\right) dx$$

$$= \frac{1}{\pi}\left[-\frac{\cos(1+n)x}{1+n} - \frac{\cos(1-n)x}{1-n}\right]_0^\pi$$

$$= \frac{1}{\pi}\left[\left\{-\frac{\cos(1+n)\pi}{1+n} - \frac{\cos(1-n)\pi}{1-n}\right\} - \left\{-\frac{\cos 0}{1+n} - \frac{\cos 0}{1-n}\right\}\right]_0^\pi$$

$$= \frac{1}{\pi}\left[-\frac{(-1)^{1+n}}{1+n} - \frac{(-1)^{1-n}}{1-n} + \frac{1}{1+n} + \frac{1}{1-n}\right]$$

$$= \frac{1}{\pi}\left[-(-1)^{1+n}\left\{\frac{1}{1+n} + \frac{1}{1-n}\right\} + 1\left\{\frac{1}{1+n} + \frac{1}{1-n}\right\}\right]$$

$$= \frac{1}{\pi}\left[-(-1)^{1+n}\frac{2}{1-n^2} + \frac{2}{1-n^2}\right]$$

$$= \frac{-2}{\pi}\left[\frac{(-1)^{1+n} - 1}{1-n^2}\right]$$

if n is odd than, $(-1)^{1+n} = 1$ $a_n = 0$

if n is even than $(-1)^{1+n} = -1$

$$a_n = -\frac{2}{\pi}\left[\frac{(-1)^{1+n}-1}{1-n^2}\right]$$

$$= \frac{2}{\pi}\left[\frac{-2}{1-n^2}\right] \Rightarrow$$

$$a_n = \frac{4}{\pi(1-n^2)}$$

$$\boxed{f(x) = \frac{2}{\pi} + \sum_{n=1}^{\infty} -\frac{4}{\pi(n^2-1)} \cos nx,}$$

PART - C

1) Define a periodic Function for $f(x)$ and given Example.

A function $f(x)$ is said to be periodic if $f(x+T) = f(x)$ for all $x \in R$ where T is a positive constant. This number T is called its period. The least positive period T of a function $f(x)$ is known as the primitive period of $f(x)$.

Examples:

1) $\sin x$, $\cos x$, $\sec x$, $\operatorname{cosec} x$, are periodic with 2π as period

2) $\tan x$, $\cot x$ with periodic π

3) $\sin nx$ is periodic function with $\frac{2\pi}{n}$ as period

4) $\tan nx$ is periodic function with $\frac{\pi}{n}$ as period.

2) State even and odd function $f(x)$

1) A function $f(x)$ is said to be even function if $f(-x) = f(x)$ for all x

2) A function $f(x)$ is said to be odd function if $f(-x) = -f(x)$ for all x.

3) Find whether the following functions are even or odd

i) $x\sin x + \cos x + x^2\cosh x$

$$-x\sin(-x) + \cos(-x) + (-x)^2\cosh(-x)$$

$$-x(-\sin x) + \cos x + x^2\cosh x$$

$$x\sin x + \cos x + x^2\cosh x$$

Since $f(x) = f(-x)$ the function is even function.

ii) $x\cosh x + x^3\sinh x$

$$(-x)\cosh(-x) + (-x)^3\sinh(-x)$$

$$(-x)\cosh x + x^3\sinh x$$

Given function is neither even nor odd.

4) Find the primitive periods of the functions $\sin 3x$, $\tan 5x$, $\sec 4x$.

The primitive period of $\sin 3x = 2\pi/3$

The primitive period of $\tan 5x$ is $\pi/5$.

The primitive period of $\sec 4x$ is $\pi/2$

Explain the Euler's Formula in the interval $(\alpha, a+2\pi)$

suppose $f(x)$ is a function defined in the interval $(a, a+2L)$ Then the fourier series for $f(x)$ is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

3) Write the Half-Range fourier sin and cosine series in $(0, l)$.

Half Range cosine series

a_0, a_n $b_n = 0$

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)$$

$$a_0 = \frac{2}{L} \int_0^L f(x)\, dx$$

$$a_n = \frac{2}{L} \int_0^L f(x) \cos\left(\frac{n\pi x}{L}\right) dx$$

Half Range sine series

$a_0 = 0$ $a_n = 0$ b_n

$$f(x) = \sum_{n=1}^{\infty} b_n \cos\left(\frac{n\pi x}{L}\right)$$

$$b_n = \frac{2}{L} \int_0^L f(x) \sin\left(\frac{n\pi x}{L}\right) dx$$

7) Write the examples of periodic function?

Sine function with period of 2π

cosine function with period of 2π.

8) Explain the Dirichlet's conditions for the existance of Fourier series of a function $f(x)$ in the interval $(\alpha, \alpha+2\pi)$

Dirichlet's Condition

Any function $f(x)$ can be developed as a Fourier series.

