Production

- production process: transform inputs or factors of production into outputs
- common types of inputs:
  - capital (K): buildings and equipment
  - labor services (L)
  - materials (M): raw goods and processed products

Production function

Relationship between quantities of inputs used and maximum quantity of output that can be produced, given current knowledge about technology and organization.

Production function with 2 inputs

A production function that uses only labor and capital:

$$q = f(L, K)$$

to produce the maximum amount of output given efficient production

Variability of inputs over time

- firm can more easily adjust its inputs in the long run (LR) than in the short run (SR)
- *short run*: a period of time so brief that at least one factor of production is fixed
- *fixed input*: a factor that cannot be varied practically in the SR
- *variable input*: a factor whose quantity can be changed readily during the relevant time period
- *long run*: lengthy enough period of time that all inputs can be varied

Short-run production

- one variable input: Labor (*L*)
- one fixed input: Capital (*K*)
- thus, firm can increase output only by using more labor

Example

- service firm assembles computers for a manufacturing firm
- manufacturing firm supplies it with the necessary parts, such as computer chips and disk drives
- assembly firm's capital is fixed: eight workbenches fully equipped with tools, electronic probes, and other equipment for testing computers can vary labor

Total Product, Marginal Product, and Average Product of Labor with Fixed Capital				
Capital, $\bar{K}$	Labor, L	Output, Total Product of Labor, Q	Marginal Product of Labor, $MP_L = \Delta Q/\Delta L$	Average Product of Labor, $AP_L = Q/L$
8	0	0		
8	1	5	5	5
8	2	18	13	9
8	3	36	18	12
8	4	56	20	14
8	5	75	19	15
8	6	90	15	15
8	7	98	8	14
8	8	104	6	13
8	9	108	4	12
8	10	110	2	11
8	11	110	0	10
8	12	108	–2	9
8	13	104	–4	8

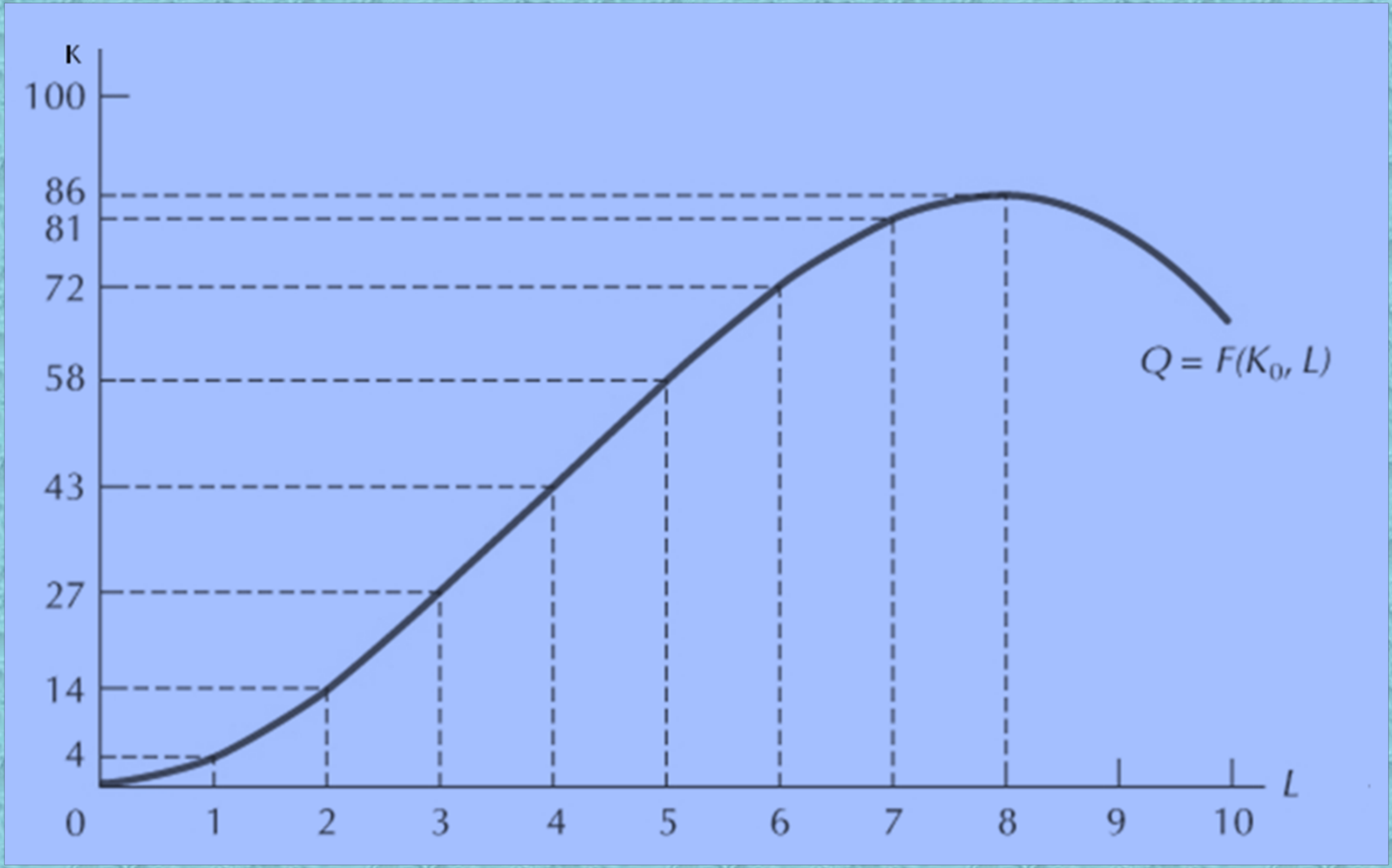
Marginal product of labor (MP_L)

- should firm hire another worker?
- want to know marginal product of labor:
  - change in total output, Δq, resulting from using an extra unit of labor, $\Delta L = 1$, holding the other factor (K) constant
  - $MP_L = \Delta q/\Delta L$

Average product of labor (AP_L)

- want to know average product of labor:
  - ratio of output to the number of workers used to produce that output
  - $AP_L = q/L$

Graphical Relationships



Graphical relationships

- total product: q
- marginal product of labor: $MP_L = \Delta q/\Delta L$
- average product of labor: $AP_L = q/L$

Effect of extra labor

- AP_L
  - rises and then falls with labor
  - slope of line from the origin to point on total product curve
- MP_L
  - first rises and then falls
  - cuts the AP_L curve at its peak
  - is the slope of the total product curve

Cobb-Douglas

- one of the most widely estimated production functions is the Cobb-Douglas:

$$q = AL^{\alpha} K^{\beta}$$

- A, α, β are positive constants

Given that the production function is Cobb-Douglas, what is the

- average product of labor
- marginal product of labor
- relationship between the MP_L and the AP_L?

1. *determine average product of labor by dividing output by labor:*
2. *differentiate the production function with respect to labor:*

$$AP_L = q / L = AL^{\alpha}K^{\beta} / L = AL^{\alpha-1}K^{\beta}$$

$$MP_L = \frac{\partial q}{\partial L} = \alpha AL^{\alpha-1}K^{\beta} = \alpha q / L$$

3. *take the ratio of the MP_L* and the *AP_L*:

$$\frac{MP_L}{AP_L} = \frac{\alpha A L^{\alpha-1} K^{\beta}}{A L^{\alpha-1} K^{\beta}} = \alpha$$

Law of diminishing marginal returns (product)

as a firm increases an input, holding all other inputs and technology constant,

- the corresponding increases in output will become smaller eventually
- that is, the marginal product of that input will diminish eventually

Long-run production: Two variable inputs

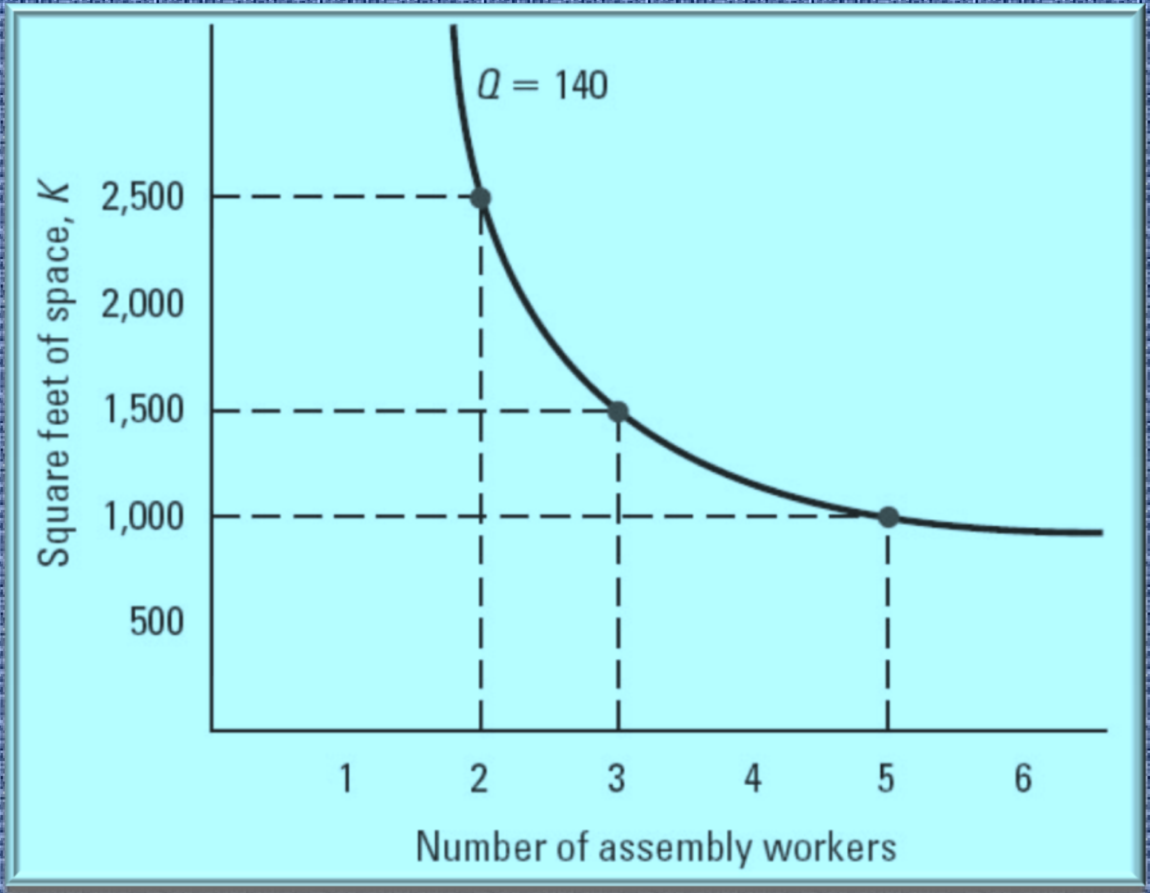
- both capital and labor are variable
- firm can substitute freely between L and K
- many combinations of L and K produce a given level of output:
- $q = f(L, K)$

Isoquant

- curve that shows efficient combinations of labor and capital that can produce a single (iso) level of output (*quant*ity):

$$\bar{q} = f(L, K)$$

Isoquant Example



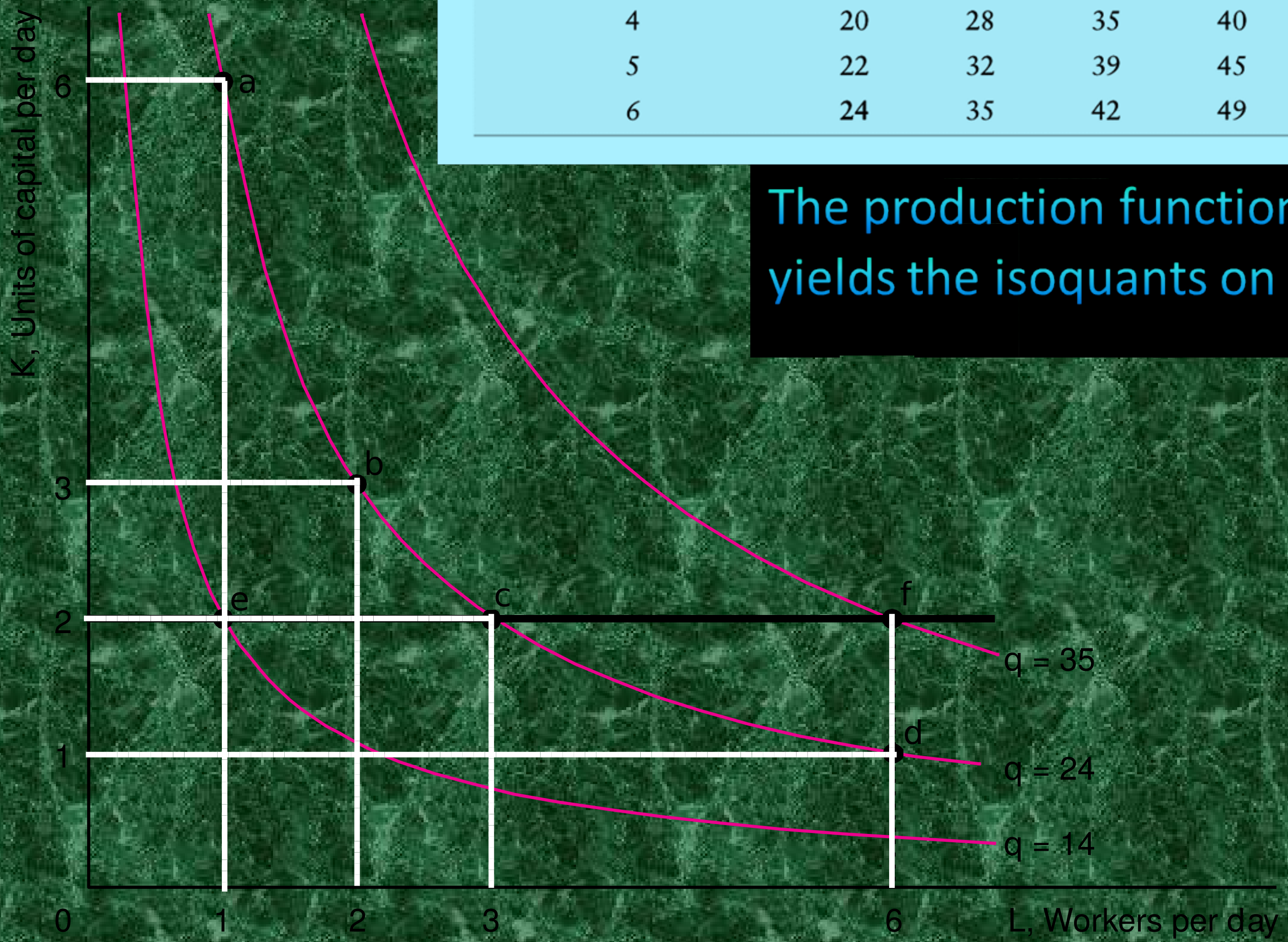
Productive Inputs Principle: Increasing the amounts of all inputs increases the amount of output. So, an isoquant must be negatively sloped

Output Produced with Two Variable Inputs

	Labor, L					
Capital, K	*1*	*2*	*3*	*4*	*5*	*6*
1	10	14	17	20	22	**24**
2	14	20	**24**	28	32	35
3	17	**24**	30	35	39	42
4	20	28	35	40	45	49
5	22	32	39	45	50	55
6	**24**	35	42	49	55	60

Capital, K	Labor, L 1	2	3	4	5	6
1	10	14	17	20	22	**24**
2	14	20	**24**	28	32	35
3	17	**24**	30	35	39	42
4	20	28	35	40	45	49
5	22	32	39	45	50	55
6	**24**	35	42	49	55	60

The production function above yields the isoquants on the left.



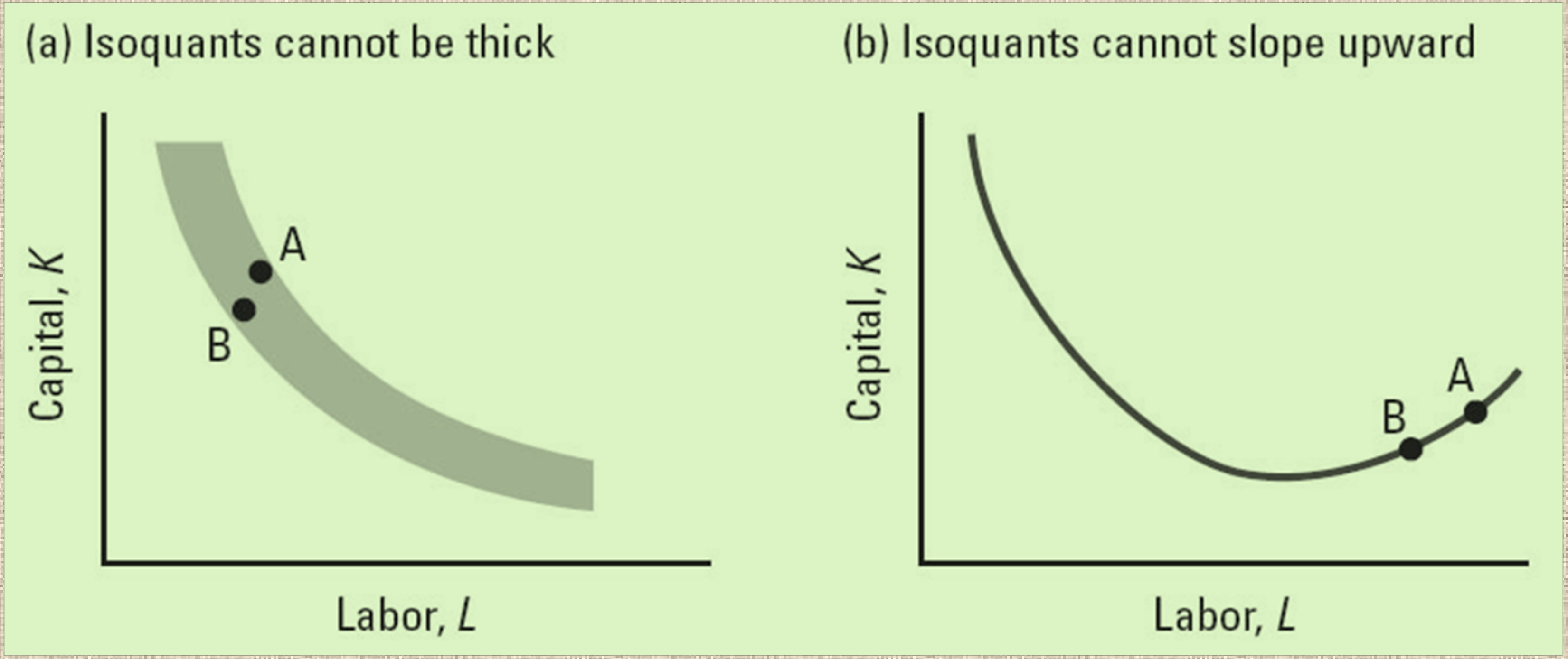
Isoquants and indifference curves

- have most of the same properties
- biggest difference:
  - isoquant holds something measurable, quantity, constant
  - indifference curve holds something that is unmeasurable, utility, constant

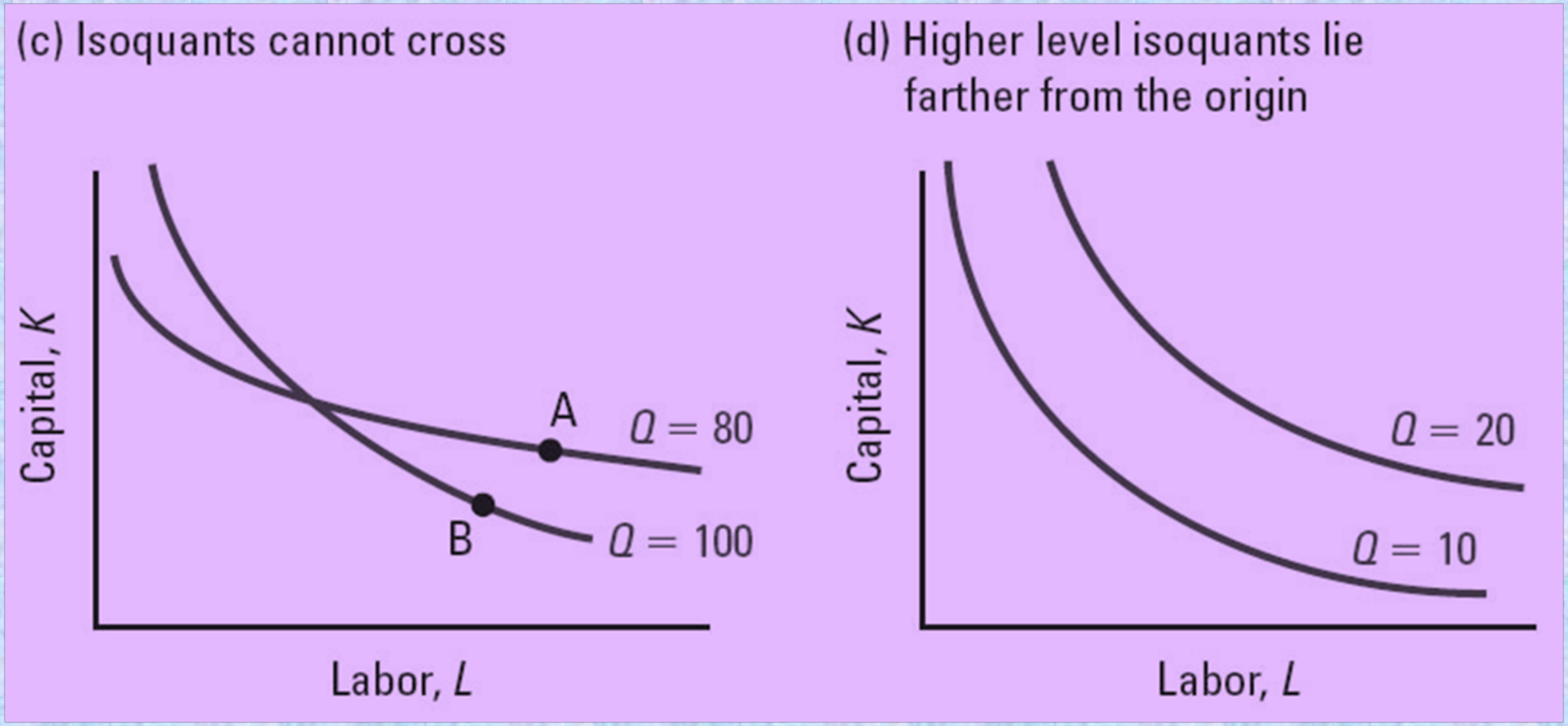
Properties of Isoquants

- Isoquants are thin
- Do not slope upward
- Two isoquants do not cross
- Higher-output isoquants lie farther from the origin

Properties of Isoquants



Properties of Isoquants



Substituting inputs

slope of an isoquant shows the ability of a
firm to substitute one input for another
while
holding output constant

Marginal Rate of Technical Substitution

- ***Marginal Rate of Technical Substitution for labor with capital* (MRTS$_{LK}$)*:*** the amount of capital needed to replace labor while keeping output unchanged, per unit of replaced labor
  - Let ΔK be the amount of capital that can replace ΔL units of labor in a way such that total output — $Q = F(L,K)$ — is unchanged.
  - Then, $MRTS_{LK} = - \Delta K / \Delta L$, and
  - $\forall$ $\Delta K / \Delta L$ is the slope of the isoquant at the *pre-change* inputs bundle.
  - Therefore, $MRTS_{LK}$ = - slope of the isoquant

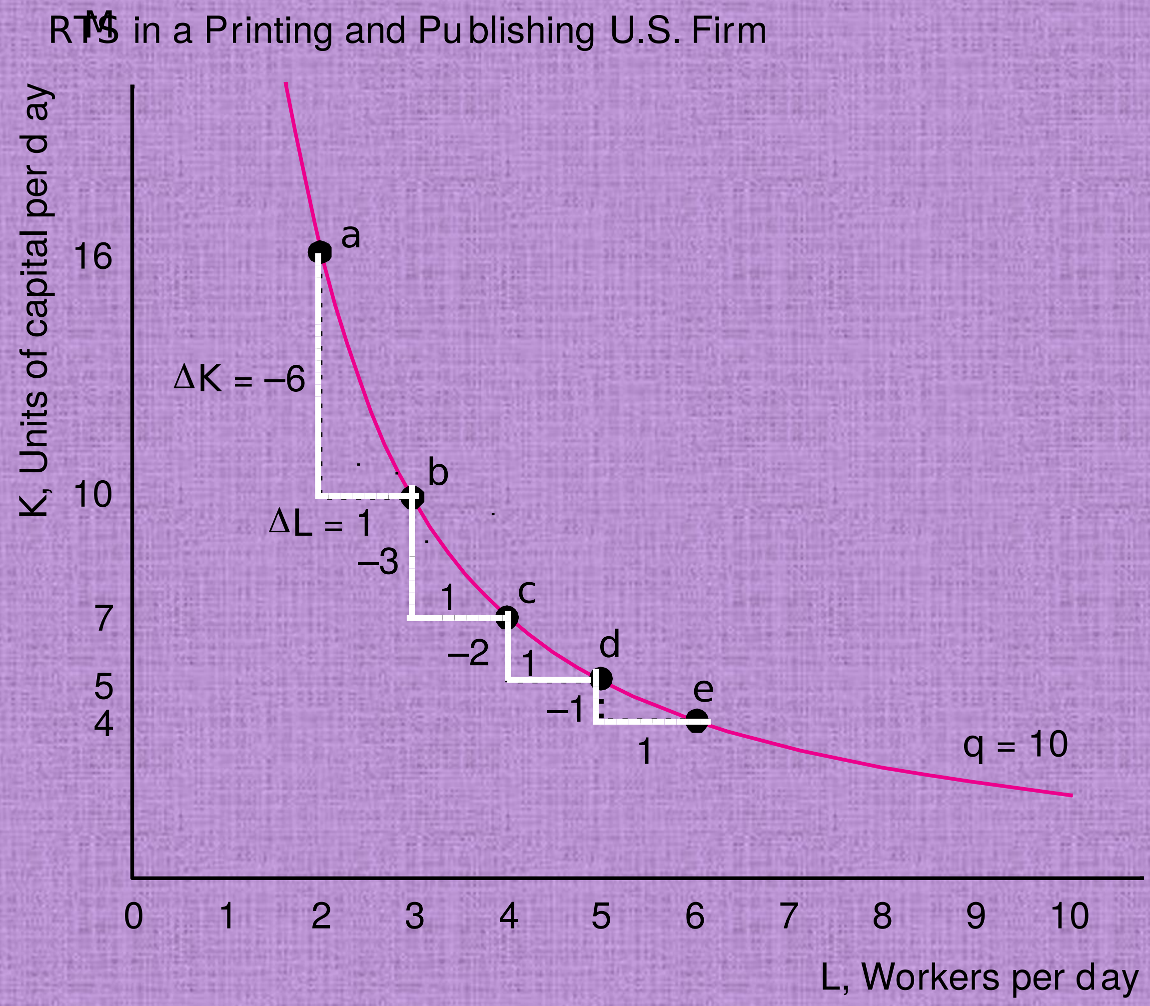
Marginal Rate of Technical Substitution

- **marginal rate of technical substitution (MRTS) -** the number of extra units of one input needed to replace one unit of another input that enables a firm to keep the amount of output it produces constant

$$-MRTS = \frac{\text{increase in capital}}{\text{increase in labor}} = \boxed{\frac{\Delta K}{\Delta L}}$$

Slope of Isoquant!

How the Marginal Rate of Technical Substitution Varies Along an Isoquant



Substitutability of Inputs and Marginal Products.

- Along an isoquant output doesn't change ($\Delta q = 0$), or:

$$(\underbrace{MP_L}_{\text{Increase in } q \text{ per extra unit of labor}} \times \overbrace{\Delta L}^{\text{Extra units of labor}}) + (\underbrace{MP_K}_{\text{Increase in } q \text{ per extra unit of capital}} \times \overbrace{\Delta K}^{\text{Extra units of capital}}) = 0.$$

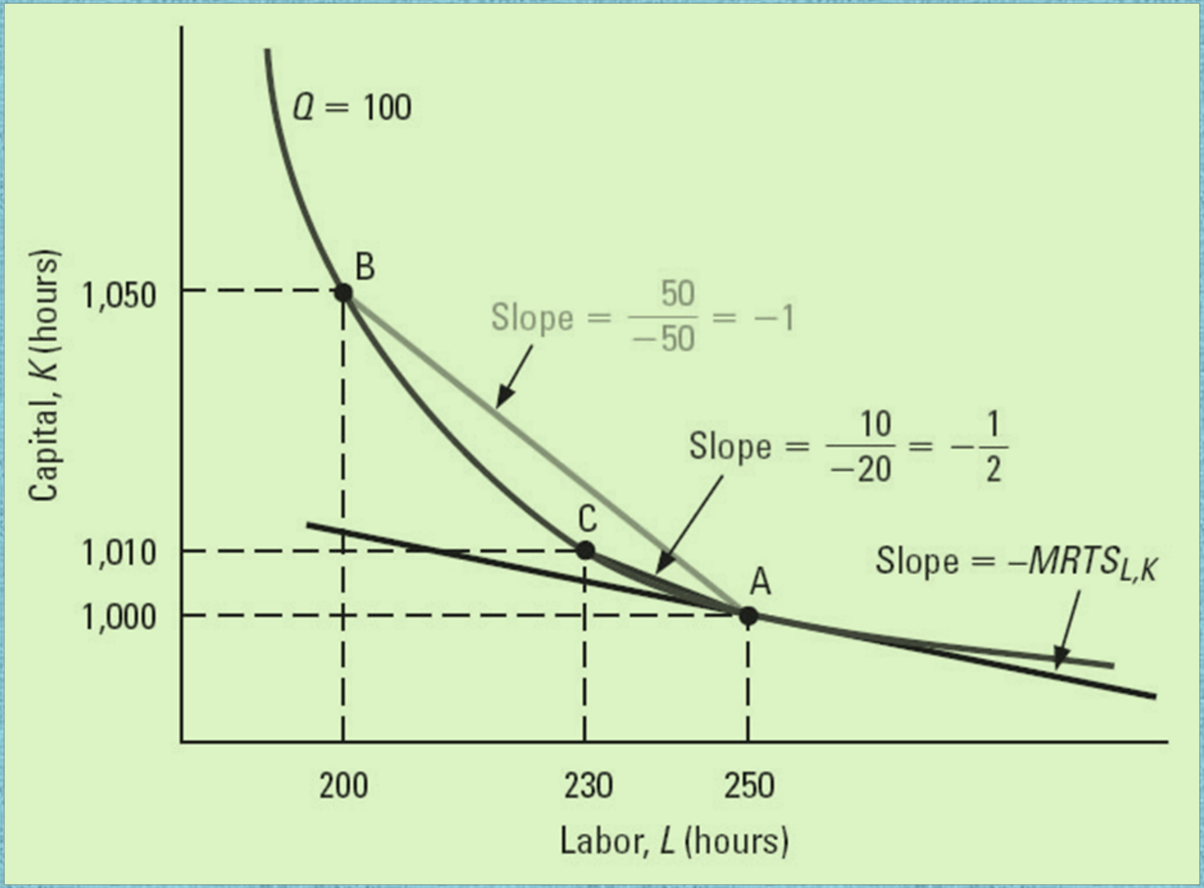
- or

$$MP_K \times \Delta K = 0 - MP_L \times \Delta L$$

$$\Delta K = -\frac{MP_L}{MP_K} \times \Delta L$$

$$\frac{\Delta K}{\Delta L} = -\frac{MP_L}{MP_K} = -MRTS = \text{slope of isoquant}$$

MRTS



Q = 100
B
Slope = 50/−50 = −1
Slope = 10/−20 = −1/2
C
A
Slope = −MRTS_{L,K}
Capital, K (hours)
1,050
1,010
1,000
200
230
250
Labor, L (hours)

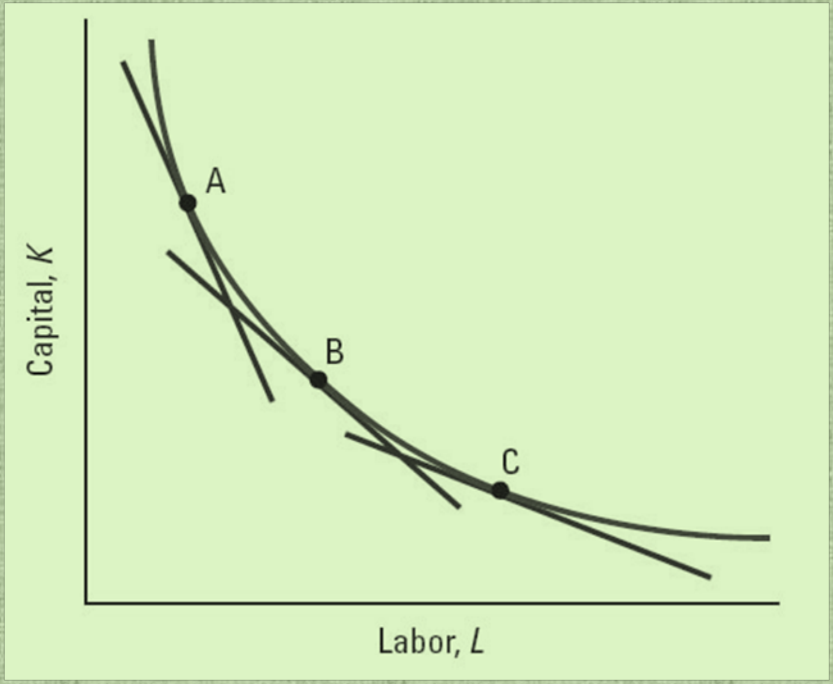
Why *MRTS* falls as we substitute *L* for *K*

$$\frac{MP_L}{MP_K} = -\frac{\Delta K}{\Delta L} = MRTS$$

- equation explains why *MRTS* diminishes as we replace capital with labor: move to right along isoquant
- less equipment per worker, so each remaining piece of capital is more useful and MP_L falls so $MRTS = MP_L/MP_K$ falls

