# Network Theory Homework 2

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Homework is not to be submitted. If you prepare a solution and upload it to Moodle, you may be eligible for extra credit, as per the course rules.

"Valkenburg" is the book "Network Analysis, 3rd Edition" by M. E. Van Valkenburg. It is available for purchase on Amazon, and is not very expensive.

1. Define the following:
   (a) Homomorphism of graphs
   (b) The line graph $L_n$ and the undirected line graph $\overline{L_n}$.
   (c) The cycle graph $C_n$ and the undirected cycle graph $\overline{C_n}$.
   (d) Directed path in a graph
   (e) Undirected path in a graph
   (f) An undirected graph
   (g) A connected graph
   (h) A strongly connected graph
   (i) A tree
   (j) An undirected cycle in a graph
   (k) A directed cycle in a graph
   (l) A spanning tree

2. Let $(N, E)$ be a graph. Define the map $\delta$. What are the domain and codomain of this map? Prove/ disprove: $\delta$ is a linear map. If $N = \{1, 2, 3, 4\}$ and $E = \{(1, 2), (2, 3), (3, 4), (4, 2)\}$ then write down the matrix corresponding to $\delta$. Define the map $\delta^T$. What are the domain and codomain of this map?

3. Define $W_{KCL}$ and $W_{KVL}$. Prove or disprove: A vector $v \in W_{KVL}$ iff $v$ satisfies Kirchhoff's voltage law. (Hint: Prove forward direction and backward direction separately. For backward direction, define a potential function $V$ by "integration" and show path-independence of $V$. For forward direction, take an arbitrary cycle and show that KVL holds.)

4. Prove/ disprove: If $v \in W_{KVL}$ and $i \in W_{KCL}$ then the "Power" $\sum_{e \in E} i_e v_e = 0$.

5. Prove/ disprove: The following are equivalent for a connected undirected graph $G$:
   (a) $G$ is a tree, i.e., for every pair of nodes $n_1, n_2$ in $G$, there is exactly one path from $n_1$ to $n_2$.
   (b) $G$ has $n-1$ edges.
   (c) $G$ has no undirected cycles.
6. Prove/ disprove: If $v \in W_{KVL}$ and $i \in \mathbb{R}^E$ and $\sum_{e\in E} i_e v_e = 0$ then $i \in W_{KCL}$.
7. Prove/ disprove: $W_{KCL} \cap W_{KVL} = \{0\}$.
8. Consider the graph with $N = \{1, 2, 3, 4\}$ and $E = \{(1, 2), (2, 3), (3, 4), (4, 2)\}$. What is the dimension of $W_{KCL}$? What is the dimension of $W_{KVL}$? Find a basis for $W_{KCL}$ and for $W_{KVL}$ and explain the meaning of the basis elements in terms of the graph. How many equations will node variable analysis on this graph give? How many equations will loop variable analysis on this graph give?
9. Prove/ Disprove: For an arbitrary graph $G$ with $n$ nodes and $|E|$ edges and $k$ connected components, the dimension of $W_{KVL}$ is $n-k$ and the dimension of $W_{KCL}$ is $|E| - n + k$.
10. Prove/ disprove: The dimension of $W_{KVL}$ equals the number of equations from node variable analysis. (Hint: Don't forget the case where the graph is not connected.) The dimension of $W_{KCL}$ equals the number of equations from loop variable analysis.