Tutorial IV

1. A power series with center at the origin and positive radius of convergence, has a sum $f(z)$. If it known that $f(\bar{z}) = \overline{f(z)}$ for all values of z within the disc of convergence, what conclusions can you draw about the power series ?

2. Evaluate the following integrals:

   (i) $\int_{|z|=1} \frac{z}{(z-2)^2} dz$;

   (ii) $\int_{|z|=2} \frac{e^z}{z(z-3)} dz$;

   (iii) $\int_{|z|=2} \frac{e^z}{z(z-1)} dz$;

   (iv) $\int_{|z|=4} \frac{\sin z}{(z-2)^2} dz$.

3. Let $z_1, z_2 \in \mathbb{C}$, $R > \max\{|z_1|, |z_2|\}$ and $f : \mathbb{C} \to \mathbb{C}$ be a holomorphic function. Show that
$$\int_{|z|=R} \frac{f(z)}{(z-z_1)(z-z_2)} dz = 2\pi\imath \frac{f(z_2) - f(z_1)}{z_2 - z_1}.$$

4. Let f and g be two holomorphic functions on an open set containg a simple closed contour γ and its interior. If $f(z) = g(z)$ for all z on γ, what can be said about f and g in the interior of γ ?