

Assignment 2: Predicate Logic

Mathematical Logic - A (MSH1B3)

First Term 2016-2017

Instructions:

1. This assignment is due **Thursday October 6 at 5:00 p.m.**. Please submit your work at School of Computing academic roster (*roster akademik Fakultas Informatika*), room A203A (building A room A203A). Do not forget to write your identity on the space provided. You may submit this assignment as of **Monday September 26 at 8:00 a.m.**
2. In order to prevent any academic misconduct, you also need to submit a readable scan or photograph of this assignment to the provided submission slot in IDEA. Please contact your class instructor for more detailed information. The due date of this online submission is the same as the hardcopy. **Please make sure that your file size do not exceed the maximum file size allowed.**
3. To save paper, you may print and reproduce this assignment on both sides of a paper.
4. Your answers should be handwritten. You may use: HB or 2B pencil, or pen with blue or black ink.
5. All problems in this assignment are adapted from the textbooks. **The problems are written in English.** If you are a student in a regular class, you may answer the problems in Bahasa. However, if you are a student in international class, your answers must be written in English – otherwise your assignment will not be graded. You may ask your class instructor or teaching assistant for helping you understanding the problem, but you should not ask them to give the solution of any problem.
6. Write your solutions on the space provided. If you need more space, you may use additional A4 papers and attach them to your assignment.
7. Be neat and write legibly. You will be graded not only on the correctness of your answers, but also on the clarity with which you express them.
8. This assignment consists of **8** problems, each but Problems 7 and 8 worth **10 points**. Problems 7 and 8 worth **20** points each.
9. Please retain yourself from copying answers from elsewhere without understanding the steps. This assignment is an individual evaluation.
10. **Important:** late submission without reasonable explanation will not be graded.

Problem 1 (2 points each) Let $C(x) :=$ “ x has a cat”, $D(x) :=$ “ x has a dog”, and $F(x) :=$ “ x has a ferret”. Express each of these statements in terms of $C(x)$, $D(x)$, $F(x)$ by using quantifiers and also logical connectives while the domain consist of all students in your class.

- (a). A student in your class has a cat, a dog, and a ferret.

ANSWER:

- (b). All students in your class have a cat, a dog, or a ferret.

ANSWER:

- (c). Some students in your class have a cat and a ferret, but not a dog.

ANSWER:

- (d). No student in your class has a cat, a dog, and a ferret.

ANSWER:

- (e). For each of the three animals: cats, dogs, and ferrets, there is a student in your class who has one of these animals as a pet.

ANSWER:

Problem 2 Let $Q(x) := x + 1 > 2x$. If the domain consists of all integers, what are the truth value of these formulas?

- (a). [1 pt] $Q(0)$

ANSWER:

- (b). [1 pt] $Q(-1)$

ANSWER:

- (c). [1 pt] $Q(1)$

ANSWER:

- (d). [1.5 pts] $\exists x.Q(x)$

ANSWER:

- (e). [1.5 pts] $\forall x.Q(x)$

ANSWER:

- (f). [2 pts] $\exists x.\neg Q(x)$

ANSWER:

- (g). [2 pts] $\forall x.\neg Q(x)$

ANSWER:

Problem 3 Suppose that the domain of the predicate $P(x)$ consists of $-2, -1, 0, 1$, and 2 . Write out each of the following predicate logic formulas in propositional logic formulas using disjunctions, conjunctions, negations, or their combinations.

(a). [1 pt] $\exists x.P(x)$

ANSWER:

(b). [1 pt] $\forall x.P(x)$

ANSWER:

(c). [2 pts] $\exists x.\neg P(x)$

ANSWER:

(d). [2 pts] $\forall x.\neg P(x)$

ANSWER:

(e). [2 pts] $\neg\exists x.P(x)$

ANSWER:

(f). [2 pts] $\neg\forall x.P(x)$

ANSWER:

Problem 4 (2.5 points each) Let $P(x)$, $Q(x)$, $R(x)$, and $S(x)$ be the statements “ x is a duck”, “ x is one of my poultry”, “ x is an officer”, and “ x is willing to waltz”, respectively. Express each of these statements using quantifiers, logical connectives, and also using the predicates above.

(a). No ducks are willing to waltz.

ANSWER:

(b). No officers ever decline to waltz.

ANSWER:

(c). All my poultry are ducks.

ANSWER:

(d). My poultry are not officers.

ANSWER:

Problem 5 (2 points each) Translate each of these statements into logical expression using only the specified domain and predicates.

- (a). “Someone in your school has visited Uzbekistan”.

The domain is $D := \{x \mid x \text{ is a human being}\}$ and the predicates are $P(x) := “x \text{ is a person in your school}”$ and $U(x) := “x \text{ has visited Uzbekistan}”$.

