Tutorial - 2

Q1) $u_x = v_y$

$u_y = -v_x$

$f(z) = e^x(\cos y + i \sin y)$

$u = e^x \cos y$ $\quad$ $u_x = e^x \cos y$ $\quad$ $v_y = e^x \cos y$

$v = e^x \sin y$ $\quad$ $u_y = -e^x \sin y$ $\quad$ $v_x = e^x \sin y.$

CR equation is Satisfied

and u_x, u_y, v_x, v_y are continuous then it is holomorphic

Q2) $\frac{du}{dr} = \frac{du}{dx} \cdot \frac{dx}{dr}$

$v_r = \frac{1}{r} v_\theta$

$v_\theta = -\frac{1}{r} u_\theta$

$u_r = \frac{\partial u}{\partial x} \cdot \frac{\partial x}{\partial r} + \frac{du}{dy} \times \frac{dy}{dr}$

$u_\theta = \frac{\partial u}{\partial x} \times \frac{\partial x}{d\theta} + \frac{\partial u}{\partial y} \cdot \frac{dy}{d\theta}$

Q3) v is harmonic conjugate of u

$\Rightarrow u + iv$

u is a harmonic

u is harmonic conjugate of v $\quad$ $f(z) = u + iv$ as

$v + ui$ is holomorphic = $\quad$ a constant function

CR eq on 1 $\quad$ CR eq on 2 $\quad$ $u =$ constant

$v =$ Constant

$u_x = v_y$ $\quad$ $v_x = u_y$

$u_y = -v_x$ $\quad$ $v_y = -u_x$

$u_x = v_y$

$v_y = -v_y$ $\quad$ $v_y = 0$

$u_y = v_x$

$v_x = -u_y$

$v_x = v_y = -v_y = 0$ as $v_y = -v_y$ in $v_y = 0$

$u_y = u_x = -v_x = 0$

Q-4) i) $u(x,y) = xy + 3x^2y - y^3$

$u_x = y + 6xy$

$u_{xx} = 6y$

$u_y = x + 3x^2 - 3y^2$

$u_{yy} = -6y$

$u_{xx} + u_{yy} = 0$

let v be its harmonic conjugate

$u + iv \Rightarrow$ holomorphic.

$u_x = v_y$

$u_y = -v_x$

$v = \int u_x \, dy$

$= \int (y + 6xy) dy$

$= \frac{y^2}{2} + \frac{6xy^2}{2} + f(x)$

$v(x,y) = \frac{y^2}{2} + 3xy^2 + f(x)$

$v_x = 0 + 3y^2 + f'(x) = -(x + 3x^2 - 3y^2)$

$f'(x) = -(x + 3x^2)$

$f(x) = -\left(\frac{x^2}{2} + \frac{3x^3}{3}\right) + c$

$v(x,y) = \frac{y^2}{2} + 3xy^2 + \left(\frac{x^2}{2} + x^3\right) + c$

$f(z) = u + iv$

5.) $R = \lim_{n \to \infty} \left| \frac{a_n}{a_{n+1}} \right| = R$.

$R = \frac{1}{\sup \lim_{n \to \infty} \sqrt[n]{|a_n|}}$

Power series only.

a_n is coefficient of series

Sequence is decreasing.

1) $\sum_{k=1}^{\infty} k z^k$

$\lim_{n \to \infty} \left| \frac{a_n z^n}{a_{n+1} z^{n+1}} \right| > 1$

$|z| < \left| \frac{a_n}{a_{n+1}} \right|$

$R = \lim_{n \to \infty} \left| \frac{a_n}{a_{n+1}} \right| = \left| \frac{n}{n+1} \right|$

5)(ii) $\sum_{P \text{ Prime}} z^P$

$a_n = 1$ if n is Prime
$= 0$ if n is composite

we not apply ratio test because many ratio is a_{n+1} is zero

$R = \frac{1}{\lim \sup_{n \to \infty} \sqrt[n]{a_n}} = \frac{1}{\sqrt[n]{1}} = 1$

iii)
$$\sum_{K=1}^{\infty} \frac{K!}{K^K} Z^K$$

using

Ratio test

$$R = \lim_{K \to \infty} \left| \frac{a_n}{a_{n-1}} \right| = \left(\frac{\frac{K!}{K^K}}{\frac{(K+1)!}{(K+1)^{K+1}}} \right)$$

$$= \frac{(K!)\,(K+1)^{(K+1)}}{(K+1)!\;K^K}$$

$$= \frac{K\,(K+1)^{K+1}}{(K+1)\;K^{K+1}}$$

$$= \left(\frac{K}{K+1}\right)\left(\left(1+\frac{1}{K}\right)\right)^{K+1}$$

$$\frac{1}{k+1} \cdot 1+\frac{1}{k}$$

$$\lim_{n \to \infty} \left(\frac{n}{n+1}\right)\left(1+\frac{1}{n}\right)^{n+1}$$

$$R = e$$

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