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Tut 1

1) $P_n = a_n x^n + a_{n-1} x^{n-1} \cdots a_0 x^0$

FTA → Any complex poly has atleast 1 root

$P_1 = a_1 x + a_0$ → (every non constant, single variable polynomial with complex coefficients has atleast one complex root)

This has 1 root $\boxed{\text{(FTA)}}$

Assume that a polynomial of degree $n-1$ has $n-1$ roots

Taking $P_n = a_n x^n + a_{n-1} x^{n-1} \cdots a_0$

we can factorise this as

$(x-\alpha)\left(b_{n-1} x^{n-1} + b_{n-2} x^{n-2} \cdots b_0\right)$

bcoz $x = \alpha$ is a root of P_n
By FTA

So p_n has $1 + n - 1 = n$ roots (By induction)

2) Real poly is irreversible, then it has a degree ≤ 2

$P_n = a_n x^n + a_{n-1} x^{n-1} \cdots a_0$, $a_n, a_{n-1} \cdots a_0 \in R$

$n \geq 3$

Prove that p_n is reducible → real factors of P_n exist

Case I — Complex root

By FTA → assume 1 root $= \alpha$

So another root $= \bar{\alpha}$ (as coeff are Real)

we can factorise P_n as

$$P_n = ((x-\alpha)(x-\bar{\alpha})q_{n-2})$$

q_{n-2} is a $n-2$ degree polynomial

So p_n is reducible

as $(x-\alpha)(x-\bar{\alpha}) = (x^2+|\alpha|^2 - (\alpha+\bar{\alpha})x)$

$\in R$

Case II

Real root of P_n exist

Let root be $= x_0$

$$P_n = (x-x_0)(q_{n-1})$$

P_n is reducible.

Q3 $f'(z_0) = \lim\limits_{z \to z_0} \frac{f z - f(z_0)}{z - z_0}$

iii) z^n $\quad n \in N$

$$f'(z_0) = \lim_{\substack{h \to 0 \\ h \in C}} \frac{(z_0 + h)^n - z_0^n}{h}$$

$$\Rightarrow \lim_{h \to 0} \frac{z_0^n \left(1 + \frac{h}{z_0}\right)^n - z_0^n}{h}$$

$$\Rightarrow \lim_{h \to 0} z_0^n \left(1 + n\frac{h}{z_0}\right) - z_0^n$$

$$\Rightarrow \lim_{h \to 0} \frac{z_0^n \left(1 + n\frac{h}{z_0}\right) - z_0^n}{h}$$

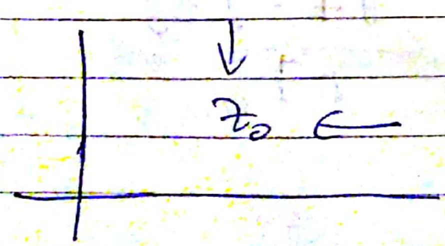
$$\cancel{\Rightarrow 0}$$

$$\Rightarrow n z_0^{n-1}$$

diff, holomorphic

iv) $f(z) = Re(z)$

$$f'(z_0) = \lim_{h \to 0} \frac{Re(z_0 + h) - Re(z_0)}{h_x}$$



I behind a limit along x axis $\to h \in R$

$$f'(z_0) = \frac{Re(z_0) + h - Re(z_0)}{h} = 1$$

taking $R \in Im$

$$f'(z_0) = \frac{Re(z_0) - Re\, z_0}{h} \quad (\text{as } Re(z_0 + h) = Re\, z_0)$$

$$= 0$$

So as limit do not match,

derivative does not exist for any z_0

$\mathcal{P}$

V(I) $f(z_0) = \bar{z}$

$$f'(z_0) = \lim_{h \to 0} \frac{(\overline{z_0 + h}) - \bar{z_0}}{h}$$

(1) $\to f'(z_0) =$

~~[illegible]~~ $h \lim \frac{z - z_0}{h} = 1$

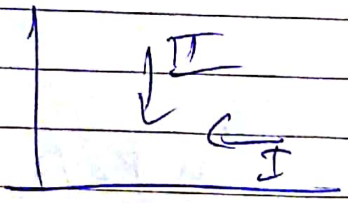
(2) $f'(z_0) = \frac{\bar{h} + \bar{z_0} - \bar{z}}{h} = \frac{-h}{h} = -1$

Q4 $f(z)$ is real valued $z \in C$

$f(z) \in R$

$f'(z) = 0$ or DNE

$$f'(z_0) = \lim_{z \to z_0} \frac{f(z) - f(z_0)}{z - z_0}$$



I $f'(z_0) = \frac{f(z) - f(z_0)}{z - z_0} \in R$ (as $z - z_0 \in R$) lim along x axis

II $f'(z_0) = \frac{f(z) - f(z_0)}{z - z_0} \in$ Im as $f(z) - f(z_0) \in R$, $z - z_0 \in$ Im

$$f'(z_0) = \lim_{z \to z_0} \frac{f(z) - f(z_0)}{z - z_0}$$

$f'(z)$ to be holomorphic

$f'(z) = 0$ everywhere

also as $f(z) = \lim_{z \to z_0} f'(z_0) \times (z - z_0) + f(z_0)$

$= 0 \times (z - z_0) + f(z_0)$

$f(z) = f(z_0)$

$\Rightarrow z$ is constant

if $f(z)$ is holomorphic