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx$$

where a_0, a_n, b_n are constants.

i) $f(x)$ is periodic, single valued and finite

ii) $f(x)$ has a finite number of ~~discontinuous~~ discontinuitities ~~an~~ in any one period

iii) $f(x)$ has a finite number of maxima and minima.

9) What are the conditions for expansion of a function in the interval $(-l, l)$ fourier series?

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx$$

is called Fourier series expansion of $f(x)$ and a_0, a_n, b_n $(n = 1, 2 \ldots)$ are called fourier coefficients of $f(x)$

(10) If $f(x)$ is an odd function in the interval $(-1, 1)$ then what are the value of a_0, a_n.

for any odd function the values of a_0 and a_n are zero

(11) If $f(x) = x^2$ in $(-1, 1)$ then find b_1?

Given function is Even function

a_0 a_n $b_n = 0$

for any Even function the value of $\boxed{b_1, b_2, b_3, \ldots b_n = 0}$

(12) what is the Fourier sine series for $f(x) = x$ in $(0, \pi)$?

$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx$$

$$b_n = \frac{2}{\pi}\int_0^{\pi} f(x) \sin nx \, dx$$

$$= \frac{2}{\pi}\int_0^{\pi} x \sin nx \, dx$$

$$= \frac{2}{\pi}\left[(x)\left(\frac{-\cos nx}{n}\right) - (1)\frac{-\sin nx}{n^2}\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[-x\left(\frac{\cos nx}{n}\right)\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[-\pi\frac{(-1)^n}{n} - 0\right]$$

$$= \frac{-2\pi(-1)^n}{n\pi}$$

$$\boxed{b_n = \frac{-2(-1)^n}{n}}$$

$$f(x) = \sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{n} \sin nx$$

13) State Fourier series of a function $f(x)$ in the interval $(C, C+2\pi)$?

periodic function $f(x)$ defined in $(C, C+2l)$

To change the length $2l$ to 2π

The fourier series is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) + b_n \sin\left(\frac{n\pi x}{l}\right)$$

14) Define Fourier series of a function $f(x)$ in the interval $(-l, l)$?

In the interval $(-l, l)$

i) If the function is even function

$$a_0 = \frac{2}{l}\int_0^l f(x)\,dx$$

$$a_n = \frac{2}{l}\int_0^l f(x)\cos\frac{n\pi x}{l}\,dx$$

ii) If the function is odd function

$$b_n = \frac{2}{l}\int_0^l f(x)\sin\left(\frac{n\pi x}{l}\right)dx$$

iii) If the function is Niether function

$$a_0 = \frac{1}{l}\int_{-l}^{l} f(x)\,dx$$

$$a_n = \frac{1}{l}\int_{-l}^{l} f(x)\cos\left(\frac{n\pi x}{l}\right)dx$$

$$b_n = \frac{1}{l}\int_{-l}^{l} f(x)\sin\left(\frac{n\pi x}{l}\right)dx$$

15) If $f(x) = x^2 - x$ in $(-\pi, \pi)$ then what is a_0.

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} (x^2 - x)\,dx$$

$$= \frac{1}{\pi}\left[\frac{x^3}{3} - \frac{x^2}{2}\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[\frac{\pi^3}{3} - \cancel{\frac{\pi^2}{2}} + \frac{\pi^3}{3} + \cancel{\frac{\pi^2}{2}}\right]$$

$$= \frac{2\pi^3}{3\cancel{\pi}}$$

$$\boxed{a_0 = \frac{2\pi^2}{3}}$$

16) Explain the Fourier Series for even function

If the function is said to be even function Hence $f(-x) = f(x)$

a_0, a_n $b_n = 0$

17) Write about the Fourier series for odd function.

If the function is said to be odd function Hence $f(-x) = -f(x)$

$a_0 = 0$ $a_n = 0$ b_n.

18) If $f(x) = x$ in $(0, \pi)$ the find the fourier coefficient a_0?

$f(x) = x$

$$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi} x\,dx$$

$$= \frac{2}{\pi}\left[\frac{x^2}{2}\right]_0^{\pi}$$

$$= \frac{1}{\pi}\left[\pi^2\right]$$

$$\boxed{a_0 = \pi}$$

Q19) If $f(x) = \cos x$ in $(-\pi, \pi)$ then find the Fourier coefficient a_0

Given function is even function

$$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi} \cos x$$

$$= \frac{2}{\pi}\left(\sin x\right)_0^{\pi}$$

$$\boxed{a_0 = 0}$$

Q20) Find the Fourier series of Periodicity in

If $f(x) = x^3$ in $(-\pi, \pi)$ then what is a_0

$$a_0 = \frac{2}{\pi}\int_{-0}^{\pi} f(x)\,dx$$

Given function is odd function

the value of $\boxed{a_0 = 0}$