Carrier Phase & Frequency Recovery and Compensation in Communication Links

EE 340: Prelab Reading Material for Experiment 10

November 7, 2016

Carrier Phase & Frequency Synchronization

In many of the modern digital communication modulation techniques, the message signal modulates the phase of the carrier wave (in addition modulation of the amplitude, if any). Such signals are commonly demodulated using coherent techniques, which require the receiver to know or derive the phase and frequency of the carrier wave precisely. However, the local oscillator at the receiver uses a separate reference oscillator for generating the carrier frequency, which has a non-zero frequency (and hence phase) offset. When the RF signal is down-converted using this carrier frequency, the baseband signal will contain the undesired carrier frequency (and phase) offset.

There are a few techniques that can be used for removing these phase and frequency offsets from the signal. In this document, the Costas loop based carrier phase/frequency recovery and compensation technique for QPSK signals and the Viterbi-Viterbi algorithm for carrier phase estimation has been described.

Costas Loop

The Costas loop is basically a phase locked loop (PLL) with a phase detector that can be used for obtaining the carrier phase error for digital modulation schemes, such as BPSK, QPSK, 8-PSK and 16-QAM. The phase detector structure depends on the modulation format. The phase detector used for QPSK signals is shown in Figure 1. It is assumed that the phase error lies within $\pm\pi/4$. The comparator shown in the figure has a threshold at zero and clips the output to $\pm\sqrt{1/2}$.

How the phase detector for QPSK signals works:
Note that the message phase in QPSK signals $\phi_m \epsilon \{\pm\pi/4, \pm 3\pi/4\}$. Therefore, when the phase error ϕ_{err} is within $\pm\pi/4$, the comparator outputs in Figure 1 do not depend on the amount of error (and only depend on the sign of the input signal). The upper comparator decides if ϕ_m lies in the right half of the complex plane (for which the output is $+\sqrt{1/2}$), or in the left half plane (for which the output is $-\sqrt{1/2}$). Therefore, in both cases, the comparator output value is equal to cos(ϕ_m), where $\phi_m \epsilon \{\pm\pi/4, \pm 3\pi/4\}$. Similarly, the lower comparator output value is sin(ϕ_m). Thus, the output of the phase detector is

$$sin(\phi_m + \phi_{err})cos(\phi_m) - cos(\phi_m + \phi_{err})sin(\phi_m) = sin(\phi_{err}) \approx \phi_{err}(when \phi_{err} \ll 1) \tag{1}$$

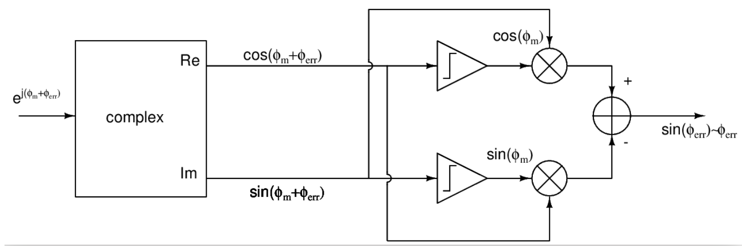

Figure 1: Phase Detector for QPSK signals used in a Costas Loop.

If the phase error exceeds $\pm\pi/4$ the phase detector still assumes that error is within $\pm\pi/4$ and an ambiguity of $\pm n\pi/2$ gets added to ϕ_m. However, this doesnt change the constellation diagram for the QPSK symbols. Once the symbols are demodulated to bits after synchronization etc., mapping can be done between the transmitted symbol constellation and received symbol constellation using some known bit sequences (that are sent at the beginning of the transmission). This operation is called demapping and is used for removing the $\pm n\pi/2$ phase ambiguity.

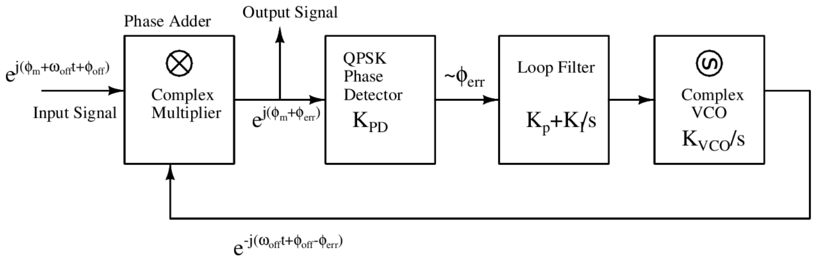

Figure 2: Costas Loop for removing phase and frequency offsets from the incoming signal. In steady state(i.e. when the phase and frequency offsets are fixed), the phase error in the loop becomes zero because of the integrator in the loop filter.

Operation of the Costas Loop:
The figure of the Costas loop is shown in Figure 2. The Costas loop generates the phase error, which tries to adjust the VCO (Voltage Controlled Oscillator) frequency and phase to match it with the offset frequency and phase in the incoming message signal. The resultant frequency and phase are subtracted from that of the incoming signal frequency and phase using a complex multiplier to obtain the desired signal. Please note that in the steady state (i.e. when phase and frequency offsets are not changing), the phase error (ϕ_{err}) becomes zero because of the integrator in the feedback loop.

