

Quiz

MA 205: Complex Analysis

Instruction: Use only results discussed in the class to answer the questions.
Full marks 25

Time 1 hour

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1. Find the points of holomorphicity of the function $f(z) = e^{|z|^2}$ [6]
 2. Compute the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n z^n}{n^2}$ [6]
 3. State the Cauchy Integral Formula. Use it to compute $\int_{|z-1|=2} \frac{dz}{z^5 - z^4 + 30z^2 - 30z}$ [7]
 4. Let $f(z)$ be an entire function such that there exists a real constant C and a non-negative integer n such that $|f(z)| \leq C|z|^n$ for any $z \in \mathbb{C}$ with $|z|$ sufficiently large. Show that $f(z)$ is a polynomial of degree less than or equal to n . [6]

$$F(z_0) = \frac{1}{2\pi i} \int_{|z|=r} \frac{f(z)}{(z-z_0)} dz$$

$f(z)$

$\frac{1}{f(z)}$ holomorphic in the same domain.

Mid-Semester Exam

MA 205: Complex Analysis

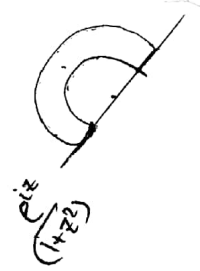
Instructions :

1. Use only results discussed in the class to answer the questions.
2. Calculators and mobile phone are PROHIBITED in the exam hall.

Full marks 50

Time 2 hours

1. Find the Laurent expansions for the function $f(z) = \frac{2(z-1)}{z^2-2z-3}$ about 0 valid on the annuli :
(i) $1 < |z| < 3$
(ii) $|z| > 3$. [6]
2. Let $f(z) = u + iv$ be an entire function satisfying the identity:
 $u^7 v^2 + 2u^2 v - u + 2v + 1 = 0$ at all points in \mathbb{C} . Show that $f(z)$ is a constant. (Give complete details) [6]
3. Determine the nature of the singularities at 0 with proper justification. [12]
a) $\frac{e^{(1/z^2)}}{z^2}$ b) $\frac{e^z - z - 1}{z}$ c) $\tan(\frac{1}{z})$
4. State the Cauchy Residue Theorem and use it to compute $\int_0^\infty \frac{\cos(x) dx}{(1+x^2)^2}$ using contour integration. [9]
5. Define harmonic conjugate of a harmonic function defined on an open subset of \mathbb{C} . Give an example with proper justification of an open subset of \mathbb{C} and a harmonic function on it which doesn't admit any harmonic conjugate. (Give complete details) [7]
6. State Rouché's theorem. Use it to compute the number of roots of $z^5 + z^2 - 6z + 3$ in the annulus $\frac{1}{3} \leq |z| \leq 1$. [7]
7. Does there exist an entire function which vanishes on the subset $S = \{\frac{1}{n}; n \in \mathbb{N}\} \subset \mathbb{R}$ but is not identically zero? Justify. [3]



$\frac{e^{i\pi}}{1+z^2}$

$\lim_{x \rightarrow 0} \frac{e^{i\pi} - 1}{x}$

$\frac{e^{i\pi} - 1}{0}$

Handwritten calculations for problem 6:

$$6 + \sqrt{36 - 4 \cdot 3 \cdot 1} = 6 + \sqrt{36 - 12} = 6 + \sqrt{24} = 6 + 2\sqrt{6}$$
$$2z^2 - 6z + 3 = 0 \implies z = \frac{6 \pm \sqrt{36 - 24}}{4} = \frac{6 \pm 2\sqrt{6}}{4} = \frac{3 \pm \sqrt{6}}{2}$$

Handwritten calculations for problem 7:

$$u_x = v_y$$
$$v_y = -u_x$$
$$u_x = y e^{x+y} + c$$
$$v_x = y e^{x+y} + c$$
$$v = y e^{x+y} + c x + d$$
$$u = y e^{x+y} + c x + d$$

Metting - left hand side... new monomorphic in...

Handwritten calculations for problem 4:

$$\lim_{n \rightarrow \infty} \frac{e^{i\pi} - 1}{n} = 0$$
$$\lim_{n \rightarrow \infty} \frac{e^{i\pi} - 1}{n} = 0$$
$$\lim_{n \rightarrow \infty} \frac{e^{i\pi} - 1}{n} = 0$$