Assignment 2 :

Due Date: 20 January, 2020

1. Generate a set of points around a line $y = ax + b$

   (a) Choose $a = 2$ and $b = 3$

   (b) Select the range for x as $[-10, 10]$ and generate $n = 100$ values for x in that interval.

   (c) Compute the values of y for each x as $y_i = 2x_i + 3$.

   (d) Plot the line $y = 2x + 3$ in black color.

   (e) Generate a set of n points around the line using the equation

   $$y_i = 2x_i + 3 + \sigma\mathcal{N}(0, 1) \tag{1}$$

   where σ is the standard deviation and $\mathcal{N}(0, 1)$ is the zero-mean unity-variance normal distribution

   (f) Show the scatter plot of these noisy points (in red color) on the same graph generated in step (d).

2. Plot the average error surface E for different values of a and b in the interval of $[-10 : 0.1 : 10]$.

   (a) Vary both a and b in steps of 0.1 in the interval $[-10, 10]$

   (b) Compute the element-wise error as $e_i = y_i - \hat{y}_i$ where $\hat{y}_i = ax_i + b$ and y_i is computed using equation 1

   (c) Compute the average error as

   $$E = \frac{1}{n}\sum_{i=i}^{n} {e_i}^2 \tag{2}$$

   (d) Compute the average error values for all combinations of a and b.

   (e) Plot the error surface with the values of a along x-axis, that of b along y-axis and E along z-axis.

3. Solve for a and b using Pseudo-inverse based approach on the points generated in question 1.

4. Solve for a and b using the Gradient Descent approach where the values of $\mathbf{p} = (a, b)^T$ in the $(k+1)^{th}$ iteration is updated as

$$\mathbf{p}_{k+1} = \mathbf{p}_k - \eta \nabla_{\mathbf{p}} E|_{\mathbf{p}=\mathbf{p}_k} \tag{3}$$

Vary the update rate η and the initial values (a_0, b_0) and note the final solution after 100 iterations. Plot the trajectory of the solutions (a_k, b_k) for varying (a_0, b_0, η) on the contour plot of E on (a, b) plane.

5. Consider the multi-modal function given by

$$z = 1.7 * exp\left[-\left\{\frac{(x-3)^2}{10} + \frac{(y-3)^2}{10}\right\}\right] + exp\left[-\left\{\frac{(x+5)^2}{8} + \frac{(y+5)^2}{8}\right\}\right] + 2 * exp\left[-\left\{\frac{x^2}{4} + \frac{y^2}{5}\right\}\right] + 1.5 * exp\left[-\left\{\frac{(x-4)^2}{18} + \frac{(y+4)^2}{16}\right\}\right] + 1.2 * exp\left[-\left\{\frac{(x+4)^2}{18} + \frac{(y-4)^2}{16}\right\}\right] \tag{4}$$

Display the surface plot and contour plot of the above function in the search space given by $\mathbf{S}_{min} = [x_{min}, y_{min}]^T = [-10, -10]^T$ and $\mathbf{S}_{max} = [x_{max}, y_{max}]^T = [10, 10]^T$.

6. Find the maxima of the above function (Equation 4) using stochastic search. Display the *Scatter Plot* of the entire population obtained in each iteration on the contour plot (obtained from first question) and show how the solutions are moving towards the optimal values in each iteration. Write the following function in Python

   stochasticSearch($\mathbf{S}_{min}$, $\mathbf{S}_{max}$, popSize ,radius , nRRI , nRLC , maxItr)

   where, S_{min} is the minimum limit of the search space, S_{max} is the maximum limit of the search space, **popSize** is the population size, **radius** is used for generating children around the parent population, **nRRI** is the number of samples generated using random re-initialization, **nRLC** is the number of samples generated using random linear combination and **maxItr** is the maximum number of iterations. Experiment with different values of stochastic search parameters.