Declining MRTS

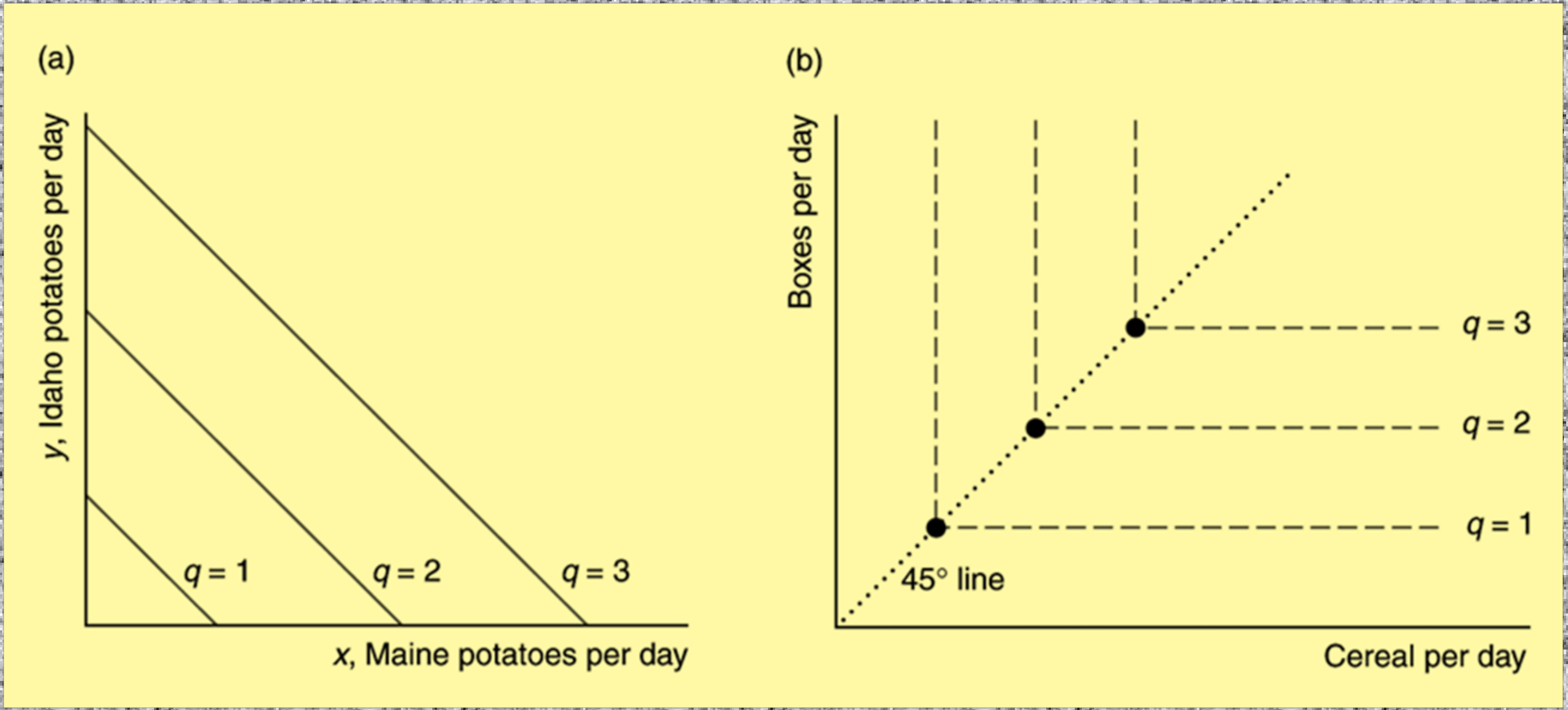
- We often assume that $MRTS_{LK}$ decreases as we increase L and decrease *K*
- Why is this a reasonable assumption?



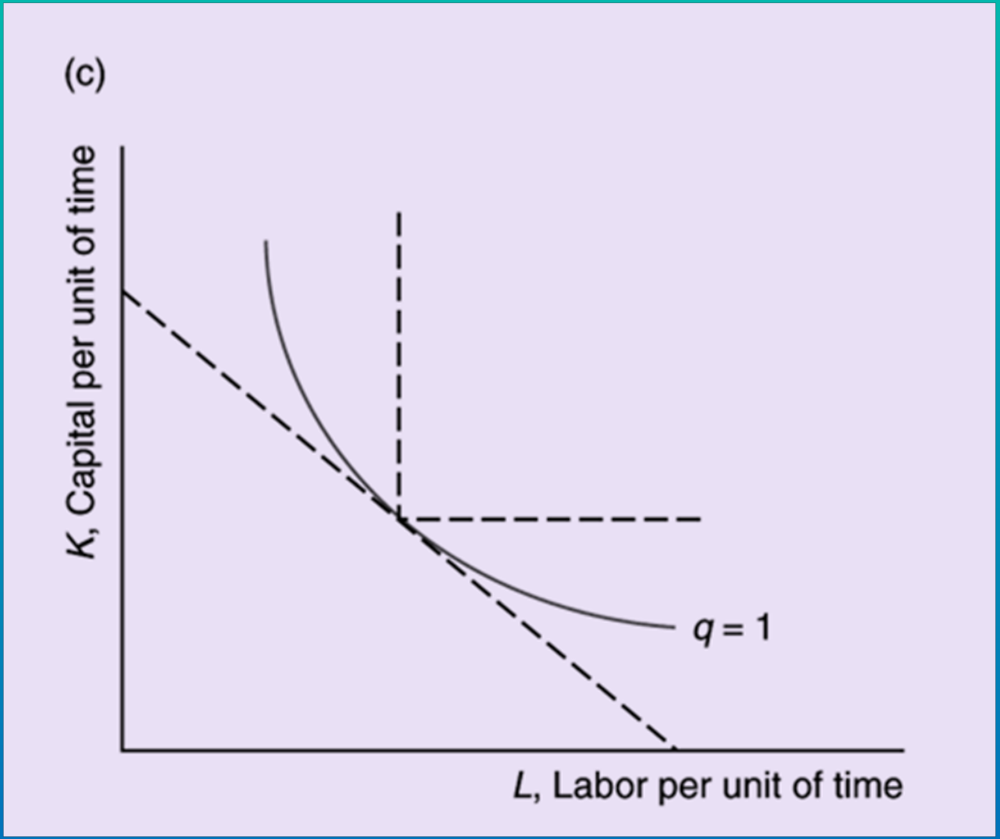
Extreme Production Technologies

- Two inputs are ***perfect substitutes*** if their functions are identical
  - Firm is able to exchange one for another at a fixed rate
  - Each isoquant is a straight line, constant MRTS
- Two inputs are ***perfect complements*** when
  - They must be used in ***fixed proportions***
  - Isoquants are L-shaped

Substitutability of Inputs



Substitutability of Inputs



Returns to Scale

Types of Returns to Scale	Proportional change in ALL inputs yields…	What happens when all inputs are doubled?
Constant	Same proportional change in output	Output doubles
Increasing	Greater than proportional change in output	Output more than doubles
Decreasing	Less than proportional change in output	Output less than doubles

Returns to Scale

- **Constant returns to scale (*CRS*) -** property of a production function whereby when all inputs are increased by a certain percentage, output increases by that same percentage.

$$f(2L, 2K) = 2f(L, K).$$

Returns to Scale (cont).