ANSWER:

- (b). “Everyone in your class has studied calculus and C++”.

The domain is $D := \{x \mid x \text{ is a human being}\}$ and the predicates are $C(x) := “x \text{ is in your class}”$, $Q(x) := “x \text{ has studied calculus}”$, and $R(x) := “x \text{ has studied C++}”$.

ANSWER:

- (c). No one in your school owns both a bicycle and a motorcycle.

The domain is $D := \{x \mid x \text{ is a human being}\}$ and the predicates are $S(x) := “x \text{ is a person in your school}”$, $T(x) := “x \text{ owns a bicycle}”$, and $U(x) := “x \text{ owns a motorcycle}”$.

ANSWER:

- (d). There is a person in your school who is not happy.

The domain is $D := \{x \mid x \text{ is a human being}\}$ and the predicates are $S(x) := “x \text{ is a person in your school}”$ and $H(x) := “x \text{ is happy}”$.

ANSWER:

- (e). Everyone in your school was born in the twentieth century.

The domain is $D := \{x \mid x \text{ is a human being}\}$ and the predicate are $S(x) := “x \text{ is a person in your school}”$ and $T(x) := “x \text{ was born in the twentieth century}”$.

ANSWER:

Problem 6 (2 points each) Express each of these statements using quantifiers and specified predicates. Then form the negation of the statement so that no negation is to the left of a quantifier. Next, express the negation in simple English sentence.

(a). All dogs have fleas.

The domain is $D = \{x \mid x \text{ is a dog}\}$ and the predicate is $\text{Fleas}(x) := "x \text{ has fleas}"$.

ANSWER:

- Predicate formula:

- Negation of the formula:

- Sentence in English of negated formula:

(b). There is a horse that can add.

The domain is $D := \{x \mid x \text{ is a horse}\}$ and the predicate is $\text{Add}(x) := "x \text{ can add}"$.

ANSWER:

- Predicate formula:

- Negation of the formula:

- Sentence in English of negated formula:

(c). Every koala can climb.

The domain is $D := \{x \mid x \text{ is a koala}\}$ and the predicate is $\text{Climb}(x) := "x \text{ can climb}"$.

- Predicate formula:

- Negation of the formula:

- Sentence in English of negated formula:

(d). No monkey can speak French.

The domain $D := \{x \mid x \text{ is an animal}\}$ and the predicates are $\text{Monkey}(x) := "x \text{ is a monkey}"$ and $\text{French}(x) := "x \text{ speaks French}"$.

- Predicate formula:
- Negation of the formula:
- Sentence in English of negated formula:

(e). There exists a pig that can swim and catch fish.

The domain is $D := \{x \mid x \text{ is a pig}\}$ and the predicates are $\text{Swim}(x) := "x \text{ can swim}"$ and $\text{Catch}(x) := "x \text{ can catch fish}"$.

ANSWER:

- Predicate formula:
- Negation of the formula:
- Sentence in English of negated formula:

Name:

NIM:

Class:

Problem 7 (5 points each) Verify whether each of these arguments is valid or not. Explain which rules of inference are used for each step.

- (a). “Linda, a student in this class, owns a red convertible. Everyone who owns a red convertible has gotten at least one speeding ticket. Therefore, someone in this class has gotten a speeding ticket.”

ANSWER:

- (b). “Each of five roommates, Melissa, Aaron, Ralph, Veneesha, and Keeshawn, has taken a course in discrete mathematics. Every student who has taken a course in discrete mathematics can take a course in algorithms. Therefore, all five roommates can take a course in algorithms next year.”

ANSWER:

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- (c). “All movies produced by John Sayles are wonderful. John Sayles produced a movie about coal miners. Therefore, there is a wonderful movie about coal miners.”

ANSWER:

- (d). **[5 pts]** “There is someone in this class who has been to France. Everyone who goes to France visit the Louvre. Therefore, someone in this class has visited the Louvre.”

ANSWER:

Name:

NIM:

Class:

Problem 8 For each of these arguments determine whether the argument is correct or incorrect and explain why.

- (a). “Everyone enrolled in the university has lived in a dormitory. Mia has never lived in a dormitory. Therefore, Mia is not enrolled in the university.”

ANSWER:

- (b). “A convertible car is fun to drive. Isaac’s car is not a convertible. Therefore, Isaac’s car is not fun to drive.”

ANSWER:

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- (c). “Quincy likes all action movies. Quincy likes the movie ‘*Eight Men Out*’. Therefore, ‘*Eight Men Out*’ is an action movie.”

ANSWER:

- (d). “All lobstermen set at least a dozen traps. Hamilton is a lobsterman. Therefore, Hamilton sets at least a dozen traps.”

ANSWER: