Quiz

MA 205: Complex Analysis

Instruction: Use only results discussed in the class to answer the questions.
Full marks 25 Time 1 hour

---

1. Find the points of holomorphicity of the function $f(z) = e^{|z^2|}$ [6]

2. Compute the radius of convergence of the power series $\sum_{1}^{\infty} \frac{(-1)^n z^n}{n^2}$ [6]

3. State the Cauchy Integral Formula. Use it to compute $\int_{|z-1|=2} \frac{dz}{z^5 - z^4 + 30z^2 - 30z}$ [7]

4. Let $f(z)$ be an entire function such that there exists a real constant C and a non-negative integer n such that $|f(z)| \leq C|z|^n$ for any $z \in \mathbb{C}$ with $|z|$ sufficiently large. Show that $f(z)$ is a polynomial of degree less than or equal to n. [6]

$$F(z_0) = \frac{1}{2\pi i} \int_{|z|=r} \frac{f(z)\,dz}{(z - z_0)}$$

$f(z)$

$\frac{1}{f(z)}$ holomorphic in the same domain.

Mid-Semester Exam

MA 205: Complex Analysis

Instructions :

1. Use only results discussed in the class to answer the questions.
2. Calculators and mobile phone are PROHIBITED in the exam hall.

Full marks 50 Time 2 hours

---

1. Find the Laurent expansions for the function $f(z) = \frac{2(z-1)}{z^2-2z-3}$ about 0 valid on the annuli : [6]
   (i) $1 < |z| < 3$
   (ii) $|z| > 3$.

2. Let $f(z) = u + iv$ be an entire function satisfying the identity: $u^7v^2 + 2u^2v - u + 2v + 1 = 0$ at all points in $\mathbb{C}$. Show that $f(z)$ is a constant. (Give complete details) [6]

3. Determine the nature of the singularities at 0 with proper justification. [12]
   a) $\frac{e^{(1/z^2)}}{z^2}$ b) $\frac{e^z - z - 1}{z}$ c) $\tan(\frac{1}{z})$

4. State the Cauchy Residue Theorem and use it to compute $\int_0^\infty \frac{\cos(x)dx}{(1+x^2)^2}$ using contour integration. [9]

5. Define harmonic conjugate of a harmonic function defined on an open subset of $\mathbb{C}$. Give an example with proper justification of an open subset of $\mathbb{C}$ and a harmonic function on it which doesn't admit any harmonic conjugate. (Give complete details) [7]

6. State Rouche's theorem. Use it to compute the number of roots of $z^5 + z^2 - 6z + 3$ in the annulus $\frac{1}{3} \leq |z| \leq 1$. [7]

7. Does there exist an entire function which vanishes on the subset $S = \{\frac{1}{n};\ n \in \mathbb{N}\} \subset \mathbb{R}$ but is not identically zero ? Justify. [3]

$\frac{e^{iz}}{(1+z^2)^2}$

$u_x = v_y$

$v_x = y e^x + c'$

$(\cos(\frac{\pi}{2}) - i\sin(-\frac{\pi}{2}))e^{-1}$

$a_{-2} + a_{-1}(z-i) + a_{-2}$