EE 223

Tut-1

1.

(a) min $\sum (x_i - c)^2$

$$\sum x_i^2 + \sum c^2 - 2\sum x_i c .$$

diff. it w.r.t c.

$$2\sum c - 2\sum x_i = 0.$$

$$\sum c = \sum x_i$$

$$cn = \sum x_i$$

$$c = \frac{\sum x_i}{n}$$

$$\therefore c = \bar{x}$$

(b) min. $\sum |x_i - c|$

there are two cases.

$$\sum_{i \in A} (x_i - c) + \sum_{i \in B} (c - x_i).$$

$$\sum x_i + \sum_{i \in B} c - \sum_{i \in A} c$$

when $|A| + |B|$ = even. both $\sum c$ cancels out

when $|A| + |B|$ = odd then $\pm c$ lifts.

hence c is median

one-sided Chebyshev inequality $S_k = \{i : x_i - \bar{x} \geq ks\}$.

show that $\frac{|S_k|}{n} \leq \frac{1}{1+k^2}$ for $k \geq 1$

$$y_i = x_i - \bar{x}$$

$$\sum_{i=1}^{n} (y_i+b)^2 \geq \sum_{y_i \geq ks} (y_i+b)^2$$

$$\geq \sum_{y_i \geq ks} (ks+b)^2$$

$$\sum_{i=1}^{n} (y_i+b)^2 \geq (ks+b)^2 \, |S_k| \qquad \text{—(i)}$$

$$\because \sum_{i=1}^{n} (y_i+b)^2 = \sum y_i^2 + 2b \sum y_i + \sum b^2$$

$$= s^2(n-1) + 0 + nb^2$$

from(i)

$$\therefore |S_k| \leq \frac{s^2(n-1) + nb^2}{(ks+b)^2}$$

$$\frac{|\xi_k|}{n} \le \frac{b^2+\xi^2}{k(k\xi+b)^2}$$

now, the RHS attains its min. at $b = \xi/k$.

$$\therefore \frac{|\xi_k|}{n} \le \frac{1+k^2}{(1+k^2)^2}$$

$$\therefore \frac{|\xi_k|}{n} \le \frac{1}{1+k^2}$$

H.P

3.

$X = \{x_1, x_2, \ldots, x_n\}$ $\quad Y = \{y_1, y_2 \ldots y_n\}$

$$r = \frac{\Sigma (x_i - \bar{x})(y_i - \bar{y})}{(n-1)\, s_x\, s_y}.$$

$$s_x = \sqrt{\frac{\Sigma (x_i - \bar{x})^2}{n-1}} \qquad s_y = \sqrt{\frac{\Sigma (y_i - \bar{y})^2}{n-1}}$$

$$\therefore r = \frac{\Sigma (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\Sigma (x_i - \bar{x})^2\, \Sigma (y_i - \bar{y})^2}}$$

C.1 Assuming $\Sigma (x_i - \bar{x})(y_i - \bar{y}) > 0$.

$$\therefore |r| = \sqrt{\frac{(\Sigma (x_i - \bar{x})(y_i - \bar{y}))^2}{\Sigma (x_i - \bar{x})^2\, \Sigma (y_i - \bar{y})^2}}$$

From cautchy swartz inequality.

$$\Sigma (x_i - \bar{x})^2\, \Sigma (y_i - \bar{y})^2 > \left(\Sigma (x_i - \bar{x})(y_i - \bar{y})\right)$$

$\therefore \quad r < 1$

C.2 for $\Sigma (x_i - \bar{x})(y_i - \bar{y}) < 0$.

$\therefore$

(b) $\sum y_i = an + b\sum x_i$

$\bar{y} = a + b\bar{x}$

$y_i - \bar{y} = a + bx_i - a - b\bar{x}$

$= b(x_i - \bar{x})$.

$$r = \frac{\sum (x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\sum(x_i-\bar{x})^2 \sum(y_i-\bar{y})^2}}$$

$$= \frac{b\sum(x_i-\bar{x})^2}{\sqrt{b^2\sum(x_i-\bar{x})^2}} = \frac{b}{|b|} \frac{\cancel{\sum(x_i-\bar{x})^2}}{\cancel{\sum(x_i-\bar{x})^2}}$$

$\therefore r = \frac{b}{|b|}$

$\therefore r \in (-1, 1)$

$\therefore$ r depends on sign of b.

$\therefore r = sgn(b)$

4.

Subset of N

	1	2	3	4	5	
S_1	1	0	1	1	0	. . .

$[0 - 1]$.

$P(N)$

Let S be any subset of N define $g : P(N) \to \{0,1\}^\infty$

~~Define $b_K = 1$ if K~~

Define $g_K(S)$ be K^{th} bit of seq $g(S)$, where

$$g_K(S) = 1 \quad ; \text{ if } K \in S$$
$$= 0 \quad \text{otherwise.}$$

$$h : \{0,1\}^\infty \to \mathbb{R}$$

$$h(g(S)) = \sum_{K=1}^{\infty} g_K(S)\, 2^{-K}$$

7. $\{A_1, A_2, \dots\} \subseteq \mathcal{F}$

$$\limsup_{n\to\infty} A_n = \bigcap_{n=1}^{\infty} \underbrace{\bigcup_{k=n}^{\infty} A_k}_{B_n}$$

$x_1, x_2, x_3, \dots$

$\limsup y_n$

$y_1 = \sup\{x_1, \dots\}$

$y_2 = \sup\{x_2, \dots\}$

$y_3 =$

$\therefore y_1 \geq y_2 \geq y_3 \dots$

$w \in \limsup_{n\to\infty} A_n$ if

$\forall n, \exists k \geq n$ s.t $w \in A_k$

$$\liminf A_n = \bigcup_{n=1}^{\infty} \bigcap_{k=n}^{\infty} A_k \to C_n$$

$w \in \liminf A_n$ iff $\quad C_1 \subseteq C_2 \subseteq C_3 \subseteq \dots$

$\exists n \geq 1$ s.t $\forall k \geq n$

$w \in A_k$

$$P(\limsup A_n) = P\left(\bigcap_{n=1}^{\infty} B_n\right) = \lim_{n\to\infty} P(B_n).$$

continuity from above for prob. measure.

$$P(\liminf A_n) = P\left(\lim_{n\to\infty} C_n\right)$$

$$P(C_n) \le P(A_n) \quad \forall n$$

every limit point of $\{P(A_n)\}_{n\geq 1} \geq \lim_{n\to\infty} P(C_n)\}$

$$= P\left(\lim_{n\to\infty} C_n\right) \le \liminf P(A_n).$$