- **Increasing returns to scale (*IRS*) -** property of a production function whereby output rises more than in proportion to an equal increase in all inputs

$$f(2L, 2K) > 2f(L, K).$$

Returns to Scale (cont).

- **Decreasing returns to scale (*DRS*) -** property of a production function whereby output increases less than in proportion to an equal percentage increase in all inputs

$$f(2L, 2K) < 2f(L, K).$$

Productivity Differences and Technological Change

- A firm is ***more productive*** or has ***higher productivity*** when it can produce more output use the same amount of inputs
  - Its production function shifts upward at each combination of inputs
  - May be either general change in productivity of specifically linked to use of one input
- Productivity improvement that leaves MRTS unchanged is ***factor-neutral***

The Cobb-Douglas Production Function

- It one is the most popular estimated functions.

$$q = AL^{\alpha}K^{\beta}$$

Cobb-Douglas Production Function

$$Q = F(L, K) = AL^{\alpha}K^{\beta}$$

- *A* shows firm's general productivity level
- α and β affect relative productivities of labor and capital

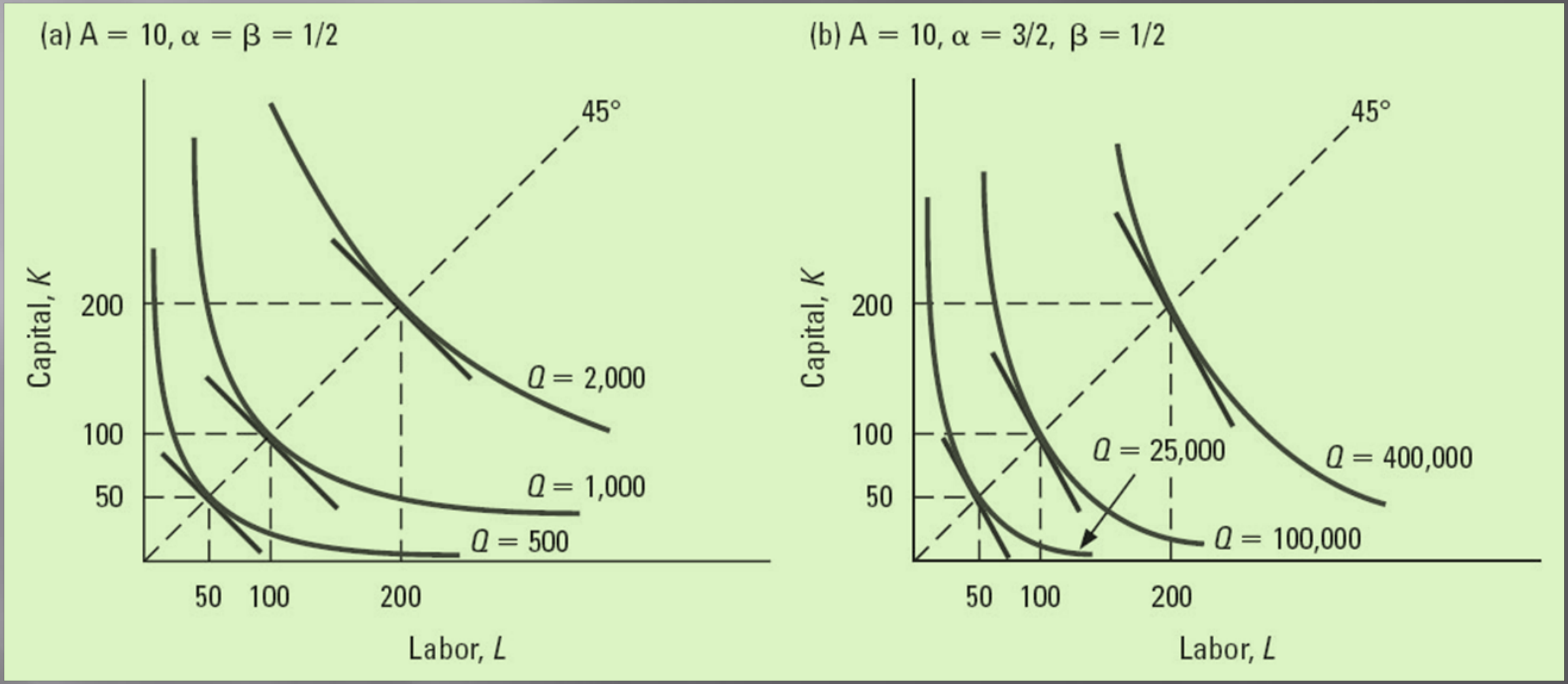
$$MP_L = \alpha AL^{\alpha-1}K^{\beta}$$

$$MP_K = \beta AL^{\alpha}K^{\beta-1}$$

- Substitution between inputs:

$$MRTS_{LK} = \left(\frac{\alpha}{\beta}\right)\left(\frac{K}{L}\right)$$

Cobb-Douglas Production Function



The degree of substitutability of factor L for factor K, resulting exclusively from the change in relative factor prices is called the elasticity of technical substitution and is measured by

$$(e\ subst.)_{LK} = \frac{\Delta \frac{K}{L} \Big/ \frac{K}{L}}{\Delta MRTS_{LK} \big/ MRTS_{LK}}$$