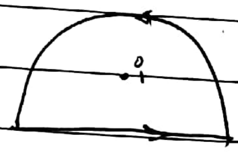
# Tutorial 7

Q2)



Use Jordans lemma & residue thm

Q3) $\alpha \in \mathbb{D}$ ($|\alpha|<1$), $\psi_\alpha : \overline{\mathbb{D}} \to \mathbb{C}$

$\overline{\mathbb{D}}$ is the closed disc

$\psi_\alpha = \dfrac{z-\alpha}{1-\bar{\alpha} z}$

$\mathbb{D}$ is the open disc

(i) $|z|=1$

$$\psi_\alpha = \frac{z-\alpha}{z\bar{z}-\bar{\alpha} z} = \frac{(z-\alpha)}{(z)(\bar{z}-\bar{\alpha})}$$

$$\therefore |\psi_\alpha| = \frac{|z-\alpha|}{|z|\,|\overline{z-\alpha}|} = 1 \qquad \forall z \neq \alpha$$

but $|z|=1$ but $|\alpha| \neq 1$

(ii) We are trying to prove $\psi_\alpha(\mathbb{D}) \subseteq \mathbb{D}$

$\Leftrightarrow$ codomain on $\mathbb{D}$ is $\mathbb{D}$

$\Leftrightarrow$ $|\psi_\alpha(z)| < 1 \quad \forall\ |z|<1$

Now on our open domain (domain is always open), we apply ~~Schwarz~~ 2 MMT. $\because$ $\psi_\alpha(z)$ is holomorphic hai

The maximum value that our function takes is 1

$\therefore$ maximum value is never attained

$\psi_\alpha = \dfrac{\text{Poly } A}{\text{poly } B}$ when poly B has no zero inside our domain.

(iii) $$\psi_\alpha(\psi_{-\alpha}(z)) = \frac{\dfrac{z+\alpha}{1+\bar{\alpha} z} - \alpha}{1-\bar{\alpha}\left(\dfrac{z+\alpha}{1+\bar{\alpha} z}\right)} = \frac{z-|\alpha|^2 z}{(1-|\alpha|^2)} = z$$

Scanned by CamScanner

Date
Page

Q4) $f$ is analytic on $\mathbb{D}$

$|f| < M \quad f(a) = 0 \quad a \in \mathbb{D}$

TP $\quad |f| \leq M \dfrac{|z-a|}{|1-\bar{a}z|} \quad \forall\, z \in \mathbb{D}$

we define $\quad g(z) = \dfrac{1}{M}\,(f \circ \psi_{(-a)})(z) \quad ; \quad g(0) = 0$

$\therefore$ By Schwartz lemma

$|g(z)| \leq |z| \quad \forall\, z \in \mathbb{D}$

$g(\psi_a(z)) = \dfrac{1}{M} f(z) \quad$ substitute $z = \psi_a$ $\quad [\because \psi_a \in \mathbb{D}]$

$\therefore |f(z)| \leq M\, |\psi_a(z)|$

Q5) $g(z) = \phi_{b_1} \circ f \circ \phi_{-a_1} \qquad g(0) = 0 \qquad ; \ |g(z)| \leq 1$

$\therefore |g(z)| \leq |z| \quad \forall\, z \in \mathbb{D}$

put $z = \phi_{a_1}$

$|(\phi_{b_1} \circ f)(z)| \leq |\phi_{a_1}(z)|$

put $z = a_2$

$|(\phi_{b_1} \circ f)(a_2)| \leq |\phi_{a_1}(a_2)|$

but $f(a_2) = b_2$

$\therefore |\phi_{b_1}(b_2)| \leq \left|\dfrac{a_2 - a_1}{1 - \bar{a}_1 a_2}\right|$

$$\left|\frac{b_2 - b_1}{1 - \bar{b}_1 b_2}\right| \leq \left|\frac{a_2 - a_1}{1 - \bar{a}_1 a_2}\right|$$

Scanned by CamScanner

classmate
Date
Page

Q2) $$\int_0^{2\pi} \frac{d\theta}{a^2+1-2a\cos\theta} \qquad z = e^{i\theta} \qquad dz = iz\,d\theta$$

$$\int_{CR} \frac{(-i)\,dz}{(z)\left[a^2+1-a\left(z+\frac{1}{z}\right)\right]}$$

$$= \int_{CR} \frac{-i\,dz}{[a^2z+z-az^2-a]} = \int_{CR} \frac{-i\,dz}{(z-a)(1-az)}$$

$$= \frac{1}{a}\int_{CR} \frac{i\,dz}{(z-a)(z-1/a)}$$

$$= \frac{i}{a}\int \frac{(z-a)-(z-1/a)}{(z-a)-(z-1/a)}\,dz \cdot \frac{1}{\frac{1}{a}-a}$$

$$= \frac{i}{(1-a^2)}\left\{\int_{CR} \frac{dz}{(z-1/a)} - \int_{C} \frac{dz}{z-a}\right\}$$

either of them is $\emptyset$ ~~or~~ $2\pi i$ $\quad a<1$

$\therefore \quad 1/a > 1$

$$= \frac{i}{(1-a^2)} - (2\pi i) = \frac{2\pi}{1-a^2}$$

Scanned by CamScanner