Loop dynamics of the Costas Loop:
In a conventional feedback loop, the loop gain is *H(s)G(s)*, where *H(s)* is the feedforward gain and *G(s)* is the feedback gain. In the case of Costas Loop shown in Figure 2, if the VCO output is considered as the system output, $H(s) = K_{PD}(K_P + \frac{K_I}{s})\frac{K_{VCO}}{s}$ and*G(s)=1* (please revisit Lab 7 document on PLLs and your analog circuits lectures). The loop filter shown in the figure implements a zero at angular frequency K_I/K_P and a pole at $\omega = 0$. It has to be ensured that this loop gain has sufficient gain and phase margins for stability.

Based on this loop gain, the closed loop transfer function is: $H_{CL}(s) = \frac{H(s)}{1+H(s)G(s)}$. This closed looptransfer function has two poles as the denominator can be expressed as $s^2 + 2\zeta\omega_n + \omega_n{}^2$. To ensure good stability, the damping factor $\zeta > 0.5$. The time constant for settling of the loop is given by $\tau = \zeta\omega^{-1}$ where ω_n is called the natural frequency of the closed loop transfer function.

Implementation of the Costas Loop in discrete time

The Costas loop (including the loop filter) can be implemented in discrete time, in which the signals are represented by discrete samples, as in the case of GNU Radio. In discrete time, one sample delay (for sampling period=T) is represented as $z^{-1} = e^{-j\omega T}$ (explain why the transfer function $e^{-j\omega T}$ represents a delay=T ?). For frequencies $\ll$ *1/T* , we can approximate $z^{-1} \approx 1 - j\omega T$(using the Taylor series expansion of the exponential function). In your lab assignment, you will be using this approximation to implement the loop filter as an IIR filter with the following transfer function:

$$H_{LF}(z) = \frac{1.0001 - z^{-1}}{1 - z^{-1}} \tag{2}$$

For the above mentioned Costas loop and transfer function, calculate the following:

- The K_P and K_I values corresponding to the loop filter transfer function $H_{LF}(z)$ if sampling rate

= 32kHz (using the approximation mentioned above for the z-transform).

• The approximate value of K_{PD} if the phase detector output, after some scaling factor inside the phase detector block, changes from -1V to +1V as phase error varies from $-\pi/4$ to $+\pi/4$.

• The value of K_{VCO} if the sensitivity of the VCO is 4 Hz/V.

• The gain and phase margins calculated for the above loop parameters.

• The values of ζ, ω_n and settling time constant τ calculated for the above loop parameters.

The Viterbi-Viterbi Algorithm

The Viterbi-Viterbi Algorithm is a non-data aided mechanism for estimating the carrier phase of M-PSK signals. Consider a M-PSK signal $e^{(j\frac{2\pi n}{M}+\phi_{err})}$ with phase error given by ϕ_{err}. The Viterbi-Viterbi algorithm estimates the phase error $\hat{\phi}_{err}$ as

$$\hat{\phi}_{err} = \frac{1}{M}\arctan(e^{j(\frac{2\pi n}{M}+\phi_{err})M}) \tag{3}$$

Figure 3 shows the block diagram describing the algorithm. The M-PSK signal is taken to its M^{th} power, which removes its the M-ary modulation. This leads to a M-fold multiplication in the phase error. The phase detector takes the argument of the signal i.e. the $arctan(\frac{Img}{Real})$, giving the estimated phase error $\hat{\phi}_{err}$. Appropriate scaling is performed and the output is given to a phase modulator followed by a complex multiplier to obtain the original signal back. You can observe that the phase detector described earlier can also be implemented using this approach.

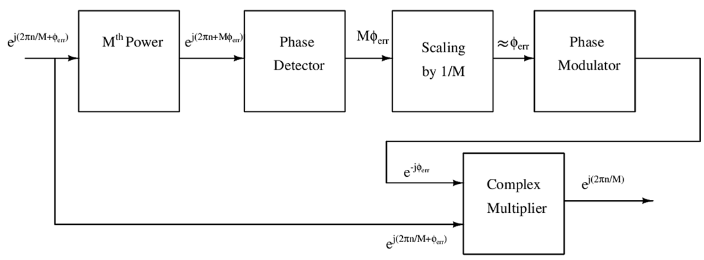

Figure 3: Block Diagram of Viterbi-Viterbi Algorithm.

The maximum allowable phase error is within $\pm\pi/M$. When the phase error exceeds $\pm\pi/M$, the constellation point drifts to the neighbouring decision region. This drift leads to an error when the symbols are demodulated. To resolve this issue, the differential of the phase may be demodulated. Recall that in FM demodulation, you multiply the received signal with its delayed and phase conjugated copy before before demodulating the phase. The argument of this output is the differential of phase,magnitude of which is $\ll \pi$ (if the noise power is not too high and the sample rate is sufficiently high).